Planning Against Disasters in Dynamic Production Networks

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OK Boomer

Consensus on RBC circa early 90s:

- Poor amplification:
    - requires large & mysterious aggregate technology shocks in data
- Log-linearized solutions are accurate:
    - endogenous quantities and prices are quite linear in state variables
- Small welfare cost of business cycles

OK Zoomer?

Consensus on RBC circa early 90s:

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- Log-linearized solutions as accurate
- Small welfare cost of business cycles

Consensus on Production Networks now:

- Powerful amplification:
  - i.i.d micro-shocks cascade via supply chains generate large aggregate fluctuations
- Non-linearities galore:
  - complementarities generate endogenous disasters and negative skewness in aggregates.
- Welfare cost of business cycles maybe large?

Intro

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- Log-linearized solutions as accurate
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Revisit (multisector) stochastic growth model with nonlinear networks

This Paper

1) Analytically in a simple two-period/two-sector model:

- optimal capital allocation across sectors under uncertainty $\Longrightarrow$ excess investment (relative to DSS) in upstream sectors.
  - planner manipulates capital allocation in order to minimize nonlinear cascades and consumption disasters.
- efficient strategy averts disasters but reduces the average level of consumption
  - potential for high welfare costs of business cycles in stochastic growth model via level effects

This Paper

2) Quantitatively: deep-learning technique on large-scale, dynamic, nonlinear production networks.

- ergodic distribution features higher mean capital levels in key upstream sectors.
- lower (than DSS) mean levels of macro aggregates.
- welfare cost of business cycles: 1%.

This Paper

2) Quantitatively: deep-learning technique on large scale, dynamic, nonlinear production networks.

- ergodic distribution features higher mean capital levels in key upstream sectors.
- lower (than DSS) mean levels of macro aggregates.
- welfare cost of business cycles: 1%.

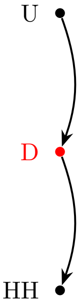
TLDR: Planner avoids disasters through strategic capital allocation.

→ The stochastic growth model does features high welfare cost of business cycles, but it hits through low average consumption, not volatility.

Plan of Talk

1. **Simple Analytics**
    - Capital allocation under uncertainty in 2-sector/2-period nonlinear environment
    - Pre-allocation, aggregate consumption and welfare cost of business cyle
2. Quantitative Environment
3. Quantitative Results

The Simplest 2 x 2 x 2 Economy: Structure



- **Structure: 2 x 2 x 2**
  - 2 sectors: Upstream and Downstream
  - 2 inputs:
    - Capital
    - Intermediate Input
  - 2 periods

The Simplest 2 x 2 x 2 Economy: Second period

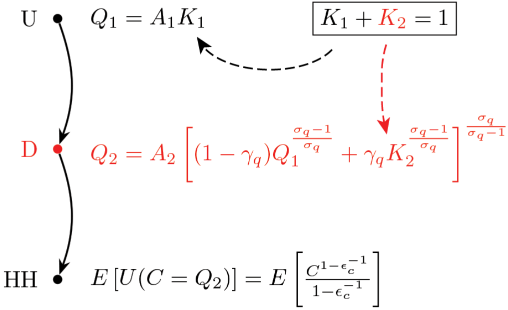
U $\bullet$ $Q_1 = A_1 K_1$

D $\bullet$ $Q_2 = A_2 \left[(1-\gamma_q) Q_1^{\frac{\sigma_q - 1}{\sigma_q}} + \gamma_q K_2^{\frac{\sigma_q - 1}{\sigma_q}} \right]^{\frac{\sigma_q}{\sigma_q - 1}}$

HH $\bullet$ $U(C = Q_2) = \frac{C^{1-\epsilon_c^{-1}}}{1-\epsilon_c^{-1}}$

- **Period 2: Given a (K_1,K_2) allocation**:
  - TFP shocks realize, production and consumption take place.
  - symmetric shocks: high (A_H) or low (A_L).
  - Upstream: Q_1 CRS with capital, K_1, s.t. A_1.
  - Downstream: Q_1: CRS-CES combination of capital K_2 and upstream good Q_1, s.t. A_2.
  - HH: CRRA over downstream good.

