Flight to Safety and New Keynesian Demand Recessions

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Overview

- This paper: merge
  1. safe asset framework of Brunnermeier, Merkel, Sannikov (2024)
  2. New Keynesian (NK) price setting models

  to study transmission of uncertainty shocks & flight to safety to real economy

- Why interesting?
  - large literature emphasizes role of uncertainty for aggregate fluctuations
  - one prominent channel: uncertainty $\rightarrow$ aggregate demand

    (e.g. Christiano, Motto, Rostagno 2014; Basu, Bundick 2017; Caballero, Farhi 2018; Caballero, Simsek 2020)
  - but abstracts from safe nominal government debt; potentially problematic omission:
    - (a) we observe large debt stocks & flight to safety when measures of uncertainty spike
    - (b) real value of nominal debt is tightly linked to nominal goods prices

- Key lessons:
  - flight to safety at the core of transmission of uncertainty shocks to real economy
  - this mechanism matters for model predictions, including power of monetary policy

Key Model Element: Portfolio Choice with Nominal Safe Assets

	conventional NK model	safe asset NK model
aggregate demand	**IS curve**: consumption-saving • forward-looking • relates demand to future $r_t - r_t^*$-gaps	**safe asset demand**: portfolio choice btw. capital (risky) and nominal bonds (safe) • forward-looking **safe asset supply**: debt accumulation equation • backward-looking: gradually adjusts to inflation and interest rates
aggregate supply	**Phillips curve** • forward-looking • relates inflation to future output gaps	

Some Implications

1. Portfolio choice not intertemporal substitution matters
   - uncertainty shocks increase desire to save (precautionary motive)
   - but relevant for aggregate demand only if there is flight to safety
2. Interest Rate Policy Ineffectiveness
   - powerful in conventional IS logic except at ZLB
   - much less powerful in affecting portfolios; cannot fix demand recessions, regardless of ZLB
   - optimal policy: underreacts & utilizes inflationary revaluation of safe asset supply
3. Asset Pricing: overshooting of capital prices
   - sticky goods prices $\Rightarrow$ sluggish adjustments in real bond value
   - capital prices "overreact" to shocks, generates asymmetric volatility

Related Literature

- NK models:
  - uncertainty shocks in RANK: Born, Pfeifer (2014); Christiano, Motto, Rastagno (2014); Ilut, Schneider (2014); Fernández-Villaverde, Guerrón-Quintana, Kuester, Rubio-Ramírez (2015); Leduc, Liu (2016); Basu, Bundick (2017); Caballero, Simsek (2020)

    This paper: safe assets in pos. net supply, role of flight to safety, policy ineffectiveness, ...
  - ... and quantitative HANK: Bayer, Lütticke, Pham-Dao, Tjaden (2019); Schaab (2020)

    This paper: analytical results, isolate aggregate effects
  - analytical HANK: Werning (2015); Acharya, Dogra (2020); Ravn, Sterk (2020); Bilbiie (2020, 2024)

    This paper: novel tractable framework with portfolio choice and safe assets
- Safe assets (selective):
  - flight to safety: Brunnermeier, Merkel, Sannikov (2024)

    This paper: sticky prices, capital price overshooting
  - safety trap: Caballero, Farhi (2018); Acharya, Dogra (2022)

    This paper: (ir)relevance of ZLB
- Government debt as nominal anchor (FTPL/Ricardian non-equivalence): e.g., Leeper (1991); Sims (1994); Woodford (1995, 2001); Cochrane (2017, 2023); Caramp, Silva (2021); Bénassy (2000, 2005); Leith, von Thadden (2008); Hagedorn (2021); Angeletos, Lian, Wolf (2024)

  This paper: portfolio choice with government bonds as safe assets

Outline

1. Model
2. Transmission of Uncertainty Shocks
3. Implications
   - Implications for Interest Rate Policy and Asset Pricing
   - Comparison to Models without Safe Assets
4. Long-term Bonds and (Optimal) Interest Rate Policy
   - Long-term Bonds
   - Optimal Policy

