Information Span in Credit Market Competition [*]

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Abstract

We develop a credit market competition model that distinguishes between the information span (breadth) and signal precision (quality), capturing the emerging trend in fintech/non-bank lending where traditionally subjective ("soft") information becomes more objective and concrete ("hard"). In a model with multidimensional fundamentals, two banks equipped with similar data processing systems possess hard signals about the borrower's hard fundamentals, and the specialized bank, who further interacts with the borrower, can also assess the borrower's soft fundamentals. Increasing the span of the hard information hardens soft information, enabling the data processing systems of both lenders to evaluate some of the borrower's soft fundamentals. We show that hardening soft information levels the playing field for the non-specialized bank by reducing its winner's curse. In contrast, increasing the precision or correlation of hard signals often strengthens the informational advantage of the specialized bank.

JEL Classification: G21, L13, L52, O33, O36

Keywords: Banking competition, Winner's curse, Specialized lending, Information technology, Big data

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As a crucial intermediary sector in modern economies, commercial banks serve as the main conduit between savers and creditworthy borrowers, leveraging a broad spectrum of information including customer financial data, collateral assessments, market and economic trends, and advanced data analytics. One of the recent technology developments enhancing the quality of information available to lenders and improving their screening capabilities is big data technology. This innovation transforms qualitative, subjective assessments into quantifiable, objective metrics—a process known as "hardening soft information" (Liberti and Petersen, 2019; Hardik, 2023). In this paper, we are interested in how the hardening of soft information affects credit market equilibrium and what differentiates this trend from previous technological innovations.

The farming industry provides an illustrative example of how technology can transform traditional lending practices. In the past, farm loans required extensive in-person visits from specialized loan officers, who leveraged their expertise to evaluate the borrower's abilities and farm infrastructure quality. This hands-on approach was necessary to evaluate these "soft" fundamentals, as the officers needed to directly observe factors like crop rotation techniques, pest management strategies, and barn conditions to accurately assess loan risks. Today, satellite imaging and AI-enabled data analysis allow lenders to gather some of these insights remotely via computerized "hard" data. Although on-site assessments still provide important insight, new technologies have expanded access to farm data, demonstrating how technology can expand the information span of "hard signals" without completely disrupting specialized but "soft" human expertise.

As the example above highlights, the remarkable recent and ongoing technological advancements have the potential to alter the information available to participants in the credit market and significantly impact the industrial landscape of the banking sector. We offer an economic framework to analyze the welfare implications of this technological advancement by incorporating a novel information structure into an otherwise conventional credit market competition model. In the model, the quality of the borrower depends on multiple fundamental states, which broadly belong to two categories—"hard" states and "soft" states as distinguished by the type of information technology capable of assessing these states. Before making lending decisions, lenders can access private signals about these two categories. We refer to a signal that reflects the borrower's hard states as a "hard-information-based signal" or simply a "hard signal," and likewise, a signal that reflects the borrower's soft states as a "soft-information-based signal" or a "soft signal." Crucially, hard states might overlap with soft states, so hard and soft signals might be correlated. This correlation and its potential implications for credit market competition are the main innovation relative to the model in our companion paper Blickle, He, Huang, and Parlatore (2024).

Our framework highlights the difference between the breadth (information span) and quality (signal precision) of data. When hard states cover more fundamental states that are critical to the borrower's quality, the information span of the hard signal expands. This expansion captures the core idea of "hardening soft information" in the context of credit market competition. In contrast, increasing the precision of hard signal improves the accuracy in the assessment of the existing hard characteristics. Although both of these improvements are associated with technological advances that reduce the overall uncertainty faced by lenders, we show that changes in the span and the precision of hard information have vastly different impacts on credit market outcomes.

In our model of credit market competition, as we outline in Section 1, a specialized bank competes with a non-specialized bank. Each lender has a *private* hard signal about the hard fundamental states that stems from data processing. Additionally, the specialized lender has access to a *private* soft signal about the borrower's soft states. We assume that the hard signal is binary and decisive in that each lender makes an offer only if it receives a positive realization. The soft signal—which differentiates our paper from existing models such as Broecker (1990) and Marquez (2002)—is continuous and guides the fine-tuned interest rate offering of the specialized bank. In addition to analytical convenience, this loan-making rule of the specialized bank matches the observed lending practices well. Essentially, in our model, the specialized bank acquires two signals, one being "principal" while the other being "supplementary." The former determines whether to lend and the latter affects the detailed pricing terms.[1]

Section 2 fully characterizes the competitive credit market equilibrium with specialized lending in closed form. As in Blickle, He, Huang, and Parlatore (2024), our model has a unique equilibrium, which can fall into two distinct categories depending on whether the non-specialized bank makes zero profits. In the "zero-weak" equilibrium, the winner's curse faced by the non-specialized "weak" bank causes it to randomly withdraw from competition upon receiving a positive hard signal and earn zero profits. In the "positive-weak" equilibrium, the Winner's Curse is less severe so the non-specialized weak bank always participates upon a positive hard signal and earns positive profits.

Our main analysis in Section 3 examines how the span of hard information affects the equilibrium in the credit market. In our model, the information technology available to lenders affects their screening and their beliefs about the competitor's available information (through strategic considerations), which determines the severity of the winner's curse. In general, an expansion in the information span of the hard signal reduces Type II errors (making bad loans) from hard-information-based screening for both specialized and non-specialized lenders. This economic force, however, is stronger for the non-specialized lender who now learns (partially) about its specialized opponent's soft signal. Put differently, a greater span of hard information increases the overlap between hard and soft states, thereby leveling the playing field by reducing the winner's curse faced by the non-specialized bank due to the specialized opponent's soft signal. As a result, the non-specialized lender starts making positive profits when the information span is sufficiently large.

As one of the main results of the paper, we contrast an increase in the span of hard information with two other types of information technology advancement: an increase in the precision of each

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[1] Alternatively, the principal signal represents the result of a credit screening test, while the supplementary signal serves the role of internal ratings (of borrowers who are qualified for credit).

hard signal and an increase in the correlation between the two hard signals. These technological improvements improve the overall screening ability of banks, but affect competition in the credit market differently. We show that while an increase in the span of hard information levels the playing field for the non-specialized bank, an increase in the precision of hard signal or the correlation of hard private signals amplifies the informational edge of the specialized bank, especially when the hard signal is sufficiently informative.

Intuitively, there are two effects that differentiate broader versus more precise hard information. The first effect pertains the event in which Bank B receives a positive hard signal while Bank A receives a negative one. Given the symmetric structure of the hard signals, a disagreement in their realization has no informational content. Therefore Bank B can potentially profit in such scenarios when it is the one making an offer and winning the borrower. While increases in the span of information increase the likelihood of this disagreement event, a higher hard signal precision decreases such profitable opportunities for Bank B as the two lenders' hard signals become more correlated.

The second effect regards the soft characteristics that Bank B is uninformed about but can partially infer when both banks receive positive hard signals. From Bank B's perspective, an increase in the span of hard information reduces the residual uncertainty about the soft fundamental. In contrast, as discussed above, an increase in the precision of hard information raises the likelihood that both lenders receive favorable hard signals and compete. This means that Bank B suffers more frequently from the winner's curse as these residual soft fundamentals can only be assessed by the specialized Bank A through its soft signal.

It is also worth highlighting that these distinctions emerge from our examination of credit market competition with specialized lending, which is a practically relevant setting demonstrated by Blickle, He, Huang, and Parlatore (2024).[2] Both information precision and information span affect the posteriors of some fundamentals that each lender would like to learn, but they have drastically different implications on the posterior distribution of the opponent's private information especially when both lenders are asymmetrically informed. Improving the hard signal's precision allows both lenders to have a more precise evaluation of borrower quality, while enlarging the information span provides the non-specialized lender direct insights into its opponent's pricing strategy: the former tends to reinforce the position of specialized lender, while the latter "levels the playing field."

Why is it important to differentiate between the various aspects of information technologies? We stress that the recent significant advancement in information technology should have improved the hard-information-based screening technology for both types of lender equally; specialized and established banks can adopt these technologies just as effectively as non-specialized banks and new fintech challengers. However, the fast-growing empirical literature on fintechs (see, e.g. Berg,

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[2] Blickle, He, Huang, and Parlatore (2024) show that banks with asymmetric private information are needed to match the empirical patterns of a lower loan pricing and lower non-default rate among loans granted by specialized lenders.

Fuster, and Puri, 2022) seems to suggest that new technologies have helped relatively weaker (fintech) lenders catch up, intensifying competition in the credit market. Building a model with asymmetric lenders but symmetric technological improvement, our theory clarifies that extending the information span, rather than the mere improvement in the precision of existing signals, can deliver the observed empirical patterns in a robust way. As elucidated in our opening motivating example of loans to the farming industry, big data technology empowers non-specialized lenders to utilize "hardened soft information."

The process of "hardening soft information," which expands the span of hard information, has important implications for the equilibrium credit allocation and the resulting welfare. We formally prove that total welfare, measured as the expected surplus from projects that are funded, is always increasing in the span of hard information. Besides, we also show that when the hard signal precision is relatively low, the specialized bank's profits could also increase in the parameter range of positive-weak equilibrium. This highlights the modelling feature that information span reflects the improvement of information technology in the entire banking industry, which benefits both specialized and non-specialized lenders.

Throughout the paper, we model the hard information technology as generating one binary hard signal from hard fundamental characteristics that may expand. As an extension of the model, we consider an alternative way of modeling the hardening of soft information by introducing an additional signal on hardened soft fundamentals. We analytically show that this alternative modeling delivers similar economic implications to our baseline model.

Literature Review

Lending market competition and common-value auctions. Our paper is built on Broecker (1990) which studies lending market competition with screening tests with symmetric lenders (i.e., with the same screening abilities). Hauswald and Marquez (2003) study the competition between an inside bank that can conduct credit screenings and an outside bank without such access. He, Huang, and Zhou (2023) consider competition between asymmetric lenders with different screening abilities under open banking when borrowers control access to data, and Goldstein, Huang, and Yang (2022) highlight the endogenous response from banks' liability side once the incumbent bank's borrower data become open to a challenger bank, where maturity transformation of using short-term funding to support long-term loans plays an important role. Asymmetric credit market competition can also arise naturally from the bank-customer relationship, since a bank knows its existing customers better than a new competitor does.[3] In these models, for analytical tractability it is often assumed that private screening yields a binary signal and lenders participate in bidding only following the positive signal realization.

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[3] This idea was explored by a two-period model in Sharpe (1990) where asymmetric competition arises in the second period (with the corrected analysis of a mixed-strategy equilibrium offered by Von Thadden (2004)). A similar analysis is present in Rajan (1992).

Building on the framework established in our companion paper Blickle, He, Huang, and Parlatore (2024), our paper considers competition between asymmetric lenders with multiple sources of information. In both papers a non-specialized lender has access to a private "hard" signal about the borrower's credit quality, while the specialized lender receives not only an independent private "hard" signal but also a "soft" one, both of which are informative about the borrower's quality. The distinction is that in Blickle, He, Huang, and Parlatore (2024), hard (soft) signals reflect *independent* borrower characteristics that drive the loan quality. This paper, however, allows these underlying states to overlap, resulting in correlated hard and soft signals. Naturally, the span of hard information has a tight link to correlation, which allows us to study its implications of "hardening soft information" on credit market competition.

In a closely related paper, Karapetyan and Stacescu (2014) argue that sharing borrower's "hard" information (say default history) in fact increases the incumbent bank's incentive to further acquire "soft" information regarding borrower's quality.[4] Although their model also involves the stronger bank having more than one private signal, one important difference is that in Karapetyan and Stacescu (2014) there is always a strict Blackwell ordering of information between two lenders, simply because the hard information becomes public after sharing. In contrast, conditionally independent hard signals in our model allow for the possibility of having a profitable weaker lender, yielding much richer empirical predictions and welfare outcomes.

Specialization in lending. There is a growing literature documenting specialization in bank lending; the early work includes Acharya, Hasan, and Saunders (2006). Paravisini, Rappoport, and Schnabl (2023) shows that Peruvian banks specialize their lending across export markets benefiting borrowers who obtain credit from their specialized banks. Based on data for US stress-tested banks, Blickle, Parlatore, and Saunders (2023) documents that specialization is linked with lower interest rates and better performance in the industry of specialization, pointing to a strong link between specialization in lending and informational advantages. Our paper contributes to this literature by providing a framework that can rationalize these patterns, allowing us to understand the economic mechanisms behind them and their implications more deeply.

The nature of soft/hard information in bank lending. The existing literature on soft and hard information (e.g., Stein, 2002; Liberti and Petersen, 2019) emphasizes that the latter is easily verifiable and hence transferable (within an organization); for instance, Bertomeu and Marinovic (2016) and Corrao (2023) model "soft" information via a cheap talk game a la Crawford and Sobel (1982) where the messages are soft and carry no intrinsic meaning themselves.[5] Since we do

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[4] In that paper, the hard information that banks are sharing is not that soft information that banks acquire at a cost. If sharing leads incumbent banks to lose their edge, they should have a stronger incentive to acquire soft information (which cannot be shared).

[5] For related empirical studies, see Liberti and Mian (2009), Paravisini and Schoar (2016). Recently, based on Harte Hanks data, He, Jiang, Xu, and Yin (2023) shows a significant rise in IT investment within the U.S. banking sector over the past decade, particularly among large banks. They also establish a causal link between communication

not explicitly model communications within or across banks, whether the information is verifiable is irrelevant to the core economics that our model aims to capture. However, complementing the traditional way of modeling hard/soft information which focuses on communication (e.g., Bertomeu and Marinovic, 2016; Corrao, 2023), our paper highlights the novel concept of "information span" that is necessary to understand the recent phenomenon where certain soft information becomes hardened. Furthermore, similar to Karapetyan and Stacescu (2014) where hard information can be shared, as hard information is transferable and can be analyzed by anyone, once soft information becomes hardened into verifiable data, it also becomes accessible to nonspecialists. This levels the playing field for non-specialized lenders, a development that improves welfare in our analysis.

Fintech. Our paper connects to the growing literature on fintech disruption.[6] Empirical studies document the use of alternative data in fintech lending, which is consistent with our emphasis on the increasing span of hard information.[7] In particular, Huang, Zhang, Li, Qiu, Sun, and Wang (2020) shows that unconventional data from the Alibaba platform, such as business transactions, customer ratings, and consumption patterns improve credit assessment. Our paper emphasizes that the recent development of cashless payments increases the scope of firms that could be assessed by hard information (Ghosh, Vallee, and Zeng, 2022), and perhaps more importantly, the combination of payments and big data technology enlarges the span of hard information.

1 Model Setup

We consider a credit market competition model with two dates and one good. There are two ex-ante symmetric lenders (banks), indexed by $j \in \{A, B\}$ and one borrower firm; everyone is risk neutral.

1.1 The Setting

Project. At $t = 0$, the firm needs to borrow one dollar to invest in a (fixed-scale) risky project that pays a random cash flow y at $t = 1$. The cash flow realization y depends on the project's quality denoted by $\theta \in \{0, 1\}$. For simplicity, we assume that

$$y = \begin{cases} 1 + \overline{r} & \text{when } \theta = 1 \\ 0 & \text{when } \theta = 0, \end{cases} \tag{1}$$

IT investments and banks' capacity to generate and transmit soft information, which motivates our modeling of the soft signal as the outcome of interactions with borrowers.

[6] See Berg, Fuster, and Puri (2022); Vives (2019), e.g. for a review of bank-fintech competition.

[7] Examples of alternative data include phone device and spelling (Berg, Burg, Gombović, and Puri, 2020), mobile phone logs (Agarwal, Alok, Ghosh, and Gupta, 2020). Along the line of our model with different dimensions of information, Huang (2023) develops a theoretical framework wherein the importance of information concerning underlying qualities varies between collateral-backed bank lending and revenue-based fintech lending such as Square.

where $\overline{r} > 0$ is exogenously given so only the good project pays off. We will later refer to $\overline{r}$ as the interest rate cap or the return of a good project. The project's quality θ is the firm's private information at $t = 0$, and the prior probability of a good project is $q \equiv \mathbb{P}(\theta = 1)$. Later we will use "project success," "good project" and/or "good borrower" interchangeably to refer to $\theta = 1$. The project quality θ is unobservable.

Hard and soft states. The project success $\theta \in \{0, 1\}$ depends on two fundamental states, one being "*h*ard" denoted by θ_h and the other being "*s*oft" denoted by θ_s. We assume that both fundamental states are binary so that $\theta_h \in \{0, 1\}$ and $\theta_s \in \{0, 1\}$, with

$$q_h \equiv \mathbb{P}(\theta_h = 1), \text{ and } q_s \equiv \mathbb{P}(\theta_s = 1).$$

Importantly, θ_h and θ_s are potentially correlated, and the correlation is related to the span of hard information technology.

Multi-dimensional fundamental states and information span Following the O-ring theory of economic development (Kremer, 1993), we model the hard and soft states by a setting with multi-dimensional fundamental states. As a main contribution of our paper, this offers a novel way to study the "span" of the information available to banks.

More specifically, suppose that the success of the project θ depends on N characteristics in the following multiplicative way:

$$\theta = \prod_{n=1}^{N} \theta_n = \rlap{$\overbrace{\phantom{\prod_{n=1}^{N_h^h} \theta_n \cdot \prod_{n=N_h^h+1}^{N_h^h+N_s^h} \theta_n}}^{\theta_h}$}\prod_{n=1}^{N_h^h} \theta_n \cdot \underbrace{\prod_{n=N_h^h+1}^{N_h^h+N_s^h} \theta_n \cdot \prod_{n=N_h^h+N_s^h+1}^{N} \theta_n}_{\theta_s}. \tag{2}$$

We assume that $\{\theta_n\}$ follow independent Bernoulli distributions, i.e., $\theta_n = 1$ with probability $q_n \in [0, 1]$ for all $n = 1, ..., N$; they capture "(unobservable) characteristics" that are critical to the ultimate success of the project, such as product quality, market and funding conditions, the regulatory environment, etc. As shown in (2), the hard state θ_h covers the first $N^h \equiv N_h^h + N_s^h$ characteristics while the soft state covers the last $N - N_h^h$. Importantly, hard and soft states overlap across the middle N_s^h characteristics, which leads to correlated fundamental states. Later we will vary N_s^h—i.e., the span of hard information—and study the implication of this on credit market competition.

Since the order of characteristics plays no role in the analysis, it is without loss of generality to analyze a simplified setting with three independent fundamental states as follows:

$$\theta = \mathrlap{\overbrace{}^{\theta_h}}\theta_h^h \cdot \underbrace{\theta_s^h \cdot \theta_s^s}_{\theta_s}, \tag{3}$$

with priors denoted by

$$q_h^h \equiv \mathbb{P}\left(\theta_h^h = 1\right), \quad q_s^h \equiv \mathbb{P}\left(\theta_s^h = 1\right), \quad \text{and} \quad q_s^s \equiv \mathbb{P}\left(\theta_s^s = 1\right).$$

As we will explain shortly, θ_h^h in (3) captures those fundamental states that are already "hard" even before the technology progress, θ_s^h captures those states that were used to be "soft" but now can be covered by hard information thanks to the technology progress, while θ_s^s captures those states that remain soft. When $\theta_h^s = 1$ for sure (i.e., $q_h^s = 1$ or $N_s^h = 0$ in Eq. (2)), this model degenerates into independent hard and soft fundamental states as in Blickle, He, Huang, and Parlatore (2024).

Credit market competition. At date $t = 0$, given its private information about the borrower's project quality (see Section 1.2), each bank j can choose to make a take-it-or-leave-it offer to the borrower firm or to make no offer (i.e., exit the lending market). An offer consists of a fixed loan amount of one and an interest rate r. The borrower firm accepts the offer with the lowest rate if it receives multiple offers.

1.2 Information Technology and Information Span

Information technology corresponds to mappings from some fundamental states to signals. We will introduce two types of signals, each modeled as a specific mapping from its corresponding fundamental state θ_h or θ_s to a bank-specific signal realization. To capture specialized lending, we assume that both lenders have *hard-information*-based private signal h^j for $j \in \{A, B\}$ about θ_h while only specialized bank A has the *soft-information*-based private signal s about θ_s. Figure 1 provides a summary of information technology.

Hard signals. We assume that both lenders have access to "hard" data (including financial and operating data in the past as well as "alternative data" that become available following the big data technology), which they can process to produce a *hard-information*-based private signal h^j about the firm's fundamental state θ_h. We call them "hard" signals, and for tractability we assume these hard signals to be binary, i.e., $h^j \in \{H, L\}$, with H (L) being a positive (negative) signal of θ_h. (Binary hard signal is related to the assumption that hard signals are "decisive;" see Section 1.4.) Conditional on the (relevant) state, hard signals are independent across lenders.[8]

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[8] In the companion paper Blickle, He, Huang, and Parlatore (2024), we consider a general (binary) information technology where hard signals are potentially correlated.

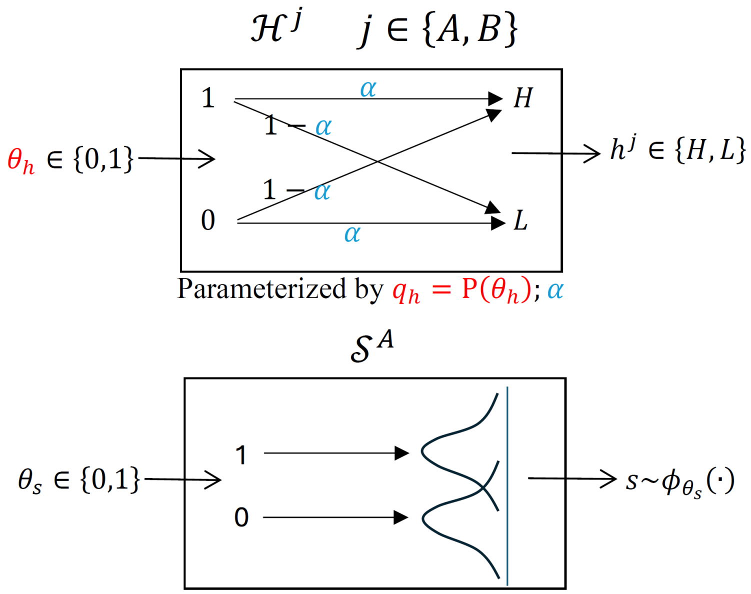

Figure 1: Information Technology, Hard (top panel) and Soft (bottom panel)

More specifically, the hard signal technology $\mathcal{H}^j$ takes the binary fundamental state $\theta_h \in \{0,1\}$—which could vary as the information span changes as will be introduced next—as input and generates a binary signal $h^j \in \{H, L\}$ as output. Following most of the literature with exogenous symmetric information technology (Broecker, 1990; Marquez, 2002), we assume that

$$\mathbb{P}\left(h^j = H \,|\theta_h = 1\right) = \mathbb{P}\left(h^j = L \,|\theta_h = 0\right) = \alpha \text{ for } j \in \{A, B\}\,. \tag{4}$$

As illustrated in the top panel of Figure 1, $\alpha \in \left(\frac{1}{2}, 1\right)$ measures the precision of the hard signal, and captures equal probabilities of Type I and Type II errors. Given the binary fundamental state θ_h, the hard signal technology $\mathcal{H}^j$ thus can be summarized by two parameters: the prior of input $q_h = \Pr\left(\theta_h\right)$, and the signal's precision α given in (4).

Span (of hard) information Define

$$\eta \equiv 1 - \Pr(\theta_s^h = 1) = 1 - q_s^h > 0. \tag{5}$$

We call η the information span (of hard signals). Corresponding to a larger N_s^h in (2) (or θ_s^h becomes more important in (3)), an expansion of the coverage of θ_h leads to a smaller q_s^h and hence a greater η. All else equal, the larger η, the broader the span of hard information h^j's, and the greater the hard signal's information content (and capturing more of information that was soft previously, i.e., θ_s^h).

As a key distinction between our paper and the existing literature (Broecker, 1990; Marquez, 2002), the span of hard information η controls the input θ_h of the hard signal technology $\mathcal{H}^j$. More specifically, before soft information gets hardened the input is $\theta_h = \theta_h^h$ with a prior of $q_h = q_h^h$ while after the technology improvement the input becomes $\theta_h = \theta_h^h \theta_s^h$ with a prior of $q_h = q_h^h q_s^h = (1-\eta) q_h^h$; see Equation (3). Importantly, from the perspective of any hard signal technology $\mathcal{H}^j$, this only changes the prior of the binary input θ_h, i.e., $q_h(\eta) = (1-\eta) q_h^h$, while keeping the precision α constant.[9]

The binary structure of the hard signal captures the coarseness with which much of the hard information is used in practice,[10] and the main insight that the information span stemming from the big data technology differs from the signal precision is robust to a more general non-binary hard signal structure. In addition, we intentionally assume that both lenders have the same hard information technology because we are interested in how different aspects of information technology improvement affect the relative market power when both lenders enjoy the technology advancement symmetrically.[11]

Soft signal. Additionally, we endow Bank A with a signal s, which captures the bank being "specialized" in the firm. Similar to Blickle, He, Huang, and Parlatore (2024) we assume that the signal s is continuous. Our preferred interpretation of this additional signal is as a *soft-information*-based private signal, collected after due diligence or face-to-face interactions with the borrower after on-site visits. Besides mathematical convenience, the continuous distribution captures soft information resulting from research tailored to the particular borrower and, therefore, allows for a more granular assessment of the borrower's quality.

