When Do Endogenous Portfolios Matter for HANK?

Adrien Auclert Stanford
Matthew Rognlie Northwestern
Ludwig Straub Harvard
Tomáš Ťapák Stanford

Portfolios in heterogeneous-agent macro

- Large part of het-agent macro literature assumes **exogenous portfolios**
  - Agents choose consumption & savings s.t. idiosyncratic & aggregate risk
  - May save in *accounts* of differing liquidity… [liquidity vs. return]
  - … but cannot choose the *mix* of assets in those accounts [risk vs. return]
- Almost all "HANK" literature makes this assumption. Some findings:
  - deficit-financed transfers have large & persistent output effects
  - nominal asset exposures matter for aggregate effects of monetary policy
- **Q:** what changes when agents can **choose portfolios** to hedge aggregate risk?

"By comparison, perturbation is done around a point of no aggregate shocks (the steady state), where the portfolio decision is indeterminate.

Hence, perturbation is usually inapplicable for portfolio-choice models."

– Fernandez-Villaverde and Levintal (2024)

This paper

- New method to solve for endogenous portfolios in the **sequence space**
  [Auclert-Bardóczy-Rognlie-Straub 2021; vs. recent Bhandari-Bourany-Evans-Golosov in state space]
- With enough assets, second-order perturbation analysis delivers **aggregate risk-sharing condition** across agents:
  - to first order, expected marginal utility varies w/shock by same proportion
- With this condition, can solve for first-order impulse responses:
  - computation uses same objects as exogenous-portfolio method
  - just add simple "correction" to sequence-space Jacobian!
  - can back out implied portfolios and risk premia
- Can extend method to case with fewer assets, portfolio restrictions, etc.

Application to HANK

- Take a "canonical" HANK model (Auclert, Rognlie, Straub JPE/ARE)
- Let households optimally choose assets, compare with exogenous portfolios
- When do endogenous portfolios matter?
  - Sometimes **not at all**
    [monetary policy shock example: exogenous portfolios are a natural hedge]
  - Sometimes **not**, but only **provided we constrain portfolios**
    [deficit-financed shock example: hedging portfolios are implausible]
  - Sometimes **a lot**, and with **reasonable optimal portfolios**
    [nominal bonds example: hedging achievable with real bonds]
- Key question: can high-MPC agents hold large gross positions?

Risk-sharing in a general setting with risk and heterogeneity

General setting

- Heterogeneous households i can allocate wealth a_i to $K+1$ assets
- Asset k has supply A^k; stochastic payoff $x^k(\epsilon)$, $\epsilon \equiv (\epsilon_1, \ldots, \epsilon_Z)$ (Z shocks)
- Assume $\epsilon_z = \sigma\bar{\epsilon}_z$, with $\bar{\epsilon}_z$ independent, mean 0, var $\bar{\sigma}_z^2$; common scaling σ
- Given value function W_i, prices p^k, problem of household i is:

$$\max_{\{a_i^k\}} \mathbb{E}_\epsilon \left[W_i \left(\sum_{k=0}^{K} x^k(\epsilon) a_i^k, \epsilon \right) \right] \text{ s.t. } \sum_{k=0}^{K} p^k a_i^k = a_i$$

- Can embed in larger dynamic problem (letting everything vary with σ)

Deriving first-order condition

- Problem is:

$$\max_{\{a_i^k\}} \mathbb{E}_\epsilon \left[W_i \left(\sum_{k=0}^{K} x^k(\epsilon) a_i^k, \epsilon \right) \right] \text{ s.t. } \sum_{k=0}^{K} p^k a_i^k = a_i$$

- Implies classic first-order condition:

$$\mathbb{E}_\epsilon \left[\frac{x^k(\epsilon)}{p^k} \frac{W_i'(\epsilon)}{\gamma_i} \right] = 1 \quad \forall i, k$$

- Expectation of asset-specific return times household-specific SDF is always 1

Perturbation around $\sigma = 0$

- Given σ, equilibrium is a_i^k, p^k s.t. FOCs hold and asset markets clear
- At $\sigma = 0$ (no risk), we get

$$\frac{x^k}{p^k} = \frac{\gamma_i}{W_i'} \equiv R$$

- Rates of return on all assets equalized at R, portfolio choice **indeterminate**!
- Now consider **perturbation** of model around $\sigma = 0$
  - to first order in σ, no effect on a_i^k, p^k, γ_i [symmetry: σ and $-\sigma$ identical]
  - but second order in σ, get risk premia and **well-defined** $a_i^k(\sigma)$ as $\sigma \to 0$

[as in Tille-van Wincoop 2010, Devereux-Sutherland 2011, Coeurdacier-Rey 2013, etc.]

Second-order perturbation

- Subtract FOC for asset 0 from asset k to get

$$\mathbb{E}_\epsilon \left[\left(\frac{x^k(\epsilon)}{p^k} - \frac{x^0(\epsilon)}{p^0} \right) \frac{W_i'(\epsilon)}{\gamma_i} \right] = 0 \quad \forall i, k$$

- Then differentiate twice with respect to σ:

$$\sum_{z=1}^{Z} \left(\frac{dx^k/x^k}{d\epsilon_z} - \frac{dx^0/x^0}{d\epsilon_z} \right) \frac{dW_i'/W_i'}{d\epsilon_z} \bar{\sigma}_z^2 = -(r^k - r^0)$$

- [r^k is one-half 2nd derivative of expected return wrt σ, over R]

(Locally) Complete markets case

- From last slide

$$\sum_{z=1}^{Z}\left(\frac{dx^k/x^k}{d\epsilon_z}-\frac{dx^0/x^0}{d\epsilon_z}\right)\frac{dW_i'/W_i'}{d\epsilon_z}\bar{\sigma}_z^2=-(r^k-r^0)$$

- Collect parentheses in $\mathbf{X}=[X_{kz}]$, define $[\boldsymbol{\lambda}_i]_z=d\log W_i'/d\epsilon_z$, $\Sigma=\text{diag}(\bar{\sigma}_z^2)$:

$$\mathbf{X}'\Sigma\boldsymbol{\lambda}_i=-\mathbf{r}$$

- If enough assets ($K=Z$) and $\mathbf{X}$ has full rank, then $\boldsymbol{\lambda}_i$ is same for all i!
- **Risk-sharing condition:** $d\log W_i'/d\epsilon_z$ equals common λ_z for all agents
    - can use to **test** for portfolio optimality, or **solve** for optimal portfolios