Outline

Model Setup

- Continuous time, infinite horizon, one consumption good (= final output good)
- Agents
  - households: hold capital (idiosyncratically risky) and government bonds (nominally safe)
  - intermediate goods firms: rent capital, produce differentiated intermediate goods
  - final goods firms: combine intermediate goods outputs (CES technology)
- Government
  - issues nominal bonds
  - taxes capital, sets nominal interest rate
- Frictions
  - financial friction (incomplete markets): households cannot trade idiosyncratic risk
  - price setting friction: intermediate goods firms face price adjustment cost
- Aggregate shock: stochastic fluctuations in volatility of idiosyncratic shocks

Selected Formal Details (simplified model)

- Household preferences ($i \in [0,1]$ agent index):

▸ household problem

$$\mathbb{E}\left[\int_0^\infty e^{-\rho t}\left(\log c_t^i - \frac{(u_t^i)^{1+\varphi}}{1+\varphi}\right) dt\right]$$

- Manages capital k_t^i:
  - capital services (rented out): $u_t^i k_t^i dt$
  - capital tax by government: $\tau_t k_t^i dt$
  - capital evolution:

▸ full production side

$$\frac{dk_t^i}{k_t^i} = \underbrace{d\Delta_t^{k,i}}_{\text{trading}} + \underbrace{\tilde{\sigma}_t d\tilde{Z}_t^i}_{\text{idio. shocks}}$$

- Government bonds
  - nominal face value $\mathcal{B}_t$: $d\mathcal{B}_t/\mathcal{B}_t = \mu_t^{\mathcal{B}} dt$
  - floating nominal interest i_t (extension: long-term bonds)
  - government flow budget constraint

$$i_t = \mu_t^{\mathcal{B}} + \check{s}_t, \qquad \check{s}_t := \frac{\mathcal{P}_t \tau_t K_t}{\mathcal{B}_t}$$

Notation: Assets Values

- Assets in positive net supply: capital & bonds
  - capital: aggregate supply $K_t \equiv 1$, value q_t^K
  - bonds: real value of bond stock $q_t^B := \frac{\mathcal{B}_t}{\mathcal{P}_t}$
- Also define total wealth $q_t := q_t^B + q_t^K$
- Share of bond wealth

$$\vartheta_t := \frac{\mathcal{B}_t/\mathcal{P}_t}{q_t^K + \mathcal{B}_t/\mathcal{P}_t} = \frac{q_t^B}{q_t}$$

- In equilibrium:
  - all households choose identical portfolios
  - ϑ_t is also individual portfolio weight in bonds
- Note: $\mathcal{B}_t$ is slow-moving (only drifts, no jumps or Brownian shock loadings)

  $\rightarrow$ when $\mathcal{P}_t$ is sticky, then so is q_t^B

$\mathcal{P}_t$-dynamics

Remark: Nested Models

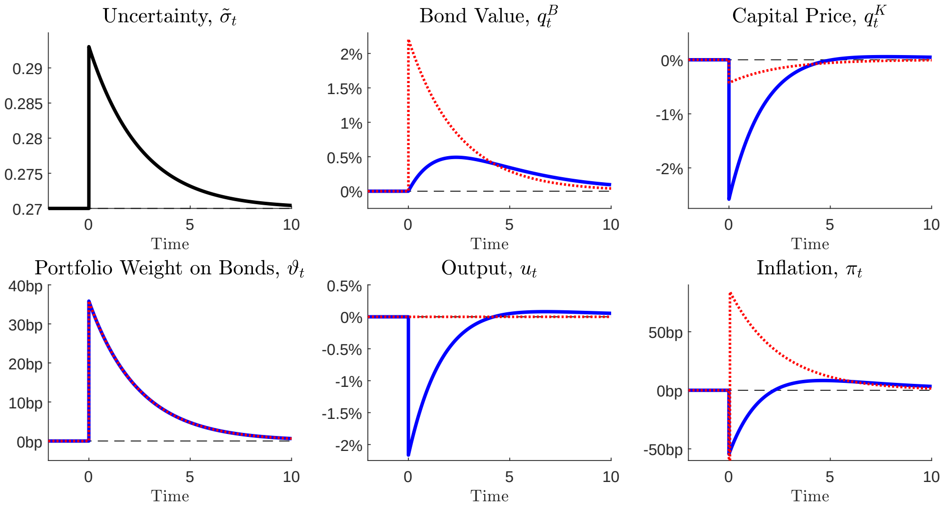
Special cases of our model are (essentially) isomorphic to the following models:

