Business Cycles with Pricing Cascades

Mishel Ghassibe A

Anton Nakov

CREi, UPF & BSE

European Central Bank

NBER Summer Institute 2025 Workshop on Methods and Applications for Dynamic Equilibrium Models July 10th 2025

The views expressed here are the responsibility of the authors only, and do not necessarily coincide with those of the ECB or the Eurosystem.

• Recent events have brought new evidence regarding the drivers and dynamics of inflation:

• Recent events have brought new evidence regarding the drivers and dynamics of inflation:

i Possibility of large inflationary swings in advanced economies



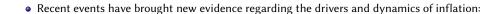
• Recent events have brought new evidence regarding the drivers and dynamics of inflation:

i Possibility of large inflationary swings in advanced economies



ii Fluctuations in the **frequency of price adjustment** (Montag and Villar, 2023; Cavallo et al., 2024)





i Possibility of large inflationary swings in advanced economies



ii Fluctuations in the frequency of price adjustment (Montag and Villar, 2023; Cavallo et al., 2024)



iii Importance of sector-specific drivers of inflation (Schneider, 2023; Rubbo, 2024)



• Recent events have brought new evidence regarding the drivers and dynamics of inflation:

Possibility of large inflationary swings in advanced economies



ii Fluctuations in the frequency of price adjustment (Montag and Villar, 2023; Cavallo et al., 2024)



▶ Show

iii Importance of sector-specific drivers of inflation (Schneider, 2023; Rubbo, 2024)



Develop a **dynamic quantitative** general equilibrium model that features: a number of **sectors** interconnected by networks with state-dependent pricing that is solved fully non-linearly

• Recent events have brought new evidence regarding the drivers and dynamics of inflation:

- i Possibility of large inflationary swings in advanced economies

 Challenge: NKPC with a realistic slope requires implausibly large shocks (L'Huillier and Phelan, 2024)

 ii Fluctuations in the frequency of price adjustment (Montag and Villar, 2023; Cavallo et al., 2024)

 Challenge: a fixed menu cost model matches that at the cost of an implausibly steep NKPC (Blanco et al., 2024)

 iii Importance of sector-specific drivers of inflation (Schneider, 2023; Rubbo, 2024)

 Challenge: need to allow for large sector-specific shocks in a setting with menu costs
- Develop a dynamic quantitative general equilibrium model that features: a number of sectors interconnected by networks with state-dependent pricing that is solved fully non-linearly

New cyclical mechanism: interaction of **networks** and pricing **cascades**

• **Interaction** of our model ingredients creates pricing **cascades**: large movements in aggregates trigger additional price adjustment decisions at the extensive margin

New cyclical mechanism: interaction of **networks** and pricing **cascades**

- **Interaction** of our model ingredients creates pricing **cascades**: large movements in aggregates trigger additional price adjustment decisions at the extensive margin
- Demand shocks Networks slow down the extensive margin adjustment: cascades dampening
 - i Networks slow down the desired price changes, hence firms are less willing to pay the cost of adjustment
 - ii Quantitatively, delivers a "global flattening" of the Phillips Curve, implying strong monetary non-neutrality even following very large shocks

New cyclical mechanism: interaction of **networks** and pricing **cascades**

- **Interaction** of our model ingredients creates pricing **cascades**: large movements in aggregates trigger additional price adjustment decisions at the extensive margin
- Demand shocks Networks slow down the extensive margin adjustment: cascades dampening
 - i Networks slow down the desired price changes, hence firms are less willing to pay the cost of adjustment
 - ii Quantitatively, delivers a "global flattening" of the Phillips Curve, implying strong monetary non-neutrality even following very large shocks
- Supply shocks (Agg./sectoral) Networks speed up the extensive margin adjust.: cascades amplification
 - i Networks amplify the desired price changes, hence firms are more willing to pay the cost of adjustment
 - ii Quantitatively, creates frequency increases and inflationary spirals following aggregate TFP/markup shocks, or TFP/markup shocks to sectors that are **major and concentrated** suppliers to the rest of the economy

MODEL

Model overview

• **Timing**: infinite-horizon setting in discrete time, indexed by t = 0, 1, 2, ...

• Households: continuum of identical households; consume output and supply labor

• **Firms**: continuum of monopolistically competitive firms, each belongs to one of N sectors, indexed $i \in \{1, 2, ..., N\}$; there is a measure one of firms in each sector

• Factors: firms use labor and intermediate inputs purchased from other firms

• **Government Policy**: central bank sets the level of money supply M_t

Households

• The representative household maximizes expected lifetime utility:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\log C_t - L_t \right]$$

subject to a standard budget constraint

• Households are also subject to a **cash-in-advance** constraint: $P_t^C C_t \leq M_t$

Households

The representative household maximizes expected lifetime utility:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} eta^t \left[\log C_t - L_t
ight]$$

subject to a standard budget constraint

• Households are also subject to a **cash-in-advance** constraint: $P_t^C C_t \leq M_t$

• Aggregate consumption: $C_t = \iota^C \prod_{i=1}^N C_i^{\overline{\omega}_i^C}, \quad \sum_{i=1}^N \overline{\omega}_i^C = 1, \quad \overline{\omega}_i^C \ge 0, \forall i$

