Sufficient Statistics for Measuring Forward-Looking Welfare

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  “The futures prices needed... are simply unavailable. So it [is] difficult to make operational the theorists’ desired measures.”
- Literature takes two approaches to the problem:
  1. Compute welfare inside fully-specified dynamic model.
  2. Use net-present value of real wealth (discounting future cashflows & prices).
- Both require taking stance on state-contingent prices, cashflows, returns, probabilities, plans.

What We Do

- Develop sufficient-stats for dynamic welfare side-stepping knowledge of future.
- Key assumption: preferences are separable between the present and the future.
- Use changes in savings behavior to learn about changes in expectations about the future.
- Method allows for incomplete markets, idiosyncratic risk & borrowing constraints.
- Application using the PSID between $2005-2019$:
  - Dynamic measures of growth and cost-of-living very different to static.
- Measure can be used to estimate dynamic welfare treatment effects:
  - e.g. job loss associated with 20% reduction in welfare.

Selected Related Literature

- Basic theory of intertemporal welfare measures:
  Samuelson (1961), Alchian and Klein (1973), Pollack (1975).
  - Fully-specified models:
    Reis (2005), Aoki and Kitahara (2010), Jones and Klenow (2016), etc.
  - Discounting future:
    Hulten (1979), Goodhart (2001), Basu et al. (2022), Fagereng et al. (2022), Del Canto et al. (2023).
- Static price index literature: Feenstra (1994), Hamilton (2001), Costa (2001), Almås (2012), Atkin et al. (2024), Jaravel and Lashkari (2024), Baqaee et al. (2024).

Agenda

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Inferring Welfare in Stripped-down Example

Inferring Welfare in General Environment

Empirical Illustration Using PSID data

Conclusion

Example with Complete Markets

- Consumers with horizon J living at date τ solve following problem:

$$V(\tau, w) = \max_{\mathbf{c},\mathbf{a}} \sum_{j=0}^{J} \sum_{s_j} \beta^j \pi(s^j|\tau) \frac{c(s^j|\tau)^{1-1/\sigma}}{1-1/\sigma}.$$

- Subject to sequence of budget constraints

$$p(s^0|\tau)c(s^0|\tau) + \sum_{k \in S_1} a_k(s^0|\tau) = w,$$

$$p(s^j|\tau)c(s^j|\tau) + \sum_{k \in S_{t+1}} a_k(s^j|\tau) = R_k(s^{j-1}|\tau)a_k(s^{j-1}|\tau),$$

$$p(s^J|\tau)c(s^J|\tau) \leq R_k(s^{J-1}|\tau)a_k(s^{J-1}|\tau).$$

- Hold J (age of consumer) fixed, so suppress dependence on J.

Measuring Welfare

- Measure welfare at (τ, w) using money-metric: wealth in τ_0 that makes consumer indifferent

$$V(\tau, w) = V(\tau_0, m(\tau, w|\tau_0)).$$

- Use $m(\tau, w|\tau_0)$ to measure growth (by varying τ & w) or cost-of-living (by varying τ_0).
- Write $m(\tau, w)$ instead of $m(\tau, w|\tau_0)$ from now (hold base year τ_0 constant).

Towards Solution

- With intertemporal budget constraint, problem is static w/ CES demand over dates/states.
- Hence, money-metric is total wealth divided by CES ideal price index.

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- Need beliefs about future probabilities, prices & returns.
- Illustrates Samuelson's problem: require knowledge of future prices.