- Brunnermeier, Merkel, Sannikov (2024): without price setting friction
- Gali (2008, Chapter 3) textbook NK model: for $\tilde{\sigma} = 0$
  (reinterpret utilization as labor and capital as labor productivity)
- Caballero, Simsek (2020): for $\mathcal{B} = q^B = 0$ (no safe assets)
  (they have aggregate instead of idiosyncratic capital shocks, but irrelevant for conclusions)

Remark II: Relationship to HANK

Outline

Impulse Responses



Transmission Preliminaries I: Separation of Portfolio Choice

- "Bond valuation equation": ϑ_t satisfies in equilibrium

$$\vartheta_t = \mathbb{E}_t \left[\int_t^\infty e^{-\rho(s-t)} \vartheta_s \left(\check{s}_s + (1-\vartheta_s)^2 \tilde{\sigma}_s^2 \right) ds \right].$$

  - depends only on fiscal instrument $\check{s}_t$ and idiosyncratic risk $\tilde{\sigma}_t$
  - not on aggregate output or price setting frictions

- Separation: if $\check{s}_t$ is function of $(\tilde{\sigma}_t, \vartheta_t)$ only, then $\vartheta_t = \vartheta(\tilde{\sigma}_t)$ does not depend on bond valuation state q_t^B

  $\rightarrow$ portfolios adjust "fast" (as under flexible prices)

- *Remark*: separation condition satisfied for conventional linear fiscal reaction rules

$$S_t/Y_t = \alpha + \beta q_t^B/Y_t \qquad \Rightarrow \qquad \check{s}_t = \alpha \frac{\rho}{\vartheta_t} + \beta$$

- Flight to Safety: Unless $\check{s}$-policy leans strongly against it, rise in σ_t leads to increase in ϑ_t

Transmission Preliminaries II: Asset Valuation and Aggregate Demand

- Goods market clearing relates real activity to *level of asset valuations*

$$u_t = \underbrace{\rho q_t = \rho(q_t^B + q_t^K)}_{=\text{aggr. cons. demand}}$$

- Portfolio choice (ϑ_t) determines *relative asset valuations*

$$q_t = q_t^B + q_t^K = \frac{1}{\vartheta_t} q_t^B$$

- Combining the previous:

$$\boxed{u_t = \rho \frac{q_t^B}{\vartheta_t}}$$

Shock Transmission: Impact Effect

$$\underbrace{u_t}_{\text{supply}} = \underbrace{\rho \frac{q_t^B}{\vartheta_t}}_{\text{demand}}$$

	uncertainty shock $\tilde{\sigma}_t \uparrow$	
	flexible prices	sticky prices
portfolio choice	$\vartheta_t \uparrow$	$\vartheta_t \uparrow$
demand for given q_t^B	$\downarrow$	$\downarrow$
equilibrium adjustment	$u_t \rightarrow$, $q_t^B \uparrow$	$u_t \downarrow$, $q_t^B \rightarrow$
required price adjustment	$\mathcal{P}_t \downarrow$	$\mathcal{P}_t \rightarrow$

Shock Transmission: Adjustment Dynamics under Sticky Prices

- After shock, gradual inflation/deflation slowly adjusts q_t^B towards flexible-price value ("Pigou effect") (Pigou, 1943; Patinkin, 1956)
- Dynamics guided by two equations

  ‣ closed-form solutions (under simplifying assumptions)

  - Bond value evolution (backward-looking):

$$dq_t^B = (\underbrace{i_t - \check{s}_t}_{=\mu_t^{\mathcal{B}}} - \pi_t) q_t^B dt$$

