Technology Adoption and Optimal Industrial Policy

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Introduction and Summary

- Optimal "Industrial Policy" in set up often used for Big-Push
- *Dynamic* economy with complementarities, tech. adoption after paying fixed cost (non-convexities), market power & heterogeneity.

Results:

1. Economy is inefficient: $\overbrace{\text{misallocation}}^{\text{static}}$ and $\overbrace{\textbf{technology adoption}}^{\textbf{dynamic}}$
2. Static inefficiency stems from underproduction/use of intermediate goods: corrected by directly (or indirectly) subsidizing its use
3. Dynamic Inefficiency stems from firms valuation of tech. adoption too low relative to its cost: corrected by subsidizing adoption
4. If complementarity are large enough, multiple steady states/BGP
5. Optimal policy started at Laissez-faire BGP without tech adoption

   - Either stays there because adoption is too costly

   - Or start transition to high adoption steady state/BGP (a Big Push?)

   - No role equilibrium selection unless intertemporal elasticity is high

Selected Related Literature

- Big Push: Murphy, Schleifer and Vishny (1989), and many others
- Round-about production as complementarities: Ciccone (1996,2002), Jones (2011)
- Dynamics, multiple equilibrium paths:
  Matsuyama (1991) and Krugman (1991)
- Dynamics of optimal allocation w/non-convexities:
  Skiba (1978), Dechert and Nishimura (1983), Stachurski, Venditti and Yano (2012)
- Replacement Effect: Arrow (1962), Tirole (1988)
- Vintage Capital: Chari and Hopenhayn (1991), Bertolotti and Lanteri (2024), and many others

A MODEL WITH A GROWING FRONTIER

Set Up

- Technology frontier grows: $e^{\gamma t}$ (firms can adopt a new tech. after paying a fixed cost)
- Gap g : log of TFP distance of frontier, in time units
- At t operate technologies with gap $g \leq G(t)$ (optimal to adjust at threshold $G(t)$)
- Poisson rate q: free adoption opportunity
- Distribution (density) of Firms at time t indexed by gaps $m(g, t)$
- Law of motion for $m(g, t)$: # firms w/gap g, for $0 \leq g \leq G(t)$

$$m(g + dg, t + dt) - m(g, t)(1 - dt\, q) = 0 \quad \text{(discrete time)}$$
$$\implies \quad m_t(g, t) + m_g(g, t) + q\, m(g, t) = 0 \quad \text{(continuous time)}$$

- Mass preservation, $1 = \int_0^{G(t)} m(g, t)dg$, for all $t > 0$

$$\Rightarrow \underbrace{m(0, t)}_{\text{adoption}} = \underbrace{m\,(G(t), t)}_{\text{reach } G(t)} + \underbrace{q}_{\text{free}} - \underbrace{m\,(G(t), t)\, G'(t)}_{\text{change } G}$$

Feasibility: adoption

- Consumption $C(t)$ of aggregate good
- Costly adoption: $\kappa(t)$ units of of aggregate good; $\kappa(t) = \kappa\, e^{\frac{\gamma}{1-\nu}t}$
- Feasibility, $C(t) = Y(t) - \kappa(t)\, \left[m(0,t) - q \int_0^{G(t)} m(g,t)dg\right]$
- Preferences: $\int_0^{\infty} e^{-\rho t}\, \frac{C(t)^{1-\theta}-1}{1-\theta}\, dt$

Period t technology

- Cobb-Douglas output of differentiated good w/TFP $e^{(t-g)\gamma}$

$$e^{(t-g)\gamma}\, b\, x(g,t)^{\nu}\, n(g,t)^{1-\nu}$$

(ν share of intermediate input, $1-\nu$ labor share, b constant)

- $Y(t)$: net agg. output & $X(t)$: Intermediate Aggregate

$$\overbrace{Y(t)+X(t)}^{\equiv Q(t)} = \left[\int_0^{G(t)} \left(e^{(t-g)\gamma}\, b\, x(g,t)^{\nu} n(g,t)^{1-\nu}\right)^{1-\frac{1}{\eta}} m(g,t)\, dg\right]^{\frac{1}{1-1/\eta}}$$

$$X(t) = \int_0^{G(t)} x(g,t)\, m(g,t)\, dg$$

- Exogenous labor supply normalized to 1, so: $1 = \int_0^{G(t)} n(g,t) m(g,t) dg$

