The Welfare of Nations: Social Preferences and the Macroeconomy

François Le Grand   Xavier Ragot   Diego Rodrigues*

PRELIMINARY, July 2024

Abstract

We present a Bewley-type theory of the Social Welfare Function (SWF): agents have heterogeneous Individual Welfare Functions (or political views) shaped by their life experiences and the planner then constructs the SWF by the aggregation of these individual views. We then consider a Ramsey problem in an heterogeneous-agent economy to impose restrictions on the SWF. We then use the inverse optimal approach for the model to replicate a chosen fiscal system and allocation. We apply our methodology to France and the United States. France's SWF places greater emphasis on individuals with lower incomes, contrasting with the United States' SWF, which assigns greater weight to individuals with higher income. Finally, we simulate the fiscal system of the United States under the assumption of adopting the French SWF. Our findings indicate that the SWF significantly shapes the optimal steady-state fiscal system and equilibrium inequality.

Keywords : Social Welfare Function, Inequality, Fiscal systems.

JEL codes : E61, E62, E32.

*LeGrand: emlyon business school and ETH Zurich; legrand@em-lyon.com. Ragot: SciencesPo, OFCE, and CNRS; xavier.ragot@gmail.com. Rodrigues: SciencesPo; diego.desousarodrigues@sciencespo.fr

1 Introduction

Countries differ widely in terms of their fiscal system: The levels of social transfers, labor and capital taxes are very different across OECD countries. For instance, the average mandatory levies in France were 43% of its GDP from 1995 to 2007, while they were around 26% in the US for the same period. Various explanations can rationalize these international differences. First, production technologies can be different across countries, and so do the distortions generated by taxation. Second, in addition to facing different trade-offs, preferences can be heterogeneous. Individual preferences over consumption and leisure, as well as social preferences regarding redistribution, can differ across countries. Third, even if technology and preferences are the same, the political system that selects and implements actual policies can differ across countries, generating outcomes that are observationally equivalent to alternative social preferences.

The goal of this paper is to identify Social Welfare Functions (SWFs) from observed fiscal policy and redistributive outcomes in both France and the US. We first provide a Bewley-type theory of the SWF, assuming that agents can have heterogeneous Individual Welfare Functions (IWFs), which reflect their moral or political views. Consistent with the Bewley view of heterogeneity, it is assumed that this heterogeneity is the outcome of idiosyncratic history. The SWF is the stable outcome of an aggregation procedure depending on political institutions. This theory generates a simple SWF that can be estimated. The motivation for this assumption is that political views about redistribution are the outcome of each agent's own history, which is consistent with the theoretical construction of steady-state equilibrium in heterogeneous agent models (where initial distributions do not affect the equilibrium one). As these histories are exogenous variables for individual agents, such SWFs are theoretically consistent. In addition, following theoretical results in Social Choice (Sen (1970)), we impose a minimum set of restrictions on this SWF. In particular, we impose a weaker restriction than the Pareto Principle, which is indeed very often rejected in empirical work (Fehr and Schmidt (2006) or Schokkaert and Tarroux (2021) for surveys). We first present a simple model to introduce definitions and concepts within a structured framework.

Second, we show that income and wealth inequalities can be closely reproduced in a heterogeneous-agent model (Bewley-Huggett-Aiyagari type), with a realistic fiscal system, considering a rich set of fiscal instruments : a consumption tax, non-linear labor tax, capital tax and public debt. If one considers that these instruments are optimally chosen by a government, one can determine the set of possible Social Welfare Functions for which the value of instruments is optimal for the observed allocation. More formally, we identify the SWFs for which the steady-state market equilibrium with the given fiscal tools is a Ramsey steady state. This strategy is often called the inverse optimal approach, and it has been used in environment assuming ex ante heterogeneity (see Bargain and Keane 2010, Bourguignon and Amadeo 2015, Heathcote and Tsujiyama 2021 and LeGrand and Ragot 2023a). Using this strategy in an full-fledged heterogeneous-agent model with capital raises conceptual and methodological challenges. We build upon LeGrand and Ragot (2023a) to estimate the SWF, extending the methodology to ex post heterogeneity consistent with the Bewley-type theory of the SWF. We use the Lagrangian approach to derive the first-order conditions of the Ramsey problem with commitment. Subsequently, we use the sequential representation of heterogeneous-agent models, along with a truncation procedure to obtain a finite state space (as in LeGrand and Ragot, 2022a).

In this methodology, admittedly, we identify the SWF assuming that the observed market equilibrium is the steady-state of a Ramsey allocation with commitment. This may seem demanding, as many details of political institutions, such as the voting process, the bargaining power of some parties, lobbying activities, and so on, could influence how the same SWF translates into different fiscal outcomes. All of these effects are captured in the estimated SWFs. We do not claim that all political institutions result from choices reflecting social preferences. Rather, our claim is that the estimated SWF serves as a useful summary measure of the actual outcome of the entire political process.

Finally, since the number of instruments is finite and there exists a vast set of SWFs that could rationalize the fiscal outcomes, we adopt two identification strategies. First, we choose among the possible SWFs the one that is closest to the Utilitarian SWF, which assigns the same weight to all agents. This conservative assumption is sufficient to identify the SWF. Second, we use a parametric representation of the SWFs based on a specification similar to Heathcote and Tsujiyama (2021), ensuring exact identification of the parameters. This approach ensures that the estimated SWFs are closely aligned.

We apply this methodology to identify the SWFs of France and the US, focusing initially on their fiscal systems before the financial crisis. This selection is motivated by the substantial differences in fiscal policies between these two OECD countries. We choose the period from 1995 to 2007 to exclude the financial crisis and the subsequent Covid-19 crisis, which led to significant (transitory) changes in fiscal structures.

This estimation generates two sets of results. First, the SWFs in France and in the US are very different from each other. The US SWF is mostly increasing with income and puts the largest weight on rich and high-income agents. The weights for middle-class agents are lower and somewhat similar, and low-income agents have the lowest weight. The French SWF is different. It assigns the highest weight to low-income agents, with weights decreasing with income and then increasing again for high-income agents. In both a simple model and a quantitative model, we define an Egalitarian and a Libertarian SWF. Unsurprisingly, we find that the US is more Libertarian than Egalitarian, while France is more Egalitarian than Libertarian. Second, to understand the role of the SWF, we simulate the optimal US fiscal system if the US were to adopt the French SWF. We find that the Gini coefficient of wealth would decrease from 78% to 63%, approaching the French Gini coefficient of wealth of 68%. Consequently, social preferences are a primary driver of the fiscal system and household inequality.

Related literature [To be Completed]. Our paper is related to three streams of literature, social choice, public Finance and quantitive macro.

First we contribute to the optimal fiscal policy literature, and to Inverse Optimal Taxation Problem, which estimate SWFs from actual fiscal policies. Most of this literature estimates the SWF, using only labor tax in economies without capital (Bargain and Keane, 2010; Bourguignon and Amadeo, 2015). Chang et al. (2018) use an heterogeneous-agent model with capital to estimate a parameter of the SWF, which is the concern for inequality. They focus on a single fiscal instrument. As we derive the first-order conditions of the Ramsey problem, we can estimate the whole set of Pareto weights using four fiscal instruments. Heathcote and Tsujiyama (2021) also estimate the SWF with one degree of freedom. Their focus is slightly different from ours since their framework is static, but features private information à la Mirrlees.

Second, this paper contributes to the new literature on optimal policies in heterogeneous-agent models. Some papers have studied the effects of given fiscal experiments in heterogeneous-agent frameworks, such as Heathcote (2005), who considers aggregate shocks, and Kaplan and Violante (2014), who consider a fiscal transfer. Heathcote and Perri (2017) analyze equilibrium multiplicity in an economy without capital. In economies without aggregate shocks, Aiyagari (1995) shows that the steady-state capital tax can be non-negative. Aiyagari and McGrattan (1998) compute the optimal steady-state level of public debt. Dávila et al. (2012) show that the steady-state capital stock can be too low, solving for a constrained-efficient allocation.

Some recent papers solves a Ramsey problem to obtain the steady-state fiscal policy and level of public debt. Dyrda and Pedroni (2022) maximize welfare over all transition paths to determine optimal tax system. Açikgöz et al. (2022) use the first-order conditions of the Ramsey problem to derive the optimal tax system. These two papers consider the Utilitarian SWF.

Finally, there is a vast literature on social preferences, considering fairness principles. In this literature, we follow the weighted-utilitarianism approach, which is flexible enough to incorporate these concerns (Fleurbaey and Maniquet, 2018). This approach although simpler than the recent contribution of Saez and Stantcheva (2016) allows us to estimate SWF in general equilibrium.

The paper proceeds as follows. In Section 2 we present motivating evidence. We provide the basics of our Bewley construction of SWFs in Section 3 in the context of a simple two-period model. The construction is generalized to an infinite horizon model in Section 4. Section 5 presents the environment in which we will conduct our estimation of SWF weights. The Ramsey program in this setup is presented in Section 6 explain the general methodology. Finally, the quantitative investigation is presented in Section 7, while Section 8 concludes.

2 The fiscal structure in France and in the US

We report key statistics about the French and US fiscal systems. As explained in the introduction, these two countries have the particularity of being among the most different OECD countries in terms of total taxation. On the one hand, France is one of the countries with the highest mandatory levies, while the US is one of the countries with the lowest levies. This is confirmed in Figure 1, which reports government spending on final goods as a share of GDP and tax revenues as a share of GDP for the two countries. A first observation from Figure 1 is that France and the US drastically differ with respect to the size of their governments. Both government spending and tax revenues are significantly higher in France than in the US. Second, the gap between tax revenues and government spending is larger in France than in the US. This reflects that within-country redistribution – measured as the difference between tax revenues and government spending – is of larger magnitude in France than in the US.

We now turn to the details of the taxation system within each country. We focus on the average tax system from 1995 to 2007, before the 2008-crisis –which we will use as a benchmark for calibrating our initial stationary equilibrium. We acknowledge that the numbers are quantitatively

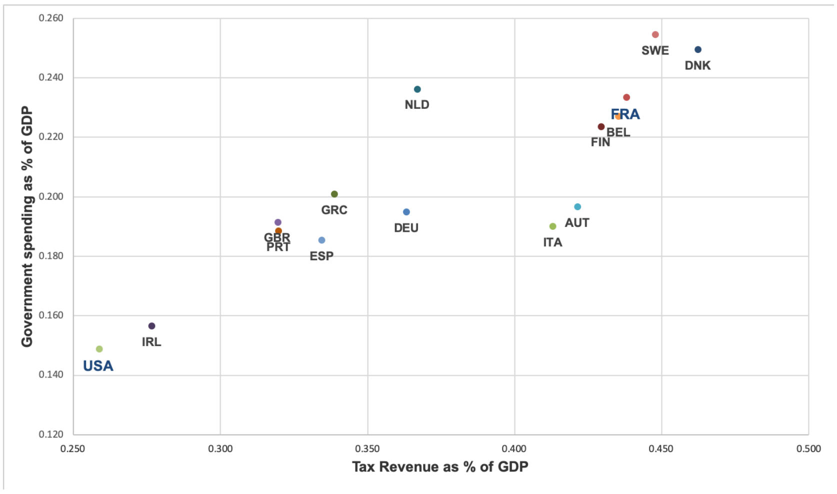


Figure 1: Government spending and tax revenues average from 1995 to 2021 (as a share of GDP). Source: own calculations.

very close if we consider the period from 1995 to 2021, as shown in Figure 1, but consider a period before macroeconomic shocks appears more relevant. We use the results of Trabandt and Uhlig (2011), who provide estimates for the period 1995-2007. Results for France and the US are gathered in Table 1, which also includes some elements related to inequalities.

	Total taxes (%GDP)	$\tau^K(\%)$	$\tau^L(\%)$	$\tau^c(\%)$	B (%GDP)	G (%GDP)	Gini before redist.	Gini after redist.	Gini wealth
France	40	35	46	18	60	24	0.48	0.28	0.68
United States	26	36	28	5	63	15	0.48	0.40	0.77

Table 1: Summary of fiscal systems and inequalities in the US and and in France. Total taxes, public debt B and public spending G in percentage of GDP; tax rates τ^K, τ^L and τ^c in percent; Gini indices unitless.

The first column reports the total mandatory levies as a share of GDP for the two countries. Following the literature, we decompose these total levies into three components: capital tax, labor tax, and consumption tax. Since Mendoza et al. (1994), this decomposition is widely used to compare the tax structure across countries (OECD, Eurostat). These three taxes are reported in columns 2–4. The second column shows the implicit capital tax, calculated as tax receipts on capital income divided by the capital tax. The third column provides the same statistic for the labor tax , while the fourth column reports the implicit tax on consumption.

We can observe that overall taxes are 50% higher in France than in the US. Although capital taxes are very close in both countries, the labor and consumption taxes differ significantly. The difference in labor tax partly stems from the financing of the French welfare system, which covers public pensions, unemployment benefits, health care, and family allowances. It mostly relies on social contributions based on the wage bill, which are considered as labor tax. Regarding consumption tax, it is much higher in France compared to the US, although this high value is comparable to those in other European countries.

Tax revenues are used to finance public spending, which includes both public consumption and public investment. Public spending is approximately one-fourth higher in France than in the US. This difference is partly explained by larger investments in public infrastructure in France.

The results in Table 1 considers linear tax for labor, however, progressive labor income tax is commonly used as a fiscal instrument for countries and current evidence suggests that the progressivity rate differ between France and the US. Comparing the progressivity of labor income tax across countries is challenging due to the complex tax schedules and deductions that are specific to each country. One approach to make this comparison tractable is to use a parametric form for the tax function. To compute this, we employ the log-linear tax function currently used in the literature (e.g. Benabou (2002) and Heathcote et al. (2017)).

$$\begin{aligned} \text{Tax:} \qquad & T(y_i) = y_i - \kappa y_i^{1-\tau}. \\ \text{Disposable income:} \qquad & D(y_i) = \kappa y_i^{1-\tau}. \\ & \log D(y_i) = \log \kappa + (1-\tau) \log y_i, \end{aligned}$$

where y_i denotes the labor income of the country i, the parameter τ reflect the level of progressivity, and κ the average level of taxation. Notice that the higher the τ, the more progressive the tax system. Using the Luxembourg Income Study (LIS) database for France and the US for 2005, we estimate the tax progressivity for labor income ($\hat{\tau}$). To estimate this parameter, we restrict our attention to the heads of households and their spouses aged between 25 and 60 who were employed. We define *labor income* as the sum of wage income, self-employment income, and private transfers. Using the estimates from Mendoza et al. (1994), we can deduce from the capital income the value of capital income tax and we subtract from the total income tax this last variable. This allow us to obtain an estimate of the labor income tax, and then we define *disposable income* as *labor income* minus the *labor income tax.* Finally, we regress the log of disposable income on the log of labor income.

Table 2 reports our estimates for France and the US. One can notice that France has a much more progressive labor tax than the US. Our estimate of progressivity for the US is 0.16, which closely alligns with values used in the literature. Our value is lower than the 0.181 value estimated by Heathcote et al. (2017), as we focused solely on estimating the progressivity of labor income and did not consider progressivity estimates for labor and capital income combined.

	$\hat{\tau}$	SE	Obs	R^2
France	0.23	0.0056	5289	0.855
United States	0.16	0.0019	38111	0.942

Table 2: Estimate of the progressivity of the labor income tax in the US and and in France for 2005 using the LIS database.

All this heterogeneity in redistribution is also reflected in the evolution of income inequality before and after taxation. We proxy income inequality using the average Gini index between 1995 and 2007 (included), as reported in the OECD Income Inequality Database.[1] Note that the Gini indices barely vary over the period, and the picture would not have been different if we had reported the 2007 data only. The before-tax Gini indices for income are roughly similar in France

[1] See `https://stats.oecd.org/index.aspx?queryid=66670`.

and in the US. This value for France stems from the accounting of the (high) public pensions in France, which are counted as transfers and not as income. Consequently, this contributes to increasing the before-tax inequalities. However, the after-tax Gini indices are very different in the two countries, which is a consequence in terms of inequalities of the high transfers to households in France. While redistribution diminishes the Gini index for income by less than 10 points in the US, the reduction is twice as large in France and amounts to 20 points.

The second-to-last element we report is the public debt-to-GDP ratio, which appears to be comparable in France and in the US at around 60%.

The last column reports the Gini index for wealth. The data for France come from the Household Finance and Consumption Survey (HFCS) for the 2010 wave, which is the closest wave to our benchmark years. We have checked that the Gini index remains highly similar in the other waves. The wealth Gini index for the US is taken from the PSID in 2006.[2] As is standard, wealth inequalities in each country are higher than for income. The wealth Gini index in each country is approximately 30 points higher for wealth than for post-tax income. The comparison between the US and France yields a similar result as the income Gini index did. It confirms that inequalities—for wealth here—are sharper in the US than in France.

We will use the elements of Table 1 and the progressivity of labor income estimate in Table 2 to calibrate our heterogeneous-agent model below – and in particular the social weights.

3 A Bewley theory of the SWF: A two-period case

The objective of this section is to provide an overview of our approach to SWF in a simple environment. We first show how the SWF can be constructed as the aggregation of the individual welfare functions. Second, we explain how the SWF can be identified from the observed allocation. Finally, we then discuss the possible political interpretations of the weights along a couple of polar cases, such as the Utilitarian, Egalitarian and Libertarian SWFs.

3.1 The setup

We consider a two-period economy where a single good is available in the second period period. The economy is populated by ex-ante identical agents of mass 1, who consume only in period 2.[3] The agents are endowed by a common period utility function u that is assumed to satisfy standard properties: $u' > 0$, $u'' < 0$, and $u'(0) = \infty$.

Agents are ex ante identical, but face an uninsurable risk both in period 1 and in period 2. The uninsurable risk can take two values denoted 1 and 2. The probability to switch from state i in period 1 to state j in period 2 is $\Pi_{ij} \in [0, 1]$, with $\Pi_{i1} + \Pi_{i2} = 1$ for $i, j = 1, 2$. We assume that the transition matrix $\mathbf{\Pi} = (\Pi_{ij})_{i,j=1,2}$ admits a unique stationary distribution denoted $\boldsymbol{\pi} = [\pi_1, \pi_2]$ that verifies $\boldsymbol{\pi}\Pi = \boldsymbol{\pi}$ and $\pi_1 + \pi_2 = 1$. We assume that the initial distribution of agents in period 1 follows the stationary distribution and the share of type-i agents is π_i, $i = 1, 2$. By definition of the stationary distribution, the distribution of agents in the second period is identical to $\boldsymbol{\pi}$. The type determines the endowment in the second period that can take two

[2] In the 2007 SCF, the wealth Gini index was found to be 0.78, which is very close to the PSID value.

[3] We follow Green (1994) and assume that the law of large number holds.

values, $y_2 \neq y_1$. As a normalization, we assume that the average endowment is equal to one: $\pi_1 y_1 + \pi_2 y_2 = 1$.

A benevolent planner has the objective to choose the allocation that maximizes the aggregate welfare, subject to a feasibility constraint. The planner can transfer resources across agents, but it faces a quadratic redistribution cost. This cost aims at capturing all distortions generated by distribution, and is scaled by a parameter $\kappa > 0$. To implement a consumption c_i for an agent of type $i = 1, 2$ receiving the endowment y_i generates the cost a destruction of resources equal to $\frac{\kappa}{2}(c_i - y_i)^2$. To consider meaningful solutions, we assume that the redistribution cost is not too high and verifies $\kappa y_i < 1$, which formally guarantees that the resource constraint is always increasing in consumption. Focusing on the symmetric equilibrium where all agents of the same type i receive the same consumption c_i, the feasibility constraint can be written as:

$$\sum_{i=1}^{2} \pi_i \left(c_i + \frac{\kappa}{2}(c_i - y_i)^2 \right) \leq \sum_{i=1}^{2} \pi_i y_i = 1. \tag{1}$$

3.2 Individual welfare functions

Agents are assumed to have their own perception of how the planner should value their own welfare, as well as the one of other agents, depending on the realization of the period 1 shock. Following the literature and the early papers of Harsanyi (1955) and Sen (1977), we assume that agents have their own "ethical preferences", which may differ from their individual preferences. The latter solely rank the allocation an individual is privately concerned about, irrespective other individuals. In our setup of heterogeneous histories, this means that individual preferences only care about the histories the individual is likely to experience. The ethical preferences – or "political values" in the wording of Harsanyi (1955) – are defined in Harsanyi (1955) by "what he prefers only in those possibly rare moments when he forces a special impartial and impersonal attitude on himself." The ethical preferences therefore involve the welfare of other agents and thus in our setup of idiosyncratic histories the individual may not experience. Individual and ethical preferences may thus differ from each other.

In our setup, individual preferences are assumed to be of the expected utility kind and individuals therefore weights the different states using the objective probabilities of the different idiosyncratic states. By construction, these probabilities are heterogeneous and dependent on the individual idiosyncratic state. We assume that this is also the case for the ethical preferences that are allowed to be heterogeneous. This heterogeneity is a pervasive observation in empirical works (see Schokkaert and Tarroux, 2021 for a recent survey). Consistently with our setup of ex-ante homogeneity but ex-post heterogeneity, and dependent on the individual idiosyncratic state. We also see this assumption as being consistent with the Bewley tradition. Furthermore, this approach offer the flexibility through the the definition of the idiosyncratic shock. In the remainder, we will call the Individual Welfare Function (IWF) the functional representation of the individual ethical preferences.

Furthermore, we make the assumption that the IWF is additive in the period utility. This follows the welfarist tradition of Harsanyi (1955). More general non-linear aggregators could be considered but our estimation procedure does not allow us to estimate such general functions. For this reason, we restrict our attention to a linear aggregator, in which the weights are seen as marginal weights in the sense of Saez and Stantcheva (2016). Formally, we denote by IWF^i the IWF of an agent, who is of type i in date 1. Denoting by ω_j^i the weight on the utility of agent j, the IWF of agent i can be expressed as follows:

$$IWF^i = \omega_1^i u(c_1) + \omega_2^i u(c_2). \tag{2}$$

The key parameters of the IWFs are the weights $(\omega_j^i)_{i,j=1,2}$, which offer flexibility in the context of the additive functional form. First, they include the case where the ethical preferences are identical to the individual preferences. Individual then only care about their own welfare and are thus *self-seeking* in the sense of Sen (1977). This corresponds to the case where the weights are identical to the probabilities: $\omega_j^i = \pi_{ij}$ for all i, j. Second, we do not impose specific restrictions on these weights. The weights are not required to sum to 1. The reason is that since we aggregate IWFs together, the IWF weights have both an ordinal and a cardinal meaning. Their normalization is not innocuous,as it would remove the possible heterogeneity in the intensity of ethical preferences (using the wording of Arrow, 1951). Finally, we do not restrict the sign of the weights $(\omega_j^i)_{i,j=1,2}$ that are allowed to be negative. This may reflect the fact that the well-being of an individual may be negatively affected by the consumption of other agents. In other words, the consumption of others can be a negative externality for the welfare of an individual. Such externalities have been modelled in asset pricing or macroeconomics to reflect the idiom that households want "to keep up with the Joneses" (see Abel, 1990 or Galí, 1994 among others). Such externalities have also received support in experimental studies (see Fehr et al., 2013 among others), where such a behavioral trait has been called spitefulness. Fehr and Schmidt (2006) describe this behavior as follows: "A spiteful person always values the material payoff of relevant reference agents negatively. Such a person is, therefore, always willing to decrease the material payoff of a reference agent at a personal cost to himself".

3.3 Social welfare functions

3.3.1 Constructing SWFs as the aggregation of IWFs.

We construct the SWF as the additive aggregation of the IWFs. Our assumption is that the planner equally weights the IWFs of all individuals to build the SWF. The aggregation can thus be seen as the result of democratic process, in which individuals vote with their IWFs and all individuals have one vote. By construction, the SWF remains additive in the period utility of all economic agents. The planner performs the aggregation in the first period once the agents have formed their own IWF. Since the economy is populated in the first period by π_i agents of type $i = 1, 2$, the SWF, denoted SWF, reflects the aggregation of IWFs using these population weights:

$$SWF = \pi_1 IWF_1 + \pi_2 IWF_2. \tag{3}$$

Using the expression (2) of IWFs, we obtain that the following expression for the SWF:

$$SWF = (\pi_1 \omega_1^1 + \pi_2 \omega_1^2) u(c_1) + (\pi_1 \omega_2^1 + \pi_2 \omega_2^2) u(c_2). \tag{4}$$

The SWF therefore also weights the period utility functions of the different agents' types, with weights that are state-dependent. The expression (4) becomes:

$$SWF = \sum_{i=1}^{2} \omega_i^S u(c_i), \tag{5}$$
$$\text{where } \omega_i^S = \pi_1 \omega_i^1 + \pi_2 \omega_i^2 \text{ for } i = 1, 2.$$

While the weights of the IWFs have a cardinal interpretation, this is not the case of the SWF weights. The SWF weights are determined up to a positive scalar. This mans that an optimal allocation corresponding to the pair of weights (ω_1^S, ω_2^S) will also correspond to any pair of weights $(\lambda\omega_1^S, \lambda\omega_2^S)$ where $\lambda > 0$.

Admittedly, we could have considered other aggregation procedures that would reflecting different voting procedure, heterogeneous turnouts, or lobbying procedures. However, our estimation strategy is not be able to identify these aspects meaning that any deviation from the democratic aggregation will be captured in the estimated SWF weights.

3.3.2 Inverse optimal approach

The inverse optimal approach consists in identifying social preferences from the observed allocation. The assumption is that the observed allocation results from the optimal choice of the planner. The planner is assumed to choose the allocation (c_1, c_2) that maximizes aggregate welfare, represented by the SWF of equation (5) subject to the resource constraint of equation (1). For given social weights (ω_1^S, ω_2^S), The allocation is characterized by the following FOC:

$$\frac{\omega_1^S}{\pi_1} \frac{u'(c_1)}{1 + \kappa(c_1 - y_1)} = \frac{\omega_2^S}{\pi_2} \frac{u'(c_2)}{1 + \kappa(c_2 - y_2)}, \tag{6}$$

together with the constraint (1). Note that the condition $\kappa y_i < 1$ ensures that the FOC is well defined for all consumption levels. The interpretation of the allocation is rather intuitive. The quantity $\frac{u'(c_i)}{1+\kappa(c_i-y_i)}$ is the marginal utility of an extra unit of consumption for agent i divided by the marginal cost of distributing one unit of good to type i. The higher the weight of a given type, the higher the consumption of that type. Similarly, the higher the redistribution cost κ, the smaller the gap between the consumption c_i and the endowment y_i for $i = 1, 2$.

The inverse optimal approach relies on the same FOC (6) but takes different perspective. Instead of computing the allocation for given weights, it computes the weights for a given allocation. Formally, the FOC (6) can be written as:

$$\frac{\omega_1^G}{\omega_2^G} = \frac{u'(c_2)}{u'(c_1)} \frac{1 + \kappa(c_1 - y_1)}{1 + \kappa(c_2 - y_2)}, \tag{7}$$

that shows that the weights (ω_1^S, ω_2^S) are determined as a function of an allocation (c_1, c_2) satisfying the resource constraint (1). Since the weights of the SWF have only an ordinal meaning, they are determined up to a positive scalar. In other words, if the inverse optimal approach identifies the weights (ω_1^S, ω_2^S), then any pair of weights $(\lambda\omega_1^S, \lambda\omega_2^S)$ (for all $\lambda > 0$) will also be a solution of the inverse optimal approach.

The inverse optimal approach of equation (7) requires knowing the redistribution cost κ, as well as the marginal utilities of the observed allocation. This typically require making assumptions about the period utility function or being able to directly measure marginal utilities.

3.3.3 Political interpretations of SWFs

While our discussion about IWFs referred to individual behavior, our approach to the SWF offers a possible interpretation in terms of approaches discussed in the political sciences and moral philosophy. In particular, we can mention four SWFs.

