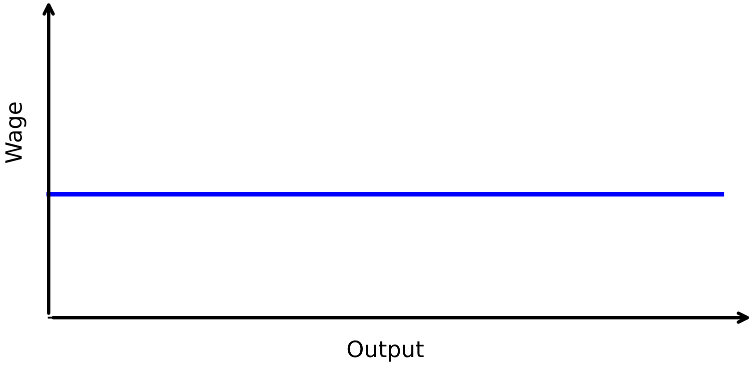
Incentives and Selection

Henrique Castro-Pires
University of Miami

George Georgiadis
Northwestern University

Lazear (2000): Performance Pay and Productivity



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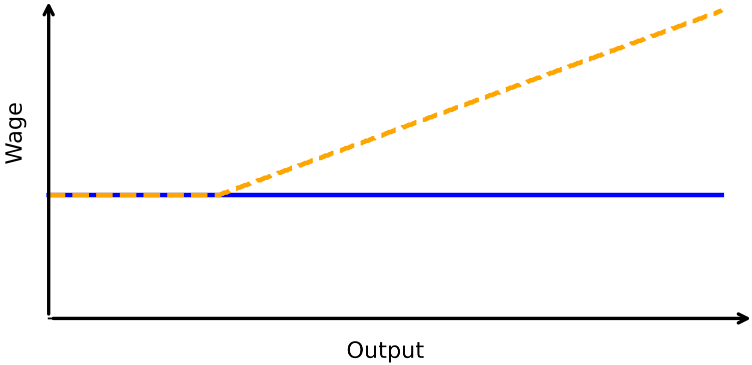

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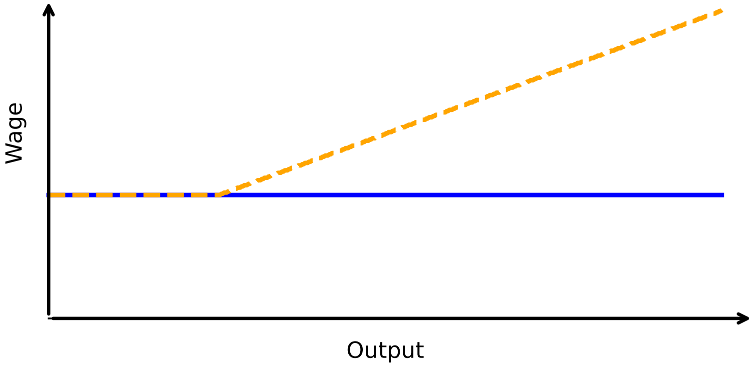

Productivity ↑ 44%

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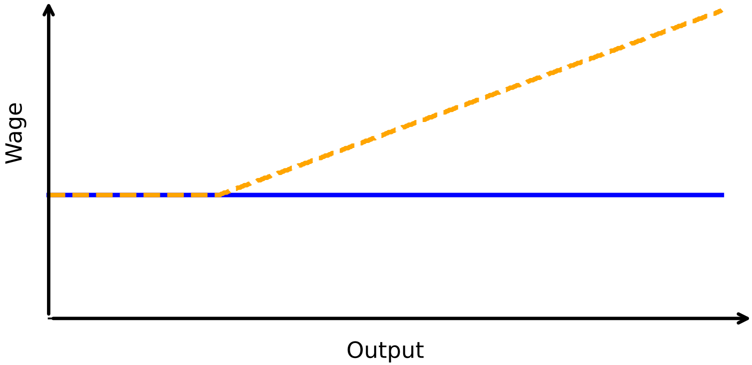

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1/2 due to higher effort

1/2 due to better selection

This paper in a nutshell

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   - Characterize model primitives under which selection improves or worsens
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Prescriptive:

3. How to optimize contract accounting for selection?
   - Find the best direction of improvement
   - *Trade-off:* insurance, incentives, and shifting payments to improve selection

Model: Players & Timing

- There is a principal and a unit mass of agents
- The principal wants to hire a fixed number of agents and motivate them to exert effort
- Each agent has type $t \in \{l, h\}$, decides whether to apply and, if hired, chooses effort

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Timing:

1- The principal posts a wage scheme $w(x)$ *(not a menu)*

2- Each agent draws his outside option and decides whether to apply for job

3- The principal screens applicants and hires at random among those who pass the test

