Are New Keynesian Models Useful When Trend Inflation is Not Low?*

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Abstract

The equilibrium in the standard New Keynesian (NK) model with Calvo-pricing becomes explosive at low levels of trend inflation (between 4 to 7 percent). Even halfway before that threshold, optimal prices, price dispersion and costs rise fast to very large levels, and output plummets. We show that the root of these issues is not Calvo pricing as commonly assumed, but rather the popular Dixit-Stiglitz demand structure in NK models. Considering models with general firms' demand functions, we provide two important results: (i) regardless of the price setting behavior, i.e. time- or state-dependent, marginal costs rapidly increasing with trend inflation is a direct consequence of demand functions that fast rise at low relative prices; and (ii) under Calvo pricing, the condition for NK models to always have a stable equilibrium, independently of the level of trend inflation, is that the demand function does not increase unboundedly as relative prices decrease. The Dixit-Stiglitz demand structure fails to satisfy the latter condition. We then propose a model with price wedges to augment any existing demand structure and make them in line with those conditions. Using Dixit-Stiglitz and Kimball-demand aggregators, we show that the generalized NK model with price wedges allows price dispersion to rise slowly with trend inflation and avoids output plummeting to zero. In addition, the implied demand function with price wedges has relatively superior properties, better aligned with the micro and macro evidence.

Keywords: New Keynesian models, Calvo pricing, trend inflation, steady state problem, demand functions.

JEL Codes: E31, E32, E52

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1 Introduction

The literature on positive trend inflation has long recognized that the standard New Keynesian (NK) model with Calvo (1983) price setting does not have a stable solution if trend inflation exceeds some low single-digit threshold (in the 4% to 7% range).[1] Even half-way before that threshold where the steady state ceases to exist is reached, optimal prices, price dispersion and costs all rise fast to very large levels, and output plummets. These problems mean that NK framework cannot be used readily during periods of sustained high inflation as experienced recently in the U.S. and many advanced countries. Most of the existing literature agrees that these properties arise from the assumption of Calvo pricing which implies that a forward-looking firm might not receive the exogenous signal to re-optimize its price for a long period of time, even though with low probability. At the same time, Calvo pricing to characterize nominal rigidity remains popular for monetary policy analysis. So ad hoc remedies like indexation or mechanically increasing price-adjustment frequency are typically adopted to avoid the issues, both of these have little empirical support. Thus, the non-existence of the steady state—the *steady state problem*—remains embedded at the core of NK models.

In this paper, we show that the steady state problem under trend inflation arises not because of the Calvo pricing assumption but due to another modelling assumption commonly used in macroeconomic models, namely, the Dixit and Stiglitz (1977) constant-elasticity-of-substitution (CES) consumption aggregator. This assumption leads to a tractable constant-elasticity demand function for all goods in the economy and allows for adding monopolistic competition to macroeconomic models. However, it is actually in its implied demand function that the root of the steady state problem lies, as it diverges to infinite relative demand when relative prices approach zero. This property simultaneously creates several problems for the NK model: First, it creates a threshold (an upper bound) in the level of trend inflation consistent with the existence of a steady state for plausible model parameters; second, it causes the marginal costs of non-readjusting firms and price dispersion to rise sharply when trend inflation approaches the upper bound; third, it makes the output level fall rapidly (almost vertically) as trend inflation approaches this limit.

To demonstrate the role of the popular CES demand structure in generating the steady state problem, we start by considering a model with a general demand structure, in the spirit of Gagliardone,

[1] While the threshold is somewhat higher (7-9%) in models with strategic substitutability in pricing decisions such as King and Wolman (1996) and Ascari (2004), when strategic complementarities are present, as recommended in Woodford (2003), Bakhshi, Burriel-Llombart, Khan and Rudolf (2007) show that the threshold becomes quite low, about 4-5%, especially if output growth is also taken into consideration. See the first part of Section 4 for an analytical discussion on this threshold. Cogley and Sbordone (2008) find that in the US, the time-varying trend in inflation was never above 5% (their Figure 1) between 1960 and 2003, and hence the condition for the existence of the steady state is satisfied for this low trend inflation period.

Gertler, Lenzu and Tielens (2023). With this general structure, we show that the marginal costs of non-readjusting firms do not increase rapidly with trend inflation as relative prices decrease. Most importantly, we show that this result is independent of the price-setting nature, i.e. time- or state-dependent.

Next, when considering Calvo pricing, we prove that the condition for steady-state equilibrium to always exist *independently* of the level of trend inflation is that general demand structures remain finite when relative prices approach zero. The CES demand structure fails to satisfy this condition, which is the source of the steady state problem. This is the main theoretical result of our paper.[2]

We then show that our result can be applied to any preference structure or demand aggregator. In models with the Kimball (1995) aggregator, for example, Kurozumi and Van Zandweghe (2016, 2024) note that positive trend inflation does not cause the steady state problem. We provide the underlying reason why the Kimball aggregator avoids the steady state problem — it does so only when the curvature is sufficiently large. But the empirical evidence does not support large curvature, as shown in Klenow and Willis (2016) and Dossche, Heylen and Van Den Poel (2010). Assuming a large curvature leads to an additional issue. It truncates the distribution support of the relative prices and, therefore, cannot match the price distribution observed in microdata. Thus, simply replacing the Dixit-Stiglitz demand structure with the Kimball aggregator does not offer a compelling solution to the steady state problem.

We, therefore, propose a novel approach to augment existing demand function derivations to make NK models consistent with *any* level of trend inflation. The method can be applied to any demand structure, including commonly used ones like the Dixit and Stiglitz (1977) or the Kimball (1995). Our premise is that agents never face infinite demand, as consuming requires extra costs that creates a wedge between the sticker price and the effective price. Those costs can rise either from direct monetary causes or from efforts, which then can be translated into a monetary price. And more importantly, they might be resilient even if the sticker price is set at zero. For instance, apple trees can be very tall, requiring an effort to pick apples even when they are free. And an orange tree about the same height requires the same effort, even though being a different good. Since there is only so much fruit indi-

[2] Our solution strongly departs from the two usual remedies to mechanically resolve the steady state problem. First, by assuming full price indexation to trend inflation. Empirical evidence from macro and microdata, however, suggests that there is very small indexation on individual prices (see e.g., Bils and Klenow (2004), Cogley and Sbordone (2005), Klenow and Kryvtsov (2008), Klenow and Malin (2010), and Levin et al. (2005)). So, this is not a satisfactory resolution. Second, by mechanically increasing the Calvo probability of price adjustment with trend inflation, that is, by introducing some state-dependence with respect to trend inflation. But, as Bakhshi, Burriel-Llombart, Khan and Rudolf (2007) show, the elasticity of the Calvo probability with respect to trend inflation needs to be very high. Put differently, one has to assume essentially that prices are near-flexible even at single-digit inflation rates, rendering the New Keynesian model not useful for any monetary policy analysis since nominal rigidity is essential to account for the effects of monetary policy on an economy.

viduals can carry down the trees, the extra costs should increase with consumed fruit volumes, as consuming more fruit requires more climbing. This price wedge prevents individuals from consuming infinite amounts of goods, as it keeps the effective price at a strictly positive level even if the sticker price reduces to zero.

Applying price wedges to Kimball aggregation, the elasticities and superelasticities decrease, aligning more closely with micro evidence. In the simpler Dixit-Stiglitz aggregation, price wedges make superelasticities rise to positive levels, inducing the demand function to have a smoothed-out kinked form that does not diverge to infinity. This feature allows the augmented demand function to be used with the Calvo model for all levels of trend inflation. Importantly, this property holds for any level of price wedges, no matter how small. We also find that price wedges strongly attenuate welfare losses and the increase in price dispersion as trend inflation rises, making them more in line with the findings of Nakamura, Steinsson, Sun and Villar (2018) and Sheremirov (2020). We embed the augmented demand function based on Dixit-Stiglitz with price wedges in a textbook general equilibrium NK model as a proof of concept and study the dynamics at large rates of trend inflation. When the level of trend inflation exceeds about 10%, we find that inflation becomes less responsive to monetary policy, more persistent, and harder to tame, as it lingers much longer and requires a much greater output sacrifice to bring it down. These properties are in line with recent empirical results found by Canova and Forero (2024). The authors estimate a Markov-Switching model for the US with two states (high and low inflation) from 1960 to 2023. They find that, after contractionary monetary policy shocks, inflation rates do not fall as much and become more persistent in high-inflation states when compared to low-inflation states. These results align well with the experience of many countries whose inflation rates that have faced high double-digit inflation rates.

Using Calvo pricing to characterize nominal rigidity remains popular not only in the academic literature on NK models but also in models used at central banks for monetary policy analysis. Besides the well-known theoretical elegance in modelling nominal rigidity, the Calvo model also does a decent job of matching empirical micro evidence. In addition, the time-dependent Calvo model is also shown to be equivalent to a large class of state-dependent pricing models. Klenow and Kryvtsov (2008) show that the Calvo model matches six of the eight stylized facts in the microdata underlying the Consumer Price Index, being even better than some state-dependent models. In line with this result, Costain and Nakov (2011, 2023) build and test a model nesting both Calvo (time-dependent) and Golosov and Lucas (2007) fixed menu costs (state-dependent) models. They find that the parameterization that best fits microdata has low state dependence, implying a Phillips curve closer but not the same as the one implied by the Calvo model. Similarly, Gautier and Le Bihan (2022) estimate a industry-specific

Calvo Plus model (based on Nakamura, Steinsson, Sun and Villar (2018) hybrid model with time-and state-dependent pricing) with French micro data on prices and find that 60% of price changes are triggered by the Calvo mechanism. Previously, Bakhshi, Burriel-Llombart, Khan and Rudolf (2007) showed that the Calvo model approximates the inflation dynamics generated from the Dotsey, King and Wolman (1999) state-dependent model. More recently, Auclert, Rigato, Rognlie and Straub (2024) show that in a broad class of menu cost models, the first-order dynamics of aggregate inflation is first-order equivalent to a mixture of two time-dependent models (e.g. the Calvo model), reflecting the extensive and intensive margins of price adjustment.

The remainder of the paper is organized as follows: Section 2 reviews related literature on micro and macro evidence. In Section 3, we present the model with a general preference structure, assess how marginal costs increase with trend inflation and discuss how Calvo (1983) price setting is affected in this general framework. Section 4 presents the main result of our paper, in the form of a theorem that describes the general conditions that demand functions must satisfy so that there always exists a determinate steady-state equilibrium independently of the level of trend inflation. Section 5 discusses demand functions compatible with the theorem, assessing models such as Kimball aggregation (Section 5.1) and presenting another contribution of our paper, i.e., models with price wedges (Section 5.2). Section 6 presents simulations, and Section 7 concludes.

2 Micro and Macro Empirical Evidence

Before presenting the formal model, in which we consider a generic functional form for demand functions, we present some micro evidence. This evidence gauges the features demand functions should comprise and the macro predictions they should imply when used in NK models.

There is a vast empirical literature, based on micro data, strongly suggesting that actual demand functions are "kinked", in the sense that superelasticity (curvature) is positive. For context, often-used Dixit and Stiglitz (1977) demand functions have constant positive elasticity and zero superelasticity. Recent empirical literature using large scanner data generally finds relatively low, but still positive, values for elasticities (ξ) and superelasticities (η). Micro evidence shown below suggests that price elasticities likely range in $\xi^{micro} \in [1.0, 5.0]$, while superelasticities lie in the narrower interval of $\eta^{micro} \in [1.5, 2.0]$.

For the US, Burya and Mishra (2022) find the representative elasticity and superelasticity to be about $\xi = 4.8$ and $\eta = 1.8$.[3] For Europe, Dossche, Heylen and Van Den Poel (2010) find the median

[3] See Figures 1 and 3 in Burya and Mishra (2022). Values were retrieved using the median Herfindahl-Hirschman Index

elasticity and superelasticity across all products and sectors are rather small, at $\xi = 1.4$ and $\eta = 0.8$,[4] whereas Beck and Lein (2015) find that the median (weighted mean) elasticities and superelasticities are $\xi = 3.6(2.1)$ and $\eta = 1.4(1.5)$.[5]

As for relative prices in the US, Kaplan and Menzio (2015) results suggest that the empirical distribution of relative prices is approximately symmetric, leptokurtic (fat-tailed), and has large dispersion, even when controlling for exactly the same product (same UPC barcode - Universal Product Code) or allowing for strong substitutability.[6] Under both Brand Aggregation[7] and Brand and Size Aggregation,[8] products have at least the same features and the same size, aligning with what economists usually consider commodity goods. For them, the authors find that the empirical standard deviations of relative prices, relative to the sample average price, are 0.25 (Brand Aggregation) and 0.36 (Brand and Size).[9] Moreover, the reported empirical quantile ratios for relative prices, i.e., $r_{(0.90,0.50)} = 1.38$ (Brand) and $r_{(0.90,0.50)} = 1.55$ (Brand and Size), allow us to conclude that the 90% quantile for US prices can be 38% or even 55% larger than the average price. These statistics are important as they shed light on the distribution support of relative prices that demand functions used in economic models should be consistent with. We highlight that the authors' empirical histograms show that relative prices in both aggregation types have non-negligible masses even when prices are twice as large as the average price.

Turning to macro cross-sectional facts, we also present evidence on the effects of trend inflation that models should comply with. In short, we present evidence on how trend inflation affects price dispersion. While standard trend-inflation NK models with Dixit and Stiglitz (1977) aggregation predict that price dispersion strongly increases with trend inflation, ultimately inducing welfare to plunge at low levels of long-run inflation, recent literature finds that this relationship is actually weak, even though still positive. That is, price dispersion slightly rises for larger levels of trend inflation.

(HHI) level in their sample (0.14). The authors use weekly firm-level data on prices and quantities (2007 to 2015) from a subsample of the ACNielsen Retail Scanner Database (35,000 US stores).

[4]The authors use bi-weekly scanner data with 15,000 items from Jan 2002 to Apr 2005.

[5]The authors use scanner data from Belgium (2,000 households), Germany (12,000 households), and the Netherlands (4,000 households), provided by AiMark (Advanced International Marketing Knowledge), with about 190,000 products and 2 million individual shopping trips from 2005 through 2008.

[6]The dataset contains 300 million transactions by 50,000 households for 1.4 million goods in 54 US geographic markets.

[7]In their Brand aggregation, products share the same features and the same size, but may have different brands and different Universal Product Code (UPC).

[8]In their Brand and Size aggregation, products are only required to share the same features, even though they might have different sizes, brands, or UPCs.

[9]Using a different dataset, Klenow and Willis (2016) find the standard deviation of relative prices to be 0.14 (regular prices) and 0.19 (posted prices: regular and sales). They also recognize that the standard deviation is higher if including price changes due to product turnover, seasonal changeovers, and temporary stockouts. The dataset the authors consider is a subset of the CPI Commodities and Services Survey (US Bureau of Labor Statistics), which includes 85,000 items per month. Specifically, they assess 14,000 price items from the three New York, Los Angeles, and Chicago from January 1988 to December 2004.

For instance, while Nakamura, Steinsson, Sun and Villar (2018) find that the size of price changes did not increase in response to the Great Inflation of the late 1970s and early 1980s in the United States, Sheremirov (2020) finds that the positive relationship between price dispersion and inflation is only significant for regular prices. Sale prices, which are included in analyses with all prices, actually dampen this effect.

International evidence from different countries suggests that trend inflation is negatively correlated (not causal) with per capita consumption levels (e.g., Bleaney (1999)). Even though standard trend-inflation NK models with Dixit and Stiglitz (1977) aggregation also predict a fall in consumption as trend inflation rises,[10] the predicted fall is implausibly strong.

In the next section, we consider a formal model based on generic demand functions and staggered price setting, and show the conditions the former must satisfy for the general equilibrium to exist independently of the level of trend inflation. We aim to find demand functions that simultaneously satisfy those conditions and are consistent with the micro and macro evidence summarized in this section.

3 The Model

Following textbook expositions as in Woodford (2003) and Walsh (2017), we describe the standard NK model with Calvo (1983) price setting and flexible wages. The economy consists of a representative infinitely-lived household that consumes an aggregate bundle and supplies differentiated labor to a continuum of differentiated firms indexed by $z \in [0, 1]$. Firms produce and sell goods in a monopolistic competition environment. We depart from this structure by considering a broader class of demand functions.

3.1 Households

The representative household consumes $c_t(z)$ units of each differentiated good $z \in (0, 1)$ at price $p_t(z)$. Consumption over all differentiated goods is aggregated into a bundle C_t. Prices across all firms are aggregated into a consumption price index P_t, which is defined as $P_t C_t \equiv \int_0^1 p_t(z)\, c_t(z)\, dz$.

The household supplies $h_t(z)$ hours of labor to each firm z, at a differentiated nominal wage $W_t(z) = P_t w_t(z)$, where $w_t(z)$ is the real wage. Disutility over hours is $v_t(z) \equiv \chi h_t(z)^{1+\nu} / (1+\nu)$, where ν^{-1} is the Frisch elasticity of labor supply. The household's aggregate disutility function is

[10] See e.g., Ascari (2004), Levin, Lopez-Salido and Yun (2007), Yun (2005), Bakhshi, Burriel-Llombart, Khan and Rudolf (2007), Ascari and Sbordone (2014), Alves (2014, 2018), and Khan, Phaneuf and Victor (2020)

$v_t \equiv \int_0^1 v_t(z)\,dz$. The aggregate consumption bundle C_t provides utility $u_t \equiv \epsilon_t \left(C_t^{1-\sigma} - 1\right) / (1-\sigma)$, where σ^{-1} is the intertemporal elasticity of substitution and ϵ_t is a preference shock. The household's instantaneous utility is $u_t - v_t$.

Financial markets are complete. We consider a general budget constraint

$$P_t C_t + E_t q_{t+1} S_{t+1} + B_t \leq S_t + I_{t-1} B_{t-1} + P_t \int_0^1 w_t(z)\, h_t(z)\, dz + d_t \tag{1}$$

where E_t is the time-t expectations operator, S_t is the state-contingent value of the portfolio of financial securities held at the beginning of period t, B_t is the stock of government-issued bonds held at the end of period t, d_t denotes nominal dividend income, $I_t = (1+i_t)$ is the gross nominal interest rate at period t, i_t is the riskless one-period nominal interest rate, and q_{t+1} is the stochastic discount factor from $(t+1)$ to t.

The household chooses the sequence of C_t, $h_t(z)$, B_t, and S_{t+1} to maximize its welfare measure $\mathcal{W}_t \equiv \max E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} (u_\tau - v_\tau)$, subject to the budget constraint and a standard no-Ponzi condition, where $\beta \in (0,1)$ denotes the subjective discount factor. In equilibrium, the Lagrange multiplier λ_t on the budget constraint and the optimal labor supply function satisfy $\lambda_t = u'_t / P_t$ and $w_t(z) = v'_t(z) / u'_t$, where $u'_t \equiv \partial u_t / \partial C_t$ is the marginal utility of consumption, $v'_t(z) \equiv \partial v_t(z) / \partial h_t(z)$ is the marginal disutility of hours.[11] The optimal consumption plan and the dynamics of the stochastic discount factor, which satisfies $E_t q_{t+1} = 1/I_t$, are described by the following Euler equations

$$1 = \beta E_t \left(\frac{u'_{t+1}}{u'_t} \frac{I_t}{\Pi_{t+1}} \right) \quad ; \; q_t = \beta \frac{u'_t}{u'_{t-1}} \frac{1}{\Pi_t} \tag{2}$$

where $\Pi_t \equiv \frac{P_t}{P_{t-1}} = 1 + \pi_t$ is the gross inflation rate at period t.

3.1.1 General Demand Functions

Recall that P_t is the average price of the household's expenditure basket. In this regard, let $\wp_t(z) \equiv \frac{p_t(z)}{P_t}$ denote the relative price of firm z. For demand considerations, it is also convenient to define an additional vector of sufficient statistics $P_{s,t}$, describing the state of prices in the economy. It can be implicitly defined as a weighted average of individual prices, with state-dependent weights:

$$P_{s,t} \equiv \int_0^1 g\left(\wp_t(z), \wp_{s,t}\right) p_t(z)\, dz \tag{3}$$

[11] As usual, an equilibrium is defined as the equations describing the first-order conditions of the representative household and firms, a transversality condition $\lim_{T\to\infty} E_T q_{t,T} S_T = 0$, where $q_{t,T} \equiv \Pi_{\tau=t+1}^{T} q_\tau$, and the market clearing conditions.

where $\wp_{s,t} = \frac{P_{s,t}}{P_t}$ is the relative price of $P_{s,t}$ and $g(\wp(z), \wp_s)$ are weights, satisfying $g(1,1) = 1$, $g(\wp(z), \wp_s) \in (0,1)$, and $\int_0^1 g(\wp(z), \wp_s)\, dz = 1$. For instance, after considering a particular case of Kimball (1995) consumption aggregation, Dotsey and King (2005), Levin, Lopez-Salido and Yun (2007), Harding, Linde and Trabandt (2022), and Kurozumi and Van Zandweghe (2024) find a utility-based demand function that depends not only on the aggregate price P_t but also the simple arithmetic average of prices $P_{s,t} = \int_0^1 p_t(z)\, dz$. In this particular case, $g(\wp t(z), \wp s, t) = 1$ for all $\wp_t(z)$ and $\wp_{s,t}$.

In the spirit of Gagliardone, Gertler, Lenzu and Tielens (2023), we consider a general class of relative demand functions $\frac{c_t(z)}{C_t} = f(\wp_t(z), \wp_{s,t})$, where $f(\wp, \wp_s)$ is continuous and differentiable, satisfying $f(\wp, \wp_s) \geq 0$, $f(1,1) = 1$ and $f_1(\wp, \wp_s) \leq 0$, $\forall (\wp, \wp_s)$ in its domain, where $f_1(\wp, \wp_s) \equiv \frac{\partial f(\wp, \wp_s)}{\partial \wp}$. A particular case of $P_{s,t}$ is the mean price $P_{m,t} \equiv \int_0^1 p_t(z)\, dz$.

Since the aggregate price satisfies $P_t = \int_0^1 p_t(z) \frac{c_t(z)}{C_t} dz$, we obtain a general formulation for P_t:

$$P_t \equiv \int_0^1 p_t(z)\, f(\wp_t(z), \wp_{s,t})\, dz \tag{4}$$

where we assume that $P_{s,t}$ and P_t grow at the same rate in the steady state. Finally, firm z's price elasticity $\xi_t(z) \equiv -\frac{p_t(z)}{c_t(z)} \frac{\partial c_t(z)}{\partial p_t(z)}$ and the superelasticity of demand $\eta_t(z) \equiv \frac{p_t(z)}{\xi_t(z)} \frac{\partial \xi_t(z)}{\partial p_t(z)}$ are:

$$\xi_t(z) = -\frac{f_1(\wp_t(z), \wp_{s,t})}{f(\wp_t(z), \wp_{s,t})} \wp_t(z) \quad ; \eta_t(z) = 1 + \xi_t(z) + \frac{f_{11}(\wp_t(z), \wp_{s,t})}{f_1(\wp_t(z), \wp_{s,t})} \wp_t(z) \tag{5}$$

3.2 Price Setting

Each firm $z \in [0,1]$ produces a differentiated good using the technology $y_t(z) = \mathcal{A}_t h_t(z)^{\varepsilon}$, where $h_t(z)$ is its demand for labor, $\mathcal{A}_t$ is the aggregate technology shock and $\varepsilon \in (0,1)$. The market clearing condition $c_t(z) = y_t(z)$, $\forall z$, implies that the aggregate output across all firms satisfies $Y_t = C_t$.

Since firm-specific hours $h_t(z)$ are the only production input, the firm's real payroll cost is $co_t(z) = w_t(z)\, h_t(z)$. Taking wages as given, the firm's real marginal cost $mc_t(z) \equiv \frac{\partial co_t(z)}{\partial y_t(z)}$ is:

$$mc_t(z) = w_t(z) \frac{\partial h_t(z)}{\partial y_t(z)} = \frac{\chi}{\varepsilon} \frac{(Y_t)^{(\sigma+\omega)}}{\epsilon_t (\mathcal{A}_t)^{(1+\omega)}} [f(\wp_t(z), \wp_{s,t})]^{\omega}$$

where $\omega \equiv \frac{(1+\nu)}{\varepsilon} - 1$ is a composite parameter. As for the firm's real payroll cost, it can be written as $co_t(z) = \frac{(Y_t)^{(1+\sigma+\omega)}}{\epsilon_t (\mathcal{A}_t)^{(1+\omega)}} [f(\wp_t(z), \wp_{s,t})]^{(1+\omega)}$.

