

Ads in Conversations

Market Thickness and Match Quality

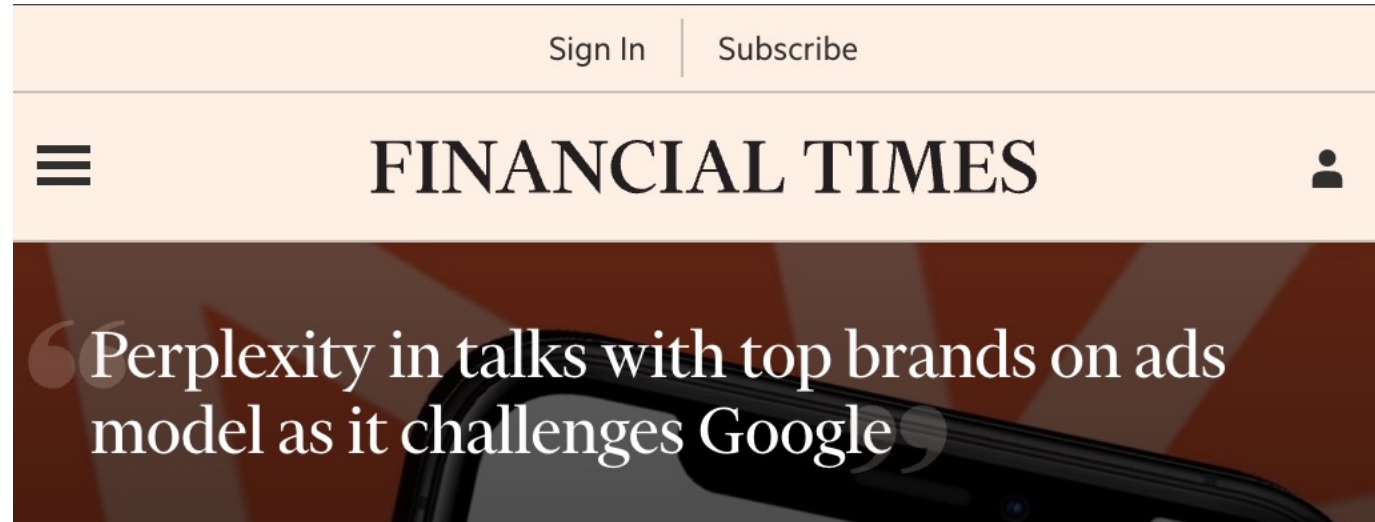
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Bocconi, IGER, and Google Research

Joint work with **A. Mehta, A. Perloth**

NBER Market Design - October 18th, 2024

LLM chatbots consider ad monetization



The Verge

Menu +

GOOGLE / TECH / ARTIFICIAL INTELLIGENCE

Google's AI search summaries officially have ads / The search giant will now stick relevant products into its AI Overviews.

Search Advertising

For each query, search engine runs an auction

Advertisers pay only if their ad is clicked

Winner is bidder with largest **expected** bid
product of bid and *click-through rate*

Query arrives, ads are shown and the user leaves: *one-shot* interaction.

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Refine results

- On sale
- Nearby
- Get it fast
- Small busine...

Sort by

- Price: low to high
- Price: high to low

Department

- Men's
- Women's
- Kids'
- Girls'
- Boys'

Color

- Black
- White
- Red
- Blue

See more

Athletic Style









- Running
- Basketball
- Tennis
- Cross Training

Type

- Sneakers






- Men's
- Women's
- Kids'
- Girls'
- Boys'

Sponsored

 In store lululemon Casual... \$138.00 lululemon	 Balenciaga Men's... \$925.00 Bergdorf Goo... Free shipping	 SALE Allbirds Men's Couriers, Beig... \$49.00 \$98 Allbirds ★★★★★ (64)	 In store lululemon Chargefeel 2... \$89.00 lululemon ★★★★★ (331)	 New Balance Men's 806 V1... \$119.95 Amazon.com Free shipping	 In store lululemon Casual... \$138.00 lululemon	 White Leather Premier Low... \$129.00 Thursday Boo... ★★★★★ (6k+)	 In store lululemon Chargefeel 2... \$99.00 lululemon ★★★★★ (128)
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Athletic Shoes

Designed for specific sports and activities, providing support, cushioning, and traction.