Using Consumption-Savings Decisions to Back-Out Future Prices

- Use consumption-wealth ratio, B^P, to back out intertemporal $P(\tau)$:

$$B^P(\tau, w) = \frac{p\left(s^0|\tau\right) c\left(s^0|\tau, w\right)}{w} = B^P(\tau).$$

- By CES demand, change in consumption-wealth ratio satisfies

$$\frac{B^P(\tau)}{B^P(\tau_0)} = \left[\frac{p(s^0|\tau\ )/P(\tau\ )}{p(s^0|\tau_0)/P(\tau_0)}\right]^{1-\sigma}.$$

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- Using equation above (Feenstra, 1994 logic for value of new goods applied to value of future goods)

$$\log m(\tau, w) = \underbrace{\log w - \log \frac{p\left(s^0|\tau\right)}{p\left(s^0|\tau_0\right)}}_{\text{static “real wealth”}} + \underbrace{\frac{1}{1-\sigma} \log \frac{B^p(\tau)}{B^P(\tau_0)}}_{\text{adjustment for future}}.$$

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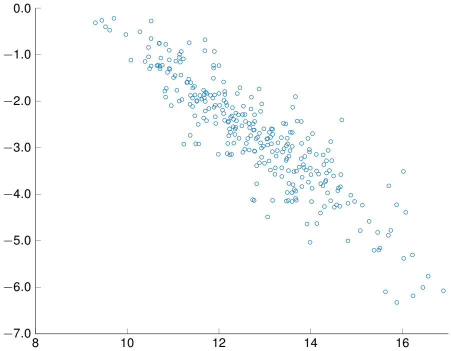
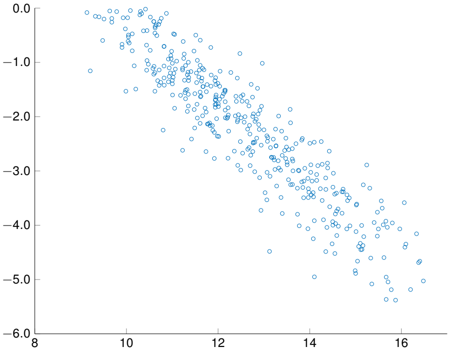
Non-homotheticities in consumption-wealth ratio

- Method requires using changes in consumption-wealth ratio.
- But, consumption-wealth ratio strongly declining in permanent wealth (as in Straub 2019).



(a) 2005



(b) 2019

- Need to extend to non-homothetic case; whose changes in consumption-wealth to use?

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$$U = D(\mathbf{c}, \boldsymbol{\pi}, U),$$

where D is homogeneous of degree one in $\mathbf{c}$.

- Assume this representation is separable between present and future

$$U = D(\underbrace{P(\mathbf{c}(s^0), U)}_{\text{present bundle}}, \underbrace{F(\{\mathbf{c}(s^j)\}_{j>0}, \{\pi(s^j)\}_{j>0}, U)}_{\text{future bundle}}, U),$$

If preferences are homothetic, then can drop U from RHS.

- Implies spending on i relative to j in same block only function of prices in that block and U.

Examples with Time Separability

- Some examples assuming one good per period (to keep notation light).

- e.g. Non-homothetic patience

$$U = \frac{c(s^0)^{1-1/\sigma}}{1-1/\sigma} + \sum_{j=1}^{J} \beta_j(U) \frac{c(s^j)^{1-1/\sigma}}{1-1/\sigma}.$$

- e.g. Non-homothetic risk-aversion

$$U = \frac{c(s^0)^{1-1/\sigma}}{1-1/\sigma} + \sum_{j=1}^{J} \beta^j \frac{\left[\sum_{s^j} \pi(s^j) c(s^j)^{\gamma(U)}\right]^{\frac{1-1/\sigma}{\gamma(U)}}}{1-1/\sigma}.$$

- e.g. Non-homothetic intertemporal elasticity of substitution

$$U = \frac{c(s^0)^{1-1/\sigma(U)}}{1-1/\sigma(U)} + \sum_{j=1}^{J} \beta^j \frac{c(s^j)^{1-1/\sigma(U)}}{1-1/\sigma(U)}.$$

Constraints

- First period budget constraint:

$$p(s^0|\tau)c(s^0|\tau) + \sum_{k \in K} a_k(s^0|\tau) = w.$$

- Each subsequent history s^j, receive capital and labor income $y(s^j|\tau)$:

$$p(s^j|\tau)c_n(s^j|\tau) + \sum_{k \in K} a_k(s^j|\tau) = \sum_{k \in K} R_k(s^j|\tau)a_k(s^{j-1}|\tau) + y(s^j|\tau).$$

- Borrowing constraints $\sum_k a_k(s^j|\tau) \geq -X(s^j|\tau)$. No-ponzi requires $X(s^J) = 0$.

