Disagreement About the Term Structure of Inflation Expectations

Hie Joo Ahn[1] Leland E. Farmer[2]

[1]Federal Reserve Board

[2]University of Virginia

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Contributions

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   - Disagreement as a measure of anchored expectations
3. Identify three sources of disagreement: **prior beliefs, private information, and heterogeneous responses to public information**
   - Heterogeneous responses to public information: reducible by monetary policy
4. Investigate the role of disagreement and its sources in the transmission of monetary policy

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4. When **public information** plays a large role, 1) Fed communication is effective at reducing disagreement, and 2) traditional monetary policy **has a larger effect on real outcomes, but with a delay**, and **is not effective at stabilizing inflation.**

Outline

1. Data
2. Model
3. Estimation Results
4. Three Sources of Disagreement
5. Implications for Policy
6. Conclusion

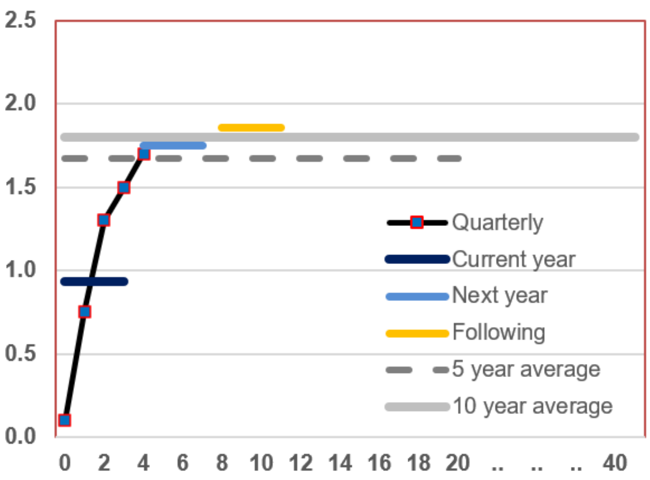
Data

Survey of Professional Forecasters

- Quarterly survey which is filled out in the middle of each quarter
- We focus on forecasts of seasonally adjusted **CPI** inflation ▸ CPI Inflation
- Sample period : 1991:Q4 – 2023:Q3
  - Long-run expectations (10-year average expectations) are available from 1991:Q4.
- There are 38 forecasters in a given quarter on average.
- Forecasters appear in the data set for 27 quarters on average. ▸ Forecasters
- Drop forecasters who appear for fewer than 12 quarters (Patton & Timmermann (2010))
- Total of 101 forecasters who report an average of 44 forecasts

Incomplete Picture of the Term Structure

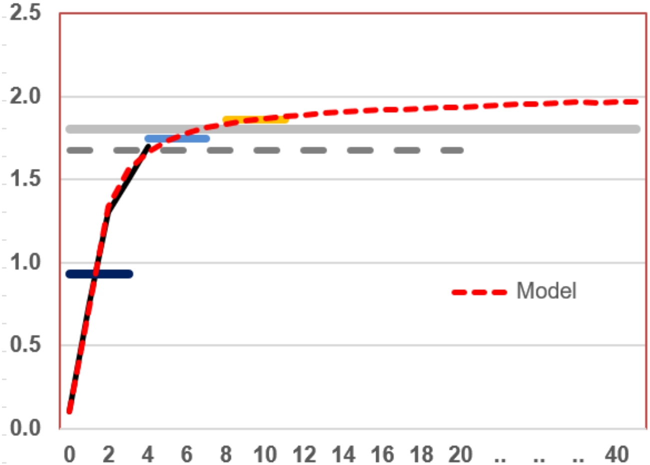
In quarter t, forecaster i submits a set of



- **Fixed-horizon forecasts:** current – 4-quarter ahead
- **Fixed-event forecasts**: current, next, and the following year
- **Fixed-event forecasts:** Averages of 5 years and 10 years including the current year

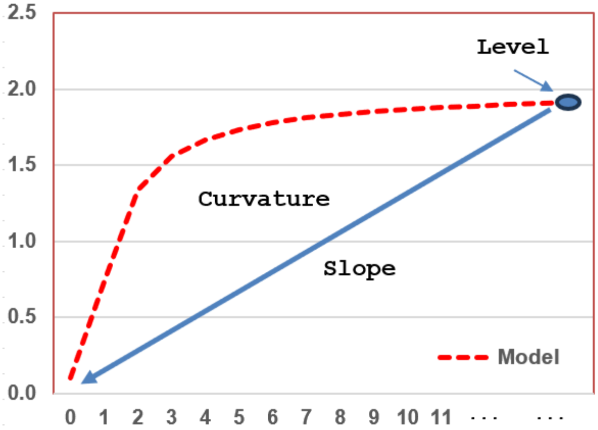
Model

Individual Term Structure of Inflation Expectations



- Infer the term-structure of annualized inflation forecasts from 10 total observations of each forecaster at each point in time
- Rich patterns in inflation expectations across different participants including downward sloping curves

Parsimonious and Flexible Characterization: Nelson-Siegel Model

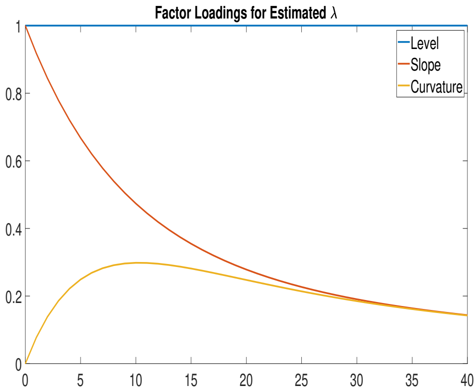


- **Level** : long-end forecast
- **Slope** : difference between long-end forecast and nowcast (current quarter)
- **Curvature** : nonlinearity not fully captured with the slope

Nelson-Siegel Model

Aruoba (2020) for average inflation expectations between t and $t+h$.

$$\pi_{i,t\to t+h|t} = L_{i,t} - \left(\frac{1-e^{-\lambda_i h}}{\lambda_i h}\right) S_{i,t} + \left(\frac{1-e^{-\lambda_i h}}{\lambda_i h} - e^{-\lambda_i h}\right) C_{i,t}$$



- $\pi_{i,t\to t+h|t}$: forecaster i's annualized forecast of continuously compounded inflation between t and $t+h$ given time t information
- $L_{i,t}$, $S_{i,t}$, and $C_{i,t}$ are forecaster-specific level, slope, and curvature components
- λ_i: the peak of the curvature loading
- Curve fitting → panel datasets of factors

Modeling the Individual Factors

- Following Diebold *et al.* (2008), we model the individual factors as

$$L_{i,t} = \alpha_{i,L} + \beta_{i,L}\mathbf{L_t} + \varepsilon_{i,L,t}$$
$$S_{i,t} = \alpha_{i,S} + \beta_{i,S}\mathbf{S_t} + \varepsilon_{i,S,t}$$
$$C_{i,t} = \alpha_{i,C} + \beta_{i,C}\mathbf{C_t} + \varepsilon_{i,C,t}$$

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- The *common* and *idiosyncratic* factors are allowed to follow general VAR dynamics.
- We estimate the state space model with standard Bayesian methods (Gibbs sampling)

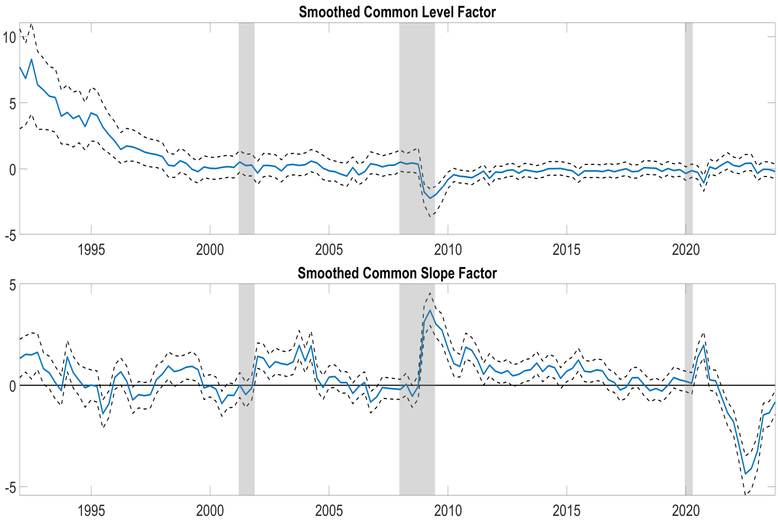
▸ State-space model

Simplifying Assumptions in Baseline Model

- We omit the curvature factor given the large number of forecasters we are estimating parameters for.
- Common and idiosyncratic factors are assumed to follow independent AR(1) processes.
- Parameters same across i
  1. Shape parameter $\lambda_i = \lambda$ (Diebold *et al.* (2008))
  2. The dynamics of the idiosyncratic components: auto-correlation and variance parameters
  3. The variance of measurement errors
- Still 431 parameters to estimate
- Results are robust to a model which includes curvature factors and allows for AR(3) dynamics in the factors.