Similar to the hard signal, the soft signal technology should be viewed as a mapping $\mathcal{S}^A$ from the soft fundamental state $\theta_s \in \{0,1\}$ to a variable s that is correlated with θ_s, as shown in the bottom panel of Figure 1. It is without loss of generality to directly work with the posterior probability of the soft state being good $\theta_s = 1$ given the soft signal realization, i.e.,

$$s \equiv \Pr[\theta_s = 1|s] \in [0,1]. \tag{6}$$

We denote the density function of s by $\phi(s)ds \equiv \mathbb{P}(s \in (s, s+ds))$, which satisfies $\int_0^1 \phi(s)\, ds = 1$ and the prior consistency $\int_0^1 s\phi(s)\, ds \equiv q_s$.

For later exposition purposes, our numerical examples consider $s = \Pr[\theta_s = 1|\theta_s + \epsilon] =$

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[9] Many papers that adopt the binary-fundamental-binary-signal structure, including Marquez (2002), conduct the comparative statics on the prior of the project quality, with the implicit assumption that the signal precision can be kept at a constant.

[10] For example, credit scores are binned in five ranges even though scores are computed at a much granular level and go from 300 to 850.

[11] In the companion paper Blickle, He, Huang, and Parlatore (2024), we consider a general (binary) information technology that is potentially asymmetric between lenders.

$\mathbb{E}[\theta_s|\theta_s+\epsilon]$ where $\epsilon\sim\mathcal{N}(0,1/\tau)$ with τ capturing the signal-to-noise ratio of Bank A's soft information technology. This soft signal precision τ captures similar economics as α, and we stress that it has different implications from the information span parameter η.

In light of Figure 1, one can derive the density of s conditional on $\theta_s=1$, which we denote by $\phi_1(s)\equiv\phi(s|\,\theta_s=1)$. Using the short-hand notation $s\in ds$ for $s\in(s,s+ds)$, we have

$$\phi_1(s)\equiv\frac{1}{ds}\mathbb{P}(s\in ds|\theta_s=1)=\frac{\mathbb{P}(\theta_s=1|s\in ds)\cdot\frac{1}{ds}\mathbb{P}(s\in ds)}{\mathbb{P}(\theta_s=1)}=\frac{s\cdot\phi(s)}{q_s}. \tag{7}$$

Similarly, we can calculate

$$\phi_0(s)\equiv\phi(s|\,\theta_s=0)=\frac{(1-s)\phi(s)}{1-q_s}.$$

As s is the posterior expectation of θ_s and a higher value of s is "good news" (Milgrom, 1981), these two densities, i.e., $\phi_1(\cdot)$ and $\phi_0(\cdot)$, satisfy the strict Monotone Likelihood Ratio Property (MLRP).

1.3 Discussions on Modelling and Related Literature

Our model departs from the literature in several ways that warrant discussion.

Hardening soft information. The concept of information span η allows us to model "hardening soft information." To see this, consider Eq. (3) in Section 1.1. There, the first term θ_h^h captures those fundamental states that are already "hard" even before the information technology progresses; we call them "always hard" fundamentals. The second term θ_s^h captures those states that were used to be "soft" but now can be covered by hard signals thanks to the technology progress; the coverage of these "hardened soft" fundamentals grows with information span η. Finally, θ_s^s captures those states that remain soft; and we call them "always soft" fundamentals. Essentially, technological advancement (e.g. big data and machine learning) enables lenders to acquire pertinent hard objective data points, i.e., hard signals h^j for both lenders, about these "hardened soft" fundamentals θ_h^s, which previously could only be collected through human interactions and were accessible only to the specialized lender.

Throughout the paper, we use the hardening of soft information as an example of technological change that can increase the span of hard information. We do this for two reasons: first, to fix ideas and provide a concrete setting in which our model applies; and second, this application is practical relevant in the context of the current "Big Data" environment. As we explain below, verifiability does not play a role in the modeling of "hard" and "soft" information in our framework. Instead, our results are broader and apply to any circumstance in which access to information is democratized and characteristics previously accessible only to a monopolist are now "learnable" by all market participants.

Connection to the literature of soft/hard information. The literature on soft and hard information (e.g., Stein, 2002; Liberti and Petersen, 2019) often emphasizes that the latter is easily verifiable and hence transferable (within an organization); for example, Bertomeu and Marinovic (2016) and Corrao (2023) model "soft" information via a cheap talk game a la Crawford and Sobel (1982) where the messages are soft and carry no intrinsic meaning themselves. Since we do not explicitly model communications within or across banks, whether the information is verifiable or not is irrelevant to the core economics that our model aims to capture.

Nevertheless, our information technology discussed above, i.e., hard signals are available for both lenders while only the specialized lender has access to the soft signal, connects to this traditional view of soft information. Exactly due to the non-verifiable nature of soft information, loan officers often need to collect it individually and possess the expertise to interpret it, whereas verifiable hard information can be processed by anyone in a rather routine way. What is more, when soft information becomes hardened so that the IT equipment and software can analyze it from data, naturally some soft information becomes verifiable and hence available to non-specialists. This is exactly the logic in Karapetyan and Stacescu (2014) where hard information can be shared while soft cannot.

Hard information technology. In general, an information technology corresponds to mappings from some fundamental states to signals, and, as usual, there are potentially important modeling choices when specifying the details of the (hard) information technology.[12] As the top panel of Figure 1 illustrates, the hard information technology takes the entire binary hard fundamental θ_s as input and generates a binary signal as output. But this is not the only way to model this in a setting with multi-dimensional fundamental states. Given our hard fundamental $\theta_h = \theta_s^h \theta_s^s$, another natural modeling is to keep the original hard and soft signals (h^j's and s) and introduce additional signals of the hardened soft fundamental θ_h^s. Section 4.1 considers this alternative and shows that our economic implications are qualitatively similar to our baseline modeling.

Information span versus signal precision. The information span η is a key parameter in our analysis. By incorporating multi-dimensional information, our model highlights the distinction between the information span η and information precision (α for h-signal and τ for s-signal). Take α as an example. Recall that α measures the quality/precision of hard information while η measures the breadth/span of hard information. Both are significant aspects affected by the astonishing technological advancement in the past decades, but feature important differences. When the computer was introduced, it was faster and easier to process and compile bank statements. This improvement in processing made information more precise but did not change its scope much. However, the use of "big data," a distinctive trend in information technology during the last decade,

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[12] Information design along the line of Kamenica and Gentzkow (2011); Bergemann and Morris (2016) addresses this issue but is beyond the scope of this paper.

has changed what can be digitized as hard information (think of Amazon predicting consumer preferences). As many scholars have argued, big data technology has expedited the process of "hardening soft information" by converting subjective or qualitative data (soft information) into more objective or quantifiable (hard) metrics; for recent evidence in the banking industry, see, for example, Hardik (2023). By incorporating multi-dimensional information, our model allows us to distinguish between these two aspects, which, as we explain below, have distinctive economic implications on credit market competition.

Endogenous information structure. Throughout we take the lenders' asymmetric information technologies as given. Blickle, He, Huang, and Parlatore (2024) endogenize this asymmetric information technology in a symmetric setting with two firms, a and b, where Bank A (B) endogenously becomes specialized by acquiring both hard and soft signals for firm a (b), while non-specialized Bank B (A) only acquires the "hard" signal of the firm a (b). There, we highlight a key difference when acquiring these two types of signals: a one-time investment—for example, installing IT equipment and software—enables lender j to receive two hard signals, one for each firm, whereas soft information must be collected separately for each firm. This is connected to our point regarding the modeling of soft/hard information.

1.4 Decisive Hard Signals and Parametric Assumptions.

For tractability reasons, we assume that the hard signal is "decisive" for participation: Bank j participates if and only if it receives $h^j = H$. For the specialized Bank A, the hard signal serves as "pre-screening," in the sense that the bank rejects the borrower upon receiving an L signal, while upon an H signal it makes a pricing decision based on its soft signal s. In other words, for the specialized lender, the "principal" signal is the one that determines whether to lend, and the "supplementary" one helps its loan pricing.[13] We therefore impose the following parameter restrictions to ensure hard signal being "decisive."

Assumption 1. ***(Decisive Hard Signals)***

1. *Bank A rejects the borrower upon an L hard signal, regardless of any soft signal s:*

$$q_h (1-\alpha)\,\overline{r} < (1-q_h)\,\alpha; \tag{8}$$

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[13] Alternatively, the principal signal represents the result of a credit screening test, while the supplementary signal serves the role of internal ratings (of borrowers who are qualified for credit). This ranking portrays the key role played by hard information for large banks when dealing with new borrowers. Indeed, as documented on page 1677 of Crawford, Pavanini, and Schivardi (2018), Italian large banks list the factors they consider in assessing any new loan applicant's creditworthiness, with the following order of importance: i) hard information from the central bank (financial statement data); ii) hard information from Credit Register; iii) statistical-quantitative methods; iv) qualitative information (i.e., bank-specific soft information codifiable as data); v) availability of guarantees; and vi) first-hand information (i.e., branch-specific soft information).

2. Bank B is willing to participate if and only if its hard signal $h^B = H$:

$$q\alpha\overline{r} > (q_h - q)\,\alpha + (1 - q_h)\,(1 - \alpha)\,. \tag{9}$$

Assumption 1 says that the hard signal has to be sufficiently strong (informative) to serve as pre-screening of loan applications for both lenders. Condition (8) states that it is not profitable for Bank A to compete upon receiving a hard signal L even when the soft signal reveals that the soft fundamental θ_s is good with certainty; this then implies that Bank B with hard signal only also chooses not to compete upon $h^B = L$. Condition (9) states that upon $h^B = H$, Bank B is willing to lend at the highest possible interest rate if it is the monopolist lender. This condition also implies that Bank A with an additional soft signal is willing to lend at $\overline{r}$ if it is the monopolist lender upon $h^A = H$ and some favorable enough soft signal realizations.

1.5 Credit Market Equilibrium Definition

We now formally define the credit market equilibrium with specialized lending, along the line of Blickle, He, Huang, and Parlatore (2024).

Bank strategies. Conditional on the hard signal, we define the space of interest rate offers to be $\mathcal{R} \equiv [0, \overline{r}] \cup \{\infty\}$. Here, $\overline{r}$ is the exogenous maximum interest rate (or project return, see Section 1.1) and ∞ captures the strategy of not making an offer.

For Bank A, we denote its pure strategy by $r^A(s) : \mathcal{S} \to \mathcal{R}$, which induces a distribution of its interest offerings denoted by $F^A(r) \equiv \Pr\left(r^A \leq r\right)$ according to the underlying distribution of the soft signal. (At this point we take as given that Bank A uses pure strategy, though we formally prove this result in Proposition 1). And, the endogenous support of the equilibrium interest rates when making an offer will be shown to be a sub-interval of $[0, \overline{r}]$. Therefore, with a slight abuse of terminology, we refer to that subinterval as the "support" of the interest rate distribution even though loan rejection ($r = \infty$) could also occur along the equilibrium path.

Bank B randomizes its interest rate offerings conditional on a positive hard signal in equilibrium, with an endogenous cumulative distribution $F^B(r) \equiv \Pr\left(r^B \leq r\right)$. Since domain of offers includes $r = \infty$ which captures rejection, it is possible that $F^B(\overline{r}) = \mathbb{P}\left(r^B < \infty | h^B = H\right) \leq 1$.

The borrower picks the lower rate from two competing offers. For instance, conditional on $h^A = h^B = H$, if Bank B quotes $r^B < \infty$, then its winning probability $1 - F^A\left(r^B\right)$ equals the probability that Bank A with soft signal s offers a rate that is higher than r^B; note that this includes the event that Bank A rejects the borrower ($r^A(s) = \infty$), presumably because of an unfavorable soft signal.[14] When $r^A = r^B = \infty$, the borrower fails to get the loan.

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[14] Upon ties, i.e. $r^A = r^B < \infty$, borrowers randomly choose the lender with probability one-half, although the details of the tie-breaking rule do not matter as ties occur as zero-measure events in equilibrium.

Definition 1. (Credit market equilibrium) A competitive equilibrium in the credit market (with decisive hard signals) consists of the following lending strategies and borrower choice:

1. A lender j rejects the borrower or $r^j = \infty$ upon $h^j = L$ for $j \in \{A, B\}$; upon $h^j = H$,
   i) Bank A offers $r^A(s) : [0, 1] \rightarrow \mathcal{R} \equiv [0, \overline{r}] \cup \{\infty\}$ to maximize its expected lending profits given $h^A = H$ and s, which induces a distribution function $F^A(r) : \mathcal{R} \rightarrow [0, 1]$;
   ii) Bank B offers $r^B \in \mathcal{R}$ to maximize its expected lending profits given $g^B = H$, which induces a distribution function $F^B(r) : \mathcal{R} \rightarrow [0, 1]$;
2. The borrower chooses the lowest offer $\min\{r^A, r^B\}$.

As standard, there exists an endogenous lower bound of interest rate $\underline{r} > 0$, so that the two distributions $F^j(\cdot)$, $j \in \{A, B\}$ share a common support $[\underline{r}, \overline{r}]$ (besides ∞ as rejection). The following lemma is standard in the literature and shows that resulting equilibrium strategies in our setting are well-behaved.

Lemma 1. ***(Equilibrium Structure)*** *In any credit market equilibrium*

a. The two lenders' interest rate distributions $F^j(\cdot)$, $j \in \{A, B\}$ are smooth over $[\underline{r}, \overline{r})$, i.e. no gap and atomless, so that they admit well-defined density functions

b. At most only one lender can have a mass point at $\overline{r}$.

2 Credit Market Equilibrium Characterization

We now solve for the credit market equilibrium with specialized lending and potentially overlapping information spans. Blickle, He, Huang, and Parlatore (2024) characterize the credit market equilibrium under two key conditions: i) binary signals are decisive, and ii) the two binary and one continuous signals are conditionally independent when success (i.e., independent conditional on the project's success). Our setting with arbitrary information span satisfies both conditions and therefore can be viewed as a special case of the general information structure in Proposition 4 in Blickle, He, Huang, and Parlatore (2024). For this reason, our exposition of this section will be less formal and instead focus on illustrating the key properties of the equilibrium, especially the differences from the special case of $\eta = 1$ in Blickle, He, Huang, and Parlatore (2024).

2.1 Bank Profits and Optimal Strategies

Joint Distributions of Signals and Posterior

We first define the joint and posterior probabilities of project success $\theta = 1$ for a collection of certain events. Denote by the ordered subscript $\{h^A h^B\} \in \{HH, HL, LH, LL\}$ to the events of the corresponding hard signal realizations, where HL stands for Bank A's (B's) hard signal being H (L). Denote by $\overline{p}_{h^A h^B}$ the joint probability of any collection of hard signal realizations; here, the "bar" indicates "taking the average over all possible soft signal realizations." For instance,

$$\overline{p}_{HH} \equiv \mathbb{P}\left(h^A = H, h^B = H\right) = q_h \alpha^2 + (1-q_h)(1-\alpha)^2. \tag{10}$$

Similarly, we denote by $\overline{\mu}_{h^A h^B}$ the posterior of project success conditional on $h^A h^B$; for instance

$$\overline{\mu}_{HH} \equiv \frac{\mathbb{P}\left(h^A = H, h^B = H, \theta = 1\right)}{\mathbb{P}\left(h^A = H, h^B = H\right)} = \frac{q_h \alpha^2}{q_h \alpha^2 + (1-q_h)(1-\alpha)^2} q_s^s. \tag{11}$$

Competing lenders also need to assess the probabilities of hard signals together with the soft signal. Denote by $p_{h^A h^B}(s)\, ds \equiv \mathbb{P}\left(h^A, h^B, s \in ds\right)$ the joint probability of the two hard signals being $h^A h^B$ and $s \in ds$ (i.e., the soft signal s falls in the interval $(s, s+ds)$). Similarly, $\mu_{h^A h^B}(s)$ denotes the posterior probability of project success (i.e. $\theta = 1$), conditional on the hard signal realizations and the soft signal:

$$\mu_{h^A h^B}(s) = \mathbb{P}\left(\theta = 1 \middle|\, h^A, h^B, s\right) = \frac{\mathbb{P}\left(\theta = 1, h^A, h^B, s \in ds\right)}{\mathbb{P}\left(h^A, h^B, s \in ds\right)}. \tag{12}$$

And, under the multiplicative structure in Eq. (3), project success $\theta = 1$ implies that $\theta_h = \theta_s = 1$, which allows us to derive the joint probability of $\mathbb{P}\left(\theta = 1, h^A, h^B, s \in ds\right)$ as

$$p_{h^A h^B}(s)\, \mu_{h^A h^B}(s) = \underbrace{\mathbb{P}(\theta = 1)}_{q} \cdot \mathbb{P}\left(h^A \,|\theta_h = 1\right) \cdot \mathbb{P}\left(h^B \,|\theta_h = 1\right) \cdot \underbrace{\phi\left(s|\, \theta_s = 1\right)}_{\phi_1(s)}. \tag{13}$$

This result points to conditional independence when success, i.e., all signals, including hard and soft, are independent conditional on project success $\theta = 1$. We will come back to this later when we derive the equilibrium.

Bank A's Strategy

Consider Bank A when it observes a positive hard signal $h^A = H$ and a soft signal s. If Bank A chooses to exit the lending market by quoting $r = \infty$, its expected profits are given by $\pi^A(r = \infty, s) = 0$. If Bank A participates in the lending market by offering $r \in [\underline{r}, \overline{r}]$, its expected profits are

$$\pi^A(r, s) \equiv \underbrace{p_{HH}(s)}_{h^A = h^B = H, s} \underbrace{\left[1 - F^B(r)\right]}_{A \text{ wins}} \left[\mu_{HH}(s)(1+r) - 1\right] + \underbrace{p_{HL}(s)}_{h^A = H, h^B = L, s} \left[\mu_{HL}(s)(1+r) - 1\right], \tag{14}$$

where the first term takes into account the expected payoff conditional on winning the borrower when Bank B participates and the second term accounts for the likelihood that Bank B receives a negative hard signal (as Bank A cannot observe Bank B's hard signal h^B). With probability $p_{HH}(s)$, both banks get favorable hard signals, and Bank A with offer r wins with probability $\left[1 - F^B(r)\right]$, whereas with probability p_{HL} Bank B withdraws and Bank A faces no competition for the borrower. Since Bank B randomizes its strategy upon $h^B = H$, from Bank A's perspective winning the price competition is not informative about the borrower's quality. But, whether Bank B participates in the loan market or not affects Bank A's expected quality of the borrower; this economic force is captured by $\mu_{HH}(s)$ and $\mu_{HL}(s)$ given in (12).

Given the profit function defined above, Bank A's optimal interest rate offering is $r^A(s) \equiv \arg\max_{r \in \mathcal{R}} \pi^A(r, s)$. As shown in Blickle, He, Huang, and Parlatore (2024), Bank A's equilibrium pricing strategy $r^A(s)$ is decreasing in s, hits the interest rate cap $\overline{r}$ when the soft signal worsens, and in general will jump to ∞ for sufficiently low s. Formally, $\hat{s} \equiv \sup\left\{ s | \, r^A(s) = \overline{r} \right\}$; that is to say, $\hat{s}$ is the highest realization of the soft signal such that Bank A quotes $\overline{r}$.[15] And, we define $x \leq \hat{s}$ as the threshold such that $\pi^A(\overline{r}, x) = 0$; that is to say, Bank A rejects the borrower for all $s < x$ so that $x \equiv \sup\left\{ s | \, r^A(s) = \infty \right\}$. Note that $x = \hat{s}$ could occur along the equilibrium path. We therefore can define the inverse function (correspondence) of $r^A(s)$ to be

$$s^A(r) \equiv \begin{cases} r^{A(-1)}(r), & \text{when } r \in [\underline{r}, \overline{r}), \\ [x, \hat{s}), & \text{when } r = \overline{r}, \\ [0, x), & \text{when } r = \infty. \end{cases} \tag{15}$$

And, it is easy to see that $r^A(s) = \overline{r}$ for $s \in [x, \hat{s})$, and $r^A(s) = \infty$ for $s \in [0, x)$. We take the convention that $r^A(x) = \overline{r}$ when $\hat{s}$ coincides with x.

Bank B's Strategy

For the non-specialized lender B a standard winner's curse ensues because the outcome of competition against the specialized Bank A is informative about θ_s. More specifically, besides the possibility of the competitor's unfavorable hard information faced by Bank A, the non-specialized lender B who wins the price competition also infers that $r^A(s) > r^B$, which implies $s < s^A\left(r^B\right)$. Taking these unfavorable inferences into account, Bank B's lending profits when quoting r are

$$\pi^B(r) \equiv \int_0^{s^A(r)} \underbrace{p_{HH}(t)}_{h^A = h^B = H, t} \left[\mu_{HH}(t)(r+1) - 1\right] dt + \underbrace{\overline{p}_{LH}}_{h^A = L, h^B = H} \left[\overline{\mu}_{LH}(r+1) - 1\right]. \tag{16}$$

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[15] Throughout the paper we adopt the convention that $\sup\{\emptyset\} = \inf\{[0,1]\} = 0$.

The first term in Eq. (16) accounts for the event in which Bank A competes and the second term considers the case in which Bank A receives a negative hard signal and does not participate. Note that Bank B infers the project's quality based on the event of "winning the borrower"—this occurs when Bank A receives an unfavorable soft signal realization $t < s^A(r)$. Importantly, since the span of hard and soft information can overlap, the updating is not only about the soft fundamental θ_s (as in Blickle, He, Huang, and Parlatore (2024)), but also about the hard fundamental θ_h.

Therefore, Bank B's strategy $F^B(\cdot)$ maximizes its expected payoff

$$\max_{F^B(\cdot)} \int_{\mathcal{R}} \pi^B(r)\, dF^B(r). \tag{17}$$

With mixed strategies, profit-maximizing Bank B is indifferent between any action on its support.

2.2 Credit Market Equilibrium

We characterize the credit market equilibrium in the proposition below.

Proposition 1. ***(Credit Market Equilibrium)*** *In the credit market equilibrium, Bank A follows a pure strategy as in Definition 1. In this unique equilibrium, lenders reject borrowers upon a negative hard signal realization $h^j = L$ for $j \in \{A, B\}$. Otherwise (i.e., when $h^j = H$), their strategies are characterized as follows:*

1. *Bank A with soft signal s offers*

$$r^A(s) = \begin{cases} \min\left\{ \frac{\pi^B + \int_0^s p_{HH}(t)dt + \overline{p}_{LH}}{\int_0^s p_{HH}(t)\cdot\mu_{HH}(t)dt + \overline{p}_{LH}\overline{\mu}_{LH}} - 1, \overline{r} \right\}, & for \quad s \in [x, 1] \\ \infty, & for \quad s \in [0, x). \end{cases} \tag{18}$$

   The equation pins down $\underline{r} = r^A(1)$, For $s \in (\hat{s}, 1]$ where $\hat{s} = \sup s^A(\overline{r})$, $r^A(\cdot)$ is strictly decreasing with its inverse function $s^A(\cdot) = r^{A(-1)}(\cdot)$.

2. *Bank B makes an offer with cumulative probability given by ($\mathbf{1}_{\{X\}} = 1$ if X holds)*

$$F^B(r) = \begin{cases} 1 - \frac{\int_0^{s^A(r)} t\phi(t)dt}{q_s}, & for \quad r \in [\underline{r}, \overline{r}), \\ 1 - \mathbf{1}_{\{\pi^B = 0\}} \cdot \frac{\int_0^{\hat{s}} t\phi(t)dt}{q_s}, & for \quad r = \overline{r}. \end{cases} \tag{19}$$

   When $\pi^B = 0$, $F^B(\overline{r}) = F^B(\overline{r}^-) \leq 1$ is the probability that Bank B makes the offer (and with probability $\frac{1}{q_s}\int_0^{\hat{s}} t\phi(t)\, dt$ it withdraws by quoting $r^B = \infty$); when $\pi^B > 0$, $F^B(\overline{r}) = 1$ and there is a point mass of $\frac{1}{q_s}\int_0^{\hat{s}} t\phi(t)\, dt$ at $\overline{r}$.

