# A Theory of Price Caps on Non-Renewable Resources<sup>\*</sup>

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July 2, 2024

#### Abstract

What is the optimal response by a producer of an exhaustible resource faced with sanctions in the form of a cap on the sale price? Motivated by the recent price cap on the sales of Russian oil, this paper provides a dynamic framework to answer this question and explore the broader implications. Within this model, a financially constrained exporter exercises market power and the resource's price is subject to stochastic fluctuations. Our analysis reveals that the introduction of a binding price cap may prompt the exporter to ramp up extraction efforts. This effect is particularly pronounced when the exporter wields significant market power; a comprehensive and permanent price cap on sales can effectively curtail the use of this power, potentially leading to lower and more stable global prices. However, the cap's efficacy is greatly reduced under conditions that have in fact been observed in the recent episode, that is when the enforcement is lax, policy is perceived as temporary, or the sanctioned entity can leverage alternative, non-compliant distribution networks.

JEL: F51, L13, L71, Q41.

<sup>&</sup>lt;sup>\*</sup>We thank Mark Aguiar, Ben Moll, Morten Ravn, Elina Ribakova, Jose-Victor Rios-Rull, Stephen Salant and participants at several seminars and conferences for comments. We also thank the members of the Yermak-McFaul International Working Group on Russian Sanctions for useful discussions.

*Disclaimer:* Both Johnson and Wolfram were involved in the design and implementation of the price cap policy, Johnson as an informal advisor in various policy forums and Wolfram as the Deputy Assistant Secretary for Climate and Energy at the U.S. Treasury during 2021-22. Rachel has been a member of the Stanford Group on Sanctions since 2022. During this time this group has advocated for the price cap policy.

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# 1 Introduction

On December 5, 2022, the Price Cap Coalition, consisting of the G7, the European Union (EU), and Australia, responded to the continuing Russian invasion of Ukraine by imposing a cap on the price of seaborne Russian oil sold into global markets. Companies based in coalition countries are currently allowed to provide services that support Russian oil sales, including shipping, insurance and trade finance, but only if the price paid to Russia does not exceed \$60 per barrel. The Coalition's goal has been to reduce Russian revenue from oil sales, while also ensuring the uninterrupted flow of Russian oil to global markets, hence preventing a negative supply shock that could have adverse short-term consequences for the rest of the world.<sup>1</sup>

The development of the price cap policy can be viewed as a significant development in the realm of international economic policy. It represents a novel approach to sanctions in an era of globalization when some markets are dominated by few large autocratic producers. An effective price cap would mean that no country is too large to escape the consequences of sanctions. However, as we discuss below, the policy has not been adequately enforced, in part due to concerns that, if implemented fully, the cap would ultimately lead to reductions in Russian extraction and supply and a potentially damaging global oil supply shock. Whether these concerns are valid remains an open question, and the one we take up in this paper.

This episode has highlighted the urgent need for a new analytical framework to study policies such as the price cap. The existing models are inadequate for several reasons.

Firstly, static models commonly used in policy analysis can only provide a partial picture. Analyzing the price cap requires a dynamic framework where resource prices fluctuate due to both endogenous and exogenous factors. Given these fluctuations, a price cap might be binding today but expected to be non-binding in the future, influencing current policy effects. Additionally, a dynamic setting is essential to address issues related to the credibility of such policies.

Secondly, models for analyzing price cap policies must align with key empirical findings,

<sup>&</sup>lt;sup>1</sup>The price policy was developed in the context of the EU's "6th Sanctions Package", which was adopted in early June 2022. These measures included an embargo on the purchase of Russian oil from December 5, 2022, along with a ban on EU countries providing services in support of Russian oil exports. A similar ban on Russian oil products and services was slated for February 5, 2023. Since western services were used for a large share of Russian exports – over 70% by most accounts in the case of Russian seaborne crude trade (see Craig Kennedy, forthcoming) – there were concerns that this EU policy package could keep large volumes of Russian crude and product out of the market, effectively squeezing global supply and sharply raising oil prices everywhere. The price cap mechanism was designed to maintain the supply of Russian oil to world markets while squeezing Russian government revenue and sustaining the EU embargo.

including very low supply elasticities (close to zero or negative, implying vertical or slightly downward-sloping supply curves). The canonical frictionless model of Hotelling (1931) inaccurately predicts a perfectly elastic supply. Any deviation from the Hotelling Rule (prices increasing at the rate of interest) results in drastic changes in extraction rates, either to zero or full extraction.

The dynamic models of resource extraction at the frontier of the literature point to adjustment costs as the primary reason for short-run inelasticity of supply. While these costs are undoubtedly important in the day-to-day functioning of the market, there are at least two reasons why they cannot tell the whole story in our context. First, adjustment costs are likely asymmetric: it might be easier to decrease output than to increase it. Since policymakers are mainly concerned with supply cuts by sanctioned producers, adjustment costs may be less significant. Second, models of adjustment costs predict inaction regions: small shocks may not elicit a response, but large shocks could. Since we are interested in potentially large environmental changes for the extractor, we focus on economic incentives without adjustment costs.

Given our focus on the behavior of a national producer under sanctions, our model should incorporate several key features that are critical in the context in which these agents operate. Firstly, it is essential to consider that countries or governments often have access to income sources other than resource extraction, such as general taxation. Additionally, our model should align with findings from the literature on the resource curse, particularly the notion that volatility adversely impacts growth. Furthermore, considering the geopolitical context of our study, it is important to account for the potential complementary effects of other sanctions (e.g., financial sanctions) in conjunction with restrictions on commodity sales.

Our dynamic model fills these important gap. The framework focuses on the decision problem of a state exporter of a exhaustible resource. Sales of this resource fund a part of the producer's consumption, and financial frictions mean that the volatility in the path of its income matters for the time-path of the producer's consumption.<sup>2</sup> Furthermore, the price of the commodity varies stochastically over time, reflecting demand, supply or sentiment shocks. The final key element in our framework – which we add after we develop the baseline model – is that the producer has market power.<sup>3</sup>

<sup>&</sup>lt;sup>2</sup>These frictions are driven, in part, by both the (ex-ante) anticipated possibility of future sanctions and by the imposition of sanctions. Russia has substantial official foreign reserves, but these were frozen by the G7 immediately after the February 2022 Russian invasion of Ukraine. Since that initial freeze, Russia has been allowed to sell oil, and some other commodities, accumulating foreign assets in Gazprombank and other "private" entities. Russian authorities may be concerned about potential future freezes of those assets.

<sup>&</sup>lt;sup>3</sup>This is realistic in the current context, given that Russia is one of the the world's leading oil producers,

This approach yields several novel findings about the producer's behavior and the effects of the price cap. Using continuous time methods from macroeconomics and finance, we characterize the extraction policy function, considering the level of reserves and the ongoing commodity price. We demonstrate that the optimal supply curve is inelastic, even without physical adjustment costs. <sup>4</sup>

The close-to-vertical supply schedule emerges as a confluence of three forces. First, because the price of the resource varies over time, the producer has the incentive to *time the market*: extract and sell more when prices are high, and vice-versa when prices are low. This effect thus points to the usual upward slope in the supply curve. Second, since the producer lacks perfect insurance mechanisms and it dislikes price volatility, it attempts to self-insure by *smoothing revenues:* extracting more when prices are low. This acts to rotate the supply curve, making it steeper. Third, access to income unrelated to oil extraction – such as general taxation – means that the producer extracts the commodity faster on average and particularly so when the resource is worth relatively little. Thus, when prices are persistently low, the extraction rate is high. We refer to this as *non-oil income* effect. As a result of this force, the supply schedule rotates further.

The relative strength of these forces is governed by the degree of financial frictions. More severe frictions strengthen the revenue smoothing motive and weaken the desire to time the market.

In our full model, we consider a producer with sufficient size to influence global markets. Market power in this context is determined by the producer's market share, similar to the principles of Cournot competition. Consequently, the degree of market power is endogenous, evolving dynamically based on previous extraction decisions.

Our findings indicate that market power induces the producer to adopt a more conservationist approach, extracting resources more slowly than they would without such influence. This behavior exerts upward pressure on global prices.

Our analysis points to three important insights about the economics of the price cap. We start by analyzing a perfect price cap – one that applies to all of the sanctioned producer's sales and is permanent – and then turn to study the effects of an imperfect cap that might be leaky or non-credible.

oil prices spiked immediately after the 2022 invasion began, and a principal rationale for implementing the price cap policy was that a complete EU embargo – refusing to buy Russian oil and effectively blocking sales to third countries – could lead to a contraction in world oil supply and a spike in world prices of oil.

<sup>&</sup>lt;sup>4</sup>This finding is consistent with the evidence of a negative correlation between the price of oil and Russian extraction that we present in Section 2, and with the observation that Russian production has changed little in the face of large fluctuations in the oil price over the past few years.

Imposing a perfect price cap effectively changes the stochastic process for the price that the producer receives for its commodity: the cap eliminates the upside of high prices, making the stock of reserves less valuable and reducing uncertainty. We show that, as long as some alternative source of income is available, this leads to a more rapid depletion of reserves, all else equal. Thus, the supply curve shifts outwards: the price cap makes the producer extract more, not less. This finding - which goes against some of the intuition held by policymakers is driven by the non-oil income effect we described above: the reserves are worth less, and so are used up faster. Furthermore, the supply curve becomes close-to-vertical at prices above the cap.

If the producer has market power, implementing a binding, perfect price cap significantly diminishes the incentives to exercise market power in equilibrium. The logic behind this is simple: when the price cap is binding, curbing supply leads to lower volumes but unaltered prices, thereby rendering the use of market power ineffective. Thus, in the model with market power, there is an additional force that l;eads the supply curve to shift outwards.

This finding has important implications for the impact of the price cap. Most notably, and against the concern that is pervasive among policymakers, a binding price cap can actually *drive down* world oil prices and act as a *stabilizer* of the global oil market. Such positive effects are stronger the greater is the degree of market power of the producer. Overall, our study demonstrates that a price cap can be a potent tool and suggests that its benefits might actually be greater if it is applied to the exports of a producer with significant power in a market for a given commodity. Importantly though, these results hold when the producer has no ability to bypass the price cap regime.

Our final set of results concerns the effects of an imperfect price cap, i.e., a cap that applies to only a share of a country's sales of a commodity and/or is expected to be temporary. This analysis is important because monitoring and enforcement of any cap is likely to be imperfect and the sanctioning coalition is likely to be able to impact only a certain part of exporter's sales. Moreover, if the cap is expected to be temporary, the producer might respond very differently to when it is expected to be essentially permanent. Our findings show that if world prices are high, so that sufficient revenues can be generated through sales outside of the price cap regime, the producer may have strong incentives to "shut-in" production and instead sell the reduced quantities only outside of the regime. This is because, with an imperfect cap, the incentives to exercise market power remain. This dampens the stabilizing effect of the policy. However, our simulations also suggest that shutting in production can be costly in terms of contemporaneous profits, and that the impact on world prices is manageable. We also find that the expectation that the price cap is lifted at some point in the future increase the incentive to shut-in today in response to the price cap.

These results have important policy implications. They emphasize the importance of effective enforcement of the price cap and highlight the benefits of credible commitment to keep the policy in place for a long time.