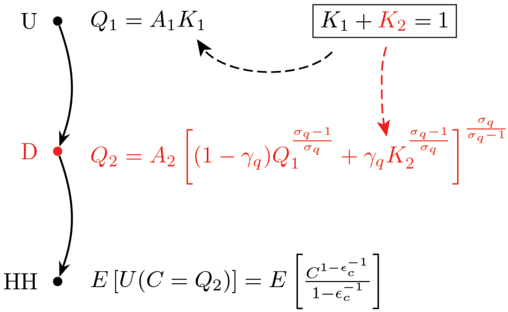
The Simplest 2 x 2 x 2 Economy: First period



- Period 2: Production + Consumption, given capital allocation & shocks.
- **Period 1: Planner picks (K_1,K_2) allocation**
  - to maximize expected utility in period 2

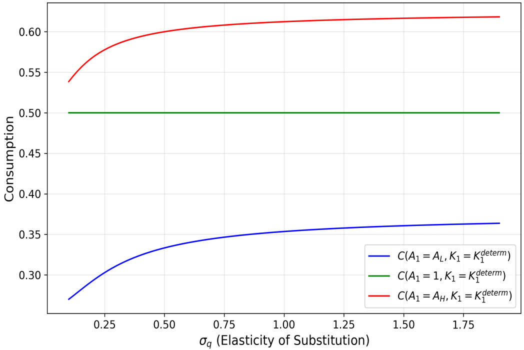
    s.t. $K_1 + K_2 = 1$.

The Simplest 2 x 2 x 2 Economy: Question



- Period 2: Production + Consumption, given capital allocation & shocks.
- Period 1: Planner picks (K_1, K_2) allocation.
- **Question: Does the planner deviate from deterministic K-allocation?**
  - Insurance benefit: Allocating more K upstream minimizes nonlinear cascades.
  - Insurance cost: Allocating more K upstream generates lower expected consumption.

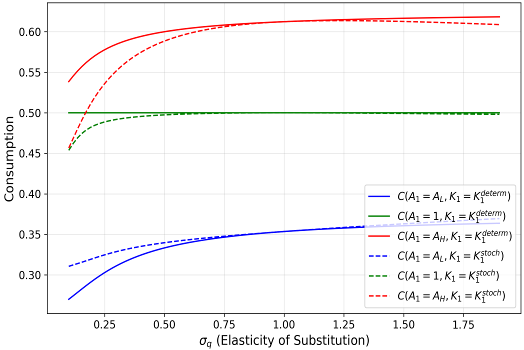
Insurance Benefits in Nonlinear Economies



Consumption levels vs elasticity of substitution

- Symmetric shocks upstream $\Longrightarrow$ asymmetric aggregate fluctuations
    - also, for given negative shock upstream, aggregate contraction nonlinear in σ_q
    - Possible consumption disasters near Leontieff
- Q: what happens if we allocate more capital to the upstream sector?

Insurance Benefits in Nonlinear Economies

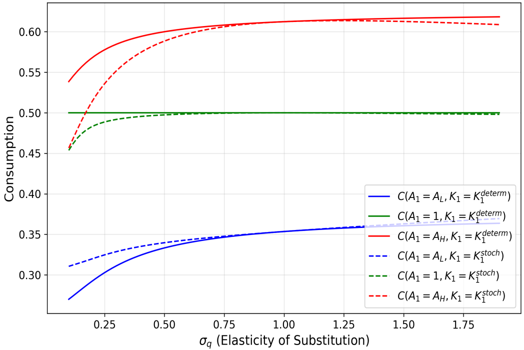


Consumption levels vs elasticity of substitution

- Symmetric shocks upstream $\implies$ asymmetric aggregate fluctuations
  - also, for given negative shock upstream, aggregate contraction nonlinear in σ_q
  - Possible consumption disasters near Leontieff

- **Lemma 1:**
  If inputs are complements, excess capital allocation to upstream (relative to K_{ss}) $\implies$ lower impact of negative upstream shocks on $log(C)$