 40% OFF	 360	 360	 360	 360
adidas Men's Runfalcon 5 Running Shoes \$39.00 \$65 adidas 4.8 ★★★★★ (33)	Men's Under Armour Charged Surge 4 Running... \$54.99 Under Armour Free \$99+ 4.5 ★★★★★ (662)	On Men's Cloudrunner \$100.00 On Free by 10/22 4.3 ★★★★★ (397)	Nike Women's Interact Run Running Shoes \$85.00 Nike Free by 10/24 4.6 ★★★★★ (450)	Skechee Joy \$44.86 Ama 4.7 ★★

Conversational AI Assistants change search

AI Assistant/LLM chatbot can **direct** search more than a search engine

A conversation with ChatGPT/Gemini/Claude **clarifies** the user intent

☰ Gemini Advanced ▾



Hi, I need shoes



Shopping Assistant · Custom Gem

Hello there! I'm your personal shopping assistant, here to help you find the perfect pair of shoes. Could you tell me a bit more about what you're looking for, like what kind of activities you'll be using them for and what style you prefer?





I am looking for **running shoes**



Shopping Assistant • Custom Gem

That's great! There are so many great running shoes available, so let's narrow down your options based on your needs. Could you tell me:

- **What type of runner are you?** Are you a beginner, intermediate, or advanced runner?
- **What kind of terrain will you be running on?** Are you planning to run on roads, trails, or a combination?
- **What is your foot type?** Do you have high arches, flat feet, or neutral feet?
- **What is your budget?** How much are you willing to spend on a pair of running shoes?



☰ Gemini Advanced ▾



I'll mostly run trails, and I have some overpronation



Shopping Assistant • Custom Gem


Got it! You're looking for trail running shoes that provide stability for overpronation.


To make sure I recommend the best options, could you tell me a bit more about your running experience and the type of trails you'll be tackling?


For example, are you a beginner trail runner or do you have experience running on technical trails? This will help me determine the level of cushioning and support you need.





☰ Gemini Advanced + 

 I'm an intermediate runner, I do some technical trails in the alps.

 Shopping Assistant • Custom Gem

Okay, an intermediate runner tackling technical trails in the Alps! That's exciting. You'll need a shoe with good stability, excellent traction, and enough protection for those rocky paths. Here are a few options that might be a good fit for you:

1. Salomon Speedcross 6: These shoes are known for their aggressive grip, making them ideal for technical trails. They provide good stability for overpronation and have a snug fit that keeps your foot secure on uneven terrain.

Recommendations:

AI Assistant refines search

For each query, assistant asks more questions.

In an *AI-user conversation*, the user's intent becomes very clear

The conversation is a funnel: from many candidate answers, AI refines to a small set of good fits.



Assistant learns the quality of a match

Further down the funnel

→ *click-through rates* get more precise

In the limit, assistant gets a perfect match



Click-Through Rate

The click-through rate can be thought of as a measure of ad quality.

Ad quality treated as **static** score.

As a user spends time in the conversation, platform gets better informed about click-through rates.

The platform **learns** the ad quality.

In this paper

We model online advertising on conversational AI assistants

Auctioneer **learns** the ad quality

The user is shown **one ad**

The auctioneer commits to the auction format **but not to its timing**

Auctioneer chooses **auction format** and **auction timing**.

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More formally

We study the equilibria of auctions with limited commitment, where

- The auctioneer chooses precision of ad quality predictions **after** observing the advertisers' bids
- The advertisers anticipate this decision and bid accordingly

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When should the auctioneer run the auction?

Does the auction format matter?