Literature and contribution. The price cap on a non-renewable resource such as oil is a new and live policy and there is little direct literature on this topic, which motivates this project. Early analysis of the economics of the price cap on Russian oil appears in Wolfram et al. (2022) and in Johnson et al. (2023). In a recent paper that complements ours with the empirical analysis of the cap, Babina et al. (2023) use customs data to provide evidence on the effectiveness of the cap imposed by the G7 on Russia. They find that sanctions have led to a fragmentation of the oil market, with the oil that was destined to Europe trading at steep discounts and below the cap, while the oil sold elsewhere trading at close to global prices. In a complementary theoretical contribution, Salant (2023) studies the effects of pre-announcing the price cap. Sappington and Turner (2023) investigate the impact of a price cap in a static two producer Cournot model. Wachtmeister et al. (2023) consider what different price cap levels imply for net losses of Russia. Baumeister (2023) provides a broader overview of the developments in the oil market over since the Covid-19 pandemic. Price caps have also been examined in other contexts, in the industrial organization or urban economics literatures – see e.g. Bulow and Klemperer (2012) and Leautier (2018) and references within. More broadly, this paper contributes to the rapidly expanding literature on geoeconomics, which studies the interplay between economic relationships, international politics and power (see Clayton et al. (2023) and references within).

The framework we employ in the analysis builds on the classic work by Hotelling (1931), but appends it with the stochastic price of oil as studied in the finance literature (see Cox et al. (1985), Longstaff and Schwartz (1992), Chen and Scott (1993), Duffie and Kan (1996) for models of interest rates, and Schwartz and Smith (2000) and Pindyck (1999) for models of commodity prices) and develops a tractable way to think about market power. The Hotelling framework has been studied and extended in numerous studies.<sup>5</sup> A notable contribution is that of Anderson et al. (2018) who study the role of capacity constraints and drilling decisions

<sup>&</sup>lt;sup>5</sup>Classic references include Solow (1974), Stiglitz (1976), Dasgupta and Heal (1974), Pindyck (1980), Arrow and Chang (1982). For an overview of work in the 50 years after the publication of Hotelling's article, see Devarajan and Fisher (1981). Recent work includes van der Ploeg and Withagen (2012), Newell and Prest (2017), Salant (2012) and Gaudet (2013) and most recently Harstad (2023), who considers the dynamic game between successive governments controlling an exhaustible resource.

- both margins which we abstract from – and an earlier work by Salant (1976) who studies the extraction problem in a framework with realistic industrial organization structure of the world market. Our paper analyzes the impact of the price cap on the extraction decisions and world oil prices, stopping short of analyzing the general equilibrium impact on the global economy. A complementary paper by Bornstein et al. (2023) develops a quantitative general equilibrium macroeconomic model with oil production sector, and uses it to study the advent of fracking. For broader overview of forces that drive oil prices, see Hamilton (2009).

**Structure.** The rest of the paper is structured as follows. Section 2 describes Russia's oil sector, including its costs, the prices it faces and typical export volumes and routes, and provides some institutional context on the price cap that is relevant to our model. Section 3 presents the baseline model without market power, and Section 4 studies the effects of the price cap in this setting. In Section 5 we construct our equilibrium model with market power, and in Section 6 we study the effects of the price cap on the degree of market power exercised in equilibrium. Section 7 considers the case where the producer can partially bypass the price cap. Finally, Section 8 concludes with a discussion of policy implications and directions for future research.

# 2 Motivating facts and background on price cap policy

#### 2.1 Oil extraction in Russia historically

In the 1970s and 1980s, Russia was the world's largest oil producer, but with the fall of the Soviet regime, oil production dropped to as low as 6 million barrels of oil per day compared to a high of over 11 million barrels of oil per day in the late 1980s (left panel of Figure 1). Major investment beginning in the mid-1990s, along with access to western oil field services, helped to restore production to more than 10 millions barrels per day by 2019, which made Russia the third largest oil producer in the the world (after the US and Saudi Arabia). In recent years, most Russian production has been exported (7.5-8 million barrels per day, out of production of 10-10.5 million barrels per day), making Russia the world's top exporter of crude oil and product combined.<sup>6</sup> The right panel of Figure 1 plots monthly production in the last 5 years, highlighting the major disruption around the pandemic and

<sup>&</sup>lt;sup>6</sup>https://www.iea.org/reports/russian-supplies-to-global-energy-markets/ oil-market-and-russian-supply-2



Figure 1: Russia's oil extraction historically: annual, 1970-2020 (left panel) and monthly, January 2018-March 2023 (right panel). Source: CEIC (https://www.ceicdata.com) (left) and U.S Energy Information Administration (right).

the recovery gradual recovery since then. Note that the drop in extraction that coincided with the invasion of Ukraine in February 2022 was relatively small and short-lived.

Of the 7.5 million barrels per day exported by Russia in 2021, crude accounted for 4.7 million barrels and refined products for the remaining 2.8 million barrels.<sup>7,8</sup>

Most Russian oil is produced in Western Siberia and transported by pipeline to refineries and shipping facilities in Russia's Western ports. Before the war, Russia's largest oil customer was the European Union, which received 0.7 million barrels of crude oil per day by pipeline and 1.5 million barrels by sea in 2021. The EU also bought 1.2 million barrels of oil product, almost all of which arrived by sea. Overall, the EU imported almost half of Russia's total oil exports. Most of the tankers carrying these fossil fuels to the EU departed from three sets of ports: in the Black Sea, the Baltic Sea, and Murmansk in the far north.

China was also an important customer, and received 1.6 million barrels of crude per day

<sup>&</sup>lt;sup>7</sup>https://iea.blob.core.windows.net/assets/9aea25c1-5450-49db-8e1f-a67c0212720c/ -16MAR2022\_0ilMarketReport.pdf

<sup>&</sup>lt;sup>8</sup>A single barrel of crude oil can be processed to produce multiple refined products such as gasoline, diesel, jet fuel, and other derivatives of oil. Refineries can be designed to produce different mixes of refined products. The scope to change this is limited, especially in the short run. As of 2021, Russia's refining industry had the capacity to serve domestic gasoline demand and the country exported the remaining products. Substituting between exporting crude and exporting refined products is possible to some degree, but the infrastructure differs and there are pipeline and port constraints.

in 2021, half by pipeline and half by sea. China did not previously buy a significant quantity of Russia's refined product.

## 2.2 Russian oil exports since the start of the war

We now discuss the recent developments in Russia's oil trade, focusing first on quantities and then on prices.

Figure 3 plots Russia's seaborne crude oil exports by destination from January 2022 to September 2023.<sup>9</sup> Figure 4 plots Russia's oil product exports by destination from January 2022 to September 2023, almost all of which travels by ship.

Both figures paint a consistent picture. Shortly after the invasion of Ukraine in February 2022, Russia's export to the US and the UK quickly collapsed: these countries swiftly implemented embargoes. However, the US and the UK were never the main destination markets for Russian oil. Exports to the EU, Russia's largest customer, diminished much more gradually, and reached practically zero only after the implementation of the embargo on crude oil in December 2022 and on oil product in February 2023. The overall level of exports has remained steady, however, with significant substitution away from the western markets towards buyers from Asia, most notably India, which has previously imported very little oil from Russia.

We discuss the implementation of the price cap policy below in more detail. It is worth noting at this point, however, that the steady level of exports from Russia to the global market has been the intended outcome of the policy mix implemented by the G7 and other coalition countries. These countries have aimed to reduce revenues from oil sales *without taking Russian supply off the global market*, thus avoiding the risk of a damaging global oil supply shock.

While the quantities of Russian oil exported have been high, prices received for these exports have declined. Immediately after the invasion, some shipping companies and customers declined to do business with Russia, resulting in a stigma that lowered the price paid for Russian oil. Consequently, Russian oil sold for a discount from the world benchmark price. News agencies report prices for "Urals" oil, which describes the mix typically sold by Russia, including oil from fields in the Urals, Volga and Western Siberian regions. Figure 2 plots the Urals discount (Urals price minus Brent price) since just before the invasion through September 2023. Urals prices are not based on publicly posted transactions but are

<sup>&</sup>lt;sup>9</sup>It does not reflect the approximately 1.5 million barrels of crude oil per day exported via pipeline, roughly half of which used to go to the EU and half to China.



Figure 2: Russian Urals price minus Brent price January 2022 - September 2023. Source: Thomson Reuters.

collected by reporting services, like Argus Media and S&P Global, who request quotes from traders. There is some doubt about the accuracy and representativeness of the prices that the reporting services are able to collect, particularly after the price cap was enacted. With these caveats in mind, we note that before the war, the Urals discount was usually small, reflecting the market value of Russia's blend of primarily heavy sour oil. The discount was largest at nearly \$40 in mid-April 2022, and then declined before increasing again in early December, as the price cap and the EU embargo were imposed. Oil sold out of Russia's Eastern ports, primarily to China, is priced relative to the benchmark "ESPO" price, referencing the Eastern Siberia-Pacific Ocean oil pipeline. ESPO prices are even less transparent than Urals prices, but historically traded close to Urals and have been discounted less than Urals since the war began. Specifically, estimates from the IEA suggest that in December 2022, when the EU embargo and the price cap came into force, the ESPO discount increased from \$6 to circa \$11 (Shapoval et al. (2024)).



Feb 2022 Apr 2022 Jun 2022 Aug 2022 Oct 2022 Dec 2022 Feb 2023 Apr 2023 Jun 2023 Aug 2023

Figure 3: Russia's seaborne crude oil exports by destination, January 2022 - September 2023. Dashed line indicates the start of the price cap policy for crude oil on December 5, 2022. Source: CREA.

## 2.3 Structural features of Russia's oil extraction

This paper's main objective is to develop a dynamic framework for analysis of behavior of a commodity producer under sanctions. While this aim is general and extends to any entity on whom commodity exports price controls are implemented, we are directly motivated by the experience of the past two years and the sanctions imposed on Russia. Given this, we now discuss several features that inform our modelling choices: storage and shut-in costs, marginal costs of production, the importance of oil in the federal budget and the decision-making power of the state, market power, and the historical correlation between prices and extraction volumes.

**Storage capacity and shut-in costs.** Aside from re-routing its exports, a country such as Russia has, in principle, two other options in response to sanctions: extract a given volume of oil and store it, or reduce extraction.

In terms of the first option, Russia has limited on-shore storage available, and most of this was already full when the 2022 invasion of Ukraine started.<sup>10</sup> Storage "on the sea" is

<sup>&</sup>lt;sup>10</sup>https://www.energyintel.com/0000017f-6982-d580-a37f-f99bdebb0000.



Feb 2022 Apr 2022 Jun 2022 Aug 2022 Oct 2022 Dec 2022 Feb 2023 Apr 2023 Jun 2023 Aug 2023

Figure 4: Russia's oil product exports by destination, January 2022 - September 2023. Dashed line indicates the onset of the price cap policy for oil products on February 5, 2023. Source: CREA.

available but costly: it requires chartering and insuring ships for the duration, as is thus unlikely to be a quantitatively meaningful option, especially beyond the very near term. For these two reasons, we abstract from this margin of adjustment in our model.

The other option is often referred to as "shut-in" of production, meaning closing down wells. Doing so might entail adjustment costs, and can create uncertainty regarding the costs of restarting extraction, as some wells that are once shut down might be costly or impossible to re-open. This is a particular concern for Russia, as some of its oil fields are old and access to advanced western technologies – which would likely be required to re-open closed wells or open up new ones – might be curtailed due to a host of import sanctions.<sup>11</sup> Nonetheless, as Figures 3, 4 and ?? in this Section show, a certain degree of adjustment of extraction volumes – to the order of 20% or so – is historically common.

