Characteristics of a Sufficient Statistic to Measure City Housing Prices

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Presentation Plan

1. Problems with conventional index construction
2. Sufficient statistic requirements of a Laspeyres index considering units and prices vary with distance
3. Propositions from urban spatial models
4. Demonstration aggregating median indexes
5. Demonstration aggregating weighted repeat sales indexes
6. Conclusions

Preview: Appreciation should be measured as a function of distance from city center and weighted by fraction of housing at each distance interval

Problems

Sample Selection

- Tenure status not spatially random
- Rent growth and price appreciation rates vary spatially
- Need to apply geographic × compositional weights
- Conventional measures weight when aggregating to national level, not metro

Problems

Index Base

1. Housing units vs. housing services
   - Unit size not spatially random
   - Unit-level prices may diverge from price PSF or per room
2. Rents vs. prices
   - Rents not observed for OO housing
   - Asset prices not generally available for NOO

Laspeyres Index

Levels

$$r_t^* = \sum_k \frac{r_t(k)h_t(k)}{H_t(k)}$$

$$H_t(k) = \sum_k h_t(\mathrm{k})$$

Changes

$$\frac{\Delta r_t^*}{r_{t-1}} = \frac{\sum_k \frac{r_t(k)h_{t-1}(k)}{H_{t-1}(k)}}{\sum_k \frac{r_{t-1}(k)h_{t-1}(k)}{H_{t-1}(k)}}$$

Sufficient Statistic Requirements:

Need distributions of housing prices (r) and quantities (h) over distances (k)

DEMANDING

Laspeyres Index

Results Preview:
Essentially Never

When can we ignore space?

- If relative change in housing price is constant over distance from city center

$$r_t(k) \Big/ r_{t-1}(k) = \theta$$

- Spatial distribution can vary, not consequential
- Two cases
  - Small cities with flat gradients
  - Specific (unrealistic) assumption about household preferences

What about Urban Models?

Gradients

- Land rent
- Structure rent
- Population density
- Structural density (FAR)

Proposition 1

- Constant price change implies quasi-linear utility
- But no evidence IED = 0 for housing

Proposition 2

- Appreciation must be measured as a function of distance from city center and weighted by fraction of housing at each distance interval

Aggregating Median Value Indexes (AHS-MS Data)

Sampling Strata

1. HUD-assisted
2. Trailer or mobile home
3. Owner-occupied SFD
4. Owner-occupied 2+ units
5. Renter-occupied SFD
6. Renter-occupied 2+ units
7. Vacant SFD
8. Vacant 2+
9. Other units

Control Counts

1. Population
2. Black population
3. Population +65
4. Hispanic population
5. Units in HUD programs
6. Occupied units
7. Vacant units

Problem

- Constant sampling rate, but shares vary by location
- Census does not stratify below the metro level
- Not spatially random

Table 1: Compound annual growth rates in median values (2015–2019)

