How to Sell Public Debt in Uncertain Times

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Introduction

- Wide variation in the auction protocols used to sell government bonds.
    - Most commonly: uniform price (UP), discriminatory-price (DP).

- No clear rationale for which is better and whether protocols can be improved upon.
    - Mirrors lack of theoretical consensus on optimal multi-unit auctions in "realistic" settings.

- We study bond auction design using a model which allows for key macro/finance aspects:
    - Risk averse bidders with CRRA preferences $\rightarrow$ Risk premia and downward-sloping demand.
    - Asymmetric information about (common-value) default risk and supply shocks $\rightarrow$ winner's curse.

Overview

- Key tradeoff under decreasing marginal utility and common value uncertainty:

Inframarginal surplus extraction (favors DP)

versus

Bidder discouragement through the winner's curse (favors UP)

- Based on this trade-off **propose a simple modified protocol** that can do better than UP and DP.

- Paper: implications for information acquisition and revelation, and tests in Mexican data.

Relationship to the Literature

- Macro literature on sovereign bond pricing. We focus on the design of primary markets.

- Auction theory. Add risk aversion and asymmetric information about common values.
  - We study "large auctions" with many bidders + divisible good: $\approx$ price-taking.

Model

Setting

- One country, one good (the numeraire), unit mass of investors and two dates, $t = 1, 2$.
- At date 1, **Government** needs to raise ψD (e.g., to roll over debt) by selling bonds.
  - Promises to repay 1 per unit of bond, but pays 0 if it defaults.
  - Probability of default is κ and $\delta = 1$ denotes default, $\delta = 0$ repay.
- κ is a **quality shock** and ψ is a **quantity shock** (also interpretable as demand shock.)
- **Investors** are risk-averse and ex-ante identical with fixed per-capita wealth W,
  - CRRA preferences over date 2 consumption $u(c)$
  - can invest in government's risky debt or risk-free asset with gross return 1.
  - Investors are prohibited from shorting either asset

Information environment

- All investors know baseline funding need D, but are initially uninformed about shocks κ and ψ.
- We then consider two groups of investors: **informed** and **uninformed**.
  - Fraction of informed denoted by $n \in [0, 1]$. (For most of the talk, treat n as a parameter).
- Simple structure: informed investors know either the quantity shock *or* the quality shock.
- Later also consider endogenous costly information acquisition.

Auction rules

- Government sells bonds using sealed-bid multi-unit auctions.
- Investors can submit multiple **bids** $=$ non-negative quantity and price.
- A bid is a **commitment** to buy at a price determined by protocol **if** government accepts bid.
- Government treats bids independently and executes them in descending order of prices.
- Government stops when it generates the required revenue ψD.
- **Marginal price** $P(s)$ is lowest accepted price in **state** $\boldsymbol{s} = (\kappa, \psi)$.

Primary Auction

Focus mainly on two protocols widely used in large multi-unit auctions of common-value goods:

1. **discriminatory-price** (DP) auction in which all accepted bids are executed at the bid price
2. **uniform-price** (UP) auction in which all accepted bids are executed at the lowest accepted price.

Later: propose a convex combination of these protocols with *partial discrimination*.

The "Walrasian" Approach to Auctions

- Continuum of investors plus perfectly divisible bonds leads to **price-taking**.

- Investors have rational expectations about the **set** of potential marginal prices.
- Choose bids at potential marginal prices under uncertainty about which state will be realized.

$$\underbrace{[B(P(s)), P(s)]}_{\text{bid}} \Rightarrow B(s)\text{: bid quantity in state } s \text{ at associated marginal price } P(s).$$

- Informed investors know that some states (and thus marginal prices) will **not** be realized.
- Information is valuable because you can bid only at *relevant* marginal prices.