Utilitarian SWF. This SWF for instance to the aggregation of self-seeking IWFs, where individual weights are equal to objective probabilities. The SWF weights are then equal to the date-1 population shares of the different types of agents, or formally to: $\omega_i^{Ut} = \pi_i$ for $i = 1, 2$. The allocation is such that the marginal utility divided by the marginal resource cost is equalized across agents:

$$\frac{u'(c_1)}{1 + \kappa(c_1 - y_1)} = \frac{u'(c_2)}{1 + \kappa(c_2 - y_2)}.$$

While the utilitarian SWF is based on explicit weights that determine the allocation, the weights of the other SWFs are determined by the inverse optimal approach applied to a specific allocation.

Egalitarian SWF. The weights of the Egalitarian SWFs are determined as those corresponding to an allocation featuring perfect equality across agents: $c_1 = c_2 = c$, where where c solves the resource constraint: $\sum_{i=1}^{2} \pi_i(c + \frac{\kappa}{2}(c - y_i)^2) = 1$. The weights $(\omega_1^{Eg}, \omega_2^{Eg})$ are then determined by the inverse optimal approach as the solution of:

$$\frac{\omega_1^{Eg}}{\omega_2^{Eg}} = \frac{1 + \kappa(c - y_1)}{1 + \kappa(c - y_2)}.$$

We can observe that if $y_2 > y_1$, we will have $\omega_1^{Eg} > \omega_2^{Eg}$. The Egalitarian planner puts a larger weight on the agents having the lower initial resources.[4]

Libertarian SWF. The Libertarian objective involves a minimal governmental intervention, which means in our simple setup simply precludes any redistribution. The Libertarian allocation therefore implies that agents consume exactly their endowment: $c_i = y_i$ for $i = 1, 2$. The inverse optimal approach then allows us to compute the weights $(\omega_1^{Li}, \omega_2^{Li})$ of the Libertarian SWF, which verify:

$$\frac{\omega_1^{Li}}{\omega_2^{Li}} = \frac{u'(y_2)}{u'(y_1)}. \tag{8}$$

What about Pareto deviations? In general, our construction of the SWF does not ensure that the planner chooses Pareto-optimal allocations. This is for instance the case of the allocation chosen by a Libertarian planner. As explained above, a Libertarian planner (who is in our construction the democratic consequence of Libertarian agents) chooses a total absence of

[4]The Egalitarian SWF can be seen as the representation in our setup of the preferences of a Rawlsian planner having a max-min objective, i.e., whose program is $\max_{(c_1,c_2)} \min(u(c_1), u(c_2))$, subject to (1).

redistribution. Any insurance mechanism would reduce aggregate welfare, as measured by the Libertarian SWF corresponding to weights of equation (8). However, some redistribution is ex-ante optimal from the individual point of view of each agent, even when the period-1 uncertainty is revealed. Hence, the Libertariance allocation is not Pareto optimal. For an Egalitarian planner, an allocation offering a marginally highr consumption to some agents (even if Pareto-improving) would not be socially desirable as this would be the social allocation further aways from the Egalitarian allocation. More generally, more elaborated restrictions on the SWF functions can generate deviation from the Pareto principle, as is known since the claim of Sen (1970) about the impossibility of a Paretian Liberal.

As a consequence, we will not require the Pareto principle to be fulfilled by the SWFs we estimate. We will however consider a weaker restriction (see Definition 1).

4 A Bewley theory of the SWF: The intertemporal case

We extend the previous construction of the SWF to a general intertemporal framework. Our construction relies on the sequential representation of the heterogeneous agent model, which is the most adapted to our normative analysis.

4.1 The setup

We generalize the two-period setup of Section 3.1 to an infinite-horizon model with incomplete financial markets. Time is discrete, indexed by $t \geq 0$. As in the two-period case, the economy is still populated by a continuum of size 1 of ex-ante identical agents.

The sequential representation. Agents are also affected by an idiosyncratic risk still denoted y that is assumed to take values in a finite set $\mathcal{Y}$ of arbitrary cardinal Y (and not 2 anymore). The idiosyncratic risk still follows a first-order Markov chain with transition matrix $\mathbf{\Pi} = (\Pi_{yy'})_{y,y' \in \mathcal{Y}}$. The matrix $\mathbf{\Pi}$ is assumed to be irreducible and aperiod, which ensures that it admits a unique stationary distribution denoted as $\boldsymbol{\pi} = (\pi_y)_{y \in \mathcal{Y}}$ normalized such that $\sum_{y \in \mathcal{Y}} \pi_y = 1$. We denote by y_t^i the realization of the idiosyncratic risk of agent i in period t and by $y^{i,t} = \{y_0^i, \ldots, y_t^i\}$ the history of agent i up to date t.

It can be shown that we can construct a probability space related to the set of infinite histories of idiosyncratic shocks, $\mathcal{Y}^\infty$ (LeGrand and Ragot, 2022a). The construction can be summarized as follows. An element $y^\infty \in \mathcal{Y}^\infty$ can be described as a left-infinite sequence that can be written as follows:

$$y^\infty = (\ldots, y_{-k}(y^\infty), \ldots, y_{-1}(y^\infty), y_0(y^\infty)),$$

where each $y_{-k} : \mathcal{Y}^\infty \to \mathcal{Y}$ is a coordinate function returning the idiosyncratic state k periods ago. We define $L(y^\infty)$ the past of history y^∞ – which discards the current state $y_0(y^\infty)$:

$$L(y^\infty) = (\ldots, y_{-k}(y^\infty), \ldots, y_{-1}(y^\infty)),$$

We can then define the cylinder sets $C_k(A)$ for any $k \geq 1$ and any $A \subset \mathcal{Y}^k$ as:

$$C_k(A) = \{y^\infty \in \mathcal{Y}^\infty : (y_{-k+1}(y^\infty), \ldots, y_0(y^\infty)) \in A\}.$$

The cylinder set $C_k(A)$ is the subset of $\mathcal{Y}^\infty$ containing all idiosyncratic histories, whose truncation of length k belongs to A. We can then define $\mathcal{C}_0$ as the set of all cylinder sets, which can be shown to be an algebra. We denote by $\mathcal{F} = \sigma(\mathcal{C}_0)$ the cylindrical σ-algebra generated by $\mathcal{C}_0$ and we define the set function $\mu_\infty : \mathcal{C}_0 \to \mathbb{R}$ from the matrix $\mathbf{\Pi}$ and the vector $\boldsymbol{\pi}$, such that for any $k \geq 2$ and any $A \subset \mathcal{Y}^k$:

$$\mu_\infty(C_k(A)) = \sum_{(s_{-k+1},\ldots,s_0)\in A} \pi_{s_{-k+1}} \Pi_{s_{-k+1}s_{-k+2}} \ldots \Pi_{s_{-1}s_0}$$

and $\mu_\infty(C_1(A)) = \sum_{y_0\in A} \pi_{y_0}$, for any $A \subset \mathcal{Y}$. For any $y^\infty \in \mathcal{Y}^\infty$, we will denote by $\mu(y^\infty) \geq 0$ (instead of $\mu(\{y^\infty\})$) the measure of agents experiencing history y^∞ in a given period.

Finally, the triplet $(\mathcal{Y}^\infty, \mathcal{F}, \mu_\infty)$ can be shown to be a probability space (LeGrand and Ragot, 2022a, Lemma 3 in Section B.3). In particular, we have $\mu(\mathcal{Y}^\infty) = 1$, or $\int_{y^\infty\in\mathcal{Y}^\infty} \mu(dy^\infty) = 1$.

We now define some transition probabilities between histories (and not states). Consider two histories $y^\infty, \tilde{y}^\infty \in \mathcal{Y}^\infty$. The probability to switch for an history $\tilde{y}^\infty$ in one period to another history y^∞ in the next period is simply the probability to switch from state $y_0(\tilde{y}^\infty)$ to state $y_0(y^\infty)$ if y^∞ is a continuation of $\tilde{y}^\infty$ (i.e., if $L(y^\infty) = \tilde{y}^\infty$) and 0 otherwise. We denote this probability $\mu_1(y^\infty|\tilde{y}^\infty)$ that is formally defined as: $\mu_1(y^\infty|\tilde{y}^\infty) = \Pi_{y_0(\tilde{y}^\infty)y_0(y^\infty)} 1_{L(y^\infty)=\tilde{y}^\infty}$, where $1_{L(y^\infty)=\tilde{y}^\infty} = 1$ if $L(y^\infty) = \tilde{y}^\infty$ and 0 otherwise. We then define by induction the probability to switch for an history $\tilde{y}^\infty$ in one period to another history y^∞ t periods ahead as: $\mu_t(y^\infty|\tilde{y}^\infty) = \Pi_{y_0(\tilde{y}^\infty)y_0(y^\infty)} \mu_{t-1}(L(y^\infty)|\tilde{y}^\infty)$. This definition holds for all $t \geq 1$ if we define $\mu_0(y^\infty|\tilde{y}^\infty) = 1_{y^\infty=\tilde{y}^\infty}$ (the probability to switch from $\tilde{y}^\infty$ to y^∞ in 0 period is either 1 if the two histories are identical or 0 otherwise.

Initial distribution. To simplify the notation, we assume that the economy starts at date 0 from a state that is fully characterized by the measure μ_∞. In other words, agents are endowed with an history $y^\infty \in \mathcal{Y}^\infty$ and an initial wealth that is only function of y^∞. The initial distribution of the histories y^∞ follows the measure μ_∞. This assumption encompasses among others the case where all agents have the same wealth or the steady-state wealth. This assumption is not key for our results and only streamlines the presentation and notation. Our construction of SWFs indeed allows for more general initial distributions over the product space of history set cross the asset choice set. However, this makes the presentation unnecessarily complex, as we need to keep track of the joint distribution.

Since we will consider the evolution of the distribution from one period to another, we will use time subscripts for histories. For instance if $y_t^\infty \in \mathcal{Y}^\infty$ is the date-t history of an agent and $\tilde{y}_{t-1}^\infty \in \mathcal{Y}^\infty$ the date-$t-1$ history of the same agent, by definition, we must that $y_t \succeq \tilde{y}_{t-1}$.

Choices. In each period, agents can consume a vector of goods. The consumption vector is denoted $\boldsymbol{x}$ and is assumed to belong to a set $X \subset \mathbb{R}^k$. This consumption vector is possibly very general and can include private goods, public goods, or labor/leisure. Our SWF construction is not specific to any set X. In the sequential representation, the period allocation is a function $\boldsymbol{x}_t : \mathcal{Y}^\infty \to X$ that maps an history y^∞ into a consumption vector $\boldsymbol{x}_t(y_t^\infty)$. We denote by $\boldsymbol{X}$ the set of such functions. Convergence to a steady state implies the existence of a function $\boldsymbol{x}_\infty$, such that $\boldsymbol{x}_\infty = \lim_{t\to\infty} \boldsymbol{x}_t$.[5] An allocation is the collection of the period allocations for all dates and denoted $\boldsymbol{x}^\infty := \{\boldsymbol{x}_t\}_{t=0,\ldots,\infty}$. The set of all allocations is thus $\boldsymbol{X}^\infty$.

An allocation $\boldsymbol{x}^\infty \in \boldsymbol{X}^\infty$ will be the outcome of the optimal choices of individuals, given some choices of the planner (e.g., fiscal policy) and some constraints (prices clearing the market). This part of the setup is not detailed here but in Section 5, as our SWF construction is independent of these elements. The only element we need to assume is that the individual (intertemporal) preferences are time-separable, with a discount factor $\beta \in (0,1)$ and a period utility function U. The function $U : \mathcal{Y}^\infty \times \boldsymbol{X} \to \mathbb{R}$ is assumed to depend on the history y^∞ and the period allocation $\boldsymbol{x}_t$. We assume that $U(\cdot, \boldsymbol{x})$ is integrable on $\mathcal{Y}^\infty$ for all $\boldsymbol{x} \in \boldsymbol{\mathcal{X}}$. This specification encompasses the standard case where the agent solely enjoys felicity from the consumption over X. In that case, we would have $U(y^\infty, \boldsymbol{x}_t) := u(\boldsymbol{x}_t(y^\infty))$ for some standard utility function $u : X \to \mathbb{R}$. However, our specification is more general and also allows for public good consumption or taste shocks (that would depend on y^∞). The precise functional dependencies of U are not relevant for our SWF construction.

Representation of individual preferences. Similarly to the period utility function, we assume that the utility function representing individual intertemporal preferences depends on the agent's history and the allocation. We denote by $V : \mathcal{Y}^\infty \times \boldsymbol{X}^\infty \to \mathbb{R}$ this utility function, such that $V(y_0^\infty, \boldsymbol{x}^\infty)$ is the date-0 (intertemporal) utility associated to having current history y_0^∞ and the allocation $\boldsymbol{x}^\infty$. Formally, we have:

$$V(y_0^\infty, \boldsymbol{x}^\infty) = \sum_{t=0}^{\infty} \int_{y_t^{\infty,\prime} \in \mathcal{Y}^\infty} \beta^t U(y_t^{\infty,\prime}, \boldsymbol{x}_t) \mu_t(y_t^{\infty,\prime} | y_0^\infty) \mu_\infty(dy_t^{\infty,\prime}). \tag{9}$$

More generally, at any date t, the intertemporal utility corresponding to the history y_t^∞ and the allocation $\boldsymbol{x}_t^\infty$ can be written as follows:

$$V(y_t^\infty, \boldsymbol{x}_t^\infty) = \sum_{s=0}^{\infty} \int_{y_{t+s}^\infty \in \mathcal{Y}^\infty} \beta^{s-t} U(y_{t+s}^{\infty,\prime}, \boldsymbol{x}_{t,s}) \mu_s(y_{t+s}^{\infty,\prime} | y_t^\infty) \mu_\infty(dy_{t+s}^{\infty,\prime}). \tag{10}$$

We now turn to the recursive representation of V. To do so, we define for any $\boldsymbol{x}^\infty = (\boldsymbol{x}_t)_{t=0,1,\ldots,\infty} \in \boldsymbol{X}^\infty$, $F(\boldsymbol{x}^\infty) = (\boldsymbol{x}_t)_{t=1,\ldots,\infty}$ the "future" of $\boldsymbol{x}^\infty$, dropping the current value. This is analogous to $L(\cdot)$ but for moving sequences forward instead of backward. We then state the following result regarding the recursive representation of V.

Result 1 *For any $y_t^\infty \in \mathcal{Y}^\infty$ and $\boldsymbol{x}_t^\infty = (\boldsymbol{x}_{t,s})_{s=0,1,\ldots,\infty} \in \boldsymbol{X}^\infty$, the utility V of equation (10) can be recursively written as:*

$$V(y_t^\infty, \boldsymbol{x}_t^\infty) = U(y_t^\infty, \boldsymbol{x}_{t,0}) + \beta \mathbb{E}_{y_{t+1}^\infty} \left[V(y_{t+1}^\infty, \boldsymbol{x}_{t+1}^\infty) | y_t^\infty \right], \tag{11}$$

where we defined $\boldsymbol{x}_{t+1}^\infty = F(\boldsymbol{x}_t^\infty)$ and for any integrable random variable X the expectation with respect to future history y_{t+1}^∞ conditional on the current history being y_t^∞:

$$\mathbb{E}_{y_{t+1}^\infty} \left[X(y_{t+1}^\infty) | y_t^\infty \right] = \int_{y_{t+1}^\infty \in \mathcal{Y}^\infty} X(y_{t+1}^\infty) \mu_1(y_{t+1}^\infty | y_t^\infty) \mu_\infty(dy_{t+1}^\infty). \tag{12}$$

[5] Note that if the initial wealth distribution is the steady-state one, then $\boldsymbol{x}_t = \boldsymbol{x}_\infty$ for all t.

The proof can be found in Appendix A.1. Result 1 characterizes the recursive representation of the intertemporal utility function V.

4.2 Constructing the SWF

We extend construction of the SWF as the democratic aggregation of IWFs presented in Section 3 to an intertemporal framework. In the previous two-period framework, agents in state i had their own subjective valuation of how the utility of agents in state $j \neq i$ should be valued by the planner. We generalize this to the intertemporal framework. In each period t, agents with history $\tilde{y}_t^\infty$ have their own view of how the planner should value other histories, now and in the future, even in the case of histories they cannot experience. This corresponds to the ethical preferences of agents. To provide an example, middle-class US workers has a view on how the utility of the super-rich (e.g., Musk & Co) and of the very poor (e.g., hobos) should be valued by the planner, even if they have a zero probability to belong (now or latter) to one of the categories. A very poor person may have a different view on how these two categories should be accounted for by the planner. We thus consider a general form ethical preferences, where the whole idiosyncratic history affects the how the planner should value utility in other present and future histories.

4.2.1 Constructing the IWF

Our construction of the representation of the ethical preferences of a given agent is conducted in two steps. First, the agent forms a subjective appreciation of the individual preferences of the other agents. This results in a subjective valuation of the utility of other agents. Second, the agent's ethical preferences are built as the aggregation of the subjective perception of the preferences of other agents. In terms of representation, this aggregation of ethical utilities means that the IWF representing the agent's ethical preferences is built as the weighted sum of these subjective valuations.

Constructing the subjective valuation of the utility of another agent. For the formal construction, we consider an allocation $\boldsymbol{x}_t^\infty$ and two agents characterized by their history $y_t^\infty \in \mathcal{Y}^\infty$ and $\tilde{y}_t^\infty \in \mathcal{Y}^\infty$ at some date t. The tilde agent individually values the allocation in s periods in history $\tilde{y}_{t+s}^\infty$ as $U(\tilde{y}_{t+s}^\infty, \boldsymbol{x}_{t,s})$, which implies the discounted utility $\beta^{s-t}U(\tilde{y}_{t+s}^\infty, \boldsymbol{x}_{t,s})\mu_s(\tilde{y}_{t+s}^\infty|\tilde{y}_t^\infty)$. The non-tilde agent has possibly a different perception: they weight the utility $U(\tilde{y}_{t+s}^\infty, \boldsymbol{x}_{t,s})$ of the tilde agent by a loading factor that we denote $\hat{\omega}(y_t^\infty, \tilde{y}_{t+s}^\infty)$. This loading factor depends on the history y_t^∞ of the non-tilde agent, as well as on the history under consideration, $\tilde{y}_{t+s}^\infty$. We further assume that the loading function $\hat{\omega} : \mathcal{Y}^\infty \times \mathcal{Y}^\infty \to \mathbb{R}$ is integrable. This therefore implies that the subjective valuation of the utility of the tilde agent by the non-tilde agent, denoted by $\hat{V}(y_t^\infty, \tilde{y}_t^\infty, \boldsymbol{x}_t^\infty)$ is:

$$\hat{V}(y_t^\infty, \tilde{y}_t^\infty, \boldsymbol{x}_t^\infty) = \sum_{s=0}^{\infty} \int_{\tilde{y}_{t+s}^{\infty,\prime} \in \mathcal{Y}^\infty} \beta^{s-t}\hat{\omega}(y_t^\infty, \tilde{y}_{t+s}^{\infty,\prime})U(\tilde{y}_{t+s}^{\infty,\prime}, \boldsymbol{x}_{t,s})\mu_s(\tilde{y}_{t+s}^{\infty,\prime}|\tilde{y}_t^\infty)\mu_\infty(d\tilde{y}_{t+s}^{\infty,\prime}), \tag{13}$$

which is direct modification of the utility $V(\tilde{y}_t^\infty, \boldsymbol{x}_t^\infty)$ with the inclusion of the weights $\hat{\omega}(y_t^\infty, \tilde{y}_{t+s}^\infty)$. Note that if the weights are constant, we obtain that $\hat{V}(y_t^\infty, \tilde{y}_t^\infty, \boldsymbol{x}_t^\infty)$ and $V(\tilde{y}_t^\infty, \boldsymbol{x}_t^\infty)$ are proportional and represent the same individual preferences.

Constructing the IWF. The IWF of a given agent is then constructed as the aggregation of her subjective valuation of the utility of all other agents. We assume that the aggregation weights the subjective valuation by the share of the history that is considered. Formally, the IWF of an agent with history y_t^∞ and allocation $\boldsymbol{x}_t^\infty$ is defined as follows:

$$IWF(y_t^\infty, \boldsymbol{x}_t^\infty) = \int_{\tilde{y}_t^\infty \in \mathcal{Y}^\infty} \hat{V}(y_t^\infty, \tilde{y}_t^\infty, \boldsymbol{x}_t^\infty) \mu_\infty(d\tilde{y}_t^\infty). \tag{14}$$

Note that in the case of constant weights $\hat{\omega}(y_t^\infty, \tilde{y}_{t+s}^\infty)$, the IWF is proportional to $V(\tilde{y}_t^\infty, \boldsymbol{x}_t^\infty)$ and the IWF also represents the individual preferences of the agent.

4.2.2 Aggregating IWFs to obtain the SWF

With the IWFs of all agents in the population in hand, the planner constructs the SWF as the sum of all these IWFs. In a sense, this construction follows a principle of democratic aggregation, where all agents exactly have the same weight in the final social decision criterion. Formally, for any allocation $\boldsymbol{x}_t^\infty$, the aggregate SWF, denoted SWF, is defined as:

$$SWF(\boldsymbol{x}_t^\infty) = \int_{y_t^\infty \in \mathcal{Y}^\infty} IWF(y_t^\infty, \boldsymbol{x}_t^\infty) \mu_\infty(d\tilde{y}_t^\infty). \tag{15}$$

We have the following result.

Proposition 1 *The representation of the SWF can be expressed as follows.*

1. *Defining for any $\tilde{y}_{t+s}^\infty \in \mathcal{Y}^\infty$:*

$$\omega(\tilde{y}_{t+s}^\infty) = \int_{y_t^\infty \in \mathcal{Y}^\infty} \pi_{y_0(\tilde{y}_{t+s}^\infty)} \hat{\omega}(y_t^\infty, \tilde{y}_{t+s}^\infty) \mu_\infty(dy_t^\infty),$$

   the expression (15) of the SWF becomes:

$$SWF(\boldsymbol{x}_t^\infty) = \sum_{s=0}^{\infty} \int_{\tilde{y}_{t+s}^\infty \in \mathcal{Y}^\infty} \beta^{s-t} \omega(\tilde{y}_{t+s}^\infty) U(\tilde{y}_{t+s}^\infty, \boldsymbol{x}_{t,s}) \mu_\infty(d\tilde{y}_{t+s}^\infty), \tag{16}$$

$$= \int_{\tilde{y}_t^\infty \in \mathcal{Y}^\infty} \beta^{s-t} \omega(\tilde{y}_t^\infty) U(\tilde{y}_t^\infty, \boldsymbol{x}_{t,0}) \mu_\infty(d\tilde{y}_t^\infty) + \beta\,, SWF(\boldsymbol{x}_{t+1}^\infty), \tag{17}$$

   where $\boldsymbol{x}_{t+1}^\infty = F(\boldsymbol{x}_t^\infty)$.

2. *If we further assume that $\tilde{y}_{t+s}^\infty \mapsto \hat{\omega}(y_t^\infty, \tilde{y}_{t+s}^\infty)$ only depends on the current productivity level $y_0(\tilde{y}_{t+s}^\infty)$ of $\tilde{y}_{t+s}^\infty$, we further have that $\tilde{y}_{t+s}^\infty \mapsto \omega(\tilde{y}_{t+s}^\infty)$ also only depends on the current productivity level $y_0(\tilde{y}_{t+s}^\infty)$.*

The proof can be found in Appendix A.2. Proposition 1 provides two main results for the SWF. First, in the general case, we can find period weights $\omega : \mathcal{Y}^\infty \to \mathbb{R}$that depend on the current history such that the SWF expresses as the discounted sum over all dates and histories of the utility of that date and history, weighted by the factor ω. In other words, this twists the standard utilitarian SWF by weighting period utilities by a factor depending on the period history – the utilitarian SWF corresponding to $\omega(\cdot) = 1$. This is the sequential representation result of equation (16). The SWF further admits a recursive representation (equation (17)), which can be seen as the extension of the recursive representation of the utilitarian SWF to history-dependent weights.

The second result is a refinement of the previous sequential and recursive representation results. The subjective weighting function is assumed to be such that the appreciation by any agent of the utility of any other agent only depends on the current idiosyncratic state of the latter agent (instead of the current full history). The result is this restriction also holds for the period weights, that with a slight abuse of notation can be written as $\omega : \mathcal{Y} \to \omega(y)$ that maps idiosyncratic states to weights. Obviously, the SWF admits both a sequential and a recursive representation, similar to equations (16) and (17), with the appropriate restriction on the weighting scheme ω.

The goal of the quantitative part of the model is to estimate the weights $\omega : \mathcal{Y} \to \omega(y)$ of the SWF from the data using an inverse optimal approach, and characterize it using the notations of Section (3.3.3).[6]

Properties of the SWF. A discussed above, we will not restrict the SWF to satisfy the Pareto principle. We however impose a weaker restriction.

Definition 1 *A SWF* $SWF : \boldsymbol{\mathcal{X}}^\infty \to \mathbb{R}$*, associated to a period utility* $U : \mathcal{Y}^\infty \times \boldsymbol{\mathcal{X}} \to \mathbb{R}$*, is said to be element-wise monotone if for any two allocations* $\boldsymbol{x}^\infty$ *and* $\boldsymbol{x}^{\infty,\prime}$ *such that* $\boldsymbol{x}^\infty$ *element-wise dominates* $\boldsymbol{x}^{\infty,\prime}$ *(in the sense that* $U(\cdot, \boldsymbol{x}_t^\infty) \geq U(\cdot, \boldsymbol{x}_t^{\infty,\prime})$ *for all* t*), we have* $SWF(\boldsymbol{x}_t^\infty) \geq SWF(\boldsymbol{x}_t^{\infty,\prime})$*.*

This definition states that with a monotone SWF, if an allocation is in every period better (in the sense of the period utility) than another one, the former will always be preferred in the sense of the SWF to the latter. This property is similar to element-wise monotonicity for utility functions. Obviously, this is a weaker requirement than the Pareto principle. This would require that if $\boldsymbol{x}^\infty$ is preferred to $\boldsymbol{x}^{\infty,\prime}$ (in the sense of the intertemporal utility: $V(\cdot, \boldsymbol{x}^\infty) \geq V(\cdot, \boldsymbol{x}^{\infty,\prime})$), then $\boldsymbol{x}^\infty$ will be socially preferred to $\boldsymbol{x}^{\infty,\prime}$. In particular, if an allocation reduces the dispersion compared another allocation (and hence providing some form of insurance), the first may be individually preferred to the second, but this may not be the case socially. Indeed, the risk-reducing allocation involves higher utility in some states (typically the "bad" ones) and lower utility in others (typically the "good" ones). This is not consistent with our element-wise monotonicity of Definition 1.

We have the following (obvious result).

Result 2 *Consider a SWF* $SWF : \boldsymbol{\mathcal{X}}^\infty \to \mathbb{R}$ *admitting the representation (16) with weights* ω*. The SWF is element-wise monotone if and only if the weight function is positive:* $\forall y^\infty \in \mathcal{Y}^\infty,\ \omega(y^\infty) \geq 0$*.*

The proof of the result is straightforward. Our baseline estimation of the weighting scheme assumes that the SWF is element-wise monotone and hence constraints the weights to be positive.

[6] We have estimated the SWF weights assuming that they depend on a truncated representation of history, instead of only the current state. The shape of the estimated SWF did not depend on the truncation length and the model predictions were mostly unaffected. We thus restrict here to the simplest representation.

We indeed see the restriction of Definition 1 to be rather minimal. However, we will also relax the positivity restriction and estimate unconstrained weights to assess the importance of the constraint.

