HANK's Response to Aggregate Uncertainty in an Estimated Business Cycle Model

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Motivation

- Time-varying uncertainty is key for the study of business cycles and asset prices
- Both aggregate and idiosyncratic uncertainty matter, but are studied in isolation so far
- This is for technical reasons: With expected utility preferences, changes in uncertainty have only second-order effects on utility and choice
- HANK models are typically solved at first-order

This Paper: Shocks to Aggregate + Idiosyncratic Uncertainty

- Develop & estimate two-asset HANK model with time variation in **both**
  aggregate uncertainty about TFP & idiosyncratic income risk
  - aggregate uncertainty = ambiguity, modeled using multiple priors preferences
  - very tractable: ambiguity has first-order effects on utility,
    is reflected in equations for the steady state and linear dynamics

- Uncertainty affects households' savings and portfolio choice, asset prices
- Allow for (first-order) effect of aggregate uncertainty on intertemporal choices by firms:
  e.g. precautionary motives in price & wage setting

This Paper: Main Findings

- Aggregate uncertainty shocks interact with heterogeneity to generate powerful comovement
- Ambiguity about TFP jointly explains more than 60% of cyclical variation in key macroeconomic aggregates as well as in the excess return on capital and the real interest rate
- HANK frictions allow the model to fit investment and excess return on capital much better than a RANK model with aggregate uncertainty shocks
- Mechanism: capital owners' countercyclical substitution away from capital, an asset that is not only uncertain (4.7% premium) but also illiquid (1.4% premium on average)

Ambiguity

Preferences: Ambiguity Aversion

- Exogenous state for household i: vector $s_{i,t} \in S$, with history $s_i^t = (s_{i,1}, ..., s_{i,t}) \in S^t$
- Consumption plan (over goods and leisure) $C_i = C_{i,t}(s_i^t)$
- Recursive multiple-priors utility (Epstein and Schneider, 2003)

$$U_t\left(C_i; s_i^t\right) = u\left(C_{i,t}\left(s_i^t\right)\right) + \beta \min_{p \in \mathcal{P}_t\left(s_i^t\right)} E^p\left[U_{t+1}\left(C_i; s_i^t, s_{i,t+1}\right)\right]$$

- Primitives
  - felicity u (eg. GHH), discount factor β, one-step-ahead belief sets $\mathcal{P}_t\left(s_i^t\right)$
  - larger $\mathcal{P}_t\left(s_i^t\right) \rightarrow$ more ambiguity about $s_{i,t+1}$
  - state dependence of $\mathcal{P}_t\left(s_i^t\right)$ captures e.g. arrival of information
- Why this functional form?
  - worst case belief endogenous – depends on C_i

Ambiguity about Aggregate TFP

- Parameterize one-step ahead belief sets $\mathcal{P}_t\left(s_i^t\right)$ by mean of TFP innovations

$$\begin{aligned} \log Z_{t+1} &= \rho_z \log Z_t + \mu_t + \epsilon^Z_{t+1}; \quad \epsilon^Z \sim i.i.dN(0, \sigma_z) \\ \mu_t &\in [-a_t, a_t] \end{aligned}$$

- Higher a_t $\rightarrow$ larger belief set $\rightarrow$ more ambiguity about TFP in $t+1$
- Stochastic process for a_t:

$$a_t - \bar{a} = \rho_a(a_{t-1} - \bar{a}) + \epsilon^a_t$$

  - long run mean $\bar{a} > 0$, persistence $0 \leq \rho_a < 1$, and $\epsilon^a_t \sim i.i.d\ N(0, \sigma_a)$

Ambiguity in Equilibrium

- Perception of endogenous variables
  - have defined ambiguity about exogenous TFP shocks
  - agents understand law of motion of economy, as usual
  - also perceive ambiguity about wages, returns etc.

