# Housing Privatization as Intergenerational Redistribution*

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**Abstract**

This paper argues that in economies with temporary fast-growing wage incomes, housing purchase subsidy to initial cohorts serves as an effective tool of redistribution from the future towards current generations. Using China's housing reforms in the 1990s as a policy experiment, we show quantitatively that housing purchase subsidies to cohorts entering the labor market before 1994 increased substantially their homeowner rates in the initial years, which later on allowed them to enjoy the capital gain from selling their houses to future generations. Our welfare analysis suggests that, despite welfare loss for cohorts born in the 1980s, housing purchase subsidy is more desirable for future generations than alternative social security reforms as an intergenerational transfer scheme, as the latter involves high social security tax burdens for those generations experiencing a slowdown in wage growth.

**Keywords:** Housing; Intergenerational transfer; Social security
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# 1 Introduction

Many emerging economies, such as Korea, Taiwan and other East Asian Miracle economies, had experienced rapid growth of wage incomes, followed by an eventual slowdown. The fast wage growth during a period of economic transition might benefit the young more than the old when the transition starts, since the latter may not be able to enjoy the full benefit of rising incomes before they retire at a mandatory age. A social planner, therefore, may desire intergenerational redistribution from the future generations toward the old generation at the early phases of the rapid economic growth, so that they can also share the benefits of the fast growth of the economy. A conventional policy instrument for intergenerational transfers is the Pay-as-You-Go (PAYGO) social security system, which could potentially transfer resources from the future young generations to the initial old ones by offering a generous pension replacement rate. However, given the eventually slowdown of wage income growth, such a redistributive policy instrument may necessarily imposes a heavy tax burden on future generations. Adding to such challenge is the issue of population aging, which implies higher old-age dependence ratio faced by future taxpayers. Thus, a PAYGO pension system as an intergenerational transfer scheme is not financially sustainable in the *long* run when the income growth slows down, unless of course the pension system is reformed, e.g., reducing the pension replacement rate or raising the retirement age or both, to reflect the new slow growth reality in the future. This has often proved politically infeasible.

This paper we argue that during a transition stage featuring fast wage growth, subsidizing the housing purchases by the old during the housing market privatization can also play the role of intergenerational transfer from the young to the initial old generations, who are poor in income when the transition starts. The idea is as follows. Housing purchase subsidy allows those initial generations to get cheap housing when housing privatization starts. It is akin to the initial old being offered a subsidy to purchase the "initial public offering" (IPO) of the "housing" before it is "publicly listed", i.e., before the housing market becomes fully commercialized. As the income

rapidly grows, the commercialized housing market also rapidly appreciates, which raises the value of "housing IPO" the initial old generations purchased with the government subsidies at the beginning of the housing privatization. In addition, the access of initial cohorts to owner-occupied houses also help drive up the growth in housing prices as they trade up for larger commercial houses later on. When the initial generations retire, they can potentially sell the houses to reap the capital gains to finance retirement consumption.

A particularly attractive feature of using housing subsidy to the initial old generation as a way to transfer wealth from future generations is that housing prices will incorporate and adjust to the future income growth: when the future income growth is fast, the housing price will rise; and when the income growth slows down, the housing price will flatten. That is, housing prices as an instrument for intergenerational tansfer has an "automatic" adjustment property that adapts the tranfer intensity to the wage growth rates of the transitional economy and the demographic changes. As such, one can avoid the politically difficult reforms that are necessary for the PAYGO pension system as the intergenerational transfer instrument.

China offers an ideal setting to study such a policy experiment. Since the early 1990s, China has experienced fast growth in wage incomes until recent years. Between 1992 and 2012, the urban real wage grew at an average rate of 10.8%; since then, wage growth has started to slow down, averaged around 5.5% from 2013 to 2022, including a growth rate of 4.63% in 2022. The wage growth rate is expected to further slowdown to 2% beyond 2050. Accompanied with China's fast wage growth was housing market privatization, featured by two key reforms, which we will refer to as thr "94 reform" and the "98 reform" resectively. In 1994, the Chinese government allowed employees in the state-owned sector to purchase the full or partial property rights to their current apartment units at subsidized prices, the so-called "reformed housing." In 1998, the central government further approved the use of mortgage loans for commercial housing purchases and required housing provision by the work units as in-kind benefits to be terminated. Instead, employers had to include the provision of all implicit and in-kind

housing benefits in the salaries of their workers.[1]

In this paper, we quantify the role of China's housing market privatization as intergenerational transfer. We develop a dynamic housing model with household heterogeneity and incorporate the two major housing market reforms, the so called the "94 reform" and the "98 reform" described above. A key model ingredient is the introduction of two types of housing: government housing and commercial housing, with government housing smaller in size than the latter. In the initial steady states, all households rent government houses at discounted rental rates. When the transition with housing reforms starts, households born in the initial steady state are eligible to purchase government housing at discounted prices, or to purchase commercial housing at market prices, while having the option to continue to rent government houses. In contrast, households who were born after the initial steady state but before the "98 reform" lose the option to purchase government housing at discounted prices, and those born after the "98 reform" lose both the options of purchasing and renting government housing at discounted prices. The model also features a Pay-As-You-Go social security system as in China, which as we explain below, also has some built-in features of intergenerational transfer.

We calibrate the model to the Chinese economy during 1994-2014 to target key aggregate moments such as the growth rates of wages and house prices, as well as the aggregate ownership rates of different types of housing. Our quantitative model can replicate the ownership rate of government housing across different age groups and skill types along the transition path reasonably well. We then conduct counter-factual analysis to quantify the impacts of the 94 housing reform, i.e., price discounts (e.g, subsidies) to purchase government owned houses, on various cohorts alive in 1994. Our analysis suggests that the "94 reform" significantly increased the ownership rate for government housing, especially among those skilled households, in the 1990s, and that

[1]The two decrees issued by the Chinese State Council are "A Decision on Deepening the Urban Housing Reform (July, 1994)" and "Notice on Furthering the Urban Housing System Reform and Accelerating Housing Construction (1998)."

for commercial housing during the first decade of the 2000s. The key mechanism is that housing purchase subsidy facilitates the initial cohorts to become homeowners, who later on trade up to purchase commercial housing.

What would be the distributive effects of housing purchase subsidy compared with those of social security reforms that serve a similar role of intergenerational transfer? To address this question, we construct a counterfactual economy in which the pension replacement rates of the initial cohorts are chosen so that they are as well off living in the counterfactual economy as in our benchmark economy with housing reforms. We set the contribution rate for all cohorts that start to work after the "94 reform" to balance the intertemporal government budget constraint. We find that while cohorts entering the labor force during 1994-2010 are better off in the counterfactual economy with the social security reform than in our benchmark economy, all future cohorts are worse off in the counterfactual economy than in the benchmark economy. The intuition is as follows. In our benchmark economy, cohorts entering the labor force during 1994-2010 need to purchase housing at a higher price, since the trade-up demand of the initial cohorts pushes up the total housing demand. Therefore, they are worse off in the benchmark economy than in the counterfactual economy, despite lower payroll tax rates for these cohorts in the benchmark economy. For cohorts starting to work after 2010, however, they face similar housing price dynamics between the two economies, but the payroll tax burden is much heavier in the counterfactual economy with social security reform than in the benchmark economy, since in the former economy, their payroll tax revenue are used to finance the intergenerational transfer to initial cohorts, despite the slowdown in wage growth during their working life.

Our paper is close in spirit to Glover et al. (2020). Both papers study the role of asset prices in intergenerational redistribution, though in different contexts. Glover et al. (2020) show that during the Great Recession, a decline in asset prices benefits the younger generations who do not yet have much existing asset holdings but can now buy these assets at low prices, potentially compensating them for the fall in earnings they experience. By contrast, old households experienced large welfare loss, as they rely on

sales of risky assets to finance consumption. Our paper shares a similar message on the intergenerational redistributive effects of asset price dynamics. However, it differs from Glover et al. (2020) in several dimensions. First, our paper explores the redistributive effects of asset prices in the context of an emerging economy, which features temporarily fast growth in wage incomes and increasing asset prices, while Glover et al. (2020) study the redistributive effects of asset prices during Great Recession that features large declines in labor incomes and asset prices. Second, the source for asset prices to redistribute intergenerationally is very different. In our paper, the redistribution works through housing purchase subsidy during a period of housing market privatization.[2] In contrast, in Glover et al. (2020), aggregate risks, which temporarily dampened asset prices, are the main driver for intertemporal redistribution. Accordingly, the direction for intergenerational transfers via asset prices are different. In our paper, intergenerational transfer happens from the young generations to the old, while in Glover et al. (2020) the intergenerational redistribution goes the opposite directions.

Our paper also contributes to the emerging literature on China's housing reforms and its macroeconomic impacts. Wang (2011) is the first to study the effects of China's 1994 privatization of state-owned housing, and argues that the impacts of such reforms on housing prices depends on the degree of housing misallocation prior to the reform. The focus of the paper, however, is not the role of housing privatization as intergenerational redistribution, which is the key of the paper. Fang et al. (2010) study the winners of privatization of the public housing in 1994, with a focus on the distributional effects *among* the initial generations. Existing studies in the literature also explore the impacts of housing prices on different households. For example, Fang et al. (2016) argue that in anticipation of the rising housing prices, potential home buyers may be forced to purchase housing early in life with a high down payment, which can

[2]Since households are borrowing constrained, we show that without housing purchase subsidies the incentives for the initial cohorts to become homeowners would be much weaker. With a similar logic, Hur (2018) finds that the welfare benefit of declining asset prices during the Great Recession is much dampened because the young generations are often subject to binding borrowing constraints.

suppress other consumption. Furthermore, for mortgage borrower, purchasing housing early in life would increase their monthly mortgage payment as a fraction of income.[3] All these findings are in line with our model's prediction on the welfare effects of rising housing prices on cohorts entering the labor force in the 2000s.

Our paper also contributes to the discussion of alternative social security reforms as intergenerational transfer in China.[4] Song et al. (2015) is the first to explore social security reforms as intergenerational transfer in a fast-growing economy like China.[5] They find that in such an environment, a delayed reform of the current pension system, which involves paying generous pensions to the generations who are currently working or already retired, and negative pensions to subsequent generations, is a second-best policy. In this paper, we show that housing purchase subsidy for initial cohorts can provide an alternative approach to transfer to the initial old from future cohorts via the dynamics of housing prices. Our paper takes seriously that political constraints faced by the government in reducing the social security replacement rate in the future when income growth slows down and/or the dependency ratio increases. We show that under such constraints, the high payroll tax rates paid by future generations as intergenerational transfers would make social security less desirable for these later generations as an intergenerational redistributive scheme than housing purchase subsidy to initial cohorts when housing privatization starts.

The remainder of the paper is structured as follows. In Section 2 we describe the institutional background for the evolution of the Chinese housing system, particularly the 1994 and 1998 housing reforms; in Section 3 we develop a full-blown multi-period OLG framework and incorporate two housing reforms occurring in China in the 1990s;

[3]See also Chen and Wen (2017) and Jiang et al. (2022), which study the welfare effects of housing bubble bursts in a general equilibrium model.

[4]See, for example, Song et al. (2015), Imrohoroglu and Zhao (2018b) and Deng et al. (2023), among others. For an overview of China's pension system, see Fang and Feng (2020).

[5]Similarly, for the U.S., Peterman and Sommer (2019) quantifies the role of the social security system in the U.S. as intergenerational transfer when they were initially established in the 1930s. The government establish the social security system to tax the young and transfer to the old by exempting the initial old from paying payroll taxes.

in Sections 4 we calibrate the model to match key aggregate and cross-sectional moments; in Section 5 we use the calibrated model to quantify the distributive effects of housing privatization on households of different generations; in Section 6 we compare the welfare effects of housing purchase subsidy to initial cohorts on future generations with those of a counterfactual security reform that serves a similar role of intergenerational transfer; finally in Section 7, we conclude.

# 2 Institutional Background

## 2.1 Pre-1994 Housing System

From 1949 to 1978, housing in China was allocated through a work-unit-employee linkage as a form of in-kind compensation. The size and location of the housing were determined by factors such as the household size and the employee's length of service in the work unit (Fang et al., 2016).

Between 1978 and 1987, China conducted limited housing reform experiments in selected cities. These trial reforms included encouraging sales of newly built housing priced according to the construction costs, subsidizing the sales of existing public housing units, and increasing the rents charged for public housing (Garriga et al., 2017). However, during this period, most housing market participants were employers, not individual households. Employers (the so-called "work units") purchased housing and then provided these units to their employees at rents that were substantially below the market-based rental rates.

## 2.2 Housing Reforms in the 1990s

It was not until the mid-1990s did China launch its massive housing market privatization reform. In July 1994, the State Council issued "A Decision on Deepening the Urban Housing Reform," which allowed employees in the state-owned sector to purchase at subsidized prices the full or partial property rights to their current apartment units,

which were often referred to as the "*reformed housing*." The same document also required employers to gradually terminate the in-kind housing provision system for their workers.

During this stage, the price upon which the discount was based upon was called the "cost price," which was lower than the market price applicable to the high-income household who purchased houses from the market. The cost price, however, adjusted every year upward toward the market prices.[6] Evidence shows that most households who purchased housing during this period (1994-2000) at the cost prices and received price subsidy from the government. For example, using 2000 census data, Fang et al. (2010) find that among China's eight most-populous cities, the average discounted price of housing relative to the market price was as high as 38 percent.

In July 1998, the State Council issued the "Notice on Furthering the Urban Housing System Reform and Accelerating Housing Construction"(the "No. 23 Decree") . According to this decree, employers were no longer allowed to develop or purchase new housing units for their employees. Instead, employers had to include the provision of all implicit and in-kind housing benefits in the salaries of their workers. The purpose of the notice was to establish a market-based housing sector priced according to what high-, medium-, and low-income households could afford to pay. High- income households were expected to buy commercial housing, whereas those households that could not afford commercial housing could either buy economically affordable housing or rent the relatively cheap housing from the public housing system. To promote commercial housing, in August 2003, the State Council issued "Notice on Promoting a Sustainable and Healthy Development of Real Estate Market," in which the commercial housing units were established as a primary form of housing provision.

Along with the housing privatization was the establishment of mortgage market.

[6]Another pricing rule was called the "standard price," under which the household only enjoyed partial ownership right. However, according to Wang (2011), only 18 percent of the Chinese households had partial ownership. The Chinese government explicitly mandated that the standard price should only be temporary and should be replaced by the break-even cost price by 2000.

In April 1998, China's central bank, the People's Bank of China (PBOC), issued "Notice on Individual Mortgage Loan," which for the first time allows commercial banks to offer residential mortgage loans to qualified buyers of commercial residential housing. To encourage house purchases, the PBOC set aside 100 billion RMB mortgage loan quota; the mortgage loan term can be 20 years at maximum; and the maximum loan-to-value ratio was set to be 70%. In February, 1999, the PBoC issued "Guideline for Developing Individual Consumer Dredit", to further promote house purchases with mortgage borrowing.

