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Abstract

We show how legal uncertainty can enable simple legal standards to produce socially useful differentiation in incentives that better accommodates heterogeneity. First, legal uncertainty smooths out the discontinuities in incentives that coarse legal standards would otherwise produce. Second, individuals rationally form beliefs about what legal standards require in part by projecting their own circumstances. We apply our analysis to a range of issues in legal design, including the optimal degree of legal complexity, the choice between rules and standards, and the choice between "sanctions" and "prices."

Keywords: differentiation, legal uncertainty, standards, reasonable person, false consensus
JEL codes: D02; D83; K10; K13, K40

1 Introduction

A core challenge for the design of legal rules meant to channel individual behavior is to provide differentiated incentives that accommodate heterogeneity. This challenge is particularly acute for legal strategies involving "sanctions," which combine a legal norm for what behavior is required with a penalty for noncompliance (Cooter, 1984). Sanctions

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strategies like the negligence rule are a highly centralized mode of legal ordering—they involve the state micromanaging individuals by specifying behavioral norms—and therefore require the state to have a great deal of information about socially appropriate behavior in a myriad of circumstances. In contrast, pricing strategies like strict liability are much more hands off, letting individuals make their own choices so long as they pay for any resulting external costs. Because of their informational demands, sanctions strategies in practice inevitably differentiate less than would an ideal system. To give one prominent example, the reasonable person standard in tort law is generally understood to be an objective standard, based on the care that would be reasonable for the "average person," taking only limited account of variation in individuals' abilities, costs, and other characteristics.[1]

A further barrier to explicit legal differentiation stems from uncertainty among individuals subject to the regime about the precise legal consequences that would follow from alternative courses of action. Such uncertainty is endemic and arises for a range of reasons, including incomplete information available to the law enforcement system as well as individuals' lack of information about the substantive content of law. Even if the state's information costs are low, and it can devise a finely tailored set of legal rules, such a regime will not achieve the desired results if individuals do not learn the rules (Kaplow and Shavell, 1996). Further, even when the expected legal standard of behavior is equal to the socially optimal level of behavior, legal uncertainty can distort the incentives produced by law (Craswell and Calfee, 1986). On the one hand, legal uncertainty provides individuals with an incentive to "over-comply" with the expected standard of behavior to reduce the chance of being sanctioned. On the other hand, high levels of uncertainty can dilute legal incentives, resulting in under-compliance.

In this paper, we develop a very different perspective on legal uncertainty, showing how it can serve as a valuable lubricant for the legal system. Legal uncertainty can enable

[1] Restatement (Second) of Torts § 283; van Dam (2014).

simple legal standards to harness decentralized information by producing differentiated incentives that accommodate heterogeneity without incurring the costs of explicit legal differentiation. Somewhat paradoxically, the key problem that results from the informational burden faced by the state in using the more centralized "sanctions" mode of legal ordering—namely the inevitable use of non-differentiated legal standards of behavior—can in turn be ameliorated by the lack of information of individuals about how such a system of sanctions functions. Put more simply, the incompleteness of individuals' information in navigating the legal order can mitigate the incompleteness of the state's information in designing the legal order.

Legal uncertainty enables simple legal standards to produce more differentiated incentives through two basic mechanisms, which we call the *smoothing channel* and the *projection channel*. The smoothing channel operates by smoothing out the discontinuities in incentives that coarse behavioral standards would otherwise produce. The projection channel operates through individuals forming rational beliefs about the substantive content of simple legal standards based in part on their own circumstances.

We develop this perspective by extending the canonical economic model of the reasonable person standard of tort law (Brown, 1973; Shavell, 1987; Landes and Posner, 1987). In the standard setup, potential injurers vary in their costs of care and face a negligence rule based on a common standard of care. That legal rule results in a discontinuity in injurers' liability costs at the standard of care; at levels of care below the standard of care, injurers are responsible for the full cost of any accidents they cause, whereas at levels of care equal to or above the standard of care, they face no liability. In turn, this discontinuity in liability costs produces a discontinuity in injurers' private marginal benefits of care. As a result, in the absence of legal uncertainty, injurers with a wide range of costs of care bunch at the level of due care in order to avoid liability. The optimal common standard of care balances the costs of requiring too great a level of care from some against the costs of requiring too little care from others. The reason the law does not individualize the standard of care is understood to be due to "the impossibility of nicely measuring a man's powers and limitations" (Holmes, 1881), i.e., information costs.

When potential injurers are uncertain about the liability they face, however, they are provided much more differentiated incentives. First, consider the case in which, from the perspective of injurers, the legal system is subject to what we might call "legal noise": random errors in the finding of liability that are orthogonal to the characteristics of any particular case. This noise could be due to a range of sources, including injurers' lack of information about what legal standard will be used to evaluate their level of care and measurement error in courts' assessments of their level of care. As a result, there is no longer a level of care at which injurers' expected marginal liability costs fall discontinuously to zero, which is what causes bunching at the level of due care in the absence of uncertainty. Rather, under legal noise, individuals' expected marginal liability costs vary smoothly with their level of care so that individuals with higher costs of care will choose lower levels of care. As a result of this smoothing channel, legal uncertainty can increase the efficiency of, rather than undermine, the incentives produced by a sanctions-based legal regime.

The second mechanism, which we call the projection channel, functions through the formation of individuals' beliefs about the legal consequences of alternative courses of action. Such beliefs are the proximate cause of the incentive effects of law on individual behavior. When we say that individuals are deterred from engaging in some socially destructive behavior, what we mean is that individuals do not engage in the behavior because they believe that they face a less attractive lottery over legal consequences if they do so than if they do not. It's the individuals' beliefs about the law that are doing the incentive work, not the law *per se*. If this point seems pedantic, that's only because under the dominant paradigm in the economic analysis of law, systematic variation in legal beliefs is driven only by variation in the actual legal treatment of cases. But we identify a simple mechanism that, in the absence of explicit legal differentiation, leads Bayesian individuals to form beliefs about what the law requires that are correlated with what the law would ideally require from each of them.

In particular, consider again the canonical model of the reasonable person standard where due care is set at the optimal level of care for an injurer with cost of care equal to the average cost in the population. Suppose that potential injurers know that they are required to take reasonable care but they do not know what is reasonable. They thus form Bayesian beliefs about the level of due care based on their information about the distribution of costs of care in the population. To keep things simple, suppose injurers have a common prior about the cost distribution and that each has a single draw from the distribution, namely, their own cost of care. As a result, injurers with lower costs of care believe that the average cost of care in the population is lower—and therefore that the level of care required under the reasonable person standard is higher—than do injurers with higher costs of care. This differentiation in beliefs about the substantive content of simple legal standards in turn can produce socially useful differentiation in behavior.

At the heart of the projection mechanism is a sort of parochiality of even rational, Bayesian individuals. Much of the information individuals have about the human condition stems from their own capacities, preferences, experiences, and opportunities. They naturally and rationally form beliefs about the situations of other individuals in society based in important part on their own situation. In a sense, they *overgeneralize* (rationally!) from their own experiences. That's what Bayesian updating requires, at least to some extent. Note, moreover, that most people spend their lives interacting primarily with a subset of their national community that is more like themselves than the rest, a phenomenon sociologists refer to as "homophily" (Lazarsfeld and Merton, 1954). The beliefs of individuals about others in society in turn inform their beliefs about the content of simple legal standards, like "reasonableness." Since their beliefs about others' capacities, preferences, situations, and so forth are shaped by their own self-knowledge, they will typically believe that the law is closer to what is appropriate for themselves than it in fact is.

These two channels through which legal uncertainty can produce differentiated incentives—the smoothing channel and the projection channel—can operate together, as in the reasonable person standard, or on their own. The smoothing channel is operative only when there would be discontinuities in marginal legal incentives in the absence of uncertainty and even when all individuals have the same beliefs about the law. In contrast, the projection channel is operative only when an individual's own type is informative about the content of the relevant legal standards, which can occur even when there are no discontinuities in marginal legal incentives.

These ways legal uncertainty can enable simple standards to produce socially useful differentiation in incentives arguably help explain the abiding role of such standards in modern legal systems. The statute books and the Code of Federal Regulations certainly grow longer by the year, detailing ever more precisely the commands of law. One might think that it is inevitable that the broad standards of conduct characteristic of the common law approach will become obsolete and give way in the face of this rising tide of positive law. And yet simple—and vague—standards endure. They remain central to many bodies of law, from the fiduciary duties of corporate officers and directors, to the Sherman Act's prohibition on "contracts, combinations, and conspiracies in restraint of trade," to the obligation each of us is under to act as a reasonable person in like circumstances would to avoid visiting harm on others. Even the Internal Revenue Code, commonly thought to be the paradigmatic system of rules, is replete with standards (Weisbach, 1999).

Our analysis contributes to an existing literature analyzing the optimal degree of differentiation in law. Kaplow (1995) identifies two key considerations in determining how "complex" law should be: the extent of heterogeneity (in the relevant sense) in the regulated activity and information costs. Information costs can result in complex rules failing to achieve the desired differentiation in behavior, for example because individuals do not learn the legal consequences of alternative choices (Kaplow and Shavell, 1996). But as our model shows, legal differentiation is not the only way to produce socially useful variation in incentives and, moreover, information costs are not only a barrier to the legal strategy of explicit differentiation; they also create opportunities for the state to differentiate incentives without incurring the costs of legal differentiation.

What we call the smoothing channel was first introduced in the law-and-economics literature in a debate over the efficiency of comparative negligence rules. Rubinfeld (1987) first identified the basic mechanism at work in the smoothing channel in a model in which a comparative negligence rule smooths out the discontinuity in injurers' marginal expected liability costs by providing a degree of sharing of liability with the victim that varies continuously with the injurers' care, arguing that this feature of comparative negligence strengthened the argument for its efficiency over the standard negligence rule. Bar-Gill and Ben-Shahar (2003) contests this argument for the efficiency of comparative negligence by pointing out that the differentiation mechanism that Rubinfeld (1987) attributed to comparative negligence can also arise under other types of negligence rules. For example, the contributory negligence rule results in victims bearing all of the accident costs so that victims' incentives are smooth, resulting in differentiated levels of care. As well, they show that evidentiary uncertainty can smooth out the discontinuity in injurers' incentives under a negligence rule. Tax scholars have similarly recognized how uncertainty about a standard governing the classification of an activity for tax purposes can smooth out discontinuities in incentives that would exist under a bright-line rule (Weisbach, 1999; Fox and Goldin, 2019). Our analysis of the smoothing mechanism goes beyond this existing literature by providing a more general analysis of how different forms of legal uncertainty can be affirmatively socially valuable through the smoothing of incentives, as well as through the projection channel, including by clarifying the conditions under which the smoothing mechanism is operative, characterizing the resulting welfare effects, and analyzing the implications for a range of issues in legal design, such as the choice between rules and standards.

Our analysis is also related to prior work analyzing potential beneficial effects of legal uncertainty. Lang (2017) develops an adverse selection model of an enforcement authority determining whether firms may lawfully take a specified action and shows that an increase in firms' uncertainty about how they will be evaluated can result in more efficient screening of the firms that take the action. Ederer et al. (2018) analyze how uncertainty about the weights on performance measures used in an incentive scheme to determine an agent's reward can mitigate the problem of gaming of the incentive scheme. Baker and Raskolnikov (2017) show that in an environment in which agents seek preapproval for a proposed action from a regulator, an increase in legal uncertainty can raise the welfare of the agent if the regulator is more likely to grant preapproval if the action is deemed to greatly surpass the legal standard.

The paper proceeds as follows. In Section 2, we analyze a model of the reasonable person standard and characterize the differentiation in injurers' care levels generated without uncertainty and under different forms of legal uncertainty. In Section 3, we discuss the implications of our analysis for the optimal degree of complexity and personalization of law, the choice between rules and standards, and the choice between sanctions and prices. Section 4 concludes. All proofs are in Appendix A.