5 The general model

The previous Section has constructed a general SWF, which can be estimated from the data. This is done in two steps. First we present the set of tools of the planner (which is our representation of fiscal policy) and then we derive the first-order conditions of a Ramsey planner for a given SWF, to derive the Ramsey Steady state. Second, we use observed allocation to identify the set of SWFs from the restrictions of the Ramsey steady-state.

5.1 Risk structure

The risk structure is the same as in the previous section. More precisely, we model idiosyncratic risk as productivity risk, uninsurable idiosyncratic labor productivity shock y_t that can take Y distinct values in the set $\mathcal{Y} \subset \mathbb{R}_+$. At each period t, agents have an idiosyncratic history s^t which is $s^t = \{..., y_{t-1}, y_t\} \in \mathcal{Y}^\infty$. The productivity shock y_t follows a first-order Markov process with transition matrix $M \in [0,1]^{Y \times Y}$. As before $\mu(s^t)$ corresponds to the share of agents with history y^t at date t.

Remark 1 (Simplifying Notation) *If an agent has an idiosyncratic history $s_i^{i,t}$, at period t, we will then denote the realization in state $s_i^{i,t}$ of any random variable $X_t : \mathcal{Y}^\infty \to \mathbb{R}$ simply by X_t^i, instead of $X_t\left(s^{i,t}\right)$.*

As a consequence, the aggregation of the variable X_t at period t over all agents will be written as $\int_i X_t^i \ell(di)$, instead of $\int_{s^t \in \mathcal{Y}^\infty} X_t\left(s^t\right) \mu\left(s^t\right)$.

5.2 Production and government

Production. In any period t, a production technology with constant returns to scale (CRS) transforms capital K_{t-1} and labor L_t into $F(K_{t-1}, L_t,)$ units of output. The production function is smooth in K and L and satisfies the standard Inada conditions. Furthermore, it features constant-return to scale and is homogeneous of degree 1. Capital must be installed one period before production, and the state of the world may potentially affect productivity through a TFP shock. This formulation allows for capital depreciation, which is subsumed by the production function F. Labor L_t is the total labor supply measured in efficient units. The good is produced by a unique profit-maximizing representative firm. We denote by $\tilde{w}_t$ the real before-tax wage rate in period t and by $\tilde{r}_t$ the real before-tax rental rate of capital in period t. Profit maximization yields in each period $t \geq 1$:

$$\tilde{r}_t = F_K(K_{t-1}, L_t) \quad \text{and} \quad \tilde{w}_t = F_L(K_{t-1}, L_t). \tag{18}$$

Government. A benevolent government has to choose a path stream of public spending, denoted by G and to finance it. Several instruments are available. First, the government can levy one-period public debt B_t. We assume the existence of an enforcement technology that makes the public debt default-free. As there is no aggregate risk, public debt and capital are perfectly substitute and they payoff the same pre-tax interest rate $\tilde{r}_t$. Second, the government can raise a number of distortionary taxes, which concern consumption, labor income, and capital revenues. The tax on labor income, denoted by $\mathcal{T}_t(\tilde{w}yl)$ for a labor income $\tilde{w}yl$, is assumed to be non-linear, and possibly time-varying. We follow Heathcote et al. (2017) (henceforth, HSV) and assume that $\mathcal{T}_t$ is defined as follows:

$$\mathcal{T}_t(\tilde{w}yl) := \tilde{w}yl - \kappa_t(\tilde{w}yl)^{1-\tau_t}, \tag{19}$$

where κ captures the level of labor taxation and τ the progressivity. Both parameters are assumed to be time-varying and will be planner's instruments. When $\tau_t = 0$, labor tax is linear with rate $1 - \kappa_t$. Oppositely, the case $\tau_t = 1$ corresponds to full income redistribution, where all agents earn the same post-tax income κ_t. Functional form (19), combined with a linear capital tax, allows one to realistically reproduce the actual US system and its progressivity (see Ferriere and Navarro, 2023).[7]

Consumption and capital taxes are linear and are denoted by τ_t^c, and τ_t^K at date t. All of these taxes are proportional taxes and imply a total governmental revenue equal to $\tau_t^c C_t + \int_i \mathcal{T}_t(\tilde{w}_t y_t^i l_t^i)\ell(di) + \tau_t^K \tilde{r}_t(K_{t-1} + B_{t-1})$, where C_t is the aggregate consumption and $K_{t-1} + B_{t-1}$ is aggregate savings in period t (i.e., capital plus public debt). From now on the aggregate savings will be denoted by A_{t-1} such that $\tau_t^K \tilde{r}_t(K_{t-1} + B_{t-1})$ are capital revenues in period t.

With these elements, the governmental budget constraint can be written as follows:

$$G + (1 + \tilde{r}_t)B_{t-1} = \tau_t^c C_t + \int_i \mathcal{T}_t(\tilde{w}_t y_t^i l_t^i)\ell(di) + \tau_t^K \tilde{r}_t A_{t-1} + B_t. \tag{20}$$

We define post-tax rates r_t and w_t, as follows:[8]

$$r_t := (1 - \tau_t^K)\tilde{r}_t, \quad w_t := \kappa_t(\tilde{w}_t)^{1-\tau_t}. \tag{21}$$

Using the property of constant-return-to-scale for F and the definition of post-tax rates (21), the governmental budget constraint can be written as:

$$G + (1 + r_t)B_{t-1} + w_t \int_i (y_t^i l_t^i)^{1-\tau_t}\ell(di) + r_t K_{t-1} = \tau_t^c C_t + F(K_{t-1}, L_t, s_t) + B_t. \tag{22}$$

5.3 Households preferences and program

Preferences. Agents are expected-utility maximizers with a time-separable utility function, whose constant discount factor is denoted by $\beta \in (0, 1)$. Their period utility function $U(c, l, G)$ over private consumption c, labor supply l, and public good consumption G is assumed to be

[7]The literature uses either the combination of a linear tax and of a lump-sum transfer (e.g., Dyrda and Pedroni, 2022, Açikgöz et al., 2022) or the HSV structure. Heathcote and Tsujiyama (2021) show that the HSV structure is quantitatively more relevant. Opting for the HSV tax structure enables us to discuss the dynamics of optimal tax progressivity, following a public spending shock.

[8]To simplify the notation, r and w denote the *after-tax* wage and interest rates.

separable. We thus do not consider preference shock in this quantitative Section:

$$U(c,l,G) := u(c) - v(l) + u_G(G). \tag{23}$$

The functions $u, u_G : \mathbb{R}_+ \to \mathbb{R}$ are twice continuously derivable, strictly increasing, and strictly concave, with $u'(0) = u'_G(0) = \infty$, while $v : \mathbb{R}_+ \to \mathbb{R}$ is twice continuously derivable, strictly increasing, and strictly convex, with $v'(0) = 0$. The path of public good path $(G_t)_{t\geq 0}$ is exogenous, and it is not a choice variable for the government.

Agents' program. We consider an agent $i \in \mathcal{I}$. Her resources are made of labor income and saving payoffs. The post-tax labor income of an agent with productivity y_t^i and supplying the labor effort l_t^i amounts to $\tilde{w}_t y_t^i l_t^i - \mathcal{T}_t(\tilde{w}_t y_t^i l_t^i) = w_t (y_t^i l_t^i)^{1-\tau_t}$. Since public debt and capital shares are perfectly substitutes and pay the same post-tax interest rate r_t, savings payoffs are equal to $(1 + r_t)a_{t-1}^i$ where a_{t-1}^i is the end-of-period-$t-1$ saving of agent i. The agent uses these resources to save and to consume. Consumption is taxed with rate τ_t^c.

$$\max_{\{c_t^i, l_t^i, a_t^i\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(u(c_t^i) - v(l_t^i) \right), \tag{24}$$

$$(1 + \tau_t^c)c_t^i + a_t^i \leq w_t (y_t^i l_t^i)^{1-\tau_t} + (1 + r_t)a_{t-1}^i, \tag{25}$$

$$a_t^i \geq -\overline{a}, c_t^i > 0, l_t^i > 0, \tag{26}$$

where $\mathbb{E}_0$ an expectation operator (with respect to aggregate and idiosyncratic risks), and where the initial state $(s^{0,i}, a_{-1})$ is given. In what follows, as the initial capital is the same across agent, we omit it from the policy rules.

At date 0, the agent decides her consumption $(c_t^i)_{t\geq 0}$, her labor supply $(l_t^i)_{t\geq 0}$, and her saving plans $(a_t^i)_{t\geq 0}$ that maximize her intertemporal utility of equation (24), subject to a budget constraint (25) and the previous borrowing limit (26). These decisions are functions of the initial state y_0^i, of the history of idiosyncratic shocks s_i^t . Thus, there exist sequence of functions defined over $\mathcal{Y}^\infty$ and denoted by $(c_t, l_t, a_t)_{t\geq 0}$, such that the agent's optimal decision can be written as:

$$c_t^i = c_t(s_i^t), l_t^i = l_t(s_i^t), a_t^i = a_t(s_i^t).$$

In what follows we simplify the notation and keep the $i-$ index following the notation of Remark 1.[9]

The first-order conditions (FOCs) associated to the agent's program (24)–(26) can be written as follows:

$$u'(c_t^i) = \beta \mathbb{E}_t \left[(1 + r_{t+1}) \frac{1 + \tau_t^c}{1 + \tau_{t+1}^c} u'(c_{t+1}^i) \right] + \nu_t^i, \tag{27}$$

$$v'(l_t^i) = \frac{(1 - \tau_t)}{(1 + \tau_t^c)} w_t y_t^i (y_t^i l_t^i)^{-\tau_t} u'(c_t^i), \tag{28}$$

where the quantity $\beta^t \nu_t^i$ denotes the Lagrange multiplier on agent i's credit constraint.

[9] The existence of those functions can be found in Açikgöz (2018), Cheridito and Sagredo (2016), and Miao (2006).

Market clearing and resources constraints. The clearing conditions for capital and labor markets can be written as follows:

$$\int_i a_t^i \ell(di) = A_t = B_t + K_t, \quad \int_i y_t^i l_t^i \ell\,(di) = L_t. \tag{29}$$

where $y_t\,(s)$ is period $t$productivity of an agent having history s.

The economy-wide resource constraint can be written as:

$$G_t + C_t + K_t - K_{t-1} = F(K_{t-1}, L_t), \tag{30}$$

where $C_t = \int_i c_t^i \ell(di)$ is the aggregate consumption.

Equilibrium definition. Our market equilibrium (ME) definition can be stated as follows.

Definition 2 (Sequential equilibrium) *A sequential competitive equilibrium is a collection of individual allocations $(c_t^i, l_t^i, a_t^i, \nu_t^i)_{t\geq 0, i\in\mathcal{I}}$, of aggregate quantities $(K_t, L_t, Y_t)_{t\geq 0}$, of price processes $(w_t, r_t, \tilde{w}_t, \tilde{r}_t)_{t\geq 0}$, and of fiscal policies $(\tau_t^c, \tau_t^K, \tau_t, \kappa_t, B_t, G_t)_{t\geq 0}$, such that, for initial conditions and for initial values of capital stock and public debt verifying $K_{-1} + B_{-1} = \int_i a_{-1}^i \ell(di)$, we have:*

1. *given prices, the functions $(c_t^i, l_t^i, a_t^i, \nu_t^i)_{t\geq 0, i\in\mathcal{I}}$ solve the agent's optimization program in equations (24)–(26);*

2. *financial, labor, and goods markets clear at all dates: for any $t \geq 0$, equation (29) hold;*

3. *the government budget is balanced at all dates: equation (22) holds for all $t \geq 0$;*

4. *factor prices $(w_t, r_t, \tilde{w}_t, \tilde{r}_t)_{t\geq 0}$ are consistent with condition (18), and post-tax definitions (21).*

A steady-state Market equilibrium (SSME) is a ME for which the joint distribution (c, l, a, ν), and aggregate quantities K, L, Y, prices $w, r, \tilde{w}, \tilde{r}$, and of fiscal policy $\tau^c, \tau^K, \tau, \kappa, B$ are time-invariant.

6 The Ramsey problem and the identification of weights

Following the construction of Proposition 1, the period 0 SWF is:

$$W_0 := \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \int_i \omega(y_{i,t}) \left(u(c_t^i) - v(l_t^i) + u_G(G) \right) \ell(di) \right]. \tag{31}$$

In the Ramsey program, the planner aims to determine the fiscal policy corresponding to the competitive equilibrium – as specified in Definition 2 – that maximizes aggregate welfare according to the criterion in equation (31), while satisfying the government's budget constraint.

We now provide a formulation of the Ramsey allocation. Following Chamley (1986) and some equivalence results in the literature, the program can be simplified through an appropriate change of variables. For instance, when the planner can impose linear taxes on consumption and capital, a progressive tax on labor, and issue one-period public debt, the consumption tax becomes redundant with other fiscal instruments. We summarize the results in the next Proposition and provide the proof in Appendix B.

Proposition 2 *The Ramsey program can be written in as:*

$$\max_{\left(W_t,R_t,\tau_t,\hat{B}_t,\tilde{A}_t,K_t,L_t,(c_t^i,l_t^i,\tilde{a}_t^i,\nu_t^i)_i\right)_{t\geq 0}} W_0, \tag{32}$$

$$G + W_t \int_i (y_t^i l_t^i)^{1-\tau_t} \ell(di) + (R_t - 1)\tilde{A}_{t-1} + \hat{B}_{t-1} = F(K_{t-1}, L_t, s_t) + \hat{B}_t, \tag{33}$$

$$\textit{for all } i \in \mathcal{I}\textit{:}\ c_t^i + \tilde{a}_t^i = W_t (y_t^i l_t^i)^{1-\tau_t} + R_t \tilde{a}_{t-1}^i, \tag{34}$$

$$\tilde{a}_t^i \geq -\tilde{\bar{a}},\ \nu_t^i(\tilde{a}_t^i + \tilde{\bar{a}}) = 0,\ \nu_t^i \geq 0, \tag{35}$$

$$u'(c_t^i) = \beta \mathbb{E}_t \Big[R_{t+1} u'(c_{t+1}^i) \Big] + \nu_t^i, \tag{36}$$

$$v'(l_t^i) = (1 - \tau_t) W_t y_t^i (y_t^i l_t^i)^{-\tau_t} u'(c_t^i), \tag{37}$$

$$K_t + \hat{B}_t = \tilde{A}_t = \int_i \tilde{a}_t^i \ell(di),\ L_t = \int_i y_t^i l_t^i \ell\,(di)\,. \tag{38}$$

This reformulation is based on a change of variables for public debt and individual saving. It simplifies the derivation of the first-order conditions of the planner. The discussion of these FOCs in a similar environment is developed in LeGrand and Ragot (2023a). We provide the first-order conditions of the planner in Appendix C.

A Ramsey Steady-State (RSS) is a set of constant values of $W, R, \tau, \hat{B}, \tilde{A}, K, L$, and a constant joint distribution of $(c, l, \tilde{a}, \nu)$, which is a solution to the Ramsey problem. From these values, using the change in variables provided in Appendix B, we can recover a tax system $\left(\tau^c, \tau^K, \tau, \kappa, B\right)$, and the relevant distributions (c, l, a) that characterize the RSS.

This tax system is not unique, as consumption taxes are redundant. However, these taxes are crucial for identifying the SWF, as explained below.

Identification of weights

The goal of our exercise is to estimate the Social Welfare Function (SWF) based on a market equilibrium, assuming it represents the optimal outcome for the planner. In other words, an estimated SWF is a SWF for which a SSME is a RSS. More formally:

Definition 3 (Consistent SW) *Consider a SSME with a fiscal policy $(\tau^c, \tau^K, \tau, \kappa, B)$. An estimated increasing SWF is a mapping $\omega : \mathcal{Y} \to \mathbb{R}^+$, with $\sum_{y\in\mathcal{Y}} \pi_y \omega\,(y) = 1$, for which the fiscal policy $(\tau^c, \tau^K, \tau, \kappa, B, G)$ is the optimal fiscal policy of RMSS, where the weights ω are used in the objective (31).*

Some remarks are in order. First, we impose in the definition that the weights are positive. As a consequence, we focus on an increasing SWF in the sense of Definition 1. Second, we impose a population-weighted normalization $\sum_{y\in\mathcal{Y}} \pi_y \omega\,(y) = 1$ in the definition, as SWF are obviously identified up to a positive multiplicative scalar (recall that π_y is the measure of the population having productivity $y \in \mathcal{Y}$). Third, the previous definition does not imply the uniqueness of an estimated SWF for a given SSME. Indeed, as can be expected, a large set of consistent social weights are compatible with a given fiscal policy that includes only those fiscal instruments. Social weights will thus be generically under-identified.

Indeed, an outcome of the investigation of the first-order conditions of the planner (and the normalization of the SWF) is that although the state has five instruments $(\tau^c, \tau^K, \tau, \kappa, B)$, a SSME being a RMSS imposes only 3 restrictions on the SWF. Specifically, the capital tax is redundant, and both the budget of the state and financial market equilibrium impose independent restrictions on the planner's tool. As a consequence, as soon as the size of the finite set $\mathcal{Y}$ is larger than three (which is the case in quantitative work), the SWF will generically be under-identified. To allow for identification, we follow two distinct strategies.

The first strategy consists of selecting, among the set of consistent social weights, those that are closest – in terms of the Euclidean norm – to the standard Utilitarian social welfare function. Indeed, since the work of Aiyagari (1995), the Utilitarian social welfare function has been the standard benchmark in the heterogeneous agent literature, as discussed in the literature review above. More formally, we define the *identified* consistent social weights as those that minimize the following distance in each period:

$$\omega(y)_{y\in\mathcal{Y}} = \arg\min_{\tilde{\omega}(.)} \sum_{y\in\mathcal{Y}} \pi_y(\tilde{\omega}(y) - 1)^2, \tag{39}$$

subject to the constraint that the weights are consistent in the sense of Definition 3.

The second identification strategy consists of estimating a parametric functional form for $\omega(\cdot)$ – similar to Heathcote and Tsujiyama (2021) – with as many degrees of freedom in the functional form as the number of fiscal policy constraints. More precisely, we adopt the following function form:

$$\log\omega(y) := \omega_0 + \omega_1 \log(y) + \omega_2(\log(y))^2, \tag{40}$$

where ω_0, ω_1, and ω_2 are three free parameters that are exactly identified with the fiscal policy restrictions of Definition 3.

We can now provide our algorithm to estimate the SWF.

1. Calibrate parameters and choose a fiscal system $(\tau^c, \tau^K, \tau, \kappa, B)$ to align the SSME as closely as possible with the empirical allocation and fiscal system of the country under consideration.

2. Identify the SWF using either method (39) or (40).

Step 1 is standard in heterogeneous-agent model, as well-calibrated models can effectively replicate observed inequality, as demonstrated below. Step 2 is the challenging aspect, as the FOCs of the planner imply a joint distribution of various Lagrange multipliers, which are very hard to compute. Therefore, to identify the weights, we employ techniques to reduce the state space, initially developed in LeGrand and Ragot (2022a) and subsequently refined in more recent works (LeGrand and Ragot, 2023b, LeGrand and Ragot, 2022b and LeGrand and Ragot, 2023a). Basically this method allows us to compute the steady-state optimal allocation and derive a finite number of equations. The idea is to build an aggregation of the Bewley model (under a specific policy and no aggregate shock), where agents sharing the same history over last N (where N is a fixed horizon) periods are aggregated into a single "agent". This method implies that the "aggregate" agent is endowed with the average wealth and average allocation of all individuals with this N-period history.

This finite state space representation of the Bewley model is particularly well-suited for the Lagrangian method, as the identification of Pareto weights can be accomplished through simple matrix algebra. We provide all the construction and derivation in Appendix D.

7 Quantitative investigation

As explained above, we first provide two calibrations to reproduce the tax system and the wealth distribution in both the US and France for the period 1995-2007. We then estimate the SWFs in both countries.

7.1 Calibration

The US calibration

The estimation parameters are gathered in Table 3, and we detail below our calibration strategy.

Preference parameters. The period is a quarter. The discount factor is set to $\beta = 0.992$ to match a realistic capital-to-output ratio. The period utility is specified as $U(\cdot) = \frac{c^{1-\gamma}-1}{1-\gamma} + \frac{1}{\chi}\frac{l^{\frac{1}{\varphi}+1}}{\frac{1}{\varphi}+1}$ with $\gamma = 1.8$. This risk-aversion is set to match a realistic wealth inequality for the targeted capital-to-output ratio. Furthermore, the Frisch elasticity for labor is set to $\varphi = 0.5$, which is recommended by Chetty et al. (2011) for the intensive margin. We set the labor-scaling parameter to $\chi = 0.0477$, which implies normalizing the aggregate labor supply to 0.33.

Technology. The production function is of the Cobb-Douglas form and subsumes capital depreciation: $F(K, L, s) = sK^{\alpha}L^{1-\alpha} - \delta K$. The capital share is set to the standard value, $\alpha = 36\%$, while the depreciation rate is set to $\delta = 2.5\%$.

Idiosyncratic labor risk. Various estimations of the idiosyncratic process can be found in the literature. The productivity follows an AR(1) process: $\log y_t = \rho_y \log y_t + \varepsilon_t^y$, with $\varepsilon_t^y \sim_{\mathrm{IID}} \mathcal{N}(0, \sigma_y^2)$. The calibration features an autocorrelation $\rho_y = 0.99$ and a standard deviation $\sigma_y = 0.0995$, which is close to the estimates of Krueger et al. (2018). We discretize this AR(1) process using the Tauchen (1986) procedure, with 10 states. This calibration implies a Gini index of post-tax and transfers of 0.40, which is the same as that reported in Table 1. We obtain a Gini for wealth of 0.78. The model does a good job in matching the income and wealth distribution in the US.

Taxes and government budget constraint. Fiscal parameters are calibrated based on the computations by Trabandt and Uhlig (2011) reported in Table 1, with the exception of the progressivity of the labor tax, which was computed using our own calculations based on the Luxembourg Income Study Database (LIS) and summarized in Table 2. We recall that their estimations for the US in the period 1995-2007 yield a capital tax of $\tau^K = 36\%$ and a consumption tax of $\tau^c = 5\%$. In our estimation for the progressivity parameter, we obtain $\tau = 0.16$, which is close to the estimates in the literature.[10] We consider the years 1995-2007 to examine a period unaffected by major macroeconomic shocks.

Finally, we estimate the parameter κ such that it matches the public spending over GDP ratio. By doing this, we use a value of $\kappa = 0.85$, which is close to the estimates in Ferriere and Navarro (2023). With this fiscal system, the model generates a public-debt-to-GDP ratio equal to 63%. The model also implies a public-spending-to-GDP ratio of 15%. These two values are reported in Table 1.

Additionally, the model performs well in replicating the ratios of consumption over GDP and investment over GDP. The model predicts a consumption-to-GDP ratio of 58%, very close to its empirical counterpart of 60% for the period 1995-2007. The investment-to-GDP ratio generated by the model amounts to 27%, close to the empirical value of 25%. Finally, regarding inequalities, the model generates a Gini index for post-tax income equal to 0.40, identical to its empirical counterpart in Table 1. The Gini index for wealth is found to be 0.78, very close to its empirical value of 0.77 in Table 1.

We gather the model implications in Table 4. These implications show that our tax system provides a good approximation of the redistributive effects of the actual tax system. This confirms the results of Heathcote et al. (2017) and Dyrda and Pedroni (2022).

French calibration

The calibration for France shares a number of similarities with the one for the US. We use the same period and the same functional forms. For the sake of clarity, we mimic the structure of the US calibration in Section 7.1, even though our presentation is more streamlined. The calibration parameters can be found, as those for the US, in Table 3.

Preference parameters. The discount factor is set to $\beta = 0.996$ and the Frisch elasticity for the labor supply is still equal to $\varphi = 0.5$. We fix the scaling parameter to $\chi = 0.0228$, which implies an aggregate labor supply normalized to 0.29. It happens that the same risk-aversion parameter $\gamma = 1.8$ is consistent with French statistics.

Technology and TFP shock. The production function is of the Cobb-Douglas form and subsumes capital depreciation: $F(K, L, s) = sK^{\alpha}L^{1-\alpha} - \delta K$. The capital share is set to the standard value, $\alpha = 36\%$, while the depreciation rate is set to $\delta = 2.5\%$.[11]

Idiosyncratic risk. The AR(1) productivity process is calibrated using $\rho_y = 0.99$ and $\sigma_y = 0.0646$. These values are in line with the estimates of Fonseca et al. (2023). We discretize this

[10] To estimate this parameter, we use the dataset for the US in 2005 and restrict our attention to the heads of households and their spouses aged between 25 and 60 who were employed. We define labor income as the sum of wage income, self-employment income, and private transfers. We then define a variable net tax as the income tax and contributions minus public transfers and occupational pensions. Capital income includes interest, dividends, rental income, and private pensions. We apply the estimate of Trabandt and Uhlig (2011) of $\tau^K = 36\%$ to the capital income and obtain the capital income tax. Finally, the labor income tax is simply the variable net tax minus the capital income tax. We then proceed by defining disposable income as labor income minus the labor income tax, and we regress the log of disposable income on the log of labor income. The results are presented in Table 2.

[11] We are keeping the same values as in the United States to emphasize more the differences in the SWFs due to differences in the fiscal systems of both countries.

		US		France	
Parameter	Description	Value	Target or ref.	Value	Target or ref.
Preference parameters					
β	discount factor	0.992	$K/Y = 2.7$	0.996	$K/Y = 3.1$
u	utility function	·	$\gamma = 1.8$	·	$\gamma = 1.8$
φ	Frisch elasticity	0.5	Chetty et al. (2011)	0.5	Chetty et al. (2011)
χ	hours worked	0.33	Penn World Table	0.29	Penn World Table
α	capital share	36%	Profit Share, NIPA	36%	Profit Share, INSEE
δ	depreciation rate	2.5%	Chetty et al. (2011)	2.5%	Own calc., INSEE
Productivity parameters					
σ^y	std. err. prod.	0.10	Gini for income	0.06	Fonseca et al. (2023)
ρ^y	autocorr. prod.	0.99	Gini for income	0.99	Fonseca et al. (2023)

Table 3: Parameter values.

AR(1) process using the Tauchen (1986) procedure, with 10 states. With the process, the model is also able to replicate a realistic level of wealth inequalities. The Gini coefficient of income after taxes and transfers in the model is 0.28, matching the value reported in Table 1, while the Gini coefficient of wealth is estimated to be 0.68.

Taxes and government budget constraint. We use the values summarized in Table 1 for the French taxes, with exception for the labor tax that in our model is progressive. We consider a capital tax of $\tau^K = 35\%$, a parameter $\tau = 0.23$ (estimated in the same way as estimated for the US using the dataset of France for the year 2005), and a consumption tax of $\tau^c = 18\%$. This tax system has realistic implications for the model. In terms of public finance, we use $\kappa = 0.728$ to match the empirical public-spending-to-GDP of 24 %. The public-debt-to-GDP ratio amounts to 60%, as the value of Table 1. Regarding private consumption and investment, the model generates aggregate private consumption equal to 44% of GDP, which is close to the empirical counterpart of 45% estimated by Trabandt and Uhlig (2011) for the period 1995-2007, while investment amounts to 31% of GDP , compared to 31% for its empirical counterpart. Finally, in terms of inequalities, the model implies a Gini coefficient for post-tax income equal to 0.28, the same as the empirical value reported in Table 1. The Gini index for wealth generated by the model amounts to 0.68, which is also the same as the empirical counterpart. These elements confirm that the tax system, comprising a linear tax for capital and progressive tax for labor, is very relevant from an empirical point of view.

Model outcomes

We gather the model implications for both France and the US in Table 4.