## 2.3 Housing Market Boom in the 2000s

China experienced an enormous and sustained housing boom in the decade since 2003. For example, Fang et al. (2016) find an average annual real growth rate of 13.1 percent in the four first-tier cities, 10.5 percent in the second-tier cities, and 7.9 percent in the third-tier cities between 2003 and 2013. Wu et al. (2014) show that the national real housing price indices for the 35 major cities increased 17 percent per year between 2006:Q1 and 2010:Q4. In comparison, the annual house price growth in first-tier U.S. cities between 1996 and 2006 was slower than the annual house price growth that occurred in third-tier Chinese cities between 2003 and 2013, and was only 40 percent of the annual price house growth of first-tier Chinese cities during this period (Glaeser et al., 2017). More recently, Liu and Xiong (2020) show that following a temporary slowdown between late 2013 and 2014, housing prices in both the first- and second-tier cities experienced a remarkable pickup during 2015 and 2016, before prices stabilized in 2017. The persistent increase in housing prices in major Chinese cities between 2003 and 2016 has been referred to as the Great Chinese Housing Boom (Chen and Wen (2017)).

The nationwide housing market privatization that began in 1994 has resulted in China having one of the world's highest home ownership rates, surging from 42 percent in 1995 to 78 percent in 2002 and going further up to 88.7 percent by 2007. Since then,

China's home ownership rate has remained above 85 percent.

At least in the early part of the Chinese housing reform, the reformed housing played an important role in raising China's home ownership rate before the commercial housing development significantly increased from the 2000s. Sato et al. (2013), using data from the Chinese Household Income Project ("CHIP" henceforth), report that the fraction of households owning privatized public housing increased from 27 percent in 1995 to 61 percent (out of a total national home ownership rate of 78 percent) in 2002, whereas during this same seven-year period, the fraction of households owning commercial housing increased only modestly, from 1.3 percent to 7.4 percent. Figure 1 plots the share of various types of housing from 2002-2009, computed using the data from Chinese Urban Household Survey (UHS). In 2002, the reformed houses constituted more than 60% of all types of housing, in contrast to a less than 10% share for commercial housing. During 2002-2009, the share of reformed houses decreased steadily, to about 43% in 2009, whereas the share of commercial houses increased to 36% in 2009. This suggests that households who purchased reformed housing initially later trade up their reformed housing for larger commercial housing. Nonetheless, reformed housing remained a significant component of China's overall housing stock.

The trade-up from reformed housing to commercial housing by the initial cohorts who purchased such government housing with subsidies increased not only housing consumption, but also non-housing consumption. For example, using panel data from the China Health and Nutrition Survey ("CHNS" henceforth), Wang (2011) finds that households living in state-owned housing units prior to the reform were consuming approximately 15 percent less housing services than they would have chosen in the private market. Using the CHNS data, Yin and Gan (2009) find that after the reforms, households which had purchased public housing had a significantly higher rate of durable goods consumption a few years afterwards.

# 3 The Model

In this section we specify a multi-period overlapping-generations economy, which features economic transition following housing reforms that mimic those in China during the 1990s. For simplicity, we assume all agents have perfect foresight when the transition starts.

## 3.1 Household

**Demographic Structure.** In each period, a continuum of households is born, and the new-born population grows at an exogenous rate $n_t \geq 0$. Households live a maximum of $J$ periods, with each period representing one year. They enter the economy as adults and work as workers until retirement age $J_w$. All households face a probability $S_j$ of surviving up to age $j$, and they die at age $J$ with certainty, where $S_j = \prod_{k=1}^{j} \psi_k$, and $\psi_k$ is the conditional survival probability from age $k-1$ to age $k$. The fraction of households of age $j \in \{2, ..., J\}$ at the calendar time $t$, denoted by $\mu_{t,j}$ is

$$\mu_{t,j} = \frac{\psi_j}{1+n_t}\mu_{t-1,j-1}, \tag{1}$$

and $\mu_{1,j} = 1 - \sum_{j=2}^{J} \mu_{t,j}$.

**Preference.** All agents have identical preference over an aggregation of goods consumption, denoted by $c$ and housing service, denoted by $s$. For a household born at period $t$, his/her lifetime utility is

$$U_t = \sum_{j=1}^{J} \beta^{j-1} S_j \big[\log(c_{t+j-1,j}) + \phi \log(s_{t+j-1,j} + \underline{s})\big] + \iota \log(B^n_{t+J}), \tag{2}$$

where $\underline{s} > 0$ could be interpreted as the minimum housing services provided by the government, so that price elasticity for housing service is larger than one. In (2), we simplify the bequest utility as follows: $B^n_{t+J}$ is the accidental bequest left by

household for newborns, which equals the sum of financial and housing wealth, and $\iota > 0$ captures the strength of the bequest motive; for simplicity we assume that the accidental bequests, regardless of the time period at which the household actually dies, are collected by the government and redistributed to the new born of the same household type (i.e., skill type) in period $t + J$.[7]

Household $i$'s after-tax income in period $t$ at age $j$, denoted by $y^i_{t,j}$, is given by:

$$y^i_{t,j} = \begin{cases} (1 - \tau^{ss}_t - \tau_t) w_t e^i_{t,j} & \text{for } j \leq J_w \\ b^i_{t,j} & \text{for } j > J_w \end{cases} \tag{3}$$

where $w_t$ is the wage rate per unit of efficiency labor supply; $e^i_{t,j}$ is $i$'s efficiency units in period $t$ at age $j$ which is described below in Eq. (4); $b^i_{t,j}$ is the social security benefit for a household $i$ who is age $j$ in period $t$, which we describe below in Eq. (6); $\tau_t$ and $\tau^{ss}_t$ are respectively the labor income tax rate and the social security tax rate in period $t$. $\tau^{ss}_t$ is determined to balance the PAYGO in period $t$, which we describe in Eq. (8).

The efficiency unit of household $i$'s labor supply in period $t$ at age $j$ is specified as:

$$e^i_{t,j} = \lambda^i \varepsilon_j z^i_t \epsilon^i_t, \tag{4}$$

where $\lambda^i$ is skill-specific component to capture skill premium; $\varepsilon_j$ is life-cycle income profile reflecting, e.g., the experience effect;$z^i_t$ is a persistent shock that follows an AR(1) process

$$\log(z^i_t) = \rho \log(z^i_{t-1}) + \nu^i_t, \tag{5}$$

where $\nu^i_t$ follows a standard normal distribution $N(0, \sigma_\nu)$; additionally, $\epsilon_{t,j}$ is a transitory shock drawn randomly from a log-normal distribution in each period.

[7] We are currently working on alternative formulations of bequest process.

**Social Security.** The government runs a PAYGO social security system. For individual household $i$, the social security benefit each period is determined as

$$b^i_{t,j} = \theta[0.6y^i_{t-j+J_w,J_w} + 0.4\bar{y}_t], \tag{6}$$

where $\theta > 0$ is the pension replacement rate at the time of retirement, $J_w$ is the maximum working age. $y^i_{t-j+J_w,J_w}$ is the before-tax wage earnings of the household $i$ at its retirement age $J_w$, and $\bar{y}_t$ is the average yearly earning of all workers, known as the "social average wage", in period $t$, which is given by

$$\bar{y}_t = \frac{\sum_{j=1}^{J_w} w_t \mu_{t,j} \int_i e^i_{t,j} di}{\sum_{j=1}^{J_w} \mu_{t,j}} \tag{7}$$

Note that the above formula only allows the partial indexation of pension benefits to the current period earnings, which implies *partial* inter-generational redistribution. We assume that the social security program is self-financing where the social security payroll tax rate $\tau^{ss}_t$ each period is endogenously determined to balance the social security budget. In other words, the social security tax rate is determined as

$$\tau^{ss}_t = \frac{\sum_{j=J_w+1}^{J} \mu_{t,j} \int_i b^i_{t,j} di}{\sum_{j=1}^{J_w} \mu_{t,j} w_t \int_i e^i_{t,j} di}. \tag{8}$$

## 3.2 Housing Reforms

**Pre-reform State** We assume that, prior to the housing reforms, the government assigns each household a house whose size is determined according to the household's skill type $\lambda^i$: households of skill type $\lambda^i$ are assigned a house of size $h^i_g$ by the government; the households do not own the assigned house but pays a discounted rental rent with the rental subsidy determined by the government. We assume that more skilled households are assigned larger houses, i.e., $h^i_g$ is weakly increasing in $\lambda^i$.

We assume before period 1 –which corresponds to the year 1994 in reality– the economy is in the pre-reform steady state, and the government budget is in balance.

Households' idiosyncratic shocks are assumed to be perfectly insured in the pre-reform stage.

We assume that the government uses land and labor to produce housing and then rent it out to households at a given rental rate. The equilibrium outcome in the initial steady state determines the initial distribution we observe in the data when housing reform started in period 1. Appendix A.1 specifies the households' optimization problem in the pre-reformed steady state.

**Housing Privatization** No households born before period 1 own any housing. Assume at the beginning of period 1 at the beginning of period 1 when the housing privatization starts. From period 1 onward, all households can also choose to rent an apartment $h_a \in \mathcal{H}_a = \{h_a^1, h_a^2, ...h_a^{N_a}\}$ that provide housing service $s = h_a$ at market rental rate $\rho_t$, or purchase an owner-occupied houses $h \in \mathcal{H} = \{h^1, h^2, ...h^N\}$ at market price $p_t$. To capture the fact that there may be additional utility from home ownership, we assume that an owner-occupied housing provides housing service $s = \zeta h$, with $\zeta \geq 1$. Owner-occupied houses carry a per-period maintenance cost, $\delta_h p_t h_{t,j}^i$, expressed in units of the numeraire goods, and we assume that the maintenance fully offsets any physical depreciation of the dwelling. When a household sells its home of size $h$ in period $t$, it incurs a transaction cost $\kappa_0^j + \kappa_h p_t h$, where $\kappa_0^j$ is a fixed transation cost that may depend on age of the houeshold $j$, and $\kappa_h p_t h$ is a variable transaction cost that is proportional to the transacted housing value. For renters who purchase commercial housing or government housing (if qualified), they incur a fixed cost of initiating the mortgage, $\bar{m}_{t,j}$, which is time and age dependent.

**Subsidized Purchase of Reformed Houses.** Those households who were alive before period 1, whom we call the "initial cohorts" throughout this paper, are eligible to purchase the house they rented in the pre-reformed state at the subsidized house price $p^g t, j$, the so-called "reformed houses". In addition, they can keep renting the housing at the discounted rental rates, which we assume to be a fraction $\omega$ of the

market rental rate $\rho_t$. Alternatively, households can purchase or rent housing from the market, which gives them the option to choose the size of housing they desire. However, if they choose to move out of the government rental house and move into a commercial housing, they become ineligible to purchase the reformed house throughout the rest of their life.

We assume that the discounted price to purchase government housing is given by:

$$p_{gt,j} = p_t \left[ 1 - \frac{\min\{\text{Age in 1994}, J_w\}}{J_w} \times 65 \times 0.9\% \right] \tag{9}$$

which is a simplified rule of the discounted price in practice, where $p_t$ is the market price of housing in period $t$. According to this formula, the house purchase discount is age-specific: if a household was retired before 1994, then he can enjoy a 58.5% (65*0.9%) price discount.[8] The longer an household worked before 1994, the more discount he or she can enjoy. This reflects the government's desire to transfer resources from future cohorts to these cohorts who entered the labor force before 1994. Note that according to the formula, households born at or after period 1 (i.e., entering the labor force at or after 1994) do not enjoy any price discount since their working years before period 1 is zero.

**Elimination of Government Rental Houses.** Corresponding to the 1998 housing reform, we assume that for household born after period 5, the option to rent government houses at the discount rent is eliminated. However, all households born no later than period 5 can keep renting the government-assigned rental housing until they move out to a commercial housing. Households born before period 1 have an additional option of purchasing rental housing with a discounted price, and the option is valid as long as they live in government-assigned rental housing.

[8] In reality, the total effective maximum working years of the household in the computation of the house price discount is 65 (35 years for the husband, plus 30 years for the wife, to be consistent with the fact that male retries at age 60 and female retires at age 55 at that time.)

**Access to Mortgage Market** We assume that the mortgage market opens starting from period 5, which corresponds to the year 1998. All mortgages are long-term, subject to a fixed mortgage origination cost, and amortized over the remaining life of the buyer at the common real interest rate $r_m$.

At the time of mortgage origination, households are subject to the loan-to-value ratio limits. The initial mortgage balance $d$ must be less than a fraction of $\gamma$ of their housing purchase prices. After the mortgage loan origination, for a household of age $j$ with outstanding mortgage balance $d$, the per-period minimum mortgage payment $m$ is determined by the constant amortization formula as follows:

$$m_j \geq m \equiv \frac{r_m(1+r_m)^{J+1-j}}{(1+r)^{J+1-j}-1}d \tag{10}$$

Accordingly, the principal evolves according to $d' = d(1+r_m) - m_j$.

## 3.3 Household Decision

At each period, a household may start the period at three possible states: 1) renting government housing; 2) renting commercial housing; 3) owning a house. Denote the index for the beginning of period housing status as $I \in \{N_g, N, H\}$. Depending on their birth period, which corresponds to the year they start to work in reality, households of different cohorts have different choice sets on housing. In this section, we describe the household decisions for three types of cohorts. Appendix A.1 and A.2, respectively contains a full description of the household problem in recursive form and a formal definition of equilibrium.

**Initial Cohorts.** Initial cohorts are eligible for purchasing reformed houses at subsidized prices. At the beginning of a period, if the household rents government houses (i.e., $I = N_g$), he/she has four options: i) keep renting the government house; ii) purchase the government house which he previous rented; iii) rent a commercial house; iv) purchase a commercial house. Those who remain as renters or owners of government housing

choose the quantity of nondurable goods to consume, and how much to save in the liquid asset.[9] Those who select to become renters or homeowners of commercial housing also choose the size of house. Home buyers also choose the value of the mortgage they wish to take out, and make an initial down payment. For households who rent commercial houses (i.e., $I = N$), they can only rent or purchase commercial houses. For both types of renters, since they do not own any collateral, they cannot borrow. On the other hand, for a homeowner (i.e., $I = H$), whether owning a reformed house or commercial house, he/she only has two options: a) keep the current house; b) sell the current house, after which the household can only buy or rent a commercial house. Once the household sell a house, he/she needs to pay down all existing mortgage balance, in addition to the aforementioned transaction cost. Diagram I summarizes the households' choice.

Diagram I: Beginning-of-period Housing Status and Housing Choices for Initial Cohorts

$$
N_g \longrightarrow \begin{cases} \text{rent reformed housing} \rightarrow N_g \\ \text{buy reformed housing} \rightarrow H \\ \text{buy commercial housing} \rightarrow H \\ \text{rent commercial housing} \rightarrow N \end{cases}
$$

$$
N \longrightarrow \begin{cases} \text{buy commercial housing} \rightarrow H \\ \text{rent commercial housing} \rightarrow N \end{cases}
$$

$$
H \longrightarrow \begin{cases} \text{keep housing} \rightarrow H \\ \text{sell housing} \longrightarrow \begin{cases} \text{buy commercial housing} \rightarrow H \\ \text{rent commercial housing} \rightarrow N \end{cases} \end{cases}
$$

[9] We assume free conversion between consumption goods and rental apartment. Therefore, if households choose to move out of the reformed house that they were initially assigned to rent, those houses are then converted to consumption goods at no cost.