2 The Model

Our model builds on the canonical economic model of the reasonable person standard of Shavell (1987). We begin in Section 2.1 by laying out the basic setup and characterizing the first-best outcome. In Section 2.2 we analyze the model under the assumption of no uncertainty, briefly recapitulating standard results in the literature. In Section 2.3 we assume instead that injurers are uncertain about how their behavior will be evaluated by the legal system but have common, unbiased beliefs and analyze the operation of the smoothing mechanism. In Section 2.4 we consider a different form of legal uncertainty in which injurers do not know the standard of care and form Bayesian beliefs about it based on their own information and analyze the operation of the projection mechanism.

2.1 Basic setup

Consider a population of potential injurers who can choose levels of care (precaution) $x \geq 0$ at cost cx in order to reduce the expected cost of accidents $l(x)$, which satisfies the standard assumptions that care reduces the accident loss at a decreasing rate, i.e., $l'(x) < 0$ and $l''(x) > 0$, with $\lim_{x\to 0} l'(x) = -\infty$ and $\lim_{x\to\infty} l'(x) = 0$. Injurers vary in their cost of care $c \in (0, \infty)$. We assume that c is distributed according to a probability density function (PDF) $f(c)$ with associated cumulative distribution function (CDF) $F(c)$.

We take the social objective to be to minimize the total costs of accidents, which are given by

$$L = \int_0^\infty \left[\left(l\left(x(c)\right) + cx(c)\right] f(c)dc, \tag{1}$$

where $x(c)$ is the care taken by an injurer of type c. The first-best level of care for an injurer of type c, denoted $x^{FB}(c)$, minimizes the integrand in (1) for each injurer's type and hence is implicitly defined by the first-order condition:[2]

$$l'\left(x^{FB}(c)\right) + c = 0. \tag{2}$$

By the implicit function theorem, we have:

$$\frac{dx^{FB}(c)}{dc} = -\frac{1}{l''(x)} < 0. \tag{3}$$

The first-best level of care is thus differentiated: it decreases monotonically in c.

As is well known, a "pricing" regime like strict liability can implement the first-best levels of care across heterogenous injurers (Cooter, 1984). In particular, in this model,

[2]Note that the second order condition is satisfied and the solution is unique.

under strict liability injurers would pay the full cost of accidents they cause, so that they would choose care to minimize their total social costs, $l(x)+cx$, resulting in each choosing $x^{FB}(c)$. However, we focus on the negligence rule, which is an example of the more general class of "sanctions" legal strategies that specify a standard of behavior and impose a sanction for failure to comply, because sanctions strategies potentially implicate both the smoothing channel and the projection channel. In this section, we analyze the degree to which the reasonable person standard of the negligence rule differentiates incentives under alternative assumptions about injurers' information. We consider the implications of our analysis for strict liability—and more generally for "pricing" legal strategies—in the discussion in Section 3.

2.2 Reasonable person standard with no uncertainty

Suppose that the injurers' activity is subject to a negligence rule under which injurers who cause an accident would be found negligent, and thus have to pay liability equal to the accident costs they cause, if and only if they choose a level of care less than the reasonable person standard of care, s.[3] We assume that courts cannot observe any individual injurer's cost of care but do know the distribution of injurers' costs of care in the population. Suppose that the level of due care is set at the optimal level of care for the average-cost injurer in the population, $s = x^{FB}(\bar{c})$, as in the standard formulation of the reasonable person standard in the literature (see, e.g., Shavell, 2007).[4] For now we assume that injurers know the standard of care, s.

In equilibrium, each injurer of type c chooses their level of care to minimize the sum

[3]We exclude consideration of activity levels and of victim care levels to develop our analysis of differentiation through uncertainty in the simplest setting possible in which to elucidate the smoothing channel and the projection channel. But our basic analysis would also apply in a richer model that included other margins of behavior.

[4]The basic mechanisms we identify in this paper would also apply if the standard of care were set at some other function of the cost distribution—e.g., the median-cost injurer's first-best level of care—as long as our informational assumptions hold.

of their costs of care and their expected liability costs:

$$\min_{x\geq 0}\begin{cases} cx & \text{if} \quad x\geq s \\ l\left(x\right)+cx & \text{if} \quad x<s \end{cases}$$

The solution to this problem depends on c, with relatively low-cost injurers choosing to take the due level of care s, and high-cost injurers "opting out" of compliance to take their first-best levels of care.

Proposition 1. *Under a known standard of care $s = x^{FB}(\bar{c})$, there is a cutoff level of the cost of care, $\tilde{c} > \bar{c}$, such that in equilibrium injurers choose the following levels of care:*

$$x^{*}\left(c\right)=\begin{cases} s & \textit{if} \quad c\leq\tilde{c} \\ x^{FB}\left(c\right) & \textit{if} \quad c>\tilde{c} \end{cases}$$

Proposition 1 restates a well-know result (see, e.g., Shavell, 1987, pp. 86 - 89) that shows that a negligence rule that sets a known, common standard of care does not handle heterogeneity well. The bulk of the distribution of injurers, those with $c \leq \tilde{c}$, bunch at the standard of care, s. Injurers with $c < \bar{c}$ are choosing less than their socially optimal level of care because they only have incentives to use the minimum care necessary to avoid liability. Higher-cost injurers with $c \in (\bar{c}, \tilde{c}]$ also choose $x = s$, which is greater than their socially optimal level of care.[5] The differentiation in care induced by a known reasonable person standard of care in this model is only among very high cost injurers—those with $c > \tilde{c}$—for whom the costs of complying with the standard outweigh the avoided liability costs.[6] These injurers face in effect a strict liability rule and thus are induced to take first-

[5]Injurers with $c = \tilde{c}$ are indifferent between s and $x^{FB}(c)$, which we arbitrarily resolve in favor of $x^{*}(\tilde{c}) = s$. In the interest of brevity, we will similarly resolve other ties below without further comment.

[6]For example, under the two assumptions that we make in Section 2.4—c distributed $\mathcal{LN}\left(\theta, \sigma^2\right)$ and $l(x) = \frac{1}{x}$—fewer than 5% of injurers have costs higher than the cutoff $\tilde{c}$. In particular, in this case we have $\tilde{c} = 4\bar{c}$ and $F\left(4\bar{c}\right) = \Phi\left(\frac{\log[4]}{\sigma} + \frac{\sigma}{2}\right) \geq \Phi\left(\sqrt{2\log\left[4\right]}\right) = 0.952$, where Φ is the CDF of the standard normal distribution and $\min_{\sigma}\left[\frac{\log[4]}{\sigma} + \frac{\sigma}{2}\right] = \sqrt{2\log\left[4\right]}$. While this specific quantitative result rests on the lognormality assumption, since the cutoff $\tilde{c}$ is always greater than the average cost of care $\bar{c}$, the percentage of injurers taking first best care will typically be small.

best care, as illustrated by Figure 1 below.

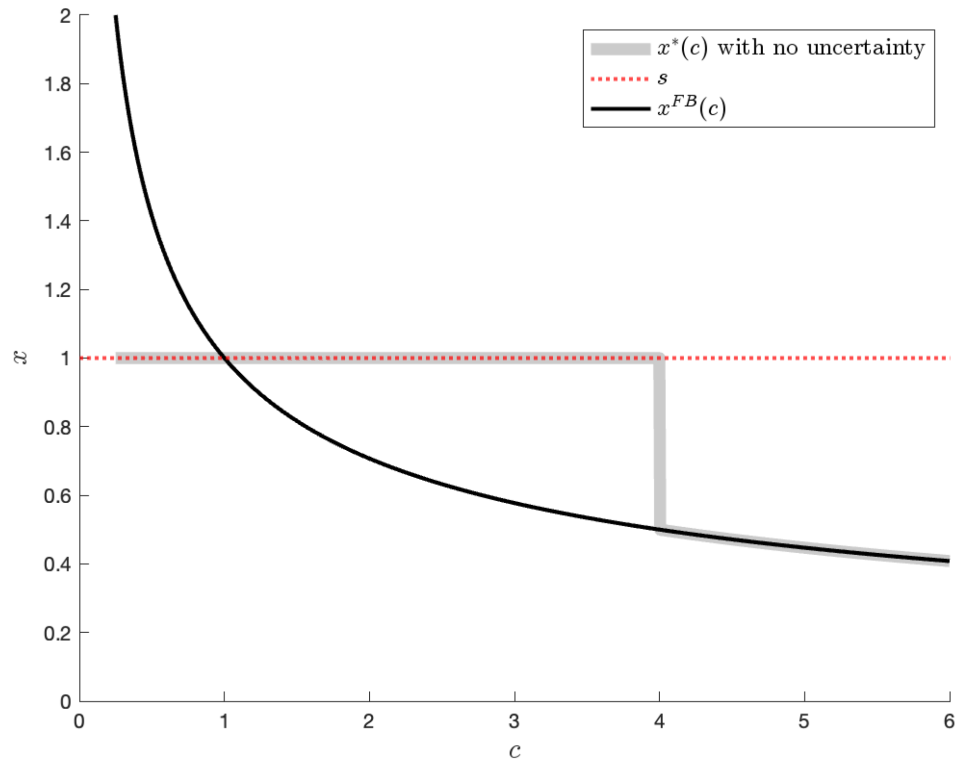


Figure 1: **Care taken with no uncertainty.** Assumptions: $l(x) = \frac{1}{x}$ and $\bar{c} = 1$ so that $s \equiv x^{FB}(\bar{c}) = 1$ and $\tilde{c} = 4$.

2.3 Reasonable person standard with legal noise

Suppose instead that injurers are uncertain about how the legal system will evaluate their behavior but have common, unbiased beliefs. We assume in particular that all injurers view the standard of care s as a random variable with mean $\bar{s}_{\varepsilon} = x^{FB}(\bar{c})$ and variance σ_{ε}^2. We will refer to this as the "legal noise" model. This is the typical way legal uncertainty is modeled in the law-and-economics literature on the reasonable person standard (see, e.g., Craswell and Calfee, 1986). In this subsection we are agnostic about the precise sources of this uncertainty. As argued by Shavell (1987), under appropriate assumptions, the case in which injurers are uncertain about the level of due care, and the case in which injurers are uncertain about how the court will measure their level of care, can be modeled isomorphically.[7]

Denote the CDF of injurers' common beliefs about s by $F_\varepsilon(\cdot)$, with corresponding PDF $f_\varepsilon(\cdot)$. An injurer of type c chooses their level of care by solving the following problem:

$$\min_{x \geq 0} \Big[\big(1 - F_\varepsilon(x)\big) l\left(x\right) + cx \Big], \tag{4}$$

where $1 - F_\varepsilon\left(x\right)$ is the probability that the standard of care is greater than the level of care chosen by the injurer, which results in liability. Denote the solution to this problem for type c by $x^*_\varepsilon\left(c\right)$. In the proof of Proposition 2 provided in Appendix A we show that this solution must be interior and hence satisfies the first-order condition:

$$-\Big[1 - F_\varepsilon\big(x^*_\varepsilon(c)\big)\Big]\, l'\big(x^*_\varepsilon(c)\big) + f_\varepsilon\big(x^*_\varepsilon(c)\big) l\big(x^*_\varepsilon(c)\big) = c. \tag{5}$$

On the left-hand side of (5) is the private marginal benefit of care, which comes from the reduction in expected liability costs. On the right-hand side is the marginal cost of care, which varies across injurers. As in Craswell and Calfee (1986), for any given type of injurer c, uncertainty about the legal standard has two effects. The first term on the left-hand side of (5) captures how uncertainty dilutes injurers' incentives. To see this, suppose that the second term were equal to 0. Then the only difference between the private first-order condition in (5) and the first-order condition for socially optimal care in (2) above would be that marginal external costs $l'\big(x^*_\varepsilon(c)\big)$ are being given less than full weight because the injurer believes the probability they will have to bear those costs through the liability system is only $1 - F_\varepsilon\big(x^*_\varepsilon(c)\big) < 1$. This dilution effect tends to reduce the injurer's level of care compared to the first best. The second term on the left-hand side of (5), on the other hand, captures the reduction in the probability of liability if the injurer

[7]We ignore the possibility that individuals could obtain additional information at a cost in order to reduce legal uncertainty, but all of our positive analysis would go through unchanged if we allowed for that so long as there remained residual legal uncertainty in equilibrium.

increases x. This leads to an incentive to take greater care. The balance of these two effects determines how the level of care chosen by the injurer compares to the level of due care, s—that is, whether the injurer will over- or under-comply—and to the injurer's first-best level of care, $x^{FB}(c)$—that is, whether the injurer will be over- or under-deterred.

