# WHAT DETERMINED THE EVOLUTION OF GLOBALIZATION 1995-2017?

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## 1. INTRODUCTION

- The period 1995-2017 is widely recognized as a period of "hyperglobalization" in the sense of deepening trade relationship among countries.
- What determined the evolution of globalization during this period?
- We measure the change in the degree of globalization of a country by the change in gains from trade (GFT).
- In this paper, we aim to
  - quantitatively account for the contributions of global changes in factors such as trade costs and technology stocks to the change in GFT of each country.
  - calculate a matrix of the contribution of changes in each country to the GFT of foreign countries through trade.

- estimate the contribution of changes in China to the rest of the world through trade during the sample period.
- find out whether changes in developing countries or developed countries were more important drivers of the evolution of globalization.

## Our approach

- We construct a GE multi-sector Eaton-Kortum Ricardian model with inputoutput linkages.
- We distinguish between production functions and trade flows of intermediate goods and those of final goods.
- We run gravity regressions, making use of the data of trade flows in intermediate and final goods/services.
  - This allows us to estimate the trade elasticities as well as changes in technology, trade costs and GFT of each country in intermediate and final goods/services.
- After calibrating the model, we carry out two sets of counterfactual exercises.

- In the first set of exercises, we estimate the impact of *global changes* in each exogenous variable on the domestic GFT of each country.
- In the second set of exercises, we estimate the impact of the change of each exogenous variable *in each country* on *its own domestic GFT*.
- Thus, we are able to compute the contribution of changes in the rest of the world to the change in domestic GFT of each country.

### Our contribution

- We provide a detailed understanding of the contributions of global and domestic changes in various exogenous factors (not just trade cost reduction) to the degree of globalization of each country.
- Our calibration deviates from the standard RP model by distinguishing between production functions and trade flows of intermediate goods and those of final goods, thus providing better estimates.
- Methodologically,
  - We overcome the difficulty of estimating the contribution of each factor to the GFT of a country due to the existence of interactive effects among the various factors.
  - It turns out that the average of the pure effect and total effect of each factor

is a good proxy for the contribution of that factor.

• We compute a matrix of the contribution of the changes in the technology stocks and trade costs of each country to the GFT of foreign countries. Thus, we identify "friends" and "enemies" in trade relationship among countries from a unique angle, as distinct from, for example, that of Kleinman, Liu and Redding (2024).

### PREVIEW OF RESULTS

- There is convergence in the technology stocks and trade costs across countries for intermediate goods/services and final goods/services.
- Global changes in trade costs contribute most to the changes in GFT, but global changes in technology stocks also play an important role. Yet the previous literature tends to overlook the latter aspect.
  - global changes in trade costs and technology stocks explain about 97% of the variance of changes in GFT of nations.
  - global reduction in trade costs contribute positively to domestic change in GFT (117.6%)
  - global increases in technology stocks contribute negatively to domestic change in GFT (-21.1%)

- The above decomposition result is driven mainly by developing countries, reflecting the faster rates of changes of technology stocks and trade costs in the developing countries during the sample period 1995-2017 (due to convergence)
- A model that does not distinguish between the production functions and trade flows of intermediate and those of final goods underestimates the change in GFT by 53% on average.
- Changes in technology stocks and trade costs of foreign countries contribute to 65% to the change in domestic GFT on average.
  - Foreign technological changes (i.e. foreign export-biased growth) contribute on average 48% to the change in domestic GFT, while that of foreign changes in trade costs is 17%.

Table 1: Effects of Domestic, Global and Foreign Changes in Each factor on Change in Domestic GFT

Effects on Domestic GFT	$T_{int}$	$T_{fnl}$	$ au_{int}$	$ au_{fnl}$	NX	Total Effect
Row A: Domestic Changes	-4.393%	0.159%	3.866%	2.744%	-0.251%	2.126%
Row B: Foreign Changes	2.678%	0.223%	0.764%	0.145%	0.130%	3.939%
Row C: Global Changes	-1.715%	0.382%	4.630%	2.889%	-0.121%	6.065%

Note: Row C is equal to Row A plus Row B.