The Bidding Problem

Bidding Mechanics: Uninformed investors

Assume 4 states with prices $P_j > P_{j+1}$. Uninformed investors' **state-contingent expenditures** on bonds are:

$$\text{Uniform Protocol:} \qquad \mathbf{X^U_{UP}} = \begin{bmatrix} P_1 & 0 & 0 & 0 \\ P_2 & P_2 & 0 & 0 \\ P_3 & P_3 & P_3 & 0 \\ P_4 & P_4 & P_4 & P_4 \end{bmatrix} \begin{bmatrix} B_1^U \\ B_2^U \\ B_3^U \\ B_4^U \end{bmatrix}$$

$$\text{Discriminatory protocol:} \qquad \mathbf{X^U_{DP}} = \begin{bmatrix} P_1 & 0 & 0 & 0 \\ P_1 & P_2 & 0 & 0 \\ P_1 & P_2 & P_3 & 0 \\ P_1 & P_2 & P_3 & P_4 \end{bmatrix} \begin{bmatrix} B_1^U \\ B_2^U \\ B_3^U \\ B_4^U \end{bmatrix}$$

- Bid execution is **random**. For both UP and DP, **executed bid sets** $\mathcal{E}^U$ contain all states with lower prices.
- Difference across protocols: execution prices determine the cost of a state-contingent bond profile.

Bidding Mechanics: Informed Bidders

Informed bidders have access to an information partition: **bid only at feasible marginal prices.**

For **UP** in which partition creates **nonoverlapping** schedule $\{P_1, P_2\}\&\{P_3, P_4\}$, expenditures are

$$\mathbf{X}^{\mathbf{I}}_{\mathbf{UP}} = \begin{bmatrix} P_1 & 0 & 0 & 0 \\ P_2 & P_2 & 0 & 0 \\ 0 & 0 & P_3 & 0 \\ 0 & 0 & P_4 & P_4 \end{bmatrix} * \begin{bmatrix} B^I_1 \\ B^I_2 \\ B^I_3 \\ B^I_4 \end{bmatrix}$$

For **DP** in which partition creates **overlapping** schedule $\{P_1, P_3\}\&\{P_2, P_4\}$, expenditures are

$$\mathbf{X}^{\mathbf{I}}_{\mathbf{DP}} = \begin{bmatrix} P_1 & 0 & 0 & 0 \\ 0 & P_2 & 0 & 0 \\ P_1 & 0 & P_3 & 0 \\ 0 & P_2 & 0 & P_4 \end{bmatrix} \begin{bmatrix} B^I_1 \\ B^I_2 \\ B^I_3 \\ B^I_4 \end{bmatrix}$$

Decision Problem

- Let $\mathbf{Y}_j^i = \mathbf{B}_j^i - \mathbf{X}_j^i$ denote investor i's **net bond payoff** after repayment in protocol j.

- Let (s) denote the element associated with state s. Investor i chooses a bidding strategy to maximize

$$\sum_s \left\{ \underbrace{u(W - X_j^i(s))\kappa(s)}_{default} + \underbrace{u(W + Y_j^i(s))(1 - \kappa(s))}_{repay} \right\} Prob\{s\}$$

Leads to an intricate portfolio choice problem characterized by set of simultaneous FOCs.

- In the uniform protocol, this system has a tractable recursive structure.
- In the discriminatory protocol, all bids must be solved simultaneously.

Market clearing

For protocol j, the **market clearing condition** in state s is

$$nX_i^I(s) + (1-n)X_j^U(s) = \psi(s)D$$

- **Quantity** shocks affects equilibrium even if no investor is informed about ψ.
- **Quality** shocks affect prices only if some investors are informed.

Simple Information Structure

Simple Benchmark: Asymmetric Information about Two States

- There are two quality shocks, $\kappa \in \{\kappa_b, \kappa_g\}$, where $\kappa_g < \kappa_b$, and a known quantity sock ψ.
- Share $n \geq 0$ of investors are informed about the realized quality shock, rest is uninformed.
- If $n > 0$, there will be two marginal prices $P(\kappa_b) < P(\kappa_g)$.