The decision on whether to purchase a reformed house or commercial house is based on the following trade-off. On the one hand, reformed housing is cheaper than the commercial house. On the other hand, purchase of commercial house does not have size restriction, while the household can only purchase the reformed house he/she previously rented. Therefore, the high-income households may choose to purchase commercial housing for more desirable home size and for more capital gain. Middle-income household, whose optimal housing size is small, may choose to purchase the reformed housing at the initial stage; after they accumulate enough savings for down payment for commercial houses, they may sell their reformed houses and trade up for commercial houses. Low-income households may choose to continue to rent the government house at the subsidized rental rates and save for the down payment for the reformed houses.

**Cohorts Born between Period 1 and 5.** For a household born between period 1 and period 5, their problem is similar to households born before period 1. The only difference is that even if they are renters of government houses (i.e., $I = N_g$) , they do not have the option of purchasing reformed houses. Diagram II summarizes their choice sets.

Diagram II: Beginning-of-period Housing Status and Housing Choices for Cohorts Born between Period 1 and 5.

$$
\begin{aligned}
N_g \longrightarrow & \begin{cases} \text{rent reformed housing} \rightarrow N_g \\ \text{buy commercial housing} \rightarrow H \\ \text{rent commercial housing} \rightarrow N \end{cases} \\
N \longrightarrow & \begin{cases} \text{buy commercial housing} \rightarrow H \\ \text{rent commercial housing} \rightarrow N \end{cases}
\end{aligned}
$$

$$H \longrightarrow \left\{ \begin{array}{l} \text{keep housing} \rightarrow H \\ \text{sell housing} \longrightarrow \left\{ \begin{array}{l} \text{buy commercial housing} \rightarrow H \\ \text{rent commercial housing} \rightarrow N \end{array} \right. \end{array} \right.$$

Similar to initial cohorts, low-income households born during period 1 and 5 tend to choose to continue to rent government houses rather than rent commercial houses, tolerating the distortion on housing sizes in favor of the lower rental rate of the government houses. When they accumulate more wealth, a household may start renting commercial houses, followed by purchasing commercial houses, when he/she accumulates enough savings for down payment.

**Cohorts Born after Period 5.** For a household born after period 5, the only difference with the cohorts born between period 1 and 5 is that they do not have the option to rent the government houses. Accordingly, they face the standard trade-off between buying or renting a commercial house. The costs of buying include the initial down payment, and the benefits of owning a house is that it yields an extra housing service flow, as well as the capital gains from house price appreciation.

Diagram III: Beginning-of-period Housing Status and Housing Choices for Cohorts Born at or after Period 5.

$$N \longrightarrow \left\{ \begin{array}{l} \text{buy commercial housing} \rightarrow H \\ \text{rent commercial housing} \rightarrow N \end{array} \right.$$

$$H \longrightarrow \left\{ \begin{array}{l} \text{keep housing} \rightarrow H \\ \text{sell housing} \longrightarrow \left\{ \begin{array}{l} \text{buy commercial housing} \rightarrow H \\ \text{rent commercial housing} \rightarrow N \end{array} \right. \end{array} \right.$$

## 3.4 Production

Following Kaplan et al. (2020), there are two production sectors in the economy: a final goods sector, which produces nondurable consumption (the numeraire good of the economy); and a construction sector, which produces new houses. Labor is perfectly mobile across sectors.

The consumption goods sector is competitive and operates a constant return-to-scale technology

$$Y_t = A_t N_{ct}, \tag{11}$$

where $A_t$ denotes labor productivity and $N_{ct}$ is the total units of labor services (in terms of efficiency units) used to produce consumption goods in period $t$. Therefore, the wage rate per efficiency unit of labor supply is given by

$$w_t = A_t. \tag{12}$$

We assume $A_t$ evolves according to an exogenous process.

The construction sector operates the production technology to produce new houses:

$$Y_{ht} = (A_t N_{ht})^{\alpha} (L_t)^{1-\alpha}, \alpha \in (0, 1) \tag{13}$$

where $N_{ht}$ is the quantity of labor services (in terms of efficiency units) employed in the construction sector, and $L_t$ is the amount of new available buildable land. We assume that in each period the government issues new permits equivalent to $L_t$ units of land, and these permits are sold at a competitive market price to developers.[10] This implies that all rents from land ownership accrue to the government and the construction sector makes zero profits in equilibrium.

Solving housing developer's profit maximization problem gives the supply of new

[10]The land sales revenue received by the government is currently assumed to be exclusively spent on infrastructure investment, which we do not model in this paper. As a result it does not show up in the government budget constraint (16) below.

houses as

$$Y_{ht} = (\alpha p_t)^{\frac{\alpha}{1-\alpha}} L_t. \tag{14}$$

## 3.5 Rental Sector

To simplify the rental market, we assume that there is a representative rental developer. The developer utilizes a reversible technology $Y_{at} = A_{at} S_t$ that converts consumption goods $S_t$ into apartment space that depreciates at rate $\delta_a$. Rental firms can either sell this space at price $p_t^a$ or lease to households at rent $\rho_t$. Profit maximization for rental developer implies:

$$p_t^a = \frac{1}{A_{at}} = \rho_t + \frac{1-\delta_a}{1+r} p_{t+1}^a. \tag{15}$$

## 3.6 Government

Government outlays includes both housing purchase subsidies and rental subsidies to those who either buy or rent reformed houses. Government revenue includes the income tax, as the social security budget is ear-marked. We assume the government maintains a balanced intertemporal budget constraint by imposing payroll tax $\tau$:

$$\begin{aligned} B_0 + \sum_{t=1} \frac{1}{(1+r)^t} \sum_{j=1}^{J_W} \sum_i \tau w_t N_{t,j} = \sum_{t=1} \frac{1}{(1+r)^t} \Bigg[ \sum_{j=1}^{J} \sum_i \int (p_t - p_{gt,j}) h_g^i \mathbb{I}_{\text{buy}_t(N_g,j,y,z)} d\mu_t^i(N_g,j,y,z) \\ + \sum_{j=1}^{J} \sum_i \int (1-\omega) \rho_t h_g^i (1 - \mathbb{I}_{\text{buy}_t(N_g,j,y,z)}) d\mu_t^i(N_g,j,y,z) \Bigg], \end{aligned} \tag{16}$$

where $B_0$ denotes the government's initial asset position at the beginning of period 1. $\mathbb{I}_{\text{buy}_t(N_g,j,y,z)}$ is an indication function of the buying status for renters of the reformed houses with state variables $(N_g, j, y, z)$. $\mu_t^i(N_g, j, y, z)$ is the mass of skill-type $\lambda^i$ renters of reformed houses with state variables $(N_g, j, y, z)$ at the beginning of period $t$ before the purchase decision is made.

The left-hand-side of equation (16) is the present value of the government revenue, discounted to period 1, which includes the income tax revenue.[11] The right-hand-side of (16) is the present discount value of the government outlays, which includes housing purchase subsidy, captured by the first argument, and rental subsidy, which is captured by the second argument. This intertemporal government budget constraint implies that all transfer to cohorts born before period 1 need to be financed by future generations.

# 4 Calibration

## 4.1 Model Parameters

We calibrate the benchmark economy to match the Chinese economy during 2002-2009, a period that data from UHS is available. Each period corresponds to one year in the data.

**Demography.** In our model, $j = 1$ corresponds to age 21 in real life, and $J_w = 40$ so that agents retire at age 60 in real life. We set $J = 70$, corresponding to a maximum lifetime of 90 years.

The top panel of Figure 2 plots the dynamics of population growth rates $n_t$.[12] In the initial steady state, the population growth rate is set to be 2.4%, which was the average growth rate of the working-age population growth rat until 1995; between 1995 and 2020, the population growth rate each year is taken from the data on working-age population growth rate by the United Nations; between 2021 and 2045, the projected growth rate is as low as 0.088 percent; after 2045, the projected population growth rate is set to be zero.

[11] For simplicity, we assume all government revenues by selling the land are consumed by the government in the current period.

[12] Note that since 2010, China has started to experience population aging. The growth rate of working-age population decreased from 1.73% on average between 2005-2010 to 0.66% on average between 2010-2015.

The age-specific conditional survival probabilities $\{\psi_j\}_{j=1}^{J}$ are taken from Imrohoroglu and Zhao (2018a).

**Process for Labor Supply Efficiency Units.** The components in the process for labor supply efficiency units (4) are calibrated as follows. For the skill-specific component, we normalize $\lambda^L = 1$. The average wage rate of high school graduate is approximately 1.79 times that of high school dropout. So $\lambda^H = 1.79$. The life-cycle income profile, $\epsilon_j$ is taken from He et al. (2017) who use the CHNS data to estimate them. The persistent shock $z_t^i$ (until the retirement) follows an AR(1) process shown in equation 5; we follow He et al. (2017), and take $\rho = 0.84$ and $\sigma_v^2 = 0.055$. We then discretize this process into a seven-state Markov chain using the Tauchen method. The transitory shock, $\epsilon_t^i$, is assumed to follow a log-normal distribution with zero mean and a standard deviation at $\sigma_\epsilon^2 = 0.03$ from Fan et al. (2010).

**Preference.** The discount factor, $\beta$, is set to be 0.95. Utility parameter for housing service $\phi$ is set to match the average share of housing consumption in total consumption expenditure, which is 20%. The government housing service $\underline{s}$ is chosen to match a price elasticity of housing demand being -0.5. We calibrate the home service flow premium for owning a house relative to renting, $\zeta$, to match a home-ownership rate in 2002 of 82.4 percent, we we calculated from the UHS data.

**Housing.** Housing depreciation rate, $\delta_h$ is set to be 2%, which is the estimated depreciation rate on China's urban owner-occupied housing by OECD. We set the down payment requirement to be 100 percent during 1994-1998, and 30 percent afterward to be consistent with the timing of the establishment of the mortgage market. The subsidized rent to the renters of reformed houses is set to be 20% of the market rental rate. The annual deposit interest rate, $r = 0.02$, matches the real interest rate in 1994. The mortgage interest rate $r_m$ is set to be 5 percent.

We normalize the size of smaller reformed house ($h_g^L$) to be 1, and the size of the

larger reformed house ($h_g^H$) is chosen so that the expenditure share on housing rents remain roughly the same between skilled and unskilled in the initial steady-state. The rent rate chargedby the government in the initial steady state is set so that the rental expenditure share is about 20 percent. We let commercial house take six different sizes. The smallest size for commercial house is chosen to match the ownership of commercial house in 2002 at 11 percent. The largest commercial house is chosen to be five times the size of the smallest one. The remaining four sizes of commercial houses are evenly distributed between the smallest and largest one.

**Production Sector.** The construction technology parameter $\alpha$ is set to be 0.8, so that price elasticity of new housing supply equals 4, about the average value by Wang et al. (2012) for the 35 major cities in China. We calibrate the TFP growth rate in the pre-reform steady state to be 4.2%, which is the average growth rate of urban wage rate growth rate between 1980-1993. TFP process for non-durable goods, $\{A_t\}_{t=1}^T$ along the transition is set to match the urban hourly wage growth rate. In particular, between 1994-2010, the wage growth rate is 10.8%; between 2010-2020, the wage growth rate is 8.1%; between 2021-2050, the wage growth is 5.0 percent; after 2050 we assume that the wage growth will be 2 percent per year (the bottom panel of Figure 2 ).

**Government sector.** The pension replacement rate $\theta$ is calibrated to be 0.6, which is the replacement rate for the retirees between 1997 and 2011 covered by the system according to Song et al. (2015). We calibrate the government's initial asset holding to match an average of 20% labor income tax rate in the data.

Along the transition path, we feed in the urban residential land supply for residential use from the data up to 2017[13]. Since 2018, the land supply is set to mimic the simulated housing prices, whose annual growth rate is set to be 5 percent during 2018-2028, and 2 percent afterward.

[13]The land supply also takes into consideration the deterioration of the land quality. Following Jiang et al. (2022), we assume land quality diminishes by 8% per year.

Table 1 summarizes the calibrated parameter values and their targeted moments. Table 2 compares the targeted moments generated by the model and the data. Overall, our calibrated model can match the targeted data moments fairly well.

## 4.2 Model Predictions on Home Ownership Rates

We now compare the cross-sectional moments generated by the model with those by the data, which are untargeted in calibration. Table 3 compares the home ownership rate for all houses, including both reformed and commercial ones, by age and education group between the model and the data. To compare, we pick two years, 2002 and 2009, which are the starting and ending years of our UHS data.[14] We can see that the model can replicate home ownership rates for all age groups, with the exception for those aged between 21 and 30 in these two years.[15] In addition, our model can replicate the home ownership rate for households in both skill groups reasonably well. Overall, the home ownership in our model is 75.8% in 2002, slightly lower than its counterpart of 82% in the data.

Table 4 compares the model generated ownership rate of the reformed house with the data. Again, the model has matched the ownership rates across different age groups and skill groups reasonably well. It is noted that even for the age group 21-30, the model simulated ownership rates of reformed houses are close to the data counterparts in both years. Accordingly, the model replicates reasonably well the overall ownership rate for reformed houses. For example, in 2002, in our model 71.7% of households owned reformed houses, as compared with 78.2% in the data.

Figure 3 plots the simulated house price-to-income ratios for the whole economy

[14]Since period 1 in our model maps into the year 1994, these two years correspond to period 9 and 16 in our model.

[15]The home ownership rates for the age group 21-30 in the data are significantly higher than our model predictions. In our model, this age group is constrained by the mortgage down payment requirements. In reality this age group often rely on inter vivos transfer from their parents to overcome down payment requirement, a mechanism that we do not yet incorporate in our model. We are currently working on this modification.

and for households of different percentiles in income distribution. The median price to income ratio in our model is 11 in 2002 and steadily increases to 15 in 2009. For households in the bottom 10 percentile, the price-to-income ratio was 18 in 2002 and increased to 25 in 2009, while for households in the top 10 percentile, the number increases slightly from 5 in 2002 to 6 in 2009. The faster-than-income growth rate of house prices generated by our model is consistent with the empirical findings in the literature (Wu et al., 2014; Fang et al., 2016).