Estimation Results

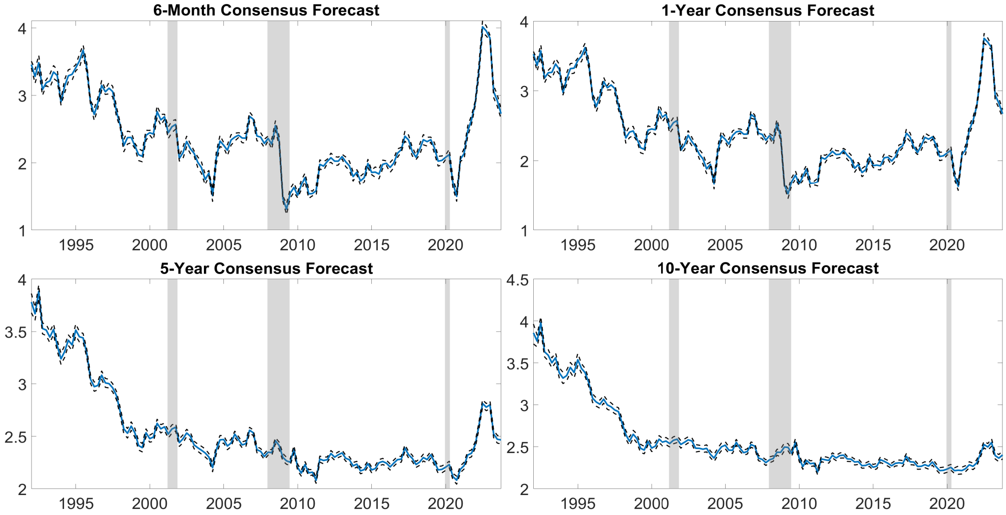
Factor Estimates ($\hat{L}_{t|T}$ and $\hat{S}_{t|T}$)



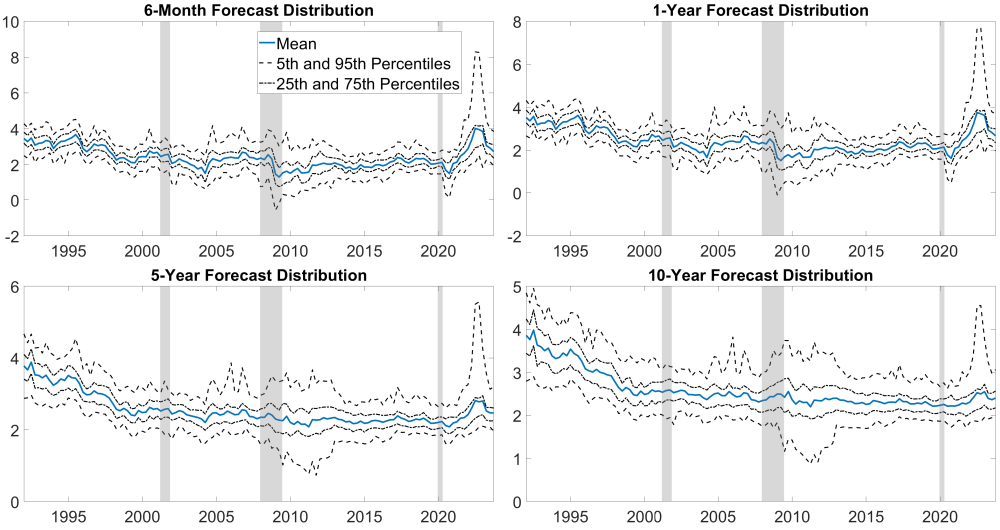
- **Common level ($\hat{L}_{t|T}$) :** inflation trend, stable during the pandemic
- **Common slope ($\hat{S}_{t|T}$) :** transitory changes, positive (upward), negative (downward)

Mean Forecast Estimates

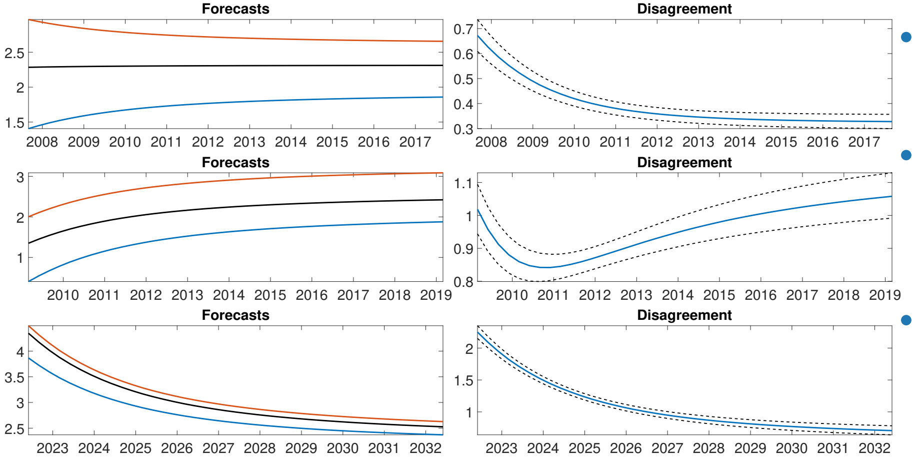
Long-run inflation expectations seem to be well anchored. **Really?**



Disagreement Paints a More Nuanced Picture



The Term Structure Across Forecasters and Over Time



- **2007 :**
  ⇑ Short run
- **2009 :**
  ⇑ Both short and long run
- **2022 :**
  ⇑ Short run

Three Sources of Disagreement

Three Sources of Disagreement: Factors

- We decompose an individual factor, e.g. $L_{i,t}$, into three components (same for $S_{i,t}$):

$$L_{i,t} = \underbrace{\alpha_{\mathbf{i,L}}}_{\text{prior beliefs}} + \underbrace{\beta_{\mathbf{i,L}}\mathbf{L_t}}_{\text{common}} + \underbrace{\varepsilon_{\mathbf{i,L,t}}}_{\text{idiosyncratic}} = \mathbf{L_i^{pb}} + \mathbf{L_{i,t}^{c}} + \mathbf{L_{i,t}^{id}}.$$

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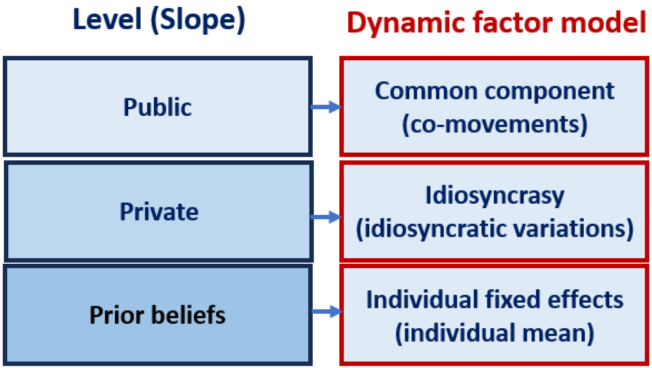
- Plug the three components back into the Nelson-Siegel model → Decompose the forecasts themselves into the three components

$$\begin{aligned}
\pi_{i,t\to t+h|t} &= L_{i,t} - \left(\frac{1-e^{-\lambda h}}{\lambda h}\right) S_{i,t} \\
&= \underbrace{L_i^{pb} - \left(\frac{1-e^{-\lambda h}}{\lambda h}\right) S_i^{pb}}_{\text{prior belief component}} + \underbrace{L_{i,t}^{c} - \left(\frac{1-e^{-\lambda h}}{\lambda h}\right) S_{i,t}^{c}}_{\text{public}} + \underbrace{L_{i,t}^{id} - \left(\frac{1-e^{-\lambda h}}{\lambda h}\right) S_{i,t}^{id}}_{\text{private}} \\
&= \pi_{i,t\to t+h|t}^{pb} + \pi_{i,t\to t+h|t}^{c} + \pi_{i,t\to t+h|t}^{id}
\end{aligned}$$

