Sufficient Statistics for Measuring Forward-Looking Welfare

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Measuring Welfare

- Standard welfare measure converts income at τ into equivalent income in τ_0 dollars.
- For a static decision maker, income deflated by average price change.
- Foundation for real quantities in national accounts (e.g. real income) & cost of living.
- Dynamic stochastic decision maker's welfare depends on future uncertain outcomes.
- Requires future state-contingent constraints (prices & incomes), probabilities, plans.
- Such welfare measures typically calculated using fully specified structural models.

Theory Ahead of Measurement

- Sufficient statistics formulas of index number theory unusable for dynamic problems.
- This tension is a classic problem dating back to Fisher & Pigou.
- For example, Samuelson (1961) concludes with:

  "The futures prices needed for making the requisite wealth-like comparisons are simply unavailable. So it would be difficult to make operational the theorists' desired measures."

What We Do: Theory

- Develop sufficient-statistics method under two key assumptions:

  1. preferences are "separable" between the present and future.
  2. there exist "rentiers" that don't face idiosyncratic undiversifiable risk.

- Allow for incomplete markets, borrowing constraints, life-cycle motives, non-exponential time discounting, non-parametric & non-homothetic preferences.

- Without specifying beliefs or preferences about state-contingent prices and returns.

What We Do: Empirical Illustration

- Apply our methodology to the US using the PSID.
- Compute dynamic welfare and cost-of-living measures between $2005-2019$:
  - Dynamic welfare measures can be very different to static ones.
  - Dynamic non-homotheticicities an order of magnitude more important than static ones.
- Welfare measure can be used as outcome in reduced-form work:
  - Job loss associated with 20% reduction in welfare.

Agenda

Related Literature

- Static literature: Feenstra (1994), Hamilton (2001), Costa (2001), Almås (2012), Atkin et al. (2024), Jaravel and Lashkari (2024), Baqaee et al. (2024).
- Older dynamic literature: Samuelson (1961), Alchian and Klein (1973), Pollack (1975), Hulten (1979).
- More recent literature: Reis (2005), Aoki and Kitahara (2010), Jones and Klenow (2016), Basu et al. (2022), Del Canto et al. (2023), Fagereng et al. (2022).

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Stripped-down Example

General Environment

Empirical Illustration Using PSID data: Construction of Money-Metric

Empirical Illustration Using PSID data: Dynamics Treatment Effects

Conclusion

Complete markets with CRRA preferences

- Intertemporal preferences:

$$\mathcal{U}(\mathbf{c}, \boldsymbol{\pi}) = \sum_{t=0}^{T} \sum_{s_t} \beta(s^t) \pi(s^t) \frac{c(s^t)^{1-1/\sigma}}{1-1/\sigma}, \qquad \text{where} \qquad \sigma \neq 1.$$

- Subject to

$$\begin{aligned} p(s^0)c(s^0) + \sum_{k \in S_1} a_k(s^0) &= w, \\ p(s^t)c(s^t) + \sum_{k \in S_{t+1}} a_k(s^t) &= R_{s_t}(s^{t-1}) a_{s_t}(s^{t-1}), \\ p(s^T)c(s^T) &\leq R_{s_T}(s^{T-1}) a_{s_T}(s^{T-1}). \end{aligned}$$

- Value function $V(\{p, R, \boldsymbol{\pi}\}, w)$.

Money-Metric Utility

- There are cohorts with horizon T at each date τ facing different $\mathbf{p}$, $\mathbf{R}$, and $\boldsymbol{\pi}$.
- Index each cohort's problem by start date τ, i.e. value function is $V(\tau, w)$.
- Money-metric utility $u(\tau, w|\tau_0)$ is wealth in τ_0 that gives same utility:

$$V(\tau, w) = V(\tau_0, u(\tau, w|\tau_0)).$$

- Use $u(\tau, w|\tau_0)$ to measure growth (by varying τ or w) or inflation (by varying τ_0).
- Write $u(\tau, w)$ instead of $u(\tau, w|\tau_0)$ from now (hold base year τ_0 constant).