7.2 Estimation of the SWFs

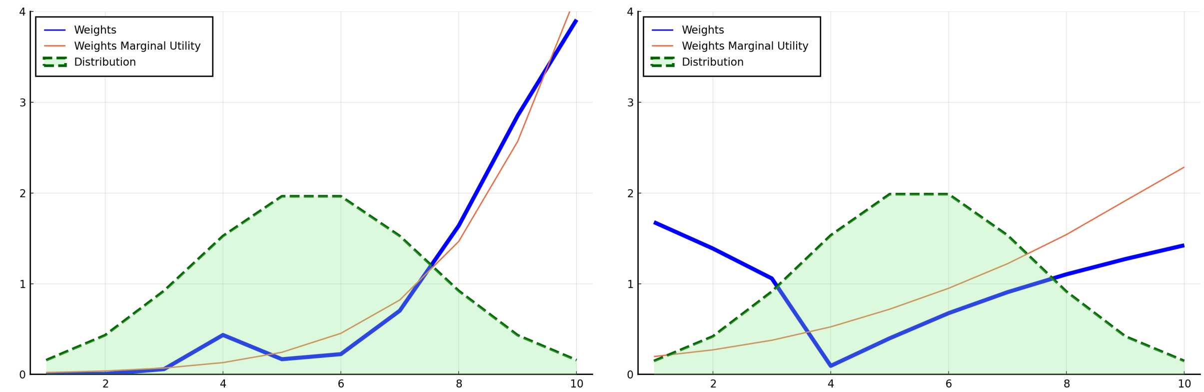
The estimation procedure follows the algorithm presented in Section 6 and the algebra of Appendix D. For the exercises below, we use a truncation length of $N = 5$, although the main characteristic of the results do not change when we consider a longer truncation length (see Appendix G).

		US		France	
Parameter	Description	Model	Data	Model	Data
Public finance aspects					
B/Y	Public debt (%GDP)	63%	63%	60%	60%
G/Y	Public spending (%GDP)	15%	15%	25%	24%
	Total tax revenues (%GDP)	16%	26%	25%	40%
Aggregate quantities					
C/Y	Aggregate consumption (%GDP)	58%	60%	44%	45%
I/Y	Aggregate investment (%GDP)	27%	25%	31%	31%
Inequality measures					
	Gini for post-tax income	40%	40%	28%	28%
	Gini for wealth	78%	77%	68%	68%

Table 4: Model implications for key variables. Empirical values are discussed in Section 2 and summarized in Table 1.

As we consider 10 idiosyncratic productivity levels, this implies $N^{tot} = 10^5 = 100000$ different truncated histories.

We first plot in Figure 2 the period weights as a function of the productivity level. The weight for a given productivity level y_k is computed using the explanation above, while accounting for the population distribution.



(a) United States (b) France

Figure 2: Period weights as a function of the productivity levels for the US and France.

Figure 2 plots the period weights along the productivity dimension for the agents. We observe that in the US, the period weights increase with productivity level, whereas for France they exhibit a U-shaped pattern, assigning higher weights to low-productivity agents compared to those at the top of the productivity distribution. The positivity constraint on weights is only slightly binding for low productivity agents in the US. In Section 7.5, we provide the unconstrained estimation of weights for the US. In this graph, we also report the inverse of the average marginal utility for agents of each productivity level k, denoted as ω_k^{MU}. More formally, for each productivity level $k = 1, \ldots, 10$, it is defined as $\omega_k^{MU} = \int_i \mathbf{1}_{y^i = y_k} \frac{1}{u'(c^i)} \ell(di)$. This measure captures the heterogeneity in marginal utility across productivity levels at the equilibrium allocation.[12]

In the US, agents with the highest weight in the population are those with high productivity. These weights are about 3.9 times larger than the weights of the agents with the lowest level of productivity. In France, low-productivity agents have a higher weight than those with high productivity. Table 5 contains some summary statistics for the period weights computed for France and the US.

	US	**France**
Mean	1.00	1.00
Std. deviation	1.37	0.49
Min.	0.006	0.095
Max.	3.91	1.68
Bottom 10 %	0.006	0.37
Median	0.33	1.08
Top 10%	2.96	1.45

Table 5: Summary statistics for the period weights of the US and France.

As discussed in Section 6, we also compute a parametric version of the period weights to check the robustness of an alternative identification strategy. These weights are obtained such that the FOCs of the planner are exactly identified. To calculate those weights we consider the following functional form:

$$\log \omega(y) := \omega_0 + \omega_1 \log(y) + \omega_2 (\log(y))^2,$$

where y represents the productivity states. More details about this calculation can be found in the Appendix D, but basically, the idea is that the FOCs of the planner will give us three sets of conditions to be met, and we use those conditions to exactly identify the three parameters above.[13] This allows us to avoid pinning down the identification by choosing the closest weights to the utilitarian ones.

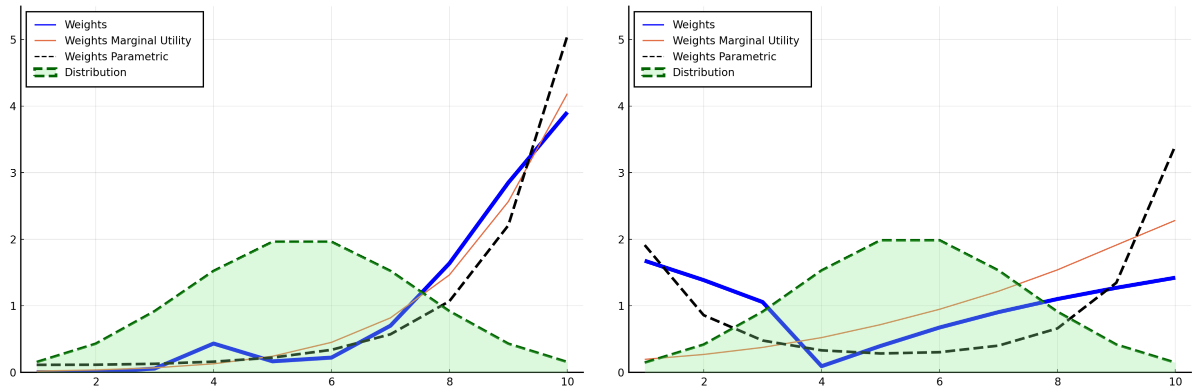
We obtain the following parametric function for the US and France, respectively:

$$\begin{aligned}\log \omega(y)^{us} &= -0.25 + 1.06 \log(y) + 0.22(\log(y))^2,\\ \log \omega(y)^{fr} &= -0.51 + 0.62 \log(y) + 1.44(\log(y))^2.\end{aligned}$$

In Figure 3, we plot the parametric weights together with the weights we had previously plotted. Notice that the parametric weights have the same shape as those obtained through direct estimation of the weights. However, for the US, the difference in weights between the high productivity and low productivity agents becomes more pronounced. For France, we observe that the weights for the high productivity agents are now higher than those for the low productivity ones, although we still observe a U-shaped format and similar weights for the low-productivity agents.

[12] The rationale for these weights is that if the planner were to use them, resources would not be transferred across agents if the planner could implement productivity-specific transfers.

[13] We use the restrictions in the problem stated in Appendix D, equation (D.47), to exactly identify the parameters ω_0, ω_1, and ω_2.



(a) United States (b) France

Figure 3: Parametric period weights as a function of productivity for the US and France.

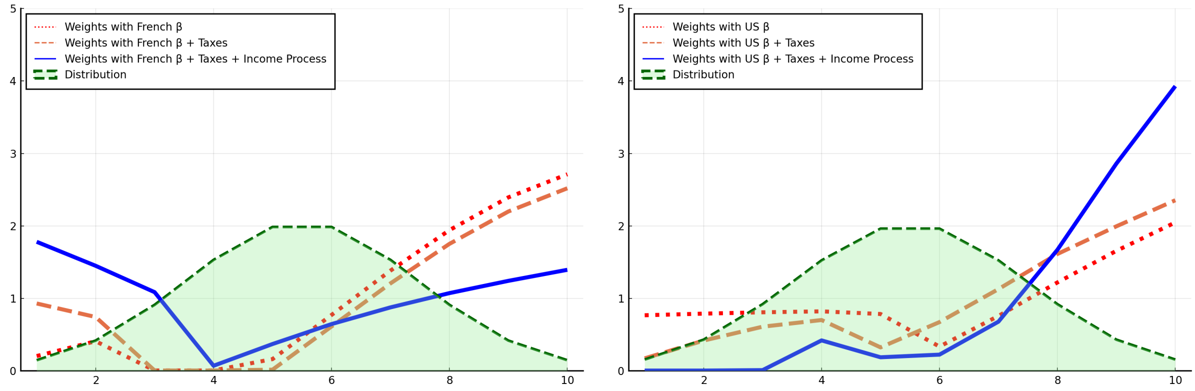
7.3 Investigating the drivers of weight differences between the United States and France

We use the previous methodology to investigate the drivers behind the differing weights assigned to agents in the United States and France. Our objective is to determine if we can replicate the weights of one country using fundamentals of the other. Initially, we examine the role of preferences by observing the behavior of the US economy when it adopts the French discount factor, β.

In panel (a) of Figure 4, the red dashed line represents the weights as a function of productivity for the US, adopting the French discount factor. Compared to the original weights, there is a slight increase in the weights for lower productivity agents, but the overall trend remains: higher weights are given to agents with higher productivity levels. In panel (b), the red dashed line shows the same case for France adopting the US preference parameter. We observe that the weights for low-productivity agents in France decrease, while those for high-productivity agents increase. However, the discount factor alone does not fully account for the differences in weights between the two countries.

Next, we analyze the impact of fiscal systems. We examine how the US economy behaves under the French fiscal system, characterized by higher labor tax progressivity. This system, which is more aligned with reducing inequality, contrasts with the US system, which is more libertarian as we will discuss below. Panel (a) of Figure 4 shows the US weights with the French tax system (orange dashed line). The weights for lower productivity agents increase at the expense of those for higher productivity agents. Conversely, for France adopting the US tax system, the weights for low-productivity agents decrease, while those for high-productivity agents increase (panel (b) of Figure 4).

Appendix I, Figure 22 illustrates the changes in period weights, utility, labor, and capital income when the US adopts the French tax system and vice versa. Adopting the French tax system in the US reduces labor income for high-productivity agents due to decreased labor supply incentives. Heavier taxation reduces their utility and weights assigned by the social planner. Conversely, the progressive tax system boosts consumption and utility for low-productivity agents, increasing their weights.



(a) United States (b) France

Figure 4: Change in the period weights for the United States and France due to different preference parameters, tax systems, and income processes.

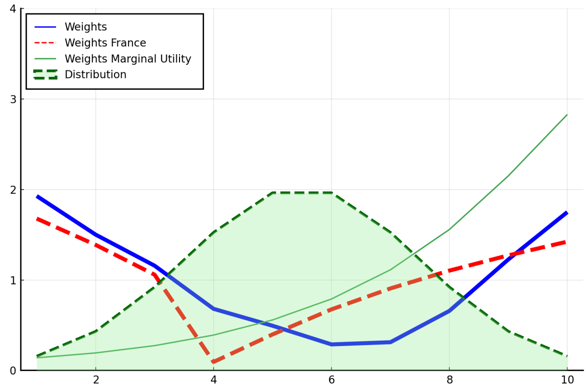
This experiment demonstrates that consistent changes in the tax system can rationalize changes in period weights. For the US to increase weights for low-productivity agents and decrease weights for high-productivity agents, adopting a more progressive labor tax is effective. The relatively low labor tax in the US favors high-income/high-productivity agents.

Finally, to fully uncover the differences in weights, we incorporate the French income process into the US economy. This allows us to completely replicate the French weights (blue line in panel (a) of Figure 4). Similarly, incorporating the US income process into the French economy allows us to replicate the US weights (panel (b)). This highlights that weights depend on preference parameters, taxes, and income processes, underscoring the significant role of taxes and income processes in explaining weight differences. As we compare steady states, these are long-run effects, not considering the welfare effects of transitioning from the benchmark fiscal system to a new one.

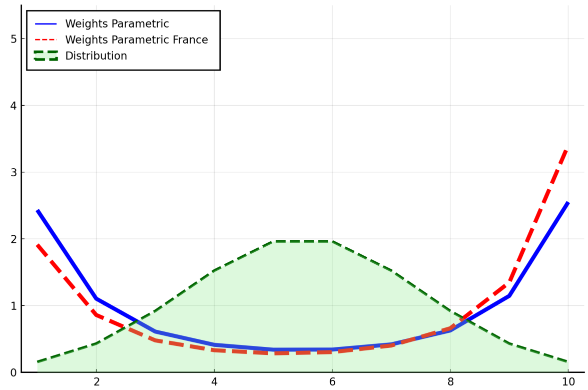
7.4 A world where the US have the weights of France

In this section, we computed the US fiscal system that makes social weights as close as possible to those of France. The goal is to find a fiscal system in the US where the Euclidean distance between the weights in the modified US economy and the initial France economy is minimized. This exercise aims to understand the role of social preferences in shaping the actual tax system, distinct from the influence of technology and individual preferences.

We conduct the experiment as follows. Once the capital-to-output ratio has been set to its steady-state value, we iterate over every combination of the capital tax rate and the progressivity of the labor tax, adjusting κ to keep the government spending-to-output ratio constant. This allows us to match the main macroeconomic ratios. Specifically, all model parameters and primary macroeconomic ratios remain unchanged. The only variable adjusted in each case is the vector representing the fiscal system. $(\tau_K, \tau_c, \tau, \kappa, B)$. For each combination, we calculate the weights and, by using interpolation on the productivity states of the target economy (i.e., in this case, France), we can obtain the weights on the union of the grids that represent the whole set of productivity states. Once we obtain these weights, we can calculate the distance between the vectors using standard measures such as the Euclidean distance. We then consider, as in our



(a) Period weights



(b) Period weights parametric

Figure 5: Period weights for France (red dashed line) and for the United States with the tax system that minimizes the distance to the French weights (blue line). Panel (a) is non-parametric, Panel (b) is parametric.

benchmark case, that this new fiscal system, which represents the minimum distance between the weights, results from the optimal planner's decisions.

In this experiment, we are considering the distance between the weights on the union of the grids in the productivity states. Another experiment we also conducted was to calculate the distance between the weights for the possible histories $k = 1, \ldots, 100000$, and the results were very similar.

Figure 5 plots the weights as a function of productivity for the US with the new fiscal system and for France. In panel (a), we have the weights considering the technique we discussed previously and the marginal-utility weights ω^{MU} implied by the equilibrium allocation. In panel (b) we show the parametric version. As can bee seen, our procedure allows us to find a fiscal system, such the weights in the US with this modified fiscal system are quite close to the French weights.

We report in Table 6 the values of the new fiscal system that allows the weights in the US with this modified fiscal system to minimize the distance to the French weights. We also report the fiscal system values for the US benchmark economy and the French benchmark economy. The column Gini post-tax income represents the Gini of income after taxes and transfers, and the last one represents the Gini for wealth.

	Public debt (%GDP)	τ_k (%)	τ (%)	κ (%)	Gini post-tax income	Gini wealth
US	63	36	16	85	40	78
France	60	35	23	72.8	28	68
US with French SWF	299	9	57	71	27	63

Table 6: Comparison between the benchmark economies and the US economy with the French SWF.

The first observation is that the distribution of income and wealth for the United States is now much closer to their French counterparts, and therefore less unequal. This mostly comes from a much higher progressivity that is needed to increase the weights on low-productivity/low-income agents. The progressivity indeed increases from 16% to 57%. We recall that the other policy parameters, as well as the main macro ratios (e.g., consumption-to-GDP, government spending-to-GDP, investment-to-GDP), remain the same as in the benchmark economy. In particular, the pre-tax interest rate is kept at its optimal value, which is the inverse of the discount factor. Because of the labor taxation, the labor supply falls, which means that the capital also falls to keep the capital-to-labor ratio constant. However, agents are still patient, and more than in the French economy, which means that they overall save more. This requires an increase in public debt to absorb the excess savings. The decrease in capital tax from 36% to 9% is needed for low productivity agent to save more, as their weight is now higher.

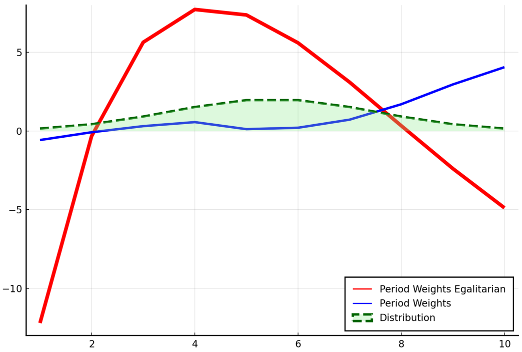
This increase in public debt and the decrease in capital taxes requires an increase in the labor tax to compensate for the loss in tax returns. Moreover, the increase in the labor tax was expected since the relatively low labor tax in the United States favors high-income/high-productivity agents. This decreases the estimated weights for high-labor-income agents. Admittedly, as we compare steady states, these are long-run effects that do not consider the additional welfare effects of the transition from the benchmark fiscal system to the new one.

7.5 Egalitarian, Libertarian or Utilitarian?

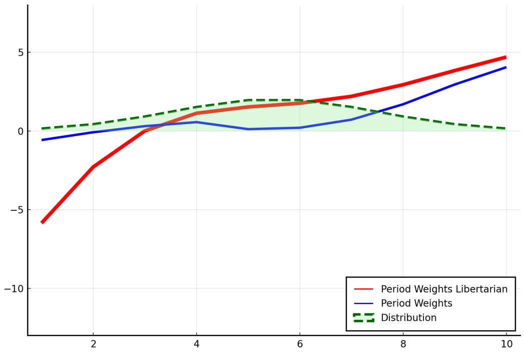
We now want to compare the SWF of the US and France to benchmark SWFs such as the Egalitarian, Libertarian and Utilitarian ones, discussed in Section 3.3.3. The Utilitarian SWF is easy to identify, as it corresponds to $\omega_k = 1$ for $k = 1, \ldots, 10$. In this environment, the identification of Egalitarian and Libertarian SWFs is more difficult as it depends on the allocation, as shown in the simple example of Section 3. Hence, we use the following strategy for each country. First, we select a fiscal system that can be classified as Egalitarian (or Libertarian), keeping everything else constant. Second, we compute the implied allocation of the SSME. Third, we identify Egalitarian weights using the same procedure as in Section 7. This exercise provides Egalitarian and Libertarian weights for France and the US.

- The Libertarian fiscal system is defined as $\tau = 0$ (no progressivity in labor tax), and $\tau^k = 5\%$, (a very small capital tax). We iterate on κ (slope of the linear labor tax) for the budget of the state to be balanced. Note that in this exercise, public debt is endogenous, as it must clear the financial markets.
- The Egalitarian fiscal system is defined as high capital tax $\tau^K = 100\%$ such that there is no wealth inequality, and a high value of τ in each country to balance the budget of the state.

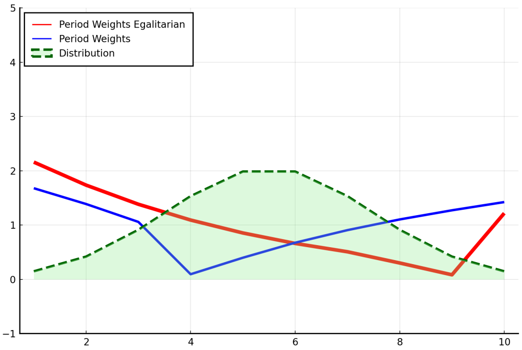
Indeed, other fiscal systems could be used, but these systems capture the Egalitarian and Libertarian ideas of redistribution. These fiscal systems generate rather extreme distributions. Capital inequality is very high for the Libertarian fiscal system (both for France and the US), and agents do not save in the Egalitarian fiscal system. Consequently, public debt is negative, and the state owns all the capital stock. This allocation is then close to the popular idea of communism. For this reason, the estimated SWF has many negative values, making the positivity constraint on weights very binding. Therefore, we now proceed with the estimation strategy without the positivity constraint on weights. Below, we present the Egalitarian and Libertarian cases for the United States and France, respectively.



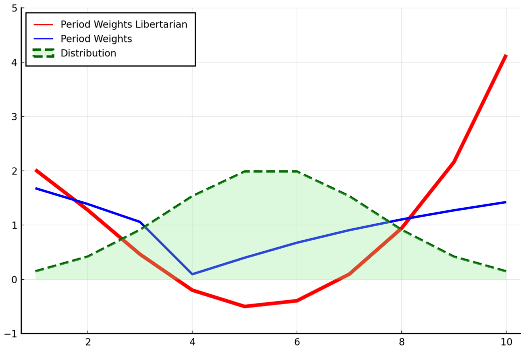
(a) Period weights in the Egalitarian case for the United States



(b) Period weights in the Libertarian case for the United States



(c) Period weights in the Egalitarian case for France



(d) Period weights in the Libertarian case for France

Figure 6: Period weights for France and the US in the Egalitarian and Libertarian cases.

Table 7 presents the summary statistics for both cases.

8 Conclusion

We derive a methodology to identify the Social Welfare Function (SWF) of a government that is compatible with the empirical wealth and income distributions given the actual tax structure. Using this methodology, we calculate the social weights for France and the US. To estimate these weights, we selected among the large set of possible SWFs the closest one to the Utilitarian SWF, which attributes the same weights to all agents. Using four fiscal instruments—consumption, capital and progressive labor taxes, and public debt—we estimated the SWFs of the two countries and showed that they differ from each other. The SWF for France gives a higher weight to low-productivity agents and is less heterogeneous than that of the US, while the US SWF has an increasing shape in income with more weight to high-income agents. Our methodology allows for computing the optimal allocation with aggregate shocks.

	United States	France
Egalitarian		
Public Debt (%GDP)	−270	−310
τ_k (%)	1	1
τ (%)	65	47
κ (%)	85	73
Gini post-tax income	16	13
Gini Wealth	0	0
Libertarian		
Public Debt (%GDP)	1289	2116
τ_k (%)	5	5
τ (%)	0	0
κ (%)	21	26
Gini post-tax income	36	28
Gini wealth	39	34

Table 7: Comparison of economic metrics between the US and France for the Egalitarian and Libertarian cases.

References

Abel, A. B. (1990): "Asset Prices under Habit Formation and Catching up with the Joneses," *The American Economic Review*, 80, 38–42.

Açikgöz, O., M. Hagedorn, H. Holter, and Y. Wang (2022): "The Optimum Quantity of Capital and Debt," Working Paper, University of Oslo.

Açikgöz, O. T. (2018): "On the Existence and Uniqueness of Stationary Equilibrium in Bewley Economies with Production," *Journal of Economic Theory*, 173, 18–55.

Aiyagari, S. R. (1995): "Optimal Capital Income Taxation with Incomplete Markets, Borrowing Constraints, and Constant Discounting," *Journal of Political Economy*, 103, 1158–1175.

Aiyagari, S. R. and E. R. McGrattan (1998): "The Optimum Quantity of Debt," *Journal of Monetary Economics*, 42, 447–469.

Arrow, K. (1951): *Social Choice and Individual Values*, New York: John Wiley.

Auclert, A., B. Bardóczy, M. Rognlie, and L. Straub (2021): "Using the Sequence-Space Jacobian to Solve and Estimate Heterogeneous-Agent Models," *Econometrica*, 89, 2375–2408.

Bargain, O. and C. Keane (2010): "Tax-Benefit-Revealed Redistributive Preferences Over Time: Ireland 1987-2005," *Labour*, 24, 141–167.

Bayer, C., R. Luetticke, L. Pham-Dao, and V. Tjaden (2019): "Precautionary Savings, Illiquid Assets, and the Aggregate Consequences of Shocks to Household Income Risk," *Econometrica*, 87, 255–290.

Benabou, R. (2002): "Tax and Education Policy in a Heterogeneous-Agent Economy: What Levels of Redistribution Maximize Growth and Efficiency?" *Econometrica*, 70, 481–517.

Boppart, T., P. Krusell, and K. Mitman (2018): "Exploiting MIT Shocks in Heterogeneous-Agent Economies: The Impulse Response as a Numerical Derivative," *Journal of Economic Dynamics and Control*, 89, 68–92.

Bourguignon, F. and S. Amadeo (2015): "Tax-Benefit Revealed Social Preferences," *Journal of Economic Inequality*, 1, 75–108.

Chamley, C. (1986): "Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives," *Econometrica*, 54, 607–622.

Chang, B., Y. Chang, and S.-B. Kim (2018): "Pareto Weights in Practice : Income Inequality and Tax Reform," *Review of Economic Dynamics*, 28, 181–204.

Cheridito, P. and J. Sagredo (2016): "Existence of Sequential Competitive Equilibrium in Krusell-Smith Type Economies," Working Paper, ETH Zurich.

Chetty, R., A. Guren, D. Manoli, and A. Weber (2011): "Are Micro and Macro Labor Supply Elasticities Consistent? A Review of Evidence On the Intensive and Extensive Margins," *American Economic Review*, 101, 471–475.

Dávila, J., J. H. Hong, P. Krusell, and J.-V. Ríos-Rull (2012): "Constrained Efficiency in the Neoclassical Growth Model With Uninsurable Idiosyncratic Shocks," *Econometrica*, 80, 2431–2467.

Dyrda, S. and M. Pedroni (2022): "Optimal Fiscal Policy in a Model with Uninsurable Idiosyncratic Shocks," *Review of Economic Studies*, 90, 744–780.

Fehr, E., D. Rutzler, and M. Sutter (2013): "The Development of Egalitarianism, Altruism, Spite and Parochialism in Childhood and Adolescence," *European Economic Review*, 64, 369–383.

Fehr, E. and K. Schmidt (2006): "The Economics of Fairness, Reciprocity and Altruism - Experimental Evidence and New Theories," Elsevier, vol. 1, chap. 08, 615–691, 1 ed.

Ferriere, A. and G. Navarro (2023): "The Heterogeneous Effects of Government Spending: It's All About Taxes," *Review of Economic Studies*, forthcoming.

Fleurbaey, M. and F. Maniquet (2018): "Optimal Taxation Theory and Principles of Fairness," *Journal of Economic Literature*, 56.

Fonseca, R., F. Langot, P.-C. Michaud, and T. Sopraseuth (2023): "Understanding Cross-Country Differences in Health Status and Expenditures: Health Prices Matter," *Journal of Political Economy*, 131, 1949–1993.

Galí, J. (1994): "Keeping up with the Joneses: Consumption Externalities, Portfolio Choice, and Asset Prices," *Journal of Money, Credit and Banking*, 26, 1–8.

Green, E. (1994): "Individual-Level Randomness in a Nonatomic Population," Working Paper, University of Minnesota.

Harsanyi, J. (1955): "Cardinal Welfare, Individualistic Ethics, and Interpersonal Comparisons of Utility," *Journal of Political Economy*, 63, 309–321.

Heathcote, J. (2005): "Fiscal Policy with Heterogeneous Agents and Incomplete Markets," *Review of Economic Studies*, 72, 161–188.

Heathcote, J. and F. Perri (2017): "Wealth and Volatility," *Review of Economic Studies*, forthcoming.

Heathcote, J., K. Storesletten, and G. L. Violante (2017): "Optimal Tax Progressivity: An Analytical Framework," *Quarterly Journal of Economics*, 132, 1693–1754.

Heathcote, J. and H. Tsujiyama (2021): "Optimal Income Taxation: Mirrlees Meets Ramsey," *Journal of Political Economy*, 129, 3141–3184.

Kaplan, G. and G. L. Violante (2014): "A Model of the Consumption Response to Fiscal Stimulus Payments," *Econometrica*, 82, 1199–1239.

Krueger, D., K. Mitman, and F. Perri (2018): "On the Distribution of the Welfare Losses of Large Recessions," in *Advances in Economics and Econometrics: Volume 2, Eleventh World Congress of the Econometric Society*, ed. by B. Honoré, A. Pakes, M. Piazzesi, and L. Samuleson, Cambridge University Press, 143–184.

LeGrand, F. and X. Ragot (2022a): "Managing Inequality Over the Business Cycle: Optimal Policies with Heterogeneous Agents and Aggregate Shocks," *International Economic Review*, 63, 511–540.