Equilibrium

- Household borrow and save, own firms, supply labor
- Monopolistic competitive firms:
  1. "Static": set prices, hire labor, buy intermediate aggregate
  2. "Dynamic": pay fixed cost $\kappa(t)$ & adopt frontier technology ($g = 0$)
- Prices:

  Differentiated good w/gap $g : p(g, t)$; Aggregate final good $P(t)$
  Wages $w(t)$; Interest rate $r(t)$
- Policy instruments - lump sum from household, $T(t)$

  Revenue subsidy, s_r; Intermediate inputs subsidy, s_x; Labor subsidy, s_l
  Adoption subsidy, s_a; Operating profits subsidy, s_π

Households

- Budget constraint

$$0 = \int_0^\infty e^{-\int_0^t r(s)ds} \left[P(t)C(t) - \Pi(t) - w(t) + T(t)\right] dt \ ,$$

- $\Pi(t)$ profits, $T(t)$ transfers, $w(t)$ wages

- Euler equation

$$r(t) = \rho + \theta \frac{\dot{C}(t)}{C(t)} + \frac{\dot{P}(t)}{P(t)} \ .$$

Monopolistic Competitive Firm

- $\pi(g,t) = s_\pi \hat{\pi}(g,t)$ after subsidy profits of firm g where t

$$\hat{\pi}(g,t) \equiv \max_p \left[\frac{p}{P(t)}\right]^{-\eta} Q(t) \left[s_r\, p - \overbrace{e^{\gamma(g-t)} \left(\frac{w(t)}{s_l}\right)^{1-\nu} \left(\frac{P(t)}{s_x}\right)^{\nu}}^{\text{marginal cost}} \right],$$

- Markup over marginal cost: $p(g,t) = \frac{1}{s_r} \frac{\eta}{\eta-1} e^{\gamma(g-t)} \left(\frac{w(t)}{s_l}\right)^{1-\nu} \left(\frac{P(t)}{s_x}\right)^{\nu}$

- Adoption problem, value function $V(g,t) \implies G(t)$:

$$r(t)V(g,t) = \max \begin{cases} r(t)\left[V(0,t) - \kappa(t)\frac{P(t)}{s_a}\right] & \text{optimal if } g \geq G(t) \\ s_\pi\, \hat{\pi}(g,t) + V_g(g,t) + V_t(g,t) + q\left(V(0,t) - V(g,t)\right) & \end{cases}$$

(solved using VM and SP details)

Temporal Equilibrium, given $m(\cdot, t)$

- Labor allocation: independent of subsidies.
- Detrended aggregate productivity:

$$Z(t) \equiv \left[\int_0^{G(t)} e^{-\gamma g(\eta-1)} m(g,t) dg\right]^{\frac{1}{\eta-1}}$$

- Profits, (numeraire) in terms wages:

$$\pi(g,t) = s_\pi \frac{1}{(\eta-1)(1-\nu)} \frac{e^{-\gamma g(\eta-1)}}{Z(t)^{\eta-1}}$$

- Profits, (real) in terms of final goods:

$$\frac{\pi(g,t)}{P(t)} = e^{\frac{\gamma}{1-\nu} t} \frac{s_\pi}{s_x} \frac{\left[s_r s_x \left(\frac{\eta-1}{\eta}\right)\right]^{\frac{1}{1-\nu}}}{(1-\nu)(\eta-1)} \frac{e^{-\gamma g(\eta-1)}}{Z(t)^{\frac{(\eta-1)(1-\nu)-1}{1-\nu}}}$$

- Real profits increasing in $Z(t)$ if $(\eta-1)(1-\nu) < 1$

$s_e = s_r\, s_x$ is a sufficient statistic for temporary equilibrium

- To simplify consider model with m concentrated in one value g.
- Obviously allocation is labor is efficient, normalize $w/s_l = 1$.
- Firms optimal price $p = \frac{\eta}{\eta-1}\frac{1}{s_r}(P/s_x)^{\nu}$
- Equilibrium $p = P$ so $P/s_x = \left(\frac{\eta}{\eta-1}\frac{1}{s_r s_x}\right)^{1/(1-\nu)}$
- Optimal choice of input: $\frac{(P/s_x)x}{(w/s_l)n} = \frac{\nu}{1-\nu}$
- In equilibrium $n = 1$ and $(w/s_l) = 1$ hence $x = \frac{1}{(P/s_x)}\frac{\nu}{1-\nu}$
- Thus, equilibrium value of x is monotone increasing in $s_e \equiv s_r s_x$
- There is a finite efficient value of x, achieved with $s_e \equiv \frac{\eta}{\eta-1}$