3. The equilibrium Bank B's profit is given by

$$\pi^B = \left[\hat{\pi}^B\left(\overline{r}; s^A\left(\overline{r}\right) = s_A^{be}\right)\right]^+, \tag{20}$$

where s_A^{be} *satisfies* $\hat{\pi}^A\left(\overline{r}, s_A^{be}; F^B\left(\overline{r}\right) = \int_{s_A^{be}}^1 \frac{s\phi(s)dt}{q_s} ds\right) = 0$ *with auxiliary functions* $\hat{\pi}^B(\cdot;\cdot)$ *and* $\hat{\pi}^A\left(\cdot,\cdot;\cdot\right)$ *defined in Appendix.*

Similar to Milgrom and Weber (1982), it is relatively easy to solve for Bank A's equilibrium strategy by invoking Bank B's indifference condition, which says that Bank B makes the same profit π_B across all rates on the support $[\underline{r}, \overline{r}]$. Plugging in $r = r^A(s)$ in Bank B's profit in (16), we have

$$\pi^B\left(r\right) = \underbrace{\left[\int_0^s p_{HH}(t)\mu_{HH}\left(t\right) + \overline{p}_{LH}(t)\overline{\mu}_{LH}\right]}_{\text{borrowers who repay}}\left(1 + r^A(s)\right) - \underbrace{\left(\int_0^s p_{HH}(t)dt + \overline{p}_{LH}\right)}_{\text{lending amount}}. \tag{21}$$

Solving for $r^A(s)$ yields (18) in Proposition 1 which further takes into account of necessary truncation on interest rate cap $\overline{r}$.

Although the derivation of Bank B's equilibrium strategy is more involved, conceptually it is quite simple: B's equilibrium strategy needs to support $r^A\left(\cdot\right)$ in (18) to be Bank A's optimal strategy. Specifically, as shown below, (22) gives Bank A's first-order condition (FOC) that balances the lower probability of winning against a higher payoff from served borrowers, and this holds for $s^A(r)$ so that r is optimal at this signal:

$$F^{B\prime}\left(r\right)\underbrace{p_{HH}(s^A(r))\left[\mu_{HH}(s^A(r))\left(1+r\right)-1\right]}_{A\text{'s marginal borrowers}} = \underbrace{\left[1 - F^B\left(r\right)\right]p_{HH}(s^A(r))\mu_{HH}(s^A(r)) + p_{HL}(s^A(r))\mu_{HL}(s^A(r))}_{A\text{'s existing borrowers}}. \tag{22}$$

On the other hand, to maximize (16), Bank B' FOC is

$$\left[-s^{A\prime}\left(r\right)\right]\cdot\underbrace{p_{HH}\left(s^A(r)\right)\left[\mu_{HH}\left(s^A\left(r\right)\right)\left(1+r\right)-1\right]}_{B\text{'s marginal borrowers}} = \underbrace{\int_0^{s^A(r)} p_{HH}\left(t\right)\mu_{HH}\left(t\right)dt + \overline{p}_{LH}\overline{\mu}_{LH}}_{B\text{'s existing borrowers}}. \tag{23}$$

Similarly, when Bank B marginally cuts its quote, it gets $(-s^{A\prime}\left(r\right))dr$ additional borrowers of quality $\mu_{HH}(s^A\left(r\right))$ given competition (which occurs with probability $p_{HH}(s^A(r))$), and this is exactly offset by the marginal lower payoff from Bank B's existing borrowers.

Two key further steps allow us to derive $F^B(r)$ based on (22)-(23). First, note that both lenders are competing on the same marginal borrower (type), i.e., $p_{HH}(s^A(r))\left[\mu_{HH}\left(s^A(r)\right)(1+r)-1\right]$; so we can cancel this term. Second, conditional independence when success, i.e., all signals are independent conditional on project success $\theta = 1$,[16] implies that

$$p_{HL}\left(s^A(r)\right)\mu_{HL}\left(s^A(r)\right) = \frac{1-\alpha}{\alpha} p_{HH}\left(s^A(r)\right)\mu_{HH}\left(s^A(r)\right), \tag{24}$$

so the second term on right hand side of (22) only depends on the event of $\left\{h^A = h^B = H, s = s^A(r)\right\}$. Applying these two steps, we obtain

$$F^{B\prime}(r)\left[\int_0^{s^A(r)} p_{HH}(t)\,\mu_{HH}(t)\,dt + \overline{p}_{LH}\overline{\mu}_{LH}\right] = -s^{A\prime}(r)\left[\frac{1}{\alpha} - F^B(r)\right] p_{HH}\left(s^A(r)\right)\mu_{HH}\left(s^A(r)\right),$$

which can be simplified further to[17]

$$\frac{d}{dr}\overbrace{\left[\frac{\frac{1}{\alpha} - F^B(r)}{\int_0^{s^A(r)} p_{HH}(t)\,\mu_{HH}(t)\,dt + \overline{p}_{LH}\overline{\mu}_{LH}}\right]}^{\text{equals a constant}} = 0. \tag{25}$$

Therefore the term inside the bracket of (25) has to be independent of r, allowing us to derive Bank B's equilibrium strategy (19) in Proposition 1 after imposing proper boundary conditions.

Finally, the equilibrium characterization point 3) in Proposition 1 highlights a key difference between the two types of equilibrium: one with $\pi^B = 0$—we call it the zero-weak equilibrium—and the other with $\pi^B > 0$ so that only Bank B places a positive mass on the interest rate cap $\overline{r}$—we call it the positive-weak equilibrium as the weak bank earns positive profits. In the zero-(positive-)weak equilibrium, only Bank A (Bank B) places a positive mass on the interest rate cap $\overline{r}$. This captures the competition at the interest rate cap $\overline{r}$, exactly reflecting point c) in Lemma 1—otherwise, lenders will undercut each other at this point. We will soon show that, as information span η increases (due to the hardening of soft information), the non-specialized lender benefits more and a positive-weak equilibrium is more likely to arise.

3 Credit Market Competition Equilibrium

Our model is designed to understand how changes in different aspects of information technology affect the equilibrium in the credit market, highlighting the differences between hardening soft

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[16] Formally, because of the multiplicative structure in (2), we have

$$\mathbb{P}\left(h^A, h^B, s \in ds \middle| \theta = 1\right) = \mathbb{P}\left(h^A, h^B, s \in ds \middle| \theta_h = \theta_s = 1\right) = \mathbb{P}\left(h^A \middle| \theta_h = 1\right)\cdot\mathbb{P}\left(h^B \middle| \theta_h = 1\right)\cdot\mathbb{P}\left(s \in ds \middle| \theta_s = 1\right).$$

Relating to (13), it implies that $\frac{p_{HL}(s)\mu_{HL}(s)}{p_{HH}(s)\mu_{HH}(s)} = \frac{\mathbb{P}(h^B = L|\theta=1)}{\mathbb{P}(h^B=H|\theta=1)} = \frac{1-\alpha}{\alpha}$, i.e., (24). Intuitively, conditional on success ($\theta = 1$), for Bank A seeing signal s does not affect the likelihood ratio of its opponent to receive H or L hard signals. This is not true conditional on project failure.

[17] Based on the intuition that two asymmetrically informed lenders are competing on the same marginal borrower and conditional independence when success (which are the two key steps mentioned above), Blickle, He, Huang, and Parlatore (2024) offer a detailed explanation on why this key ODE (25) holds in equilibrium.

information (an increase in the span of hard information) and shifts in other characteristics of information technology, such as the precision of signals. In this section, we first illustrate how the information span η affects the equilibrium strategies and profits of both lenders using several numerical examples. We then contrast how bank profits respond to an increase in the span of hard information and to an increase in its precision, and offer an explanation of these differences by exploring the distinct effects of two technology parameters on the inference problem of Bank B about its opponent's information advantage. Finally, we focus on the implications that an increase in the span of the hard signal has on the allocation of credit and welfare.

3.1 Information Span and Equilibrium Illustration

Figure 2 illustrates how the credit market equilibrium responds to changes in the span of hard information η. For ease of exposition, we assume that Bank A's soft signal s is obtained from observing a noisy version of θ_s, i.e., $\theta_s + \epsilon$, so that

$$s = \mathbb{E}\left[\theta_s | \theta_s + \epsilon\right]. \tag{26}$$

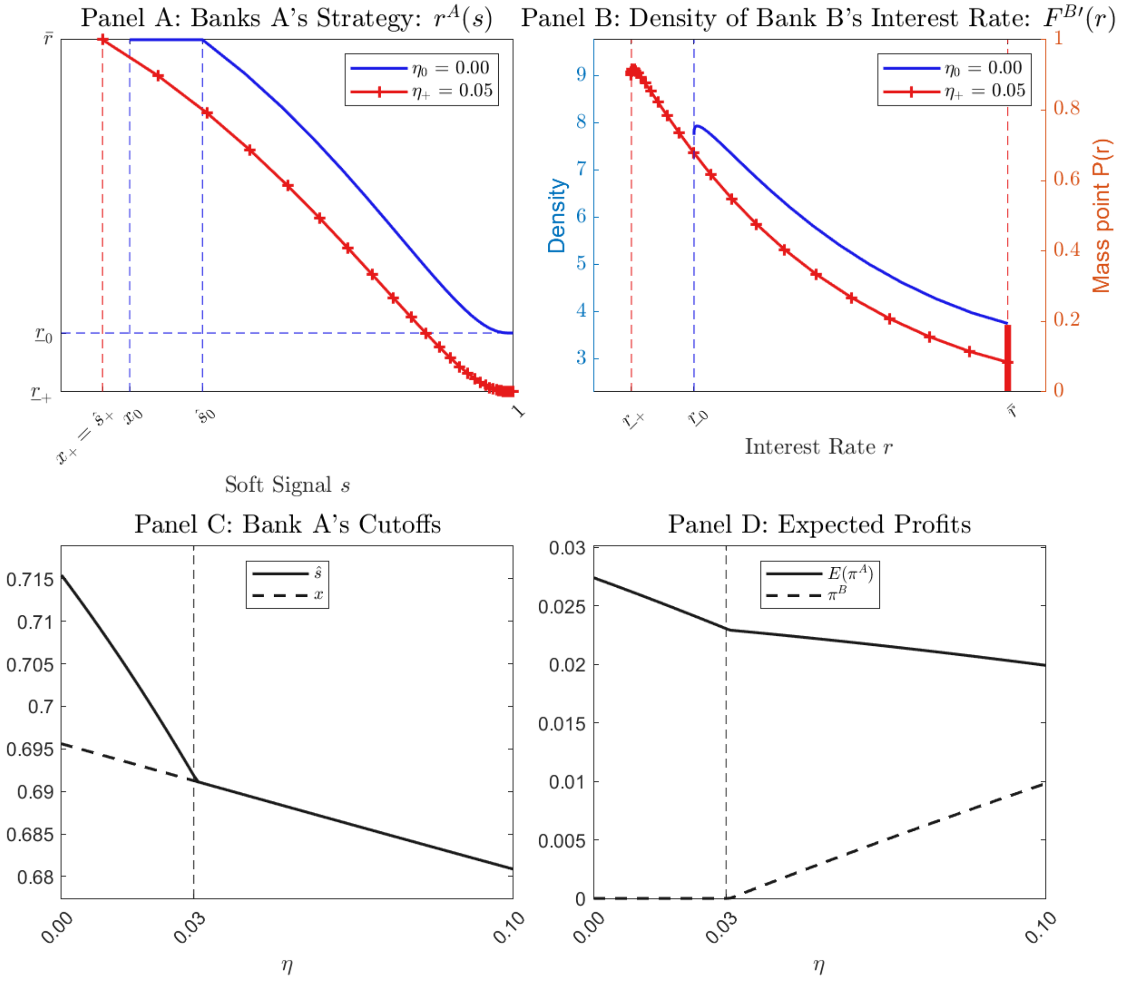
Here, $\epsilon \sim \mathcal{N}\left(0, 1/\tau\right)$ indicates white noise, with the precision parameter τ capturing the signal-to-noise ratio of Bank A's soft information technology.

The top two panels in Figure 2 plot both lenders' pricing strategies conditional on making an offer, with Panel A plotting Bank A's $r^A(s)$ as a function of s and Panel B the density $F^{B\prime}(r)$ as a function of r for Bank B. We plot equilibrium pricing strategies for two levels of information span η: the baseline $\eta_0 = 0$, and a higher $\eta_+ = 0.05$. As suggested by Proposition 1, we will show that a positive-(zero-) weak equilibrium arises when η is relatively high (low); and this is why we use the subscript "+" for the larger η.

Overall, consistent with the premise that hardening soft information levels the playing field for the non-specialized lender in our model, Bank B with a greater η becomes more aggressive as its distribution of offered rates shifts downward (panel B), resulting in a lower equilibrium lower bound $\underline{r}_+ < \underline{r}_0$. We observe that the entire curve $r^A(s)$—which decreases in s as the specialized Bank A with a more favorable soft signal bids more aggressively——shifts downward in response to the more aggressive bidding by Bank B.

Panel C plots the two soft signal cut-offs for the specialized Bank A, i.e., $\hat{s}$ at which it starts quoting $\overline{r}$ and x at which it starts rejecting the borrower. For a sufficiently large η, $\hat{s}$ and x coincide reflecting a zero probability mass on the interest rate cap $\overline{r}$. Finally, Panel D plots the expected profits—$\mathbb{E}(\pi^A)$ and π^B—for two lenders; when η increases, the non-specialized lender becomes relatively stronger, leading to a strictly positive π^B as shown in Panel D.

To piece all panels together, consider the competition at the interest rate $\overline{r}$. As shown in Panel A-B, for a low information span $\eta_0 = 0$ so that $\pi^B = 0$ in equilibrium, Bank A has a point mass at


Panel A: Banks A's Strategy: $r^A(s)$

Panel B: Density of Bank B's Interest Rate: $F^{B\prime}(r)$

Panel C: Bank A's Cutoffs

Panel D: Expected Profits

Figure 2: **Equilibrium strategies and profits for information span** η**.** Panel A depicts $r^A(s)$ as a function of s and Panel B plots $F^{B\prime}(r)$ as a function of r; strategies for $\eta_+ = 0.05$ are depicted in red with markers while strategies with $\eta_0 = 0$ are depicted in blue. Panel C depicts Bank A's thresholds $\hat{s} = \sup s^A(\bar{r})$ and $x = \sup s^A(\infty)$, and Panel D depicts the expected profits for two lenders, both as a function of η. Priors q_h, q_s^s and information span η satisfy $q_h = q/q_s \cdot (1 - \eta)$ and $q_s^s = q_s/(1-\eta)$. Parameters: $\bar{r} = 0.36$, $q = 0.72$, $q_s = 0.9$, $\alpha = 0.7$, and $\tau = 1$.

$\bar{r}$ (corresponding to $s \in (x, \hat{s})$ as in Panel C) but Bank B does not, while for a high $\eta_+ = 0.05$ so that $\pi^B > 0$, the opposite holds. Intuitively, thanks to the big data technology that hardens soft information, a sufficiently large η leads to a positive-weak equilibrium where the non-specialized Bank B places a point mass on $\bar{r}$, enjoying some "local monopoly power" as it is the only lender when Bank A rejects the borrower upon $s < \hat{s} = x$. Importantly, this is still profitable for Bank B: for a sufficiently large η, the non-specialized Bank B faces a relatively minor winner's curse due to the opponent's soft signals (see later discussion in this section). In contrast, for a smaller span η, we are in a zero-weak equilibrium, where the specialized Bank A places a point mass on this

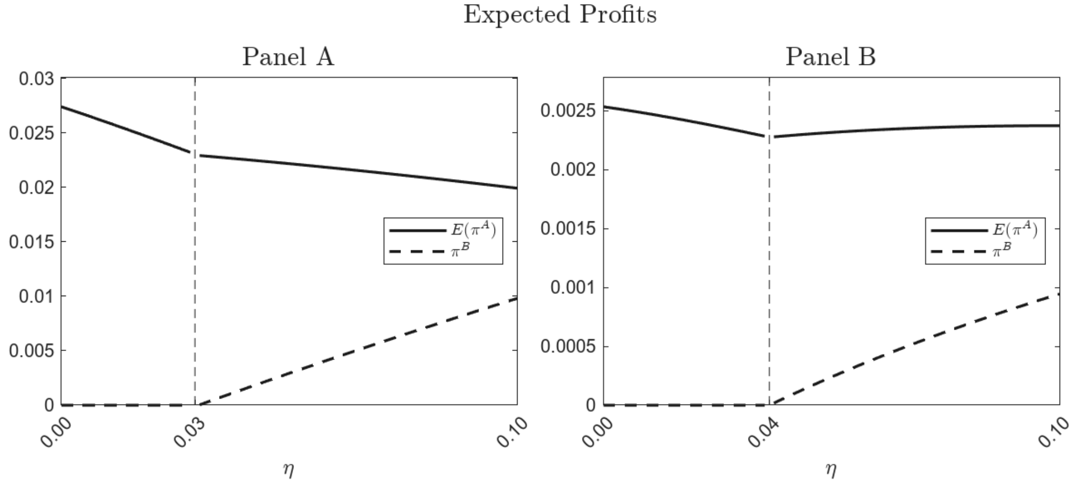


Figure 3: **Expected lender profits.** Panel A and Panel B show expected lender profits as a function of the span of hard information η under different primitive settings. The solid lines correspond to Bank A while the dashed lines correspond to Bank B. Priors q_h, q_s^s and information span η satisfy $q_h = q/q_s \cdot (1-\eta)$ and $q_s^s = q_s/(1-\eta)$. Parameters: Panel A, $\bar{r} = 0.36$, $q = 0.72$, $q_s = 0.9$, $\alpha_u = \alpha_d = \alpha = 0.7$, $\tau = 1$; Panel B, $\bar{r} = 0.33$, $q = 0.72$, $q_s = 0.9$, $\alpha_u = \alpha_d = \alpha = 0.6$, $\tau = 0.1$.

interest rate (when $s \in (x, \hat{s})$, as shown in Panel C) while the non-specialized Bank B withdraws.

Last but not least, it is important to recognize that Bank A's profits can also increase with the information span η in the range of positive-weak equilibrium parameters. Figure 3 shows such an example in which Bank A's expected profits increase with η (Panel B) and contrasts it with the case in which the opposite holds (Panel A, which we copy from Panel D in Figure 2). This example highlights the same technological improvement for the specialized and non-specialized banks. Comparing the parameters that lead to these two cases, $\mathbb{E}(\pi^A)$ is more likely to increase with η when the precision of hard information is low. When the precision of the hard signal α is relatively low, the credit market is less competitive as lenders bid less aggressively due to the high uncertainty in screening. When the precision of the soft signal τ is relatively low, Bank A, who initially has an imprecise soft signal about the soft fundamental θ_s, benefits more because hardening of the soft information also helps to learn about θ_s. Hence, Bank A's profits increase as technological improvement dominates the intensified competition from Bank B. Our welfare analysis later in Section 3.3 show that in this scenario likely every agent in the entire economy enjoys a higher surplus (hence a Pareto improvement).

3.2 Bank Profits: Information Span vs. Information Precision

A key advantage of our model is that it allows us to distinguish between different aspects of information technology. In this section, we first explain how the span of hard information, i.e., η, affects the Type I and Type II errors faced by each bank—especially from the perspective of the relatively weak Bank B. We then show that in general the information span η improves Bank B's equilibrium profit, and compare this to that of changes in the precision of hard signals (α). Finally, we illustrate that in credit market competition, (hard) information span η differs from (hard) information precision α because they have drastically different implications given asymmetrically informed competitors.

Span of Hard Information

The information span η determines the extent of overlap between the hard and soft fundamentals, which determines the correlation between hard and soft signals. When $\eta = 0$, zero overlap between hard and soft fundamentals implies independence between hard and soft signals, while hard and soft fundamentals (and the signals about them) become correlated when $\eta > 0$. For illustration, we consider the effect of η on the belief of weak Bank B' by focusing on two particular events.

Two positive hard signals. We start with the event $\{H, H, s\}$, where two lenders receive positive hard signals, (potentially) competing against each other, and Bank A receives a soft signal s. The probability of this event is

$$p_{HH}(s) = \underbrace{q\alpha^2\phi_1(s)}_{\theta=1} + \underbrace{\left(1-q_h^h\right)(1-\alpha)^2\phi(s)}_{\theta_h^h=0} + \underbrace{q_h^h}_{\theta_h^h=1}\left[\underbrace{\left(1-q_s^h\right)}_{\theta_s^h=0}(1-\alpha)^2 + \underbrace{\left(q_s^h-q_s\right)}_{\theta_s^h=1,\theta_s^s=0}\alpha^2\right]\phi_0(s). \tag{27}$$

The first term captures the probability of the event $\{H, H, s\}$ when the project is good ($\theta = 1$; this necessarily implies that $\theta_h^h = \theta_s^h = \theta_s^s = 1$), which occurs with probability q. This event is independent of the information span η as hard and soft signals are conditionally independent when the project is good (see footnote 16). The remaining two terms refer to the cases in which the project is bad ($\theta = 0$), which can occur when one of the states θ_h^h, θ_s^h, or θ_s^s takes a value of zero.

The second term captures the events with $\theta_h^h = 0$ and is independent of the span η. This is because, given the multiplicative structure, $\theta_h^h = 0$ already determines that the hard fundamental state is $\theta_h = 0$, and broader hard signals that assess more characteristics do not add information content. Note that while the likelihood of HH when $\theta = 1$ or $\theta_h^h = 0$ (i.e., the first or second term in (27)) is independent of η, both terms depend on the precision of the hard signal because α determines the conditional probability of receiving a hard signal H.

The third term in Eq. (27) captures the novelty of our modeling, i.e., how the hardening of soft information affects the likelihood that the two banks compete in the credit market. Given $\theta_h^h = 1$, this term includes two scenarios: i) when $\theta_s^h = 0$, both the hard fundamental and the soft fundamental fail $\theta_h = \theta_s = 0$, so HH occurs with probability $(1-\alpha)^2$ and the soft signal density is $\phi_0(s)$; and ii) when $\theta_s^h = 1$ but $\theta_s^s = 0$, the hard fundamental succeeds $\theta^h = 1$ (so HH occurs with probability α^2) but the soft fundamental fails (the soft signal density is $\phi_0(s)$).

Using $\eta = 1 - q_s^h$ and simplifying terms, we can rewrite the joint probability of $\{H, H, s\}$ as:

$$p_{HH}(s) = q\alpha^2 \phi_1(s) + \left(1 - q_h^h\right)(1-\alpha)^2 \phi(s) + \underbrace{\left[(1 - q_s)\alpha^2 - \eta(2\alpha - 1)\right]}_{\downarrow \text{ in } \eta \text{ as } \alpha > \frac{1}{2}} q_h^h \phi_0(s). \tag{28}$$

The last term in Eq. (28) delivers the key message that a broader information span reduces the probability of the competition scenario HH when soft fundamentals are unsuccessful $\theta_s = 0$. Before soft information hardens, i.e., when $\eta = 0$, the state θ_s is partially discernible only through the soft signal s. Technological advances that harden soft information, i.e., increases in η, allow lenders to learn about θ_s from hard signals. When competing in the credit market, an increase in η affects how lenders update their beliefs, especially for the non-specialized lender B who does not observe a direct signal of s but understands that competition occurs in the event of HH. As the overlapping state θ_s^h generates a positive correlation between soft and hard signals, two positive hard signal realizations (the event of competition under HH) lead Bank B to update its beliefs about the opponent lender's soft signal distribution upward when soft information is hardened.

To further illustrate this point, we compute $\phi(s|HH)$, i.e., the conditional density of s given HH, i.e., the event where lenders compete. To make the point clearer, we set $q_h^h = 1$ so that $q = q_s$ (so hard signals only reflect the overlapping state θ_s^h), and the resulting conditional density of the soft signal s is

$$\begin{aligned}\phi(s|HH) &= \frac{q_s\alpha^2\phi_1(s) + \left[\left(1 - q_s^h\right)(1-\alpha)^2 + \left(q_s^h - q_s\right)\alpha^2\right]\phi_0(s)}{(1 - q_s^h)(1-\alpha)^2 + q_s^h\alpha^2} \\ &= \phi_0(s) + \overbrace{\frac{\alpha^2}{\alpha^2 - (2\alpha - 1)\eta}}^{\uparrow \text{ in } \eta \text{ as } \alpha > \frac{1}{2}} \cdot q_s\left[\phi_1(s) - \phi_0(s)\right].\end{aligned} \tag{29}$$

Without hardening soft information ($\eta \to 0$) as in Blickle, He, Huang, and Parlatore (2024), independent hard and soft signals imply that

$$\phi(s|HH) = (1 - q_s)\phi_0(s) + q_s\phi_1(s) = \phi(s). \tag{30}$$

When hard information broadens, $\phi(s|HH)$ puts more weight on the favorable distribution $\phi_1(s)$, as suggested by the higher coefficient $\frac{\alpha^2}{\alpha^2 - (2\alpha-1)\eta}$ in (29). Put differently, given the monotone likelihood ratio property, we know $\phi_1(s) - \phi_0(s) > 0$ for relatively high soft signals. Hence, the conditional density $\phi(s|HH)$ increases with η for high values of s implying that a favorable soft signal is more likely to arise (upon HH) when the span of hard information is broader. In contrast, for relatively low soft signals with $\phi_1(s) - \phi_0(s) < 0$, the opposite occurs, so the conditional density

$\phi(s|HH)$ decreases with η. (This MLRP property will be used formally in the next section.)