# 5 Benchmark Results

## 5.1 Housing Trade-up

We now characterize the extent of housing trade-up among the initial cohorts. The upper panel of Table 5 reports the ownership rate of reformed houses for each initial cohorts, categorized in ten-year bins. The results indicate that, despite the 100% down payment requirement (because mortgage loans were not yet introduced in this period), a significant fraction of the initial cohorts of different ages bought their reformed houses during 1994-1998 . Moreover, during 1994-1998, the ownership rate of reformed houses increases in age (except for those born during 1905-1914), because of a positive relationship between the price discount and the years worked in pre-reform state. Since 1998, the ownership rate of reformed houses has started to decline for almost all cohorts except those born in 1965-1974, which suggests the emergence of a trade-up pattern among those initial cohorts from reformed to commercial houses. This trade-up pattern is evident in the bottom panel of Table 5, which reports the ownership rate of commercial houses. In contrast to the secular pattern of the ownership rates of the reformed houses, the ownership rate of commercial houses has gone up steadily at the aggregate level and seeprately for most age groups. For example, while the years 1999-2003 witnesses a 8% decline in ownership rate of the reformed houses, the overall ownership rate of commercial houses among initial cohorts increased by 29%

(31.8%-2.8%).

Table 6 reports the loan-to-value (LTV) ratio at origination when those initial cohorts upgrade from reformed to commercial houses. Initially, a dominant share of the upgraders chose to take mortgages with an LTV ratio between 50 and 70 percent. Over time, more and more households chose mortgages with lower LTV ratio when they switch from reformed houses to commercial houses.

To quantify the size of capital gains from selling the reformed houses purchased by the those early cohorts, we calculate in Table 7 the ratio of the capital gain to the average labor earning of each specific age-group in a given period.[16] The table shows that during those periods where trade-up were prevalent (i.e., 1999-2013), the realized capital gain was more than 100 percent of the average earnings for both skilled and unskilled households.

Figure 4 plots the age profile of ownership rate of reformed houses and commercial houses by four different initial cohorts, those born in 1940s, 1950s, 1960s and 1970s.[17] For each cohort, we see that the ownership rate of reformed houses declines over the life cycle, while the opposite is true for the ownership rate of commercial houses. For example, for those born during the 1960s, the ownership rate of reformed houses decreases from 50% in their 30 to almost 0 when they reach age 80. By contrast, the ownership rate of commercial houses increases from 50% to 100% over the life cycle. These patterns suggest a clear pattern of trading up over the life cycle by various initial cohorts from reformed to commercial houses.

Figure 5 shows among those initial cohorts, the age or skill composition of trade-up. Panel (a) suggests that in 2000 both the young and middle-age households among those initial cohorts are the main contributors for trade-up. A few of households who were very young in 1994 also caught the wave of trading up when they turned middle age in 2020. Panel (b) implies that both skilled and unskilled households contributed roughly

[16] Among the retiree the average labor earning refers to the average pension.

[17] For each cohort, the plot starts from their specific age in period 1.

equally to the trade-up.

Figure 6 plots the share of trade-up demand in the total demand for commercial houses in both volumes and population in selective years.[18] In 2000, trade-up contributes to more than 80% of demand for commercial houses in both volumes and ownership rates. The contribution of trade-up to the demand for commercial houses dropped to about 50% in 2010. In 2020, less than 10% of the demand for commercial houses was attributable to housing trade-up, suggesting that first-time homebuyers have become the dominant drivers the demand for commercial houses over time.

## 5.2 Drivers of House Price Booms

In our model, two main factors contribute to the fast growth of housing prices along economic transition: the first is the capital gains when households who purchase reformed houses trade up for larger commercial houses, and home buyers may buy houses for anticipated capital gains; the other is the fast wage income growth during the transition. To quantify the importance of these two factors, we conduct two counterfactual experiments. One is to impose 100% capital gain taxes when households sell their existing reformed or commercial houses. In such a counterfactual environment, investment motive for housing demand (for its capital gains) would be absent. Thus the gap of housing price growth between this counterfactual economy and the benchmark economy captures the importance of capital gains (or investment, or speculative, motives) for housing demand. The other is to reduce the growth rate of TFP along the transition path by half. Accordingly, the gap between this counterfactual economy and the economy benchmark economy captures the effect of access to reformed houses.

Figure 7 plots the dynamics of housing prices under these two counterfactual economies, together with their benchmark counterparts. To compare the growth rate, we normalize the level of house price in 1994 to be one in all three economies. Housing

[18] We only keep track of those who sell their reformed houses and purchase commercial houses within the same period; that is, we do not record those who may purchase commercial houses a few periods later than selling the reformed houses.

prices in the economy without capital gain growth significantly slower than their counterpart in the benchmark economy, especially before 2000. In contrast, the gap of housing prices in the economy with slower wage growth and those in the benchmark economy is much smaller, especially in the years before 2000. This suggest that the capital gain enjoyed by existing homeowners when they trade up their homes is a key driver of the strong housing demand along economic transition.

## 5.3 The Role of Housing Reforms

Our analysis suggests that the capital gain enjoyed by initial cohorts facilitates their trade up from reformed houses to commercial houses, which in turns contribute to the increasing housing demand along economic transition. In this section, we quantity the role of the 1994 housing reform for the housing demand.

The "94 reform" affect the ownership rates of initial cohorts via two potential channels. First, it allows these cohorts to purchase government houses, which are smaller in size than commercial houses. Second, it provides purchase subsidy for government houses, which allows initial cohorts to purchase government houses at a cheaper prices.

To evaluate the quantitative importance of these two channels, we conduct two counterfactual exercises. In the first one, we remove the housing purchase subsidy to those initial cohorts, but they are still allowed to purchase the government house that they were initially assigned to at the market price. We call such a counterfactual economy the economy without purchase subsidy. In the second exercise, we further eliminate the option by those initial cohorts to purchase the reformed houses which they previously resided. As a result, if they plan to own a house, they have to purchase a commercial house. We call this counterfactual economy the economy without 94 reform. The difference of home ownership rate between our benchmark economy and the economy without purchase subsidy captures the effects of purchase subsidy, while the difference of home ownership rate between the economy without purchase subsidy

and the one without 94 reform captures the effects of the availability of government houses.

In Figure 8 we plot the dynamics of housing prices in the three scenarios, respectively. Housing price is initially higher in the benchmark economy than the other two scenarios. This suggests that the housing purchase subsidy to initial cohorts and the provision of reformed houses play important role to facilitate initial cohorts to become homeowners. However, during 2010-2020, housing prices appear to be highest in the scenario without housing purchase subsidy. This is likely due to the fact that even without purchase subsidy initial cohorts still purchase the reformed house at a later point in life. Compared with the case without 94 reform, the provision of reformed houses, which are smaller in size that commercial houses, facilitates the initial cohorts to become homeowners. In addition, when there is no purchase subsidy, the government is able to levy a lower payroll tax to balance the budget. This positive income effect tends to trigger a stronger demand for commercial houses among younger cohorts, who tend to be borrowing constrained, than those in the benchmark economy. Housing prices tend to be the lowest in the scenario when purchasing reformed house is not allowed. In this case, not only the demand for reformed house among initial cohorts is absent, but also the trade-up motive is missing. The younger cohorts are also unlikely to afford a commercial house in the initial years.

How does the 1994 housing reform contributes to the owner-occupation of reformed and commercial houses? In Figure 9, we compare the evolution of home ownership rate among different groups of households and different types of houses. The top two panels show the dynamics of home ownership rates for the unskilled and the skilled households. For both skill types, home ownership rate in the benchmark economy is the highest during the first decade since the reform starts. In particular, in the mid-1990s, the gap of home ownership rate between the benchmark and the economy without housing purchase subsidy is twice as much as the gap between the latter and the economy without the 94 reform. This suggests that the provision of house purchase subsidy plays the main role for initial cohorts to become homeowners initially. This is

especially the case for the skilled households: Without housing purchase subsidy, the home ownership rate is less than 20% in the mid-1990s, 30% lower than the benchmark.

The bottom two panels capture the role of the 94 housing reform on the ownership rate of two types of houses. It is clear that before 2000, the ownership rate of reformed houses is significantly higher in the benchmark economy that in the economy without housing purchase subsidy.[19] Without housing purchase subsidy, the ownership rate of reformed house is less than half of the benchmark economy in the 1990s. This suggests less trade-up takes place when initial cohorts are not given purchase subsidy. Later on, ownership rate in the benchmark economy declined at a much faster speed than that in the economy without housing purchase subsidy. This suggests that the provision of housing purchase subsidy, which allow initial cohorts to enjoy capital gain, facilitates their trading up to commercial houses. The bottom right panel supports this conjecture: while the ownership of commercial houses differs little between the benchmark economy and the economy without housing purchase subsidy, the gap between these two became much wider in the first decade of the 2000s.

# 6 Welfare Effects of Housing Reforms v.s. Pension Reforms

In this section, we compare the roles of intergenerational transfer between housing purchase subsidy and social security. To this end, we construct a counterfactual economies without housing purchase subsidy for initial cohorts, but with an alternative social security system that serves as intergenerational transfer from the later generations to earlier generations. We then explore the welfare impacts of such an alternative social security system, in comparison with housing reforms, on younger cohorts.

We use consumption equivalent variation (CEV) to measure the welfare effects of a

[19]By construction, the ownership rate of the reformed houses is zero in the economy without 94 reform.

particular redistributive policy. Specifically, CEV is defined as the uniform percentage change in expected consumption in each period over the remainder of an individual's lifetime that makes the individual from cohort $s$, i.e., born in period $s$, indifferent between the benchmark and the counterfactual scenario.

$$E\Big[u(c_s^B, h_s^B) + \sum_{j=1}^{J-s}\big[u(c_{s+j}^B, h_{s+j}^B)\big]\Big] = E\Big[u\big(c_s^C(1+\frac{CEV}{100}), h_s^C\big) + \sum_{j=1}^{J-s}\big[u\big(c_{s+j}^C(1+\frac{CEV}{100}), h_{s+j}^C\big)\big]\Big]$$

To compare the effects of housing privatization with those of social security on cohorts born after 1994, in the following exercise, we construct a counterfactual economy, in which we adjust the age-specific replacement rate to match the average utility level of each given age group among initial cohorts in the benchmark economy. Specifically, for those initial cohorts, we choose three possible values for pension replacement rate so that the average welfare among young-age, middle-age and old-age individuals in 1994 match their benchmark counterparts. In the meantime, we waive the payroll tax for those initial cohorts. For all those cohorts born after period 1, The replacement rate is kept at 0.6. The purpose is to have a social security reform that makes those initial cohorts be well off those in the benchmark economy with housing reform. We then evaluate how such social security reform affects cohorts born since period 1, in comparison with the benchmark economy.

$$\theta_{t,j} = \begin{cases} \theta^y & \text{if } 1 \le 1994-(t-j) \le 30 \\ \theta^m & \text{if } 31 \le 1994-(t-j) \le 50 \\ \theta^o & \text{if } 1994-(t-j) > 51 \end{cases}$$

and

$$\tau_{t,j}^{ss} = \begin{cases} 0, \text{ if } & t-j+1 < 1994 \\ \tau^{ss}, \text{ if } & t-j+1 >= 1994 \end{cases}$$

Finally, $\tau^{ss}$ is set to balance the government's intertemporal budget constraint:

$$\sum_{t=1}\frac{1}{(1+r)^t}\left[\sum_{j=1}^{J}\sum_{i}\int d\rho_t h_g^i(1-\mathbb{I}_{\text{buy}_{t,j}^g(y,z)})d\mu_{t-1,j-1}^{i,rg}(y,z)+\sum_{j=J_w+1}^{J}\sum_{i}b_{t,j}^i\mu_{t,j}^i\right]$$
$$=B_0+\sum_{t=1}\frac{1}{(1+r)^t}\sum_{j=1}^{J_W}\sum_{i}\tau^{ss}w_t^i\mu_{t,j}^i,$$

Figure 10 shows that in our counterfactual economy, the payroll tax paid by the cohorts entering the labor force after 1994 is uniformly higher than their counterparts in the benchmark economy. In particular, cohorts entering the labor force after 2020 have to bear a heavy tax burden, despite the slowdown of their wage income growth. Figure 11 shows that house prices in our benchmark economy are significantly higher than in the economy with social security reforms until 2015, after which these two economies share similar house price growth rates.

In Figure 12 we compare the home ownership rates for different skill groups and housing types with their benchmark counterparts. The patterns of home ownership rates in the economy with social security reforms resemble those in the counterfactual economy without housing purchase subsidy. Specifically, without housing purchase subsidy, households start to become homeowners much later. Accordingly, the peak of the ownership rate of reformed houses is also later, as shown by panel (c). The results suggest that fewer households trade-up their reformed houses in the economy with social security reform. Accordingly, the ownership rate of commercial house is also lower than the benchmark counterpart (Panel d).

In Figure 13 we plot the welfare effects of social security reforms for those cohorts born since period 1, mapped into those entering the labor force since 1994 in reality. A positive number indicates newborn is better off in the economy with social security reform. For both skilled and unskilled, cohorts entering the labor force between 1994-2010 is strictly better off in the economy with social security reform than the benchmark economy. However, the opposite is true for cohorts entering the labor force after 2010, which correspond to those born since the 1990s. Intuitively, for those

entering the labor forces during 1994-2010 (i.e., born in the late 70s and 1980s), they suffer from high house prices growth in the benchmark economy, despite lower payroll taxes. However, for those born in the 1990s and after (i.e., entering the labor force in 2010 and after), their payroll tax burden is significantly higher in the economy with social security reform for intergenerational transfer, and house price growth slows down when they become first-time home buyers. Hence, they are strictly better off in the economy with housing purchase subsidy as intergenerational transfer.

# 7 Conclusion

Many emerging economies experienced fast growing but eventually declining wage income growth, which makes intergenerational transfer from future generations to the initial generations challenging via conventional redistributive policies such as social security. This paper uses China's housing reforms in the 1990s as a policy experiment to show that in such economies housing purchase subsidy to initial cohorts when housing privatization starts can serve as an alternative intergenerational redistributive scheme. By developing a quantitative general equilibrium model and calibrating it to Chinese economy, we find that housing purchase subsidy to these initial cohorts facilitates them, especially the skilled workers, to become homeowners, who later on reap the capital gain from selling their houses to future generations. Our welfare analysis suggests that for a majority of future generations, housing purchase subsidy is more desirable than an alternative social security reform that redistribute from future generations to the initial ones, as the latter involves high social security tax burdens for those generations experiencing slowing down in wage growth. This is despite the fact that cohorts born in the 1980s would prefer social security as intergenerational transfer to avoid higher housing prices caused by housing purchase subsidy to initial cohorts. Our findings provides a concrete step towards future research on how a combination of housing purchase subsidy to earlier generations with social security reforms can jointly redistribute from future towards current generations for economies that experience fast

growing wage incomes during their economic transition.