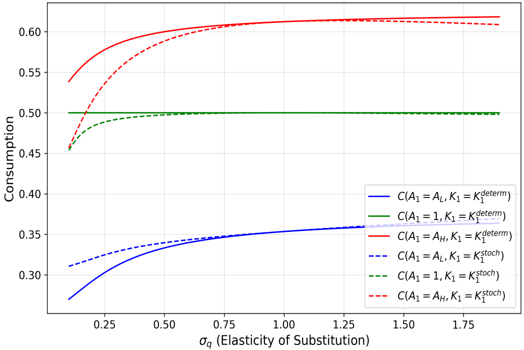
Insurance Costs in Nonlinear Economies



Consumption levels vs elasticity of substitution

- Insuring upstream cascades is costly
  - reallocating capital to a relatively unproductive use (low productivity upstream) and away from more productive (high productivity downstream)
- Q: What is the result of a reallocation of capital to the upstream sector?

Insurance Costs in Nonlinear Economies



Consumption levels vs elasticity of substitution

- Insuring upstream cascades is costly
  - reallocating capital to a relatively unproductive use (low productivity upstream) and away from more productive (high productivity downstream)

- **Lemma 2:**
  If inputs are complements and upstream sector is not too small, excess capital allocation to upstream $\implies$ expected consumption lower than deterministic C_{ss}

K-Allocation and Expected Consumption

Theorem

If inputs are complements ($\sigma_q < 1$), the planner preallocates capital to the upstream sector (that is, $K_1^* > K_1^{determ}$) whenever risk aversion is large enough (and always if > 1). Furthermore, preallocation is larger if shocks are more volatile.

- Risk aversion > 1 resolves the tradeoff between insurance benefits vs. costs.
  (1) planner coping with uncertainty deliberately over-invests in upstream resilience to avert final demand disasters
  (2) this comes at a 'level' cost in terms of average consumption due to capital misallocation

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- Risk aversion > 1 resolves the tradeoff between insurance benefits vs. costs.
- Welfare cost implications:
  - not from consumption disasters (planner avoids them!)
  - not from fluctuations around deterministic steady state (cf Lucas)
  - via permanently lower average consumption?

Plan of Talk

1. Simple(st) Analytics
2. **Quantitative Environment**
   - Multi-sector stochastic growth model: CES nests everywhere & I-O linkages for intermediate and investment goods
   - Standard calibration & Deep Learning solution method
3. Quantitative Results

Quantitative Model

- **∞-lived HH with GHH preferences over consumption and labor bundles**: model
    - CES aggregator over $j = 1, .., N$ sector goods, elasticity σ_c
    - Time invariant preferences for each good, ξ_j
    - CES aggregator over hours worked in each N sector: σ_l controls degree of labor reallocation across sectors.
- **Representative firm in sector j, produces gross output Q_{jt} with CRS technology:**
    - CES aggregator over primary and intermediate inputs: elasticity σ_c
    - Primary input is CES bundle of capital and labor, σ_y
    - Intermediate input bundle M_{jt} is CES nest of sector $j = 1, ..., N$ goods; elasticity σ_m
      $\implies$ "Intermediate Input Network"
    - Value-added TFP shocks A_{jt}: AR(1) with sector specific AR and VCOV unrestricted

Quantitative Model

- **Firms accumulate capital via industry-specific investment good I_{jt}**
  - investment subject to quadratic capital adjustment costs (I/K) with sector-specific depreciation rates
  - Each sector's investment good is produced via CES bundle of investment goods in other industries; elasticity σ_I
    $\implies$ "Investment Network"
- **Resource Constraint**
  - Gross output of each sector satisfies final demand consumption by HH, intermediate input demand, and investment good demand by other sectors

Calibration

1. **Calibrated to match US data** (37 sectors, 1948-2008).
   - Intensity shares (ξ_j, μ_j, α_j) and networks (Γ^m, Γ^I).
   - TFP process (ρ_j, Σ_A).
   - Capital adjustment (ϕ) and labor reallocation (σ_l) costs.
2. **Elasticities of substitution**.
   - Set based on estimates: $\sigma_m = 0.1$, an $\sigma_y = 0.8$.
   - Set to intermediate levels (0.5): σ_c, σ_I, and σ_q.
3. **Standard parameters in the literature**
   - Intertemporal elast. of subs. (ϵ_c), Frisch elasticity (ϵ_l), discount factor (β).