Aggregate Production Function, given $m(\cdot, t)$

- Aggregate output at t depends only on $m(\cdot, t)$ and $\boxed{s_e \equiv s_r s_x}$

$$\underbrace{e^{\frac{\gamma}{1-\nu}t}}_{\text{trend}} \underbrace{Y(m(\cdot,t), s_e)}_{\text{detrended output}} \equiv e^{\frac{\gamma}{1-\nu}t} \underbrace{A(s_e)}_{\text{Misallocation}} \underbrace{F(m(\cdot,t))}_{\text{Prod Function}}$$

  - Loss on TFP: Static 'misallocation' (stems from distortions)

$$A(s_e) \equiv \frac{1}{1-\nu}\left[\frac{1}{s_e}\frac{\eta}{\eta-1} - \nu\right]\left[s_e\frac{\eta-1}{\eta}\right]^{\frac{1}{1-\nu}}$$

  - Aggregate production function

$$F(m(\cdot,t)) \equiv Z(t)^{\frac{1}{1-\nu}} = \left[\int_0^{G(t)} e^{-\gamma g(\eta-1)} m(g,t)dg\right]^{\frac{1}{(\eta-1)(1-\nu)}}$$

- Curvature parameter (compl. vs subs. or convex vs conc.)

$$\zeta \equiv \frac{1}{(\eta-1)(1-\nu)} \gtrless 1$$

Static Efficient Allocation $\mathcal{Y}(m)$

- Fix m. Maximize net detrended aggregate output $\mathcal{Y}(m)$:

  choice of date t allocation s.t. mkt clearing intermediate, labor & prod. functions
  $\Longrightarrow \mathcal{Y}(m) = Z(t)^{\frac{1}{1-\nu}}$

- If $\nu = 0$, then $\mathcal{Y}(m) = Y(m, s_e)$

- If $\nu > 0$, then $\mathcal{Y}(m) \geq Y(m, s_e)$ with equality if $s_e = s_e^* \equiv \frac{\eta}{\eta-1}$

- m^ϵ: m perturbed so that ϵ density is moved from g_2 to g_1:

  1. $\mathcal{Y}(m^\epsilon)$ is concave in ϵ and $m \iff \zeta \equiv \frac{1}{(\eta-1)(1-\nu)} \leq 1$
  2. $\frac{d\mathcal{Y}(m^\epsilon)}{d\epsilon}\Big|_{\epsilon=0} = \frac{\pi(g_1,t)-\pi(g_2,t)}{s_\pi\, P(t)/s_x} \left[\frac{1}{s_e}\left(\frac{\eta}{\eta-1}\right)\right]^{\frac{1}{1-\nu}}$

     $\Longrightarrow$ Social marginal benefit $\propto$ (temp. eqbm) real profits

Efficient Allocation - Two period Mickey-Mouse model

- At an Equilibrium, adoption gives:

$$\frac{\pi(0,t)-\pi(G(t),t)}{P(t)/s_a}=\kappa(t)$$

- For the planner efficient adoption gives:

$$\left.\frac{d\mathcal{Y}(m(\cdot,t))}{d\epsilon}\right|_{\epsilon=0}=\kappa(t)$$

- Previous results gives

$$\underbrace{\frac{d\mathcal{Y}(m(\cdot,t))}{d\epsilon}}_{\text{social benefit}}=\left[\frac{1}{s_e}\left(\frac{\eta}{\eta-1}\right)\right]^{\frac{1}{1-\nu}}\frac{s_e}{s_r s_a s_\pi}\times\underbrace{s_a\frac{\pi(0,t)-\pi(G(t),t)}{P(t)}}_{\text{private benefit}}$$

- Define $\boxed{s_d=s_\pi\, s_a\, s_r}$, decentralize efficient $s_e^*=s_d^*=\frac{\eta}{\eta-1}$

- Result extends exactly the same to infinite horizon model.