To summarize, that $\phi(s\,|HH\,)$ increases in η implies that a larger information span reduces the winner's curse for Bank B from Bank A's private soft signal, suggesting the former to benefit more from technological advancements that harden soft information. We will come back to this point later in this section when we compare information span η to information precision α.

Opposite hard signals. What if two lenders receive opposite hard signals, i.e., when one bank's hard signal is positive while the opponent lender's signal is negative? As illustrated in the lenders' profits π^A in (14) and π^B in (16), these events represent the critical economic force behind the "Winner's Curse" in models with hard signals only (Broecker, 1990; He, Huang, and Zhou, 2023).[18] Going through steps similar to (27), one can calculate the probabilities for these events as

$$p_{HL}(s) = p_{LH}(s) = \alpha(1-\alpha)\,\phi(s)\,. \tag{31}$$

Integrating over s, we have that the probability of opposite hard signals is $\overline{p}_{HL} = \overline{p}_{LH} = \alpha(1-\alpha)$, which depends only on the signal precision α, but not the information span η.

The effect of α is natural: when α increases, hard signals become more precise, h^A and h^B become more correlated, and opposite hard signals $h^A \neq h^B$ are less likely to arise. The observation that the information span η does not enter (31) relies on the symmetry of the hard information technology (i.e., same Type I and II errors). Intuitively, this symmetry implies that independent of the realization of θ^h_s, the probability of $h^A \neq h^B$ is always $\alpha(1-\alpha)$. Because no information about the fundamental is revealed from the disagreement events HL or LH, the distribution of the soft signal conditional on opposite hard signals remains the unconditional one:

$$\phi(s\,|HL\,) = \phi(s\,|LH\,) = \phi(s)\,. \tag{32}$$

This property facilitates our analytical proof and sharpens our core economic mechanism.

Information Span and Bank Profits

We now formally show that an enlarged information span—i.e., a greater η—levels the playing field by benefiting the non-specialized Bank B relatively more than the specialized Bank A. The following proposition states our result.

Proposition 2. ***(Hardening Soft Information on Equilibrium Profits)***

1. *The equilibrium profits of the non-specialized lender π^B are increasing in η.*

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[18] In our model, as in Blickle, He, Huang, and Parlatore (2024), the non-specialized lender B faces a winner's curse even in the event of HH because of the soft signal received by the specialized lender only.

2. *In the region of positive-weak equilibrium, the impact of η on Bank B's profits dominates that on Bank A's profits:*

$$\frac{d\pi^B}{d\eta} > \frac{d}{d\eta}\mathbb{E}\left[\pi^A\right]. \tag{33}$$

Part 1) in Proposition 2 suggests that there exists a cutoff $\hat{\eta}$ so that when $\eta > \hat{\eta}$ the credit market features a positive-weak equilibrium with $\pi^B > 0$. When the information span is limited ($\eta < \hat{\eta}$), Bank A maintains a substantial information advantage and enjoys local monopoly power (bidding $\overline{r}$ when $s \in (x, \hat{s})$). In this range, when η increases, Bank B's equilibrium profits stay at zero as heightened competition exactly offsets the gains from technology. Once η rises above the threshold $\hat{\eta}$, Bank A's information advantage shrinks to the extent that it loses the local monopoly power and becomes the break-even lender when receiving a sufficiently low signal. In this case, for Bank B, the technological advancement dominates the increase in competition and starts making positive profits.

Part 2) Proposition 2 concerns the differential effect of η on two lenders' profits. To better connect with the key economic intuition illustrated by $\phi\left(s\,|HH\right)$ in (29), we provide a key proof step here. In Appendix A.3, we show that it is sufficient to study the profit wedge between Bank B and Bank A conditional on $s \in [\hat{s}, 1]$:

$$\Delta\pi(s;\eta) \equiv \pi^B\left(r^A\left(s\right)\right) \cdot \overbrace{\phi_1\left(s\right)}^{\frac{d\left[1-F^B\left(r^A(s)\right)\right]}{ds}=\frac{s\phi(s)}{q_s}} - \pi^A\left(r^A\left(s\right), s\right) \tag{34}$$

$$= -\left[\phi_1\left(s\right)\int_0^s p_{HH}\left(t\right)dt - p_{HH}\left(s\right)\int_0^s \phi_1\left(t\right)dt\right] - \left[\phi_1\left(s\right)\overline{p}_{LH} - p_{HL}\left(s\right)\right]. \tag{35}$$

In (34) we have multiplied Bank B's profit π^B (evaluated at $r^A(s)$) by the density of its equilibrium offer in Eq. (19); this way, it is comparable to Bank A's profit (14) which incorporates the density of s. The gist of Eq. (35) is that the profit differential depends on η only through the banks' lending amounts (which enter profits negatively and are related to Type II errors); in the Appendix A.3, we show lending revenues of both lenders are equal, because under profit maximization, a lender's existing revenue equals the value of the marginal borrower for whom it is competing, and lenders compete for the same marginal borrower in equilibrium.

As explained after Eq. (31), η does not affect the probability of lenders receiving opposite hard signals, captured by the second term in parentheses in Eq. (35). Therefore, we focus on the first term that captures the event of both lenders receiving positive hard signals and (potentially) competing for the borrower. Using $p_{HH}\left(t\right) = \phi\left(t|HH\right)\overline{p}_{HH}$ and $\phi\left(t|HH\right)$ in (29), we derive that

$$p_{HH}\left(s\right)\int_0^s \phi_1\left(t\right)dt - \phi_1\left(s\right)\int_0^s p_{HH}\left(t\right)dt = K\left(\eta\right) \cdot \underbrace{\int_0^s \phi_0\left(s\right)\phi_0\left(t\right)\left[\frac{\phi_1\left(t\right)}{\phi_0\left(t\right)} - \frac{\phi_1\left(s\right)}{\phi_0\left(s\right)}\right]dt}_{\text{independent of } \eta\text{, negative because of MLRP}}. \tag{36}$$

Appendix A.3 shows that $K(\eta) \equiv \overline{p}_{HH}(\eta) \cdot \left[1 - \frac{\alpha^2 q_s}{\alpha^2-(2\alpha-1)\eta}\right]$; it captures Type II errors and can be decomposed into the hard signal part—$\overline{p}_{HH}(\eta)$ where we emphasize its dependence on η, and the remaining soft signal part—$\phi\left(s|HH\right)$'s loading on unfavorable $\phi_0(s)$ given in Eq. (29). Intuitively, a broader hard information span (higher η) reduces Type II errors. Specifically, both terms in $K(\eta)$ decrease in η because lenders screen out more lemons in the competition case—a smaller $\overline{p}_{HH}$, and competition itself suggests the Bank A's soft signal is more likely to be associated with good fundamentals—$\phi\left(s|HH\right)$ puts less weight on $\phi_0(s)$. Finally, the second term in Eq. (36) is independent of η and negative thanks to MLRP of $\phi_1(\cdot)$ with respect to $\phi_0(\cdot)$. This term suggests that, the non-specialized Bank B who is subject to the winner's curse due to the opponent's soft signal, benefits more from Type II error reduction. Therefore, (35) increases with η and Bank B benefits more from hardening soft information.

Information Precision and Bank Profits

The economic implications of changes in information precision in our model are drastically different from those coming from changes in the span of hard information. Two parameters capture the information precision, one being the hard signal precision α and the other the soft signal precision τ. It is quite transparent that an increase in the soft signal precision τ gives a greater advantage to the specialized lender, opposite to the effect of a greater η which levels the playing field for the non-specialized lender.

The effect of a hard signal precision α is a bit more involved and in general non-monotone. To understand the non-monotonicity, it is useful to consider two extreme cases. In an auction setting with asymmetric bidders, the uninformed bidder makes zero profit (Milgrom and Weber, 1982). When $\alpha = 0.5$ so that the hard signal is completely uninformative,[19] the model is identical to Milgrom and Weber (1982) where the uninformed lender B ignores the realization of h^B, randomizes its bids, and makes zero profit in equilibrium. On the other extreme case $\alpha = 1$, hard information becomes a public signal and we are back to Milgrom and Weber (1982) again upon the realization $h^A = h^B = H$ and updated prior, and Bank B still makes zero profits. In general, for intermediate values of $\alpha \in (0, 1)$, a positive-weak equilibrium (with $\pi^B > 0$) could arise.

This non-monotonicity with respect to α is qualitatively different from the monotonicity with respect to η shown in Proposition 2. The following proposition provides a formal counterpart to Proposition 2 in the vicinity of $\alpha = 1$. We focus on this extreme case not only because it is more analytically tractable but more importantly because our analysis rests on the assumption that hard signals are decisive (Assumption 1).

Proposition 3. ***(Hard Signal Precision on Bank Profits.)*** *When $\alpha \to 1$, we have*

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[19] Although this limiting case violates Assumption 1 which requires hard signals to be sufficiently strong, we have a well-defined equilibrium in this case where both lenders ignore the hard signals.

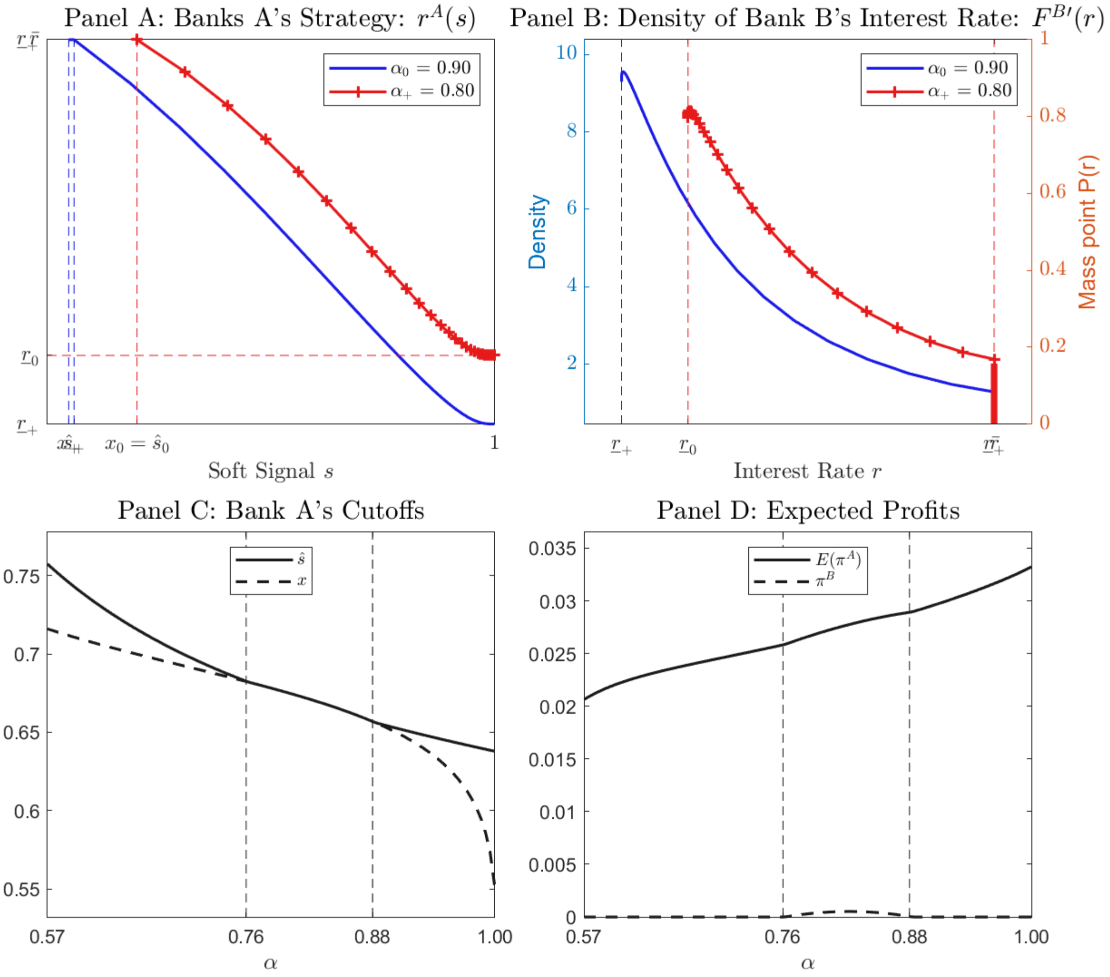

Figure 4: **Equilibrium strategies and profits for hard signal precision α.** Panel A depicts $r^A(s)$ as a function of s and Panel B plots $F^{B\prime}(r)$ as a function of r; strategies for $\alpha_+ = 0.8$ are depicted in red with markers while strategies with $\alpha_0 = 0.9$ are depicted in blue. Panel C depicts Bank A's thresholds $\hat{s} = \sup s^A(\bar{r})$ and $x = \sup s^A(\infty)$, and Panel D depicts the expected profits for two lenders, both as a function of α. Parameters: $\bar{r} = 0.36$, $q = 0.72$, $q_s = 0.9$, $\eta = 0.02$, and $\tau = 1$.

1. *The equilibrium profit of non-specialized lender $\pi^B \to 0$, i.e., a zero-weak equilibrium arise.*

2. *Suppose that $q_h > 0.5$. In the vicinity of $\alpha \to 1$, the impact of α on Bank A's profit dominates that on Bank B's profit:*

$$\frac{d}{d\alpha}\mathbb{E}\left[\pi^A\right] > \frac{d\pi^B}{d\alpha} = 0. \tag{37}$$

In the above proposition, part 1) naturally follows from Milgrom and Weber (1982) given the discussion above. Part 2) makes a further theoretical point: in the vicinity of $\alpha = 1$, an increase in hard signal precision helps Bank A gain more profits. To see this, consider the profit wedge as in Eq. (35). Suppose that the hard fundamental prior is relatively high and $q_h > 0.5$, which is empirically relevant.[20] When α increases so hard signals become more precise, lenders are more likely to compete ($h^A = h^B = H$) than disagree and not compete ($h^A \neq h^B$). This tilting towards competition effectively increases the winner's curse that Bank B suffers from due to Bank A's soft signal. Hence, Bank A benefits more from increases in hard signal precision and its equilibrium profit improves.

Figure 4 displays the same variables as Figure 2, plotting the comparative statics on the precision of the hard signal α. First, Panels A and B illustrate the lenders' equilibrium pricing strategies, showing that lenders set more aggressive rates (lower rates) for $\alpha_+ < \alpha_0$. When α increases from $\alpha_+ = 0.8$ to $\alpha_0 = 0.9$, both lenders are competing more fiercely by quoting lower interest rates, so the equilibrium turns from positive-weak to a zero-weak (this is why we call the larger α as α_0). However, as illustrated in Panel D, the non-specialized lender B's profits π^B are non-monotone in α. This aligns with the discussion preceding Proposition 3 that $\pi^B = 0$ at the two limiting cases, $\alpha = \frac{1}{2}$ or 1. In Panel C, the cutoff strategies of Bank A generally decrease as α increases; this reflects the standard learning effect—Bank A, receiving a more accurate positive signal, withdraws at a weaker soft signal. Notably, $\hat{s}$ and x coincide for mid-values of α, which is consistent with the non-monotonicity of π^B.

Span vs. Precision: How do They Affect Lenders' Beliefs?

The information technology in any information economics model translates signals into posterior distributions of unobservables, as these serve as sufficient statistics of any signals received by the agents. In our setting of credit market competition, besides learning about the fundamentals (various θ's) from their own signals, each lender—especially the relatively weaker, non-specialized Bank B—is also concerned with its stronger opponent's superior information regarding the fundamentals. Importantly, this shapes the Winner's Curse and hence the equilibrium bidding strategies. Because the information span η and the information precision α affect each lender's signals and their correlations differently, these two information technology parameters drive each lender's beliefs about fundamentals and their corresponding distributions in drastically different ways.

To highlight the winner's curse, our discussion primarily focuses on the perspective of the non-specialized Bank B who has received $h^B = H$ and therefore consider bidding for the loan. We first consider the "disagreement" event in which Bank A receives a negative hard signal and therefore withdraws from competition. Conditional on $h^B = H$, the probability of this event, which we denote by $\mathbb{P}(LH|\, h^B = H)$, is

$$\mathbb{P}\left(LH|h^B = H\right) = \frac{\alpha(1-\alpha)}{q_h\alpha + (1-q_h)(1-\alpha)} = \frac{1}{q_h^h(1-\eta)\frac{(2\alpha-1)}{\alpha(1-\alpha)} + \frac{1}{\alpha}}. \tag{38}$$

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[20] This parameter is empirically relevant because, in the data, the non-performing loan rate—which is about 5% as documented in Blickle, He, Huang, and Parlatore (2024)—is quite low.

The top two panels of Figure 5 plot (38) as functions of η and α, respectively.

Importantly, the symmetric structure of hard information implies that Bank B's belief about the project quality, conditional on $\{LH\}$, remains at its prior $\mathbb{E}[\theta] = q$, independent of either η or α. However, these two technology parameters have opposite economic effects on the likelihood of competition that Bank B faces when receiving $h^B = H$. As shown in Panel I_{η}^{B}, Bank B knows that as η increases, it becomes more likely that Bank A receives a negative hard signal and withdraws. This comparative static result can be shown formally.[21] Intuitively, when η increases so that hard signals cover more fundamentals, the multiplicative structure implies a lower probability of a positive signal, so lenders are less likely to compete. Taking into account that a higher η in equilibrium leads Bank B more profitable (Proposition 2), this effect potentially helps Bank B in the credit market.

The economic effects of changes in precision α are different. As screening becomes more precise about the underlying state, lenders' hard signals become more unconditionally correlated, and they are less likely to disagree. This force leads Bank B, when receiving $h^B = H$, to know that competition from Bank A is more likely, which contrasts with the comparative statics in Panel I_{η}^{B}.[22]

Now consider the "agreement" event in which Bank A competes after receiving $h^A = H$, along with a soft signal s. For Bank B, what is the implied fundamental when taking into account the opponent's potential information? We study these beliefs about two fundamental states: i) the overall fundamental θ, which determines the ultimate loan repayment probability; and ii) the "always soft" fundamental θ_s^s, which only Bank A can assess with a soft signal. Speficically, in these events, the posterior mean of θ_z (i.e., either θ or θ_s^s) conditional on $\left\{h^A = h^B = H, s\right\}$, can be calculated as (with detailed expressions given in Appendix A.5).

$$z(HHs) \equiv \mathbb{E}\left[\theta = 1 \middle| h^A = h^B = H, s\right] = \frac{\mathbb{P}\left(\theta = 1, h^A = h^B = H, s\right)}{p_{HH}(s)}, \tag{39}$$

$$z_s^s(HHs) \equiv \mathbb{E}\left[\theta_s^s = 1 \middle| h^A = h^B = H, s\right] = \frac{\mathbb{P}\left(\theta_s^s = 1, h^A = h^B = H, s\right)}{p_{HH}(s)}. \tag{40}$$

For illustration, we plot $z(HHs)$ in (39) as a function of s for various levels of η and α in Panels II_{η}^{A} and II_{α}^{A}, respectively. These posterior means of project quality are observed by Bank A depending on the soft signal s it receives (and taking into account of Bank B's hard signal conditional on competing). Not surprisingly, $z(HHs)$ increases with the soft signal s. We emphasize that both technology parameters improve Bank A's learning, in that fixing any signal s, the larger the η or α, the higher the posterior quality of the project.

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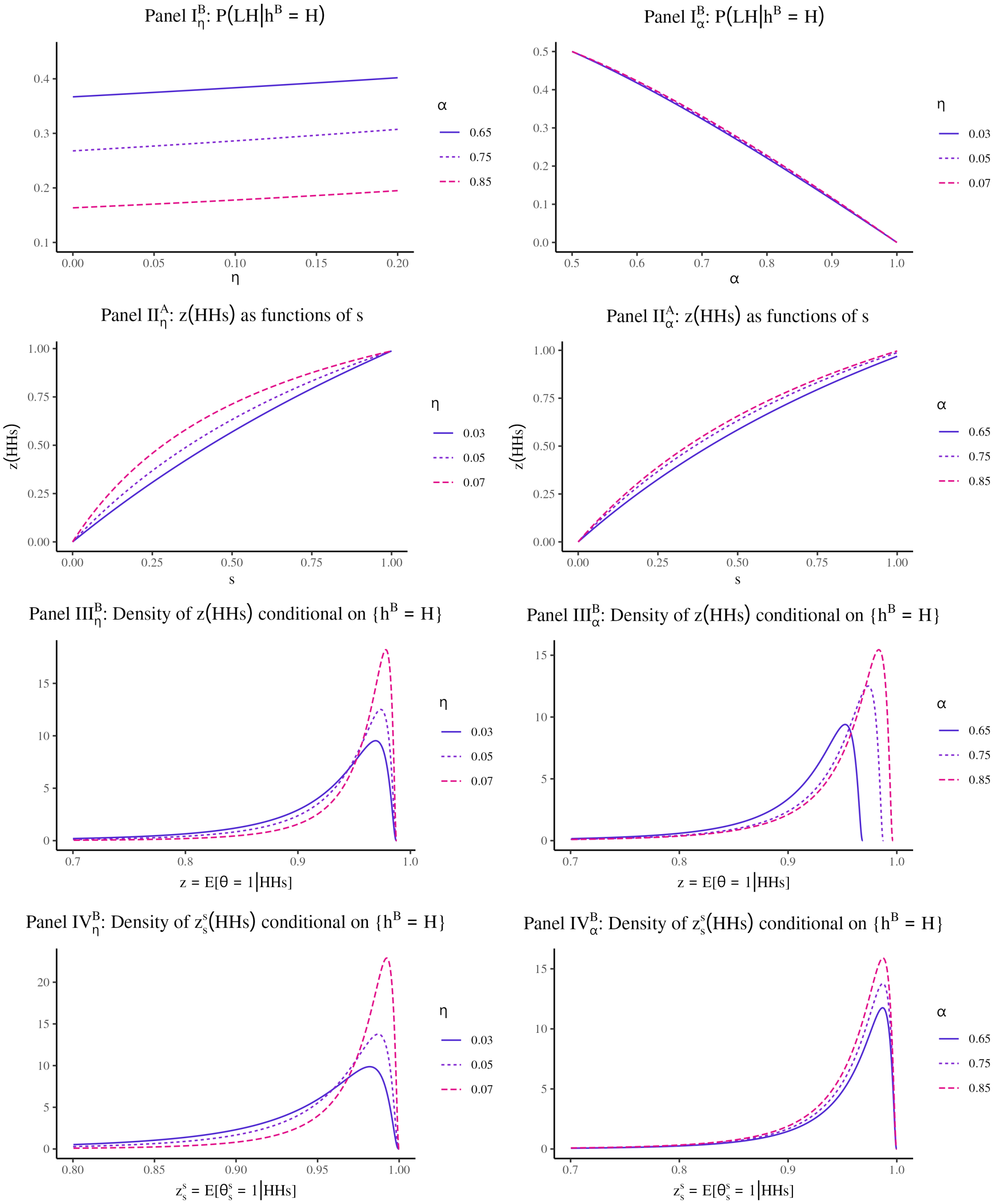
[21] It is easy to show that $\mathbb{P}(h^B = H) = q_h\alpha + (1-q_h)(1-\alpha)$ increases in q_h (so it decreases with η) as $\alpha > 0.5$. Because we have shown in (31) that the probability of disagreement events is independent of η, it implies $\mathbb{P}\left(LH|h^B = H\right)$ increases with η.

[22] Formally, we need a relatively high prior quality ($q_h > \frac{1}{2}$) so that lenders are more likely to receive H when α increases. Combining this with a lower chance of disagreement implies that $\mathbb{P}\left(LH|h^B = H\right)$ decreases with α.



Panel I^{B}_{η}: $P(LH|h^{B} = H)$

Panel I^{B}_{α}: $P(LH|h^{B} = H)$

Panel II^{A}_{η}: z(HHs) as functions of s

Panel II^{A}_{α}: z(HHs) as functions of s

Panel III^{B}_{η}: Density of z(HHs) conditional on $\{h^{B} = H\}$

Panel III^{B}_{α}: Density of z(HHs) conditional on $\{h^{B} = H\}$

Panel IV^{B}_{η}: Density of z^{s}_{s}(HHs) conditional on $\{h^{B} = H\}$

Panel IV^{B}_{α}: Density of z^{s}_{s}(HHs) conditional on $\{h^{B} = H\}$

Figure 5: **Conditional probability and density of posterior means of fundamentals.** Panel I^{B}_{η} and I^{B}_{α} depict $\mathbb{P}\left(LH|h^{B} = H\right)$ as functions of η and α, for three different levels of α and η respectively. Panel II^{A}_{η} and II^{A}_{α} depict posteriors z as functions of s. Panel III^{B}_{η} and III^{B}_{α} depict the density of posterior z, and Panel IV^{B}_{η} and IV^{B}_{α} depict the density of posterior z^{s}_{s}. For all panels, different levels of α's and η's are plotted: $\eta = 0.03$ ($\alpha = 0.65$) in blue solid line, $\eta = 0.05$ ($\alpha = 0.75$) in purple dotted line, and $\eta = 0.07$ ($\alpha = 0.85$) in rose-red dash line. Baseline parameters: $\alpha = 0.75, \eta = 0.05, \tau = 1, q^{h}_{h} = 0.9$, and $q_{s} = 0.9$.