Under flexible prices, all firms set the same price when maximizing profits $p_t(z)\, y_t(z) - P_t co_t(z)$. Optimal pricing requires $\left(1 - \frac{1}{\xi_t^n(z)}\right) \wp_t^n(z) = mc_t^n(z)$, where superscript '$n$' denotes natural equilibrium and $\xi_t^n(z)$ is the firm price-demand elasticity. Since all optimal prices are the same, $\xi_t^n(z) = \xi^n$ is constant and we have $\wp_t^n(z) = 1$, $\wp_{s,t}^n = 1$, and $f\left(\wp_t^n(z), \wp_{s,t}^n\right) = 1$. Therefore, the monopolistic static markup $\mu \equiv \frac{\wp_t^n}{mc_t^n}$ under flexible prices is:

$$\mu = \frac{1}{\left(1 - \frac{1}{\xi^n}\right)} \tag{6}$$

In addition, under flexible prices, all firms produce the same level in equilibrium $y_t^n = Y_t^n$, where Y_t^n is the natural output:

$$\left(Y_t^n\right)^{(\sigma+\omega)} = \frac{1}{\mu}\frac{\varepsilon}{\chi}\epsilon_t\left(\mathcal{A}_t\right)^{(1+\omega)} \quad ; \xi^n = -f_1(1,1) > 0 \tag{7}$$

This result also implies that, in general price settings, the total real cost and marginal cost, respectively, are:

$$\begin{aligned} co_t(z) &= \left(\frac{1}{\mu}\frac{\varepsilon}{\chi}\right)^{\frac{(1+\sigma+\omega)}{(\sigma+\omega)}} \left(\epsilon_t\left(\mathcal{A}_t\right)^{(1+\omega)}\right)^{\frac{1}{(\sigma+\omega)}} \left(X_t\right)^{(1+\sigma+\omega)} \left[f\left(\wp_t(z), \wp_{s,t}\right)\right]^{(1+\omega)} \\ mc_t(z) &= \frac{1}{\mu}\left(X_t\right)^{(\sigma+\omega)}\left[f\left(\wp_t(z), \wp_{s,t}\right)\right]^{\omega} \end{aligned} \tag{8}$$

where $X_t \equiv \frac{Y_t}{Y_t^n}$ is the gross output gap.

Now, we highlight the role of the demand function in defining how the firm's marginal cost fast increases in an environment with positive trend inflation. Before defining a particular price setting structure, i.e. regardless of whether it is time- or state-dependent, consider a general case in which firm z had last reset its price to $p_{t-j}^*(z)$ at period $(t-j)$ and kept it unchanged up to period t. In this case, the firm's current relative price is $\wp_t(z) = \frac{p_{t-j}^*(z)}{P_t} = \frac{1}{\Pi_{t-j,t}}\wp_{t-j}^*(z)$, where $\Pi_{t-j,t} \equiv \frac{P_t}{P_{t-j}}$ is the cumulated gross inflation from period $(t-j)$ to t. It implies that its current sticky price marginal cost is $mc_{t-j,t}^*(z) = \frac{1}{\mu}\left(X_t\right)^{(\sigma+\omega)}\left[f\left(\frac{1}{\Pi_{t-j,t}}\wp_{t-j}^*(z), \wp_{s,t}\right)\right]^{\omega}$. We highlight that this result does not depend on price-setting structure, i.e. time- or state-dependent.

For simplification, consider that the steady-state exists – see Section 4 for a general result on steady-state existence. In this case, the marginal cost after j periods without optimal readjustments can be written as $\overline{mc}_j^*(z) = \frac{1}{\mu}\left(\bar{X}\right)^{(\sigma+\omega)}\left[f\left(\frac{1}{(\bar{\Pi})^j}\bar{\wp}^*(z), \bar{\wp}_s\right)\right]^{\omega}$, where barred variables indicate steady-state levels. Under positive trend inflation, i.e. $\bar{\Pi} > 1$, the firm's optimal relative price $\frac{1}{(\bar{\Pi})^j}\bar{\wp}^*(z)$ erodes toward zero as the number of periods j without readjustments increases, and $f\left(\frac{1}{(\bar{\Pi})^j}\bar{\wp}^*(z), \bar{\wp}_s\right)$ rises. Since $\omega \geq 0$, the marginal cost increases faster than the demand function as trend inflation rises.

Therefore, the general relative demand function $f(\wp, \wp_s)$ is the main driver of marginal costs in between price readjustments. For illustration, consider the standard Dixit and Stiglitz (1977) aggregation. In this case, the relative demand function is $\frac{y_t(z)}{Y_t} = f(\wp_t(z)) = (\wp_t(z))^{-\theta}$, where $\theta > 1$ is the elasticity of substitution between goods, satisfying $\theta = \frac{\mu}{(\mu-1)}$. Since this demand function grows fast and unboundedly as the relative price decreases, so does the marginal cost between readjustments as trend inflation rises. That is, given $\bar{\wp}^*(z)$, $\overline{mc}_j^*(z) = \frac{1}{\mu}(\bar{X})^{(\sigma+\omega)}\left(\frac{1}{(\bar{\Pi})^j}\bar{\wp}^*(z)\right)^{-\theta\omega}$ fast shoots to infinity as trend inflation rises. This effect is amplified for larger values of $\theta\omega$.

In a nutshell, the marginal cost increases fast with trend inflation when the demand function increases fast as relative prices decrease. This effect is amplified in economies with large curvatures, captured by $\omega \geq 0$. And this result is independent of the price-setting nature, i.e. time- or state-dependent.

For the remainder of this paper, we consider the particular time-dependent Calvo price setting before we formally address the steady state problem under general demand functions. With standard Calvo (1983) pricing, with probability $(1-\alpha)$, the firm optimally readjusts its price to $p_t(z) = p_t^*$. With probability α, the firm sets the price with partial indexation according to $p_t(z) = p_{t-1}(z)\Pi_t^{ind}$, where $\Pi_t^{ind} \equiv \Pi_{t-1}^{\gamma}$ is the gross indexation rate, $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ is the gross inflation rate, and $\gamma \in (0,1)$.[12] When optimally readjusting at period t, price $p_t(z) = p_t^*$ maximizes the present value of nominal profit flows $E_t \sum_{j=0}^{\infty} \alpha^j q_{t,t+j}\left[\Pi_{t,t+j}^{ind} p_t(z) y_{t+j}(z) - P_{t+j} co_{t+j}(z)\right]$, given the demand function and the price setting structure, where $q_{t,t+j}$ is the cumulated nominal stochastic discount factor from period $(t+j)$ to t, recursively defined as $q_{t,t} = 1$, $q_{t,t+1} = q_{t+1}$, and $q_{t,t+j} \equiv q_{t+1}q_{t+1,t+j}$ for $j \geq 1$.

If firm z has last optimally readjusted its price at period t, its marginal cost and demand functions at $(t+j)$ are $mc_{t,t+j}(z) = \frac{1}{\mu}(X_{t+j})^{(\sigma+\omega)}\left[f\left(\frac{\Pi_{t,t+j}^{ind}}{\Pi_{t,t+j}}\frac{p_t(z)}{P_t}, \wp_{s,t+j}\right)\right]^{\omega}$ and $\frac{y_{t+j}(z)}{Y_{t+j}} = f\left(\frac{\Pi_{t,t+j}^{ind}}{\Pi_{t,t+j}}\frac{p_t(z)}{P_t}, \wp_{s,t+j}\right)$,[13] where $\Pi_{t,t+j}$ and $\Pi_{t,t+j}^{ind}$, for $j \geq 1$, are the cumulated gross inflation and indexation rates from period t to $(t+j)$, recursively defined as $\Pi_{t,t} = \Pi_{t,t}^{ind} = 1$, $\Pi_{t,t+1} = \Pi_{t+1}$, $\Pi_{t,t+1}^{ind} = \Pi_{t+1}^{ind}$, $\Pi_{t,t+j} \equiv \Pi_{t+1}\Pi_{t+1,t+j} = \Pi_{t,t+j-1}\Pi_{t+j}$, and $\Pi_{t,t+j}^{ind} \equiv \Pi_{t+1}^{ind}\Pi_{t+1,t+j}^{ind} = \Pi_{t,t+j-1}^{ind}\Pi_{t+j}^{ind}$. Most importantly, note that $mc_{t,t+j}(z)$ is not the marginal cost $mc_{t+j}(z)$, as the former depends on the state at period t and cumulated rates from t to $(t+1)$.

In this context, all optimally readjusting firms have the same first order condition for $p_t(z) = p_t^*$

[12] We allow for price indexation even though empirical evidence from macro and micro data suggest that there is very small indexation on individual prices. Full indexation is the particular case in which $\gamma = 1$.

[13] That is, considering cumulative indexation, the relative price is $\frac{\Pi_{t,t+j}^{ind} p_t(z)}{P_{t+j}} = \frac{\Pi_{t,t+j}^{ind}}{\Pi_{t,t+j}}\frac{p_t(z)}{P_t}$.

in equilibrium. Therefore, it can be conveniently written as in the following system:

$$\begin{aligned} 0 = \quad & E_t \sum_{j=0}^{\infty} \alpha^j q_{t,t+j} \mathcal{G}_{t,t+j} \Pi^{ind}_{t,t+j} f\left(\frac{\Pi^{ind}_{t,t+j}}{\Pi_{t,t+j}} \wp^*_t, \wp_{s,t+j}\right) \\ + \quad & E_t \sum_{j=0}^{\infty} \alpha^j q_{t,t+j} \mathcal{G}_{t,t+j} \Pi^{ind}_{t,t+j} \left(\frac{\Pi^{ind}_{t,t+j}}{\Pi_{t,t+j}} \wp^*_t\right) f_1\left(\frac{\Pi^{ind}_{t,t+j}}{\Pi_{t,t+j}} \wp^*_t, \wp_{s,t+j}\right) \\ - \quad & E_t \sum_{j=0}^{\infty} \alpha^j q_{t,t+j} \mathcal{G}_{t,t+j} \Pi^{ind}_{t,t+j} f_1\left(\frac{\Pi^{ind}_{t,t+j}}{\Pi_{t,t+j}} \wp^*_t, \wp_{s,t+j}\right) mc^*_{t,t+j} \\ mc^*_{t,t+j} = \quad & \frac{1}{\mu} \left(X_{t+j}\right)^{(\sigma+\omega)} \left[f\left(\frac{\Pi^{ind}_{t,t+j}}{\Pi_{t,t+j}} \wp^*_t, \wp_{s,t+j}\right)\right]^{\omega} \end{aligned} \tag{9}$$

where $\wp^*_t \equiv \frac{p^*_t}{P_t}$, $\mathcal{G}_t \equiv \frac{Y_t}{Y_{t-1}}$ denotes the gross output growth rate, and $\mathcal{G}_{t,t+j}$ is the cumulated gross growth rate, defined as $\mathcal{G}_{t,t} = 1$, $\mathcal{G}_{t,t+1} = \mathcal{G}_{t+1}$, and $\mathcal{G}_{t,t+j} \equiv \mathcal{G}_{t+1}\mathcal{G}_{t+1,t+j}$ for $j \geq 1$.

Note that infinite sums involving $f\left(\frac{\Pi^{ind}_{t,t+j}}{\Pi_{t,t+j}} \wp^*_t, \wp_{s,t+j}\right)$, $f_1\left(\frac{\Pi^{ind}_{t,t+j}}{\Pi_{t,t+j}} \wp^*_t, \wp_{s,t+j}\right)$ and $mc^*_{t,t+j}$ do not generally allow for recursive representations, and so steady state computations must be done numerically after considering a finite sum $j = \{0, ..., J\}$, for a large J. This is true even in commonly used models based on Kimball aggregation. Lastly, price aggregations (3) and (4) imply

$$\wp_{s,t} = (1-\alpha) \sum_{j=0}^{\infty} \alpha^j g\left(\frac{\Pi^{ind}_{t-j,t}}{\Pi_{t-j,t}} \wp^*_{t-j}, \wp_{s,t}\right) \frac{\Pi^{ind}_{t-j,t}}{\Pi_{t-j,t}} \wp^*_{t-j} \tag{10}$$

$$1 = (1-\alpha) \sum_{j=0}^{\infty} \alpha^j f\left(\frac{\Pi^{ind}_{t-j,t}}{\Pi_{t-j,t}} \wp^*_{t-j}, \wp_{s,t}\right) \frac{\Pi^{ind}_{t-j,t}}{\Pi_{t-j,t}} \wp^*_{t-j} \tag{11}$$

3.3 Quarterly Benchmark Calibration

We calibrate the model parameters at the quarterly frequency. As in Cooley and Prescott (1995), we set the subject discount factor at $\beta = 0.99$ and the elasticity to hours at the production function at $\varepsilon = (1 - 0.36)$. We set $\alpha = 0.60$ as the degree of price stickiness, which is consistent with micro and macro evidence.[14] Since empirical evidence from macro and micro data suggest that there is non-existent or very small indexation on individual prices, we set $\gamma = 0$.[15] Using central estimates (the modes of the posterior distributions) obtained by Smets and Wouters (2007), we set the reciprocal of the elasticity of intertemporal substitution at $\sigma = 1.39$. As for the reciprocal of the Frisch elasticity,

[14]Nakamura and Steinsson (2008), using microdata from 1988 to 2005, estimate the median duration between price changes at roughly 4.5 months (including sales) and 10 months (excluding sales). Their findings are similar to those obtained in Bils and Klenow (2004). The median durations τ_m are consistent with $\alpha = 0.63$ and $\alpha = 0.81$ in quarterly fequency, using $\tau_m = -\log(2) / \log(\alpha)$. As for the macro evidence, Cogley and Sbordone (2008), for instance, report $\alpha = 0.588$ as their median estimate, while Smets and Wouters (2007) report $\alpha = 0.65$ as the mode estimate, using the full sample period from 1966:1 to 2004:4.

[15]For instance, this evidence is found in Bils and Klenow (2004), Cogley and Sbordone (2005), Cogley and Sbordone (2008), Klenow and Kryvtsov (2008), Klenow and Malin (2010), Levin et al. (2005) and Smets and Wouters (2007).

we set it at $\nu = 1$ for a compromise between micro estimates and macro evidence on total hours fluctuation over the business cycle.[16] Finally, based on median estimates from Cogley and Sbordone (2008) and Ascari and Sbordone (2014), we set the monopolistic static markup of $\mu = 1.12$.[17]

4 Steady State Convergence

For notation purposes, barred variables stand for steady state levels. We want to assess which conditions ensure a determinate steady-state equilibrium with no output growth, i.e. $\bar{\mathcal{G}} = 1$, for different levels of trend inflation $\bar{\Pi} = (1 + \bar{\pi})$. We define determinate steady-state equilibria as those in which all infinite summations in the steady-state equation implied by (9) converge.

Except for the general demand function, the model previously described is otherwise a typical example of the standard New Keynesian framework with Calvo staggered price setting: it has monopolistic competition, standard functional forms, only one source of nominal rigidity and shocks to preferences and technology.

Under those circumstances, the generally accepted paradigm in the literature on trend inflation is that there is a low upper limit for trend inflation consistent with a determinate steady-state equilibrium (see e.g. Ascari and Sbordone (2014)). In the standard NK model, given a trend inflation level $\bar{\Pi}$, the steady state equilibrium with no output growth only exists if $\bar{\Pi} < \min\left[\left(\frac{1}{\alpha}\right)^{\frac{1}{(\theta-1)(1-\gamma)}}, \left(\frac{1}{\alpha\beta}\right)^{\frac{1}{\theta(1+\omega)(1-\gamma)}}\right]$. Using the benchmark calibration, the annualized upper limit for trend inflation is 7.40%. If we had assumed a calibration more compatible with micro evidence in the labor market,[18] with $\nu = \frac{1}{0.59} = 1.69$, the annualized upper limit would be much smaller, at 5.44%.

We formally show below, this inflation upper bound in standard New Keynesian models arises because the usual Dixit and Stiglitz (1977) demand function $f\left(\wp_t(z), \wp_{s,t}\right) = \left(\wp_t(z)\right)^{-\theta}$ and its derivative $f_1\left(\wp_t(z), \wp_{s,t}\right) = -\theta\left(\wp_t(z)\right)^{-(\theta+1)}$ have a singularity point at $\wp_t(z) \to 0$. And so, if the relative price approaches zero, this relative demand diverges to infinite. This is exactly what happens in between optimal price readjustments in case of $\gamma < 1$. After j periods since last optimal readjustment, with probability α^j, we have $\wp_{t+j}(z) = \frac{\Pi^{ind}_{t,t+j}}{\Pi_{t,t+j}}\wp^*_t$. If inflation is positive on average, $\Pi_{t,t+j}$ grows faster than $\Pi^{ind}_{t,t+j}$ as j gets larger, which in turn causes $\wp_{t+j}(z)$ to approach zero. As a consequence, if $\bar{\Pi}$

[16] In this regard, even though Chetty et al. (2011) finds a smaller value for ν^{-1} (i.e. a larger value for ν) on the micro side, recent evidence suggests that earlier estimates of micro elasticities for ν^{-1} might be downwardly biased, as their inference approaches did not account for important features in households composition between: (i) male and female workers; (ii) age; and (iii) primary and secondary earners. See, for example, Keane and Rogerson (2012), Peterman (2016), and Bredemeier, Gravert and Juessen (2023)

[17] In a Dixit and Stiglitz (1977) aggregation model, with the elasticity of substitution set at $\theta = 9.5$, the markup is $\mu = \frac{\theta}{(\theta-1)} = 1.12$.

[18] See Table 1 in Chetty et al. (2011).

is not sufficiently small, relative demand grows faster than the probability-adjusted discounting rate, causing all infinite summation terms to diverge.

As discussed below, the convergence at any level of trend inflation requires a weak restriction in the relative demand function. Intuitively, if the relative demand function is never infinite (in absolute value), including at zero relative prices $\wp_t(z) \to 0$, then the infinite sums in the price setting system converge in the steady state for virtually all levels of trend inflation. This idea is formalized in the Theorem 1 presented below.

Assumptions: Under the Calvo staggered price setting ($\alpha > 0$) with partial indexation ($\gamma < 1$), as previously described, consider the generic relative demand function $\frac{y}{Y} = f(\wp, \wp_s)$ described in Section 3.1.1, where $\wp_s \equiv \frac{P_s}{P}$, where P_s and P grow at the same rate in any steady state, such that $f(\wp, \wp_s)$ is a non-negative, continuous and differentiable function in $(\wp, \wp_s) \in (\mathbb{R}_+^* \times \mathbb{R}_+^*)$ and non-increasing in $\wp \in R_+^*$. Let $f_1(\wp, \wp_s) \equiv \frac{\partial f(\wp,\wp_s)}{\partial \wp}$ denote the partial derivative of f with respect to $\wp$.

Theorem 1 *If $f(\wp, \wp_s)$ and $\wp \cdot f_1(\wp, \wp_s)$ are finite and defined at all their domain, including at $\wp \to 0$ and $\wp \to \infty$,[19] there always exists a steady state equilibrium (with no output growth) for any level of trend inflation ($\bar{\Pi} = 1 + \bar{\pi}$), provided that it is not extremely negative, i.e. $\bar{\Pi} > (\alpha)^{\frac{1}{(1-\gamma)}}$. For any other level of trend inflation, including all positive values, both the optimal relative price and the output-gap converge to finite steady state levels.*

The proof is shown in Appendix A.

Note that the requirement that the threshold level of inflation be greater than a certain negative value, $\bar{\pi} > \left[(\alpha)^{\frac{1}{(1-\gamma)}} - 1\right]$, is easily satisfied. We do not observe extremely negative levels of trend inflation in the data. For instance, if $\alpha = 0.60$, $\beta = 0.99$, and $\gamma = 0$, steady state levels cease to exist if $\bar{\pi} < (\alpha\beta - 1) = -40\%$ in quarterly frequency (-87% annually). Therefore, the feasibility inequality in Theorem 1 does not pose a practical restriction. Of course, the existence of a steady-state equilibrium does not preclude it to be inconsistent with economic reasoning in terms of sign and scale of equilibrium levels of all endogenous variables. For instance, we must still impose usual restrictions on parametrization to ensure positive levels of output.

Hahn (2022) adopts an alternative approach to cope with the fact that the Dixit and Stiglitz (1977) demand function diverges to infinity when relative prices approaches zero. His approach, however, is to keep the standard demand function while allowing firms not to satisfy demand all the times. In

[19]Formally, $0 \leq \lim\limits_{\wp \to 0} f(\wp, \wp_s) < \infty$, $0 \leq \lim\limits_{\wp \to \infty} f(\wp, \wp_s) < \infty$, $-\infty < \lim\limits_{\wp \to 0} \wp \cdot f_1(\wp, \wp_s) \leq 0$, and $-\infty < \lim\limits_{\wp \to \infty} \wp \cdot f_1(\wp, \wp_s) \leq 0$.

a continuous time model with sticky prices, the author introduces an optimal rationing mechanism, by curtailing supply to its optimal level.

By contrast, our approach is to investigate the root of the steady state problem, and propose conditions for demand functions in the standard approach used in the literature of supply meeting demand at any price level in equilibrium.[20]

At this point, once ensuring that a meaningful economic steady state exists, we can also assess what the general demand function $\frac{y_t(z)}{Y_t} = f\left(\wp_t\left(z\right), \wp_{s,t}\right)$ implies for real rigidity in price setting. In the natural equilibrium, i.e. flexible prices ($\alpha = 0$), system (9) simplifies into

$$f\left(\wp_t^n, \wp_{s,t}^n\right) + \left(\wp_t^n\right) f_1\left(\wp_t^n, \wp_{s,t}^n\right) = \left(1 - \frac{1}{\xi^n}\right) f_1\left(\wp_t^n, \wp_{s,t}^n\right) \left[f\left(\wp_t^n, \wp_{s,t}^n\right)\right]^{\omega} \left(X_t\right)^{(\sigma+\omega)}$$

Using the fact that $f(1,1) = 1$, the last relation is easily log-linearized about the economy steady-state with flexible prices, in which $\overline{\wp}^n = \overline{\wp}_s^n = 1$:

$$\widehat{\wp}_t^n = -\frac{\left[\frac{f_2(1,1)+f_{12}(1,1)}{1+f_1(1,1)} - \frac{f_{12}(1,1)}{f_1(1,1)} - \omega f_2(1,1)\right]}{\left[\frac{2f_1(1,1)+f_{11}(1,1)}{1+f_1(1,1)} - \frac{f_{11}(1,1)}{f_1(1,1)} - \omega f_1(1,1)\right]} \widehat{\wp}_{s,t} + \frac{(\sigma+\omega)}{\left[\frac{2f_1(1,1)+f_{11}(1,1)}{1+f_1(1,1)} - \frac{f_{11}(1,1)}{f_1(1,1)} - \omega f_1(1,1)\right]} \hat{x}_t$$

where $\widehat{\wp}_t^n$, $\widehat{\wp}_{s,t}$ and $\hat{x}_t$ are log-deviations from steady state levels.

Following Ball and Romer (1990) approach, we compute the content $\psi_{\text{real}} \equiv \frac{1}{\kappa_{\text{real}}}$ of real rigidities in this model, where $\kappa_{\text{real}} \equiv \frac{\partial \widehat{\wp}_t^n}{\partial \hat{x}_t}$ is the pass-through from output gap to prices. If prices are rigid ($\alpha > 0$), κ_{real} is part of the output-gap coefficient in the Phillips curve. Evaluating equations (5) in the steady-state equilibrium with flexible prices, we obtain a simple result to general demand-driven real rigidities[21] as a function of the natural elasticity ξ^n and superelasticity η^n:[22]

$$\kappa_{\text{real}} = \frac{(\sigma+\omega)}{1+\frac{\eta^n}{(\xi^n-1)}+\omega\xi^n} \quad ; \psi_{\text{real}} = \frac{1}{\kappa_{\text{real}}} \tag{12}$$

Therefore, the demand structure is a relevant source of real rigidities. As for the role of changes in ξ^n and η^n, notice that $\frac{\partial(\psi_{\text{real}})}{\partial \eta^n} = \frac{1}{(\sigma+\omega)} \frac{1}{(\xi^n-1)}$ and $\frac{\partial(\psi_{\text{real}})}{\partial \xi^n} = -\frac{1}{(\sigma+\omega)} \left(\frac{\eta^n}{(\xi^n-1)^2} - \omega\right)$. We conclude

[20] However, if we were to consider the more general case in which demand is not necessarily met all the time, the proof of a theorem for steady state existence would follow the same lines as the proof of Theorem 1, and so we will not assess this case in this paper. Basically, it is enough to consider instead that firms have production schedules $\bar{\mathfrak{f}}\left(\wp, \wp_s\right) \leq f\left(\wp, \wp_s\right)$ that can be smaller than demand. And so, optimal pricing will be a composite function of $\bar{\mathfrak{f}}\left(\wp, \wp_s\right)$ instead of $f\left(\wp, \wp_s\right)$. In this case, Hahn (2022) approach could be seen as a particular case of using production schedules, as the author consider that production matches the minimum between optimal supply and demand.

[21] For that, we easily compute $\kappa_{\text{real}} \equiv \frac{\partial \widehat{\wp}_t^n}{\partial \hat{X}_t} = \frac{(\sigma+\omega)}{\frac{2f_1(1,1)+f_{11}(1,1)}{1+f_1(1,1)} - \frac{f_{11}(1,1)}{f_1(1,1)} - \omega f_1(1,1)}$, and apply the definitions in (5).

[22] In Burya and Mishra (2022), the authors derive a similar but simpler result in a model with linear production function, log utility to consumption and no disutility to work, which implies $\omega = 0$. The authors show the pass-through $\kappa_p \equiv \frac{\kappa_{\text{real}}}{(\sigma+\omega)}$ from marginal costs to prices. When $\omega = 0$, their inverse real rigidity metrics is then $\kappa_{pt} = \frac{(\xi^n-1)}{(\xi^n-1)+\eta^n}$.

that, no matter the form of the demand function, there must be the case that: (i) increases in natural superelasticity η^n leads to larger (smaller) real rigidity ψ_{real} if natural elasticity ξ^n is larger (smaller) than unity; and (ii) increases in natural elasticity ξ^n leads to larger (smaller) real rigidity ψ_{real} if natural superelasticity η^n is smaller (larger) than $\omega\,(\xi^n - 1)^2$.

Given its importance for both steady-state determinacy under trend inflation and as a source of real rigidity, the question is whether there are utility-based demand functions that simultaneously satisfy: (i) micro and macro empirical support for demand functions, relative prices, and macroeconomic relations; and (ii) Theorem 1 conditions for steady-state existence under trend inflation.