4- Each hired agent choose effort a

5- Each worker's output $x \sim f(\cdot|a)$ and payoffs are realized

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- Each type-t agent draws outside option $\overline{u} \sim G_t(\cdot)$ and applies iff $u_t(w) \geq \overline{u}$
  - *Assumption:* High types have better outside options; i.e., $G_h(\cdot) \succeq^{fosd} G_l(\cdot)$

Model: The Principal

- The ex-ante share of high types in the population is p

Screening test:

- Each type-t agent passes the test with probability $1 - r_t$. (*Assume:* $r_h < r_l$)
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Principal's payoff (per worker):

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Main objects of interest: How does $q(w)$ and $\pi(w)$ change with w?

Building Blocks

Given contract w**:**

- Each type-t agent applies with probability $G_t := G_t(u_t(w))$
- The probability that each agent is a high type is

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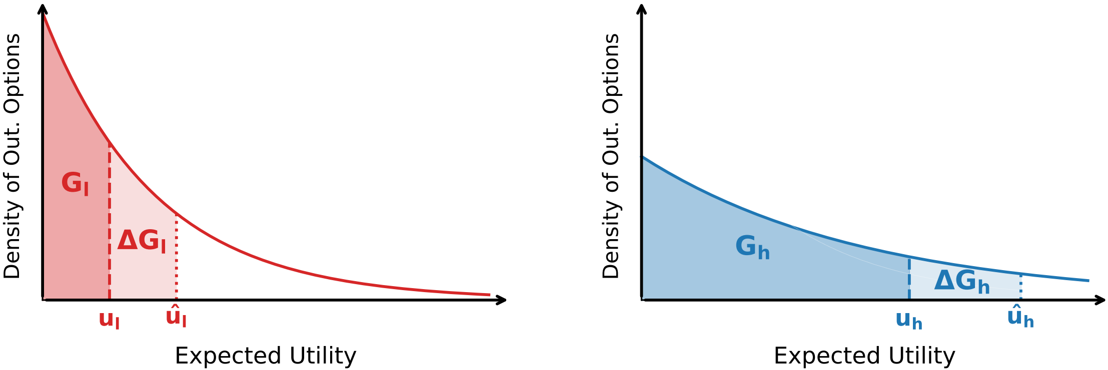
Definition

A change in w improves selection if it causes $q(w)$ to rise (and vice versa)

Remark 1

A change in w improves selection if and only if it causes G_h/G_l to rise.

An Illustrative Example



Given status quo contract w, each low and high type applies w.p G_l and G_h, respectively

If we replace w with $\hat{w}$, both types' payoffs and their probabilities of applying will change

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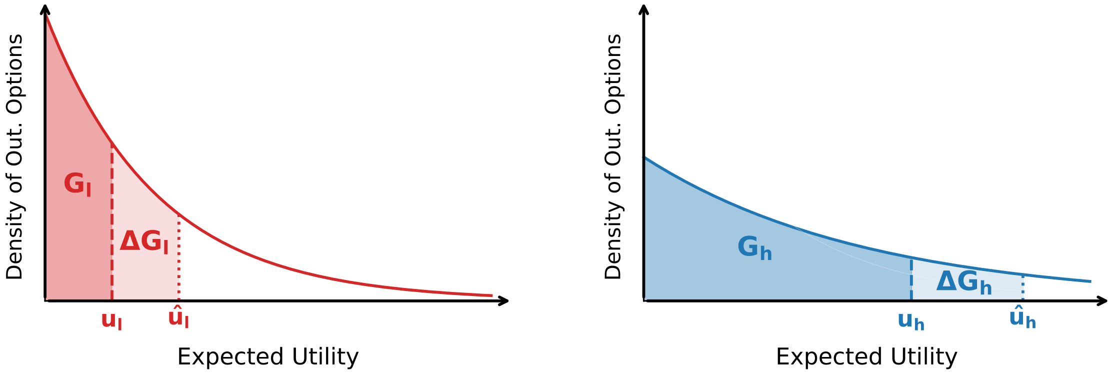

Selection improves if and only if

$$\frac{\Delta G_h}{G_h} > \frac{\Delta G_l}{G_l} \quad \textit{i.e., iff} \quad \%\Delta\,\Pr\{\text{high type applies}\} > \%\Delta\,\Pr\{\text{low type applies}\}$$

Local modifications to w

- We evaluate the effects of small contract changes in arbitrary directions.

  i.e., we replace $w(x)$ by $w(x) + \varepsilon\ell(x)$ for some small ε.