We further calculate the distribution of z and z_s^s conditional on $h^B = H$ (see Appendix A.5). Essentially, we are asking, from the perspective of Bank B, what is the distribution of fundamentals given $h^B = H$ if taking into account the opponent's information? In Figure 5, Panel III_η^B and III_α^B plots the density of z when varying η and α, respectively. Similarly, Panels IV_η^B and IV_α^B plot the corresponding densities for z_s^s when we vary η and α.

The comparative statics with respect to the two technology parameters are quite transparent for z's distribution. Both types of technology advancement improve the assessment of project quality θ. It is clear from Panel III_α^B that z conditional on $\left\{h^A = h^B = H, s\right\}$ increases as the hard signal precision α increases, and a similar pattern is observed in Panel III_η^B when η increases, reflecting better screening as hard information becomes broader.

Turning to the posterior mean of θ_s^s (i.e., z_s^s) from the perspective of Bank B, the effects of two technology parameters differ significantly. When the span of hard information η increases, we observe a distributional shift from low beliefs to high beliefs about the "always soft" fundamental θ_s^s, for Bank B with $h^B = H$. Intuitively, the residual uncertainty that stems from the "always soft" fundamental θ_s^s shrinks. Consistent with discussion around Eq. (29), this suggests that from the perspective of non-specialized Bank B, the private information of its opponent A is more likely to be favorable when η increases, suggesting a weakened winner's curse from the residual uncertainty about "always soft" fundamental θ_s^s.

In contrast, when the signal precision α increases, the competition event HH is more likely for Bank B who receives $h^B = H$. This explains the larger probability mass of z_s^s in Panel IV_α^B as α increases.[23] Bank B who suffers from the winner's curse from the "always soft" fundamental θ_s^s is particularly concerned about the left-tail events of z_s^s, which become more likely for a larger α. This is opposite to the pro-competitive implications of an increase in the information span η.

3.3 Credit Allocation and Welfare

We now analyze how information span affects the allocation of credit and welfare. After presenting some comparative static results on aggregate markers of credit market health as a function of η we formally prove that welfare always improves when information span on hard signals increase.

Comparative statics of information span on credit market outcomes. We focus on three aggregate markers of credit market health: loan approval rate, non-performance rate, and probability of funding for high- and low-quality borrowers. We also investigate the expected NPV of a funded project as a measure of total welfare in the banking sector.

Three effects govern Figure 6 which plot the comparative statics of equilibrium outcomes as a function of the span of hard information η. First of all, a higher η represents a better screening technology. Second, an increase in η decreases the probability of getting a positive hard signal as

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[23] It is important to note that θ_s^s itself does not change when α varies.

there are more fundamental states covered by the hard signal and any one of them failing makes the loan quality low. Finally, a larger η alleviates the winner's curse faced by the non-specialized lender, leading to more aggressive bidding in equilibrium.

Panel A depicts the expected loan approval rates for two lenders. For Bank A who receive both hard and soft signals, a higher η implies a better screening overall, and therefore it lends more often given a relatively high prior of borrower quality ($q = 0.72$) in our numerical example. For Bank B, the effect of an increase in η on its approval rate (dashed line) depends on whether it makes zero or positive profits in equilibrium. In a zero-weak equilibrium, the reduction in the winner's curse for Bank B increases the likelihood of Bank B competing for the borrower after receiving $h^B = H$, pushing the approval rate upwards. In a positive-weak equilibrium, Bank B always participates and the effect of a lower winner's curse is dampened. In Panel A, the effect of η on the winner's curse for Bank B dominates for values of $\eta < 0.03$ (zero-weak) while the opposite holds for $\eta > 0.03$ (positive-weak). The jump in Bank B's loan approval rate when switching from a zero-weak to a positive-weak equilibrium mirrors the jump in Bank B's participation upon receiving $h^B = H$, which goes from being less than one when $\pi^B = 0$ to being one when $\pi^B > 0$.

Panel B shows the non-performing rates of loans made by Bank A (solid line) and Bank B (dashed line), which decrease in the information span for both lenders (within one equilibrium type). A higher η improves the screening technology (reduces Type II errors) and increases the average loan quality. The jump in Bank B's non-performing rate follows from the jump in Bank B's participation when the equilibrium switches from zero-weak to positive-weak (and starts quoting $\bar{r}$ in point mass); this explains all the jumps in Figure 6. Consequently, Bank B's incremental borrowers are of relatively low quality because it only wins competition when the opponent receives low soft signals $s < \hat{s}$.

Panel C plots the probability of good (solid line) and bad (dashed line) borrowers receiving funding in equilibrium. As a higher η represents a better screening technology, one would expect the probability of funding good loans to rise while that of bad loans to fall. This is indeed the case in Panel C when the equilibrium is in the positive-weak regime for $\eta > 0.03$. In a zero-weak equilibrium, the effect of lowering winner's curse leads Bank B more likely to compete, and hence its higher probability extending loans to good and bad borrowers. This effect dominates and the probability of bad loans being funded increases with η for $\eta < 0.03$.

Information span and credit market welfare. We can formally prove that welfare increases with the span of hard information.

Proposition 4. *Total welfare, which is measured as expected net present value (NPV) of funded projects, strictly increases in information span η.*

Panel D shows aggregate welfare measured as the expected NPV of a funded project, as well as the surplus to each agent in this sector. In the zero weak equilibrium, the welfare effect of lender

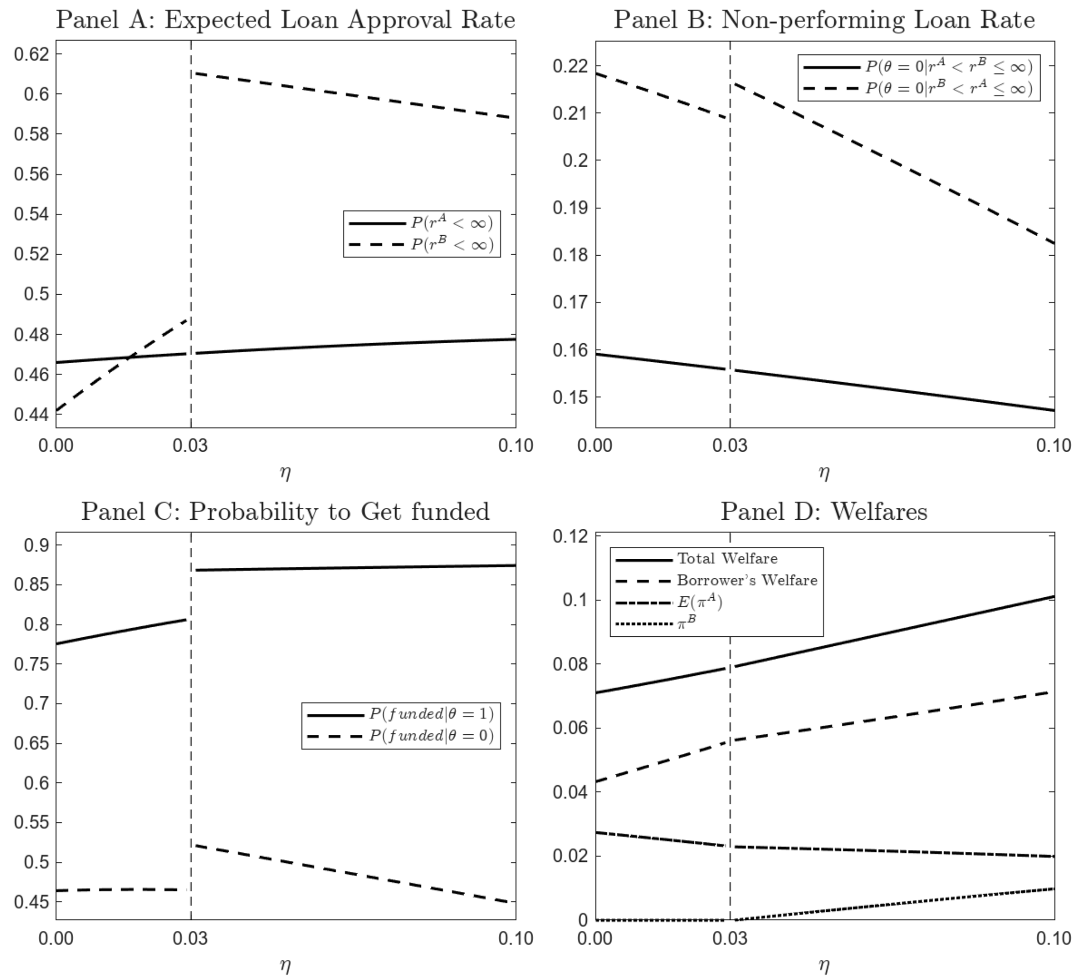

Figure 6: **Credit allocation and welfare.** Panel A and Panel B show the expected loan approval and non-performing rates, respectively. The solid lines correspond to Bank A while the dashed lines correspond to Bank B. Panel C depicts the probability of getting funded for a high-quality borrower (solid line) and a low-quality borrower (dashed line). Panel D illustrates aggregate welfare (solid line), borrower surplus (dashed line), and lender profits. All variables are depicted as a function of the span of hard information η. Priors q_h, q_s^s and information span η satisfy $q_h = q/q_s \cdot (1 - \eta)$ and $q_s^s = q_s/(1 - \eta)$. Parameters: $\bar{r} = 0.36$, $q = 0.72$, $q_s = 0.9$, $\tau = 1$ (top two panels) and $\alpha_u = \alpha_d = \alpha = 0.7$ (bottom two panels).

participation is zero, so the total welfare increases due to improved screening. To see this, as a lender adjusts its participation upon H in response to a broader span of hard information, the marginal borrower may gain or lose credit access. As a result, the impact on the marginal borrower is exactly the same as that from the planner's perspective. This means that the welfare effect of lender participation is exactly equal to the lender's profits on its marginal borrower, which is zero given the lender's optimality condition. In the positive weak equilibrium, however, the welfare effect of lender participation is no longer zero, as there is a gap between the marginal borrower's value to Bank A and to aggregate welfare. Specifically, in the case of competition (HH), Bank A earns profits by winning this marginal borrower from Bank B, but this is a transfer between lenders leaving total welfare unchanged. Nevertheless, we are able to show that the benefit of improved screening dominates and total welfare increases even in the regime of a positive-weak equilibrium.

Note in Panel D welfare is continuous in η when we switch from a zero-weak to a positive-weak equilibrium. At the knife-edge parameter of equilibrium type switching, both Bank B and the borrower make zero profits.[24] Hence, despite a jump in quantity, these additional loans are zero NPV projects on average, implying that total welfare increases continuously as η widens.

Moving on to surplus of each agent, recall that as lender screening becomes more efficient and competition intensifies, good-type borrowers are more likely to be funded (Panel C) and receive lower rates. (We normalize the surplus of bad-type borrowers to zero).[25] When $\eta < \hat{\eta} = 0.03$, the equilibrium is zero-weak and Bank A's expected profits decrease according to Proposition 2. In this case, the improvement in total welfare all accrues to borrowers, and there is additional transfer from banks to borrowers. When $\eta \geq 0.03$, the equilibrium is positive-weak and Bank B also enjoys a higher surplus from an increase in the information span η.

Finally, recall that in Panel D of Figure 6, all welfare goes up except the specialized Bank A in the range of positive-weak equilibrium. Is it possible that an increase in information span leads to a Pareto improvement for all agents in this sector? The answer is yes. As shown in Panel B of Figure 3 in Section 3.1, Bank A's profits could also increase in information span. Highlighting the feature that we directly model technology improvement, both the specialized and nonspecialized lenders enjoy the same technology improvement, especially when signal precisions before hardening soft information are low. In sum, broader hard information, which is an important form of information technology improvement in the recent decade, could lead to a Pareto improvement of all sectors and everyone enjoys a higher surplus.

4 Model Extensions and Discussions

This section considers several model extensions. First, the open banking initiative (He, Huang, and Zhou, 2023) implies that lenders' hard signals are likely to become more and more correlated; our model can be easily adapted to incorporate this aspect of change in information technology. Second, so far we have adopted one particular hard information technology as illustrated in Figure 1. As robustness, we explicitly model the signal on hardened soft fundamental θ_s^h (potentially generated by Big Data technology) and show that both the equilibrium characterization and the

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[24] The discrete jump of loans is made with an interest rate of $\bar{r}$, so that borrowers receive no surplus from these loans. Recall we rule out non-pledgeable income of borrowers; otherwise, there will be an upward jump in total welfare which includes the borrower's non-pledgeable income.

[25] See He, Huang, and Zhou (2023) for an analysis that includes the welfare of both good and bad types of borrowers in the context of open banking regulation.

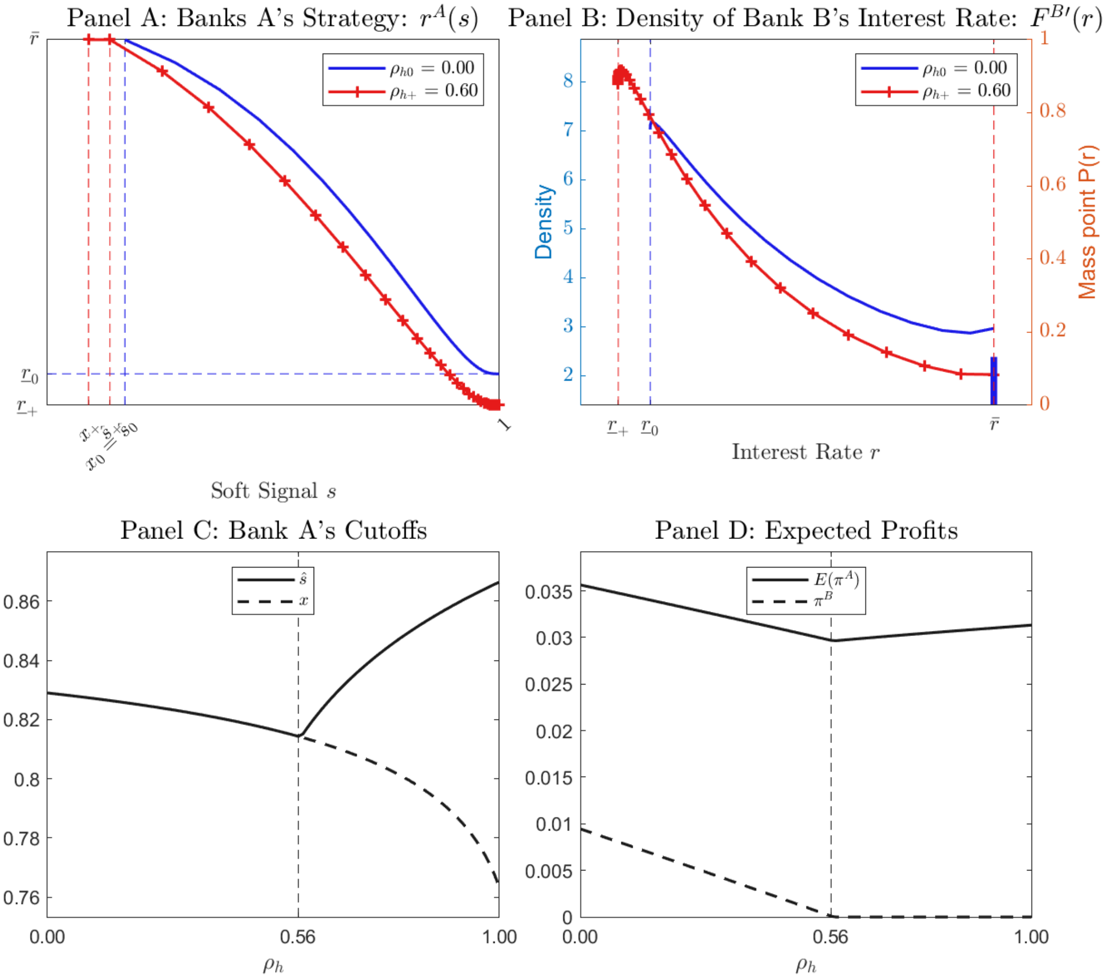

Figure 7: **Equilibrium strategies and profits for hard signal correlation ρ^h.** Panel A depicts $r^A(s)$ as a function of s and Panel B plots $F^{B\prime}(r)$ as a function of r; strategies for $\rho^{h+} = 0.6$ are depicted in red with markers while strategies with $\rho^{h0} = 0$ are depicted in blue. Panel C depicts Bank A's thresholds $\hat{s} = \sup s^A(\bar{r})$ and $x = \sup s^A(\infty)$, and Panel D depicts the expected profits for two lenders, both as a function of ρ_h. Parameters: $\bar{r} = 0.45$, $q_h = 0.8$, $q_s = 0.9$, $\eta = 0$, $\alpha = 0.7$, and $\tau = 1$.

key economic takeaways are robust to this alternative modeling of the hard information technology.

4.1 Correlated Hard Signals

Another widely acknowledged aspect of information technology advancement is that lenders' hard information signals become more correlated. For example, the open banking initiative enables sharing financial data with potential lenders under customer consent (He, Huang, and Zhou, 2023; Babina, Buchak, De Marco, Foulis, Gornall, Mazzola, and Yu, forthcoming), making lenders' assessments to be more alike. We extend our model to capture this effect, and show that making signals more "public" differs from hardened soft information in affecting credit market equilibrium.

We modify the hard information technology as follows. Suppose that with probability $\rho_h \in [0,1]$ lenders receive the same binary signal realization $h^c \in \{H, L\}$, while with probability $(1-\rho_h)$ each lender receives an independent binary hard signal. We solve the model extension (detailed derivations are available in Appendix A.6) and plot the key comparative statics in Figure 7 with respect to the correlation $\rho_h \in [0,1]$ of hard signals across two lenders. We observe in the bottom two panels on Figure 7 that a larger ρ_h leads to a zero-weak equilibrium more likely to occur. In the extreme case in which $\rho_h = 1$, the hard signal becomes a public signal, and Bank B who becomes effectively uninformed ends up with zero profit (Milgrom and Weber, 1982, as discussed in Section 3.2). From this perspective, it is interesting to observe that the economic implications of ρ_h, which is more about data sharing, are qualitatively similar to that of changes in signal precision studied in Section 3.2 but opposite to information span highlighted in this paper.

4.2 Signals on Hardened Soft Fundamental θ_s^h

Information technology corresponds to mappings from some fundamental states to signals, and as usual, there are potentially important modeling choices in specifying the details of the (hard) information technology. As illustrated in the top panel of Figure 1, we have adopted a technology that takes the entire hard fundamental θ_h as input and produces a binary signal as output. Does our core mechanism depend on this modeling choice?

More specifically, given our hard fundamental $\theta_h = \theta_s^h \theta_s^s$, another natural way to model "hardening soft information" is to keep the original hard and soft signals the same but introduce additional signals of the hardened soft fundamental θ_s^h. Denote by $h_s^j \in \{H, L\}$ the lender j's binary signal of θ_h^s, which we call hardened soft signal. For tractability, we assume that they are also decisive just as in Section 1.4, so that both lenders reject the borrower if $h_s^j = L$.

We can generally allow for any correlation ρ_s^h between two hardened soft signals, as modeled in Section 4.1. For illustration purposes, however, we assume that $\rho_s^h = 1$; essentially, the hardened soft signal becomes public. (Appendix A.7 provides a full analysis for any general $\rho_s^h \in (0,1)$.) More specifically, we assume $h_s^A = h_s^B = h_s^c$ where h_s^c takes a value of H (L) with probability $\alpha_s \in (\frac{1}{2}, 1)$ conditional on $\theta_s^h = 1$ $(\theta_s^h = 0)$. In practice, the signals generated by Big Data technology are indeed increasingly correlated across users, and this assumption captures this trend in its stark form. In fact, in the limiting case $\alpha_s \to 1$, h_s^j which reveals θ_s^h perfectly will be the same across two lenders for any ρ_s^h.

That $h_s^A = h_s^B = h_s^c$ is public, together with the assumption that h_s^j's are decisive, simplifies the analysis greatly. Conceptually, because lenders understand that they compete only when $h_s^c = H$ which is informative about the hardened soft fundamentals θ_s^h, this changes the effective distribution of soft signal s to $\phi\left(s|h_s^c = H\right)$. This in turn affects the credit market equilibrium outcome.

Similar to Section 2.1, we introduce $p_{HHH}(s) \equiv \mathbb{P}\left(h^A = H, h^B = H, h_s^c = H, s \in ds\right)$ as the joint probability of all three hard signals (h^A, h^B, h_s^c) being H and the soft signal s falling in the interval $(s, s+ds)$; we can define analogously $p_{HLH}(s)$, and finally $\overline{p}_{LHH}$ the joint probability of $\left\{h^A = L, h^B = h^c_s = H\right\}$. We define $\mu_{HHH}(s)$ and $\overline{\mu}_{LHH}$ analogously. Then, Bank B's lending profits when quoting r and $h^B = h^B_s = H$, is similar to (16):

$$\pi^B(r) = \int_0^{s^A(r)} \underbrace{p_{HHH}(t)}_{h^A=h^B=h^c_s=H,t} \left[\mu_{HHH}(t)(r+1)-1\right] dt + \underbrace{\overline{p}_{LHH}}_{h^A=L,h^B=h^c_s=H} \left[\overline{\mu}_{LHH}(r+1)-1\right]. \quad (41)$$

And, Bank A's profit when quoting r and $\left\{h^A = H, h^c_s = H, s\right\}$ is similar to (14):

$$\pi^A(r,s) = \underbrace{p_{HHH}(s)}_{h^A=h^B=h^c_s=H,s} \underbrace{\left[1-F^B(r)\right]}_{A \text{ wins}} \left[\mu_{HHH}(s)(1+r)-1\right] + \underbrace{p_{HLH}(s)}_{h^A=h^c_s=H,h^B=L,s} \left[\mu_{HLH}(s)(1+r)-1\right]. \quad (42)$$

Appendix A.7 shows that the above profits are isomorphic (up to a constant) to those in Blickle, He, Huang, and Parlatore (2024) with independent fundamentals (and signals), once we replace the relevant distributions—say $\theta_s = 1$ or s—to be those conditional on $h^c_s = H$. This allows us to fully characterize the credit market equilibrium under the alternative modeling.

The alternative modeling of hardened soft signal delivers quite similar economics as in our baseline. In Section 3.1 we have illustrated that the key mechanism of hardening soft information is to help the non-specialized lender alleviate the winner's curse against Bank A's unfavorable soft signal. Under the alternative modeling, one can calculate the distribution of soft signal conditional on positive realization of h^c_s, i.e.,

$$\phi\left(s \middle| h^c_s = H\right) = \phi_0(s) + \overbrace{\frac{\alpha_s}{\alpha_s - (2\alpha_s - 1)\eta}}^{\uparrow \text{ in } \eta \text{ as } \alpha_s > \frac{1}{2}} \cdot q_s \left[\phi_1(s) - \phi_0(s)\right]. \quad (43)$$

Comparing it to $\phi\left(s \middle| h^A = h^B = H\right)$ in (29), the only difference arises from the perfectly correlated hardened soft signal h^c_s introduced here, versus the conditionally independent h^A, h^B in our baseline model (if we further set $\theta^h_h = 1$). Essentially, observing a positive (public) hardened soft signal helps both lenders update the belief about s upward, and this effect is stronger for non-specialized lender who was particularly concerned about the winner's curse caused by Bank A's soft signal s. This economic insight is robust even if the hardened soft signal is independent; see a full analysis in Appendix A.7.

5 Concluding Remarks

One of the main roles of banks in the economy is producing information to allocate credit. In this paper, we show that the nature of the banks' information technology affects the credit market equilibrium and the degree of competition among banks. More specifically, we explore how the recent trend in Big Data technology that transforms qualitative or subjective assessments into quantifiable and objective metrics, known as hardening soft information, affects credit market outcomes in the presence of specialized lenders.

It is important to note that a priori, the significant advance in information technology should benefit all lenders, including specialized and established banks, as well as non-specialized lenders and new fintech challengers; in fact, large banks might front-run in their IT investment in the past decade (He, Jiang, Xu, and Yin, 2023). However, the fast-growing empirical literature on fintechs (see, e.g. Berg, Fuster, and Puri, 2022) seems to suggest that the new technology has helped relatively weaker (fintech) lenders catch up, intensifying the credit market competition.

We build a novel model with asymmetric lenders but symmetric technology improvement to study information span, and its implications on credit market competition. Our model highlights the crucial difference between information span, which captures the "breadth" of information, and the precision of information, which captures its "quality." This distinction is crucial in understanding the changing landscape in the credit market due to technological advances related to data gathering and processing that lead to the hardening of soft information. Our theory clarifies that it is enlarging the information span, not the mere improvement of precision, that can deliver the desired empirical pattern in a robust way; in fact, the former tends to reinforce the position of specialized lender while the latter "levels the playing field."

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A Technical Appendices

A.1 Credit Competition Equilibrium

Proof of Lemma 1

Proof. Note that the property of no gap implies common support $[\underline{r}, \overline{r}]$, because if a bank's interest rate offering has a larger lower bound or a smaller upper bound interest rate than its competitor's, this is one example of gaps in the first bank's support.

