Uncovering the Costs of High Inflation

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Motivation for Geeks :-)

let $x \equiv \log(p/p^*)$ be price gap, μ be inflation

In NK models cost of inflation shows up in: $\chi \propto \text{Var}(x)$; study $\frac{\partial \log \chi}{\partial \log \mu}$

Economist observes distribution Δx but not of x; need a model to connect the two

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Cost of inflation varies a lot across models (examples)

	Calvo model	Sheshinsky-Weiss $\sigma^2/\mu \to 0$	Golosov-Lucas $\sigma^2/\mu \to \infty$	
$\frac{\partial \log \chi(\mu)}{\partial \log \mu} \approx$	2	$2/3$	≈ 0	at small μ

Match with data selects model with "reasonable" σ^2/μ

This paper: welfare costs of inflation from a NK perspective

Costs: “inefficient price dispersion” and wasteful “price setting activities”

- Today: add an information friction to a canonical sticky price model
  - firms engage in info-collection (research) and price adjustment activities

- In the model higher inflation leads firms to
  - pay less attention to own idiosyncratic info (wider inaction region, a “SW effect”)
  - choose more dispersed markups (new channel for price dispersion)

- Calibrate model using granular data from Turkey (moderate to high inflation)
  - analyze an episode where inflation tripled
  - costs of inflation not so elusive, steep inflation gradient

Related NK literature

- empirical: Nakamura et al (QJE18) “Elusive cost of inflation”, ongoing work by Adam-Alexandrov-Weber
- modeling sticky price w. inattention
  Mackoviack-Wiederholt (AER09), Alvarez-Lippi-Paciello (QJE11)
- Inverse inference problem: recovering distr. x from Δx
  Alvarez-Lippi-Oskolkov (QJE 22), Bailey-Blanco (RES 23)
- menu cost model for high inflation countries
  Gagnon (QJE09), Alvarez-Beraja-Neumeyer et al (QJE19)
- “rockets and feathers” w asymmetric profit function
  Fernandez-Villaverde et al (AER15), Cavallo-Lippi-Miyahara (AERI24)

Simple Model of Demand and Production

- Monopolistic competition with CES demand $A_i c_i = \left(\frac{p_i/A_i}{P}\right)^{\eta} C$
- Production: CRS in labor $y_i = h_i/Z_i$ where $Z_i = \exp(\underbrace{z_i}_{\text{STD}\,\sigma})$
- Definition: **Price gap** $x_i \equiv \log p_i - \underbrace{\log \frac{\eta}{\eta - 1} Z_i W}_{\equiv p_i^*}$
- constant money growth μ and $A_i = Z_i$ keeps math simple
- gap's law of motion: $dx = -\mu dt + \sigma dZ_i$
- No complementarities between firms' decisions & steady state analysis

The Firm's price setting problem

- Firm knows inflation μ but does not know marginal costs Z_i

  Price-setting requires costly communication with production plant

- Info discoveries (about Z_i) arrive at chosen hazard: ω

- Price adjustment opportunities arrive at chosen hazard: α

- uncontrolled evolution of expected gap $\bar{x}(t) \equiv \mathbb{E}(x(t))$, for firm with t_0 info

$$\bar{x}(t) = x(t_0) - \mu \cdot (t - t_0) \quad , \quad \tau(t) \equiv t - t_0 \quad , \quad x(t) \sim \mathcal{N}(\bar{x}(t), \sigma^2 \tau(t))$$

- The firm's state: $\{\bar{x}, \tau\}$ is the *predicted gap* and the *information age*

The Firm's problem

- Flow cost $F(x)$: forgone profits (w. CES demand system) due to $x \neq 0$

- Firm's value function solves

$$
\begin{aligned}
v(\bar{x}(\tau), \tau) = \min_{\alpha,\omega} \mathbb{E}\Big(\int_0^{\min\ \{\tau^a, \tau^r\}} & e^{-\rho s} \left[F(x(\tau+s)) + \kappa_a \alpha(s)^2 + \kappa_r \omega(s)^2 \right] ds \\
& + \mathbf{1}^a \cdot e^{-\rho \tau^a} \min_{x^*} v(x^*, \tau + \tau^a) \\
& + (1 - \mathbf{1}^a) \cdot e^{-\rho \tau^r} v(x(\tau + \tau^r), 0) \ \Big|\ \bar{x}(\tau), \tau \Big)
\end{aligned}
$$

- indicator function $\mathbf{1}^a$ equals 0 if new info arrives before adjustment

The Firm's problem (in recursive form)

