How Good is International Risk Sharing?

Stepping outside the Shadow of the Welfare Theorems

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VERY PRELIMINARY!

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International Risk Sharing

- **Efficient allocation** requires

$$\frac{U_C^*}{U_C} = MRT \equiv \tilde{Q}$$

  - **MRT** how many units of C it takes to increase C^* by one unit

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- Good reasons to be sceptical that $MRT \stackrel{?}{=} \mathcal{Q}$:

  — **macro**: exchange rates disconnected from TFP, output...

  — **micro**: alphabet soup of goods market frictions (PtM, PCP, LCP, DCP)

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   - resource constraints + functional forms
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2. **Mapping** between $\tilde{\mathcal{Q}}$ and $\mathcal{Q}$:
   - — BS wedge is neither necessary nor sufficient for distorted risk sharing

3. **Apply** to the data:
   - — on average, risk-sharing wedge is small and $\mathrm{cor}(c - c^*, \tilde{q}) \approx 0.6$

Relation to the Literature

- **International (mis)allocation**:
  - Consumption efficiency: Backus, Kehoe & Kydland (1992), Mendoza (1992), Backus & Smith (1993), Kollmann (1995), van Wincoop (1994, 1999), Lewis (1996), Aguiar & Gopinath (2007), Corsetti, Dedola & Leduc (2008), Bai & Zhang (2010, 2012), Fitzgerald (2012), Gourinchas & Jeanne (2013), Heathcote & Perri (2014), Ohanian, Restrepo-Echavarria & Wright (2018), Corsetti et al (2023)
  - Asset prices and portfolios: Brandt, Cochrane & Santa-Clara (2006), French & Poterba (1991), Baxter & Jermann (1997), Cole & Obstfeld (1991), Heathcote & Perri (2013), Coeurdacier & Gourinchas (2016), Farhi & Werning (2016), Coeurdacier & Rey (2013), Lewis & Liu (2023)
  - Wedge accounting: Chari, Kehoe & McGrattan (2007), Hsieh & Klenow (2009), Capelle & Pellegrino (2023), Kleinman, Liu & Redding (2023)
- **Exchange rates and risk sharing**:
  - Financial markets: Alvarez, Atkeson & Kehoe (2002), Jeanne & Rose (2002), Kollmann (2005), Gabaix & Maggiori (2015), Fornaro (2021), Itskhoki & Mukhin (2021, 2023)
  - Goods markets: Rogoff (1996), Engel (1999, 2011), Devereux & Engel (2003), Atkeson & Burstein (2008), Bianchi (2011), Corsetti, Dedola & Leduc (2018), Gopinath et al (2020), Amiti, Itskhoki & Konings (2019)

ENVIRONMENT

Environment

- **Two regions**: Home and Foreign (RoW)
- **Endowments**:
  - Armington model with country-specific goods/inputs
  - focus on efficiency of allocation given output

$$C_H + C_H^* = Y$$

$$C_F + C_F^* = Y^*$$

- **Preferences**:

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t \frac{C_t^{1-\sigma}}{1-\sigma}, \qquad C = \left[(1-\gamma)^{\frac{1}{\theta}}\ C_H{}^{\frac{\theta-1}{\theta}} + \gamma^{\frac{1}{\theta}}\ C_F{}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}$$

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t \frac{C_t^{*1-\sigma}}{1-\sigma}, \qquad C^* = \left[(1-\gamma^*)^{\frac{1}{\theta}}\, C_F^*{}^{\frac{\theta-1}{\theta}} + \gamma^{*\frac{1}{\theta}}\, C_H^*{}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}$$

Efficient Allocation

- Planner's problem:

$$\max_{\{C_{Ht}, C_{Ft}, C^*_{Ht}, C^*_{Ft}, C_t, C^*_t\}} \quad \omega U(\{C_t\}) + U(\{C^*_t\})$$

$$\text{s.t.} \quad C(C_{Ht}, C_{Ft}) = C_t$$

$$C^*(C^*_{Ht}, C^*_{Ft}) = C^*_t$$

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1. **"Static" efficiency**:

Edgeworth

$$\frac{\gamma}{1-\gamma}\frac{C_H}{C_F} = \left(\frac{\nu}{\nu^*}\right)^{-\theta} = \frac{1-\gamma^*}{\gamma^*}\frac{C_H^*}{C_F^*}$$