  - Phillips curve (forward-looking):

$$\mathbb{E}_t[d\pi_t] = \left(\rho\pi_t - \kappa\left(\left(\rho\frac{q_t^B}{\vartheta_t}\right)^{1+\varphi} - 1\right)\right) dt$$

Outline

I. Intertemporal Substitution versus Portfolio Choice

- Standard NK story: intertemporal substitution drives aggregate demand
  - key equation: IS equation (in terms of wealth q_t)

$$\mathbb{E}_t[dq_t] = (i_t - \pi_t - r_t^*)\, q_t dt$$

  - relates *level* of wealth to *level of interest rate*
  - usual interpretation: future q_T fixed (e.g., by "anchored beliefs"), q_0 adjusts
  - if $i_t - \pi_t > r_t^*$ for a while: q_0 falls (demand recession)
- This model: portfolio demand for nominal safe assets drives aggregate demand
  - recall: $q_t = q_t^B/\vartheta_t$ fully determined by ϑ_t and safe asset state q_t^B
  - portf. choice determines *relative* asset values ϑ_t
  - "level component" in $q_t = q_t^B/\vartheta_t$ is backward-looking state variable q_t^B

Conclusion: Portfolio choice and flight to safety are key for impact (demand) effect of shocks

II. Interest Rate Policy Ineffectiveness

- How does i_t affect aggregate demand?
  1. Portfolio separation: portfolio demand for safe assets (ϑ_t) unaffected by i_t
  2. Safe asset value q_t^B is slow-moving state: affected by i_t only gradually over time

  $\Rightarrow$ Impact effect of shock on aggregate demand does not depend on i_t

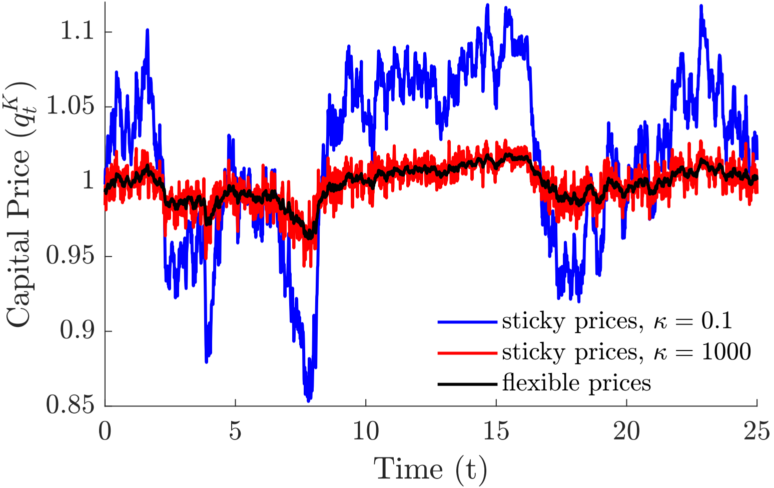
- Conclusion: interest rate policy cannot eliminate aggregate demand recession
  (in contrast to standard NK models)

- *Remark*: "no impact effect" can be overturned with long-term bonds, broader conclusion is robust

III. Capital Price Overshooting & Flight-to-Safety Volatility

- Portfolio separation: ϑ_t rises as fast as under flexible prices
- Stickiness of bond value: q_t^B unaffected by shock, whereas $q_t^{B,flex} \uparrow$
- Consequence: capital price *overshoots* relative to flexible price response
  - $q_t^K = (1 - \vartheta_t)/\vartheta_t \cdot q_t^B$ falls by more under sticky prices
- Corrects major shortcoming of flexible price model (Brunnermeier, Merkel, Sannikov 2024)
  - in that model: bond market (q^B) more volatile than stock market (q^K)
  - this paper: *any* degree of price stickiness shifts *all* relative volatility into q^K fluctuations
- Reminiscent of Dornbusch's (1976) overshooting model
  - original: sticky domestic price $\rightarrow$ volatile exchange rate
  - here: sticky bond value $\rightarrow$ volatile capital price