Ad Quality vs Market Thickness

Running the auction late → no misallocation, high-q ad wins

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Running the auction late

→ no misallocation, high-q ad wins

→ competition is weak, low prices

Running the auction early

→ misallocation, low-q ad may win

→ competition is fierce, high prices

Preview of Results:

Different auction formats trade off **ad quality** and **market thickness** differently:

Without reserve prices, second-price revenue dominates first-price

With reserves, the relationship flips

Model

Preliminaries

$\theta = (\theta_1, \theta_2)$ with $\theta_i \sim_{i.i.d.} \text{Bernoulli}(p)$ – **quality** of ad i

User only clicks on ads with $\theta_i = 1$

Preliminaries

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Auctioneer chooses one ad to show to the user

Conversation

User interacts with auctioneer over a **conversation** in continuous time
 $t \in \mathbb{R}_+$

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Auctioneer receives informative signals about ad quality over time.

News about θ_i arrives according to stochastic process $(\theta^t)_{t \geq 0}$

Beliefs

The auctioneer forms beliefs

$$\mu_t^i = \mathbb{E}[\theta_i | \mathcal{F}_t]$$

about the quality of both ads.

Assumption: news process $(\theta^t)_{t \geq 0}$ is such that $\mathbb{P} \left(\lim_{t \rightarrow \infty} \mu_t^i = \theta^i \right) = 1$

Allocation

Focus on two mechanisms:

First-Price Auction:

$$x_i(b, \mu) = 1 \text{ if } b_i \mu_t^i > b_j \mu_t^j$$

$$p_i(b, \mu) = b_i \text{ if } \theta_i = 1$$

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Second-Price Auction:

$$x_i(b, \mu) = 1 \text{ if } b_i \mu_t^i > b_j \mu_t^j$$
$$p_i(b, \mu) = \frac{b_j \mu_t^j}{\mu_t^i} \text{ if } \theta_i = 1$$

Objectives

The auctioneer maximizes revenue by choosing an auction timing

$$\max_{\tau} \mathbb{E}_0[Rev_{\tau}(b, \mu)] = \max_{\tau} \mathbb{E}_0 \left[\sum_i \theta_i p_i(b, \mu_{\tau}) \right]$$

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The advertiser's expected payoff is

$$u_i(v_i; b_i) = \mathbb{E}_0[\theta_i(x_i(b, \mu_{\tau})v_i - p_i(b, \mu_{\tau}))]$$

Timing

Auctioneer
commits to
auction
format

$t = 0$



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Auctioneer
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Advertisers
submit bids
 b_1, b_2 ;
the quality θ
is realized
(but unknown)

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$\tau(b, \mu) \geq 0$
Auction “runs”;
Ad is shown and
payments are made

Assumption: Limited Commitment

Formally, fix an auction format $(x, p) \in \{(x_{FP}, p_{FP}), (x_{SP}, p_{SP})\}$.

Assumption: The auctioneer **cannot commit** to running the auction at time t . Instead, she solves

$$\max_{t \leq \tau} \mathbb{E}_t[\text{Rev}_\tau(b, \mu)]$$

for all t , where τ is a stopping time wrt the natural filtration $\mathbb{F} = \{\mathcal{F}_t\}_{t \in \mathbb{R}_+}$

Auction as a real option

The auctioneer solves a real options problem

When she stops, the ad is chosen and shown according to (x, p)

Bids are set in advance, so pure stopping problem

Definition 1: The auctioneer *exercises the auction at τ* if τ is a stopping time that solves the auctioneer's problem.

Results

Second-Price Auction

If the format is (x_{SP}, p_{SP}) , auctioneer solves

$$\max_{\tau} \mathbb{E}_0 \left[\min_{i=1,2} \{ \mu_{\tau}^i b_i \} \right]$$

Lemma 1: In a second-price auction, the auctioneer exercises the auction at $\tau_{SP}^* = 0$.