Applying the risk-sharing condition

- Suppose $\bar{a}_i^k$ is exogenous portfolio; let t_i be excess return from other portfolio:

$$t_i \equiv \sum_{k=0}^{K} x^k(\epsilon)(a_i^k - \bar{a}_i^k) \quad \text{and} \quad \bar{W}_i(t_i, \epsilon) \equiv W_i\left(\sum_{k=0}^{K} x^k(\epsilon)\bar{a}_i^k + t_i, \epsilon\right)$$

- Now can write risk-sharing condition as:

Direct exposure to shocks under exogenous portfolios →

$$\frac{d\bar{W}_i'/\bar{W}_i}{d\epsilon_z} + R\frac{\bar{W}_i''}{\bar{W}_i'}\frac{dt_i}{d\epsilon_z} = \lambda_z$$

← "Transfer" exposure to shocks under endogenous portfolios, to achieve aggregate risk-sharing

- Think of this as solving for **transfers** $dt_i/d\epsilon_z$ contingent on shocks
- Market clearing requires aggregate $dt_i/d\epsilon_z$ to be zero, use this to solve λ_z & $dt_i/d\epsilon_z$

Where we stand now

- In "complete markets case" where assets span shocks, **risk-sharing condition**
- Given exogenous-portfolio exposures $d \log \bar{W}_i'/d\epsilon_z$, can solve jointly for:
  - Shock-contingent transfers $dt_i/d\epsilon_z$
  - Common post-transfer exposure $d \log W_i'/d\epsilon_z = \lambda_z$
- If desired (and if we have $\mathbf{X}$) can also back out:
  - Actual portfolios from $[p^k(a_i^k - \bar{a}_i^k)]_{k=1}^K = \mathbf{X}^{-1}\mathbf{t}$
  - Asset risk premia from $[r^k - r^0]_{k=1}^K = -\mathbf{X}'\Sigma\boldsymbol{\lambda}$
    - (actual risk premium of k over 0 will be $(r^k - r^0)\sigma^2$)

Implementation in heterogeneous-agent models

What's left to do

- Can solve for transfers $dt_i/d\epsilon_z$ given exposures $d\log \bar{W}_i'/d\epsilon_z$
- But in a fully-articulated equilibrium setting:
  - $d\log \bar{W}_i'/d\epsilon_z$ are determined endogenously in GE …
  - this will affect transfers $dt_i/d\epsilon_z$ …
  - … but transfers $dt_i/d\epsilon_z$ will matter for GE and thus $d\log \bar{W}_i'/d\epsilon_z$!
- How do we resolve this apparent **fixed-point problem**?
  - Solution: make this feedback part of our **sequence-space Jacobians**

Modifying the sequence-space Jacobian

- Intertemporal MPC matrix $\mathbf{M}$:
  - M_{ts} gives agg consumption response at t to e.g. wage change at s…
  - … where risk is realized at date 0
  - other sequence-space Jacobians analogous, just different inputs / outputs
- Now, wage change at s is shock that affects all $\bar{W}_i'$, implies date-0 transfers
  - these transfers change distribution of assets coming into date 0
  - which has implications for consumption at future t, implying a **correction** $\mathbf{M}^{corr}$ and a **corrected** intertemporal MPC matrix $\widetilde{\mathbf{M}} \equiv \mathbf{M} + \mathbf{M}^{corr}$

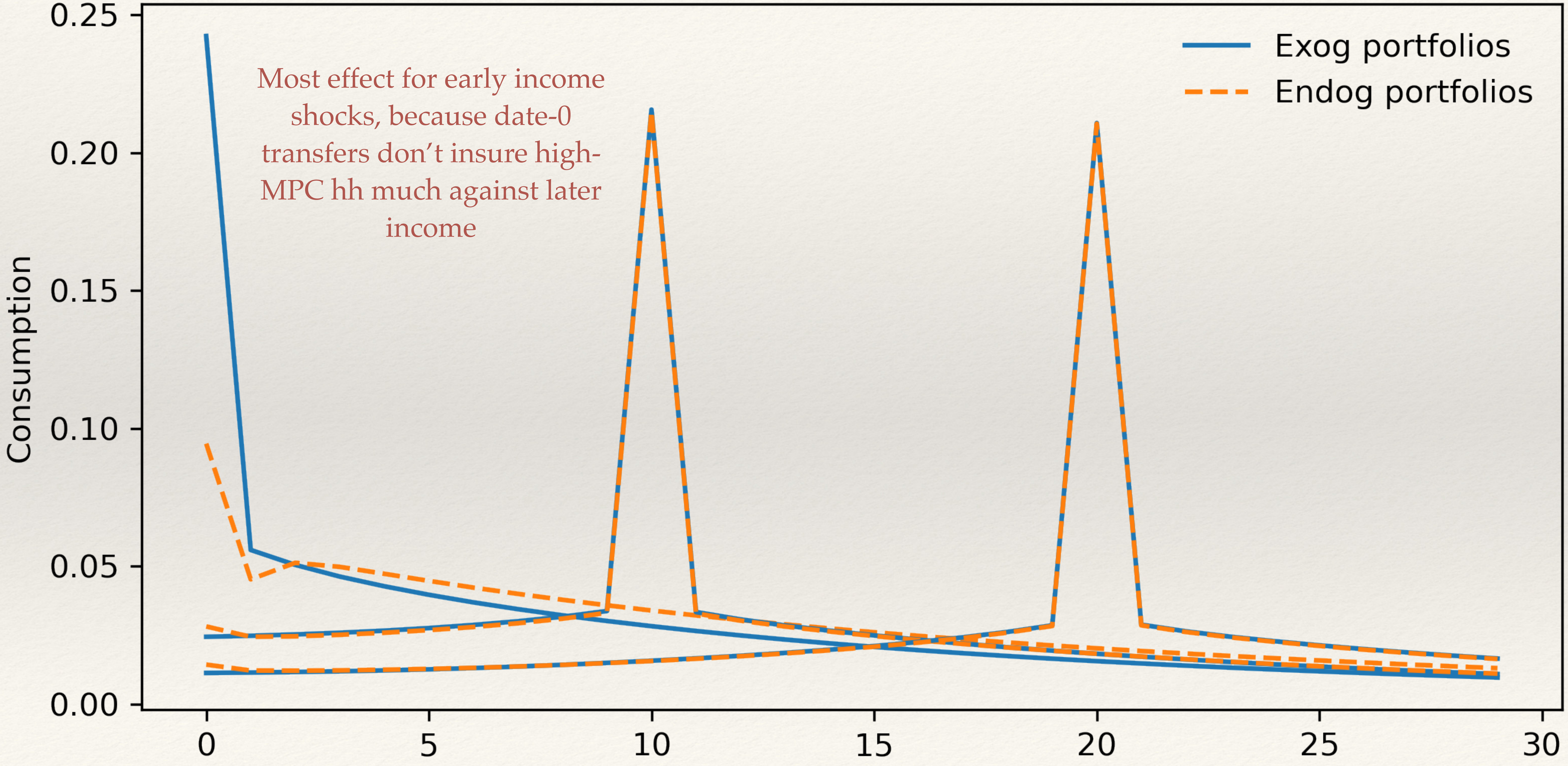
Obtaining corrected sequence-space Jacobian