Three Sources of Disagreement – Dynamic Factor Decomposition



- Sung (2023): An individual's memory (cognitive noise) makes the interpretation of public information different across forecasters → This dynamic factor structure creates over- or under- reactions to public information across forecasting horizons

Disagreement Decomposition by Information Source

$$\pi_{i,t\to t+h|t} = = \pi^{pb}_{i,t\to t+h|t} + \pi^{c}_{i,t\to t+h|t} + \pi^{id}_{i,t\to t+h|t}$$
$$Var_i(\pi_{i,t\to t+h|t}) \approx Var_i(\pi^{pb}_{i,t\to t+h|t}) + Var_i(\pi^{c}_{i,t\to t+h|t}) + Var_i(\pi^{id}_{i,t\to t+h|t})$$

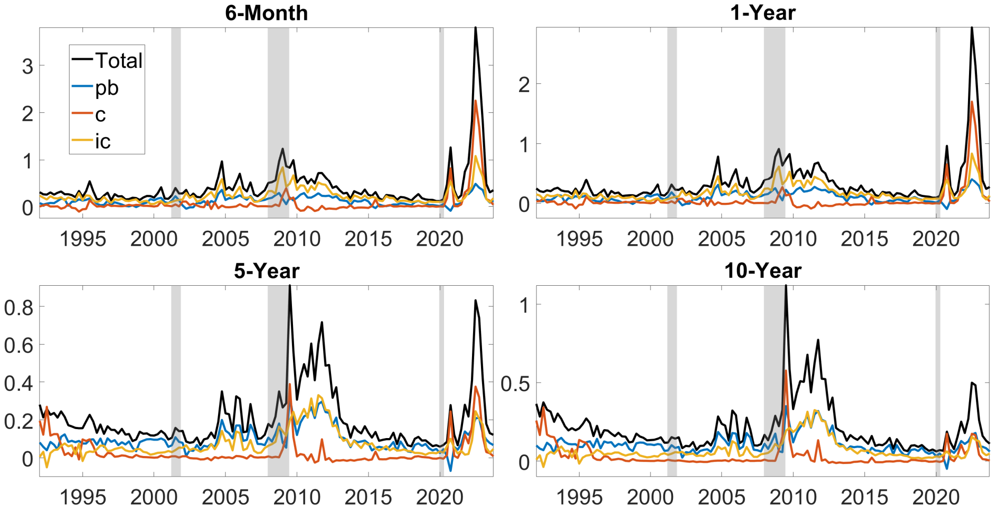
Taking the covariance of both sides with $\pi_{i,t\to t+h|t}$ and dividing through by the variance of $\pi_{i,t\to t+h|t}$, we obtain the following expression:

$$1 = \beta^{pb}_{h,t} + \beta^{c}_{h,t} + \beta^{id}_{h,t},$$

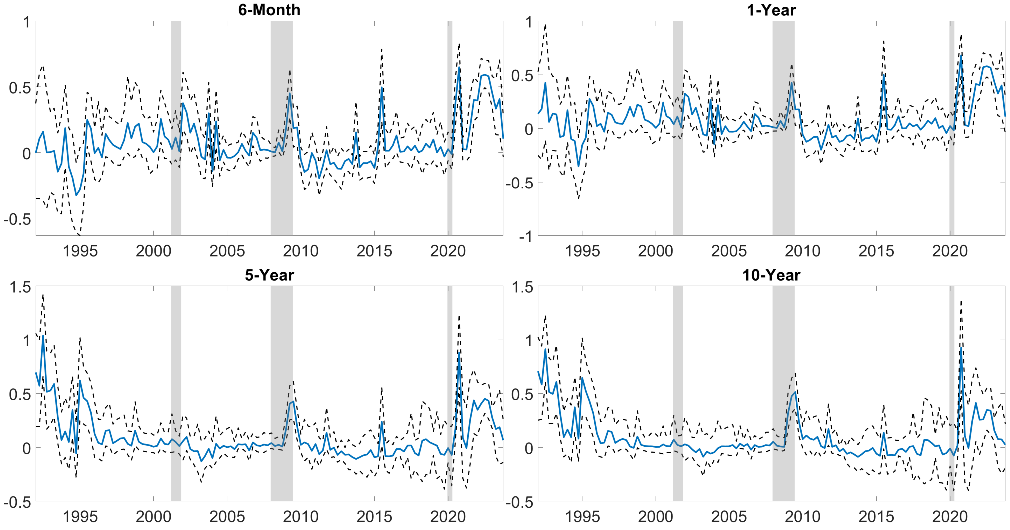
$$\beta^{pb}_{h,t} = \frac{\text{Cov}_i\left(\pi_{i,t\to t+h|t}, \pi^{pb}_{i,t\to t+h|t}\right)}{\text{Var}_i(\pi_{i,t\to t+h|t})} \qquad \beta^{c}_{h,t} = \frac{\text{Cov}_i\left(\pi_{i,t\to t+h|t}, \pi^{c}_{i,t\to t+h|t}\right)}{\text{Var}_i(\pi_{i,t\to t+h|t})}$$
$$\beta^{id}_{h,t} = \frac{\text{Cov}_i\left(\pi_{i,t\to t+h|t}, \pi^{id}_{i,t\to t+h|t}\right)}{\text{Var}_i(\pi_{i,t\to t+h|t})}.$$

Disagreement shares: The sensitivity of disagreement to three information sources

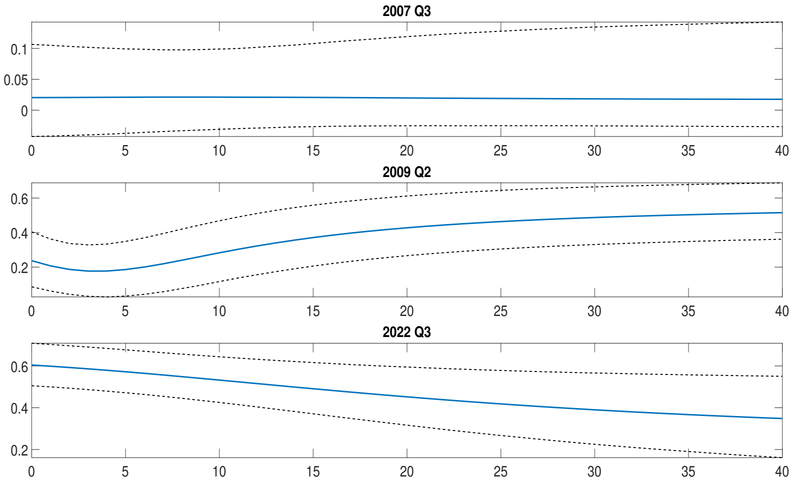
Prior Beliefs (Long-Run) and Private Information (Short-Run)



Disagreement Share of Public Information



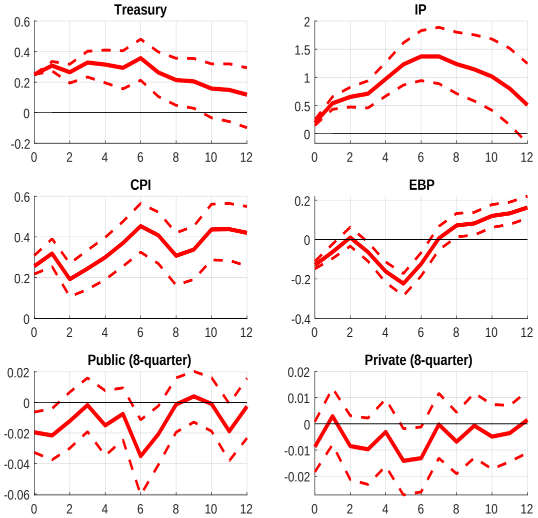
Disagreement Share of Public Information: 2009 and 2022



- **2007** :
  Flat and close to zero
- **2009** :
  $\Uparrow$ Both short and long run, upward sloping
- **2022** :
  $\Uparrow$ Both short and long run, downward sloping

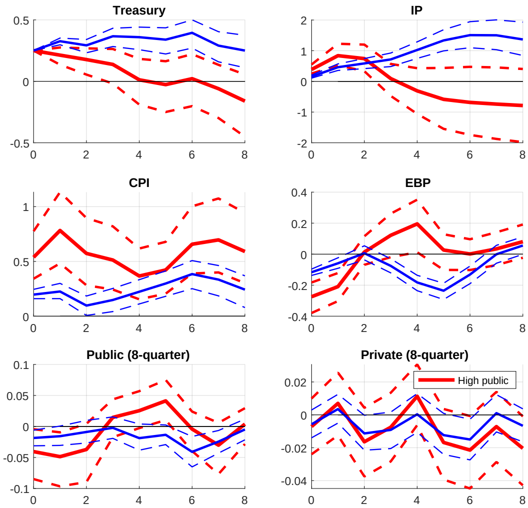
Implications for Monetary Policy

Fed's Response to News Reduces Disagreement About Public Info.