- *Rentiers* are households with no labor income: $y(s^j|\tau) = 0$ for all s^j.

Dynamic Money Metric Utility

- Consumer behaves as-if future plans are followed (relax in extensions).

- Value function is

$$V(\underbrace{\{\mathbf{p}, \mathbf{R}, \boldsymbol{\pi}, \mathbf{X}\}}_{\text{indexed by } \tau}, w, \mathbf{y}) = \max_{\mathbf{c}, \mathbf{a}} \{\text{utility } : \text{constraints satisfied}\}.$$

- No single intertemporal budget constraint, so many alternative ways to define money metric.

- We use equivalent wealth as rentier.

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- Consider a consumer living at τ with wealth w and income $\mathbf{y}$. How much wealth would he need, as a rentier, at date τ_0 to be indifferent? This is $m(\tau,w,\mathbf{y})$ that solves

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- Consider special cases that build to general result.

Special Case I: homothetic rentiers

Proposition

Suppose homoth. preferences, one consumption good per period, and constant EIS. For rentiers:

$$\log m(\tau, w, \mathbf{0}) = \underbrace{\log w - \log \frac{p(s^0|\tau)}{p(s^0|\tau_0)}}_{\text{static "real wealth"}} + \underbrace{\frac{\log\left(B^P(\tau, w, \mathbf{0})/B^P(\tau_0, w, \mathbf{0})\right)}{1-\sigma}}_{\text{adjustment for future}}.$$

- Generalizes simple example for incomplete markets & more general preferences (e.g. EZ).

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Special Case II: allowing non-homotheticity

- If preference non-homoth., then consumption-wealth ratio could change for two reasons:
  1. Because cost of present consumption relative to future changed.
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  (logic of Baqaee et al. (2024) applied to dynamic)

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General Case for Rentiers

- Fully general case, with multiple goods and variable EIS.

Proposition

Money metric for rentiers is solution to the fixed point problem:

$$\log m(\tau, w) = \underbrace{\log w - \int_{\tau_0}^{\tau} \sum_{n \in N} B_n(t, w_t^*) \frac{d \log p_n}{dt} dt}_{\text{static ``real wealth''}} + \underbrace{\int_{\tau_0}^{\tau} \frac{d \log B^P(t, w_t^*)/dt}{1 - \sigma(t, w_t^*)} dt}_{\text{adjustment for future}},$$

where for each $t \in [\tau_0, \tau]$, w_t^* satisfies the equation

$$m(t, w_t^*) = m(\tau, w).$$

Money Metric for Non-Rentiers

- Separability implies budget shares in present, b_n, depend **only** on prices in present and U.
- If $b_n(\mathbf{p}, U)$ is one-to-one in U, can use budget shares to infer U for non-rentiers.
- e.g. dentist & landlord with same budget shares in τ on same indifference curve.
- To obtain this inverse:
  1. Regress wealth on budget shares and time for rentiers.
  2. Use fitted relationship to impute money metric wealth for non-rentiers.

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Logic of Hamilton (2001) applied in the cross-section rather than over time.

