Sustainable Social Security

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Motivation

Sustainability of Social Security is challenging due to:

1. Demographic changes:

   - Rising dependency ratio (65+/working-age) driven by increasing longevity and declining fertility

   - Demographic projections by government actuaries are highly uncertain $\Rightarrow$ exposure to macro longevity risk

Motivation

Sustainability of Social Security is challenging due to:

1. Demographic changes:
   - Rising dependency ratio (65+/working-age) driven by increasing longevity and declining fertility
   - Demographic projections by government actuaries are highly uncertain $\Rightarrow$ exposure to macro longevity risk
2. Limited enforcement arising from intergenerational conflicts:
   - Retirees (Old) demand preservation of promised entitlements
   - Workers (Young) oppose higher fiscal burdens

Research Question

How should a Sustainable Social Security rule be structured in the presence of demographic changes (macro longevity risk) and limited enforcement?

Pure (no public reserves) vs **Partially Funded** (with public reserves) Paygo systems

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Preview of Result

The sustainable rule is **non-linear** and **history-dependent**.

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Preview of Result

The sustainable rule is **non-linear** and **history-dependent**.

Previous Literature

Optimal parametric rules
Nishiyama & Smetters (2007)
Huggett & Parra (2010)
Golosov et al. (2013)

Full enforcement or info frictions
Conesa & Garriga (2008)
Hosseini & Shourideh (2019)
Berriel & da Costa (2025)

Dynamic voting (inefficient)
Cooley & Soares (1999)
Bassetto (2008)
Gonzalez-Eiras & Niepelt (2008)

Framework

- Discrete-time OLG model, two-period-lived agents: **Young** (N_t^y) and **Old** (N_t^o)
- Growth rate of Young is constant $\frac{N_t^y}{N_{t-1}^y} = 1$
- Stochastic survival probability: $\phi_t = \frac{N_t^o}{N_{t-1}^y} \in \{\phi_L, \phi_H\}$, with $\phi_L < \phi_H$ and π_{ij} the probability of transiting from ϕ_i to ϕ_j
- Young with w and Old endowed with $\alpha < w$
- Young pay contributions τ_t to finance pensions p_t to the Old
- Preferences: time-separable utility $u(\cdot)$, with discount factor $\beta \in (0, 1]$

$$u(w \cdot (1 - \tau_t)) + \beta \cdot \mathbb{E}_t \left[\phi_{t+1} u(\alpha + p_{t+1}) \right]$$

Social Planner Problem

A benevolent planner maximizes the sum of expected discounted utility of all generations, weighting future generations by $\delta \in (0,1)$, subject to a balanced budget (pure Paygo) each period:

$$\tau_t \cdot w \cdot N_t^y = p_t \cdot N_t^o \quad \Rightarrow \quad p_t = \frac{w}{\phi_t} \cdot \tau_t$$

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- The optimal policy may depend on the history of shocks, but recursive structure means current p_t summarizes relevant past information

- Given current state (p, ϕ_i), planner chooses:

$$\begin{aligned} &\tau(p, \phi_i) && \Leftarrow \text{current contribution} \\ &p_j(p, \phi_i) && \Leftarrow \text{next-period contingent pension} \end{aligned}$$

Full Enforcement

For $i \in \{L, H\}$, the Intergenerational Pareto Frontier is

$$V_i(p) = \max_{\tau,(p_j)_{j\in\{L,H\}}} \left\{ \underbrace{\frac{\beta}{\delta} \cdot \phi_i}_{\text{Old's weight}} \cdot \underbrace{u\left(\alpha + \frac{w}{\phi_i} \cdot \tau\right)}_{\text{Old's payoff}} + \underbrace{u\left(w \cdot (1-\tau)\right)}_{\text{Young's payoff}} + \underbrace{\delta \sum_{j\in\{L,H\}} \pi_{ij} \cdot V_j(p_j)}_{\text{Expected future value}} \right\}$$

s.t. the Promise-Keeping (PK) Constraint:

$$u\left(\alpha + \frac{w}{\phi_i} \cdot \tau\right) \geq u(\alpha + p) \qquad \underbrace{\left(\frac{\beta}{\delta} \cdot \phi_i \cdot \lambda\right)}_{\text{Multiplier}}$$

Full Enforcement (PK does not bind)

Constant ratio of marginal utilities:

$$\frac{u'\left(w - \phi_i \cdot p\right)}{u'\left(\alpha + p\right)} = \frac{\beta}{\delta}$$

First-best policy features:

$p_H^* < p_L^*$ and $\tau_H^* > \tau_L^*$

$\Rightarrow$ Perfect, history-independent risk sharing

Full Enforcement (PK binds)

There is a wedge that distorts the ratio of marginal utilities above (β/δ):

$$\frac{u'(w-\phi_i \cdot p)}{u'(\alpha+p)} = \frac{\beta}{\delta} \cdot (1+\lambda(p,\phi_i))$$