- Just a slight tweak to **"fake news algorithm"** [Auclert-Bardóczy-Rognlie-Straub 2021]
- As we iterate backward, calculate (under exogenous portfolios) effect of income at date s on date-0 $\bar{W}_i'$
- Calculate implied transfers and effect $\mathscr{D}_s$ on date-0 distribution
- Use expectation functions $\mathscr{E}_t$ to find effect on later consumption:

$$\mathbf{M}^{corr} = \begin{pmatrix} \mathscr{E}_0'\mathscr{D}_0 & \mathscr{E}_0'\mathscr{D}_1 & \cdots \\ \mathscr{E}_1'\mathscr{D}_0 & \mathscr{E}_1'\mathscr{D}_1 & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

Flexibility here: can limit transfers to subset of agents (e.g. no low-wealth agents), enforcing limited participation

Example: how does this change M matrix?



Algorithm with corrected sequence-space Jacobians

- Same as standard sequence-space Jacobian algorithm, but where Jacobians are recalculated to include "complete-market corrections" like $\mathbf{M}^{corr}$
- Solve all first-order GE impulse responses using these Jacobians
  - then, with these calculated, can back out implied transfers and λ_z…
  - and given asset returns (also determined in GE), further back out underlying asset portfolios, risk premia
- Altogether, we're solving for:

$$\text{0th order portfolios} \longleftrightarrow \text{1st order impulses} \longleftrightarrow \text{2nd order risk premia}$$

Application to HANK

"Canonical" HANK model

- Households face uninsurable risk to e_{it}, solve problem:

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_{it}) - v(n_{it}) \qquad \text{s.t.} \qquad p_t s_{it} + b_{it} \geq 0$$

$$c_{it} + p_t s_{it} + b_{it} \leq (p_t + d_t) s_{it-1} + (1 + r_{t-1}) b_{it-1} + e_{it}(1 - \tau_t) w_t n_{it}$$

- s_{it} stocks (price p_t, dividends d_t), b_{it} bonds, assume EIS = 1
- Production from labor $Y_t = N_t$, constant markups, so constant wage and dividend shares of income
- Can embed any period of hh problem into framework from earlier, we'll consider impulse to shock at date 0