To show that the distributions have no gap, suppose that, say, the support of F^B has a gap $(r_1, r_2) \subset [\underline{r}, \overline{r}]$.[26] Then F^A should have no weight in this interval either, as any $r^A(s) \in (r_1, r_2)$ will lead to the same demand for Bank A and so a higher r will be more profitable. At least one lender does not have a mass point at r_1 (it is impossible that both distributions have a mass point

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[26] The same argument follows if the support of F^A has a gap in the conjectured equilibrium, and then for Bank B, any quotes within the gap lead to the same demand of the same posterior quality of customers, where Bank B updates its belief from Bank A's strategy.

at $\overline{r}_1$), under which its competitor has a profitable deviation by revising r_1 to $r \in (r_1, r_2)$ instead. Contradiction.

Regarding point mass, suppose that one distribution, say F^B has a mass point at $\tilde{r} \in [\underline{r}, \overline{r})$. Then Bank A would not quote any $r^A(s) \in [\tilde{r}, \tilde{r}+\epsilon]$ and it would strictly prefer quoting $r^A = \tilde{r} - \epsilon$ instead. In other words, the support of F^A must have a gap in the interval $[\tilde{r}, \tilde{r}+\epsilon]$. This contradicts the property of no gaps which we have shown. Finally, it is impossible that both distributions have a mass point at $\overline{r}$.

□

A.2 Proof of Proposition 1

Proof. This part proves that Bank A's equilibrium interest rate quoting strategy as a function of soft signal $r^A(s)$ is always decreasing; this implies that the FOC that helps us derive Bank A's strategy also ensures the global optimality.

Write Bank A's value $\Pi^A(r,s)$ as a function of its interest rate quote and soft signal, in the event of $h^A = H$ and s. (We use π to denote the equilibrium profit but Π for any strategy.) Recall that Bank A solves the following problem:

$$\max_r \Pi^A(r,s) = \underbrace{p_{HH}(s)}_{h^A=H,h^B=H,s} \underbrace{\left[1-F^B(r)\right]}_{A\text{ wins}} \left[\mu_{HH}(s)(1+r)-1\right] + \underbrace{p_{HL}(s)}_{h^A=H,h^B=L,s} \left[\mu_{HL}(s)(1+r)-1\right] \tag{44}$$

with the following FOC:

$$\begin{aligned} 0 &= \Pi_r^A(r(s), s) \\ &= \underbrace{p_{HH}(s)\left[-\frac{dF^B(r)}{dr}\right]}_{\text{lost customer}} \left[\underbrace{\left[\mu_{HH}(s)(1+r)-1\right]}_{\text{customer return}}\right] + \underbrace{p_{HH}(s)\left[1-F^B(r)\right]}_{\text{customer}} \underbrace{\mu_{HH}(s)}_{\text{MB of customer}} + p_{HL}(s)\,\mu_{HL}(s). \end{aligned} \tag{45}$$

One useful observation is that on the support, it must hold that $\mu_{HH}(s)(1+r)-1>0$; otherwise, $\mu_{HL}(s)(1+r)-1 < \mu_{HH}(s)(1+r)-1 \leq 0$, implying that Bank A's profit is negative (so it will exit).

Lemma 2. *Consider s_1, s_2 in the interior domain with corresponding interest rate quote r_1 and r_2. The marginal value of quoting r_2 for type $s = s_1$, i.e. $\Pi_r^A(r_2, s_1)$, has the same sign as $s_2 - s_1$.*

Proof. Evaluating the FOC of type s_1 but quoting r_2:

$$\begin{aligned}\Pi_r^A\left(r_2, s_1\right) = & p_{HH}\left(s_1\right)\left[-\frac{dF^B\left(r_2\right)}{dr}\right]\left[\mu_{HH}\left(s_1\right)\left(1+r_2\right)-1\right] \\ & + p_{HH}\left(s_1\right)\left[1-F^B\left(r_2\right)\right]\mu_{HH}\left(s_1\right)+p_{HL}\left(s_1\right)\mu_{HL}\left(s_1\right).\end{aligned} \tag{46}$$

FOC at type s_2 yields

$$\begin{aligned}0=\Pi_r^A\left(r_2, s_2\right) = & p_{HH}\left(s_2\right)\left[-\frac{dF^B\left(r_2\right)}{dr}\right]\left[\mu_{HH}\left(s_2\right)\left(1+r_2\right)-1\right] \\ & + p_{HH}\left(s_2\right)\left[1-F^B\left(r_2\right)\right]\mu_{HH}\left(s_2\right)+p_{HL}\left(s_2\right)\mu_{HL}\left(s_2\right),\end{aligned}$$

or

$$\frac{dF^B\left(r_2\right)}{dr}=\frac{p_{HH}\left(s_2\right)\left[1-F^B\left(r_2\right)\right]\mu_{HH}\left(s_2\right)+p_{HL}\left(s_2\right)\mu_{HL}\left(s_2\right)}{p_{HH}\left(s_2\right)\left[\mu_{HH}\left(s_2\right)\left(1+r_2\right)-1\right]}. \tag{47}$$

Plugging in this term to (46), $\Pi_r^A\left(r_2, s_1\right)$ becomes

$$\begin{aligned}\Pi_r^A\left(r_2, s_1\right) & =\left[\phi_1\left(s_1\right)-\frac{p_{HH}\left(s_1\right)}{p_{HH}\left(s_2\right)}\cdot\frac{\mu_{HH}\left(s_1\right)\left(1+r_2\right)-1}{\mu_{HH}\left(s_2\right)\left(1+r_2\right)-1}\cdot\phi_1\left(s_2\right)\right]\left\{\overline{p}_{HH}\overline{\mu}_{HH}\left[1-F^B\left(r_2\right)\right]+\overline{p}_{HL}\overline{\mu}_{HL}\right\} \\ & =\frac{p_{HH}\left(s_1\right)\phi_1\left(s_2\right)-\phi_1\left(s_1\right)p_{HH}\left(s_2\right)}{p_{HH}\left(s_2\right)\left[\mu_{HH}\left(s_2\right)\left(1+r_2\right)-1\right]}\left\{\overline{p}_{HH}\overline{\mu}_{HH}\left[1-F^B\left(r_2\right)\right]+\overline{p}_{HL}\overline{\mu}_{HL}\right\},\end{aligned} \tag{48}$$

where $\overline{p}_{h^A h^B}\equiv\mathbb{P}\left(h^A, h^B\right)$, $\overline{\mu}_{h^A h^B}\equiv\mathbb{P}\left(\theta=1\middle|h^A, h^B\right)$ are defined in Section 2.1 and $\phi_1(s)\equiv\phi\left(s\left|\theta_s=1\right.\right)=\frac{s\phi(s)}{q_s}$ is the conditional density of soft signal (also $\phi_0(s)\equiv\phi\left(s\left|\theta_s=0\right.\right)=\frac{(1-s)\phi(s)}{q_s}$). Hence, the sign of $\Pi_r^A\left(r_2, s_1\right)$ depends on $p_{HH}\left(s_1\right)\phi_1\left(s_2\right)-p_{HH}\left(s_2\right)\phi_1\left(s_1\right)$ because the denominator is positive as we noted right after Eq. (45).

Last, we argue that the sign of $p_{HH}\left(s_1\right)\phi_1\left(s_2\right)-p_{HH}\left(s_2\right)\phi_1\left(s_1\right)$ depends on s_2-s_1. Note that the event HH correlates with the soft signal s only via θ_s^h which affects θ_s, we have

$$p_{HH}(s)=\mathbb{P}\left(\theta_s=1, HH\right)\phi_1(s)+\mathbb{P}\left(\theta_s=0, HH\right)\phi_0(s),$$

i.e., two positive hard signals HH are no longer informative about the soft signal s once we condition on the soft state θ_s. Using this observation,

$$p_{HH}\left(s_1\right)\phi_1\left(s_2\right)-p_{HH}\left(s_2\right)\phi_1\left(s_1\right)=\mathbb{P}\left(\theta_s=0, HH\right)\phi_0\left(s_1\right)\phi_0\left(s_2\right)\left[\frac{\phi_1\left(s_2\right)}{\phi_0\left(s_2\right)}-\frac{\phi_1\left(s_1\right)}{\phi_0\left(s_1\right)}\right], \tag{49}$$

which shares the same sign as s_2-s_1. ☐

Lemma 2 has three implications. First, as long as $r^A(\cdot)$ is (strictly) increasing in some segment, then Bank A would like to deviate in this segment. To see this, suppose that $r_1>r_2$ when $s_1>s_2$

for s_1, s_2 arbitrarily close. Because Lemma 1 has shown that Bank A's strategy is smooth, r_2 is arbitrarily close to r_1. Then $\Pi_r^A(r_2, s_1) < 0$, implying that the value is convex and the Bank A at s_1 (who in equilibrium is supposed to quote r_1) would like to deviate further.

Second, the monotonicity implied by Lemma 2 helps us show that Bank A uses a pure strategy. To see this, for any $s_1 > s_2$ that induce interior quotes $r_1, r_2 \in [\underline{r}, \overline{r})$, however close, in equilibrium we must have $\sup r^A(s_1) < \inf r^A(s_2)$ by monotonicity. Combining this with Part 3 of Lemma 1, i.e., the induced distribution $F^A(\cdot)$ is atomless except for at $\overline{r}$ and has no gaps, we know that Bank A must adopt a pure strategy in the interior of $[\underline{r}, \overline{r})$, or for $s \le \hat{s}$. Finally, the following argument shows pure strategy for $s < \hat{s}$: i) randomize over $s = 0$ is a zero-measure set; and ii) on $s > \hat{s}$ Bank A can either quote $\overline{r}$ or ∞, which, generically, gives different values (and hence rules out randomization).

Third, if $r^A(\cdot)$ is decreasing globally over $\mathcal{S}$, then the FOC is sufficient to ensure global optimality. Consider a type s_1 who would like to deviate to $\check{r} > r_1$; then

$$\Pi^A(\check{r}, s_1) - \Pi^A(r_1, s_1) = \int_{r_1}^{\check{r}} V_r^A(r, s_1)\, dr.$$

Given the monotonicity of $r(s)$, we can find the corresponding type $s(r)$ for $r \in [r_1, \check{r}]$. From Lemma 2 we know that

$$\Pi_r^A(r, s_1) \propto \frac{\phi_1(s(r))}{\phi_0(s(r))} - \frac{\phi_1(s_1)}{\phi_0(s_1)}$$

which is negative given $s(r) < s_1$. Therefore the deviation gain is negative. Similarly, we can show a negative deviation gain for any $\check{r} < r_1$.

Next we show that $r^A(\cdot)$ is single-peaked over the space of $\mathcal{S} = [0, 1]$.

Lemma 3. *Given any exogenous $\pi^B \ge 0$, $r^A(\cdot)$ single-peaked over $\mathcal{S} = [0, 1]$ with a maximum point.*

Proof. When $r \in [\underline{r}, \overline{r})$, the derivative of $r^A(s)$ with respect to s is

$$\frac{dr^A(s)}{ds} = \frac{p_{HH}\phi(s)\left(\overbrace{\int_0^s p_{HH}(t)\left[\mu_{HH}(t) - \mu_{HH}(s)\right] dt}^{M_1(s)<0,\text{and} M_1'(s)<0} + \overbrace{\overline{p}_{LH}\overline{\mu}_{LH} - \left(\pi^B + \overline{p}_{LH}\right)\mu_{HH}(s)}^{M_2(s)?0,\ \text{but} M_2'(s)<0}\right)}{\left(\int_0^s p_{HH}(t)\cdot\mu_{HH}(t)\, dt + \overline{p}_{LH}\overline{\mu}_{LH}\right)^2}.$$

As $\mu_{HH}(t) < \mu_{HH}(s)$ for $t \in [0, s)$, the first term in the bracket $M_1(s) < 0$, and

$$M_1'(s) = -\frac{\partial\mu_{HH}(s)}{\partial s}\int_0^s p_{HH}(t)\, dt < 0.$$

For $M_2(s) = \overline{p}_{LH}\overline{\mu}_{LH} - \left(\pi^B + \overline{p}_{LH}\right)\mu_{HH}(s)$, it has an ambiguous sign, but is decreasing in s.

This implies that $M_1(s) + M_2(s)$ decreases with s. It is easy to verify that $M_1(0) + M_2(0) > 0$ and $M_1(1) + M_2(1) < 0$. Therefore $r^A(s)$ first increases and then decreases, therefore single-peaked. $\square$

Suppose that the peak point is $\tilde{s}$; then Bank A should simply charge $r(s) = \tilde{r}$ for $s < \tilde{s}$ for better profit. This is the standard "ironing" technique and we therefore define the following ironed strategy formally (here, we also take care of the capping $r \leq \overline{r}$):

$$r^A_{ironed}(s) \equiv \sup_{t \in [s,1]} \min\left(r^A(t), \overline{r}\right).$$

By definition $r^A_{ironed}(s)$ is monotone decreasing.

We now argue that in equilibrium, π^B and $\underline{r}$ adjust so that $r^A(\cdot)$ is always monotonely decreasing over $[x, 1]$. (Since we define $r^A(s) = \infty$ for $s < x$, monotonicity over the entire signal space $[0, 1]$ immediately follows.) There are two subcases to consider.

1. Suppose that $\tilde{r} = \overline{r}$. In this case, $r^A(s) = \frac{\pi^B + \int_0^s p_{HH}(t)dt + \overline{p}_{LH}}{\int_0^s p_{HH}(t) \cdot \mu_{HH}(t)dt + \overline{p}_{LH}\overline{\mu}_{LH}} - 1$ used in Lemma 2 and 3 does not apply to $s < \tilde{s}$ because the equation is defined only over the left-closed-right-open interval $[\underline{r}, \overline{r})$. Instead, $r^A(s)$ in this region is determined by Bank A's optimality condition: as r^A does not affect the competition from Bank B (which equals $F^B(\overline{r}^-)$), Bank A simply sets the maximum possible rate $r^A(r) = \overline{r}$ unless it becomes unprofitable (for $s < x$). (This is our zero-weak equilibrium with $\pi^B = 0$, and there is no "ironing" in this case.)

2. Suppose that $\tilde{r} < \overline{r}$; then bank A quotes $\tilde{r}$ for all $s < \hat{s}$. But this is not an equilibrium—Bank A now leaves a gap in the interval $[\tilde{r}, \overline{r}]$, contradicting with point 3) in Lemma 1 (there, we rule out gaps in equilibrium). Intuitively, Bank B always would like to raise its quotes inside $[\tilde{r}, \overline{r}]$ to $\overline{r}$; there is no "ironing" in this case. (This is our positive-weak equilibrium with $\pi^B > 0$.)

Therefore, we have shown that Bank A uses a pure strategy $r^A(s)$ that decreases in s. Bank A's equilibrium strategy Eq. (18) is then derived from Bank B's indifference condition.

Equilibrium strategy F^B In this part, we derive equilibrium strategy taking π^B and $\hat{s}$ as given.

Bank A's equilibrium strategy Eq. (18) in $s \in [\hat{s}, 1]$ maximizes its profit and so satisfies the following first order condition (FOC):

$$p_{HH}(s)\left(-\frac{dF^B(r)}{dr}\right)\left[\mu_{HH}(s)(r+1) - 1\right] + \left\{p_{HH}(s)\left[1 - F^B(r)\right]\mu_{HH}(s) + p_{HL}(s)\mu_{HL}(s)\right\} = 0. \tag{50}$$

Bank B's equilibrium quotes $r \in [\underline{r}, \overline{r})$ maximizes its expected profits and satisfy the following

FOC:

$$\underbrace{\left(-s^{A\prime}\left(r\right)\right)p_{HH}\left(s^{A}\left(r\right)\right)}_{\text{B's additional borrowers}}\left[\mu_{HH}\left(s\right)\left(r+1\right)-1\right]=\underbrace{\int_{0}^{s^{A}\left(r\right)}p_{HH}\left(t\right)\mu_{HH}\left(t\right)dt+\overline{p}_{LH}\overline{\mu}_{LH}}_{\text{B's existing borrowers}}.$$

Plug this condition into Bank A's optimality condition (50), and we have

$$\left(-\frac{dF^{B}\left(r\right)}{dr}\right)\frac{\int_{0}^{s^{A}\left(r\right)}p_{HH}\left(t\right)\mu_{HH}\left(t\right)dt+\overline{p}_{LH}\overline{\mu}_{LH}}{-s^{A\prime}\left(r\right)}+p_{HH}(s)\left[1-F^{B}\left(r\right)\right]\mu_{HH}(s)+p_{HL}(s)\mu_{HL}(s)=0,$$

which could be rearranged as

$$-\frac{d}{ds}\left\{\frac{1-F^{B}\left(r\right)}{\int_{0}^{s}p_{HH}\left(t\right)\mu_{HH}\left(t\right)dt+\overline{p}_{LH}\overline{\mu}_{LH}}\right\}=-\frac{\frac{d}{ds}\left\{\int_{0}^{s}p_{HL}(t)\mu_{HL}(t)dt\right\}}{\left[\int_{0}^{s}p_{HH}\left(t\right)\mu_{HH}\left(t\right)dt+\overline{p}_{LH}\overline{\mu}_{LH}\right]^{2}}.$$

Using the conditional independence of signals, the right-hand-side of the above equation is

$$\begin{aligned}-\frac{\frac{d}{ds}\left\{\int_{0}^{s}p_{HL}(t)\mu_{HL}(t)dt\right\}}{\left[\int_{0}^{s}p_{HH}\left(t\right)\mu_{HH}\left(t\right)dt+\overline{p}_{LH}\overline{\mu}_{LH}\right]^{2}}&=-\frac{\frac{d}{ds}\left\{\int_{0}^{s}\frac{p_{HL}(t)\mu_{HL}(t)}{p_{HH}(t)\mu_{HH}(t)}p_{HH}\left(t\right)\mu_{HH}\left(t\right)dt\right\}}{\left[\int_{0}^{s}p_{HH}\left(t\right)\mu_{HH}\left(t\right)dt+\overline{p}_{LH}\overline{\mu}_{LH}\right]^{2}}\\&=\frac{1-\alpha}{\alpha}\frac{d}{ds}\left\{\frac{1}{\int_{0}^{s}p_{HH}\left(t\right)\mu_{HH}\left(t\right)dt+\overline{p}_{LH}\overline{\mu}_{LH}}\right\}.\end{aligned}$$

Hence, Bank B's equilibrium strategy satisfies the following key ordinary differential equation,

$$\frac{d}{ds}\left[\frac{\frac{1-\alpha}{\alpha}+1-F^{B}\left(r\right)}{\int_{0}^{s}p_{HH}\left(t\right)\mu_{HH}\left(t\right)dt+\overline{p}_{LH}\overline{\mu}_{LH}}\right]=0, \tag{51}$$

so there exists a constant C so that

$$\frac{\overline{p}_{HH}\overline{\mu}_{HH}\left[1-F^{B}\left(r\right)\right]+\overline{p}_{HL}\overline{\mu}_{HL}}{\int_{0}^{s}p_{HH}\left(t\right)\mu_{HH}\left(t\right)dt+\overline{p}_{LH}\overline{\mu}_{LH}}=C.$$

Using the boundary condition $F^{B}\left(\underline{r}\right)=0$, we solve for the constant

$$C=\frac{\overline{p}_{HH}\overline{\mu}_{HH}+\overline{p}_{HL}\overline{\mu}_{HL}}{\overline{p}_{HH}\overline{\mu}_{HH}+\overline{p}_{LH}\overline{\mu}_{LH}}=1.$$

Therefore, for $r\in\left[\underline{r},\overline{r}\right)$, we have

$$F^{B}\left(r\right)=1-\frac{\int_{0}^{s(r)}t\phi\left(t\right)dt}{q_{s}},$$

and if $\pi^B > 0$ ($\pi^B = 0$), we have $F^B(\overline{r}) = 1$ ($F^B(\overline{r}) = 1 - \frac{\int_0^{\hat{s}} t\phi(t)dt}{q_s}$).

π^B and boundary condition $\hat{s}$ We define the following auxilary functions

$$\hat{\pi}^B\left(\overline{r}; s^A(\overline{r}) = \check{s}\right) = \int_0^{\check{s}} p_{HH}(t)\left[\mu_{HH}(t)(\overline{r}+1) - 1\right] dt + \overline{p}_{LH}\left[\overline{\mu}_{LH}(\overline{r}+1) - 1\right], \tag{52}$$

which is Bank B's profits when assuming that quoting $r^B = \overline{r}$ wins Bank A of type $s \in [0, \check{s}]$ in competition (HH). We define s_B^{be} as the threshold where Bank B's auxilary profits break even,

$$\hat{\pi}^B\left(\overline{r}; s^A(\overline{r}) = s_B^{be}\right) = 0. \tag{53}$$

In addition, we define the following auxilary profit function for Bank A,

$$\begin{aligned}&\hat{\pi}^A\left(\overline{r}, \check{s}; F^B(\overline{r}) = \int_{\check{s}}^1 \frac{s\phi(s)}{q_s} ds\right)\\ =&p_{HH}(\check{s}) \underbrace{\int_0^{\check{s}} \frac{s\phi(s)}{q_s} ds}_{=1-F^B(\overline{r})} \left[\mu_{HH}(\check{s})(1+\overline{r}) - 1\right] + p_{HL}(\check{s})\left[\mu_{HL}(\check{s})(1+r) - 1\right],\end{aligned} \tag{54}$$

which assumes that Bank A receiving soft signal $\check{s}$ wins with probability $\int_0^{\check{s}} \frac{s\phi(s)}{q_s} ds$ when quoting $\overline{r}$. We define s_A^{be} as the threshold where Bank A's auxilary profits break even,

$$\hat{\pi}^A\left(\overline{r}, s_A^{be}; F^B(\overline{r}) = \int_{s_A^{be}}^1 \frac{s\phi(s)}{q_s} ds\right) = 0. \tag{55}$$

First, we argue that equilibrium $\hat{s} \equiv \arg\sup_s \left\{s : r^A(s) \geq \overline{r}\right\}$ either equals s_A^{be} or s_B^{be}. To see this, if $\pi^B = 0$, we have $\hat{s} = s_B^{be}$ by construction. If $\pi^B > 0$, then Bank B always makes an offer upon H, i.e., $F^B(\overline{r}) = 1$. We also know that $F^B(\overline{r}^-) = 1 - \frac{\int_0^{s^A(r)=\hat{s}^+} t\phi(t)dt}{q_s} < 1$, because Bank A must reject the borrower when s realizes as close to 0 and $\hat{s} > 0$. Hence, $F^B(r)$ has a point mass at $\overline{r}$. It follows that $F^A(r)$ is open at $\overline{r}$: $\hat{s} = x$ and $\pi^A\left(r^A(\hat{s})|\hat{s}\right) = 0$, which is exactly the definition of s_A^{be} and so $\hat{s} = s_A^{be}$.

Now we prove the claim in this lemma. In the first case of $s_B^{be} < s_A^{be}$, we have $\hat{s} \leq s_A^{be}$ and thus Bank A's probability of winning when quoting $r^A = \overline{r}$ is at most $\frac{\int_0^{s_A^{be}} t\phi(t)dt}{q_s} \geq \frac{\int_0^{\hat{s}} t\phi(t)dt}{q_s} = 1 - F^B(\overline{r}^-)$. The definition of s_A^{be} says that Bank A upon s_A^{be} breaks even when quoting $r^A\left(s_A^{be}\right) = \overline{r}$ and facing this most favorable winning probability $\frac{\int_0^{s_A^{be}} t\phi(t)dt}{q_s}$. Then upon a worse soft signal $s_B^{be} < s_A^{be}$, Bank A must reject the borrower because offering $\overline{r}$ leads to losses, which rules out $\hat{s} = s_B^{be}$. According to our earlier observation of $\hat{s} = s_B^{be}$ or s_A^{be}, we have $\hat{s} = s_A^{be}$ and $\pi^B > 0$ in this case, where π^B is the same as Eq. (52).