• China's contribution to the change in welfare of foreign countries through trade was by far larger than that of any other country (31% of world total).



Figure 1: Contribution of change in technology stock of each country to the gains from trade of foreign countries.



Contribution of change of domestic trade cost to foreign countries

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Figure 2: Contribution of change in importing trade cost of each country to the gains from trade of foreign countries.



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Figure 3: The average contribution (in terms of percentage point increase in GFT) to foreign countries of each country's change in technology stock (left panel), importing trade costs (middle panel) and the total change (right panel).

- The countries contributing most to foreign countries through trade were all developing countries (e.g. the top five are China, Russsia, India, Poland and South Korea). Thus, developing countries accounted for the lion's share of the evolution of globalization during 1995-2017.
- Domestic changes in technology and trade costs pertaining to intermediate goods trade are more important to home GFT than those pertaining to final goods trade. (Elasticities of GFT w.r.t. intermediate goods technology and trade cost are 2 to 3 times those of final goods.)

## RELATED LITERATURE:

- Technological convergence and implications on trade: Levchenko and Zhang (2016)
- GVCs, global sourcing, vertical specialization, multi-stage production: Johnson and Noguera (2012), Koopman, Wang and Wei (2014), Antras and de Gortari (2020), Antras and Chor (2013, 2021).
- The Quantitative trade model and GFT: Eaton and Kortum (2002) (EK) no distinction between intermediate and final goods, one sector, with round-able production (RP), no GE. Arkolakis, Costinot and Rodriguez-Clare (2012) (ACR), Costinot and Rodriguez-Clare (2014).
- Deviation from the restrictions of ACR Melitz and Redding (2015), Ossa (2015).

- Welfare gains determined by technological change and change in GFT: Donaldson (2018).
- Vertical specialization vs. standard RP: Alexander (2021).
- GE, multi-sector EK, I-O linkages, with standard RP: Caliendo and Parro (2015).
- Our paper: GE multi-sector EK, I-O linkages, modified RP (we distinguish between trade flows and production functions of intermediate goods from those of final goods).

### 2. THE MODEL

#### The General Setting

- $\bullet$  We extend Eaton-Kortum's (2002) model.
- N countries, K sectors, specializing into K = 12.
- 12 sectors: i = 1 is primary goods, i = 2, ..., 11 are manufactured goods, i = 12 is services sector.
- Markets for goods and services are perfectly competitive.
- Goods and services are tradable.
- There are input-output linkages.
- General equilibrium model.

• We distinguish between production functions and trade flows of intermediate goods & services and those of final goods & services.

#### Preferences.

$$U_{n} = \prod_{i=1}^{K} \left\{ \left[ \int_{0}^{1} \left( \widetilde{Q_{n}^{i}} \left( \omega \right) \right)^{\frac{\widetilde{\sigma}^{i}-1}{\widetilde{\sigma}^{i}}} d\omega \right]^{\frac{\widetilde{\sigma}^{i}}{\widetilde{\sigma}^{i}-1}} \right\}^{\alpha^{i}} \quad \text{for } n = 1, 2, ..., N$$
(1)

Technology and Market Structure.

• The production function of an input bundle in country n is given by:

$$y_{n}^{i} = \left(M_{n}^{i}\right)^{1-\beta_{n}^{i}} \left(l_{n}^{i}\right)^{\beta_{n}^{i}}, \text{ with } 0 < \beta_{n}^{i} < 1,$$
 (2)

#### (A) Intermediate Goods

• The production function of the composite intermediate good:

$$M_n^i = \prod_{h=1}^K \left\{ \left[ \int_0^1 \left( Q_n^{ih}(\omega) \right)^{\frac{\sigma^{h-1}}{\sigma^h}} d\omega \right]^{\frac{\sigma^h}{\sigma^{h-1}}} \right\}^{\gamma_n^{ih}}, \tag{3}$$