- Firm's value function v solves HJB: appdx: derivation from sequence prob

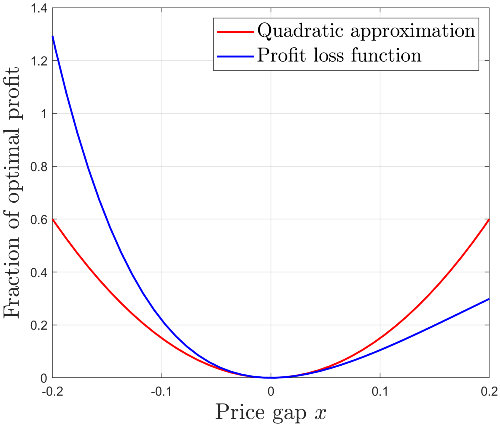
$$\rho\, v(\bar{x},\tau) = \mathbb{E}\left[F(x)|\bar{x},\tau\right] - \mu\partial_{\bar{x}} v(\bar{x},\tau) + \partial_{\tau} v(\bar{x},\tau)$$
$$+ \min_{\alpha\geq 0,\, \bar{x}^*} \{\alpha\cdot[v(\bar{x}^*,\tau) - v(\bar{x},\tau)] + \kappa_a\,\alpha^2\}$$
$$+ \min_{\omega\geq 0}\{\omega\cdot[\mathbb{E}\left[v(x,0)|\bar{x},\tau\right] - v(\bar{x},\tau)] + \kappa_r\,\omega^2\}$$

- $\mathbb{E}$ with respect to $\mathcal{N}(\bar{x},\sigma^2\tau)$, where $\bar{x}(t) = x(t_0) - \mu\cdot(t-t_0)$, $\tau(t) = t - t_0$

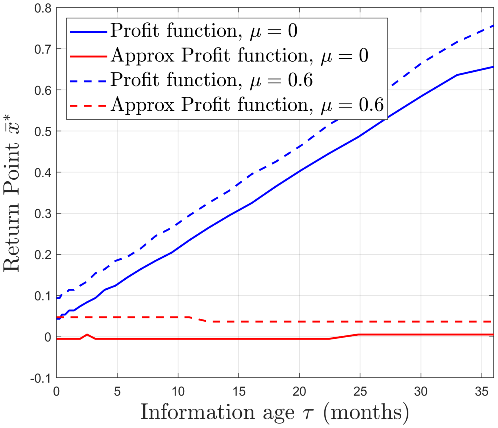
$\Longrightarrow$ Policy: Hazard functions $\alpha(\bar{x},\tau)$, $\omega(\bar{x},\tau)$ and return point $\bar{x}^*(\tau)$

Forgone Profits and Optimal Return Point

(a) Forgone Profit Function $F(x)$



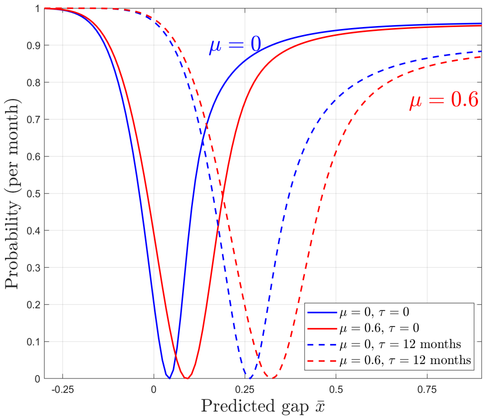
(b) Return points $\bar{x}^*(\tau)$ for $\mu \in \{0, 0.6\}$



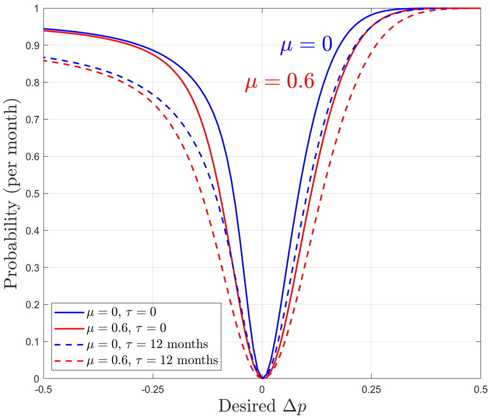
– *hedging motive* as information ages (high τ, as in Fernandez-Villaverde et al paper)