- Need to find (endogenous) equilibrium belief together with optimal choices
- This model: worst case belief is always low mean TFP

- Given equilibrium law of motion, characterize path of variables under true DGP

$$\log Z_t = \rho_z \log Z_{t-1} + \epsilon_t^Z$$

Ambiguity and Decision Rules

- Objective of the firm
  - Linearization leads to risk-neutrality without ambiguity
  - Ambiguity is about the mean: *as if* risk-neutral owner with $\mu_t^* = -a_t$
  - all agents share that common belief = objective of the firm well defined
  - precautionary motive in firm's intertemporal decisions
- Correlated wedges: precautionary motive in *all* intertemporal decisions
  - households save & choose portfolios *as if* future expected wages & returns are low
    - interest rate reflects benefit of safety
    - capital premium reflects compensation for uncertainty
  - firms invest & set prices *as if* future expected cost is high

Estimating a two-asset HANK model with aggregate and idiosyncratic uncertainty

Application: A two-asset HANK model (Bayer et al., 2024) ▸ Details

Households		Production	Government
Obtain income	Trade Assets	Produce and differentiate goods	Monetary & fiscal authority
Wages • idiosyn. risk • taxes and transfers • sticky wages **Interest on bonds** • set by monetary authority **Illiquid capital** • earns net MPK **All non-wage rents** • go to rich entrepreneurs			

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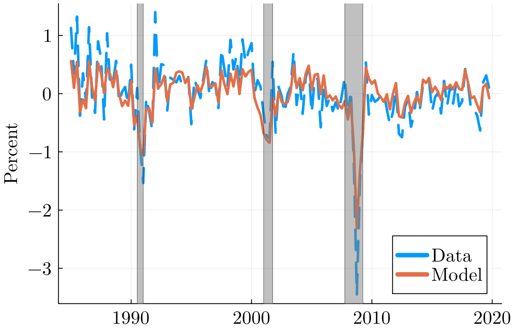
Estimation

- Shocks
  - TFP, ambiguity, idiosyncratic income risk, monetary policy, inflation target

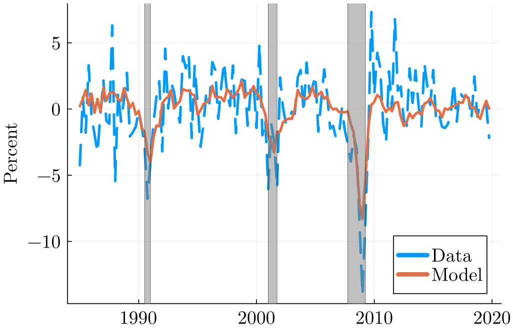
- Infer parameters from joint effect on stochastic steady state & dynamics
  1. Solve steady-state + first-order dynamics with ambiguity about TFP, which yields the ergodic distribution of the model with aggregate uncertainty
  2. Do Bayesian estimation based on dynamics

- Observables (1985-2019)
  - long run averages: wealth/output, liquid/illiquid wealth, capital premium
  - demeaned time series: $\Delta logC, \Delta logH, \Delta logI$, nominal rate, inflation, capital premium

Model Fit



Consumption Growth



Investment Growth

Identification: Long Run

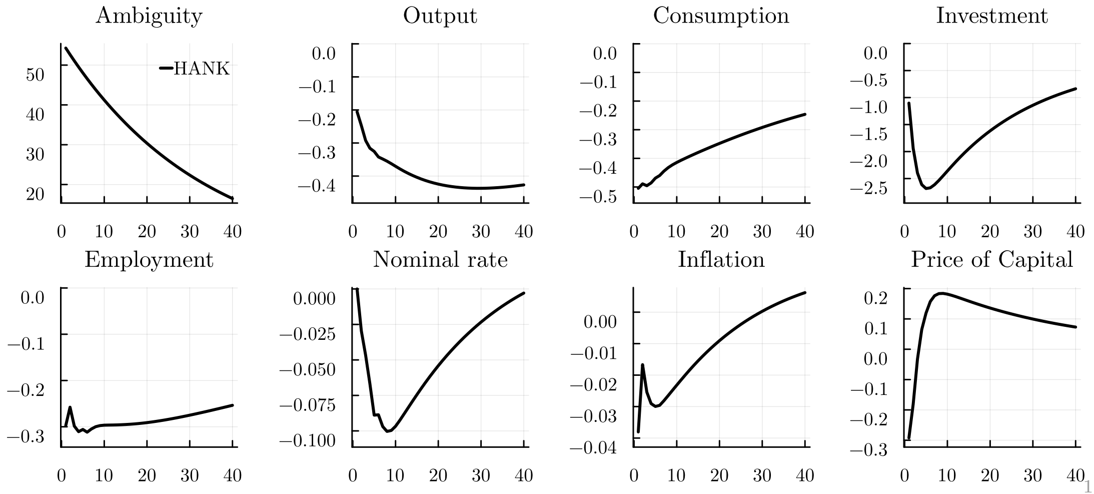
- 6% avg capital premium: compensation for trading friction or uncertainty?
- Both forces generate premium, but different portfolio effects ▸ Portfolios
  - more trading friction shifts portfolio → higher liquid/illiquid wealth ratio
  - more uncertainty increases savings, capital → lower liquid/illlquid wealth ratio
- Results
  - 1.4% premium from trading friction, 4.7% from uncertainty;
    estimated $\underline{Z} = .99$: agents plan as if TFP 1% lower in long run ▸ More
  - trading friction much lower than estimates without ambiguity (5% trading prob.),
    estimation also recovers standard investment adjustment cost values
  - 21% hand-to-mouth households, only 5% if ambiguity is turned off
    → interaction between standard HANK effects & aggregate uncertainty