- LP with external shock with Fed's response to news (Bauer & Swanson, 2022) ▸ Shocks ▸ Model
- The news component reduces the disagreement about inflation 8 quarters ahead attributable to public information
- but **not** the portion attributatle to private information and prior beliefs
- Robust with SVAR-IV ▸ SVAR-IV

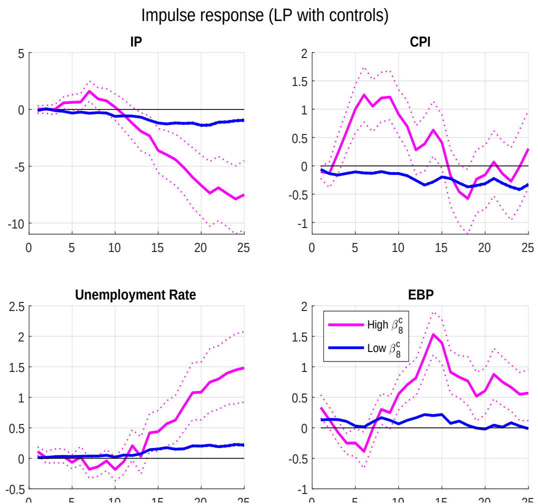
Larger Effects When Attention to Public Information is High



- State dependence: interaction of Fed's response to news with the fraction of public information out of the sum of dispersions driven by the three sources ($h = 8$)
- Larger effects (red) when the contribution of public information to disagreement increases → High attention to public information.

SVAR-IV

High Attention to Public Information Affects Transmission



- LP with orthogonalized MP shocks (Bauer & Swanson, 2022) ▸ Shocks ▸ Model
- Interaction of shocks with $\hat{\beta}^c_{8,t}$ (1-$\hat{\beta}^c_{8,t}$) to capture the MP effects when the sensitivity of disagreement to public (private) information is dominant.
- The real effects are larger with higher $\hat{\beta}^c_{8,t}$, but with a delay. ▸ Level
- No stabilization of inflation

Conclusion

1. We develop a new parametric model of the individual term structure of inflation expectations and recover the term structure of disagreement over time.
2. We decompose disagreement into three sources: prior beliefs, heterogeneous responses to public information, and private information.
3. Prior beliefs and private information explain the bulk of disagreement since the early 1990s, but in periods of high inflation uncertainty, heterogeneous responses to public information are the primary driver of disagreement.
4. Fed communication can reduce the portion of disagreement driven by public information.
5. Traditional monetary policy has large real effects, with a delay, in times when public information is the primary driver of disagreement, but is not effective at stabilizing inflation.

Appendix

Robustness

1. **Time-varying loadings** : The common component (loading $\times$ factor) is identified (Dempster *et al.*, 1977).
   - Time-varying loadings are not ruled out.
   - Loadings on the common factors ($\beta_{i,L}$, $\beta_{i,S}$, and $\beta_{i,C}$) are estimated for each individual i.

2. **Covid-19 pandemic** : The Covid shock could potentially distort model estimates.
   - The estimates prior to the pandemic are stable even after the Covid observations are folded in.

3. **Non-parametric model** : Detail
   - Model the individual term structure with polynomials.
   - Estimate the common factor with a dynamic factor model (Banbura & Modugno, 2014)

Estimation : State space model + Bayesian method

State Space Model

Goal: Estimate the common factors and the idiosyncratic component that are dynamic unobserved (latent) variables along with time-invariant parameters.

State equation: Describes the dynamics of latent variables

$$\mathbf{x}_t = \mathbf{F}\mathbf{x}_{t-1} + \mathbf{e}_t \qquad \mathbf{e}_t \sim N(0, \mathbf{Q})$$

Measurement equation: Maps the unobserved variables to observables

$$\mathbf{Y}_t = \mathbf{H}\mathbf{x}_t + \mathbf{r}_t \qquad \mathbf{r}_t \sim N(0, \mathbf{R})$$

Inference on the unobserved variables via Kalman filter, maximum likelihood estimation.

Factor Decomposition

State Equation (1)

We assume that the common factors follow a VAR(1) process:

$$\begin{bmatrix} L_t \\ S_t \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} L_{t-1} \\ S_{t-1} \end{bmatrix} + \begin{bmatrix} u_{L,t} \\ u_{S,t} \end{bmatrix}. \tag{1}$$

Since the levels of the common factors and the factor loadings are not separately identified, we normalize the shocks to the common factors $u_{L,t}$ and $u_{S,t}$, to have unit variance. We assume the shocks are uncorrelated:

$$\begin{bmatrix} u_{L,t} \\ u_{S,t} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \tag{2}$$

We assume that the idiosyncratic factors follow VAR(1) processes so that

$$\begin{bmatrix} \varepsilon_{i,L,t} \\ \varepsilon_{i,S,t} \end{bmatrix} = \begin{bmatrix} b_{i,11} & 0 \\ 0 & b_{i,22} \end{bmatrix} \begin{bmatrix} \varepsilon_{i,L,t-1} \\ \varepsilon_{i,S,t-1} \end{bmatrix} + \begin{bmatrix} u_{i,L,t} \\ u_{i,S,t} \end{bmatrix} \tag{3}$$

Return

State Equation (2)

In our baseline specification we assume that all of the VAR coefficients and covariance matrices are the same across forecasters, and that the covariance matrix is diagonal:

$$\begin{bmatrix} u_{i,L,t} \\ u_{i,S,t} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_L^2 & 0 \\ 0 & \sigma_S^2 \end{bmatrix} \right) \tag{4}$$

Let the state vector x_t be defined as

$$x_t = \begin{bmatrix} L_t, S_t, \varepsilon_{1,L,t}, \varepsilon_{1,S,t}, \cdots, \varepsilon_{n,L,t}, \varepsilon_{n,S,t} \end{bmatrix}$$

Define the transition matrix F to be

$$F = \begin{bmatrix} a_{11} & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & a_{22} & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & b_{11} & 0 & 0\dots & 0 & 0 \\ 0 & 0 & 0 & b_{22} & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & b_{11} & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & b_{22} \end{bmatrix}$$

State Equation (3): Full Representation

Define the state equation shocks to be

$$u_t = \left[\ u_{L,t}, u_{S,t}, u_{1,L,t}, u_{1,S,t}, \cdots, u_{n,L,t}, u_{n,S,t} \ \right]$$

The covariance matrix of the shocks is given by:

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \sigma_L^2 & 0 & 0\dots & 0 & 0 \\ 0 & 0 & 0 & \sigma_S^2 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & \sigma_L^2 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & \sigma_S^2 \end{bmatrix}$$

State Equation:

$$x_t = F x_{t-1} + u_t, \quad u_t \sim N(0, Q)$$

Measurement Equation (1)

The observation vector in any period, y_t is given by

$$
\begin{aligned}
y_t^{fh} &= \left[\ \pi_{1,t\to t+1|t}, \pi_{1,t+1\to t+2|t}, \pi_{1,t+2\to t+3|t}, \pi_{1,t+3\to t+4|t}\ \right]' \quad \text{fixed-horizon} \\
y_t^{fe(short)} &= \left[\ \begin{array}{r} \pi_{1,t+3\to t+7|t}, \pi_{1,t+2\to t+6|t}, \pi_{1,t+1\to t+5|t}, \pi_{1,t\to t+4|t}, \\ \pi_{1,t\to t+7|t+11}, \pi_{1,t+6\to t+10|t}, \pi_{1,t+5\to t+9|t}, \pi_{1,t+4\to t+8|t} \end{array}\ \right]' \quad \text{fixed-event (short)} \\
y_t^{fe(long)} &= \left[\ \begin{array}{r} \pi_{1,t\to t+19|t}, \pi_{1,t\to t+18|t}, \pi_{1,t\to t+17|t}, \pi_{1,t\to t+16|t}, \\ \pi_{1,t\to t+39|t}, \pi_{1,t\to t+38|t}, \pi_{1,t\to t+37|t}, \pi_{1,t\to t+36|t}, \\ \cdots, \pi_{n,t\to t+37|t}, \pi_{n,t\to t+36|t} \end{array}\ \right]' \quad \text{fixed-event (long)} \\
y_t &= \left[\ y_t^{fh}{}', y_t^{fe(short)}{}', y_t^{fh(long)}{}'\ \right]'
\end{aligned}
$$

