A Disaggregated Economy with Optimal Pricing Decisions

Alessandro Ferrari — *University of Zurich & CEPR*

Mishel Ghassibe — *CREi, UPF & BSE*

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  (iii) **State-dependent** probability of price adjustment (Cavallo et al., 2024)

- Need a unified framework to conduct both positive and normative analysis of aggregate business cycles, as well policy stablization

This paper

1. Develop an analytically-tractable **multi-sector** model with fully general **production networks** and **endogenous price rigidity** due to menu costs
   - Both the *intensive* and the *extensive* margins of price adjustment are optimally chosen, and take into account the decisions of other firms in the economy

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2. Provide a closed-form link between first-order changes in **aggregate** variables and **micro** shocks, network **centralities** and **pricing statistics**
    - Study aggregation under fixed menu costs and log-linear preferences (Golosov and Lucas, 2007): $\mathcal{U}(C, L) = \log C - L$

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3. **Quantify** the effect of sectoral shocks on aggregate variables and welfare using 6-digit US data

Key result: Aggregation

1 **Aggregate GDP**: a first-order change following a productivity shock to sector k:

$$\Delta \log C = \tilde{\lambda}_k \times \Delta \log A_k = \Delta \log C^{flex} \qquad \text{(Hulten, 1978)}$$

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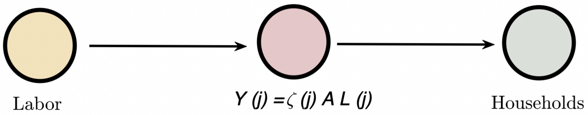
Aggregate Employment: a Simple Example



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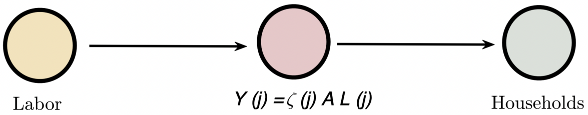

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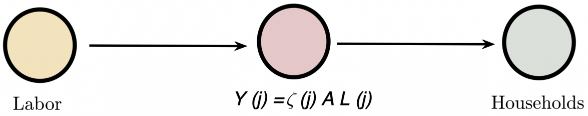

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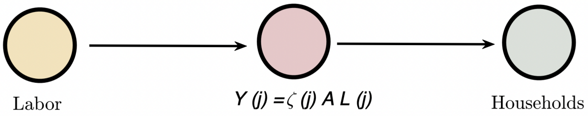

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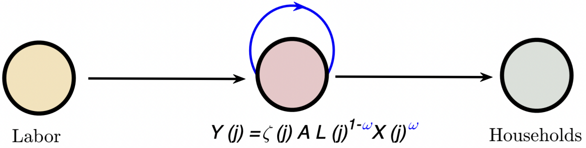

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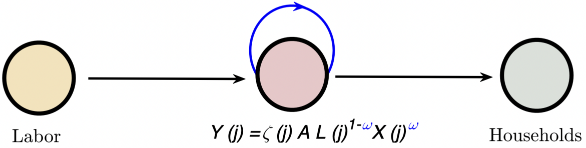

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where $\mathcal{S}$ is the **supplier-of-suppliers (SS) centrality**, which is a function of the Leontief Inverse Ψ and final consumption shares $\overline{\boldsymbol{\omega}}_{\boldsymbol{c}}$:

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3 **Aggregate TFP/Welfare**: first-order loss following a productivity shock to sector k:

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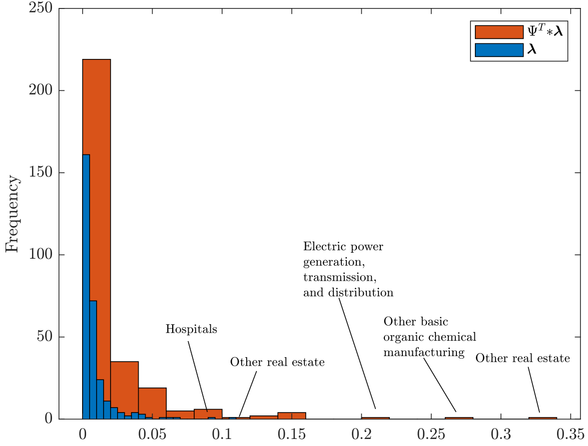
SS-centrality vs Sales shares (United States)