	Metrowide		Central City		Suburbs		Difference	
	Value [1]	Rent [2]	Value [3]	Rent [4]	Value [5]	Rent [6]	Value [7]	Rent [8]
Atlanta	13.6	5.7	24.5	7.8	11.3	5.7	13.3	2.0
Boston	8.2	6.2	11.7	4.5	8.2	8.8	3.5	-4.3
Chicago	5.7	3.8	9.7	4.1	4.7	3.4	5.1	0.8
Cincinnati	4.7	5.7	7.0	7.5	5.4	8.3	1.6	-0.9
Cleveland	3.2	2.0	0.0	1.8	4.1	6.9	-4.1	-5.1
Dallas	11.0	6.1	10.1	6.1	10.1	6.7	0.0	-0.6
Denver	9.2	7.5	12.5	4.8	10.4	6.2	2.1	-1.4
Detroit	7.5	2.7	10.7	3.5	7.0	2.7	3.6	0.7
Houston	9.3	3.9	9.5	4.3	8.1	5.1	1.4	-0.8
Kansas City	6.2	3.4	5.6	3.4	5.2	2.7	0.3	0.7
Los Angeles	5.5	6.7	6.2	7.4	6.8	6.2	-0.6	1.2
Memphis	9.2	3.9	5.1	3.9	6.8	3.6	-1.7	0.3
Miami	6.9	4.5	8.8	5.6	7.5	4.3	1.3	1.3
Milwaukee	5.0	2.7	5.1	1.9	6.1	5.4	-1.0	-3.6
New Orleans	4.1	1.6	5.6	1.6	5.7	3.2	-0.1	-1.6
New York City	4.3	4.1	4.7	3.8	4.0	4.5	0.6	-0.7
Philadelphia	5.3	4.1	7.5	5.6	4.7	4.2	2.9	1.4
Phoenix	11.1	7.0	11.5	7.5	10.7	4.6	0.8	3.0
Pittsburgh	5.3	2.0	13.6	7.7	5.1	2.1	8.5	5.6
Portland	9.4	7.9	9.8	10.1	9.3	7.5	0.5	2.6
Raleigh	4.7	4.9	1.5	9.3	8.0	6.1	-6.5	3.2
Riverside	6.8	6.8	7.5	8.7	6.6	5.7	0.8	3.0
San Francisco	8.7	6.6	9.3	7.0	9.5	8.5	-0.2	-1.5
Seattle	10.7	7.2	13.6	6.8	10.0	7.4	3.7	-0.6
Washington DC	1.4	2.7	1.0	1.6	2.7	2.7	-1.6	-1.1
Average	7.1	4.8	8.5	5.5	7.1	5.3	1.4	0.1
Deviation	2.8	1.9	4.9	2.4	2.3	1.9	3.8	2.4

- Appreciation not spatially invariant
- Central city > suburbs (during this period)
- Stratified metro sample unlikely to approach randomized ideal

Table 2: Compound annual growth rates in median values (2015–20

Metrowide Value [1]	Metrowide Rent [2]		Simple Avg Value [1]	Simple Avg Rent [2]	Laspeyres Value [3]	Laspeyres Rent [4]	Difference Value [5]	Difference Rent [6]
13.6	5.7	Atlanta	17.91	6.75	12.96	5.99	-4.95	-0.75
8.2	6.2	Boston	9.92	6.64	9.03	7.72	-0.89	1.09
5.7	3.8	Chicago	7.20	3.77	6.65	3.69	-0.55	-0.08
4.7	5.7	Cincinnati	6.20	7.90	5.69	8.18	-0.51	0.28
3.2	2.0	Cleveland	2.03	4.32	3.05	5.60	1.02	1.28
11.0	6.1	Dallas	10.13	6.37	10.13	6.36	0.00	-0.01
9.2	7.5	Denver	11.41	5.53	11.23	5.65	-0.18	0.12
7.5	2.7	Detroit	8.85	3.10	8.25	2.98	-0.60	-0.12
9.3	3.9	Houston	8.83	4.74	8.71	4.82	-0.13	0.07
6.2	3.4	Kansas City	5.40	3.07	5.37	3.01	-0.03	-0.06
5.5	6.7	Los Angeles	6.50	6.77	6.46	6.85	-0.04	0.08
9.2	3.9	Memphis	5.99	3.79	5.97	3.79	-0.02	0.00
6.9	4.5	Miami	8.12	4.93	7.78	4.59	-0.34	-0.34
5.0	2.7	Milwaukee	5.58	3.65	5.62	3.80	0.04	0.15
4.1	1.6	New Orleans	5.67	2.40	5.67	2.41	0.00	0.00
4.3	4.1	New York City	4.35	4.13	4.32	4.16	-0.03	0.03
5.3	4.1	Philadelphia	6.09	4.86	5.45	4.54	-0.64	-0.31
11.1	7.0	Phoenix	11.11	6.05	11.19	6.37	0.08	0.32
5.3	2.0	Pittsburgh	9.38	4.92	6.29	2.87	-3.09	-2.05
9.4	7.9	Portland	9.57	8.81	9.54	8.64	-0.03	-0.17
4.7	4.9	Raleigh	4.73	7.71	4.88	7.64	0.15	-0.07
6.8	6.8	Riverside	7.05	7.24	6.87	6.61	-0.17	-0.63
8.7	6.6	San Francisco	9.44	7.78	9.43	7.72	-0.01	-0.06
10.7	7.2	Seattle	11.79	7.10	11.46	7.16	-0.33	0.06
1.4	2.7	Washington DC	1.85	2.14	2.24	2.40	0.40	0.25
7.1	4.8	Average	7.80	5.38	7.37	5.34	-0.43	-0.04
2.8	1.9	Deviation	3.30	1.80	2.70	1.90	1.15	0.58