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Table 1: Summary of Model Parameters

| Description | Parameter | Value | Sources |
|---|---|---|---|
| Depreciation rate | $\delta_h$ | 0.02 | OECD estimates |
| Reformed H rental discount | $d$ | 0.2 | Wang (2012) |
| Replacement rate | $\theta$ | 0.6 | Song et. al (2015) |
| Discount Factor | $\beta$ | 0.90 | standard |
| Minimum Down Payment Ratio | $\gamma$ | 0.3 | government policy |
| Interest Rate for Savings | $r$ | 0.02 | Government policy |
| Mortgage Interest Rate | $r_m$ | 0.06 | Government policy |
| Land share | $\alpha$ | 0.7 | Wang, Chan and Xu (2012) |
| TFP growth rate | $g_w$ | 0.0713 | urban wage growth 1994-2050 |
| Conditional survival prob | $\{\psi\}_j$ | | Imrohoroglu and Zhao (2018) |
| Life-cycle income profile | $\{\varepsilon\}_j$ | | He et.al. (2018) |
| Skill premium | $\lambda^i$ | {1,1.79} | Population census |
| Initial gov assets | $B_0$ | 5.3 | 10-percent income tax rate |
| Minimum reformed house Size | $h_q^1$ | 0.8 | ownership of gov. house in 2002[a] |
| Minimum commercial house Size | $h^1$ | 2.4 | ownership of com. house in 2009[a] |
| Minimum rental apartment size | $h_a^1$ | 1.6 | average house size of owners to renters[b] |
| Initial TFP level | $A_1$ | 1.3 | price-to-income ratio in 2003[c] |
| Initial land supply | $\ell_0$ | 98.2 | normalized initial housing price |
| Weight on $c$ | $\phi$ | 0.35 | expenditure share on rents[b] |
| Bequest motive | $\iota$ | 10.0 | median NW at age 75 to median NW at age 50[b] |
| Floor of housing | $\underline{h}$ | 0.08 | average earnings of owners to renters[b] |
| Owning premium | $\overline{\zeta}$ | 2.3 | ownership rate in 2004[a] |
| Commercial house Size | $\{h^i\}$ | | ratio of net housing wealth to net worth[b] |
| Land supply | $\ell_t$ | | annual housing price growth rate[c] |

[a] Urban Household Survey; [b] China Household Finance Survey; [c] Fang et al. (2016).

Table 2: Targeted Moments in the Calibration

| Description | Data | Model |
|---|---|---|
| Price-to-rent ratio | 15 | 15 |
| Price-to-income ratio in 2006 | 8.4 | 8.4 |
| Expenditure share on rents | 0.2 | 0.2 |
| Median NW at age 75 to median NW at age 50* | 0.81 | 0.80 |
| Ownership rate in 2004 | 0.85 | 0.85 |
| Average earnings of owners to renters* | 2.1 | 2.1 |
| Average house size of owners to renters* | 1.5 | 1.5 |
| Ratio of net housing wealth to net worth: 10 th percentile* | 0.61 | 0.63 |
| Ratio of net housing wealth to net worth: 50 th percentile* | 0.93 | 0.93 |
| Ratio of net housing wealth to net worth: 90 th percentile* | 1 | 1 |

*Final equilibrium.

Table 3: Model Fit: Distribution of the Overall Ownership rate

| | 2002 | | 2009 | |
|---|---|---|---|---|
| | Data | Model | Data | Model |
| 21-30 | 0.736 | 0.107 | 0.819 | 0.246 |
| 31-40 | 0.827 | 0.759 | 0.906 | 0.737 |
| 41-50 | 0.813 | 0.903 | 0.897 | 0.980 |
| 51-60 | 0.824 | 0.969 | 0.907 | 0.995 |
| 61-70 | 0.862 | 0.994 | 0.926 | 0.995 |
| | By edu group | | | |
| unskilled | 0.750 | 0.769 | 0.861 | 0.782 |
| skilled | 0.839 | 0.799 | 0.910 | 0.855 |
| Overall | 0.820 | 0.758 | 0.903 | 0.792 |

Table 4: Model Fit: Distribution of Government House Ownership rate

| | 2002 | | 2009 | |
|---|---|---|---|---|
| | Data | Model | Data | Model |
| 21-30 | 0.531 | 0.437 | 0.192 | 0.203 |
| 31-40 | 0.684 | 0.733 | 0.325 | 0.345 |
| 41-50 | 0.730 | 0.799 | 0.488 | 0.488 |
| 51-60 | 0.745 | 0.801 | 0.586 | 0.566 |
| 61-70 | 0.783 | 0.890 | 0.641 | 0.710 |
| | By edu group | | | |
| unskilled | 0.672 | 0.744 | 0.523 | 0.554 |
| skilled | 0.729 | 0.705 | 0.496 | 0.397 |
| Overall | 0.7820 | 0.717 | 0.491 | 0.441 |

Table 5: Ownership rate of Reformed and Commercial Houses among Initial Cohorts

| Year | All | 1965-74 | 1955-64 | 1945-54 | 1935-44 | 1925-34 | 1915-24 | 1905-14 |
|---|---|---|---|---|---|---|---|---|
| | Reformed House Ownership rate | | | | | | | |
| 1994-1998 | 0.545 | 0.167 | 0.408 | 0.603 | 0.739 | 0.791 | 0.710 | 0.400 |
| 1999-2003 | 0.461 | 0.363 | 0.387 | 0.317 | 0.266 | 0.439 | 0.672 | 0.783 |
| 2004-2008 | 0.309 | 0.192 | 0.143 | 0.085 | 0.195 | 0.529 | 0.711 | - |
| 2009-2013 | 0.195 | 0.098 | 0.073 | 0.039 | 0.068 | 0.367 | 0.526 | - |
| 2014-2018 | 0.121 | 0.031 | 0.017 | 0.019 | 0.176 | 0.362 | - | - |
| 2019-2023 | 0.073 | 0.014 | 0.008 | 0.004 | 0.059 | 0.281 | - | - |
| | Commercial House Ownership rate | | | | | | | |
| 1994-1998 | 0.028 | 0.004 | 0.018 | 0.042 | 0.063 | 0.046 | 0.020 | 0.001 |
| 1999-2003 | 0.318 | 0.152 | 0.270 | 0.463 | 0.603 | 0.488 | 0.242 | 0.010 |
| 2004-2008 | 0.484 | 0.407 | 0.607 | 0.745 | 0.696 | 0.386 | 0.059 | - |
| 2009-2013 | 0.556 | 0.489 | 0.628 | 0.779 | 0.796 | 0.494 | 0.153 | - |
| 2014-2018 | 0.637 | 0.653 | 0.801 | 0.854 | 0.672 | 0.207 | - | - |
| 2019-2023 | 0.671 | 0.694 | 0.797 | 0.881 | 0.763 | 0.219 | - | - |

**Notes:** This is the simulated share of households who own reformed or commerical houses in each cohort during the specific time period.

Table 6: Distribution of LTV upon purchase among upgraders

| Year | $(0, 10\%]$ | $(10\%, 30\%]$ | $(30\% - 50\%]$ | $(50\%, 70\%]$ |
|---|---|---|---|---|
| 1994-1998 | - | - | - | - |
| 1999-2003 | 0.016 | 0.041 | 0.183 | 0.761 |
| 2004-2008 | 0.036 | 0.049 | 0.211 | 0.703 |
| 2009-2013 | 0.022 | 0.034 | 0.301 | 0.642 |
| 2014-2018 | 0.006 | 0.030 | 0.447 | 0.517 |
| 2019-2023 | 0.002 | 0.224 | 0.290 | 0.484 |

**Notes:** This is the distribution of LTV among those households who upgrade to commercial houses immediately after selling reformed houses. Each row should sum up to 1.

Table 7: Ratio of Capital Gain to Average Labor Earning

| Year | All | 1965-74 | 1955-64 | 1945-54 | 1935-44 | 1925-34 | 1915-24 | 1905-14 |
|---|---|---|---|---|---|---|---|---|
| | Unskilled | | | | | | | |
| 1999-2003 | 1.097 | 1.484 | 1.221 | 1.009 | 0.895 | 1.130 | 1.022 | 0.916 |
| 2004-2008 | 1.163 | 1.274 | 1.029 | 0.895 | 1.400 | 1.266 | 1.114 | - |
| 2009-2013 | 1.246 | 1.215 | 1.036 | 0.888 | 1.531 | 1.503 | 1.302 | - |
| 2014-2018 | 1.333 | 1.097 | 0.930 | 1.534 | 1.665 | 1.440 | - | - |
| 2019-2023 | 1.181 | 0.939 | 0.838 | 1.298 | 1.479 | 1.352 | - | - |
| | Skilled | | | | | | | |
| 1999-2003 | 0.983 | 1.186 | 0.975 | 0.806 | 0.715 | 1.146 | 1.066 | 0.983 |
| 2004-2008 | 1.119 | 1.018 | 0.822 | 0.715 | 1.500 | 1.395 | 1.263 | - |
| 2009-2013 | 1.248 | 0.971 | 0.827 | 0.709 | 1.687 | 1.744 | 1.550 | - |
| 2014-2018 | 1.427 | 0.876 | 0.743 | 1.694 | 2.028 | 1.793 | - | - |
| 2019-2023 | 1.279 | 0.750 | 0.670 | 1.418 | 1.824 | 1.733 | - | - |

**Notes:** This is the ratio of capital gain from selling the reformed houses of unskilled to the average unskilled labor earning in the current period of each specific birth cohort.

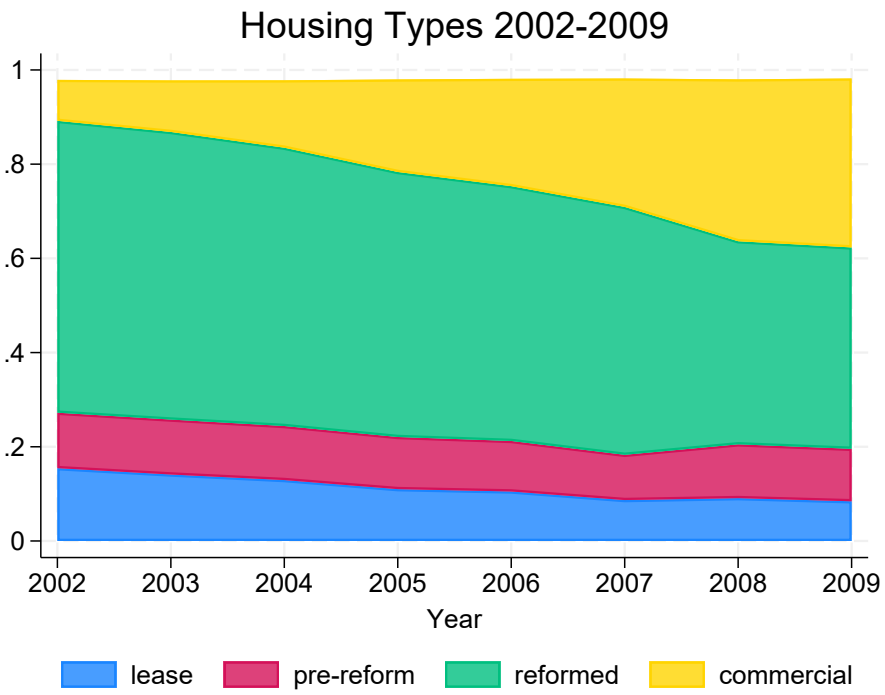



Figure 1: Housing Types: 2002-2009

Notes: The data source is China Urban Household Survey in Various Years.

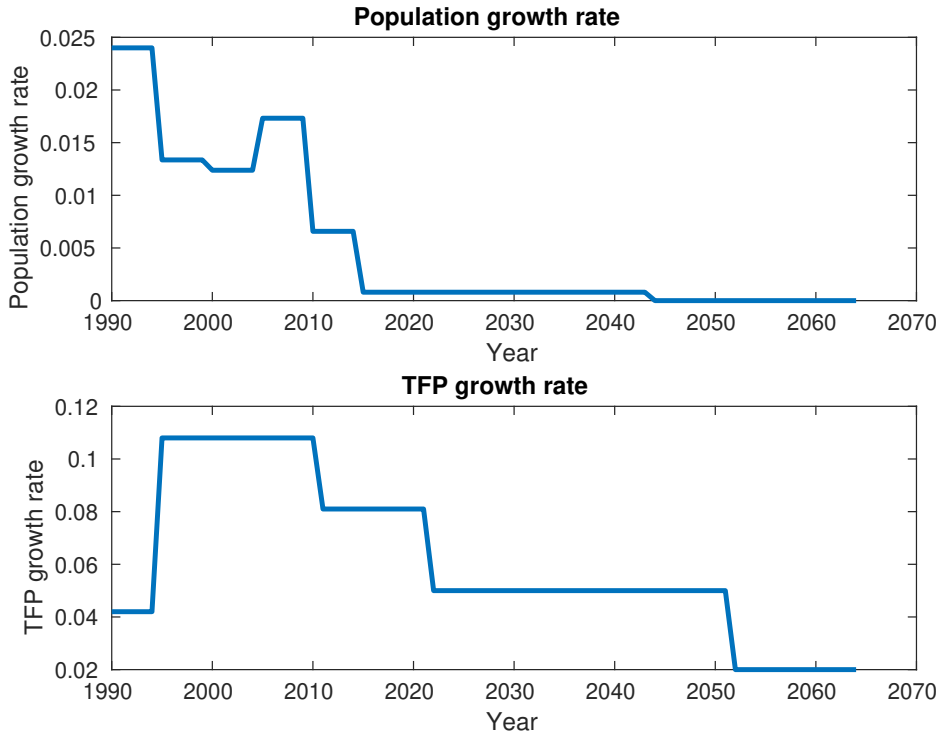



Figure 2: Growth Rates of Population and TFP during Economic Transition

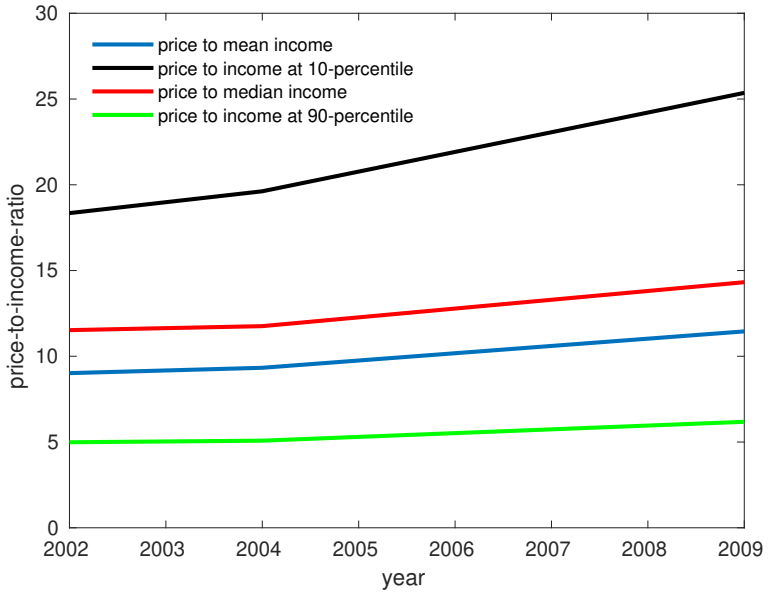
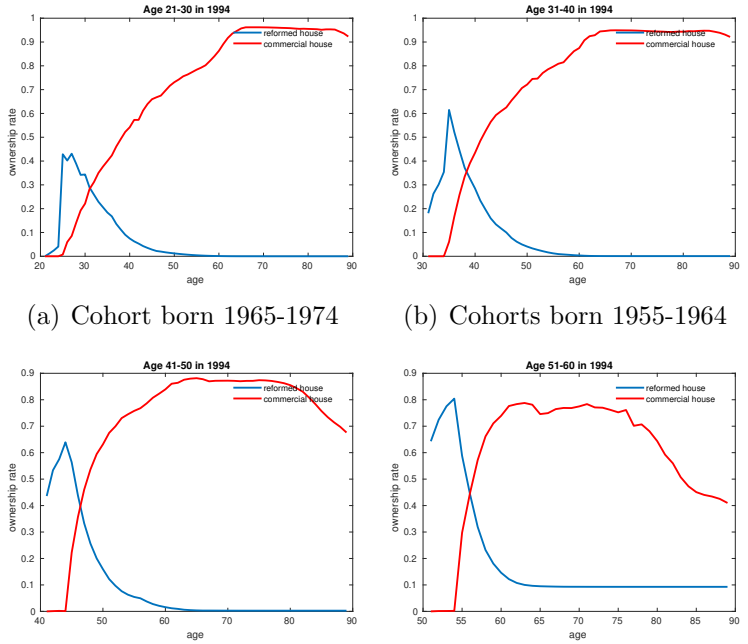