Our primary interest, however, is not in the level of care taken by any particular type of injurer c but rather in the degree to which injurers' levels of care are differentiated based on their cost of care c in a socially useful way. In particular, under legal noise, injurers with higher costs of care take strictly lower levels of care, unlike in the absence of legal uncertainty, as the following proposition states more formally.

Proposition 2. *Under the reasonable person standard with legal noise:*

1. *The equilibrium level of care $x_{\varepsilon}^{*}(c)$ is continuous and strictly monotonically decreasing in the injurer's cost of care, c;*

2. *Injurers with costs of care below a threshold, $c < \hat{c}$, over-comply with the standard, $x_{\varepsilon}^{*}(c) > s$, while injurers with costs of care above the threshold, $c > \hat{c}$, under-comply, $x_{\varepsilon}^{*}(c) < s$;*

3. *If the variance of the distribution of s is not too large, then this threshold is above the average cost of care, $\hat{c} > \overline{c}$, so that the average injurer over-complies.*

Figure 2 illustrates the greater degree of differentiation under uncertainty by showing equilibrium levels of care as a function of c implemented by the liability system under various degrees of legal noise, measured by the standard deviation of each injurer's beliefs about s, σ_{ε}. Starting from the case with no uncertainty, the introduction of a small amount of legal noise means that injurers have to take somewhat greater care in order to substantially eliminate the risk of liability. For this reason, the threshold of injurers' cost of care c at which injurers effectively decide to not try to comply with the standard of care falls, as illustrated by the care taken with $\sigma_{\varepsilon} = 0.003$ relative to the care taken under no uncertainty. As well, at low levels of uncertainty, relatively low-cost injurers "over-comply" by taking somewhat greater care than the expected standard of care. At greater levels of legal uncertainty, care becomes smoothly differentiated throughout the domain of c and, for intermediate levels of uncertainty (e.g., $\sigma_\varepsilon = 1$) equilibrium care tracks first-best care fairly closely. Finally, as the amount of legal uncertainty becomes large (e.g., $\sigma_\varepsilon = 3$), legal incentives become diluted, resulting in all injurer types taking substantially less than first-best care.

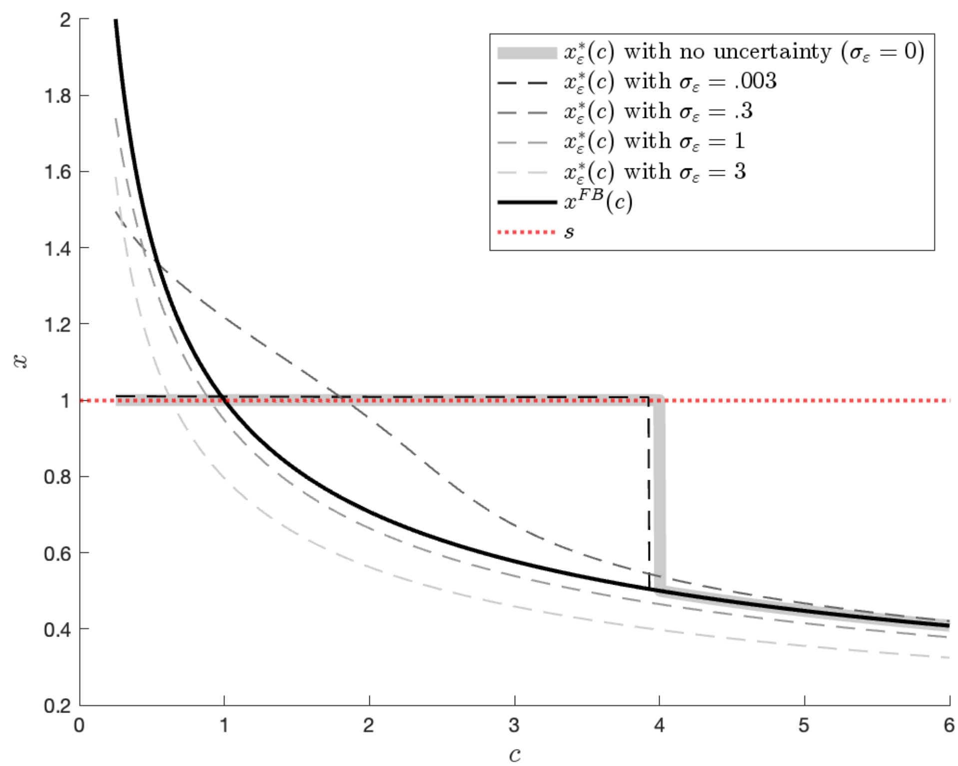


Figure 2: **Care taken with legal noise.** Assumptions: $l(x) = \frac{1}{x}$ and $\bar{c} = 1$ so that $\tilde{c} = 4$ and $s \equiv x^{FB}(\bar{c}) = 1$, with injurers' beliefs about s distributed $\mathcal{N}\left(1, \sigma_\varepsilon^2\right)$ for varying levels of σ_ε.

To build intuition for what is driving the differentiation in behavior in this model, Figure 3 shows the private marginal benefit of care for varying degrees of legal uncertainty along with the social marginal benefit of care. The social marginal benefit of care stems from the reduction in expected accident costs from greater care, or $-l'(x)$. In contrast, the private marginal benefit of care is the reduction in the injurer's expected liability from an increase in their level of care. With no uncertainty, at $x < s$ the injurer will be liable for the full social costs of accidents so that the injurer's private marginal benefit of care equals the social marginal benefit of care. But the private marginal benefit of care drops discontinuously to 0 at the standard of care, above which the injurer is not liable. This discontinuity in the private marginal benefit of care is why injurers bunch at a corner solution right at $x = s$. In contrast, under uncertainty the private marginal benefit of care is continuous through the standard of care so that all injurers are at an interior optimum that equates their private marginal benefit of care with their marginal cost of care. This is why legal noise results in equilibrium care levels being smoothly differentiated with respect to injurers' costs of care, c.

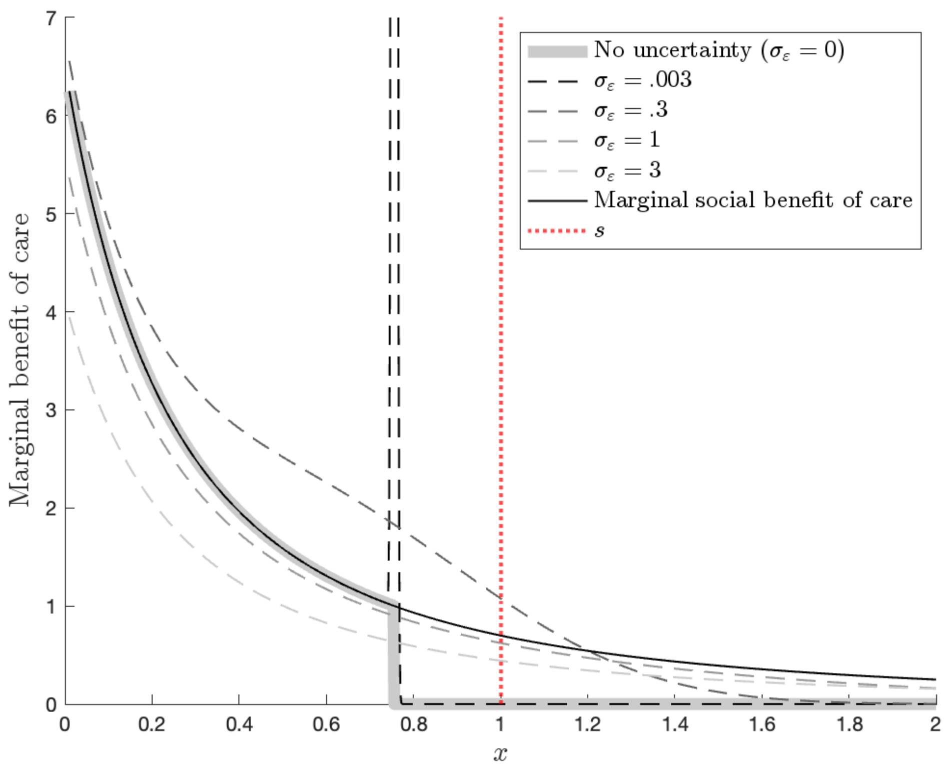


Figure 3: **Marginal private benefit of care with legal noise.** Assumptions: $l(x) = \frac{1}{x}$ and $\bar{c} = 1$ so that $\tilde{c} = 4$ and $s \equiv x^{FB}(\bar{c}) = 1$, with injurers' beliefs about s distributed $\mathcal{N}\left(1, \sigma_\varepsilon^2\right)$ for varying levels of σ_ε.

For care levels below the standard of care, $x < s$, legal uncertainty generally distorts incentives relative to the first-best. For relatively low amounts of uncertainty (e.g., $\sigma_\varepsilon = .3$ in this example), uncertainty pushes the marginal private benefit of care above the marginal social benefit of care. But for higher levels of legal uncertainty (e.g., $\sigma_\varepsilon = 1$ and $\sigma_\varepsilon = 3$), legal uncertainty dilutes legal incentives, pushing the private marginal benefit of care below the social marginal benefit of care, potentially by a substantial amount. For care levels above the standard of care, the private marginal benefit of care tracks the social marginal benefit of care more closely under uncertainty than under certainty. Because under uncertainty the implemented care level will be at the level of care at which the private marginal benefit of care equals an injurer's marginal cost of care (measured by the vertical axis), uncertainty results in the equilibrium levels of care tracking socially optimal care more closely for relatively low-cost injurers, as illustrated in Figure 2.

The smoothing channel is potentially operative only when, in the absence of legal uncertainty, there are discontinuities in marginal legal incentives as individuals' behavior changes, as illustrated in the case of the negligence rule in Figure 3. When such discontinuities exist, then any form of legal uncertainty that results in individuals' expected legal outcome varying smoothly with their behavior will produce more differentiated incentives. These potentially include forms of substantive legal uncertainty, in which individuals do not know the legal thresholds that generate the discontinuities, as well as forms of procedural legal uncertainty, in which individuals are uncertain about how the legal system will measure their behavior to determine the appropriate legal outcome. Sanctions regimes that delineate legally permissible behavior from legally prohibited behavior are a primary example of discontinuities in marginal legal incentives that implicate the smoothing channel, as the model of the reasonable person standard in Section 2 illustrates. Another class of legal regimes in which the smoothing channel may be operative are those that use thresholds of behavior to determine the classification of the activity for regulatory or tax purposes, such as whether a financial instrument will be taxed as equity or as debt.

Note, however, that the smoothing mechanism operates by making expected legal outcomes *differentiable*, not by affecting the continuity of legal outcomes *per se*, contra existing accounts in the literature.[8] As one way to see this point, and its implications, consider the cause-in-fact requirement of tort law, which is a common law device that might reduce or even eliminate the discontinuity in expected legal outcomes produced by the negligence rule (Grady, 1987; Kahan, 1989). But even if injurers are only liable for accidents that would not have occurred had they taken due care, so that their expected liability costs are $l(x) - l(s)$ for $x < s$ and therefore continuous through $x = s$, there is a discontinuity in their private *marginal* benefit from care at $x = s$ and bunching will occur at $x = s$ for injurers with $x < \bar{c}$. As a result, both the smoothing mechanism and the projection mechanism will be potentially operative in the presence of legal uncertainty. We provide an analysis of the model of negligence with incremental damages in Appendix B.