- Use notion of a directional derivative. Define the Gateaux differential in direction ℓ:

$$\mathcal{D}h(w,\ell) := \lim_{\varepsilon\downarrow 0} \frac{h(w+\varepsilon\ell) - h(w)}{\varepsilon}$$

Definition

Modifying w in direction ℓ improves selection if $\mathcal{D}q(w,\ell) > 0$ (and harms selection if < 0).

Key Lemma

Lemma 1

Modifying the contract w in direction ℓ improves selection if and only if

$$\mathcal{D}q(w,\ell) =^s \rho_h(u_h) \times \mathcal{D}u_h(w,\ell) - \rho_l(u_l) \times \mathcal{D}u_l(w,\ell) > 0,$$

The key determinants of selection are:

1. The reverse hazard rates $\rho_h := g_h/G_h$ and $\rho_l := g_l/G_l$

   $\rho_t \simeq$ %increase in type-t applicants if they get an extra util

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Theorem 1

If $\mathcal{D}u_h(w,\ell) \geq 0 \geq \mathcal{D}u_l(w,\ell)$, then selection **improves** *(and vice versa)*

"Steepening" incentives may harm selection

Def: Modifying w in direction ℓ "steepens" incentives if both types obtain stronger incentives

Theorem 2

- *Consider a steepening of w in direction* ℓ *such that* $\mathcal{D}u_h \times \mathcal{D}u_l > 0$.
- *There exist* G_h *and* $G_l \prec_{fosd} G_h$ *such that this modification harms selection.*

Takeaway: For any marginal modification for which the selection effect is nontrivial, there exist outside option distributions such that this modification *harms* selection.

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Proof idea:

- Selection effect depends on the *utility gains* and the *reverse hazard rates*
- Construct G_h and G_l such that their reverse hazard rates decrease fast enough

A sufficient statistic for improved selection

Denote by $A(w)$ the *fraction* of applicants who pass the screening test

Theorem 4

Modifying the contract w in direction ℓ improves selection if and only if $\mathcal{D}A(w, \ell) > 0$.

Key: High types pass the screening test more often than low types

If pass rate $\uparrow$, modified contract must have attracted proportionally more high than low types

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- Valuable information before principal trains the workers and put them to work (costly!)

How (local) changes to w impact selection

Assumption 1: The principal knows:

- Mass of total and rejected applicants, and screening technology (r_h, r_l)
- Agents' marginal utility $v'(\cdot)$
- Output distributions $f(\cdot|a_l(w))$ and $f(\cdot|a_h(w))$; *i.e.*, can identify high types in workforce

Experiment 1: Principal observes output data from a local modification of w in direction $\hat{\ell}$

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Selection depends on:

- Utility gains $\leftarrow$ Assumption 1
- Reverse hazard rate $\leftarrow$ Assumption 1 + Experiment 1

Optimal local modifications

Principal's profit:

$$\pi(w) = \int \left[x - w(x)\right] \left[q(w) f\left(x|a_h\right) + (1 - q(w)) f\left(x|a_l\right)\right] dx$$

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We are interested in solving the following problem:

$$\max_{\ell :\ \|\ell\| \leq 1} \mathcal{D}\pi(w, \ell) \tag{PP}$$

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Modifying w in direction ℓ has 3 effects:

1- **Direct Effect:** Direct cost of changing payments

2- **Selection Effect:** Effect of changes in selection

3- **Incentive Effect:** Effect of changes in efforts

Prescriptive problem

The principal solves

$$\max_{\ell} \text{(direct effect)} + \text{(selection effect)} + \text{(incentive effect)}$$

Information needed to compute each effect:

- *Direct effect:* Observational data under contract w
- *Selection effect:* Per Theorem 4, need Assumption 1 + Experiment 1
- *Incentive effect:* Need one more assumption + one more experiment

Prescription: Replace w with $w(x) + \epsilon\ell^*(x)$ for some small $\varepsilon > 0$

Computing incentives effect: Condition + Experiment

Experiment 2: Post-hiring, offer an unannounced increase in wages in direction ℓ'

- Observe how the output distribution responds holding $q(w)$ constant
- Allows us to identify the incentive effect; *i.e.*, effort response holding selection fixed

Assumption 2: $f(x|a)$ is affine in a

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Theorem 6

Assumptions 1+2 and Experiments 1+2 suffice to solve (PP)

Trade-off: Insurance *vs.* incentives *vs.* shifting payments to outputs that improve selection

- Explicit characterization is in the paper

Limitations

Non-local modifications

- To extrapolate, additional assumptions on $c_t(\cdot)$ and $G_t(\cdot)$ are needed

Endogenous screening

- How to jointly optimize the wage scheme and the screening technology?

Miscellaneous

- Binary types
- Binary screening technology
- Positions are scarce, workers are abundant; *i.e.*, filling positions is never a problem
- Outside option distributions are assumed to be exogenous