Identification: Dynamics

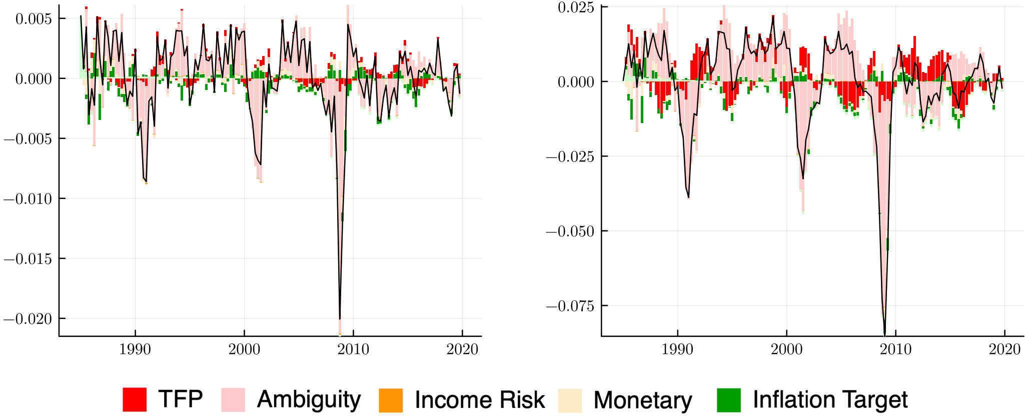
- What drives the business cycle: income risk or ambiguity about aggregate TFP?
- Ambiguity shock activates correlated wedges in recession
  1. lower consumption demand drives down consumption, hours, output, interest rate
  2. lower capital demand drives down investment & price of capital, increases capital premium
  3. cautious price/wage setting: dampens effect on inflation, lower hours
- Income risk shock: more precautionary savings, hard time explaining investment
- Ambiguity shock accounts for bulk of business cycle variation
  - 60% of output, 70% of investment, 80% of consumption, 60% of excess capital returns