Efficient Allocation - Infinite Horizon Model

- Given initial $m_0(g)$ all g, maximize

$$\int_0^\infty e^{-\bar{\rho}t} \frac{c(t)^{1-\theta} - 1}{1-\theta} \, dt \, , \text{ by choosing path of adoption } G'(t) \text{ s.t.}$$

  - Law of motion of entire distribution

$$0 = m_t(g,t) + m_g(g,t) + q\, m(g,t) \, , \text{ all } g \in [0, G(t)], \; t \geq 0$$

  - Resource constraint: $c(t) = \mathcal{Y}(m(\cdot,t)) - \kappa\,(m(0,t) - q)$

- Implementation of efficient allocation
  - Eqbm and nec. conditions coincide
    - If $\nu = 0$, $\boxed{s_d \equiv s_\pi s_r s_a = \frac{\eta}{\eta-1}}$
    - If $\nu > 0$, $\boxed{s_e \equiv s_r s_x = \frac{\eta}{\eta-1}}$ and $\boxed{s_d \equiv s_\pi s_r s_a = \frac{\eta}{\eta-1}}$
  - Possible multiple equilibrium paths under optimal policy
    - unique path: if $\zeta \leq 1$
    - if multiple eqbm path: role of coordination

Equivalence Eqbm & necessary conditions

Solving for a BGP: fixed point

Economy grows at rate $\frac{\gamma}{1-\nu}$, $G(t) = G^*$ and interest rates are constant

Aggregation: $G \to Z^*$

$$Z^* = \left[\int_0^G e^{-\gamma g(\eta-1)} \frac{qe^{-qg}}{1-e^{-qG}} dg\right]^{\frac{1}{\eta-1}}$$

Higher aggregate adoption $\implies$ higher "TFP" Z

Optimization of a firm: $Z \to G^*$

$$\overbrace{\zeta \left[Z^{\eta-1}\right]^{\zeta-1} R(G^*)/(q+\bar{\rho})}^{\text{Net discounted gain}} = \overbrace{\kappa \frac{s_e}{s_d} \left(\frac{1}{s_e}\frac{\eta}{\eta-1}\right)^{\frac{1}{1-\nu}}}^{\text{Adjusted fixed cost}}$$

where $R(G) = 1 - e^{-\gamma(\eta-1)G} - \frac{\gamma(\eta-1)}{q+\rho+\gamma(\eta-1)}\left[1 - e^{-(q+\rho+\gamma(\eta-1))G}\right]$

Higher TFP Z: has two effects on adoption

1. pro-competitive effect (lower mkt share) $\implies$ lower adoption incentives
2. lower price of adoption good $\implies$ higher adoption incentives

Strength of Complementarities and BGPs

$\zeta \leq 1$: *one* BGP

(pro-competitive effect dominates)

If $\bar{\kappa}$ 'large', then without costly adoption

Otherwise, then with costly adoption

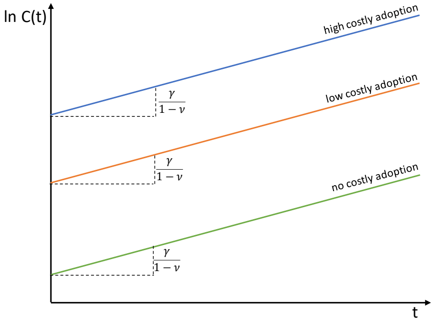
$\zeta > 1$: *multiple* BGPs are possible

(lower price adoption dominates)

1 without costly adoption

1 with infrequent costly adoption

1 with frequent costly adoption



A MODEL WITH A STATIC FRONTIER

Setup

- Frontier normalized to 1. No free adoption, $q = 0$
- Firm with gap $z \rightarrow$ productivity $e^{-z} < 1$
- Pay fixed cost & jump to frontier; can recoup fixed cost & get back to z
  $V(z, t)$
- Define $K(t) =$ mass of firms at frontier ; m_0 constant through t

$$K = 1 - \int_0^{\hat{G}(K)} m_0(z)\, dz \implies \dot{K}(t) = -m_0(G(t))\, \dot{G}(t)$$

- Feasibility: $\kappa \dot{K}(t) + C(t) = A(s_e) F(K(t))$
- Aggregate Production: $F(K) = \left[\int_0^{\hat{G}(K)} e^{-z(\eta-1)} m_0(z)\, dz + K \right]^{\zeta}$

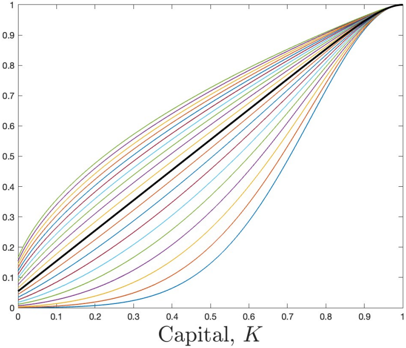
$\implies$ Akin to Neoclassical Growth Model

- K: capital stock
- Same law of motion for K
- Difference: $F(K)$ is not necessarily concave!

