Elephants in Equity Markets

Hélène Rey (LBS), Adrien Rousset Planat (LBS), Vania Stavrakeva (LBS), Jenny Tang (Boston Fed)

June 2024

Main Contribution

- **Novel decomposition of equity price growth rates in terms of equilibrium equity holdings , based on market-clearing conditions.**
- **Asset managers are like elephants in equity markets.**
  - With **only 5% coverage** in our sample, our reconstructed equilibrium holdings accounts for **89%** of the monthly variation in equity price growth rates for over **20,000 individual stocks** and **96%** of the fluctuations for **33 aggregate stock markets**.

Main Contribution

- Express *individual* level and *aggregate* stock market price growth rates in terms of equilibrium equity holdings and its sub-components:
  1. exchange rate valuation effects
  2. net portfolio returns
  3. portfolio weight changes
  4. final fund flows

- Document novel facts about the importance of the equity holdings sub-components for equity price determination.

Literature Review

- **Importance of mutual funds' demand for asset price determination** (equities and exchange rates): Koijen and Yogo (2019), Gabaix and Koijen (2022), Richmond, Koijen and Yogo (2023), Koijen and Yogo (2024), Hau and Rey (2004, 2006) and Camanho, Hau and Rey (2022), Rey, Stavrakeva and Tang (2024) .
- **Some of the most-relevant intermediation-based asset pricing models**: Basak and Pavlova (2011), Kashyap, Kovrijnykh, Li and Pavlova (2021, 2023), Gourinchas, Ray, Vayanos (2022), Greenwood, Hanson, Stein and Sunderam (2020), Buffa, Vayanos and Woolley (2023)

Road Map

1. Stock Market Price Growth Rate Decomposition
2. Stylized Facts

Stock Market Price Growth Rate Decomposition

Market Clearing Conditions

- The market clearing condition for asset j (ISIN) denominated in currency $c^j = l$:

$$\underbrace{\sum_{i \in I} \omega_t^{i,j} W_t^i S_t^{l/c^i}}_{\text{Nominal holdings}} = \underbrace{P_t^j Q_t^j}_{\text{Nominal Supply}} \quad \text{where } c^j = l$$

- c^i – currency of investor i (currency of the main region of sale (ROS))
- W_t^i – AUM of investor i in units of c^i
- $\omega_t^{i,j}$ – portfolio share invested in asset j by investor i
- S_t^{l/c^i} – nominal exchange rate; units of currency l per one unit of currency c^i
- P_t^j and Q_t^j – price of asset j in currency c^j and shares issued

Market Clearing Conditions

Since the growth rate of AUM is a function of net-of-fee portfolio returns and final fund inflows/outflows,...

$$\Delta w_t^i = \frac{W_t^i - W_{t-1}^i}{W_{t-1}^i} = \underbrace{\left(R_t^{i,NF} - 1\right)}_{r_t^{i,NF}} + \underbrace{\frac{Flow_t^i}{W_{t-1}^i}}_{flow_t^i}$$

Market Clearing Conditions

We linearize market clearing conditions to decompose the growth rate of the equity price of stock j as:

$$\Delta p_t^j = \sum_{i \in I} \frac{\widehat{W^i} \widehat{S}^{I/c^i} \widehat{\omega}^{i,j}}{\widehat{P^j Q^j}} \left(\Delta s_t^{I/c^i} + \frac{\Delta \omega_t^{i,j}}{\widehat{\omega}^{i,j}} + r_t^{i,NF} + flow_t^i \right) - \Delta q_t^j$$

- $\frac{\widehat{W^i} \widehat{S}^{I/c^i} \widehat{\omega}^{i,j}}{\widehat{P^j Q^j}}$ **– sample average holding of stock** j **by fund** i **relative to the sample average market cap of stock** j.

Aggregation Methodology

- To construct the sub-components of equilibrium holdings, we need to observe the holdings of **all investors** holding ISIN j.
- We circumvent this problem by aggregating our observed holdings in the Morningstar Direct mutual funds data as if we have a representative sample of equity investors.

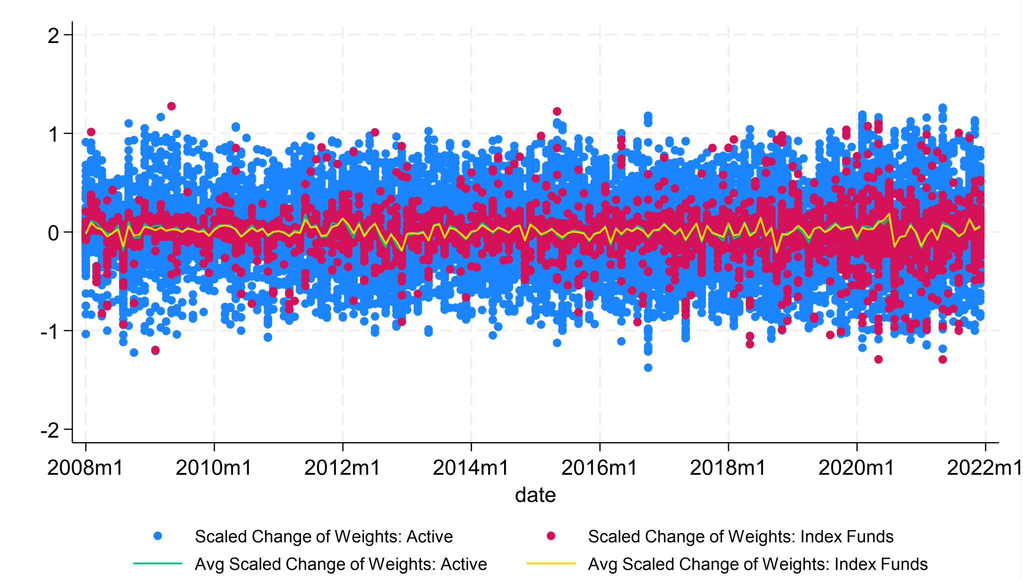
Aggregation Methodology

Step 1: We decompose the scaled portfolio weight changes for a given ISIN by fund *i* into an unweighted average within a type of funds and a residual.

$$\frac{\Delta\omega_t^{i,j}}{\widehat{\omega}^{i,j}} = \sum_{k\in\tau_i'} \frac{1}{|\tau_i'|} \frac{\Delta\omega_t^{k,j}}{\widehat{\omega}^{k,j}} + \varepsilon_t^{\omega,i,j},$$

The investor group is represented by $\tau' \in \Upsilon'$, where
$\Upsilon' = \mathit{Active} \times \mathit{Broad\ Strategy} \times \mathit{Freq\ Rebalance} \times \mathit{ROS\ Local\ Currency}$ and
$\tau_i' = \{k \in \tau' | i \in \tau'\}$

Apple – Portfolio Weight Changes $\frac{\Delta\omega_t^{i,j}}{\widehat{\omega}^{i,j}}$



Aggregation Methodology

Similarly, we decompose the net-of-fee returns and flows for a given fund as an unweighted average within a type of funds and a residual:

$$flow_t^i = \sum_{k \in \tau_i} \frac{flow_t^k}{|\tau_i|} + \varepsilon_t^{f,i} \tag{1}$$

$$r_t^{i,NF} = \sum_{k \in \tau_i} \frac{r_t^{k,NF}}{|\tau_i|} + \varepsilon_t^{r,i}, \tag{2}$$

The investor group is represented by $\tau \in \Upsilon$, where $\Upsilon = Active \times Size \times Broad\ Strategy \times Narrow\ Strategy \times Freq\ Rebalance \times ROS\ Currency$ and $\tau_i = \{k \in \tau | i \in \tau\}$ Graphs

Aggregation Methodology

Step 2: Our aggregation methodology assumes that:

- **the sample average** portfolio weight changes, flows and net-of-fee returns for all investors that belong to a certain type, and which we observe in the Morningstar Direct data, **capture well the population averages**,
- and that we have **"representative holdings ratios"** in our sample for each sub-type of fund.
  - **"representative holdings ratio"** = for each type of fund, τ, we assume that the ratio of average-over-time holdings of ISIN j we observe in our sample relative to the average-over-time population holdings of ISIN j is the same across all types of funds.