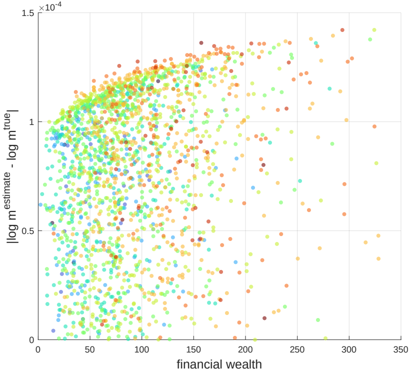
Testing Method using Monte Carlo

- Consider parametric example

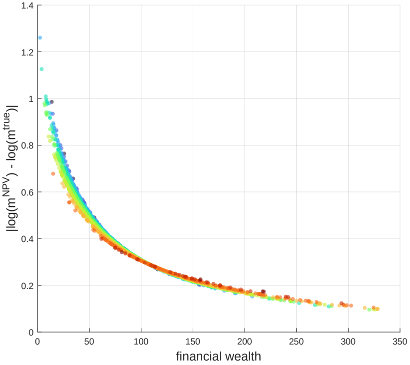
$$U = \frac{1}{1 - 1/\sigma} \mathbb{E}_0 \sum_{j=0} \beta^j C_j^{1-\frac{1}{\sigma}}, \quad \text{where} \quad C_j = \left[\sum_n \omega_n^{\frac{1}{\gamma}} \left[\frac{c_{nj}}{U^{\varepsilon_n}} \right]^{\frac{1-\gamma}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}} .$$

- Simulate Bewley households with borrowing constraint & risk-free bond.
- Off-the-shelf AR(1) calibration of income process.
- Use $\beta = 0.96$, $\sigma = 0.1$ (Best et al. 2024), $\gamma = 0.25$.
- Compare error against NPV calculation using risk-free discount rate.

Money Metric Against NPV of Total Wealth



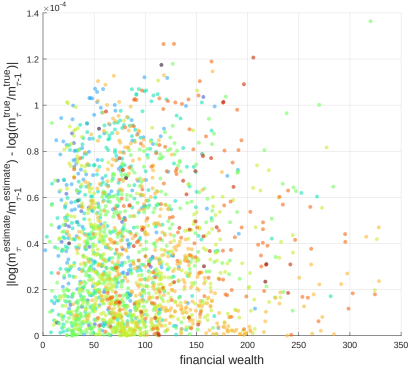
(a) Our method



(b) NPV

- NPV of income overstates money metric utility, especially for poor households.

Changes Due to Income Shock



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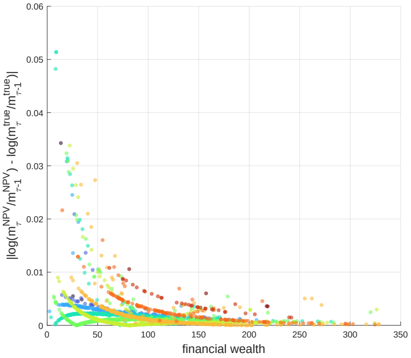

(b) NPV

- NPV performs less badly in response to shocks, but errors still large for low wealth.

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3. No commitment with time inconsistency: method applies if time separability holds.
4. Changes in mortality: method extends but need WTP to reduce probability of death.
5. What about non-time-separable non-homotheticity?

Non-Separable Non-homotheticity

- A common functional form, due to Comin et al. (2020), is

$$U = \frac{1}{1-1/\sigma}\mathbb{E}_0 \sum_{j=0} \beta^j C_j^{1-\frac{1}{\sigma}}, \quad \text{where} \quad C_j = \left[\sum_n \omega_n^{\frac{1}{\gamma}} \left[\frac{c_{nj}}{C_j^{\varepsilon_n}}\right]^{\frac{1-\gamma}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}}.$$

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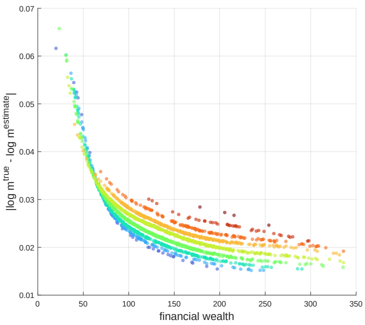
- Non-homotheticity depends on C_j not U, so does not satisfy our assumption.
- But method still works in practice:
  - As EIS tends to zero, we show method works exactly for both rentiers and non-rentiers.
  - Under mild assumption about shocks, method exact for rentiers even if EIS far from zero.
- Quantitative illustration using EIS $\sigma = 0.1$.