Proof

Let the revenue process be $Rev_t(b) = \min_{i=1,2} \{b_i \mu_t^i\}$

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2. For each i and for all $t' > t$

$$\mathbb{E}_t \left[\min_{i=1,2} \{b_i \mu_{t'}^i\} \right] \leq \mathbb{E}_t [b_i \mu_{t'}^i] = b_i \mu_t^i$$

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3. Then,

$$\mathbb{E}_t [Rev_{t'}(b)] = \mathbb{E}_t \left[\min_{i=1,2} \{b_i \mu_{t'}^i\} \right] \leq \min_{i=1,2} \{b_i \mu_t^i\} = Rev_t(b)$$

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$Rev_t(b)$ is a super-martingale. By Doob's OST, $\tau_{SP}^* = 0$. □

Equilibrium

Bidders anticipate that auctioneer will stop at $\tau_{SP}^* = 0$

Second-price auction is **truthful**

Highest bidder wins, gets a click with probability p

Efficient Outcome

$$b_1 > b_2 \Rightarrow v_1 > v_2$$

	$\theta_2 = 0$	$\theta_2 = 1$
$\theta_1 = 0$	$\frac{1}{2}$	2
$\theta_1 = 1$	1	1

SPA stopped at 0

$$b_1 > b_2 \Rightarrow v_1 > v_2$$

Inefficient

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Usual FPA differential eq

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ODE for bid function:

$$\underbrace{\beta'(v)F(v) + \beta(v)f(v)}_{\text{Usual FPA differential eq}} = vf(v) - \underbrace{\frac{1-p}{p}\beta'(v)}_{\text{Extra term because competition may be absent}}$$

Usual FPA differential eq

Extra term because
competition may be absent

Equilibrium

Advertisers anticipate that auctioneer will stop at $\tau_{FP}^* = \infty$

$$\beta_{FP}(v) = \frac{1}{\frac{1-p}{p} + F(v)} \left[\int_0^v y f(y) dy \right]$$

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Equilibrium

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$$\beta_{FP}(v) = \frac{1}{\frac{1-p}{p} + F(v)} \left[\int_0^v y f(y) dy \right]$$

Naturally, $\frac{d}{dp} \beta_{FP}(v) > 0$ (more competition \rightarrow higher bids)

FPA stopped at ∞

$\theta_2 = 0$

$\theta_2 = 1$

$\theta_1 = 0$

$1/2$

2

$b_1 > b_2 \Rightarrow v_1 > v_2$

Reaches the efficient outcome

$\theta_1 = 1$

1

1

	$\theta_2 = 0$	$\theta_2 = 1$
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Summary

In the equilibrium of the FPA bidders shade their bids to account for the reduced competition

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In the equilibrium of the FPA bidders shade their bids to account for the reduced competition

However, the auctioneer allocates efficiently. Instead in a SPA the auctioneer misallocates often

But bids are higher in a SPA. In fact, agents are truthful

Expected equilibrium revenue depends on the type and value distribution

Next step: compare revenue across formats

Result: revenue is ordered

Theorem 1: The equilibrium revenue π_{SP}^* from the optimally-stopped SPA dominates the equilibrium revenue π_{FP}^* from the optimally-stopped FPA. In particular,

$$\frac{\pi_{SP}^*}{\pi_{FP}^*} = \frac{1}{p}$$

Proof

Instead of proving directly, notice:

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Proof

Instead of proving directly, notice:

1. SPA is truthful so long as τ is independent of v_1, v_2
2. \Rightarrow SPA at $\tau_{FP}^* = \infty$ is truthful and allocates as FPA
3. $\Rightarrow Rev_{FPA}(\tau_{FP}^*) = \pi_{FP}^*$ is the same as $Rev_{SPA}(\tau_{FP}^*)$
4. But $Rev_{SPA}(t)$ is a super-martingale, so

$$\mathbb{E}_0[Rev_{SPA}(\tau_{FP}^*)] < Rev_{SPA}(0) = \pi_{SP}^*$$

Revenue Bound

For the second part, envelope theorem:

$$\begin{aligned}\pi_{FP}^* &= p^2 \mathbb{E}_{v_1, v_2} [\max\{\psi(v_1), \psi(v_2)\}] + 2p(1 - p) \mathbb{E}_v [\psi(v)] \\ &= \mathbf{p}^2 \mathbb{E}_{v_1, v_2} [\max\{\psi(v_1), \psi(v_2)\}]\end{aligned}$$