5 Demand Functions Consistent With Theorem 1

Now the question is whether there are utility-based demand functions that simultaneously satisfy: (i) Theorem 1 conditions, especially those related to finite demand and slope at zero relative price; and (ii) micro and macro empirical support, as described in Section 2.

Under monopolistic competition models with a continuum of firms, as we consider in this paper, the class of Kimball (1995) demand functions, especially the one proposed by Dotsey and King (2005), has been often used in recent literature.[23] Using Theorem 1, we show that having a sufficiently large curvature parameter is the necessary condition for Kimball demand functions to be consistent with all levels of trend inflation. However, under large curvature, Kimball demand functions have three features that are at odds with what micro and macro evidence suggests (see Section 2): (i) superelasticities become much larger than the micro evidence range; (ii) they fail to accommodate a sizable mass of relative prices found in the US empirical distribution; and (iii) if used in NK macroeconomic models with Calvo pricing in lieu of Dixit and Stiglitz (1977) demand functions, Kimball-based NK models predict that the distorted output (due to nominal rigidities) becomes much larger than the flexible-prices output as trend inflation rises. These facts are in line with the recent critiques and findings on Kimball-based NK models found in the literature (see e.g., Dossche, Heylen and Van Den Poel (2010), Beck and Lein (2015), Klenow and Willis (2016), and Kurozumi and Van Zandweghe (2016, 2024)).

[23] If we were to extend our modelling approach to also consider oligopoly models with a finite number of firms, instead of only monopolistic competition models with an infinite number of firms in the continuum $z \in (0,1)$, we have a broad set of demand functions satisfying Theorem 1 conditions, as oligopoly demand functions are typically bounded. In this regard, the Atkeson and Burstein (2008) and Wang and Werning (2022) oligopoly models with N firms are strong candidates to be applied to NK models with trend inflation in future extensions.

5.1 Issues with the Kimball Aggregator

Within the broad class of Kimball (1995) consumption aggregation, Dotsey and King (2005) propose a particular functional form that has been frequently used in the literature (e.g. Levin et al. (2007), Harding et al. (2022) and Kurozumi and Van Zandweghe (2016, 2024)). As we better detail in Appendix B, the implied demand function is:

$$\frac{c_t(z)}{C_t} = f(\wp_t(z), \wp_{s,t}) = \begin{cases} \frac{1}{(1+\varphi)}\left(\frac{\wp_t(z)}{\wp_{k,t}}\right)^{\varpi} + \frac{\varphi}{(1+\varphi)} & ;\text{if } \left(\frac{\wp_t(z)}{\wp_{k,t}}\right) \leq (-\varphi)^{\frac{1}{\varpi}} \\ 0 & ;\text{if } \left(\frac{\wp_t(z)}{\wp_{k,t}}\right) > (-\varphi)^{\frac{1}{\varpi}} \end{cases}$$

where $\wp_{k,t} \equiv (1+\varphi) - \varphi\wp_{s,t}$, $\wp_{k,t} \equiv \frac{P_{k,t}}{P_t}$ is an auxiliary composite relative price, $P_{s,t} \equiv \int_0^1 p_t(z)\,dz$ is the average price, and P_t is the aggregate price, implicitly defined by $1 = \int_0^1 \left(\frac{\wp_t(z)}{\wp_{k,t}}\right)^{(1+\varpi)} dz$. The composite parameters are $\varpi \equiv \frac{\mu_k(1+\varphi)}{(1-\mu_k)}$ and $\mathfrak{m} \equiv \frac{\mu_k(1+\varphi)}{(1+\mu_k\varphi)}$, where $\mu_k \geq 1$ is the elasticity parameter, which matches the implicit markup rate μ under flexible prices, and $\varphi \leq 0$ sets the aggregation curvature. If $\varphi = 0$, the demand curve simplifies into the standard Dixit and Stiglitz (1977) form.

As for Theorem 1 conditions, note first that $f(\wp, \wp_s)$ and $\wp f_1(\wp, \wp_s)$ are zero when $\left(\frac{\wp}{\wp_k}\right) > (-\varphi)^{\frac{1}{\varpi}}$. Therefore, this function trivially converges to a finite and defined level when $\wp \to \infty$. It remains to verify the conditions for convergence when $\wp \to 0$. Notice that its power ϖ flips into a positive value only when $\varphi < -1$. Only in this case, i.e. when its curvature is sufficiently large, the relative demand $f(\wp, \wp_s)$ and $\wp f_1(\wp, \wp_s)$ converge to finite and defined levels at $\wp \to 0$, i.e. $\lim_{\wp\to 0} f(\wp, \wp_s) = f(0,1) = \frac{\varphi}{(1+\varphi)}$ and $\lim_{\wp\to 0} \wp f_1(\wp, \wp_s) = \frac{\varpi}{(1+\varphi)}\left(\frac{0}{\wp_k}\right)^{\varpi} = 0$.

When $\varphi < 0$, Kimball demand function has positive superelasticities (see Appendix B), which makes it qualitatively in line with micro evidence, as described in Section 2. However, quantitatively, Kimball elasticities and superelasticities in estimated/calibrated macroeconomic models tend to be much larger than their empirical microdata counterparts, i.e. $\xi^{micro} \in [1.0\,,\,5.0]$ and $\eta^{micro} \in [1.5\,,\,2.0]$. To illustrate this property, consider those levels under flexible prices, i.e. $\xi^n = \frac{\mu_k}{(\mu_k-1)}$ and $\eta^n = (-\varphi)\frac{\mu_k}{(\mu_k-1)}$. Since $\varphi < -1$ for the existence of the steady state, and μ_k matches the static markup $\mu = 1.12$ under flexible prices, we obtain $\xi^n = 9.3$ and $\eta^n > 9.3$.

In macroeconomic models, extremely negative values for φ have been estimated/calibrated the literature. For the US, the parameters is usually estimated/calibrated in the range $\varphi \in [-16, -2]$.[24] If the static markup is set at the usual low levels of $\mu_k = \mu = 1.12$, even at the smallest curvature in the

[24] Some typical values for the US are the following: (*i*) $\varphi = -12.2$ in Harding et al. (2022); (*ii*) $\varphi = -2.6$ in Kurozumi and Van Zandweghe (2024); (*iii*) $\varphi = -8$ in Levin et al. (2007); and (*iv*) $\varphi = -3.79$ in Smets and Wouters (2007). In addition, obtaining a better marginal likelihood statistics for model comparison, Harding et al. (2022) re-estimate Smets and Wouters (2007) model with a different prior distribution and obtain $\varphi = -16.37$.

range, the implied superelasticity $\eta^n = 18.6$ is much larger than what microevidence suggests.

In addition, the large macro curvatures imply that the upper limit for relative prices $(-\varphi)^{\frac{1}{\varpi}}$ is just slightly larger than unity. In this case, there is no demand for prices that are set slightly above the average price, and the distribution of relative prices is strongly asymmetric to the left. For illustration, Klenow and Willis (2016) show that about 15% of goods end up with zero relative demand when the demand function is Kimball-based with large curvature. Relative to actual economies, to have a mass of price setting firms with zero demand is an odd implication. The Kimball aggregator implied distribution of relative prices is, therefore, not in line with empirical evidence when the curvature is large. See a detailed discussion in Appendix B.3, in which we propose an alternative approach to test the plausibility of Kimball's upper limit on relative prices.

In light of those results, we propose in the next section a remedy to attenuate the issues induced by Kimball demand functions.

5.2 Sticker and Effective Prices

Between purchasing a good and consuming it within a specific period, it is not uncommon for individuals to face extra costs that creates a wedge between the sticker price and the effective price. Those costs can rise either from direct monetary causes or from efforts, which then can be translated into a monetary price. And more importantly, they might be resilient even if the sticker price is set at zero. For instance, apple trees can be very tall, requiring an effort to pick apples even when they are free. And an orange tree about the same height requires the same effort, even though being a different good. Since there is only so much fruit individuals can carry down the trees, the extra costs should increase with consumed fruit volumes, as consuming more fruit requires more climbing.

We can indirectly compute the extra price added to the sticker price by quantifying the effort (energy, abilities, etc.) needed to climb the tree in every period we want to consume a fruit. And we highlight that acquiring them has a complementary nature with consuming the fruit, as individuals would not "buy" more effort goods and less fruit unities if effort becomes relatively cheaper than fruits.

Sometimes, the costs can be directly measured in monetary units, for consuming the good might require post-purchase accompanying extra cost from handling, shipping and storing the goods within the period. Again, even if the good's sticker price is set at zero, those extra costly activities still remain. And their cost in many cases depend on good volumes and weights, rather than good types. Those properties again characterize a complementarity rather than a substitutability between consumed goods and the extra cost sources.

Of course, features such as rarity, fragility and perishability also matters. The extra costs might also vary across different individuals and across time. In this paper, for simplicity, we abstract from those possibilities and do not specify any particular source of the realistic nature of extra costs. In all cases, extra costs prevent individuals from consuming infinite amounts of goods, even though that is what they would like to if sticker prices were to approach zero in the absence of extra costs.[25] For short, we use the term "price wedges" to characterize this class of extra-cost models. Here, we consider a simple structure to make the case that this class of models can be used to assess the economy at all levels of trend inflation.

And lastly, the extra costs might be simply wasted (deadweight loss) or might be recovered somehow into the economy. Notice that the first type of extra costs generates more distortions than the second type, as there are no firms or individuals able to accrue the losses individuals bear.

Notice that extra costs can be also be generated if, for consuming goods, individuals are required to buy extra services or goods that do not reflect extra utility-bearing consumption, in the spirit of Michaillat and Saez (2015) when they model a case in which consuming one service unit requires buying buy a total of $(1+\tau)$ service units. For instance, household storage rooms and refrigerators can generate this effect, as their associated costs are related to volumes and not to the specific goods they store. As our alternative case, we could assume that $\delta c_t(z)$ represents the storage volume required to keep $c_t(z)$ units of utility-bearing goods, and there is no price wedge. And so, paralleling Michaillat and Saez (2015) results, consuming $c_t(z)$ units would require buying a total of $(1+\delta)\,c_t(z)$ units. Even though this alternative approach also embeds the complementary nature between utility-bearing and non-utility bearing consumption, it requires changing the market clearing condition to account for both types of produced goods. In this paper, for simplicity, we will follow the price wedge approach and let the extra costs to be recovered by firms in order to minimize implied distortions.

As will be clear from the steps shown in the next section, applying price wedges for any general preference framework is straightforward. In this paper, for simplicity, we derive the implied price wedges demand function under the general Kimball (1995) framework and show that, for a given curvature parameter φ, increasing price wedges pushes down the implied elasticities and superelasticities, making them more in line with what microdata suggest. Larger price wedges also push the relative prices threshold further away, giving more room to accommodate empirical distributions. In addition, even when considering the extreme case of $\varphi = 0$ (standard Dixit and Stiglitz (1977) aggregation), the resulting demand function satisfies Theorem 1 conditions for convergence under any level

[25] Since firms are assumed to satisfy any demand level, and all individuals face the same price, there is no incentive for over purchasing goods intended for reselling.

of trend inflation and any level of price wedge, no matter how small the latter is. We also highlight that, when applied to Dixit and Stiglitz (1977) aggregation, price wedges push up the elasticities and superelasticities, towards microdata estimates.

5.2.1 Price Wedges

Building upon the general framework described in Section 3.1, we assume that consuming $c_t(z)$ units of good z at sticker price $p_t(z)$ requires paying a extra price wedge $\delta P_{s,t}$ to an intermediate representative firm for processing, handling and storing, where $\delta \geq 0$ is the wedge rate and $P_{s,t}$ is the normalized price wedge. As the surcharge only depends on volumes, independently of the good type, each unit has the same price wedge $\delta P_{s,t}$. Therefore, the household's total expenditure is $\int_0^1 (p_t(z) + \delta P_{s,t}) c_t(z) \, dz$.[26]

In this context, let us initially define the aggregate price P_t and consumption C_t as in $(1+\delta) P_t C_t \equiv \int_0^1 (p_t(z) + \delta P_{s,t}) c_t(z) \, dz$. As we show below, in equilibrium the aggregate price also satisfies $P_t C_t \equiv \int_0^1 p_t(z) c_t(z) \, dz$, which is in line with the typical definition we mention in Section 3.1.

The intermediate firm's nominal revenue is $R_{s,t} = \int_0^1 (\delta P_{s,t}) c_t(z) \, dz = \delta P_{s,t} C_{s,t}$, where $C_{s,t} \equiv \int_0^1 c_t(z) \, dz$ is the average consumption (arithmetic mean). For processing, handling and storing, we assume that the intermediate firm pays $\delta p_t(z)$ per unit to each firm $z \in (0,1)$, so its nominal cost is $Cost_{s,t} = \int_0^1 (\delta p_t(z)) c_t(z) \, dz = \delta \int_0^1 p_t(z) c_t(z) \, dz$. This firm sets its price at a zero-profit condition $R_{s,t} = Cost_{s,t}$. Coupling this condition with the aggregate price definition $(1+\delta) P_t C_t \equiv \int_0^1 (p_t(z) + \delta P_{s,t}) c_t(z) \, dz$ allows us to obtain two important results (see Appendix C):

$$P_t C_t = \int_0^1 p_t(z) c_t(z) \, dz \quad ; \; P_{s,t} = P_t \frac{C_t}{C_{s,t}} \tag{13}$$

where $C_{s,t} \equiv \int_0^1 c_t(z) \, dz$. Note that those results do not depend on any particular preference structure or consumption aggregation.

We need small changes to adapt the general results shown in Section 3. On the household side, after substituting the total expenditure $(1+\delta) P_t C_t$ for $P_t C_t$ in the budget constraint, the optimal labor supply curve becomes $w_t(z) = (1+\delta) v_t'(z) / u_t'$. Firms results also change a little bit, as we show further on in Section 5.2.3. We first assess the consequences of having price wedges under Kimball and Dixit-Stiglitz aggregation. In this context, Section 5.2.2 below derives the resulting demand curves and studies their properties.

[26] Here, $P_{s,t}$ does not have the same meaning as in Kimball's approach, detailed in Section 5.1. In Kimball demand, $P_{s,t}$ is the average price.

5.2.2 Demand function under price wedges

If the representative household is subject to price wedges and has Kimball-type preferences, as described in Section B, total expenditure minimization gives us the implied demand function and price aggregation:

$$\frac{c_t(z)}{C_t} = \begin{cases} \frac{1}{(1+\varphi)}\left(\frac{\wp_t(z)+\delta\wp_{s,t}}{\wp_{k,t}+\delta\wp_{ss,t}}\right)^{\varpi} + \frac{\varphi}{(1+\varphi)} & ;\text{if } \left(\frac{\wp_t(z)+\delta\wp_{s,t}}{\wp_{k,t}+\delta\wp_{ss,t}}\right) \leq (-\varphi)^{\frac{1}{\varpi}} \\ 0 & ;\text{if } \left(\frac{\wp_t(z)+\delta\wp_{s,t}}{\wp_{k,t}+\delta\wp_{ss,t}}\right) > (-\varphi)^{\frac{1}{\varpi}} \end{cases} \tag{14}$$

where $\wp_{k,t} \equiv (1+\varphi) - \varphi\wp_{sk,t}$, $\wp_{ss,t} \equiv (1+\varphi) - \varphi\wp_{s,t}$, $P_{sk,t} \equiv \int_0^1 p_t(z)\,dz$ is the average price, and P_t is the aggregate price, implicitly defined by $1 = \int_0^1 \left(\frac{\wp_t(z)+\delta\wp_{s,t}}{\wp_{k,t}+\delta\wp_{ss,t}}\right)^{(1+\varpi)} dz$. The composite parameters are again $\varpi \equiv \frac{\mu_k(1+\varphi)}{(1-\mu_k)}$ and $\mathfrak{m} \equiv \frac{\mu_k(1+\varphi)}{(1+\mu_k\varphi)}$, where $\mu_k \geq 1$ is the elasticity parameter, and $\varphi \leq 0$ sets the aggregation curvature. Paralleling the notation used in Section 3.1.1, we define $\wp_t(z) \equiv \frac{p_t(z)}{P_t}$ as the relative price of firm z, and $\wp_{s,t} = \frac{P_{s,t}}{P_t}$ as the a relative price of price wedges. In addition, we also define $\wp_{sk,t} \equiv \frac{P_{sk,t}}{P_t}$.

Note that $\wp_{s,t}$, $\wp_{sk,t}$, $\wp_{ss,t}$, and $\wp_{k,t}$ are not affected by individual prices $p_t(z)$. Therefore, in case of non-zero price wedges ($\delta \neq 0$), the term $(\wp_t(z) + \delta\wp_{s,t})$ is always finite and defined for any relative price $\wp_t(z)$. In addition, note that $\lim_{\wp_t(z)\to 0} (\wp_t(z) + \delta\wp_{s,t}) = \delta\wp_{s,t}$ and $\lim_{\wp_t(z)\to\infty} (\wp_t(z) + \delta\wp_{s,t}) = \infty$. Therefore, $f(\wp, \wp_s, \wp_{sk})$ and $\wp f_1(\wp, \wp_s, \wp_{sk})$ always satisfy the Theorem 1 conditions. It implies that now we can use any curvature to assess the NK model at all levels of trend inflation, as long as δ is not zero.

When $\varphi = 0$, we have the standard Dixit and Stiglitz (1977) aggregation $C_t = \left(\int_0^1 c_t(z)^{\frac{\theta-1}{\theta}} dz\right)^{\frac{\theta}{\theta-1}}$, where $\theta = \frac{\mu_k}{(\mu_k-1)} > 1$ is the elasticity of substitution between goods. In this case, the demand function with price wedges is $\frac{c_t(z)}{C_t} = (1+\delta)^{\theta}(\wp_t(z) + \delta\wp_{s,t})^{-\theta}$, where $(1+\delta)^{1-\theta} = \int_0^1 (\wp_t(z) + \delta\wp_{s,t})^{1-\theta} dz$.[27] Even though assuming price wedges seems to pose only small changes into the demand function and price aggregation, when compared to those obtained under standard Dixit-Stiglitz aggregation, it allows the demand function $\frac{c_t(z)}{C_t} = (1+\delta)^{\theta}(\wp_t(z) + \delta\wp_{s,t})^{-\theta}$ to be quasi-kinked and more in line with empirical micro-evidence, as presented in Section 2. In addition, this demand function is always positive for all levels of relative prices, and has no relative-price threshold beyond which the demand is zero.

As better detailed further on, Figure 1 depicts the demand function (log-log), price elasticities and price superelasticities for different levels of price wedge rates $\delta \in [0, 0.50]$ and curvature parameters

[27] In line with general definition (3), we can also rewrite $P_{s,t}$ as a weighted average $P_{s,t} = \int_0^1 g(\wp_t(z), \wp_{s,t})\, p_t(z)\,dz$ of individual prices $p_t(z)$, where $g(\wp_t(z), \wp_{s,t}) = \frac{(\wp_t(z)+\delta\wp_{s,t})^{-\theta}}{\int_0^1 (\wp_t(z)+\delta\wp_{s,t})^{-\theta} dz}$. The proof is in Appendix C.1.

$\varphi \in \{0, -2.0\}$, keeping the static markup fixed at $\mu = 1.12$.

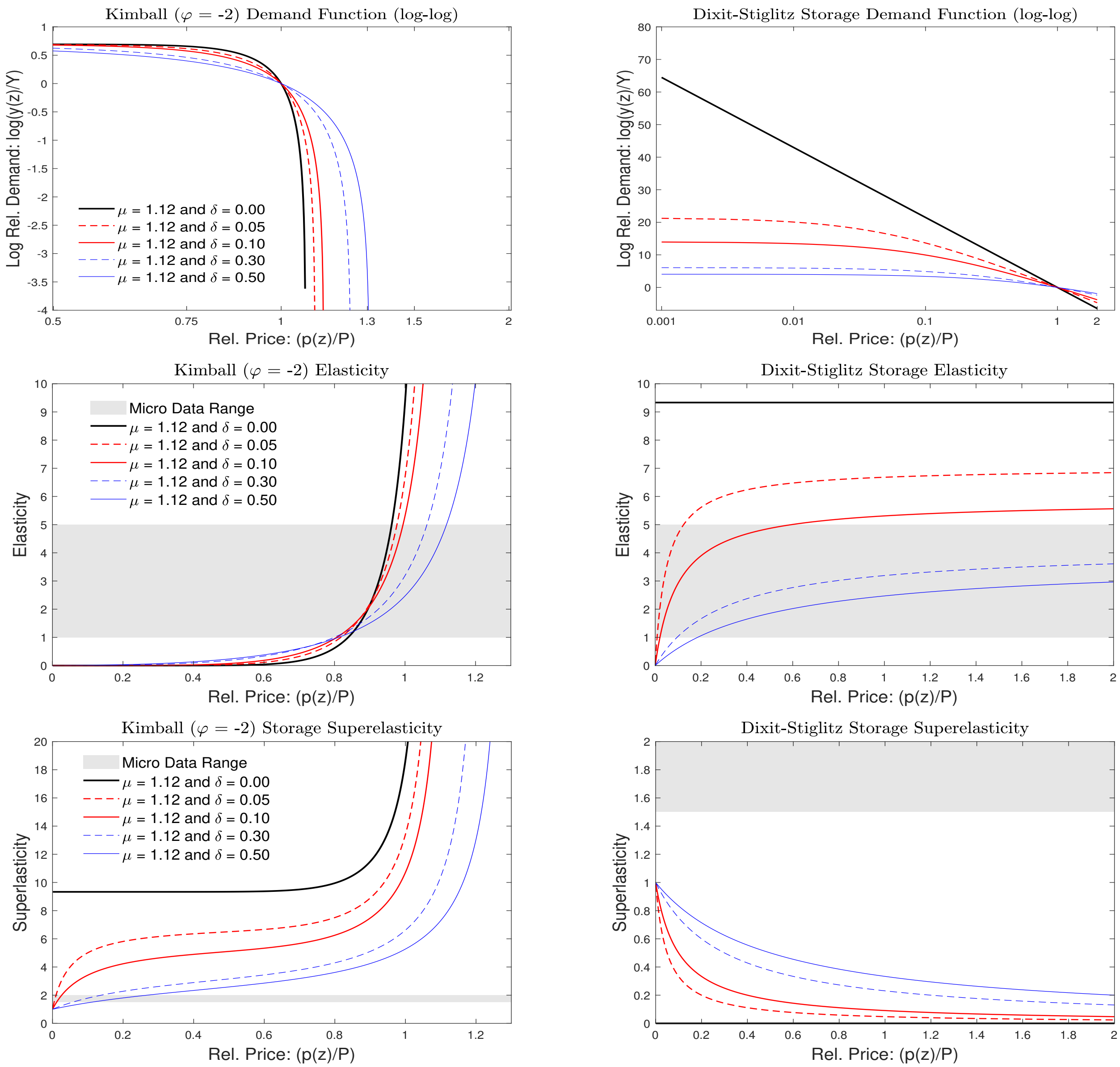

Figure 1: Demand, Elasticities and Superelasticities - Price Wedges

Notes: In top panels, the demand function is plotted using $\log(y(z)/Y)$.and $\log(p(z)/P)$. For simulations, we fix $\wp_s = \wp_k = 1$ and recompute μ_k for each value of δ in order to keep the static markup at $\mu = 1.12$.

For each relative price $\wp_t(z)$, the firm z's price elasticity and superelasticity are: (i) $\xi_t(z) = -\frac{\varpi\left(\frac{\wp_t(z)}{(1+\delta)\wp_{k,t}}\right)\left(\frac{\wp_t(z)+\delta\wp_{s,t}}{(1+\delta)\wp_{k,t}}\right)^{(\varpi-1)}}{\left[\left(\frac{\wp_t(z)+\delta\wp_{s,t}}{(1+\delta)\wp_{k,t}}\right)^{\varpi}+\varphi\right]}$ and $\eta_t(z) = \frac{1}{(\wp_t(z)+\delta\wp_{s,t})}\left[\frac{\varphi\varpi\wp_t(z)}{\left[\left(\frac{\wp_t(z)+\delta\wp_{s,t}}{(1+\delta)\wp_{k,t}}\right)^{\varpi}+\varphi\right]}+\delta\wp_{s,t}\right]$, if $\left(\frac{\wp_t(z)+\delta\wp_{s,t}}{(1+\delta)\wp_{k,t}}\right) \leq (-\varphi)^{\frac{1}{\varpi}}$; or (ii) $\xi_t(z) = 0$ and $\eta_t(z) = 0$, if $\left(\frac{\wp_t(z)+\delta\wp_{s,t}}{(1+\delta)\wp_{k,t}}\right) > (-\varphi)^{\frac{1}{\varpi}}$. In Figure 1, we recompute μ_k for each pair (δ, φ), in order to keep the static markup fixed at $\mu = 1.12$. For that, we use $\mu = \frac{\mu_k}{(1+\delta)[1-\delta(\mu_k-1)]}$, obtained from equation (6), applied to the model's elasticity under flexible prices $\xi^n = -\frac{1}{(1+\delta)}\frac{\mu_k}{(1-\mu_k)}$.