In the second case of $s_B^{be} \geq s_A^{be}$, we have $\hat{s} \leq s_B^{be}$. When Bank B quotes $r^B = \overline{r}$, the marginal type that Bank B wins over is at most s_B^{be}. The definition of s_B^{be} says that Bank B breaks even when quoting $r^B = \overline{r}$ and facing this most favorable winning probability at marginal type s_B^{be}. Then if the competition from A were more aggressive, say the marginal type quoting $r^A = \overline{r}$ is $s_A^{be} \leq s_B^{be}$, Bank B would make a loss when quoting $\overline{r}$, so $\hat{s} = s_A^{be}$ cannot support an equilibrium. Hence, in this case, $\hat{s} = s_B^{be}$ and $\pi^B = 0$. From the definition of s_A^{be}, Bank A's equilibrium break-even condition $0 = \pi^A\left(\overline{r}|x\right)$, and the fact that $s_B^{be} \geq s_A^{be}$ in this case, we have

$$\begin{aligned}
0 =& \frac{\int_0^{s_A^{be}} p_{HH}\left(s\right)ds}{q_s}\left[\mu_{HH}\left(s_A^{be}\right)\left(1+\overline{r}\right)-1\right] + p_{HL}\left[\mu_{HL}\left(s_A^{be}\right)\left(1+\overline{r}\right)-1\right] \\
=& \frac{\int_0^{s_B^{be}} p_{HH}\left(s\right)ds}{q_s}\left[\mu_{HH}\left(x\right)\left(1+\overline{r}\right)-1\right] + p_{HL}\left[\mu_{HL}\left(x\right)\left(1+\overline{r}\right)-1\right] \\
\geq& \frac{p_{HH}\int_0^{s_A^{be}} t\phi\left(t\right)dt}{q_s}\left[\mu_{HH}\left(x\right)\left(1+\overline{r}\right)-1\right] + p_{HL}\left[\mu_{HL}\left(x\right)\left(1+\overline{r}\right)-1\right].
\end{aligned}$$

Hence, $x \leq s_A^{be} \leq s_B^{be} = \hat{s}$. ☐

A.3 Proof of Proposition 2

Lemma 4. *The break-even soft signals* s_A^{be} *and* s_B^{be} *defined in Eq. (55) and (53) satisfy*

$$\frac{\partial s_A^{be}}{\partial \eta} < 0, \quad \frac{\partial s_B^{be}}{\partial \eta} < 0.$$

Proof. The definition of s_A^{be} or $\hat{\pi}^A\left(\overline{r}, s_{be}^A; \frac{\int_0^{s_{be}^A} t\phi(t)dt}{q_s}\right) = 0$ corresponds to an implicit function of η and s_{be}^A,

$$\begin{aligned}
\hat{\pi}^A\left(\eta, s\right) \equiv\ & p_{HH}\left(s\right)\frac{\int_0^s t\phi\left(t\right)dt}{q_s}\left[\mu_{HH}\left(s\right)\left(1+\overline{r}\right)-1\right] + p_{HL}\left(s\right)\left[\mu_{HL}\left(s\right)\left(1+\overline{r}\right)-1\right] \\
=\ & \underbrace{\frac{\int_0^s t\phi\left(t\right)dt}{q_s}}_{\text{soft info, indept of }\eta}\left[\underbrace{p_{HH}\left(s\right)\mu_{HH}\left(s\right)}_{\text{Type 1, indept of }\eta}\left(1+\overline{r}\right) - \underbrace{p_{HH}\left(s\right)}_{\text{decrease in }\eta}\right] + \underbrace{p_{HL}\left(s\right)\mu_{HL}\left(s\right)}_{\text{Type 1, indept of }\eta}\left(1+\overline{r}\right) - p_{HL}\left(s\right).
\end{aligned}$$

We first analyze the key terms' monotonicity in η. Note that the joint events of signal realizations and good project (Type 1 error) is independent of η,

$$p_{HH}(s)\,\mu_{HH}(s) = q\alpha^2 \cdot \frac{s\phi(s)}{q_s},$$
$$p_{HL}(s)\,\mu_{HL}(s) = q\alpha(1-\alpha) \cdot \frac{s\phi(s)}{q_s}.$$

In addition, $p_{HH}(s)$ decreases with η as shown in Eq. (28). The remaining term $p_{HL}(s)$ is independent of η as shown in Eq. (31).

Taken together,

$$\frac{\partial \hat{\pi}^A(\eta, s)}{\partial \eta} > 0;$$

Combining with the fact that $\frac{\partial \hat{\pi}^A(\eta,s)}{\partial s} > 0$, the implicit function theorem shows

$$\frac{\partial s_A^{be}(\eta)}{\partial \eta} < 0.$$

Part 2. The definition of s_B^{be} corresponds to an implicit function

$$0 = \hat{\pi}^B(\eta, s) = \int_0^s p_{HH}(t)\left[\mu_{HH}(t)(\overline{r}+1) - 1\right] dt + \overline{p}_{LH}\left[\overline{\mu}_{LH}(\overline{r}+1) - 1\right].$$

Similar as in the previous argument, Type I mistakes are constant in the span η due to the multiplicative-characteristic setting:

$$\int_0^s p_{HH}(t)\,\mu_{HH}(t)\,dt = q\alpha^2 \Phi(s|\,\theta_s = 1) = q\alpha^2 \cdot \frac{\int_0^s t\phi(t)\,dt}{q_s},$$
$$\overline{p}_{LH}\overline{\mu}_{LH} = q\alpha(1-\alpha).$$

In addition, the probability of disagreement in hard signals is also independent of η as $\alpha^A = \alpha^B$,

$$\overline{p}_{LH} = \alpha(1-\alpha).$$

The probability that Bank B wins in competition $\int_0^s p_{HH}(t)\,dt$ decreases with η.

Taken together, we have $\frac{\partial \hat{\pi}^B(\eta,s)}{\partial \eta} > 0$. Combining with $\frac{\partial \hat{\pi}^B(\eta,s)}{\partial s} > 0$, the implicit function theorem implies that

$$\frac{\partial s_B^{be}(\eta)}{\partial \eta} < 0.$$

□

Proof of Proposition 2

Proof. The two lenders' expected profits in equilibrium can be written as:

$$\mathbb{E}[\pi^A] = \int_0^1 \pi^A\left(r^A(s), s\right) ds = \underbrace{\int_0^{\hat{s}} \left[\pi^A(\overline{r}, s)\right]^+ ds}_{\text{non-competing case}} + \underbrace{\int_{\hat{s}}^1 \pi^A\left(r^A(s), s\right) ds}_{\text{compete against Bank } B}, \tag{56}$$

$$\pi^B = \int_{\underline{r}}^{\overline{r}} \pi^B dF^B(r) = \underbrace{\pi^B \cdot \left[1 - F^B(\overline{r}^-)\right]}_{\text{non-competing case}} + \underbrace{\int_{\underline{r}}^{\overline{r}} \underbrace{\pi^B(r)}_{\text{constant } \pi^B} \ \phi_1\left(s^A(r)\right)\left(-s^{A\prime}(r)\right) dr}_{\text{compete against Bank } A}. \tag{57}$$

Note, (56) takes $\pi^A\left(r^A(s), s\right)$ in (14) as given which includes the density $\phi(s)$ already, and (57) uses the expression of equilibrium $F^B(r)$ in (19). The second term in both equations represents profits when lenders engage in direct competition by offering interest rates $r \in [\underline{r}, \overline{r})$. Since $ds = \left[-s^{A\prime}(r)\right] dr$, we should compare the integrand $\pi^B\left(r^A(s)\right)\phi_1(s)$ against $\pi^A\left(r^A(s), s\right)$, where the adjustment of $\phi_1(s)$ for π^B reflects Bank B's equilibrium probability density (i.e., $F^{B\prime}(r) = \frac{s\phi(s)}{q_s} = \phi_1(s)$) when compared against Bank A with realization s.

We study the following object for $s \in [\hat{s}, 1]$:

$$\Delta\pi(s; \eta) \equiv \pi^B\left(r^A(s)\right)\phi_1(s) - \pi^A\left(r^A(s), s\right). \tag{58}$$

We aim to show that for every s, we have $\frac{d\Delta\pi(s;\eta)}{d\eta} > 0$, i.e., the impact of η on density-adjusted π^B always dominates that of π^A.

First, we make the key observation that, in equilibrium, both lenders make the same revenue but face different costs (i.e., the probability of lending). Specifically, we have

$$\begin{aligned}\pi^A\left(r^A(s), s\right) = & \underbrace{\left\{\left[1 - F^B\left(r^A(s)\right)\right] p_{HH}(s)\,\mu_{HH}(s) + p_{HL}(s)\,\mu_{HL}(s)\right\}}_{\text{borrowers who repay}}\left(1 + r^A(s)\right) \\ & - \underbrace{\left\{p_{HH}(s)\left[1 - F^B\left(r^A(s)\right)\right] + p_{HL}(s)\right\}}_{\text{lending amount}},\end{aligned} \tag{59}$$

$$\pi^B\left(r^A(s)\right)\phi_1(s) = \phi_1(s)\underbrace{\left[\int_0^s p_{HH}(t)\,\mu_{HH}(t)\,dt + \overline{p}_{LH}\overline{\mu}_{LH}\right]}_{\text{borrowers who repay}}\left(1 + r^A(s)\right) - \phi_1(s)\underbrace{\left[\int_0^s p_{HH}(t)\,dt + \overline{p}_{LH}\right]}_{\text{lending amount}}. \tag{60}$$

Using Bank B's equilibrium strategy $F^B\left(r^A(s)\right) = \frac{\int_s^1 t\phi(t)dt}{q_s} = \int_s^1 \phi_1(t)dt$ and the property of conditional independence, both lenders make the same revenue in competition (when HH realizes)

$$\underbrace{\int_0^s \phi_1(t)dt}_{\text{A wins}} \cdot \underbrace{p_{HH}\left(s\right)\mu_{HH}\left(s\right)}_{\text{A's good borrower}} = \underbrace{\phi_1(s)}_{\text{measure adjustment}} \underbrace{\int_0^s}_{\text{B wins}} \underbrace{p_{HH}\left(t\right)\mu_{HH}\left(t\right)}_{\text{B's good borrower}} dt = \overline{p}_{HH}\overline{\mu}_{HH}\phi_1(s)\int_0^s \phi_1(t)dt.$$

In addition, because lenders have the same technology to process hard information, they also make the same revenue when the other bank rejects the borrower upon L,

$$\underbrace{p_{HL}\left(s\right)\mu_{HL}\left(s\right)}_{\text{A's good borrower}} = \overline{p}_{HL}\overline{\mu}_{HL}\phi_1(s) = \underbrace{\phi_1(s)\overline{p}_{LH}\overline{\mu}_{LH}}_{\text{B's good borrower}},$$

where the first equality follows from conditional independence and the second one is from the symmetric hard information technology.

Therefore, $\Delta\pi\left(s;\eta\right)$ defined in Eq. (58) becomes

$$\Delta\pi(s;\eta) = p_{HH}\left(s\right)\int_0^s \phi_1\left(t\right)dt - \phi_1\left(s\right)\int_0^s p_{HH}\left(t\right)dt + \left[p_{HL}\left(s\right) - \phi_1\left(s\right)\overline{p}_{LH}\right].$$

The bracketed term about the lending costs in disagreement events is a constant in η. We rearrange the remaining terms about the lending costs in the competition event HH,

$$\begin{aligned} p_{HH}\left(s\right)\int_0^s \phi_1\left(t\right)dt - \phi_1\left(s\right)\int_0^s p_{HH}\left(t\right)dt &= \overline{p}_{HH}\int_0^s \left[\phi\left(s|HH\right)\phi_1\left(t\right) - \phi\left(t|HH\right)\phi_1\left(s\right)\right]dt \\ &= \overline{p}_{HH}\int_0^s \left\{\phi_0\left(s\right) + \frac{\alpha^2 q_s}{\alpha^2 - \left(2\alpha-1\right)\eta}\left[\phi_1\left(s\right) - \phi_0\left(s\right)\right]\right\}\phi_1\left(t\right)dt \\ &- \overline{p}_{HH}\int_0^s \left\{\phi_0\left(t\right) + \frac{\alpha^2 q_s}{\alpha^2 - \left(2\alpha-1\right)\eta}\left[\phi_1\left(t\right) - \phi_0\left(t\right)\right]\right\}\phi_1\left(s\right)dt \\ &= \overline{p}_{HH}\left(1 - \frac{\alpha^2 q_s}{\alpha^2 - \left(2\alpha-1\right)\eta}\right)\int_0^s \phi_0\left(s\right)\phi_0\left(t\right)\left[\frac{\phi_1\left(t\right)}{\phi_0\left(t\right)} - \frac{\phi_1\left(s\right)}{\phi_0\left(s\right)}\right]dt \\ &= K(\eta)\int_0^s \phi_0\left(s\right)\phi_0\left(t\right)\left[\frac{\phi_1\left(t\right)}{\phi_0\left(t\right)} - \frac{\phi_1\left(s\right)}{\phi_0\left(s\right)}\right]dt \end{aligned}$$

where $K(\eta) \equiv \overline{p}_{HH}\left(1 - \frac{\alpha^2 q_s}{\alpha^2-(2\alpha-1)\eta}\right)$ decreases in η as both $\overline{p}_{HH}$ and $1 - \frac{\alpha^2 q_s}{\alpha^2-(2\alpha-1)\eta}$ decrease in η, and $\frac{\phi_1(s)}{\phi_0(s)} - \frac{\phi_1(t)}{\phi_0(t)}$ is negative due to MLRP and $s > t$. Therefore, for each $s \in [\hat{s}, 1]$, $\Delta\pi\left(s;\eta\right)$ increases in information span η.

We now show the first part of the Proposition holds. We aim to argue that η weakly increases Bank B's profits

$$\frac{d\pi^B}{d\eta} \geq 0.$$

In addition, the inequality is strict if and only if $\pi^B > 0$, so there exists a threshold information span above which equilibrium is positive weak. When $\pi^B = 0$, from

$$\frac{d\pi^A\left(r\left(s\right),s\right)}{d\eta} < 0 = \frac{d\left[\pi^B\left(r\left(s\right)\right)\frac{s\phi(s)}{q_s}\right]}{d\eta}, \quad \text{where } s \in [\hat{s}, 1]$$

When $\pi^B > 0$, $\hat{s} = s^A_{be}$. From Lemma 4, for any $\eta_1 < \eta_2$,

$$\hat{s}\left(\eta_1\right) = s^A_{be}\left(\eta_1\right) > s^A_{be}\left(\eta_2\right) = \hat{s}\left(\eta_2\right).$$

Then when $\eta = \eta_2$, Bank A breaks even upon soft signal $\hat{s}\left(\eta_2\right)$ and makes profits upon a better soft signal $\hat{s}\left(\eta_1\right)$, i.e.,

$$\pi^A\left(r^A\left(\hat{s}\left(\eta_1\right)\right), \hat{s}\left(\eta_1\right); \eta_2\right) > 0 = \pi^A\left(r^A\left(\hat{s}\left(\eta_1\right)\right), \hat{s}\left(\eta_1\right); \eta_1\right).$$

This is to say, conditional on soft signal type $\hat{s}\left(\eta_1\right)$ for Bank A, broader information span from η_1 to η_2 increases its profits. This implies that Bank B should benefit more from broader hard information,

$$\frac{\hat{s}\left(\eta_1\right)\phi\left(\hat{s}\left(\eta_1\right)\right)}{q_s}\left[\pi^B\left(\hat{s}\left(\eta_1\right);\eta_2\right) - \pi^B\left(\hat{s}\left(\eta_1\right);\eta_1\right)\right] > \pi^A\left(r\left(\hat{s}\left(\eta_1\right)\right), \hat{s}\left(\eta_1\right);\eta_2\right) - \pi^A\left(r^A\left(\hat{s}\left(\eta_1\right)\right), \hat{s}\left(\eta_1\right);\eta_1\right) > 0.$$

As Bank B makes a constant profit,

$$\pi^B\left(\eta_2\right) > \pi^B\left(\eta_1\right).$$

This holds for any $\eta_1 < \eta_2$ when the resulting equilibrium is positive weak, so

$$\frac{d\pi^B\left(\eta\right)}{d\eta} > 0\left(=0\right)$$

if $\pi^B\left(\eta\right) > 0\left(=0\right)$.

For the second part of the proposition, in a positive-weak equilibrium the first non-competing term in (56) is zero for Bank A, it follows that $d\pi^B/d\eta > d\mathbb{E}\left[\tilde{\pi}^A\right]/d\eta$, which is our desired claim.[27] Note, as we can show that the dominance holds point-wisely, it is stronger than the statement on expectation in point 2) in Proposition 2. □

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[27] The total effect of $d\pi^B/d\eta$ should also take into account the first non-competing term for Bank B; but point 1) in Proposition 2 shows that this term is positive.

A.4 Proof of Proposition 3

Proof. First, we argue that Bank A benefits more when $s \in [\hat{s}, 1]$, i.e. $\left.\frac{\partial \Delta \pi(s;\alpha)}{\partial \alpha}\right|_{\alpha \to 1} > 0$ in the vicinity of $\alpha = 1$. To see this,

$$\frac{\partial \Delta \pi (s; \alpha)}{\partial \alpha} = \int_0^s \left[\frac{s\phi(s)}{q_s} \frac{\partial p_{HH}(t)}{\partial \alpha} - \frac{\partial p_{HH}(s)}{\partial \alpha} \frac{t\phi(t)}{q_s} \right] dt + \left[\frac{\partial \overline{p}_{LH}}{\partial \alpha} \frac{s\phi(s)}{q_s} - \frac{\partial p_{HL}(s)}{\partial \alpha} \right].$$

Recall that

$$p_{HH}(s) = \left[\underbrace{\left(1 - q_h^h\right)}_{\theta_h^h = 0} \phi(s) + \underbrace{q_h^h \left(1 - q_s^h\right)}_{\theta_h^h = 1, \theta_s^h = 0} \phi_0(s) \right] (1-\alpha)^2 + \left[\underbrace{q_h^h q_s^h (1 - q_s^s)}_{\theta_h^h = \theta_s^h = 1, \theta_s^s = 0} \phi_0(s) + \underbrace{q}_{\theta = 1} \phi_1(s) \right] \alpha^2,$$

and then

$$\frac{\partial p_{HH}(s)}{\partial \alpha} = -2(1-\alpha)\left[\left(1 - q_h^h\right)\phi(s) + q_h^h\left(1 - q_s^h\right)\phi_0(s)\right] + 2\alpha\left[q_h^h q_s^h (1 - q_s^s)\phi_0(s) + q\phi_1(s)\right].$$

Hence, when $\alpha \to 1$, we have

$$\frac{s\phi(s)}{q_s} \frac{\partial p_{HH}(t)}{\partial \alpha} - \frac{\partial p_{HH}(s)}{\partial \alpha} \frac{t\phi(t)}{q_s} \to 2\alpha q_h^h q_s^h (1 - q_s^s)(\phi_1(s)\phi_0(t) - \phi_0(s)\phi_1(t)).$$

In addition,

$$\frac{\partial \overline{p}_{LH}}{\partial \alpha} \frac{s\phi(s)}{q_s} - \frac{\partial p_{HL}(s)}{\partial \alpha} = (1 - 2\alpha)\left[\frac{s\phi(s)}{q_s} - \phi(s)\right].$$

Using these terms, we have

$$\left.\frac{\partial \Delta \pi (s; \alpha)}{\partial \alpha}\right|_{\alpha \to 1} = \frac{2q_h^h q_s^h (1 - q_s^s)}{q_s (1 - q_s)} \phi(s) \int_0^s (s - t)\phi(t)\, dt - \left[\frac{s\phi(s)}{q_s} - \phi(s)\right].$$

Under the primitive condition of $\frac{2q_h^h q_s^h (1-q_s^s)}{1-q_s} > 1$,

$$\left.\frac{\partial \Delta \pi (s; \alpha)}{\partial \alpha}\right|_{\alpha \to 1} > \int_{\hat{s}}^1 \phi(s) \left[\frac{1}{q_s} \int_0^s (s - t)\phi(t)\, dt - \left(\frac{s}{q_s} - 1\right)\right] ds.$$

For the integrand, note that when $s = 1$,

$$\left.\frac{1}{q_s} \int_0^s (s - t)\phi(t)\, dt - \left(\frac{s}{q_s} - 1\right)\right|_{s=1} = \frac{1}{q_s}(1 - q_s) - \left(\frac{1}{q_s} - 1\right) = 0;$$

addition, the integrand decreases in s,

$$\frac{\partial\left[\frac{1}{q_s}\int_0^s (s-t)\,\phi(t)\,dt-\left(\frac{s}{q_s}-1\right)\right]}{\partial s}=\frac{1}{q_s}\left[\Phi(s)-1\right]<0,$$

so it is positive when $s\in[\hat{s},1)$.

Therefore, we have shown the first part that when $s\in[\hat{s},1]$, $\left.\frac{\partial\Delta\pi(s;\alpha)ds}{\partial\alpha}\right|_{\alpha\to1}>0$. Because the equilibrium is zero-weak and α has no effect on Bank B's equilibrium profits, we have

$$\left.\frac{\partial\pi(s;\alpha)}{\partial\alpha}\right|_{\alpha\to1}>0.$$

In addition, similar as in Lemma 4, we have $\frac{d\hat{s}}{d\alpha}<0$. Hence, if $\frac{dx}{d\alpha}<0$, Bank A's expected equilibrium profits $\int_x^1\pi^A ds$ increases in α.

In a zero-weak equilibrium, we have $\hat{s}=s_B^{be}$. For Bank A, the break-even threshold x satisfies

$$0=\pi^A(\overline{r},x)=p_{HH}(x)\frac{\int_0^{s_{be}^B}t\phi(t)\,dt}{q_s}\left[\mu_{HH}(x)(1+\overline{r})-1\right]+p_{HL}(x)\left[\mu_{HL}(x)(1+\overline{r})-1\right].$$

Define the Implicit function $F(\alpha,x)\equiv\pi^A(\overline{r},x)=0$. We calculate

$$\frac{\partial F}{\partial\alpha}=\frac{d\pi^A(\overline{r},x)}{d\alpha}=\frac{\partial\pi^A(\overline{r},x)}{\partial\alpha}+\frac{\partial s_{be}^B}{\partial\alpha}\frac{\partial\pi^A(\overline{r},x)}{\partial s_{be}^B}.$$

When $\alpha\to1$, we can solve for x and s_B^{be}. The threshold s_B^{be} is defined from

$$0=\hat{\pi}^B\left(\overline{r};s_{be}^B\right)=\int_0^{s_{be}^B}p_{HH}(t)\left[\mu_{HH}(t)(\overline{r}+1)-1\right]dt\underbrace{+\overline{p}_{LH}\left[\overline{\mu}_{LH}(\overline{r}+1)-1\right]}_{\to0\text{ when }\alpha\to1},$$

Hence, $\frac{\int_0^{s_{be}^B}t\phi(t)dt}{\int_0^{s_{be}^B}\phi(t)dt}=\frac{1}{1+\overline{r}}$, and similarly, $x(\alpha\to1)=\frac{1}{1+\overline{r}}$. It follows that

$$\begin{aligned}\frac{\partial\pi^A(\overline{r},x)}{\partial s_{be}^B}&=\frac{s_{be}^B\phi\left(s_{be}^B\right)}{q_s}p_{HH}(x)\left[\mu_{HH}(x)(1+\overline{r})-1\right]\\&\to\frac{s_{be}^B\phi\left(s_{be}^B\right)}{q_s}p_{HH}(x)\left[x(1+\overline{r})-1\right]\to0.\end{aligned}$$

Hence, at $x(\alpha)$ where Bank A breaks even, the only effect of α is via the direct change in technology,

$$\frac{\partial F}{\partial \alpha} = \frac{\partial \pi^A(\overline{r}, x)}{\partial \alpha} + \frac{\partial s_{be}^B}{\partial \alpha} \underbrace{\frac{\partial \pi^A(\overline{r}, x)}{\partial s_{be}^B}}_{\to 0} \to \frac{\partial \pi^A(\overline{r}, x)}{\partial \alpha}.$$

Note that

$$\frac{\partial \pi^A(\overline{r}, x)}{\partial \alpha} = \underbrace{\frac{\int_0^{s_{be}^B} t\phi(t)\,dt}{q_s} \phi(x) \left\{2q_h \alpha x(1+\overline{r}) + 2(1 - q_h - q_h)\right\}}_{\to 0} - (2\alpha - 1)\phi(x) \left[\underbrace{q_h^0 x(1+\overline{r}) - 1}_{-}\right] > 0.$$

For Bank A, α has no effect in the case of competition upon HH, because the profit conditional on winning is close to zero. Instead, α reduces the winner's curse to Bank A with signal x—the case where competitor Bank B receives $h^B = L$.