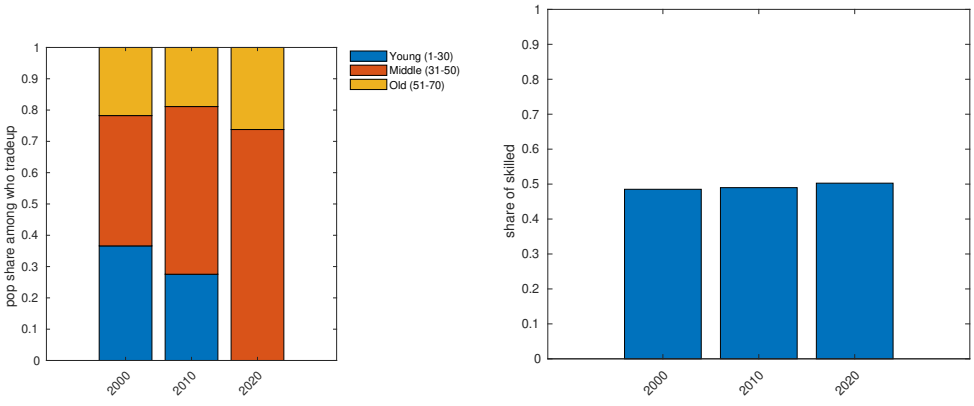

Figure 3: Simulated House Price to Income Ratios for Households of Different Income Levels in the Benchmark Economy




(a) Cohort born 1965-1974 (b) Cohorts born 1955-1964

(c) Cohort born 1945-1954 (d) Cohort born 1935-1944

Figure 4: Ownership Rate of Reformed vs. Commercial Houses by Four Initial Cohorts.




(a) age group (b) education group

Figure 5: Age and Education Distribution of Upgraders

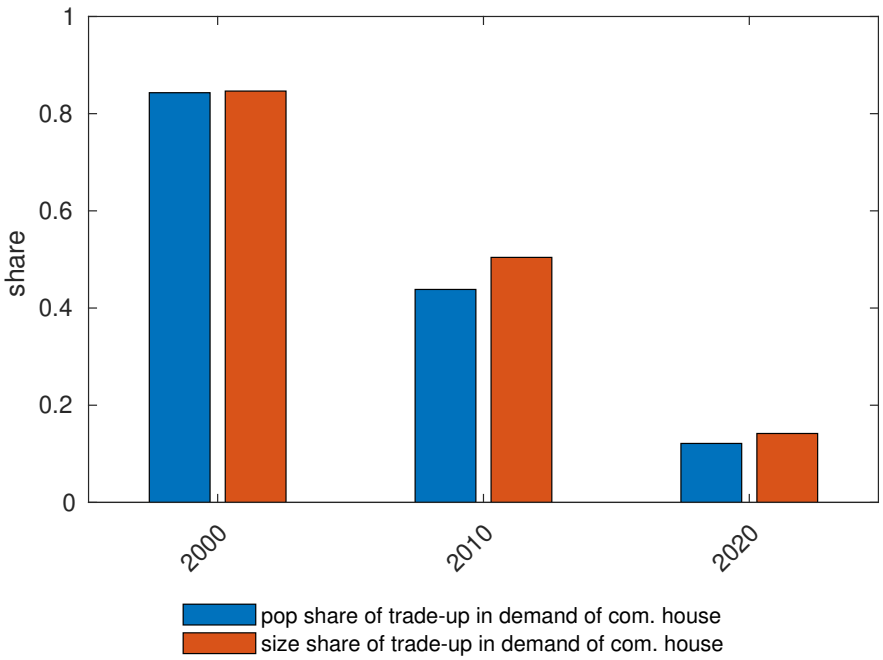



Figure 6: The Contribution of Upgraders to the Demand for Commercial Houses

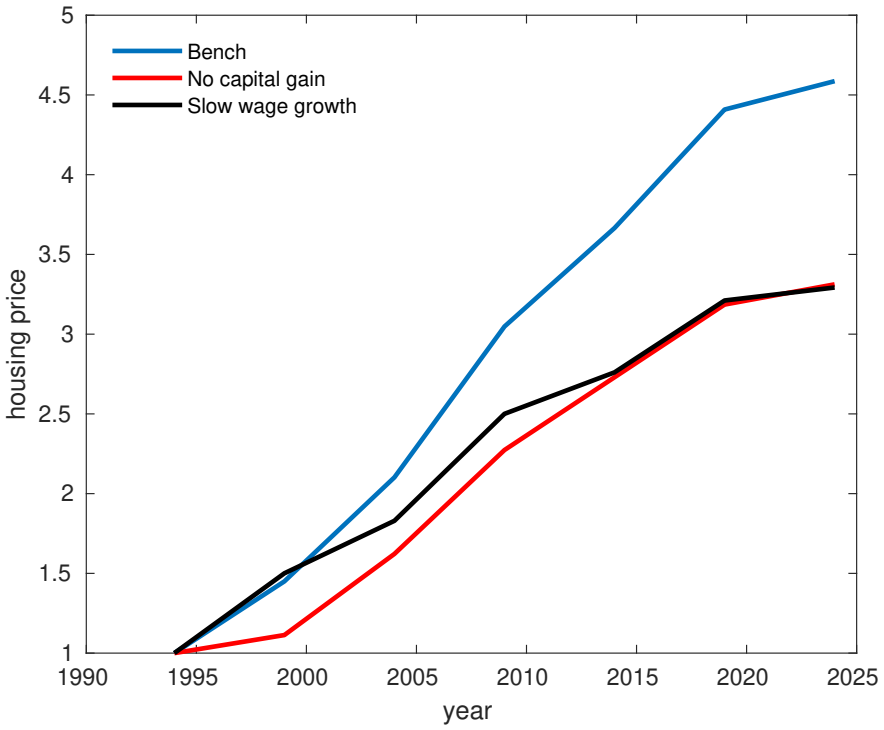



Figure 7: The Contribution of Capital Gain vs. Wage Growth to Housing Boom

Notes: "Bench" denotes the benchmark economy. "No capital gain" denotes the counterfactual economy with 100% capital gain taxes on housing sales. "Slow wage growth" denotes the counterfactual economy in which the TFP (wage) growth rate each period is half of its counterpart in the benchmark economy.

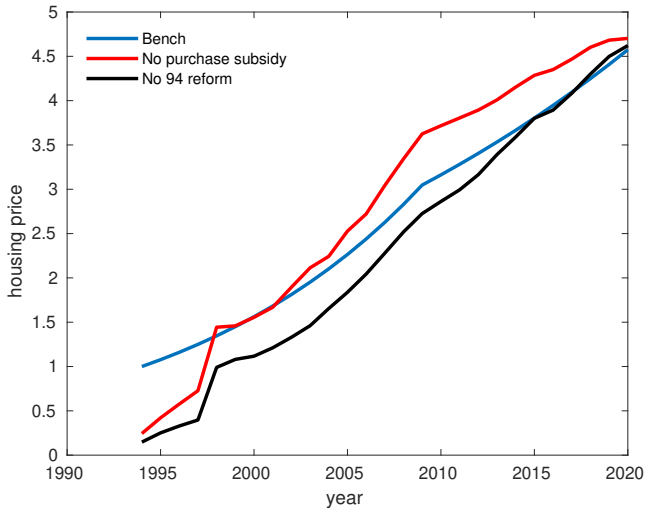
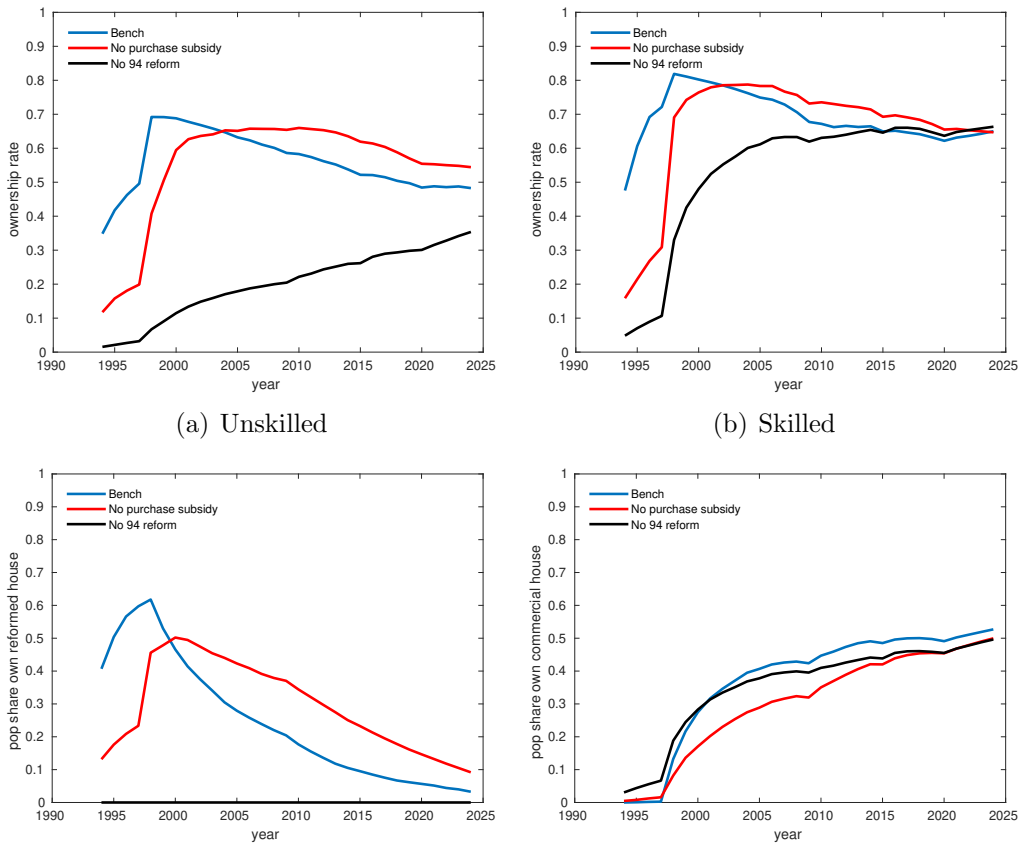



Figure 8: The Dynamics of House Prices under Different Scenarios

Notes: "Bench" denotes the benchmark economy. "No purchase subsidy" denotes the counterfactual economy without housing purchase subsidy. "No 94 reform" denotes the counterfactual economy without access to government houses.




(a) Unskilled (b) Skilled

(c) Reformed House (d) Commercial House

Figure 9: Home Ownership Rate under Different Scenarios

Notes: "Bench" denotes the benchmark economy. "No purchase subsidy" denotes the counterfactual economy without housing purchase subsidy. "No 94 reform" denotes the counterfactual economy without access to government houses. Panel (a) and (b) plot the home ownership rates for households of skilled and unskilled types. Panel (c) and (d) plot the home ownership rates for reformed and commercial houses.

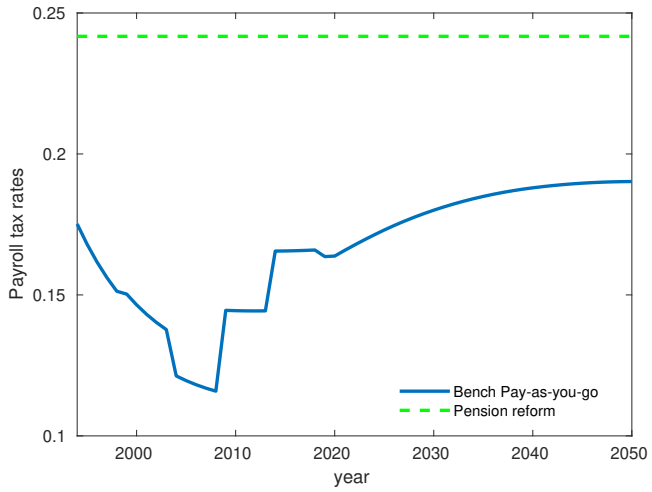



Figure 10: Payroll Tax Rates in the Benchmark Economy vs. the Counterfactual Economy with Alternative Social Security System.

Notes: The horizontal axis denotes the calendar year, the vertical axis denotes the social security tax rate. "Bench" denotes the benchmark economy. "Pension reform" denotes the counterfactual economy without housing purchase subsidy, but with age-specific replacement rates for initial cohorts.