Towards Solution

- Define $c(s^t|\tau, w)$ to be consumption of cohort τ with initial wealth w in state s^t.
- Money-metric is also Lucas-number relative to consumption in τ_0 with unit wealth:

$$\mathcal{U}(u(\tau, w) \times \mathbf{c}(\cdot|\tau_0, 1)) = \mathcal{U}(\mathbf{c}(\cdot|\tau, w)).$$

- Combine with Euler equations and intertemporal budget constraint:

$$u(\tau, w) = w \Bigg/ \left[\frac{\sum_{t=0}^{T} \sum_{s_t} \left(\beta(s^t)\pi(s^t|\tau)\right)^{\sigma} p(s^t|\tau)^{1-\sigma} \prod_{l=0}^{t} R_{s_{l+1}}(s^l|\tau)^{\sigma-1}}{\sum_{t=0}^{T} \sum_{s_t} \left(\beta(s^t)\pi(s^t|\tau_0)\right)^{\sigma} p(s^t|\tau_0)^{1-\sigma} \prod_{l=0}^{t} R_{s_{l+1}}(s^l|\tau_0)^{\sigma-1}} \right]^{\frac{1}{1-\sigma}}.$$

- Need beliefs about discounting, probabilities, prices & returns.

Using Consumption-Savings Decisions to Back-Out Future Prices

- Denote consumption-wealth ratio by

$$B^P(\tau, w) = B^P(\tau) = \frac{p\left(s^0|\tau\right) c\left(s^0|\tau, w\right)}{w}.$$

- Consumption-wealth ratio satisfies

$$B^P(\tau) = \frac{p(s^0)^{1-\sigma}}{\sum_{t=0}^{T} \sum_{s_t} \left(\beta\left(s^t\right)\pi\left(s^t|\tau\right)\right)^{\sigma} p\left(s^t|\tau\right)^{1-\sigma} \prod_{l=0}^{t} R_{s_{l+1}} \left(s^l|\tau\right)^{\sigma-1}}.$$

- Plug into previous expression to get

$$\log u(\tau, w) = \underbrace{\log w - \log \frac{p\left(s^0|\tau\right)}{p\left(s^0|\tau_0\right)}}_{\text{“real wealth”}} + \underbrace{\frac{1}{1-\sigma} \log \frac{B^p(\tau)}{B^P(\tau_0)}}_{\text{adjustment for future}}.$$

- If consumption-wealth ratio rises and $\sigma < 1 \Rightarrow$ future is brighter.

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Conclusion

Time Separable Preferences

- Preferences rank consumption streams over multiple goods **c** with beliefs $\boldsymbol{\pi}$.
- Write utility function in implicit form, $D(\mathbf{c}, \boldsymbol{\pi}, U) = 1$, where D is H.O.D 1 in **c**.
- *Time separable* if

$$D(P(\mathbf{c}(s^0), U), F(\{\mathbf{c}(s^t)\}_{t>0}, \boldsymbol{\pi}, U), U) = 1,$$

  where P, F are scalar H.O.D 1 in **c** and D is H.O.D 1 in first two arguments.
- If homothetic, U separates from rest of D and can be moved to other side.
- Relative budget shares today not function of future prices (& vice versa), given U.
  - e.g. interest rates do not affect present relative budget shares, given U.

Time Separability Example

- Intertemporal non-homothetic CES:

$$\sum_{t=0}^{T} \beta^t U^{\varepsilon_t} \sum_{s_t} \pi(s^t) C\left(s^t\right)^{\frac{\sigma-1}{\sigma}} = 1,$$

$$C\left(s^t\right) = \left(\sum_n \omega_{nt} U^{\varepsilon_n} c_n(s^t)^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}}.$$

- Extends to non-homothetic Epstein and Zin (1989), where EIS $\neq$ risk aversion.