As our analysis so far has shown, legal uncertainty has both benefits and costs—it potentially improves the care of many injurers, in particular relatively low-cost injurers, by inducing them to take more differentiated levels of care rather than bunch at the standard of care, but it generally erodes the incentives of the very high-cost injurers who would "opt-out" of a known standard of care to take first-best care. As well, the relative importance of these competing effects of legal uncertainty on the total costs of accidents depends on the distribution of types, making it difficult to say anything in general about the net effect of legal uncertainty on welfare. But to illustrate the overall welfare effects, Figure 4 shows the total costs of accidents as a function of the amount of legal uncertainty in the legal noise model under a set of specific functional-form assumptions, including $l(x) = \frac{1}{x}$ and c distributed lognormally.

Total costs of accidents are non-monotonic in the degree of legal uncertainty. Starting

[8]See, e.g., Bar-Gill and Ben-Shahar (2003, pp. 454 - 458) (arguing that the "self-selection mechanism" identified by Rubinfeld (1987) functions by removing discontinuities in expected legal outcomes and noting that, under the incremental damages model of Grady (1987) and Kahan (1989), "the negligence regime does *not* in fact set a discontinuous payoff structure"); Weisbach (1999, pp. 872 - 873) (analyzing "discontinuous" rules in which "[m]oving one step to the left will cause a large change in tax consequences"); and Fox and Goldin (2019, p. 244) (focusing on "the discontinuous relationship between legal inputs (e.g., taxpayer characteristics) and legal outputs (e.g., tax liability)."). In contrast, Rubinfeld (1987, p. 388)'s analysis of comparative negligence as a smoothing device focused correctly on continuity of *marginal* legal incentives, not on continuity of legal outcomes.

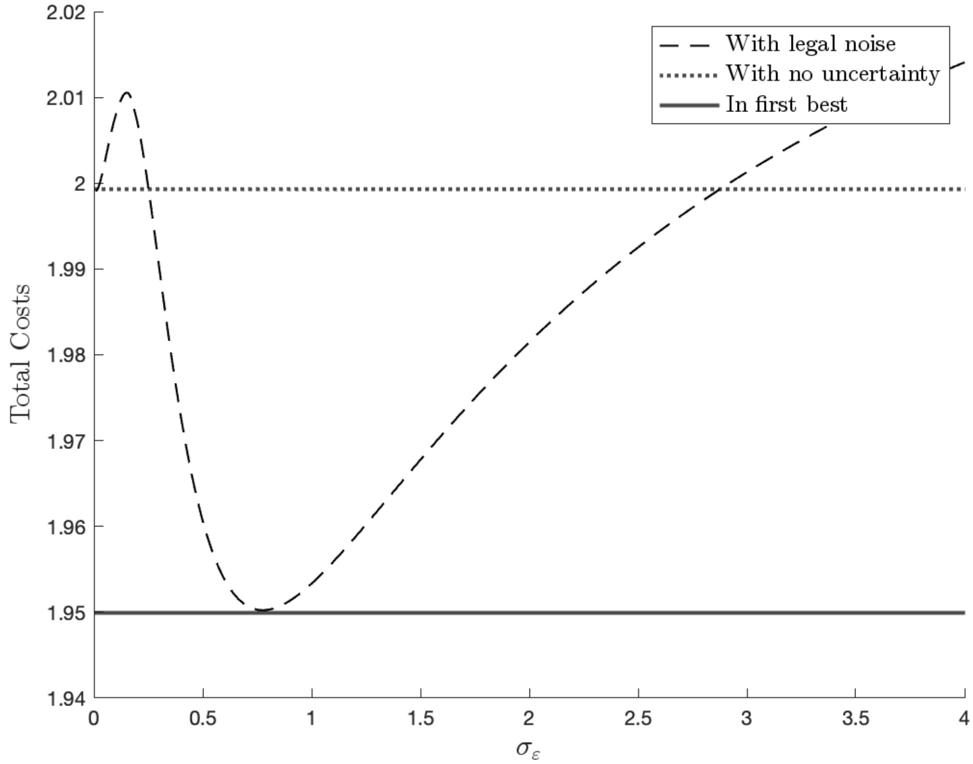


Figure 4: **Total social costs of accidents with legal noise.** Assumptions: $l(x) = \frac{1}{x}$ and $c \sim \mathcal{LN}\left(\theta, \sigma^2\right)$, with $\theta = -\frac{1}{2}\sigma^2$ and $\sigma = 0.45$ so that $\bar{c} = 1$, $\tilde{c} = 4$, and $s \equiv x^{FB}(\bar{c}) = 1$, with injurers' beliefs about s distributed $\mathcal{N}\left(1, \sigma_\varepsilon^2\right)$ for varying levels of σ_ε.

from the case with no legal uncertainty, introducing legal uncertainty initially raises the total costs of accidents by inducing injurers to over-comply with the expected standard of care, distorting care upward. But as the amount of legal uncertainty increases, the differentiation benefits of legal uncertainty kick in, dramatically reducing the total costs of accidents, down almost to their level in the first-best. High levels of legal uncertainty ultimately dilute injurers' incentives to take care, eventually to the degree that total costs rise back above their level with no legal uncertainty.

2.4 Reasonable person standard with updating

Consider now a different form of legal uncertainty that stems specifically from injurers not knowing the standard of care and forming Bayesian beliefs about the unknown standard based on their information. In particular, suppose that injurers do not know the standard of care s, or the average cost of care in the population $\bar{c}$, but that they understand that they are required to take "reasonable care," meaning the optimal level of care for an average-cost injurer, $s = x^{FB}(\bar{c})$. Injurers thus form beliefs about s based on their beliefs about $\bar{c}$.

To keep the problem tractable, we make three functional-form assumptions. First, we assume that $l(x) = \frac{1}{x}$, which will be used to prove the monotonicity of injurers' care choices in Proposition 4 and to construct the numerical examples we use in all of the figures. Under this assumption, the first-best level of care that solves the first-order-condition in (2) is $x^{FB}(c) = c^{-\frac{1}{2}}$.

Second, we assume that the costs of care in the population follow a lognormal distribution, which restricts injurers' costs of care to be non-negative. That is, we assume that $c = e^{\tau}$ where τ is a normally distributed "personal characteristic"—such as, for instance, experience, expertise, reaction time, economies of scope or scale, or access to technology—with mean θ and variance σ^2. Injurers' costs c thus have mean $\bar{c} = e^{\theta + \frac{\sigma^2}{2}}$. Note that, since there is a bijective relationship between c (the injurer's cost of care) and τ (the injurer's personal characteristic), we can refer to the injurer's type as c or τ interchangeably.

Third, we assume that injurers share a common, unbiased prior about θ, distributed $\mathcal{N}(\theta, \sigma_0^2)$. This ensures that injurers' prior and posterior beliefs about θ have the same functional form (i.e., are conjugate distributions).[9] For simplicity we assume that the remaining parameter of the type distribution, σ^2, and the overall structure of the model (including functional forms) are common knowledge, so that the only relevant unknown for the injurers is θ.

Each injurer's information set about $\bar{c}$ consists of their prior and a single draw from

[9] Assuming a normal (instead of lognormal) distribution of the costs of care would also result in conjugate distributions for beliefs and in some ways simplify the analysis, but at the cost of introducing negative costs of care, which would pose a problem—and hence require additional assumptions—when constructing the social objective function. Additionally, allowing priors to be biased—that is, distributed $\mathcal{N}(\mu_0, \sigma_0^2)$ for some μ_0 possibly different from θ—would only add a layer of analysis without affecting our results qualitatively.

the distribution of c, namely their own type. To derive injurers' posterior beliefs about the standard of care, it will be useful to work first with their beliefs about θ, the underlying mean of $\tau = \log(c)$. This lets us draw on standard results on Bayesian beliefs about a normally distributed variable with a normally distributed signal to derive injurers' beliefs about θ, which in turn pin down their beliefs about $\bar{c} = e^{\theta + \frac{\sigma^2}{2}}$ and thus about $s = x^{FB}\left(\bar{c}\right) = e^{-\frac{1}{2}\left(\theta + \frac{\sigma^2}{2}\right)}$.

In particular, an injurer of type c has a signal of θ, $\tau = \log(c)$, which is distributed $\mathcal{N}\left(\theta, \sigma_0^2\right)$. Their posterior beliefs about θ follow the well-known normal signal updating rule and are distributed normally with mean $\mu_1\left(c\right)$ and variance σ_1^2, given by:

$$\mu_1\left(c\right) = \frac{\sigma^2}{\sigma_0^2 + \sigma^2}\theta + \frac{\sigma_0^2}{\sigma_0^2 + \sigma^2}\log\left(c\right), \tag{6}$$

and

$$\sigma_1^2 = \frac{\sigma_0^2\sigma^2}{\sigma_0^2 + \sigma^2}. \tag{7}$$

As is intuitive, equation (6) implies that injurers with a higher cost of care c have higher expectations of θ and therefore of the mean cost of care in the population, $\bar{c} = e^{\theta + \frac{\sigma^2}{2}}$. Accordingly, they have lower expectations about the standard, $s = e^{-\frac{1}{2}\left(\theta + \frac{\sigma^2}{2}\right)}$. We refer to this as the "projection channel": each injurer's own cost of care c informs his beliefs about the reasonable-person standard. The resulting variation in legal incentives is correlated with injurers' costs of care in a socially useful way, as we will show below.

The greater the variation across injurers in their beliefs about the mean cost of care, the more differentiated are injurers' incentives and behavior. A simple way to measure the degree of this heterogeneity in beliefs is in terms of the variance of the mean of the posterior beliefs about θ, $\mu_1(c)$, across injurers in the population, which is given by:

$$\text{Var}\left(\mu_1\left(c\right)\right) = \frac{\sigma^2}{\left(\frac{\sigma^2}{\sigma_0^2} + 1\right)^2}. \tag{8}$$

Equation (8) shows that the extent of differentiation in injurers' expectations about θ depends on the information structure. The greater is injurers' uncertainty about the population's average characteristic under the prior (that is, the higher σ_0^2), the more weight injurers put on their signal relative to the prior in (6), and therefore the greater is the variance in expectations about θ in the population. The extent of differentiation in injurers' expectations about θ also depends on the variance in the signals they receive, σ^2, both indirectly through its effect on the information structure and directly through its effect on the degree of heterogeneity across injurers. First, the greater the variance of the signal, the less injurers update based on their signals, leading to less differentiation in injurers' beliefs (this channel corresponds to the σ^2 in the denominator of (8)). But on the other hand, the greater the heterogeneity of injurers in the signals they receive, the more differentiated they are in their expectations about θ (this corresponds to the σ^2 in the numerator of (8)). On net it can be shown that the variance of $\mu_1(c)$ across injurers in the population is increasing in σ^2 if and only if $\sigma^2 < \sigma_0^2$.

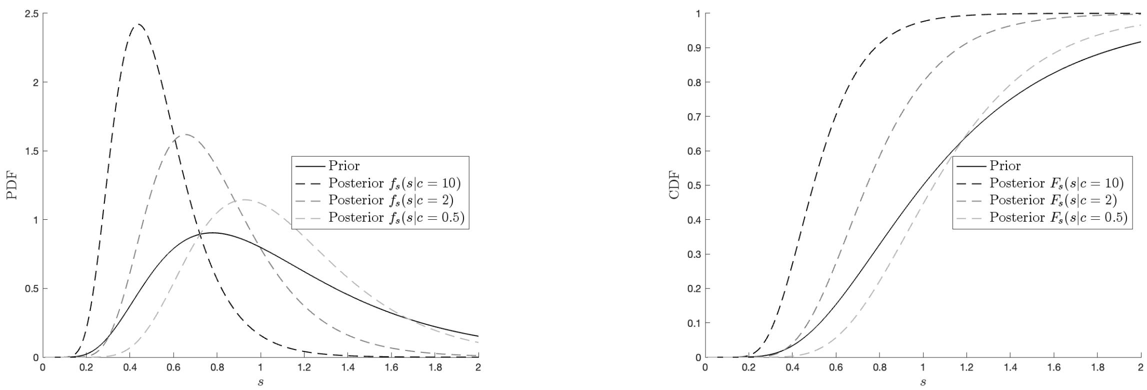
Denote by $f_s(\cdot|c)$ and $F_s(\cdot|c)$ the PDF and CDF, respectively, of injurers' posterior beliefs about the standard s, conditional on their type c. We now have our first key result for the projection channel, characterizing these posterior beliefs and showing that injurers with lower c have greater beliefs about the standard of care in a first-order stochastic dominance (FOSD) sense.