MODEL

Firms: Production

- Each firm j in a sector i has access to the following production function:

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$$Z_i(j) \equiv \zeta_i(j)^{\frac{1}{\epsilon - 1}} \times A_i$$

where A_i is a sectoral productivity, $\epsilon \geq 2$ is within-sector elasticity of substitution, and $\zeta_i(j)$ is firm-level idiosyncratic shock drawn from uniform distribution:

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- Cost-minimizing choice of inputs delivers the marginal cost function:

$$MC_i(j) = \zeta_i(j)^{\frac{1}{1-\epsilon}} \times \mathcal{Q}_i(W, P_1, ..., P_N; A_i)$$

where $\mathcal{Q}_i$ rises in input prices and falls in A_i

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$$\Pi_i(j) = (1 - \tau_i) P_i(j) Y_i(j) - MC_i(j) Y_i(j) - W \upsilon_i \chi_i(j)$$

where

$$\chi_i(j) = \begin{cases} 1 & \text{if} \quad P_i(j) \neq P_{i,0} \\ 0 & \text{if} \quad P_i(j) = P_{i,0} \end{cases}$$

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- We set $1 - \tau_i = \frac{\epsilon}{\epsilon - 1}$ and consider $P_{i,0} = 1, \forall i$ (flex-price deterministic steady state)

Firms: Pricing (II)

- The loss function associated with the price adjustment is given by:

$$\mathcal{L}\left[\zeta_i(j)\right] = \left[\Pi_i(j)|\chi_i(j) = 1\right] - \left[\Pi_i(j)|\chi_i(j) = 0\right], \quad \mathcal{L}'' > 0$$

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- For tractability, assume that the adjustment decision is based on an **approximate loss function**:

$$\hat{\mathcal{L}}\left[\zeta_i(j)\right] \equiv \mathcal{L}\left[1\right] + \mathcal{L}'\left[1\right]\left(\zeta_i(j) - 1\right) + \frac{1}{2}\mathcal{L}''\left[1\right]\left(\zeta_i(j) - 1\right)^2$$

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- The approximate loss function delivers analytic expressions for the sector-specific **adjustment bands** ζ_i^L and ζ_i^H in terms of aggregate and sectoral variables, and an *Ss* rule for price adjustment:

$$\text{If} \quad \zeta_i(j) \leq \zeta_i^L \quad \text{or} \quad \zeta_i(j) \geq \zeta_i^H : \text{adjust}$$

$$\text{If} \quad \zeta_i(j) \in \left(\zeta_i^L, \zeta_i^H\right) : \text{do not adjust}$$

Adjustment bands

Lemma : Adjustment bands

Under the approximate loss function $\hat{\mathcal{L}}$, the adjustment bands are:

$$\zeta_i^L = 1 + \mathbf{\Gamma_1^i} - \left[(2 + \Gamma_1^i)\Gamma_1^i + \mathbf{\Gamma_2^i}\right]^{\frac{1}{2}},$$

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- The analytic adjustment bands also deliver a tractable expression for the sectoral price index:

$$P_i^{1-\epsilon} = \mathcal{Q}_i^{1-\epsilon}\left(1 - \frac{\zeta_i^H - \zeta_i^L}{b_i} \times \underbrace{\frac{\zeta_i^L + \zeta_i^H}{2}}_{\text{Midpoint}}\right) + \underbrace{\frac{\zeta_i^H - \zeta_i^L}{b_i}}_{\text{Width}}$$

Households, Monetary Policy and Equilibrium Misallocation

- *Households' utility*: $\mathcal{U}(C, L) = \log C - L, \quad C = \iota^C \prod_{i=1}^{N} C_i^{\overline{\omega}_i^C}, \quad \iota^C \equiv \prod_{i=1}^{N} \overline{\omega}_i^{C^{-\overline{\omega}_i^C}}$