Households

• The representative household maximizes expected lifetime utility:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\log C_t - L_t \right]$$

subject to a standard budget constraint

- Households are also subject to a **cash-in-advance** constraint: $P_t^C C_t \leq M_t$
- Aggregate consumption: $C_t = \iota^C \prod_{i=1}^N C_i^{\overline{\omega}_i^C}, \quad \sum_{i=1}^N \overline{\omega}_i^C = 1, \quad \overline{\omega}_i^C \geq 0, \forall i$
- Sectoral consumption: $C_{i,t} = \left\{ \int_0^1 \left[\zeta_{i,t}(j) C_{i,t}(j) \right]^{\frac{\epsilon-1}{\epsilon}} dj \right\}^{\frac{\epsilon}{\epsilon-1}}, \quad \epsilon > 1$ where $\zeta_{i,t}(j)$ is a **firm-level quality** process:

$$\log \zeta_{i,t}(j) = \log \zeta_{i,t-1}(j) + \sigma_i \varepsilon_{i,t}(j)$$

Firms: production

• Any firm *j* in sector *i* has access to the following production technology:

$$Y_{i,t}(j) = \iota_i \frac{1}{\zeta_{i,t}(j)} \times A_{i,t} \times L_{i,t}(j)^{\overline{\alpha}_i} \prod_{k=1}^N X_{i,k,t}(j)^{\overline{\omega}_{ik}},$$

where $A_{i,t}$ is a **sectoral productivity** process, $L_{i,t}(j)$ is firm-level labor input, $X_{i,k,t}(j)$ is firm-level intermediate input demand for sector k's goods and $\overline{\alpha}_i + \sum_{k=1}^N \overline{\omega}_{ik} = 1, \quad \overline{\alpha}_i \geq 0, \overline{\omega}_{ik} \geq 0, \ \forall i, k$

Firms: production

• Any firm *j* in sector *i* has access to the following production technology:

$$Y_{i,t}(j) = \iota_i \frac{1}{\zeta_{i,t}(j)} \times A_{i,t} \times L_{i,t}(j)^{\overline{\alpha}_i} \prod_{k=1}^N X_{i,k,t}(j)^{\overline{\omega}_{ik}},$$

where $A_{i,t}$ is a **sectoral productivity** process, $L_{i,t}(j)$ is firm-level labor input, $X_{i,k,t}(j)$ is firm-level intermediate input demand for sector k's goods and $\overline{\alpha}_i + \sum_{k=1}^N \overline{\omega}_{ik} = 1$, $\overline{\alpha}_i \geq 0$, $\overline{\omega}_{ik} \geq 0$, $\forall i, k$

Cost-minimization delivers the following marginal cost:

$$MC_{i,t}(j) = \zeta_{i,t}(j) \times \frac{1}{A_{i,t}} \times W_t^{\overline{\alpha}_i} \prod_{k=1}^N P_{k,t}^{\overline{\omega}_{ik}}.$$

• Price resetting involves paying a sector-specific menu cost $\kappa_{i,t}$ measured in labor hours

• Price resetting involves paying a sector-specific **menu cost** $\kappa_{i,t}$ measured in labor hours

• Let
$$p_{i,t}(j) \equiv \log \tilde{P}_{i,t}(j) = \log \frac{P_{i,t}(j)}{\zeta_{i,t}(j)M_t}$$
 be the quality-adjusted \log real price

• Price resetting involves paying a sector-specific menu cost $\kappa_{i,t}$ measured in labor hours

• Let $p_{i,t}(j) \equiv \log \tilde{P}_{i,t}(j) = \log \frac{P_{i,t}(j)}{C_{i,t}(j)M_t}$ be the quality-adjusted \log real price

• The value of a firm in sector *i* that has set a quality-adjusted real price *p*:

$$V_{i,t}(p) = \tilde{\mathcal{D}}_{i,t}(p) + \beta \mathbb{E}_t \left[\left\{ 1 - \eta_{i,t+1} \left(p - \sigma_i \varepsilon_{i,t+1} - m_{t+1} \right) \right\} \times V_{i,t+1} \left(p - \sigma_i \varepsilon_{i,t+1} - m_{t+1} \right) \right]$$

$$+\beta \mathbb{E}_{t} \left[\underbrace{ \eta_{i,t+1} \left(\mathbf{p} - \sigma_{i} \varepsilon_{i,t+1} - m_{t+1} \right)}_{\text{Pr. of adjustment}} \times \left(\max_{\mathbf{p}'} V_{i,t+1} \left(\mathbf{p}' \right) - \kappa_{i,t} \right) \right]$$

- Price resetting involves paying a sector-specific menu cost $\kappa_{i,t}$ measured in labor hours
- Let $p_{i,t}(j) \equiv \log \tilde{P}_{i,t}(j) = \log \frac{P_{i,t}(j)}{C_{i,t}(j)M_t}$ be the quality-adjusted *log* real price
- The value of a firm in sector *i* that has set a quality-adjusted real price *p*:

$$V_{i,t}(p) = \tilde{\mathcal{D}}_{i,t}(p) + \beta \mathbb{E}_t \left[\left\{ 1 - \eta_{i,t+1} \left(p - \sigma_i \varepsilon_{i,t+1} - m_{t+1} \right) \right\} \times V_{i,t+1} \left(p - \sigma_i \varepsilon_{i,t+1} - m_{t+1} \right) \right]$$

$$+\beta \mathbb{E}_{t} \left[\underbrace{\eta_{i,t+1} \left(\mathbf{p} - \sigma_{i} \varepsilon_{i,t+1} - m_{t+1} \right)}_{\text{Pr. of adjustment}} \times \left(\max_{\mathbf{p}'} V_{i,t+1} \left(\mathbf{p}' \right) - \kappa_{i,t} \right) \right]$$

• Following Golosov and Lucas (2007), we assume the following **adjustment hazard** $\eta_{i,t}(.)$:

$$\eta_{i,t}(p) = \mathbf{1}(L_{i,t}(p) > 0) = \mathbf{1}\left(\max_{p'} V_{i,t}\left(p'\right) - V_{i,t}(p) > \overline{\kappa}_i\right)$$