Proposition 3. *Under the reasonable person standard with updating:*

1. *Injurers' posterior beliefs about the standard s are distributed $\mathcal{LN}(\mu_s(c), \sigma_s^2)$ where $\mu_s(c) = -\frac{1}{2}\left(\mu_1(c) + \frac{\sigma^2}{2}\right)$ and $\sigma_s^2 = \frac{\sigma_1^2}{4}$.*

2. *The family of posterior distributions of s parameterized by c satisfies the FOSD property with respect to c: for all $c_1 < c_2$, $F_s(x|c_1) < F_s(x|c_2)$ for all $x \in (0, \infty)$.*

This result is illustrated in Figure 5, which shows the PDF's and CDF's of injurers' beliefs about the standard s for different draws of c. The differentiation in injurers' beliefs about the standard of care is intuitive: injurers with higher costs of care rationally believe that the average cost of care in the population is higher and therefore that the reasonable person standard will require less of them.



(a) Posterior PDF of s

(b) Posterior CDF of s

Figure 5: **Posterior beliefs about the standard.** Parameters: $\sigma^2 = 1$ and $\theta = -\frac{1}{2}$, so that $\bar{c} = 1$, and $\sigma_0 = 1$.

This phenomenon is akin to a set of empirical findings in psychology that show that individuals form judgments about the broader population in part by *projecting* their own characteristics onto the population (Krueger, 2000). For example, survey respondents tend to overestimate the extent to which their own opinions are shared by others (Ross et al., 1977), a phenomenon that has been coined the "false consensus effect." Much of the literature on social projection characterizes its underlying mechanisms as reflecting various types of biases, such as availability bias or motivated reasoning. But Dawes (1989) argues that social projection is in fact required by Bayes' rule when one views oneself as a draw from the population distribution, in much the same way we model here.

Figure 6 shows injurers' expected standard of care as a function of c, denoted $\overline{s}\,(c)$, for different degrees of background legal uncertainty, as measured by σ_0 (the standard deviation of the prior distribution), along with injurers' first-best level of care as a function of c. For low levels of legal uncertainty (e.g., $\sigma_0 = 0.5$), injurers only slightly update their beliefs based on their type, so that $\bar{s}(c)$ is relatively flat. But for high degrees of legal uncertainty (e.g., $\sigma_0 = 2$ and $\sigma_0 = 4$), each injurer updates strongly about $\bar{c}$ based on their own type c and therefore expects a standard of care $s = x^{FB}(\bar{c})$ that is very close to their own first-best level of care, $x^{FB}(c)$, illustrating the projection mechanism at work.[10]

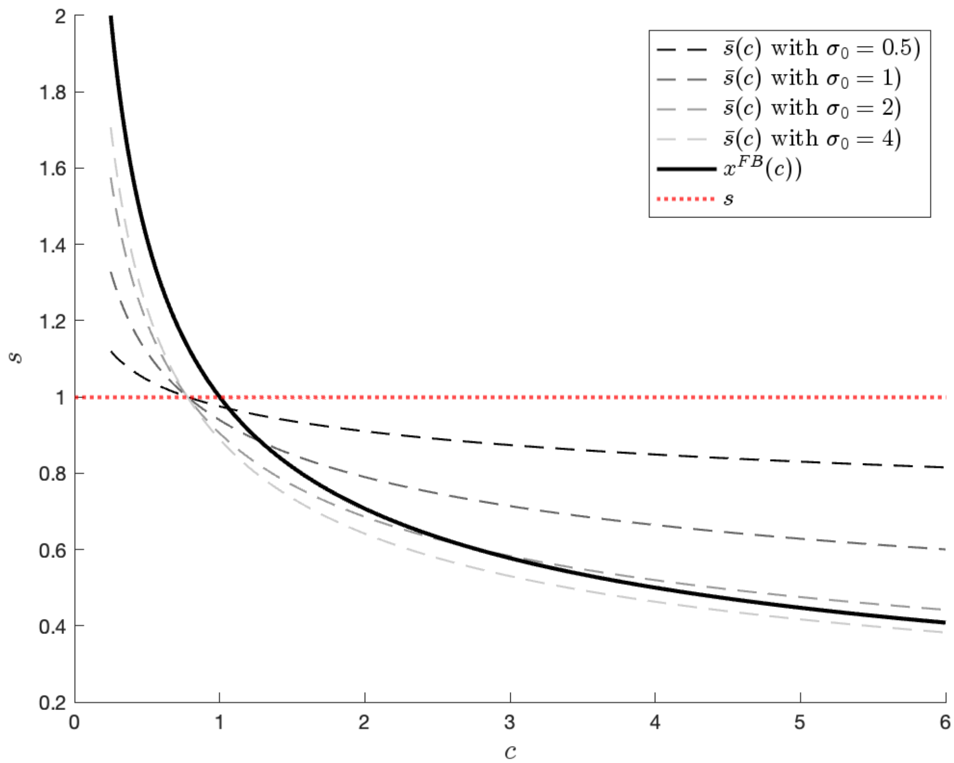


Figure 6: **Injurers' expectations of s with updating.** Assumptions: $\sigma^2 = 1$ and $\theta = -\frac{1}{2}$, so that $\bar{c} = 1$ and $s = 1$, for varying levels of σ_0.

The differences in beliefs about the standard of care of different types of injurers in turn produce differentiated incentives to take care. An injurer of type c chooses the level of care x that solves,

$$\min_{x \geq 0} \Big[\big(1 - F_s(x|c)\big) l(x) + cx \Big]. \tag{9}$$

[10]An observant reader might notice that the $\bar{s}(c)$ functions for different levels of σ_0 all cross at $\bar{s}(0.77881) = s = 1$. This particular signal $c = 0.77881$ leads injurers to not update their unbiased mean prior beliefs and as a result leaves them with an unbiased posterior expectation about the standard, $\overline{s}(c) = s$. With an unbiased posterior expectation, we have $e^{\mu_s(c) + \frac{\sigma_s^2}{2}} = e^{-\frac{1}{2}\left(\theta + \frac{\sigma^2}{2}\right)}$, which—after replacing $\mu_s(c)$ and σ_s^2 and simplifying—yields $c = e^{-\frac{1}{4}\sigma^2}\bar{c}$. It is easy to verify that this implies that the curves in Figure 6 cross at $s = 1$ for $c = e^{-\frac{1}{4}} = .77881$. This signal that results in unbiased posterior expectations is less than $\bar{c}$ because of our lognormal distributional assumption.

The injurer's choice of care, $x_s^*(c)$, is implicitly defined by the first-order condition:[11]

$$-\Big[1 - F_s\big(x_s^*(c)|c\big)\Big] l'\big(x_s^*(c)\big) + f_s\big(x_s^*(c)|c\big) l\big(x_s^*(c)\big) = c. \tag{10}$$

The first-order condition under substantive legal uncertainty with updating has the same basic form as the first-order condition of the legal noise model, (5). The difference is that the density and distribution functions of injurers' beliefs are now conditional on their type, c. Thus, in this model, both the projection channel and the smoothing channel are operating to produce differentiated incentives.

While it is intuitive that both channels will result in a strictly monotonic negative relationship between injurers' costs of care and levels of care, to prove this result raises some challenges. One way to see why is to note that, in the absence of uncertainty about the standard of care, s, injurers' care is not monotonically increasing with respect to s. The reason is that while at low levels of s, injurers will choose $x = s$ to avoid liability, when s becomes sufficiently high, injurers will prefer to choose their socially optimal level of care $x^{FB}(c) < s$, even though they are then liable for any accidents they cause, because the cost of complying with the standard is too great (Shavell, 1987).

At a more technical level, the objective function in the absence of uncertainty, $\mathbb{I}_{\{x<s\}} l(x) + cx$, does not satisfy the single-crossing property in s that would be sufficient for monotone comparative statics with respect to s (Milgrom and Shannon, 1994). In turn, this means that we cannot rely on the standard results in Athey (2002) to guarantee monotone comparative statics in response to FOSD shifts in beliefs about s, nor is the standard approach to comparative statics based on the implicit function theorem (also employed by Edlin and Shannon, 1998) tractable. To prove the monotonicity asserted in the following proposition, we show instead that the injurers' objective function under uncertainty satisfies the Interval Dominance Order property of Quah and Strulovici (2009) with respect to c.

[11]This follows from the fact that the solution must be interior. The proof is analogous to the case with noise and is omitted.

Proposition 4. *Under the reasonable person standard with updating, injurers' levels of care are differentiated:* $\frac{\partial x_s^*(c)}{\partial c} < 0$.

Figure 7 illustrates the effect of legal uncertainty on injurers' equilibrium care levels with updating. Consider first the outcome under low levels of legal uncertainty, illustrated by the case with $\sigma_0 = 0.5$. Injurers' levels of care are more smoothly differentiated than in the absence of uncertainty, as expected. Relatively low cost injurers take greater care than they would with no uncertainty for two reasons. First, very low cost injurers expect a standard of care above the true standard of care, due to the projection mechanism. Second, legal uncertainty creates incentives to "over-comply" with the expected standard of care in order to further reduce the probability of liability. For higher degrees of legal uncertainty, the projection mechanism operates more strongly, which pulls injurers' equilibrium care close to their own first-best level of care.

The projection channel is operative only when an individual is uncertain about the substantive content of law and the individual's own type is informative about the content of the relevant legal standards. By "type" we mean not just characteristics of the individuals but also any characteristics of the circumstances in which they make decisions that are informative about the general legal standard. The most natural case is when the legal standard is set to be appropriate for the "typical" case, but individuals do not know what is typical, as in our model of the reasonable person standard above. This situation strikes us as not at all unusual. Individuals will often have more information about the costs and benefits of alternative courses of action than the state has. That is, information about optimal conduct is decentralized, to a greater or lesser degree in different contexts. And on the other hand, the state will often have better information than individuals about the broader scope of a problem. In such a case, the optimal legal strategy might be to use a simple vague legal standard, taking advantage of the projection channel to nonetheless differentiate incentives at least to some extent.