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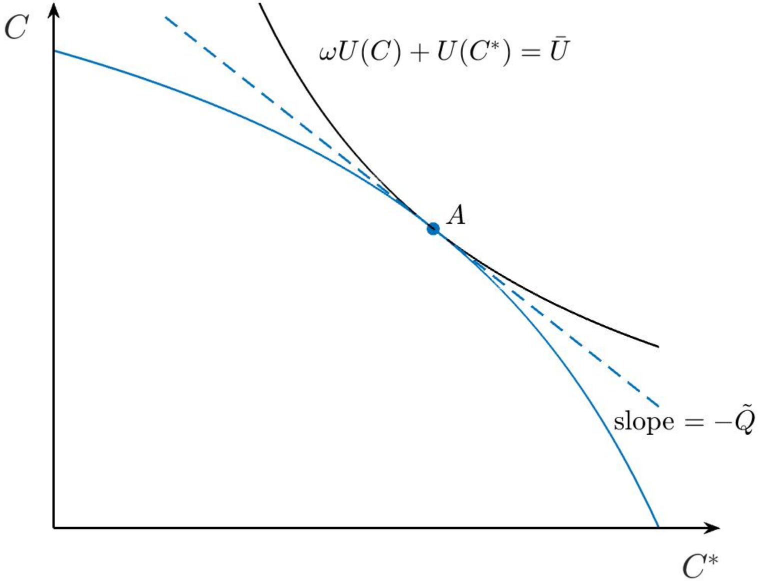

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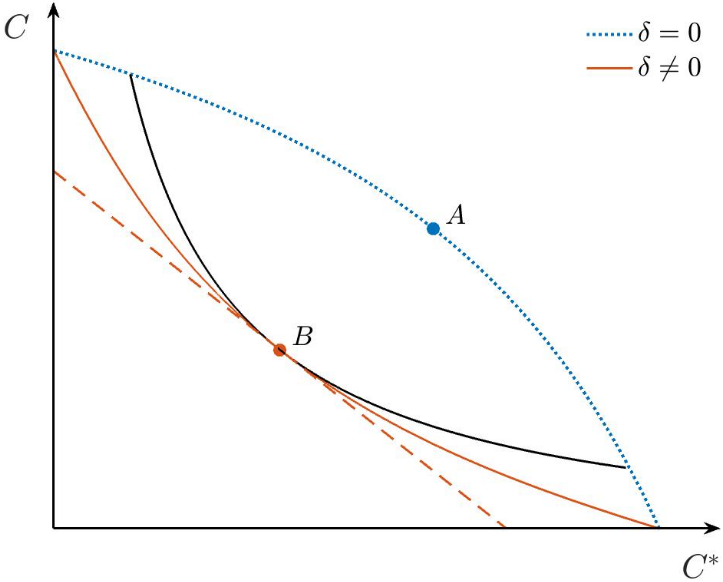

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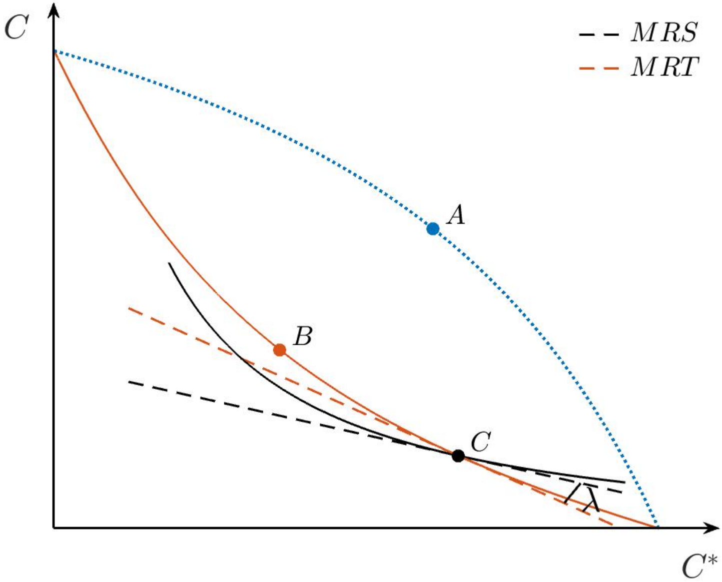

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- Solve for **shadow prices**, not allocations:

CD case

$$Y, Y^* \text{ and } \delta, \lambda \quad \Longrightarrow \quad C, C^* \text{ and } \tilde{Q}$$

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DECENTRALIZED EQUILIBRIUM

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- **Goods markets**:

$$P_H = W^{1-\alpha_H} P^{\alpha_H} \qquad P_F = (W^*\mathcal{E})^{1-\alpha_F} P^{\alpha_F}$$

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3. **DCP** $\alpha = 0, \alpha^* = 1$:

$$1 + \lambda = \left[(1+\psi)\frac{Y}{Y^*}\right]^{\frac{1}{2(1-\gamma)}}, \qquad 1 + \delta = \left(\frac{1}{1+\psi}\right)^{\frac{1-2\gamma}{1-\gamma}} \left(\frac{Y}{Y^*}\right)^{\frac{1}{1-\gamma}}$$