STATIC SETUP

Intuition in a simplified setup

ullet Consider the static limit of the model (eta=0) and focus on the initial time period (t=0)

Intuition in a simplified setup

• Consider the static limit of the model ($\beta = 0$) and focus on the initial time period (t = 0)

• For a firm *j* in sector *i*, the real profit gain from price adjustment satisfies:

$$\tilde{D}_i^*(j) - \tilde{D}_i(j) \quad \stackrel{\infty}{\sim} \quad [\tilde{p}_i(j)]^2$$

where $\tilde{p}_i(j) \equiv \log \tilde{P}_i(j) - \log \tilde{P}_i^*$ is the firm-level real **price gap**

Intuition in a simplified setup

ullet Consider the static limit of the model (eta=0) and focus on the initial time period (t=0)

• For a firm *j* in sector *i*, the real profit gain from price adjustment satisfies:

$$\tilde{D}_i^*(j) - \tilde{D}_i(j) \quad \stackrel{\sim}{\sim} \quad [\tilde{p}_i(j)]^2$$

where $\tilde{p}_i(j) \equiv \log \tilde{P}_i(j) - \log \tilde{P}_i^*$ is the firm-level real **price gap**

• The real sectoral optimal reset price is $\tilde{P}_i^* \equiv P_i^*/M = \frac{1}{A_i} \prod_k \tilde{P}_k^{\overline{\omega}_{ik}}$, hence normalizing $\sigma_i = 1$:

$$\widetilde{p}_{i}(j) = \underbrace{-\varepsilon_{i}(j) - m}_{\text{"Erosion"}} + \underbrace{a_{i} - \sum_{k=1}^{N} \overline{\omega}_{ik} \log \widetilde{p}_{k}}$$

where m is money supply, a_i is sectoral TFP shock and $\log \tilde{P}_k \equiv (\log P_k - m)$ is real sectoral price

Monetary shock: cascades dampening by networks

• Inaction region: a firm will not adjust if it draws an innovation in

$$(\kappa_{i,t} = \overline{\kappa}_i [\tilde{P}_{i,t}/\tilde{P}_{i,t}^*]^{\epsilon-1} \lambda_{i,t})$$

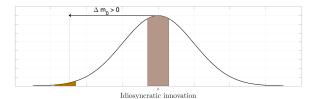
$$[\underline{\varepsilon}_i, \ \overline{\varepsilon}_i] = -m - \sum_{k=1}^N \overline{\omega}_{ik} \log \tilde{P}_k \pm \sqrt{\frac{2\overline{\kappa}_i}{\epsilon - 1}}$$

Monetary shock: cascades dampening by networks

• Inaction region: a firm will not adjust if it draws an innovation in $(\kappa_{i,t} = \overline{\kappa}_i / \tilde{p}_{i,t}^* / \tilde{p}_{i,t}^* / \tilde{p}_{i,t}^*)^{\epsilon-1} \lambda_{i,t})$

$$(\kappa_{i,t} = \overline{\kappa}_i [\tilde{P}_{i,t}/\tilde{P}_{i,t}^*]^{\epsilon-1} \lambda_{i,t})$$

$$[\underline{\varepsilon}_{i}, \ \overline{\varepsilon}_{i}] = -m - \sum_{k=1}^{N} \overline{\omega}_{ik} \log \tilde{P}_{k} \pm \sqrt{\frac{2\overline{\kappa}_{i}}{\epsilon - 1}}$$

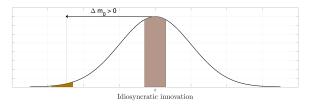


Monetary shock: cascades dampening by networks

• Inaction region: a firm will not adjust if it draws an innovation in $(\kappa_{i,t} = \overline{\kappa}_i / \tilde{p}_{i,t}^* / \tilde{p}_{i,t}^* / \tilde{p}_{i,t}^*)^{\epsilon-1} \lambda_{i,t})$

$$(\kappa_{i,t} = \overline{\kappa}_i [\tilde{P}_{i,t}/\tilde{P}_{i,t}^*]^{\epsilon-1} \lambda_{i,t})$$

$$[\underline{\varepsilon}_i, \ \overline{\varepsilon}_i] = -m - \sum_{k=1}^N \overline{\omega}_{ik} \log \tilde{P}_k \pm \sqrt{\frac{2\overline{\kappa}_i}{\epsilon - 1}}$$



• Incomplete monetary pass-through: $\log \tilde{P}_k = (\log P_k - m) < 0, \forall k$

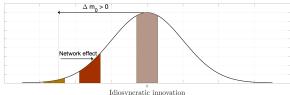
$$\log \tilde{P}_k = (\log P_k - m) < 0, \forall k$$

cascades dampening by networks Monetary shock:

• Inaction region: a firm will not adjust if it draws an innovation in $(\kappa_{i,t} = \overline{\kappa}_i [\tilde{P}_{i,t}/\tilde{P}_{i,t}^*]^{\epsilon-1} \lambda_{i,t})$

$$(\kappa_{i,t} = \overline{\kappa}_i [\tilde{P}_{i,t}/\tilde{P}_{i,t}^*]^{\epsilon-1} \lambda_{i,t})$$

$$[\underline{\varepsilon}_i, \ \overline{\varepsilon}_i] = -m - \sum_{k=1}^N \overline{\omega}_{ik} \log \tilde{P}_k \pm \sqrt{\frac{2\overline{\kappa}_i}{\epsilon - 1}}$$



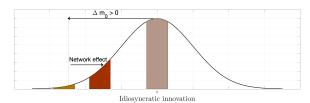
• Incomplete monetary pass-through: $\log \tilde{P}_k = (\log P_k - m) < 0, \forall k$