Only four of the final sixteen elements of y_t are observed in any given quarter. These final sixteen elements correspond to fixed event forecasts, where each group of four corresponds, in order, one calendar year-ahead average inflation, two calendar yea-ahead average inflation, five-year average inflation and ten-year average inflation including the current calendar year. Return

Measurement Equation (2)

For the final eight elements, which correspond to forecasts of average inflation over five and ten year periods including the current calendar year, we must adjust them to directly match our model's output. Specifically

$$
\begin{aligned}
\textbf{Q1} \quad \pi_{i,t\to t+19|t} &= \frac{4}{19}\left(5\pi_{i,t-1\to t+19|t} - \frac{1}{4}\pi_{i,t-1\to t|t}\right) \\
\pi_{i,t\to t+39|t} &= \frac{4}{39}\left(10\pi_{i,t-1\to t+19|t} - \frac{1}{4}\pi_{i,t-1\to t|t}\right) \\
\textbf{Q2} \quad \pi_{i,t\to t+18|t} &= \frac{4}{18}\left(5\pi_{i,t-2\to t+18|t} - \frac{1}{4}\pi_{i,t-1\to t|t} - \frac{1}{4}\pi_{i,t-2\to t-1|t}\right) \\
\pi_{i,t\to t+38|t} &= \frac{4}{38}\left(10\pi_{i,t-2\to t+38|t} - \frac{1}{4}\pi_{i,t-1\to t|t} - \frac{1}{4}\pi_{i,t-2\to t-1|t}\right) \\
\textbf{Q3} \quad \pi_{i,t\to t+17|t} &= \frac{4}{17}\left(5\pi_{i,t-3\to t+17|t} - \frac{1}{4}\pi_{i,t-1\to t|t} - \frac{1}{4}\pi_{i,t-2\to t-1|t} - \frac{1}{4}\pi_{i,t-3\to t-2|t}\right) \\
\pi_{i,t\to t+37|t} &= \frac{4}{37}\left(10\pi_{i,t-3\to t+17|t} - \frac{1}{4}\pi_{i,t-1\to t|t} - \frac{1}{4}\pi_{i,t-2\to t-1|t} - \frac{1}{4}\pi_{i,t-3\to t-2|t}\right) \\
\textbf{Q4} \quad \pi_{i,t\to t+16|t} &= \frac{4}{16}\left(5\pi_{i,t-4\to t+16|t} - \frac{1}{4}\pi_{i,t-1\to t|t} - \frac{1}{4}\pi_{i,t-2\to t-1|t} - \frac{1}{4}\pi_{i,t-3\to t-2|t} - \frac{1}{4}\pi_{i,t-4\to t-3|t}\right) \\
\pi_{i,t\to t+36|t} &= \frac{4}{36}\left(10\pi_{i,t-4\to t+16|t} - \frac{1}{4}\pi_{i,t-1\to t|t} - \frac{1}{4}\pi_{i,t-2\to t-1|t} - \frac{1}{4}\pi_{i,t-3\to t-2|t} - \frac{1}{4}\pi_{i,t-4\to t-3|t}\right)
\end{aligned}
$$

Measurement Equation (3)

The loading function on the slope factor for forecasts of inflation between $t + h_1$ and $t + h_2$ as

$$f_S(h_1, h_2) = \frac{e^{-\lambda h_1} - e^{-\lambda h_2}}{\lambda(h_2 - h_1)}$$

Final state space system

$$y_t = \mu_y + H x_t + v_t, \quad v_t \sim N(0, R)$$

Measurement Equation (4)

$$
y_t := \begin{bmatrix}
\pi_{1,t\to t+1|t} \\
\pi_{1,t+1\to t+2|t} \\
\pi_{1,t+2\to t+3|t} \\
\pi_{1,t+3\to t+4|t} \\
\pi_{1,t+3\to t+7|t} \\
\pi_{1,t+2\to t+6|t} \\
\pi_{1,t+1\to t+5|t} \\
\pi_{1,t\to t+4|t} \\
\pi_{1,t\to t+7|t+11} \\
\pi_{1,t+6\to t+10|t} \\
\pi_{1,t+5\to t+9|t} \\
\pi_{1,t+4\to t+8|t} \\
\pi_{1,t\to t+19|t} \\
\pi_{1,t\to t+18|t} \\
\pi_{1,t\to t+17|t} \\
\pi_{1,t\to t+16|t} \\
\pi_{1,t\to t+39|t} \\
\pi_{1,t\to t+38|t} \\
\pi_{1,t\to t+37|t} \\
\pi_{1,t\to t+36|t} \\
\vdots \\
\pi_{n,t\to t+37|t} \\
\pi_{n,t\to t+36|t}
\end{bmatrix}
\quad
\mu_y := \begin{bmatrix}
\alpha_{1,L} - \alpha_{1,S}fS(0,1) \\
\alpha_{1,L} - \alpha_{1,S}fS(1,2) \\
\alpha_{1,L} - \alpha_{1,S}fS(2,3) \\
\alpha_{1,L} - \alpha_{1,S}fS(3,4) \\
\alpha_{1,L} - \alpha_{1,S}fS(3,7) \\
\alpha_{1,L} - \alpha_{1,S}fS(2,6) \\
\alpha_{1,L} - \alpha_{1,S}fS(1,5) \\
\alpha_{1,L} - \alpha_{1,S}fS(0,4) \\
\alpha_{1,L} - \alpha_{1,S}fS(7,11) \\
\alpha_{1,L} - \alpha_{1,S}fS(6,10) \\
\alpha_{1,L} - \alpha_{1,S}fS(5,9) \\
\alpha_{1,L} - \alpha_{1,S}fS(4,8) \\
\alpha_{1,L} - \alpha_{1,S}fS(0,19) \\
\alpha_{1,L} - \alpha_{1,S}fS(0,18) \\
\alpha_{1,L} - \alpha_{1,S}fS(0,17) \\
\alpha_{1,L} - \alpha_{1,S}fS(0,16) \\
\alpha_{1,L} - \alpha_{1,S}fS(0,39) \\
\alpha_{1,L} - \alpha_{1,S}fS(0,38) \\
\alpha_{1,L} - \alpha_{1,S}fS(0,37) \\
\alpha_{1,L} - \alpha_{1,S}fS(0,36) \\
\vdots \\
\alpha_{n,L} - \alpha_{n,S}fS(0,37) \\
\alpha_{n,L} - \alpha_{n,S}fS(0,36)
\end{bmatrix}
$$