Impulse Responses to Ambiguity Shock

Idiosyncratic Risk

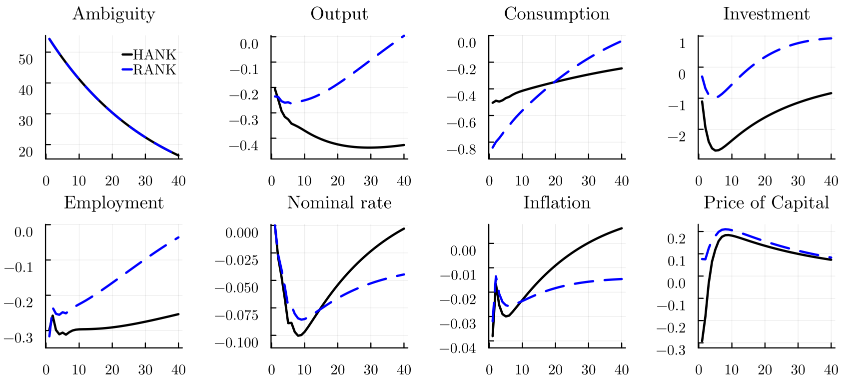


Historical Decomposition of Consumption & Investment



The Role of HANK frictions

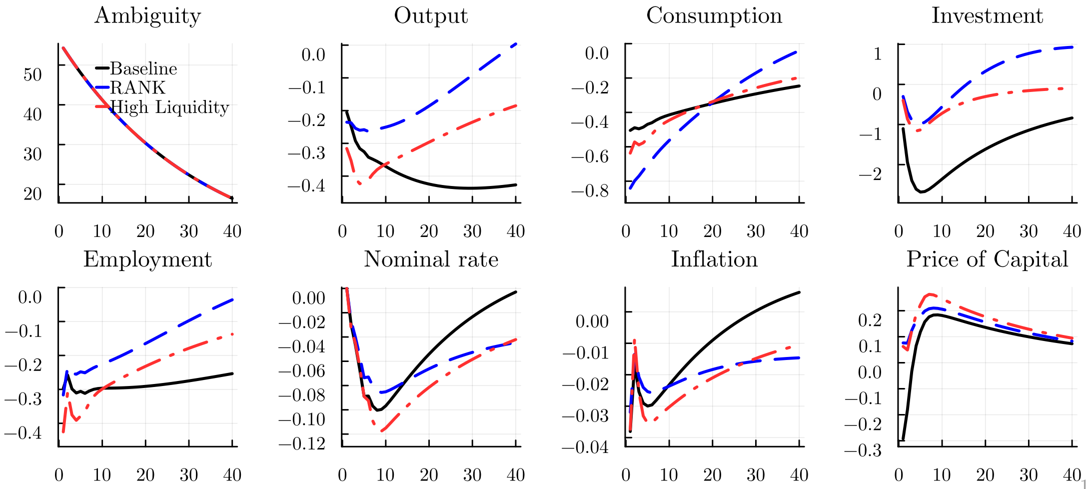
- Counterfactual RANK version: shut down income risk & trading friction
  - RANK model misses volatility in investment & capital premium
  - ambiguity shock explains only 7% of investment (vs 72% in HANK)



The Role of HANK frictions

- Key feature of HANK: portfolio adjustment of the rich
  - rich households hold most capital, drive investment dynamics Cross-section
  - rich have relatively little labor income, try to sell capital
  - ambiguity shock has large effect on investment & capital premium
- RANK representative agent instead worries about both capital return & labor income
  - not just substitution, but also precautionary savings in capital
  - ambiguity shock moves consumption & interest rate, not investment & capital premium
- Liquidity frictions amplify & propagate ambiguity shocks
  - capital less attractive for the rich $\rightarrow$ stronger substitution away after shock

Counterfactual: High Liquidity



The Role of Cautious Price & Wage Setting

- Counterfactual version that shuts down effect of ambiguity on price & wage setting
  - eliminates shifts in price & wage Phillips curves due to ambiguity
  - retains other wedges, e.g. consumption demand

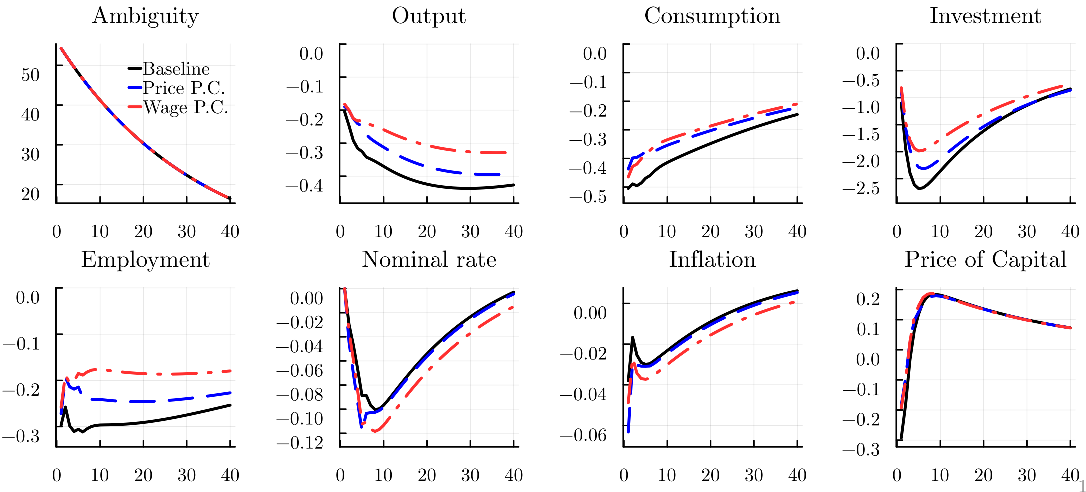
- Right after ambiguity shock, recession with more deflation (about 50%)
  - firms do not worry about future cost increases, set lower prices

- In medium run, higher employment (about 50%)
  - firms do not worry, set lower wages, hire more workers