ISIN Level Equity Price Growth Rates Decomposition ⏩ Derivations

Steps 1 and 2 imply that we can scale up the averages of our observed equilibrium holdings sub-components, appropriately scaled by the importance of the funds for the ISIN, by the inverse of the coverage ratio, to obtain what we call the **"common" holdings sub-components**:

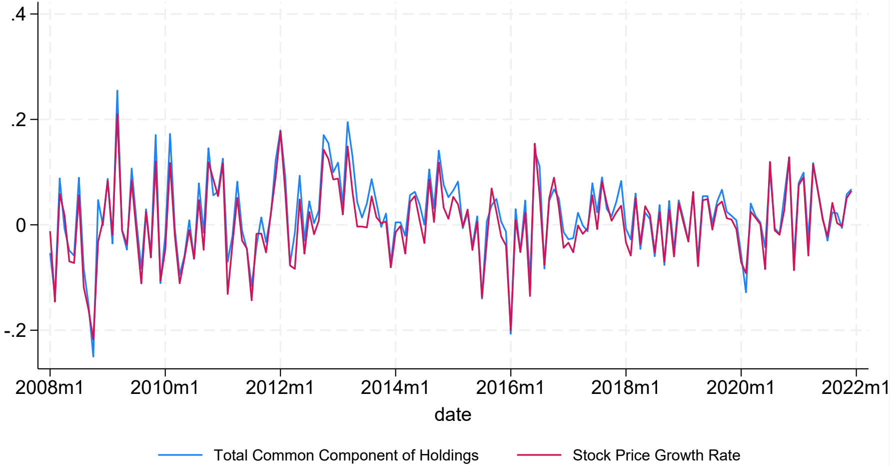
$$\Delta p_t^j = \underbrace{\Delta d_t^{s,j} + \Delta d_t^{f,j} + \Delta d_t^{\omega,j} + \Delta d_t^{r^{NF},j}}_{\Delta d_t^j} + d_t^{Resid,j} - \Delta q_t^j$$

- $\Delta d_t^{s,j}$ – component of holdings due to exchange rate valuations effects
- $\Delta d_t^{f,j}$ – "common" component of holdings due to final fund flows
- $\Delta d_t^{\omega,j}$ – "common" component of holdings due to portfolio weight changes
- $\Delta d_t^{r^{NF},j}$ – "common" component of holdings due to net-of-fee portfolio returns

Data

- Morningstar mutual fund data: Start with about 36,500 funds with clean data, ISIN level positions and additional information on fund flows, AUM, Region of Sale, Fund Type, etc.
- Refinitiv/Eikon – stock prices, market cap and various ISIN level characteristic such as industry of the firm, currency of the issued asset, type of asset, main region of operation of the issuer.
- Total AUM over 50 trillion USD (quarterly series) and close to 25 trillion USD (monthly series).
- 33 stock markets, associated with 33 currencies.
- Focus on the period from Jan 2008 till Dec 2021 and for most of the analysis study monthly frequency.
- Average correlation between our weighted stock market growth rates and equity growth rates from well-known stock market indices for the respective country is 96 percent. Details

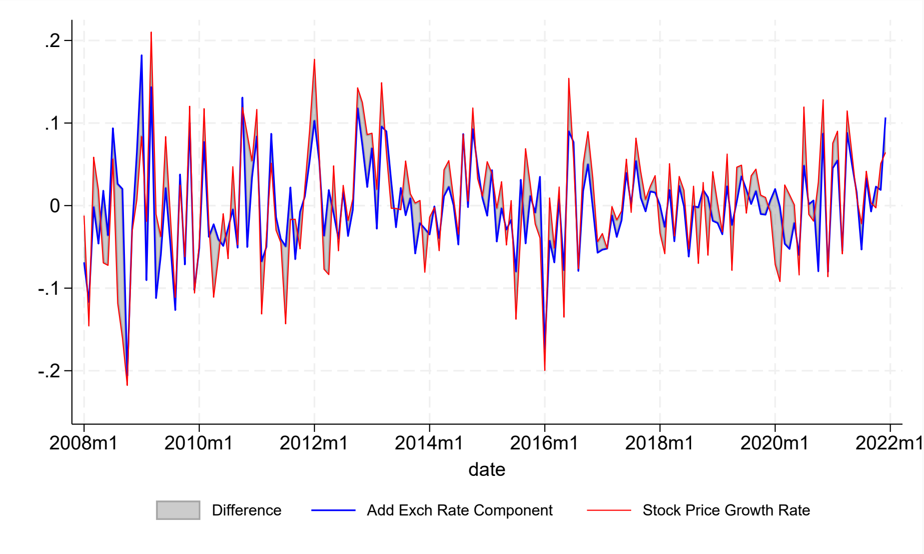
ISIN Level Equity Price Growth Rate Decomposition: TOYOTA



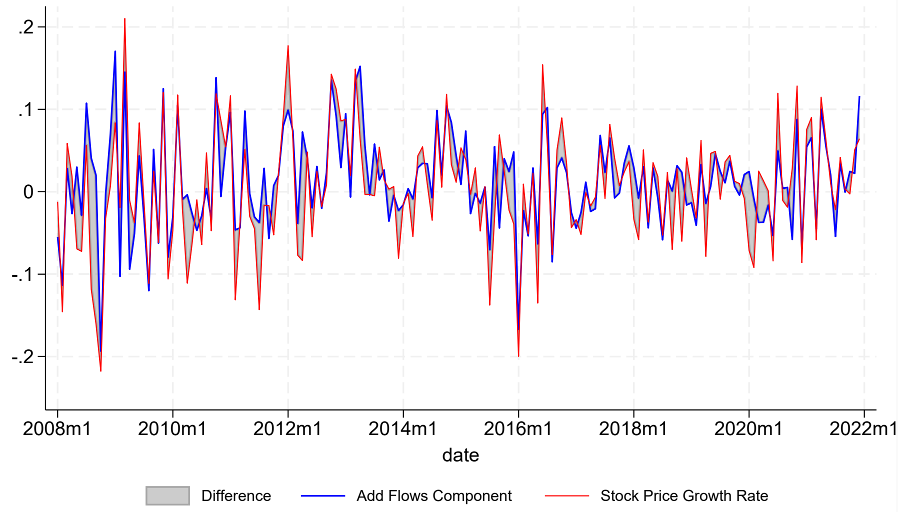
Avg coverage: 0.050; Avg number of funds holding the ISIN per month: 451.30
ISIN: JP3633400001; ISIN Market Cap USD bil: 193.64; Name: Toyota

ISIN Level Equity Price Growth Rate Decomposition: TOYOTA

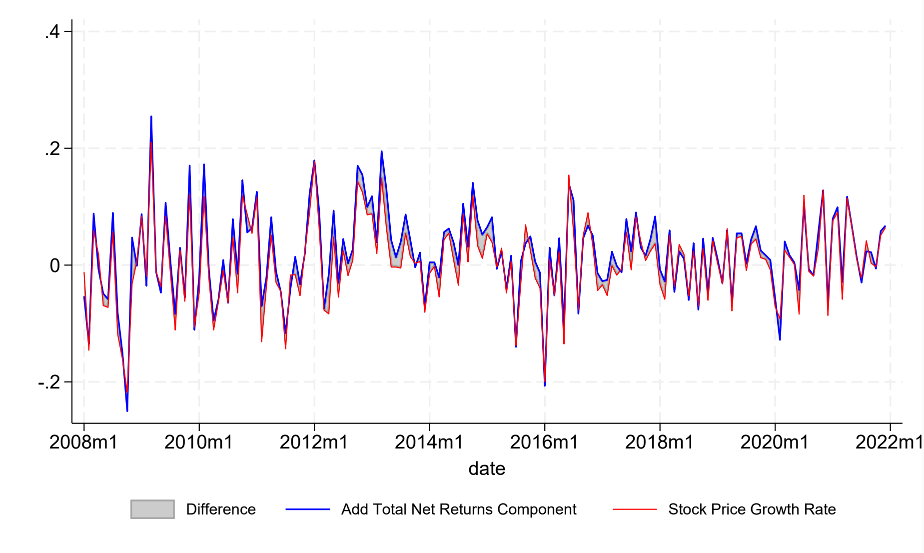
ISIN Level Equity Price Growth Rate Decomposition: TOYOTA