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$$\pi_{SP}^* = \mathbf{p} \mathbb{E}_{v_1, v_2} [\max\{\psi(v_1), \psi(v_2)\}]$$

Market Thickness vs. Information

SPA relies on market thickness -> stops early

FPA instead collapses market thickness at time 0 (in bidders' decision)

Reserve Price

Reserve prices act against market thickness (or lack thereof)

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Reserve prices act against market thickness (or lack thereof)

Proposition 1. The optimal mechanism can be implemented as a first-price auction with reserve price R such that $\psi(R) := R - \frac{1-F(R)}{f(R)} = 0$.

In particular, such a first-price auction stops at $\tau_{FP}^* = \infty$.

Optimal Mechanism

To see this, note that the optimal mechanism solves

$$\max_{x_1, x_2} \text{ s.t. } \sum_i x_i \leq 1 \quad \{x_1 \theta_1 \psi(v_1), x_2 \theta_2 \psi(v_2)\}$$

Allocate to the highest positive quality-weighted virtual value.

Not allocate only if $\psi(v) < 0 \rightarrow$ reserve R is sufficient

SPA with reserve

Instead, SPA cannot implement the optimal auction.

Proposition 2. There exist processes $(\theta^t)_{t \geq 0}$ and distributions F such that no second-price auction with reserves implements the optimal mechanism.

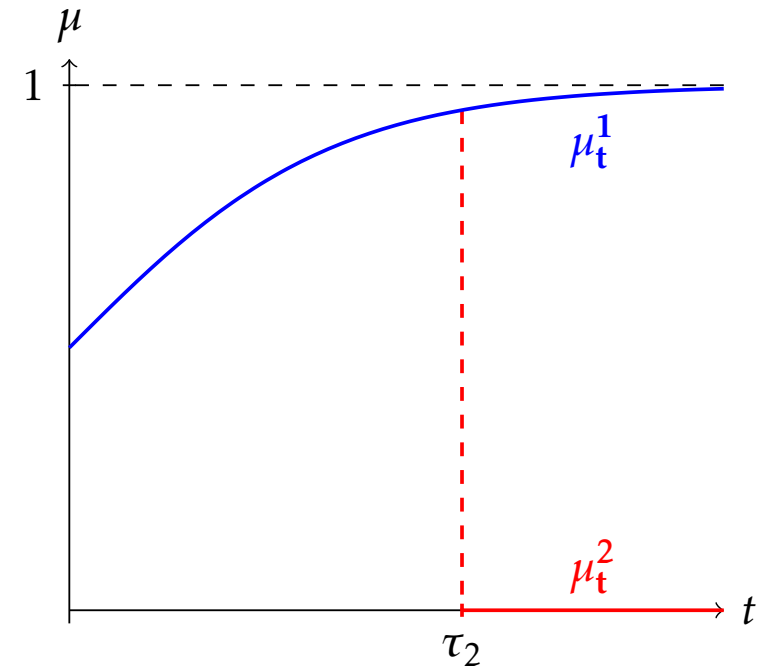
The auctioneer will generally still have an incentive to avoid delay.

Counterexample

Bad news model, arrives at rate $\lambda \rightarrow$ until news arrives, beliefs drift upward:

$$\dot{\mu}_t^i = \lambda \mu_t^i (1 - \mu_t^i)$$

$$\mu_0^i = p$$



The auctioneer stops early

Lemma 4: Suppose that advertisers bid $b_1 > b_2 > 2R$, where R is the reserve price of the SPA. Then, $\tau_{SP}^*(b_1, b_2) = 0$ for any realization of the process μ .