With the standard Dixit and Stiglitz (1977) aggregation, i.e. when $\varphi = 0$ and $\theta = \frac{\mu_k}{(\mu_k - 1)}$, we obtain $\xi_t(z) = \theta \frac{\wp_t(z)}{(\wp_t(z)+\delta\wp_{s,t})} \geq 0$ and $\eta_t(z) = \delta \frac{\wp_{s,t}}{(\wp_t(z)+\delta\wp_{s,t})} \geq 0$. Note that, if $\delta \neq 0$, the demand elasticity $\xi_t(z)$ and superelasticity $\eta_t(z)$ are price-dependent even if $\varphi = 0$. As Figure 1 shows, accounting for price wedges allows demand functions derived from Dixit and Stiglitz (1977) aggregation to be quasi-kinked.

In the remainder of the paper, we assess the dynamic properties of price wedge models when firms have sticky prices. For that, we consider the simpler Dixit-Stiglitz aggregation with price wedges as a proof of concept to study the implied NK model for small and large levels of trend inflation.

5.2.3 Firms

As a proof of concept, consider the case with Dixit-Stiglitz aggregation and price wedges. Using the market clearing condition $y_t(z) = c_t(z)$, $\forall z$, the aggregate and average levels of output satisfy $Y_t = C_t$ and $Y_{s,t} = C_{s,t}$. Therefore, firm z's demand function is $\frac{y_t(z)}{Y_t} = f(\wp_t(z), \wp_{s,t}) = (1+\delta)^{\theta}(\wp_t(z) + \delta\wp_{s,t})^{-\theta}$.

The firm's revenue is now $(1+\delta)\, p_t(z)\, c_t(z)$. As we mentioned before, optimal labor supply curve under price wedges is $w_t(z) = (1+\delta)\frac{\chi}{\epsilon_t} h_t(z)^{\nu}(Y_t)^{\sigma}$. Adapting the results shown in Section 3.2, optimal pricing under flexible prices now requires $\frac{(1+\delta)[\theta-(1+\delta)]}{\theta}\wp_t^n = mc_t^n$, where $mc_t^n = \frac{(1+\delta)^{(1+\theta\omega)}[\theta-(1+\delta)]}{\theta}\left(\wp_t^n + \delta\wp_{s,t}^n\right)^{-\theta\omega}$ is the marginal cost under flexible prices. Therefore, we conclude that the natural output evolves according to:

$$(Y_t^n)^{(\sigma+\omega)} = \frac{1}{(1+\delta)\,\mu}\frac{\varepsilon}{\chi}\epsilon_t(\mathcal{A}_t)^{(1+\omega)} \tag{15}$$

where $\mu \equiv \frac{\wp_t^n}{mc_t^n}$ is the static markup under flexible prices and price wedges:

$$\mu = \frac{\mu_\delta}{(1+\delta)}\ ;\ \mu_\delta \equiv \frac{\theta}{\theta-(1+\delta)} \tag{16}$$

This last result allows us to design a strategy to calibrate θ as a function of markup μ and price wedge rate δ: $\theta = \frac{\mu(1+\delta)^2}{[\mu(1+\delta)-1]}$. Note that, for a given steady state markup μ, the elasticity of substitution θ decreases with δ, when it is not unreasonably large. In particular, at the benchmark low markup level $\mu = 1.12$, the elasticity of substitution can be as low as $\theta = 5$ even for a small price wedge rate of $\delta = 0.10$. For larger price wedge rates, θ falls to about $\theta = 3.6$ when $\delta \in [0.5, 1.0]$. Those low levels for the elasticity of substitution θ are consistent with microdata estimates in Broda and Weinstein

(2006). The authors find that consumers have low elasticities of substitution across similar goods in most categories, with the median elasticity being estimated at about $\theta = 3$.

With Calvo price setting under price wedges, we can simplify firm z's optimal pricing equation (9) into the following system:

$$\begin{aligned}
1 =& \quad \mu_\theta \tfrac{N_t^*}{D_t^*} \\
N_t^* =& \quad E_t \sum_{j=0}^{\infty} \alpha^j q_{t,t+j} \mathcal{G}_{t,t+j} \Pi_{t,t+j}^{ind} \left(z_{t,t+j}^*\right)^{-(1+\theta)} \left(\tfrac{mc_{t,t+j}^*}{(1+\delta)} + \delta \wp_{s,t+j}\right) \\
D_t^* =& \quad E_t \sum_{j=0}^{\infty} \alpha^j q_{t,t+j} \mathcal{G}_{t,t+j} \Pi_{t,t+j}^{ind} \left(z_{t,t+j}^*\right)^{-\theta} \\
mc_{t,t+j}^* =& \quad \tfrac{(1+\delta)^{\theta\omega}}{\mu} \left(X_{t+j}\right)^{(\sigma+\omega)} \left(z_{t,t+j}^*\right)^{-\theta\omega} \\
z_{t,t+j}^* =& \quad \tfrac{\Pi_{t,t+j}^{ind}}{\Pi_{t,t+j}} \wp_t^* + \delta \wp_{s,t+j}
\end{aligned} \tag{17}$$

where again $\wp_t^* \equiv \frac{p_t^*}{P_t}$ and $X_t = \frac{Y_t}{Y_t^n}$. Here, $\mu_\theta \equiv \frac{\theta}{(\theta-1)}$ is the markup without price wedges. Note that we also use three additional auxiliary variables, i.e. N_t^*, D_t^* and $z_{t,t+j}^*$.

In this framework, price aggregation $(1+\delta)^{1-\theta} = \int\limits_0^1 \left(\frac{p_t(z)}{P_t} + \delta \frac{P_{s,t}}{P_t}\right)^{1-\theta} dz$, average output and wedge pricing evolve according to:

$$\begin{aligned}
&(1+\delta)^{1-\theta} = (1-\alpha) \textstyle\sum_{j=0}^{\infty} \alpha^j \left(z_{t-j,t}^*\right)^{1-\theta} \\
&Y_{s,t} = (1+\delta)^\theta Y_t \left(\wp_{\delta,t}\right)^{-\theta} \\
&\left(\wp_{\delta,t}\right)^{-\theta} = (1-\alpha) \textstyle\sum_{j=0}^{\infty} \alpha^j \left(z_{t-j,t}^*\right)^{-\theta} \\
&\wp_{s,t} = \tfrac{Y_t}{Y_{s,t}}
\end{aligned} \tag{18}$$

Now, the infinite sum is on $z_{t-j,t}^* = \left(\frac{\Pi_{t-j,t}^{ind}}{\Pi_{t-j,t}} \wp_{t-j}^* + \delta \wp_{s,t}\right)$ instead of $z_{t,t+j}^* = \left(\frac{\Pi_{t,t+j}^{ind}}{\Pi_{t,t+j}} \wp_t^* + \delta \wp_{s,t+j}\right)$. Most importantly, $z_{t,t+j}^*$ and $z_{t-j,t}^*$ enter systems (17) and (18) raised to non-positive integer powers. This fact prevents the equations to have recursive forms. In order to cope with that, we present a precise approximation in next section, allowing those terms to have recursive forms in log-linearizations.

5.2.4 Aggregates and Welfare

Let $h_t \equiv \int_0^1 h_t(z)\, dz$ denote the aggregate working hours. Given the production function $y_t(z) = \mathcal{A}_t h_t(z)^\varepsilon$ and demand function $\frac{y_t(z)}{Y_t} = (1+\delta)^\theta \left(\wp_t(z) + \delta \wp_{s,t}\right)^{-\theta}$, we conclude that aggregate hours evolve according to $h_t = (1+\delta)^{\frac{\theta}{\varepsilon}} \left(\frac{Y_t}{\mathcal{A}_t}\right)^{\frac{1}{\varepsilon}} \Lambda_{y,t}$, where $\Lambda_{y,t} \equiv \int_0^1 \left(\wp_t(z) + \delta \wp_{s,t}\right)^{-\frac{\theta}{\varepsilon}} dz$. Therefore, following the vast literature of price dispersion, we can write the aggregate output as $Y_t = \frac{1}{\eth_{y,t}} \mathcal{A}_t \left(h_t\right)^\varepsilon$, where $\eth_{y,t} \equiv (1+\delta)^\theta \left(\Lambda_{y,t}\right)^\varepsilon$ is the production-relevant metric of price dispersion. Using Calvo price setting, note that $\Lambda_{y,t} = (1-\alpha)\sum_{j=0}^{\infty}\alpha^j\left(z^*_{t-j,t}\right)^{-\frac{\theta}{\varepsilon}}$.

As for welfare considerations, recall that $\mathcal{W}_t \equiv (u_t - v_t)$ is the relevant instantaneous welfare metric, where $u_t \equiv \epsilon_t \frac{(Y_t)^{(1-\sigma)}-1}{(1-\sigma)}$ is the consumption utility and $v_t \equiv \int_0^1 v_t(z)\,dz$ is the aggregate disutility of working hours, in which $v_t(z) \equiv \frac{\chi}{(1+\nu)} h_t(z)^{(1+\nu)}$. Given the production and demand functions, we can write the aggregate disutility as $v_t = \mathfrak{d}_{v,t}\frac{\chi}{(1+\nu)}(h_t)^{(1+\nu)}$, where $\mathfrak{d}_{v,t} = \frac{\Lambda_t}{\left(\Lambda_{y,t}\right)^{(1+\nu)}}$ is the welfare-relevant metric of price dispersion, $\Lambda_t \equiv \int_0^1 \left(\wp_t(z) + \delta\wp_{s,t}\right)^{-\theta_1} dz$, and $\theta_1 = \theta(1+\omega)$ is a composite parameter. Under Calvo price setting, note that $\Lambda_t = (1-\alpha)\sum_{j=0}^{\infty}\alpha^j\left(z^*_{t-j,t}\right)^{-\theta_1}$.

In the equilibrium with flexible prices ($\alpha = 0$), we obtain $\Lambda^n_{y,t} = (1+\delta)^{-\frac{\theta}{\varepsilon}}$, $\Lambda^n_t = (1+\delta)^{-\theta_1}$, and $\mathfrak{d}^n_{y,t} = \mathfrak{d}^n_{v,t} = 1$. In this equilibrium, the instantaneous welfare evolves according to $\mathcal{W}^n_t \equiv (u^n_t - v^n_t) = \epsilon_t\frac{(Y^n_t)^{(1-\sigma)}-1}{(1-\sigma)} - \frac{\chi}{(1+\nu)}\left(\frac{Y^n_t}{\mathcal{A}_t}\right)^{(1+\omega)}$, where $u^n_t = \epsilon_t\frac{(Y^n_t)^{(1-\sigma)}-1}{(1-\sigma)}$, $v^n_t = \frac{\chi}{(1+\nu)}(h^n_t)^{(1+\nu)}$, and $h^n_t = \left(\frac{Y^n_t}{\mathcal{A}_t}\right)^{\frac{1}{\varepsilon}}$.

Therefore, following Schmitt-Grohe and Uribe (2007), we can compute the consumption-equivalent welfare metric as a distorted output level Y^{eq}_t that would prevail in a equilibrium with flexible prices in order to keep the welfare level as the one obtained with sticky prices ($\mathcal{W}_t$). That is, Y^{eq}_t satisfies:

$$\epsilon_t\frac{\left(Y^{eq}_t\right)^{(1-\sigma)}-1}{(1-\sigma)} - \frac{\chi}{(1+\nu)}\left(\frac{Y^{eq}_t}{\mathcal{A}_t}\right)^{(1+\omega)} = \mathcal{W}_t = \epsilon_t\frac{(Y_t)^{(1-\sigma)}-1}{(1-\sigma)} - \mathfrak{d}_{v,t}\frac{\chi}{(1+\nu)}\left(\mathfrak{d}_{y,t}\frac{Y_t}{\mathcal{A}_t}\right)^{(1+\omega)}$$

In this regard, we define $X^{eq}_t \equiv \frac{Y^{eq}_t}{Y^n_t}$ as the consumption-equivalent output gap.

5.2.5 Steady State Properties

Using the steady state relations shown in Appendix C.3, Figure 2 shows how relevant steady state levels vary with different levels of trend inflation $\bar{\pi}$ and different price wedge rates δ. For this, we use the benchmark calibration defined in Section 3.3, and use (16) recompute θ for each value of δ to keep the static markup at $\mu = 1.12$.

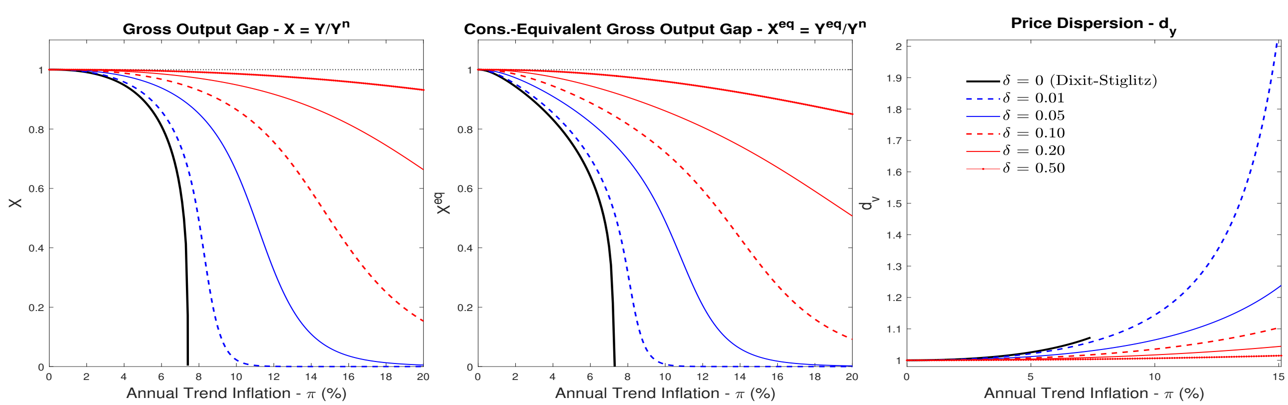

Figure 2: Steady State Levels - Price Wedges

As predicted, steady state levels now exist for all levels of trend inflation as long as $\delta > 0$. In the left panel, note that the gross output gap falls as trend inflation rises. It is interesting to note that using larger values for δ makes output smoothly decline with respect to the natural output, avoiding the sharp fall observed under $\delta = 0$ (Dixit-Stiglitz). If δ is very small, the model is able to present a seamless continuation of what standard NK models (Dixit-Stiglitz) predict for the steady state, but now without the upper limit on trend inflation, which is 7.40% using the benchmark calibration. In the middle panel, the consumption-equivalent output gap behaves similarly to the output gap shown in the left panel. However, due to the effect of disutility of hours, $\bar{X}^{eq}$ is always smaller than $\bar{X}$. Finally, in the right panel, note that the presence of price wedges strongly attenuates the price dispersion caused by trend inflation. We highlight these results as recent micro evidence on price dispersion suggests that it only weakly increases as inflation rises (e.g., Nakamura, Steinsson, Sun and Villar (2018) and Sheremirov (2020)).

6 Simulations

In this section, we assess impulse responses using the log-linearized model presented at Appendix C.4. Using the benchmark calibration, recall that the annualized upper limit for trend inflation is 7.40% under standard Dixit and Stiglitz (1977) preferences and zero price wedges ($\delta = 0$).[28] However, as we show in Section 5.2.2, the demand function function under price wedges satisfies Theorem 1, and so setting $\delta > 0$ is a sufficient condition for the existence of steady state equilibrium at any level of trend inflation. Therefore, we set $\delta = 0.03$ and consider impulse responses for a range of different levels of annual trend inflation, from 0% to 20%.[29] Setting a low value for δ, as we do, implies that the dynamics are very similar to those obtained under standard NK models when trend inflation are smaller than the upper limit. However, as $\delta > 0$, it allows us to explore the dynamics at larger long-run inflation rates, past the usual upper limit.

For dynamic simulations, we assume that the central bank has a mandated gross inflation target $\bar{\Pi} \geq 1$ (or $\bar{\pi} \geq 0$) and follows a flexible Taylor-type rule to achieve it. We consider the specification $\left(\frac{I_t}{\bar{I}}\right) = \epsilon_{\mathrm{i},t} \left(\frac{I_{t-1}}{\bar{I}}\right)^{\phi_{\mathrm{i}}} \left[\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi_\pi} \left(\frac{\Pi_t}{\Pi_{t-1}}\right)^{\phi_{\Delta\pi}} \left(\frac{X_t}{\bar{X}}\right)^{\phi_x} \left(\frac{X_t}{X_{t-1}}\right)^{\phi_{gx}} \left(\frac{Y_t}{\bar{Y}}\right)^{\phi_y} \left(\frac{Y_t}{Y_{t-1}}\right)^{\phi_{gy}} \right]^{1-\phi_{\mathrm{i}}}$, where $\epsilon_{\mathrm{i},t}$ is the monetary policy shock, $\phi_{\mathrm{i}} \in (0,1)$ is the policy smoothing parameter, and the response parameters ϕ_π, ϕ_x, ϕ_{gx}, ϕ_y and ϕ_{gy} are consistent with stability and determinacy in equilibria with rational expectations.

Reacting to output gap growth or output growth is in line with the findings of Coibion and Gorod-

[28] See Section 4.

[29] Recall that equation (16) allows us to calibrate θ as a function of markup μ and price wedge rate δ.

nichenko (2011) and Khan, Phaneuf and Victor (2020) as it generates more stabilizing properties when the trend inflation is not zero. In addition, reacting to output growth is in line with Walsh (2003), Orphanides and Williams (2007), and Coibion and Gorodnichenko (2011), and is empirically relevant. In addition, reacting to inflation variation helps reducing short-run inflation accelerations.

In order to minimize any inertial dynamics coming from shocks and policy, we consider all shocks as being pure white noise, without any AR component, and set $\phi_{\mathrm{i}} = 0$ in the Taylor rule. This rule is completed by setting $\phi_\pi = 2.05$, $\phi_{\Delta\pi} = 0.50$ and $\phi_{gx} = 1.65$. The remaining policy parameters are set at zero. Since $\phi_{\mathrm{i}} = 0$, the calibrated values for ϕ_π, $\phi_{\Delta\pi}$ and ϕ_{gx} are set so responses to monetary policy shocks (annualized nominal interest rates) at zero trend inflation are roughly in line with empirical evidence for the US: a less than one-to-one response in annualized inflation, and a more than one-to-one response in aggregate output. Impulse responses to utility $\hat{\epsilon}_t$ and technology $\widehat{\mathcal{A}}_t$ are not too different from those obtained in the trend inflation literature. But we do find that responses to monetary policy shocks $\hat{\epsilon}_{\mathrm{i},t}$ give us important insights as trend inflation is not low.

Figure 3 shows the impulse responses for different levels of trend inflation. The top two rows show the responses under low annual trend inflation, from 0 to 11%. And the bottom two rows assesses responses under high trend inflation, from 11 to 20%. Since we are considering a range of different trend inflation levels, we normalize the shocks amplitude in order to make comparisons easier. That is, in each level of trend inflation, the amplitude of monetary policy shocks $\hat{\epsilon}_{\mathrm{i},t}$ is such that the annualized nominal interest rate $\hat{\imath}_t$ has unit response at period 1, when the one-off shock hit.

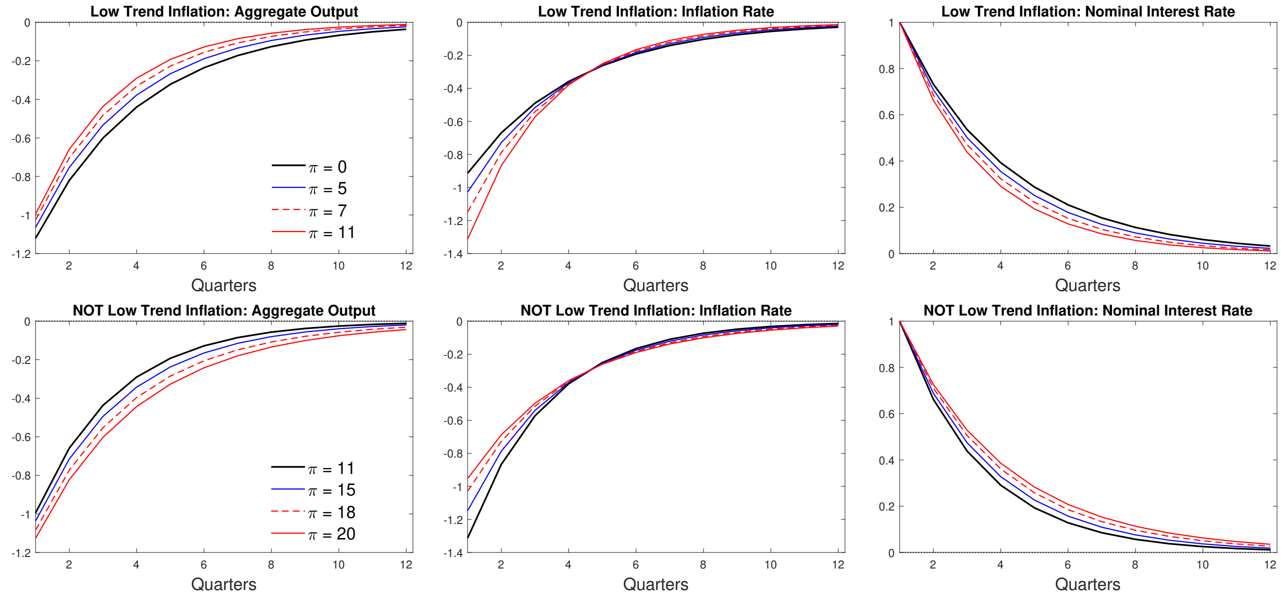

Figure 3: Impulse responses to monetary policy shocks

Notes: Impulse responses to Monetary Policy shock $\hat{\epsilon}_{i,t}$ with $\delta = 0.03$, in the Dixit-Stiglitz model with Price Wedges. In the top row, we consider low levels of trend inflation ($0 \leq \bar{\pi} \leq 11$). In the bottom row, we consider "not low" levels of trend inflation ($11 \leq \bar{\pi} \leq 20$). Shocks are normalized so that nominal interest rate $\hat{\imath}_t$ has unit response at period 1 for all levels of trend inflation.

When trend inflation rise up to about the 11%, responses behave similarly to what we observe in standard NK models with trend inflation, in the sense that the amplitude of inflation rate responses increase with trend inflation, whereas output has its response amplitudes decreased. However, we see that there is a reversal in this pattern at high levels of trend inflation. From this point on, amplitudes of inflation responses decrease, while that of output increase, as trend inflation gets higher. It means that it becomes harder for central banks to curb inflation hikes and bring it down, when the average inflation sits above the 11% level. We highlight that these properties are in line with recent empirical results found by Canova and Forero (2024). The authors estimate a Markov-Switching model for the US with two states (high and low inflation) from 1960 to 2023. They find that, after contractionary monetary policy shocks, inflation rates do not fall as much and become more persistent in high-inflation states when compared to low-inflation states.

7 Conclusion

We provide a resolution to a well-known issue: the steady state of the widely-studied New Keynesian (NK) models based on Calvo-pricing does not exist beyond a low single-digit trend inflation threshold, rendering them not useful for monetary policy analysis when trend inflation is not low. This 'steady state problem', where the relative price and price dispersion shoot to infinity and output goes to zero, can be mechanically resolved by assuming price indexation or increasing the price-adjustment frequency with trend inflation. These resolutions are, however, unsatisfactory and not supported by evidence. The main contribution of the paper is to establish that the root of the steady state problem is not Calvo pricing, as commonly assumed, but rather the popular Dixit-Stiglitz demand structure in NK models. We consider a general demand structure with the feature that demand remains finite when relative prices increase and show that the steady state always exists with Calvo pricing for any trend inflation level. Using this framework, we assess the properties of the Kimball-demand aggregator, which avoids the steady state problem but creates new ones. We then present a model with price wedges to augment the Dixit-Stiglitz and Kimball-demand aggregators and show that it resolves the steady state problem. Our findings show that modification of the demand structure can ensure that NK models are useful in evaluating alternative monetary policies for reducing inflation when trend inflation is not low.

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A Proof of Theorem 1

Assumptions: Under the Calvo staggered price setting ($\alpha > 0$) with partial indexation ($\gamma < 1$), as previously described, consider the generic relative demand function $\frac{y}{Y} = f(\wp, \wp_s)$ described in Section 3.1.1, where $\wp_s \equiv \frac{P_s}{P}$, where P_s and P grow at the same rate in any steady state, such that $f(\wp, \wp_s)$ is a non-negative, continuous and differentiable function in $(\wp, \wp_s) \in (\mathbb{R}_+^* \times \mathbb{R}_+^*)$ and non-increasing in $\wp \in R_+^*$. Let $f_1(\wp, \wp_s) \equiv \frac{\partial f(\wp, \wp_s)}{\partial \wp}$ denote the partial derivative of f with respect to $\wp$.

Theorem 1 *If $f(\wp, \wp_s)$ and $\wp \cdot f_1(\wp, \wp_s)$ are finite and defined at all their domain, including at $\wp \to 0$ and $\wp \to \infty$,*[30] *there always exists a steady state equilibrium (with no output growth) for any level of trend inflation ($\bar{\Pi} = 1 + \bar{\pi}$), provided that it is not extremely negative, i.e. $\bar{\Pi} > (\alpha)^{\frac{1}{(1-\gamma)}}$. For any other level of trend inflation, including all positive values, both the optimal relative price and the output-gap converge to finite steady state levels.*

Proof. Consider that all shocks are kept at their means, i.e. $\epsilon_t = \bar{\epsilon}$ and $\mathcal{A}_t = \overline{\mathcal{A}}$, at all periods, so that there are no stochastic uncertainties. Also consider that gross trend inflation is kept constant at $\bar{\Pi} = 1 + \bar{\pi}$. Since we assume that P_s and P grow at the same rate in any steady state, it must be the case that $\bar{\wp}_s$ is independent of j.