Adjustment Hazard

(a) Hazard of adjustment (uncentered)
$\alpha(\bar{x}, \tau; \mu)$

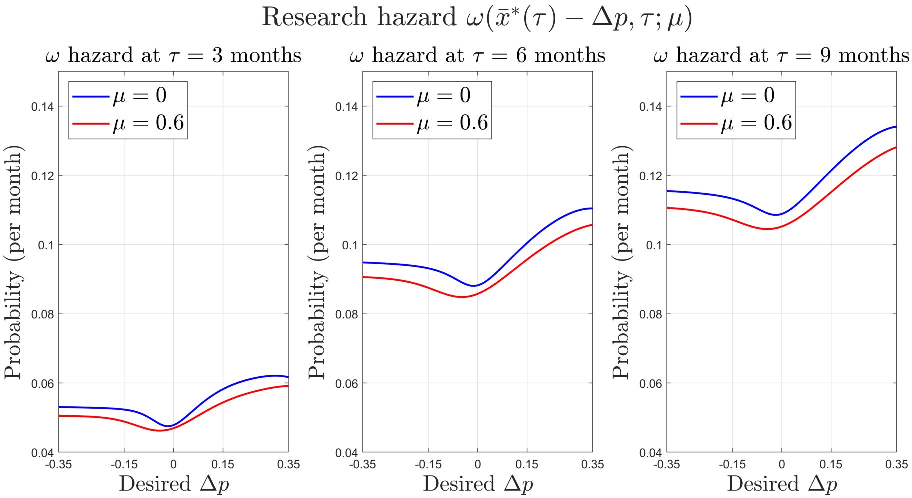


(b) Hazard of adjustment (centered)
$\alpha(\bar{x}^*(\tau) - \Delta p, \tau; \mu)$



– *wider inaction* as information ages (reduces the importance of idiosyncratic shocks)

Research Hazard



– Large gradient w.r.t. τ, small gradient w.r.t μ

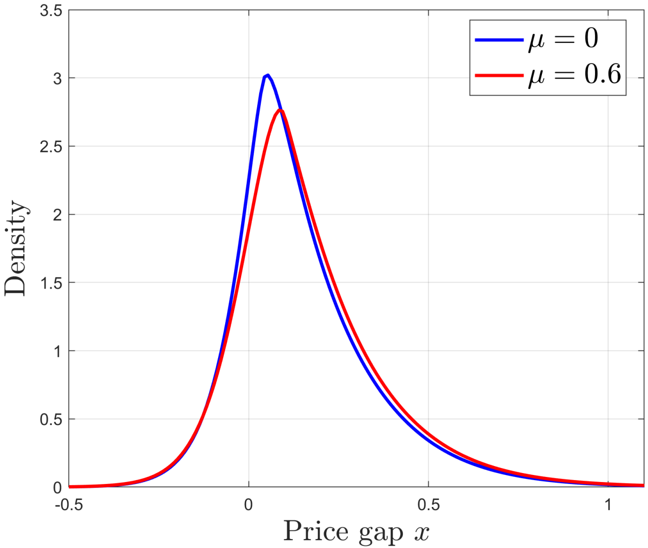
Cross-section aggregation of firms

- Aggregation: Kolmogorov equation $\bar{m}(\bar{x}, \tau)$ (omit arguments)

$$(\alpha + \omega)\, \bar{m} = \mu\, \partial_{\bar{x}} \bar{m} - \partial_{\tau} \bar{m}$$

- using $x \sim \mathcal{N}(\bar{x}, \sigma^2 \tau)$ gives distribution of actual gaps and info age $m\,(x, \tau)$

Aggregation: Marginal distribution of x



Mapping Model to Observables

- Frequency of price adjustment and research

$$N_a = \int_0^\infty \int_{-\infty}^\infty \alpha(\bar{x},\tau)\cdot \bar{m}(\bar{x},\tau)\, d\bar{x}\, d\tau$$
$$N_r = \int_0^\infty \int_{-\infty}^\infty \omega(\bar{x},\tau)\cdot \bar{m}(\bar{x},\tau)\, d\bar{x}\, d\tau$$

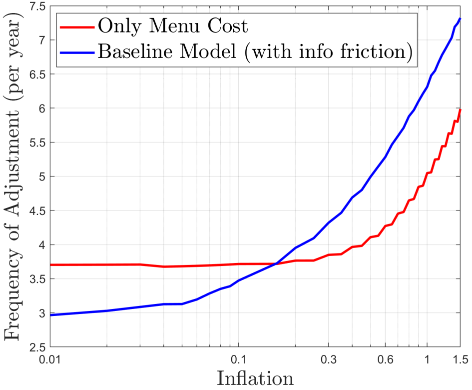
- Distribution of price changes

$$q(\Delta p) = \int_0^\infty \int_{-\infty}^\infty \mathbb{1}(\bar{x},\tau;\Delta p)\cdot \alpha(\bar{x},\tau)\cdot \bar{m}(\bar{x},\tau)\, d\bar{x}\, d\tau$$