Monetary shock: **cascades dampening** by networks

• **Inaction region**: a firm will **not** adjust if it draws an innovation in $(\kappa_{i,t} = \overline{\kappa}_i [\tilde{P}_{i,t}/\tilde{P}_{i,t}^*]^{\epsilon-1} \lambda_{i,t})$

$$(\kappa_{i,t} = \overline{\kappa}_i [\tilde{P}_{i,t}/\tilde{P}_{i,t}^*]^{\epsilon-1} \lambda_{i,t})$$

$$[\underline{\varepsilon}_i, \ \overline{\varepsilon}_i] = -m - \sum_{k=1}^N \overline{\omega}_{ik} \log \tilde{P}_k \pm \sqrt{\frac{2\overline{\kappa}_i}{\epsilon - 1}}$$



• Incomplete monetary pass-through: $\log \tilde{P}_k = (\log P_k - m) < 0, \forall k$

Proposition (Cascades dampening: monetary shocks)

For a monetary shock **m**, under incomplete pass-through, networks **lower the probability** of adjustment ρ_i for any firm in any sector i, and the dampening increases in the sectoral customer centrality C_i

$$\Delta\sqrt{\varrho_i(\mathbf{m})} \quad \stackrel{\sim}{\sim} \quad \mathbf{C}_i \quad \equiv \quad \sum_{i=1}^N (I-\overline{\Omega})_{ij}^{-1} - 1$$

• **Inaction region**: a firm will **not** adjust if it draws an innovation in $(\kappa_{i,t} = \overline{\kappa}_i [\tilde{P}_{i,t}/\tilde{P}_{i,t}^*]^{\epsilon-1} \lambda_{i,t})$

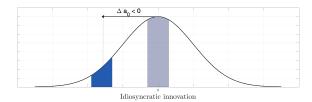
$$(\kappa_{i,t} = \overline{\kappa}_i [\tilde{P}_{i,t}/\tilde{P}_{i,t}^*]^{\varepsilon-1} \lambda_{i,t})$$

$$[\underline{\varepsilon}_i, \ \overline{\varepsilon}_i] = a_i - \sum_{k=1}^N \overline{\omega}_{ik} \log \widetilde{P}_k \pm \sqrt{\frac{2\overline{\kappa}_i}{\epsilon - 1}}$$

• Inaction region: a firm will not adjust if it draws an innovation in $(\kappa_{i,t} = \overline{\kappa}_i [\tilde{P}_{i,t}/\tilde{P}_{i,t}^*]^{\epsilon-1} \lambda_{i,t})$

$$(\kappa_{i,t} = \overline{\kappa}_i [\tilde{P}_{i,t}/\tilde{P}_{i,t}^*]^{\varepsilon-1} \lambda_{i,t})$$

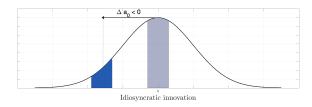
$$[\underline{\varepsilon}_i, \ \overline{\varepsilon}_i] = a_i - \sum_{k=1}^N \overline{\omega}_{ik} \log \tilde{P}_k \pm \sqrt{\frac{2\overline{\kappa}_i}{\epsilon - 1}}$$



• Inaction region: a firm will not adjust if it draws an innovation in $(\kappa_{i,t} = \overline{\kappa}_i [\tilde{P}_{i,t}/\tilde{P}_{i,t}^*]^{\epsilon-1} \lambda_{i,t})$

$$(\kappa_{i,t} = \overline{\kappa}_i [\tilde{P}_{i,t}/\tilde{P}_{i,t}^*]^{\varepsilon-1} \lambda_{i,t})$$

$$[\underline{\varepsilon}_i, \ \overline{\varepsilon}_i] = a_i - \sum_{k=1}^N \overline{\omega}_{ik} \log \widetilde{P}_k \pm \sqrt{\frac{2\overline{\kappa}_i}{\epsilon - 1}}$$

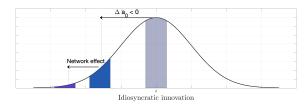


• Inflationary TFP deterioration: $\log \tilde{P}_k > 0, \forall k$

• Inaction region: a firm will not adjust if it draws an innovation in $(\kappa_{i,t} = \overline{\kappa}_i [\tilde{P}_{i,t}/\tilde{P}_{i,t}^*]^{\epsilon-1} \lambda_{i,t})$

$$(\kappa_{i,t} = \overline{\kappa}_i [\tilde{P}_{i,t}/\tilde{P}_{i,t}^*]^{\varepsilon-1} \lambda_{i,t})$$

$$[\underline{\varepsilon}_i, \ \overline{\varepsilon}_i] = a_i - \sum_{k=1}^N \overline{\omega}_{ik} \log \widetilde{P}_k \pm \sqrt{\frac{2\overline{\kappa}_i}{\epsilon - 1}}$$

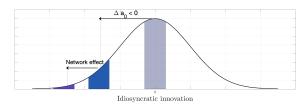


• Inflationary TFP deterioration: $\log \tilde{P}_k > 0, \forall k$

• **Inaction region**: a firm will **not** adjust if it draws an innovation in $(\kappa_{i,t} = \overline{\kappa}_i [\tilde{P}_{i,t}/\tilde{P}_{i,t}^*]^{\epsilon-1} \lambda_{i,t})$

$$[\kappa_{i,t} = \overline{\kappa}_i [\tilde{P}_{i,t}/\tilde{P}_{i,t}^*]^{\epsilon-1} \lambda_{i,t})$$

$$[\underline{\varepsilon}_i, \ \overline{\varepsilon}_i] = a_i - \sum_{k=1}^N \overline{\omega}_{ik} \log \tilde{P}_k \pm \sqrt{\frac{2\overline{\kappa}_i}{\epsilon - 1}}$$