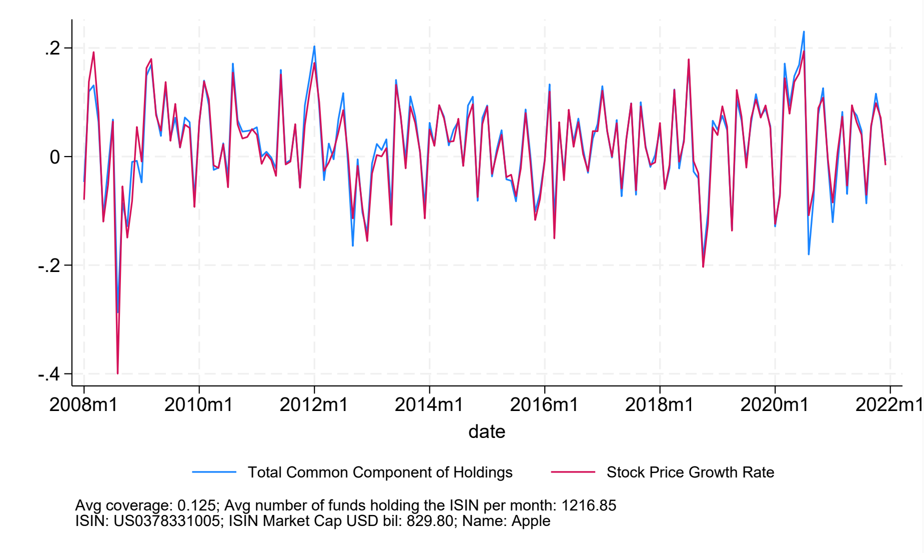
ISIN Level Equity Price Growth Rate Decomposition: TOYOTA



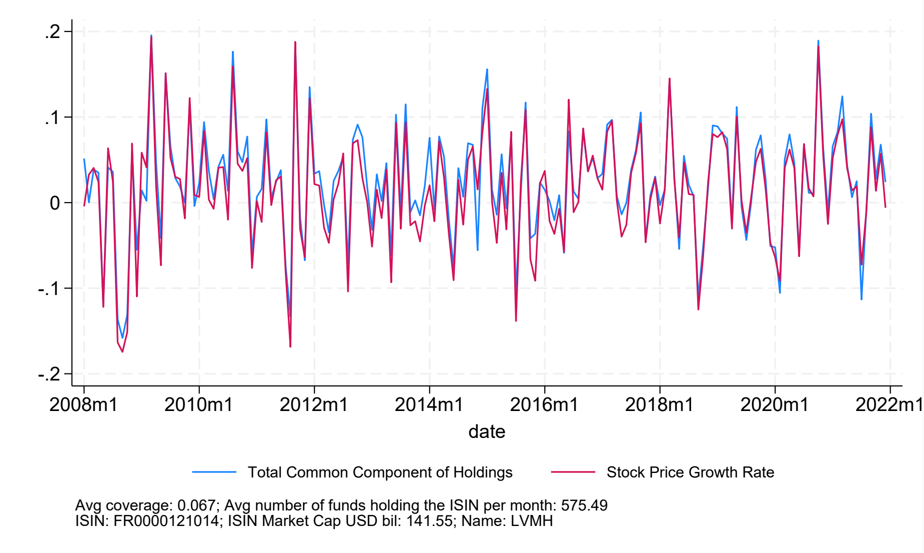
ISIN Level Equity Price Growth Rate Decomposition: TOYOTA



ISIN Level Equity Price Growth Rate Decomposition: APPLE



ISIN Level Equity Price Growth Rate Decomposition: LVMH



ISIN Level Equity Price Growth Rate Decomposition: VCV Decomposition

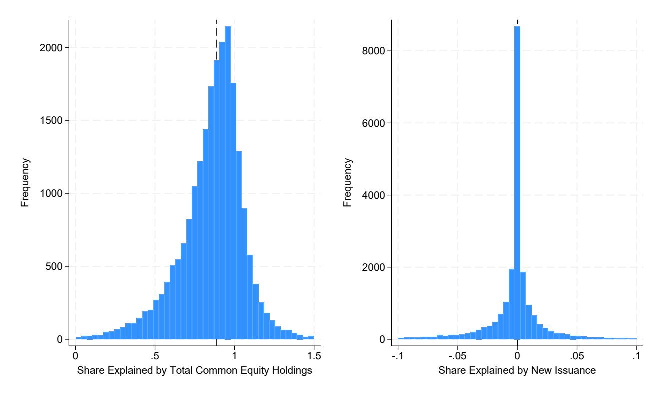
$$1 = \underbrace{\beta^{p,\Delta d^{s,j}} + \beta^{p,\Delta d^{f,j}} + \beta^{p,\Delta d^{\omega,j}} + \beta^{p,\Delta d^{r^{NF},j}}}_{\beta^{p,\Delta d^{j}}} + \beta^{p,d^{Resid,j}} - \beta^{p,\Delta q^{j}},$$

$$\text{where } \beta^{p,x} \equiv \frac{Cov\left(x_t, \Delta p_t^j\right)}{Var\left(\Delta p_t^j\right)}.$$

We estimate $\beta^{p,x}$ by regressing x_t on Δp_t^j at the ISIN level.

ISIN Level Equity Price Growth Rate Decomposition: VCV Decomposition

The "common" equity holdings sub-components, on average, account for 89% of the variation of all 20,378 ISIN.

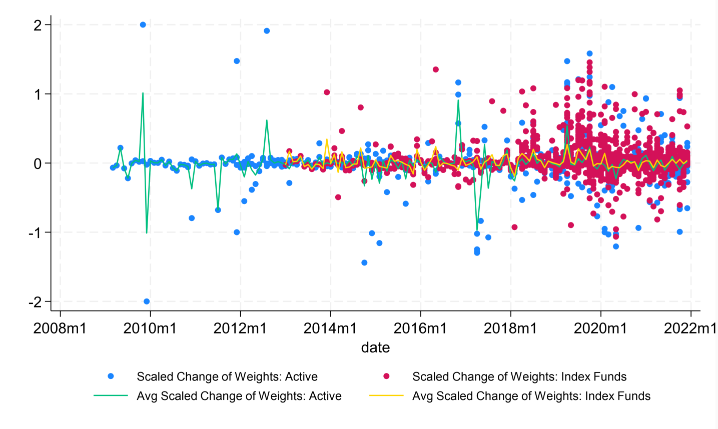


$0.5 \leq \frac{Cov\left(\Delta d^j, \Delta p_t^j\right)}{Var\left(\Delta p_t^j\right)} \leq 1.5$ for 20,738 of the 22,381 ISINs in our sample.

When Does the Aggregation Fail?