Environment

- First period budget constraint:

$$\sum_{n\in N} p_n(s^0|\tau)c_n(s^0|\tau) + \sum_{k\in K} a_k(s^0|\tau) = w.$$

- Each subsequent history s^t:

$$\sum_{n\in N} p_n(s^t|\tau)c_n(s^t|\tau) + \sum_{k\in K} a_k(s^t|\tau) = \sum_{k\in K} R_k(s^t|\tau)a_k(s^{t-1}|\tau) + y(s^t|\tau),$$

  where $y(s^t|\tau)$ is payment from assets that cannot be traded.

- Borrowing constraints

$$\sum_k a_k(s^t|\tau) \geq -X(s^t|\tau).$$

  No-ponzi requires $X(s^T) = 0$ for every s^T.

- *Rentiers*: $y(s^t|\tau) = 0$ for all s^t. (Extends to risk free $y(s^t|\tau)$ with more assumptions.)

Dynamic Money-Metric Utility

- Value function is

$$V(\underbrace{\{\mathbf{p}, \mathbf{R}, \boldsymbol{\pi}, \mathbf{X}\}}_{\text{indexed by } \tau}, w, \mathbf{y}) = \max_{\mathbf{c}, a} \{\mathcal{U}(\{\mathbf{c}, \boldsymbol{\pi}\}) : \text{constraints satisfied}\}.$$

- With incomplete markets, no single budget constraint, many possible compensations.

- Money-metric of decision problem $(\tau, w, \mathbf{y})$ in τ_0 dollars is $u(\tau, w, \mathbf{y})$ solving

$$V(\tau, w, \mathbf{y}) = V(\tau_0, u(\tau, w, \mathbf{y}), \mathbf{0}).$$

Equivalent lump sum in τ_0, rentier thereafter, to make agent indifferent.

Notation and Data Requirements

- Budget share of good n relative to current expenditures:

$$B_n(\tau, w, y) = \frac{p_n(s^0|\tau)c_n(s^0|\tau, w, y)}{\sum_{n \in N} p_n(s^0|\tau)c_n(s^0|\tau, w, y)}.$$

- Current consumption to wealth ratio:

$$B^P(\tau, w, y) = \frac{\sum_{n \in N} p_n(s^0|\tau)c_n(s^0|\tau, w, y)}{w}.$$

- EIS — log change in $\frac{B^P}{1-B^P}$ to uniform change in present prices: $\sigma(\tau, w, y)$.
- We observe cross-section of w, B^P, B_n, and $p(s^0|t)$ for each $t \in [\tau_0, \tau]$.
- Rentier households at date t are denoted by $(t, w, \mathbf{0})$.
- Consider special cases that build to general result.

Special Case 1: Homothetic Rentiers

Proposition

For homothetic rentiers with constant EIS,

$$\log u(\tau, w, \mathbf{0}) = \underbrace{\log w - \int_{\tau_0}^{\tau} \sum_n B_n(t) \frac{d \log p_n}{dt} dt}_{\text{“real wealth”}} + \underbrace{\frac{\log \left(B^P(\tau, \mathbf{0}) / B^P(\tau_0, \mathbf{0}) \right)}{1 - \sigma}}_{\text{adjustment for future}}.$$

- Extends example to allow for non-parametric static preferences & incomplete markets.
- Next: introduce non-homotheticities but make preferences static.

Special Case 2: Static Non-Homothetic Preferences

Proposition

Assume myopic households. Money-metric is solution to a fixed point problem:

$$\log u(\tau, w, \mathbf{0}) = \underbrace{\log w - \int_{\tau_0}^{\tau} \sum_{n \in N} B_n(t, w_t^*, \mathbf{0}) \frac{d \log p_n}{dt} dt}_{\substack{\textit{wealth deflated} \\ \textit{w/ compensated spending}}}, \tag{1}$$

where for each $t \in [\tau_0, \tau]$, w_t^* *satisfies the equation*

$$u(t, w_t^*, \mathbf{0}) = u(\tau, w, \mathbf{0}). \tag{2}$$

- Guess $u(\tau, w, \mathbf{0})$ — obtain w_t^* using (2), update guess using (1).
- Infer compensated demand from observed demand of matched household w_t^* (Baqaee et al., 2024).