This distortion is temporary: lasts at most one period

$$V_j'(p_j(p,\phi_i)) = 0 \quad \Rightarrow \quad p_j^* = \sup\left\{p \,\middle|\, V_j'(p) = 0\right\} \qquad \text{(FOC w.r.t. } p_j\text{)}$$

$$V_i'(p) = -\frac{\beta}{\delta} \cdot \phi_i \cdot u'(\alpha+p) \cdot \lambda(p,\phi_i) \qquad \text{(Envelope w.r.t. } p\text{)}$$

$$\Rightarrow \quad \lambda\left(p_j(p,\phi_i),\phi_j\right) = 0 \qquad \forall j$$

Limited Enforcement

The planner is subject to the Limited Enforcement (Participation) Constraint:

$$u(w \cdot (1-\tau)) - u(w) + \beta \cdot \sum_{j \in \{L,H\}} \pi_{ij} \cdot \phi_j \cdot \Big[u(\alpha + p_j) - u(\alpha) \Big] \geq 0 \quad \Rightarrow \underbrace{(\mu)}_{\text{Multiplier}}$$

- Deviation is deterred by the threat of autarky (worst payoff) to sustain the best allocation $\Rightarrow$ it can be relaxed

- A non-trivial sustainable (i.e., satisfying participation) social security system exists, improving upon autarky iff:

$$-u'(w) + \beta \cdot u'(\alpha) > 0 \quad \Rightarrow \quad \text{dynamic inefficiency}$$

Limited Enforcement

An additional wedge distorts the marginal utility ratio downward:

$$\frac{u'\left(w-\phi_i\cdot p\right)}{u'\left(\alpha+p\right)}=\frac{\beta}{\delta}\cdot\frac{(1+\lambda(p,\phi_i))}{(1+\mu(p,\phi_i))}$$

If first-best is unsustainable, i.e., some participation constraint is violated, the distortion is permanent: it persists even in the long-run

$$V_j'(p_j(p,\phi_i))=-\frac{\beta}{\delta}\cdot\phi_j\cdot u'(\alpha+p_j(p,\phi_i))\cdot\mu(p,\phi_i) \qquad \text{(FOC w.r.t. } p_j\text{)}$$

$$V_i'(p)=-\frac{\beta}{\delta}\cdot\phi_i\cdot u'(\alpha+p)\cdot\lambda(p,\phi_i) \qquad \text{(Envelope w.r.t. } p\text{)}$$

$$\Rightarrow \quad \underbrace{\lambda(p_j(p,\phi_i),\phi_j)=\mu(p,\phi_i)}_{\text{Updating Rule}} \qquad \forall i,j$$

Limited Enforcement

Illustrative Case:

- Young are *constrained* when ϕ_H for any relevant p
- Young are *unconstrained* when ϕ_L for any relevant p

Dynamics Implications:

- If ϕ_L: the planner sets future pensions equal to

$$p_j^0 = \sup \left\{ p \,\middle|\, V_j'(p) = 0 \right\} \leq p_j^* \quad \text{with} \quad p_L^0 = p_L^* \quad \text{and} \quad p_H^0 < p_H^*$$

- If ϕ_H: the planner promises $p_j > p_j^0$ to relax participation constraints
- But a higher p_j increases contributions from next-period Young $\Rightarrow$ if ϕ_H persists, their participation constraint tightens over time

$\Rightarrow$ intertemporal efficiency–incentive trade-off

Social Security Rule

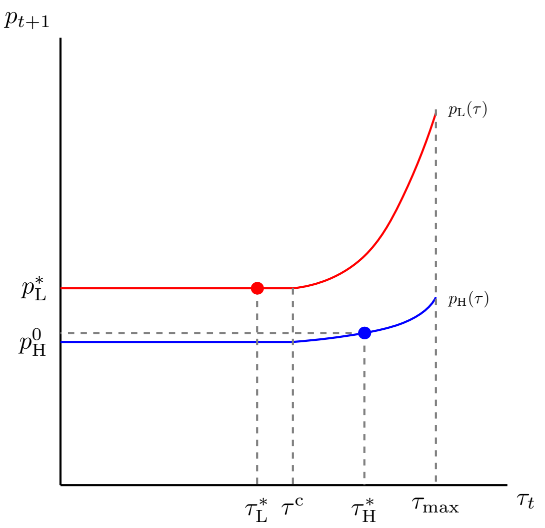
The optimal rule $p_j = h_j(\tau, \phi_i)$ is *non-linear* and *history-dependent*