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- *Misallocation*: let $\lambda_i \equiv \frac{P_i Y_i}{P^C C}$ be the **Domar weight** of sector i, then goods market clearing implies

$$\lambda_i = \overline{\omega}_i^C + \sum_{k=1}^N \overline{\omega}_{ki} \lambda_k \mu_k^{-1}, \qquad \mu_k \equiv \left(\int_0^1 \frac{1}{\mu_k(j)} \frac{P_k(j) Y_k(j)}{P_k Y_k} dj \right)^{-1}$$

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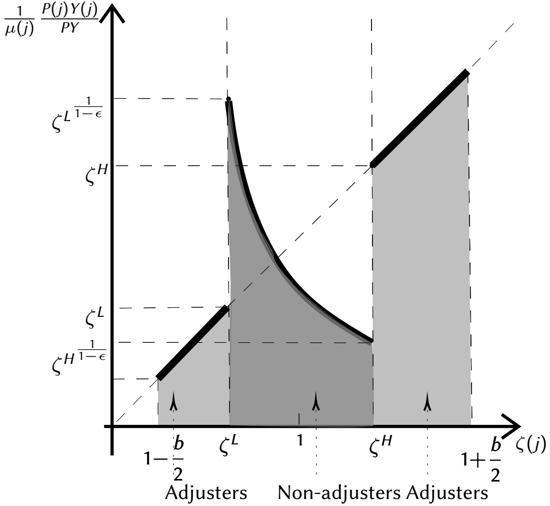
where μ_k^{-1} is the sales-weighted **harmonic average** of firm-level markups, can be expressed as

$$\mu_k^{-1} = \left(\underbrace{\int_{\zeta_k(j) \leq \zeta_k^L} \zeta_k(j) dj + \int_{\zeta_k(j) \geq \zeta_k^H} \zeta_k(j) dj}_{\text{Adjusters}} + \overbrace{\int_{\zeta_k^L < \zeta_k(j) < \zeta_k^H} \zeta_k(j)^{\frac{1}{1-\epsilon}} \mathcal{Q}_k^{\epsilon} dj}^{\text{Non-adjusters}} \right) \left(\frac{P_k}{\mathcal{Q}_k} \right)^{\epsilon - 1}$$

Within-sector misallocation



FIRST-ORDER PERTURBATIONS

Baseline Equilibrium:

$$\overline{P}_i = \overline{Q}_i = 1, \qquad \overline{\zeta}_i^{H,L} = 1 \pm \varepsilon\sqrt{\rho_i}, \quad \rho_i = \frac{\overline{W}\upsilon_i}{\overline{P}_i\overline{Y}_i}$$

Sectoral Variables

Sectoral prices

- Denote by $\alpha_i \equiv \frac{\overline{\zeta}_i^H - \overline{\zeta}_i^L}{b_i}$ the baseline sector-specific frequency of non-adjustment, then:

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The elasticity between the price index of sector i and a productivity shock to sector k is given by:

$$\frac{d \log P_i}{d \log A_k} = \frac{d \log \mathcal{Q}_i}{d \log A_k} = -\Psi_{ik}$$

*where Ψ_{ik} is the (i,k) entry of the **cost-based Leontief inverse**:*

$$\Psi \equiv (I - \overline{\Omega})^{-1}, \qquad \text{where} \quad [\overline{\Omega}]_{i,j} = \overline{\omega}_{ij}.$$

Within-sector misallocation

Proposition : Within misallocation

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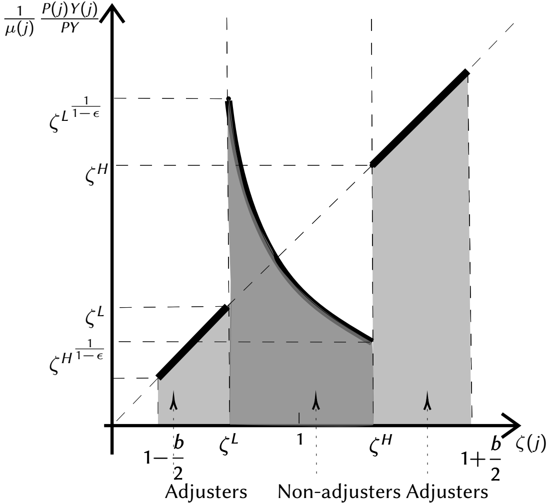