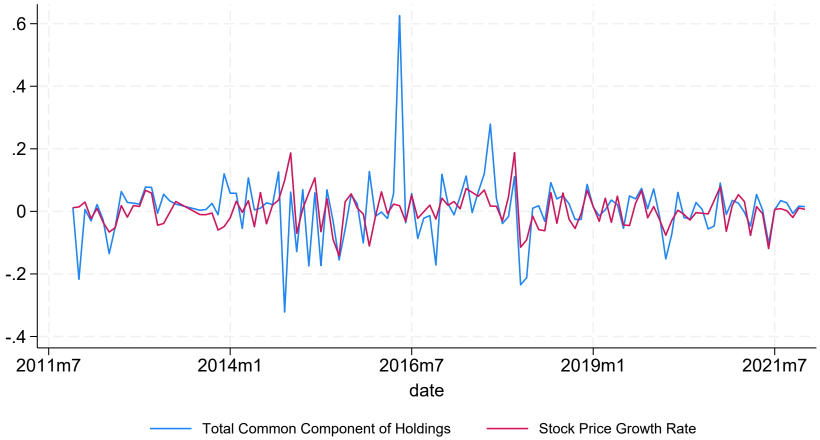
Portfolio Weights Changes $\frac{\Delta \omega_t^{i,j}}{\widehat{\omega}^{i,j}}$

Industrial and Commercial Bank of China Ltd



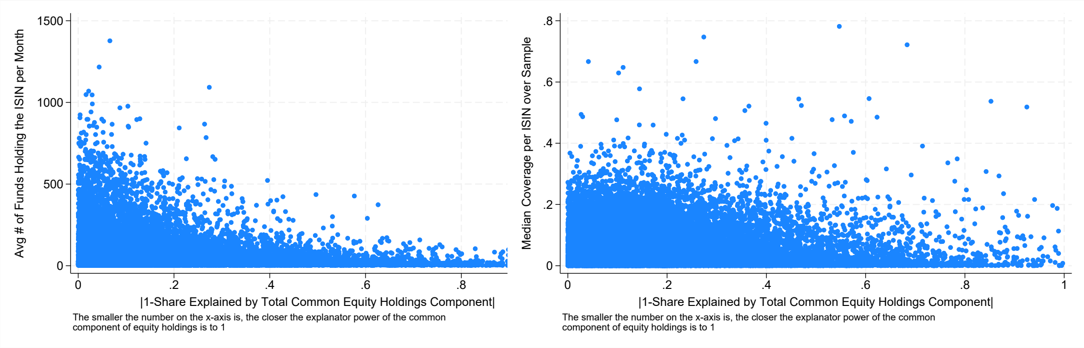
When Does the Aggregation Fail?

Figure 1: Industrial and Commercial Bank of China Ltd



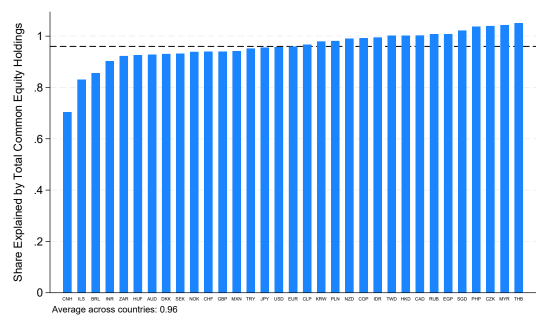
Avg coverage: 0.001; Avg number of funds holding the ISIN per month: 37.97
ISIN: CNE000001P37; ISIN Market Cap USD bil: 189.32; Name: Industrial and Commercial Bank of China Ltd

Higher coverage and more funds improve, but do not fully explain, the fit.

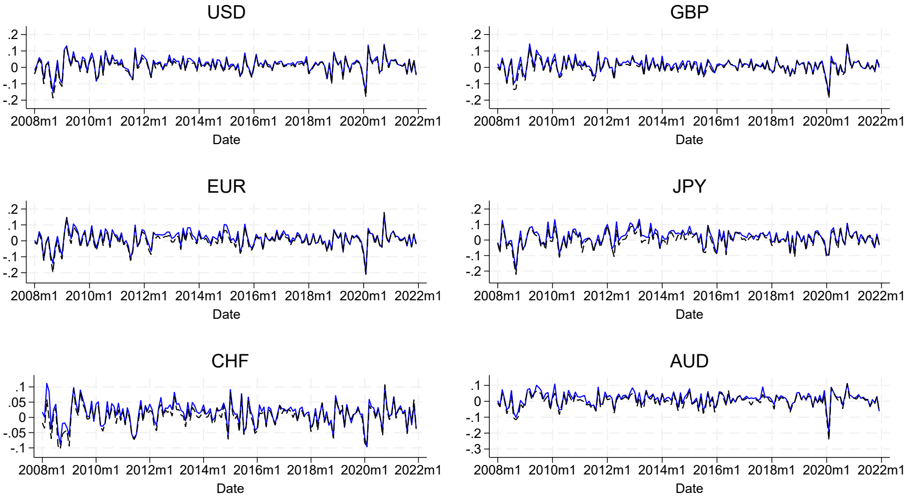


Stock Market Price Growth Rate Decomposition: VCV Decomposition

- We aggregate our ISIN-level market clearing decomposition to the level of the stock market of a given country and perform a similar variance covariance decomposition for stock market price growth rates.
- **The "common" equity holdings sub-components account for 96% of the variation of all stock markets, on average.**



"Common" equity holdings, built from mutual funds data, tracks aggregate stock market



Robustness Checks

Results are robust to:

1. *Performing the decomposition at quarterly frequency (double the AUM).*

2. *Using only the holdings of the marginal traders – i.e. funds that change the shares held.* – ⏩ Deriv

Stylized Facts

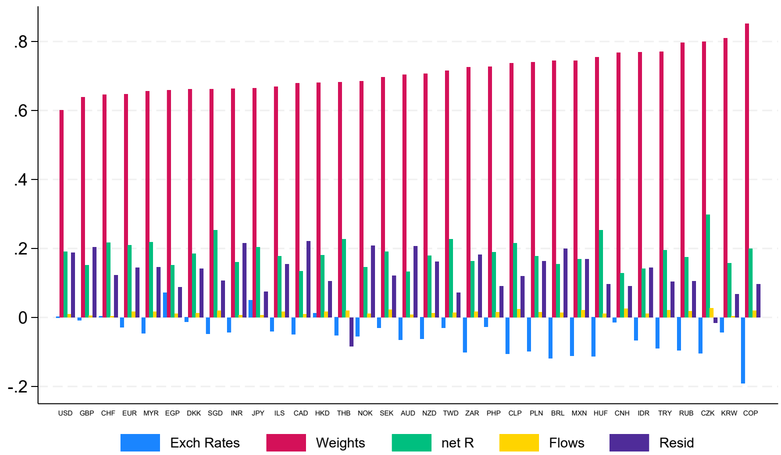
Main Stylized Facts

1. Micro is not like macro:
   - Portfolio weight changes explains most of the variation of individual stock prices.
   - Aggregate wealth effects are relatively more important for the overall stock market but significant cross-country heterogeneity.
   - *Explanation: Heterogeneous within- and cross- "currency borders" re-balancing.*

2. Exchange rates play a key role in equilibrating all stock markets:
   - Higher equity holdings by foreign investors is associated with an increases of local stock market prices and an appreciation of the local currency, except for the "safe haven" currencies: USD and JPY
     $\Longrightarrow$ exchange rate fluctuations dampen overall local stock market volatility, unconditionally, except for the "safe haven" currencies.