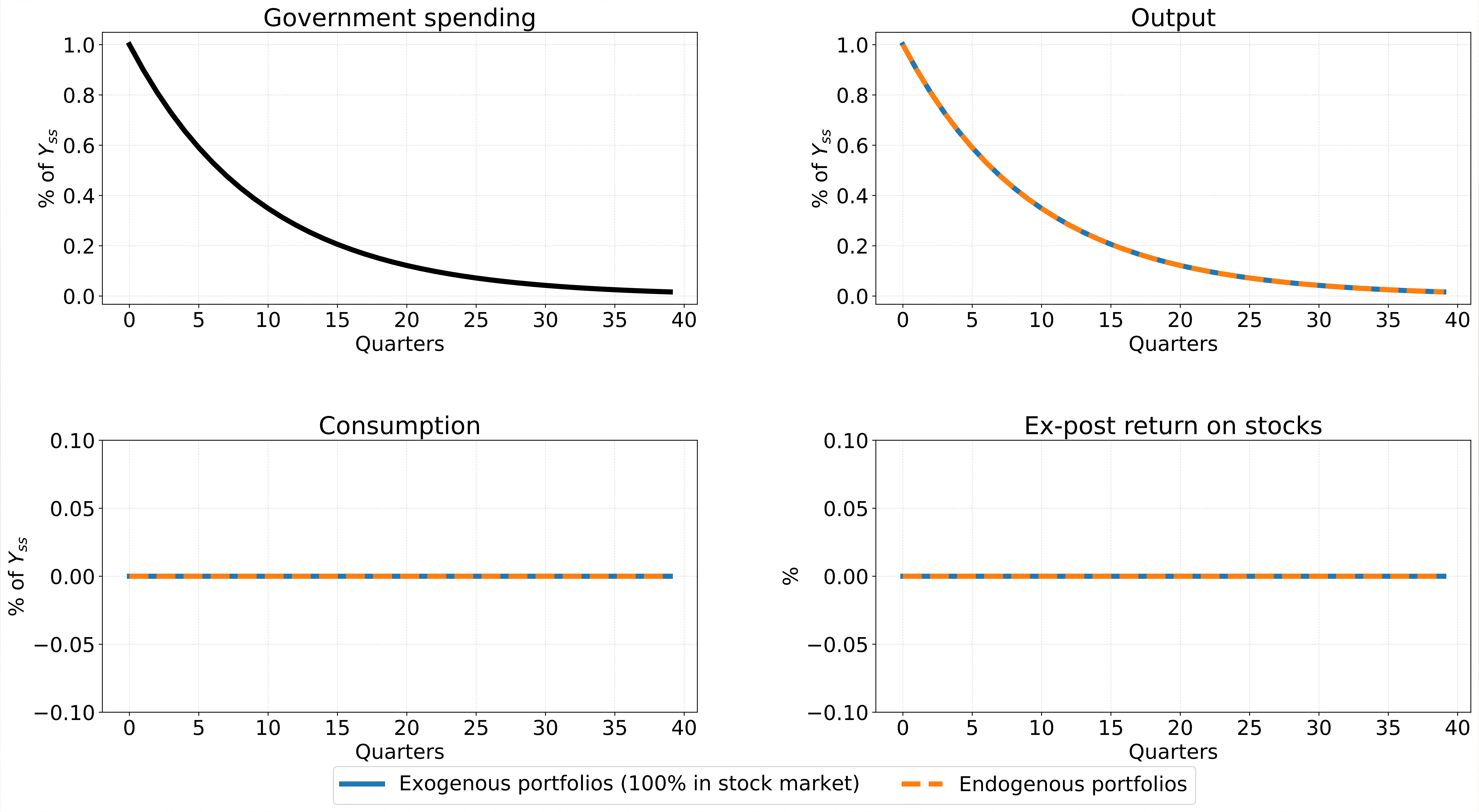
Model continued

- Fiscal policy sets τ_t, spends G_t and has debt B_t, with

$$B_t = (1 + r_{t-1})B_{t-1} + G_t - \tau_t Y_t$$

- Sticky nominal wages, implying:
  - Labor rationed, equal allocation rule $n_{it} = N_t = Y_t$
  - Phillips curve for inflation π_t (not needed to solve for real quantities)
- Monetary policy sets real rate r_t, using nominal rule $i_t = r_t + \pi_{t+1}$
- Assume steady state with $B = 0$, exogenous portfolios = 100% stocks
- Consider shocks to $\{G_t\}$, $\{B_t\}$, $\{r_t\}$

Example 1: balanced-budget $\{G_t\}$ shock

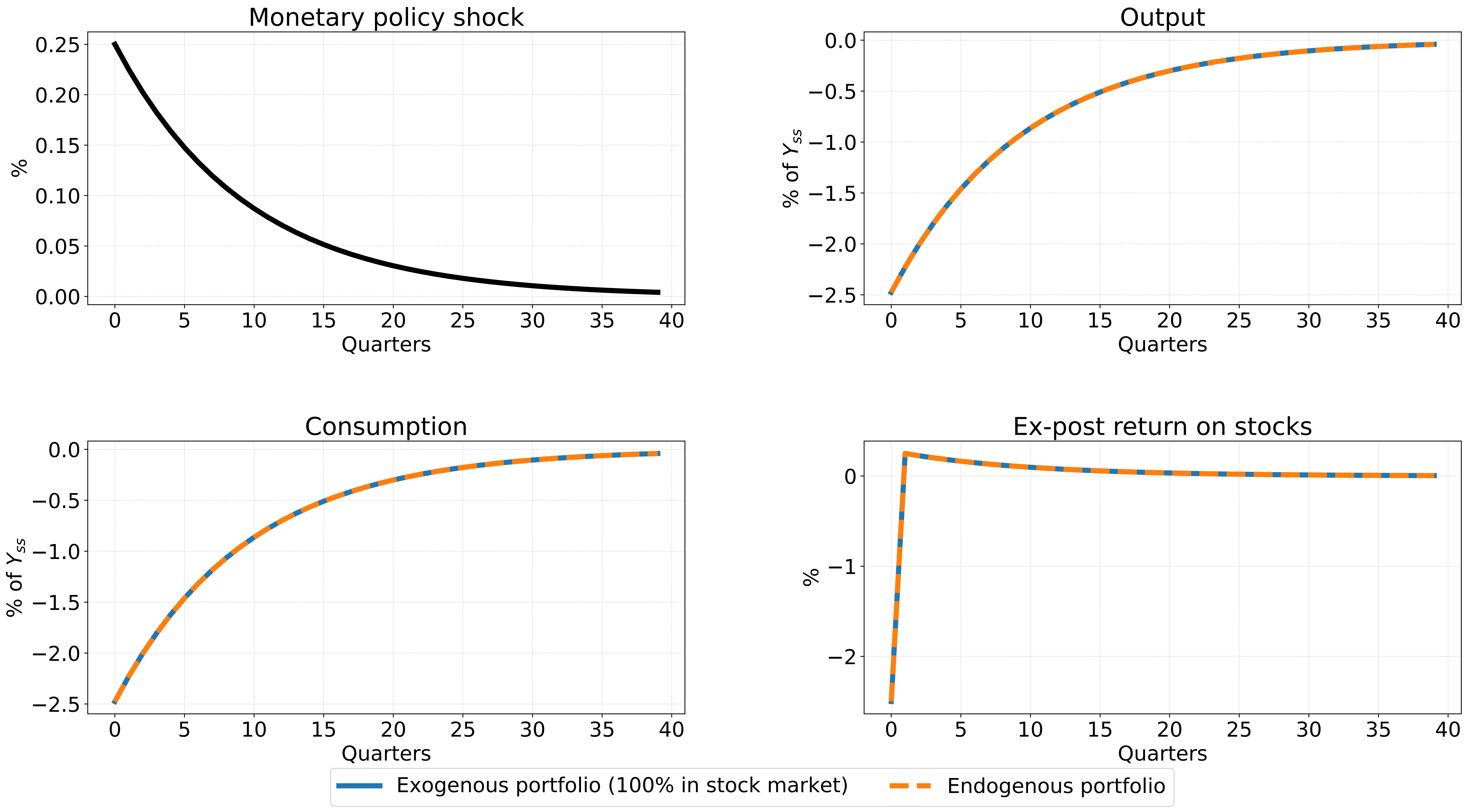


No effect from portfolio choice!

Why? Risk-sharing condition already holds with exogenous portfolios, since everyone's consumption unchanged (see Intertemporal Keynesian Cross).