• The market share of sector-i intermediate goods in n imported from l:

$$\pi_{nl}^{i} = \frac{T_{l}^{i} \left(c_{l}^{i} \tau_{nl}^{i}\right)^{-\theta}}{\sum_{m=1}^{N} T_{m}^{i} \left(c_{m}^{i} \tau_{nm}^{i}\right)^{-\theta}}$$
(4)

- Define  $D_l^i \equiv T_l^i (c_l^i)^{-\theta}$  as country *l*'s competitiveness in supplying sector-*i* intermediate goods.
- Define  $\Phi_n^j \equiv \sum_{m=1}^N T_m^i \left( c_m^i \tau_{nm}^i \right)^{-\theta}$  as country n's "global access" to sector-*i* intermediate goods.

#### (B) Final Goods

• The market share of sector-i final goods in n imported from l:

$$\widetilde{\pi}_{nl}^{i} = \frac{\widetilde{T}_{l}^{i} \left(c_{l}^{i} \cdot \widetilde{\tau_{nl}^{i}}\right)^{-\widetilde{\theta}}}{\sum_{m=1}^{N} \widetilde{T}_{m}^{i} \left(c_{m}^{i} \cdot \widetilde{\tau_{nm}^{i}}\right)^{-\widetilde{\theta}}}$$
(5)

• Define  $\widetilde{D_l^i} \equiv \widetilde{T_l^i} (c_l^i)^{-\widetilde{\theta}}$  as the competitiveness of country l in supplying sector-i final goods.



Production of Sector - *i* Intermediate and Final Goods in Country *n* 

#### Market Clearing

- For each country *n* and each sector *i*, the supply of goods is equal to the demand for intermediate goods plus the demand for final goods.
- For each country n and each sector i, expenditure on goods is equal to expenditure on final goods, plus expenditure on intermediate goods.
- For each country n, net exports is equal to output minus expenditure.

### 3. EMPIRICAL ESTIMATION

#### (A) Estimating Trade Elasticities

• Equation (4) implies that, for intermediate goods trade, for i = 1, ..., 11 (primary and manufacturing sectors)

$$\ln\left(\frac{X_{nlt}^{i}}{X_{nnt}^{i}}\right) = \ln D_{lt}^{i} - \ln D_{nt}^{i} - \theta_{i} \ln \tau_{nlt}^{i} , \qquad (6)$$

• 
$$\tau^i_{nlt} = d_{nlt} + b_{nlt} + lang_{nlt} + legal_{nlt} + colonial_{nlt} + RTA_{nlt} + im^i_{nt} + Tariff^i_{nlt} + v^i_{nlt}$$

- d, b, lang, legal, colonial, RTA are dummy variables (bilateral controls)
- We can rewrite equation (6) as

$$\ln\left(\frac{X_{nlt}^{i}}{X_{nnt}^{i}}\right) = \underbrace{\ln D_{lt}^{i}}_{\text{Exporter Fixed Effect}} - \underbrace{\left(\ln D_{nt}^{i} + im_{nt}^{i}\right)}_{\text{Importer Fixed Effect}} - \underbrace{\theta_{i} \cdot \operatorname{Tariff}_{nlt}^{i}}_{\text{Tariff Effect}} - \operatorname{bilateral controls}_{- \underbrace{\theta_{i} v_{nlt}^{i}}_{\text{Error Term}}}$$

- $\ln D_{lt}^i$  is the intermediate goods competitiveness of country l
- Equation (5) implies that for final goods trade, for i = 1, ..., 11,

$$\ln\left(\frac{\widetilde{X_{nlt}^i}}{\widetilde{X_{nnt}^i}}\right) = \ln\widetilde{D_{lt}^i} - \ln\widetilde{D_{nt}^i} - \widetilde{\theta_i} \cdot \ln\widetilde{\tau_{nlt}^i}$$
(7)

•  $\widetilde{\tau_{nlt}^i} \equiv d_{nlt} + b_{nlt} + lang_{nlt} + legal_{nlt} + colonial_{nlt} + RTA_{nlt} + \widetilde{im}_{nt}^i + \widetilde{Tariff}_{nlt}^i + \widetilde{v}_{nlt}^i$ 