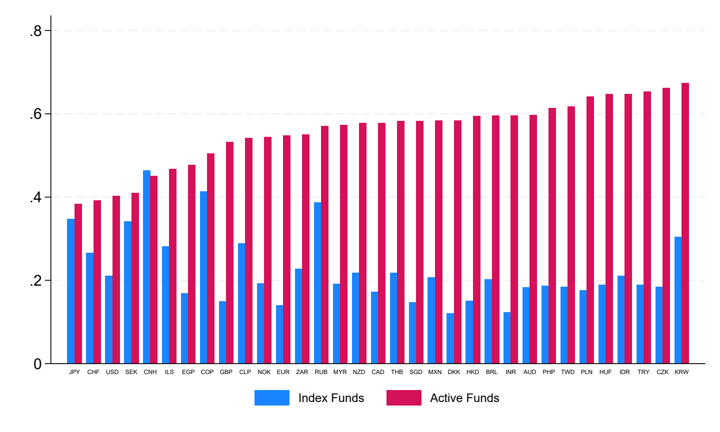
Micro: Portfolio Weight Changes Accounts for Most Equity Price Growth Rate Variation

ISIN Level VCV Decomposition: Panel Regressions: All Sub-Components



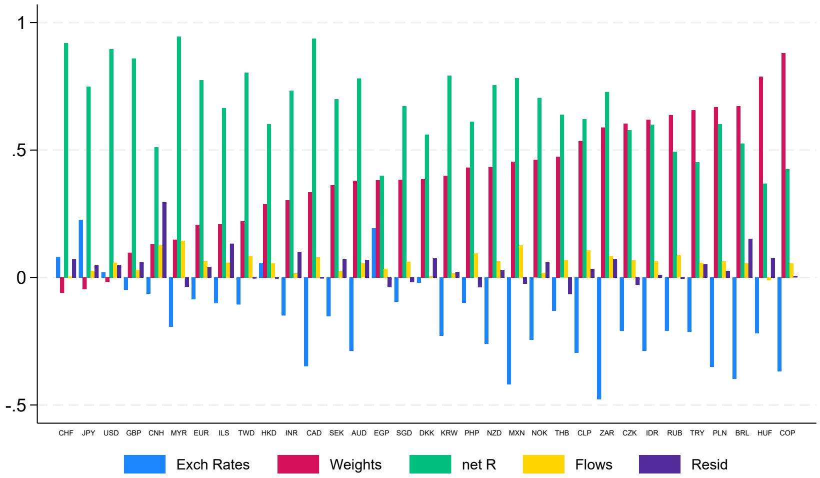
Micro: Active Funds Relatively More Important

ISIN Level VCV Decomposition: Portfolio Weight Changes: Index Funds vs Active Investors; Panel Regressions



Macro is not like Micro

Aggregate Stock Market Level VCV Decomposition: All Sub-Components



Heterogeneous Across vs Within "Currency Borders" Re-balancing ▸▸ Deriv

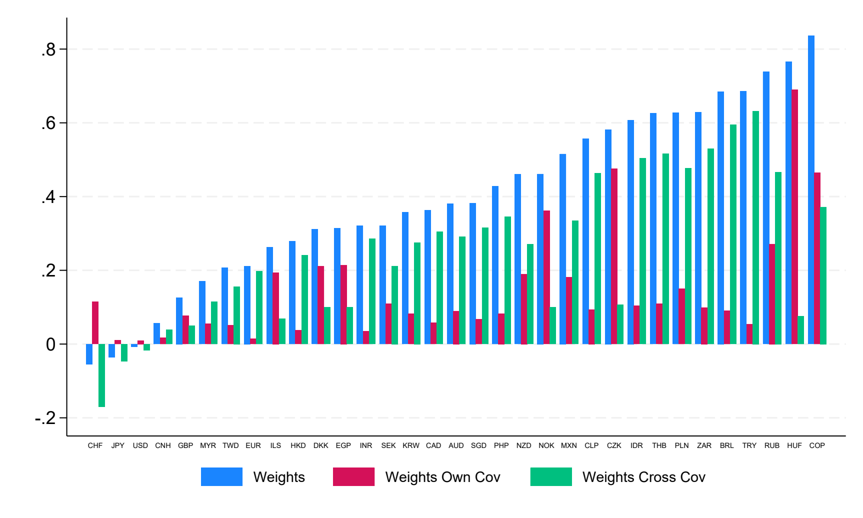
Own vs Cross-Covariance

$$\frac{Cov\left(\sum_j \nu^j \Delta p_t^j, \sum_j \nu^j d_t^{j,\omega}\right)}{Var\left(\sum_j \nu^j \Delta p_t^j\right)} = \underbrace{\sum_j \left(\nu^j\right)^2 \tilde{\sigma}^j \text{own covariance}}_{\beta^{\omega}_{OwnElast}} + \underbrace{\sum_j \sum_{k \neq j} \nu^j \nu^k \tilde{\sigma}^k \text{cross covariance}}_{\beta^{\omega}_{CrossElast}},$$

ν^j - stock market size weight; $\tilde{\sigma}^j$ - relative equity price growth rate variance for stock j

$$\frac{Cov\left(d_t^{j,\omega}, \Delta p_t^j\right)}{Var(\Delta p_t^j)} - \text{own covariance} \qquad \frac{Cov\left(d_t^{j,\omega}, \Delta p_t^k\right)}{Var(\Delta p_t^k)} - \text{cross covariance}$$

Heterogeneous Across vs Within "Currency Borders" Re-balancing



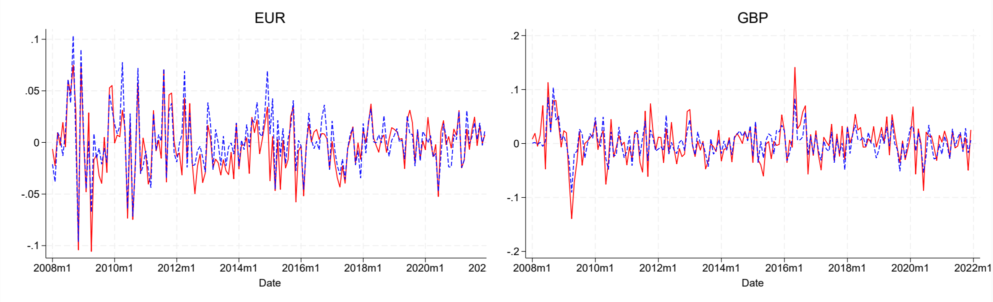
"Currency Centrality in Equity Markets, Exchange Rates and Global Financial Cycles" by Rey, Stavrakeva and Tang

Based on the same market clearing conditions for equities, we...

- decompose exchange rate changes in terms of currency-specific **equity net supplies** (*growth rate of the stock market capitalization less the change in equilibrium holdings, denominated in the investors' currencies*), scaled by **observed elasticities** that capture investor currency "centrality",
- **show the observed components also account for the vast majority of monthly variation in exchange rate changes**,
- we use our decomposition to explain the striking comovement between the USD, global equity markets, and the Global Financial Cycle, as captured by US macro news and risk aversion news

⏩ Details

"Currency Centrality in Equity Markets, Exchange Rates and Global Financial Cycles" by Rey, Stavrakeva and Tang



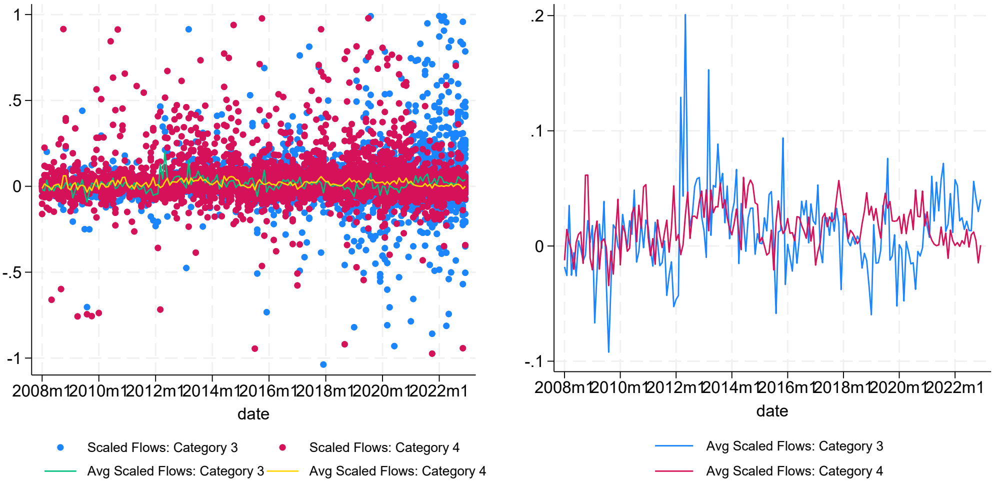
dashed blue line – realized exchange rate changes (USD Base)

red line – observed net supplies, scaled by currency elasticties (USD Base)