With **iid** shocks, $p_j = h_j(\tau)$:

i. If $\tau \leq \tau^c$: $h_j(\tau)$ is constant in τ
$\Rightarrow$ Minimum guaranteed benefits

ii. If $\tau > \tau^c$: $h_j(\tau)$ increases in τ
$\Rightarrow$ Contribution-based component

iii. $h_j(\tau)$ decreases in ϕ_j
$\Rightarrow$ Longevity risk adjustment



Public Reserve Funds

- With public reserve funds $a \geq 0$, the planner can partially fund pensions subject to the budget constraint:

$$\tau w - \phi_i\, p = S \equiv q\, a_+ - a \quad \text{with} \quad a_+ \geq 0$$

  where $q \geq \delta$ is the fixed exogenous price of a risk-free asset

- The surplus S can be: $S > 0$ (reserve accumulation); $S < 0$ (reserve drawdown)

- Given current state (a, p, ϕ_i), the planner chooses:

$$\begin{aligned}
&\tau(a, p, \phi_i) && \Leftarrow \text{current contribution} \\
&p_j(a, p, \phi_i) && \Leftarrow \text{next-period contingent pension} \\
&a_+(a, p, \phi_i) && \Leftarrow \text{next-period risk-free asset}
\end{aligned}$$

Public Reserve Funds

Marginal utility wedge: Affected by asset accumulation:

$$\frac{u'\left(w(1-\tau(a,p,\phi_i))\right)}{u'(\alpha+p)} = \frac{\beta}{\delta}\cdot\frac{1+\lambda(a,p,\phi_i)}{1+\mu(a,p,\phi_i)}$$

Euler condition:

$$u'(w(1-\tau(a,p,\phi_i))) \geq \frac{\beta}{q}\sum_{j\in\{L,H\}} u'(\alpha+p_j(a,p,\phi_i)) \quad \text{and} \quad a_+ \geq 0$$

with complementary slackness

Double feedback effect: Participation constraints $\Leftrightarrow$ public reserve funds

i. Participation constraint $\Rightarrow$ additional precautionary saving motive
ii. Public reserve funds affects MU wedge and pension–contribution link

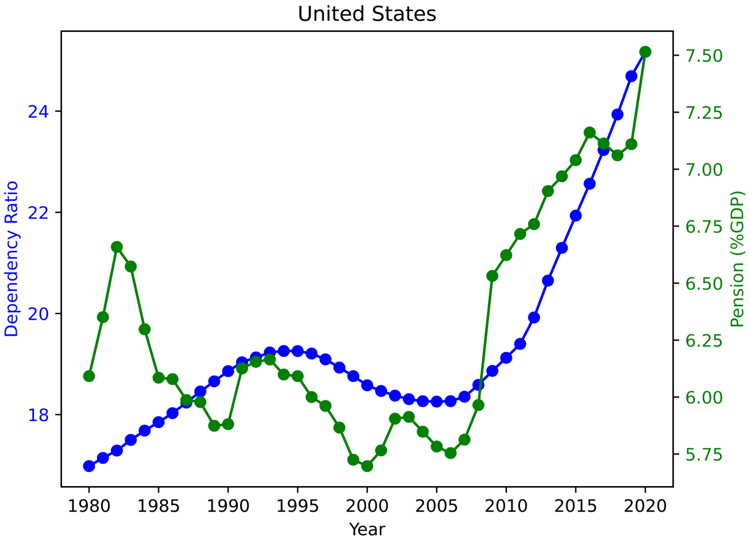
Takeaway

We develop a simple theory of optimal social security under limited enforcement and demographic risk

- ✓ The pension system exhibits history dependence
- ✓ Pension benefits must be linked to contributions when enforcement is limited
- ✓ Reserve funds weaken the benefit-contribution link and full depletion can be optimal

Appendix

Dependency Ratio and Pension Share



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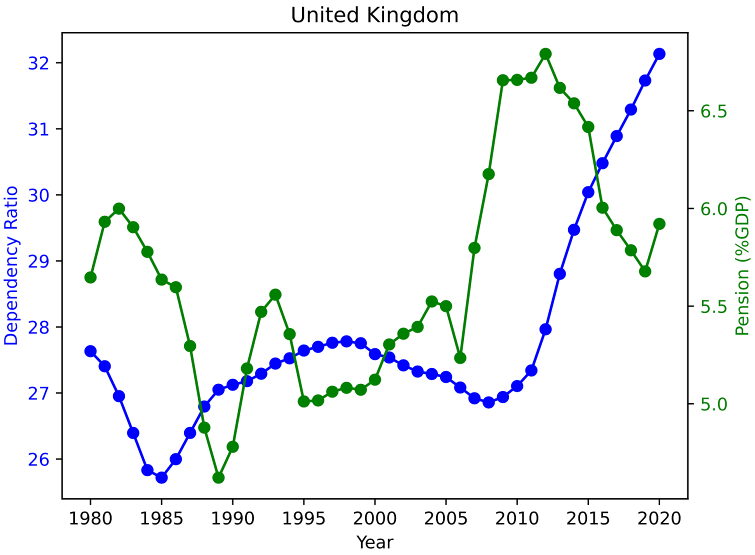
Dependency Ratio and Pension Share