— state wedge δ can arise from financial shocks ψ

EMPIRICAL RESULTS

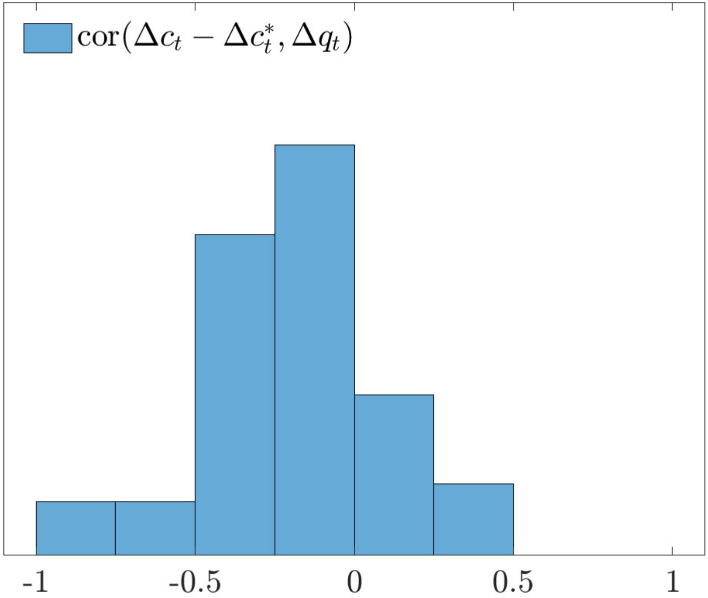
Data and Calibration

- **Data**:
  - $Y_t, IM_t, EX_t, C_t, \mathcal{Q}_t$ from WDI
  - balanced panel from 2000-2019 for about 60 countries
  - analysis for each country against the RoW

- **Calibration**:
  - $\theta = 4$, $\sigma = 2$, $\beta = 0.96$ (annual)
  - γ, γ^* from trade shares in base year ▸ details
  - caveat: real quantities not observed in levels, calibrate base-year ω, δ

- **Estimation**:
  - generalize model to allow for absorption $C + I + G = GDP - NX$
  - back out $\{C_{Ht}, C_{Ft}, C^*_{Ht}, C^*_{Ft}\}$ from $\{Y_t, C_t\}$ ▸ details
  - compute $\{\delta_t, \lambda_t, \tilde{\mathcal{Q}}_t\}$ from planner's problem ▸ details

Backus-Smith Correlation: RER vs MRT



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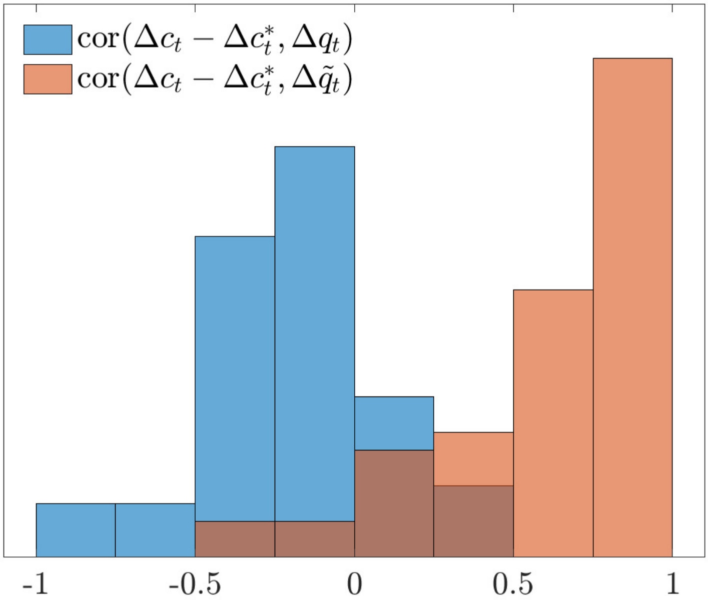

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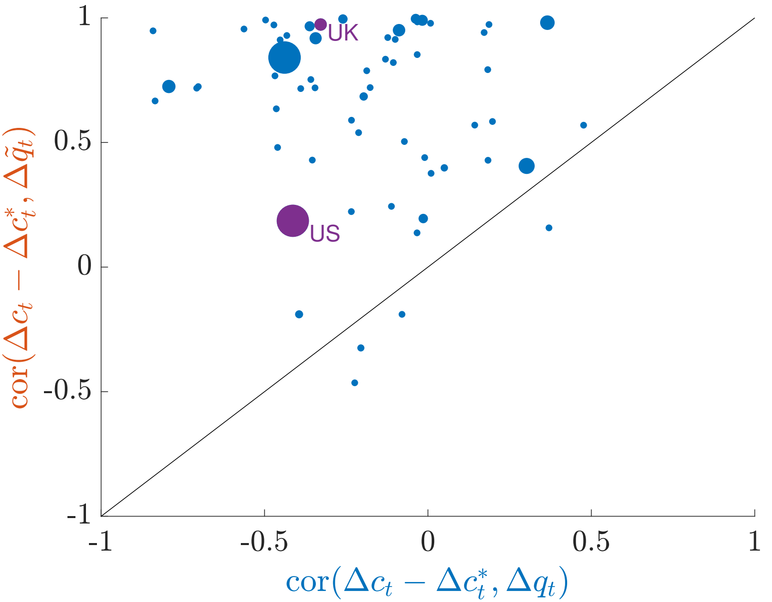