For simplicity sake, let us define the function $\tilde{f}(\wp, \wp_s) \equiv \wp \cdot f_1(\wp, \wp_s)$, where $f_1(\wp, \wp_s) \equiv \frac{\partial f(\wp, \wp_s)}{\partial \wp}$.

Now, since we assume in Section 3.1.1 that the weight function is bounded, i.e. $g(\wp, \wp_s) \in (0, 1)$, we must have that $g(0, \wp_s)$ and $\lim_{\wp \to \infty} g(\wp, \wp_s)$ exist and are both finite. Given the theorem assumptions, for a fixed value $\bar{\wp}_s$, let $f_0 \equiv \lim_{\wp \to 0} f(\wp, \bar{\wp}_s)$, $f_\infty \equiv \lim_{\wp \to \infty} f(\wp, \bar{\wp}_s)$, $\tilde{f}_0 \equiv \lim_{\wp \to 0} \tilde{f}(\wp, \bar{\wp}_s)$, $\tilde{f}_\infty \equiv \lim_{\wp \to \infty} \tilde{f}(\wp, \bar{\wp}_s)$, $g_0 \equiv \lim_{\wp \to 0} g(\wp, \bar{\wp}_s)$, and $g_\infty \equiv \lim_{\wp \to \infty} g(\wp, \bar{\wp}_s)$ denote the implied finite limits, i.e. $0 \leq f_0 < \infty$, $0 \leq f_\infty < \infty$, $-\infty < \tilde{f}_0 \leq 0$, $-\infty < \tilde{f}_\infty \leq 0$, $0 < g_0 < 1$ and $0 < g_\infty < 1$.

In addition, $\bar{q}_{t,t+j} = \left(\frac{\beta}{\bar{\Pi}}\right)^j$, $\bar{\Pi}_{t,t+j}^{ind} = (\bar{\Pi}^\gamma)^j$, and $\bar{\mathcal{G}}_{t,t+j} = 1$. Therefore, if existent, the pricing steady state relations implied by the system in equations (9), (10) and (11) are:

$$
\begin{aligned}
0 = &\sum_{j=0}^{\infty} \left(\frac{\alpha\beta}{\bar{\Pi}^{(1-\gamma)}}\right)^j f\left(\frac{\bar{\wp}^*}{\bar{\Pi}^{(1-\gamma)j}}, \bar{\wp}_s\right) - \sum_{j=0}^{\infty} \left(\frac{\alpha\beta}{\bar{\Pi}^{(1-\gamma)}}\right)^j \left[-\tilde{f}\left(\frac{\bar{\wp}^*}{\bar{\Pi}^{(1-\gamma)j}}, \bar{\wp}_s\right)\right] \\
&+ \sum_{j=0}^{\infty} (\alpha\beta)^j \left(1 + \frac{1}{\xi^n}\right) \frac{(\bar{X})^{(\sigma+\omega)}}{\bar{\wp}^*} \left[-\tilde{f}\left(\frac{\bar{\wp}^*}{\bar{\Pi}^{(1-\gamma)j}}, \bar{\wp}_s\right)\right] \left[f\left(\frac{\bar{\wp}^*}{\bar{\Pi}^{(1-\gamma)j}}, \bar{\wp}_s\right)\right]^\omega \\
\wp_s = &(1-\alpha) \sum_{j=0}^{\infty} \left(\frac{\alpha}{\bar{\Pi}^{(1-\gamma)}}\right)^j (\bar{\wp}^*) g\left(\frac{\bar{\wp}^*}{\bar{\Pi}^{(1-\gamma)j}}, \bar{\wp}_s\right) \\
1 = &(1-\alpha) \sum_{j=0}^{\infty} \left(\frac{\alpha}{\bar{\Pi}^{(1-\gamma)}}\right)^j (\bar{\wp}^*) f\left(\frac{\bar{\wp}^*}{\bar{\Pi}^{(1-\gamma)j}}, \bar{\wp}_s\right)
\end{aligned}
$$

[30] Formally, $0 \leq \lim_{\wp \to 0} f(\wp, \wp_s) < \infty$, $0 \leq \lim_{\wp \to \infty} f(\wp, \wp_s) < \infty$, $-\infty < \lim_{\wp \to 0} \wp \cdot f_1(\wp, \wp_s) \leq 0$, and $-\infty < \lim_{\wp \to \infty} \wp \cdot f_1(\wp, \wp_s) \leq 0$.

where

$$\tilde{f}\left(\frac{\bar{\wp}^*}{\bar{\Pi}^{(1-\gamma)j}}, \bar{\wp}_s\right) = \left(\frac{\bar{\wp}^*}{\bar{\Pi}^{(1-\gamma)j}}\right) f_1\left(\frac{\bar{\wp}^*}{\bar{\Pi}^{(1-\gamma)j}}, \bar{\wp}_s\right)$$

It is not hard to recognize this result as a system involving five different non-negative power series, whose format is $g_b\left(\rho_b | \frac{\bar{\wp}^*}{\bar{\Pi}^{(1-\gamma)j}}, \bar{\wp}_s\right) \equiv \sum_{j=0}^{\infty} (\rho_b)^j\, b_b\left(\frac{\bar{\wp}^*}{\bar{\Pi}^{(1-\gamma)j}}, \bar{\wp}_s\right)$, where $\rho_b \geq 0$ and $b_b\left(\frac{\bar{\wp}^*}{\bar{\Pi}^{(1-\gamma)j}}, \bar{\wp}_s\right) \geq 0$. Of course, each one of those power series has specific parameter ρ_b and function $b_b(\cdot,\cdot)$, i.e. $\rho_b \in \left\{\frac{\alpha\beta}{\bar{\Pi}^{(1-\gamma)}}, \alpha\beta, \frac{\alpha}{\bar{\Pi}^{(1-\gamma)}}\right\}$ and $b_b(\cdot,\cdot)$ depends on different combinations of $f\left(\frac{1}{\bar{\Pi}^{(1-\gamma)j}}\bar{\wp}^*, \bar{\wp}_s\right)$, $\tilde{f}\left(\frac{\bar{\wp}^*}{\bar{\Pi}^{(1-\gamma)j}}, \bar{\wp}_s\right)$ and $g\left(\frac{\bar{\wp}^*}{\bar{\Pi}^{(1-\gamma)j}}, \bar{\wp}_s\right)$. Note that, fixing the values of $\bar{\wp}^*$ and $\wp_s$, $b_b\left(\frac{\bar{\wp}^*}{\bar{\Pi}^{(1-\gamma)j}}, \bar{\wp}_s\right)$ is a well-defined, finite and non-negative sequence in j.

All we have to do is to show that all those five non-negative power series converge. If $\bar{\Pi}^{(1-\gamma)} = 1$, we have the trivial case in which all power series in the system are actually geometric series. In this case, given the assumption that $f(\wp, \wp_s)$, $f(\wp, \wp_s)$ and $g(\wp, \wp_s)$ are finite and defined at all their domain, convergence is always ensured, for $0 \leq \alpha < 1$ and $0 \leq \beta < 1$ in the model.

When $\bar{\Pi}^{(1-\gamma)} > 1$ (positive trend inflation), or $\bar{\Pi}^{(1-\gamma)} < 1$ (negative trend inflation), we will use the Ratio test. But before, some considerations are necessary. Since the relative demand and weight functions are general, there is nothing precluding $b_b\left(\frac{\bar{\wp}^*}{\bar{\Pi}^{(1-\gamma)j}}, \bar{\wp}_s\right)$ to be zero at some points. Therefore, we resort to an auxiliary power series $g_\zeta(\rho_b) \equiv \sum_{j=0}^{\infty} (\rho_b)^j \zeta$, defined for an arbitrary fixed and strictly positive value $\zeta > 0$. Obviously, $g_\zeta(\rho_b)$ converges as long as $|\rho_b| < 1$, which is the case whenever $\alpha\beta < 1$, $\alpha\beta < \bar{\Pi}^{(1-\gamma)}$ and $\alpha < \bar{\Pi}^{(1-\gamma)}$. Since $0 \leq \alpha < 1$ and $0 \leq \beta < 1$, the restriction is simplified to $\alpha < \bar{\Pi}^{(1-\gamma)}$.

Let us consider an augmented power series $\breve{g}_b\left(\rho_b | \frac{\bar{\wp}^*}{\bar{\Pi}^{(1-\gamma)j}}, \bar{\wp}_s\right) \equiv g_\zeta(\rho_b) + g_b\left(\rho_b | \frac{\bar{\wp}^*}{\bar{\Pi}^{(1-\gamma)j}}, \bar{\wp}_s\right)$. By construction, all terms in this power series are strictly positive. It implies that convergence can be verified using the sufficient Ratio test, which states that the power series $\breve{g}_b\left(\rho_b | \frac{\bar{\wp}^*}{\bar{\Pi}^{(1-\gamma)j}}, \bar{\wp}_s\right) = \sum_{j=0}^{\infty} (\rho_b)^j \left[\zeta + b_b\left(\frac{\bar{\wp}^*}{\bar{\Pi}^{(1-\gamma)j}}, \bar{\wp}_s\right)\right]$ converges if the limiting ratio $T_{ratio} \equiv \lim_{j\to\infty} \left| \frac{(\rho_b)^{(j+1)}\left[\zeta + b_b\left(\frac{\bar{\wp}^*}{\bar{\Pi}^{(1-\gamma)(j+1)}}, \bar{\wp}_s\right)\right]}{(\rho_b)^j\left[\zeta + b_b\left(\frac{\bar{\wp}^*}{\bar{\Pi}^{(1-\gamma)j}}, \bar{\wp}_s\right)\right]} \right|$ is smaller than unity, i.e. if $T_{ratio} < 1$, and diverges if $T_{ratio} > 1$. At the boundary $T_{ratio} = 1$, the test is mute, as the power series can either converge or diverge, depending on its functional form.

Note that $\lim_{j\to\infty} \tilde{f}\left(\frac{\bar{\wp}^*}{\bar{\Pi}^{(1-\gamma)j}}, \bar{\wp}_s\right) = \tilde{f}_0$ when $\bar{\Pi}^{(1-\gamma)} > 1$, whereas $\lim_{j\to\infty} \tilde{f}\left(\frac{\bar{\wp}^*}{\bar{\Pi}^{(1-\gamma)j}}, \bar{\wp}_s\right) = \tilde{f}_\infty$ when $\bar{\Pi}^{(1-\gamma)} < 1$. Since f_0, f_∞, $\tilde{f}_0$, $\tilde{f}_\infty$, g_0 and g_∞ are all finite and defined, it is easy to verify that the limiting ratio is $T_{ratio} = |\rho_b|$ for all five augmented power series, regardless of whether $\bar{\Pi}^{(1-\gamma)} > 1$ or $\bar{\Pi}^{(1-\gamma)} < 1$. And so, the ratio test predicts that all of them converge if $|\rho_b| < 1$. Since the auxiliary power series $g_\zeta(\rho_b)$ converges, it implies that all five original power series also converge under the same conditions.

Therefore, we conclude that the system implies a convergent relation and provides an implicit solution for the steady state levels $\bar{\wp}^*$, $\bar{\wp}_s$ and $\bar{X}$, as long as $\alpha < \bar{\Pi}^{(1-\gamma)}$.

Since $\bar{\Pi} = 1 + \bar{\pi}$, the condition is always satisfied for non-negative trend inflation ($\bar{\Pi} \geq 1$). Only for some negative trend inflation, the condition can be violated. That is, convergence is only achieved if trend inflation $\bar{\pi}$ is not extremely negative, i.e. when $\bar{\pi} > (\alpha)^{\frac{1}{(1-\gamma)}} - 1$. ■

B Kimball Aggregator

In Kimball (1995), consumption over all differentiated goods $c_t(z)$ are aggregated into a bundle C_t, according to $1 = \int_0^1 G\left(\frac{c_t(z)}{C_t}\right) dz$, where function $G(\varkappa)$ satisfies $G(1) = 1$, $G'(\varkappa) > 0$, and $G''(\varkappa) < 0$, for all $\varkappa \geq 0$. In this context, Dotsey and King (2005) propose the particular functional form $G\left(\frac{c_t(z)}{C_t}\right) = \frac{\mathfrak{m}}{1+\varphi}\left[(1+\varphi)\frac{c_t(z)}{C_t} - \varphi\right]^{\frac{1}{\mathfrak{m}}} + 1 - \frac{\mathfrak{m}}{1+\varphi}$, where $\mathfrak{m} \equiv \frac{\mu_k(1+\varphi)}{(1+\mu_k\varphi)}$ is a composite parameter, $\mu_k \geq 1$ is the elasticity parameter, which matches the implicit markup rate μ under flexible prices, and $\varphi \leq 0$ sets the aggregation curvature. If $\varphi = 0$, $G(\cdot)$ simplifies into the standard Dixit and Stiglitz (1977) aggregation form. Allowing for smooth-kinked demand function, it has also been used by Levin et al. (2007), Harding et al. (2022) and Kurozumi and Van Zandweghe (2024).

According the notation used in Section 3.1, this model sets $\delta = 0$. The literature typically derives the utility-based demand function by choosing $c_t(z)$ to minimize expenditure $P_tC_t \equiv \int_0^1 p_t(z)\, c_t(z)\, dz$, subject to only one restriction, the Kimball aggregation $1 = \int_0^1 G\left(\frac{c_t(z)}{C_t}\right) dz$. The implied demand function and implied price aggregation are:

$$\begin{gathered}\frac{c_t(z)}{C_t} = f\left(\wp_t(z), \wp_{s,t}\right) = \begin{cases} \frac{1}{(1+\varphi)}\left(\frac{\wp_t(z)}{\wp_{k,t}}\right)^{\varpi} + \frac{\varphi}{(1+\varphi)} & ;\text{if } \left(\frac{\wp_t(z)}{\wp_{k,t}}\right) \leq (-\varphi)^{\frac{1}{\varpi}} \\ 0 & ;\text{if } \left(\frac{\wp_t(z)}{\wp_{k,t}}\right) > (-\varphi)^{\frac{1}{\varpi}} \end{cases} \\ \wp_{k,t} \equiv (1+\varphi) - \varphi\wp_{s,t} \quad ;\, P_{s,t} \equiv \int_0^1 p_t(z)\, dz \quad ;\, 1 = \int_0^1 \left(\frac{\wp_t(z)}{\wp_{k,t}}\right)^{(1+\varpi)} dz\end{gathered} \tag{19}$$

where $\varpi \equiv \frac{\mu_k(1+\varphi)}{(1-\mu_k)} = -\frac{\mathfrak{m}}{(\mathfrak{m}-1)}$, $P_{k,t}$ is an auxiliary composite price aggregate, $P_{s,t}$ is the average price. Paralleling the notation used in Section 3.1.1, we define $\wp_t(z) \equiv \frac{p_t(z)}{P_t}$ as the relative price of firm z, and $\wp_{s,t} = \frac{P_{s,t}}{P_t}$ as the average relative price. We also set $\wp_{k,t} \equiv \frac{P_{k,t}}{P_t}$ as the auxiliary composite relative price. In addition, it is straightforward to verify that the price aggregation $1 = \int_0^1 \left(\frac{\wp_t(z)}{(1+\varphi)-\varphi\wp_{s,t}}\right)^{(1+\varpi)} dz$ is equivalent to $P_t = \int_0^1 p_t(z)\, f\left(\wp_t(z), \wp_{s,t}\right) dz$.

Under this type of Kimball aggregation, the firm z's price $p_t(z)$ elasticity and superelasticity are: (i) $\xi_t(z) = -\varpi\left(\frac{\wp_t(z)}{\wp_{k,t}}\right)^{\varpi}\left[\left(\frac{\wp_t(z)}{\wp_{k,t}}\right)^{\varpi} + \varphi\right]^{-1}$ and $\eta_t(z) = \varpi\varphi\left[\left(\frac{\wp_t(z)}{\wp_{k,t}}\right)^{\varpi} + \varphi\right]^{-1}$, if $\left(\frac{\wp_t(z)}{\wp_{k,t}}\right) \leq (-\varphi)^{\frac{1}{\varpi}}$; or (ii) $\xi_t(z) = 0$ and $\eta_t(z) = 0$, if $\left(\frac{\wp_t(z)}{\wp_{k,t}}\right) > (-\varphi)^{\frac{1}{\varpi}}$.

In macroeconomic models, the way to generate empirically observed persistent non-neutrality in aggregate output is to combine real and nominal rigidities. However, empirical evidence suggest that price stickiness is not so large.[31] Therefore, macroeconomists tend to use theoretical models with large real rigidities (see e.g. Ball and Romer (1990), Basu (1995), Blanchard and Gali (2007)). In this regard, Kimball's implied real rigidity can be easily computed using (12), evaluated in the steady state equilibrium with flexible prices:[32]

$$\kappa_{\text{real}}^{Kimball} = \frac{(\sigma+\omega)}{1-\mu\varphi+\frac{\mu}{(\mu-1)}\omega} \quad ; \; \psi_{\text{real}}^{Kimball} = \frac{1}{\kappa_{\text{real}}}$$

Therefore, for a given preferences/production structure represented by σ and ω, a large degree of large real rigidity $\psi_{\text{real}}^{Kimball}$ can be achieved with a convenient balance between a large demand curvature ($\varphi << 0$) and a appropriate markup $\mu > 1$.

B.1 Kimball NK Model

In general equilibrium, based on the generic model shown in Section 3, we have:

$$\begin{aligned}
1 &= \beta E_t \left(\frac{\epsilon_{t+1}}{\epsilon_t} \left(\frac{Y_t}{Y_{t+1}} \right)^{\sigma} \frac{I_t}{\Pi_{t+1}} \right) \\
q_t &= \beta \frac{\epsilon_t}{\epsilon_{t-1}} \left(\frac{Y_{t-1}}{Y_t} \right)^{\sigma} \frac{1}{\Pi_t} \\
\left(\frac{I_t}{\bar{I}} \right) &= \epsilon_{\text{i},t} \left(\frac{I_{t-1}}{\bar{I}} \right)^{\phi_{\text{i}}} \left[\left(\frac{\Pi_t}{\bar{\Pi}} \right)^{\phi_\pi} \left(\frac{X_t}{\bar{X}} \right)^{\phi_x} \left(\frac{Y_t}{Y_{t-1}} \right)^{\phi_{gy}} \left(\frac{Y_t}{\bar{Y}} \right)^{\phi_y} \right]^{(1-\phi_{\text{i}})} \\
(Y_t^n)^{(\sigma+\omega)} &= \frac{1}{\mu} \frac{\varepsilon}{\chi} \epsilon_t \left(\mathcal{A}_t \right)^{(1+\omega)}
\end{aligned}$$

$$\begin{array}{llllll}
X_t = & \frac{Y_t}{Y_t^n} & ; \; q_{t,t+j} = & q_{t+1} q_{t+1,t+j} & \text{for } j \geq 1 \text{ and } q_{t,t} = 1 \\
\mathcal{G}_t = & \frac{Y_t}{Y_{t-1}} & ; \; \Pi_{t,t+j} = & \Pi_{t+1} \Pi_{t+1,t+j} & \text{for } j \geq 1 \text{ and } \Pi_{t,t} = 1 \\
\Pi_t^{ind} = & \Pi_{t-1}^{\gamma} & ; \; \Pi_{t,t+j}^{ind} = & \Pi_{t+1}^{ind} \Pi_{t+1,t+j}^{ind} & \text{for } j \geq 1 \text{ and } \Pi_{t,t}^{ind} = 1 \\
 & & ; \; \mathcal{G}_{t,t+j} = & \mathcal{G}_{t+1} \mathcal{G}_{t+1,t+j} & \text{for } j \geq 1 \text{ and } \mathcal{G}_{t,t} = 1
\end{array}$$

[31] As in e.g. Bils and Klenow (2004) and Nakamura and Steinsson (2008), estimated median duration between price changes ranges from about 4.5 months, when sales are included, to 10 months, when they are excluded.

[32] Considering that μ here is the gross markup rate, the component $(1 - \mu\varphi)$ is the same found in Levin, Lopez-Salido and Yun (2007) and Harding, Linde and Trabandt (2022), as their models implies $\omega = 0$.

$$
\begin{aligned}
\wp_{s,t} &= (1-\alpha)\,\wp_t^* + \alpha \frac{\Pi_t^{ind}}{\Pi_t} \wp_{s,t-1} \\
(1+\varphi) &= \wp_{k,t} + \varphi \wp_{s,t} \\
(\wp_{k,t})^{-\frac{1}{(\mathfrak{m}-1)}} &= (1-\alpha)\,(\wp_t^*)^{-\frac{1}{(\mathfrak{m}-1)}} + \alpha \left(\frac{\Pi_t^{ind}}{\Pi_t}\right)^{-\frac{1}{(\mathfrak{m}-1)}} (\wp_{k,t-1})^{-\frac{1}{(\mathfrak{m}-1)}} \\
\wp_t^* &= \varphi\,(\mathfrak{m}-1)\,(\wp_t^*)^{\left(1+\frac{\mathfrak{m}}{(\mathfrak{m}-1)}\right)} \frac{N_{1,t}}{D_t} + \frac{\mathfrak{m}}{\mu} \frac{N_{2,t}}{D_t} \\
D_t &= (\wp_{k,t})^{\frac{\mathfrak{m}}{(\mathfrak{m}-1)}} + \alpha E_t q_{t+1} \mathcal{G}_{t+1} \Pi_{t+1} \left(\frac{\Pi_{t+1}}{\Pi_{t+1}^{ind}}\right)^{\frac{1}{(\mathfrak{m}-1)}} D_{t+1} \\
N_{1,t} &= 1 + \alpha E_t q_{t+1} \mathcal{G}_{t+1} \Pi_{t+1}^{ind} N_{1,t+1} \\
N_{2,t} &= \mu E_t \textstyle\sum_{j=0}^{\infty} q_{t,t+j} \alpha^j \mathcal{G}_{t,t+j} \Pi_{t,t+j} \left(\frac{\Pi_{t,t+j}}{\Pi_{t,t+j}^{ind}}\right)^{\frac{\mathfrak{m}}{(\mathfrak{m}-1)}} (\wp_{k,t+j})^{\frac{\mathfrak{m}}{(\mathfrak{m}-1)}} mc_{t,t+j}^* \\
mc_{t,t+j}^* &= \frac{1}{\mu} \left(X_{t+j}\right)^{(\sigma+\omega)} \left[\frac{1}{(1+\varphi)} \left(\frac{\Pi_{t,t+j}^{ind}}{\Pi_{t,t+j}} \frac{\wp_t^*}{\wp_{k,t+j}}\right)^{-\frac{\mathfrak{m}}{(\mathfrak{m}-1)}} + \frac{\varphi}{(1+\varphi)}\right]^{\omega}
\end{aligned}
$$

Since power ω in the equation for $mc_{t,t+j}^*$ is not a positive integer, we cannot write $N_{2,t}$ in a finite recursive way. Therefore, simulations are to be carried out using the same approximation we use for the price wedge model.

B.2 Steady state

Given an exogenous level of trend inflation $\bar{\Pi}$, the steady state levels can be numerically obtained as follows. First, we compute $\bar{I}$, $\bar{q}$, and $\bar{Y}^n$:

$$
\bar{I} = \frac{\bar{\Pi}}{\beta} \quad ; \; \bar{q} = \frac{\beta}{\bar{\Pi}} \quad ; \; (\bar{Y}^n)^{(\sigma+\omega)} = \frac{1}{\mu} \frac{\varepsilon}{\chi} \bar{\epsilon} \left(\overline{\mathcal{A}}\right)^{(1+\omega)}
$$

Next, we use a numerical code to solve the following non-linear system for relative prices $\bar{\wp}^*$, $\bar{\wp}_s$, and $\bar{\wp}_k$:

$$
\bar{\wp}^* = \frac{(1+\varphi)}{\left[\left(\frac{1-\alpha}{1-\bar{\alpha}_{k1}}\right)^{-(\mathfrak{m}-1)} + \varphi \frac{(1-\alpha)}{(1-\bar{\alpha}_{k3})}\right]} \quad ; \; \bar{\wp}_s = \frac{(1-\alpha)}{(1-\bar{\alpha}_{k3})} \bar{\wp}^* \quad ; \; \bar{\wp}_k = (1+\varphi) - \varphi \bar{\wp}_s
$$

where

$$
\bar{\alpha}_{k1} = \alpha \left(\bar{\Pi}\right)^{\frac{(1-\gamma)}{(\mathfrak{m}-1)}} \quad ; \; \bar{\alpha}_{k2} = \alpha \bar{\Pi}^{\frac{\mathfrak{m}(1-\gamma)}{(\mathfrak{m}-1)}} \quad ; \; \bar{\alpha}_{k3} = \alpha \bar{\Pi}^{-(1-\gamma)}
$$

The following step is to find the gross output gap $\bar{X}$:

$$
\begin{aligned}
&\bar{S}_d \equiv \sum_{j=0}^{\infty} (\bar{\alpha}_{k2}\beta)^j \left[\frac{1}{(1+\varphi)} \left(\frac{\bar{\alpha}_{k2}}{\alpha}\right)^j \left(\frac{\bar{\wp}^*}{\bar{\wp}_k}\right)^{-\frac{\mathfrak{m}}{(\mathfrak{m}-1)}} + \frac{\varphi}{(1+\varphi)}\right]^{\omega} \quad ; \; \bar{D} = \frac{(\bar{\wp}_k)^{\frac{\mathfrak{m}}{(\mathfrak{m}-1)}}}{(1-\bar{\alpha}_{k1}\beta)} \quad ; \; \bar{N}_1 = \frac{1}{(1-\bar{\alpha}_{k3}\beta)} \\
&\bar{N}_2 = \frac{\mu}{\mathfrak{m}} (\bar{\wp}^*) \left(\bar{D} - \varphi\,(\mathfrak{m}-1)\,(\bar{\wp}^*)^{\frac{\mathfrak{m}}{(\mathfrak{m}-1)}} \bar{N}_1\right) \quad ; \; (\bar{X})^{(\sigma+\omega)} = \frac{\bar{N}_2}{(\bar{\wp}_k)^{\frac{\mathfrak{m}}{(\mathfrak{m}-1)}}} \frac{1}{\bar{S}_d} \quad ; \; \bar{Y} = \bar{X}\bar{Y}^n
\end{aligned}
$$

If $\frac{\bar{\alpha}_{k2}}{\alpha} \leq 1$ the infinite sum $\bar{S}_d$ converges when $(\bar{\alpha}_{k2}\beta) < 1$. If $\frac{\bar{\alpha}_{k2}}{\alpha} > 1$, it converges when $(\bar{\alpha}_{k2}\beta) \left(\frac{\bar{\alpha}_{k2}}{\alpha}\right)^{\omega} <$

1.