Simple Average
>
Laspeyres
>
Metrowide

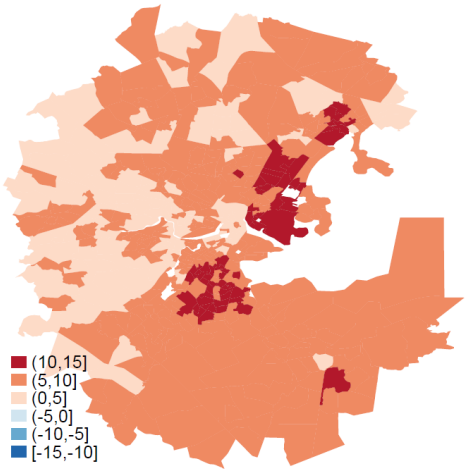
Aggregating WRS Indexes

Census tract appreciation rates from Contat & Larson (2024) using four weighting strategies

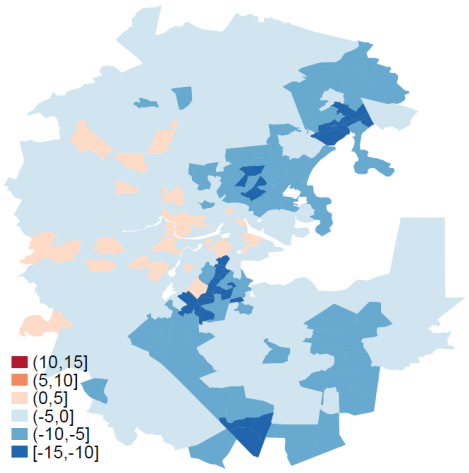
1. Equal
2. By unit share
3. By size (rooms) share
4. By value share

Examine Boston vs Houston for today (more in final paper)

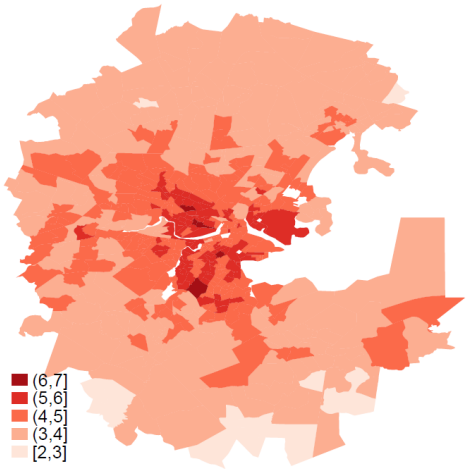
Figure 1: Within-city appreciation differences (annual average)