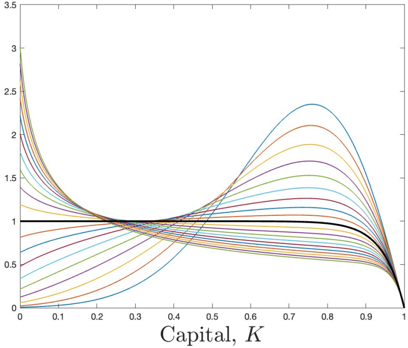
Shape of Production Function $F(K)$

- Properties of $F(K)$ as function of ζ
  1. If $\zeta \leq 1$, globally concave
  2. Allays concave near $K = 1$
  3. If $\zeta > 1$ and regularity, then $F(\cdot)$ is S-shaped, $F'(\cdot)$ inverse U

$F(K)$



$F'(K)$



Equilibrium: Neoclassical Growth model w/tax!

- Fix s_e, s_d and $K(0) = K_0$
- Nec. and suff. conditions for interior eq. is that $\{C(t), K(t)\}$ solve

$$C(t) + \kappa \dot{K}(t) = A(s_e)F(K(t))\ ,\ \theta\frac{\dot{C}(t)}{C(t)} = B(s_e, s_d)A(s_e)F'[K(t)]/\kappa - \rho$$

where $B(s_e, s_d) \equiv \left(\frac{1-\nu}{\frac{1}{s_e}\frac{\eta}{\eta-1}-\nu}\right)\frac{s_d}{s_e}$ & $0 = \lim_{T\uparrow\infty} e^{-\rho T}C(T)^{-\theta}A(s_e)F[K(T)]$

- Interpretation: NGM with $1 - B(1,1)$ tax on capital returns

  ($B(1,1) = 1 - \frac{1}{\eta}$ when $\nu = 0$)

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  ($B(1, 1) = 1 - \frac{1}{\eta}$ when $\nu = 0$)

Interior SS
Solves $B(s_e, s_d)A(s_e)F'(K^*)/\kappa = \rho$

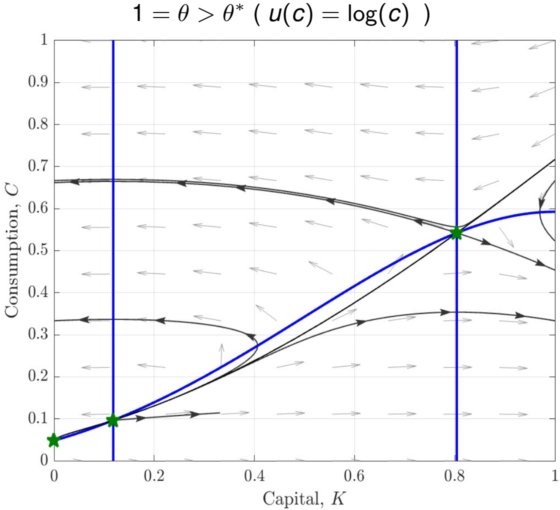
- If $\zeta \leq 1$: at most one
- If F is S-shaped & ζ large enough
  K_L^*: source, or spiral source (θ^*)
  K_H^*: saddle
  $K_L^* < K_H^*$

SS with No Adoption
If $B(s_e, s_d)A(s_e)F'(0)/\kappa < \rho$

- $K^* = 0$ and $C^* = A(s_e)F(0)$
- Locally stable (if $\theta > \theta^*$)
- Convergence in finite time