- Ambiguity in Phillips curves dampens deflationary effect from low demand

More

Counterfactual: No Ambiguity in Phillips Curves



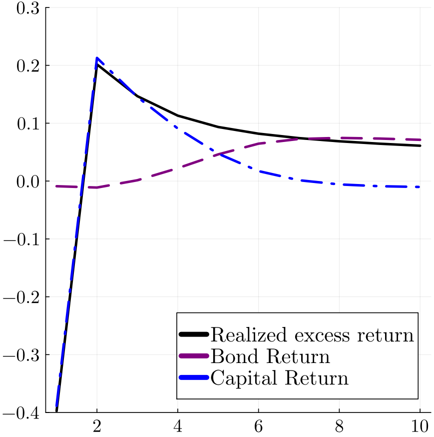
Asset Pricing

- After ambiguity shock price of capital falls, then recovers
  - predictably high capital premium in recessions, as in data

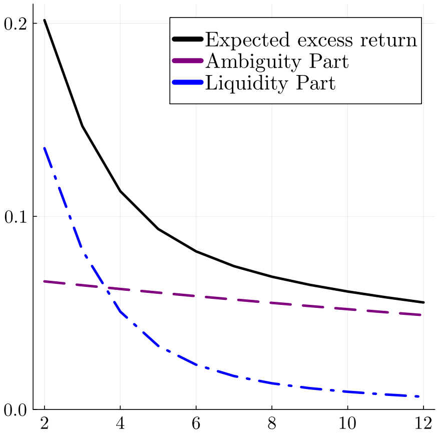
- Why is excess return on capital predictably high after recessions?
  - short run: compensation for trading frictions as investors try to sell capital
  - medium run: compensation for aggregate uncertainty

- HANK frictions are crucial
  - concentrated ownership + liquidity component of premium spikes up after shock
  - capital premium flat in RANK counterfactual

Results: Capital Premium

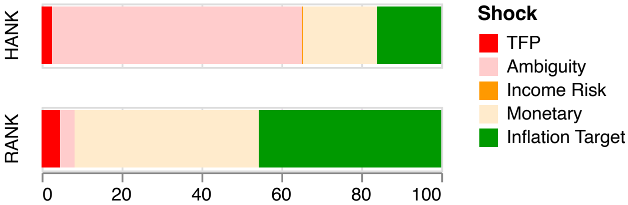


Price Decomposition



Mechanism Decomposition

Results: Variance Decomposition



Capital Premium $\frac{r+q}{q^-} - \frac{RB}{\pi}$

Conclusion

Conclusion

HANK's response to aggregate uncertainty is key for short- and long-run outcomes

- Capital premium mainly reflects compensation for aggregate uncertainty, but illiqudity amplifies volatility
- Aggregate uncertainty generates HtM households with less portfolio frictions

HANK frictions amplify the business cycle consequences of aggregate uncertainty

- Ambiguity about TFP jointly explains more than 60% of cyclical variation
- Strong substitution distinguishes aggregate from idiosyncratic uncertainty shocks

Additional Slides

Utility Specification

- Utility function:

$$u\left(c_{it} - h_{it}\frac{n_{it}^{1+\gamma}}{1+\gamma}\right)$$

- FOC wrt labor supply yields:

$$h_{it}G'(n_{it}) = w_t h_{it}.$$

- Demand for x_{it} can be rewritten as:

$$x_{it} = c_{it} - h_{it}G(N_t) = c_{it} - \frac{w_t h_{it} N_t}{1+\gamma}.$$

Household Optimization Problem

- Value function V^a for the case where the household adjusts its capital holdings, the function V^n for the case in which it does not adjust, and the expected continuation value, $\mathbb{W}$, over both,

$$V_t^a(b,k,h) = \max_{b'_a, k'} u[x(b,b'_a,k,k',h)] + \beta \mathbb{E}_t \mathbb{W}_{t+1}(b'_a,k',h') \ ,$$
$$V_t^n(b,k,h) = \max_{b'_n} u[x(b,b'_n,k,k,h)] + \beta \mathbb{E}_t \mathbb{W}_{t+1}(b'_n,k,h') \ ,$$
$$\mathbb{W}_{t+1}(b',k',h') = \lambda V_{t+1}^a(b',k',h') + (1-\lambda) V_{t+1}^n(b',k',h') \ .$$