Illustration: Overshooting & Flight-to-Safety Volatility



Outline

An Economy without Nominal Bonds

- Consider economy with $\mathcal{B}_t \equiv 0 \Rightarrow \vartheta_t = q_t^B \equiv 0$
- Goods market clearing equation (note $q_t = q_t^K$)

$$u_t = \rho q_t$$

  - pricing wealth $\Leftrightarrow$ pricing capital: (without aggregate shocks)

$$\underbrace{\rho + \mathbb{E}_t[dq_t]/(q_t dt)}_{=\mathbb{E}_t[dr_t^K]/dt} = \underbrace{i_t - \pi_t}_{=r_t} \quad + \quad \underbrace{\tilde{\sigma}_t^2}_{=\text{risk premium}}$$

$\rightarrow$ Aggregate demand without bonds is *purely forward-looking* and follows IS equation logic

1. no sticky bond value state ($q_t^B \equiv 0$) & no nominal anchor
2. previous equation

$$\mathbb{E}_t[dq_t]/(q_t dt) = i_t - \pi_t - \underbrace{(\rho - \tilde{\sigma}_t^2)}_{=r_t^*}$$

determines *level* of asset values as function of *level* of returns

▸ Illustration

Demand Recessions & Power of Monetary Policy

Conclusions from such models (e.g. Basu, Bundick 2017; Caballero, Farhi 2018; Caballero, Simsek 2020):

1. if insufficient reduction in $r_t = i_t - \pi_t$, also these models predict shortfall in demand
2. but this is not necessary: sufficient reduction in r_t can prevent demand shortfall
   - e.g. natural rate policy $i_t = r_t^* := \rho - \tilde{\sigma}_t^2$ & appropriate equilibrium selection
   - leads to divine coincidence: $\pi_t = 0$, $u_t = u^*$
   - more generally: only task of policy is *expectations management*
3. corollary: demand recessions are (mostly) a problem at ZLB

Outline

How Can Policy Stabilize Aggregate Demand on Impact?

1. Manage *safe asset demand* by distorting portfolio choice
   - use policy instrument $\breve{s}_t$ (by adjusting taxes)
   - mitigates flight to safety, but not optimal (in richer model)
     (safe asset services more valuable when $\tilde{\sigma}$ is large, higher ϑ beneficial)

2. Manage *safe asset supply* by introducing safe asset whose value is not (fully) sticky
   a. lump-sum transfers
      - PV of lump-sum transfers acts as implicit safe asset (if aggr. risk markets complete)
      - use dynamic adjustments of transfers to absorb variations in safe asset demand
      - issue: works in theory but difficult in practice
   b. long-term bonds
      - i-policy affects (flexible) nominal bond price through expected future rates
      - *but*: cannot control i_t and q_t^B independently, insufficient to prevent demand recession

      $\rightarrow$ generates interesting policy problem

Model Extension with Long-term Bonds

- In baseline model: bonds have infinitesimal duration
  - there is no relative price between "money" (sticky unit of pricing) and nominal bonds
- Extension: bonds are long-term with geometric maturity structure
  - nominal face value $\mathcal{B}_t$ as before
  - each period: government must make payments $\lambda \mathcal{B}_t dt$, $\lambda > 0$
  - $\mathcal{P}_t^B$ is the nominal price of one nominal unit of bonds
  - note: $\lambda \to \infty$: short-term bonds, $\lambda \to 0$: perpetuities
- Proposition: all model equations are as before except
  - $q_t^B = \mathcal{P}_t^B q_t^{B,0}$ and only $q_t^{B,0} := \mathcal{B}_t / \mathcal{P}_t$ is a state due to stickiness
  - $\mathcal{P}_t^B = \frac{\lambda}{\lambda + i_t}$
  - i_t is the long-term interest rate (fully controlled by controlling short rate i_t^0)