• We can rewrite equation (7) as



•  $\ln D_{lt}^i$  is the final goods competitiveness of country l

Trade Elasticities in different sectors	Interm.	Final
Primary sector	2.318	1.505
Food products, beverages and tobacco[ISIC 10-12]	1.159	1.015
Textiles, textile products, leather and footwear [ISIC 13-15]	5.057	3.139
Wood and products of wood and cork[ISIC 16]	5.325	3.034
Paper products and printing[ISIC 17-18]	5.314	4.143
Chemical products & Pharmaceuticals [ISIC 20-21]	6.618	6.538
non-metallic mineral products [ISIC 22-23]	4.055	3.432
Basic metals & Fabricated metal products [ISIC 24-25]	8.650	4.784
Machinery and equipment [ISIC 26-28]	6.254	5.442
Transport equipment [ISIC 29-30]	2.203	2.470
Other manufacturing [ISIC 31-33]	1.778	2.229
Mean	4.430	3.430

Note: The trade elasticity for the services sector is assumed to be the average

elasticity of the non-service sectors.

- (B) Estimating Technology Stocks  $\widehat{T_n^i}$  and  $\widehat{\widetilde{T_n^i}}$  are computed from the estimated  $\widehat{D_n^i}$ ,  $\widehat{w_n}$  and  $\widehat{\Phi}_n^j$  ( $\equiv \sum_{m=1}^N T_m^i (c_m^i \tau_{nm}^i)^{-\theta}$ ).
- (C) Estimating Trade Barriers τ<sup>i</sup><sub>nl</sub> and τ<sup>i</sup><sub>nl</sub> are backed out from equations
   (6) and (7).

# CONVERGENCE IN TECHNOLOGY STOCKS AND TRADE COSTS

- On average, developing countries' technology stock increases by a factor which is more than 3.2 times that of developed countries.
- On average, developing countries' trade cost decreases by a factor which is less than 0.84 times that of developed countries.



Figure 4: Convergence in Technology



Figure 5: Convergence in Trade Costs

### 4. CALCULATING CHANGE IN GFT

• Define:  $GFT_n \equiv \ln (W_n / W_n^A)$ .

• Change in GFT:  $\Delta GFT_n = (GFT_n)' - GFT_n$ .

$$\Delta GFT_n = \ln \left\{ \prod_{i=1}^{K} \left( \widehat{\pi_{nn}^i} \right)^{-\alpha_n^i / \widetilde{\theta_i}} \right\} + \ln \left\{ \prod_{i=1}^{K} \prod_{j=1}^{K} \left( \widehat{\pi_{nn}^j} \right)^{-\left(\alpha_n^i \cdot \delta_n^{ij}\right) / \theta_i} \right\}$$
(8)  
Change in GFT in final goods

• The average change in GFT in intermediate goods is distinctly larger than (1.5 times) that in final goods. The pattern is more pronounced in the developing countries (2.0 times).

# Table 2: Model Fit concerning the average change in GFT in intermediate goods and final goods

Change in GFT	All Countries		Develop	ed Countries	Developing Countries		
	Actual	Estimated	Actual	Estimated	Actual	Estimated	
Total	6.047%	6.065%	7.044%	6.874%	5.291%	5.520%	
Intermediate Goods	3.667%	3.802%	3.845%	3.719%	3.532%	3.864%	
Final Goods	2.379%	2.263%	3.200%	3.065%	1.757%	1.656%	



Figure 6: Model Fit for Intermediate Goods and Final Goods

# 5. COUNTERFACTUAL EXERCISES — GLOBAL CHANGES ON DOMESTIC GFT

- Exogenous factors:(a) intermediate and final goods technology stocks in all sectors, (b) intermediate and final goods trade costs in all sectors, (c) net exports.
- Variance decomposition method: We regress the value of the contribution of each of the exogenous factors on the estimated total change in GFT based on our model.
- For each country, the contribution of an exogenous factor is obtained from counterfactual exercises, by taking the average of:
  - 1. Pure effect: Allowing only that factor (in all countries) to change while shutting down all other factors (in all countries).