Equilibrium w/Laissez-Faire ($s_e = s_d = 1$), $\zeta > 1$,

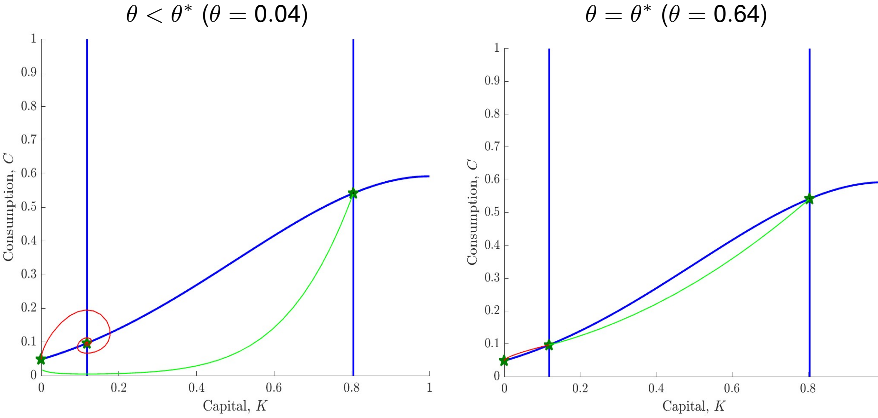
- 3 steady states (green stars), middle one unstable.



- The case of $\zeta < 1$ is just like the Neoclassical Growth Model

Multiplicity of Eqbm Paths for $\zeta > 1$ and low θ

- Let $s_e = s_d = 1$



- Low θ case has multiple equilibrium path for $K(0) \in [0, 1.9]$

Planner's Problem

$$\max_{C(\cdot)} \int_0^\infty e^{-\rho t} U(C(t))dt \text{ subject to } \kappa \dot{K}(t) = F(k(t)) - C(t)$$

Necessary conditions:

1. Euler eq. and Transversality condition hold
2. K^* is an optimal steady state if $F(K^*) = C^*$ and $\rho = F'(K^*)/\kappa$

- If $\zeta > 1$ these are *only* necessary. When F is S-shaped there can be interior solutions $K_L^* < K_H^*$
  1. K_L^* cannot be stable
  2. If $\rho < F'(0)/\kappa$, K_H^* from any $K(0)$ is locally stable (saddle)
  3. If $\theta < \theta^*$ multiple paths satisfying EE + TC.
- Decentralization: eliminate both distortions: $s_e^* = s_d^* = \frac{\eta}{\eta-1} \implies A(s_e^*) = B(s_d^*) = 1$

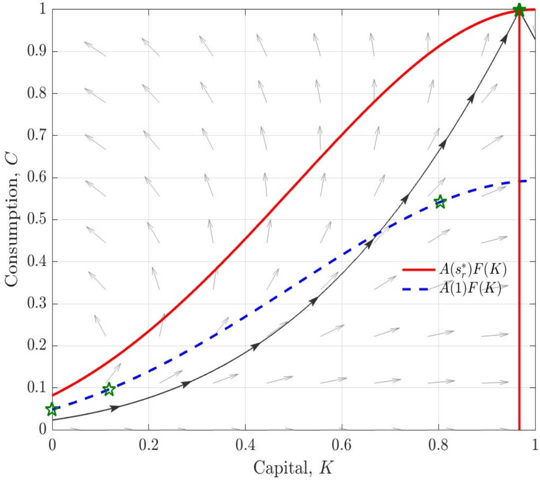
Trap or No trap?

Consider an economy that starts at SS $K^* = 0$ w/no adoption

- If $\left(\frac{\eta-1}{\eta}\right)^{\frac{1}{1-\nu}} F'(0) < \kappa\rho < F'(0)$
  - Only one interior SS w/high adoption survives with subsidy
  - Long transition from $K^* = 0$ to interior SS w/high adoption (i.e. implements a Big Push)
  - Laissaz Faire SS w/no adoption is a TRAP, optimal policy moves the economy away from it
  - See Figure
- If $F'(0) < \kappa\rho$?
  - The three SS remain even w/optimal policy
  - Economy remains in the SS w/no adoption (but with no static misallocation)
  - The SS with no adoption is NOT A TRAP

Optimal exit of trap: $s_e^* = s_d^* = \eta/(\eta - 1)$

- Assume that $\zeta > 1$ and that $\left(\frac{\eta-1}{\eta}\right)^{\frac{1}{1-\nu}} F'(0) < \kappa\rho < F'(0)$



Optimal policy pushes the economy out of the 'trap', which converges to the higher steady state, far away from no adoption SS.