While the differentiation produced by the reasonable person standard in our model is

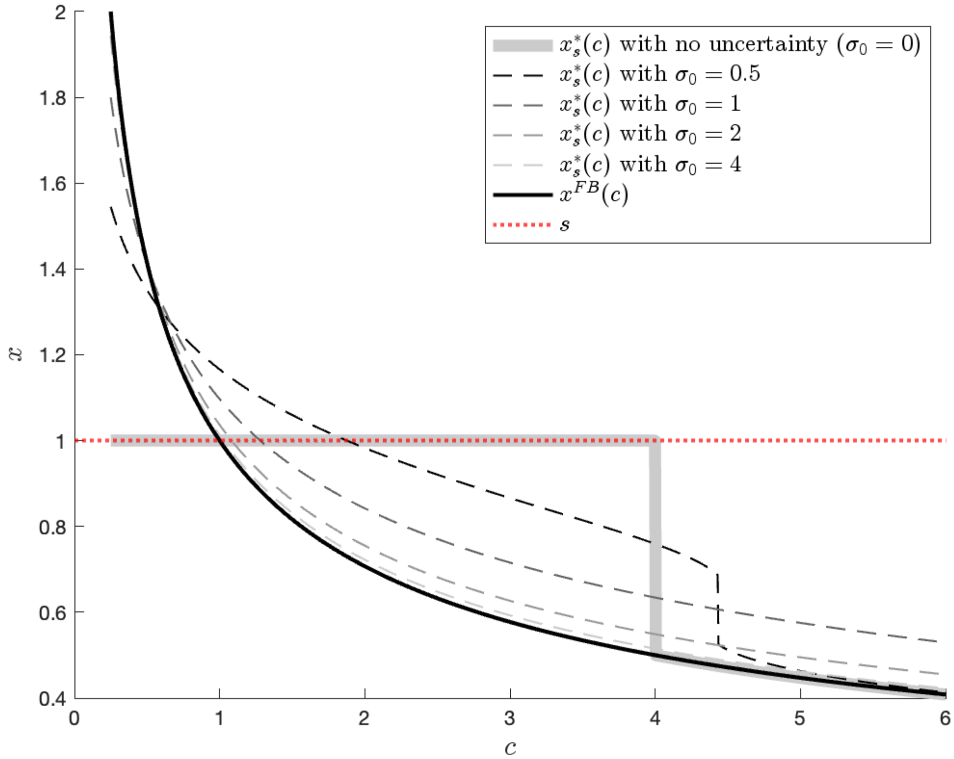


Figure 7: **Care taken with updating.** Assumptions: $\sigma^2 = 1$ and $\theta = -\frac{1}{2}$, so that $\bar{c} = 1$ and $s = 1$, for varying levels of σ_0.

in a socially useful direction, in that it results in lower cost injurers taking greater care, in other settings the projection channel might operate in a dysfunctional manner. Consider, for example, a standard that subjects "abnormally dangerous" activities to sanction. Each injurer might then form beliefs about what constitutes "normal" vs. "abnormal" degrees of dangerousness based on their own activity's level of dangerousness, resulting in individuals underestimating the likelihood of their activity being sanctioned.

Note as well that we have assumed that individuals form beliefs about the content of the standard in a rational, Bayesian manner. Deviations from this standard model of belief formation might fundamentally change the nature of the projection channel. Consider, for example, a setting in which individuals are subject to "self-serving bias," in which they conflate what is legal with what benefits themselves (Feldman, 2018). Such a bias also entails a form of projection but one that could produce perversely differentiated beliefs about what the law requires.

A more general limitation of the projection channel is that it requires individuals to act on the basis of an understanding of the content of law that is incorrect. Experience with the law might disabuse individuals of these misconceptions over time. In contrast, procedural legal uncertainty—and therefore the smoothing channel—might be more durable. Still, we think there is substantial room for persistence of substantive legal uncertainty, and therefore the projection mechanism, in the long run. A moment's introspection reveals to us that we ourselves remain ignorant of the content of all but a small fraction of the laws that regulate our behavior in the jurisdictions in which we live and work. Much of human affairs, we submit, is regulated by individuals' best guesses of what the law requires and little else of a legal nature.

In the reasonable person standard model with updating, both the smoothing channel and the projection channel operate. But in other settings only one of the two channels is operative. The model of the reasonable person standard with noise in Section 2.3 above provides an example of a setting in which only the smoothing channel, and not the projection channel, is operative. But there are also situations in which the converse is true. For example, consider the case in which a strict liability rule is used to control the releases of a certain class of hazardous chemicals. Suppose that the chemicals in the class cause varying levels of harm, h, and that the government knows only the average level of harm of the chemicals in the class, $\bar{h}$, not the individual harm levels of each chemical. Individuals are thus held strictly liable for the average harm of chemicals in the class, not their specific harm. Suppose further that the individuals subject to this regime know only their own chemical's harm, h, not $\bar{h}$. In this setting the projection channel would be operative: individuals would form beliefs about the legal sanction using their own harm h as a signal of the average harm $\bar{h}$. Individuals' posterior beliefs about $\bar{h}$ would vary according to their individual signals, h, and potential injurers would calibrate their behavior accordingly. But in this setting, the smoothing channel does not operate—there is no discontinuity in marginal legal incentives with respect to behavior for uncertainty to smooth.

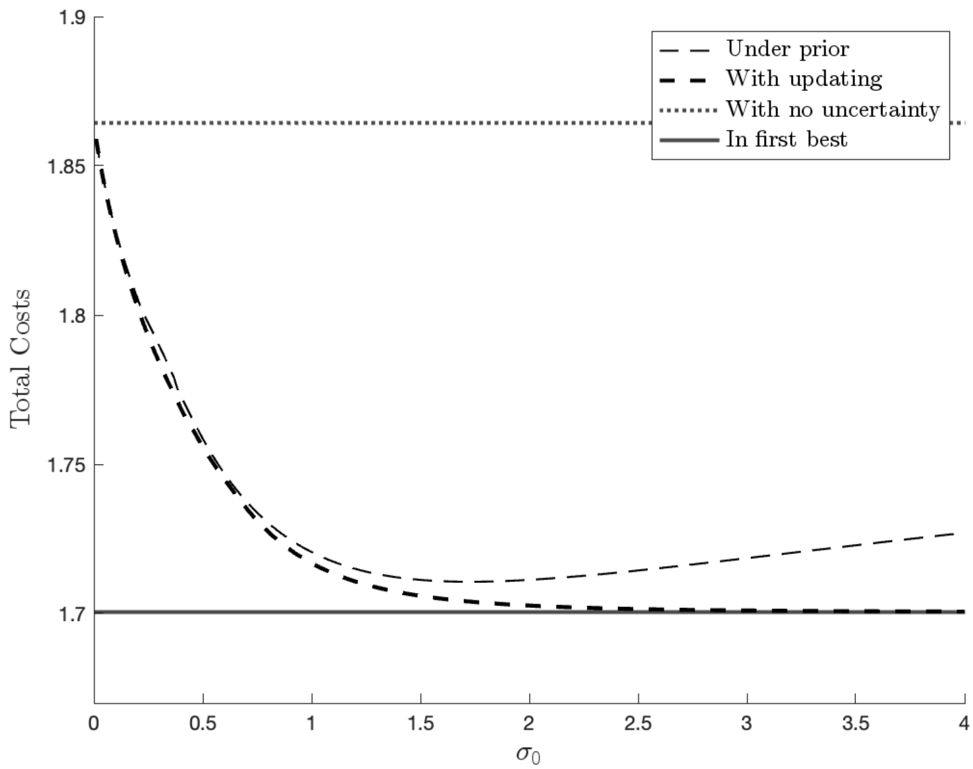


Figure 8: **Total social costs of accidents with updating.** Assumptions: $\sigma^2 = 1$ and $\theta = -\frac{1}{2}$, so that $\bar{c} = 1$ and $s = 1$, for varying levels of σ_0.

As the care functions in Figure 7 suggest, the greater differentiation in incentives produced by the projection mechanism can dramatically reduce the total costs of accidents. This is shown in Figure 8, which depicts the total social costs of accidents as a function of σ_0 under the reasonable person standard with updating along with three benchmarks: total costs with no uncertainty, total costs in the first-best, and total costs if injurers chose care levels with their beliefs held fixed at their common prior. We include total costs based on beliefs held at the common prior as a way to characterize the incremental benefit of the projection mechanism on top of the smoothing mechanism. At low levels of legal uncertainty, the main benefit of legal uncertainty in this model stems from the smoothing mechanism, as reflected by the total costs with updating tracking closely the total costs under the prior for σ_0 below about 0.75. But at higher levels of σ_0, the incremental benefits of legal uncertainty under the prior slow down and eventually reverse, for reasons discussed in the legal noise model in Section 2.3 above. In contrast, with updating, total social costs continue to fall as σ_0 increases until they approximate the total costs under the first-best.

3 Applications to legal design

We now consider the implications of our analysis for three classic issues in legal design: the optimal degree of complexity and personalization of law, the choice between rules and standards, and the choice between prices and sanctions.

3.1 Complexity and personalization

Our analysis of legal uncertainty has implications for how "complex" law should be, in the sense of how finely the law on the books distinguishes among different acts. Kaplow (1995) shows that the greater is the extent of heterogeneity (in the relevant sense) in the regulated activity, and the lower the state's and individuals' information costs, the greater is the optimal degree of legal complexity. A closely related issue is the optimal degree of accuracy in enforcement and adjudication of the law, meaning the precision of the law as applied. The two are closely related since ultimately explicit legal differentiation will produce differentiated incentives only if individuals expect the law as applied to result in differentiated legal outcomes. More recently Ben-Shahar and Porat (2016) argue that the fall in information costs over time implies that the law should differentiate the legal treatment of cases more finely than it has done so historically. But as our model shows, legal differentiation is not the only way to produce socially useful variation in incentives. In the presence of legal uncertainty, simple legal standards can nonetheless produce differentiated incentives, yet spare the information costs that legal complexity entails.

These two differentiation strategies—explicit legal differentiation and differentiation through legal uncertainty—are, to a significant extent, alternatives that the law must choose between rather than deploy simultaneously. As Kaplow (1995) and Kaplow and Shavell (1996) emphasize, explicit legal differentiation requires individuals to be able to predict the legal outcomes of alternative courses of action in order to produce differentiated incentives. In other words, it requires low levels of legal uncertainty, generally attenuating the alternative mechanisms for differentiation we analyze, which rely on legal uncertainty.

Ben-Shahar and Porat (2016, p. 634) argue that individuals are often better informed about what is reasonable for them to do than about what would be reasonable for the "average person." They take this as part of the case for greater personalization of standards of care, since individuals would be able to predict their personalized level of due care better than a uniform standard of care. But in our model individuals' ignorance of what would be reasonable for the average person, combined with their knowledge of their own idiosyncratic characteristics, opens the door for uniform standards to provide differentiated incentives on the cheap, without the state having to bear the costs of determining individuals' types, through the projection channel and the smoothing channel. In general then, our analysis suggests that lower levels of legal complexity are optimal than recent work on legal complexity implies.

3.2 Rules versus standards

Our analysis also provides a new perspective on a classic issue in legal design: the choice between rules and standards (Kaplow, 1992). All else equal, it is generally easier for individuals to learn a rule prior to making decisions than to learn what is required under a standard. One implication of our analysis, then, is that the greater substantive legal uncertainty under standards potentially provides a cheap way to induce differentiated behavior.

Our results thus provide a way to reconcile the old idea that standards have a comparative advantage over rules in differentiating behavior (Ehrlich and Posner, 1974; Kennedy, 1975; Schauer, 1991) with the trenchant critique of Kaplow (1992). Kaplow pointed out that this conventional wisdom was based on the implicit assumption that standards are more complex than rules. In principle, however, the complexity of a legal norm can be varied independently of whether it is a rule or a standard, and Kaplow cast doubt on the view that standards are generally more complex than rules. Even standards that ostensibly admit consideration of detailed nuances of the facts may not actually do so in operation. One reason is that, because standards apply case-by-case, it is often optimal ex post for an adjudicator or enforcement body to simplify and consider only the factors most likely to be important, since costly efforts to process information to identify more precisely the optimal legal consequences for the case will have little social benefit when the application of the standard (in its purest form) governs just the instant case. In contrast, in the case of a rule that will govern numerous cases going forward, it might be worthwhile to invest a great deal more in fine-tuning the legal regime, resulting in rule systems often being more complex and nuanced than standards are (in operation), contra the standard account contrasting simple rules with complex standards.

The relevant choice for legal design, then, is often between *simple* standards, on the one hand, and either complex or simple rules, on the other. In analyzing this choice, Kaplow (1992) assumes both that it is cheaper for individuals to learn about the content of rules than of standards, and that it is socially desirable for individuals to become informed since that results in behavior more in line with legal norms. An implication of these assumptions is that, in settings in which it is highly desirable for the state to differentiate incentives, typically the best way to do so is to deploy a complex rule.