Marginal costs. Naturally, marginal costs are an important element in the decision of the producer. Current estimates peg marginal costs at most Russian fields at \$10 to \$40 per

<sup>&</sup>lt;sup>11</sup>https://www.consilium.europa.eu/en/policies/sanctions/restrictive-measures-against-russia-over-ukrassanctions-against-russia-explained/.

barrel, with the high end generally reflecting longer run marginal costs of developing new fields.<sup>12</sup> For example, Osintseva (2021) estimates marginal costs across countries, and reports Russia's cost of \$19 per barrel. At its low point at the beginning of the COVID pandemic, the price of oil was around \$20-\$25 per barrel, which anecdotally has been above the marginal cost of production.<sup>13</sup> Since we are primarily interested in the immediate response to the price cap policy, we think the short-run cost estimate is the relevant one. Thus, we assume that the cost is \$19 in our central calibration.<sup>14</sup>

The assumption of roughly \$15 - \$20 per barrel marginal cost is also broadly consistent with the behavior of fiscal authorities in Russia. According to analysis by CREA (a Finnish research institute) of Russia's government's decisions in 2022, it appears that the Russian fiscal authorities adjust tax rates so that producers received (post-tax) around \$25 per barrel.

State decision-making power, and the federal budget. The above discussion highlights another important feature of oil extraction in Russia: the fact that ultimately the extraction sector is under a very strong influence of the central government. To reflect the power of the Russian state over its oil companies and its ability to require payment of expost profit taxes in our framework, we find it most natural to analyze the problem of a national-level decision maker in our model.

This conclusion is only strengthened by a very significant dependence of the Russia's fiscal budget on oil revenues. In 2021, oil (crude and product) was Russia's largest export by category, followed by natural gas and coal.<sup>15</sup> In total, energy accounted for over 50% of all export revenues, with oil accounting for 75% of energy exports. Oil and gas sales are a significant source of federal budget revenues, with the oil and gas share in total revenues generally between 40 and 50% over the past decade.<sup>16</sup>

**Financial constraints.** There are at least three reasons for why financial constraints likely play a major role in the decision-making of a sanctioned commodity producer. All of these

<sup>&</sup>lt;sup>12</sup>See the S&P Global estimates for long-run marginal costs: https://www.spglobal.com/ commodityinsights/en/ci/research-analysis/global-crude-oil-curve-shows-projects-break-even-through-204 html

<sup>&</sup>lt;sup>13</sup>At the time, media reported that a presentation to investors by Rosneft, Russia's largest state-owned oil company, indicated that this price still covered short-run marginal cost of circa \$15 per barrel.

<sup>&</sup>lt;sup>14</sup>A constant marginal cost is a conservative assumption in the context of our model, which finds inelastic (i.e. steep) supply curves. An increasing marginal cost would be an additional force driving an upward sloping supply curve.

<sup>&</sup>lt;sup>15</sup>https://oec.world/en/profile/country/rus

 $<sup>^{16}</sup>$ See e.g. Figure 4 in Chanysheva et al. (2021).

are applicable to the case of Russia.

First, even without any policy response at home or abroad, during time of materialization of geopolitical risks and heightened uncertainty, risk-averse international investors are almost inevitably reluctant to lend to a state that is being sanctioned.

Second, a price cap policy is likely to be employed as a result of major act of aggression, and wars are expensive to run. According to its Ministry of Finance, Russia has dramatically increased the outlays on military spending. Grozovski (2023) reports that in 2023 Russia has dedicated 40% of its budget to military needs. Guriev (2023) estimates military spending to be in excess of 6% of GDP. Such high spending on war puts significant pressure on the budget.

Third, a price cap policy might be coupled with financial sanctions, which can diminish the financial war chest and effectively cut off the exporter from international financial markets. This is precisely what occurred in the case of Russia in 2022. Western countries have frozen the \$300bn of the central bank reserves. In April 2022 US Treasury Department banned Russia from withdrawing funds held in US banks to pay off its debt obligations. Russian default followed (Itskhoki and Muhkin (2023), Bianchi and Sosa-Padilla (2024), Lorenzoni and Werning (2023)). As a result, borrowing from abroad is essentially impossible. And past saving turns out to be of limited use – corresponding to an ex-post rate of return that is negative.

For these reasons we view the fact that the producer is cash strapped as an integral part of the analysis. In our model, lower revenues translate into lower welfare, as the state is unable to fully isolate welfare-relevant consumption from revenue fluctuations. This is motivated by and consistent with the literature that has quantified a strong link between sectoral concentration, sectoral shocks, and macroeconomic volatility (Koren and Tenreyro (2007), van der Ploeg and Poelhekke (2009), Aghion et al. (2009)).

Market power. As the world's largest exporter of crude and product combined, Russia has a significant degree of market power. It can thus exert short-run influence over oil prices through its announcements and actions. The 2022 invasion of Ukraine, for example, pushed world oil prices up by nearly 40 percent from the end of February to June 2022, presumably at least in part because participants in the oil market were concerned about potential disruptions to Russian supply. In addition, Russia belongs to the OPEC Plus cartel, which periodically sets production quotas and has considerable influence on world prices. Correspondingly, we make the market power of the producer a central tenet of our

framework.

**Price volatility and price-extraction correlation patterns.** One of the main uncertainties facing a producer of a commodity is the price of that commodity in the global market. For example, prices of oil fluctuate on a daily basis. This is important for the analysis of the price cap: in a dynamic setting, even if the price cap does not bind today, because the price is low, it might bind in the future. In other words, an appropriately enforced and credible price cap might reduce the upside from future upward movements in the price of the commodity. To make sure these considerations are reflected in our framework, we put the stochastic properties of the price centre-stage.

#### 2.4 Price cap: implementation details and enforcement challenges

The price cap operates by setting terms and conditions on the provision of western financial and shipping services. Specifically, services can only be provided for shipping Russian oil by companies located in price cap coalition countries if the price paid to Russia is at or below the cap.<sup>17</sup> The caps were initially set at \$60 per barrel for crude, \$100 per barrel for high-value refined products (including diesel, gasoline and kerosene) and \$45 per barrel for low-value refined products (including fuel oil and naphtha).<sup>18</sup>

It is important to stress that the price cap was implemented in response to the EU's 6th sanctions package, which would have banned the provision of services for shipments of Russian oil altogether and could have reduced the supply of Russian oil to world markets considerably. The price cap effectively allows for an exception to that outright ban.

Several ex post analyses examine some of the impacts of the price cap, including Harris (2023), Hilgenstock et al. (2023), O'Toole et al. (2023), Rosenberg and Van Nostrand (2023), and Kilian et al. (2024).

The cap appears to have been largely successful at keeping the supply of Russian oil on the market, as documented above. As we discuss below, in the initial phases the cap policy

<sup>&</sup>lt;sup>17</sup>In addition to the G7, EU and Australia, Albania, Bosnia and Herzegovina, Iceland, Liechtenstein, Montenegro, North Macedonia, Norway, Switzerland and Ukraine have all pledged to follow EU sanctions against Russia.

<sup>&</sup>lt;sup>18</sup>This design of the policy means that if an entity, e.g., in India, buys crude at or below the cap, it is allowed to sell the refined product at world prices. This arrangement is expected to encourage the flow of Russian oil and helps explain why Russian deliveries to the world market are largely unchanged. But who earns the rents from the difference (world price minus capped price) remains shrouded in some mystery. As one example, a *Wall Street Journal* article in April 2023 cited evidence that Saudi Arabia and the United Arab Emirates were importing Russian oil products at low prices and earning high profits (Faucon and Said (2023)), but no systematic accounting of where the rents have gone exists.

applied to large volumes of Russian oil trade. Consistent with that, the implementation of the price cap and the EU embargo has coincided with an increase in the discounts on Russian oil (more so for Urals, less so for ESPO).

More recently, however, several important developments appear to have limited the effectiveness of the price cap.

First, the price cap has not been strictly enforced. While data from CREA suggest that in April 2023, about 60% of crude oil shipments and 75% of product shipments from Russia's ports were covered by insurers from the EU, G7 or Norway, lack of clear verification procedures has meant that, during the periods when the price cap was binding (i.e. when the market price of oil was above the cap), a significant share of exports have been sold at prices above the cap. Shapoval et al. (2024) report that, in the fourth quarter of 2023, up to 95% of all Russian seaborne crude oil exports took place above the \$60 per barrel threshold, indicating that some actors break the rules imposed by the regime.

Furthermore, Russia has increased its capacity to transport its oil. Based on industry data, Shapoval et al. (2024) assess that the share of the oil carried by tankers from outside of the sanctioning countries has increased from around a fifth in early 2022 to two-thirds and one-third for crude and product, respectively (see also Kennedy (2023)). The same report argues that a significant share of this capacity are old tankers that are likely unfit to pass through international waters, e.g. through the territorial waters of Finland, Estonia and Denmark in the Baltic Sea. Stronger enforcement of environmental and safety standards such as those imposed by the UN's International Maritime Organization would therefore indirectly strengthen the degree to which the price cap is binding.

One potential reason for why the enforcement of the price cap policy has appeared to be relatively timid is the concern that a tighter sanctions system might result in Russia strategically responding by limiting its supply. In this light, in the rest of this paper we provide a framework that helps to tease out the economic incentives of a large, financially constrained producer facing sanctions. Such a framework is a necessary step to assess the risks and address the concerns that might have limited the degree to which the price cap is being enforced.

# 3 Model of a price-taking producer

We begin by studying a decision problem a state producer of a commodity who takes the price of the commodity as given. This framework offers interesting insights in its own right, and is an important input into our analysis of equilibrium with market power in the next Section.

## 3.1 Producer's problem

We study a dynamic problem of an agent – e.g. a government of a country – endowed with  $x_0$  amount of natural exhaustible resource, such as oil. We normalize  $x_0 = 1$ .

The state producer uses the proceeds from the sales of the commodity to (partially) finance its consumption. We think of state consumption here as a broad concept, encompassing the state's own consumption of publicly provided goods and services, as well as the share of consumption of citizens of the state that is financed from the public purse (e.g. through public sector employment, transfers, or revenues from procurement contracts). Most directly, our consumption concept includes the consumption of the ruling elite. In any given period, higher state consumption yields greater utility. In a dynamic setting we consider here, the state's overarching objective is intertemporal economic welfare (i.e. maximization of the present discounted value of utility of consumption).

However, the producer might face financial frictions, and as a result might be constrained in its ability to trade financial claims inter-temporally. For example, the producer might face a low, possibly negative, interest rate on its financial savings, and a high, possibly infinite, interest rate on its borrowing. Because of this, the timing and volatility in the income flow matters for welfare.

To capture the trade-offs and frictions in a tractable and transparent way, we focus on the extraction decision, while keeping the consumption-saving problem in the background. Specifically, we assume that at each instant the producer's payoff function u takes as an argument the profits from the sales of the commodity plus the net revenues unrelated to oil sales, which we denote with  $\tau$ . The profits are  $\pi_t := (p_t - \mathcal{M})y_t$ , where  $y_t$  denotes the amount of oil extracted at time t and  $\mathcal{M}$  is marginal cost of extraction (which we assume to be constant).

The producer's problem is:<sup>19</sup>

$$\max_{y_t} \mathbb{E}_0\left[\int_0^\infty e^{-\rho t} u(y_t, p_t) dt\right] \text{ subject to } dx_t = -y_t dt, \ x_t \ge 0, \ y_t \ge 0,$$
(1)

<sup>&</sup>lt;sup>19</sup>If  $p_t = p \forall t$ , this becomes a canonical cake-eating problem in continuous time. An agent has a cake of size  $x_0 = 1$  and decides on the optimal way of eating the cake, given time-separable preferences and the instantaneous utility u over consumption of cake  $y_t$  and a discount rate  $\rho$ .  $x_t$  is the size of the cake at t;  $dx_t = -y_t dt$  simply says that the size of the cake gets smaller with each bite.

and the stochastic Markov process for  $p_t$ .  $\rho$  is a discount rate, and the constraints say that the stock of reserves  $x_t$  diminishes by the amount extracted  $y_t$ , and that reserves and extraction must be non-negative. We assume that  $u(y_t, p_t) := u((p_t - \mathcal{M})y_t + \tau)$  is increasing and concave in output:  $u_y > 0$ ,  $u_{yy} \le 0$ .