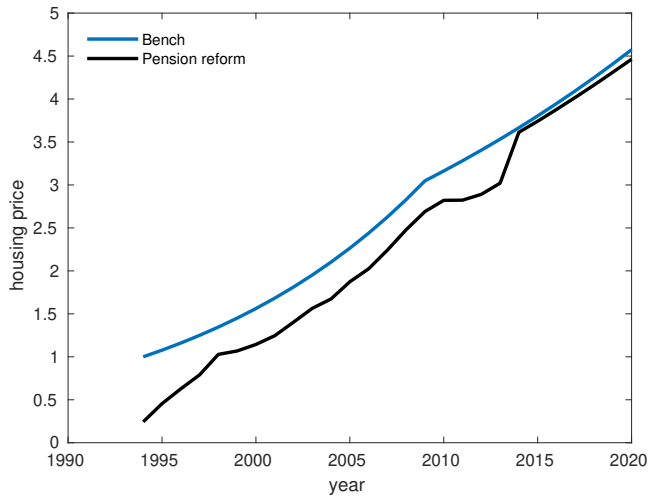
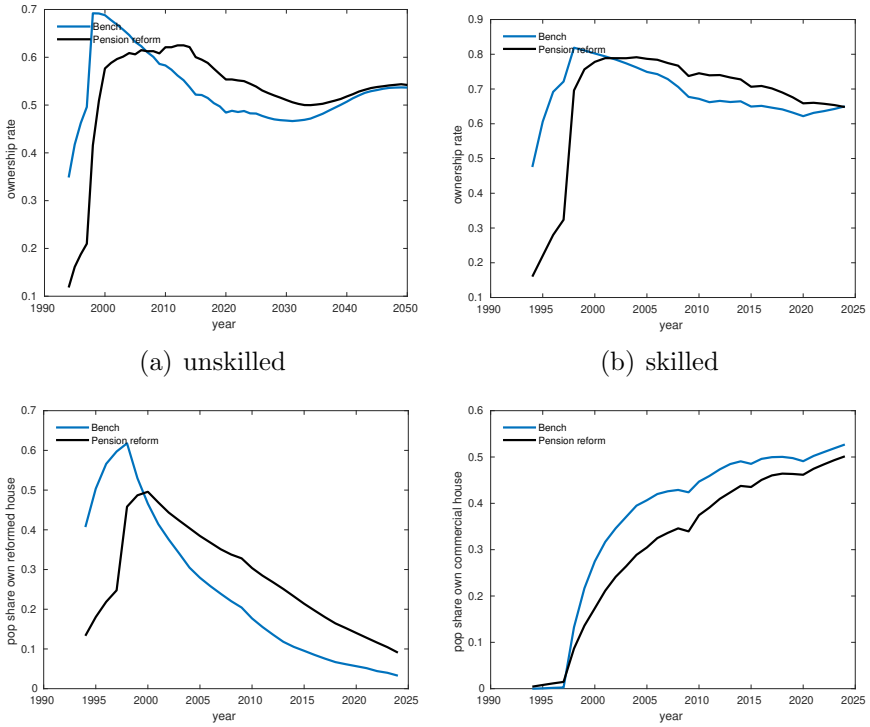



Figure 11: House Prices in the Benchmark Economy vs. the Counterfactual Economy with Alternative Social Security System

Notes: "Bench" denotes the benchmark economy. "Pension reform" denotes the counterfactual economy without housing purchase subsidy, but with age-specific replacement rates for initial cohorts.

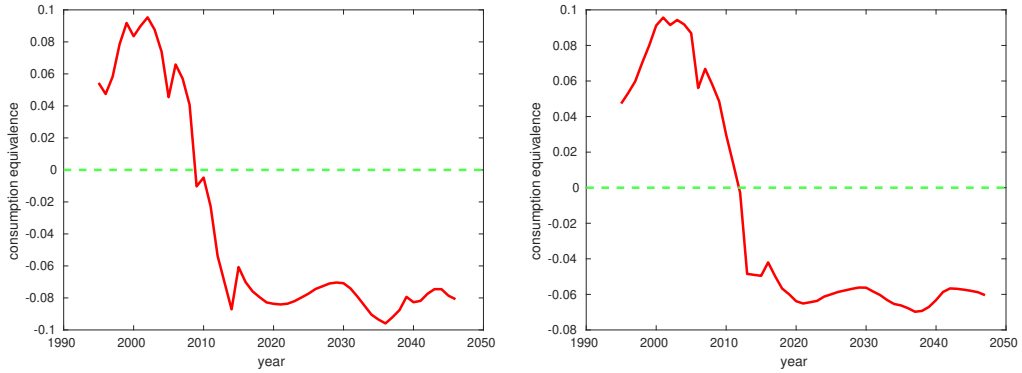



(a) unskilled (b) skilled

(c) reformed house (d) commercial house

Figure 12: Home Ownership Rate: Benchmark Economy v.s. Social Security Reform

Notes: In each plot, "Bench" denotes the benchmark economy. "Pension reform" denotes the counterfactual economy without housing purchase subsidy, but with age-specific replacement rates for initial cohorts. Panel (a) and (b) plot the home ownership rates for households of skilled and unskilled types. Panel (c) and (d) plot the home ownership rates for reformed and commercial houses.




(a) unskilled (b) skilled

Figure 13: Welfare Effects of Alternative Social Security System

Notes: The horizontal axis denotes the years, individual cohorts entered the labor market. The vertical axis denotes the consumption equivalent variation for each individual cohort in the counterfactual economy with alternative social security system relative to their counterparts in the benchmark economy.

# Online Appendix (Not For Publication)

# A Appendix

## A.1 Households' Optimization Problem

**Pre-reformed Steady State** For a household born in the pre-reformed state, their problem is

$$\begin{aligned} V_j^{i,rg}(y) &= \max u\left(c_j^i, h_g^i\right) + \beta\psi_{j+1} E\left[V_{j+1}^{i,rg}(y')\right], \\ \text{s.t.} \quad & c_j^i + b_{j+1}^i + \rho_0 h_g^i = y + b_j^i(1+r), \end{aligned}$$

where $\rho_0$ denotes the rental price set by the government.

**Economic Transition** Households can be divided into renters, buyers, and owners of either reformed or commercial houses. We now characterize the optimization problem in recursive form for households of different groups. The state variables for a renter typically contain age $j$, skill type $i$, persistent shock $z$, and cash-in-hand, which we denoted as $y$. In addition to these, for an owner the state variables also include size of houses they own ($h$) and their outstanding mortgage debt ($m$).

Specifically, for a household renting reformed house (rg) at the beginning of the period, the value function can be expressed as

$$\begin{aligned} V_{t,j}^{i,rg}(y,z) = \max_{c_{t,j}, b_{t+1,j+1}} & u(c_{t,j}, h_g^i) + \beta\psi_{j+1}\mathbb{E}\Big[\max\big\{V_{t+1,j+1}^{i,rg}(y',z'), \\ & V_{t+1,j+1}^{i,bg}(y',z'), V_{t+1,j+1}^{i,rc}(y',z'), V_{t+1,j+1}^{i,bc}(y',z')\big\}\Big] \\ \text{s.t.} \quad & c_{t,j} + b_{t+1,j+1} + \omega * \rho_t h_g^i = y \\ & y' = (1 - \tau_t^{ss} - \tau_t)\lambda^i w_{t+1}\varepsilon_{j+1} z' \epsilon_{t+1,j+1} + b_{t+1,j+1}(1+r) \end{aligned}$$

Renters of the reformed houses must be those households born before 1998, and

thus, their rental expenditures are subsidized by the government. $\rho_t$ denotes the unit rental price. $\omega \in (0,1)$ denotes the discount the households receive from renting the reformed house. The differences between the market price and the subsidized price are paid by the government, whose budget constraint will be discussed in xx. Other than rental payments, households make standard consumption and saving decisions given after-tax income. At the beginning of the next period, they choose whether to continue renting the current reformed house ($rg$). If they do not want to rent anymore, they can choose among the renter ($rc$), the buyer ($bc$) of the commercial house, and the buyer of the reformed house ($bg$).

The other group of renters will be those renting commercial housing (rc). Their value function can be expressed as

$$
\begin{aligned}
V_{t,j}^{i,rc}(y,z) = \max_{c_{t,j}, b_{t+1,j+1}, h_a \in \mathcal{H}_a} & u(c_{t,j}, h_a) + \beta \psi_{j+1} \mathbb{E}\Big[ \max\big\{ V_{t+1,j+1}^{i,rc}(y',z'), \\
& V_{t+1,j+1}^{i,bc}(y',z') \big\} \Big] \\
\text{s.t.} \quad & c_{t,j} + b_{t+1,j+1} + \rho_t h_a = y \\
& y' = (1 - \tau^{ss} - \tau)\lambda^i w_{t+1} \varepsilon_{j+1} z' \epsilon_{t+1,j+1} + b_{t+1,j+1}(1+r)
\end{aligned}
$$

Different from renters of reformed houses, renters of commercial houses can choose housing sizes among available types ($\mathcal{H}_a$). However, they can no longer choose to rent a reformed house even though they were born before 1998. So at the beginning of the next period, they choose between continuing to be a commercial house renter ($rc$) and buying a commercial house ($bc$).

Now, we switch to characterize households who are buyers of either reformed or commercial houses at the beginning of the period. For a household that chooses to buy

a reformed house, his optimization problem is

$$V_{t,j}^{i,bg}(y,z) = \max_{c_{t,j},b_{t+1,j+1},d_{t+1}} u(c_{t,j},\zeta h_g^i) + \beta\psi_{j+1}\mathbb{E}\Big[\max\big\{V_{t+1,j+1}^{i,og}(y'_{\text{own}},z,\zeta h_g^i,d_{t+1}),$$
$$V_{t+1,j+1}^{i,rc}(y'_{\text{sell}},z'),V_{t+1,j+1}^{i,bc}(y'_{\text{sell}},z')\big\}\Big]$$

$$\text{s.t.}\quad c_{t,j} + b_{t+1,j+1} + p_{gt,j}h_g^i = y + d_{t+1}$$
$$y'_{\text{own}} = (1-\tau^{ss}-\tau)\lambda^i w_{t+1}\varepsilon_{j+1}z'\epsilon_{t+1,j+1} + b_{t+1,j+1}(1+r)$$
$$y'_{\text{sell}} = (1-\tau^{ss}-\tau)\lambda^i w_{t+1}\varepsilon_{j+1}z'\epsilon_{t+1,j+1} + b_{t+1,j+1}(1+r) + p_{t+1}h_g^i - d_{t+1}(1+r_m)$$
$$d_{t+1} \le (1-\gamma)p_{gt,j}h_g^i$$

First, if the household is a buyer of the reformed house, then he was born before 1994 and thus subjected to a purchase subsidy. The difference between market price ($p_t$) and the price he paid ($p_{gt}$) is specified in equation xx, and paid by the government, whose budget constraint will be discussed in xx. Households can take mortgages and borrow up to a fraction $(1-\gamma)$ of the purchase price. At the beginning of the next period, a household may choose to become an owner of the house ($og$) he just bought or sell the house and become either a renter ($rc$) or a buyer ($bc$) of a commercial house.

Note that when a household ever sells the reformed house, he sells it at the market price. This gives rise to a potential capital gain from the difference between the low-purchased and high-sales prices.

The value function for a buyer of a commercial house can be similarly represented

as

$$V^{i,bc}_{t,j}(y,z) = \max_{c_{t,j},b_{t+1,j+1},d_{t+1},h_c\in\mathcal{H}_c} u(c_{t,j},\zeta h_c) + \beta\psi_{j+1}\mathbb{E}\Big[\max\big\{V^{i,oc}_{t+1,j+1}(y'_{\text{own}},z',\zeta h_c,d_{t+1}),$$
$$V^{i,rc}_{t+1,j+1}(y'_{\text{sell}},z'),V^{i,bc}_{t+1,j+1}(y'_{\text{sell}},z')\big\}\Big]$$

$$\text{s.t.}\quad c_{t,j} + b_{t+1,j+1} + p_t h_c = y + d_{t+1}$$
$$y'_{\text{own}} = (1-\tau^{ss}-\tau)\lambda^i w_{t+1}\varepsilon_{j+1}z'\epsilon_{t+1,j+1} + b_{t+1,j+1}(1+r)$$
$$y'_{\text{sell}} = (1-\tau^{ss}-\tau)\lambda^i w_{t+1}\varepsilon_{j+1}z'\epsilon_{t+1,j+1} + b_{t+1,j+1}(1+r) + p_{t+1}h_c - d_{t+1}(1+r_m)$$
$$d_{t+1} \le (1-\gamma)p_t h_c$$

The problem for buyers of commercial houses only differs from that for buyers of reformed houses in two aspects. First, buyers of commercial houses no longer receive purchase subsidies offered by the government, so they buy at the market price. Second, buyers get to choose among available sizes of houses ($\mathcal{H}_c$).

Lastly, we characterize the optimization problem for owners. For an owner of a reformed house, his value function is

$$V^{i,og}_{t,j}(y,z,h^i_g,d) = \max_{c_{t,j},b_{t+1,j+1},m_{t,j}} u(c_{t,j},\zeta h^i_g) + \beta\psi_{j+1}\mathbb{E}\Big[\max\big\{V^{i,og}_{t+1,j+1}(y_{\text{own}},z',\zeta h^i_g,d'),$$
$$V^{i,rc}_{t+1,j+1}(y_{\text{sell}},z'),V^{i,bc}_{t+1,j+1}(y_{\text{sell}},z')\big\}\Big]$$

$$\text{s.t.}\quad c_{t,j} + b_{t+1,j+1} + \delta_h p_t h^i_g + m_{t,j} = y$$
$$m_{t,j} \ge \frac{r_m(1+r_m)^{J+1-j}}{(1+r)^{J+1-j}-1}d$$
$$d' = d(1+r_m) - m_{t,j}$$
$$y'_{\text{own}} = (1-\tau^{ss}-\tau)\lambda^i w_{t+1}\varepsilon_{j+1}z'\epsilon_{t+1,j+1} + b_{t+1,j+1}(1+r)$$
$$y'_{\text{sell}} = (1-\tau^{ss}-\tau)\lambda^i w_{t+1}\varepsilon_{j+1}z'\epsilon_{t+1,j+1} + b_{t+1,j+1}(1+r) + p_{t+1}h^i_g - d'(1+r_m)$$

State variable $d$ denotes the owner's outstanding mortgage debt. Each period, the

owner needs to decide the debt repayment level $m$, subject to a minimum repayment requirement. In addition, some housing maintenance costs are incurred in each period to make up for the housing depreciation. At the beginning of the next period, the owner can choose to continue owning the current reformed house or sell the reformed house to become either a renter or buyer of a commercial house. We skip the value function for an owner of a commercial house since it is almost identical to that of a reformed house.

## A.2 Equilibrium

Given policies $\{r, r_m, \theta, \gamma, \omega\}$, the discount price function (9) and the sequence of exogenous variables $\{A_t, L_t, N_t\}_{t=1}^{\infty}$, a dynamic equilibriumis quantities $\{N_{ct}, N_t, S_t\}$, prices $\{p_t, \rho_t, w_t\}$, taxation policies $\{\tau_t, \tau_t^{ss}\}$, household value functions $\{V_t^{rural}, V_{t,j}^{i,rg}, V_{t,j}^{i,rc}$, $V_{t,j}^{i,bg}, V_{t,j}^{i,bc}, V_{t,j}^{i,og}, V_{t,j}^{i,oc}\}$ and associated policy functions, and end-of-period distributions $\{\mu_{t,j}^{i,rg}, \mu_{t,j}^{i,rc}, \mu_{t,j}^{i,og}, \mu_{t,j}^{i,oc}\}$ that satisfy the following conditions.

1. Given the prices and government policies, the value functions solve the recursive problem of the households and the associated policy functions.

2. Construction sector firms maximize profit with associated labor demand and housing investment function $(N_c, I_h)$

3. The labor market clears at wage rate $w = A$.

4. The rental market clears at price $\rho$.

5. Housing market clears at price $p$.

$$\sum_{j=1}^{J} \sum_{i} \left[ \int h_c d\mu_{t,j}^{i,oc} + \int h_g^i d\mu_{t,j}^{i,og} \right] = H_{t-1} + Y_{ht}.$$

6. The payroll tax $\{\tau_t^{ss}\}_{t=1}^{\infty}$ is such that the social security system is balanced each period.

7. Government maintains a balanced intertemporal budget constraint by imposing labor income tax $\{\tau_t\}_{t=1}^{\infty}$.