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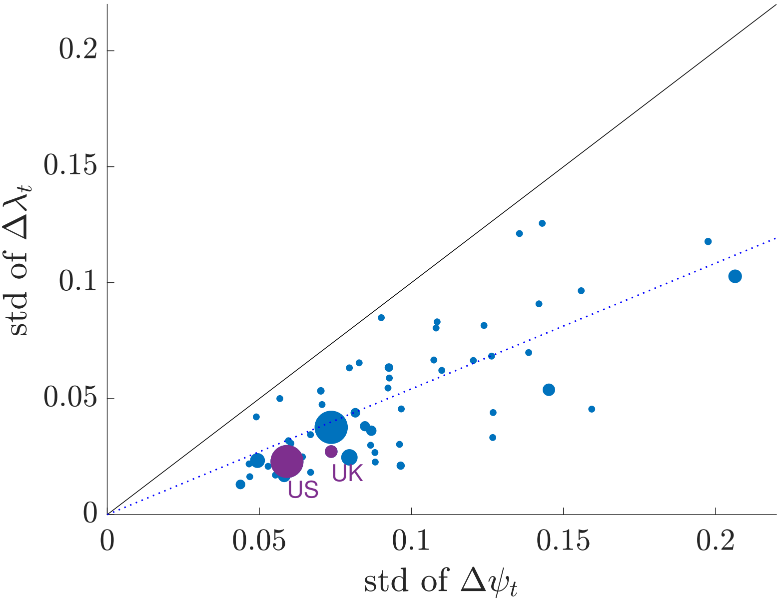

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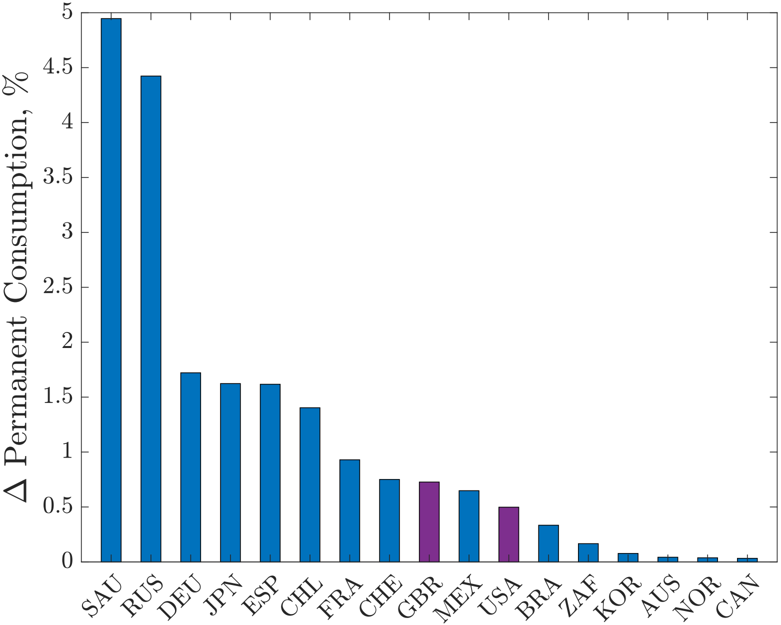

Conclusion

- How good is international risk sharing?
- Backus-Smith is a poor measure
- Propose a simple alternative
- **Better than one might think!**

APPENDIX

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- Optimality conditions:

$$\underbrace{\frac{\partial C/\partial C_F}{\partial C/\partial C_H}}_{MRS_{HF}} = \underbrace{\frac{\nu^*}{\nu}}_{\tilde{S}} = \underbrace{\frac{\partial C^*/\partial C_F^*}{\partial C^*/\partial C_H^*}}_{MRS_{HF}^*}, \qquad \underbrace{\frac{U_C^*}{\omega U_C}}_{MRS_{CC^*}} = \underbrace{\frac{\mu^*}{\mu}}_{\tilde{Q}} = \underbrace{\frac{\nu \frac{C_H^*}{C^*} + \nu^* \frac{C_F^*}{C^*}}{\nu \frac{C_H}{C} + \nu^* \frac{C_F}{C}}}_{MRT_{CC^*}}$$