- Expectations about the continuation value are taken with respect to all stochastic processes conditional on the current states. Maximization is subject to the corresponding budget constraint. The distribution Θ_t then evolves according to

$$\Theta_{t+1}(b',k',h') = \lambda \int_{b'=b^*_{a,t}(b,k,h),k'=k^*_t(b,k,h)} \Phi(h,h') d\Theta_t(b,k,h)$$
$$+ (1-\lambda) \int_{b'=b^*_{n,t}(b,k,h),k'=k} \Phi(h,h') d\Theta_t(b,k,h) \ .$$

Embedded in an otherwise standard NK model

- Factor Prices equal marginal products

$$w_t^F = \alpha mc_t Z_t \left(\frac{u_t K_t}{N_t}\right)^{1-\alpha}, \qquad r_t^F + q_t^F \, \delta(u_t) = u_t(1-\alpha) mc_t Z_t \left(\frac{N_t}{u_t K_t}\right)^{\alpha},$$

where $\delta\left(u_t\right) = \delta_0 + \delta_1\left(u_t - 1\right) + \delta_2/2\left(u_t - 1\right)^2$

- Capital Price equals costs of production of capital

$$\begin{aligned} 1 =& q_t^F \left[1 - \frac{\phi}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 - \phi\left(\frac{I_t}{I_{t-1}} - 1\right)\frac{I_t}{I_{t-1}}\right] \\ &+ \beta q_{t+1}^F \phi\left(\frac{I_{t+1}}{I_t} - 1\right)\left(\frac{I_{t+1}}{I_t}\right)^2 \end{aligned}$$

Back

Embedded in an otherwise standard NK model

- Phillips Curve under quadratic price adjustment costs

$$\log\left(\frac{\pi_t}{\bar{\pi}}\right) = \beta E_t\left[\left(\frac{\pi_{t+1}}{\bar{\pi}}\right)\frac{Y_{t+1}}{y_t}\right] + \kappa_y\left(mc_t - \frac{1}{\mu_t^y}\right)$$

- Wage Phillips Curve under quadratic price adjustment costs

$$\log\left(\frac{\pi_t^w}{\bar{\pi}^w}\right) = \beta E_t\left[\left(\frac{\pi_{t+1}^w}{\bar{\pi}^w}\right)\frac{N_{t+1}w_{t+1}^F}{N_t w_t^F}\right] + \kappa_w\left(\frac{w_t}{w_t^F} - \frac{1}{\mu_t^w}\right)$$

Back

Government

- Monetary policy follows Taylor rule

$$\log \frac{R^b_{t+1}}{\bar{R}^b} = \rho_{TR} \log \frac{R^b_t}{\bar{R}^b} + (1-\rho_{TR})\theta_\pi \log \frac{(\prod_t^{t-3} \pi_t)^{1/4}}{\bar{\pi}_t} + (1-\rho_{TR})\theta_y \log \frac{Y_t}{\bar{Y}} + \varepsilon_t^R$$

- Government debt accumulation rule as in Woodford (1995):

$$\Delta \log B_{t+1} \quad = \quad \gamma_B \log \frac{B_t}{\bar{B}} + \gamma_Y \log \frac{Y_t}{\bar{Y}}$$

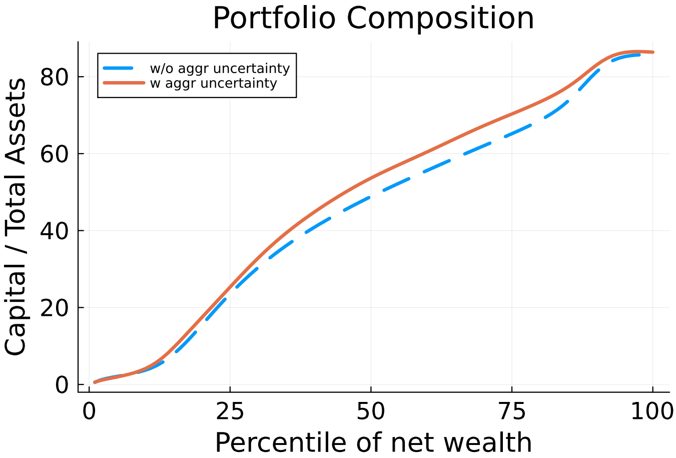
- Government spending determined by government budget constraint