8. The end-of-period distributions are induced by the exogenous stochastic processes and all the decision rules, and they are consistent with individual behavior.

## A.3 Algorithm

We have two algorithms here. One algorithm is to solve the pre-reform steady state. The other is to solve jointly the steady state after 2013 and the transition path from some initial state of the economy to the steady state after 2013. For example, in the benchmark, we need to use transition path algorithm twice. Firstly, we solve the transition path from the pre-reform to the steady state after 2013. Secondly, we use the economy in year 1998 as the initial state and solve for the transition path again.

**Pre-reform steady state:**

To solve for the pre-reform steady state, we assume households taking the assigned housing service as given, and government plans for the life cycle consumption profile across permanent efficiency shock groups.

Given government policy, $\tau, b$, and interest rate $r$ and wage rate $w$ in pre-reform steady state.

1. Parameterize the model, and calculate the density of retired workers in the population, $\mu_t$, $t = J_{w+1}, ..., J$.

2. Given the government expenditure, $\bar{g}$, guess the pre-reform rental rate $R_c$;

3. Given the policy function (analytical) of households,

   (a) guess the initial bequest.

   (b) simulate the optimal path for consumption and saving for the new born generation by forward induction given the initial bequest.

(c) aggregate household's decision, and calculate the bequest leftover on the path.

(d) update the guess of the initial bequest until it converges. (Guass-Seidel method)

4. Aggregate government's tax revenue, renting revenue, and pension expenditure. Check whether government's intertemporal budget is balanced, and update the guess of the rental rate $R_c$.

5. Check whether $R_c$ match the calibration target, if not, update $\bar{g}$. Derive the allocation of $\overline{H}$.

**Transition path:**

We need to find the equilibrium path of housing price and the tax rate in the final steady state. Assume we know the state of the economy at $t = 1$, and the economy reaches the final steady state after some periods $T$. ($T$ is larger than the three times of the maximum lifespan.)

To solve the transition path, we have the following steps:

Given government policy, $\{\tau_t, b_t\}_{t=1}^{\infty}$, discount housing policy, land supply, $\{H_t\}_{t=1}^{\infty}$, and interest rate $\{r_t\}_{t=1}^{\infty}$ and wage rate $\{w_t\}_{t=1}^{\infty}$ on the path, and the initial distribution of household on the state space (initial state).

1. Choose the number of transition periods $T$.

2. De-trend the economy by the time $T$ variables.

3. Provide an initial guess for tax rate in the steady state, $\tau$.

4. Given all policy variables, solve for the final steady state housing price that clear the housing market by bisection method.

5. Provide an initial guess for housing price on the path, $\{p_t\}_{t=0}^{T}$, and solve household's problem backwards:

At period $t$, compute the value functions and policy functions for the new born at $t$, which has a perfect foresight.

6. Compute the transition path: Compute the optimal path for consumption, housing, and saving by forward induction given the initial state in period $t = 1$. In initial state, households receive assignment of public housing from the government, $\bar{H}$. The bequests for period $t$ newborn are collected from the household passing away at period $t$.

7. Aggregate household's net housing demand each period. Check if housing market in each period is clear. If not, update the guess of $\{p_t\}_{t=1}^{T}$, and go to step 5.

8. Aggregate government tax revenue, housing sale revenue, pension expenditure on the path. Combined with government's deficit/surplus in the steady state, check whether government's intertemporal budget is balanced. If not, update the guess of $\tau$, and go to step 4.

9. Check whether $p_T$ is close enough with the final steady state housing market price. If not, increase T, and go to step 2.

**Long run equilibrium:**

The price adjustment step. Because it is the long run equilibrium

$$I_t = \delta H$$

$$p_0 \Longrightarrow H(p_0) + \widetilde{H}(p_0) \Longrightarrow I_t(p_0) \Longrightarrow p_1 = 0.2 * \frac{1}{\alpha}\left(\frac{I_t(p_0)}{L}\right)^{\frac{1-\alpha}{\alpha}} + 0.8 * p_0$$

## A.4 Detrending

This section describe how we detrend the individual and aggregate variables. Since the initial steady state and final steady state have different trend growth rates for

productivity and population, we detrend the initial steady state and transitional paths separately. After we solve for the initial steady state, we back out the original values for the initial steady state and detrend it with the growth rate of the final steady state as the initial values in the transition path.

### A.4.1 Transitional path and final steady state

Denote $g$ as the balance-growth rate of the wage rate, $g_p$ and $g_I$ as the growth rate of housing prices and housing investment at the balance growth rate in the final steady state. Note that according to (**??**), since there is no secular growth of land at the steady state, the growth rate of housing investment at balance growth path is

$$1 + g_I = [(1 + g)(1 + n)]^{\alpha}$$

or

$$g_I = \alpha(g + n)$$

Then, according to (**??**), the growth rate of housing price at balance growth path is

$$1 + g_p = [(1 + g)(1 + n)]^{1-\alpha}$$

or

$$g_p = (1 - \alpha)(g + n)$$

For individual housing growth, notice that at balance growth path

$$\sum_{j=1}^{J} N_j \sum_i h_j^i = H$$

Therefore, the balance growth rate of $h_j^i$, denoted as $g_h$ is simply as

$$g_h = g_I - n$$

Note that

$$1+g=\left(1+g_h\right)\left(1+g_p\right)$$

For individual variables, we detrend as follows

$$\begin{aligned}
\widehat{c}_{t,j} &= c_{t,j}(1+g)^{T+j-t-1} \\
\widehat{a}_{t+1,j+1} &= a_{t+1,j+1}(1+g)^{T+j-t-1} \\
\widehat{h}_{t+1,j+1} &= h_{t+1,j+1}(1+g_h)^{T+j-t-1} \\
\widehat{s}_{t,j} &= s_{t,j}(1+g_h)^{T+j-t-1} \\
\widehat{\widetilde{h}}_{t+1,j+1} &= \widetilde{h}_{t+1,j+1}(1+g_h)^{T+j-t-1} \\
\widehat{b}_{t,j} &= b_{t,j}(1+g)^{T+j-t-1} \\
\widehat{y}_{t,j} &= y_{t,j}(1+g)^{T+j-t-1}
\end{aligned}$$

For aggregate variables, we detrend as follows

$$\begin{aligned}
\widehat{w}_t &= w_t(1+g)^{T-t} \\
\widehat{N}_{et} &= N_{et}(1+n)^{T-t} \\
\widehat{N}_{ht} &= N_{ht}(1+n)^{T-t} \\
\widehat{p}_t &= p_t(1+g_p)^{T-t} \\
\widehat{p}^g_{t,j} &= p^g_{t,j}(1+g_p)^{T-t} \\
\widehat{I}_t &= I_t(1+g_I)^{T-t} \\
\widehat{L}_t &= L_t \\
\widehat{H}_t &= H_t\,(1+g_I)^{T-t} \\
\widehat{\rho}_h &= \rho_h(1+g_p)^{T-t} \\
\widehat{R}_{c,t} &= R_{c,t}(1+g_p)^{T-t} \\
\widehat{\tilde{H}} &= \tilde{H}'\,(1+g_I)^{T-t} \\
\widehat{Y}_t &= Y_t\left[(1+g)\,(1+n)\right]^{T-t} \\
\widehat{z}_t &= z_t(1+g)^{T-t}
\end{aligned}$$

Finally, we assume that the operating cost $\psi$ grows at a constant rate $g_I$ along transition and at the final steady state. Therefore we have the detrended version of the above

equations as

$$
\begin{aligned}
\widehat{c}^i_{t,j} &= \widehat{y}^i_{j,t} + \widehat{a}^i_{t,j} - (q\widehat{a}^i_{t+1,j+1} + \widehat{R}_c\widehat{s}^i_{t,j}(1+g_p)^{j-1}) \\
\widehat{s}^i_{t,j} &= \widehat{\widehat{h}}^i_{1,1+j-t}; \widehat{s}^i_{t,j} = \widehat{h}^i_{t+1,j+1} \\
\widehat{y}^i_{j,t} &= \begin{cases} (1-\tau_t-\tau^{ss}_t)\widehat{w}_t(1+g)^{j-1}e^i_{t,j} \text{ for } j \leq J_w \\ \widehat{b}^i_{t,j} \text{ for } j > J_w \end{cases} \\
\widehat{c}^i_{t,j} &= \widehat{y}^i_{j,t} + \widehat{a}^i_{t,j} - (q\widehat{a}^i_{t+1,j+1} + \widehat{\rho}_{h,t}\widehat{\widehat{h}}^{i\prime}_{t+1,j+1}(1+g_p)^{j-1}), \\
\widehat{c}^i_{t,j} &= \widehat{y}^i_{j,t} + \widehat{a}^i_{t,j} - (q\widehat{a}^i_{t+1,j+1} + \widehat{p}^g_{t,j}\widehat{h}^i_{t+1,j+1}(1+g_p)^{j-1}) \\
\widehat{c}^i_{t,j} &= \widehat{y}^i_{j,t} + \widehat{a}^i_{t,j} - (q\widehat{a}^i_{t+1,j+1} + \widehat{p}_t\widehat{h}^i_{t+1,j+1}(1+g_p)^{j-1}), \\
\widehat{a}^i_{t+1,j+1} &\geq -\Upsilon\widehat{p}^g_{t,j}\widehat{h}^i_{t+1,j+1}(1+g_p)^{j-1} \\
\widehat{a}^{i,n}_{t,j} &= \widehat{a}^i_{t,j} + (1-\delta_h-\kappa)\widehat{p}_t\widehat{h}^i_{t,j}(1+g_p)^{j-1} \\
\widehat{\rho}_{h,t}(\Phi) &= \psi + \widehat{p}_t(\Phi) - \frac{(1-\delta_h)(1+g_p)}{1+r}E_{\Phi'}[\widehat{p}_{t+1}(\Phi)|\Phi] \\
\widehat{Y}_t &= \widehat{z}_t\widehat{N}_{ct} \\
\widehat{I}_t &= \left(\widehat{z}_t\widehat{N}_{ht}\right)^{\alpha}\left(\widehat{L}_t\right)^{1-\alpha} \\
\widehat{I}_t &= (\alpha\widehat{p}_t)^{\frac{\alpha}{1-\alpha}}\widehat{L}_t \\
\widehat{N}_{ht} &= (\alpha\widehat{p}_t)^{\frac{1}{1-\alpha}}\frac{L_t}{\widehat{\widehat{z}}_t} \\
(1+g_I)\left(\widehat{\widehat{H}}_{t+1} + \widehat{H}_{t+1}\right) &= (1-\delta_h)\left(\widehat{\widehat{H}}_t + \widehat{H}_t\right) + \widehat{I}_t \\
(1+g_{Ipre})(h_{pre}) &= (1-\delta_h)(h_{pre}) + \widehat{I}_t \\
\widehat{A}_0 &= \sum_{t=1}^{\infty} R^{-t} \left[\begin{array}{c} \sum_{j=t+1}^{J}\mu_{j,t}\int_{I\in bg}\left(\widehat{p}_t - \widehat{p}^g_{t,j}\right)\widehat{h}^i_{1,1+j-t}/\left((1+g)^{T-t}(1+g_h)^{j-1}\right)\Phi_t( \\ \sum_{j=t+1}^{J}\mu_{j,t}\int_{I\in rg}\left(\widehat{\rho}_t - \widehat{R}_{c,t}\right)\widehat{h}^i_{1,1+j-t}/\left((1+g)^{T-t}(1+g_h)^{j-1}\right)\Phi_t \\ \sum_{j=1}^{t}\mu_{j,t}\int_{I\in rg}\left(\widehat{\rho}_t - \widehat{R}_{c,t}\right)\widehat{h}^i_{1,j}/\left((1+g)^{T-t}(1+g_h)^{j-1}\right)\Phi_t(s) \\ -\sum_{j=1}^{J_w}\mu_{j,t}\tau_t\widehat{w}_t/(1+g)^{T-t}\int_s e\Phi_t(s)ds \end{array}\right. \\
\widehat{b}^i_{t,j} &= \theta(0.6\widehat{\widetilde{y}}^j_{t-j+J_w,J_w} + 0.4\widehat{\bar{y}}_t(1+g)^{j-1}) \\
\widehat{\widetilde{y}}^j_{t-j+J_w,J_w} &= \widehat{w}_{t-j+J_w}(1+g)^{J_w-1}e_{t-j+J_w,J_w} \\
\widehat{\bar{y}}_t &= \frac{\sum_{j=1}^{J_w}\sum_i \mu_{j,t}\widehat{w}_t e^i_{t,j}}{\sum_{j=1}^{J_w}\mu_{j,t}} \\
\tau^{ss}_t &= \frac{\sum_{j=1}^{J_w}\sum_i \mu_{j,t}\widehat{b}^i_{t,j}(1+g)^{1-j}}{\sum^{J_w}\sum\mu_{j,t}\widehat{w}_t e^i_{t,j}}
\end{aligned}
$$

### A.4.2 Pre-reform Steady State

Denote $g_0$ as the balance-growth rate of the wage rate, $g_{p0}$ and $g_{I_0}$ as the growth rate of housing prices and housing investment at the balance growth rate in the initial steady state. Similar to the final steady state, we have

$$\begin{aligned} 1+g_{I0} &= [(1+g_0)(1+n_0)]^{\alpha} \\ 1+g_{p0} &= [(1+g_0)(1+n_0)]^{1-\alpha} \\ g_{h0} &= g_{I0}-n_0 \\ 1+g &= (1+g_{h0})(1+g_{p0}) \end{aligned}$$

For individual variables, we detrend as follows

$$\begin{aligned} \widehat{c}_{t,j} &= c_{t,j}(1+g_0)^{-t} \\ \widehat{a}_{t+1,j+1} &= a_{t+1,j+1}(1+g_0)^{-t} \\ \widehat{s}_{t,j} &= s_{t,j}(1+g_{h0})^{-t} \\ \widehat{y}_{t,j} &= y_{t,j}(1+g_0)^{-t} \end{aligned}$$

For aggregate variables, we detrend as follows

$$
\begin{aligned}
\widehat{w}_t &= w_t(1+g_0)^{-t} \\
\widehat{N}_{ht} &= N_{ht}(1+n_0)^{-t} \\
\widehat{I}_t &= I_t(1+g_{I0})^{-t} \\
\widehat{L}_t &= L_t \\
\widehat{H}_t &= H_t\left(1+g_{I0}\right)^{-t} \\
\widehat{H}_t^D &= H_t^D\left(1+g_{I0}\right)^{-t} \\
\widehat{R}_{c,t} &= R_{c,t}(1+g_{p0})^{-t} \\
\widehat{C}_t &= C_t\left[\left(1+g_0\right)\left(1+n_0\right)\right]^{-t} \\
\widehat{Y}_t &= Y_t\left[\left(1+g_0\right)\left(1+n_0\right)\right]^{-t} \\
\widehat{z}_t &= z_t(1+g_0)^{-t}
\end{aligned}
$$