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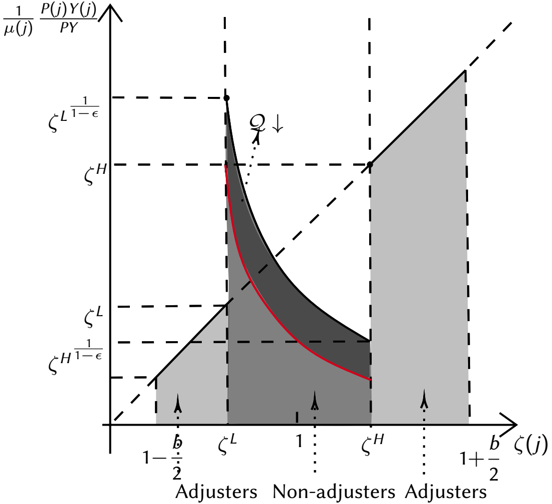

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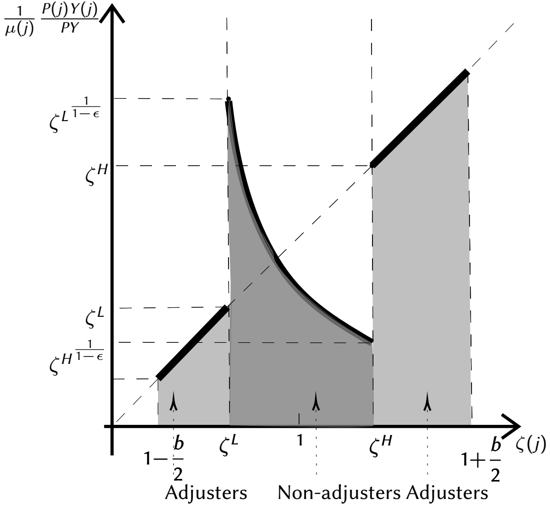

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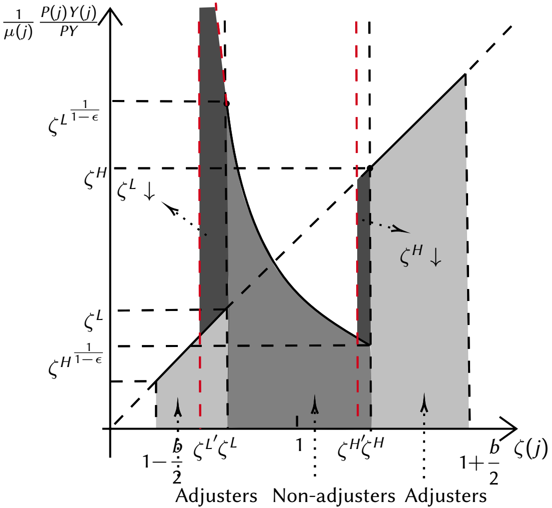

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Aggregation

Aggregate GDP

Proposition : Aggregate GDP

To a first order, the change in aggregate demand in response to shock in sector k is given by:

$$\frac{d \log C}{d \log A_k} = \tilde{\lambda}_k.$$

where $\tilde{\lambda}_k$ *is the sales share (Domar weight) in flexible-price equilibrium:*

$$\tilde{\boldsymbol{\lambda}} = (I - \overline{\Omega}^T)^{-1} \overline{\boldsymbol{\omega}}^{\boldsymbol{C}} = \Psi^T \overline{\boldsymbol{\omega}}^{\boldsymbol{C}}.$$

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- The first-order response of aggregate GDP is given by the Domar weight (sales share) in the flexible-price equilibrium
- Follows directly from the first-order full pass-through of sectoral marginal cost to the sectoral price