$$G_t = B_{t+1} + T_t - R^b_t B_t / \pi_t \ ,$$

where $T_t = \tau(N_t w_t + \Pi_t^U + \Pi_t^F)$

Back

Household Portfolio Heterogeneity



Back

Magnitude of Ambiguity

- Steady state ambiguity
  - Estimate $\underline{Z} = .99$: agents plan as if TFP 1% lower in long run
  - Implied one-step ahead ambiguity:

$$\bar{a} = (1 - \underline{Z})(1 - \rho_z) = 0.13\%$$

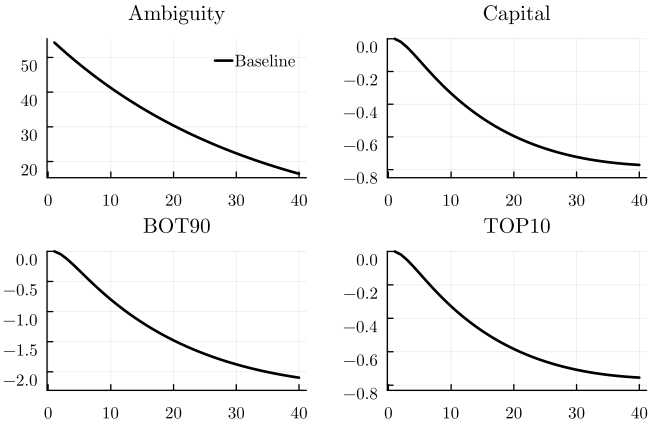
- Model-consistency bound on a_t: a consistency criterion (Ilut and Schneider, 2014)

$$\bar{a} + 2\frac{\sigma_a}{\sqrt{1 - \rho_a^2}} \leq 2\sigma_z$$

  - do not entertain forecasts outside a 95% confidence interval, centered around the long-run mean of $\log Z_{t+1}$, given its observed variation
  - bound holds given estimates

Back

Heterogeneous Capital Responses to Ambiguity Shock



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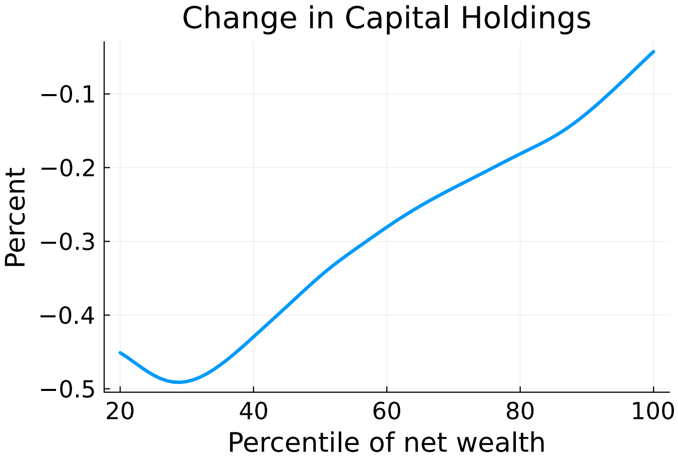
Results: Heterogeneous Capital Responses to Ambiguity Shock



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Results: Impulse Responses to Idiosyncratic Income Risk Shock

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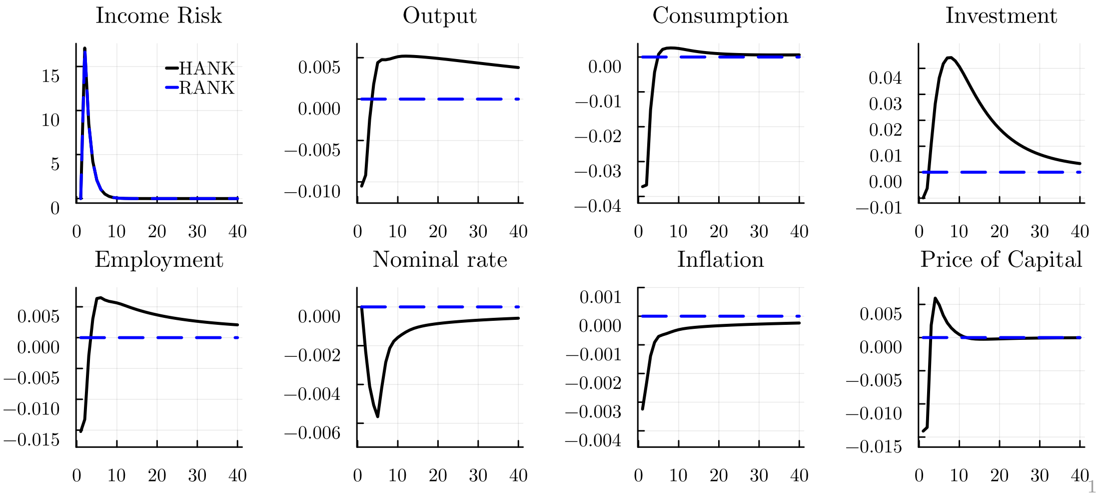
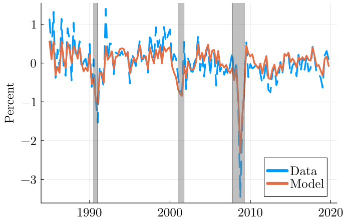
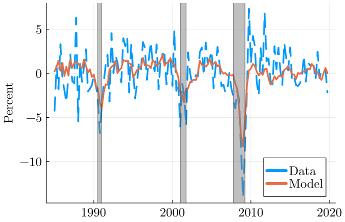