Optimality conditions: Informed Investors (who know κ)

Informed investors face a **standard risk-return trade-off** for both auction protocols

Discriminatory protocol:

$$-u'(W - X^I_{DP,g})P_g\kappa_g\pi_g + u'(W + Y^I_{DP,g})(1 - P_g)(1 - \kappa_g)\pi_g = 0 \quad \text{(High price)}$$

$$-u'(W - X^I_{DP,b})P_b\kappa_b\pi_b + u'(W + Y^I_{DP,b})(1 - P_b)(1 - \kappa_b)\pi_b = 0 \quad \text{(Low price)}$$

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$$-u'(W - X^I_{DP,b})P_b\kappa_b\pi_b + u'(W + Y^I_{DP,b})(1 - P_b)(1 - \kappa_b)\pi_b = 0 \qquad \text{(Low price)}$$

Uniform protocol has the same exact structure:

$$-u'(W - X^I_{UP,g})P_g\kappa_g\pi_g + u'(W + Y^I_{UP,g})(1 - P_g)(1 - \kappa_g)\pi_g \qquad \text{(High price)}$$

$$-u'(W - X^I_{UP,b})P_b\kappa_b\pi_b + u'(W + Y^I_{UP,b})(1 - P_b)(1 - \kappa_b)\pi_b = 0 \qquad \text{(Low price)}$$

DP optimality conditions: Uninformed Investors (who do not know κ)

In the DP auction, uninformed bidding strategies are linked across states of the world:

$$-u'(W - X^U_{DP,g})P_g\kappa_g\pi_g + u'(W + Y^U_{DP,g})(1 - P_g)(1 - \kappa_g)\pi_g \qquad \text{(High price)}$$

$$-u'(W - X^U_{DP,b})P_g\kappa_b\pi_b + u'(W + Y^U_{DP,b})(1 - P_g)(1 - \kappa_b)\pi_b = 0$$

$$-u'(W - X^U_{DP,b})P_b\kappa_b\pi_b + u'(W + Y^U_{DP,b})(1 - P_b)(1 - \kappa_b)\pi_b = 0 \qquad \text{(Low price)}$$

Problem DP: Because of the winner's curse, uninformed investors submit fewer bids at the high price.

UP optimality conditions: Uninformed Investors (who do not know κ)

The UP auction removes this disincentive, and creates a "block recursive" structure:

$$-u'(W - X^U_{DP,g})P_g\kappa_g\pi_g + u'(W + Y^U_{DP,g})(1 - P_g)(1 - \kappa_g)\pi_g \qquad \text{(High price)}$$
$$-u'(W - X^U_{DP,b})P_b\kappa_b\pi_b + u'(W + Y^U_{DP,b})(1 - P_b)(1 - \kappa_b)\pi_b = 0$$

$$-u'(W - X^U_{DP,b})P_b\kappa_b\pi_b + u'(W + Y^U_{DP,b})(1 - P_b)(1 - \kappa_b)\pi_b = 0 \qquad \text{(Low price)}$$

Perfect Replication in the UP protocol

- If informed bids are ordered by price (i.e., $B^I(\kappa_g) < B^I(\kappa_b)$), **uninformed can perfectly replicate**:

$$\underbrace{B^U(\kappa_g) = B^I(\kappa_g)}_{\text{Bid the same at the high price}} \qquad \text{and} \qquad \underbrace{B^U(\kappa_b) = B^I(\kappa_b) - B^I(\kappa_g)}_{\text{Bid only the \textbf{increment} at the low price}}$$

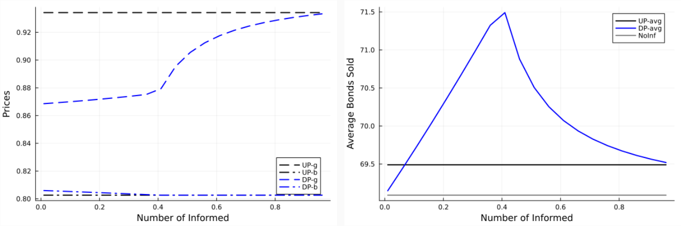
$\Longrightarrow$ No gain from being informed.

- Overlapping schedules with common price $P(\kappa_g, \cdot) = P(\kappa_b, \cdot)$ breaks this result.

- **Problem UP:** Government sells everything at the lowest accepted price – surplus goes to investors.