Untargeted Moments: Volatility of Aggregate Consumption and GDP.

Deep Learning solution method for production networks: motivation

- Classic problem: minimize loss of a system of equations with expectation terms.
- Until recently: **feasible up to $\sim$ 6 state vars. But we have 74 state vars!**
  - And we are interested in nonlinearities and effect of uncertainty.
- Classic solutions that allow for nonlinearities:
  1. **Higher order perturbation** around deterministic SS.
     - Problem: no stochastic SS, and unstable/unfeasible with high dimensionality.
  2. **Perfect foresight** solution.
     - Problem: no stochastic SS, and no impact of uncertainty.

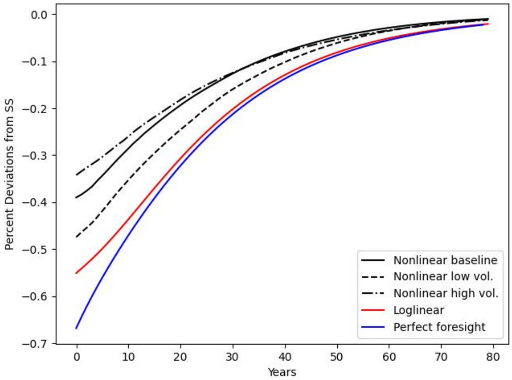
Deep Learning solution method for production networks: explanation

- Our approach: policy function iteration but with **neural net function approximation**, adapted from DEQN (Azimovic et al 2022).
- DEQN: at each optimization step, use policy function to:
  1. **Simulate** forward and get sample points of state space. Then, at each point:
  2. solve future policies for all possible realization shocks $\rightarrow$ **recover expectations**.
  3. **Construct loss**, and differentiate with respect to parameters of policy function.
- Our contribution:
  - many continuous shocks $\rightarrow$ **montecarlo simulation** to get expectation $\rightarrow$ clever **parallelization scheme on GPUs**.
  - One of the rare cases in which a high dimensional model solved using NNs **exhibits strong nonlinearities.**

Plan of Talk

Negative shocks: dampened in global solution vs. local/perfect foresight

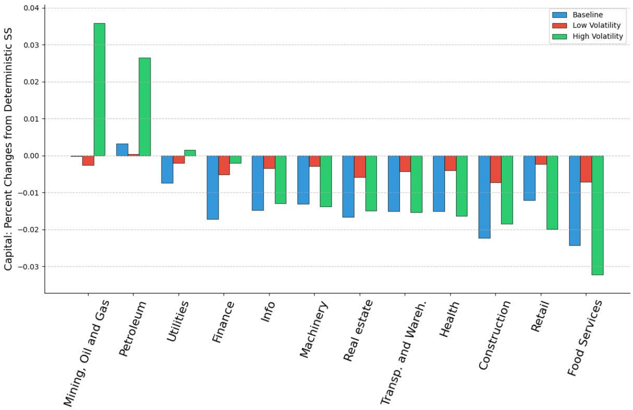
- IRF of C to a -20% shock to Mining, Oil, and Gas.
- perfect foresight vs loglinear: amplification.
- loglinear vs **fully nonlinear**: attenuation.
- Attenuation is stronger when shocks are more volatile.