# 3.2 Curvature of the payoff function u as the degree of financial constraints

In the state producer's optimization problem we specified in (1), profits from oil sales enter as an argument in the payoff function u. The key advantage of this formulation is that the curvature of the u function indexes the degree to which the exporter is subject to financial frictions, i.e. the degree to which it lives hand-to-mouth. To see why this is the case, consider an underlying utility maximization problem where the consumption of real goods and services appear as an argument in the *utility* function v(c) of the exporter. In this underlying problem, the profits from oil sales – as well as the financial frictions faced by the producer – determine the budget set.

Consider now the two extreme cases with regards to the financial context for the state producer's operations: one is a scenario of perfectly frictionless capital markets, and the other is a state of complete financial autarky.

In the former extreme, the producer has frictionless access to borrowing and lending opportunities at some interest rate r. As a result, the time path of profits becomes irrelevant for producer's decisions – only the expected net present value of profits matters as an objective in the extraction decision problem (which can be separated from the consumption decision – the Fisher (1930) separation theorem holds). This extreme therefore corresponds to no curvature in the problem we have written down – a linear u function – and the maximand is just the expected net present value of net revenues,  $\mathbb{E}_0 \int_0^\infty e^{-\rho t} (\pi_t + \tau) dt$ .

On the other extreme, if the producer cannot save or borrow at all, it must consume the proceeds from oil sales each period, implying hand-to-mouth behavior. With  $c_t = \pi_t + \tau \forall t$ , u inherits the curvature from the utility function over consumption of the underlying consumption-choice problem. The maximand is  $\int_0^\infty e^{-\rho t} u(c_t) dt$ .

In between these two extremes, the curvature of the payoff function indexes the degree of financial imperfections.<sup>20</sup>

<sup>&</sup>lt;sup>20</sup>Beyond the financial frictions interpretation, the formulation of the problem in (1) can also be motivated by the presence of dividend smoothing motives, as in the corporate finance literature (Lintner (1956), Fama and Babiak (1968), Cui (2022)), but applied to public finance. Smoother revenues from oil sales may aid

This setup has two additional advantages. The first is that the degree of hand-to-mouth of the sanctioned producer depends, in part, on the full set of sanctions that are imposed on the producer. Most directly, financial sanctions that restrict borrowing and saving in the international financial markets tighten the financial constraints. Acting on the stock of assets – e.g. freezing foreign reserves – means that the producer must rely on the flow of revenues to finance itself. Our model recognizes the importance of financial constraints in the producer's problem and therefore can be informative about the degree of interplay between financial and energy sanctions.

The second advantage of our setting is that it is consistent with a large body of literature in development economics which has found that volatility in the price of exported commodities is largely responsible for the resource curse. This literature has found that an important channel through which country's resource abundance translates into sub-par economic performance is through volatility. Our framework assumes that commodity terms of trade volatility exert a negative impact on welfare, e.g. through more volatility and lower economic growth. Furthermore, it implies that a better access to financial markets, and in particular to attractive saving vehicles, dampens this impact. Both implications are consistent with the empirical evidence across countries (Mohaddes and Raissi (2017), Cavalcanti et al. (2015)).

#### **3.3** Recursive representation

We denote with v(x, p) the value of owning x reserves when the current price of the commodity is p. The Hamilton-Jacobi-Bellman equation of the problem in (1) is:

$$\underbrace{\rho v(x,p)}_{\text{required}} = \max_{y_t} \underbrace{u(y,p)}_{\substack{\text{payoff} \\ \text{from} \\ \text{extraction}}} - \underbrace{v_x(x,p)y}_{\substack{\text{value loss} \\ \text{from} \\ \text{extraction}}} + \underbrace{v_p(x,p)\mu(p)}_{\substack{\text{value change} \\ \text{due to expected} \\ \text{price change}}} + \underbrace{\frac{1}{2} v_{pp}(x,p)\sigma(p)}_{\substack{\text{compensation} \\ \text{for risk}}}.$$

where we denoted the possibly state-dependent drift and variance of the price process with  $\mu(p)$  and  $\sigma(p)$ , respectively.

The HJB equation conveys the usual economic intuition, namely that the required return on holding the reserves must be equal to the payoff from optimally chosen extraction adjusted for the change in value and for risk.

fiscal (and war) planning and might help in achieving a smoother path for taxes that finance the budget (Barro (1979)).

The first order condition is simply

$$u_y(y) = v_x,$$

implying the optimal rate of extraction<sup>21</sup>

$$y = u_y^{-1} \left( v_x \right).$$

To characterize the behavior of the producer, we must solve the functional HJB equation – i.e. we must find the function v(x, p). Since the HJB equation does not admit an analytical solution, we proceed to find the solution numerically. To do so, we first parametrize the model.

## 3.4 Parametrization

#### 3.4.1 Stochastic process for the price of oil

A rich and complex combination of demand, supply, and market shocks result in daily fluctuations in commodity prices. In general, our framework is able to incorporate any Markov process for the price of a commodity.

With the application to the price cap on Russian oil in mind, we aim to obtain an empirically plausible model of the oil market. To do so, we model the price with the Cox–Ingersoll–Ross process (also known as a Feller square root process):

$$dp_t = D(\tilde{p} - p)dt + \varsigma \sqrt{p}dW_t \tag{2}$$

where  $W_t$  is the standard Wiener process and  $\tilde{p}$ , D, and  $\varsigma$  are (strictly positive) parameters that satisfy  $2D\tilde{p} > \varsigma^2$ . The process is mean-reverting, and parameter D determines how quickly the gap between the current price and the average price  $\tilde{p}$  closes. Parameter  $\varsigma$ determines the volatility of the price, driven by standard Brownian motion.

The process in (2) ensures that the price always stays positive: as  $p \to 0$ , the importance of Brownian noise diminishes, and mean reversion drives the price away from zero. There is no upper bound to the price: we have  $p_t \in (0, \infty) \forall t$ . Due to mean reversion, as time becomes large, the distribution of  $p_{\infty}$  will approach a Gamma distribution with the probability density

<sup>&</sup>lt;sup>21</sup>The constraints  $x \ge 0$  and  $y \ge 0$  give rise to a state boundary condition  $u_y(0) = v_x(0, p)$ . This is because at x = 0, extraction must be zero.

function

$$f(p_{\infty}; D, \tilde{p}, \varsigma) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} p_{\infty}^{\alpha-1} e^{-\beta p_{\infty}},$$

where  $\beta := \frac{2D}{\varsigma^2}$ ,  $\alpha := \frac{2D\tilde{p}}{\varsigma^2}$  and  $\Gamma(\alpha)$  is the Gamma function.<sup>22</sup>

We estimate the process in (2) using monthly data on real oil prices from 1970 until 2024.<sup>23</sup> We obtain  $\tilde{p} = \$75$  (in today's prices),  $\varsigma = 3.02$  and D = 0.19 (at the annual frequency). With these estimated values, the limiting distribution of the oil price is skewed to the right (Figure 5). Estimated standard deviation of the price at the mean is \$25. The model fits the data very well – the estimated long-run Gamma distribution follows closely the histogram of historical oil prices (the right panel of Figure 5). The estimated parameters imply a significant degree of persistence in the process for the price, with half life equal to  $\ln 2/D = 3.6$  years.

We set the marginal cost of extraction  $\mathcal{M} = \$19$  per barrel, reflecting the range of estimates of the short-run marginal cost across a range of producers (see e.g. Osintseva (2021)). We focus on the short-run estimates reflects the fact that we mostly concentrate on the immediate response of the producer to the price cap policy.

#### **3.4.2** The payoff function u

We assume that the payoff function u belongs to a HARA (hyperbolic absolute risk aversion) class:

$$u(y) = \frac{\sigma}{1 - \sigma} \left(\frac{\gamma(\pi + \tau)}{\sigma}\right)^{1 - \sigma}$$
(3)

with  $\sigma > 0$ ,  $\gamma > 0$  and  $\tau \ge 0$ . This broad class includes notable special cases such as linear, quadratic, exponential, and isoelastic utility functions. Note that, if  $\tau > 0$ , the level of utility at zero extraction u(0), and the marginal utility at that point  $u_y(0)$  are both bounded. This means that complete shut-in of production – limiting extraction all the way to zero, something that is of a concern to policymakers in the West in the context of Russia, for example – is not ruled out ex-ante by our framework.

Two useful special cases of the utility function are Constant Absolute Risk Aversion (CARA) utility, obtained by setting  $\tau = \sigma/\gamma$  and taking the limit as  $\sigma \to \infty$ :<sup>24</sup>

<sup>24</sup>We have  $\lim_{\sigma\to\infty} \frac{\sigma}{1-\sigma} \left(\frac{\gamma}{\sigma}\pi + 1\right)^{1-\sigma} = -\lim_{\sigma\to\infty} \left(\left(1 + \frac{\gamma\pi}{\sigma}\right)^{\sigma}\right)^{\frac{1-\sigma}{\sigma}}$ , since  $\frac{\sigma}{1-\sigma}$  goes to -1 as  $\sigma\to\infty$ . Using

<sup>&</sup>lt;sup>22</sup>The variance of the limiting distribution is  $\frac{2D\tilde{p}}{c^2}$ .

 $<sup>^{23}</sup>$ We obtain our data series from the FRED database. We deflate the monthly nominal oil price (code WTISPLC) by US CPI index (code CPIAUCSL) set to 1 in May 2024. We use maximum likelihood estimation, making use of the numerical implementation by Kladivko (2013).



Figure 5: The left panel shows the data on real oil prices used in the estimation of the price process, and the right panel shows that long-run distribution of the estimated process. The bars in the right panel represent the histogram of historical prices since 1970.

$$u(y) = -e^{-\gamma\pi} \tag{4}$$

and power utility, obtained by setting  $\sigma = \gamma$ :

$$u(y) = \frac{(\pi+b)^{1-\gamma}}{1-\gamma}.$$
(5)

Our baseline results assume that the u is a power function, as in (5). All our results are robust to this choice.<sup>25</sup>

We parametrize the payoff function as follows. We want to set  $\gamma$  so as to capture the degree of financial frictions that the state producer faces. This parameter can lie between 0 (no frictions) and the curvature of the underlying utility function over consumption,  $\frac{v''(c)c}{v'(c)}$ , reflecting full financial autarky and hand-to-mouth behavior. A standard calibration would set the curvature over consumption – and thus the upper bound for our parameter – of around 3-5.<sup>26</sup> Since we know that the curvature of the payoff function must be smaller than

the limit definition of the exponential, this limit equals  $-\lim_{\sigma\to\infty} (e^{\gamma\pi})^{\frac{1-\sigma}{\sigma}} = -e^{-\gamma\pi}$ .

<sup>&</sup>lt;sup>25</sup>The results under the assumption that u is CARA as in (4) are available from the authors by request.