——— (2022b): "Refining the Truncation Method to Solve Heterogeneous-Agent Models," *Annals of Economics and Statistics*, 146, 65–91.

——— (2023a): "Optimal Fiscal Policy with Heterogeneous Agents and Capital: Should We Increase or Decrease Public Debt and Capital Taxes?" Working Paper, SciencesPo.

——— (2023b): "Optimal Policies with Heterogeneous Agents: Truncation and Transitions," *Journal of Economic Dynamics and Control*, 156, 104737.

Mendoza, E., A. Razin, and L. Tesar (1994): "Effective Tax Rates in Macroeconomics: Cross-Country Estimates of Tax Rates on Factor Incomes and Consumption," *Journal of Monetary Economics*, 34, 297–323.

Miao, J. (2006): "Competitive Equilibria of Economies with a Continuum of Consumers and Aggregate Shocks," *Journal of Economic Theory*, 128, 274–298.

Reiter, M. (2009): "Solving Heterogeneous-Agent Models by Projection and Perturbation," *Journal of Economic Dynamics and Control*, 33, 649–665.

Saez, E. and S. Stantcheva (2016): "Generalized Social Marginal Welfare Weights for Optimal Tax Theory," *American Economic Review*, 106.

Schokkaert, E. and B. Tarroux (2021): "Empirical Research on Ethical Preferences: How Popular is Prioritarianism?" Working Papers 2104, Groupe d'Analyse et de Theorie Economique Lyon St-Etienne, Université de Lyon.

Sen, A. (1970): "The Impossibility of a Paretian Liberal," *Journal of Political Economy*, 78, 152–157.

——— (1977): "Rational Fools: A Critique of the Behavioral Foundations of Economic Theory," *Philosophy and Public Affairs*, 6, 317–344.

Tauchen, G. (1986): "Finite State Markov-Chain Approximations to Univariate and Vector Autoregressions," *Economics letters*, 20, 177–181.

Trabandt, M. and H. Uhlig (2011): "The Laffer Curve Revisited," *Journal of Monetary Economics*, 58, 305–327.

Werning, I. (2015): "Incomplete Markets and Aggregate Demand," Working Paper 21448, National Bureau of Economic Research.

Appendix

A Proof of the SWF construction

A.1 Proof of Result 1

To lighten notation, we conduct the proof between $t = 0$ and $t = 1$. It is obviously straightforward to extend it to arbitrary t and $t + 1$. We consider an agent whose individual preferences are represented by a utility function $V : \mathcal{Y}^\infty \times \boldsymbol{X}^\infty \to \mathbb{R}$. Let $y_0^\infty \in \mathcal{Y}^\infty$ and $\boldsymbol{x}_0^\infty \in \boldsymbol{X}^\infty$. The representation result of equation (9) becomes after splitting the sum for $t = 0$ and $t \geq 1$:

$$V(y_0^\infty, \boldsymbol{x}_0^\infty) = U(y_0^\infty, \boldsymbol{x}_{0,0}) + \beta \sum_{t=1}^{\infty} \int_{y_t^{\infty,\prime} \in \mathcal{Y}^\infty} \beta^{t-1} U(y_t^{\infty,\prime}, \boldsymbol{x}_{0,t}) \mu_t(y_t^{\infty,\prime} | y_0^\infty) \mu_\infty(dy_t^{\infty,\prime}). \tag{41}$$

Using the definition μ_t and the Bayes rule, we have for all $t \geq 1$:

$$\mu_t(y_t^{\infty,\prime} | y_0^\infty) = \int_{y_1^\infty \in \mathcal{Y}^\infty} \left(\mu_{t-1}(y_t^{\infty,\prime} | y_1^\infty) \mu_1(y_1^\infty | y_0^\infty) \mu_\infty(dy_1^\infty) \right).$$

In words it means that the probability to transit from y_0^∞ to y_t^∞ in t periods is equal to the product of the probabilities to transit from y_0^∞ to y_1^∞ (in 1 period) and from y_1^∞ to y_t^∞ (in $t - 1$ periods) summed for all possible histories y_1^∞. Using the decomposition of μ_t, the expression of V in (41) becomes:

$$\begin{aligned} V(y_0^\infty, \boldsymbol{x}_0^\infty) = U(y_0^\infty, \boldsymbol{x}_{0,0}) + \beta \sum_{t=1}^{\infty} \int_{y_t^{\infty,\prime} \in \mathcal{Y}^\infty} \beta^{t-1} U(y_t^{\infty,\prime}, \boldsymbol{x}_{0,t}) \\ \times \int_{y_1^\infty \in \mathcal{Y}^\infty} \left(\mu_{t-1}(y_t^{\infty,\prime} | y_1^\infty) \mu_1(y_1^\infty | y_0^\infty) \mu_\infty(dy_1^\infty) \right) \mu_\infty(dy_t^{\infty,\prime}). \end{aligned}$$

Since $y^\infty \mapsto U(y^\infty, \boldsymbol{x}_t)$ is integrable on $\mathcal{Y}^\infty$ and $(y^{\infty,\prime}, y^\infty) \mapsto \mu_t(y^{\infty,\prime} | y^\infty)$ is integrable on $\mathcal{Y}^\infty \times \mathcal{Y}^\infty$ for all $t \geq 0$, we can use the Fubini theorem twice, to swap the order of integrals and then of the sum and of the integral on $y_1^\infty \in \mathcal{Y}^\infty$. We obtain:

$$\begin{aligned} &V(y_0^\infty, \boldsymbol{x}_0^\infty) = U(y_0^\infty, \boldsymbol{x}_{0,0}) \\ &+\beta \int_{y_1^\infty \in \mathcal{Y}^\infty} \left\{ \sum_{t=1}^{\infty} \int_{y_t^{\infty,\prime} \in \mathcal{Y}^\infty} \beta^{t-1} U(y_t^{\infty,\prime}, \boldsymbol{x}_{0,t}) \mu_{t-1}(y_t^{\infty,\prime} | y_1^\infty) \mu_\infty(dy_t^{\infty,\prime}) \right\} \mu_1(y_1^\infty | y_0^\infty) \mu_\infty(dy_1^\infty). \end{aligned} \tag{42}$$

The term between curly braces is $V(y_1^\infty, (\boldsymbol{x}_{0,t}^\infty)_{t \geq 1})$ using (10) applied to $t = 1$. Defining $\boldsymbol{x}_1^\infty = F(\boldsymbol{x}_0^\infty)$, we obtain from (42):

$$V(y_0^\infty, \boldsymbol{x}_0^\infty) = U(y_0^\infty, \boldsymbol{x}_{0,0}) + \beta \int_{y_1^\infty \in \mathcal{Y}^\infty} V(y_1^\infty, \boldsymbol{x}_1^\infty) \mu_1(y_1^\infty | y_0^\infty) \mu_\infty(dy_1^\infty),$$

which gives the representation (11) using the notation (12) for the conditional expectation.

A.2 Proof of Proposition 1

Using the definition (14), the expression (15) of the SWF becomes:

$$SWF(\boldsymbol{x}_t^\infty) = \int_{y_t^\infty \in \mathcal{Y}^\infty} \int_{\tilde{y}_t^\infty \in \mathcal{Y}^\infty} \hat{V}(y_t^\infty, \tilde{y}_t^\infty, \boldsymbol{x}_t^\infty) \mu_\infty(d\tilde{y}_t^\infty) \mu_\infty(dy_t^\infty),$$

which can be further simplified using the expression (13) of $\hat{V}$:

$$\begin{aligned} SWF(\boldsymbol{x}_t^\infty) = & \int_{y_t^\infty \in \mathcal{Y}^\infty} \int_{\tilde{y}_t^\infty \in \mathcal{Y}^\infty} \sum_{s=0}^{\infty} \int_{\tilde{y}_{t+s}^{\infty,\prime} \in \mathcal{Y}^\infty} \beta^{s-t} \times \\ & \hat{\omega}(y_t^\infty, \tilde{y}_{t+s}^{\infty,\prime}) U(\tilde{y}_{t+s}^{\infty,\prime}, \boldsymbol{x}_{t,s}) \mu_s(\tilde{y}_{t+s}^{\infty,\prime} | \tilde{y}_t^\infty) \mu_\infty(d\tilde{y}_{t+s}^{\infty,\prime}) \mu_\infty(d\tilde{y}_t^\infty) \mu_\infty(dy_t^\infty), \end{aligned}$$

Remember that $\mu_t(y^\infty|\tilde{y}^\infty) = \Pi_{y_0(\tilde{y}^\infty) y_0(y^\infty)} \mu_{t-1}(L(y^\infty)|\tilde{y}^\infty)$ for $t > 0$ and $\mu_0(y^\infty|\tilde{y}^\infty) = 1_{y^\infty = \tilde{y}^\infty}$. We thus show by recursion that $(y^\infty, \tilde{y}^\infty) \in \mathcal{Y}^\infty \times \mathcal{Y}^\infty \to \mu_t(y^\infty|\tilde{y}^\infty)$ is integrable. Since $\hat{\omega}$ and U are also integrable (on $\mathcal{Y}^\infty \times \mathcal{Y}^\infty$ and $\mathcal{Y}^\infty$, respectively), we can use use the Fubini theorem to permute the order of integrals and obtain:

$$\begin{aligned} SWF(\boldsymbol{x}_t^\infty) = & \sum_{s=0}^{\infty} \int_{\tilde{y}_{t+s}^{\infty,\prime} \in \mathcal{Y}^\infty} \beta^{s-t} U(\tilde{y}_{t+s}^{\infty,\prime}, \boldsymbol{x}_{t,s}) \times \\ & \left\{ \left[\int_{y_t^\infty \in \mathcal{Y}^\infty} \hat{\omega}(y_t^\infty, \tilde{y}_{t+s}^{\infty,\prime}) \mu_\infty(dy_t^\infty) \right] \left[\int_{\tilde{y}_t^\infty \in \mathcal{Y}^\infty} \mu_s(\tilde{y}_{t+s}^{\infty,\prime} | \tilde{y}_t^\infty) \mu_\infty(d\tilde{y}_t^\infty) \right] \right\} \mu_\infty(d\tilde{y}_{t+s}^{\infty,\prime}). \end{aligned}$$

By definition $\int_{\tilde{y}_t^\infty \in \mathcal{Y}^\infty} \mu_s(\tilde{y}_{t+s}^{\infty,\prime}|\tilde{y}_t^\infty) \mu_\infty(d\tilde{y}_t^\infty) = \pi_{y_0(\tilde{y}_{t+s}^{\infty,\prime})}$, which is the share of agents at date $t+s$ having current productivity level $y_0(\tilde{y}_{t+s}^{\infty,\prime})$. We then define:

$$\omega(\tilde{y}_{t+s}^{\infty,\prime}) = \int_{y_t^\infty \in \mathcal{Y}^\infty} \pi_{y_0(\tilde{y}_{t+s}^{\infty,\prime})} \hat{\omega}(y_t^\infty, \tilde{y}_{t+s}^{\infty,\prime}) \mu_\infty(dy_t^\infty),$$

we deduce:

$$SWF(\boldsymbol{x}_t^\infty) = \sum_{s=0}^{\infty} \int_{\tilde{y}_{t+s}^{\infty,\prime} \in \mathcal{Y}^\infty} \beta^{s-t} \omega(\tilde{y}_{t+s}^{\infty,\prime}) U(\tilde{y}_{t+s}^{\infty,\prime}, \boldsymbol{x}_{t,s}) \mu_\infty(d\tilde{y}_{t+s}^{\infty,\prime}),$$

which concludes the proof.

B Proof of Proposition 2

In the Ramsey program, the planner aims at determining the fiscal policy corresponding to the competitive equilibrium – as specified in Definition 2 – that maximizes aggregate welfare according to the criterion of equation (31) – while satisfying the governmental budget constraint.

Formally, the Ramsey program can be stated as follows.

$$\max_{\left(w_t, r_t, \tilde{w}_t, \tilde{r}_t, \tau_t^c, \tau_t^K, \tau_t, \kappa_t, B_t, G_t, K_t, L_t, (c_t^i, l_t^i, a_t^i, \nu_t^i)_i\right)_{t\geq 0}} W_0, \tag{B.43}$$

$$G_t + (1+r_t)B_{t-1} + w_t \int_i (y_t^i l_t^i)^{1-\tau_t} \ell(di) + r_t K_{t-1} = \tau_t^c C_t + F(K_{t-1}, L_t, s_t) + B_t, \tag{B.44}$$

$$\text{for all } i \in \mathcal{I}\text{: } (1+\tau_t^c)c_t^i + a_t^i = (1+r_t)a_{t-1}^i + w_t (y_t^i l_t^i)^{1-\tau_t}, \tag{B.45}$$

$$a_t^i \geq -\bar{a}, \ \nu_t^i (a_t^i + \bar{a}) = 0, \ \nu_t^i \geq 0, \tag{B.46}$$

$$u'(c_t^i) = \beta \mathbb{E}_t \left[(1+r_{t+1}) \frac{1+\tau_t^c}{1+\tau_{t+1}^c} u'(c_{t+1}^i) \right] + \nu_t^i, \tag{B.47}$$

$$v'(l_t^i) = \frac{(1-\tau_t)}{1+\tau_t^c} w_t y_t^i (y_t^i l_t^i)^{-\tau_t} u'(c_t^i), \tag{B.48}$$

$$K_t + B_t = \int_i a_t^i \ell(di), \ L_t = \int_i y_t^i l_t^i \ell(di), \tag{B.49}$$

and subject to several other constraints (which are not reported here for space constraints): the definition (18) of pre-tax wage and interest rates $\tilde{w}_t$ and $\tilde{r}_t$, the definition (21) of post-tax rates, the positivity of labor and consumption choices, and initial conditions.

Since the Ramsey program involves selecting a competitive equilibrium, its constraints include the equations characterizing this equilibrium: individual budget constraints (B.45), individual credit constraint (and related constraints on ν_t^i) (B.46), Euler equations for consumption and labor (B.47) and (B.48), and market clearing conditions for financial and labor markets (B.49). Moreover, the fiscal policy selected by the Ramsey equilibrium should also fulfill the governmental budget constraint (B.44) – that is also a constraint.

We now provide a reformulation of the Ramsey problem, using the change of variables:

$$\tilde{a}_t^i := \frac{a_t^i}{1+\tau_t^c}, \tag{B.50}$$

$$W_t := \frac{w_t}{1+\tau_t^c}, \tag{B.51}$$

$$R_t := \frac{(1+r_t)(1+\tau_{t-1}^c)}{1+\tau_t^c}, \tag{B.52}$$

which represents the asset choices in (B.50), the wage rate in (B.51), and the interest rate in (B.52). With this notation, the agent's budget and credit constraints become:

$$c_t^i + \tilde{a}_t^i = W_t (y_t^i l_t^i)^{1-\tau_t} + R_t \tilde{a}_{t-1}^i, \tag{B.53}$$

$$\tilde{a}_t^i \geq -\frac{\bar{a}}{1+\tau_t^c} := -\tilde{\bar{a}}. \tag{B.54}$$

Since taxes and prices are considered as given by agents, we can equivalently state their optimization program using the notation (B.50)–(B.52) and the constraints (B.53) and (B.54) rather than the original notation and the constraints (25) and (26). This modifies Euler equations (27)–(28)

as follows:

$$u'(c_t^i) = \beta \mathbb{E}_t \Big[R_{t+1} u'(c_{t+1}^i) \Big] + \nu_t^i,$$
$$v'(l_t^i) = (1-\tau_t) W_t y_t^i (y_t^i l_t^i)^{-\tau_t} u'(c_t^i).$$

We now turn to the governmental budget constraint. We further define:

$$\tilde{B}_t := \frac{B_t}{(1+\tau_t^c)}, \tag{B.55}$$
$$\tilde{A}_t := \frac{A_t}{1+\tau_t^c}. \tag{B.56}$$

and

$$\hat{B}_t := (1+\tau_t^c)\tilde{B}_t - \tau_t^c \tilde{A}_t, \tag{B.57}$$

With these new definitions, the financial market equilibrium given by (B.49) holds since we have $\tilde{A}_t = \int_i \tilde{a}(i) l(di)$.

Using the government budget constraint defined in (22) we have:

$$G_t + (1+r_t)B_{t-1} + w_t \int_i (y_t^i l_t^i)^{1-\tau_t} \ell(di) + r_t K_{t-1} = \tau_t^c C_t + F(K_{t-1}, L_t, s_t) + B_t.$$

Now replace (30) into the equation above and obtain:

$$\begin{aligned} & G_t + (1+r_t)B_{t-1} + w_t \int_i (y_t^i l_t^i)^{1-\tau_t} \ell(di) + r_t K_{t-1} = \\ & \tau_t^c (F(K_{t-1}, L_t, s_t) - G_t - (K_t - K_{t-1})) + F(K_{t-1}, L_t, s_t) + B_t. \end{aligned}$$

Divide both sides of the equation above by $(1+\tau_t^c)$ and obtain:

$$\begin{aligned} & G_t + \frac{(1+r_t)}{1+\tau_t^c} B_{t-1} + \frac{w_t}{1+\tau_t^c} \int_i (y_t^i l_t^i)^{1-\tau_t} \ell(di) + \frac{r_t}{1+\tau_t^c} K_{t-1} = \\ & - \frac{\tau_t^c}{1+\tau_t^c}(K_t - K_{t-1}) + F(K_{t-1}, L_t, s_t) + \frac{B_t}{1+\tau_t^c}. \end{aligned}$$

Using the definitions (B.51), (B.52), and (B.55):

$$\begin{aligned} & G_t + \underbrace{\frac{(1+r_t)(1+\tau_{t-1}^c)}{1+\tau_t^c} \frac{B_{t-1}}{1+\tau_{t-1}^c}}_{R_t \tilde{B}_{t-1}} + W_t \int_i (y_t^i l_t^i)^{1-\tau_t} \ell(di) + \frac{r_t}{1+\tau_t^c} K_{t-1} = \\ & - \frac{\tau_t^c}{1+\tau_t^c}(K_t - K_{t-1}) + F(K_{t-1}, L_t, s_t) + \underbrace{\frac{B_t}{1+\tau_t^c}}_{\tilde{B}_t}. \end{aligned}$$

Hence,

$$G_t + R_t \tilde{B}_{t-1} + W_t \int_i (y_t^i l_t^i)^{1-\tau_t} \ell(di) + \frac{r_t}{1+\tau_t^c} K_{t-1} = -\frac{\tau_t^c}{1+\tau_t^c}(K_t - K_{t-1}) + F(K_{t-1}, L_t, s_t) + \tilde{B}_t.$$

Now using definitions in (B.55) and (B.56) we have $K_{t-1} = A_{t-1} - B_{t-1} = (1+\tau_{t-1}^c)(\tilde{A}_{t-1} - \tilde{B}_{t-1})$

and:

$$G_t + R_t\tilde{B}_{t-1} + W_t\int_i (y_t^i l_t^i)^{1-\tau_t}\ell(di) + \frac{r_t(1+\tau_{t-1}^c)}{1+\tau_t^c}(\tilde{A}_{t-1} - \tilde{B}_{t-1}) =$$
$$\frac{\tau_t^c(1+\tau_{t-1}^c)}{1+\tau_t^c}(\tilde{A}_{t-1} - \tilde{B}_{t-1}) + F(K_{t-1}, L_t, s_t) - \tau_t^c(\tilde{A}_t - \tilde{B}_t) + \tilde{B}_t.$$

Observe $\frac{r_t(1+\tau_{t-1}^c)}{1+\tau_t^c} = R_t - \frac{1+\tau_{t-1}^c}{1+\tau_t^c}$ and that $-\tau_t^c(\tilde{A}_t - \tilde{B}_t) + \tilde{B}_t = \hat{B}_t$ given by equation (B.57), which leads us:

$$G_t + R_t\tilde{B}_{t-1} + W_t\int_i (y_t^i l_t^i)^{1-\tau_t}\ell(di) + \Big(R_t - \frac{1+\tau_{t-1}^c}{1+\tau_t^c}\Big)(\tilde{A}_{t-1} - \tilde{B}_{t-1}) =$$
$$\frac{\tau_t^c(1+\tau_{t-1}^c)}{1+\tau_t^c}(\tilde{A}_{t-1} - \tilde{B}_{t-1}) + F(K_{t-1}, L_t, s_t) + \hat{B}_t.$$

Hence, we have:

$$G_t + R_t\tilde{B}_{t-1} + W_t\int_i (y_t^i l_t^i)^{1-\tau_t}\ell(di) + (R_t - (1+\tau_{t-1}^c))(\tilde{A}_{t-1} - \tilde{B}_{t-1}) =$$
$$F(K_{t-1}, L_t, s_t) + \hat{B}_t.$$

Finally, using (B.57) in period $t-1$ (i.e., $\hat{B}_{t-1} = (1+\tau_{t-1}^c)\tilde{B}_{t-1} - \tau_{t-1}^c\tilde{A}_{t-1}$) we get (33):

$$G_t + W_t\int_i (y_t^i l_t^i)^{1-\tau_t}\ell(di) + (R_t - 1)\tilde{A}_{t-1} + \hat{B}_{t-1} =$$
$$F(K_{t-1}, L_t, s_t) + \hat{B}_t.$$

Since the public debt can be freely chosen by the planner, it is equivalent for the planner to choose $\hat{B}_t$ rather than $\tilde{B}_t$. Using these expressions, the Ramsey program (B.43)–(B.49) becomes equivalent to the one in (32)–(38).

C FOCs of the planner

The Lagrangian associated to the Ramsey program (32)–(38) can be written as:

$$\begin{aligned}
\mathcal{L} = \ & \mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\int_i \omega_t^i(u(c_t^i) - v(l_t^i))\ell(di) \\
& - \mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\int_i \Big(\lambda_{c,t}^i - R_t\lambda_{c,t-1}^i\Big)\, u'(c_t^i)\ell(di) \\
& - \mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\int_i \lambda_{l,t}^i\left(v'(l_t^i) - (1-\tau_t)W_t y_t^i(y_t^i l_t^i)^{-\tau_t}u'(c_t^i)\right)\ell(di) \\
& - \mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\mu_t\left(G_t + (1-\delta)\hat{B}_{t-1} + (R_t - 1 + \delta)\int_i \tilde{a}_{t-1}^i\ell(di) + W_t\int_i (y_t^i l_t^i)^{1-\tau_t}\ell(di) - Y_t - \hat{B}_t\right).
\end{aligned}$$

where the instruments are: $\tilde{a}_t^i$, l_t^i, W_t, R_t, τ_t, and $\hat{B}_t$ and we have:

$$c_t^i = -\tilde{a}_t^i + R_t\tilde{a}_{t-1}^i + W_t(y_t^i l_t^i)^{1-\tau_t}, \tag{C.1}$$

$$Y_t = \Big(\int_i \tilde{a}_{t-1}^i \ell(di) - \hat{B}_{t-1}\Big)^\alpha \Big(\int_i y_t^i l_t^i \ell\,(di)\,\Big)^{1-\alpha}. \tag{C.2}$$

FOC with respect to public debt $\hat{B}_t$.

$$\mu_t = \beta\mathbb{E}_t\left[(1+\tilde{r}_{t+1})\mu_{t+1}\right]. \tag{C.3}$$

FOC with respect to savings choices $\tilde{a}_t^i$. We define the marginal social value of liquidity for agent i at date t as:

$$\psi_t^i := \omega_t^i u'(c_t^i) - \left(\lambda_{c,t}^i - R_t\lambda_{c,t-1}^i - \lambda_{l,t}^i(1-\tau_t)W_t(y_t^i)^{1-\tau_t}(l_t^i)^{-\tau_t}\right)u''(c_t^i), \tag{C.4}$$

and $\hat{\psi}_t^i = \psi_t^i - \mu_t$ as the marginal social value of liquidity net of the cost for planner's resources. We obtain using (C.3):

$$\hat{\psi}_t^i = \beta\mathbb{E}_t\left[R_{t+1}\hat{\psi}_{t+1}^i\right]. \tag{C.5}$$

FOC with respect to labor supply l_t^i . We define:

$$\psi_{l,t}^i := \omega_t^i v'(l_t^i) + \lambda_{l,t}^i v''(l_t^i).$$

The FOC with respect to labor supply l_t^i is:

$$\psi_{l,t}^i = (1-\tau_t)W_t(y_t^i)^{1-\tau_t}(l_t^i)^{-\tau_t}\hat{\psi}_t^i + \mu_t F_{L,t}y_t^i - (1-\tau_t)W_t(y_t^i)^{1-\tau_t}(l_t^i)^{-\tau_t}\lambda_{l,t}^i\tau_t\frac{u'(c_t^i)}{l_t^i}.$$

FOC with respect to the wage rate W_t.

$$0 = \int_j (y_t^j l_t^j)^{1-\tau_t}\left(\hat{\psi}_t^j + \lambda_{l,t}^j(1-\tau_t)u'(c_t^j)/l_t^j\right)\ell(dj).$$

FOC with respect to the interest rate R_t.

$$0 = \int_j \left(\hat{\psi}_t^j \tilde{a}_{t-1}^j + \lambda_{c,t-1}^j u'(c_t^j)\right)\ell(dj).$$

FOC with respect to progressivity τ_t.

$$\begin{aligned}0 &= \int_j (y_t^j l_t^j)^{1-\tau_t}(\hat{\psi}_t^j + \lambda_{l,t}^j(1-\tau_t)(u'(c_t^j)/l_t^j))\ln(y_t^j l_t^j)\ell(dj) \\ &+ \int_j \lambda_{l,t}^j\left((y_t^j l_t^j)^{1-\tau_t}\right)(u'(c_t^j)/l_t^j)\ell(dj).\end{aligned}$$

D Truncating the model and identification of Pareto Weights

The Ramsey problem of Section 6 involves a joint distribution across wealth and Lagrange multipliers. This high-dimensional object raises a number of difficulties for the program resolution.

For instance, this joint distribution affects the planner's instruments in a non-obvious way, which makes the methods based on perturbation of a well-identified steady-state not usable for solving such problems (as Reiter, 2009, Boppart et al., 2018, Bayer et al., 2019 or Auclert et al., 2021).

The solution we provide here builds on LeGrand and Ragot (2022a). It allows us to compute the steady-state allocation and to derive a finite number of equations that can simulate by perturbation the dynamics of the Ramsey program for small aggregate shocks. The intuition of the method can be summarized as follows. We build an aggregation of the Bewley model (thus for a given fiscal policy and no aggregate shock) in which agents with the same history over last N periods (where N is a fixed horizon) are aggregated into a unique "agent". The method implies that the "aggregate" agent is endowed with the average wealth of all individual agents with this N-period history. The wealth heterogeneity among these individual agents is captured in our aggregate model through additional parameters – that will be called "ξs".[14]

The aggregation method yields a so-called truncated model, which thanks to the "ξs" is an exact aggregation of the underlying Bewley model in the absence of aggregate shocks. In the presence of aggregate shocks, we can simulate the truncated model using standard perturbation techniques. The truncated model also allows us to solve for the Ramsey program.

D.1 The truncated model

Let $N \geq 0$ be a truncation length. The key step of the aggregation consists in assigning to all agents sharing the same idiosyncratic history over the last $N \geq 0$ periods the same wealth and the same allocation. Such a N-period history will be said to be a truncated history and for a history $y^t = \{y_0, \ldots, y_{t-N}, y_{t-N+1}, \ldots, y_{t-1}, y_t\}$, this corresponds to the N-length vector denoted $y^N = \{y^N_{-N+1}, \ldots, y^N_{-1}, y^N_0\}$. To sum up we can represent the truncated history of an agent i whose idiosyncratic history is y^t as:

$$\begin{aligned} y^t &= \{\underbrace{y_0, \ldots, y_{t-N-2}, y_{t-N-1}, y_{t-N}}_{\xi_{y^N}}, \underbrace{y_{t-N+1}, \ldots, y_{t-1}, y_t}_{=y^N}\} \\ &:= \{\underbrace{\ldots, y^t_{-N-2}, y^t_{-N-1}, y^t_{-N}}_{\xi_{y^N}}, \underbrace{y^t_{-N+1}, \ldots, y^t_{-1}, y^t_0}_{=y^N}\}, \end{aligned}$$

where the parameter ξ_{y^N} captures the residual heterogeneity for the truncated history y^N and y^t_{-k} represents the idiosyncratic variable (at date t) k periods in the past. The way to compute this parameter will be further discussed below. In what follows we will discuss the various elements to apply the aggregation procedure.