Money Metric for Non-Rentiers Against NPV of Total Wealth

- Not shown: method works very well for rentiers even with gigantic shocks.
- Method also works quite well for non-rentiers:



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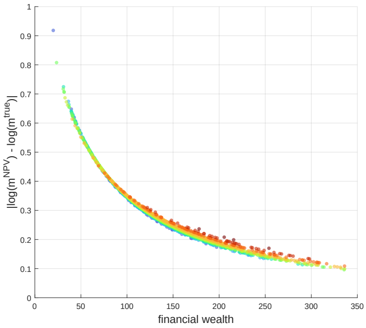
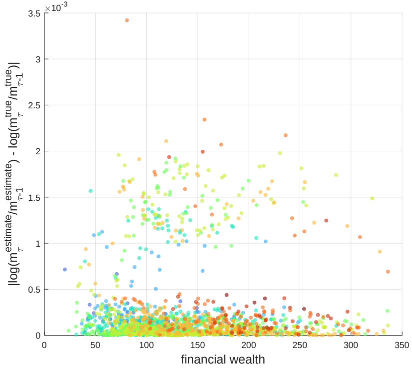


(b) NPV

- Method works much better than NPV, with much less information.

Changes Due to Income Shocks

- In response to income shocks, in changes, the errors are even smaller.



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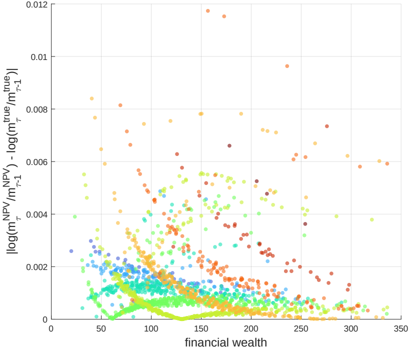

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Data

- Household survey with financial net worth, age, consumption survey, subset of rentiers.
  (We use PSID, bi-annual from $2005-2019$).
  (Group consumers by decade of life — show results for $60-69$ year olds for illustration.)

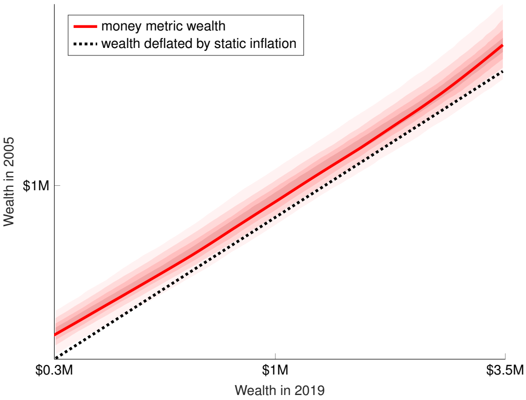
- Prices of goods and services.
  (CPI prices for seven categories in PSID).

- Elasticity of intertemporal substitution $\sigma(\tau, w)$
  (use Best, Cloyne, Ilzetzki, and Kleven 2020 of $\sigma = 0.1$.)
  (if consumption is a normal good, then compensated EIS $<$ uncompensated EIS.)

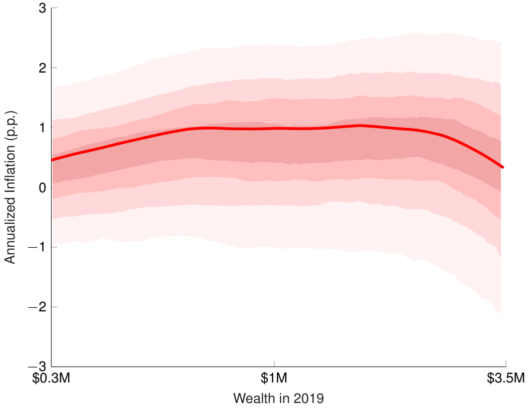
Classifying Rentiers

- Proxy wealth = net assets (including DC) + discounted labor income $+$ transfers.
- Forecast income using cross-section $+$ CBO forecast of NGDP.
- Discount future labor income and transfers by 4$\%$ real rate (Catherine et al., 2022).
- Rentiers: Net financial assets $\geq$ 90$\%$ of total wealth & not unemployed.
- Drop from rentier set if net assets are in the top and bottom 2.5$\%$.