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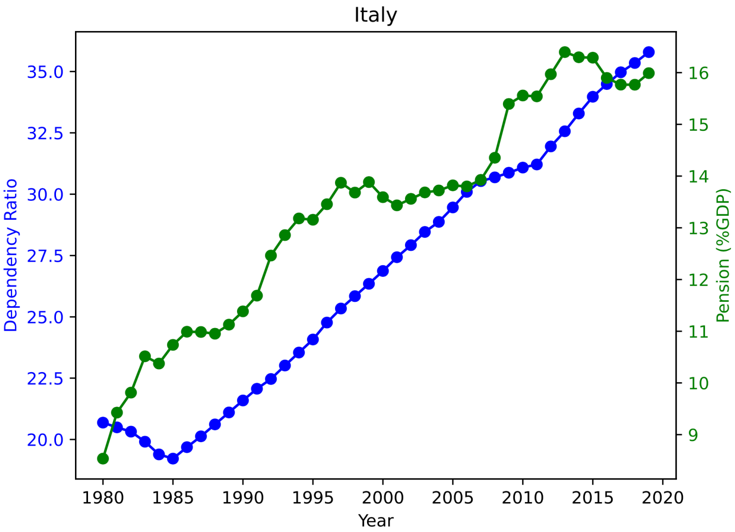
Dependency Ratio and Pension Share



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Life Expectancy Projections

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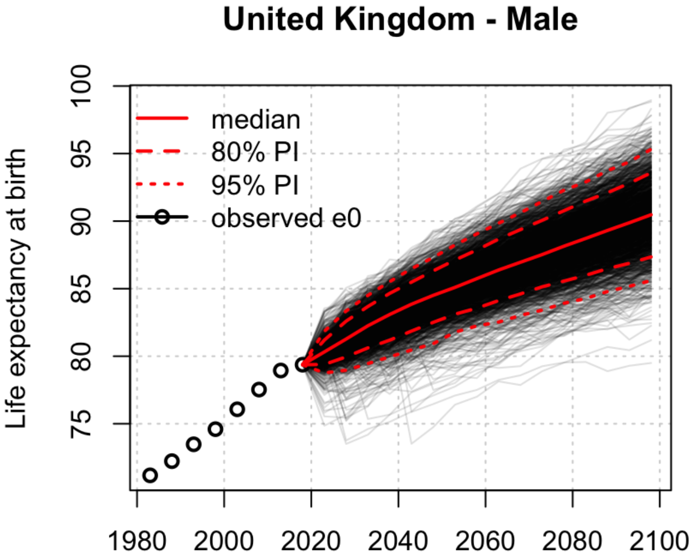

Public Reserve Funds (2023)

Country	Name of the fund or institution	Established in	USD billions	% GDP
Korea	National Pension Fund	1988	803	46.3
Japan	Government Pension Investment Fund	2006	1,595	38.3
Sweden	National Pension Funds (AP1–AP4, AP6)	2000	194	31.4
Canada	Canadian Pension Plan (CPPIB/CPP)	1965	537	24.6
New Zealand	Superannuation Fund	2001	44	16.9
Portugal	Financial Stabilisation Fund	1989	33	11.2
United States	Social Security Trust Fund	1940	2,641	9.7
France	Pension Reserve Fund	1999	218	7.0
Norway	Govt Pension Fund - Norway	2006	35	6.9
Australia	Future Fund	2006	145	7.8
United Kingdom	National Insurance Fund	1948	98	3.1
Chile	Pension Reserve Fund	2006	9	2.7
Poland	Demographic Reserve Fund	2002	16	1.9
Spain	Social Security Reserve Fund	1997	6	0.4
Mexico	IMSS Reserve	n.d.	8	0.4

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Political Constraint

$$\boldsymbol{\Psi} \overbrace{\left(u\left(w(1-\tau)\right) + \beta \sum_{j\in\{L,H\}} \pi_{ij}\phi_j\, u\left(\alpha + p_j\right)\right)}^{\text{Young}} + \left(1-\boldsymbol{\Psi}\right) \overbrace{u\left(\alpha + \frac{1}{\phi_i} w\tau\right)}^{\text{Old}}$$

$$\geq \underbrace{\boldsymbol{\Psi}\left(u(w) + \beta\bar{\phi}_i u(\alpha)\right) + (1-\boldsymbol{\Psi})u(\alpha)}_{\text{Outside Option}}$$

with Ψ equal to the **relative political weight** of Young versus Old as in **probabilistic voting** (Lindbeck and Weibull, 1987; Dovis, Golosov and Shourideh, 2024)

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Sustainable Social Security

Definition (Sustainability)

A **Sustainable Social Security** is a policy that solves the planner's recursive problem and respects all incentive and feasibility constraints over time.