Also, stock prices constant -> no difference between bonds and stocks

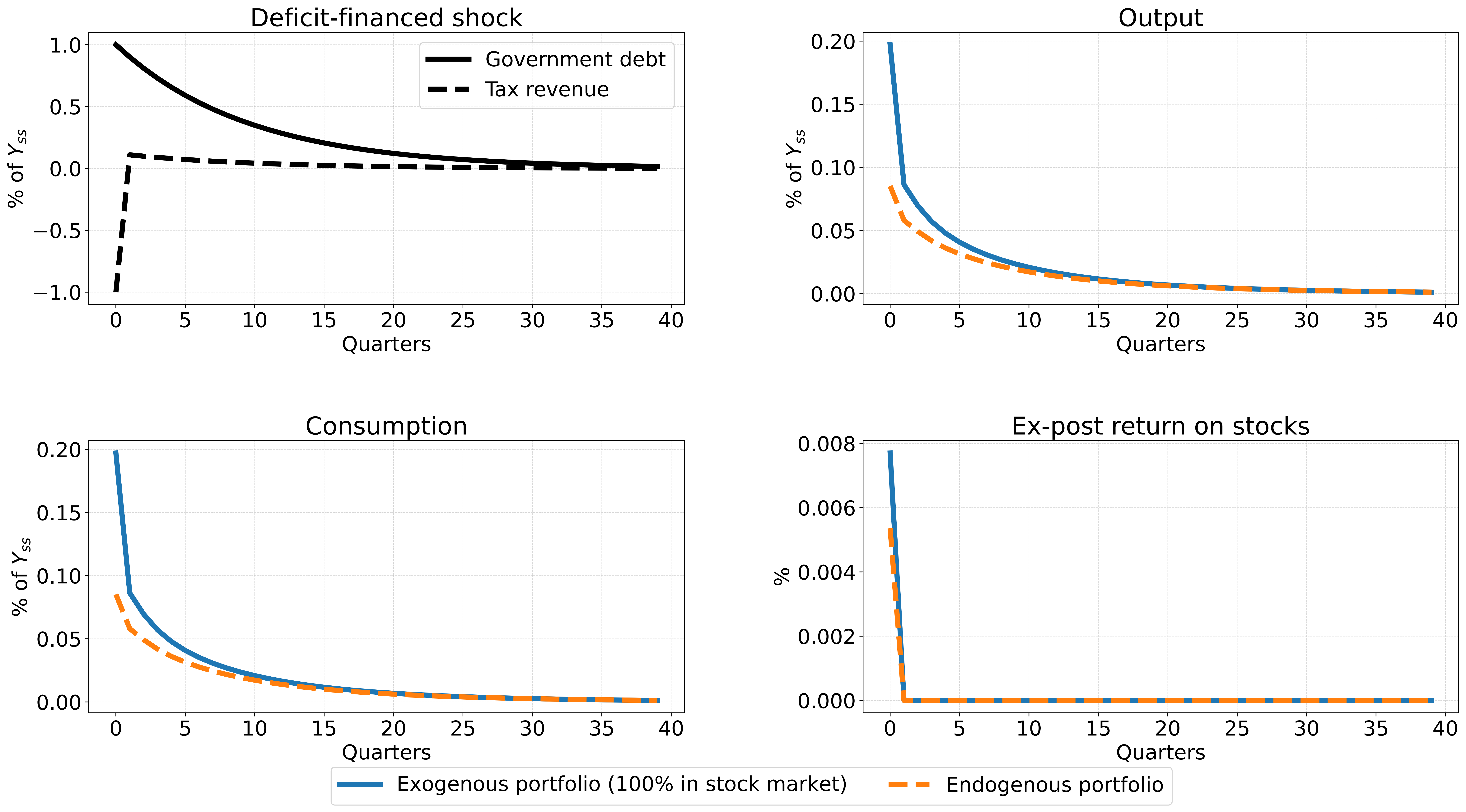
Example 2: monetary policy $\{r_t\}$ shock



No effect from portfolio choice!

Risk-sharing condition already holds with exogenous portfolios, since everyone's consumption moves by same proportion (Werning 2015 neutrality)

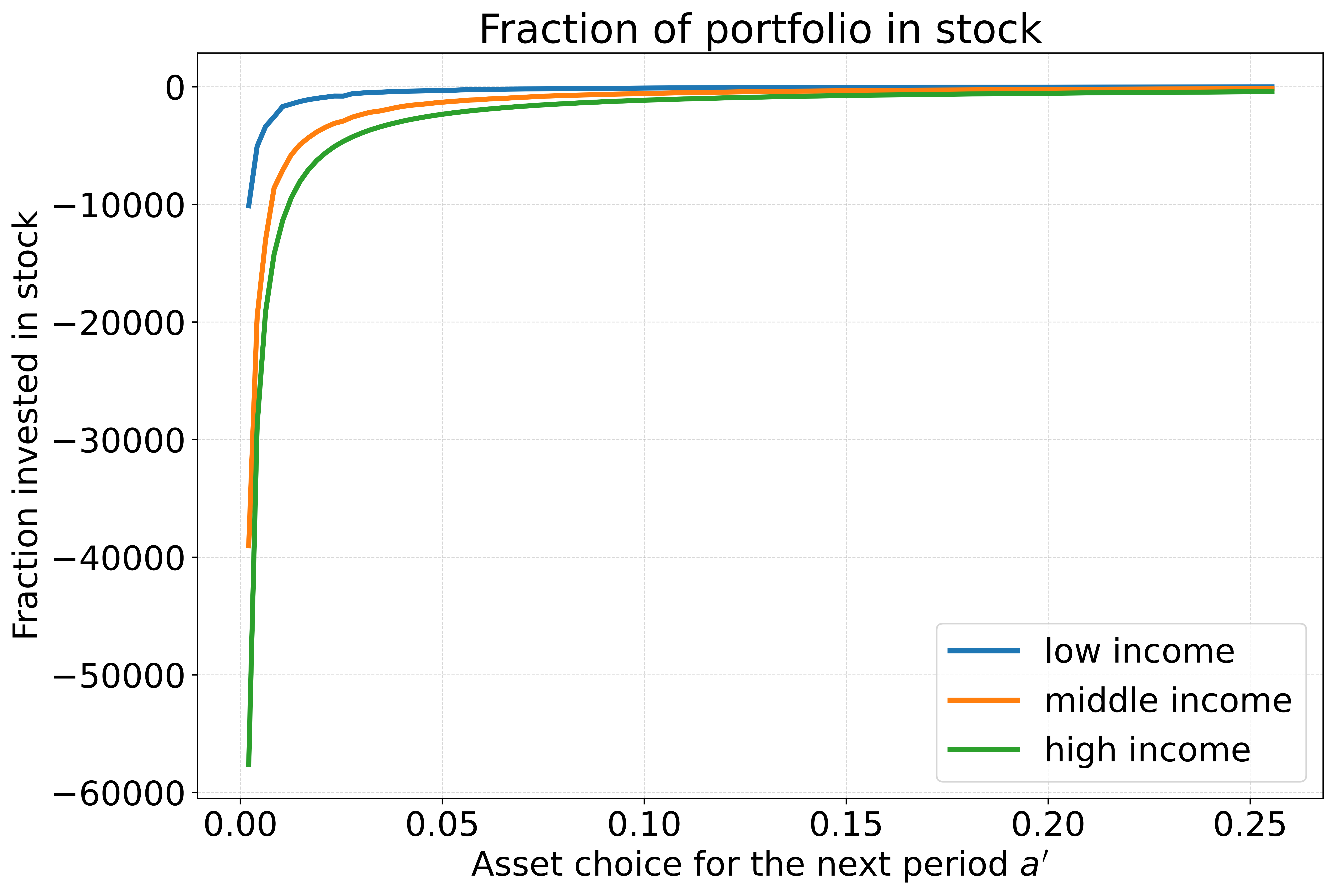
Example 3: deficit-financed transfer $\{B_t\}$ shock



Now, big effect from endogenous portfolios, with much smaller output effect.