The infinite sum $\bar{S}_d$ is generally numerically retrieved by using a finite sum in $j = \{0, 1, ..., J\}$ for a large J. In this paper, we use $J = 10000$. For numerical stability when $\frac{\bar{\alpha}_{k2}}{\alpha} > 1$, $\bar{S}_d$ is better computed using $\bar{S}_d = \sum_{j=0}^{\infty} \left(\bar{\alpha}_{k2}\beta \left(\frac{\bar{\alpha}_{k2}}{\alpha}\right)^{\omega} \right)^j \left[\frac{1}{(1+\varphi)} \left(\frac{\bar{\wp}^*}{\bar{\wp}_k} \right)^{-\frac{m}{(m-1)}} + \frac{\varphi}{(1+\varphi)\left(\frac{\bar{\alpha}_{k2}}{\alpha}\right)^j} \right]^{\omega}$.

Alternatively, if ω is a positive integer, it is feasible to derive an exact closed form solution for $\bar{S}_d$. For that, we only need to expand the term in brackets and obtain a couple of infinite sums that allow for closed form solutions. For instance, if $\omega = 2$, $\frac{(\bar{\alpha}_{k2})^2}{\alpha^2}\beta < 1$ and $\bar{\alpha}_{k2}\beta < 1$, we obtain:

$$\begin{aligned}
S \equiv & \sum_{j=0}^{\infty} (\bar{\alpha}_{k2}\beta)^j \left[\frac{1}{(1+\varphi)} \left(\frac{\bar{\alpha}_{k2}}{\alpha}\right)^j \left(\frac{\bar{\wp}^*}{\bar{\wp}_k} \right)^{-\frac{m}{(m-1)}} + \frac{\varphi}{(1+\varphi)} \right]^2 \\
= & \sum_{j=0}^{\infty} (\bar{\alpha}_{k2}\beta)^j \left[\frac{1}{(1+\varphi)^2} \left(\frac{\bar{\wp}^*}{\bar{\wp}_k} \right)^{-\frac{2m}{(m-1)}} \left(\left(\frac{\bar{\alpha}_{k2}}{\alpha}\right)^2 \right)^j + 2\frac{1}{(1+\varphi)} \left(\frac{\bar{\wp}^*}{\bar{\wp}_k} \right)^{-\frac{m}{(m-1)}} \left(\frac{\bar{\alpha}_{k2}}{\alpha}\right)^j + \frac{\varphi^2}{(1+\varphi)^2} \right] \\
= & \frac{(\bar{\wp}^*/\bar{\wp}_k)^{-\frac{2m}{(m-1)}}}{(1+\varphi)^2} \sum_{j=0}^{\infty} \left(\frac{(\bar{\alpha}_{k2})^2}{\alpha^2}\beta \right)^j + \frac{2(\bar{\wp}^*/\bar{\wp}_k)^{-\frac{m}{(m-1)}}}{(1+\varphi)} \sum_{j=0}^{\infty} \left(\frac{(\bar{\alpha}_{k2})^2}{\alpha}\beta \right)^j + \frac{\varphi^2}{(1+\varphi)^2} \sum_{j=0}^{\infty} (\bar{\alpha}_{k2}\beta)^j \\
= & \frac{1}{(1+\varphi)^2} \frac{(\bar{\wp}^*/\bar{\wp}_k)^{-\frac{2m}{(m-1)}}}{\left(1 - \frac{(\bar{\alpha}_{k2})^2}{\alpha^2}\beta\right)} + \frac{2}{(1+\varphi)} \frac{(\bar{\wp}^*/\bar{\wp}_k)^{-\frac{m}{(m-1)}}}{\left(1 - \frac{(\bar{\alpha}_{k2})^2}{\alpha}\beta\right)} + \frac{\varphi^2}{(1+\varphi)^2} \frac{1}{(1-\bar{\alpha}_{k2}\beta)}
\end{aligned}$$

B.3 Constrained Demand

Given the extra demand kink at $\left(\frac{p_t(z)}{P_{k,t}}\right) = (-\varphi)^{\frac{1}{\omega}}$, this particular case of Kimball's aggregation implies firms will typically not set any price $p_t(z)$ larger than $(-\varphi)^{\frac{1}{\omega}} [(1+\varphi) P_t - \varphi P_{s,t}]$, as would lead to zero demand. If $\varphi = 0$, in particular, the threshold $(-\varphi)^{\frac{1}{\omega}}$ is infinity. And so the restriction $\frac{c_t(z)}{C_t} \geq 0$ is never biding under Dixit and Stiglitz (1977) aggregation. If $\varphi < 0$, however, we argue that the condition for non-zero demand $\left(\frac{p_t(z)}{P_{k,t}}\right) < (-\varphi)^{\frac{1}{\omega}}$ might not always hold with real data. That is, an empirical test for this type of aggregation is to verify whether empirical values of relative prices $\frac{p_t(z)}{P_{k,t}}$ are smaller than $(-\varphi)^{\frac{1}{\omega}}$.

Here, as it is not the main scope of this paper, we do not propose a sophisticated formal econometric test. Rather, we propose a simple approach. So, the question is whether we can find a way to compute P_t and $P_{s,t}$ using typical moments from price samples, which would provide us with an estimate for $P_{k,t} \equiv [(1+\varphi) P_t - \varphi P_{s,t}]$. And here lies a slight caveat. While $P_{s,t} \equiv \int_0^1 p_t(z)\, dz$ is the simple average price, which can be easily estimated using sample price means, P_t has no obvious empirical counterpart. Therefore, P_t is not easily empirically retrievable without relying on a general equilibrium model.

In order to tackle this issue, we propose a second-order approximation approach. Consider the typical price aggregation into P_t, abstracting from the relative price threshold. It can be written as

$(P_{k,t})^{-\frac{1}{(\mathfrak{m}-1)}} = \int_0^1 (p_t(z))^{-\frac{1}{(\mathfrak{m}-1)}} dz$. Note that a second-order approximation of $(p_t(z))^{-\frac{1}{(\mathfrak{m}-1)}}$, about the average price $P_{s,t} \equiv \int_0^1 p_t(z)\, dz$, is:

$$(p_t(z))^{-\frac{1}{(\mathfrak{m}-1)}} \approx (P_{s,t})^{-\frac{1}{(\mathfrak{m}-1)}} - \frac{(P_{s,t})^{-\frac{\mathfrak{m}}{(\mathfrak{m}-1)}}}{(\mathfrak{m}-1)} (p_t(z) - P_{s,t}) + \frac{1}{2} \frac{\mathfrak{m}(P_{s,t})^{-\frac{2\mathfrak{m}-1}{(\mathfrak{m}-1)}}}{(\mathfrak{m}-1)^2} (p_t(z) - P_{s,t})^2$$

It implies that

$$(P_{k,t})^{-\frac{1}{(\mathfrak{m}-1)}} = \int_0^1 (p_t(z))^{-\frac{1}{(\mathfrak{m}-1)}} dz \approx (P_{s,t})^{-\frac{1}{(\mathfrak{m}-1)}} \left[1 + \frac{1}{2} \frac{\mathfrak{m}}{(\mathfrak{m}-1)^2} \int_0^1 \left[\frac{p_t(z)}{P_{s,t}} - P_{s,t} \right]^2 dz \right]$$

Therefore, we obtain the following relation between $P_{k,t}$ and $P_{s,t}$:

$$\begin{aligned} P_{k,t} \approx & \left[1 + \frac{1}{2} \frac{\mathfrak{m}}{(\mathfrak{m}-1)^2} s_{s,t}^2 \right]^{-(\mathfrak{m}-1)} P_{s,t} \\ s_{s,t}^2 \equiv & \int_0^1 \left[\frac{p_t(z)}{P_{s,t}} - 1 \right]^2 dz \end{aligned} \tag{20}$$

where $P_{k,t} \equiv [(1+\varphi) P_t - \varphi P_{s,t}]$. Since $P_{s,t} \equiv \int_0^1 p_t(z)\, dz$ and $\int_0^1 \frac{p_t(z)}{P_{s,t}} dz = 1$, $s_{s,t}$ is the cross-section standard deviation of relative prices $\frac{p_t(z)}{P_{s,t}}$, which is a measure of relative price dispersion.

Recalling that $\mathfrak{m} \equiv \frac{\mu_k(1+\varphi)}{(1+\mu_k\varphi)}$, it is not hard to verify that: (i) $\frac{p_t(z)}{P_{s,t}} < \frac{p_t(z)}{P_{k,t}} < \frac{p_t(z)}{P_t}$, if $\varphi \in (-1, 0]$; and (ii) $\frac{p_t(z)}{P_{k,t}} \leq \frac{p_t(z)}{P_{s,t}} < \frac{p_t(z)}{P_t}$, if $\varphi \leq -1$. In both cases, all three relative prices are very close to each other whenever price dispersion is $s_{s,t}$ small.

Since the cross section average relative price is unity, i.e. $\int_0^1 \frac{p_t(z)}{P_{s,t}} dz = 1$, we can reasonable conclude that $\int_0^1 \frac{p_t(z)}{P_{k,t}} dz$ is also close to unity. And there lies a potential empirical issue with this type Kimball's demand function. Recall that its relative price constraint for non-zero demand is $\left(\frac{p_t(z)}{P_{k,t}} \right) \leq (-\varphi)^{\frac{1}{\varpi}}$, for $\varpi \equiv \frac{\mu_k(1+\varphi)}{(1-\mu_k)}$.

In the literature, the upper limit $(-\varphi)^{\frac{1}{\varpi}}$ for relative prices is generally very close to unity when φ is set, or implied, using common values estimated or calibrated for the US. In order to verify this property, consider first that μ_k matches the implicit markup rate μ under flexible prices. In this case, some typical calibrations for the US are the following ones: (i) $\mu = 1.10$, $\varphi = -12.2$ and $(-\varphi)^{\frac{1}{\varpi}} = 1.021$ in Harding, Linde and Trabandt (2022); (ii) $\mu = 1.17$, $\varphi = -8$ and $(-\varphi)^{\frac{1}{\varpi}} = 1.043$ in Levin, Lopez-Salido and Yun (2007);[33] and (iii) $\mu = 1.61$ (estimated), $\varphi = -3.79$ and $(-\varphi)^{\frac{1}{\varpi}} = 1.198$ in Smets and Wouters (2007).[34] In addition, obtaining a better marginal likelihood statistics for model

[33]In Levin, Lopez-Salido and Yun (2007), the elasticity of substitution between goods ϵ can be mapped into our notation as $\mu = \frac{\epsilon}{(\epsilon-1)}$. The authors calibrated $\epsilon = 7$, and so $\mu = \frac{7}{6}$.

[34]In Smets and Wouters (2007), the demand's curvature parameter ϵ_p can be mapped into our notation as $\epsilon_p = -\frac{\mu\varphi}{(\mu-1)}$. The authors calibrated $\epsilon_p = 10$ and estimated the gross markup rate at $\mu = 1.61$.

comparison,[35] Harding, Linde and Trabandt (2022) re-estimate Smets and Wouters (2007) model with a different prior distribution. Their new posterior modes imply $\mu = 1.34$ (estimated), $\varphi = -16.37$ and $(-\varphi)^{\frac{1}{\omega}} = 1.047$.

As for the empirical dispersion of relative prices, we consider the Kaplan and Menzio (2015) results described in Section 2. In particular, we make a conservative choice by considering the authors' Brand Aggregation, in which products have at least the same features and the same size, and so are in line with what economists usually think about commodity goods. Under this aggregation, the authors find that the empirical standard-deviation of relative prices, relatively to the sample average price $P_{a,t}$, is 0.25.[36] Notice that, under this type of Kimball aggregation, $P_s = P_a$. As depicted in Section 2, the authors' findings imply that a 80% confidence interval for empirical relative prices in the US are at least ranging from $\left(\frac{p(z)}{P_s}\right)_{0.10} = 0.68$ to $\left(\frac{p(z)}{P_s}\right)_{0.90} = 1.38$.

Using approximation (20), with standard deviation $s_s = 0.25$, and considering the authors' different calibration options for μ and φ, we are able to compute the implied 80% confidence intervals for $\left(\frac{p(z)}{P_k}\right)$ as follows:

$$\left(\frac{p(z)}{P_k}\right)_{0.10} = \left(\frac{P_s}{P_k}\right)\left(\frac{p(z)}{P_s}\right)_{0.10} \quad ; \quad \left(\frac{p(z)}{P_k}\right)_{0.90} = \left(\frac{P_s}{P_k}\right)\left(\frac{p(z)}{P_s}\right)_{0.90}$$

Therefore, considering different calibration options for μ and φ, Table 1 verifies whether the implied 80% confidence intervals for $\left(\frac{p(z)}{P_k}\right)$ are at least totally included in the feasibility region $\left(\frac{p(z)}{P_k}\right) \leq (-\varphi)^{\frac{1}{\omega}}$. Of course, this back-of-the-envelope analysis is by no means meant to be a formal hypothesis test, but the fact that all 90% quantiles surpass the theoretical Kimball's upper limit $(-\varphi)^{\frac{1}{\omega}}$ strongly suggests that an important fraction of relative prices are larger than the implied Kimball's upper limit for relative prices $(-\varphi)^{\frac{1}{\omega}}$. This conclusion is specially so for cases in which $(-\varphi)^{\frac{1}{\omega}}$ is very close to unity. This result is in line with simulations carried out by Klenow and Willis (2016), who find that about 15% of goods end up with zero relative demand when the demand function is Kimball-based with large curvature.

Table 1: Kimball's Relative Prices - Empirical confidence intervals and Kimball's upper limit

Authors	μ	φ	$\left(\frac{P_s}{P_k}\right)$	$(-\varphi)^{\frac{1}{\omega}}$	$\left(\frac{p(z)}{P_k}\right)_{0.90}$
Harding, Linde and Trabandt (2022)[a]	1.10	−12.2	0.95	1.02	1.31
Harding, Linde and Trabandt (2022)[b]	1.34	−16.4	0.93	1.05	1.28
Levin, Lopez-Salido and Yun (2007)	1.17	−8.0	0.92	1.04	1.27
Smets and Wouters (2007)	1.61	−3.8	0.88	1.20	1.21

[35] The authors obtain a marginal likelihood gain of 5 log points.

[36] Here, we are abstracting from frequency considerations the authors dealt with when using empirical data.

Notes: The empirical relative price 90% quantile is computed using Equation (20) and Kaplan and Menzio (2015) estimates. Kimball's relative price upper bound is $(-\varphi)^{\frac{1}{\varpi}}$. Harding, Linde and Trabandt (2022) first (a) calibrates $\mu = 1.10$ and $\varphi = -12.2$; and then (b) estimates $\mu = 1.34$ and $\varphi = -16.4$ using Smets and Wouters (2007) model with a different prior distribution. Again, ϖ is defined as $\varpi \equiv \frac{\mu_k(1+\varphi)}{(1-\mu_k)}$, and $\mu_k = \mu$.

C Price Wedge Model

The intermediate firm's nominal revenue is $R_{s,t} = \int_0^1 P_{s,t} c_{s,t}(z)\, dz = \delta P_{s,t} C_{s,t}$, where $C_{s,t} \equiv \int_0^1 c_t(z)\, dz$ is the average consumption (arithmetic mean), while its nominal cost is $Cost_{s,t} = \int_0^1 p_t(z)\, c_{s,t}(z)\, dz = \delta \int_0^1 p_t(z)\, c_t(z)\, dz$. As shown further on, the average consumption $C_{s,t}$ will only be the same as the aggregate consumption C_t when all goods z are perfect substitutes. This firm sets its price at a zero-profit condition $R_{s,t} = Cost_{s,t}$. Coupling this condition with the aggregate price definition $(1+\delta) P_t C_t \equiv \int_0^1 (p_t(z) + \delta P_{s,t})\, c_t(z)\, dz$ allows us to obtain two important results:

$$\begin{aligned}
(1+\delta) P_t C_t = & \int_0^1 (p_t(z) + \delta P_{s,t})\, c_t(z)\, dz = \int_0^1 p_t(z)\, c_t(z)\, dz + \delta P_{s,t} \int_0^1 c_t(z)\, dz \\
= & \int_0^1 p_t(z)\, c_t(z)\, dz + \delta P_{s,t} C_{s,t} = \int_0^1 p_t(z)\, c_t(z)\, dz + R_{s,t} \\
= & \int_0^1 p_t(z)\, c_t(z)\, dz + Cost_{s,t} = \int_0^1 p_t(z)\, c_t(z)\, dz + \delta \int_0^1 p_t(z)\, c_t(z)\, dz \\
= & (1+\delta) \int_0^1 p_t(z)\, c_t(z)\, dz \\
\therefore P_t C_t = & \int_0^1 p_t(z)\, c_t(z)\, dz
\end{aligned}$$

Since $R_{s,t} = Cost_{s,t}$, it implies that $P_{s,t} C_{s,t} = \int_0^1 p_t(z)\, c_t(z)\, dz = P_t C_t$. Therefore, we obtain two important results:

$$P_t C_t = \int_0^1 p_t(z)\, c_t(z)\, dz \quad ; \; P_{s,t} = P_t \frac{C_t}{C_{s,t}}$$

where $C_{s,t} \equiv \int_0^1 c_t(z)\, dz$.

C.1 Aggregate price as a weighted average of individual prices

Since $C_{s,t} \equiv \int_0^1 c_t(z)\, dz$ and $P_{s,t} = P_t \frac{C_t}{C_{s,t}}$, we obtain:

$$\frac{P_t}{P_{s,t}} = \int_0^1 \frac{c_t(z)}{C_t} dz = \int_0^1 (1+\delta)^{\theta} \left(\frac{p_t(z)}{P_t} + \delta \frac{P_{s,t}}{P_t} \right)^{-\theta} dz$$

Moreover, since $P_t = \int_0^1 p_t(z)\, f(\wp_t(z), \wp_{s,t})\, dz$, we obtain:

$$P_t = \int_0^1 p_t(z)\, (1+\delta)^{\theta} \left(\frac{p_t(z)}{P_t} + \delta \frac{P_{s,t}}{P_t} \right)^{-\theta} dz$$

Therefore, the normalized wedge price can be written as follows:

$$P_{s,t} = \frac{\int_0^1 \left(\frac{p_t(z)}{P_t} + \delta \frac{P_{s,t}}{P_t} \right)^{-\theta} p_t(z)\, dz}{\int_0^1 \left(\frac{p_t(z)}{P_t} + \delta \frac{P_{s,t}}{P_t} \right)^{-\theta} dz} = \int_0^1 g(\wp_t(z), \wp_{s,t})\, p_t(z)\, dz$$

where $g(\wp_t(z), \wp_{s,t}) = \frac{(\wp_t(z) + \delta \wp_{s,t})^{-\theta}}{\int_0^1 (\wp_t(z) + \delta \wp_{s,t})^{-\theta} dz}$.

C.2 General Equilibrium

The composite parameters are:

$$\omega \equiv \frac{(1+\nu)}{\varepsilon} - 1 \quad ;\ \theta_1 \equiv \theta\,(1+\omega) \quad ;\ \mu_\theta \equiv \frac{\theta}{(\theta-1)} \quad ;\ \mu_\delta \equiv \frac{\theta}{\theta-(1+\delta)} \quad ;\ \mu = \frac{\mu_\delta}{(1+\delta)}$$

The dynamic equations are:

$$\begin{aligned}
1 =& \quad \beta E_t \left(\frac{\epsilon_{t+1}}{\epsilon_t} \left(\frac{Y_t}{Y_{t+1}} \right)^\sigma \frac{I_t}{\Pi_{t+1}} \right) \\
q_t =& \quad \beta \frac{\epsilon_t}{\epsilon_{t-1}} \left(\frac{Y_{t-1}}{Y_t} \right)^\sigma \frac{1}{\Pi_t} \\
\left(\frac{I_t}{\bar{I}} \right) =& \quad \epsilon_{\mathrm{i},t} \left(\frac{I_{t-1}}{\bar{I}} \right)^{\phi_\mathrm{i}} \left[\left(\frac{\Pi_t}{\bar{\Pi}} \right)^{\phi_\pi} \left(\frac{X_t}{\bar{X}} \right)^{\phi_x} \left(\frac{Y_t}{Y_{t-1}} \right)^{\phi_{gy}} \left(\frac{Y_t}{\bar{Y}} \right)^{\phi_y} \right]^{(1-\phi_\mathrm{i})} \\
\left(Y_t^n\right)^{(\sigma+\omega)} =& \quad \frac{1}{(1+\delta)\mu} \frac{\varepsilon}{\chi} \epsilon_t \left(\mathcal{A}_t\right)^{(1+\omega)} \\
Y_{s,t} =& \quad (1+\delta)^\theta\, Y_t \left(\wp_{\delta,t}\right)^{-\theta}
\end{aligned}$$

$$\begin{aligned}
X_t =& \ \frac{Y_t}{Y_t^n} & ;\ q_{t,t+j} =& \ q_{t+1} q_{t+1,t+j} & \text{for } j \geq 1 \text{ and } q_{t,t} = 1 \\
\mathcal{G}_t =& \ \frac{Y_t}{Y_{t-1}} & ;\ \Pi_{t,t+j} =& \ \Pi_{t+1} \Pi_{t+1,t+j} & \text{for } j \geq 1 \text{ and } \Pi_{t,t} = 1 \\
\wp_{s,t} =& \ \frac{Y_t}{Y_{s,t}} & ;\ \Pi^{ind}_{t,t+j} =& \ \Pi^{ind}_{t+1} \Pi^{ind}_{t+1,t+j} & \text{for } j \geq 1 \text{ and } \Pi^{ind}_{t,t} = 1 \\
\Pi^{ind}_t =& \ \Pi^\gamma_{t-1} & ;\ \mathcal{G}_{t,t+j} =& \ \mathcal{G}_{t+1} \mathcal{G}_{t+1,t+j} & \text{for } j \geq 1 \text{ and } \mathcal{G}_{t,t} = 1
\end{aligned}$$

$$\begin{aligned}
(1+\delta)^{1-\theta} =& \quad (1-\alpha) \textstyle\sum_{j=0}^\infty \alpha^j \left(\frac{\Pi^{ind}_{t-j,t}}{\Pi_{t-j,t}} \wp^*_{t-j} + \delta \wp_{s,t} \right)^{1-\theta} \\
\left(\wp_{\delta,t}\right)^{-\theta} =& \quad (1-\alpha) \textstyle\sum_{j=0}^\infty \alpha^j \left(\frac{\Pi^{ind}_{t-j,t}}{\Pi_{t-j,t}} \wp^*_{t-j} + \delta \wp_{s,t} \right)^{-\theta} \\
1 =& \quad \mu_\theta \frac{N_t^*}{D_t^*} \\
N_t^* =& \quad E_t \sum_{j=0}^\infty \alpha^j q_{t,t+j} \mathcal{G}_{t,t+j} \Pi^{ind}_{t,t+j} \left(z^*_{t,t+j} \right)^{-(1+\theta)} \left(\frac{mc^*_{t,t+j}}{(1+\delta)} + \delta \wp_{s,t+j} \right) \\
D_t^* =& \quad E_t \sum_{j=0}^\infty \alpha^j q_{t,t+j} \mathcal{G}_{t,t+j} \Pi^{ind}_{t,t+j} \left(z^*_{t,t+j} \right)^{-\theta} \\
mc^*_{t,t+j} =& \quad \frac{(1+\delta)^{\theta\omega}}{\mu} \left(X_{t+j} \right)^{(\sigma+\omega)} \left(z^*_{t,t+j} \right)^{-\theta\omega} \\
z^*_{t,t+j} =& \quad \frac{\Pi^{ind}_{t,t+j}}{\Pi_{t,t+j}} \wp^*_t + \delta \wp_{s,t+j}
\end{aligned}$$