<sup>&</sup>lt;sup>26</sup>An important caveat is that the standard parametrization refers to the curvature of individual households rather than that of the government. Nonetheless, at some level the intertemporal preferences of the government should be aligned with those of the consumers. To this extent we inform the calibration of  $\gamma$  with

inverse IES, this gives us a range in which we could set our parameter  $\gamma$ . In our baseline calibration we set  $\gamma = 2$ , reflecting substantial degree of financial frictions (but perhaps not full hand-to-mouth behavior). Admittedly, the calibration of  $\gamma$  is exploratory, and  $\gamma$  in our setting is dependent on the set of financial circumstances (including financial sanctions) of the producer. Given this, below we explore the sensitivity of our results to the wide range of values of  $\gamma$ .

We set the real interest rate that is used to discount future payoffs to 3%, to match the level of extraction of between 1 and 3% of the resource stock per year (when the producer has market power in the model of the next section). Finally, we set  $\tau = 2$ , targeting a substantial share of state's income that comes from commodity sales. Our choice implies that income from commodity sales constitutes a substantial fraction – between 1/3 and 1/2 – of the overall income of the state.

#### 3.5 Solution

We solve the model globally (we provide details of the solution method in the Appendix).

#### 3.5.1 Policy function

The solution is a policy function y(x, p) which specifies the optimal level of extraction at each price, for any level of reserves. This policy function is depicted in Figure 6. There is a flat region at the lower range of oil prices, where the producer does not extract any oil. As prices increase, there is a steep increase in extraction. But optimal extraction is non-monotonic in the price. We now explore why this is the case by zooming in on a specific part of the policy function – the contemporaneous supply curve.

estimates from the household side. See e.g. Havranek et al. (2015) for a meta-study about this parameter across countries, and Best et al. (2020) for evidence using quasi-experimental variation from the UK.

![](_page_23_Figure_0.jpeg)

Figure 6: Optimal extraction when prices follow the estimated CIR process Notes: This Figure is useful to get a general sense of how extraction evolves over time. The next figure (Figure 7) shows the policy function for x = 1 more clearly.

#### 3.5.2 Contemporaneous supply curve

Figure 7 shows the contemporaneous supply curve – that is, the optimal extraction rate for any price of the commodity, y(1,p). In much of the state space the supply curve is inelastic, and in fact bends backward slightly at higher prices. Supply falls sharply as prices approach marginal cost. When prices are about \$30 or lower, the producer ceases to extract the commodity. Note that this cutoff price is above marginal cost, assumed to be \$19 in our analysis. At high prices, beyond \$50 per barrel, the supply curve becomes highly inelastic and eventually bends backwards: higher prices now result in lower extraction rates. What explains this shape?

#### 3.5.3 Forces shaping the supply curve

Figure 8 decomposes the supply curve, taking as a reference benchmark the supply curve of a producer who faces no uncertainty in terms of the fluctuations in the price of oil and has no access to alternative source of funds,  $\tau = 0$ . In these circumstances the problem of the producer admits a closed form solution, namely that the producer extracts a constant

![](_page_24_Figure_0.jpeg)

Figure 7: Supply curve when price is stochastic

fraction of remaining reserves:

$$y_t = \frac{\rho}{\gamma} x_t \; \forall t. \tag{6}$$

Since extraction is independent of (the constant) p, the contemporaneous supply curve is a vertical line at  $\frac{\rho}{\gamma}$  – the solid black line in the left panel of Figure 8.

Relative to this benchmark, fluctuations in oil prices induce the revenue smoothing and time-the-market effects. However, when  $\tau = 0$ , timing the market is difficult, as cutting supply at low prices has a strong impact on payoffs. In this case the revenue smoothing motive is dominant, and drives the downward slope of the supply curve (dashed blue line in the left panel). Consequently, we label these deviations form the benchmark vertical supply schedule in the right-hand panel as driven by the revenue smoothing motive.

When prices are non-stochastic and fixed forever, but the producer has access to non-oil income source, the extraction rate expands for any price, but particularly so when prices are low. The intuition for this effects is the following: with an alternative source of income, the producer extracts all of the commodity in final time (rather than asymptotically as implied by (6)). Thus, the extraction rate rises over time as the reserves are depleted (and reaches 100% when the last unit of the commodity is extracted). This same relationship between extraction rate and the value of the reserves is induced by permanently low price of the resource: low p, if it is permanent, is in a sense equivalent to low x. Following this logic,

![](_page_25_Figure_0.jpeg)

Figure 8: Forces shaping the contemporaneous supply curve

for low p the producer behaves as they are more impatient, extracting higher share of the remaining reserves. This is shown with a red dashed downward sloping line in the left panel. We label the deviations from the vertical schedule as the non-oil income effect in the panel on the right-side of Figure 8.

Finally, with both volatile prices and non-oil income, we obtain the contemporaneous supply curve of our main specification. With both of the model ingredients present, there is a strong motive to time the market – the non-oil income provides a cushion against sharp increases in the marginal utility of oil revenues. Consequently, supply responds strongly negatively as prices approach marginal cost. We label the differences between the supply curve in our model and the supply curve that would obtain from the two effects discussed above as the time the market effect ion the right panel of Figure 8.

Overall, then, the shape of the supply curve is determined by the balance of these three forces. The time the market effect is most dominant at low prices, driving the upward slope in that region of the state space. For higher prices, the effects broadly offset each other, resulting in an inelastic supply curve.

#### 3.5.4 How the results depend on the degree of financial frictions

We now consider how the balance of the forces discussed above changes as we vary  $\gamma$ , which indexes the degree of financial frictions.

Figure 9 shows our baseline case in the solid line, as well as three alternative calibrations.

![](_page_26_Figure_0.jpeg)

Figure 9: The supply curve and the degree of financial frictions

We do not view these calibrations as representing realistic alternatives – rather, they are extreme cases that illustrate the direction of the comparative statics with respect to  $\gamma$  well.

The pink-dashed line shows the effect of tightening the degree of financial frictions (corresponding to a high  $\gamma$  of 4). Since the timing of the flow of revenues matters more in this case, the smoothing effect is more powerful. It takes an even lower price for the producer to leave the commodity under the ground, and the production at high prices is significantly curtailed.

The green-dashed line illustrates what happens when the degree of financial frictions is less severe than in the baseline (we set  $\gamma = 1$  in this case). This weakens the revenue smoothing motive and strengthens the option value effect. The supply curve is now significantly flatter for a larger range of the parameter space.

In the limit, as financial frictions disappear and  $\gamma \to 0$  (the *u* function becomes linear), the supply curve becomes discontinuous, and the producer extracts all of its reserves as long as prices are high, and extracts nothing otherwise (the turquoise dashed line). Effectively, our model then collapses to the frictionless Hotelling (1931) benchmark.

## 4 Price cap

This Section incorporates a price cap policy into the framework outlined so far. The price cap we consider in this section is "perfect", in the sense that it applies to all of the exporter's

![](_page_27_Figure_0.jpeg)

Figure 10: Distribution of the oil prices faced by Russia under the cap

sales. We also assume it is permanent. We continue to assume that the producer has no market power in the global market for oil (i.e., that it faces a perfectly elastic demand). We relax both assumptions in subsequent analysis.

#### 4.1 Price that the producer receives under a price cap

A price cap limits upside exposure to the volatility in oil prices. Denoting with  $p_r$  the price actually received by the sanctioned state producer when the price cap of  $\bar{p}$  is in force, we have

$$p_{r,t} = \min\left\{p_t, \bar{p}\right\}.$$
(7)

The price that Russia receives for its oil is simply the cap  $\bar{p}$  whenever the price cap is binding, and the ongoing price when it is not. The resulting distribution of prices faced by Russia is depicted in Figure 10.<sup>27</sup>

# 4.2 How does a price cap affect supply?

There are two effects that a price cap has on optimal extraction behavior and thus on the supply schedule.

 $<sup>^{27}</sup>$  Formally, there is a Dirac point mass at  $\bar{p}.$ 

First, the price cap changes the nature of the stochastic process of the price that the producer receives for its commodity. It limits the upside from the swings in the price, effectively reducing uncertainty faced by the producer. In other words, it brings the environment that the producer is operating in closer to one without uncertainty, but with a lower average price. As a result of this fundamental change, the policy function – and so the contemporaneous supply curve – shifts towards the supply curve that would have been observed absent uncertainty – the dashed line in Figure 8.<sup>28</sup> This is an outward shift. Thus, for any level of the price, the producer tends to extract more of the commodity with the cap than without.

Second, when the price cap is binding, the fluctuations in the price do not affect the exporter's revenues. As a result, the supply curve becomes insensitive to global prices at and above the price cap.<sup>29</sup> Figure 11 shows what this intuition implies for the supply curve under a price cap. It shows the supply schedules under no cap and under three alternative caps, the \$60 cap that has been implemented, the lower \$45 cap, and the \$30 cap recommended e.g., by The International Working Group on Russian Sanctions (2023). As anticipated above, in each case the supply curve features a close to vertical segment above the price cap, as the producer's decisions become insensitive to fluctuations in p. The supply curve shifts out and to the right, more so the lower the cap. Thus, implementing a perfect price cap can lead to an increase in the quantity of the commodity supplied to the market, and a lower price cap leads to larger increases in supply. This is an important conclusion as it goes strongly against the static intuition that limiting prices will necessarily lower the quantity supplied by the sanctioned state. Of course, the price cap set below marginal cost will result in sharp cuts in the extraction rate, and policymakers ought to be mindful of this.

<sup>&</sup>lt;sup>28</sup>The precise intuition for this shift is as follows. From the producer's perspective, the price cap makes the resources buried underground less valuable. With non-homothetic u function due to alternative source of income  $\tau > 0$ , less valuable resource implies a higher extraction rate. To see the intuition why, consider for example a producer with  $\tau > 0$  who has only one last barrel of oil in the ground. This producer will exhaust all the resource in finite (and likely very short!) time, i.e. it would have a high extraction *rate* (unlike in the homothetic case with  $\tau = 0$ , where a constant fraction of a very small pool of resources will be extracted ad infinitum, implying that reserves will diminish only asymptotically). The price cap effectively maps into less valuable resource pool, raising the extraction rate through the same mechanism. Formally, with  $\tau > 0$ , the producer becomes more intertemporally elastic at lower prices.

<sup>&</sup>lt;sup>29</sup>The supply schedule is not *exactly* vertical above  $\bar{p}$ , because the expected duration of the price being above the cap is different at different levels of the reference price: if the price today is at \$200 per barrel, it will take some time to cross the  $\bar{p} =$ \$60 threshold, while if it is \$65, there is a good chance it will be below the cap soon. More formally, it is  $p_t$ , not  $p_r$ , that continues to be the state variable in the problem of the producer when the price cap is in place.

![](_page_29_Figure_0.jpeg)

Figure 11: Russia's supply curve under three price cap regimes

## 4.3 How much does a price cap hurt the producer?

Price caps are the new weapons of economic warfare. But how powerful are they? Back-ofthe-envelope calculations can give us some sense of the revenue losses, but tell us little about the dynamic *welfare* losses. To address this, we use our model to compute the loss of welfare that the producer suffers as a result of the price caps set at different levels. Figure 12 plots the model-based measure of welfare (the value function v(x, p)) under different assumptions about the price cap. The x-axis denotes the amount of reserves still in the ground, so that the right-most point corresponds to welfare from having today's level of oil reserves.

The model suggests that the impact of a \$60 cap – recall that we assume that the cap is permanent – is to reduce the welfare from oil by about 20%, equivalent to reduction in reserves of about 35%. Thus, a perfect price cap is potentially very powerful indeed. And lowering the price cap further would deal an even more significant blow. With a \$30 cap, the hit to welfare from having the commodity would be in the region of 50%, equivalent to wiping out 80% of state's reserves.