But… how is this possible when stocks move so little in response to the shock?

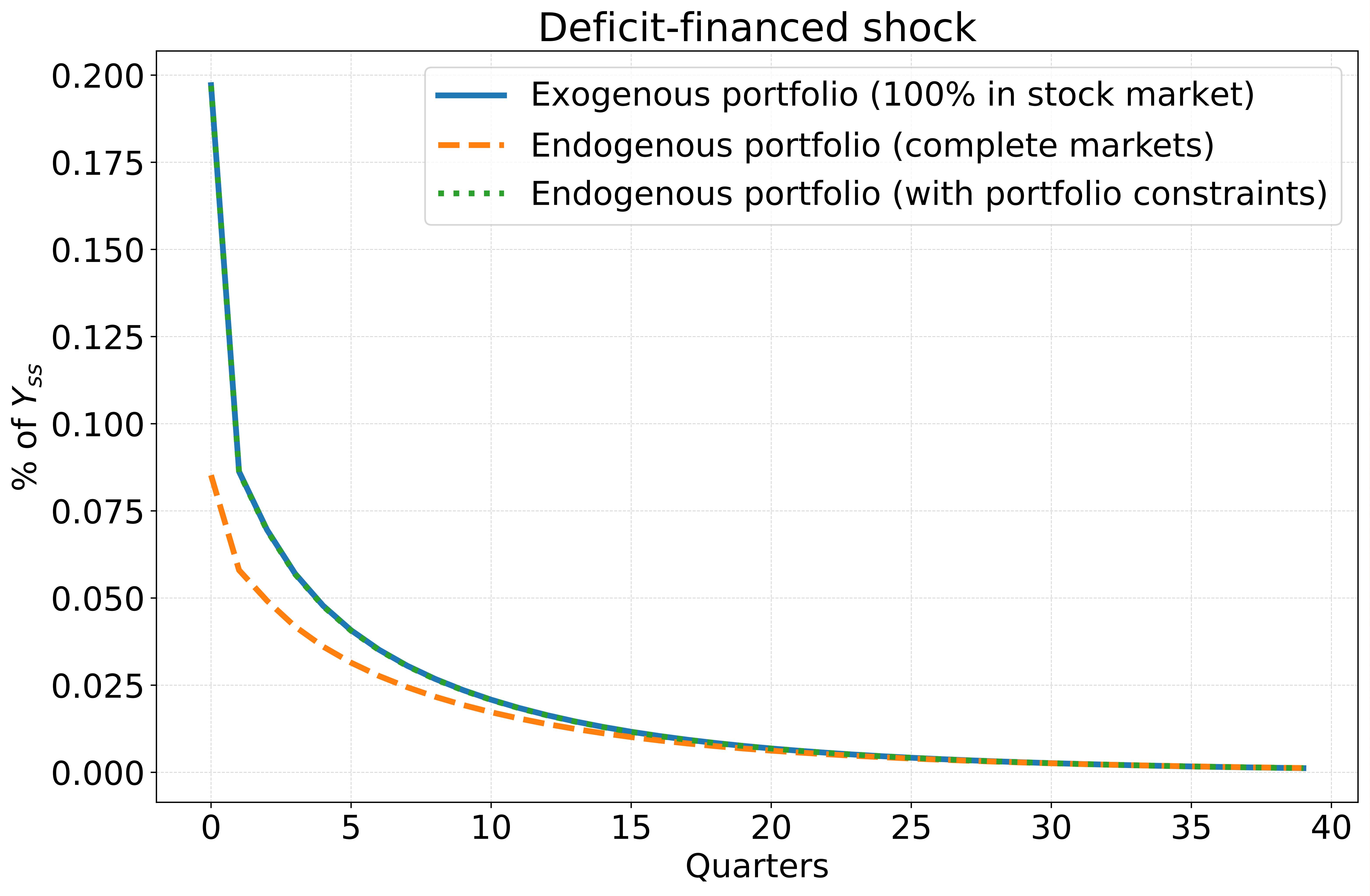
Under the hood: crazy portfolio shares!



We didn't restrict gross positions in assets: borrowing constraint applied only to net position!

So "complete markets" transfers achieved with ultra-levered short-selling by the poor.

With portfolio constraints… (no short sales, 1.5x leverage limit)