As for the remaining aggregates and welfare measures, they are:

$$\mathcal{W}_t = u_t - v_t \qquad ; \; u_t = \epsilon_t \frac{(Y_t)^{(1-\sigma)}-1}{(1-\sigma)}$$

$$h_t = \left(\mathfrak{d}_{y,t}\frac{Y_t}{\mathcal{A}_t}\right)^{\frac{1}{\varepsilon}} \qquad ; \; v_t = \frac{\chi}{(1+\nu)}\mathfrak{d}_{v,t}\left(h_t\right)^{(1+\nu)}$$

$$\mathfrak{d}_{y,t} = (1+\delta)^{\theta}\left(\Lambda_{y,t}\right)^{\varepsilon} \qquad ; \; \mathfrak{d}_{v,t} = \frac{\Lambda_t}{\left(\Lambda_{y,t}\right)^{(1+\nu)}}$$

$$\Lambda_{y,t} = (1-\alpha)\sum_{j=0}^{\infty}\alpha^j\left(z^*_{t-j,t}\right)^{-\frac{\theta}{\varepsilon}} \qquad ; \; \Lambda_t = (1-\alpha)\sum_{j=0}^{\infty}\alpha^j\left(z^*_{t-j,t}\right)^{-\theta_1}$$

$$\mathcal{W}^n_t = u^n_t - v^n_t \qquad ; \; u^n_t = \epsilon_t \frac{(Y^n_t)^{(1-\sigma)}-1}{(1-\sigma)}$$

$$h^n_t = \left(\frac{Y^n_t}{\mathcal{A}_t}\right)^{\frac{1}{\varepsilon}} \qquad ; \; v^n_t = \frac{\chi}{(1+\nu)}\left(h^n_t\right)^{(1+\nu)}$$

$$\epsilon_t \frac{\left(Y^{eq}_t\right)^{(1-\sigma)}-1}{(1-\sigma)} - \frac{\chi}{(1+\nu)}\left(\frac{Y^{eq}_t}{\mathcal{A}_t}\right)^{(1+\omega)} = \mathcal{W}_t \qquad ; \; X^{eq}_t \equiv \frac{Y^{eq}_t}{Y^n_t}$$

C.3 Steady State

For any variable χ_t, its steady state level is defined as $\bar{\chi}$. The steady state equilibrium can be numerically obtained as follows. First, we compute $\bar{I}$, $\bar{q}$, and $\bar{Y}^n$:

$$\bar{I} = \frac{\bar{\Pi}}{\beta} \quad ; \; \bar{q} = \frac{\beta}{\bar{\Pi}} \quad ; \; \left(\bar{Y}^n\right)^{(\sigma+\omega)} = \frac{1}{(1+\delta)\mu}\frac{\varepsilon}{\chi}\bar{\epsilon}\left(\overline{\mathcal{A}}\right)^{(1+\omega)}$$

Next, we use a numerical code to solve the following non-linear system for relative prices $\bar{\wp}^*$, $\bar{\wp}_s$, and $\bar{\wp}_\delta$, in which the infinite sums are retrieved by using finite sums in $j = \{0, 1, ..., J\}$ for a large J. In particular, we consider $J = 10000$:

$$(1+\delta)^{1-\theta} = (1-\alpha)\sum_{j=0}^{\infty}\alpha^j\left(\frac{\bar{\wp}^*}{(\bar{\Pi})^{(1-\gamma)j}} + \delta\bar{\wp}_s\right)^{-(\theta-1)}$$

$$(\bar{\wp}_\delta)^{-\theta} = (1-\alpha)\sum_{j=0}^{\infty}\alpha^j\left(\frac{\bar{\wp}^*}{(\bar{\Pi})^{(1-\gamma)j}} + \delta\bar{\wp}_s\right)^{-\theta}$$

$$\bar{\wp}_s = (1+\delta)^{-\theta}\left(\bar{\wp}_\delta\right)^{\theta}$$

After computing the relative prices, we pin down the following composite parameters:

$$\Sigma_{N1} \equiv \sum_{j=0}^{\infty}(\bar{\alpha}_1\beta)^j\left(\bar{z}^*_j\right)^{-(1+\theta_1)} \quad \Sigma_{N2} \equiv \sum_{j=0}^{\infty}(\bar{\alpha}_1\beta)^j\left(\bar{z}^*_j\right)^{-(1+\theta)} \quad \Sigma_{D1} \equiv \sum_{j=0}^{\infty}(\bar{\alpha}_1\beta)^j\left(\bar{z}^*_j\right)^{-\theta}$$

where $\bar{z}^*_j = \left(\frac{\bar{\wp}^*}{(\bar{\Pi})^{(1-\gamma)j}} + \delta\bar{\wp}_s\right)$, $\bar{\alpha}_1 \equiv \frac{\alpha}{\bar{\Pi}^{(1-\gamma)}}$ and $\bar{\alpha}_2 \equiv \frac{\alpha}{\bar{\Pi}^{2(1-\gamma)}}$.

Since the price wedge demand function satisfies the conditions of Theorem 1, we know that the infinite sums converge. Therefore, we can retrieve them numerically, by considering finite sums up to a very large horizon J, i.e. $j \in \{0, 1, ..., J\}$. Again, we use $J = 10000$. For avoiding numerical issues arising from dealing with very large numbers when $\delta > 0$, we proceed as follows. First, for each infinite sum of the form $\Sigma_\varphi \equiv \sum_{j=0}^{\infty}(\text{ß})^j\left(\bar{z}^*_j\right)^{-\varphi}$, where ß<1, we define its normalized peer $\tilde{\Sigma}_\varphi \equiv$

$\sum_{j=0}^{\infty} (ß)^j \left(\tilde{z}_j^*\right)^{-\varphi}$, where $\tilde{z}_j^* \equiv \frac{\bar{z}_j^*}{\delta \bar{\wp}_s} = \left(1 + \frac{1}{(\bar{\Pi})^{(1-\gamma)j}} \frac{\bar{\wp}^*}{\delta \bar{\wp}_s}\right)$. Therefore, whenever $\delta > 0$, we can accurately approximate $\tilde{\Sigma}_\varphi$ using $\sum_{j=0}^{J} (ß)^j \left(\tilde{z}_j^*\right)^{-\varphi}$. After retrieving $\tilde{\Sigma}_\varphi$, we compute $\Sigma_\varphi = (\delta \bar{\wp}_s)^{-\varphi} \tilde{\Sigma}_\varphi$.

After pinning down the gross output gap $\bar{X} = \left(\frac{\mu_\delta}{\mu_\theta} (1+\delta)^{-\theta\omega} \frac{[\Sigma_{D1} - \mu_\theta(\delta\bar{\wp}_s)\Sigma_{N2}]}{\Sigma_{N1}}\right)^{\frac{1}{(\sigma+\omega)}}$, we compute the aggregate $\bar{Y} = \bar{X}\bar{Y}^n$ and average $\bar{Y}_s = \frac{\bar{Y}}{\bar{\wp}_s}$ output levels. As for the remaining aggregates and welfare measures, they are:

$$
\begin{array}{lll}
\bar{\mathcal{W}}^n = \bar{u}^n - \bar{v}^n & \bar{\mathcal{W}} = \bar{u} - \bar{v} & ;\ \bar{\Lambda} = (1-\alpha)\,\Sigma_\Lambda \\
\bar{h}^n = \left(\frac{\bar{Y}^n}{\bar{\mathcal{A}}}\right)^{\frac{1}{\varepsilon}} & \bar{h} = \left(\bar{\mathfrak{d}}_y \frac{\bar{Y}}{\bar{\mathcal{A}}}\right)^{\frac{1}{\varepsilon}} & ;\ \bar{\Lambda}_y = (1-\alpha)\,\Sigma_y \\
\bar{u}^n = \bar{\epsilon}\frac{(\bar{Y}^n)^{(1-\sigma)}-1}{(1-\sigma)} & ;\ \bar{u} = \bar{\epsilon}\frac{(\bar{Y})^{(1-\sigma)}-1}{(1-\sigma)} & ;\ \bar{\mathfrak{d}}_y = (1+\delta)^\theta \left(\bar{\Lambda}_y\right)^\varepsilon \\
\bar{v}^n = \frac{\chi}{(1+\nu)} \left(\bar{h}^n\right)^{(1+\nu)} & ;\ \bar{v} = \frac{\chi}{(1+\nu)} \bar{\mathfrak{d}}_v \left(\bar{h}\right)^{(1+\nu)} & ;\ \bar{\mathfrak{d}}_v = \frac{\bar{\Lambda}}{\left(\bar{\Lambda}_y\right)^{(1+\nu)}}
\end{array}
$$

As for the consumption-equivalent welfare metrics, we use numerical methods to solve the following non-linear equation:

$$
\bar{\epsilon}\frac{\left(\bar{Y}^{eq}\right)^{(1-\sigma)} - 1}{(1-\sigma)} - \frac{\chi}{(1+\nu)} \left(\frac{\bar{Y}^{eq}}{\overline{\mathcal{A}}}\right)^{(1+\omega)} = \mathcal{W}_t
$$

After that, we compute $\bar{X}^{eq} = \frac{\bar{Y}^{eq}}{\bar{Y}^n}$ as the consumption-equivalent output gap.

In the particular case of $(\bar{\Pi})^{(1-\gamma)} = 1$, it is possible to obtain a closed form solution:

$$
\bar{\alpha}_1 = \bar{\alpha}_2 = \alpha \quad ;\ \bar{\wp}^* = \bar{\wp}_s = \bar{X} = 1 \quad ;\ \bar{\wp}_\delta = \bar{z}_j^* = (1+\delta)
$$

The remaining steady state levels, when $(\bar{\Pi})^{(1-\gamma)} = 1$, are then easily retrieved using the same relations previously shown.

C.4 Log-Linearized Model

Given an exogenous level of trend inflation $\bar{\Pi}$, we start by defining $\bar{z}_j^* \equiv \left(\frac{\bar{\wp}^*}{(\bar{\Pi})^{(1-\gamma)j}} + \delta \bar{\wp}_s \right)$ and the following composite parameters:

$$
\begin{aligned}
&\Sigma_{N1} \equiv \sum_{j=0}^{\infty} (\bar{\alpha}_1 \beta)^j \left(\bar{z}_j^*\right)^{-(1+\theta_1)} &&; \Sigma_{N2} \equiv \sum_{j=0}^{\infty} (\bar{\alpha}_1 \beta)^j \left(\bar{z}_j^*\right)^{-(1+\theta)} &&; \Sigma_{N3} \equiv \sum_{j=0}^{\infty} (\bar{\alpha}_2 \beta)^j \left(\bar{z}_j^*\right)^{-(2+\theta_1)} \\
&\Sigma_{N4} \equiv \sum_{j=0}^{\infty} (\bar{\alpha}_2 \beta)^j \left(\bar{z}_j^*\right)^{-(2+\theta)} &&; \Sigma_{N5} \equiv \sum_{j=0}^{\infty} (\bar{\alpha}_1 \beta)^j \left(\bar{z}_j^*\right)^{-(2+\theta_1)} &&; \Sigma_{N6} \equiv \sum_{j=0}^{\infty} (\bar{\alpha}_1 \beta)^j \left(\bar{z}_j^*\right)^{-(2+\theta)} \\
&\Sigma_{D1} \equiv \sum_{j=0}^{\infty} (\bar{\alpha}_1 \beta)^j \left(\bar{z}_j^*\right)^{-\theta} &&; \Sigma_{D2} \equiv \sum_{j=0}^{\infty} (\bar{\alpha}_2 \beta)^j \left(\bar{z}_j^*\right)^{-(1+\theta)} &&; \Sigma_{D3} \equiv \sum_{j=0}^{\infty} (\bar{\alpha}_1 \beta)^j \left(\bar{z}_j^*\right)^{-(1+\theta)} \\
&\Sigma_{s1} \equiv \sum_{j=0}^{\infty} (\bar{\alpha}_1)^j \left(\bar{z}_j^*\right)^{-\theta} &&; \Sigma_{s2} \equiv \sum_{j=0}^{\infty} \alpha^j \left(\bar{z}_j^*\right)^{-\theta} &&; \Sigma_{\delta 1} \equiv \sum_{j=0}^{\infty} (\bar{\alpha}_1)^j \left(\bar{z}_j^*\right)^{-(1+\theta)} \\
&\Sigma_{\delta 2} \equiv \sum_{j=0}^{\infty} \alpha^j \left(\bar{z}_j^*\right)^{-(1+\theta)} &&; \Sigma_{\alpha} \equiv \sum_{j=0}^{\infty} \alpha^j \left(\bar{z}_j^*\right)^{-(\theta-1)} &&; \Sigma_{\Lambda} \equiv \sum_{j=0}^{\infty} \alpha^j \left(\bar{z}_j^*\right)^{-\theta_1} \\
&\Sigma_{y} \equiv \sum_{j=0}^{\infty} \alpha^j \left(\bar{z}_j^*\right)^{-\frac{\theta}{\varepsilon}}
\end{aligned}
\tag{21}
$$

Since the price wedge demand function satisfies the conditions of Theorem 1, we know that the infinite sums converge. When $\delta > 0$, we proceed as in Appendix C.3 and accurately approximate $\tilde{\Sigma}_{\varphi}$ using $\sum_{j=0}^{J} (ß)^j \left(\tilde{z}_j^*\right)^{-\varphi}$, where $J = 10000$. After retrieving $\tilde{\Sigma}_{\varphi}$, we compute $\Sigma_{\varphi} = (\delta \bar{\wp}_s)^{-\varphi} \tilde{\Sigma}_{\varphi}$.

For computing the model loglinearized equilibrium, we also need the augment the set of composite parameters:

$$
\begin{aligned}
&\omega \equiv \tfrac{(1+\nu)}{\varepsilon} - 1 &&; \bar{\theta}_1 \equiv \theta\,(1+\omega) &&; \mu_{\theta} \equiv \tfrac{\theta}{(\theta-1)} &&; \mu_{\delta} \equiv \tfrac{\theta}{\theta-(1+\delta)} &&; \mu = \tfrac{\mu_{\delta}}{(1+\delta)} \\
&\kappa_{\delta} \equiv (1+\delta)^{(\theta\omega-1)}\,(\bar{X})^{(\sigma+\omega)} &&; \bar{\theta}_2 \equiv (\delta \bar{\wp}_s)\,\theta \tfrac{\Sigma_{\delta 2}}{\Sigma_{s2}} &&; \bar{\alpha}_1 \equiv \tfrac{\alpha}{\bar{\Pi}^{(1-\gamma)}} &&; \bar{\alpha}_2 \equiv \tfrac{\alpha}{\bar{\Pi}^{2(1-\gamma)}} &&; \mu_2 \equiv \tfrac{\theta_2}{(\theta_2-1)}
\end{aligned}
$$

For $k \in \{0, 1, 2, ...\}$ and for each composite parameter of the form $\Sigma_{\varphi} \equiv \sum_{j=0}^{\infty} (ß)^j \left(\bar{z}_j^*\right)^{-\varphi}$, as defined in (21), we consider the following k-based composite parameters:

$$
\Theta_{k,\varphi} \equiv \left(\bar{z}_k^*\right)^{-\varphi} \quad ; \Sigma_{k,\varphi} \equiv \sum_{j=0}^{\infty} (ß)^j \left(\bar{z}_{j+k}^*\right)^{-\varphi}
$$

For instance, $\Sigma_{k,N1}$ and $\Theta_{k,N1}$ are then defined as $\Sigma_{k,N1} \equiv \sum_{j=0}^{\infty} (\bar{\alpha}_1 \beta)^j \left(\bar{z}_{j+k}^*\right)^{-(1+\theta_1)}$ and$\Theta_{k,N1} \equiv \left(\bar{z}_k^*\right)^{-(1+\theta_1)}$. Note also that $\Sigma_{0,\varphi} = \Sigma_{\varphi}$.

Finally, the following composite parameters are necessary for deriving the aggregate supply system under price wedges and trend inflation:

$$\begin{aligned}
&\Omega_1 \equiv \tfrac{(1-\bar{\theta}_2)}{\theta}\Theta_{0,s1} + (\delta\bar{\wp}_s)\,\Theta_{0,\delta 1} \quad ;\ \Omega_2 \equiv \Theta_{0,s1}\tfrac{\Sigma_{1,\delta 1}}{\Sigma_{1,s1}} - \Theta_{0,\delta 1} \quad ;\ \Omega_3 \equiv \tfrac{(1-\bar{\theta}_2)}{\theta}\Sigma_{1,s1} + (\delta\bar{\wp}_s)\,\Sigma_{1,\delta 1}\\
&\Phi_{\Sigma 1,k,1} \equiv 1 + \Phi_{\Sigma,k,1}\\
&\Phi_{\Sigma,k,1} \equiv \tfrac{\mu_\theta}{\mu}\kappa_\delta \Sigma_{k,N1} - \Sigma_{k,D1} + (\delta\bar{\wp}_s)\,\mu_\theta \Sigma_{k,N2}\\
&\Phi_{\Theta,k,1} \equiv \tfrac{\mu_\theta}{\mu}\kappa_\delta \Theta_{k,N1} - \Theta_{k,D1} + (\delta\bar{\wp}_s)\,\mu_\theta \Theta_{k,N2}\\
&\Phi_{\Sigma,k,2} \equiv \tfrac{\mu_\theta}{\mu}\kappa_\delta \left(1+\bar{\theta}_1\right)\Sigma_{k,N5} - \theta\Sigma_{k,D3} + (\delta\bar{\wp}_s)\,\mu_\theta \left(1+\theta\right)\Sigma_{k,N6}\\
&\Phi_{\Theta,k,2} \equiv \tfrac{\mu_\theta}{\mu}\kappa_\delta \left(1+\bar{\theta}_1\right)\Theta_{k,N5} - \theta\Theta_{k,D3} + (\delta\bar{\wp}_s)\,\mu_\theta \left(1+\theta\right)\Theta_{k,N6}\\
&\Phi_{\Sigma,k,3} \equiv \tfrac{\mu_\theta}{\mu}\kappa_\delta \left(1+\bar{\theta}_1\right)\Sigma_{k,N3} - \theta\Sigma_{k,D2} + (\delta\bar{\wp}_s)\,\mu_\theta \left(1+\theta\right)\Sigma_{k,N4}\\
&\Phi_{\Theta,k,3} \equiv \tfrac{\mu_\theta}{\mu}\kappa_\delta \left(1+\bar{\theta}_1\right)\Theta_{k,N3} - \theta\Theta_{k,D2} + (\delta\bar{\wp}_s)\,\mu_\theta \left(1+\theta\right)\Theta_{k,N4}\\
&\Phi_{\Theta,k,4} \equiv \Phi_{\Theta,k,2} - \mu_\theta \Theta_{k,N2}
\end{aligned}$$

In general, for any variable χ_t, its loglinearized version is defined as $\hat{\chi}_t \equiv \log\left(\frac{\chi_t}{\bar{\chi}}\right)$, keeping the same case as in the original variable, e.g. $\hat{Y}_t = \log\left(\frac{Y_t}{\bar{Y}}\right)$. For gross rates, though, we represent its loglinearized version in lower cases, e.g. $\hat{\pi}_t = \log\left(\frac{\Pi_t}{\bar{\Pi}}\right)$. Usual loglinearizations from the general part of the model, i.e. comprising equations independent of pricing structure, leads to the following system:

$$\begin{aligned}
\hat{Y}_t =&\quad E_t\hat{Y}_{t+1} - \tfrac{1}{\sigma}E_t\left[(\hat{\imath}_t - \hat{\pi}_{t+1}) + (\hat{\epsilon}_{t+1} - \hat{\epsilon}_t)\right]\\
\hat{q}_t =&\quad \sigma\left(\hat{Y}_{t-1} - \hat{Y}_t\right) - \hat{\pi}_t + (\hat{\epsilon}_t - \hat{\epsilon}_{t-1})\\
\hat{\imath}_t =&\quad \phi_{\mathrm{i}}\hat{\imath}_{t-1} + (1-\phi_{\mathrm{i}})\left[\phi_\pi\hat{\pi}_t + \phi_x\hat{x}_t + \phi_{gy}\left(\hat{Y}_t - \hat{Y}_{t-1}\right) + \phi_y\hat{Y}_t\right] + \hat{\epsilon}_{\mathrm{i},t}\\
(\sigma+\omega)\,\hat{Y}^n_t =&\quad \hat{\epsilon}_t + (1+\omega)\,\widehat{\mathcal{A}}_t\\
\hat{\pi}^{ind}_t =&\quad \gamma\hat{\pi}_{t-1} \quad ; \quad \hat{x}_t = \hat{Y}_t - \hat{Y}^n_{tt-1} \quad ; \quad \hat{\mathfrak{g}}_t = \hat{Y}_t - \hat{Y}_{t-1}
\end{aligned}$$

Note that $E_t\hat{q}_{t+1} = -\hat{\imath}_t$. Lastly, as we show in Appendix C.4.1, the price setting equations presented in systems (17) and (18) imply the following loglinearized equations under price wedges and trend inflation:

$$\begin{aligned}
&\textbf{Aggregate Supply:}\\
\hat{\ell}_t =&\quad -E_t\left(\hat{\pi}_{t+1} - \hat{\pi}^{ind}_{t+1}\right) + (1-\sigma)\,E_t\left(\hat{x}_{t+1} - \hat{x}_t\right) + \hat{\xi}_t\\
\hat{\wp}^*_t =&\quad \bar{\alpha}_2\beta\Psi_{0,\alpha 2}E_t\left(\hat{\pi}_{t+1} - \hat{\pi}^{ind}_{t+1}\right) + \tfrac{1}{(\bar{\wp}^*)}\left[\Psi_{0,x}\hat{x}_t + \bar{\alpha}_1\beta\Psi_{0,\ell}\hat{\ell}_t - (\delta\bar{\wp}_s)\,\Psi_{0,\wp}\hat{\wp}_{s,t}\right]\\
+&\quad \tfrac{1}{(\bar{\wp}^*)}\bar{\alpha}_1\beta\Psi_{0,\alpha 2}\Psi_{1,x1}E_t\hat{S}_{1,\alpha 1,t+1} - \bar{\alpha}_2\beta\Psi_{0,\alpha 2}\Psi_{1,\alpha 2}E_t\hat{S}_{1,\alpha 2,t+1}
\end{aligned}$$

$$\begin{aligned}
&\textbf{Price Wedge and Price Stickiness:}\\
\hat{\wp}^*_t =&\quad \bar{\alpha}_1\psi_4\left(\hat{\pi}_t - \hat{\pi}^{ind}_t\right) + \bar{\alpha}_1\psi_5\left[\tfrac{(1-\bar{\theta}_2)}{\theta}\hat{S}_{1,s,t-1} + (\delta\bar{\wp}_s)\,\psi_6\hat{S}_{1,\delta,t-1}\right]\\
\hat{\wp}_{s,t} =&\quad -\bar{\alpha}_1\psi_1\left(\hat{\pi}_t - \hat{\pi}^{ind}_t\right) + \bar{\alpha}_1\psi_2\left[\hat{S}_{1,s,t-1} - \psi_3\hat{S}_{1,\delta,t-1}\right]\\
\hat{\wp}_{\delta,t} =&\quad \tfrac{1}{\theta}\hat{\wp}_{s,t}\\
\hat{Y}_{s,t} =&\quad \hat{Y}_t - \hat{\wp}_{s,t}
\end{aligned}$$

$$
\begin{array}{rl}
\textbf{Ancillary} & \textbf{Variables for } k=\{0,1,2,...\} \\
\widehat{\mathcal{S}}_{k,\alpha 1,t} = & \frac{\Psi_{k,x}}{\Psi_{k,x1}}\hat{x}_t + \bar{\alpha}_1\beta\frac{\Psi_{k,\ell}}{\Psi_{k,x1}}\hat{\ell}_t - \left(\delta\bar{\wp}_s\right)\frac{\Psi_{k,\wp}}{\Psi_{k,x1}}\hat{\wp}_{s,t} + \bar{\alpha}_1\beta\Psi_{(k+1),\alpha 1}E_t\hat{S}_{(k+1),\alpha 1,t+1} \\
\widehat{\mathcal{S}}_{k,\alpha 2,t} = & -\bar{\alpha}_2\beta E_t\left(\hat{\pi}_{t+1}-\hat{\pi}^{ind}_{t+1}\right) + \bar{\alpha}_2\beta\Psi_{(k+1),\alpha 2}E_t\hat{S}_{(k+1),\alpha 2,t+1} \\
\widehat{\mathcal{S}}_{k,s,t} = & \bar{\alpha}_1\left(\hat{\pi}_t-\hat{\pi}^{ind}_t\right) - \Psi_{k,\wp s}\hat{\wp}^*_t + \bar{\alpha}_1\Psi_{(k+1),s}\hat{S}_{(k+1),s,t-1} \\
\widehat{\mathcal{S}}_{k,\delta,t} = & \bar{\alpha}_1\left(\hat{\pi}_t-\hat{\pi}^{ind}_t\right) - \Psi_{k,\wp\delta}\hat{\wp}^*_t + \bar{\alpha}_1\Psi_{(k+1),\delta}\hat{S}_{(k+1),\delta,t-1}
\end{array}
$$

$$
\begin{array}{rl}
\textbf{Aggregate} & \textbf{Shock:} \\
\hat{\xi}_t = & \frac{(1+\omega)}{(\sigma+\omega)}E_t\left[\left(\hat{\epsilon}_{t+1}-\hat{\epsilon}_t\right) + (1-\sigma)\left(\widehat{\mathcal{A}}_{t+1}-\widehat{\mathcal{A}}_t\right)\right]
\end{array}
$$

where

$$
\begin{array}{llll}
\psi_1 \equiv \left(\bar{\wp}^*\right)\frac{\Sigma_{1,s1}\Omega_2}{\Sigma_{s2}\Omega_1} & ;\ \psi_2 \equiv \left(\bar{\wp}^*\right)\frac{\Theta_{0,\delta 1}\Sigma_{2,s1}}{\Sigma_{s2}\Omega_1} & ;\ \psi_3 \equiv \frac{\Theta_{0,s1}\Sigma_{2,\delta 1}}{\Theta_{0,\delta 1}\Sigma_{2,s1}} & \psi_4 \equiv \frac{\Omega_3}{\Omega_1} \\
\psi_5 \equiv \frac{\Sigma_{2,s1}}{\Omega_1} & ;\ \psi_6 \equiv \frac{\Sigma_{2,\delta 1}}{\Sigma_{2,s1}} & ;\ \Psi_{k,\wp s} \equiv \frac{\Theta_{k,s1}}{\Sigma_{(k+1),s1}} & ;\ \Psi_{k,\wp\delta} \equiv \frac{\Theta_{k,\delta 1}}{\Sigma_{(k+1),\delta 1}} \\
\Psi_{k,x} \equiv \frac{\mu_\theta}{\mu}\kappa_\delta\left(\sigma+\omega\right)\frac{\Theta_{k,N1}}{\Phi_{\Sigma,k,3}} & ;\ \Psi_{k,x1} \equiv \frac{\Phi_{\Sigma 1,(k+1),1}}{\Phi_{\Sigma,k,3}} & ;\ \Psi_{k,\ell} \equiv \frac{\Phi_{\Sigma,(k+1),1}}{\Phi_{\Sigma,k,3}} & ;\ \Psi_{k,\wp} \equiv \frac{\Phi_{\Theta,k,4}}{\Phi_{\Sigma,k,3}} \\
\Psi_{k,\alpha 1} \equiv \frac{\Phi_{\Sigma 1,(k+1),1}}{\Phi_{\Sigma 1,k,1}} & ;\ \Psi_{k,\alpha 2} \equiv \frac{\Phi_{\Sigma,(k+1),3}}{\Phi_{\Sigma,k,3}} & ;\ \Psi_{k,s} \equiv \frac{\Sigma_{(k+1),s1}}{\Sigma_{k,s1}} & ;\ \Psi_{k,\delta} \equiv \frac{\Sigma_{(k+1),\delta 1}}{\Sigma_{k,\delta 1}}
\end{array}
$$

As in Alves (2014), $\hat{\xi}_t$ is an aggregate shock term that collects the effects of the technology shock $\widehat{\mathcal{A}}_t$ and the utility shock $\hat{\epsilon}_t$. For our simulations, we truncate the infinite recursive system of ancillary variables at $\bar{k}=40$. With this approximation, we substitute $E_t\widehat{\mathcal{S}}_{40,\alpha 1,t+1}$ for $E_t\widehat{\mathcal{S}}_{41,\alpha 1,t+1}$, $E_t\widehat{\mathcal{S}}_{40,\alpha 2,t+1}$ for $E_t\widehat{\mathcal{S}}_{41,\alpha 2,t+1}$, $\widehat{\mathcal{S}}_{40,s,t-1}$ for $\widehat{\mathcal{S}}_{41,s,t-1}$, and $\widehat{\mathcal{S}}_{40,\delta,t-1}$ for $\widehat{\mathcal{S}}_{41,\delta,t-1}$.