Proposition (Existence)

There exists a non-trivial sustainable social security system that improves upon autarky iff:

$$1 > \hat{r}, \quad \textit{with} \quad \hat{r} := \frac{1}{\hat{m}} \quad \textit{and} \quad \hat{m} := \beta \cdot \frac{u'(\alpha)}{u'(w)}$$

This condition corresponds to ***dynamic inefficiency*** *à la Samuelson.*

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Full Enforcement (PK does not bind)

If CRRA utility with risk aversion coefficient γ:

$$p^*(\phi) = \frac{w - \alpha \left(\frac{\delta}{\beta}\right)^{\frac{1}{\gamma}}}{\phi + \left(\frac{\delta}{\beta}\right)^{\frac{1}{\gamma}}} \quad \text{and} \quad \tau^*(\phi) = \frac{\phi \left(w - \alpha \left(\frac{\delta}{\beta}\right)^{\frac{1}{\gamma}}\right)}{w \left(\phi + \left(\frac{\delta}{\beta}\right)^{\frac{1}{\gamma}}\right)}$$

$\Rightarrow$ An increase in ϕ reduces $p^*(\phi)$ and raises $\tau^*(\phi)$

$\Rightarrow$ Pension promises do not depend on the contributions paid

When $\beta = \delta$, $c^{o*}(\phi) = c^{y*}(\phi) = \frac{w+\alpha\phi}{1+\phi}$ for any ϕ

When $\phi_i < \phi_j$, $c_i^{y*} > c_j^{y*}$ and $c_i^{o*} > c_j^{o*}$

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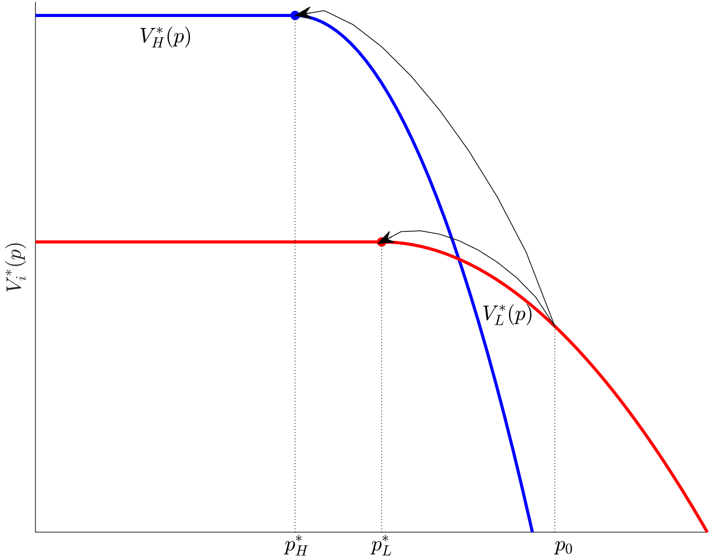
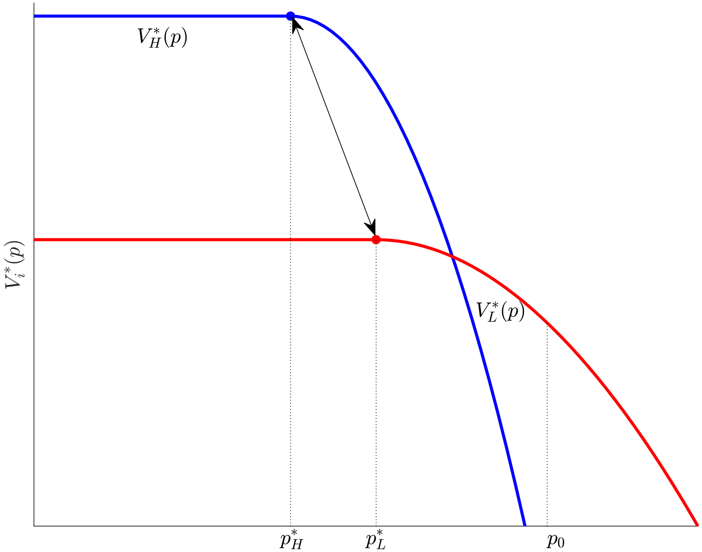
Illustrative Dynamics