Illustration

Figure 1: UP and DP Prices and Bond Issuance with Quality Uncertainty



(a) Marginal Prices **(b)** Avg. Bonds Sold

Figure 2: Prices and quantities under UP and DP protocols as a function of the share of informed investors n. Parameters: $u(c) = \log(c)$, $W = 250$, $D = 60$, equiprobable $\kappa_g = 0.05$, and $\kappa_b = 0.15$.

- P_g is higher under UP than DP because of winner's curse for uninformed.
- As $n \to 1$ everyone is informed and prices P_g converge.
- For high n uninformed do not bid at P_g in DP due to higher prices
- DP extracts more surplus only when winner's curse is mild.

Better Protocol

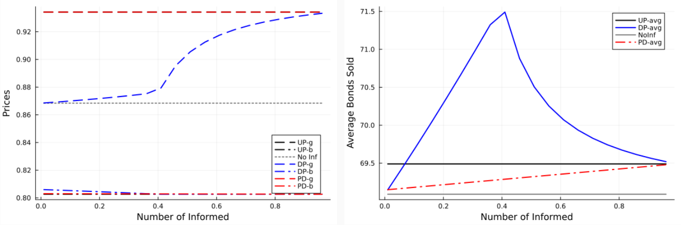
- Introduce **Partially Discriminating** protocol: Only $\mathcal{T} < B_g$ bids are at high price
- If $\mathcal{T} = 0$ then works like UP, if $\mathcal{T} > B_g$ works like DP
- With $\mathcal{T} < B_g$ in the low quality state b:

$$X^U_{PD,b} = \underbrace{(P_g - P_b)\mathcal{T}}_{\text{Penalty}} + P_b(B_g + B_b)$$

$$Y^U_{DP,b} = \underbrace{-(P_g - P_b)\mathcal{T}}_{\text{Penalty}} + (1 - P_b)(B_g + B_b)\mathcal{T}$$

- Same FOCs as UP but more rent extraction like DP.
- Easy to show improves upon both DP and UP in this case.
- Caution about setting $\mathcal{T}$ too big. Constraint set not convex. Confuse global and local optimum.

Figure 3: UP, DP and PD Prices and Bond Issuance with Quality Uncertainty



(a) Marginal Prices **(b)** Avg. Bonds Sold

Figure 4: Prices and quantities under UP and DP protocols as a function of the share of informed investors n. Parameters: $u(c) = \log(c)$, $W = 250$, $D = 60$, equiprobable $\kappa_g = 0.05$, and $\kappa_b = 0.15$.

- Surplus is small so tier is small, $\mathcal{T} = 4$.
- Tiering leads to UP (like) prices and bids with DP (like) price discrimination.

Informativeness Differences UP vs DP

Small number of informed $\implies$ overlapping (less informative) prices

Assume states 1 and 2 are (κ_g, ψ_s) and (κ_g, ψ_l) states 3 and 4 are (κ_b, ψ_s) and (κ_b, ψ_l)

$$\mathbf{X^i_{UP}} = \begin{bmatrix} P_1 & 0 & 0 & 0 \\ P_2 & P_2 & 0 & 0 \\ P_3 & P_3 & P_3 & 0 \\ P_4 & P_4 & P_4 & P_4 \end{bmatrix} * \begin{bmatrix} B^i_1 \\ B^i_2 \\ B^i_3 \\ B^i_4 \end{bmatrix}$$

$$\underbrace{(1-n)(B^U_1 + B^U_2)P_2}_{\text{Demand by Uninformed}} \uparrow \psi_l D \text{ as } n \downarrow 0 \implies (1-n)(B^U_1 + B^U_2)P_2 \approx \psi_l D > \psi_s D$$

With **replication** binds first at $(1-n)\psi_l D = \psi_s D$

Violates how auctions work. Stop at highest price that meets demand.

Small number of informed + gains from information + less informative prices with UP: $P_2 = P_3$.

DP has large gains to information $\Rightarrow$ n big to compete away rents; quality price schedules distinct.