The welfare results presented in this subsection must be interpreted with caution, since they miss important feedbacks that are due to the producer's potential to exercise market power and miss the fact that the cap does not apply to all exports and may not be perfectly enforced. We return to the welfare questions below, once we introduce these important

![](_page_30_Figure_0.jpeg)

Figure 12: Value functions with and without a price cap

Note: the value function is normalized by adding a constant so that v(0, p) = 0. The value functions are plotted assuming that current oil price is \$90 per barrel, but the current price does not affect the results, qualitatively nor quantitatively.

elements into our framework.

# 5 A model with market power

We now enrich our model by considering a state that is large enough to affect global equilibrium prices.

## 5.1 World demand for oil and producer's market power

We denote the world price of oil with  $p_{w,t}$ , and assume that the global demand for oil is isoelastic

$$p_{w,t} = \delta_t (r_t + y_t)^{-\epsilon}, \tag{8}$$

where parameter  $\epsilon \geq 0$  is the inverse of the elasticity of demand,  $r_t$  is the residual world supply, which is stochastic, and  $y_t$ , as before, is output of the state producer. Fluctuations in the rest of the world's supply  $r_t$  reflect demand, supply, or confidence shocks.

## 5.2 Producer's problem when the producer has market power

The optimization problem of the producer becomes:

$$\max_{y_t} \mathbb{E}_0 \left[ \int_0^\infty e^{-\rho t} u(\pi_t + \tau) dt \right] \text{ subject to } dx_t = -y_t dt, \ x_t \ge 0, \ y_t \ge 0$$
(9)

and the stochastic process for  $r_t$ , where now

$$\pi_t = (p_{w,t} - \mathcal{M})y_t = \left((r_t + y_t)^{-\epsilon} - \mathcal{M}\right)y_t.$$
(10)

The above problem is more complex, not least because the degree of market power changes dynamically and is endogenous to producer's actions. Specifically, extraction decisions at t affect future output and hence market power. The following Proposition derives the necessary conditions for a solution of this dynamic monopoly problem.

**Proposition 1.** The optimal extraction path satisfies the necessary condition

$$u_{\pi} \cdot (p_{w,t} \cdot (1 - \varepsilon_{D,t}) - \mathcal{M}) = v_x, \tag{11}$$

where

$$\varepsilon_{D,t} := -\frac{\partial p_{w,t}}{\partial y_t} \frac{y_t}{p_{w,t}} = \epsilon \cdot \frac{y_t}{r_t + y_t}.$$
(12)

is the effective elasticity of demand.

Equation (11) states that at the optimum the marginal utility of extraction is equal to the marginal value of reserves, and thus it accords with standard intuition in dynamic optimization. In turn, equation (12) shows that the marginal revenue depends upon the effective elasticity of demand  $\varepsilon_D$ , which depends on the parameter  $\epsilon$  as well as on relative size of the producer in world production. The intuition for why market power depends on the market share is familiar from the Cournot oligopoly model.

We can represent the problem recursively as follows:

$$\rho v(x,r) = \max_{y} u((p_w(r,y) - \mathcal{M}) \cdot y) - v_x(x,r)y + v_r(x,r)\mu(r) + \frac{1}{2}v_r(x,r)\sigma(r).$$

This HJB equation is different to the price-taker case above in two main respects. First, the stochastic variable is now rest of the world's residual demand  $r_t$ . Second, the world price is now endogenous – it depends on endogenously chosen output of the producer, as well as on

the stochastic  $r_t$ . The producer internalizes the impact its decisions have on global prices.

## 5.3 Equilibrium

An equilibrium is a policy function y(x, r) that solves producer's problem and the price function  $p_w(r, y(x, r))$  that clears the market for oil.

#### 5.4 Parametrization

The model with market power requires parametrization of the world demand elasticity and of the process for  $r_t$ .

Estimating oil demand elasticity is a subject of an extensive empirical literature. Metaanalysis in Uria-martinez et al. (2018) suggests the range for this elasticity in the short-run (around one year) is in the [0.07, 0.14] range, while the long-run elasticity (after over a decade) is within the [0.26, 0.82] range. However, these estimates are primarily based on OLS regressions, and so might suffer from the simultaneity bias. Indeed, the recent studies report elasticities that are higher in absolute value (see Baumeister and Hamilton (2019) and references within).<sup>30</sup> To reflect these considerations, we set  $1/\epsilon = 0.25$ . We discuss below how the results change as we depart from this elasticity in either direction.

In terms of the process for  $r_t$ , we estimate the model by simulated method of moments, such that the behavior of the equilibrium price  $p_{w,t}$  in the laissez-faire equilibrium follows that of the process for the oil price observed in the data (and estimated in Section 3).

All the remaining parameters are calibrated as before.

To solve the HJB equation with market power and to estimate the model, we develop a new algorithm which we describe in the Numerical Appendix.

## 5.5 Characterization

The contemporaneous supply curve of a producer with market power is plotted in Figure 13. The figure plots two equilibrium variables against each other: the extraction rate of the producer  $y_t$  and the equilibrium world price  $p_w$ .

Market power makes the producer more conservationist in our environment, except when prices are marginally above the marginal cost. The overall shape is similar to before, however. The supply curve remain inelastic over much of the range of prices.

<sup>&</sup>lt;sup>30</sup>We report the absolute value of the elasticity; of course the demand curve is downward sloping.

![](_page_33_Figure_0.jpeg)

Figure 13: Contemporaneous supply curve with market power

It is useful to contrast this result with the conclusions of the classic paper of Stiglitz and Dasgupta (1981) which studied the role of market power in resource extraction. That paper showed that in a simple benchmark model of resource extraction, market power has no effect on quantity extracted, highlighting the important difference between exhaustible resources and produced goods. Our framework differs however, because of the presence of strictly positive marginal costs, financial frictions, and non-oil income.

# 6 Price cap when the producer has market power

When the producer has market power and is subject to a price cap, the price that it receives is given by

$$p_{r,t} = \min\{\bar{p}, p_{w,t}\},$$
 (13)

where  $\bar{p}$  is the level of the price cap and  $p_{w,t}$  is the equilibrium price of oil in the world market. The difference to (7) is that  $p_{w,t}$  is now endogenous and determined by the producer's decisions (as well as by the stochastic realization of  $r_t$ ).

## 6.1 How does the price cap interact with market power?

The key insight is that the price cap limits the use of market power in equilibrium. The economics of this mechanism is straightforward: with a price cap in place, restricting quantities has no desired effect on the price, rendering the use of market power entirely ineffective.

Consequently, the supply curve with a price cap in place can be thought of as an envelope of two supply curves we have already studied in previous sections of this paper: the one with market power (and no cap) at prices well below the price cap, and the one without market power and a binding cap at prices above the price cap.

Figure 14 illustrates the main result graphically: the solid black schedule in the Figure is the supply curve with a \$60 price cap in place. The other two schedules are the same as in previous sections.

The Figure shows that when the equilibrium price is low relative to the cap,  $p_w < \bar{p}$ , the cap matters little for the producer's behavior. The producer exercises market power and the black solid line follows closely the supply curve we described in the previous section (the red line).

Conversely, when prices are above \$60 and the price cap is binding, the producer ceases to use its market power – the supply curve is shifted to the right and close to vertical. It resembles the supply curve under the cap from a model of a price taking producer.

In between, these two regions there is a smooth and continuous intermediate section. As higher prices make the price cap ever more binding, the producer gradually reduces the extent to which it uses market power in equilibrium, in such a way to keep the equilibrium price exactly at  $p_w = \bar{p}$ .

## 6.2 Effect of the price cap on equilibrium prices

Given the optimal behavior of the producer we just described, what happens to equilibrium prices as the cap is introduced?

To answer this question, it is useful to define the *reference price*  $p_t$  as the hypothetical equilibrium price under the assumption that the producer used no market power. The reference price is simply a transformation of the state variable  $r_t$ . The actual equilibrium price is higher than the reference price if the producer restricts supply relative to the no market power benchmark.

A binding price cap obviously limits the price that the sanctioned producer receives. This direct effect of the cap follows from (13) and is illustrated in the left panel of Figure 15. The Figure plots on the vertical axis the equilibrium prices  $p_r$  (left panel) and  $p_w$  (the right panel) against the reference price  $p_t$  on the horizontal axis.

A more interesting conclusion is that implementing a price cap can also lower the world equilibrium price of the commodity, especially when the reference price is high (the right

![](_page_35_Figure_0.jpeg)

Figure 14: Equilibrium supply in a model with market power with a \$60 price cap

panel of Figure 15). In other words, the cap has a *stabilizing effect* on world prices. This stabilization effect comes about precisely because when the cap is binding, the producer ceases to exercise market power, and instead has the incentive to supply large quantity of the commodity to the market as the smoothing effect does not operate.

It is important to note that these effects are more pronounced when the producer has substantial degree of market power. This is because the gap between production levels with and without market power naturally increases with the degree of market power, and it is this gap that the price cap eliminates.

We summarize these results in the following Proposition:

**Proposition 2.** When the sanctioned producer has market power, introducing a price cap that applies to all sales has the following effects:

- (1) the cap limits the extent to which the producer exercises market power in equilibrium;
- (2) a binding cap thus tends to reduce equilibrium world price  $p_w$ ;
- (3) the decline in  $p_w$  upon introduction of cap is larger the higher is reference price p;
- (4) the cap thus stabilizes equilibrium world price  $p_w$ ;
- (5) for high reference price p, the equilibrium  $p_w$  can be below p;
- (6) these effects are more powerful when the producer commands significant pricing power.

![](_page_36_Figure_0.jpeg)

Figure 15: Equilibrium prices in the model with market power, with and without a price cap

# 7 Imperfect price cap

In the analysis so far we have assumed that the price cap applies to all of the sales of the sanctioned producer and that the cap is expected to be in place forever. In reality, however, the price cap might only affect a specific portion of the exporter's oil sales, and it might not be in place indefinitely. In the case of Russia, the G7 price cap applies only to the price of seaborne oil and products that use Western services such as transportation and insurance.<sup>31</sup> Furthermore, if the price cap is not adequately enforced, a share of sales that bypass the sanctions regime could be substantial. How does such partiality of the price cap alter the analysis and the conclusions? And what if the producer puts a positive probability on the price cap to be abandoned sometime in the future? We consider these two possibilities in turn.

 $<sup>^{31}</sup>$  There is an intense debate and speculation among experts, policy makers and the media about the ability of Russia to do without these western services, including its a bility to build up such capacity – termed "shadow fleet" – over time. While there is a significant uncertainty about this, initial experts' estimates were that about 30-40% of the flow of oil exports can be legally sold outside of the price cap regime – see, for example, analysis by Craig Kennedy here: https://navigatingrussia.substack.com/p/measuring-the-shadows. On top of that, imperfect enforcement might mean that  $\kappa$  is larger still.

#### 7.1 Leaky price cap

Let us represent the percentage of the producer's current oil reserves that can be exported outside of the cap with parameter  $\kappa \in [0, 1]$ . For instance, with  $\kappa = 0.01$ , the producer can export 1% of its reserves this year without being subject to the price cap.  $\kappa = 0$  represents the case of a perfect price cap that applies to all of exports (meaning that the producer cannot sell outside the price cap regime), as described in previous sections. We assume that  $\kappa$  is fixed over time.<sup>32</sup>

With a shadow fleet of capacity  $\kappa$ , the instantaneous profits from oil sales when the price cap is in place are:

$$\pi_{t} = \begin{cases} y \cdot (p_{w}(y) - \mathcal{M}) & \text{if } y \leq \kappa \\ y \cdot (p_{w}(y) - \mathcal{M}) & \text{if } y > \kappa \text{ and } p_{w} < \bar{p} \\ \kappa \cdot (p_{w}(y) - \mathcal{M}) + (y - \kappa) \cdot (\bar{p} - \mathcal{M}) & \text{if } y > \kappa \text{ and } p_{w} > \bar{p} \end{cases}$$
(14)

where  $p_w$  is the equilibrium oil price. The first line shows the profits that the producer makes if extraction is smaller than  $\kappa$ , and hence all of the commodity is sold outside of the sanctions regime. The second line – which turns out to be the same as the first – is profits when extraction is greater than  $\kappa$ , but the price cap is not binding. The third line represents profits when extraction is above  $\kappa$  and the cap is binding. In this most interesting case, the producer receives the world equilibrium price for the quantity  $\kappa$ , and the price cap for the remaining sales.