Start from $\phi_0 = \phi_L < \phi_H$ and $p_0 > p_L^*$



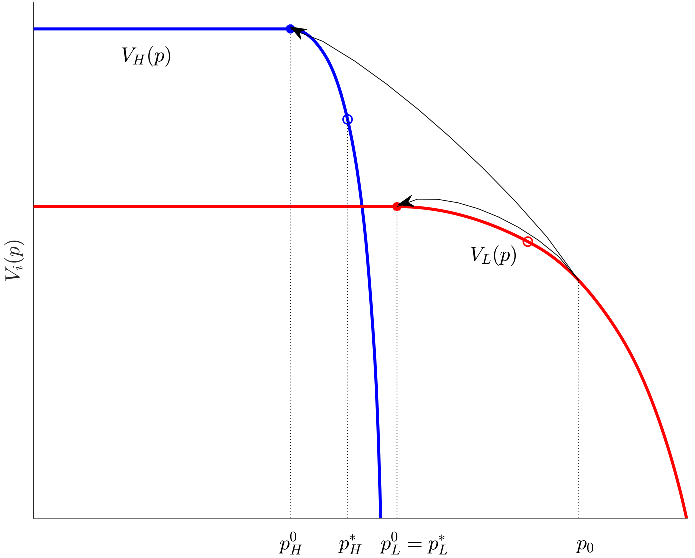
Illustrative Dynamics

For any $t \geq 1$, pension benefits oscillate between p_L^* and p_H^* ▸ Go Back



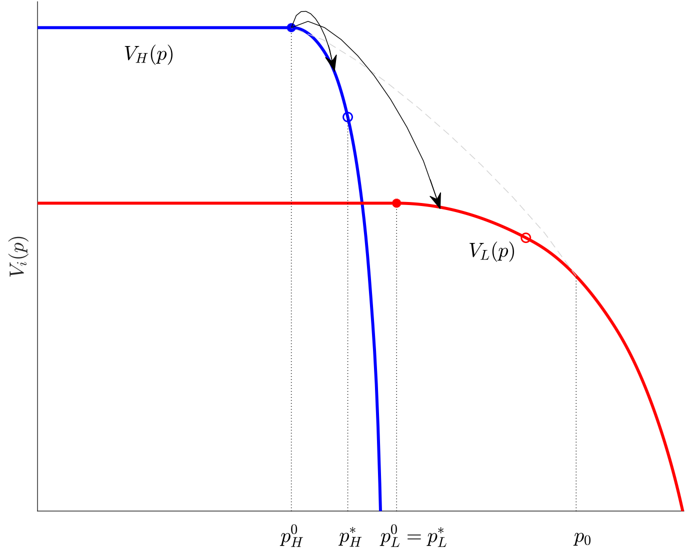
Illustrative Dynamics

Start from $\phi_0 = \phi_L$ and $p_0 > p_L^0$



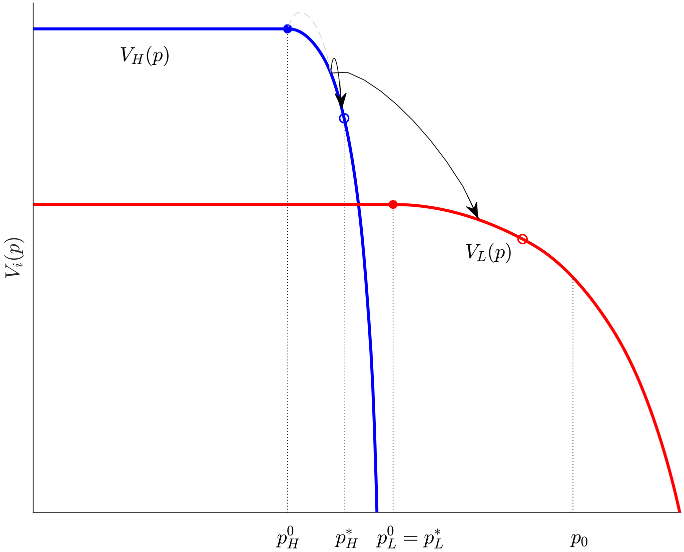
Illustrative Dynamics

If $\phi_1 = \phi_H$, promised pensions must be larger than p_j^0 to relax participation



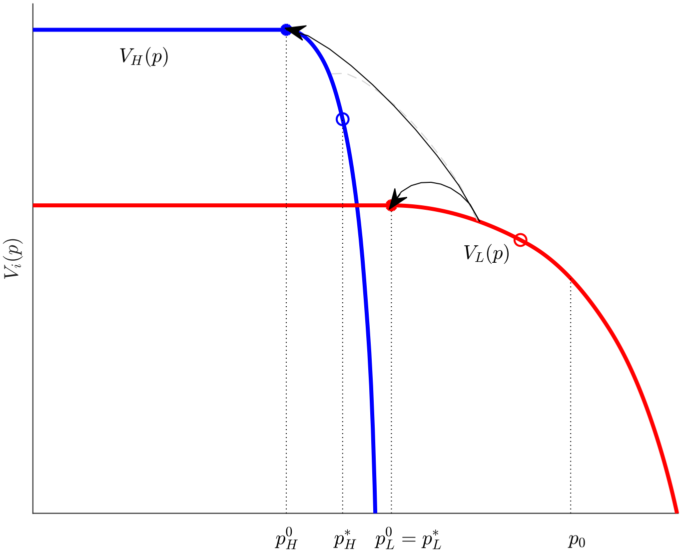
Illustrative Dynamics

If $\phi_2 = \phi_H$, promised pensions must be raised further



Illustrative Dynamics

If $\phi_3 = \phi_L$, promised pensions can be reset to p_j^0 ▸ Go Back



Long-Run Distribution

Proposition

Under limited enforcement, the optimal sustainable social security rule is history dependent, and the economy converges to a unique invariant distribution over an ergodic set with (possibly countably infinite) states (ϕ, p).