UP with common prices

- Prices are then determined as follows. Take any two states $s = [\kappa_g, \psi_l]$ and $s' = [\kappa_b, \psi_s]$ for which a binding constraint forces a common price, $P = P(s) = P(s')$. The respective auction-clearing conditions for these two states are

$$n\left(\frac{1-\kappa_g-P}{1-P}\right)+(1-n)\left(\frac{1-\tilde{\kappa}-P}{1-P}\right)=\frac{D}{W}\psi_l, \tag{1}$$

$$\text{and} \quad n\max\left[\left(\frac{1-\kappa_b-P}{1-P}\right),0\right]+(1-n)\left(\frac{1-\tilde{\kappa}-P}{1-P}\right)=\frac{D}{W}\psi_s. \tag{2}$$

- The two endogenous variables determined by these equations are the common price P and uninformed investors' inferred default probability $\tilde{\kappa}$.

Illustration 3: UP auction binary quality shocks and continuous quantity shocks

Figure 5: UP Equilibria with Quality Uncertainty

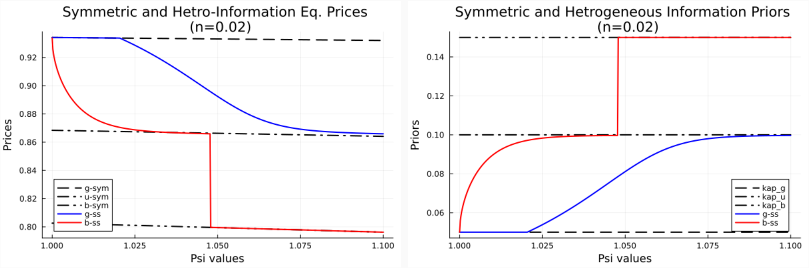
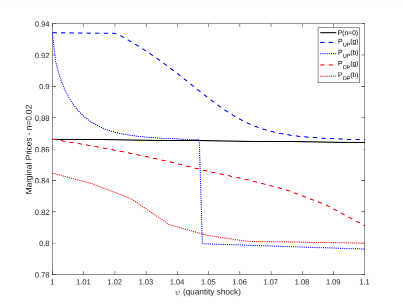


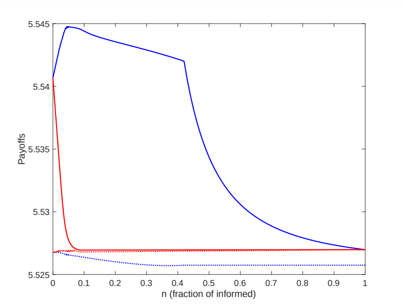
Figure 6: Parameters are: $u = \log(c)$, $\bar{\kappa} = 0.1, \kappa_g = 0.05, \kappa_b = 0.15, Pr(\kappa_g) = 0.5, W = 250, D = 60$. Supply shock ψ is uniformly distributed from $\psi = 1$ to $\psi_M = 1.1$.

- Get overlapping price schedules and hence prices are less informative.

Illustration 3: UP auction binary quality shocks and continuous quantity shocks

Figure 7: Comparing UP vs. DP with Quality Uncertainty ($n = 0.02$)





- DP has larger gains to information except for n close to 1 where rents have been compete away.
- DP has overlapping prices only for very small n. With endogenous information acquisition:
  - DP: get distinct price schedules.
  - UP: get overlapping price schedules.

How much do auction prices help predict subsequent secondary market prices?

The *marginal* R^2 is formally given by

$$\Delta R^2 = \frac{R^2_{(S_{t-1},P_t)} - R^2_{(S_{t-1})}}{1 - R^2_{(S_{t-1})}},$$

Table 1: Marginal R^2. 28-day Cetes

Auction Protocol	**DP**	**UP**
Marginal R^2	0.723	0.291
Number Auctions	*735*	*345*

Cetes are domestically-denominated zero-coupon pure discount bonds, auctioned weekly. Used a discriminatory price protocol until October 2017.

Concluding Comments

- Developed model of different auction protocols w/ heterogeneous information.
- Compared standard uniform-price and discriminating-price protocols.
- Approach suggested novel new protocol that performs better.
- Extended characterization to continuous shocks (on non-information dimension).
  - Develop different solution methods for UP and DP.
- With endogenous information acquisition, uniform-price protocol reveals less information than discriminating-price protocol.
- Validated this implication for Mexican Cetes auctions.