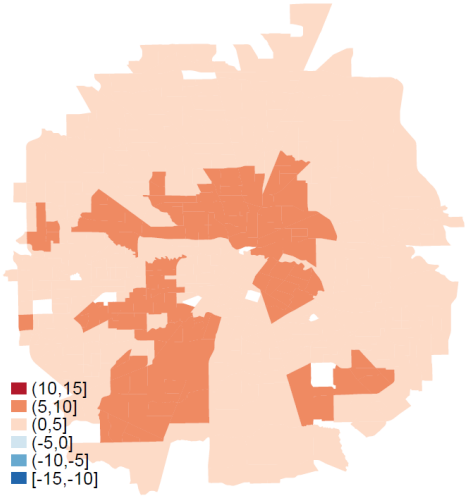
(a) Boston, MA, 2001–2006



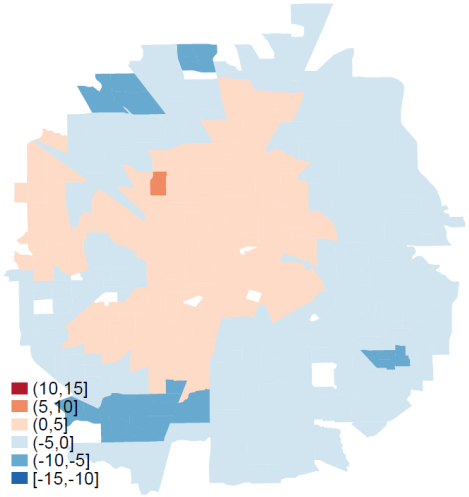
(b) Boston, MA, 2006–2011



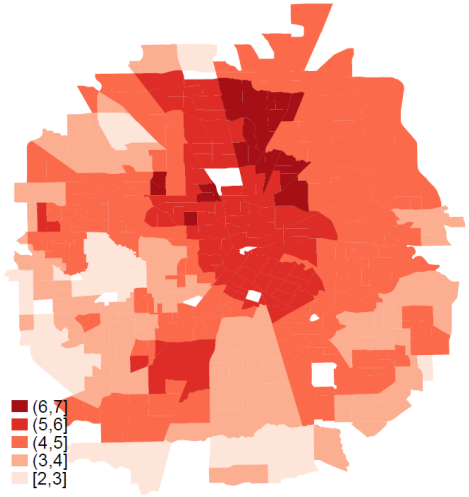
(c) Boston, MA, 1990–2020



(d) Houston, TX, 2001–2006



(e) Houston, TX, 2006–2011



(f) Houston, TX, 1990–2020

Within-City Appreciation

Substantial within-city variation in appreciation rates across census tracts

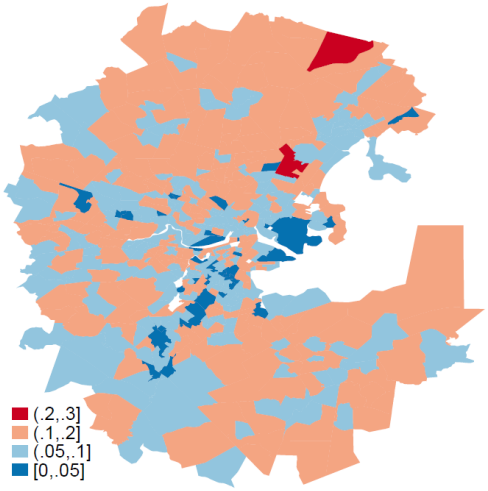
- Appears to decline with distance in both

Temporal dynamics differ

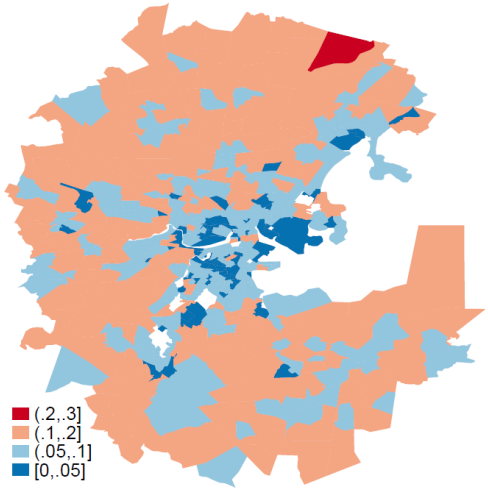
- Boston: Mean reversion
  - Consistent with relatively inelastic supply
- Houston: Persistence
  - Consistent with relatively elastic housing supply

Figure 2: Shares used in city index calculation (2010 values)