Interest Rate Policy Ineffectiveness Revisited

- Two effects of higher i_t:
  1. **debt revaluation channel**: lower $\mathcal{P}_t^B$ reduces safe asset supply q_t^B *immediately*
  2. **debt growth channel**: higher $\mu_t^{\mathcal{B}}$ raises safe asset supply q_t^B *gradually* (without need for deflation)

- First effect appears to overturn interest rate policy ineffectiveness:
  - i-policy can control q_t^B on shock impact
  - e.g. can completely eliminate (impact) output gap without any fiscal support

- *But*: interest rate policy still unable to eliminate sticky price distortions
  - second effect: lower i_t shifts deflation pressures into the future
  - i-policy cannot control level and dynamics of q_t^B independently

Outline

Benchmark: Constrained Efficient Allocation (in Model with Investment)

Proposition (Representation of Welfare Objective)

For any social welfare function, maximizing welfare is equivalent to maximizing

$$\mathbb{E}\left[\int_0^\infty e^{-\rho t}\left(\mathcal{W}_\vartheta(\vartheta_t, \tilde{\sigma}_t) + \mathcal{W}_u(u_t)\right) dt\right]$$

where $\mathcal{W}_\vartheta(\cdot, \tilde{\sigma})$ *and* $\mathcal{W}_u(\cdot)$ *are strictly quasiconcave and* $\partial_{\tilde{\sigma}\vartheta}\mathcal{W}_\vartheta > 0$.

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Conclusion: separated normative considerations concerning

1. composition of wealth into safe assets and capital assets ($\mathcal{W}_\vartheta$-term)
   - *constrained optimum*: $\vartheta_t = \vartheta^*(\tilde{\sigma}_t)$, $\vartheta^{*\prime} > 0$ $\quad\rightarrow$ some flight to safety is desirable
2. utilization of capital resources ($\mathcal{W}_u$-term)
   - *constrained optimum*: $u_t = u^*$ constant $\quad\rightarrow$ but demand recessions are undesirable

Optimal Interest Rate Underreaction

Proposition (Optimal Monetary Policy)

Suppose $\tilde{\sigma}_t$ evolves deterministically and let a path $\{\breve{s}_t\}$ for fiscal policy be given.

1. *There is precisely one initial state $q_0^{B,0} = q_0^{B,0*}$ such that interest rate policy can implement $u_t = u^*$ for all $t \geq 0$.*
2. *If $q_0^{B,0} > q_0^{B,0*}$, then the optimal interest rate policy is such that $u_t > u^*$ for all $t \geq 0$.*
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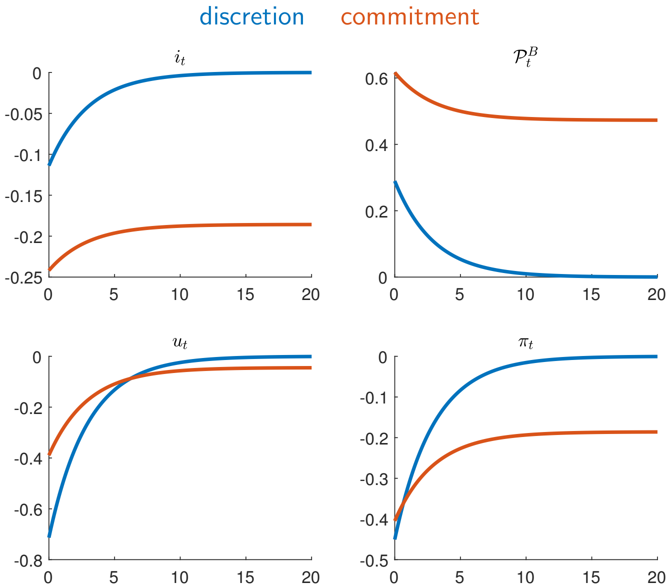
- In sum: monetary policy underreacts relative to full output gap stabilization
- Intuition for underreaction:
  - unless initial state $q_0^{B,0}$ is exactly right, (inefficient) inflation/deflation required at some point
  - concave objective $\rightarrow$ smooth resulting distortions over time