- **Proposition**: international allocation is efficient iff

$$\frac{\gamma}{1-\gamma}\frac{C_H}{C_F} = \frac{1-\gamma^*}{\gamma^*}\frac{C_H^*}{C_F^*}, \qquad \frac{1}{\omega}\left(\frac{C}{C^*}\right)^{\sigma} = \left[\frac{(1-\gamma^*)\left(\frac{1-\gamma^*}{\gamma^*}\frac{C_H^*}{C_F^*}\right)^{\frac{1-\theta}{\theta}} + \gamma^*}{1-\gamma+\gamma\left(\frac{\gamma}{1-\gamma}\frac{C_H}{C_F}\right)^{\frac{1-\theta}{\theta}}}\right]^{\frac{1}{1-\theta}}$$

back

Analytical Example

- **Special case**: $\sigma = \theta = 1$, $\gamma = \gamma^*$, $\omega = 1$
- Aggregate consumption and output pin down δ and λ:

$$C^{1+\kappa} = \frac{1+\lambda+\kappa\eta}{1+\lambda+\kappa}\left(\frac{\kappa(1+\lambda-\eta)}{1+\kappa(1+\lambda)}\right)^{\kappa} YY^{*\kappa}$$

$$C^{*1+\kappa} = \frac{1+\kappa\eta}{1+\kappa(1+\lambda)}\left(\frac{\kappa(1-\eta)}{1+\lambda+\kappa}\right)^{\kappa} Y^{\kappa}Y^{*}$$

where $\kappa \equiv \frac{\gamma}{1-\gamma}$ and $\eta = \eta(\delta) : \quad \frac{1-\eta}{1+\kappa\eta}\frac{1+\lambda-\eta}{1+\lambda+\kappa\eta} = \frac{1}{1+\delta}$

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$$MRT = \frac{\mu^*}{\mu} = \underbrace{\left(\frac{1+\frac{\kappa\eta}{1+\lambda}}{1+\kappa\eta}\right)^{1-\gamma}\left(\frac{1-\frac{\eta}{1+\lambda}}{1-\eta}\right)^{\gamma}}_{=1 \text{ when } \eta=0} \cdot \left(\underbrace{\frac{1+\kappa+\kappa\lambda}{1+\kappa+\lambda}\cdot\frac{Y}{Y^*}}_{=\nu^*/\nu}\right)^{1-2\gamma}$$

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- MRT around *undistorted SS* $\bar{\delta} = \bar{\eta} = \bar{\lambda} = 0$ (first-order approximation):

$$\mathrm{d}\log MRT \approx (1-2\gamma)(y-y^*) - (1-2\gamma)^2\lambda$$

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- **Special case**: $\sigma = \theta = 1$, $\gamma = \gamma^*$, $\omega = 1$

- Shadow values and distorted MRT:

$$MRT = \frac{\mu^*}{\mu} = \underbrace{\left(\frac{1 + \frac{\kappa\eta}{1+\lambda}}{1 + \kappa\eta}\right)^{1-\gamma} \left(\frac{1 - \frac{\eta}{1+\lambda}}{1 - \eta}\right)^{\gamma}}_{=1 \text{ when } \eta=0} \cdot \left(\underbrace{\frac{1 + \kappa + \kappa\lambda}{1 + \kappa + \lambda} \cdot \frac{Y}{Y^*}}_{=\nu^*/\nu}\right)^{1-2\gamma}$$

where $\kappa \equiv \frac{\gamma}{1-\gamma}$ and $\eta = \eta(\delta)$: $\frac{1-\eta}{1+\kappa\eta} \frac{1+\lambda-\eta}{1+\lambda+\kappa\eta} = \frac{1}{1+\delta}$

- MRT around *undistorted SS* $\bar{\delta} = \bar{\eta} = \bar{\lambda} = 0$ (first-order approximation):

$$\underbrace{c - c^*}_{mrs} = \lambda + \underbrace{(1 - 2\gamma)(y - y^*) - (1 - 2\gamma)^2 \lambda}_{mrt}$$

- Welfare (second-order approximation):

$$W(\lambda, \delta) = \log Y + \log Y^* - \gamma(1 - \gamma)\left[\lambda^2 + \frac{1}{4}\delta^2\right]$$

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- Define real absorption:

$$A \equiv C + I + G, \qquad A^* \equiv C^* + I^* + G^*$$

- Assume the same CES aggregator for C, I, G:

$$A^{\frac{\theta-1}{\theta}} = (1-\gamma)^{\frac{1}{\theta}} A_H^{\frac{\theta-1}{\theta}} + \gamma^{\frac{1}{\theta}} A_F^{\frac{\theta-1}{\theta}}$$