Conclusion

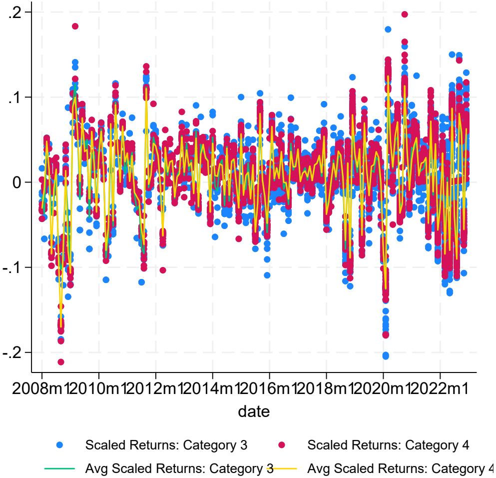
- We develop a novel decomposition of equity price growth rate based on market clearing conditions, linearization and a representativeness assumption.
- The **fit of the data for individual and aggregate stock market price growth** is exceptional. *Elephants!*
- Given the minimal number of assumptions imposed, the novel stylized facts we provide should motivate **all** theories of asset pricing.

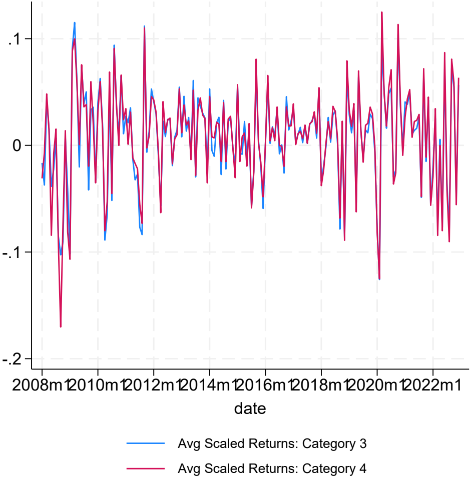
Example: Constructing the Flows Sub-Component ⏩ Resturn



Equity Funds; USD ROS currency;
Category 3 : Active: more active; Freq Rebalance: re-balancing frequently; size of fund: <=100mil; GlobalCategory: US Equity Large Cap Blend
Category 4: Active: Index Funds; Freq Rebalance: re-balancing frequently; size of fund: >1bil; GlobalCategory: US Equity Large Cap Blend

Example: Constructing the Net-Of-Fee Returns Sub-Component

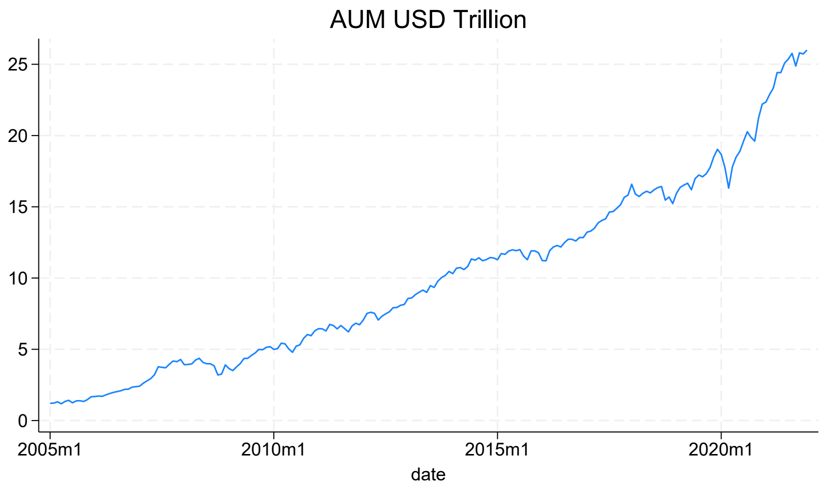




Equity Funds; USD ROS currency;
Category 3 : Active: more active; Freq Rebalance: re-balancing frequently; size of fund: <=100mil; GlobalCategory: US Equity Large Cap Blend
Category 4: Active: Index Funds; Freq Rebalance: re-balancing frequently; size of fund: >1bil; GlobalCategory: US Equity Large Cap Blend

Data

Figure 2: Total AUM USD Trillions; Monthly



Data

Figure 3: Total AUM USD Trillions by investment type and ROS; Monthly

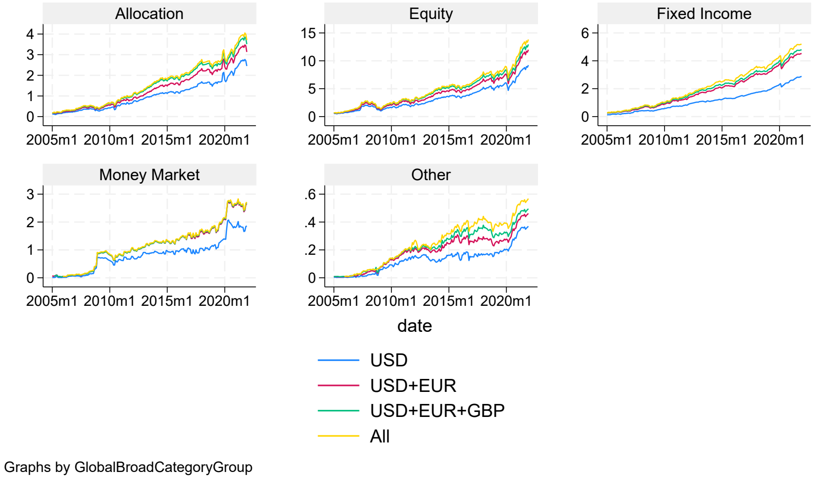


Table 1: Coverage and Market Capitalization; Monthly; ⏩ Back

Currency	*AvgCoverage*	*CoverageStart*	*CoverageEnd*	*AvgMarketCapUSDbil*	*MarketCapStartUSDbil*	*MarketCapEndUSDbil*	*ISINs*
AUD	0.04	0.03	0.06	295.33	213.67	568.89	622.00
BRL	0.04	0.02	0.04	504.58	424.73	567.80	293.00
CAD	0.04	0.03	0.06	532.88	459.98	875.06	692.00
CHF	0.07	0.04	0.10	482.63	327.11	805.25	185.00
CLP	0.02	0.00	0.02	91.35	78.02	62.15	52.00
CNH	0.00	0.00	0.01	4183.93	537.56	9725.52	1793.00
DKK	0.06	0.02	0.12	116.13	53.31	262.52	100.00
EGP	0.02	0.01	0.02	31.17	58.66	27.05	62.00
EUR	0.06	0.02	0.09	3130.43	2446.48	5590.11	1739.00
GBP	0.10	0.04	0.16	914.61	912.00	1363.11	1128.00
HKD	0.03	0.02	0.04	418.03	269.27	494.79	496.00
IDR	0.03	0.01	0.05	188.77	47.85	318.97	217.00
ILS	0.02	0.01	0.03	67.07	30.00	178.88	148.00
INR	0.03	0.02	0.04	1311.01	518.85	3189.60	1021.00
JPY	0.07	0.02	0.17	2654.40	1542.12	3781.17	2558.00
KRW	0.04	0.02	0.05	636.64	376.49	1122.56	1548.00
MXN	0.03	0.02	0.04	195.42	113.13	259.73	109.00
MYR	0.02	0.02	0.02	267.12	161.99	270.37	424.00
NOK	0.04	0.01	0.07	154.17	171.98	290.01	159.00
NZD	0.03	0.01	0.04	24.16	7.12	52.73	59.00
PHP	0.03	0.03	0.02	116.75	36.80	161.90	116.00
THB	0.01	0.00	0.01	459.03	137.25	853.41	507.00
TRY	0.03	0.02	0.03	89.14	71.91	70.11	141.00
TWD	0.05	0.02	0.07	836.05	489.56	1759.39	1144.00
USD	0.12	0.07	0.16	6073.89	4348.37	12502.75	5881.00
ZAR	0.05	0.02	0.06	157.94	129.62	181.67	109.00