2. Total effect: Find the difference between (a) allowing all factors (in all countries) to change, and (b) shutting down that factor (in all countries) while allowing all other factors (in all countries) to change.

#### • Results:

On average,

- 1. Global reduction in trade costs contribute positively to domestic change in GFT.
  - 2. Global increases in technology stocks contribute negatively to domestic change in GFT.

Table 3: Estimated average contribution of global change in each factor to the change

in	domestic GFT							
	Contribution to ave.							
	change in GFT	$T_{int}$	$T_{fnl}$	${ au}_{int}$	${ au}_{fnl}$	NX	Residual	Actual
	All Countries							
	Total	-1.715%	0.382%	4.630%	2.889%	-0.121%	-0.019%	6.047%
	Intermediate Goods	-2.113%	1.404%	5.204%	-0.676%	-0.017%	-0.135%	3.668%
	Final Goods	0.398%	-1.021%	-0.574%	3.565%	-0.104%	0.116%	2.379%

Note: This table reports the estimated average change in GFT due to the global changes in each factor, averaging over all countries.  $T_{int}$  ( $\tau_{int}$ ) is technology (trade costs) in intermediate goods;  $T_{fnl}$  ( $\tau_{fnl}$ ) is technology (trade costs) in final goods; NX is net exports.

Table 4: Variance decomposition of the contribution of global changes to the variance of domestic changes in GFT

Contribution to Variance						
of Changes in GFT	$T_{int}$	$T_{fnl}$	$ au_{int}$	$ au_{fnl}$	NX	Residual
Row A: All Countries	-0.241	0.030	0.705	0.471	0.008	0.028
Row B: Developed Countries	-0.047	0.060	0.578	0.396	-0.048	0.062
Row C: Developing Countries	-0.435	-0.001	0.832	0.546	0.063	-0.006

Note: Rows A, B and C report the contribution of global changes in each factor to the variance of the changes in GFT across all countries, developed countries and developing countries, respectively.  $T_{int}$  ( $\tau_{int}$ ) is technology (trade costs) in intermediate goods;  $T_{fnl}$  ( $\tau_{fnl}$ ) is technology (trade costs) in final goods; NX is net exports.

# 6. IMPACTS ON FOREIGN COUNTRIES AND POLICY IMPLICATIONS

### (A) Impacts on Foreign Countries

• By comparing the impacts of global changes on domestic GFT with the impacts of domestic changes on domestic GFT, we can see the impacts of foreign changes on domestic GFT. Table 5: Effects of Domestic, Global and Foreign Changes in Each factor on Change in Domestic GFT

Effects on Domestic GFT	$T_{int}$	$T_{fnl}$	$ au_{int}$	$ au_{fnl}$	NX	Total Effect
Row A: Domestic Changes	-4.393%	0.159%	3.866%	2.744%	-0.251%	2.126%
Row B: Foreign Changes	2.678%	0.223%	0.764%	0.145%	0.130%	3.939%
Row C: Global Changes	-1.715%	0.382%	4.630%	2.889%	-0.121%	6.065%

Note: Row C is equal to Row A plus Row B.

- The contributions of foreign technological changes on domestic GFT are large.
- The heat map below show that China stands out as the country that has large and positive contribution to almost all other countries.
- China has by far the largest contribution on the rest of the world through trade. On average, trading with China increases a country's GFT by about 1.2 percentage points during the period 1995-2017. This is about 4 times that of the second highest country, Russia.



Contribution of domestic technological change to foreign countries

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Figure 7: Contribution of change in technology stock of each country to the gains from trade of foreign countries.



Contribution of change of domestic trade cost to foreign countries

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Figure 8: Contribution of change in importing trade cost of each country to the gains from trade of foreign countries.