But our analysis of differentiation through legal uncertainty puts the incentive advantages of standards, identified in the prior literature Kaplow was writing against, on firmer microeconomic foundations. In short, standards that in practice operate in a relatively simple, undifferentiated manner can nonetheless produce a usefully differentiated pattern of behavior both by smoothing out discontinuities in legal incentives and by inducing variation in beliefs about the law that correlates with what the law would ideally require. To achieve the same degree of differentiation in behavior using rules, in contrast, would require costly differentiation in legal consequences, since beliefs about rules track more closely the actual content of the rules. On the other hand, the projection channel we identify has an important limitation: it operates through individuals' formation of beliefs about the law based on some announced general standard, like "reasonableness." This reveals an advantage of rules over standards: rules can convey arbitrary requirements unconstrained by what the beliefs in the population would be about intuitive standards like "reasonableness." The horserace between simple standards and complex rules, then, turns in important part on how effective simple standards would be at differentiating behavior in a particular setting, on the one hand, and the information costs of the state in formulating, and of individuals in navigating, a complex system of rules, on the other.

3.3 Prices versus sanctions

Consider now the implications of our analysis for another fundamental aspect of legal design: the choice between "sanctions" that impose a detriment for doing what is forbidden (like the negligence rule) and "prices" that specify a payment for doing what is permitted (like strict liability). In principle, either legal form could induce optimal behavior in the absence of information costs. But in his classic analysis of this issue, Cooter (1984) points out that an optimal sanctions regime requires the state to obtain information on optimal behavior whereas an optimal pricing regime requires the state to obtain information about the external costs of the activity. Accordingly, he argues that lawmakers should impose a price to govern an activity if and only if it is cheaper for the state to obtain information about its external costs than to determine optimal behavior; otherwise they should deploy a sanction.

Apropos our analysis, Cooter points to heterogeneity in private costs and benefits to the individuals engaged in the activity as one reason it might be cheaper to price behavior than to determine appropriate behavioral norms. Our analysis of legal differentiation through uncertainty suggests that sanctions might not perform as badly as one might otherwise think when, owing to information costs, heterogeneous individuals are subject to a common behavioral standard in a sanctions regime. Both the smoothing channel and the projection channel provide potential ways for the state to achieve usefully differentiated incentives through a sanction even without bearing the costs of identifying the appropriate behavioral norm for each individual subject to the regime. This suggests that sanctions are useful in a wider range of circumstances than has been previously recognized.

On the other hand, the projection channel can also enable the state to economize on the costs of collecting information about external costs in a pricing regime. As we pointed out in Section 2.4 above, if individuals know the external cost they impose but not the average cost across individuals engaged in an activity, while the state knows the average external cost but not each individual's external cost, then a vague standard setting price equal to average harm—e.g., injurers will be held "appropriately responsible" for any external costs—can potentially result in lower total costs than if the state had to actually measure the external cost each individual imposes. Such a possibility increases the attractiveness of prices as a social matter.

4 Conclusion

In this paper we develop a new perspective on legal uncertainty by showing how it can serve as a valuable lubricant for the legal system. Legal uncertainty enables simple legal standards to provide differentiated incentives "on the cheap," sparing the costs of explicit legal differentiation, through what we call the smoothing channel and the projection channel. Although we develop these ideas using the reasonable person standard of tort law as a motivating example, this is not a torts paper; these mechanisms operate in material ways, we believe, in many bodies of law.

While we view our analysis as rehabilitating legal uncertainty, from a functional perspective, to a certain extent, we do not claim that legal uncertainty is always socially beneficial once its differentiating effects are recognized. Even in the model we use to develop our analysis, in which differentiation in incentives is socially desirable, we show that legal uncertainty has costs as well as benefits. Rather, we have developed what we view as primarily a positive analysis that should be incorporated into normative theorizing about legal design in settings that implicate the mechanisms we have identified. We sketch a few such applications, focusing on several classic normative debates about fundamental aspects of legal form, but leave a more complete treatment for future work.

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Appendix A

A.1 Proof of Proposition 1

If $c \leq \bar{c}$ we have $x^{FB}(c) \geq x^{FB}(\bar{c}) = s$ and hence these injurers minimize their total liability losses by choosing $x = s$. Any greater level of care would only increase their care costs—since they are not liable if $x \geq s$—while any lower level of care would result in greater liability because $l(x) + cx$ decreases in x for $x < x^{FB}(c)$ by convexity of the total cost function. If $c > \bar{c}$, we have $x^{FB}(c) < x^{FB}(\bar{c}) = s$ and hence these injurer make a discrete choice between being liable, $x = x^{FB}(c) < s$—which minimizes their costs among all $x < s$—and abiding by the due-care standard, $x = s$—which minimizes their costs for $x \geq s$. Accordingly, an injurer c takes due care if and only if

$$l\left(x^{FB}(c)\right) + cx^{FB}(c) \geq cs \tag{A.1}$$

and chooses $x^{FB}(c)$, otherwise.

Note that at $c = \bar{c}$, the LHS of A.1 is strictly greater than the RHS. Note as well that the RHS of this inequality increases strictly and linearly in c at a rate of $s = x^{FB}(\bar{c})$ while the LHS is concave in c, increasing at a rate of $x^{FB}(c)$ with second derivative equal to $\frac{dx^{FB}(c)}{dc} < 0$ (where we make use of the envelope theorem, i.e., substitute in using the first-order condition for $x^{FB}(c)$, so that one can ignore any change in $x^{FB}(c)$ when differentiating the LHS with respect to c). Our assumptions on $l(x)$ imply that $\lim_{c\to\infty} x^{FB}(c) = 0$. Therefore, the LHS must cross the RHS from above at a finite level of c denoted $\tilde{c} > \bar{c}$. Injurers with $c \leq \tilde{c}$ face lower total costs if they abide by due care and hence do so, while injurers with $c > \tilde{c}$ violate and choose $x^{FB}(c)$.

Under the specific functional-form assumption we made in constructing the figures in the paper, $l(x) = \frac{1}{x}$, we have that $\tilde{c} = 4\bar{c}$, as can be verified by replacing $l(x) = \frac{1}{x}$ and $s = x^{FB}(\bar{c}) = \bar{c}^{-\frac{1}{2}}$ in (A.1). $\square$

A.2 Proof of Proposition 2

To show that $x^*_\varepsilon(c)$ is continuous and decreases monotonically in c, we begin by transforming the injurers' minimization problem into a maximization problem using the negation of the injurer's objective function: $\max_{x\geq 0} \Pi(x; -c)$, where

$$\Pi(x; -c) = -(1 - F_\varepsilon(x))\, l(x) - cx.$$

It is easy to verify that $\lim_{x\to 0} \frac{\partial \Pi}{\partial x} > 0$ and $\lim_{x\to\infty} \frac{\partial \Pi}{\partial x} < 0$ so that $\max_x [\Pi(x; -c)]$ must have an interior solution, and so too the injurer's problem in (4). By the Implicit Function Theorem, since $\Pi(x; -c)$ is continuously differentiable in all its arguments and $\frac{\partial \Pi(x;-c)}{\partial(-c)} = x \neq 0$ whenever $x \neq 0$, we have that $x^*_\varepsilon(c)$ exists and is continuous in c. Moreover, since $\frac{\partial^2 \Pi}{\partial x \partial(-c)} = 1 > 0$, the objective function exhibits increasing marginal returns so that this interior solution is strictly monotone increasing in $-c$ by Theorem 1 in Edlin and Shannon (1998). It follows that $x^*_\varepsilon(c) = \arg\min_{x\geq 0} [-\Pi(x; -c)]$ is strictly monotone decreasing in c.

From the injurer's problem it is evident that $\lim_{c\to 0}\left[x_\varepsilon^*(c)\right]=\infty$ and $\lim_{c\to\infty}\left[x_\varepsilon^*(c)\right]=0$. Given monotonicity of $x_\varepsilon^*(c)$ this implies that there is a cutoff level $\hat{c}$ such that for $c<\hat{c}$ injurers over-comply and for $c>\hat{c}$ they under-comply.

From Proposition 4.4. in Shavell (1987, pp. 93-96) it follows that the injurer with average cost of care will over-comply—so that $\bar{c}<\hat{c}$—if the variance of the distribution of s, σ_ε^2 is not too large and will under-comply—so that $\bar{c}>\hat{c}$—otherwise. □

A.3 Proof of Proposition 3

With c lognormally distributed with parameters θ and σ^2, the average cost can be written as $\bar{c}=e^{\frac{\sigma^2}{2}}e^\theta$. Note that, since e^θ is lognormally distributed with posterior parameters $\mu_1(c)$ and σ_1^2, defined in (6) and (7) in the main text, $\bar{c}$ is the multiple of a lognormal variable and hence, using standard formulas, is itself lognormally distributed with parameters $\mu_1(c)+\frac{\sigma^2}{2}$ and σ_1^2, where $\mu_1(c)$ is an increasing function of c. In turn, the standard, $s=x^{FB}(\bar{c})=\bar{c}^{-\frac{1}{2}}$, is the power of a lognormal variable and hence is also lognormally distributed with parameters $\mu_s(c)=-\frac{1}{2}\left(\mu_1(c)+\frac{\sigma^2}{2}\right)$ and $\sigma_s^2=\frac{\sigma_1^2}{4}$. This proves the first claim in the proposition.

Next, we have $\frac{\partial\mu_s(c)}{\partial c}=-\frac{1}{2c}\frac{\sigma_0^2}{\sigma_0^2+\sigma^2}<0$ and hence $F_s(s|c)=\Phi\left(\frac{\log(s)-\mu_s(c)}{\sigma_s}\right)$ is increasing in c, where $\Phi(\cdot)$ is the CDF of the standard normal distribution. It follows that an increase in c causes a FOSD shift in the distribution, which proves the second claim in the proposition. □

A.4 Proof of Proposition 4

We want to show that $x_s^*(c)$ is monotonically decreasing in c. To do so we will use the sufficient condition for monotone comparative statics given in Proposition 6 in Quah and Strulovici (2009). We will adopt their notation too.

Let:

$$B(x)=-\left(1-F_s(x|c)\right)l(x)$$

be the benefit of care and

$$C(x)=cx$$

be the cost of care so that we can define the objective function $\Pi(x)=B(x)-C(x)$ for $x\in(0,\infty)$. The cost and benefit functions are differentiable and $C'(x)>0$, as required. Note that the injurer's loss function defined in (9) is equal to $-\Pi(x)$ and hence $x_s^*(c)$ maximizes $\Pi(x)$. To show that $x_s^*(c)$ is monotonically decreasing in c it is sufficient to show that $\tilde{\Pi}(x)$ dominates $\Pi(x)$ according to the interval-dominance order, where $\tilde{\Pi}(x)=\tilde{B}(x)-\tilde{C}(x)$ with

$$\tilde{B}(x)=-\left(1-F_s(x|\tilde{c})\right)l(x)$$

and

$$\tilde{C}(x)=\tilde{c}x$$

for $\tilde{c} < c$. According to Proposition 6 in Quah and Strulovici (2009), if $B'(x) > 0$, then to show interval dominance it is sufficient to show that there is a positive and increasing function $\alpha(x)$ such that

$$\frac{\tilde{B}'(x)}{B'(x)} \geq \alpha(x) \geq \frac{\tilde{C}'(x)}{C'(x)}.$$

for all $x \in (0, \infty)$. Since $\frac{\tilde{C}'(x)}{C'(x)} = \frac{\tilde{c}}{c} < 1$ we can set $\alpha(x) \equiv \frac{\tilde{B}'(x)}{B'(x)}$ and show that the following three statements hold true for all $x \in (0, \infty)$:

$$B'(x) > 0, \tag{A.2}$$

$$\frac{d}{dx}\left(\frac{\tilde{B}'(x)}{B'(x)}\right) \geq 0, \tag{A.3}$$

and

$$\frac{\tilde{B}'(x)}{B'(x)} \geq 1. \tag{A.4}$$

Note that $F_s(x|c) = \Phi(z)$ and $f_s(x|c) = \frac{1}{x\sigma_s}\phi(z)$ for $z = \frac{\log(x) - \mu_s(c)}{\sigma_s}$, where ϕ and Φ are the PDF and the CDF of the standard normal distribution. Similarly, let $\tilde{z} = \frac{\log(x) - \mu_s(\tilde{c})}{\sigma_s}$. Using $\frac{dz}{dx} = \frac{1}{x\sigma_s}$ and $l(x) = \frac{1}{x}$, we can write:

$$B'(x) = [1 - \Phi(z)]\frac{1}{x^2} + \phi(z)\frac{1}{x^2\sigma_s} > 0.$$

which verifies the condition in (A.2).