The auctioneer stops early

Lemma 4: Suppose that advertisers bid $b_1 > b_2 > 2R$, where R is the reserve price of the SPA. Then, $\tau_{SP}^*(b_1, b_2) = 0$ for any realization of the process μ .

Proof: If there has been no news, stopping at time t yields revenue $b_2\mu_t$. The value function must satisfy

$$V(\mu_t) = \max\{b_2\mu_t, V(\mu_{t+\Delta})\}$$

for a small Δ .

Continuation value must be low

If she decides to continue, value must satisfy HJB

$$V'(\mu)\mu = 2(V(\mu) - \mu R)$$

Stopping at $\bar{\mu}$ + smooth pasting implies

$$\bar{\mu}b_2 = 2\bar{\mu}b_2 - 2\bar{\mu}R$$

Continuation value must be low

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Stopping at $\bar{\mu}$ + smooth pasting implies

$$\bar{\mu}b_2 = 2\bar{\mu}b_2 - 2\bar{\mu}R$$

When $b_2 > R$ there is no such belief, so either

1. $V(\mu) > \mu b_2$ and the auctioneer stops at $\bar{\mu} = 1$, or
2. $V(\mu) = \mu b_2$ and the auctioneer stops immediately.

By contradiction

Suppose that $V(\mu) > \mu b_2$ for all $\mu < 1$.

Because $\lim_{\mu \rightarrow 1} V(\mu) = b_2$, for μ sufficiently close to 1

$$V'(\mu) > 2(b_2 - R) > b_2$$

So $V(\mu) < \mu b_2$, contradiction.

Now, prove the proposition

The lemma only shows that high bids force the auctioneer to stop early.
Are high bids part of the equilibrium?

Choose F with support $[0, \bar{v}]$ such that $\bar{v} > 2R$, where $\psi(R) = 0$.

SPA with reserve R is not truthful.

Consider incentives of type $2R + \varepsilon$, when other bidder is truthful.

If other bidder bids more than $2R$, auctioneer stops immediately from Lemma 4

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Type $2R + \varepsilon$ wants to shade below $2R$, to induce auctioneer to wait

With probability $1 - p$ advertiser wins, pays R , gets payoff $(1 - p)(R + \varepsilon)$

This is better than payoff from auctioneer stopping immediately:

$$(F(2R + \varepsilon) - F(2R))\mathbb{E}[2R + \varepsilon - v | v \in (2R, 2R + \varepsilon)]$$

Symmetric Equilibrium

RAA suppose there was a symmetric equilibrium $\beta(v)$ such that SPA with reserve R implements the optimum.

Then, auctioneer stops at $\tau_{SP}^* = \infty \rightarrow$ bids must be lower than $2R$

But type $\bar{v} - \varepsilon$ has an incentive to bid $x > 2R$:

Auctioneer will still run auction at $\tau_{SP}^* = \infty$

Advertiser wins the item even when opponent is of type \bar{v} , and pays $\beta(\bar{v}) < 2R$

Conclusion

Conclusion

Advertising in a conversation hinges on ad quality vs market thickness tradeoff

This is a first step towards the design of ad auctions on conversational agents

Auction design matters: vastly different outcomes with FPA vs SPA.

Thank you!

Extensions

What about more advertisers?

Suppose there are n advertisers with qualities $(\theta_1, \dots, \theta_n)$, and suppose the belief process $(\mu_t^1, \dots, \mu_t^n)$ is an n -dimensional bad-news Poisson model.

Lemma 5: Let $K_t = |\{i: \mu_t^i \neq 0\}|$. Without reserves,

1. An optimally-exercised SPA stops at $\tau_{SP}^* = \inf\{t | K_t = 2\}$
2. An optimally-exercised FPA stops at $\tau_{FP}^* = \infty$
3. The equilibrium revenues satisfy $\pi_{SP}^* > \pi_{FP}^*$.