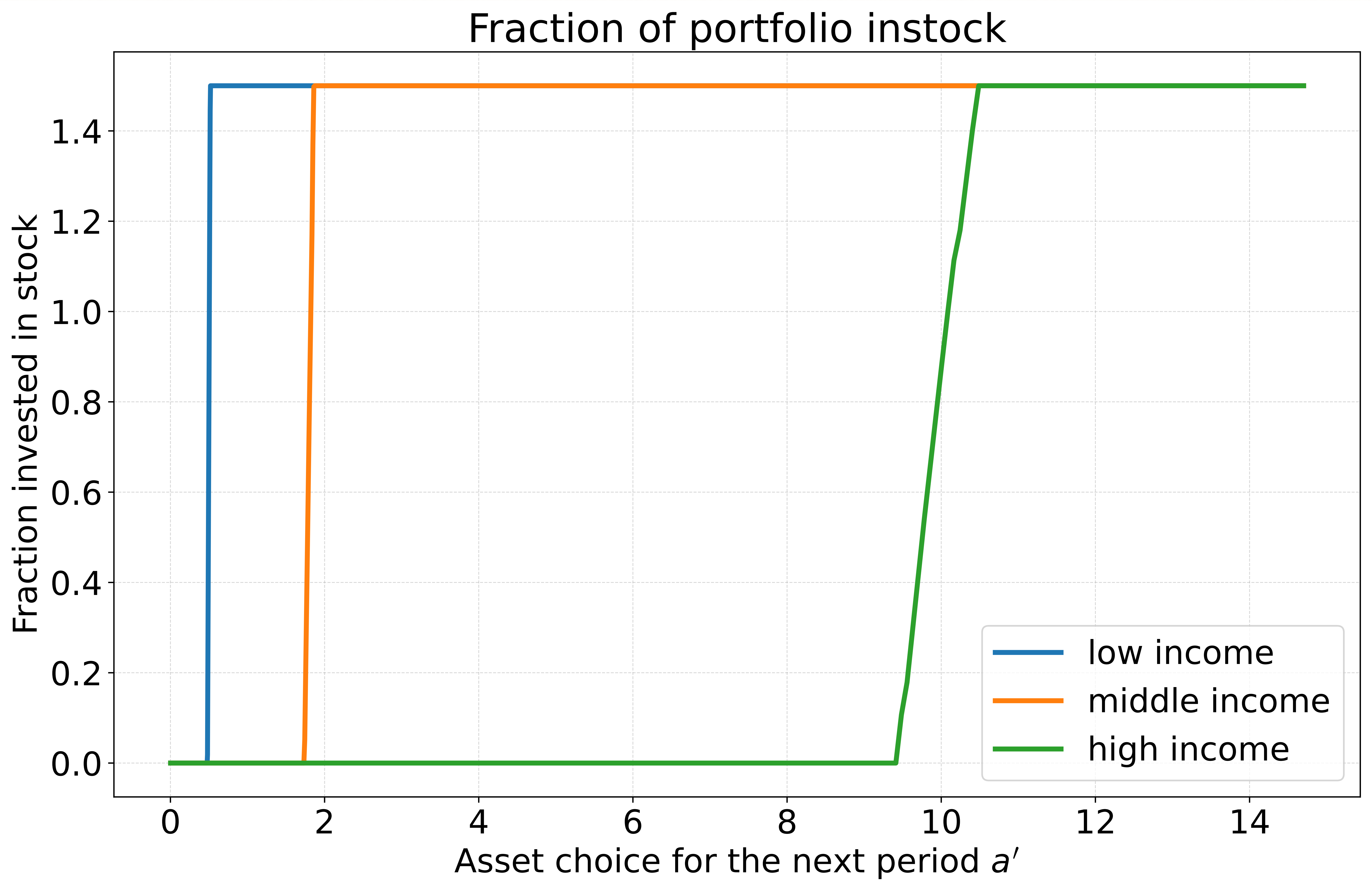


Constrained endogenous portfolio now ~same as exogenous portfolio: no effect!

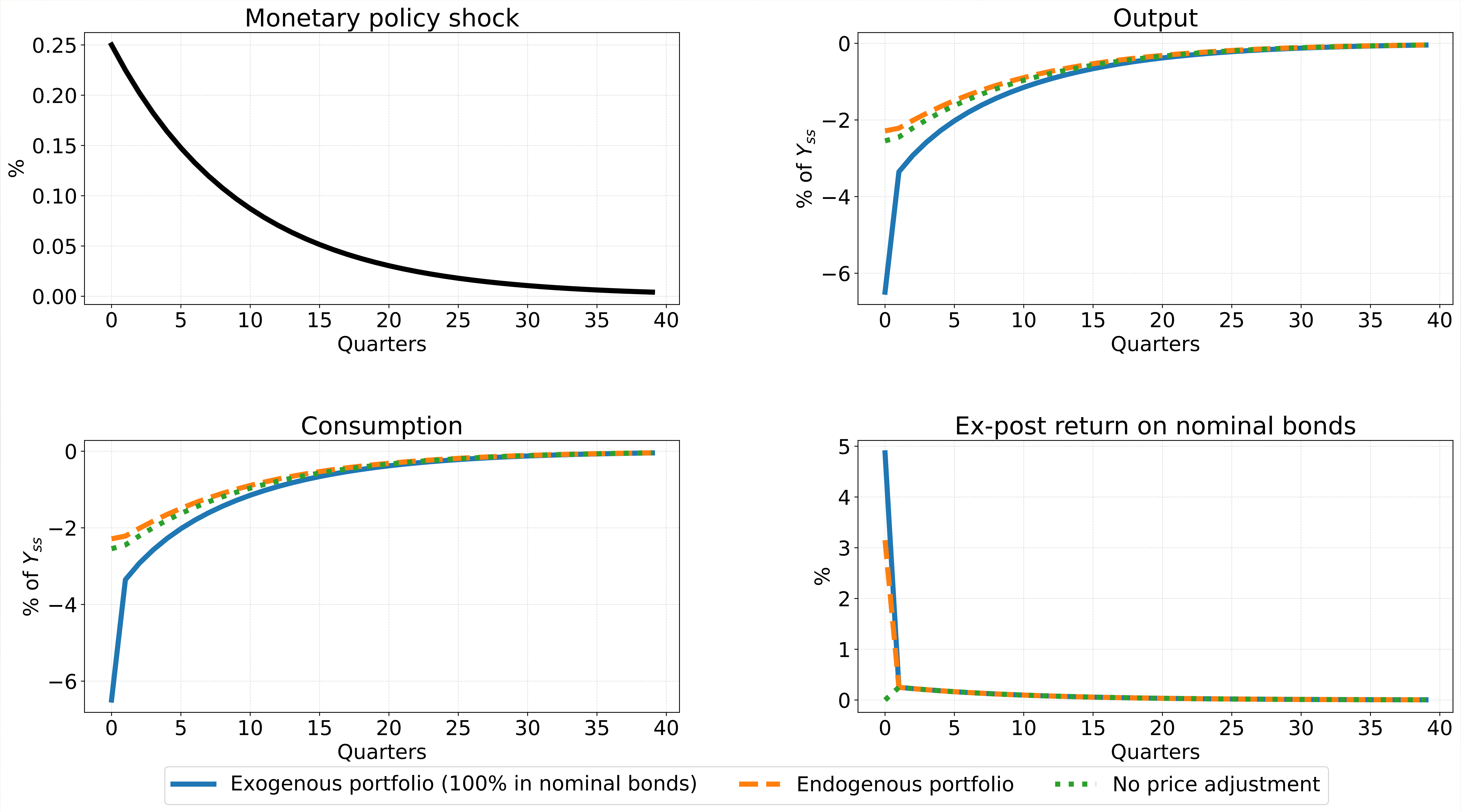
Reason: **high-MPC agents can't take much of a position either way.**

(Implemented with iterative modification to algorithm, which checks and imposes constraints.)