Numerical Example: Permanent Increase of $\tilde{\sigma}$ to Higher Level



Conclusion

- New Keynesian model with (nominal) safe assets
  - uncertainty shocks lead to flight to safety (portfolio reallocation towards bonds)
  - safe asset stock becomes a state variable
  - portfolio choice key to understand aggregate demand
- Shock transmission:
  - rigid safe asset supply and separated portfolio choice generate demand shortage
  - asset pricing: capital price overshooting
  - policy implication: interest rate policy cannot fix sticky price distortion (even absent ZLB)
- Optimal policy
  - [in paper] coordinated monetary-fiscal policy can implement constrained optimal allocation if and only if lump-sum taxes are available
  - without lump-sum taxes: interest rate underreaction on impact and optimal smoothing of demand shortfall

Household Problem

Hh i's problem: choose consumption c^i, utilization u^i, bond portfolio weight θ^i to maximize

$$\mathbb{E}\left[\int_0^\infty e^{-\rho t}\left(\log c_t^i - \frac{(u_t^i)^{1+\varphi}}{1+\varphi}\right) dt\right]$$

subject to

- net worth evolution

$$dn_t^i/n_t^i = -c_t^i/n_t^i dt + \theta_t^i dr_t^{\mathcal{B}} + \left(1-\theta_t^i\right) dr_t^{K,i}(u_t^i)$$

- return processes $dr_t^{K,i}(\cdot)$, $dr_t^{\mathcal{B}}$

$$\begin{aligned}
dr_t^{K,i}(u) &= \frac{p_t^R u + \omega_t - \tau_t}{q_t^K} dt + \frac{d(q_t^K(k_t^i - \Delta_t^{k,i}))}{q_t^K(k_t^i - \Delta_t^{k,i})} &&= \frac{\mathbb{E}_t[dr_t^{K,i}]}{dt} dt + \tilde{\sigma} d\tilde{Z}_t^i + \sigma_t^{q,K} dZ_t \\
dr_t^{\mathcal{B}} &= i_t dt + \frac{d\left(1/\mathcal{P}_t\right)}{1/\mathcal{P}_t} &&= \frac{\mathbb{E}_t[dr_t^{\mathcal{B}}]}{dt} dt + \underbrace{\sigma_t^{q,B} dZ_t}_{=0 \text{ for sticky prices}}
\end{aligned}$$

return

Full Production Side

- Aggregate capital services supplied by households:

$$\int u_t^i k_t^i di = u_t K_t$$

- Intermediate goods firms (continuum $j \in [0,1]$):
  - rent capital services at unit price p_t^R in competetive market
  - transform capital services into differentiated goods 1 for 1
  - nominal output price $\mathcal{P}_t^i$ subject to quadratic adjustment cost (rebated to households)
  - profits redistributed to households according to capital holding shares k_t^i / K_t
- Final goods firms: CES technology

$$Y_t = \left(\int (y_t^j)^{\frac{\epsilon}{\epsilon-1}} dj \right)^{\frac{\epsilon}{\epsilon-1}}$$

- Goods market clearing

$$C_t := \int c_t^i di = Y_t$$

return

Equilibrium Price Level Dynamics under Sticky Prices

- The nominal price level $\mathcal{P}_t$ is *locally deterministic* state variable (backward-looking)

$$d\mathcal{P}_t = \pi_t \mathcal{P}_t dt$$

- Instantaneous price inflation follows the Phillips curve (forward-looking)

$$\mathbb{E}_t[d\pi_t] = \left(\rho\pi_t - \kappa\left(p_t^R - p^{R,flex}\right)\right) dt$$

- In particular: π_t can react to shocks on impact but $\mathcal{P}_t$ cannot

Aside: under flexible prices, $\mathcal{P}_t$ can react to shocks on impact and $\pi_t = \frac{d\mathcal{P}_t}{\mathcal{P}_t dt}$ may not be well-defined.