• Inflationary TFP deterioration: $\log \tilde{P}_k > 0, \forall k$

Proposition (Cascades amplification: aggregate TFP shock)

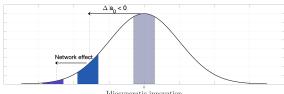
For an aggregate TFP shock a, under counter-moving sectoral prices, networks increase the probability of adjustment ρ_i for any firm in any sector i, and the amplification increases in the sectoral **customer centrality** C_i

$$\Delta\sqrt{\varrho_i(\mathbf{a})} \quad \stackrel{\sim}{\sim} \quad \mathbf{C}_i \quad \equiv \quad \sum_{i=1}^N (I-\overline{\Omega})_{ij}^{-1} - 1$$

• **Inaction region**: a firm will **not** adjust if it draws an innovation in $(\kappa_{i,t} = \overline{\kappa}_i [\tilde{P}_{i,t}/\tilde{P}_{i,t}^*]^{\epsilon-1} \lambda_{i,t})$

$$(\kappa_{i,t} = \overline{\kappa}_i [\tilde{P}_{i,t}/\tilde{P}_{i,t}^*]^{\epsilon-1} \lambda_{i,t})$$

$$[\underline{\varepsilon}_i, \ \overline{\varepsilon}_i] = a_i - \sum_{k=1}^N \overline{\omega}_{ik} \log \tilde{P}_k \pm \sqrt{\frac{2\overline{\kappa}_i}{\epsilon - 1}}$$



Idiosyncratic innovation

• Inflationary TFP deterioration: $\log \tilde{P}_k > 0, \forall k$

Proposition (Cascades amplification: sectoral TFP shock)

For a sectoral TFP shock **a**_i, under counter-moving sectoral prices, average probab. of adjustment around symmetric s.s. is:

$$\overline{\varrho}(a_i) \quad \stackrel{\propto}{\sim} \quad \mathcal{H}_i$$



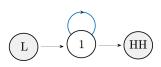
where \mathcal{H}_i is the sectoral supplier Herfindahl: $\mathcal{H}_i \equiv \left[\frac{1}{N}\sum_j(I-\overline{\Omega})_{j,i}^{-1}\right]^2 + Var(I-\overline{\Omega})_{(i)}^{-1}$.

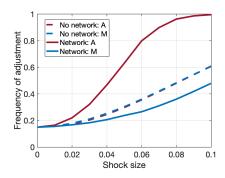
$$\mathcal{H}_i \equiv \left[\frac{1}{N}\sum_i(I-\overline{\Omega})_{j,i}^{-1}\right]^2$$

+
$$Var(I-\overline{\Omega})_{(i)}^{-1}$$
.

Toy example 1: roundabout production

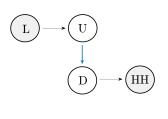
• Marginal cost: $MC(j) = \zeta(j) \times \frac{1}{A} \times M^{\overline{\alpha}} P^{1-\overline{\alpha}}$

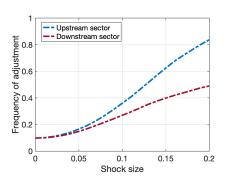




Toy example 2: two-sector vertical chain

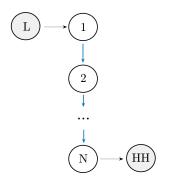
• Marginal costs: $MC_U(j) = \zeta_U(j) \times \frac{1}{A_U} \times M$, $MC_D(j) = \zeta_D(j) \times \frac{1}{A_D} \times P_U$

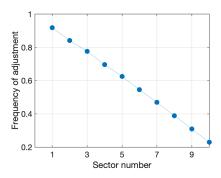




Toy example 3: *N*-sector vertical chain

• Marginal costs: $MC_i(j) = \zeta_i(j) \times \frac{1}{A_i} \times P_{i-1}$







Computation

- Steady state: solve the stationary Bellman equations and firms' price distribution on an evenly spaced grid of log quality-adjusted real prices for every sector
- Consider a **known** sequence of money supply $\{\Delta \log M_t\}_{t=0}^{\infty}$ and productivity $\{\log A_{k,t}\}_{t=0}^{\infty}$

- Assume that after a finite time period T the economy converges back to the stationary distribution
- From a guess for the sequences of aggregate and sectoral variables, follow **backward-forward iteration** until convergence:
 - ① Starting from t = T, iterate **backwards** to t = 0 to solve for the micro value functions
 - ② Starting from t = 0, iterate **forwards** to t = T to solve for price distributions and aggregate numerically

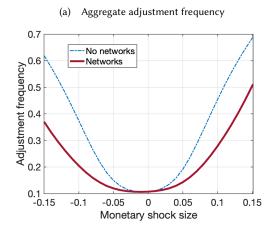
Calibration (Euro Area, monthly frequency)

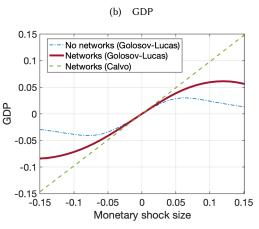
Aggregate parameters			
β	0.961/12	Discount factor (monthly)	Golosov and Lucas (2007)
ϵ	3	Goods elasticity of substitution	Midrigan (2011)
$\overline{\pi}$	0.02/12	Trend inflation (monthly)	ECB target
ho	0.90	Persistence of the TFP shock	Half-life of seven months
Sectoral parameters			
N	39	Number of sectors	Data from Gautier et al. (2024)
$\{\overline{\omega}_i^C\}_{i=1}^N$		Sector consumption weights	World IO Tables
$\{\overline{\omega}_{ik}\}_{i,k=1}^N$		Sector input-output matrix	World IO Tables
$\{\overline{\alpha}_i\}_{i=1}^N$		Sector labor weights	World IO Tables
Firm-level pricing parameters			
$\{\overline{\kappa}_i\}_{i=1}^N$		Menu costs	Estimated to fit frequency, std dev.
$\{\sigma_i\}_{i=1}^N$		Std. dev. of firm-level shocks	of Δp from Gautier et al. (2024)