General Case: Combining Special Cases

Proposition

Money-metric is solution to the fixed point problem:

$$\log u(\tau, w, \mathbf{0}) = \underbrace{\log w - \int_{\tau_0}^{\tau} \sum_{n \in N} B_n(t, w_t^*, \mathbf{0}) \frac{d \log p_n}{dt} dt}_{\substack{\text{wealth deflated} \\ \text{w/ compensated spending}}} - \underbrace{\int_{\tau_0}^{\tau} \frac{d \log B^P(t, w_t^*, \mathbf{0})/dt}{1 - \sigma(t, w_t^*, \mathbf{0})} dt}_{\substack{\text{adjustment for} \\ \text{future}}},$$

where for each $t \in [\tau_0, \tau]$, w_t^* satisfies the equation

$$u(t, w_t^*, \mathbf{0}) = u(\tau, w, \mathbf{0}).$$

- Must use compensated $\frac{d \log B^P(t, w_t^*, \mathbf{0})}{dt}$ and EIS, obtained from matched household w_t^*.

Money Metric for Non-Rentiers

- Time separability implies budget shares depend *only* on static relative prices and utility.
- If budget shares are one-to-one functions of utility, then there is mapping from budget shares and time into $u(\tau, w, y)$ for every τ, w, and y.
- In practice:
  1. Regress wealth on budget shares and time for rentiers.
  2. Use fitted relationship to impute wealth for non-rentiers.
- Worker's money-metric utility is wealth of rentier with same spending shares and time.

Extension 1: Risk free cash-flows y

- Consider a subset of households with risk-free cash flow $y(s^t|\tau) = y(t|\tau)$.
- For example, public sector employees, teachers, pensioners on defined benefits, etc.
- For these households, assume no ad-hoc borrowing constraints & access to bonds of maturities $\{1, \ldots, T\}$.
- These households' problem is isomorphic to rentier with augmented wealth

$$w(s^0|\tau) + \sum_{t=0}^{T} \frac{y(t|\tau)}{R(t|\tau)},$$

where $R(t|\tau)$ is return on bond with maturity t purchased in τ.

- Do not pursue this in empirical application (for now).

Extension 2: Changes in mortality

- Let λ_P and λ_F be prob. of reaching P and F.
- Marginal willingness to pay for increasing survival probabilities: $\Phi_P(\tau, w)$, $\Phi_F(\tau, w)$.
- Money-metric solves:

$$\log u(\tau, w, \mathbf{0}) = \underbrace{\log w - \int_{\tau_0}^{\tau} \left(\sum_{n \in N} B_n(t, w_t^*) \frac{d \log p_n}{dt} - \frac{1}{1-\sigma} \frac{d \log B^P(t, w_t^*)}{dt} \right) dt}_{\text{what we had before}}$$

$$\underbrace{- \int_{\tau_0}^{\tau} \left(\Phi_P(t, w_t^*) \frac{d \log \lambda_P(t)}{dt} + \Phi_F(t, w_t^*) \frac{d \log \lambda_F(t)}{dt} \right) dt}_{\text{compensated value of increased survival}}$$

$$\underbrace{+ \int_{\tau_0}^{\tau} \frac{\sigma}{1-\sigma} \left(1 - B^P(t, w_t^*)\right) \frac{d \log \lambda_F(t)}{dt} dt}_{\text{changes in consmption/wealth ratio due to } d\lambda_F}.$$

Extension 3: Leisure

- Results unchanged for rentiers if, conditional on observables, leisure choices do not change as a function of calendar time (e.g. labor productivity = 0, or 9-to-5 job).

- Results unchanged for non-rentiers if relative static budget shares only depend on utility and static prices of goods and services.

- Rules out non-separabilities between consumption choices and leisure.