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where import and export trade shares are given by

$$\bar{\gamma} = \gamma^{\frac{1}{\theta}} \left(\frac{\bar{C}_F}{\bar{C}}\right)^{\frac{\theta-1}{\theta}}, \quad \bar{\gamma}^* = \gamma^{*\frac{1}{\theta}} \left(\frac{\bar{C}_H^*}{\bar{C}^*}\right)^{\frac{\theta-1}{\theta}}, \qquad \bar{\chi} \equiv \frac{\bar{C}_H^*}{\bar{Y}}, \qquad \bar{\chi}^* \equiv \frac{\bar{C}_F}{\bar{Y}^*}$$

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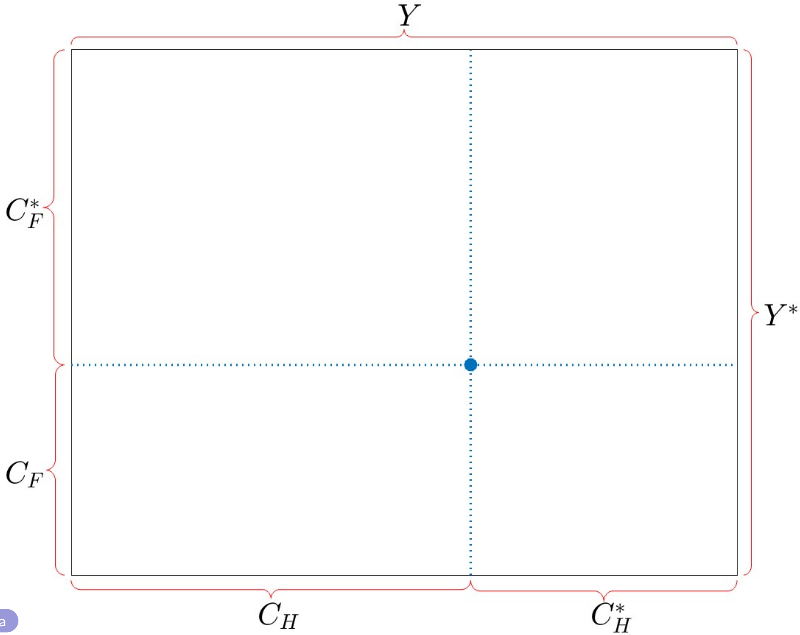
- Recover consumption components:

$$\hat{C}_H = \hat{A}_H \frac{\hat{C}}{\hat{A}}, \qquad \hat{C}_F = \hat{A}_F \frac{\hat{C}}{\hat{A}}, \qquad \hat{C}_H^* = \hat{A}_H^* \frac{\hat{C}^*}{\hat{A}^*}, \qquad \hat{C}_F^* = \hat{A}_F^* \frac{\hat{C}^*}{\hat{A}^*}$$

▸ data

Identification



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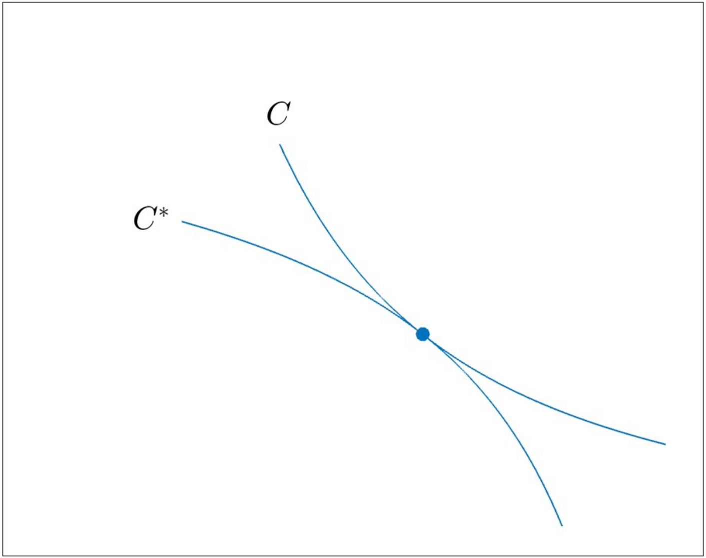