Aggregate Employment

- The aggregate employment can be expressed as:

$$L = 1 + \underbrace{\sum_i \lambda_i(\mu_i^{-1} - 1)}_{\text{Misallocation}} + \underbrace{\sum_i \rho_i \overline{\lambda}_i \left(1 - \frac{\zeta_i^H - \zeta_i^L}{b_i}\right)}_{\text{Menu cost payment}}$$

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- First-order change in aggregate employment:

$$dL = \underbrace{\sum_i d\lambda_i(\mu_i^{-1} - 1)}_{\text{①:pre-existing distortions}} - \underbrace{\sum_i \upsilon_i d\left[\frac{\zeta_i^H - \zeta_i^L}{b_i}\right]}_{\text{②: menu cost payment}}$$

$$+ \underbrace{\sum_i \lambda_i \frac{\varphi_i^P}{b_i} d\log P_i}_{\text{③: aggregate price effect}} - \underbrace{\sum_i \lambda_i \left[\frac{\varphi_i^H}{b_i} d\zeta_i^H + \frac{\varphi_i^L}{b_i} d\zeta_i^L\right]}_{\text{④: aggregate bands effect}}.$$

Aggregate Employment

- First-order change in aggregate employment near the baseline:

$$dL\left(\{\sqrt{\rho_i}\}_i\right) = \underbrace{\sum_i d\lambda_i(\overline{\mu}_i^{-1} - 1)}_{\text{①:pre-existing distortions}} - \underbrace{\sum_i \overline{\lambda}_i \rho_i d\left[\frac{\zeta_i^H - \zeta_i^L}{b_i}\right]}_{\text{②: menu cost payment}}$$

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- For perurbations near $\sqrt{\rho_i} = 0$, the **aggregate bands effect dominates**:

$$\frac{dL}{d\log A_k} = \kappa \sum_i \left[\frac{\tilde{\lambda}_i \Psi_{ik}}{b_i}\right] \rho_i^{1.5} + \mathcal{O}\left(\sum_i \rho_i^2\right).$$

where $\kappa > 0$ is a combination of structural parameters

Sufficient Statistic for Aggregate Employment

Proposition : Sufficient Statistic

Denote by $\alpha_i \equiv \frac{\bar{\zeta}_i^H - \bar{\zeta}_i^L}{b_i}$ *the sector-specific frequencies of non-adjustment, and by* $\sigma_i^2 \equiv Var(\zeta_i)$ *the sector-specific variances of idiosyncratic shocks, then:*

$$\frac{dL}{d\log A_k} \approx \quad \xi \times \mathbb{E}_{\tilde{\boldsymbol{\lambda}}}[\boldsymbol{\sigma}^2 \boldsymbol{\alpha}^3] \times \mathcal{S}_k \quad + \quad \xi \times \ell \times Cov_{\tilde{\boldsymbol{\lambda}}}[\boldsymbol{\sigma}^2 \boldsymbol{\alpha}^3, \Psi_{(k)}]$$

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where $\xi \equiv \frac{(9\epsilon-4)\epsilon}{8(\epsilon-1)^2}$, $\ell \equiv \sum_i \tilde{\lambda}_i$, $\boldsymbol{\sigma}^2\boldsymbol{\alpha}^3 \equiv [\sigma_1^2\alpha_1^3, ..., \sigma_N^2\alpha_N^3]^T$, $\Psi_{(k)}$ *is the k'th col. of* Ψ *and*

$$\mathcal{S} \equiv \Psi^T\tilde{\boldsymbol{\lambda}} = (\Psi^T)^2 \times \overline{\boldsymbol{\omega}}_{\boldsymbol{c}}$$

is the ***supplier-of-suppliers (SS) centrality****.*

Aggregate TFP and Welfare

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$$TFP \equiv \frac{C}{L}$$

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Proposition : Welfare loss

Denote δ the consumption equivalent welfare cost associated with the presence of sectoral shock of magnitude $\Delta\log A_k$ to sector k and menu costs, then

$$\delta = 1 - \exp\left\{-\frac{dL}{d\log A_k}\Delta\log A_k\right\}$$

QUANTITATIVE EXERCISES

Calibration

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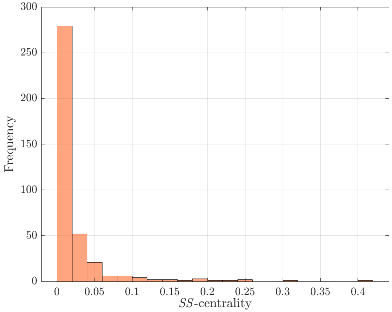
- All in all, work with 295 sectors of the US economy