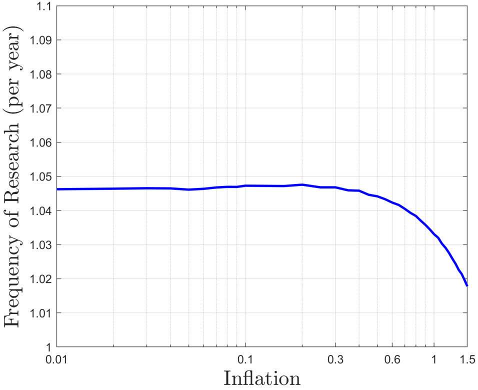
where $\mathbb{1}(\bar{x},\tau;\Delta p) \equiv \{(\bar{x},\tau):\ \bar{x}^*(\tau) - \bar{x} = \Delta p\}$

Adjustment and Research frequency wrt inflation

(a) Adjustment Frequency N_a



(b) Research frequency N_r



– menu cost flat because σ/μ large;   info friction make model more SW like

What's in a price change: Δp?

- Firm with $(\bar{x}^*(\tau_0), \tau_0)$ chooses Δp after spell of duration τ^a
- Information age at adjustment: $\tau \in [0, \tau_0 + \tau^a]$

▶ Then Δp related to (τ^a, τ) as follows:

$$\Delta p = \underbrace{\bar{x}^*(\tau) - \bar{x}^*(\tau_0)}_{\text{info age difference}} + \underbrace{\mu \tau^a}_{\text{keep up with } \mu} - \underbrace{\sigma Z(\tau_0 + \tau^a - \tau)}_{\text{new info}}$$

where Z is a Wiener process with $Z(0) = 0$. Details

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▶ We get: $N_a \tilde{\mathrm{E}}\,(\Delta p - \mu\tau^a)^2 = N_a \widetilde{\mathrm{Var}}\Big(\bar{x}^*(\tau) - \bar{x}^*(\tau_0)\Big) + \sigma^2$

Costs of inflation in the theoretical model

- Misallocation, χ, due to price gaps

$$\text{Misall. Cost}_{\mu} = \frac{\eta}{2} \text{Var}_{\mu}(x)$$

- Price management, ϕ, of adjustment and research

$$\text{Mgmt. Cost}_{\mu} = \frac{1}{\eta} E_{\mu}\left[\kappa_a(\alpha(\bar{x},\tau))^2 + \kappa_r(\omega(\bar{x},\tau))^2\right]$$

- Costs of inflation are defined in excess of levels at $\mu = 0$

$$\chi(\mu) = \text{Misall. Cost}_{\mu} - \text{Misall. Cost}_0, \qquad \phi(\mu) = \text{Mgmt. Cost}_{\mu} - \text{Mgmt. Cost}_0$$

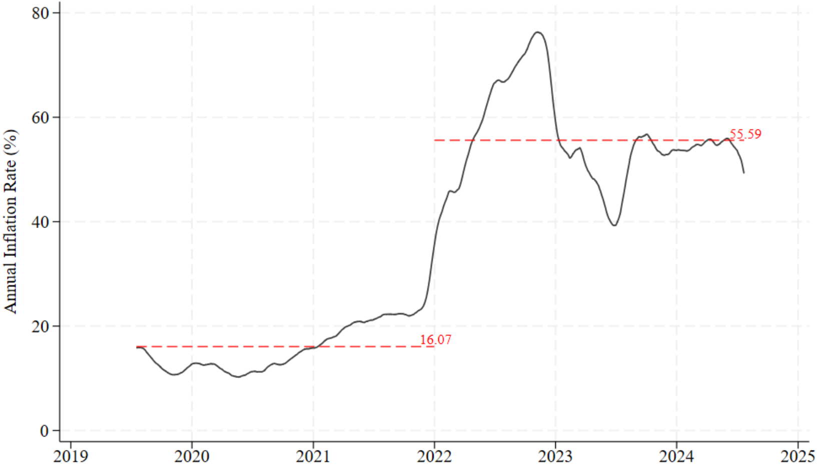
A simple exercise

- Calibrate model to Turkey with “moderate” to “high” inflation periods
- Analyze “misallocation” and “price management” costs as fct. of inflation

Data

- PriceStats data (Cavallo and Rigobon, 2016)
- Micro-data: Food and beverages sectors in Turkey (14 largest retailers)
- Sample period between June 2019 and July 2024
- Price changes and duration of completed price spells at daily frequency

Turkey: Inflation time series (source Pricestats)