Portfolios look more reasonable now…



Example 4: monetary shock, nominal bonds



Now no gov debt or markups, instead Huggett model with baseline nominal debt.

Endogenous portfolio negates effects of inflation, because high-MPC debtors switch borrowing from nominal to real debt.

Key: **high-MPC debtors hold large gross positions (debt) here.**

Recap of quantitative examples

- **Balanced-budget $\{G_t\}$ shock:**
  - households already hedged, returns the same so portfolios indeterminate
- **Monetary policy $\{r_t\}$ shock:**
  - households already hedged with uniform all-equity portfolio (Werning)
- **Deficit-financed transfer $\{B_t\}$ shock:**
  - households not hedged, want crazy positions, little effect if these not permitted
- **Monetary policy $\{r_t\}$ shock in Huggett model with default nominal bonds:**
  - households not hedged, optimal portfolios replace nominal with real debt

More shocks than assets+1:
the incomplete markets case

Incomplete markets case: the projection principle

- With incomplete markets, project complete-market transfers on **column space** of asset returns $\mathbf{X}$:

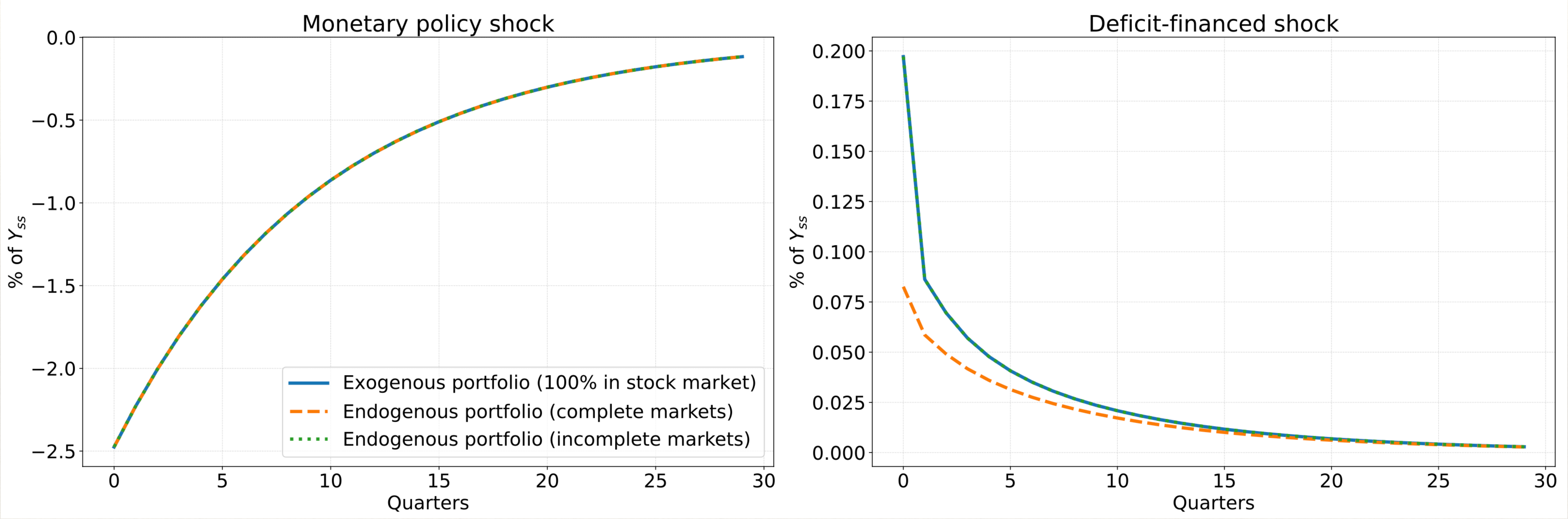
$$\mathbf{t}_i = \mathbf{X}'(\mathbf{X}'\Sigma\mathbf{X})^{-1}\mathbf{X}'\Sigma\mathbf{t}_i^{CM}$$

- Risk premia are the same as with complete-markets $\mathbf{t}_i^{CM}$
- By linearity, projection applies to corrections $\mathbf{M}^{corr}$, but need to solve impulse responses to all shocks jointly!
- Also, $\mathbf{X}$ endogenous, so there is nonlinear fixed point

Full algorithm for incomplete markets

- Precalculate all complete-market corrections $\mathbf{M}^{corr}$ (and other Jacobians)
- Given return matrix $\mathbf{X}$:
    - Calculate projection matrix $\mathbf{P}_{\mathbf{X}} \equiv \mathbf{X}'(\mathbf{X}'\Sigma\mathbf{X})^{-1}\mathbf{X}'\Sigma$
    - Calculate $\mathbf{M}^{corr,z,z'}$ for shocks z, z' by $\mathbf{M}^{corr,z,z'} = P_{\mathbf{X}}^{z,z'} \cdot \mathbf{M}^{corr}$
    - Create $(Z \times T) \times (Z \times T)$ Jacobians $\widetilde{\mathbf{M}}$, with original Jacobians $\mathbf{M}$ as main diagonal blocks, and $\mathbf{M}^{corr,z,z'}$ added to each z, z' block, and solve system
    - Update return matrix $\mathbf{X}$ and repeat until convergence
- If stacked system too large: preconditioned iterative methods work well

Example: both monetary & deficit-financed fiscal shocks



With incomplete markets, the response to deficit-financed fiscal shock returns to ~ exogenous-portfolio case.

Why? Stock returns vary much more in response to monetary policy shock, so portfolio decisions focus mostly on this shock, for which exogenous portfolios already a good hedge!

Conclusion

- Simple modification of sequence-space Jacobian algorithm gives us:
  - impulses with endogenous portfolios and second-order risk premia
  - can add portfolio constraints, incomplete markets
- In HANK, endogenous portfolios do not always matter!
  - but when exogenous portfolios are a bad hedge, and high-MPC agents can hold large gross positions, they do
- Plenty of future work! [larger quantitative examples, two-account models with endogenous portfolios in each account, 3rd-order perturbation to get time-varying portfolios…]