First of all we need to compute the measure of agents with the same history y^N. An agent with history $\tilde{y}^N$ at $t-1$ will have a different truncated history in the period t depending on the realization of the idiosyncratic variable at date t. The probability to transit from truncated history $\tilde{y}^N$ to truncated history y^N will be denoted by $\Pi_{t,\tilde{y}^N y^N}$ (with $\sum_{y^N \mathcal{Y}^N} \Pi_{t,\tilde{y}^N y^N} = 1$) and

[14] Werning (2015) also proposes an aggregation method, where the heterogeneity is captured by a change in the discount factor, while our method involve the introduction of explicit correction factors, the ξs. Using the ξs enables us to solve for the Ramsey program, as all agents are endowed with the same discount factor.

can be computed from the transition probabilities for the productivity process as:

$$\Pi_{t,\tilde{y}^N y^N} = 1_{y^N \succeq \tilde{y}^N} \Pi_{t,\tilde{y}_0^N y_0^N} \geq 0.$$

With those elements now we can compute the share of agents with truncated history y^N as S_{t,y^N}. This element will be

$$S_{t,y^N} = \sum_{\tilde{y}^N \in \mathcal{Y}^N} S_{t-1,\tilde{y}^N} \Pi_{t,\tilde{y}^N y^N}, \tag{D.1}$$

where the initial shares $(S_{-1,y^N})_{y^N \in \mathcal{Y}^N}$ is given with $\sum_{y^N \in \mathcal{Y}^N} S_{-1,y^N} = 1$.

The model aggregation then assigns to each truncated history the average choices (be it for consumption, savings, or labor supply) of the group of agents sharing the same truncated history. We consider a generic variable, denoted by $X_t(y^t, s^t)$ and we denote by X_{t,y^N} the average quantity of X assigned to truncated history y^N. Formally:

$$X_{t,y^N} = \frac{1}{S_{t,y^N}} \sum_{y^t \in \mathcal{Y}^{t+1} \big| (y^t_{-N+1},\ldots,y^t_{-1},y^t_0)=y^N} X_t(y^t,s^t)\theta_t(y^t), \tag{D.2}$$

where we remind that $\theta_t(y^t)$ is the measure of agents with history y^t. Definition (D.2) can be applied to consumption, savings, labor supply and credit-constraint Lagrange multiplier. This leads to the quantities c_{t,y^N}, $\tilde{a}_{t,y^N}$, l_{t,y^N}, and ν_{t,y^N}, respectively. Note that applying (D.2) to beginning-of-period wealth involves accounting that agents with truncated history y^N at t may have come from various truncated history at $t-1$. Indeed, this variable consists of the wealth of all agents having history y^N in period t and any other possible history in $t-1$. Formally, the beginning-of-period wealth $\tilde{\tilde{a}}_{t,y^N}$ for truncated history y^N is:

$$\tilde{\tilde{a}}_{t,y^N} = \sum_{\tilde{y}^N \in \mathcal{Y}^N} \frac{S_{t-1,\tilde{y}^N}}{S_{t,y^N}} \Pi_{t,\tilde{y}^N y^N} \tilde{a}_{t-1,\tilde{y}^N}. \tag{D.3}$$

Now, in what follows we will define the various "ξs". Observe:

$$\sum_{y^t \in \mathcal{Y}^{t+1} \big| (y^t_{-N+1},\ldots,y^t_{-1},y^t_0)=y^N} u(c_t\left(y^t\right)) = \xi^{u,0}_{y^N} u(\sum_{y^t \in \mathcal{Y}^{t+1} \big| (y^t_{-N+1},\ldots,y^t_{-1},y^t_0)=y^N} c_t\left(y^t\right)),$$

which can be write compactly as:

$$\sum_{y^N \in \mathcal{Y}^N} u(c^i_t) = \xi^{u,0}_{y^N} u(c_{t,y^N}). \tag{D.4}$$

The same procedure applied to the other variables for the Ramsey problem (32)–(38) yields:

$$\sum_{y^N \in \mathcal{Y}^N} v(l^i_t) := \xi^{v,0}_{y^N} v(l_{t,y^N}), \tag{D.5}$$

$$\sum_{y^N \in \mathcal{Y}^N} u'(c^i_t) := \xi^{u,1}_{y^N} u'(c_{t,y^N}), \tag{D.6}$$

$$\sum_{y^N \in \mathcal{Y}^N} (y^i_t l^i_t)^{1-\tau_t} := \xi^{y}_{y^N} (y^N_0 l_{t,y^N})^{1-\tau_t}. \tag{D.7}$$

We can now proceed with the aggregation of the full-fledged model. First, the aggregation of individual budget constraints (B.53) yields the following equation:

$$c_{t,y^N} + \tilde{a}_{t,y^N} = W_t \xi^y_{y^N} (l_{t,y^N} y_0^N)^{1-\tau_t} + R_t \tilde{\tilde{a}}_{t,y^N}, \text{ for } y^N \in \mathcal{Y}^N. \tag{D.8}$$

The aggregation of Euler equations for consumption (36) and labor (37) yields:

$$\xi^{u,E}_{y^N} u'(c_{t,y^N}) = \beta \mathbb{E}_t \left[R_{t+1} \sum_{\tilde{y}^N \in \mathcal{Y}^N} \Pi_{t+1,y^N\tilde{y}^N} \xi^{u,E}_{\tilde{y}^N} u'(c_{t+1,\tilde{y}^N}) \right] + \nu_{t,y^N}, \tag{D.9}$$

$$\xi^{v,1}_{y^N} v'(l_{t,y^N}) := (1-\tau_t) W_t \xi^y_{y^N} (l_{t,y^N} y_0^N)^{1-\tau_t} \xi^{u,1}_{y^N} (u'(c_{t,y^N})/l_{t,y^N}), \tag{D.10}$$

where the coefficients $(\xi^{u,E}_{y^N})_{y^N}$ for the consumption Euler equations guarantee that aggregate Euler equations yields Euler equations with aggregate consumption levels. In other words, the $(\xi^{u,E}_{y^N})_{y^N}$ are determined such that the aggregated consumption levels (for each history) verify the steady state consumption Euler equation (D.9). These coefficients are needed because Euler equations involve non-linear marginal utilities. The same idea for the coefficients $(\xi^{v,1}_{y^N})_{y^N}$ for the Euler equation for labor.

Finally, market clearing conditions can be expressed as:

$$K_t + \hat{B}_t = \sum_{y^N \in \mathcal{Y}^N} S_{t,y^N} \tilde{a}_{t,y^N}, \quad L_t = \sum_{y^N \in \mathcal{Y}^N} S_{t,y^N} y_{y^N} l_{t,y^N}. \tag{D.11}$$

Equations (D.8)–(D.11) exactly characterizes the dynamics of the aggregated variables c_{t,y^N}, $\tilde{a}_{t,y^N}$, l_{t,y^N} and ν_{t,y^N}, as well as aggregate quantities K_t, $\hat{B}_t$, and L_t. In the Appendix D.3 we have the derivation of the program formulation in the Projected model as well as the equations that characterize the social weights.

Steady-state and computation of the ξs. Steady-state allocations allow us to compute the parameters ξs as follows. Indeed, we can compute policy functions and wealth distribution of the Bewley model, as well as identify credit-constrained histories. Aggregation equations (D.2) and (D.3) can then be used to aggregate (steady-state) allocations c_{y^N}, $\tilde{a}_{y^N}$, l_{y^N} and ν_{y^N} . We then invert the consumption Euler equations (D.9) to deduce the preference parameters $(\xi^{u,E}_{y^N})_{y^N}$. The other ξs are computed as explicit by equations (D.4), (D.5), (D.6), (D.7), and (D.10).

The truncated model in the presence of aggregate shocks. We state two further assumptions that will enables us to use our truncation method in the presence of aggregate shocks. This results in the so-called truncated model.

Assumption A *We make the following two assumptions.*

1. *The preference parameters $(\xi_{y^N})_{y^N}$ remain constant and equal to their steady-state values.*
2. *The set of credit-constrained histories, denoted by $\mathcal{C} \subset \mathcal{Y}^N$, is time-invariant.*

Two properties are finally worth mentioning. First, a straightforward consequence of the construction of the ξs is that the steady-state allocations of the initial and of the truncated models are identical. Second, when the truncation length N becomes increasingly long, truncated allocations (in the presence of aggregate shocks) can be proved to converge to those of the full-fledged equilibrium. Section 7 shows that from a quantitative standpoint, the ξs efficiently capture the heterogeneity within truncated histories, even when the truncation length remains limited.

D.2 Ramsey program

Program formulation. The finite state-space representation of the truncated model allows us to solve for the Ramsey program in the presence of aggregate shocks.[15] Let $(\omega_y)_{y\in\mathcal{Y}}$ be the set period weights associated to each productivity level. The Ramsey program in the truncated economy can be written as follows:

$$\max_{\left(W_t,R_t,\tilde{w}_t,\tilde{r}_t,\tau_t^c,\tau_t^K,\tau_t,\kappa_t,\hat{B}_t,G_t,K_t,L_t,(c_{t,y^N},l_{t,y^N},\tilde{a}_{t,y^N},\nu_{t,y^N})_{y^N}\right)_{t\geq 0}} W_0, \tag{D.12}$$

where $W_0 := \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\sum_{y^N\in\mathcal{Y}^N}S_{t,y^N}\omega_{y^N}(\xi_{y^N}^{u,0}u(c_{t,y^N})-\xi_{y^N}^{v,0}v(l_{t,y^N})+u_G(G_t))\right]$ and subject to aggregate Euler equations (D.9) and (D.10), aggregate budget constraint (D.8), aggregate market clearing conditions (D.11), credit constraints $\tilde{a}_{t,y^N}\geq-\tilde{\bar{a}}$, as well as the governmental budget constraint (33), already present in the full-fledged Ramsey program.

Computing the weights $(\omega_y)_{y\in\mathcal{Y}}$. The key contribution of our paper involves estimating the weights that corresponds to different fiscal systems. More precisely, we follow the methodology of the inverse optimal taxation literature (see Bargain and Keane, 2010; Bourguignon and Amadeo, 2015; Heathcote and Tsujiyama, 2021; Chang et al., 2018, among others) and estimate weights for the first-order conditions of the Ramsey program in (32) - (38) to hold at the steady-state for the fiscal system under consideration (France or US in our quantitative exercise of Section 7). However, as we explain in Section 6, the estimation problem is under identified. We use the method of equation (39), which is however simplified in the truncated model, which feature limited heterogeneity. Formally, equation equation (39) becomes: $\omega_y = \arg\min_{(\tilde{\omega}_y)}\theta(y)\left\|(\tilde{\omega}_y)_y-\mathbf{1}\right\|_2$ subject to $\sum_y\theta(y)\tilde{\omega}_y=1$ and such that planner's first-order conditions hold. Our method also allows to estimate the weights for each history. In this case notice the same problem as argued in Section 6 applies, since we have Y^N weights (where $Y=Card\,\mathcal{Y}$) and few constraints. To estimate the weights for each history we solve the following problem $\omega_{y^N}=\arg\min_{(\tilde{\omega}_{y^N})}S_{y^N}\left\|(\tilde{\omega}_{y^N})_{y^N}-\mathbf{1}\right\|_2$ subject to $\sum_y S_{y^N}\tilde{\omega}_{y^N}=1$.[16] Notice that the only difference between the two approaches is that in the second one we will have $(\omega_{y^N})_{y^N}$ different for each $y^N\in\mathcal{Y}^N$, whereas in the first approach $(\omega_{y^N})_{y^N}=(\omega_{\tilde{y}^N})_{\tilde{y}^N}$ whenever $y_0^N=\tilde{y}_0^N$, i.e., everytime the productivity level in the first period of the truncation associated with the history y^N is the same as the productivity level associated with the history $\tilde{y}^N$ with $y^N\neq\tilde{y}^N$.

Thanks to the limited heterogeneity both computations above pin down to simple matrix algebra. In what follows we explain how to obtain those estimates.

[15]Our method involves deriving the first-order conditions of the truncated model, and not to truncate the first-order conditions of the full-fledged Ramsey model. This ensures numerical stability, as by construction the truncated model is "well-defined" for the fiscal policy under consideration.

[16]In the previous expression, $\|\cdot\|_2$ denotes the Euclidean norm, $(\omega_{y^N})_{y^N}$ is the vector of weights, and $\mathbf{1}$ is the vector of ones.

First-order conditions. We define the net social value of liquidity of history y^N as in (C.4):

$$\hat{\psi}_{t,y^N} = \omega_{y^N}\xi^{u,0}_{y^N}u'(c_{t,y^N}) - \mu_t - \left(\lambda_{c,t,y^N}\xi^{u,E}_{y^N} - R_t\tilde{\lambda}_{c,t,y^N}\xi^{u,E}_{y^N} - \lambda_{l,t,y^N}\xi^{y}_{y^N}(1-\tau_t)W_t(y_0^N)^{1-\tau_t}l^{-\tau_t}_{t,y^N}\xi^{u,1}_{y^N}\right)u''(c_{t,y^N}). \quad \text{(D.13)}$$

FOC with respect to $\tilde{a}_{t,y^N}$:

$$\hat{\psi}_{t,y^N} = \beta\mathbb{E}_t\Big[R_{t+1}\sum_{\tilde{y}^N\in\mathcal{Y}^N}\Pi_{t,y^N\tilde{y}^N}\hat{\psi}_{t+1,\tilde{y}^N}\Big] \text{ if } \nu_{y^N}=0 \text{ and } \lambda_{c,t,y^N}=0 \text{ otherwise.} \quad \text{(D.14)}$$

FOC with respect to l_{t,y^N}:

$$\frac{\omega_{y^N}\xi^{v,0}_{y^N}v'(l_{t,y^N}) + \lambda_{l,t,y^N}\xi^{v,1}_{y^N}v''(l_{t,y^N})}{(1-\tau_t)W_t\xi^{y}_{y^N}(y_0^N)^{1-\tau_t}l^{-\tau_t}_{t,y^N}} = \hat{\psi}_{t,y^N} - \lambda_{l,t,y^N}\tau_t\xi^{u,1}_{y^N}(u'(c_{t,y^N})/l_{t,y^N}) + \mu_t(1-\alpha)\frac{Y_t}{(1-\tau_t)W_t\xi^{y}_{y^N}(y_0^N)^{-\tau_t}l^{-\tau_t}_{t,y^N}L_t}. \quad \text{(D.15)}$$

FOC with respect to W_t:

$$\sum_{y^N\in\mathcal{Y}^N} S_{t,y^N}\xi^{y}_{y^N}(l_{t,y^N}y_{y^N})^{1-\tau_t}\left(\hat{\psi}_{t,y^N} + \lambda_{l,t,y^N}(1-\tau_t)\xi^{u,1}_{y^N}(u'(c_{t,y^N})/l_{t,y^N})\right) = 0. \quad \text{(D.16)}$$

FOC with respect to R_t:

$$\sum_{y^N\in\mathcal{Y}^N} S_{t,y^N}\left(\hat{\psi}_{t,y^N}\tilde{\tilde{a}}_{t,y^N} + \tilde{\lambda}_{c,t,y^N}\xi^{u,E}_{y^N}u'(c_{t,y^N})\right) = 0. \quad \text{(D.17)}$$

FOC with respect to τ_t:

$$\sum_{y^N\in\mathcal{Y}^N} S_{t,y^N}\left(\hat{\psi}_{t,y^N} + \lambda_{l,t,y^N}(1-\tau_t)\xi^{u,1}_{y^N}(u'(c_{t,y^N})/l_{t,y^N})\right)\ln\left(l_{t,y^N}y_{y^N}\right)\xi^{y}_{y^N}(l_{t,y^N}y_{y^N})^{1-\tau_t} = -\sum_{y^N\in\mathcal{Y}^N} S_{t,y^N}\lambda_{l,t,y^N}\xi^{y}_{y^N}(l_{t,y^N}y_{y^N})^{1-\tau_t}\xi^{u,1}_{y^N}(u'(c_{t,y^N})/l_{t,y^N}). \quad \text{(D.18)}$$

FOC with respect to $\hat{B}_t$:

$$\mu_t = \beta\mathbb{E}\left[\mu_{t+1}\left(1+\alpha\frac{Y_{t+1}}{K_t}-\delta\right)\right]. \quad \text{(D.19)}$$

We must furthermore have:

$$\tilde{\lambda}_{c,t,y^N} = \sum_{\tilde{y}^N\in\mathcal{Y}^N}\frac{S_{t-1,\tilde{y}^N}}{S_{t,y^N}}\Pi_{t,\tilde{y}^N y^N}\lambda_{c,t-1,\tilde{y}^N}, \quad \text{(D.20)}$$

$$\tilde{a}_{t,y^N} \geq 0 \text{ and } \tilde{\tilde{a}}_{t,y^N} = \sum_{\tilde{y}^N\in\mathcal{Y}^N}\frac{S_{t-1,\tilde{y}^N}}{S_{t,y^N}}\Pi_{t,\tilde{y}^N y^N}\tilde{a}_{t-1,\tilde{y}^N}. \quad \text{(D.21)}$$

D.3 Matrix expression

In this section, we provide closed-form formulas for preference multipliers ξs (Section D.1) and the weights ωs. We start with some notation:

- $\circ$ is the Hadamard product, $\otimes$ is the Kronecker product, $\times$ is the usual matrix product.

For any vector V, we denote by $diag\,(V)$ the diagonal matrix with V on the diagonal.

The matrix representation consists in stacking together the equations characterizing the steady-state, so as to provide a convenient matrix notation for solving the steady state. It also provides an efficient solution to compute the values for the coefficients (ξ_{y^N}) and (ω_{y^N}). The starting point is to observe that a history y^N can be seen as a N-length vector $\{y^N_{-N+1}, \ldots, y^N_0\}$ of elements of $\mathcal{Y}$. The number of histories is $N_{tot} = Y^N$. We can identify each history by an integer $k_{y^N} = 1, \ldots, N_{tot}$:

$$k_{y^N} = \sum_{k=0}^{N-1} N_{tot}^{-N+1-k}(y_k - 1) + 1, \tag{D.22}$$

which corresponds to an enumeration in base Y.

D.3.1 A closed-form formula for the ξs

Let $\mathbf{S}$ be the N_{tot}-vector of steady-state history sizes that is defined as $\mathbf{S} = (S_{k_{y^N}})_{k_{y^N}=1,\ldots,N_{tot}}$, by stacking history sizes for all histories using the enumeration given by (D.22). Similarly, let $\tilde{\mathbf{a}}$, $\mathbf{c}$, $\boldsymbol{\ell}$, $\boldsymbol{\nu}$, $\mathbf{u}'(\mathbf{c})$, $\mathbf{v}'(\mathbf{l})$ $\mathbf{u}''(\mathbf{c})$, $\mathbf{v}''(\mathbf{l})$ be the N_{tot}-vectors of end-of-period wealth, consumption, labor supply, Lagrange multipliers, marginal utilities, and derivatives of the marginal utility, respectively. These vectors are known from the steady-state equilibrium of the Bewley model. Each element is defined as the truncation of the relevant variable computed using equation (D.2). We also define:

$$\mathbf{y} = \begin{bmatrix} y_1 \\ \vdots \\ y_Y \end{bmatrix} \otimes 1_B,$$

where 1_B is a vector of 1 of length Y^{N-1}. We define $\mathbb{P}$ as the diagonal matrix having 1 on the diagonal at y^N if and only if the history y^N is not credit constrained (i.e., $\nu_{y^N} = 0$), and 0 otherwise. We similarly define $\mathbb{P}^c = \mathbf{I} - \mathbb{P}$, where $\mathbf{I}$ is the $(N_{tot} \times N_{tot})$-identity matrix. Noting Π as the transition matrix across histories, we obtain the following steady-state relationships.

D.3.2 Matrix expressions for the definition of ξs

Writing $S_{y^N} = \sum_{\tilde{y}^N \in \mathcal{Y}^N} S_{\tilde{y}^N} \Pi_{\tilde{y}^N y^N}$, (D.8) and the credit constraint yield, respectively:

$$\boldsymbol{S} = \Pi \boldsymbol{S}, \tag{D.23}$$

$$\boldsymbol{S} \circ \boldsymbol{c} + \boldsymbol{S} \circ \tilde{\boldsymbol{a}} = R\Pi\,(\boldsymbol{S} \circ \tilde{\boldsymbol{a}}) + W\boldsymbol{S} \circ \boldsymbol{\xi}^y \circ (\boldsymbol{y} \circ \boldsymbol{l})^{1-\tau}, \tag{D.24}$$

$$(\boldsymbol{I} - \boldsymbol{P})\tilde{\boldsymbol{a}} = \mathbf{0}_{N_{tot} \times 1}. \tag{D.25}$$

The Euler equation for consumption in (D.9) becomes:

$$\boldsymbol{\xi}^{u,E} \circ u'(\boldsymbol{c}) = \beta R \boldsymbol{\Pi}^{\top} \left(\boldsymbol{\xi}^{u,E} \circ u'(\boldsymbol{c})\right) + \boldsymbol{\nu},$$

where the matrix $\boldsymbol{\Pi}^{\top}$ (the transpose of Π) is used to make expectations about next period histories. Or equivalently:

$$\boldsymbol{D}_{u'(\boldsymbol{c})}\boldsymbol{\xi}^{u,E} = \beta R \boldsymbol{\Pi}^{\top} \boldsymbol{D}_{u'(\boldsymbol{c})}\boldsymbol{\xi}^{u,E} + \boldsymbol{\nu},$$

where $\boldsymbol{D}$ stands for the diagonal matrix. Finally:

$$\boldsymbol{\xi}^{u,E} = \left[\left(\boldsymbol{I} - \beta R \boldsymbol{\Pi}^{\top}\right) \boldsymbol{D}_{u'(\boldsymbol{c})}\right]^{-1} \boldsymbol{\nu}. \tag{D.26}$$

From the FOC on labor in (D.10) we obtain:

$$\boldsymbol{\xi}^{v,1} = (1-\tau) W (\boldsymbol{y} \circ \boldsymbol{l})^{1-\tau} \circ \boldsymbol{\xi}^{y} \circ \boldsymbol{\xi}^{u,1} \circ u'(\boldsymbol{c}) ./ (\boldsymbol{l} \circ v'(\boldsymbol{l})). \tag{D.27}$$

Now from the aggregation equations in (D.4)–(D.7) we get:

$$\boldsymbol{\xi}^{u,0} = \frac{\sum_{y^N \in \mathcal{Y}^N} u(c_t^i)}{u(c_{t,y^N})}, \quad \boldsymbol{\xi}^{u,1} = \frac{\sum_{y^N \in \mathcal{Y}^N} u'(c_t^i)}{u'(c_{t,y^N})}, \tag{D.28}$$

$$\boldsymbol{\xi}^{v,0} = \frac{\sum_{y^N \in \mathcal{Y}^N} v(l_t^i)}{v(l_{t,y^N})}, \quad \boldsymbol{\xi}^{y} = \frac{\sum_{y^N \in \mathcal{Y}^N} (y_t^i l_t^i)^{1-\tau}}{\left(y_0^N l_{t,y^N}\right)^{1-\tau}}. \tag{D.29}$$

Finally, we define the following variables: $\tilde{\boldsymbol{\xi}}^{u,1} := \boldsymbol{\xi}^{u,1} ./ \boldsymbol{l}$, $\tilde{\boldsymbol{\xi}}^{v,1} := \boldsymbol{\xi}^{v,1} ./ ((1-\tau) W \boldsymbol{\xi}^{y} \circ \boldsymbol{y}^{1-\tau} \circ \boldsymbol{l}^{-\tau})$, and $\tilde{\boldsymbol{\xi}}^{v,0} := \boldsymbol{\xi}^{v,0} ./ ((1-\tau) W \boldsymbol{\xi}^{y} \circ \boldsymbol{y}^{1-\tau} \circ \boldsymbol{l}^{-\tau})$.

D.3.3 Matrix expressions for the FOCs

We define the following variables: $\bar{\lambda}^l := \mathbf{S} \circ \lambda^l$, $\bar{\psi} := \mathbf{S} \circ \hat{\psi}$, $\bar{\Pi} := \mathbf{S} \circ \Pi^{\top} \circ (1/\mathbf{S})$, $\bar{\omega} := \mathbf{S} \circ \omega$, $\bar{\lambda}_c := \mathbf{S} \circ \lambda_c$, and notice that $\mathbf{S} \circ \tilde{\lambda}_c = \Pi \bar{\boldsymbol{\lambda}}_c$. With this notation, the FOCs (D.13)–(D.19) become:

$$\begin{aligned} \bar{\boldsymbol{\psi}} &= \bar{\boldsymbol{\omega}} \circ \boldsymbol{\xi}^{u,0} \circ u'(\boldsymbol{c}) - \mu \mathbf{S} && \text{(D.30)} \\ &- \left(\bar{\boldsymbol{\lambda}}_c \circ \boldsymbol{\xi}^{u,E} - R \boldsymbol{\Pi} \bar{\boldsymbol{\lambda}}_c \circ \boldsymbol{\xi}^{u,E} - (1-\tau) W \bar{\boldsymbol{\lambda}}_l \circ \boldsymbol{\xi}^{y} \circ (\boldsymbol{y} \circ \boldsymbol{l})^{1-\tau} \circ \tilde{\boldsymbol{\xi}}^{u,1}\right) \circ u''(\boldsymbol{c}), \\ \boldsymbol{P} \bar{\boldsymbol{\psi}} &= \beta R \boldsymbol{P} \bar{\boldsymbol{\Pi}} \bar{\boldsymbol{\psi}}, && \text{(D.31)} \\ (\boldsymbol{I} - \boldsymbol{P}) \bar{\boldsymbol{\lambda}}_c &= 0, && \text{(D.32)} \\ \left(\boldsymbol{\xi}^{y} \circ (\boldsymbol{y} \circ \boldsymbol{l})^{1-\tau}\right)^{\top} \bar{\boldsymbol{\psi}} &= -(1-\tau) \left(\boldsymbol{\xi}^{y} \circ (\boldsymbol{y} \circ \boldsymbol{l})^{1-\tau} \circ \tilde{\boldsymbol{\xi}}^{u,1} \circ u'(\boldsymbol{c})\right)^{\top} \bar{\boldsymbol{\lambda}}_l, && \text{(D.33)} \\ \tilde{\boldsymbol{a}}^{\top} \bar{\boldsymbol{\psi}} &= -\left(\boldsymbol{\xi}^{u,E} \circ u'(\boldsymbol{c})\right)^{\top} \Pi \bar{\boldsymbol{\lambda}}_c, && \text{(D.34)} \end{aligned}$$

$$\bar{\boldsymbol{\omega}} \circ \tilde{\boldsymbol{\xi}}^{v,0} \circ v'(\boldsymbol{l}) + \bar{\boldsymbol{\lambda}}_l \circ \tilde{\boldsymbol{\xi}}^{v,1} \circ v''(\boldsymbol{l}) = \bar{\boldsymbol{\psi}} - \tau \tilde{\boldsymbol{\xi}}^{u,1} \circ u'(\boldsymbol{c}) \circ \bar{\boldsymbol{\lambda}}_l + \mu F_L \boldsymbol{S} ./ ((1-\tau) W \boldsymbol{\xi}^{y} \circ \boldsymbol{y}^{-\tau} \circ \boldsymbol{l}^{-\tau}), \tag{D.35}$$

$$\left(\ln(\boldsymbol{y} \circ \boldsymbol{l}) \circ \boldsymbol{\xi}^{y} \circ (\boldsymbol{y} \circ \boldsymbol{l})^{1-\tau}\right)^{\top} \bar{\boldsymbol{\psi}} = -\left((\boldsymbol{1} + (1-\tau)\ln(\boldsymbol{y} \circ \boldsymbol{l})) \circ \boldsymbol{\xi}^{y} \circ (\boldsymbol{y} \circ \boldsymbol{l})^{1-\tau} \circ \tilde{\boldsymbol{\xi}}^{u,1} \circ u'(\boldsymbol{c})\right)^{\top} \bar{\boldsymbol{\lambda}}_l. \tag{D.36}$$

D.3.4 Solving the system

Equation (D.35) yields:

$$\begin{aligned}\boldsymbol{D}_{\tilde{\boldsymbol{\xi}}^{v,1}\circ v''(\boldsymbol{l})+\tau\tilde{\boldsymbol{\xi}}^{u,1}\circ u'(\boldsymbol{c})}\bar{\boldsymbol{\lambda}}_l &= \mu F_L \boldsymbol{S}./((1-\tau)W\boldsymbol{\xi}^y\circ\boldsymbol{y}^{-\tau}\circ\boldsymbol{l}^{-\tau}) + \bar{\boldsymbol{\psi}} - \boldsymbol{D}_{\tilde{\boldsymbol{\xi}}^{v,0}\circ v'(\boldsymbol{l})}\bar{\boldsymbol{\omega}},\\ \bar{\boldsymbol{\lambda}}_l &= \boldsymbol{M}_0\bar{\boldsymbol{\omega}} + \boldsymbol{M}_1\bar{\boldsymbol{\psi}} + \mu\boldsymbol{V}_0.\end{aligned} \tag{D.37}$$

with: $\boldsymbol{M}_0 := -\boldsymbol{M}_1\boldsymbol{D}_{\tilde{\boldsymbol{\xi}}^{v,0}\circ v'(\boldsymbol{l})}$, $\boldsymbol{M}_1 := \boldsymbol{D}^{-1}_{\tilde{\boldsymbol{\xi}}^{v,1}\circ v''(\boldsymbol{l})+\tau\tilde{\boldsymbol{\xi}}^{u,1}\circ u'(\boldsymbol{c})}$, and $\boldsymbol{V}_0 := F_L\boldsymbol{M}_1\boldsymbol{S}./((1-\tau)W\boldsymbol{\xi}^y\circ\boldsymbol{y}^{-\tau}\circ\boldsymbol{l}^{-\tau})$.