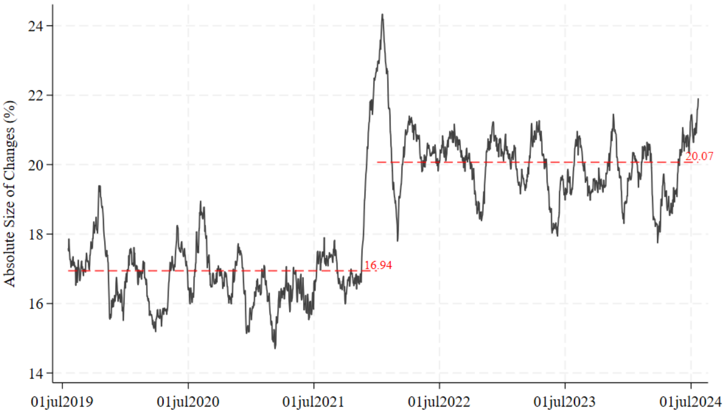


CPI vs pstats

Turkey: Frequency of price adjustment

elasticity $\theta \equiv \frac{\partial \log N}{\partial \log \mu} = 0.6$

Turkey: Absolute Size of Price Changes



This moment is Nakamura et. al (2018) measure for the cost of inflation.

Our calibration yields a similar non-targeted increase around 20% (from 13% to 16%)

Calibration

- select four parameters: $\{\mu,\ \sigma^2,\ \kappa_a,\ \kappa_r\}$

- to match four moments: $\{\tilde{\mathrm{E}}\,(\Delta\, p)\ ,\ N_a,\ \tilde{\mathrm{E}}\ (\Delta\, p - \mu\tau^a)^2\ ,\ \frac{\Delta \log N_a}{\Delta \log \mu}\}$

Useful model identities (blue is data) :

$$N_a\,\tilde{\mathrm{E}}\ (\Delta\, p - \mu\tau^a)^2 = N_a\ \underbrace{\widetilde{\mathrm{Var}}\Big(\bar{x}^*(\tau) - \bar{x}^*(\tau_0)\Big)}_{\text{Information dispersion}} + \sigma^2$$

$$\frac{1}{\tilde{\mathrm{E}}\,\tau^a} = N_a,\quad ,\quad N_a\,\tilde{\mathrm{E}}\,(\Delta\, p) = \mu\ ,\quad \text{also } \theta \equiv \frac{\Delta \log N_a}{\Delta \log \mu} \text{ is increasing in } \frac{\mu}{\sigma^2}$$

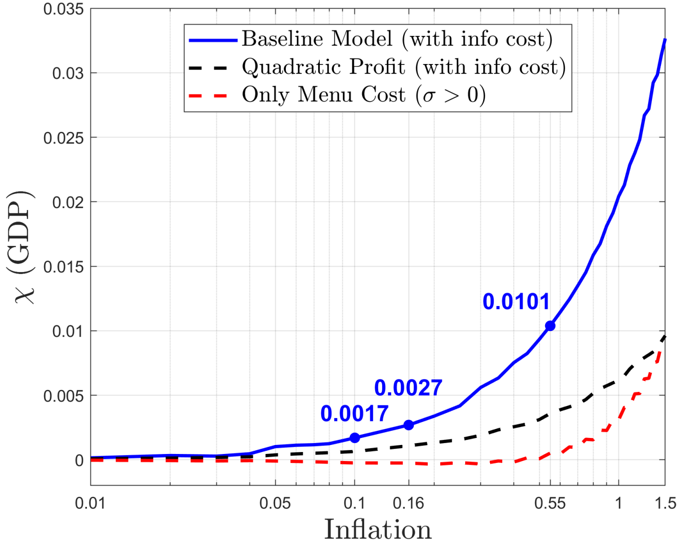
Calibration for Turkey for 2019-2021 (inflation is $\mu = 0.16$)

	Targeted moments				Parameters				Others
	N_a	$E[\Delta p]$	$N_a \tilde{E}\left[(\Delta p - \mu\tau^a)^2\right]$	Elasticity θ	σ^2	κ_a	κ_r	μ	N_r
data	3.7	0.04	0.10	0.57	–	–	–	–	–
W. info frictions	3.7	0.04	0.10	0.37	0.04	0.025^2	0.25^2	0.16	1
W/o info frictions	3.7	0.04	0.10	0.11	0.10	0.037^2	–	0.16	–