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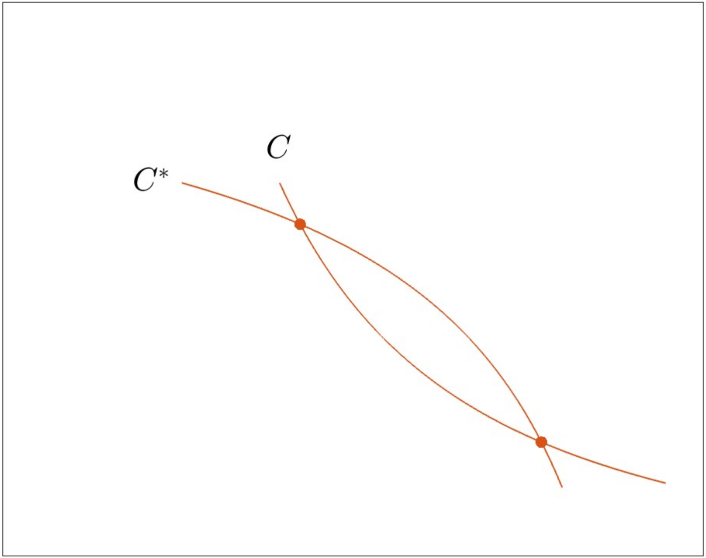

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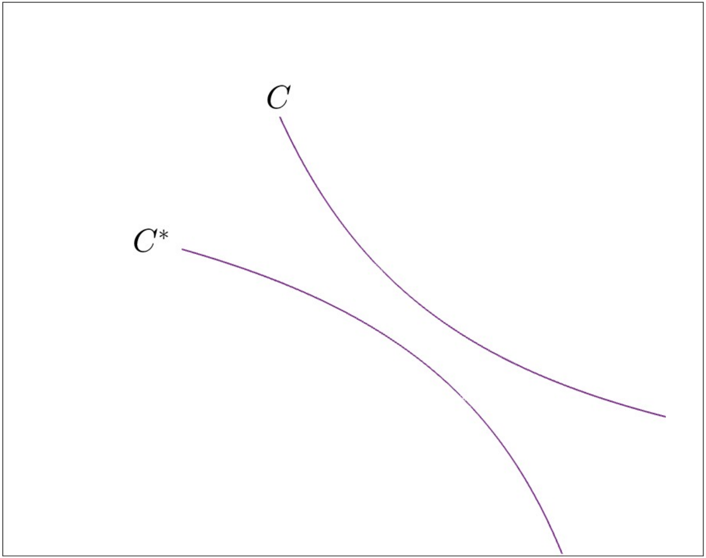

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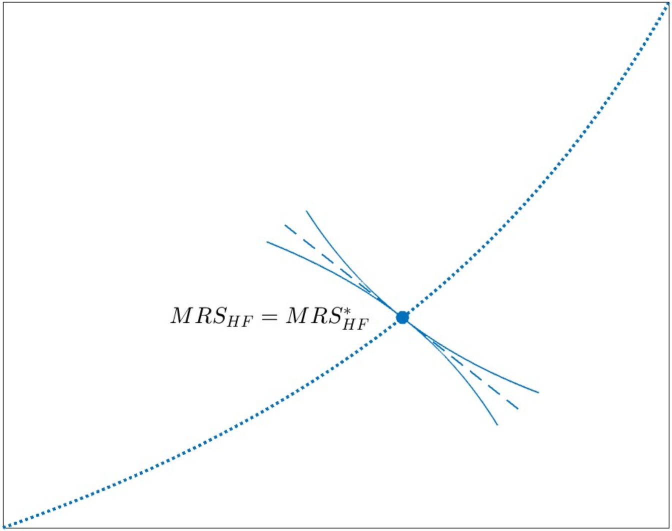
Edgeworth Box



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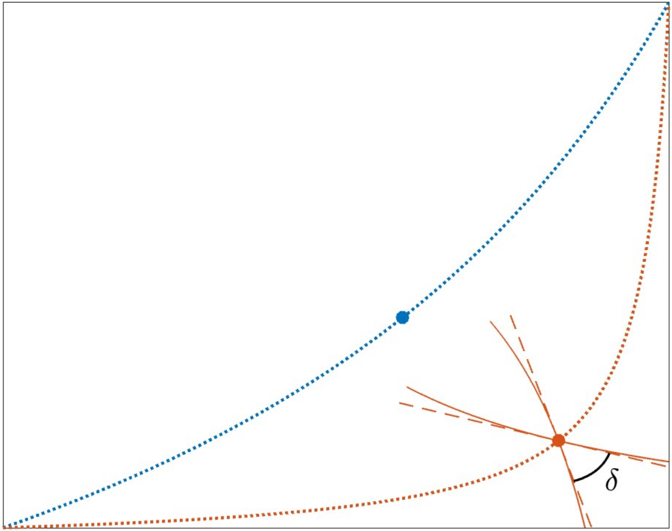
Edgeworth Box