Equation (D.30) then implies:

$$\bar{\boldsymbol{\psi}} = \hat{\boldsymbol{M}}_0\bar{\boldsymbol{\omega}} + \hat{\boldsymbol{M}}_1\bar{\boldsymbol{\lambda}}_c + \hat{\boldsymbol{M}}_2\bar{\boldsymbol{\lambda}}_l - \mu\boldsymbol{S}. \tag{D.38}$$

with: $\hat{\boldsymbol{M}}_0 := \boldsymbol{D}_{\boldsymbol{\xi}^{u,0}\circ u'(\boldsymbol{c})}$, $\hat{\boldsymbol{M}}_1 := -\boldsymbol{D}_{\boldsymbol{\xi}^{u,E}\circ u''(\boldsymbol{c})}(\boldsymbol{I}-R\boldsymbol{\Pi})$, $\hat{\boldsymbol{M}}_2 := (1-\tau)W\boldsymbol{D}_{\boldsymbol{\xi}^y\circ(\boldsymbol{y}\circ\boldsymbol{l})^{1-\tau}\circ\tilde{\boldsymbol{\xi}}^{u,1}\circ u''(\boldsymbol{c})}$.

We obtain using (D.38) and (D.37):

$$\bar{\boldsymbol{\psi}} = \boldsymbol{M}_3\bar{\boldsymbol{\omega}} + \boldsymbol{M}_4\bar{\boldsymbol{\lambda}}_c + \mu\boldsymbol{V}_1, \tag{D.39}$$

where $\boldsymbol{M}_2 := \boldsymbol{I} - \hat{\boldsymbol{M}}_2\boldsymbol{M}_1$, $\boldsymbol{M}_3 := \boldsymbol{M}_2^{-1}(\hat{\boldsymbol{M}}_0 + \hat{\boldsymbol{M}}_2\boldsymbol{M}_0)$, $\boldsymbol{M}_4 := \boldsymbol{M}_2^{-1}\hat{\boldsymbol{M}}_1$, $\boldsymbol{V}_1 := \boldsymbol{M}_2^{-1}(\hat{\boldsymbol{M}}_2\boldsymbol{V}_0 - \boldsymbol{S})$.

Furthermore, equations (D.31), (D.32), and (D.39) imply:

$$\bar{\boldsymbol{\lambda}}_c = \boldsymbol{M}_5\bar{\boldsymbol{\omega}} + \mu\boldsymbol{V}_2, \tag{D.40}$$

where $\tilde{\boldsymbol{R}}_5 := -((\boldsymbol{I}-\boldsymbol{P}) + \boldsymbol{P}(I-\beta R\bar{\boldsymbol{\Pi}})\boldsymbol{M}_4)^{-1}\boldsymbol{P}(I-\beta R\bar{\boldsymbol{\Pi}})$, $\boldsymbol{M}_5 := \tilde{\boldsymbol{R}}_5\boldsymbol{M}_3$, and $\boldsymbol{V}_2 := \tilde{\boldsymbol{R}}_5\boldsymbol{V}_1$.

Substituting (D.39) and (D.40) into (D.34), we deduce:

$$\mu = -\boldsymbol{L}_1\bar{\omega}, \tag{D.41}$$

where $\boldsymbol{C}_1 := \tilde{\bar{\boldsymbol{a}}}^\top(\boldsymbol{V}_1 + \boldsymbol{M}_4\boldsymbol{V}_2) + (\boldsymbol{\xi}^{u,E}\circ u'(\boldsymbol{c}))^\top\boldsymbol{\Pi}\boldsymbol{V}_2$ and $\boldsymbol{L}_1 := (\tilde{\bar{\boldsymbol{a}}}^\top(\boldsymbol{M}_3 + \boldsymbol{M}_4\boldsymbol{M}_5) + (\boldsymbol{\xi}^{u,E}\circ u'(\boldsymbol{c}))^\top\boldsymbol{\Pi}\boldsymbol{M}_5)/\boldsymbol{C}_1$.

We deduce from (D.39) and (D.40):

$$\bar{\boldsymbol{\lambda}}_c = (\boldsymbol{M}_5 - \boldsymbol{V}_2\boldsymbol{L}_1)\bar{\boldsymbol{\omega}}, \tag{D.42}$$

$$\bar{\boldsymbol{\psi}} = \boldsymbol{M}_6\bar{\boldsymbol{\omega}}, \tag{D.43}$$

and from (D.37):

$$\bar{\boldsymbol{\lambda}}_l = \hat{\boldsymbol{M}}_6\bar{\boldsymbol{\omega}}. \tag{D.44}$$

We have defined $\hat{\boldsymbol{M}}_6 := \boldsymbol{M}_0 + \boldsymbol{M}_1\boldsymbol{M}_6 - \boldsymbol{V}_0\boldsymbol{L}_1$ and $\boldsymbol{M}_6 := \boldsymbol{M}_3 + \boldsymbol{M}_4(\boldsymbol{M}_5 - \boldsymbol{V}_2\boldsymbol{L}_1) - \boldsymbol{V}_1\boldsymbol{L}_1$.

Constructing the constraints. The constraint of equation (D.36) becomes after the substitution of the expressions (D.43) of $\bar{\boldsymbol{\psi}}$ and (D.44) of $\bar{\boldsymbol{\lambda}}_l$:

$$\boldsymbol{L}_2\bar{\boldsymbol{\omega}} = 0, \tag{D.45}$$

where:

$$\boldsymbol{L}_2 := \left(\ln(\boldsymbol{y} \circ \boldsymbol{l}) \circ \boldsymbol{\xi}^y \circ (\boldsymbol{y} \circ \boldsymbol{l})^{1-\tau}\right)^\top \boldsymbol{M}_6 \\ + \left((\boldsymbol{1} + (1-\tau)\ln(\boldsymbol{y} \circ \boldsymbol{l})) \circ \boldsymbol{\xi}^y \circ (\boldsymbol{y} \circ \boldsymbol{l})^{1-\tau} \circ \tilde{\boldsymbol{\xi}}^{u,1} \circ u'(\boldsymbol{c})\right)^\top \hat{\boldsymbol{M}}_6.$$

The constraint (D.33) becomes after the substitution of the expressions (D.43) of $\bar{\boldsymbol{\psi}}$ and (D.44) of $\bar{\boldsymbol{\lambda}}_l$:

$$\boldsymbol{L}_3 \bar{\boldsymbol{\omega}} = 0, \tag{D.46}$$

where:

$$\boldsymbol{L}_3 := \left(\boldsymbol{\xi}^y \circ (\boldsymbol{y} \circ \boldsymbol{l})^{1-\tau}\right)^\top \boldsymbol{M}_6 + (1-\tau)\left(\boldsymbol{\xi}^y \circ (\boldsymbol{y} \circ \boldsymbol{l})^{1-\tau} \circ \tilde{\boldsymbol{\xi}}^{u,1} \circ u'(\boldsymbol{c})\right)^\top \hat{\boldsymbol{M}}_6.$$

D.3.5 Constrained weights

Now assume that there is a limited number of K weight, ω^U. Let $\boldsymbol{M}_7$ be a $N_{tot} \times Y$ and $\boldsymbol{I}_{Y \times Y}$ be a $Y \times Y$ identity matrix. Then we have

$$\bar{\boldsymbol{\omega}} = \boldsymbol{D}_{\boldsymbol{S}} \boldsymbol{M}_7 \boldsymbol{\omega}^U,$$

where $\boldsymbol{D}_S$ is the diagonal of the the variable S_{y^N} and $\boldsymbol{M}_7$ will be a matrix identifying the productivity level of each bin.

Finally, the weights for each productivity level are given as a solution of the following minimization problem:

$$\begin{aligned} \min_{\omega} \; & \boldsymbol{S}^{\boldsymbol{P}} \left\|\boldsymbol{\omega}^U - \boldsymbol{1}_Y\right\|^2, \\ \text{s.t. } & \boldsymbol{L}_2 \boldsymbol{D}_{\boldsymbol{S}} \boldsymbol{M}_7 \boldsymbol{\omega}^{U\top} = 0, \\ & \boldsymbol{L}_3 \boldsymbol{D}_{\boldsymbol{S}} \boldsymbol{M}_7 \boldsymbol{\omega}^{U\top} = 0, \\ & \boldsymbol{1}^\top \boldsymbol{D}_{\boldsymbol{S}} \boldsymbol{M}_7 \boldsymbol{\omega}^{U\top} = 1. \end{aligned} \tag{D.47}$$

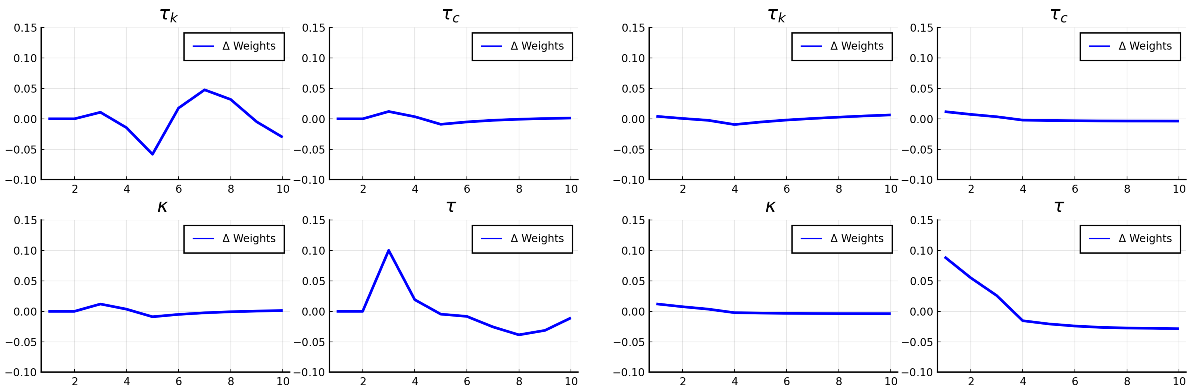
In case we want to estimate the weights for each history we just need to set $\boldsymbol{M}_7 = I$ and solve the previous problem for N^{tot} weights.

E Effects of a marginal change in fiscal systems

In Figure 7 we test the robustness of our estimates through a simple exercise. We increase the government spending to output ratio of United States by 10% and of France in 1%. We then conduct different experiments financing this increase by each one of the instruments (i.e., τ_k, τ_c, κ, and τ) such that the budget constraint of the state is still satisfied. For each one of the instruments we plot the difference in the weights using our estimation strategy discussed previously. Table 8 summarizes the changes in the instruments such that the budget of the state is satisfied after an increase in G/Y. Notice that this increase in G/Y is financed either by an increase in the tax on capital (τ_k), tax on consumption (τ_c), average tax on labor ($1-\kappa$), or progressivity (τ). It is worth noting that finance the increase in G by either τ_c or κ has the same

	US		France	
	Steady state	Increase in G/Y	Steady state	Increase in G/Y
τ_k	0.36	0.387	0.35	0.361
τ_c	0.05	0.076	0.18	0.19
κ	0.85	0.83	0.728	0.72
τ	0.16	0.22	0.23	0.25

Table 8: Changes in the fiscal instruments after an increase in G/Y for United States and France.



(a) United States (b) France

Figure 7: Change in weights by increasing G/Y for United States and France.

effect in terms of general equilibrium, which implies the effect in terms of difference in weights is the same for both experiments. This result is linked to the redundancy result we discussed in Section 6.[17]

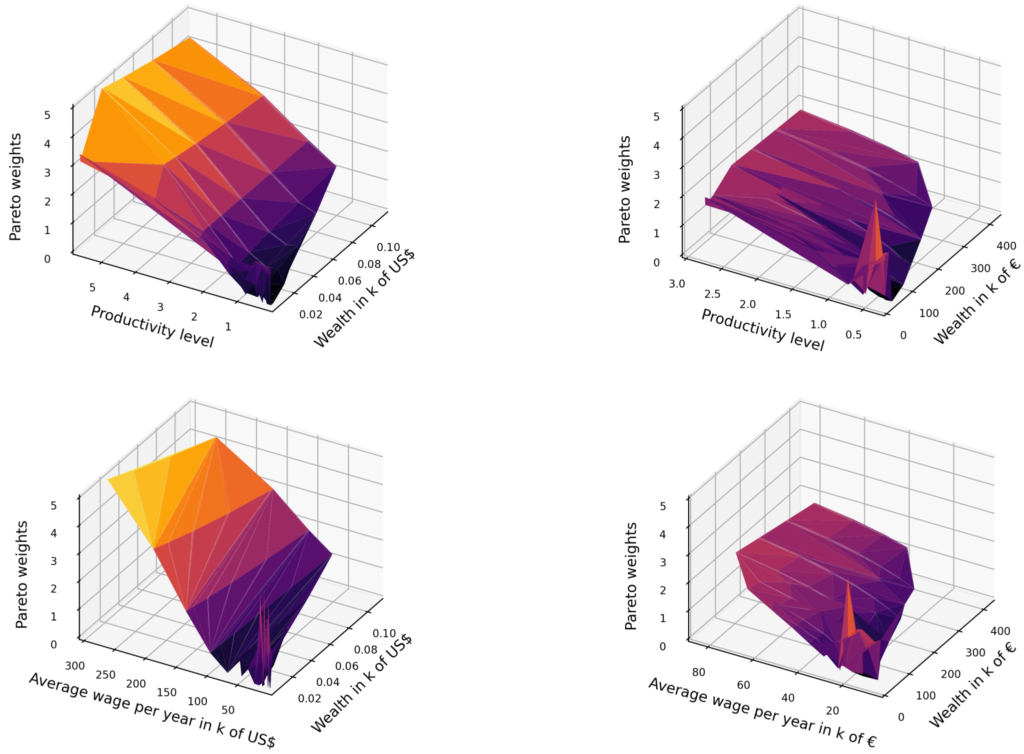
Finally, one can realize that the instrument that leads to the highest change in the weights is the progressivity of the labor tax, which corroborates to the idea that by increasing progressivity we benefit agents with low productivity shocks at the expense of those at the top, although even in this case the period weights keep the same shape as the ones we show in Figure 2. In Appendix H, Figures 20 and 21 illustrate that the shape of the period weights does not change for each experiment under consideration, being the effect almost negligible for the cases where we finance the increase in G/Y by τ_k, τ_c, and κ. This is because the effects in terms of general equilibrium are not drastic and as a result the planner does not alter the weights in a substantial way.

F Weights by History

We calculate the weights by history in Figure 8, where we represent the weights for the US in panel (a) and for France in panel (b) to ease comparison. We do the same exercise by considering the average wage per year of each agent and wealth associated to each history in the national currency of the respective country. This is also represented in Figure 8.[18]

[17] In Appendix H we present the same difference in weights for the parametric estimation, as well as the change in terms of the utility, labor, and capital income for each experiment under consideration. This result is stated in Figure 18 for US and Figure 19 for France.

[18] For the US we obtain the average hourly earnings of all employees from the *FRED* for the year 2007. This value is US$ 20.17 per hour. For France we use the value for 2006 obtained in *Eurostat*, which is also the average



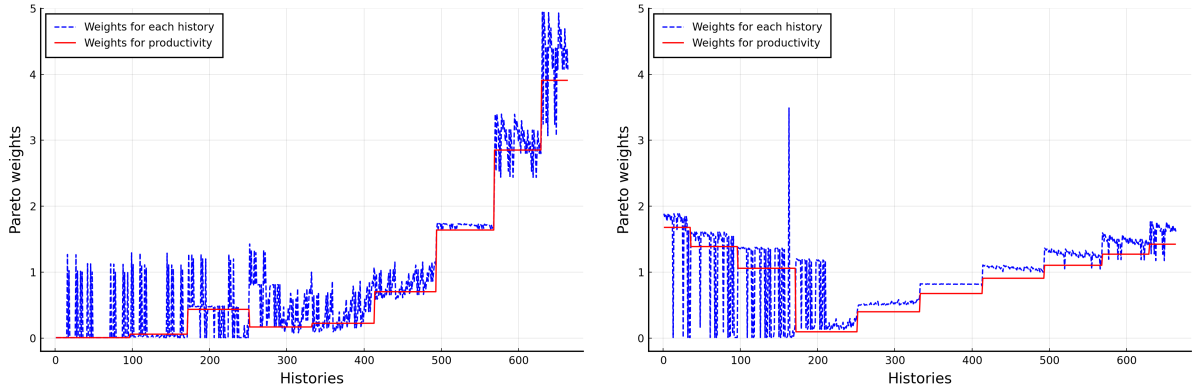
(a) United States (b) France

Figure 8: Pareto weights as a function of productivity (income per capita) and wealth for the US and France.

For the US, one can observe that the weights have a U-shape in the dimension of productivity/income (for low wealth levels) becoming more increasing as the wealth grows, and is slightly increasing in the dimension of wealth (for low productivity/income level) becoming more decreasing as the productivity/income level gets higher (in Appendix G we see the same result for a longer truncation), which means the planner puts more weight to low productivity agents as long as they are not wealthy. The relatively low progressivity tax for labor favors the high-wage and thus high-productivity agents, which implies bigger weights for these agents, however, for agents who are credit constrained the presence of the progressivity favors them and does not cause distortions, which implies the planner gives a higher weight for those agents.

For France, the shape of weights is different from the US one. Although it has also the U-shape format, one can note that it favors more the low productivity agents. The weights are also sharply decreasing with productivity and involves less heterogeneity in weights than the US distribution. This shape for weights in the income dimension result from a higher progressivity

hourly earnings of all employees. The value for France for the year 2006 is € 13.23 per hour. We then obtain the normalized wage using $\tilde{w}_t = \dfrac{\overline{w}\sum_{y^N \in \mathcal{Y}^N} S_{t,y^N}\tilde{l}_{t,y^N}}{\sum_{y^N \in \mathcal{Y}^N} \xi^y_{y^N} S_{t,y^N}(\tilde{l}_{t,y^N} y_{y^N})^{1-\tau_t}}$, where $\overline{w}$ represents the average wage. By doing this procedure we can, hence, calculate the total wage per year as being $\tilde{w}_t(\tilde{l}_{t,y^N} y_{y^N})^{1-\tau}$, where $\tilde{l}_{t,y^N} := 8760 l_{t,y^N}$ with 8760 being the total hours in a year and l_{t,y^N} the fraction of time the agent in this economy spends working. To normalize the wealth we use the fact that $\dfrac{A_t}{Y_t} = \dfrac{K_t}{Y_t} + \dfrac{B_t}{Y_t}$. Notice, we can define $A_t = GDP^i_t(\dfrac{K_t}{Y_t} + \dfrac{B_t}{Y_t})$, where the values of $\dfrac{K_t}{Y_t}$ and $\dfrac{B_t}{Y_t}$ were calculated previously. For both countries we use the GDP data measured at national currency, current prices. The data for the GDP was obtained in the OECD for the year 2007.



(a) United States (b) France

Figure 9: Period weights as a function of history for the US and France.

for the labor tax. As it was the case with the US, the heterogeneity along the wealth dimension is much more limited, although we can see it varies less than in the US case. Notice that for high levels of productivity/income, the weights along the wealth dimension become less heterogeneous. The same can be said when we analyze the evolution of the weights for higher levels of wealth, where we can notice the weights become more homogeneous as we increase the productivity/income. This result corroborates to the idea that the fiscal system in France shows a higher inequality aversion component than the one in the US.

Finally in Figure 9 we plot the weights for the US in panel (a) and France in panel (b) as a function of histories. The blue dashed line represent the history weights, whose computation is detailed in in Appendix D.3.5. In this case we have as many weights as the possible number of histories. In the solid red line we present the same weights using equation (D.47) in Appendix D. Those weights are calculated such that histories with the same productivity in the beginning of the truncation are given the same weight. In this case we will be let with 10 possible weights. The histories are organized in increasing order of the productivity level in the beginning of the truncation, which means the red solid line represents the weights showed previously in Figure 2. One can notice that although there are differences between those two ways to compute the weights, they are globally very similar.

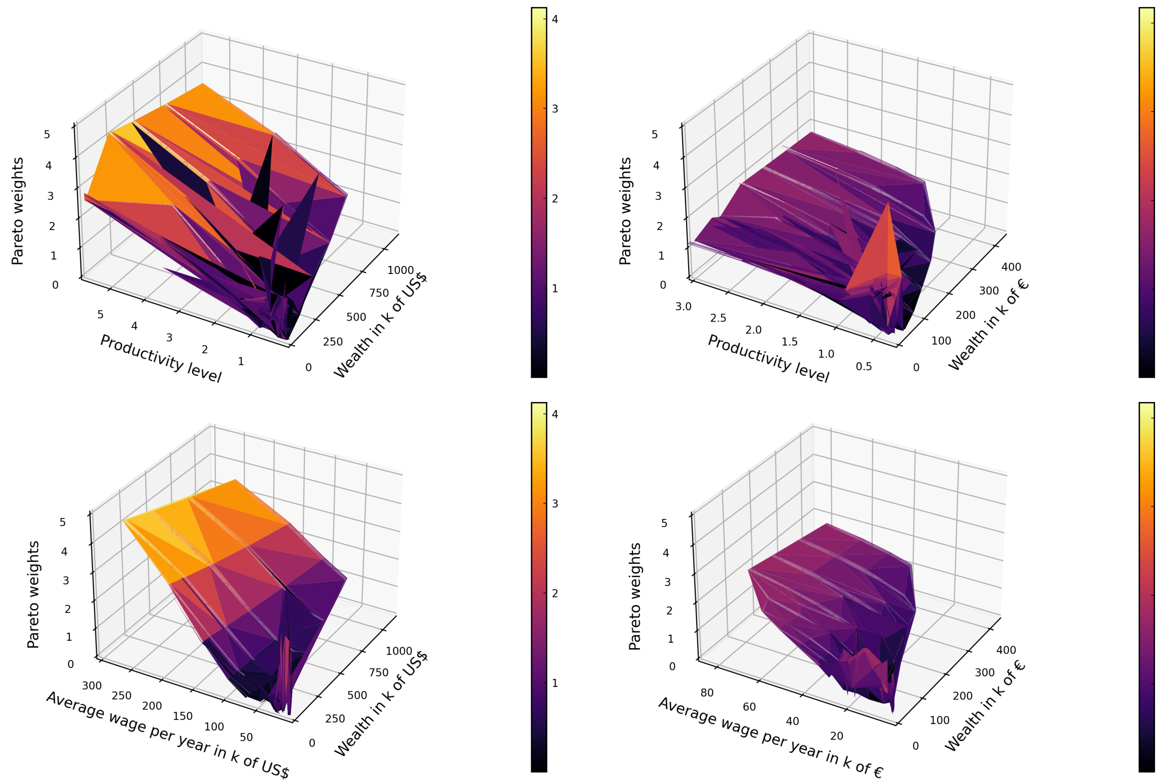
G Consistency of the results

G.1 Truncation length $N = 7$

First, we consider the case where the truncation length is $N = 7$. Figure 10 shows the same result as in Figure 8.

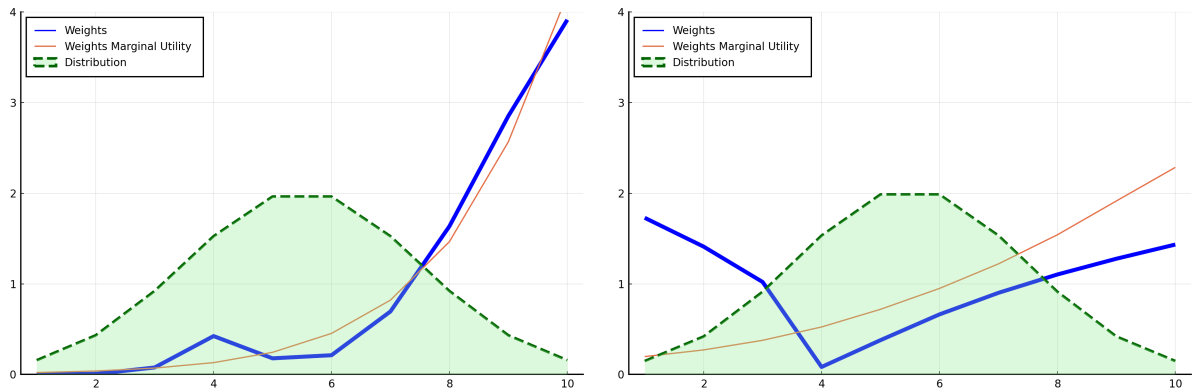
As we did in Figure 2, the weights along the productivity dimension are plotted in Figure 11. We also plot the results along the wealth dimension in Figure 12.

Table 9 shows some summary statistics for the weights in this scenario.