IRF of Agg. C to shock to Mining, Oil, and Gas

Sectoral reallocation: more capital for key upstream sectors in stochastic SS

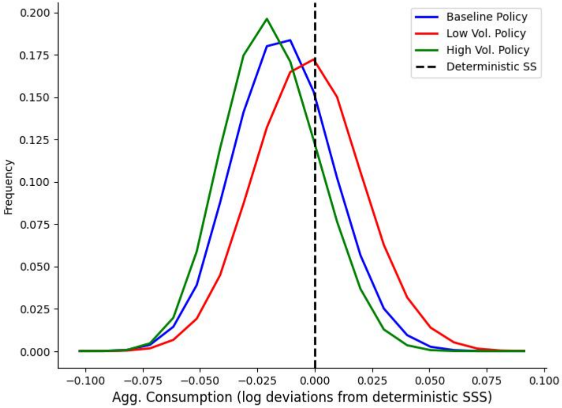
- Sectoral allocation of capital in stochastic SS as log deviations from deterministic SS.
- Volatility of shocks modify stochastic SS.
- Higher volatility → more preallocation to key upstream sectors. All sectors



Capital Allocation in Stochastic SS vs Determ SS

Ergodic distribution: lower mean consumption than DSS but no disasters

- Ergodic distribution of C under three different policies.
- Simulations: same shocks, but **policies solved under different shock volatilities.**
- Mean consumption decreases, no negative skew.



Ergodic distribution for Aggregate Consumption

Volatility increases the distance of the average/SSS from the DSS

Stochastic steady state: deviations with respect to the deterministic steady state

Policy	Consumption (%)	Labor (%)	GDP (%)	Investment (%)	Intermediates (%)	Capital (%)
Low Volatility	-0.43	-0.16	-0.28	-0.37	-0.43	-0.45
Baseline	-1.56	-0.58	-1.02	-1.32	-1.50	-1.48
High Volatility	-1.89	-0.62	-1.07	-1.14	-1.47	-1.21

Note: The stochastic steady state is calculated by sampling 1000 points from the full simulation, and simulating forward but setting shocks to zero.

- Stochastic SS features depress aggregate consumption.
- As volatility increases, aggregate consumption decreases.

Significant welfare cost of business cycles

Welfare cost of business cycle

Policy	Full Nonlinear (%)	Loglinear (%)	C fixed at DSS (%)	L fixed at DSS (%)	Mean at DSS (%)
Low Volatility	-0.46	-0.05	0.16	-0.63	-0.03
Baseline	-1.05	-0.11	0.41	-1.51	-0.06
High Volatility	-1.50	-0.19	0.57	-2.18	-0.10

Note: All values are expressed as percentage changes in consumption equivalent terms.

- Large welfare cost of business cycle ($\sim 1\%$).
- An order of magnitude larger than in the loglinear model.
- Cost manifest as lower average consumption.

Additional Insights

- In the quantitative model, we have many sources of sectoral heterogeneity affecting capital preallocation across sectors.
  - If only IO matrix heterogeneity, IO upstreamness $\rightarrow$ more capital ($\sim$ 43% correlation).
  - If only Inv. matrix heterogeneity, Inv upstreamness $\rightarrow$ more capital. ($\sim$ 60% correlation)
- IO matrix is key to get nonlinearities and welfare cost of business cycles.
  - With an identity IO matrix, nonlinearities are reduced (welfare cost $\sim$ 0.4%).
  - With uncorrelated and homogeneous sectoral shocks, you still get nonlinearities (welfare cost $\sim$ 1.5%).

Conclusions

- Simple theory of capital allocation in production networks.
- Solve globally a large-scale nonlinear RBC model with production networks.
- In both theory and quantitative exercise, we find that:
    - Efficient solution **preallocate capital** across sectors to avoid consumption disasters.
    - This comes at the cost of **lower average consumption**.
    - Higher volatility $\rightarrow$ more preallocation.
- Business cycles do not generate large variance in aggregate consumption, but instead reduce mean $\rightarrow$ **high welfare cost despite low consumption volatility**.