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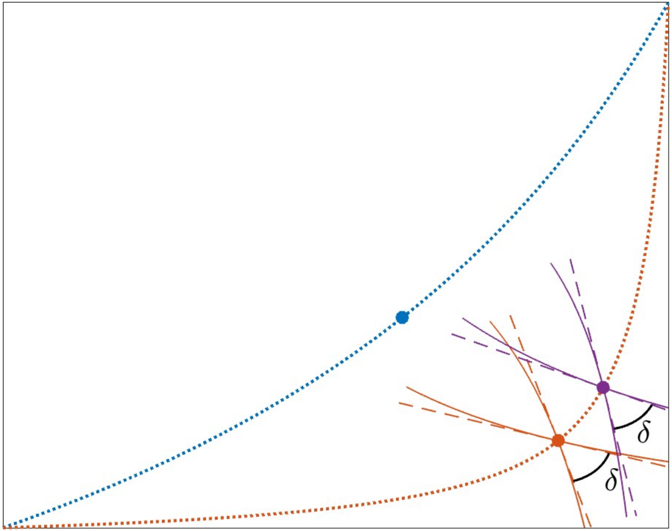
Edgeworth Box



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Estimation II

- Solve for $\lambda, \nu, \nu^*, \eta$ from planner's FOCs in growth rates:

$$(1-\bar{\gamma})\lambda \hat{C}^{1-\sigma}\left(\frac{\hat{C}_H}{\hat{C}}\right)^{\frac{\theta-1}{\theta}} = (1-\bar{\chi})\hat{C}_H\nu - \eta$$

$$\bar{\gamma}\lambda \hat{C}^{1-\sigma}\left(\frac{\hat{C}_F}{\hat{C}}\right)^{\frac{\theta-1}{\theta}} = \bar{\chi}^*\hat{C}_F\nu^* + \eta$$

$$\bar{\gamma}^*\hat{C}^{*1-\sigma}\left(\frac{\hat{C}_H^*}{\hat{C}^*}\right)^{\frac{\theta-1}{\theta}} = \bar{\chi}\hat{C}_H^*\nu + \eta$$

$$(1-\bar{\gamma}^*)\hat{C}^{*1-\sigma}\left(\frac{\hat{C}_F^*}{\hat{C}^*}\right)^{\frac{\theta-1}{\theta}} = (1-\bar{\chi}^*)\hat{C}_F^*\nu^* - \eta$$

- Distorted risk sharing:

$$\left(\frac{\hat{C}}{\hat{C}^*}\right)^{\sigma} = \lambda\frac{\bar{\chi}\frac{\hat{C}_H^*}{\hat{C}^*}\nu + (1-\bar{\chi}^*)\frac{\hat{C}_F^*}{\hat{C}^*}\nu^*}{(1-\bar{\chi})\frac{\hat{C}_H}{\hat{C}}\nu + \bar{\chi}^*\frac{\hat{C}_F}{\hat{C}}\nu^*}$$

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- Base-year system maps $\bar{\gamma}, \bar{\gamma}^*, \bar{\chi}, \bar{\chi}^*$ into base-year $\lambda, \nu, \nu^*, \eta$:

$$(1-\bar{\gamma})\lambda = (1-\bar{\chi})\nu - \eta$$
$$\bar{\gamma}\lambda = \bar{\chi}^*\nu^* + \eta$$
$$\bar{\gamma}^* = \bar{\chi}\nu + \eta$$
$$1-\bar{\gamma}^* = (1-\bar{\chi}^*)\nu^* - \eta$$

data

Estimation III

- Real variables $X \in \{Y, C, A\}$ computed in growth rates relative to base year:

$$\hat{X}_{it} = \frac{X_{it}}{\bar{X}_i}, \qquad \hat{X}^*_{it} = \frac{\sum_j w_j^X \hat{X}_{jt} - w_i^X \hat{X}_{it}}{1 - w_i^X}, \qquad w_i^X \equiv \frac{\bar{X}_i}{\sum_j \bar{X}_j}$$

- In base year, measure GDP, Exp and Imp in dollar values and compute

i) import shares (in values):

$$\bar{\gamma}_i \equiv \gamma^{\frac{1}{\theta}} \left(\frac{\bar{C}_F}{\bar{C}} \right)^{\frac{\theta-1}{\theta}} = \frac{Imp_i}{GDP_i - NX_i}, \qquad \bar{\gamma}_i^* \equiv \gamma^{*\frac{1}{\theta}} \left(\frac{\bar{C}_H^*}{\bar{C}^*} \right)^{\frac{\theta-1}{\theta}} = \frac{Exp_i}{\sum_{j \neq i} (GDP_j - NX_j)}$$

ii) export shares (in real units):

$$\bar{\chi}_i \equiv \frac{\bar{C}_H^*}{\bar{Y}} = \frac{Exp_i}{\frac{GDP_i - Exp_i}{PPP_i} + Exp_i}, \qquad \bar{\chi}_i^* \equiv \frac{\bar{C}_F}{\bar{Y}^*} = \frac{Imp_i}{\sum_{j \neq i} \frac{GDP_j - Exp_j}{PPP_j} + Imp_i}$$

data