- Example:

$$U^{\frac{\sigma-1}{\sigma}} = \tilde{P}\left(c\left(s^0\right), U\right)^{\frac{\sigma-1}{\sigma}} + \tilde{F}\left(\left\{c(s^t)\right\}_{t>0}, \boldsymbol{\pi}, U\right)^{\frac{\sigma-1}{\sigma}} + \tilde{H}\left(\left\{l(s^t)\right\}_{t\geq 0}, \boldsymbol{\pi}, U\right).$$

Assumption on Common Preferences, Prices, and Probabilities.

- Conditional on observables, rentiers and non-rentiers have same preferences relation both in the cross-section within each period and across cohorts at different times.
- Prices w/in a period can only vary as a function of observable characteristics.
- Future prices, returns and beliefs can change over time, but:
  - across rentiers, can only vary as a function of observable characteristics,
  - between non-rentiers and rentiers, can vary given other characteristics.
- We do not require that households' beliefs about the future be "objective" in any sense.

Agenda

Data Requirements

- Household survey with financial net worth, age, consumption survey, subset of rentiers.
  (We use PSID, bi-annual from $2005-2019$).
  (Group age into decade of life.)

- Prices of goods and services.
  (CPI prices).

- Elasticity of intertemporal substitution $\sigma(\tau, w)$
  (use Best, Cloyne, Ilzetzki, and Kleven 2020 of $\sigma = 0.1$.)
  (if consumption is a normal good, then compensated EIS $<$ uncompensated EIS.)

Classifying Rentiers

- Proxy wealth = net assets (including DC) + discounted labor income $+$ transfers.
- Forecast income using cross-section $+$ CBO forecast of NGDP.
- Discount future labor income and transfers by 4$\%$ real rate (Catherine et al., 2022).
- Rentiers: Net financial assets $\geq$ 90$\%$ of total wealth & not unemployed.
- Drop from rentier set if net assets are in the top and bottom 2.5$\%$.

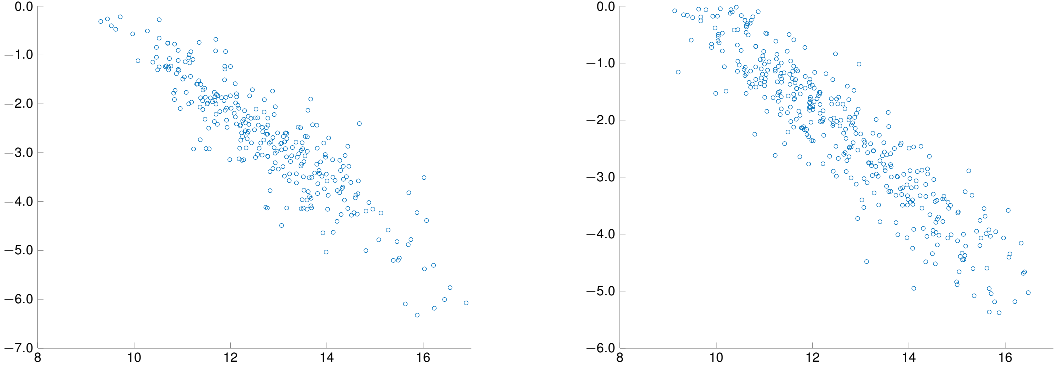
Expenditures

- Total consumption is expenditures on seven consumption categories.
  - (food, housing, transportation, education, health, clothing, and recreation)
- For home-owners, impute housing by matching owners to similar renters.
- In 2019, survey includes new question on owner-occupied housing costs.
- Regress surveyed housing costs on imputed housing costs, $\beta = 1.03$, $R^2 = 0.59$.
- Non-parametric kernel regression consumption-wealth ratio for rentiers:

$$\log B^P_{h,\tau} = \hat{B}(\log \text{wealth}_{h,\tau}, \tau, age_{h,\tau}) + \text{error}_{h,\tau}.$$

- Static budget shares regressed on flexible function of age, wealth, and date.