Money metric wealth in 2005 base prices for 60-69 year olds



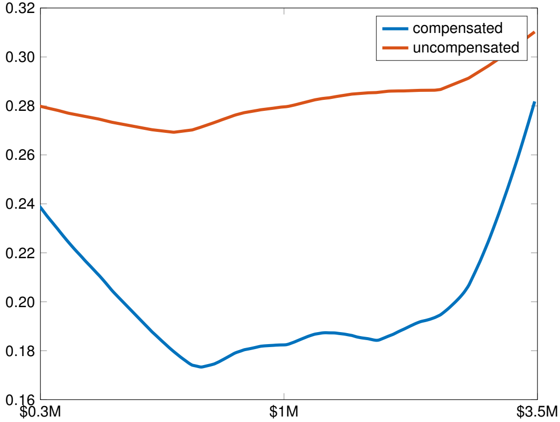
(a) Money metric



(b) Annualized inflation

- Money metric converts wealth in 2019 into equivalent in 2005 and vice versa.
- Static inflation overstates cost-of-living changes.

Change in log consumption wealth ratios 2005 – 2019 (60 – 69 year olds)



- Consumption-wealth ratios grew, so future is brighter.
- Compensated grew less than uncompensated since some changes due to wealth effects.

Dynamic Welfare Treatment Effect

- Consider a treatment that affects households over time in uncertain ways.
- For example, job training program, educational investment, interest rate policy, etc.
- Our money metric estimates can be used to study welfare treatment effect.
- Illustrate using job loss for head of household using PSID.

Percent change in money metric wealth due to job loss

	log money metric	
	(1)	(2)
Job Loss	-0.197	-0.218
	(0.031)	(0.034)
Job loss $\times$ $1(\text{age} \geq 60)$		0.180
		(0.083)
Lagged LHS	Yes	Yes
Controls	Yes	Yes
Observations	48,357	48,357
	Full Sample	

Controls: year fixed effects, age group, marital status of HH head, industry, and education level.

- Infer difference in money-metric using difference in budget shares of job-havers vs. -losers.

Agenda

Inferring Welfare in Stripped-down Example

Inferring Welfare in General Environment

Empirical Illustration Using PSID data

Conclusion

Conclusion

- Generalize money metrics to intertemporal preferences, risk, and incomplete markets.
- Use time-separability to infer it from consumption-savings for rentiers.
- Match rentiers and non-rentiers using budget shares.
- Static and dynamic different, with heterogeneity in wealth & age.
- Ingredient for policy evaluation of shocks that affect future.

Extension 1: Pseudo rentiers

- Consider a subset of households with risk-free cash flow $y(s^j|\tau) = y(j|\tau)$.

- For example, public sector employees, teachers, pensioners on defined benefits, etc.

- For these households, assume no ad-hoc borrowing constraints & access to bonds of maturities $\{1, \ldots, J\}$.

- These households' problem is isomorphic to rentier with augmented wealth

$$w(s^0|\tau) + \sum_{j=0}^{J} \frac{y(j|\tau)}{R(j|\tau)},$$

  where $R(j|\tau)$ is return on bond with maturity j purchased in τ.

- Do not pursue this in empirical application (for now).