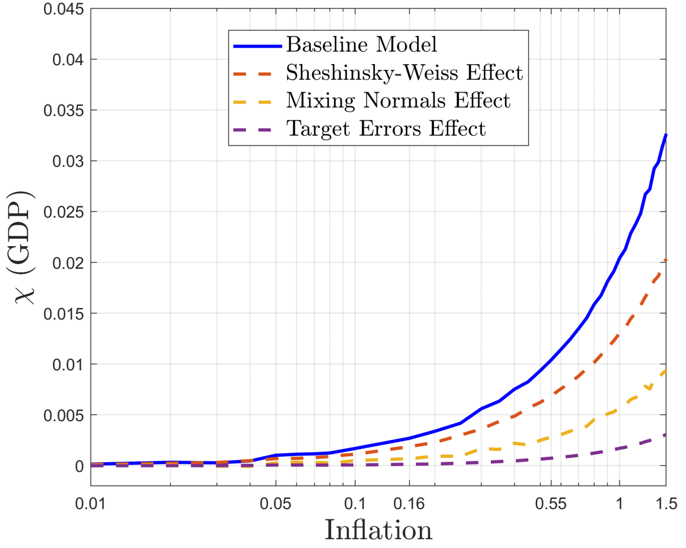
– model with info friction better at capturing extensive margin response

– w/o info friction high inflation requires larger variance of marginal costs

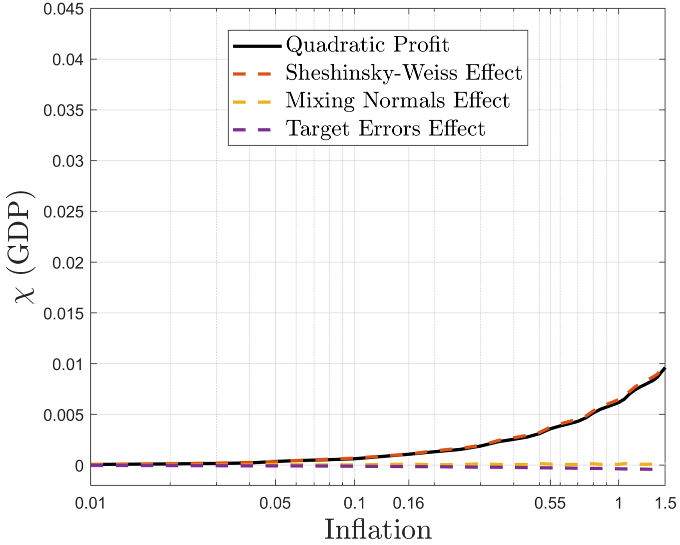
Misallocation Costs of Inflation: Models Comparison



Baseline Model Decomposition



Quadratic Profit Model Decomposition



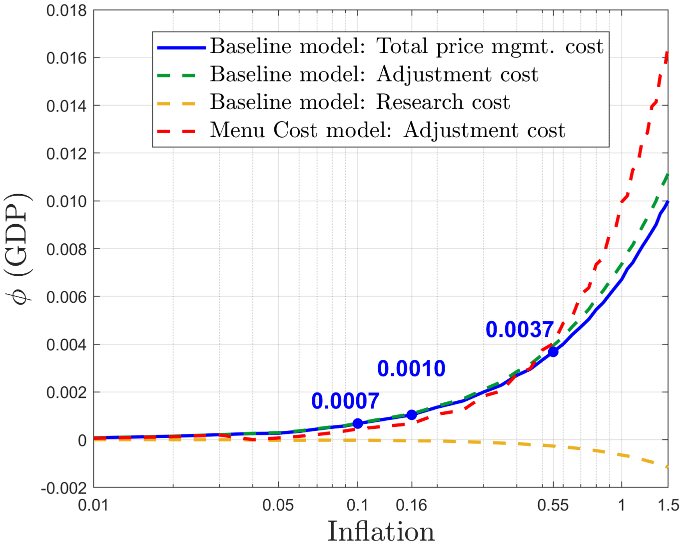
Decomposition of $\chi(0.55) - \chi(0.16) = 74$ bp

$$\frac{\eta}{2}\Delta \text{Var}(x) = \frac{\eta}{2}\Delta\left(\underbrace{E[\text{Var}(\bar{x}|\tau)]}_{\text{Sheshinski-Weiss effect}} + \underbrace{\text{Var}[E(\bar{x}|\tau)]}_{\text{mixing normals with diff. means}} + \underbrace{\sigma^2 E(\tau)}_{\text{target errors}}\right)$$

	SW	Mixing normals	Target errors	Total	N_a
$\mu = 0.55$ + “behavior fixed”	21 bp	-3 bp	-4 bp	14 bp	6
$\mu = 0.55$ + optimal $\alpha, \bar{x}^*$	43 bp	16 bp	3 bp	62 bp	5.4
$\mu = 0.55$ + optimal $\omega, \alpha, \bar{x}^*$	43 bp	23 bp	8 bp	74 bp	5.4
Only menu cost (total)	6 bp	–	–	6 bp	4.1

“behavior fixed” means $\{\omega, \alpha, \bar{x}^*\}$ fixed at $\mu = 0.16$ values

Price Management Costs: Research and Adjustment Activities



Summing up

use simple NK model to quantify costs of inflation

- imperfect info useful to fit data (boosts elast. of N to μ)
- imperfect info amplifies inflation costs
  - more action on extensive margin (wider inaction region)
  - ignorance spreads firms' return points (hedging motive)
- Non negligible inflation costs, steep gradient
  - The welfare cost of 10% inflation are 25bp of GDP in cons. equiv.
  - The welfare cost of 55% inflation are 138bp of GDP in cons. equiv.