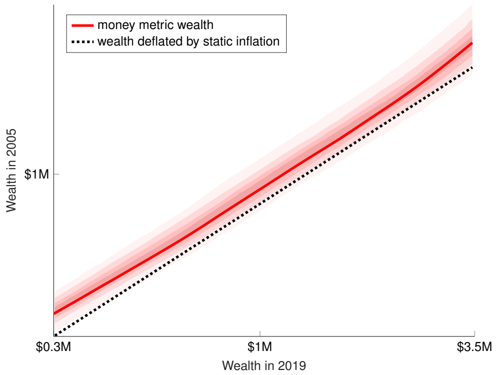
Log consumption-wealth against log wealth



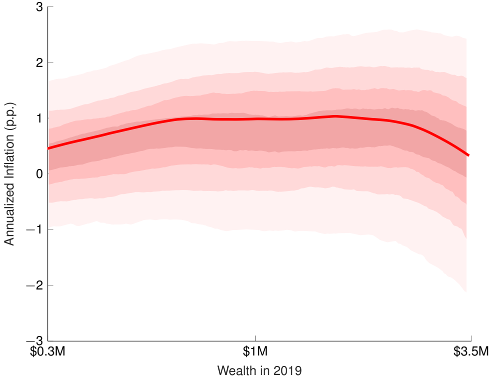
(a) 2005 (b) 2019

- Consumption-wealth ratio strongly declining in wealth (as in Straub 2019).

Money-metric wealth in 2005 base prices for 60-69 year olds



(a) Money-metric

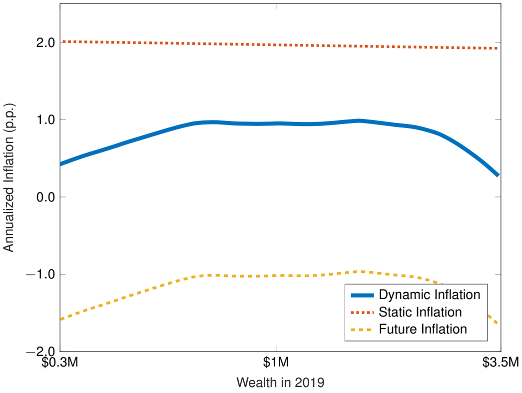


(b) Annualized inflation

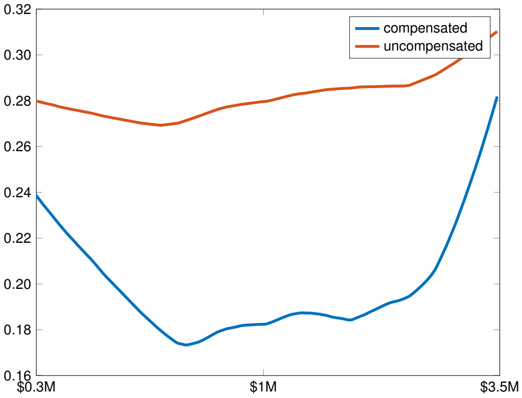
- Money-metric converts wealth in 2019 into equivalent in 2005 and vice versa.
- Black line deflates nominal wealth in 2019 by the static CPI between 2005 and 2019.

Decomposing Inflation (example for 60 – 69 year olds)

$$\log \frac{w}{u_{2005}(2019, w)} = \underbrace{\int_{2005}^{2019} \sum_{n \in N} B_n(t, w_t^*, \mathbf{0}) \frac{d \log p_n}{dt} dt}_{\text{static inflation}} - \underbrace{\frac{1}{1-\sigma} \log \left(\frac{B^P(2019, w, \mathbf{0})}{B^P(2005, w_{2005}^*, \mathbf{0})} \right)}_{\text{future relative to static inflation}}.$$



Change in log consumption wealth ratios 2005 – 2019 (60 – 69 year olds)



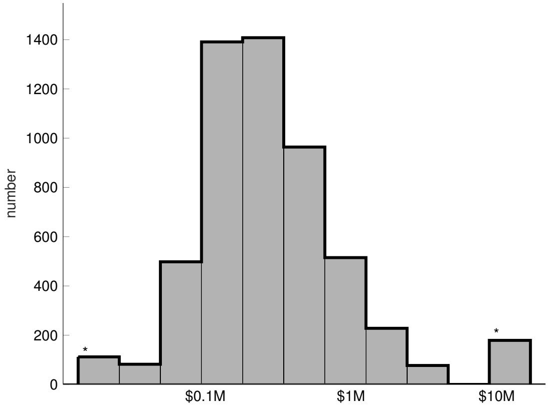
- Consumption-wealth ratios grew — but compensated less than uncompensated.