Taken together, we have $\frac{\partial F}{\partial \alpha} > 0$. Combined with $\frac{\partial F}{\partial x} = \frac{\partial \pi^A(\overline{r}, x)}{\partial x} > 0$, implicit function theorem implies $\frac{dx}{d\alpha} < 0$. □

A.5 Derivation of Lenders' Beliefs about Fundamentals

We first replicate Eq. (28) here which will be used repeatedly below,

$$p_{HH}(s) = q\alpha^2 \phi_1(s) + \left(1 - q_h^h\right)(1-\alpha)^2 \phi(s) + \left[(1 - q_s)\alpha^2 - \eta(2\alpha - 1)\right] q_h^h \phi_0(s).$$

The posterior mean of θ conditional on $\left\{h^A = h^B = H, s\right\}$, can be calculated as

$$z(HHs) \equiv \mathbb{E}\left[\theta = 1 \middle| h^A = h^B = H, s\right] = \frac{\mathbb{P}\left(\theta = 1, h^A = h^B = H, s\right)}{p_{HH}(s)} = \frac{q\alpha^2 \phi_1(s)}{p_H H(s)}.$$

Denote $z(HHs) = g(s)$ as z as the function of s and $g(\cdot)$ is monotone due to MLRP. The density of $z(HHs)$ is then

$$\begin{aligned} \Pr\left(z \in (z, z+dz), h^A = H \middle| h^B = H\right) &= \Pr\left(g^{-1}(z) \in \left(g^{-1}(g(s)), g^{-1}(g(s) + d\rho)\right), h^A = H \middle| h^B = H\right) \\ &= \Pr\left(s \in \left(s, s + g^{-1\prime} \cdot dz\right), h^A = H \middle| h^B = H\right) \\ &= \frac{p_{HH}\left(g^{-1}(z)\right)}{\mathbb{P}\left(h^B = H\right)} \frac{1}{g'\left(g^{-1}(z)\right)} dz, \end{aligned} \tag{61}$$

where

$$\frac{p_{HH}(s)}{\mathbb{P}\left(h^B = H\right)} = \frac{p_{HH}(s)}{q_h \alpha + (1 - q_h)(1 - \alpha)}. \tag{62}$$

Similarly, The posterior mean of θ_s^s conditional on $\left\{h^A = h^B = H, s\right\}$, can be calculated as

$$\begin{aligned} z_s^s\left(HHs\right) \equiv \mathbb{E}\left[\theta_s^s = 1 \middle| h^A = h^B = H, s\right] &= \frac{\mathbb{P}\left(\theta_s^s = 1, h^A = h^B = H, s\right)}{p_{HH}\left(s\right)} \\ &= \frac{\left[q_h^h \alpha^2 + \left(1 - q_h^h\right)\left(1-\alpha\right)^2\right] \cdot q_s^h q_s^s \phi_1\left(s\right) + q_s^s\left(1 - q_s^h\right)\left(1-\alpha\right)^2 \phi_0\left(s\right)}{p_{HH}(s)}. \end{aligned}$$

The density of $z_s^s(HHs)$ is

$$\frac{p_{HH}\left(g_s^{s-1}\left(z\right)\right)}{\mathbb{P}\left(h^B = H\right)} \frac{1}{g_s^{s\prime}\left(g_s^{s-1}\left(z\right)\right)} dz$$

where $g_s^s(s) = z_s^s(HHs)$ is a monotone function of s.

A.6 Derivation of Correlated Hard Signals

Another aspect of information technology advancement is that the lenders' hard information signals become more correlated. Formally, with probability ρ_h, lenders receive the same signal realization $h^c \in \{H, L\}$ and

$$\mathbb{P}\left(h^c = H \left| \theta_h = 1 \right.\right) = \mathbb{P}\left(h^c = L \left| \theta_h = 0 \right.\right) = \alpha;$$

with probability $1 - \rho_h$, each receives an independent hard signal according to Eq. (4).

With more correlated hard signals or a higher ρ^h, lenders are more likely to agree on the customer quality and so more likely to compete (the event of HH). In terms of inference, the posterior upon disagreement (that comes from the uncorrelated part of the assessment) is still the prior q_h.[28] Taken together, competition becomes fiercer, because lenders are more likely to compete but not more concerned about the winner's curse.

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[28] Upon competition (HH), lenders are less sure about a good quality borrower, i.e., $\mu_{HH}\left(\rho_h\right)$ decreases in ρ_h.

A.7 Signal on Hardened Soft Fundamental

Perfectly correlated hardened soft signal. First, given the hardened soft signal $h_s^c = H$, the conditional density of soft signal s is

$$\begin{aligned}
\phi\left(s \mid h_s^c = H\right) &= \frac{\frac{1}{ds}\mathbb{P}\left(h_s^c = H, s \in ds\right)}{\mathbb{P}\left(h_s^c = H\right)} \\
&= \frac{q_h^s q_s^s \alpha_s \phi_1(s) + q_h^s\left(1-q_s^s\right)\alpha_s \phi_0(s) + \left(1-q_h^s\right) q_s^s\left(1-\alpha_s\right)\phi_0(s) + \left(1-q_h^s\right)\left(1-q_s^s\right)\left(1-\alpha_s\right)\phi_0(s)}{q_h^s \alpha_s + \left(1-q_h^s\right)\left(1-\alpha_s\right)} \\
&= \frac{q_h^s q_s^s \alpha_s}{q_h^s \alpha_s + \left(1-q_h^s\right)\left(1-\alpha_s\right)}\left[\phi_1(s) - \phi_0(s)\right] + \phi_0(s) \\
&= \phi_0(s) + \overbrace{\frac{\alpha_s}{\alpha_s - \left(2\alpha_s - 1\right)\eta}}^{\uparrow \text{ in } \eta \text{ as } \alpha_s > \frac{1}{2}} \cdot q_s\left[\phi_1(s) - \phi_0(s)\right].
\end{aligned}$$

Note that when additionally conditioning on the good soft fundamental state $\theta_s = 1$, the hardened soft signal h_s^c and soft signal s are independent so that

$$\phi\left(s \mid h_s^c = H, \theta_s = 1\right) = \phi\left(s \mid \theta_s = 1\right) = \phi_1(s).$$

Hence, $h_s^c = H$ reveals information about s only through reducing type II errors.

Due to the independence between the original hard signals h^A, h^B and hardened soft h_s^c , soft signal s, we introduce notations to separate the events of signal realizations. Let $\hat{p}_{h^A h^B} \equiv \mathbb{P}\left(h^A, h^B\right)$ denote probability of the hard signal realizations, and $\hat{\mu}_{h^A h^B} \equiv \mathbb{P}\left(\theta_h^h = 1 \middle| h^A, h^B\right)$ denote the posterior probability of successful θ_h^h conditional on original hard signals. Let $p_{h_s^c}(t) \equiv \mathbb{P}\left(h_s^c, s \in ds\right)$ denote the joint density of hardened soft signal h_s^c and soft signal s, and $\mu_{h_s^c} \equiv \left(\theta_s = 1 \left| h_s^c, s \in ds\right.\right)$ denote the posterior probability of successful θ_s given h_s^c and s. Let $\overline{\mu}_{h_s^c} \equiv \left(\theta_s = 1 \left| h_s^c\right.\right)$ denote the posterior probability of successful θ_s given h_s^c. Then lender's payoff function could be rewritten as

$$\begin{aligned}
&\pi^A(r, s) \\
=&p_{HHH}(s)\left[1 - F^B(r)\right]\left[\mu_{HHH}(s)(1+r) - 1\right] + p_{HLH}(s)\left[\mu_{HLH}(s)(1+r) - 1\right] \\
=&\hat{p}_{HH} p_H(s)\left[1 - F^B(r)\right]\left[\hat{\mu}_{HH}\mu_H(s)(1+r) - 1\right] + \hat{p}_{HL} p_H(s)\left[\hat{\mu}_{HL}\mu_H(s)(r+1) - 1\right] \\
\propto&\hat{p}_{HH}\left[1 - F^B(r)\right]\left[\hat{\mu}_{HH}\overline{\mu}_H \phi_1(s)(1+r) - \phi\left(s \mid h_s^c = H\right)\right] + \hat{p}_{HL}\left[\hat{\mu}_{HL}\overline{\mu}_H \phi_1(s)(r+1) - \phi\left(s \mid h_s^c = H\right)\right],
\end{aligned}$$

where the "proportional to" in the last equation omits a constant $\mathbb{P}\left(h_s^c = H\right)$. Compared with the benchmark setting where hard and soft signals are independent, adding the hardening soft signal plays two roles. First, $h_s^c = H$ improves screening, and lenders are more likely to have good borrowers, as seen by the term $\overline{\mu}_H \phi_1(s)$ in the above equation versus $q_s \phi_1(s)$ in the benchmark.

Second, $h_s^c = H$ updates the distribution of the soft signal, as seen by $\phi\left(s \middle| h_s^c = H\right)$ versus $\phi(s)$ in the benchmark; note that conditional on a good project, $h_s^c = H$ is independent and uninformative about the soft signal, so under both settings its density is $\phi_1(s)$ conditional on repayment.

Similarly, for Bank B,

$$\begin{aligned}\pi^B(r) &\equiv \int_0^{s^A(r)} p_{HHH}(t)\left[\mu_{HHH}(t)(r+1)-1\right]dt + \overline{p}_{LHH}\left[\overline{\mu}_{LHH}(r+1)-1\right].\\ &= \hat{p}_{HH}\int_0^{s^A(r)} p_H(t)\left[\hat{\mu}_{HH}\mu_H(t)(r+1)-1\right]dt + \hat{p}_{LH}\overline{p}_H\left[\hat{\mu}_{LH}\overline{\mu}_H(r+1)-1\right]\\ &\propto \hat{p}_{HH}\int_0^{s^A(r)}\left[\hat{\mu}_{HH}\mu_H\phi_1(s)(r+1)-\phi\left(s \middle| h_s^c = H\right)\right]dt + \hat{p}_{LH}\left[\hat{\mu}_{LH}\mu_H(r+1)-1\right].\end{aligned}$$

A.8 Proof of Proposition 4

Proof. Recall that $r^j = \infty$ means lender $j \in \{A, B\}$ does not make an offer, and then $\min\left\{r^A, r^B\right\} < \infty$ means that the borrower is funded. Let W denote the total welfare,

$$W \equiv \mathbb{P}\left(\theta = 1, \min\left\{r^A, r^B\right\} < \infty\right)\overline{r} - \mathbb{P}\left(\theta = 0, \min\left\{r^A, r^B\right\} < \infty\right), \tag{63}$$

where funding a good borrower generates $\overline{r}$ in net social gain while funding a bad borrower costs 1 dollar. A borrower is funded when at least one lender is making an offer ($r^B < \infty$ or $s \geq x$) in the event of HH and when Bank A (B) makes an offer in the event of HL (LH). Recall that $\overline{p}_{h^A h^B} \equiv \mathbb{P}\left(h^A, h^B\right)$, $p_{h^A h^B}(s)\,ds \equiv \mathbb{P}\left(h^A, h^B, s \in ds\right)$, and $\overline{\mu}_{h^A h^B} \equiv \mathbb{P}\left(\theta = 1 \middle| h^A, h^B\right)$, $\mu_{h^A h^B} \equiv \mathbb{P}\left(\theta = 1 \middle| h^A, h^B, s\right)$, and then

$$\begin{aligned}\mathbb{P}\left(\theta = 1, \min\left\{r^A, r^B\right\} < \infty\right) &= \underbrace{\overline{p}_{HH}\overline{\mu}_{HH} - \int_0^x \left[p_{HH}(t)\,\mu_{HH}(t)\right]dt \cdot \left(1 - F^B(\overline{r})\right)}_{HH}\\ &\quad + \underbrace{\int_x^1 p_{HL}(t)\,\mu_{HL}(t)\,dt}_{HL} + \underbrace{\overline{p}_{LH}\overline{\mu}_{LH}F^B(\overline{r})}_{LH},\\ \mathbb{P}\left(\theta = 0, \min\left\{r^A, r^B\right\} < \infty\right) &= \underbrace{\overline{p}_{HH}\left[1 - \overline{\mu}_{HH}\right] - \int_0^x \left\{p_{HH}(t)\left[1 - \mu_{HH}(t)\right]\right\}dt \cdot \left(1 - F^B(\overline{r})\right)}_{HH}\\ &\quad + \underbrace{\int_x^1 p_{HL}(t)\left[1 - \mu_{HL}(t)\right]dt}_{HL} + \underbrace{\overline{p}_{LH}\left[1 - \overline{\mu}_{LH}\right]F^B(\overline{r})}_{LH}.\end{aligned}$$

When the information span η changes, there are two types of effects. The first is improvement in screening as η affects the conditional distribution of hard signals. The second is the indirect effects as η affects lender participation upon H—$x(\eta)$ for Bank A and $F^B(\overline{r}) = 1 - \mathbf{1}_{\pi^B = 0} \cdot \frac{\int_0^{\hat{s}(\eta)} t\phi(t)dt}{q_s}$ for

Bank B are both functions of η.

Zero weak equilibrium In the zero weak equilibrium, we show that the indirect effects of lender participation are zero. To see this, regarding Bank A's participation x,

$$\begin{aligned}\frac{\partial W}{\partial x} &= \Big\{-\left[p_{HH}\left(x\right)\mu_{HH}\left(x\right)\right]dt\left(1-F^{B}\left(\overline{r}\right)\right)-p_{HL}\left(x\right)\mu_{HL}\left(x\right)\Big\}\overline{r}\\ &\quad+p_{HH}\left(x\right)\left[1-\mu_{HH}\left(x\right)\right]\left(1-F^{B}\left(\overline{r}\right)\right)+p_{HL}\left(x\right)\left[1-\mu_{HL}\left(x\right)\right]\\ &= -\Big\{p_{HH}\left(x\right)\left(1-F^{B}\left(\overline{r}\right)\right)\left[\mu_{HH}\left(x\right)\left(1+\overline{r}\right)-1\right]+p_{HL}\left(x\right)\left[\mu_{HL}\left(x\right)\left(1+\overline{r}\right)-1\right]\Big\}\\ &= -\pi^{A}\left(\overline{r},x\right)=0.\end{aligned}$$

When x increases, the social marginal borrower who lost credit access is exactly Bank A's marginal borrower where it breaks even at $\overline{r}$ upon signal x, so x's effects on welfare is zero. Similarly, regarding Bank B's participation,

$$\begin{aligned}\frac{\partial W}{\partial F^{B}\left(\overline{r}\right)} &= \left\{\int_{0}^{x}\left[p_{HH}\left(t\right)\mu_{HH}\left(t\right)\right]dt+\overline{p}_{LH}\overline{\mu}_{LH}\right\}\overline{r}-\int_{0}^{x}\left\{p_{HH}\left(t\right)\left[1-\mu_{HH}\left(t\right)\right]\right\}dt-\overline{p}_{LH}\left(1-\overline{\mu}_{LH}\right)\\ &= \int_{0}^{x}p_{HH}\left(t\right)\left[\mu_{HH}\left(t\right)\left(\overline{r}+1\right)-1\right]dt+\overline{p}_{LH}\left[\overline{\mu}_{LH}\left(\overline{r}+1\right)-1\right]\\ &= \pi^{B}\left(\overline{r}\right)=0,\end{aligned}$$

i.e., the marginal borrower who gains credit access is Bank B's marginal borrower when it quotes $\overline{r}$ and breaks even, so $F^{B}\left(\overline{r}\right)$'s effects on total welfare is also zero.

Then as information span increases, total welfare is affected through screening technology only. Specifically, the Type II errors in the event of HH are reduced,

$$\begin{aligned}\frac{\partial W}{\partial \eta} &= \underbrace{-\frac{\partial \overline{p}_{HH}\left(\eta\right)}{\partial \eta}\left[1-\int_{0}^{\hat{s}}\phi_{1}\left(t\right)dt\cdot\int_{0}^{x}\phi_{0}\left(t\right)dt\right]}_{\text{screening}}+\underbrace{\frac{\partial W}{\partial x}\frac{\partial x}{\partial \eta}+\frac{\partial W}{\partial F^{B}\left(\overline{r}\right)}\frac{\partial F^{B}\left(\overline{r}\right)}{\partial \eta}}_{=0}\\ &= -\frac{\partial q_{h}}{\partial \eta}\cdot\left[\alpha^{2}-\left(1-\alpha\right)^{2}\right]\left[1-\int_{0}^{\hat{s}}\phi_{1}\left(t\right)dt\cdot\int_{0}^{x}\phi_{0}\left(t\right)dt\right]>0,\end{aligned}$$

where $\phi_{1}\left(s\right)\equiv\frac{s\phi(s)}{q_{s}}$ and $\phi_{0}\left(s\right)\equiv\frac{(1-s)\phi(s)}{1-q_{s}}$ are the conditional densities of s. To see this, Type I errors do not change due to the multiplicative structure: conditional on $\theta=1$, the distributions of hard signals are irrelevant of η. In addition, because of the symmetric hard signal precision, the disagreement events HL and LH do not have information content and are irrelevant of η either.

Positive weak equilibrium. In the positive weak equilibrium, Bank B always makes an offer upon H. The effect from Bank A's participation is no longer zero because there is a gap between the marginal borrower x's value to Bank A and the social gain. Specifically, in competition (HH), as x increases, there is transfer between the lenders as Bank B wins this marginal borrower but total welfare does not change.

First, we calculate the total welfare. In this case,

$$\mathbb{P}\left(\theta = 1, \min\left\{r^A, r^B\right\} < \infty\right) = \underbrace{q\alpha}_{\theta=1,h^B=H} + q\underbrace{\alpha\left(1-\alpha\right)}_{HL}\underbrace{\left[1 - \int_0^x \phi_1\left(t\right)dt\right]}_{s\geq x|\theta_s=1},$$

$$\begin{aligned}\mathbb{P}\left(\theta = 0, \min\left\{r^A, r^B\right\} < \infty\right) = &\underbrace{\left(1-q_h\right)\left(1-\alpha\right)^2 + q_h \cdot \left(1-q_s^s\right)\alpha^2}_{HH} + \underbrace{\left(1-q\right)\alpha\left(1-\alpha\right)}_{LH} \\ &+ \underbrace{\left(1-\alpha\right)\alpha\left\{\left(1-q_h^h\right)q_s \cdot \left[1 - \int_0^x \phi_1\left(t\right)dt\right] + \left(1-q_s\right)\left[1 - \int_0^x \phi_0\left(t\right)dt\right]\right\}}_{HL}.\end{aligned}$$

Take derivative w.r.t. η,

$$\begin{aligned}\frac{\partial W}{\partial \eta} &= -q_h'\left(\eta\right)\left[\alpha^2 - \left(1-\alpha\right)^2\right] - \left(1-\alpha\right)\alpha \cdot \frac{\partial x}{\partial \eta} \cdot \left\{\left[q\overline{r} - \left(1-q_h^h\right)q_s\right]\phi_1\left(x\right) - \left(1-q_s\right)\phi_0\left(x\right)\right\} \\ &= q_h^h \cdot \left[\alpha^2 - \left(1-\alpha\right)^2\right] - \left(1-\alpha\right)\alpha \cdot \frac{\partial x}{\partial \eta} \cdot \left\{\left[q_h^h\left(\overline{r}+1\right) - 1\right]x\phi\left(x\right) - \left(1-x\right)\phi\left(x\right)\right\} \\ &= \underbrace{q_h^h \cdot \left[\alpha^2 - \left(1-\alpha\right)^2\right]}_{\text{screening}} - \underbrace{\overbrace{\frac{\partial x}{\partial \eta}}^{-} \cdot \left(1-\alpha\right)\alpha\phi\left(x\right) \cdot \overbrace{\left[q_h^h \cdot x \cdot \left(\overline{r}+1\right) - 1\right]}^{-}}_{A\text{'s participation}} \qquad (64)\end{aligned}$$

We argue that the effect of A's participation is negative. To see this, we have shown $\frac{\partial s_A^{be}}{\partial \eta} < 0$ in Lemma 4; as $x = \hat{s} = s_A^{be}$ in the positive weak equilibrium, we have $\frac{\partial x}{\partial \eta} < 0$. In addition, Bank A's break even condition upon x is

$$\begin{aligned}0 = \pi^A\left(\overline{r}, x\right) &= p_{HH}\left(x\right)\left[1 - F^B\left(\overline{r}^-\right)\right]\left[\mu_{HH}\left(x\right)\left(1+\overline{r}\right) - 1\right] + p_{HL}\left(x\right)\left[\mu_{HL}\left(x\right)\left(1+\overline{r}\right) - 1\right] \\ &= p_{HH}\left(x\right)\int_0^x \phi_1\left(t\right)dt\left[\mu_{HH}\left(x\right)\left(1+\overline{r}\right) - 1\right] + \alpha\left(1-\alpha\right)\phi\left(x\right)\left[q_h^h x\left(1+\overline{r}\right) - 1\right]. \qquad (65)\end{aligned}$$

As $\mu_{HH}\left(x\right) > q_h^h x$ from belief updating, $q_h^h x\left(1+\overline{r}\right) - 1 < 0 < \mu_{HH}\left(x\right)\left(1+\overline{r}\right) - 1$ must hold for Bank A to break even.

We use the implicit function theorem to solve for $\frac{\partial x}{\partial \eta}$. Define $\pi^A(\eta, x) = 0$ from Eq. (65), and

$$\begin{aligned}
\frac{\partial \pi^A(\eta, x)}{\partial \eta} &= -\frac{\partial p_{HH}(x)}{\partial x} \cdot \int_0^x \phi_1(t)\, dt = q_h^h \left[\alpha^2 - (1-\alpha)^2\right] \phi_0(x) \cdot \int_0^x \phi_1(t)\, dt > 0, \\
\frac{\partial \pi^A(\eta, x)}{\partial x} &= \underbrace{p_{HH}(x)\, \phi_1(x) \left[\mu_{HH}(x)(1+\overline{r}) - 1\right]}_{+} \\
&\quad + \underbrace{\int_0^x \phi_1(t)\, dt \cdot \frac{\partial \left\{p_{HH}(x) \left[\mu_{HH}(x)(1+\overline{r}) - 1\right]\right\}}{\partial x} + \alpha(1-\alpha) \cdot \frac{\partial \left\{\phi(x) \left[q_h^h x (1+\overline{r}) - 1\right]\right\}}{\partial x}}_{+} > 0.
\end{aligned}$$

This gives us a lower bound of $\frac{\partial x}{\partial \eta}$,

$$0 > \frac{\partial x}{\partial \eta} = -\frac{\frac{\partial \pi^A(\eta,x)}{\partial \eta}}{\frac{\partial \pi^A(\eta,x)}{\partial x}} > -\frac{q_h^h \left[\alpha^2 - (1-\alpha)^2\right] \phi_0(x) \cdot \int_0^x \phi_1(t)\, dt}{p_{HH}(x)\, \phi_1(x) \left[\mu_{HH}(x)(1+\overline{r}) - 1\right]}.$$

Use this inequality in Eq. (64), we have

$$\begin{aligned}
\frac{\partial W}{\partial \eta} &= q_h^h \left[\alpha^2 - (1-\alpha)^2\right] - \frac{\partial x}{\partial \eta} (1-\alpha)\, \alpha \phi(x) \overbrace{\left[q_h^h x (\overline{r}+1) - 1\right]}^{-} \\
&> q_h^h \left[\alpha^2 - (1-\alpha)^2\right] + \frac{q_h^h \left[\alpha^2 - (1-\alpha)^2\right] \phi_0(x) \int_0^x \phi_1(t)\, dt}{p_{HH}(x)\, \phi_1(x) \left[\mu_{HH}(x)(1+\overline{r}) - 1\right]} (1-\alpha)\, \alpha \phi(x) \left[q_h^h x (\overline{r}+1) - 1\right] \\
&= q_h^h \left[\alpha^2 - (1-\alpha)^2\right] + \frac{q_h^h \left[\alpha^2 - (1-\alpha)^2\right] \phi_0(x) \int_0^x \phi_1(t)\, dt}{p_{HH}(x)\, \phi_1(x) \left[\mu_{HH}(x)(1+\overline{r}) - 1\right]} \left\{-p_{HH}(x) \int_0^x \phi_1(t)\, dt \left[\mu_{HH}(x)(1+\overline{r}) - 1\right]\right\} \\
&= q_h^h \left[\alpha^2 - (1-\alpha)^2\right] \left[1 - \frac{\phi_0(x) \left(\int_0^x \phi_1(t)\, dt\right)^2}{\phi_1(x)}\right],
\end{aligned}$$

where the second last equation holds from Bank A's break even condition at x in Eq. (65). The second bracketed term in the last equation equals

$$1 - \frac{\phi_0(x) \cdot \left(\int_0^x \phi_1(t)\, dt\right)^2}{\phi_1(x)} = 1 - \frac{(1-x) \left(\int_0^x t \phi(t)\, dt\right)^2}{q_s (1-q_s)\, x}.$$

From the Cauchy Schwartz inequality,

$$q_s (1-q_s) = \int_0^1 t \phi(t)\, dt \int_0^1 (1-t)\, \phi(t)\, dt \geq \left(\int_0^1 \sqrt{t(1-t)} \phi(t)\, dt\right)^2,$$

so

$$\begin{aligned}1-\frac{\phi_0\left(x\right)\cdot\left(\int_0^x\phi_1\left(t\right)dt\right)^2}{\phi_1\left(x\right)} &\geq 1-\frac{1-x}{x}\cdot\left(\frac{\int_0^x t\phi\left(t\right)dt}{\int_0^1\sqrt{t\left(1-t\right)}\phi\left(t\right)dt}\right)^2\\ &=1-\left(\sqrt{\frac{1-x}{x}}\frac{\int_0^x t\phi\left(t\right)dt}{\int_0^1\sqrt{t\left(1-t\right)}\phi\left(t\right)dt}\right)^2\\ &=1-\left(\frac{\int_0^x\sqrt{t}\cdot\sqrt{t-tx}\phi\left(t\right)dt}{\int_0^1\sqrt{t}\cdot\sqrt{x-tx}\phi\left(t\right)dt}\right)^2\geq 0,\end{aligned}$$

where the last inequality follows because $x\leq 1$ and $t\leq x$. Therefore, total welfare strictly increases in the case of positive weak equilibrium as well.

□