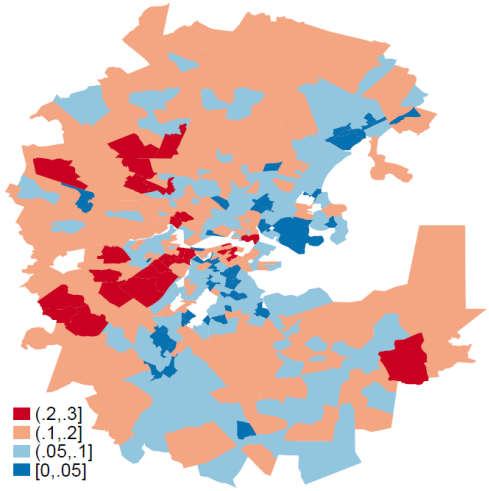
(a) Boston, MA, Housing units



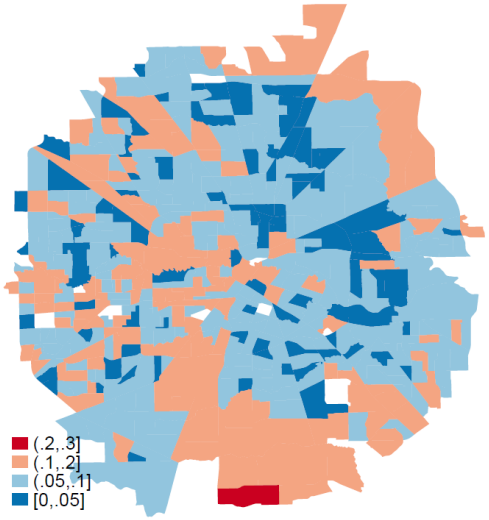
(b) Boston, MA, Rooms



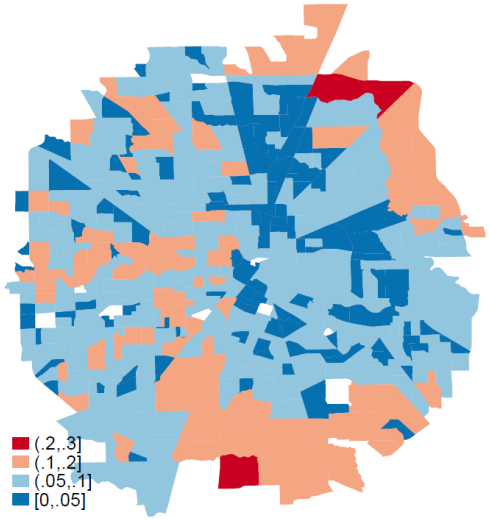
(c) Boston, MA, Value



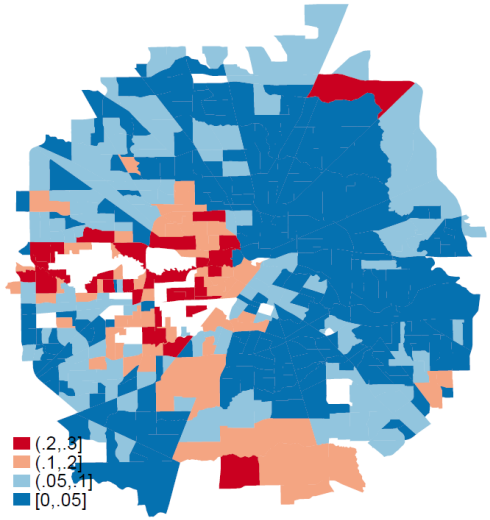
(d) Houston, TX, Housing units



(e) Houston, TX, Rooms



(f) Houston, TX, Value



Share Weights

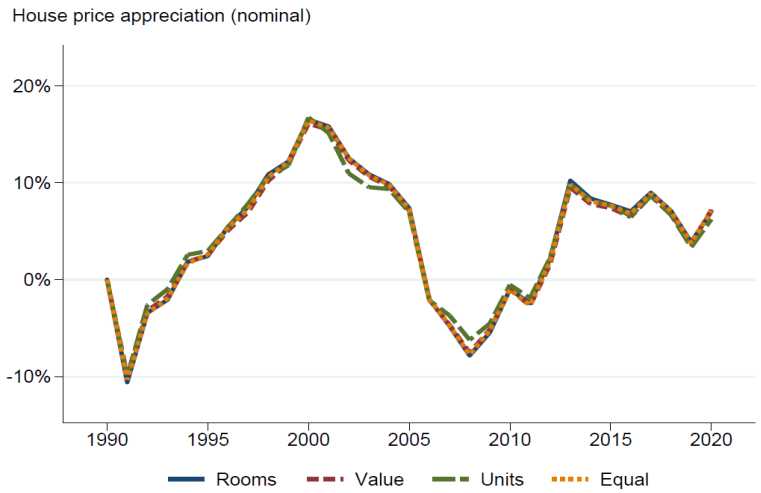
Unit and room share distributions similar