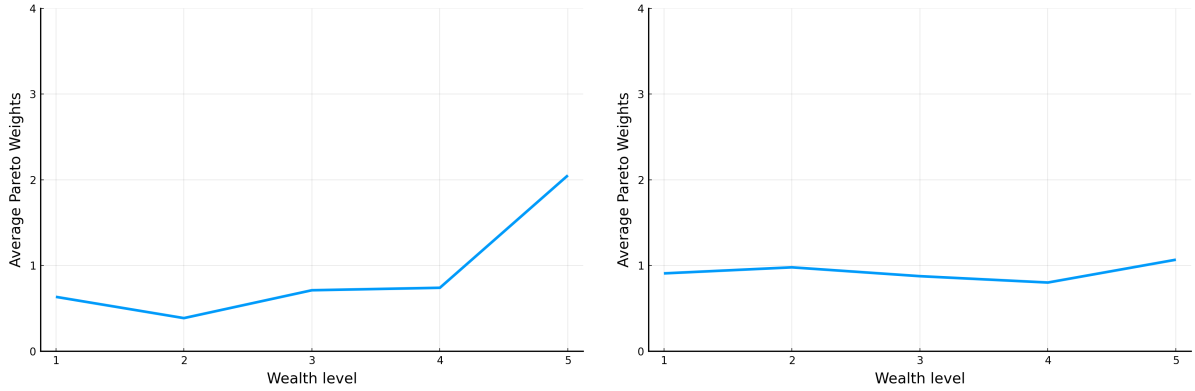
(a) United States (b) France

Figure 10: Period weights as a function of productivity (income per capita) and wealth for the US and France N=7.



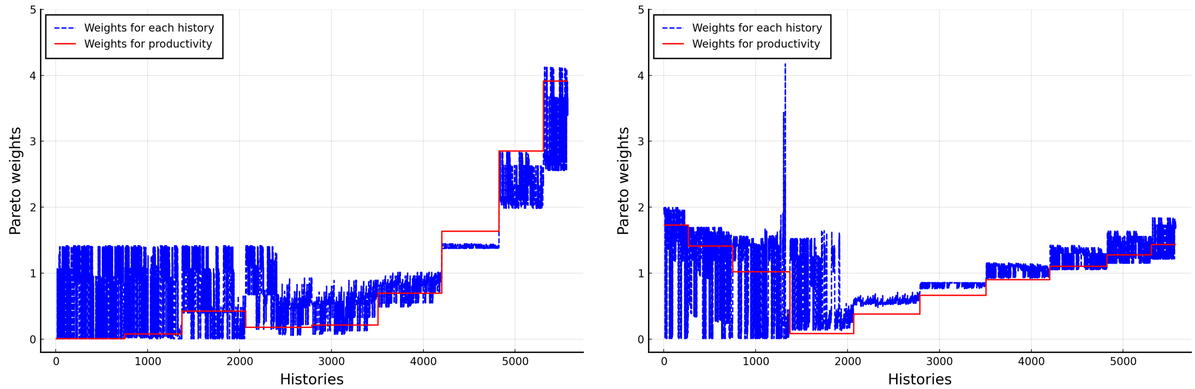
(a) United States (b) France

Figure 11: Period weights as a function of productivity for the US and France N=7.



(a) United States (b) France

Figure 12: Average weights as a function of wealth for the US and France N=7.



(a) United States (b) France

Figure 13: Period weights as a function of history for the US and France N=7.

	USA	**France**
Mean	1.00	1.00
St. deviation	1.37	0.508
Min.	0.006	0.0832
Max.	3.91	1.73
Bottom 10 %	0.006	0.35
Median	0.32	1.06
Top 10%	2.96	1.46

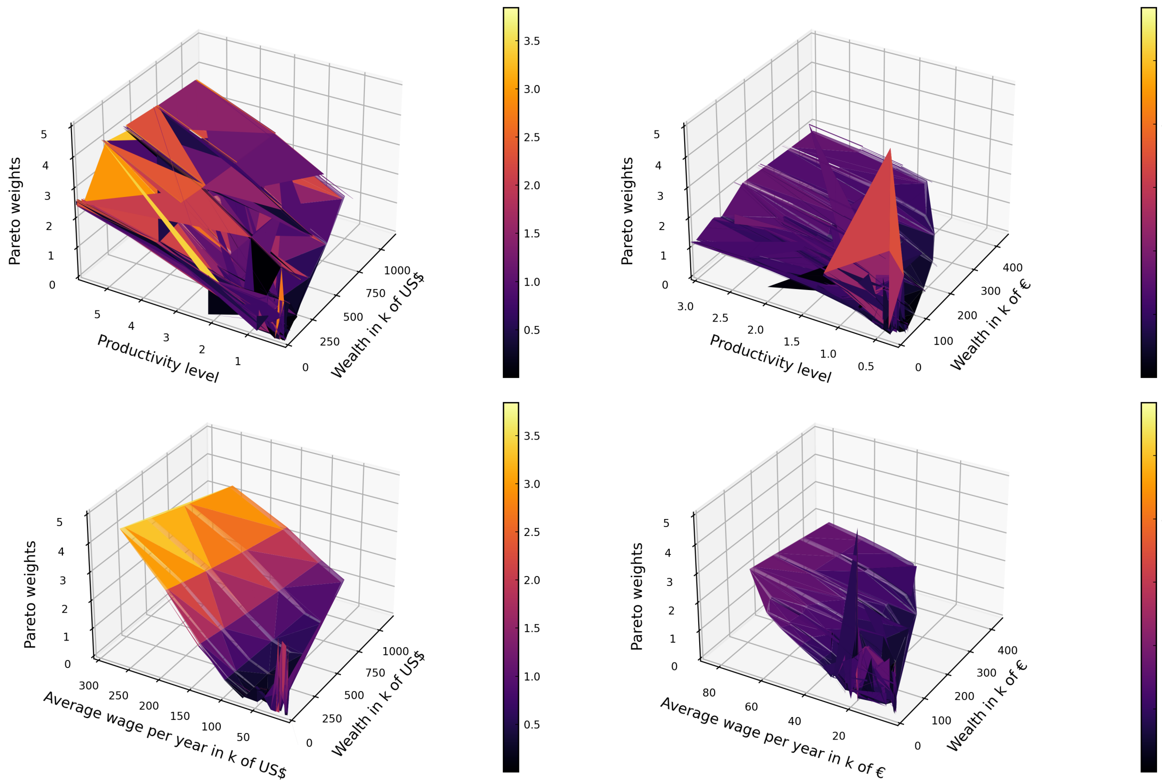
Table 9: Summary statistics for the Weights of the US and France for $N = 7$.

Lastly as in Figure 9, we plot in Figure 13 the weights for the US in panel (a) and France in panel (b) as a function of histories for truncation length of $N = 7$.

G.2 Truncation length $N = 8$

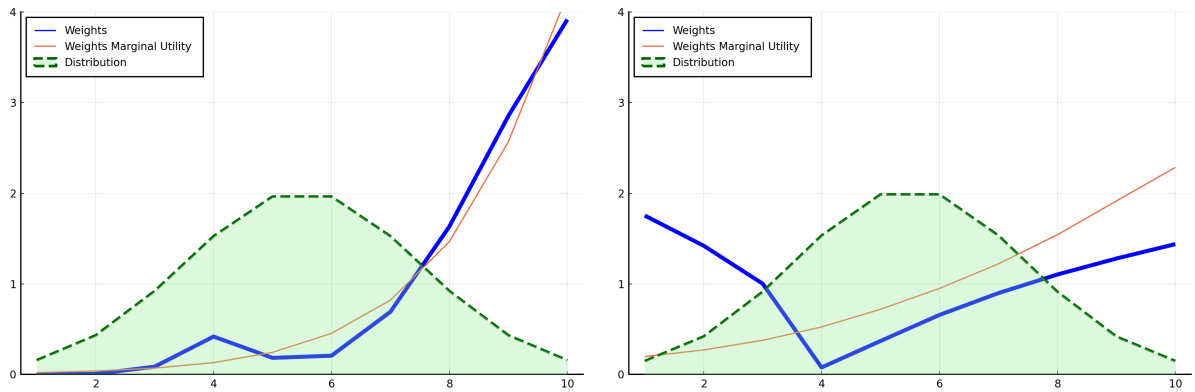
We now consider the case where the number of truncation histories is $N = 8$. Figure 14 shows the same result as in Figure 8.

As we did in Figure 2, the weights along the productivity dimension is plotted in Figure 15. We also plot the results along the wealth dimension in Figure 16.



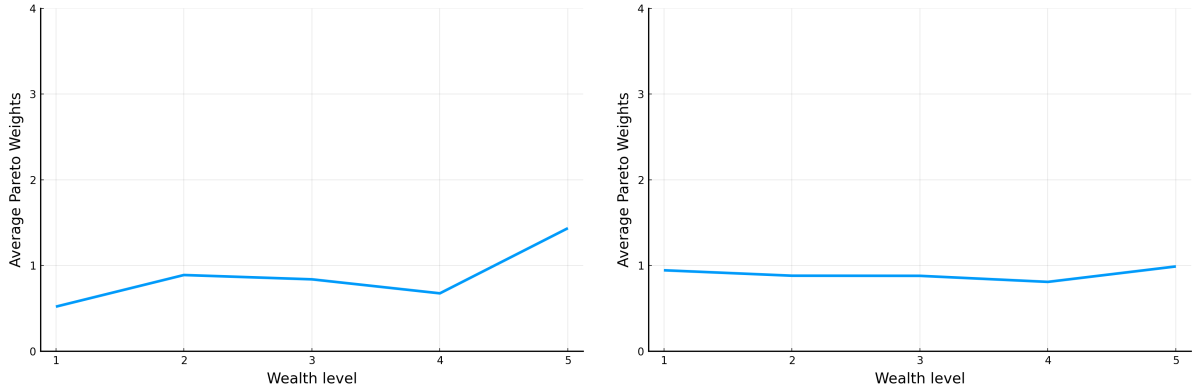
(a) United States (b) France

Figure 14: Period weights as a function of productivity (income per capita) and wealth for the US and France N=8.



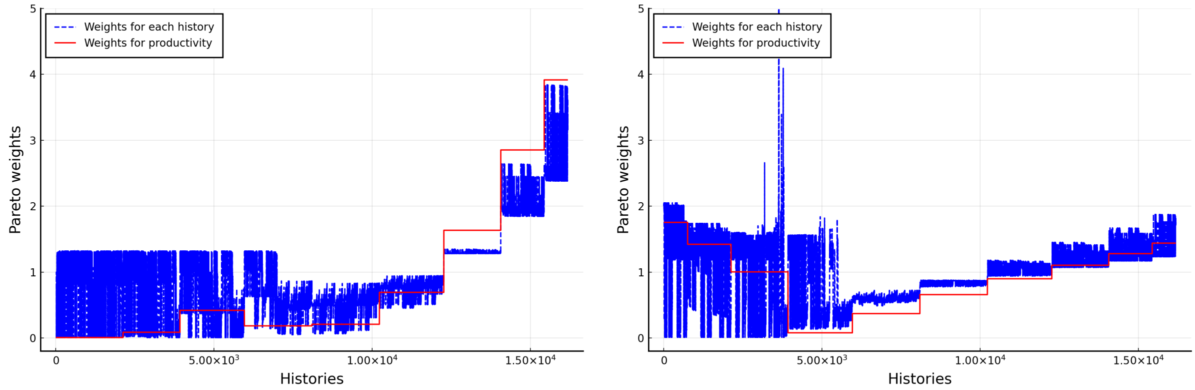
(a) United States (b) France

Figure 15: Period weights as a function of productivity for the US and France N=8.



(a) United States (b) France

Figure 16: Average Period weights as a function of wealth for the US and France N=8.



(a) United States (b) France

Figure 17: Period weights as a function of history for the US and France N=8.

Table 10 shows some summary statistics for the weights in this scenario.

	USA	**France**
Mean	1.00	1.00
St. deviation	1.37	0.517
Min.	0.006	0.0779
Max.	3.92	1.75
Bottom 10 %	0.006	0.34
Median	0.31	1.05
Top 10%	2.96	1.47

Table 10: Summary statistics for the Weights of the US and France N=8.

Lastly, as in Figure 9 we plot in Figure 17 the weights for the US in panel (a) and France in panel (b) as a function of histories for the truncation length $N = 8$.

H Changes in weights by increasing G/Y

In this Appendix we test the robustness of our estimates through a simple exercise. We increase the government spending to output ratio of United States by 10% and of France in 1%. We then conduct different experiments financing this increase by each one of the instruments (i.e., τ_k, τ_c, κ, and τ) such that the budget constraint of the state is still satisfied. For each one of the instruments we plot the difference in the weights using our estimation strategy discussed previously. This result is showed in Figure 7.

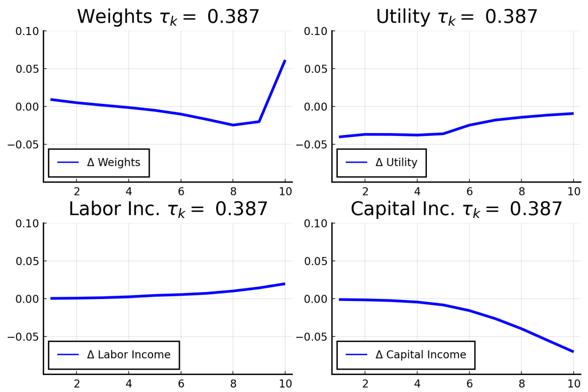
Table 8 shows the changes in the instruments such that the budget of the state is satisfied after an increase in G/Y. Notice that for each one of the cases we need to increase the tax on capital (τ_k), tax on consumption (τ_c), increase the average tax on labor $(1-\kappa)$, and increase the progressivity (τ). It is worth noting that finance the increase in G by either τ_c or κ has the same effect in terms of general equilibrium, which implies the difference in weights is the same for both experiments. This result is linked to the redundancy result we discuss in Section 6. Finally one can observe that the instrument that leads to the highest change in the weights is the progressivity of the labor tax and this result corroborates to the idea that by increasing progressivity we benefit agents with low productivity shocks at the expense of those at the top, although even in this case the weights keep the same shape as the ones showed in Figure 2.

Figure 18 illustrates the changes in weights considering its parametric version, $\log \omega(y) := \omega_0 + \omega_1 \log(y) + \omega_2(\log(y))^2$, as well as the changes in the utility of the agents, labor, and capital income for each experiment explained above. One can notice that as expected the increase in tax on capital leads to a reduction of the capital income, especially for agents with high productivity. This reduction on capital income encourages them to increase the amount of hours worked, which leads to an increase in the labor income. However, this effect is not enough to compensate the lost of revenues arising from the increase in the tax on capital and so the utility for these agents is reduced.

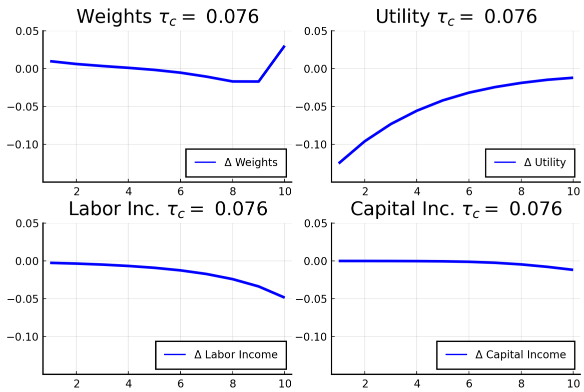
The increase in the tax on consumption has a high effect in the labor income, which is reduced. Notice that the capital income does not change much for all the agents. The reduction in utility for the low productivity agents is higher in this case, probably due to the fact that these agents are credit constrained and cannot compensate the lost in purchase power by relying on assets. The κ has the same effect of tax on consumption. For all the cases analysed so far the changes in the utility of the agents is not so drastic and as a result the planner does not alter the weights in a substantial way.

Finally notice that an increase in progressivity, increases the labor income for the agents with low values of idiosyncratic states and reduce of those with high values. In the latter case, the agents now have less incentives to save, which reduces their capital income. The overall effect is then an increase in the utility of agents at the bottom of income distribution and a decrease for those at the top. This effect is captured by the planner, which increases the weights of the low productivity agents at the expense of the high productivity ones. Figure 19 shows the same case for France and the analysis is similar to the one discussed above for the US case.

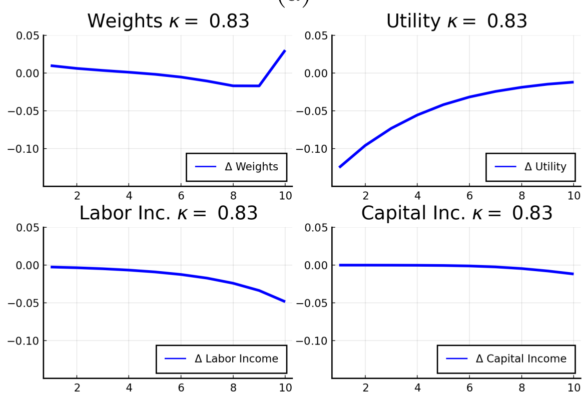
Figure 20 shows the change in the weights for the United States for the experiment we increase the value of G/Y. The blue line represents the weights in the steady state and the red line the experiment conducted above, where the panel (a) designates the weights we obtain after an increase in G financed by tax on capital, panel (b) by tax on consumption, panel (c) by the average tax on labor, and panel (d) by progressivity. Notice that although, the case where we finance the increase in G by increasing the progressivity shows the highest difference in terms of



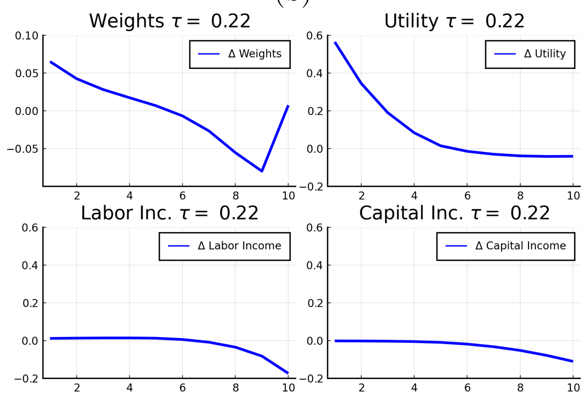
(a)



(b)



(c)



(d)

Figure 18: Difference in parametric weights, utility, labor, and capital income after an increase in G/Y financed by different instruments for United States.

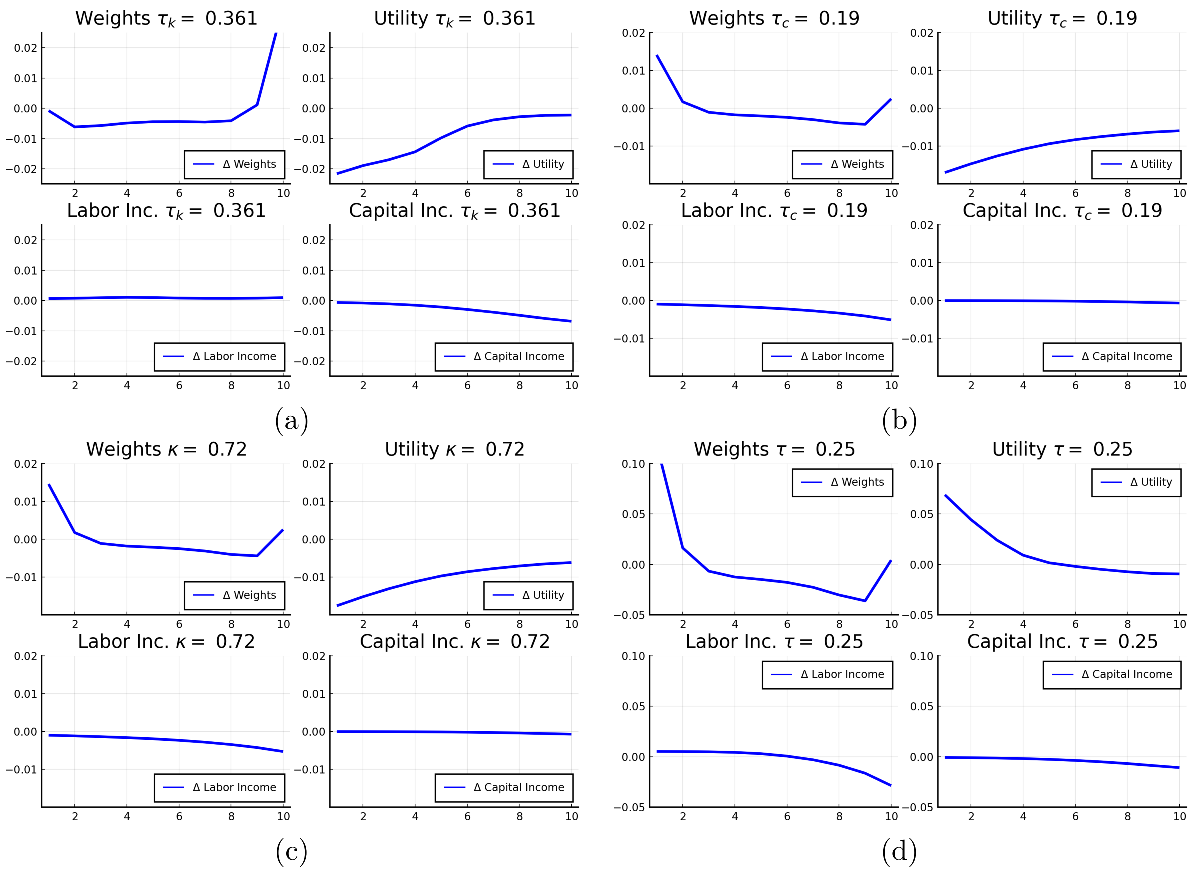


Figure 19: Difference in parametric weights, utility, labor, and capital income after an increase in G/Y financed by different instruments for United States.

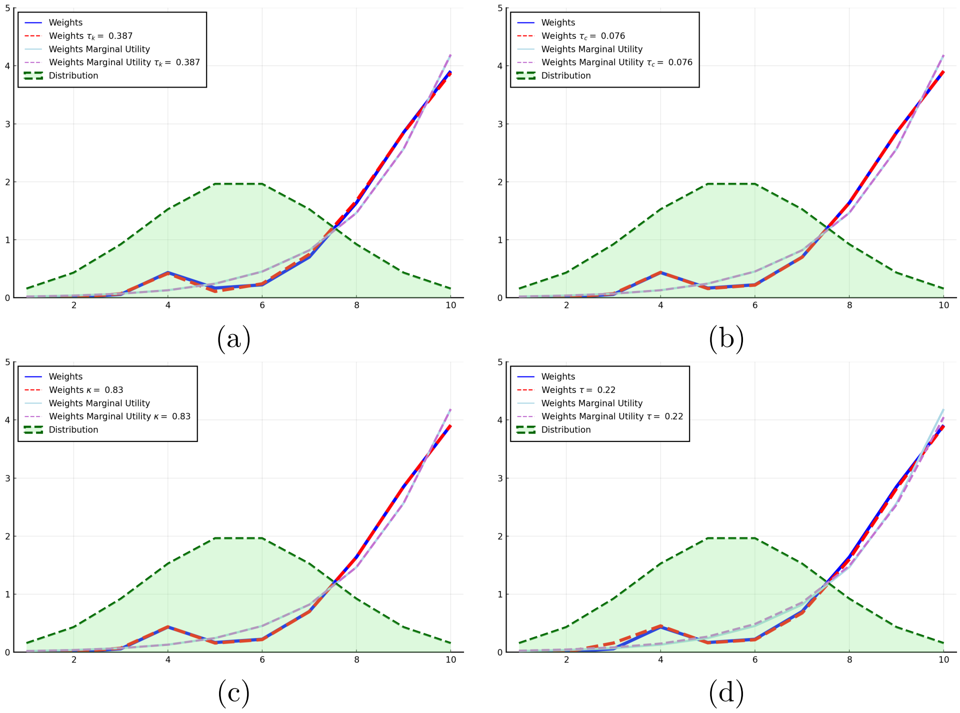


Figure 20: Period weights for United States after and increase in G/Y being financed by different instruments.

the weights, the shape is the same. Figure 21 is this analysis for France.

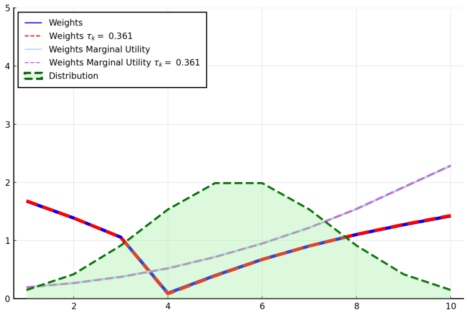
I Changes in the fiscal system

Figure 22 shows how the weights, utility, labor, and capital income change when we alter the fiscal system of each country by the other one under consideration. In panel (a) we show the case where United States adopt the tax system of France and in panel (b) is the case when France adopts the tax system of United States. As expected we can notice the effects occur in opposite directions. The experiment showed in Figure 22 is conducted as follows, once the capital-to-output ratio is set to the value in the steady state, we iterate in the value of κ such that G/Y is kept the same. By running the experiment in this way not only the model parameters are unchanged but also the main macro ratios.

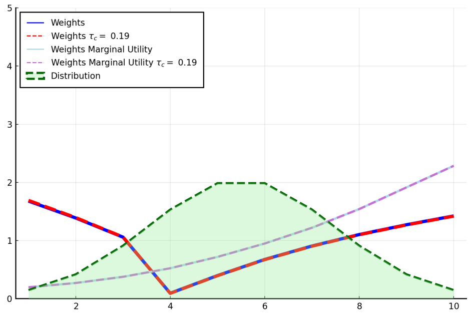
Table 11 shows the new fiscal system for each country under consideration.

	United States	France
τ_k	0.35	0.36
τ_c	0.18	0.05
κ	0.98	0.65
τ	0.23	0.16
B/Y	0.21	0.91

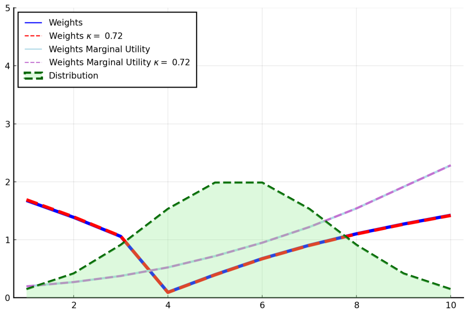
Table 11: Fiscal system for the United States and France under the experiment.



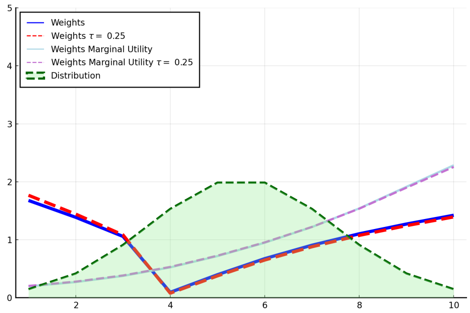
(a)



(b)

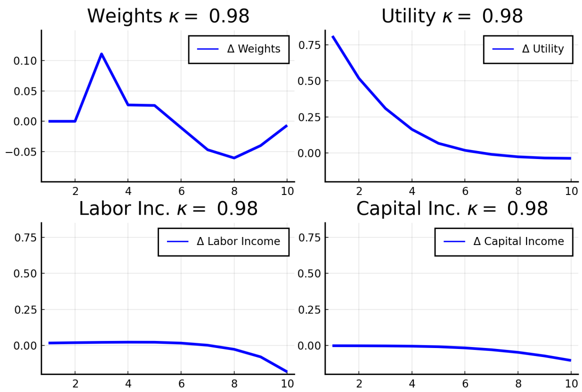


(c)

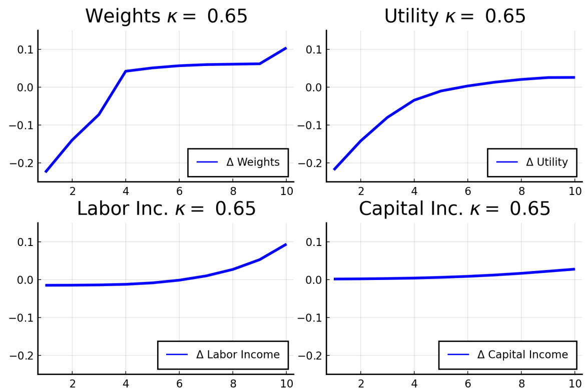


(d)

Figure 21: Period weights for France after and increase in G/Y being financed by different instruments.



(a) United States



(b) France

Figure 22: Difference between the weights between US with the French fiscal system.