Thank you!

Appendix

- How to compute joint moments of price adjustments and time between adjustments $\{\Delta p, \tau^a\}$
- Define generator of the uncontrolled state + discovery process
- From an initial condition, propagate that process forward removing mass upon adjustments
- The removed density is the joint distribution of price changes, information ages (measured at adjustment dates) and time between adjustments
- Continue in next slide...

Appendix cont'd

- Let $\mathcal{A}$ be the generator of the uncontrolled + observation process
- Let $F(\bar{x}, \tau, t)$ be a time-varying measure on $(\bar{x}, \tau)$ that keeps track of the states that have not adjusted and Q for the measure of states that have
- Let $P(\Delta p, \tilde{\tau}, \tau^a)$ be the distribution of price changes, information age (measured at times of adjustments) and time between adjustments

Appendix cont'd

- Let $F(\bar{x}, \tau, 0)$ be a density integrating to 1 of states right after an adjustment e.g. a Dirac density on $(\bar{x}^*(\tau), \tau)$
- $F(\bar{x}, \tau, t)$ solves the following PDE

$$\partial_t F = \mathcal{A}F - \alpha \cdot F$$

- Then the distributions Q and P are

$$P(\bar{x}^*(\tau) - \bar{x}, \tilde{\tau}, \tau^a) = Q(\bar{x}, \tilde{\tau}, \tau^a) = \alpha(\bar{x}, \tau) \cdot F(\bar{x}, \tilde{\tau}, \tau^a)$$

New information at price adjustment dates

- New information (mg cost shocks) at adjustment dates has three components
- τ_0: information age at start of price spell
- τ^a: information that transpired during the price spell
- τ: information age at end of price spell
- $\implies$ new info has variance $\tau_0 + \tau^a - \tau$ Return

Sequential problem

- Firm with state $(\bar{x}, \tau)$ at $t = 0$ solves

$$v(\bar{x}, \tau) = \min_{\alpha,\omega,\bar{x}^*} \mathbb{E}\left[\int_0^\infty e^{-\rho t} F[x(t)]dt \,|\, x(0) \sim \mathcal{N}(\bar{x}, \sigma^2\tau)\right]$$

$d\bar{x} = -\mu\, dt$ and $d\tau = dt$.

- For illustration assume α, ω fixed.
- Developing the RHS for the flow and the non-jump term

$$v(\bar{x}, \tau) = \min_{\bar{x}^*} \mathbb{E}_0 \left[F[x(0)]\Delta + o(\Delta) + e^{-(\rho+\alpha+\omega)\Delta} \int_\Delta^\infty e^{-\rho(t-\Delta)} F[x(t)]dt + \dots\right]$$

$$v(\bar{x}, \tau) = \min_{\bar{x}^*} \mathbb{E}_0 \left[F[x(0)]\Delta + o(\Delta) + e^{-(\rho+\alpha+\omega)\Delta} v[\bar{x}(\Delta), \tau(\Delta)] + \dots\right]$$

$$v(\bar{x}, \tau) = \min_{\bar{x}^*} \mathbb{E}_0 \left[F[x(0)]\Delta + o(\Delta) + e^{-(\rho+\alpha+\omega)\Delta} \{v(\bar{x}, \tau) + \partial_{\bar{x}} v \cdot (-\mu\Delta) + \partial_\tau v \cdot (\Delta)\} + \dots\right]$$

$$(\rho + \alpha + \omega)\Delta\, v(\bar{x}, \tau) = \min_{\bar{x}^*} \mathbb{E}_0 \left[F[x(0)]\Delta + o(\Delta) + e^{-(\rho+\alpha+\omega)\Delta} \{\partial_{\bar{x}} v \cdot (-\mu\Delta) + \partial_\tau v \cdot (\Delta)\} + \dots\right]$$

Denote the conditional expectation by $\mathbb{E}_0$

Sequential problem (cont'd)