Households

$$U = \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{1-\epsilon_c^{-1}} \left(C_t - \theta \frac{L_t^{1+\epsilon_l^{-1}}}{1+\epsilon_l^{-1}} \right)^{1-\epsilon_c^{-1}} \right] \quad \text{where} \quad \begin{cases} C_t = \left(\sum_{j=1}^{N} \xi_j^{\frac{1}{\sigma_c}} (C_{jt})^{1-\sigma_c{}^{-1}} \right)^{\frac{1}{1-\sigma_c{}^{-1}}} \\ L_t = \left(\sum_{j=1}^{N} (L_{jt})^{1+\sigma_l{}^{-1}} \right)^{\frac{1}{1+\sigma_l{}^{-1}}} \end{cases}$$

- ξ_j: time-invariant preference for good j, $\sum_{j=1}^{N} \xi_j = 1$.
- σ_c: elasticity of substitution across goods.
- σ_l: degree of labor reallocation between sectors.

Firms: Gross Output

The representative firm on industry j produces gross output Q_{jt} using the technology:

$$Q_{jt} = \left[(\mu_j)^{\sigma_q{}^{-1}} (Y_{jt})^{1-\sigma_q^{-1}} + (1-\mu_j)^{\frac{1}{\sigma_q}} (M_{jt})^{1-\sigma_q{}^{-1}} \right]^{\frac{1}{1-\sigma_q{}^{-1}}}$$

- Y_{jt}: value-added production.
- μ_j: the value-added share.
- σ_q: elasticity of substitution between value-added and materials

Firms: Value added

Value-added production is given by:

$$Y_{jt} = A_{jt}\left[(\alpha_j)^{{\sigma_y}^{-1}}(K_{jt})^{1-{\sigma_y}^{-1}} + (1-\alpha_j)^{{\sigma_y}^{-1}}(L_{jt})^{1-{\sigma_y}^{-1}}\right]^{\frac{1}{1-{\sigma_y}^{-1}}}$$

- σ_y: elasticity of substitution between labor and capital.
- A_{jt}: industry-specific shock to value-added productivity:
  - It follows the AR(1) process: $\log A_{jt+1} = \rho_j \log A_{jt} + \varepsilon^A_{jt+1}$

Firms: Capital Accumulation

Capital dynamics

Firms can accumulate capital by producing an industry-specific investment good I_{jt}:

$$K_{jt+1} = (1 - \delta_j)K_{jt} + I_{jt} - \Phi_{jt}$$

Capital Adjustment Costs

Firms are subject to capital adjustment costs:

$$\Phi_{jt} = \frac{\phi}{2}\left(\frac{I_{jt}}{K_{jt}} - \delta_j\right)^2 K_{jt}$$

- δ_j: industry-specific depreciation rate.
- ϕ: parametrizes capital adjustment costs.

Investment network

The investment good is produced by bundling goods produced by other industries:

$$I_{jt} = \left(\sum_{i=1}^{N} \left(\gamma_{ij}^{I} \right)^{\sigma_I^{-1}} \left(I_{ijt} \right)^{1-\sigma_I^{-1}-1} \right)^{\frac{1}{1-\sigma_I^{-1}-1}} \quad \text{where} \quad \sum_{i=1}^{N} \gamma_{ij}^{I} = 1$$

- γ_{ij}^{I}: use of good i in the production of the investment good for sector j.
- σ_I: elasticity of substitution between inputs of the investment bundle.

Intermediates network

The intermediate input is produced using the bundle:

$$M_{jt} = \left(\sum_{i=1}^{N} \left(\gamma_{ij}^{m}\right)^{\sigma_m{}^{-1}} \left(M_{ijt}\right)^{1-\sigma_m{}^{-1}} \right)^{\frac{1}{1-\sigma_m{}^{-1}}} \quad \text{where} \quad \sum_{i=1}^{N} \gamma_{ij}^{m} = 1$$

- γ_{ij}^{m}: : use of good i in the production of the final good for sector j.
- σ_m: elasticity of substitution between intermediate goods.

Aggregate resource Constraints

$$Q_{jt} = C_{jt} + \sum_{i=1}^{N} (M_{jit} + I_{jit})$$

Planner Problem

- The model satisfies the 1st Welfare Theorem.
- Then, it can be formulated as a planning problem in which the planner maximizes households' welfare subject to technological constraints.

Capital Allocation is Stochastic SS for all Sectors