Value share patterns differ

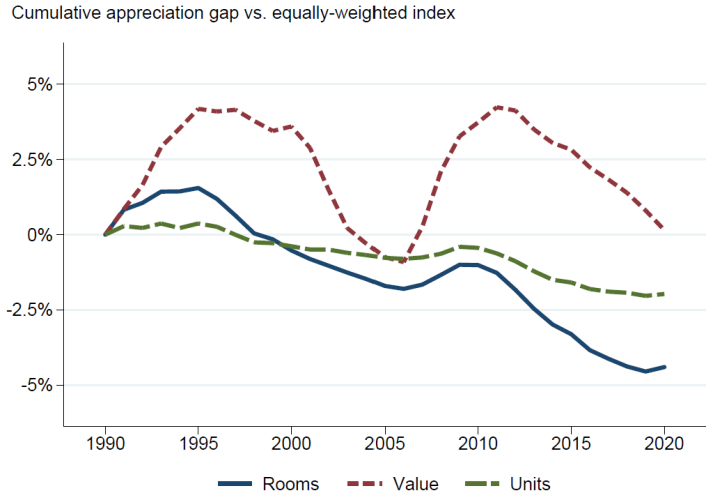
- Indexes based on value weighting may diverge from those based on physical characteristics

Figure 3: Appreciation index differences

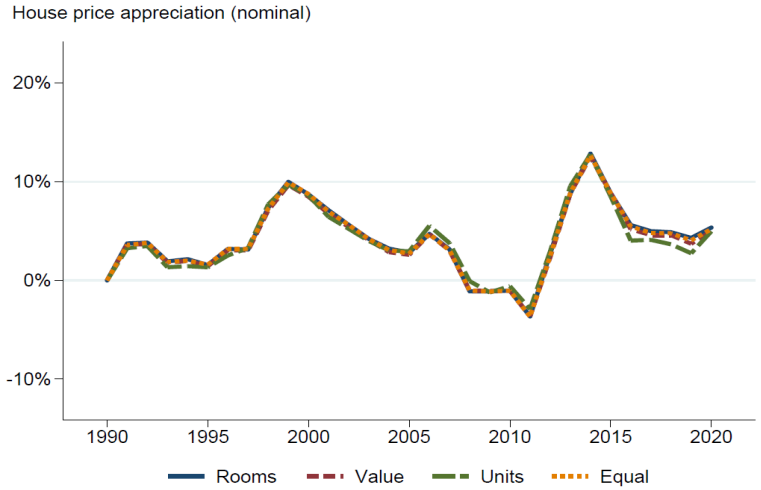
(a) Boston, MA, Annual appreciation



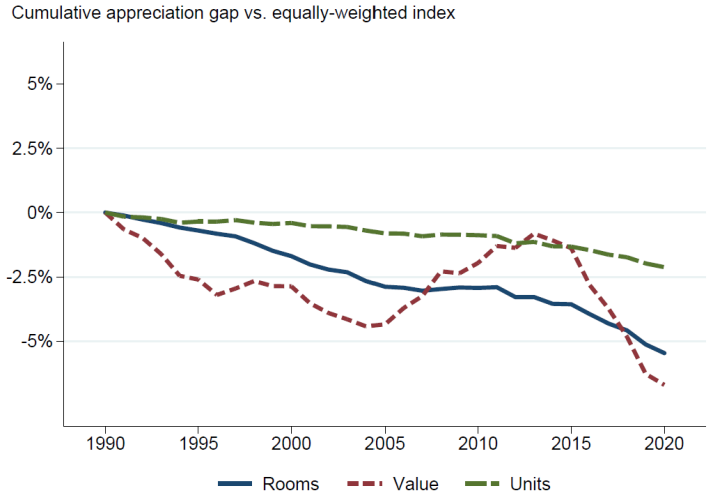
(b) Boston, MA, Cumulative difference



(c) Houston, TX, Annual appreciation



(d) Houston, TX, Cumulative difference



Appreciation Rate Differences

Using equal weighting as reference

Small differences compound over time (not noise)

Size-weighted indexes show lower cumulative appreciation.

- Larger housing units (typically newer) appreciate at a slower rate

Value-weighted indexes more volatile over time and diverge more sharply across cities

Conclusions

Appreciation and rent growth vary systematically with distance

Weighting by housing units produces different results than considering housing services

Observations should be weighted by fraction of housing services at each distance from the city center for an index to sufficiently characterize the average rate of change for that city

- City-level index not currently constructed currently in this fashion