Deriving the Equilibrium Holdings Sub-Components ⏩ Back

Express the linearized market clearing condition as:

$$
\begin{aligned}
\Delta p_t^j = \quad & \sum_m \left(\widehat{H}_{\tilde{I}}^{j,m} + \widehat{H}_{\tilde{I}^{miss}}^{j,m}\right)\left(\Delta s_t^{l/m}\right) \\
& + \sum_{\tau \in \Upsilon} \left(\widehat{H}_{\tilde{I}}^{j,\tau} + \widehat{H}_{\tilde{I}^{miss}}^{j,\tau}\right)\left(\Delta \alpha_t^{f,\tau} + \alpha_t^{\omega,\tau,j} + \bar{r}_t^{NF,\tau}\right) \\
& + \sum_{i \in I} \frac{\mu^{i,j}}{\widehat{P^j Q^j}} \left(\varepsilon_t^{r,i} + \varepsilon_t^{f,i} + \varepsilon_t^{\omega,i,j}\right) - \Delta q_t^j,
\end{aligned}
$$

- $\mu^{i,j} = \widehat{W}^i \widehat{S}^{l/c^i} \widehat{\omega}^{i,j}$
- $\tilde{I}$ is the set of funds we observe in our sample that hold ISIN j and $\tilde{I}^{miss} \equiv I \setminus \tilde{I}$ is the set of investors we do not observe.

Deriving the Equilibrium Holdings Sub-Components

where

$$\alpha_t^{f,\tau} = \sum_{k\in\tau} \frac{flow_t^k}{|\tau_i|}$$

$$\alpha_t^{\omega,\tau,j} = \sum_{k\in\tau'} \frac{1}{|\tau_i'|} \frac{\Delta\omega_t^{k,j}}{\widehat{\omega}^{k,j}} \textit{for all } \tau \subseteq \tau'$$

$$\bar{r}_t^{NF,\tau} = \sum_{k\in\tau} \frac{r_t^{k,NF}}{|\tau_i|}.$$

- $\widehat{H}_{\tilde{I}}^{j,\tau} \equiv \sum_{\{i|\ i\in\tilde{I}\cap i\in\tau\}} \frac{\mu^{i,j}}{\widehat{P^j Q^j}}$, $\widehat{H}_{\tilde{I}^{miss}}^{j,\tau} \equiv \sum_{\{i|\ i\in\tilde{I}^{miss}\cap i\in\tau\}} \frac{\mu^{i,j}}{\widehat{P^j Q^j}}$
- $\widehat{H}_{\tilde{I}}^{j,m} \equiv \sum_{\{i|\ i\in\tilde{I}\cap c^i=m\}} \frac{\mu^{i,j}}{\widehat{P^j Q^j}}$, $\widehat{H}_{\tilde{I}^{miss}}^{j,m} \equiv \sum_{\{i|\ i\in\tilde{I}^{miss}\cap c^i=m\}} \frac{\mu^{i,j}}{\widehat{P^j Q^j}}$

Deriving the Equilibrium Holdings Sub-Components

We further assume that:

$$\widehat{H}^{j,\tau}_{\tilde{I}^{miss}} = \kappa^j \widehat{H}^{j,\tau}_{\tilde{I}},$$

where the scaling parameter κ^j depends on the ISIN but not on the group of funds. Since total holdings must equal the total market capitalization of ISIN j,

$$\sum_{\tau\in\Upsilon} \left(\widehat{H}^{j,\tau}_{\tilde{I}} + \widehat{H}^{j,\tau}_{\tilde{I}^{miss}}\right) = \left(1+\kappa^j\right) \sum_{\tau\in\Upsilon} \widehat{H}^{j,\tau}_{\tilde{I}} = 1,$$

which implies

$$1+\kappa^j = \frac{1}{\sum_{\tau\in\Upsilon}\left(\widehat{H}^{j,\tau}_{\tilde{I}}\right)} = \frac{\widehat{P^j Q^j}}{\sum_{i\in\tilde{I}} \mu^{i,j}}.$$

Given that the set τ conditions on the ROS currency of the fund, $\widehat{H}^{j,m}_{\tilde{I}^{miss}} = \kappa^j \widehat{H}^{j,m}_{\tilde{I}}$

Deriving the Equilibrium Holdings Sub-Components

- Re-writing the market clearing condition:

$$\Delta p_t^j \quad = \quad \underbrace{\Delta d_t^{s,j} + \underbrace{\Delta d_t^{f,j} + \Delta d_t^{\omega,j} + \Delta d_t^{r^{NF},j}}_{\Delta d_t^{ROS,j}}}_{\Delta d_t^j} + d_t^{Resid,j} - \Delta q_t^j$$

where ...

Deriving the Equilibrium Holdings Sub-Components

Back

$$
\begin{aligned}
\Delta d_t^{s,j} &= \sum_m \frac{\sum_{\{i:\ i\in\tilde{I}\cap c^i=m\}} \mu^{i,j}}{\sum_{i\in\tilde{I}} \mu^{i,j}} \Delta s_t^{l/m}, \\
\Delta d_t^{f,j} &= \sum_{\tau\in\Upsilon} \frac{\sum_{\{i:\ i\in\tilde{I}\cap i\in\tau\}} \mu^{i,j}}{\sum_{i\in\tilde{I}} \mu^{i,j}} \alpha_t^{f,\tau}, \\
\Delta d_t^{\omega,j} &= \sum_{\tau\in\Upsilon} \frac{\sum_{\{i:\ i\in\tilde{I}\cap i\in\tau\}} \mu^{i,j}}{\sum_{i\in\tilde{I}} \mu^{i,j}} \alpha_t^{\omega,\tau,j}, \\
\Delta d_t^{r^{NF},j} &= \sum_{\tau\in\Upsilon} \frac{\sum_{\{i:\ i\in\tilde{I}\cap i\in\tau\}} \mu^{i,j}}{\sum_{i\in\tilde{I}} \mu^{i,j}} \bar{r}_t^{NF,\tau}, \\
d_t^{Resid,j} &= \sum_{i\in I} \frac{\mu^{i,j}}{\widehat{P^j Q^j}} \left(\varepsilon_t^{r,i} + \varepsilon_t^{f,i} + \varepsilon_t^{\omega,i,j}\right),
\end{aligned}
$$

Deriving the Equilibrium Holdings Sub-Components: Marginal Investor

Back

Let $X_t^{i,j}$ define the shares held by fund i of ISIN j. Fund $i's$ holdings of ISIN j can be also expressed as $P_t^j X_t^{i,j}$, where

$$P_t^j X_t^{i,j} = \omega_t^{i,j} W_t^i S_t^{l/c^i}.$$

Since $\widehat{X^{i,j}}\widehat{P^j} \approx \left(\widehat{W}^i \widehat{S}^{l/c^i} \widehat{\omega}^{i,j}\right)$, when linearized, the change in holdings can be also re-written as:

$$\Delta P_t^j X_t^{i,j} \approx \Delta p_t^j \left(\widehat{W}^i \widehat{S}^{l/c^i} \widehat{\omega}^{i,j}\right) + \widehat{P^j} \Delta X_t^{i,j}.$$