$\Rightarrow$ Proving strong convergence requires a resetting property

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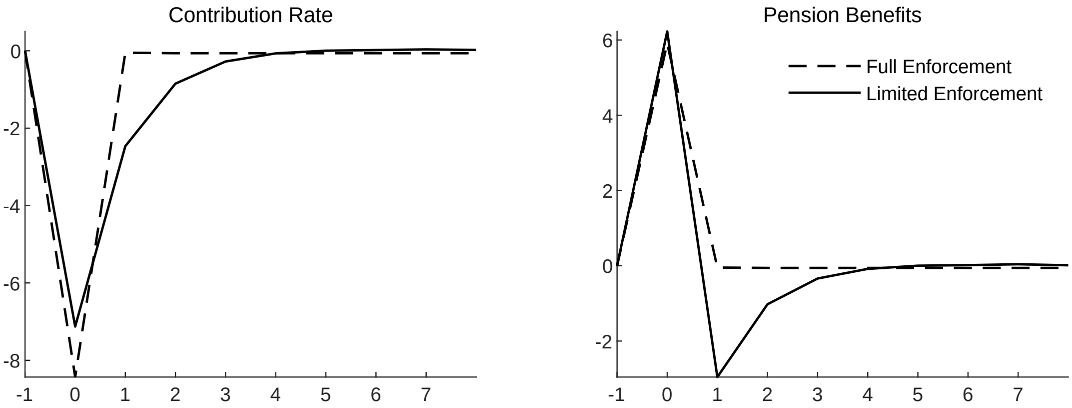
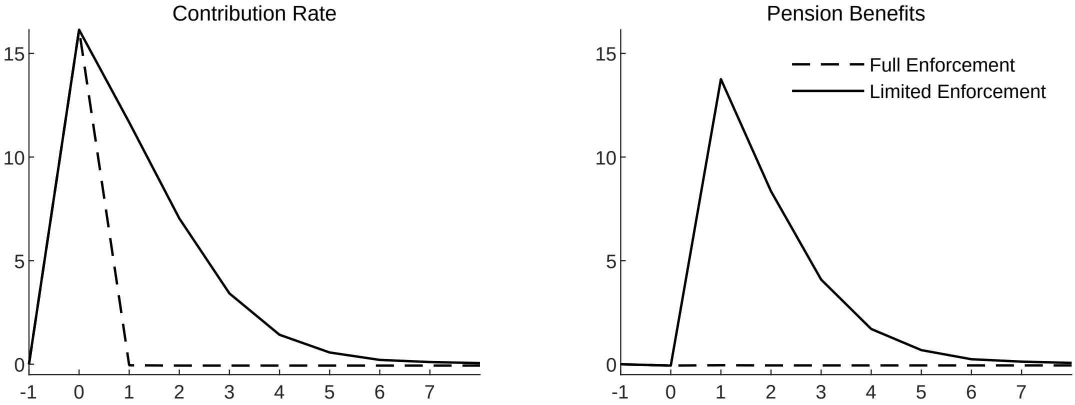
Positive Demographic Shock



Effect on average policies after a one-period unexpected increase in n_t $(+15\%)$

Negative Demographic Shock



Effect on average policies after a one-period unexpected decrease in n_t (-15%)

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Approximations

- **Approximated Rule: Piecewise Linear**

$$h_j^{AR}(\tau) = \begin{cases} p^0(j) & \text{if } \tau \leq \tau^c \\ p^0(j) + \rho^{AR}(j) \cdot (\tau - \tau^c) & \text{if } \tau > \tau^c \end{cases}$$

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- **Policy Alternatives with Longevity Risk Adjustment**

  i. **Defined-Benefit Rule**:

$$h_j^{DB}(\tau) = \rho^{DB}(j) \quad \text{for all } \tau$$

  ii. **Defined-Contribution Rule**:

$$h_j^{DC}(\tau) = \rho_0^{DC}(j) + \rho_1^{DC}(j) \cdot \tau \quad \text{for all } \tau$$

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Valuation of Pensions

- Define states $x := (\tau, \phi_i)$ and $x' := \left(\frac{h_j(\tau)\phi_j}{w}, \phi_j\right)$. The **stochastic discount factor (SDF)** is:

$$m(x, x') = \varrho \cdot \frac{\psi(x)}{\psi(x')}$$

- From the **Ross Recovery Theorem**, ϱ is the Perron root of the state-price matrix and ψ the associated eigenvector:

$$\varrho = \delta \quad \text{and} \quad \psi(x) = \frac{1}{\phi(i)u'(w(1-\tau))(1+\mu(x))}$$