The first order condition of the producer's problem becomes

$$v_{x} = \begin{cases} u_{\pi} \cdot (p_{w} (1 - \varepsilon_{D}) - \mathcal{M}) & \text{if } y < \kappa \\ u_{\pi} \cdot (p_{w} (1 - \varepsilon_{D}) - \mathcal{M}) & \text{if } y > \kappa \text{ and } p_{w} < \bar{p} \\ u_{\pi} \cdot \left(\bar{p} + \kappa \frac{\partial p_{w}}{\partial y} - \mathcal{M}\right) & \text{if } y > \kappa \text{ and } p_{w} > \bar{p} \end{cases}$$
(15)

where  $\varepsilon_D$  is the elasticity of demand. When production is low, so that all oil can be transported outside of the cap regime (the first row in (15)), the marginal utility of extracting an additional barrel is given by the marginal utility of oil profits times the world price adjusted downwards for the impact that this extraction has on the prevailing oil price. This is also true if the marginal barrel is sold using the coalition services and so under the price cap

<sup>&</sup>lt;sup>32</sup>Future work might fruitfully revisit this and explore cases with variable  $\kappa$ , reflecting, for example, Russia's potential expansion of its capacity to sell oil outside of the price cap regime.

regime, but if the price cap is not binding (the second row). Finally, when the marginal barrel is sold at a cap, the marginal benefit is just the price cap adjusted for the price impact that the sales of a marginal barrel have on the revenues from the sales of the infra-marginal  $\kappa$  barrels (the final row).<sup>33</sup>

#### 7.1.1 The effects of a leaky price cap

The combination of market power and the ability to bypass the price cap on some of its sales provides the producer with a potentially appealing strategy to deal with the sanctions: cut the production levels towards  $\kappa$ , thereby squeezing the global market and raising equilibrium prices at which the (now-lower) quantity is sold. Higher prices in part compensate for lower quantity, and as an additional benefit of this strategy, the reserves (and hence the market power) deplete more slowly. We now explore whether or not this is indeed an optimal strategy of the producer, and how this depends on the level of prices and on the capacity parameter  $\kappa$ .

Figure 16 illustrates optimal extraction and equilibrium world prices with a price cap that is imposed on the producer who has access to a shadow fleet capable of carrying 1% of its reserves, and is otherwise identical to the producer in the previous section.

The left panel displays the supply schedule. The results are striking: the supply curve features a sharp kink, meaning that for high enough prices, the producer starts restricting supply, and ultimately reduces extraction all the way to  $\kappa$  when world prices are high. Thus, the effects of the price cap are strongly state-dependent: the price cap maintains the positive effects on supply and stabilizing effects on world prices when prices are not too high, but can lead to sharp supply withdrawal by the producer when the world oil market is already tight. As a result, the cap has stabilizing effects when prices are close to its long-run average of \$75, but can have a destabilizing effect exactly when the world prices are already high.

## 7.2 Expectation that the price cap is temporary

We now investigate how the expectation that the price cap will be lifted at some point in the future affects the producer's behavior and equilibrium.

$$\tilde{u}_{\pi}\left(p_{w}\left(\frac{\bar{p}}{p_{w}}-\frac{\kappa}{y}\varepsilon_{D}\right)-\mathcal{M}\right)=v_{x}$$

 $<sup>^{33}\</sup>mathrm{Note}$  that the last line can be re-written as

![](_page_39_Figure_0.jpeg)

Figure 16: Equilibrium supply and prices under a price cap when Russia has access to a shadow fleet

We assume that the producer believes that the lifting of the cap is a Poisson event with intensity  $\lambda$ , so that the duration of the price cap is an exponentially distributed random variable and

$$Pr(\text{cap lifted before } t) = 1 - \exp(-\lambda t).$$

For concreteness, suppose that the producer perceives the probability of the cap being lifted in the first year to be 50%, implying  $\lambda = 0.69$ . How does this affect the behavior of the producer?<sup>34</sup>

Figure 17 illustrates how contemporaneous extraction responds to such expectations and what the consequences are for world prices. The expectation that the cap is temporary makes the producer more inclined to shut-in production, hence keeping more barrels of oil under ground and only extracting them when the price cap is lifted. Thus, as illustrated in the right panel of the Figure, the lack of credibility reinforces the shadow fleet mechanism in further reducing the stabilization effects of the price cap.

 $<sup>^{34}</sup>$ Technically to solve the model we must introduce another state variable which takes two values, corresponding to the cap being and not being in place. We then impose a Poisson process on the switching between the cap and the no-cap state.

![](_page_40_Figure_0.jpeg)

Figure 17: Equilibrium supply and prices when the producer expects the price cap to be temporary

#### 7.2.1 The impact of a leaky price cap on profits and welfare

We have now endowed the producer with market power and we have made it possible to partially circumvent the price cap regime by exporting oil using a shadow fleet of tankers and services. We are ready to revisit the question about the effectiveness of the price cap as a tool of economic warfare. What impact does a leaky price cap have on the producer in this environment?

Figure 18 offers an answer, both in terms of contemporaneous profits from oil sales in the left panel, as well as welfare from having oil in the ground (i.e., the value function) in the right panel.

The dashed lines in the left panel show that contemporaneous profits plummet by up to 50% as the producer turns to the "shut-in" strategy, unless the market prices are already very high. That is, our model suggests that even with a relatively highly inelastic demand which we is embedded in our calibration, the sharp reduction in exports does not generate a price response that is sufficiently strong to make the shut-in a profitable strategy in the short term. In other words, higher prices in the shut-in scenario do not compensate for the lost revenues due to lower volumes. According to the model, shutting in production to  $\kappa$  is optimal not because it raises contemporaneous profits, but because it allows for a more

![](_page_41_Figure_0.jpeg)

Figure 18: Effects of price caps on producer's contemporaneous revenues and welfare when Russia has access to a shadow fleet or the cap is imperfectly enforced Note: the right-panel assumes that the (current) reference no-market-power price of oil p is \$80.

spread out production profile over time (see also the relevant discussion in Section 5).

Indeed, the static losses are more than compensated by the dynamic gains. The righthand panel shows that welfare increases with  $\kappa$ , which is intuitive and unsurprising. The interesting result here is that the ability to circumvent the price cap regime significantly diminishes the degree to which the cap hurts the producer. Relative to a perfect cap, a leaky cap with  $\kappa = 0.01$  reduces the damage in welfare terms by about  $\frac{2}{3}$ . If a price cap is expected to be temporary, the effects on the intertemporal welfare vanbish almost completely.

# 8 Conclusions

The main contribution of this paper is a dynamic model that helps us understand the economic incentives of a financially constrained producer of a non-renewable resource. Our particular application and focus has been the on the effects of the new instrument of international policy – a price cap.

The model takes as an input an estimated flexible diffusion process for the oil price, which we embedded in an equilibrium structure where the producer has market power which changes dynamically and endogenously to the producer's decisions. The analysis uncovered interesting economic channels and forces that are at play in this setting. In particular, our model stresses the interplay between financial frictions and the dynamic optimal behavior of the producer, with an important revenue smoothing motive. It highlights the role of alternative sources of funds or other sources of non-homotheticity in producer's preferences. And it illuminates the fact that the price cap reduces the use of market power in equilibrium, which leads to a stabilizing effect of the price cap on the global commodity market.

Beyond these contributions, our analysis has important policy implications in the current context of the war in Ukraine and sanctions against the Russian Federation.

First, our economic framework supports the idea that Russia's supply curve is inelastic and may even be downward sloping, helping to explain why Russian oil production levels have remained relatively stable despite political assertions to the contrary.

Second, a binding oil price cap may increase Russia's supply to the market, stabilizing the price of oil globally. The cap may not be effective in the long run, however, if Russia can sell enough of its exports outside the price cap regime (i.e., without using western transportation and financial services). This highlights the importance of lowering the cap before Russia finds alternative ways to export its oil and the need for strict enforcement of the cap.

Third, even when a commodity producer has significant market power, this need not deter western policymakers from imposing – and lowering – the price cap. In fact, the oil price cap can effectively neutralize Russia's market power, which it already uses in equilibrium.

Finally, our simulations suggest that a lower price cap, perhaps around \$45 per barrel, could significantly impact Russia's revenue flows, and depending on the capacity of the shadow fleet, potentially also its welfare.

Our paper opens up several avenues for future research. Our setting explored the use of the price cap tool in the context of non-renewable resources. But future work might want to consider a setting in which trade of products or exchange of technologies is taking place between the sanctioning and the sanctioned state. Another useful avenue for future research would be to explicitly embed the setting developed here within a general equilibrium model of a world economy, with strategic interactions across participating states. We are excited to contribute to this exciting agenda going forward.

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# Appendix

# A Curvature of function u as an index of financial constraints

Financial frictions inhibit consumption smoothing, and hence mean that the timing, volatility and uncertainty of the income flow matter for welfare. The goal of this Appendix is to show that the curvature of a payoff function over income can be used to index the degree of the income smoothing motive. It does so in a stylized two-period model with no uncertainty. Instead, the reason why financial frictions matter is that income is distributed unevenly across periods, and the agent wants to smooth consumption.

## A.1 Frictionless model

Consider a two period model with no discounting:  $\beta = 1$ , and no financial frictions, and with r = 0 so that  $R := 1 + r = 1/\beta$ . Agent has time separable preferences, and her utility over flow of consumption in each period is CRRA. The agent's total income across two periods is Y = 1. The income is split unevenly across the two periods, with  $y_1 = \phi$  and  $y_2 = 1 - \phi$  (known with certainty). Agent solves

$$U = \max_{c_1, c_2} u(c_1) + u(c_2)$$

subject to

$$c_1 + c_2 = 1$$

Trivially, the solution is

$$c_1 = c_2 = \frac{1}{2}$$
$$\frac{dU}{d\phi} = 0$$

and

that is, the value function is independent of the timing of income. In other words, the timing is irrelevant for consumer's welfare, given the perfect financial markets.

Without loss of generality, in the remainder of the Appendix we focus on the agent who wants to save, i.e. we assume  $\phi \in (\frac{1}{2}, 1)$ , that is, more income arrives in the first period.