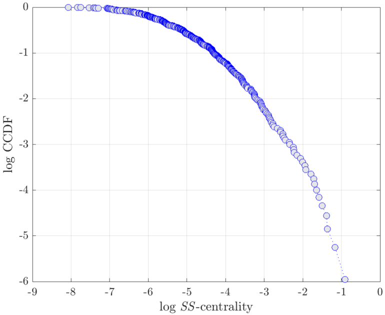
SS-centrality

Distribution of *SS*-centrality (United States)

(a) Histogram of *SS*-centrality

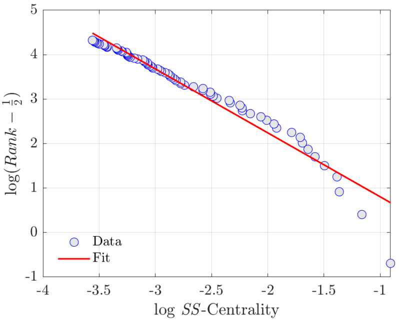


(b) Empirical C-CDF

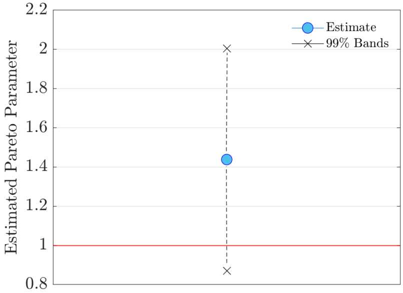


Estimated Pareto distribution for *SS*-centrality

(a) Log rank regression

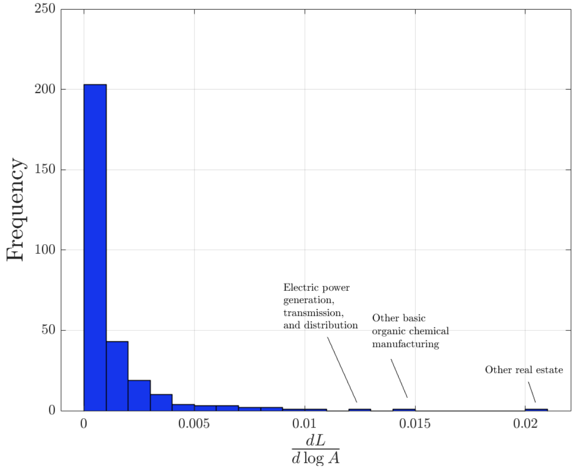


(b) Estimated Pareto parameter

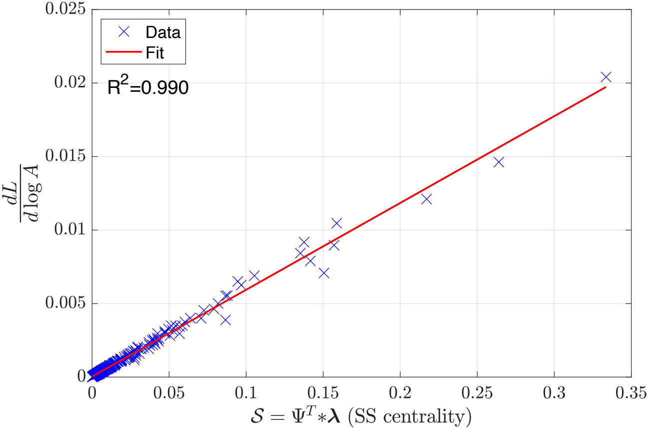


Aggregate Employment

Distribution of aggregate employment effects $\frac{dL}{d\log A_i}$

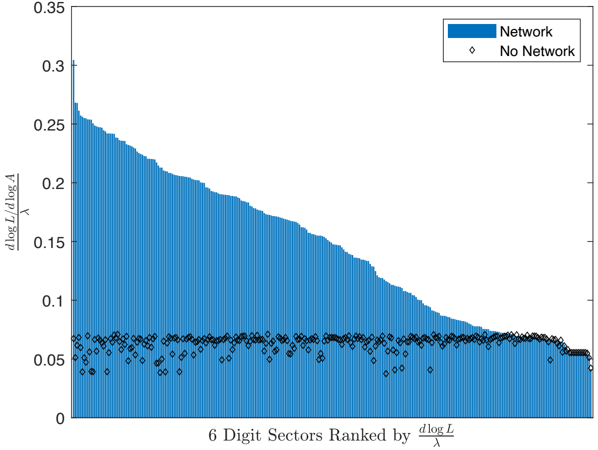


SS-centrality as a sufficient statistic



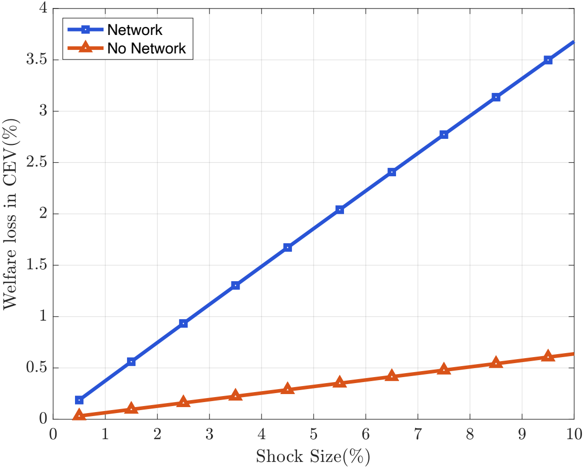
Aggregate Measured TFP Losses

Aggregated measured TFP loss: $\frac{d \log L}{d \log A_i}$ as a fraction of $\tilde{\lambda}_i$

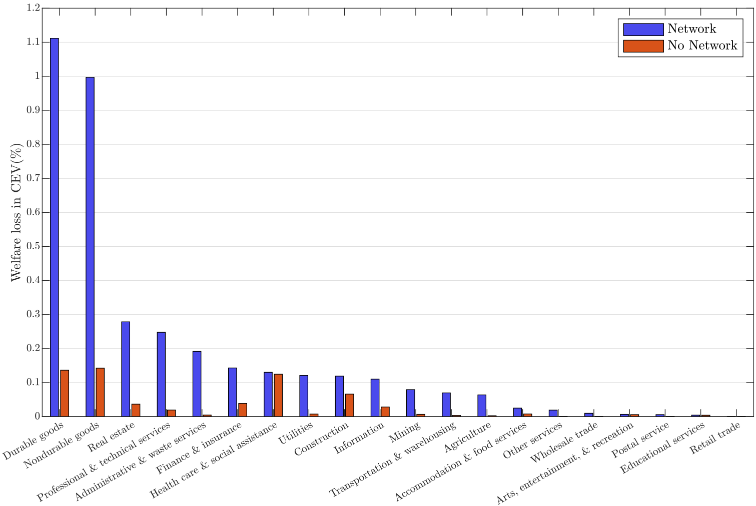


Welfare Losses

Welfare loss: aggregate productivity shocks



Welfare loss: sectoral productivity shocks



Conclusion

- Develop an analytically tractable multi-sector model with fully general input-output linkages and price changes subject to menu costs
- Obtain a novel aggregation result, which links macroeconomic variables to sectoral shocks, network centralities and pricing statistics
- While GDP is aggregated with sectoral sales shares, total employment and TFP (Welfare) loss are aggregated with the **supplier-of-suppliers (SS) centrality**

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- Future/ongoing work:
  - (i) Optimal monetary policy (Ferrari and Ghassibe, ongoing)
  - (ii) Higher-order perturbations in sectoral shocks, with implications for ex-post heterogeneity
  - (iii) Numerical analysis in a dynamic setting and under large shocks (Ghassibe and Nakov, ongoing)

APPENDIX