Imputing Money-Metric Wealth for Non-Rentiers

Fit rentiers' wealth to budget shares, squared budget shares, and age bin for each date:

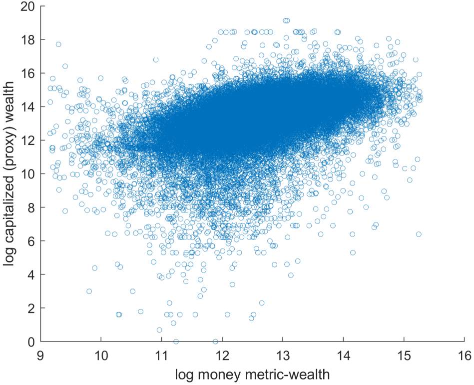
$$\log w_{h,\tau} = \boldsymbol{\alpha}'_{\tau} X_{h,\tau} + \text{error}_{h,\tau}.$$

Use fitted relationship to predict money-metric for non-rentiers.



Capitalized Wealth against Money-Metric Wealth (in logs)

(a) Rentiers



(b) Non-rentiers

Agenda

Empirical Application

- Consider a treatment that affects households in many ways.
- For example, job training program, educational investment, interest rate policy, etc.
- These treatments have dynamic effects, potentially on many relevant dimensions.
- Our money-metric estimates can be used to study welfare treatment effect.
- Example: job loss for head of household using PSID.

Percent change in money-metric wealth due to job loss

	log nominal money metric				log money metric 2019 dollars	
	(1)	(2)	(3)	(4)	(5)	(6)
Job Loss	-0.197	-0.218	-0.099	-0.108	-0.098	-0.107
	(0.031)	(0.034)	(0.031)	(0.038)	(0.031)	(0.038)
Job loss $\times$ $1(\text{age} \geq 60)$		0.180		0.050		0.045
		(0.083)		(0.080)		(0.074)
Lagged LHS	Yes	Yes	Yes	Yes	Yes	Yes
Controls	Yes	Yes	Yes	Yes	Yes	Yes
Observations	48,357	48,357	33,987	33,987	33,987	33,987
	Full Sample		Overlapping Support			

Controls: year fixed effects, age group, marital status of HH head, industry, and education level.

- Davis & von Wachter (2011): NPV of earnings fall by 12% after mass lay-offs.
- Differences, e.g. incomplete markets, risk, present bias, ex-post vs. ex-ante.

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Conclusion

- Generalize money-metrics to intertemporal preferences, risk, and incomplete markets.
- Use time-separability to infer it from consumption-savings for rentiers.
- Match rentiers and non-rentiers using budget shares.
- Static and dynamic different, with heterogeneity in wealth & age.
- Ingredient for policy evaluation of shocks that affect future.

Proof Sketch

1. There exist shadow prices q^* that "rationalize" consumer's choices:

$$c_n^*(s^t|q^*, \boldsymbol{\pi}, V(\tau, w, y)) = c_n(s^t|\tau, w, y)$$

   with shadow prices for goods in first period equal to observed prices:

$$q_n^*(s^0|\tau, w, y) = p_n(s^0|\tau).$$

2. Dual shadow prices for rentiers depend on τ and V — not the case for non-rentiers.

3. Money-metric is expressible using shadow intertemporal expenditure function:

$$u(\tau, w, \mathbf{0}) = e(q^*(\cdot|\tau_0, u(\tau, w, \mathbf{0})), \boldsymbol{\pi}(\cdot|\tau_0), u(\tau, w, \mathbf{0})).$$