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Figure 9: The average contribution (in terms of percentage point increase in GFT) to foreign countries of each country's change in technology stock (left panel), importing trade costs (middle panel) and the total change (right panel).

# (B) Policy Implications: Elasticities of Domestic GFT w.r.t. Domestic Changes

- We carry out counterfactuals to estimate the contribution of each of the domestic exogenous factors to the domestic change in GFT.
- Then we estimate the elasticity of domestic GFT with respect to (w.r.t.) domestic change in each exogenous factor in each country.

 $_{\mbox{\tiny Table 6:}}$  Mean elasticity of domestic GFT with respect to domestic change in each

#### factor

Elasticities of domestic GFT	All	Developing	Developed
w.r.t. domestic factors	Countries	Countries	Countries
Intermediate goods technology	-0.032	-0.034	-0.031
Final goods technology	-0.011	-0.010	-0.013
Intermediate goods importing trade costs	-0.199	-0.203	-0.197
Final goods importing trade costs	-0.101	-0.099	-0.107

Note: Trade cost of each country is average importing trade cost.

- Improvement in domestic technology reduces domestic GFT, as expected.
- Reduction of domestic importing trade costs increases domestic GFT, as expected.
- The mean elasticity of GFT w.r.t. intermediate goods technology (trade costs) is 2 to 3 times that w.r.t. final goods technology (trade costs)
  - One of the reasons: the multiplier effect: Reduction in intermediate goods trade costs -> reduction of prices of intermediate goods through an infinite loop. It also reduces the prices of final goods. But reduction of final goods trade costs does not have this kind of multiplier effects.
- Therefore, policy measures affecting the intermediate goods are more important to welfare than those affecting the final goods.

# 7. CONCLUSION

- The evolution of globalization during 1995-2017 can largely be explained by increases in technology stocks and reduction of trade costs of countries around the world.
- Changes in developing countries accounted for the lion's share of the deepening of globalization around the world.
- Foreign changes contributed to an average of 65 percent of the increase in the degree of globalization of countries while domestic changes accounted for only 35 percent.
- China's contribution to the rest of the world through trade, which amounts to 31% of the world's total, is by far the largest among all countries.
- Future research: 1. If reduction in domestic trade costs tends to induce faster

domestic technological improvement, then the two are not independent. We need to take this into account in future research. 2. Add dynamics by introducing capital and capital accumulation.

#### APPENDIX

## STANDARD RP MODEL VS. OUR BASELINE MODEL

$$\widehat{\overline{GFT}}_n = \prod_{i=1}^K \prod_{j=1}^K \left( \widehat{\overline{\pi_{nn}^j}} \right)^{-\left(\alpha_n^i \cdot \lambda_n^{ij}\right)/\overline{\theta_i}}$$
(9)

• On average, the standard RP model underestimates the changes in GFT of all countries by about 53%.



Figure 10: The Estimated Change in GFT in Our Model vs. the Standard RP Model for All Countries



#### **Estimating Technology Stocks**

$$\Phi_n^j = \frac{D_n^j}{\pi_{nn}^j}$$

$$\widehat{T_n^i} = \frac{\widehat{D_n^i}}{(\widehat{w_n})^{-\beta_n^i \theta} \prod_{j=1}^K \left(\widehat{\Phi}_n^j\right)^{\gamma_n^{ij} \left(1-\beta_n^i\right)}}$$

$$\widehat{\widetilde{T_n^i}} = \frac{\widehat{\widetilde{D_n^i}}}{(\widehat{w_n})^{-\beta_n^i \widetilde{\theta}} \prod_{j=1}^K \left(\widehat{\Phi}_n^j\right)^{\gamma_n^{ij} \left(1-\beta_n^i\right)}}$$

$$(11)$$

**Estimating Trade Barriers** 

$$\tau_{nl}^{i} = \frac{-1}{\theta} \left[ \ln \left( \frac{X_{nl}^{i}}{X_{nn}^{i}} \right) - \ln D_{l}^{i} + \ln D_{n}^{i} \right]$$
(12)

$$\widetilde{\tau_{nl}^{i}} = \frac{-1}{\widetilde{\theta}} \left[ \ln \left( \frac{\widetilde{X_{nl}^{i}}}{\widetilde{X_{nn}^{i}}} \right) - \ln \widetilde{D_{l}^{i}} + \ln \widetilde{D_{n}^{i}} \right]$$
(13)