Extension 2: Changes in mortality

- Let λ_P and λ_F be prob. of reaching P and F.
- Marginal willingness to pay for increasing survival probabilities: $\Phi_P(\tau, w)$, $\Phi_F(\tau, w)$.
- Money metric solves:

$$\log u(\tau, w, \mathbf{0}) = \underbrace{\log w - \int_{\tau_0}^{\tau} \left(\sum_{n \in N} B_n(t, w_t^*) \frac{d \log p_n}{dt} - \frac{1}{1-\sigma} \frac{d \log B^P(t, w_t^*)}{dt} \right) dt}_{\text{what we had before}}$$

$$+ \underbrace{\int_{\tau_0}^{\tau} \left(\Phi_P(t, w_t^*) \frac{d \log \lambda_P(t)}{dt} + \Phi_F(t, w_t^*) \frac{d \log \lambda_F(t)}{dt} \right) dt}_{\text{compensated value of increased survival}}$$

$$+ \underbrace{\int_{\tau_0}^{\tau} \frac{\sigma}{1-\sigma} \left(1 - B^P(t, w_t^*)\right) \frac{d \log \lambda_F(t)}{dt} dt}_{\text{changes in consmption/wealth ratio due to } d\lambda_F} .$$

Extension 3: Leisure

- Results unchanged for rentiers if, conditional on observables, leisure choices do not change as a function of calendar time (e.g. labor productivity = 0, or 9-to-5 job).
- Results unchanged for non-rentiers if relative static budget shares only depend on utility and static prices of goods and services.
- Rules out non-separabilities between consumption choices and leisure.
- Example:

$$U^{\frac{\sigma-1}{\sigma}} = \tilde{P}\left(c\left(s^0\right), U\right)^{\frac{\sigma-1}{\sigma}} + \tilde{F}\left(\left\{c(s^j)\right\}_{j>0}, \boldsymbol{\pi}, U\right)^{\frac{\sigma-1}{\sigma}} + \tilde{H}\left(\left\{l(s^j)\right\}_{j\geq 0}, \boldsymbol{\pi}, U\right).$$

Extension 4: Comin et al. (2020)

- Recall a common class of preferences take the form

$$U = \frac{1}{1-1/\sigma}\mathbb{E}_0 \sum_{t=0} \beta^t C_t^{1-\frac{1}{\sigma}}, \quad \text{where} \quad C_t = \sum_n \omega_{nt}^{\frac{1}{\gamma}} \left[\frac{c_{nt}}{C_t^{\varepsilon_n}}\right]^{\frac{1-\gamma}{\gamma}}.$$

- Not time separable in the way we need (unless homothetic).
- But method still works in practice:
  - In paper, prove as EIS tends to zero, method works well for both rentiers and non-rentiers.
  - Under mild assumption about shocks, method works perfectly for rentiers even if $\sigma \gg 0$.
- Illustrate reliability using quantitative examples with EIS ≈ 0.1.

Extension 4: Comin et al. (2020)

- For these preferences, the EIS is

$$\sigma(\tau, w, y) = \left[(1-\gamma) \frac{Var_{B(s^0)}(\varepsilon_n)}{\mathbb{E}_{B(s^0)}\left[\varepsilon_n\right]^2} + 1 - \frac{\left[1 - \frac{1}{\theta}\right]}{\mathbb{E}_{B(s^0)}\left[\varepsilon_n\right]} \right]^{-1}.$$

  where the variance and expectation use period 0 budget shares, denoted $B(s^0)$, as weights. Since $B(s^0)$ vary as a function of (τ, w, y), the EIS also varies.

- For rentiers, proposition holds but there is an error term.

$$\log m(\tau, w, 0) = \log w - \int_{\tau_0}^{\tau} \left(\sum_{n \in N} B_n(x, w_x^*, 0) \frac{d \log p_n}{dx} - \frac{d \log B^P(x, w_x^*, 0)/dx}{1 - \sigma(x, w_x^*, 0)} \right) dx + (1-\gamma)\text{error}.$$

Extension 4: Comin et al. (2020)