$$
H := \begin{bmatrix}
\beta_{1,L} & -\beta_{1,S}fS(0,1) & 1 & fS(0,1) & \dots & 0 & 0 \\
\beta_{1,L} & -\beta_{1,S}fS(1,2) & 1 & fS(1,2) & \dots & 0 & 0 \\
\beta_{1,L} & -\beta_{1,S}fS(2,3) & 1 & fS(2,3) & \dots & 0 & 0 \\
\beta_{1,L} & -\beta_{1,S}fS(3,4) & 1 & fS(3,4) & \dots & 0 & 0 \\
\beta_{1,L} & -\beta_{1,S}fS(3,7) & 1 & fS(3,7) & \dots & 0 & 0 \\
\beta_{1,L} & -\beta_{1,S}fS(2,6) & 1 & fS(2,6) & \dots & 0 & 0 \\
\beta_{1,L} & -\beta_{1,S}fS(1,5) & 1 & fS(1,5) & \dots & 0 & 0 \\
\beta_{1,L} & -\beta_{1,S}fS(0,4) & 1 & fS(0,4) & \dots & 0 & 0 \\
\beta_{1,L} & -\beta_{1,S}fS(7,11) & 1 & fS(7,11) & \dots & 0 & 0 \\
\beta_{1,L} & -\beta_{1,S}fS(6,10) & 1 & fS(6,10) & \dots & 0 & 0 \\
\beta_{1,L} & -\beta_{1,S}fS(5,9) & 1 & fS(5,9) & \dots & 0 & 0 \\
\beta_{1,L} & -\beta_{1,S}fS(4,8) & 1 & fS(4,8) & \dots & 0 & 0 \\
\beta_{1,L} & -\beta_{1,S}fS(0,19) & 1 & fS(0,19) & \dots & 0 & 0 \\
\beta_{1,L} & -\beta_{1,S}fS(0,18) & 1 & fS(0,18) & \dots & 0 & 0 \\
\beta_{1,L} & -\beta_{1,S}fS(0,17) & 1 & fS(0,17) & \dots & 0 & 0 \\
\beta_{1,L} & -\beta_{1,S}fS(0,16) & 1 & fS(0,16) & \dots & 0 & 0 \\
\beta_{1,L} & -\beta_{1,S}fS(0,39) & 1 & fS(0,39) & \dots & 0 & 0 \\
\beta_{1,L} & -\beta_{1,S}fS(0,38) & 1 & fS(0,38) & \dots & 0 & 0 \\
\beta_{1,L} & -\beta_{1,S}fS(0,37) & 1 & fS(0,37) & \dots & 0 & 0 \\
\beta_{1,L} & -\beta_{1,S}fS(0,36) & 1 & fS(0,36) & \dots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
\beta_{n,L} & -\beta_{n,S}fS(0,37) & 0 & 0 & \dots & 1 & fS(0,37) \\
\beta_{n,L} & -\beta_{n,S}fS(0,36) & 0 & 0 & \dots & 1 & fS(0,36)
\end{bmatrix}
$$

$$
v_t := \begin{bmatrix}
v_{1,1,t} \\ v_{1,2,t} \\ v_{1,3,t} \\ v_{1,4,t} \\ v_{1,5,t} \\ v_{1,6,t} \\ v_{1,7,t} \\ v_{1,8,t} \\ v_{1,9,t} \\ v_{1,10,t} \\ v_{1,11,t} \\ v_{1,12,t} \\ v_{1,13,t} \\ v_{1,14,t} \\ v_{1,15,t} \\ v_{1,16,t} \\ v_{1,17,t} \\ v_{1,18,t} \\ v_{1,19,t} \\ v_{1,20,t} \\ \vdots \\ v_{n,19,t} \\ v_{n,20,t}
\end{bmatrix}
\quad
R := \text{diag}\left(\begin{bmatrix}
\sigma^2_{v,1} \\ \sigma^2_{v,2} \\ \sigma^2_{v,3} \\ \sigma^2_{v,4} \\ \sigma^2_{v,5} \\ \sigma^2_{v,6} \\ \sigma^2_{v,7} \\ \sigma^2_{v,8} \\ \sigma^2_{v,9} \\ \sigma^2_{v,10} \\ \sigma^2_{v,11} \\ \sigma^2_{v,12} \\ \sigma^2_{v,13} \\ \sigma^2_{v,14} \\ \sigma^2_{v,15} \\ \sigma^2_{v,16} \\ \sigma^2_{v,17} \\ \sigma^2_{v,18} \\ \sigma^2_{v,19} \\ \sigma^2_{v,20} \\ \dots \\ \sigma^2_{v,19} \\ \sigma^2_{v,20}
\end{bmatrix}\right)
$$

Measurement Equation: Nelson-Siegel Model

- Map the individual term structure characterization to the individual-level data

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- For estimation we use 1Q-4Q ahead fixed horizon forecasts, and next year, following year, 5-year average, and 10-year average fixed event forecasts

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- For the 5-year and 10-year average forecasts, we use observed nowcasts and 1Q backcasts when available, and realized inflation of the most recent CPI vintage for 2Q and 3Q prior inflation → The fixed event forecasts are treated separately in each quarter throughout the calendar year.

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- For the 5-year and 10-year average forecasts, we use observed nowcasts and 1Q backcasts when available, and realized inflation of the most recent CPI vintage for 2Q and 3Q prior inflation → The fixed event forecasts are treated separately in each quarter throughout the calendar year.
- Each reported forecast is assumed to be observed with a measurement error, but the variances of measurement errors are same for all i.

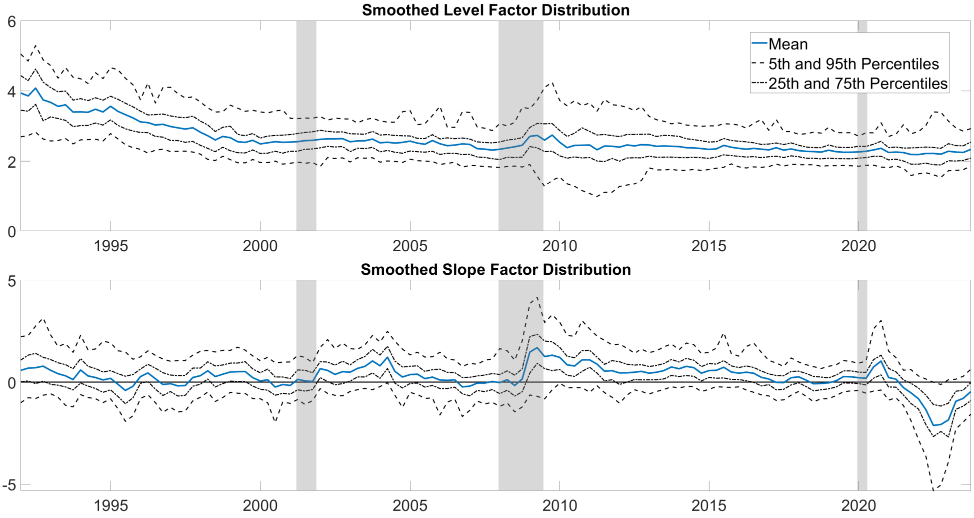
▸ Details

Parameters: Estimated with a Bayesian method (Gibbs sampling)

- Our baseline model has a total of 431 parameters consisting of
  - Forecaster-specific means $\{\alpha_{i,L}, \alpha_{i,S}\}_{i=1}^{n}$
  - Forecaster-specific factor loadings $\{\beta_{i,L}, \beta_{i,S}\}_{i=1}^{n}$
  - Factor autocorrelation parameters a_{11}, a_{22}, b_{11}, and b_{22}
  - Idiosyncratic factor conditional variances σ_L^2 and σ_S^2
  - Shape parameter λ
  - Measurement error variances $\sigma_{v,1}, \ldots, \sigma_{v,20}^2$
- The parameter vector is denoted as

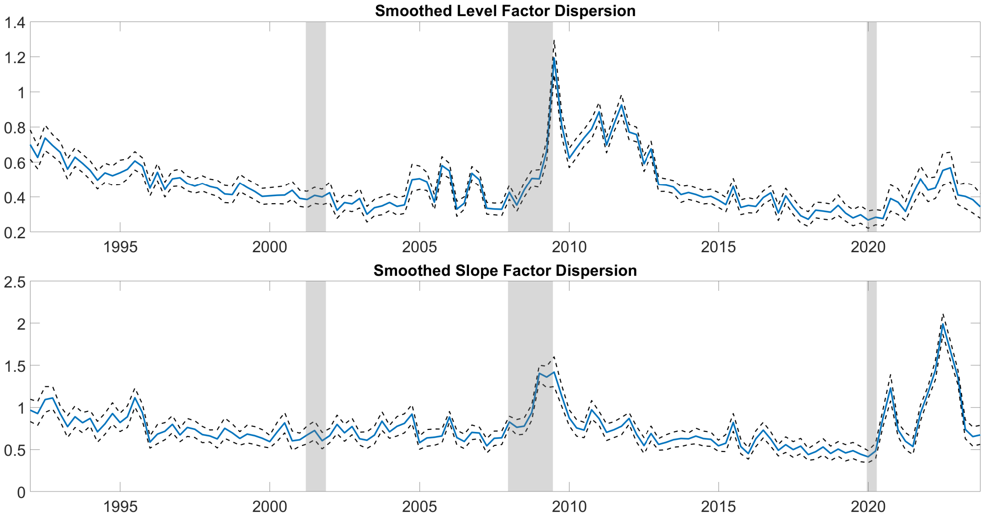
$$\theta = [\alpha_{1,L}, \ldots, \alpha_{n,S}, \beta_{1,L}, \ldots, \beta_{n,S}, a_{11}, a_{22}, b_{11}, b_{22}, \sigma_L^2, \sigma_S^2, \lambda, \sigma_{v,1}^2, \ldots, \sigma_{v,20}^2]'$$