Monetary shocks

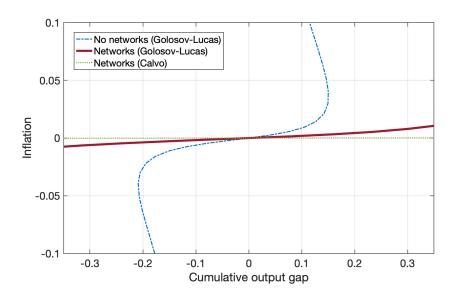
$$\log M_t = \overline{\pi} + \log M_{t-1} + \varepsilon_t^M$$

Cascades dampening following monetary shocks

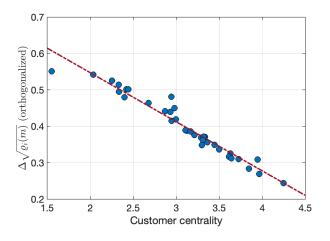




Non-linear Phillips Curves



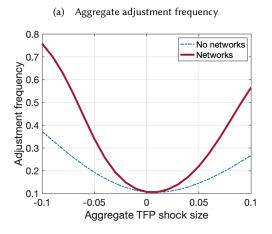
Sectoral frequency responses vs. **Customer Centrality**

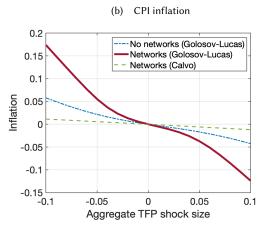


Aggregate TFP shocks

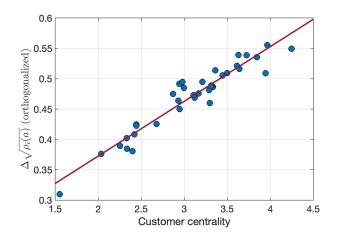
$$\log A_t = \rho \log A_{t-1} + \varepsilon_t^A$$

Cascades amplification following TFP shocks



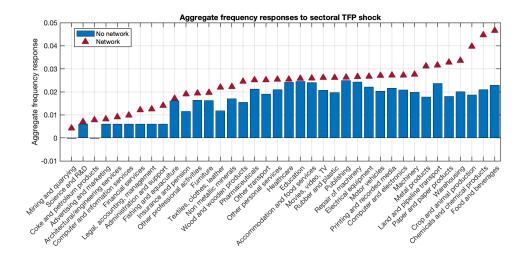


Sectoral frequency responses vs. **Customer Centrality**

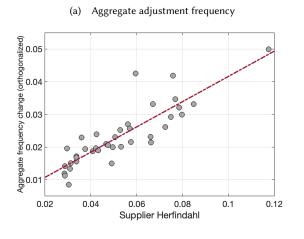


Sectoral TFP shocks

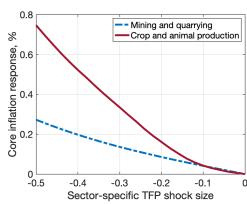
Aggregate frequency responses to sectoral TFP shocks (-20%)



Aggregate frequency responses vs. sectoral Supplier Centrality



(b) Core inflation response





Model vs. Data

• To assess the model quantitatively, we feed in observed demand and supply processes as exogenous shocks

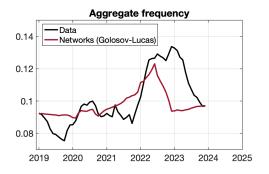
• **Aggregate demand shock**: Euro Area nominal GDP as a proxy for the $\{M_t\}_{t\geq 0}$ process

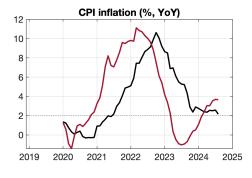
• Energy price shock: calibrate the productivity process of the "Mining and Quarrying" sector to match the IMF Global Price of Energy Index movements

• Food price shock: calibrate the productivity process of the "Crop and Animal Production" sector to match the IMF Global Price of Food Index movements

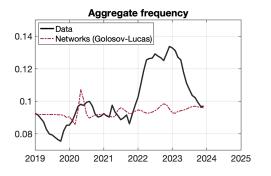
• Labor market shock: calibrate the productivity process of the labor union sector to match the hourly earnings dynamics in the Euro Area

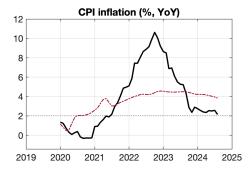
Model vs. Data: baseline setup, all shocks



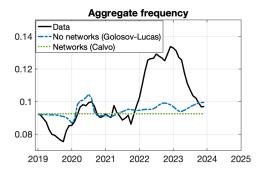


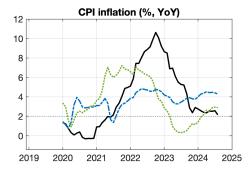
Model vs. Data: baseline setup, no commodity shocks





Model vs. Data: alternative setups, all shocks





Conclusions

Present a dynamic quantitative general equilibrium model that features: a number of sectors interconnected
 by networks with state-dependent pricing that is solved fully non-linearly

Networks slow down the extensive margin pricing response to demand shocks: cascades dampening

• Networks speed up the extensive margin response to supply shocks: cascades amplification

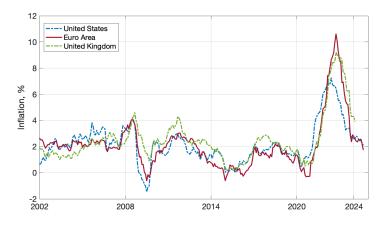
• Interaction of networks and pricing cascades crucial for quantitatively matching the observed surges in inflation and repricing frequency in the Euro Area



Golosov, Mikhail and Robert E. Lucas (2007) "Menu Costs and Phillips Curves," Journal of Political Economy, Vol. 115, pp. 171-199.