return

Remark II: Relationship to HANK Models

1. Technically, ours is a HANK model
    - but we abstract from MPC heterogeneity
      $\rightarrow$ wealth distribution does not matter for aggregates
    - deliberate choice to isolate aggregate effects from safe asset demand
    - HA component merely used to generate safe asset demand
2. HANK papers have many extra (distributional) state variables, often focus on those
    - e.g. Bayer et al. (2019): model with "flight to liquidity" after uncertainty shock
    - but discussion focused on how wealth distribution is affected (and no analytical results)
    - our point: there is something else going on that is not about redistribution

return

Closed-Form Solutions: Simplifying Assumptions

Make the following *simplifying assumptions*:

1. Assume $i_t = i$, $\breve{s}_t = \breve{s}$, $\vartheta_t = \vartheta$ are constant after the shock ($\Rightarrow \mu^{\mathcal{B}} = i - \breve{s}$ is constant)

2. Simplify the dynamic equations:

   - Case (a): replace dynamic Phillips curve with static Phillips curve

$$\pi_t = \frac{\kappa}{\rho}\left(\left(\rho\frac{q_t^B}{\vartheta_t}\right)^{1+\varphi} - 1\right)$$

   - Case (b): linearize the two equations

Closed-Form Solutions for Debt Dynamics

a. Static Phillips curve:

$$q_t^B = \left(\left(q_0^B\right)^{-(1+\varphi)} e^{-\alpha t} + \left(q_\infty^B\right)^{-(1+\varphi)} \left(1 - e^{-\alpha t}\right) \right)^{-\frac{1}{1+\varphi}},$$

where $\alpha := \frac{1+\varphi}{\rho}(\rho\mu^{\mathcal{B}} + \kappa)$, $\quad q_\infty^B := \left(1 + \frac{\rho\mu^{\mathcal{B}}}{\kappa}\right)^{\frac{1}{1+\varphi}} \frac{\vartheta}{\rho}$

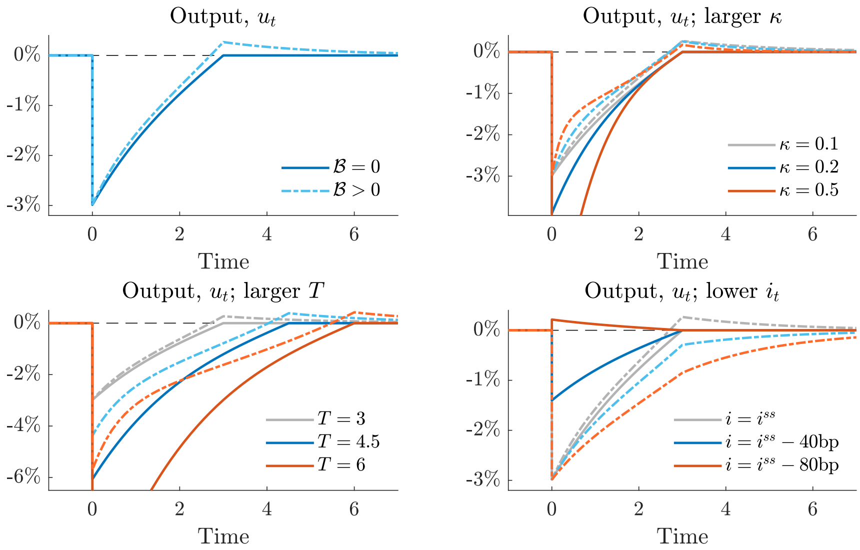
b. Linearized dynamics:

$$q_t^B = q_0^B e^{-\alpha t} + q_\infty^B \left(1 - e^{-\alpha t}\right)$$

where $\alpha := \sqrt{\left(\frac{\rho}{2}\right)^2 + (1+\varphi)\left(\rho\mu^{\mathcal{B}} + \kappa\right)} - \frac{\rho}{2}$, $\quad q_\infty^B := \left(1 + \frac{\rho\mu^{\mathcal{B}}}{\kappa}\right)^{\frac{1}{1+\varphi}} \frac{\vartheta}{\rho}$

return

Illustration: $\mathcal{B}_t > 0$ versus $\mathcal{B}_t \equiv 0$



return