Deriving the Equilibrium Holdings Sub-Components: Marginal Investor

Back

Re-writing the linearized market clearing condition for ISIN j, after splitting the equilibrium holdings into marginal and non-marginal investors' equilibrium holdings implies:

$$\left(\sum_{\{i \in I : \Delta X_t^{i,j} = 0\}} \frac{\left(\widehat{W^i} \widehat{S}^{l/c^i} \widehat{\omega}^{i,j} \right)}{\widehat{P^j Q^j}} \right) \Delta p_t^j + \sum_{\{i \in I : \Delta X_t^{i,j} \neq 0\}} \frac{\widehat{W^i} \widehat{S}^{l/c^i} \widehat{\omega}^{i,j}}{\widehat{P^j Q^j}} \left(\frac{\Delta \omega_t^{i,j}}{\widehat{\omega}^{i,j}} + \Delta w_t^i + \Delta s_t^{l/c^i} \right)$$

$$= \Delta p_t^j + \Delta q_t^j .$$

Deriving the Equilibrium Holdings Sub-Components: Marginal Investor

Back

After scaling up the observed holdings using the inverse of the coverage ratio and expressing the holdings sub-components as simple averages and residuals, we once again obtain the following expression for equity price growth rates:

$$\Delta p_t^j = \underbrace{\Delta \tilde{d}_t^{s,j} + \underbrace{\Delta \tilde{d}_t^{f,j} + \Delta \tilde{d}_t^{\omega,j} + \Delta \tilde{d}_t^{r^{NF},j}}_{\Delta \tilde{d}_t^{ROS,j}}}_{\Delta \tilde{d}_t^{j}} + \tilde{d}_t^{Resid,j} - \frac{1}{\Theta^j} \Delta q_t^j$$

where

$$\Theta^j = \left(1 - \left(\sum_{\tau \in \Upsilon} \sum_{\{i:\ i \in \tilde{I} \cap i \in \tau \cap \Delta X_t^{i,j} = 0\}} \frac{\mu^{i,j}}{\sum_{i \in \tilde{I}} \mu^{i,j}}\right)\right)$$

and ...

Deriving the Equilibrium Holdings Sub-Components: Marginal Investor

Back

$$\Delta \tilde{d}_t^{s,j} = \frac{1}{\Theta^j} \sum_{\tau \in \Upsilon} \sum_{\left\{i:\ i \in \tilde{I} \cap c^i \in m \cap \Delta X_t^{i,j} \neq 0\right\}} \frac{\mu^{i,j}}{\sum_{i \in \tilde{I}} \mu^{i,j}} \left(\Delta s_t^{l/m}\right),$$

$$\Delta \tilde{d}_t^{f,j} = \frac{1}{\Theta^j} \sum_{\tau \in \Upsilon} \sum_{\left\{i:\ i \in \tilde{I} \cap i \in \tau \cap \Delta X_t^{i,j} \neq 0\right\}} \frac{\mu^{i,j}}{\sum_{i \in \tilde{I}} \mu^{i,j}} \left(\alpha_t^{f,\tau}\right),$$

$$\Delta \tilde{d}_t^{\omega,j} = \frac{1}{\Theta^j} \sum_{\tau \in \Upsilon} \sum_{\left\{i:\ i \in \tilde{I} \cap i \in \tau \cap \Delta X_t^{i,j} \neq 0\right\}} \frac{\mu^{i,j}}{\sum_{i \in \tilde{I}} \mu^{i,j}} \left(\alpha_t^{\omega,\tau,j}\right),$$

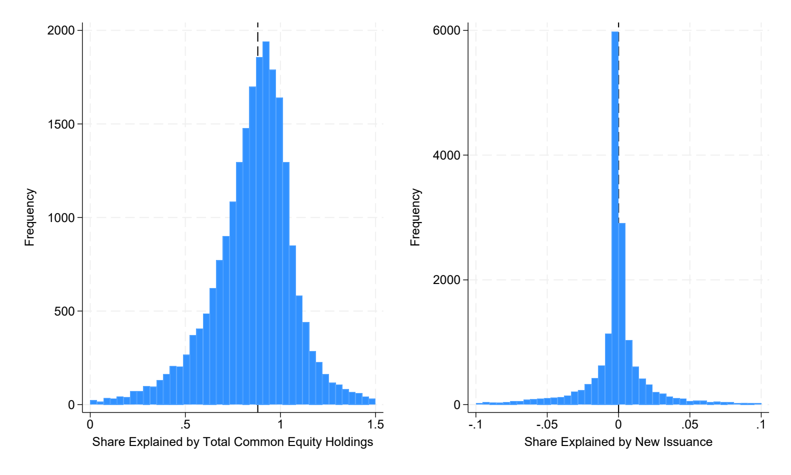
$$\Delta \tilde{d}_t^{r^{NF},j} = \frac{1}{\Theta^j} \sum_{\tau \in \Upsilon} \sum_{\left\{i:\ i \in \tilde{I} \cap i \in \tau \cap \Delta X_t^{i,j} \neq 0\right\}} \frac{\mu^{i,j}}{\sum_{i \in \tilde{I}} \mu^{i,j}} \left(\bar{r}_t^{NF,\tau}\right),$$

$$\tilde{d}_t^{Resid,j} = \frac{1}{\Theta^j} \sum_{\left\{i \in I: \Delta X_t^{i,j} \neq 0\right\}} \frac{\mu^{i,j}}{\widehat{P^j Q^j}} \left(\varepsilon_t^{r,i} + \varepsilon_t^{f,i} + \varepsilon_t^{\omega,i,j}\right),$$

Marginal Investor: ISIN Level Fit

Back

Figure 4: ISIN-Level Stock Price Contributions of Common Holdings (Marginal Traders)



Micro to Macro

Back

$$\frac{Cov\left(\sum_j \nu^{j,l}\Delta p_t^j, \sum_j \nu^{j,l} d_t^{j,\omega}\right)}{Var\left(\sum_j \nu^{j,l}\Delta p_t^j\right)} = \underbrace{\sum_j \left(\nu^{j,l}\right)^2 \frac{Var\left(\Delta p_t^j\right)}{Var\left(\sum_j \nu^{j,l}\Delta p_t^j\right)} \frac{Cov\left(\Delta p_t^j, d_t^{j,\omega}\right)}{Var(\Delta p_t^j)}}_{\beta^{\omega}_{OwnCov}} +$$

$$\underbrace{\sum_j \sum_{k\neq j} \nu^{j,l}\nu^{k,l} \frac{Cov\left(d_t^{j,\omega}, \Delta p_t^k\right)}{Var(\Delta p_t^k)} \frac{Var(\Delta p_t^k)}{Var\left(\sum_j \nu^{j,l}\Delta p_t^j\right)}}_{\beta^{\omega}_{CrossCov}},$$

$$\nu^{j,l} = \frac{\widehat{P^j Q^j}}{\sum_{\{j:c^j=l\}} \widehat{P^j Q^j}}$$

Exchange Rate Change Decomposition

Solve for the vector of currency crosses $\mathbf{\Delta s}_t^{\mathbf{l}/z}$ as a function only of net supply of all currencies. For $l \neq z$:

$$\tilde{\mathbf{A}}^z \underbrace{\begin{bmatrix} \Delta s_t^{GBP/z} \\ .. \\ \Delta s_t^{EUR/z} \end{bmatrix}}_{\mathbf{\Delta s}_t^{\mathbf{l}/z}} \approx \underbrace{\begin{bmatrix} \Delta MC_t^{GBP} - \Delta D_t^{ROS,GBP} \\ .. \\ \Delta MC_t^{EUR} - \Delta D_t^{ROS,EUR} \end{bmatrix}}_{\tilde{\mathbf{\Pi}}_t^{\mathbf{l}}} - \underbrace{\begin{bmatrix} \Delta MC_t^{z} - \Delta D_t^{ROS,z} \\ .. \\ \Delta MC_t^{z} - \Delta D_t^{ROS,z} \end{bmatrix}}_{\tilde{\mathbf{\Pi}}_t^{z}}$$

where $\tilde{\mathbf{A}}^z$ is a function of the currency centrality terms.

$$\mathbf{\Delta s}_t^{\mathbf{l}/z} \approx \left(\tilde{\mathbf{A}}^z\right)^{-1} \left(\tilde{\mathbf{\Pi}}_t^{l} - \tilde{\mathbf{\Pi}}_t^{z}\right).$$

Back