Substantial Disagreement About Level and Slope ($\hat{C}_{it|T}$ and $\hat{S}_{it|T}$)



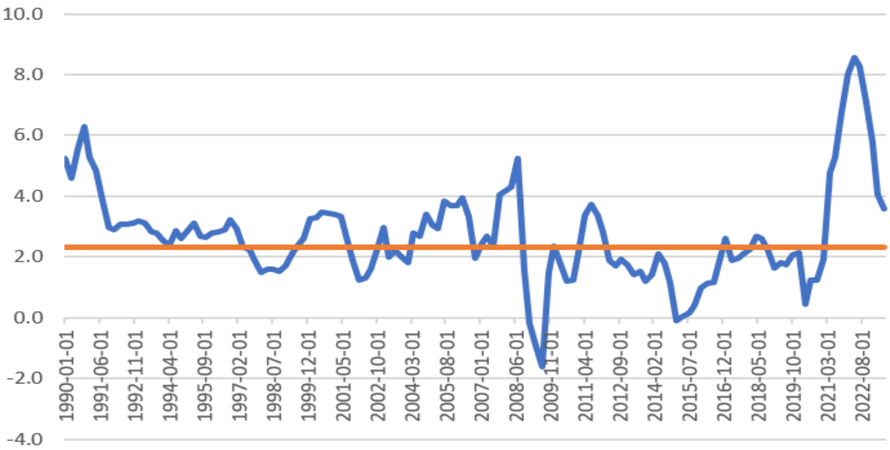
▸ Standard deviation

Disagreement About Level and Slope ($std_i(\hat{C}_{it|T})$ and $std_i(\hat{S}_{it|T})$)



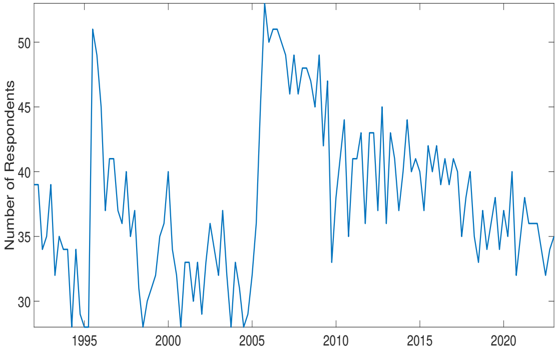
Return

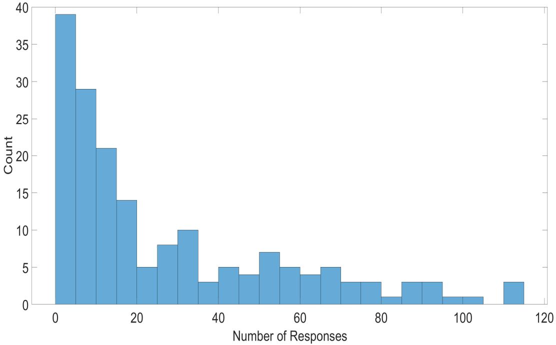
Headline CPI inflation (year-over-year % change)



Return

Survey of Professional Forecasters





Return

Average 10-year-ahead CPI Inflation (Fisher *et al.*, 2022)

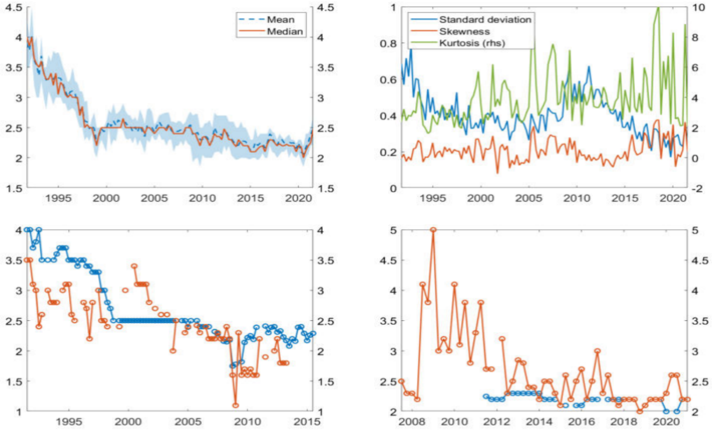


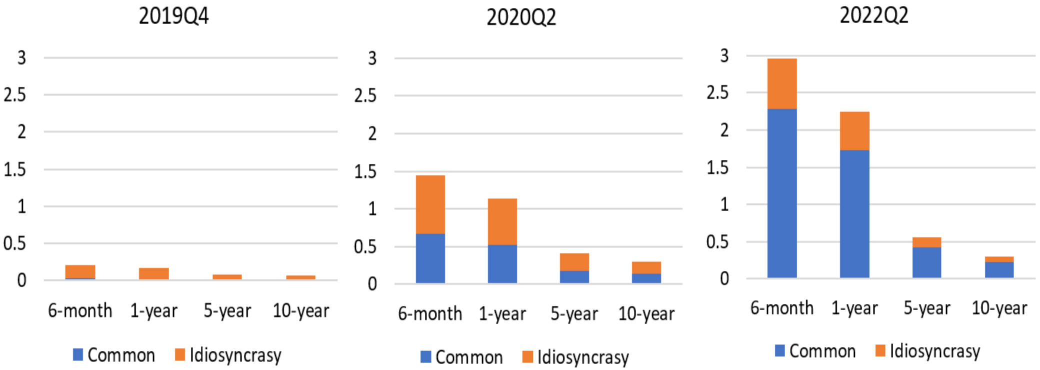
Figure 1: Time series summary of 10-year CPI inflation expectations

Notes: Top left chart shows mean and median together with the interquartile range, top right chart shows higher moments. The two bottom charts show the time series of 4 individual forecasters.

▸ Return

Evolution of Term Structure of Disagreement : Covid-19 Pandemic

- **Public** information explains the bulk of increased disagreement.
  - Disagreement about inflation persistence and inflation trend
  - Disagreement about the likelihood of recessions (soft landing vs. hard landing)



Variance Decomposition

For the variance decomposition, we compute the partial R^2.

- Compute total R^2 by regressing the forecasts on a constant, the idiosyncratic component, and the common component.
- To obtain the variance attributable to the common component, regress the forecasts on a constant and the idiosyncratic component, then I subtract the R^2 of this regression from the R^2 in the previous regression.

▸ Return

SVAR-IV with external shocks (Stock and Watson, 2018)

A reduced-form VAR:

$$Y_t = \alpha + B(L)Y_{t-1} + u_t$$

estimate the model for 1991:Q4-2019:Q4 via OLS with 4 lags, where

$$u_t = S\epsilon_t.$$

The first element of ϵ_t is ϵ_t^{mp}. The first column of S (s_1) captures the impact of ϵ_t^{mp} on Y_t.

Order the two-year Treasury yield first in Y_t and denote it by Y_t^{2y}.

$$Y_t = \tilde{\alpha} + \tilde{B}(L)Y_{t-1} + s_1 Y_t^{2y} + \tilde{u}_t.$$

Estimated with 2SLS with z_t as the instrument for Y_t^{2y}. The first element in s_1 is 1 for normalization. With the estimated s_1 and $B(L)$, we calculate the impulse responses. ▸ Return

SVAR-IV with external shocks (Stock and Watson, 2018)

Six variables

1. the log of industrial production
2. the log of the consumer price index
3. the excess bond premium from Gilchrist & Zakrajšek (2012)
4. the two-year treasury yield
5. two disagreement estimates driven by public and private information.