# THE SYSTEM OF EQUATIONS FOR ESTIMATION IN OUR BASELINE MODEL

Following Dekle, Eaton, and Kortum (2008), we use exact hat algebra to characterize the equilibrium changes:  $\hat{x} = x'/x$ . We solve for the following system of equations.

$$\widehat{c}_{n}^{i} = (\widehat{w_{n}})^{\beta_{n}^{i}} \left[ \prod_{j=1}^{K} \left( \widehat{P_{n}^{j}} \right)^{\gamma_{n}^{ij}} \right]^{1-\beta_{n}^{i}}$$

$$(14)$$

$$\widehat{P_{n}^{i}} = \left[ \sum_{m=1}^{N} \pi_{nm}^{i} \widehat{T_{m}^{i}} \left( \widehat{c_{m}^{i}} \cdot \widehat{\tau_{nm}^{i}} \right)^{-\theta_{i}} \right]^{-1/\theta_{i}}$$

$$(15)$$

$$\widehat{\widetilde{P}_{n}^{i}} = \left[\sum_{m=1}^{N} \widetilde{\pi_{nm}^{i}} \cdot \widehat{\widetilde{T}_{m}^{i}} \left(\widehat{c_{m}^{i}} \cdot \widehat{\tau_{nm}^{i}}\right)^{-\widetilde{\theta}_{i}}\right]^{-1/\theta_{i}}$$
(16)

$$\widehat{\pi_{nl}^{i}} = \widehat{T_{l}^{i}} \left( \frac{\widehat{c_{l}^{i}} \cdot \widehat{\tau_{nl}^{i}}}{\widehat{P_{n}^{i}}} \right)^{-\theta_{i}}$$

$$\widehat{\overline{\pi_{nl}^{i}}} = \widehat{\widetilde{T}_{l}^{i}} \left( \frac{\widehat{c_{l}^{i}} \cdot \widehat{\overline{\tau_{nl}^{i}}}}{\widehat{\widetilde{P_{n}^{i}}}} \right)^{-\theta_{i}}$$

$$(17)$$

$$(18)$$

$$\widehat{Y_n^i} = \sum_{l=1}^N \widehat{\pi_{ln}^i} \pi_{ln}^i \sum_{j=1}^K \gamma_l^{ji} \cdot \widehat{Y_l^j} \cdot \frac{\left(1 - \beta_l^j\right) Y_l^j}{Y_n^j} + \sum_{l=1}^N \widehat{\pi_{ln}^i} \cdot \widehat{\pi_{ln}^i} \cdot \alpha_l^i \cdot \frac{\widehat{w_l} \cdot w_l L_l - \widehat{NX_l} \cdot NX_l}{Y_n^i}$$
(19)

$$\widehat{X_n^i} = \alpha_n^i \cdot \frac{w_n L_n \widehat{w_n} - N X_n \cdot \widehat{NX_n}}{X_n^i} + \sum_{j=1}^K \widehat{Y_n^j} \cdot \frac{\gamma_n^{ji} \left(1 - \beta_n^j\right) Y_n^j}{X_n^i}$$
(20)

$$\sum_{i=1}^{K} \frac{Y_{n}^{i}}{\sum_{i=1}^{K} Y_{n}^{i}} \cdot \widehat{Y_{n}^{i}} = \sum_{i=1}^{K} \frac{X_{n}^{i}}{\sum_{i=1}^{K} Y_{n}^{i}} \cdot \widehat{X_{n}^{i}} + \frac{NX_{n}}{\sum_{i=1}^{K} Y_{n}^{i}} \cdot \widehat{NX_{n}}$$
(21)