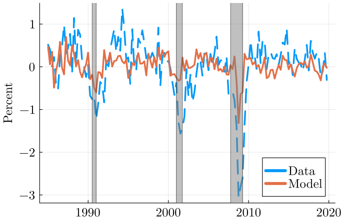
Results: Model Fit Quantities



Consumption Growth

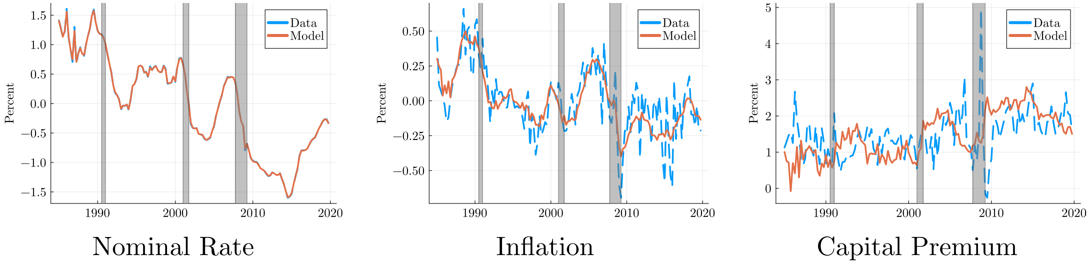


Investment Growth



Hours Growth

Results: Model Fit Prices



Counterfactuals: Shutting Down Ambiguity for Selected Decisions

- Diagnose mechanism by counterfactuals where some decisions do not react to ambiguity:
  - in steady-state all agents use the same worst-case belief
  - but away from it, some agents may not respond to time-varying ambiguity
- For example, role of ambiguity in setting goods prices:
  - In Phillips Curve: conditionally RE belief over future inflation

$$\mathbb{E}_t \widehat{\pi}_{t+1} = \mathbb{E}_t^* \widehat{\pi}_{t+1} + \varepsilon_{\pi z} a_t$$

  - undo the role of effective current pessimism about future TFP ($\mu_t^* = -a_t$)
  - over future inflation, with $\varepsilon_{\pi z}$ the original equilibrium elasticity of inflation w.r.t. TFP

▸ Back

Firm Problem under Ambiguity

- The firms' one-step ahead SDF

$$M_{t+1} = \beta \xi_{t+1} \widetilde{M}\left(s_t, s_{t+1}\right)$$

  - $\widetilde{M}\left(s_t, s_{t+1}\right) \equiv \sum_i \alpha_{i,t} \widetilde{m}_i\left(s_t, s_{t+1}\right)$, with $\alpha_{i,t}$ arbitrary weights st $\sum_i \alpha_{i,t} = 1$.
  - Standard: agent specific risk-based $\widetilde{m}_i\left(s_t, s_{t+1}\right)$ and common β
  - Ambiguity: Common ξ_{t+1} ("change of measure"): $E_t\left[\xi_{t+1}\right] = 1$ such that for any Y_{t+1}

$$E_t^*\left[Y_{t+1}\right] = E_t\left[\xi_{t+1} Y_{t+1}\right]$$

- Log-linearizing firms' optimal decisions around worst-case steady state:
  - Risk-based component $\widetilde{M}\left(s_t, s_{t+1}\right)$ does not matter to first order
  - Ambiguity about the means does, through the common change of measure

    $\Rightarrow$ *as if* risk neutral owner under worst-case beliefs

Back