- Error term is

$$error = \int_{\tau_0}^{\tau} \frac{\sigma(x, w_x^*, 0)}{\sigma(x, w_x^*, 0) - 1} \left[Cov_{B(s^0)}\left(\frac{\varepsilon_i}{\mathbb{E}_{B(s^0)}[\varepsilon_i]}, \frac{d \log p_n(s^0)}{dx}\right) - \sum_{j=0}^{J} \frac{E(s^j)/R(s^j)}{\sum_{j'} E(s^{j'})/R(s^j)} Cov_{B(s^j)}\left(\frac{\varepsilon_n}{\mathbb{E}_{B(s^j)}[\varepsilon_n]}, \frac{d \log p_n(s^j)}{dx}\right) \right] dx.$$

- As EIS goes to zero, (e.g. $\sigma \to 0$), the error goes to zero.
- Error depends on *difference* in contemporaneous and future covariance of profile of shocks with slopes of Engel curves — likely to be small for many shocks.
- e.g. interest rates shocks have zero error.

Extension 4: Comin et al. (2020)

- For non-rentiers, as $\sigma \to 0$, method also applies.

Proposition

Matching on budget shares correctly identifies money-metric utility for non-rentiers when $\sigma = 0$ (which happens if, for example, $\theta \to 0$).

Proof Sketch

1. There exist shadow prices q^* that "rationalize" consumer's choices:

$$c_n^*(s^j|q^*, \boldsymbol{\pi}, V(\tau, w, y)) = c_n(s^j|\tau, w, y)$$

   with shadow prices for goods in first period equal to observed prices:

$$q_n^*(s^0|\tau, w, y) = p_n(s^0|\tau).$$

2. Dual shadow prices for rentiers depend on τ and V — not the case for non-rentiers.

3. Money metric is expressible using shadow intertemporal expenditure function:

$$m(\tau, w, \mathbf{0}) = e(q^*(\cdot|\tau_0, m(\tau, w, \mathbf{0})), \boldsymbol{\pi}(\cdot|\tau_0), m(\tau, w, \mathbf{0})).$$

4. Manipulate to get:

$$\log m(\tau, w, \mathbf{0}) = \log w - \log \frac{e(q^*(\cdot|\tau, m(\tau, w, \mathbf{0})), \boldsymbol{\pi}(\cdot|\tau), m(\tau, w, \mathbf{0}))}{e(q^*(\cdot|\tau_0, m(\tau, w, \mathbf{0})), \boldsymbol{\pi}(\cdot|\tau_0), m(\tau, w, \mathbf{0}))}.$$

Proof Sketch

5. Using fundamental theorem of calculus:

$$\log m(\tau,w,\mathbf{0}) = \log w + \int_{\tau}^{\tau_0} \sum_{t=0}^{J} \sum_{s^j} \left(\frac{\partial \log e(q^*\left(s^j|t,m(\tau,w,\mathbf{0})\right),\boldsymbol{\pi}(s^j|t),m(\tau,w,\mathbf{0}))}{\partial \log q^*(s^j|t,m(\tau,w,\mathbf{0}))} \cdot \frac{d\log q^*\left(s^j|t,m(\tau,w,\mathbf{0})\right)}{dt} + \frac{\partial \log e(q^*\left(s^j|t,m(\tau,w,\mathbf{0})\right),\boldsymbol{\pi}(s^j|t),m(\tau,w,\mathbf{0}))}{\partial \boldsymbol{\pi}(s^j|t)} \cdot \frac{d\boldsymbol{\pi}(s^j|t)}{dt} \right) dt.$$

6. Cut through the complexity using time-separability:

$$\frac{\partial \log e(q,\boldsymbol{\pi},U)}{\partial \log q} \cdot d\log q + \frac{\partial \log e(q,\boldsymbol{\pi},U)}{\partial \boldsymbol{\pi}} \cdot d\boldsymbol{\pi} = -\frac{d\log b^P(q,\boldsymbol{\pi},U)}{1-\sigma^*(q,\boldsymbol{\pi},U)} + \sum_{n\in N} b_n(q(s^0),U)\, d\log q_n(s^0).$$

7. Substitute this back in to get desired result.

8. Idea from Baqaee et al. (2024) that compensation is fixed point.

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