External shocks capturing news component in MP

1. Fed's reactions to economic news from Bauer & Swanson (2022)

Return

Table 1 from Bauer & Swanson (2022)

Table 1: Predictive Regressions Using Macroeconomic and Financial Data

	(1)	(2)	(3)	(4)
Nonfarm payrolls	0.094	0.113	0.082	0.155
	(2.442)	(1.977)	(1.806)	(3.696)
Empl. growth (12m)	0.005	0.004	0.005	0.003
	(2.108)	(1.404)	(1.184)	(1.512)
Δ log S&P 500 (3m)	0.084	0.112	0.154	0.020
	(1.433)	(1.578)	(1.931)	(0.351)
Δ Slope (3m)	-0.010	-0.010	-0.011	-0.017
	(-1.406)	(-1.153)	(-1.049)	(-2.041)
Δ log Comm. price (3m)	0.120	0.093	0.225	0.103
	(2.392)	(1.461)	(3.527)	(1.946)
Treasury skewness	0.032	0.035	0.050	0.023
	(3.006)	(2.917)	(2.109)	(2.137)
R^2	0.161	0.173	0.192	0.163
Sample	1988:1–2019:12	1994:1–2019:12	1988:1–2007:6	1990:1–2019:6
N	322	218	216	259
Policy surprise	*mps*	*mps*	*mps*	FF4

Coefficient estimates β from predictive regressions $mps_t = \alpha + \beta' X_{t-} + \varepsilon_t$, where t indexes FOMC announcements. Columns (1)–(3) use our baseline monetary policy surprise measure *mps* described in the text, while column (4) uses the change in FF4 (also used in Gertler and Karadi, 2015). Predictors X are observed prior to the FOMC announcement: the surprise component of the most recent nonfarm payrolls release, employment growth over the last year, the log change in the S&P500 from 3 months before to the day before the FOMC announcement, the change in the yield curve slope over the same period, the log change in a commodity price index over the same period, and the option-implied skewness of the 10-year Treasury yield from Bauer and Chernov (2021). Heteroskedasticity-consistent t-statistics in parentheses. See text for details.

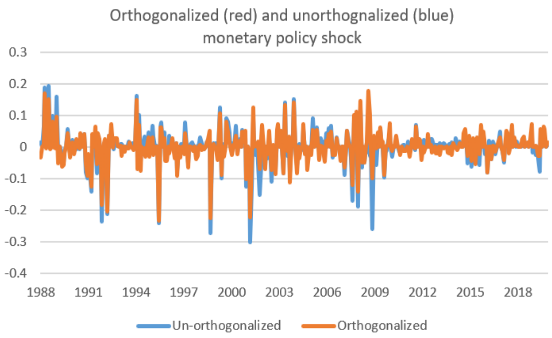
Return

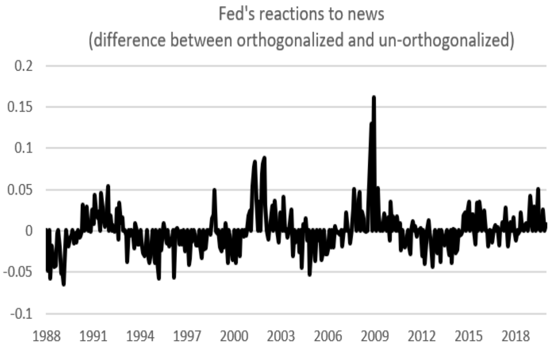
Bauer and Swanson's Orthogonalized Monetary Policy Shocks

- Unorthogonalized: High-frequency monetary policy surprises (FOMC announcements)

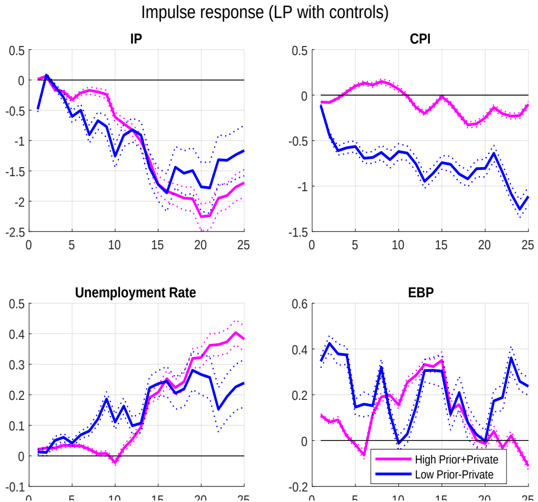
  Computed as the first principal component of the changes in euro-dollar future contracts (current to three-quarter ahead), scaled so that the impact on the three-quarter ahead contract is unity.

- Orthogonalized: *Orthogonalized* with respect to the news variables (regressing the monthly surprises on the news variables, see the next page) Return





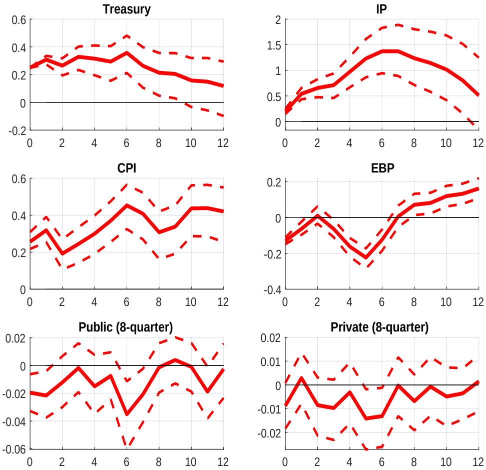
Disagreement Level Itself Does Not Create the Difference



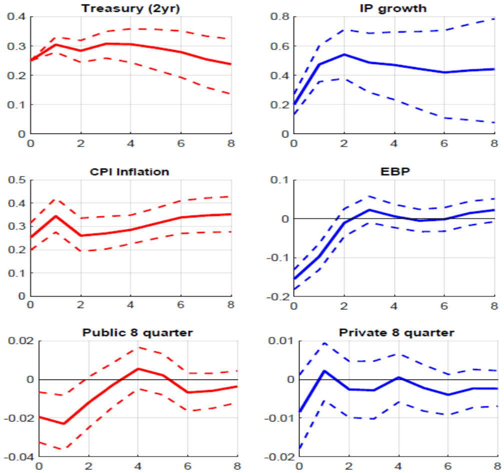
- Interacted the MP shocks with the high/low regime of total disagreement (h=8) to capture the MP effects in times of high (low) diagreement.
- The sensitivity of disagreement to public information is the driver, not the level of disagreement. Return

Effects of Fed's Response to News: LP-IV vs. SVAR-IV

LP-IV

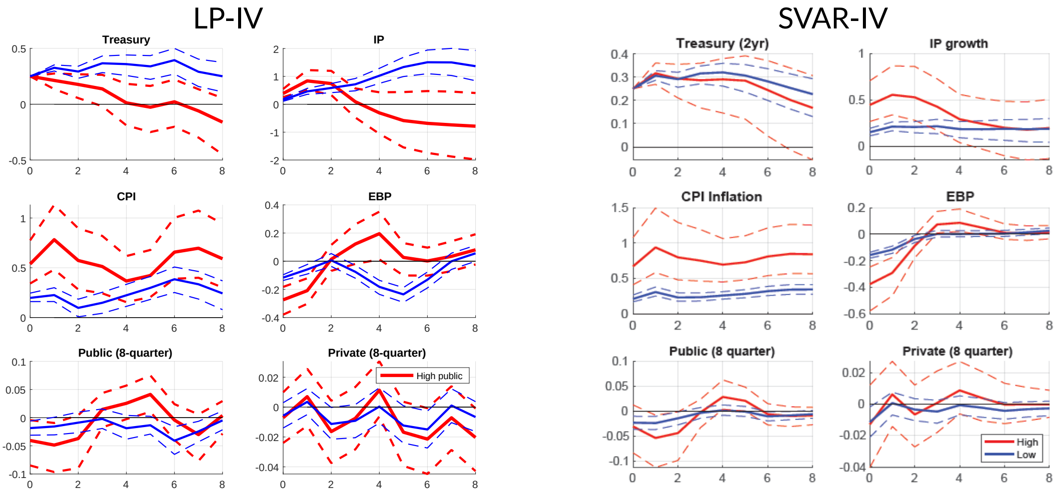


SVAR-IV



Return

Nonlinear Effects of Fed's Response to News: LP-IV vs. SVAR-IV



Return

Implications for Theory

Table: Disagreement in the models of expectation formation

	FIRE	Sticky Information	Noisy info. (Same)	Noisy info. (Different)	Disagreement about means
Scope of disagreement	X	✓	✓	✓	✓
Permanent heterogeneity	X	X	X	X	✓
Changing idiosyncratic disagreement	X	X	✓	X	X
Countercyclical common disagreement	X	✓	X	✓	X
Forecast-horizon differences	X	X	X	X	X

▸ Return

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