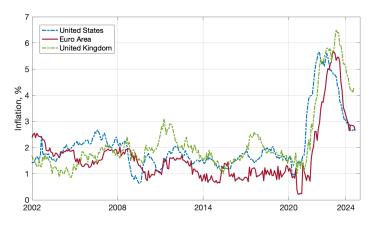
APPENDIX

Evidence I: inflation spikes in advanced economies (headline)



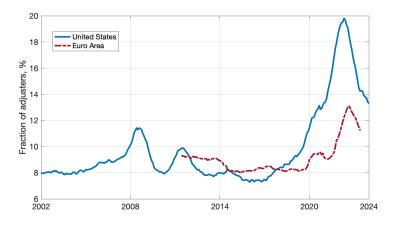
Source: FRED.

Evidence I: inflation spikes in advanced economies (core)



Source: FRED.

Evidence II: changes in frequency of price adjustment



Source: Montag and Villar (2024), Dedola et al. (2024).

Evidence III: sectoral origins of inflation



Source: Rubbo (2024).

Cascades dampening under monetary shocks

Proposition

Normalizing $p_{i,-1}(j) = \log \frac{\epsilon}{\epsilon - 1} \frac{1}{1 - \overline{\tau}_i}$, let ϱ_i be the probability that a firm in sector i decides to adjust its price. For a monetary shock m, denote by $\Delta \varrho_i(m)$ the change relative to steady-state, then:

$$\frac{1}{\phi_i}\Delta\varrho_i(\textit{m}) \quad \approx \quad \left[\textit{m} \quad + \quad \overline{\mathcal{M}}\times\mathcal{C}_i \quad + \quad \textit{N}\times\textit{Cov}\left((\overline{\Psi}-\textit{I})^{(i)},\log\boldsymbol{\mathcal{M}}\right)\right]^2$$

where $\phi_i \equiv -\Phi_i''\left(\sqrt{\frac{2\overline{\kappa}_i}{\epsilon-1}}\right) > 0$ and Φ_i is CDF of $\mathcal{N}(0, \sigma_i^2)$, $\overline{\mathcal{M}} \equiv \frac{1}{N} \sum_{i=1}^N \log \mathcal{M}_i$ and $\log \mathcal{M} \equiv [\log \mathcal{M}_1, ..., \log \mathcal{M}_N]^T$, $\overline{\Psi}^{(i)}$ is the i's row of the Leontief inverse matrix $\overline{\Psi} \equiv (I - \overline{\Omega})^{-1}$, and

$$C_i \equiv \sum_{j=1}^N \overline{\Psi}_{i,j} - 1$$

is the **customer centrality** of sector i.

▶ Back

Cascades amplification under aggregate TFP shocks

Proposition

Normalizing $p_{i,-1}(j) = \log \frac{\epsilon}{\epsilon - 1} \frac{1}{1 - \overline{\tau}_i}$. Let ϱ_i be the probability that a firm in sector i decides to adjust its price in the initial period (t = 0). For an aggregate TFP shock a, denote by $\Delta \varrho_i(a)$ the change in the adjustment probability relative to its steady-state value, then:

$$\frac{1}{\phi_i}\Delta\varrho_i(a) \quad \approx \quad \left[a \quad + \quad \left(a - \overline{\mathcal{M}}\right) \times \mathcal{C}_i \quad - \quad \mathsf{N} \times \mathsf{Cov}\left((\overline{\Psi} - \mathit{I})^{(i)}, \log \boldsymbol{\mathcal{M}}\right)\right]^2$$

where $\phi_i \equiv -\Phi_i''\left(\sqrt{\frac{2\kappa_i}{\epsilon-1}}\right) > 0$ and Φ_i is CDF of $\mathcal{N}(0,\sigma_i^2)$, $\overline{\mathcal{M}} \equiv \frac{1}{N}\sum_{i=1}^N\log\mathcal{M}_i$ is the average of sectoral markup changes and $\log\mathcal{M} \equiv [\log\mathcal{M}_1,...,\log\mathcal{M}_N]^T$, $\overline{\Psi}^{(i)}$ is the i's row of the Leontief inverse matrix $\overline{\Psi} \equiv (I-\overline{\Omega})^{-1}$, and C_i is is the customer centrality of sector i introduced in (4).