Conclusions

- Two versions of dynamics model of adoption:
  1. Growing frontier $\approx$ Vintage Capital Model
  2. Fixed frontier $\approx$ Neoclassical Growth Model
- In both cases, static inefficiency acts by reducing output.
- In both cases dynamic inefficiency acts as an tax on investment
- Optimal policy eliminates static distorsions and investment tax
  - Menu of 5 subsidies/tax to achieve efficiency
  - Optimal involves only two combinations
- Fixed frontier model: full analysis of dynamics
- Large effects due to strategic complementarities.
- No role for Eqbm selection out of a trap, unless θ low enough
- If θ small enough, temporary higher s_d used as Eqbm selection.

Efficient Allocation

- Given initial m_0, maximize

$$\int_0^\infty e^{-\bar{\rho}t} \frac{c(t)^{1-\theta} - 1}{1-\theta} \, dt$$

by choosing a time differentiable path of threshold $\{G(t)\}$

- subject to the constraints for all $t \geq 0$:

$$e^{-\bar{\rho}t}\lambda(g,t): \quad 0 = m_t(g,t) + m_g(g,t) + q\, m(g,t) \;, \text{ for } 0 \leq g \leq G(t)$$

$$e^{-\bar{\rho}t}\omega(t): \quad 0 = 1 - \int_0^{G(t)} m(g,t)dg,$$

where $e^{-\rho t}\lambda(g,t)$ and $\omega(t)$ are Lagrangian multipliers and where

$$c(t) = \frac{N}{1-\nu} Z(t)^{\frac{1}{1-\nu}} - \kappa\,(m(0,t) - q) \text{ with}$$

$$Z(t) = \left[\int_0^{G(t)} e^{-\gamma g(\eta-1)} m(g,t)dg\right]^{\frac{1}{\eta-1}}$$

Adoption problem characterization

- Given path $\{\pi(\cdot, t), P(t), r(t)\}$ solve for path of threshold $\{G(t)\}$
- For $0 \leq g \leq G(t)$:

$$r(t)V(g,t) = \pi(g,t) + V_g(g,t) + V_t(g,t) + q\,(V(0,t) - V(g,t))$$

- For $g \geq G(t)$:

$$V(g,t) = V(0,t) - s_a\,\kappa(t)P(t) \implies 0 = V_g(g,t)$$

- Value Matching:

$$V(G(t),t) = V(0,t) - \kappa(t)P(t) \text{ for all } t > 0$$

- Smooth pasting:

$$0 = V_g(G(t),t)[G'(t) - 1] \text{ for all } t > 0$$

back

Characterization of Efficient Allocation

- Multiplier for law of motion m:

$$\bar{\rho}\lambda(g,t) = c(t)^{-\theta} Z(t)^{\frac{1}{1-\nu}} \pi(g,t) + \lambda_t(g,t) + \lambda_g(g,t) \\ - \omega(t) + q\left(\lambda(0,t) - \lambda(g,t)\right) \text{ for } t \geq 0 \ \& \ g \in [0, G(t)]$$

- Boundary conditions:

$$\lambda(0,t) = c(t)^{-\theta}\kappa, \text{ for all } t > 0$$
$$\lambda(G(t),t) = 0, \text{ all } t > 0$$
$$\lambda_g(G(t),t) = 0, \text{ all } t > 0$$

- Transversality:

$$0 = \lim_{T\to\infty} e^{-\bar{\rho}T}\lambda(g,T)m(g,T) \text{ for all } 0 \leq g < \lim_{T\to\infty} G(T)$$

- These conditions + feasibility are necessary.

- If $\zeta \leq 1$ they are sufficient. back

Firm's Problem

- $V(z,t)$, the value of a z at t that has not adopted the frontier

$$V(z,t) = \max_{\tau \geq t} \int_t^{\tau} e^{-\int_t^s r(\tilde{s})d\tilde{s}} \pi(z,s)\, ds + e^{-\int_t^{\tau} r(\tilde{s})d\tilde{s}} \left[V^0(z,\tau) - s_a(\tau)\kappa P(\tau) \right]$$

- $V^0(z,t)$, the value of a z firm that has adopted the frontier

$$V^0(z,t) = \max_{\{\tau \geq t\}} \int_t^{\tau} e^{-\int_t^s r(\tilde{s})d\tilde{s}} \pi(0,s)\, ds + e^{-\int_t^{\tau} r(\tilde{s})d\tilde{s}} \left[V(\tau,z) + \kappa s_a(\tau) P(\tau) \right]$$

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