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- The SDF can be used to compute the **insurance premium**.
- The **discounted present value** of the pension claim is:

$$D^p(x) := \mathbb{E}\left[m(x, x') \cdot h_j(\tau)\phi_j \;\middle|\; x\right]$$

Expected Return and Risk Premium

- The **conditional return** on pensions and its expected value:

$$R^p(x, x') := \frac{h_j(\tau)\phi_j}{D^p(x)}, \quad \bar{R}^p(x) := \mathbb{E}\left[R^p(x, x') \mid x\right]$$

- The **conditional risk-free rate** is:

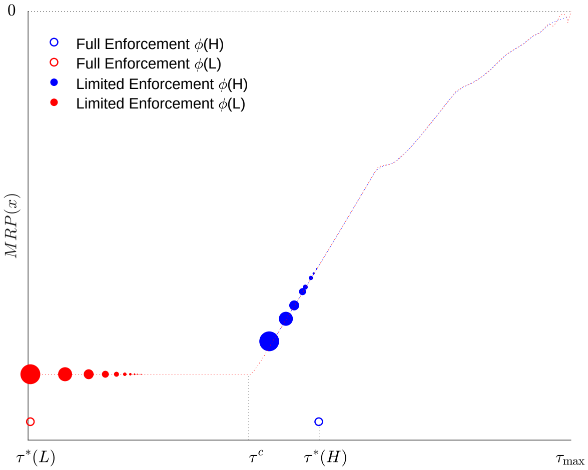
$$R^f(x) := \frac{1}{\mathbb{E}\left[m(x, x') \mid x\right]}$$

- The **multiplicative risk premium (MRP)** is:

$$MRP(x) := \frac{\bar{R}^p(x) - R^f(x)}{R^f(x)} = -\text{cov}\left[m(x, x') \cdot R^p(x, x') \mid x\right]$$

Multiplicative Risk Premium

- Since $\text{cov} > 0$, the risk premium is negative:
  - $\Rightarrow$ This implies a positive **insurance value** of the social security system.



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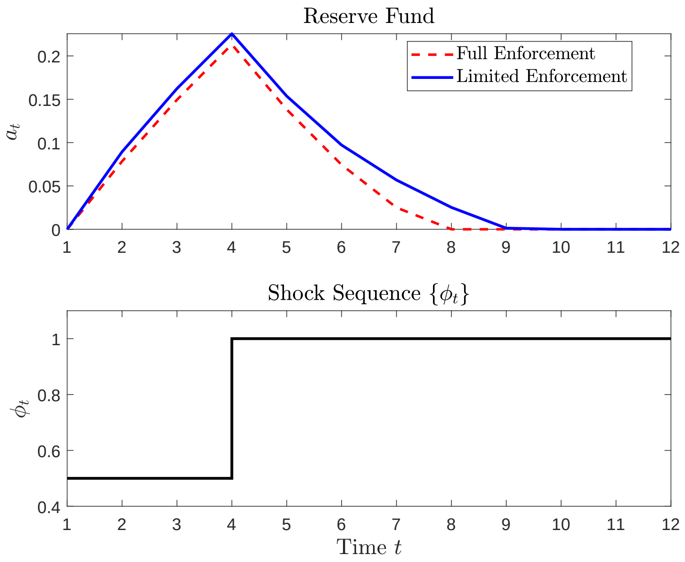
Sample Path: Asset Accumulation



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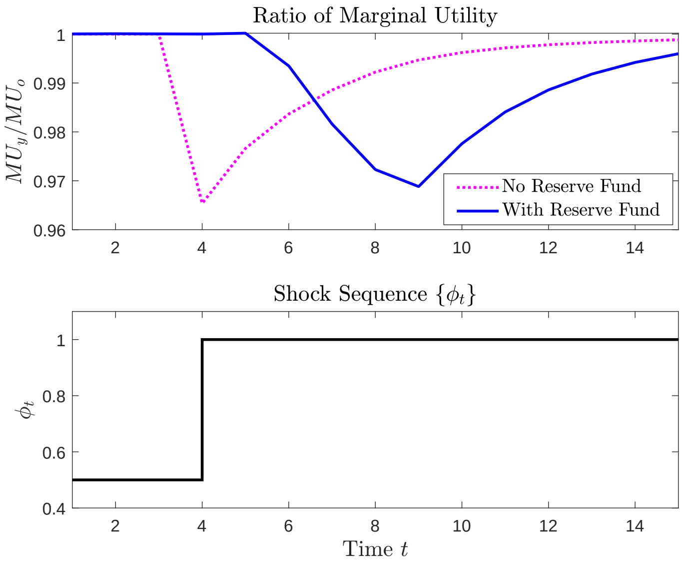
Sample Path: Marginal Utility Ratio



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Sample Path: Pensions and Contributions