- Developing the jump terms

$$\ldots = \min_{\bar{x}^*} \mathbb{E}_0 \left[\ldots + e^{-\rho\Delta} \left[(1 - e^{-\alpha\Delta}) v(\bar{x}^*, \tau(\Delta)) + (1 - e^{-\omega\Delta}) v(x(\Delta), 0) \right] \right]$$

$$\ldots = \min_{\bar{x}^*} \mathbb{E}_0 \left[\ldots + e^{-\rho\Delta} \left[(1 - e^{-\alpha\Delta}) \left[v(\bar{x}^*, \tau) + \partial_\tau v \cdot \Delta \right] + (1 - e^{-\omega\Delta}) v(x(\Delta), 0) \right] \right]$$

$$\ldots = \min_{\bar{x}^*} \mathbb{E}_0 \left[\ldots + e^{-\rho\Delta} \left[\alpha\Delta v(\bar{x}^*, \tau) + o(\Delta) + \omega\Delta v(x(0), 0) \right] \right]$$

2nd to 3rd line: drift and diffusion terms times hazards are order Δ^2

- Putting the terms together, dividing by Δ and taking the limit as $\Delta \to 0$

$$(\rho + \alpha + \omega)\, v(\bar{x}, \tau) = \min_{\bar{x}^*} \mathbb{E}_0 \left[F[x(0)] + \partial_{\bar{x}} v \cdot (-\mu) + \partial_\tau v + \alpha v(\bar{x}^*, \tau) + \omega v(x(0), 0) \right]$$

$$(\rho + \alpha + \omega)\, v(\bar{x}, \tau) = \mathbb{E}_0 F[x(0)] + \partial_{\bar{x}} v \cdot (-\mu) + \partial_\tau v + \alpha \min_{\bar{x}^*} v(\bar{x}^*, \tau) + \omega \mathbb{E}_0 v(x(0), 0)$$

Q.E.D. Return

Mapping Observables to Model Parameters: No Info Friction

- LHS: moments in the data $\{\Delta p, \tau^a\}$
- Operators $\tilde{\mathrm{E}}, \widetilde{\mathrm{Var}}$ integrate w.r.t. density of $\{\Delta p, \tau^a\}$

$$\frac{1}{\tilde{\mathrm{E}}\,\tau^a} = N_a, \quad (1)$$

$$N_a \tilde{\mathrm{E}}\,\Delta p = \mu, \quad (2)$$

$$N_a \tilde{\mathrm{E}}\,(\Delta p - \mu\tau^a)^2 = \sigma^2, \quad (3)$$

$$N_a \tilde{\mathrm{E}}\tau^a \left(\Delta p - \frac{\mu}{2}\tau^a\right) = x^* - \tilde{\mathrm{E}}x \quad (4)$$

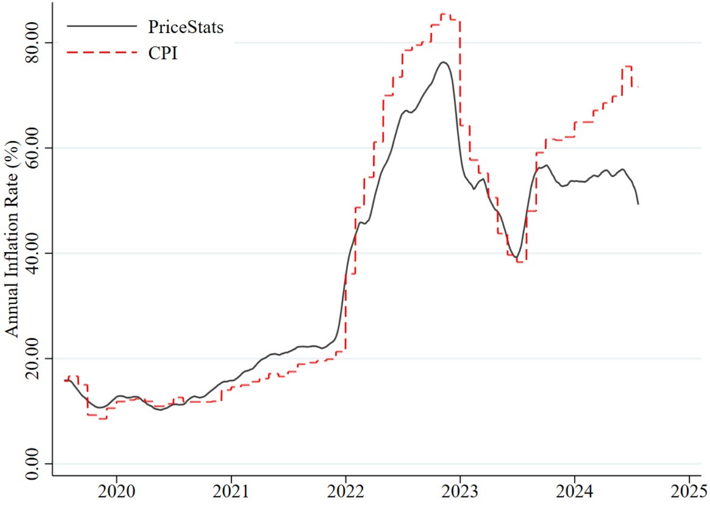
$$\frac{1}{3}\frac{N_a}{\mu}\tilde{\mathrm{E}}\,(\Delta p)^3 - (x^* - \tilde{\mathrm{E}}x)\left(\frac{\sigma^2}{\mu} + x^* - \tilde{\mathrm{E}}x\right) = \mathrm{Var}(x) \quad (5)$$

Return

Relative Entropy (skip)

- How the distribution of gaps changes with higher inflation?
- One measure is relative entropy between two densities m_1 and m_0 e.g. m_1 corresponds to 0.6 inflation and m_0 to 0.3 inflation
- Relative entropy measures $\int_{-\infty}^{\infty} \log[n(x)]n(x)\, m_0(x)dx$ where $n(x) \equiv m_1(x)/m_0(x)$
- Next figure displays $\log[n(x)]n(x)\, m_0(x)dx$ as a share of relative entropy to understand important contributors

Pricestats vs CPI data: Turkey



Return