4. Manipulate to get:

$$\log u(\tau, w, \mathbf{0}) = \log w - \log \frac{e(q^*(\cdot|\tau, u(\tau, w, \mathbf{0})), \boldsymbol{\pi}(\cdot|\tau), u(\tau, w, \mathbf{0}))}{e(q^*(\cdot|\tau_0, u(\tau, w, \mathbf{0})), \boldsymbol{\pi}(\cdot|\tau_0), u(\tau, w, \mathbf{0}))}.$$

Proof Sketch

5. Using fundamental theorem of calculus:

$$\log u(\tau,w,\mathbf{0}) = \log w + \int_{\tau}^{\tau_0} \sum_{t=0}^{T} \sum_{s^t} \left(\frac{\partial \log e(q^*\left(s^t|t,u(\tau,w,\mathbf{0})\right), \boldsymbol{\pi}(s^t|t), u(\tau,w,\mathbf{0}))}{\partial \log q^*(s^t|t,u(\tau,w,\mathbf{0}))} \cdot \frac{d \log q^*\left(s^t|t,u(\tau,w,\mathbf{0})\right)}{dt} + \frac{\partial \log e(q^*\left(s^t|t,u(\tau,w,\mathbf{0})\right), \boldsymbol{\pi}(s^t|t), u(\tau,w,\mathbf{0}))}{\partial \boldsymbol{\pi}(s^t|t)} \cdot \frac{d\boldsymbol{\pi}(s^t|t)}{dt} \right) dt.$$

6. Cut through the complexity using time-separability:

$$\frac{\partial \log e(q,\boldsymbol{\pi},U)}{\partial \log q} \cdot d\log q + \frac{\partial \log e(q,\boldsymbol{\pi},U)}{\partial \boldsymbol{\pi}} \cdot d\boldsymbol{\pi} = -\frac{d \log b^P(q,\boldsymbol{\pi},U)}{1-\sigma^*(q,\boldsymbol{\pi},U)} + \sum_{n\in N} b_n(q(s^0),U) d\log q_n(s^0).$$

7. Substitute this back in to get desired result.

8. Idea from Baqaee et al. (2024) that compensation is fixed point.

Static non-homotheticity

- Consider non-homothetic CES preferences of the form:

$$U^{\frac{\sigma-1}{\sigma}} = U^{\varepsilon_0} C\left(s^0\right)^{\frac{\sigma-1}{\sigma}} + \sum_{t=1}^{T} \beta^t U^{\varepsilon_t} \sum_{s_t} \pi(s^t) C\left(s^t\right)^{\frac{\sigma-1}{\sigma}}$$

  where

$$C\left(s^0\right) = \left(\sum_n \omega_{n0} U^{\varepsilon_n} C\left(s^0\right)^{\zeta_n} c_n(s^0)^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}}$$

  and for $t > 0$:

$$C\left(s^t\right) = \left(\sum_n \omega_{nt} U^{\varepsilon_n} c_n(s^t)^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}}.$$

- In period 0, budget shares depend on static relative prices and both wealth & expenditures.

Static non-homotheticity

- For rentiers,

$$\log u(\tau, w, \mathbf{0}) = \log w - \int_{\tau_0}^{\tau} \Bigg[\sum b_n(t, w_t^*) \frac{d \log p_{n0}}{dt} - \frac{1}{(1-\sigma)} \frac{d \log b_P(t, w_t^*)}{dt}$$
$$- \left(1 - b^P(t, w_t^*)\right) \frac{\sigma}{(1-\sigma)} \frac{d}{dt} \log \left(1 + \frac{\gamma}{1-\gamma} \mathbb{E}_{b_n}[\zeta] \right)$$
$$+ \frac{\gamma \mathbb{E}_{b_n}[\zeta]}{1 - \gamma + \gamma \mathbb{E}_{b_n}[\zeta]} \left(\frac{d}{dt} \log e_0(t, w_t^*) - \sum_n b_n(t, w_t^*) \frac{d}{dt} \log p_n \right) \Bigg] dt.$$

- First line is same as our benchmark.
- The second and third lines are new.
- Requires knowledge of ζ — not identified from Engel curves.

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