## A.2 Financial frictions

Now suppose that the budget constraint of the agent has a kink – the interest rate on savings is negative (and the rate on borrowing is positive). In particular, let  $r_S = -\chi < 0$  denote the interest rate on savings (and e.g.  $r_B = \chi > 0$  the rate on borrowing). The agent solves

$$U = \max_{c_1, c_2} u(c_1) + u(c_2)$$

subject to

$$c_1 + \frac{c_2}{1-\chi} = \phi + \frac{1-\phi}{1-\chi}.$$

The first order condition is

$$u'(c_1) = u'(c_2) (1 - \chi)$$

Thus with CRRA

$$c_2 = (1 - \chi)^{1/\gamma} c_1$$

Combining this with the budget constraint gives optimal consumption path:

$$c_{1} = \frac{\phi + \frac{1-\phi}{1-\chi}}{1 + (1-\chi)^{\frac{1-\gamma}{\gamma}}}$$
$$c_{2} = \frac{(1-\chi)^{1/\gamma} \left(\phi + \frac{1-\phi}{1-\chi}\right)}{1 + (1-\chi)^{\frac{1-\gamma}{\gamma}}}.$$

Without loss of substance, simplify by assuming that underlying utility is log:  $\gamma = 1$ . We then have

$$c_{1} = \frac{1}{2} \frac{1 - \phi \chi}{1 - \chi}$$
$$c_{2} = \frac{1 - \phi \chi}{2}.$$

Since we assumed that the consumer is saving, we must have  $c_1 \leq \phi$ . Thus we require

$$\frac{1}{2}\frac{1-\phi\chi}{1-\chi} < \phi$$

which is the case if

$$\chi < \bar{\chi} := 2 - \frac{1}{\phi}.$$

 $\bar{\chi}$  is the threshold degree of financial frictions. If financial frictions are more severe than that, the consumer is just consuming her endowment.

The value function is:

$$U(\phi) = \log\left(\frac{1}{2}\frac{1-\phi\chi}{1-\chi}\right) + \log\left(\frac{1-\phi\chi}{2}\right)$$

which is a function of  $\phi$  as long as  $\chi \neq 0$ . We have

$$\frac{\partial U}{\partial \phi} = -2\frac{\chi}{1-\phi\chi}.$$

Consider the following function: CRRA with inverse IES parameter  $\sigma$ , applied to the flow of income of the agent:

$$V(\phi) = \frac{\phi^{1-\sigma}}{1-\sigma} + \frac{(1-\phi)^{1-\sigma}}{1-\sigma}$$

The question we want to answer is: for a given income process  $\phi$ , can we map the degree of financial frictions  $\chi$  into  $\sigma$  so that the sensitivity of the value function to the timing of the endowment,  $\phi$ , is the same in the underlying problem and in the "just consume the endowment" problem? In other words, we seek  $\sigma(\chi)$  such that, for a given endowment process  $\phi$ ,

$$U'(\phi) = V'(\phi).$$

This implies:

$$-2\frac{\chi}{1-\phi\chi} = \phi^{-\sigma(\chi)} - (1-\phi)^{-\sigma(\chi)}$$

which implicitly pins down the  $\sigma(\chi)$  function.

We can deduce some properties of this function. Note that, for  $\phi > \frac{1}{2}$ ,

 $\sigma(0) = 0,$ 

and, because  $-2\frac{\bar{\chi}}{1-\phi\bar{\chi}} = \frac{1}{\phi} - \frac{1}{1-\phi}$ , we have

$$\sigma(\bar{\chi}) = 1.$$

This is intuitive: with no frictions (the former case), the timing of income does not matter, so V is linear (and  $\sigma = 0$ ). Instead, with full frictions, we have that  $\sigma = \gamma$  (and recall that we set  $\gamma = 1$ ).

Implicitly differentiating yields:

$$\frac{-2(1+\phi)}{(1-\phi\chi)^2} = -\left((\log\phi) \cdot \phi^{-\sigma(\chi)} - (\log(1-\phi)) \cdot (1-\phi)^{-\sigma(\chi)}\right) \sigma'(\chi)$$

So that

$$\sigma'(\chi) = \frac{1}{((\log \phi) \cdot \phi^{-\sigma(\chi)} - (\log(1-\phi)) \cdot (1-\phi)^{-\sigma(\chi)})} \frac{2(1+\phi)}{(1-\phi\chi)^2} > 0,$$

as long as  $\phi > 1/2$  (which holds by assumption). Thus the  $\sigma(\chi)$  function is monotonically increasing in the  $[0, \bar{\chi}]$  interval, from 0 to 1.

## A.3 Interpretation

Figure 19 shows the  $\sigma(\chi)$  function. For any  $\phi$ , this is a well-behaved monotonic function. Thus, to reflect a given degree of financial frictions in the decision problem of the agent, we pick a corresponding  $\sigma(\chi)$ , ensuring that the choices over the timing of the income stream have the impact of welfare that is consistent with the solution to (and optimal behavior in) the consumption-saving problem.

We apply this principle in the main text by focusing solely on the extraction decision of the producer and assuming that the payoff function is concave in oil revenues. To be sure, the simple model developed in this Appendix does not exactly map to the more complex framework we employ in the paper. Our model features uncertainty and infinite horizon. But the idea that, for a given properties of the price CIR process given by the triple  $\{\tilde{p}, \varsigma, D\}$ (in parallel to the single property  $\phi$  in the illustration here), there is a mapping between the degree of financial frictions and the curvature of the pay-off function.

![](_page_52_Figure_0.jpeg)

Figure 19: The  $\sigma(\chi)$  function, for different levels of unevenness of the income profile ( $\phi$ ) – the agent has log utility and there is no discounting, so that the agent wants to ideally consume the same each period

# **B** Numerical appendix

This appendix provides a summary of the numerical procedure used to solve the model.

#### B.1 Baseline case

We start with a model with no market power. We seek to solve the HJB equation (??), reproduced here:

$$\rho v(x,p) = \max_{y_t} u(\pi) - v_x(x,p)y + v_p(x,p)D(\tilde{p}-p) + \frac{1}{2}v_{pp}(x,p)\sigma^2 p.$$

We use a finite difference method; a useful reference in the macroeconomics literature is Achdou et al. (2017). We approximate the function v at I discrete points in the reserves grid,  $x_i$ ,  $i \in 1, ..., I$  with  $x_1 = 0$  and J discrete points in the price dimension,  $p_j$ ,  $j \in 1, ..., J$ . We use equispaced grids with  $\Delta x$  and  $\Delta p$  the distance between grid points. Since oil is a non-renewable resource, reserves can only stay constant or fall. Thus, the drift is always (weakly) negative. When reserves are zero, the derivative of the value function is pinned down by the boundary condition. Therefore, we approximate the derivative of the value function in the x-dimension with

$$\partial_x v_{i,j} = \begin{cases} u_y(0) & \text{if } i = 1\\ \frac{v_{i,j} - v_{i-1,j}}{\Delta x} & \text{if } i \ge 2. \end{cases}$$

The approximations of the derivatives in the p dimensions are:

$$\partial_{p,B} v_{i,j} = \frac{v_{i,j} - v_{i,j-1}}{\Delta p}$$
$$\partial_{p,F} v_{i,j} = \frac{v_{i,j+1} - v_{i,j}}{\Delta p}$$
$$\partial_{pp} v_{i,j} = \frac{v_{i,j+1} - 2v_{i,j} + v_{i,j-1}}{(\Delta p)^2}$$

We use the appropriate approximation depending on whether the price is falling or increasing. For any variable z, we use the notation  $z^+ := \max\{z, 0\}$  and  $z^- := \min\{z, 0\}$ . The finite difference approximation to the HJB equation is then:

$$\rho v_{i,j} = u(y_{i,j}) - \partial_x v_{i,j} y + \partial_{p,F} v_{i,j} \left[ D(\tilde{p} - p) \right]^+ + \partial_{p,B} v_{i,j} \left[ D(\tilde{p} - p) \right]^- + \frac{1}{2} \partial_{pp} v_{i,j} \sigma^2 p$$
$$y_{i,j} = (u_y)^{-1} (\partial_x v_{i,j}).$$

**Algorithm** The algorithm for finding the solution to the HJB equation is as follows. Guess  $v_{i,j}^0$ , i = 1, ..., I, j = 1, ..., J and for n = 0, 1, 2, ... follow

- 1. Compute  $\partial_x v_{i,j}^n$ .
- 2. Compute optimal extraction  $y^n$  assuming that the marginal barrel is priced at the cap if the cap is binding. In the model without market power, compute  $y_{i,j}^n = (u_y)^{-1}(\partial_x v_{i,j}^n)$ where  $u_y^{-1}$  is evaluated at min $\{p_j, \bar{p}\}$ .
- 3. Compute extraction as if there was no price cap,  $\tilde{y}^n$ :  $\tilde{y}^n_{i,j} = (u_y)^{-1}(\partial_x v^n_{i,j})$ , where  $u_y^{-1}$  is evaluated at  $p_j$ .
- 4. For i, j where  $y_{i,j}^n < \kappa$ , set  $y_{i,j}^n = \tilde{y}_{i,j}^n$ .
- 5. Compute  $\tilde{u}_{i,j}^n(\pi)$  with  $\pi_{i,j}^n = \min\left\{\kappa, y_{i,j}^n\right\} \cdot p_j + \max\left\{0, y_{i,j}^n \kappa\right\} \cdot \min\left\{p_j, \bar{p}\right\}$
- 6. Find  $v^{n+1}$  using the implicit method described below.
- 7. If  $v^{n+1}$  is close enough to  $v^n$ , stop. Otherwise go to step 1.

The implicit method for finding  $v^{n+1}$ . With the implicit method, we update the value function as follows. With a given step size  $\Delta$ ,  $v^{n+1}$  is defined by the following equation:

$$\frac{v_{i,j}^{n+1} - v_{i,j}^{n}}{\Delta} + \rho v_{i,j}^{n+1} = u(y_{i,j}^{n}) - \partial_x v_{i,j}^{n+1} y_{i,j}^{n} + \partial_{p,F} v_{i,j}^{n+1} \left[ D(\tilde{p} - p_j) \right]^+ + \partial_{p,B} v_{i,j}^{n+1} \left[ D(\tilde{p} - p_j) \right]^- + \frac{1}{2} \partial_{pp} v_{i,j}^{n+1} \sigma^2 p_j$$

Substituting in for the derivatives and collecting together the terms that are multiplied by the same grid point of v, this equation can be written as follows:

$$\frac{v_{i,j}^{n+1} - v_{i,j}^n}{\Delta} + \rho v_{i,j}^{n+1} = u(y_{i,j}^n) + v_{i,j}^{n+1} z_{i,j} + v_{i,j}^{n+1} \nu_{i,j} + v_{i,j-1}^{n+1} \chi_j + v_{i,j+1}^{n+1} \zeta_j$$

where  $z_{i,j} = -\frac{y_{i,j}^n}{\Delta x}$ ,  $\nu_{i,j} = \left[\frac{D(\tilde{p}-p_j)}{\Delta p}\right]^- - \left[\frac{D(\tilde{p}-p_j)}{\Delta p}\right]^+ - \frac{\sigma^2}{(\Delta p)^2}$ ,  $\chi_j = -\left[\frac{D(\tilde{p}-p_j)}{\Delta p}\right]^- + \frac{\sigma^2}{2(\Delta p)^2}$  and  $\zeta_j = \left[\frac{D(\tilde{p}-p_j)}{\Delta p}\right]^+ + \frac{\sigma^2}{2(\Delta p)^2}$ . We can now write this in a matrix form:

$$\frac{1}{\Delta}(v^{n+1} - v^n) + \rho v^{n+1} = u^n + \mathbf{A}^n v^{n+1}$$

where  $v^n$  is a vector of length  $I \cdot J$  with entries  $(v_{1,1}, ..., v_{I,1}, v_{1,2}, ..., v_{I,2}, ..., v_{I,J})$  and  $\mathbf{A}^n$  is a  $(I \times J) \times (I \times J)$  matrix that has  $z, \nu, \chi, \zeta$  as entries. Collecting terms, we get

$$\left(\frac{1}{\Delta} + \rho - \mathbf{A}^n\right)v^{n+1} = u^n + \frac{1}{\Delta}v^n.$$

This is a system Bx = b which can easily be solved numerically.