Next, we can rewrite the ratio of the marginal benefits as follows:

$$\frac{\tilde{B}'(x)}{B'(x)} = \frac{1 - \Phi(\tilde{z}) + \phi(\tilde{z})\frac{1}{\sigma_s}}{1 - \Phi(z) + \phi(z)\frac{1}{\sigma_s}}.$$

Note that $\frac{dz}{dx} = \frac{d\tilde{z}}{dx} = \frac{1}{x\sigma_s}$. So, by the quotient rule, the following condition is sufficient for $\frac{d}{dx}\left(\frac{\tilde{B}'(x)}{B'(x)}\right) \geq 0$:

$$\left[-\phi(\tilde{z}) + \frac{1}{\sigma_s}\phi'(\tilde{z})\right]\left[1 - \Phi(z) + \phi(z)\frac{1}{\sigma_s}\right] > \left[-\phi(z) + \frac{1}{\sigma_s}\phi'(z)\right]\left[1 - \Phi(\tilde{z}) + \phi(\tilde{z})\frac{1}{\sigma_s}\right].$$

Now note that $\phi'(z) = -z\phi(z)$, so that the condition becomes:

$$\phi(\tilde{z})\left[1 + \frac{\tilde{z}}{\sigma_s}\right]\left[1 - \Phi(z) + \phi(z)\frac{1}{\sigma_s}\right] < \phi(z)\left[1 + \frac{z}{\sigma_s}\right]\left[1 - \Phi(\tilde{z}) + \phi(\tilde{z})\frac{1}{\sigma_s}\right].$$

Now we multiply the LHS by $\frac{\phi(z)}{\phi(z)}$ and the RHS by $\frac{\phi(\tilde{z})}{\phi(\tilde{z})}$ and simplify the result to obtain:

$$\left[1+\frac{\tilde{z}}{\sigma_s}\right]\left[\frac{1-\Phi\left(z\right)}{\phi\left(z\right)}+\frac{1}{\sigma_s}\right]<\left[1+\frac{z}{\sigma_s}\right]\left[\frac{1-\Phi\left(\tilde{z}\right)}{\phi\left(\tilde{z}\right)}+\frac{1}{\sigma_s}\right]. \tag{A.5}$$

Since we have $\tilde{z}<z$ for $\tilde{c}<c$, we have $1+\frac{\tilde{z}}{\sigma_s}<1+\frac{z}{\sigma_s}$. Moreover, $\frac{1-\Phi(\cdot)}{\phi(\cdot)}$ is the Mill's ratio and is known to be strictly decreasing in its argument.[12] Thus, for $z>\tilde{z}$ we must have

$$\frac{1-\Phi\left(z\right)}{\phi\left(z\right)}+\frac{1}{\sigma_s}<\frac{1-\Phi\left(\tilde{z}\right)}{\phi\left(\tilde{z}\right)}+\frac{1}{\sigma_s}.$$

Combining the latter two observations proves that the inequality in (A.5) holds and hence verifies the condition in (A.3). Finally, note that, since $\lim_{x\to 0}\Phi\left(\frac{\log(x)-\mu_s(\tilde{c})}{\sigma_s}\right)=0$ and $\lim_{x\to 0}\phi\left(\frac{\log(x)-\mu_s(\tilde{c})}{\sigma_s}\right)=0$, we have:

$$\lim_{x\to 0}\left(\frac{\tilde{B}'\left(x\right)}{B'\left(x\right)}\right)=\lim_{x\to 0}\left(\frac{1-\Phi\left(\frac{\log(x)-\mu_s(\tilde{c})}{\sigma_s}\right)+\phi\left(\frac{\log(x)-\mu_s(\tilde{c})}{\sigma_s}\right)\frac{1}{\sigma_s}}{1-\Phi\left(\frac{\log(x)-\mu_s(c)}{\sigma_s}\right)+\phi\left(\frac{\log(x)-\mu_s(c)}{\sigma_s}\right)\frac{1}{\sigma_s}}\right)=1. \tag{A.6}$$

Therefore, since $\frac{\tilde{B}'(x)}{B'(x)}$ is increasing in x according to (A.3), the limit in (A.6) implies that we must have $\frac{\tilde{B}'(x)}{B'(x)}\geq 1$ for all x, which shows that the condition in (A.4) is verified and completes the proof. □

Appendix B

We show here that our main results go through qualitatively unchanged in the model of negligence with incremental damages introduced by Grady (1983) and formalized by Kahan (1989). In this model, there is no discontinuity in the injurer's cost function under certainty. Damages are equal to the harm that is caused by the injurer's negligence, that is, the harm that actually materializes minus the harm that would have occurred anyway had the injurer been nonnegligent, $l\left(x\right)-l\left(s\right)$. Yet, injurers' marginal total costs (i.e., expected liability costs plus costs of care) are still discontinuous due to the kink in the injurer's costs at $x=s$. This feature of the model dilutes incentives to take care and leads to systematically lower levels of care compared to the basic setup considered above.

B.1 Incremental damages with no uncertainty

If the standard is known, injurers face the following costs:

$$\min_{x\geq 0}\begin{cases} cx & \text{if} \quad x\geq s \\ l\left(x\right)-l\left(s\right)+cx & \text{if} \quad x<s \end{cases}$$

which leads to the following proposition.

[12] See Baricz (2008, pp. 1362-1363).

Proposition B.1. *With incremental damages, under a known standard of care s, in equilibrium injurers choose the following levels of care:*

$$x^K(c) = \begin{cases} s & \textit{if} \quad c \leq \bar{c} \\ x^{FB}(c) & \textit{if} \quad c > \bar{c} \end{cases}$$

Proof. Conditional on being negligent, the injurer minimizes $l(x) - l(s) + cx$, which results in the injurer taking care equal to $x^{FB}(c)$. If $x^{FB}(c) \geq s$—that is, if $c \leq \bar{c}$—the injurer is nonnegligent at the first-best level of care and hence the injurer's costs are minimized by reducing care to $x = s$. If instead $x^{FB}(c) < s$—that is, if $c > \bar{c}$—the injurer chooses between the standard, $x = s$, and the first-best level of care, $x = x^{FB}(c) < s$. The latter yields lower total costs if $cs > l\left(x^{FB}(c)\right) - l(s) + cx^{FB}(c)$, which can be written as $l(s) + cs > l\left(x^{FB}(c)\right) + cx^{FB}(c)$. It is easy to see that the inequality is always satisfied by definition of $x^{FB}(c)$ so that the injurer chooses $x = x^{FB}(c)$ in this case. □

Differently from the basic setup (see Proposition 1), in this model injurers either comply with the standard and are under-deterred or violate it and take first-best care. There is no over-deterrence or over-compliance in this case, and a smaller fraction of the population of injurers bunches at the due-care standard.

B.2 Incremental damages with legal noise

With legal noise injurers minimize:

$$\min_{x \geq 0} \left[\int_x^\infty \left[l(x) - l(s)\right] f_\varepsilon(s)\, ds + cx \right],$$

which yields the following FOC:

$$-\left(1 - F_\varepsilon\left(x_\varepsilon^K(c)\right)\right) l'\left(x_\varepsilon^K(c)\right) = c. \tag{B.1}$$

Proposition B.2. *With incremental damages, under the reasonable person standard with legal noise*

1. *The equilibrium level of care $x_\varepsilon^K(c)$ is strictly lower than the first-best level of care $x^{FB}(c)$, continuous, and strictly monotonically decreasing in the injurer's cost of care, c.*

2. *Injurers with costs of care below a threshold, $c < \bar{c}(1 - F_\varepsilon(s))$, over-comply with the standard, $x_\varepsilon^K(c) > s$, while injurers with costs of care above the threshold, $c > \bar{c}(1 - F_\varepsilon(s))$, under-comply, $x_\varepsilon^K(c) < s$.*

3. *The threshold $\bar{c}(1 - F_\varepsilon(s))$ is lower than $\hat{c}$ in Proposition 2 and is always below the average cost of care, $\bar{c}$, so that the average injurer under-complies.*

Proof. Note the difference between (B.1) and the corresponding FOC of the basic model in (5). Due to the continuity of the injurer's cost function, the term $f_\varepsilon(x)\, l(x)$—which produces incentives towards over-deterrence in the basic model—is missing from (B.1), which leads to under-deterrence for all injurers (i.e., $x_\varepsilon^K(c) < x^{FB}(c)$). The chosen level of care, $x_\varepsilon^K(c)$, decreases in c because the cross-partial derivative is positive,

$$\frac{dx_\varepsilon^K(c)}{dc} = -\frac{1}{(1 - F_\varepsilon(x))\, l''(x) - f_\varepsilon(x)\, l'(x)} < 0,$$

and the SOC is always satisfied. Comparing care taken with due care, there is under-compliance if

$$(1 - F_\varepsilon(s))\, l'(s) + c > 0 \tag{B.2}$$

and over-compliance otherwise. Recall that $s = x^{FB}(\bar{c})$, which implies (from the first-order condition that determines $x^{FB}(c)$) that $\bar{c} = -l'(s)$. Substituting that into (B.2) yields $c > \bar{c}(1 - F_\varepsilon(s))$. Under a known standard the threshold for under-compliance was $\bar{c}$ (see Proposition B.1), which is greater than the threshold $\bar{c}(1 - F_\varepsilon(s))$, so that, under negligence with incremental damages, uncertainty dilutes incentives.

To see that $\bar{c}(1 - F_\varepsilon(s)) < \hat{c}$ (which was the threshold in Proposition 2) note that $\hat{c}$ is such that

$$(1 - F_\varepsilon(s))\, l'(s) - f_\varepsilon(s)\, l(s) + \hat{c} = 0,$$

which compared with the left-hand side of (B.2) yields the result. □

Comparing Proposition B.2 with Proposition B.1 it is easy to see that (1) the smoothing channel is operative in the incremental damages model with legal noise, resulting in more differentiated incentives for injurers to take care; and (2) uncertainty unambiguously dilutes incentives to take care. Furthermore, comparing Proposition B.2 with the corresponding Proposition 2 in the basic model, we can appreciate that incremental damage result in systematically lower levels of care compared to full damages, which results in under-deterrence (compared to the first-best level of care) but may still yield to under- or over-compliance (compared with the due level of care under the standard).

B.3 Incremental damages with updating

The updating process is as in the basic model and hence Proposition 3 applies unchanged to the model with incremental damages. With updating, injurers minimize:

$$\min_x \left[\int_x^\infty (l(x) - l(s))\, f_s(s|c)\, ds + cx\right],$$

which leads to the FOC

$$-\left(1 - F_s\left(x_s^K(c)\,|c\right)\right) l'\left(x_s^K(c)\right) = c$$

and the following proposition, mirroring Proposition 4.

Proposition B.3. *With incremental damages, under the reasonable person standard with updating, injurers' levels of care are differentiated:* $\frac{\partial x_s^K(c)}{\partial c} < 0$.

Proof. The SOC is always satisfied and the cross-partial derivative is positive because of FOSD (See Proposition 3). Therefore, we have a unique and interior solution to the injurer's minimization problem, and monotonicity in c:

$$\frac{dx_s^K(c)}{dc} = -\frac{-\frac{d}{dc}F_s(x|c)\, l'(x) + 1}{(1 - F_s(x|c))\, l''(x) - f_s(x|c)\, l'(x)} < 0.$$

□