▶ Back

Cascades amplification under sectoral TFP shocks

Proposition

Set $p_{i,-1}(j) = \log \frac{\epsilon}{\epsilon-1} \frac{1}{1-\overline{r}_i}$, $\overline{\kappa}_i = \kappa$, $\sigma_i = \sigma$, $\forall i$ and assume $Cov\left((\overline{\Psi} - I)^{(i)}, \log \mathcal{M}\right) = 0$, $\forall i$. Let $\varrho \equiv \frac{1}{N} \sum_{i=1}^N \varrho_i$ be the average probability of adjustment in the initial period. For a TFP shock specific to sector k, a_k , denote by $\Delta \varrho(a_k)$ the change in the average adjustment probability relative to its steady-state value, then:

$$rac{1}{\phi}\Deltaarrho(a_k) \quad pprox \quad \mathcal{H}_k imes a_k^2 \quad - \quad 2\overline{\mathcal{M}} imes \left\{\overline{\mathcal{C}}\mathcal{S}_k + \mathit{Cov}(\overline{\Psi}_{(k)}, \mathcal{C})
ight\} imes a_k \quad + \quad \overline{\mathcal{M}}^2\overline{\mathcal{C}^2}$$

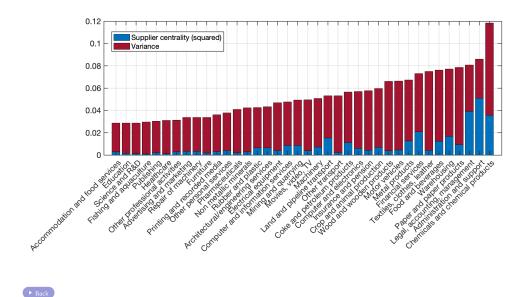
where $\phi \equiv -\Phi''\left(\sqrt{\frac{2\overline{\kappa}}{\epsilon-1}}\right) > 0$ and Φ is CDF of $\mathcal{N}(0,\sigma^2)$, $\overline{\mathcal{M}} \equiv \frac{1}{N}\sum_{i=1}^N\log\mathcal{M}_i$ is the average of sectoral markup changes, \mathcal{C}_i is is the customer centrality introduced in (4) and $\overline{\mathcal{C}} \equiv \frac{1}{N}\sum_{i=1}^N\mathcal{C}_i$, $\overline{\mathcal{C}}^2 \equiv \frac{1}{N}\sum_{i=1}^N\mathcal{C}_i^2$, $\mathcal{C} \equiv [\mathcal{C}_1,...,\mathcal{C}_N]^T$,

 $\overline{\Psi}_{(k)}$ is the k's column of the Leontief inverse matrix and

$$\mathcal{H}_k \equiv \frac{1}{N} \sum_{i=1}^N \overline{\Psi}_{i,k}^2, \qquad \qquad \mathcal{S}_k \equiv \frac{1}{N} \sum_{i=1}^N \overline{\Psi}_{i,k}$$

are, respectively, the **supplier Herfindahl** (\mathcal{H}_k) and **supplier centrality** (\mathcal{S}_k) of sector k.

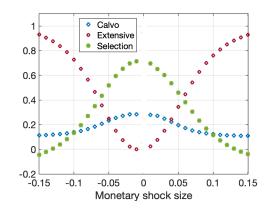
Supplier Herfindahl: a decomposition

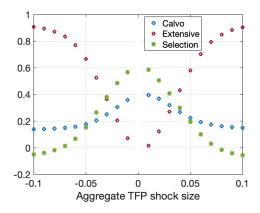


Inflation decomposition and network effects

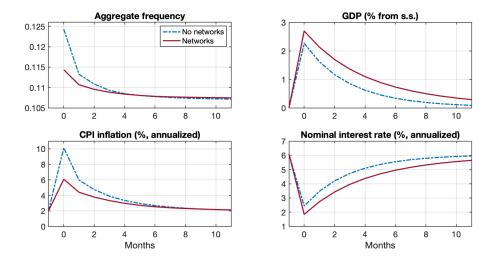
• Make use of the following inflation decomposition:

$$\Delta \pi = \Delta \pi^{\text{Calvo}} + \Delta \pi^{\text{Extensive}} + \Delta \pi^{\text{Selection}}$$

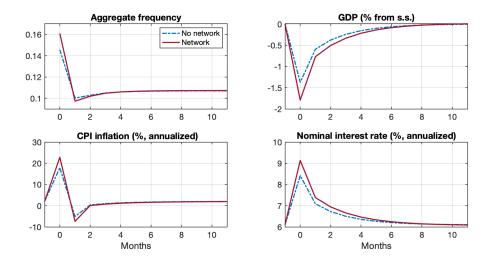




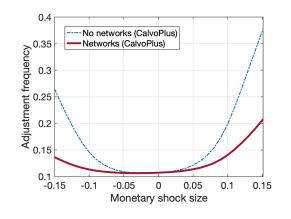
Cascades dampening following monetary shocks: Taylor rule

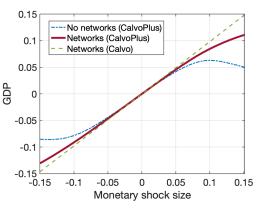


Cascades amplification following TFP shocks: Taylor rule

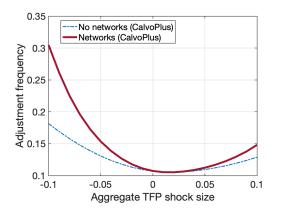


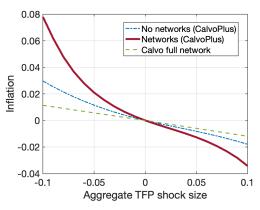
Cascades dampening following monetary shocks: CalvoPlus



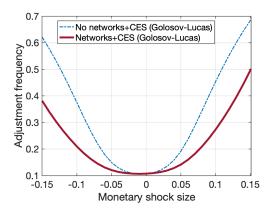


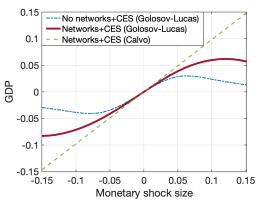
Cascades amplification following TFP shocks: CalvoPlus





Cascades dampening following monetary shocks: CES aggregation





Cascades amplification following TFP shocks: CES aggregation