C.4.1 Deriving the Log-Linearized Supply System

Since $\bar{z}^*_j = \left(\frac{\bar{\wp}^*}{\bar{\Pi}^{j(1-\gamma)}} + \delta\bar{\wp}_s\right)$, and given the steady state relations $\frac{(1+\delta)^{-(\theta-1)}}{(1-\alpha)} = \Sigma_\alpha$, $\frac{(\bar{\wp}_\delta)^{-\theta}}{(1-\alpha)} = \Sigma_{s2}$ and $\bar{\wp}_s = \frac{(1+\delta)^{-\theta}}{(1-\alpha)\Sigma_{s2}}$, direct loglinearization of the pricing systems (17) and (18) initially gives the following equations.

$$
\begin{array}{rl}
\hat{Y}_{s,t} = & \hat{Y}_t - \theta\hat{\wp}_{\delta,t} \\
\hat{\wp}_{s,t} = & \theta\hat{\wp}_{\delta,t} \\
0 = & -\left(\bar{\wp}^*\right)\hat{S}_{s1,t} + \left(\delta\bar{\wp}_s\right)\Sigma_{s2}\hat{\wp}_{s,t} \\
\hat{S}_{s1,t} \equiv & \sum_{j=0}^{\infty}\left(\bar{\alpha}_1\right)^j\left(\bar{z}^*_j\right)^{-\theta}\left(\left(\hat{\pi}_{t-j,t}-\hat{\pi}^{ind}_{t-j,t}\right)-\hat{\wp}^*_{t-j}\right) \\
\\
\Sigma_{s2}\hat{\wp}_{\delta,t} = & -\left(\bar{\wp}^*\right)\hat{S}_{\delta 1,t} + \left(\delta\bar{\wp}_s\right)\Sigma_{\delta 2}\hat{\wp}_{s,t} \\
\hat{S}_{\delta 1,t} \equiv & \sum_{j=0}^{\infty}\left(\bar{\alpha}_1\right)^j\left(\bar{z}^*_j\right)^{-(1+\theta)}\left(\left(\hat{\pi}_{t-j,t}-\hat{\pi}^{ind}_{t-j,t}\right)-\hat{\wp}^*_{t-j}\right) \\
\hline
\bar{D}\hat{D}^*_t = & \mu_\theta\bar{N}\hat{N}^*_t
\end{array}
$$

$$
\begin{aligned}
\bar{N}\hat{N}_t = & \ \tfrac{\kappa_\delta}{\mu}\hat{S}_{N1,t} + (\delta\bar{\wp}_s)\,\hat{S}_{N2,t} - \tfrac{(\bar{\wp}^*)\left(1+\bar{\theta}_1\right)\kappa_\delta}{\mu}\left(\hat{S}_{N3,t} + \Sigma_{N3}\hat{\wp}_t^*\right) \\
- & \ (1+\theta)\,(\delta\bar{\wp}_s)\,(\bar{\wp}^*)\left(\hat{S}_{N4,t} + \Sigma_{N4}\hat{\wp}_t^*\right) - \tfrac{(\delta\bar{\wp}_s)\left(1+\bar{\theta}_1\right)\kappa_\delta}{\mu}\hat{S}_{N5,t} - (1+\theta)\,(\delta\bar{\wp}_s)^2\,\hat{S}_{N6,t} \\
\hat{S}_{N1,t} \equiv & \ E_t\sum_{j=0}^{\infty}(\bar{\alpha}_1\beta)^j\left(\bar{z}_j^*\right)^{-\left(1+\bar{\theta}_1\right)}\left[\left(\hat{q}_{t,t+j} + \hat{\mathfrak{g}}_{t,t+j} + \hat{\pi}_{t,t+j}^{ind}\right) + (\sigma+\omega)\,\hat{x}_{t+j}\right] \\
\hat{S}_{N2,t} \equiv & \ E_t\sum_{j=0}^{\infty}(\bar{\alpha}_1\beta)^j\left(\bar{z}_j^*\right)^{-(1+\theta)}\left[\left(\hat{q}_{t,t+j} + \hat{\mathfrak{g}}_{t,t+j} + \hat{\pi}_{t,t+j}^{ind}\right) + \hat{\wp}_{s,t+j}\right] \\
\hat{S}_{N3,t} \equiv & \ -E_t\sum_{j=0}^{\infty}(\bar{\alpha}_2\beta)^j\left(\bar{z}_j^*\right)^{-\left(2+\bar{\theta}_1\right)}\left(\hat{\pi}_{t,t+j} - \hat{\pi}_{t,t+j}^{ind}\right) \\
\hat{S}_{N4,t} \equiv & \ -E_t\sum_{j=0}^{\infty}(\bar{\alpha}_2\beta)^j\left(\bar{z}_j^*\right)^{-(2+\theta)}\left(\hat{\pi}_{t,t+j} - \hat{\pi}_{t,t+j}^{ind}\right) \\
\hat{S}_{N5,t} \equiv & \ E_t\sum_{j=0}^{\infty}(\bar{\alpha}_1\beta)^j\left(\bar{z}_j^*\right)^{-\left(2+\bar{\theta}_1\right)}\hat{\wp}_{s,t+j} \\
\hat{S}_{N6,t} \equiv & \ E_t\sum_{j=0}^{\infty}(\bar{\alpha}_1\beta)^j\left(\bar{z}_j^*\right)^{-(2+\theta)}\hat{\wp}_{s,t+j}
\end{aligned}
$$

$$
\begin{aligned}
\bar{D}\hat{D}_t^* = & \ \hat{S}_{D1,t} - \theta\,(\bar{\wp}^*)\left(\hat{S}_{D2,t} + \Sigma_{D2}\hat{\wp}_t^*\right) - \theta\,(\delta\bar{\wp}_s)\,\hat{S}_{D3,t} \\
\hat{S}_{D1,t} \equiv & \ E_t\sum_{j=0}^{\infty}(\bar{\alpha}_1\beta)^j\left(\bar{z}_j^*\right)^{-\theta}\left(\hat{q}_{t,t+j} + \hat{\mathfrak{g}}_{t,t+j} + \hat{\pi}_{t,t+j}^{ind}\right) \\
\hat{S}_{D2,t} \equiv & \ -E_t\sum_{j=0}^{\infty}(\bar{\alpha}_2\beta)^j\left(\bar{z}_j^*\right)^{-(1+\theta)}\left(\hat{\pi}_{t,t+j} - \hat{\pi}_{t,t+j}^{ind}\right) \\
\hat{S}_{D3,t} \equiv & \ E_t\sum_{j=0}^{\infty}(\bar{\alpha}_1\beta)^j\left(\bar{z}_j^*\right)^{-(1+\theta)}\hat{\wp}_{s,t+j}
\end{aligned}
$$

Since the discounted sums do not allow for finite recursive representations, we use the following Lemmas to help us obtain simpler expressions.

Lemma 1 *Consider generic forward and backward equations* $\hat{S}_t^f \equiv E_t\sum_{j=0}^{\infty}(\beta)^j\left(\bar{z}_j^*\right)^{-\phi}\left(\hat{\varkappa}_{t,t+j}^a + \hat{\varkappa}_{t+j}^b\right)$ *and* $\hat{S}_t^l \equiv \sum_{j=0}^{\infty}(\beta)^j\left(\bar{z}_j^*\right)^{-\phi}\left(\hat{\varkappa}_{t-j,t}^a + \hat{\varkappa}_{t-j}^b\right)$, *where* $\beta\in(0,1)$ *is a discounting parameter,* $\hat{\varkappa}_{\tau_1,\tau_2}^a$ *is a cumulative variable from* τ_1 *to* τ_2, *while* $\hat{\varkappa}_\tau^b$ *is a spot variable at period* τ. *Since* $\hat{\varkappa}_{t,t+j}^a = \hat{\varkappa}_{t+1}^a + \hat{\varkappa}_{t+1,t+j}^a$, $\hat{\varkappa}_{t-j,t}^a = \hat{\varkappa}_t^a + \hat{\varkappa}_{t-j,t-1}^a$, *and* $\hat{\varkappa}_{t,t}^a = 0$, *the infinite sums lead to the following infinite recursive systems, for* $k = \{0,1,2,...,\infty\}$:

$$
\begin{aligned}
\hat{S}_t^f = & \ \hat{S}_{0,t}^f \\
\hat{S}_{k,t}^f = & \ \left(\Sigma_{k,\phi} - \Theta_{k,\phi}\right)E_t\hat{\varkappa}_{t+1}^a + \Theta_{k,\phi}\hat{\varkappa}_t^b + \beta E_t\hat{S}_{(k+1),t+1}^f \\
\hat{S}_t^l = & \ \hat{S}_{0,t}^l \\
\hat{S}_{k,t}^l = & \ \left(\Sigma_{k,\phi} - \Theta_{k,\phi}\right)\hat{\varkappa}_t^a + \Theta_{k,\phi}\hat{\varkappa}_t^b + \beta\hat{S}_{(k+1),t-1}^l \\
& \textit{where} \\
\hat{S}_{k,t}^f \equiv & \ E_t\sum_{j=0}^{\infty}(\beta)^j\left(\bar{z}_{j+k}^*\right)^{-\phi}\left(\hat{\varkappa}_{t,t+j}^a + \hat{\varkappa}_{t+j}^b\right) \\
\hat{S}_{k,t}^l \equiv & \ \sum_{j=0}^{\infty}(\beta)^j\left(\bar{z}_{j+k}^*\right)^{-\phi}\left(\hat{\varkappa}_{t-j,t}^a + \hat{\varkappa}_{t-j}^b\right) \\
\Theta_{k,\phi} \equiv & \ \left(\bar{z}_k^*\right)^{-\phi} \quad , \quad \Sigma_{0,\phi} = \Sigma_\phi \equiv \sum_{j=0}^{\infty}(\beta)^j\left(\bar{z}_j^*\right)^{-\phi} \\
\Sigma_{k,\phi} \equiv & \ \sum_{j=0}^{\infty}(\beta)^j\left(\bar{z}_{j+k}^*\right)^{-\phi} = \tfrac{1}{\beta}\left[\Sigma_{(k-1),\phi} - \Theta_{(k-1),\phi}\right]
\end{aligned}
$$

Proof. As for $\Sigma_{k,\phi} \equiv \sum_{j=0}^{\infty} (ß)^j \left(\bar{z}^*_{j+k}\right)^{-\phi}$, note that:

$$
\begin{aligned}
\Sigma_{k,\phi} \equiv &\sum_{j=0}^{\infty} (ß)^j \left(\bar{z}^*_{j+k}\right)^{-\phi} = \sum_{j=-1}^{\infty} (ß)^j \left(\bar{z}^*_{j+k}\right)^{-\phi} - (ß)^{-1} \left(\bar{z}^*_{k-1}\right)^{-\phi} \\
= &\sum_{j=0}^{\infty} (ß)^{j-1} \left(\bar{z}^*_{j-1+k}\right)^{-\phi} - (ß)^{-1} \left(\bar{z}^*_{k-1}\right)^{-\phi} = \frac{1}{ß}\left[\sum_{j=0}^{\infty} (ß)^j \left(\bar{z}^*_{j+k-1}\right)^{-\phi} - \left(\bar{z}^*_{k-1}\right)^{-\phi}\right]
\end{aligned}
$$

For the forward infinite sum, we obtain:

$$
\begin{aligned}
\hat{S}^f_{k,t} \equiv\ & E_t \sum_{j=0}^{\infty} (ß)^j \left(\bar{z}^*_{j+k}\right)^{-\phi} \left(\hat{\varkappa}^a_{t,t+j} + \hat{\varkappa}^b_{t+j}\right) \\
=\ & E_t \sum_{j=0}^{\infty} (ß)^j \left(\bar{z}^*_{j+k}\right)^{-\phi} \left(\hat{\varkappa}^a_{t,t+j}\right) + E_t \sum_{j=0}^{\infty} (ß)^j \left(\bar{z}^*_{j+k}\right)^{-\phi} \left(\hat{\varkappa}^b_{t+j}\right) \\
=\ & E_t \sum_{j=1}^{\infty} (ß)^j \left(\bar{z}^*_{j+k}\right)^{-\phi} \left(\hat{\varkappa}^a_{t,t+j}\right) + E_t \sum_{j=0}^{\infty} (ß)^j \left(\bar{z}^*_{j+k}\right)^{-\phi} \left(\hat{\varkappa}^b_{t+j}\right) \\
=\ & E_t \sum_{j=1}^{\infty} (ß)^j \left(\bar{z}^*_{j+k}\right)^{-\phi} \left(\hat{\varkappa}^a_{t+1} + \hat{\varkappa}^a_{t+1,t+j}\right) + \left(\bar{z}^*_{k}\right)^{-\phi} \hat{\varkappa}^b_{t} + E_t \sum_{j=1}^{\infty} (ß)^j \left(\bar{z}^*_{j+k}\right)^{-\phi} \left(\hat{\varkappa}^b_{t+j}\right) \\
=\ & \left(\sum_{j=1}^{\infty} (ß)^j \left(\bar{z}^*_{j+k}\right)^{-\phi}\right) E_t \hat{\varkappa}^a_{t+1} + \left(\bar{z}^*_{k}\right)^{-\phi} \hat{\varkappa}^b_{t} + E_t \sum_{j=1}^{\infty} (ß)^j \left(\bar{z}^*_{j+k}\right)^{-\phi} \left(\hat{\varkappa}^a_{t+1,t+j} + \hat{\varkappa}^b_{t+j}\right) \\
=\ & \left(\Sigma_{k,\phi} - \left(\bar{z}^*_{k}\right)^{-\phi}\right) E_t \hat{\varkappa}^a_{t+1} + \left(\bar{z}^*_{k}\right)^{-\phi} \hat{\varkappa}^b_{t} + ß E_t \sum_{j=0}^{\infty} (ß)^j \left(\bar{z}^*_{j+k+1}\right)^{-\phi} \left(\hat{\varkappa}^a_{t+1,t+1+j} + \hat{\varkappa}^b_{t+1+j}\right) \\
=\ & \left(\Sigma_{k,\phi} - \Theta_{k,\phi}\right) E_t \hat{\varkappa}^a_{t+1} + \Theta_{k,\phi} \hat{\varkappa}^b_{t} + ß E_t \hat{S}^f_{(k+1),t+1}
\end{aligned}
$$

And lastly, for the backward sum, we obtain:

$$
\begin{aligned}
\hat{S}^l_{k,t} \equiv\ & \sum_{j=0}^{\infty} (ß)^j \left(\bar{z}^*_{j+k}\right)^{-\phi} \left(\hat{\varkappa}^a_{t-j,t} + \hat{\varkappa}^b_{t-j}\right) = \sum_{j=0}^{\infty} (ß)^j \left(\bar{z}^*_{j+k}\right)^{-\phi} \left(\hat{\varkappa}^a_{t-j,t}\right) + \sum_{j=0}^{\infty} (ß)^j \left(\bar{z}^*_{j+k}\right)^{-\phi} \left(\hat{\varkappa}^b_{t-j}\right) \\
=\ & \sum_{j=1}^{\infty} (ß)^j \left(\bar{z}^*_{j+k}\right)^{-\phi} \left(\hat{\varkappa}^a_{t-j,t}\right) + \sum_{j=0}^{\infty} (ß)^j \left(\bar{z}^*_{j+k}\right)^{-\phi} \left(\hat{\varkappa}^b_{t-j}\right) \\
=\ & \sum_{j=1}^{\infty} (ß)^j \left(\bar{z}^*_{j+k}\right)^{-\phi} \left(\hat{\varkappa}^a_{t} + \hat{\varkappa}^a_{t-j,t-1}\right) + \left(\bar{z}^*_{k}\right)^{-\phi} \hat{\varkappa}^b_{t} + \sum_{j=1}^{\infty} (ß)^j \left(\bar{z}^*_{j+k}\right)^{-\phi} \left(\hat{\varkappa}^b_{t-j}\right) \\
=\ & \left(\sum_{j=1}^{\infty} (ß)^j \left(\bar{z}^*_{j+k}\right)^{-\phi}\right) \hat{\varkappa}^a_{t} + \left(\bar{z}^*_{k}\right)^{-\phi} \hat{\varkappa}^b_{t} + \sum_{j=1}^{\infty} (ß)^j \left(\bar{z}^*_{j+k}\right)^{-\phi} \left(\hat{\varkappa}^a_{t-j,t-1} + \hat{\varkappa}^b_{t-j}\right) \\
=\ & \left(\Sigma_{k,\phi} - \left(\bar{z}^*_{k}\right)^{-\phi}\right) \hat{\varkappa}^a_{t} + \left(\bar{z}^*_{k}\right)^{-\phi} \hat{\varkappa}^b_{t} + ß \sum_{j=0}^{\infty} (ß)^j \left(\bar{z}^*_{j+k+1}\right)^{-\phi} \left(\hat{\varkappa}^a_{t-1-j,t-1} + \hat{\varkappa}^b_{t-1-j}\right) \\
=\ & \left(\Sigma_{k,\phi} - \Theta_{k,\phi}\right) \hat{\varkappa}^a_{t} + \Theta_{k,\phi} \hat{\varkappa}^b_{t} + ß \hat{S}^l_{(k+1),t-1}
\end{aligned}
$$

■

The recursive systems are infinite, for $\hat{S}^l_{k,t} \left(\hat{S}^f_{k,t}\right)$ depends on $\hat{S}^l_{(k+1),t-1} \left(E_t \hat{S}^f_{(k+1),t+1}\right)$, instead of $\hat{S}^l_{k,t-1} \left(E_t \hat{S}^f_{k,t+1}\right)$, for $k = \{0, 1, 2, ..., \infty\}$. However, since coefficients $\left(\Sigma_{k,\phi} - \Theta_{k,\phi}\right)$ and $\Theta_{k,\phi}$ converges asymptotically as k rises, the equations at a conveniently chosen large level $\bar{k}$ can be approximated by finite recursions, using $\hat{S}^l_{k,t-1} \left(E_t \hat{S}^f_{k,t+1}\right)$, instead of $\hat{S}^l_{(k+1),t-1} \left(E_t \hat{S}^f_{(k+1),t+1}\right)$. In this paper, we set $\bar{k} = 40$.

From Lemma 1, note that the dicounting parameter on $E_t \hat{S}^f_{(k+1),t+1}$ and $\hat{S}^l_{(k+1),t-1}$ is always ß. Therefore, Corollary 2 derives similar results for linear combinations of the form $\hat{S}^f_{a,t} \equiv \sum_{n=1}^{N} {}_n\hat{S}^f_{n,t}$ and

$\hat{S}^l_{a,t} \equiv \sum_{n=1}^{N} {}_n \hat{S}^l_{n,t}$, where $\hat{S}^f_{n,t}$ and $\hat{S}^l_{n,t}$ are generic forward and backward ancillary variables.

Corollary 2 *Consider linear combinations of forward and backward ancillary variables of the form* $\hat{S}^f_{k,a,t} \equiv \sum_{n=1}^{N} \mathfrak{w}_{kn} \hat{S}^f_{k,n,t}$ *and* $\hat{S}^l_{k,a,t} \equiv \sum_{n=1}^{N} \mathfrak{w}_{kn} \hat{S}^l_{k,n,t}$*, where* ${}_n$ *is a n-specific real-valued parameter,* $\mathfrak{w}_k$ *is a k-specific real-valued parameter, and* $\hat{S}^f_{k,n,t}$ *and* $\hat{S}^l_{k,n,t}$ *are generic forward and backward ancillary variables, as defined in Lemma 1, with the same discounting parameter* $\text{ß} \in (0,1)$*, but with specific powers* ϕ_n*, specific cumulative variables* $\hat{\varkappa}^a_{n,\tau_1,\tau_2}$ *and specific spot variables* $\hat{\varkappa}^b_{n,\tau}$*. That is,* $\hat{S}^f_{k,n,t} \equiv E_t \sum_{j=0}^{\infty} (\text{ß})^j \left(\bar{z}^*_{j+k}\right)^{-\phi_n} \left(\hat{\varkappa}^a_{n,t,t+j} + \hat{\varkappa}^b_{n,t+j}\right)$ *and* $\hat{S}^l_{k,n,t} \equiv \sum_{j=0}^{\infty} (\text{ß})^j \left(\bar{z}^*_{j+k}\right)^{-\phi_n} \left(\hat{\varkappa}^a_{n,t-j,t} + \hat{\varkappa}^b_{n,t-j}\right)$*. The following infinite recursive systems describe the dynamics for* $\hat{S}^f_{k,a,t}$ *and* $\hat{S}^l_{k,a,t}$*, for* $k = \{0,1,2,...,\infty\}$*:*

$$
\begin{aligned}
\hat{S}^f_{a,t} &= \hat{S}^f_{0,a,t} \\
\hat{S}^f_{k,a,t} &= \sum_{n=1}^{N} \mathfrak{w}_{kn} \left(\Sigma_{k,n,\phi} - \Theta_{k,n,\phi}\right) E_t \hat{\varkappa}^a_{n,t+1} + \sum_{n=1}^{N} \mathfrak{w}_{kn} \Theta_{k,n,\phi} \hat{\varkappa}^b_{n,t} + \tfrac{\mathfrak{w}_k}{\mathfrak{w}_{(k+1)}} \text{ß} E_t \hat{S}^f_{(k+1),a,t+1} \\
\hat{S}^l_{a,t} &= \hat{S}^l_{0,a,t} \\
\hat{S}^l_{k,a,t} &= \sum_{n=1}^{N} \mathfrak{w}_{kn} \left(\Sigma_{k,n,\phi} - \Theta_{k,n,\phi}\right) \hat{\varkappa}^a_{n,t} + \sum_{n=1}^{N} \mathfrak{w}_{kn} \Theta_{k,n,\phi} \hat{\varkappa}^b_{n,t} + \tfrac{\mathfrak{w}_k}{\mathfrak{w}_{(k+1)}} \text{ß} \hat{S}^l_{(k+1),a,t-1} \\
&\textit{where} \\
\hat{S}^f_{k,a,t} &\equiv \sum_{n=1}^{N} \mathfrak{w}_{kn} \hat{S}^f_{k,n,t} \\
\hat{S}^l_{k,a,t} &\equiv \sum_{n=1}^{N} \mathfrak{w}_{kn} \hat{S}^l_{k,n,t} \\
\Theta_{k,n,\phi} &\equiv \left(\bar{z}^*_k\right)^{-\phi_n} \quad , \quad \Sigma_{0,n,\phi} = \Sigma_{n,\phi} \equiv \sum_{j=0}^{\infty} (\text{ß})^j \left(\bar{z}^*_j\right)^{-\phi_n} \\
\Sigma_{k,n,\phi} &\equiv \sum_{j=0}^{\infty} (\text{ß})^j \left(\bar{z}^*_{j+k}\right)^{-\phi_n} = \tfrac{1}{\text{ß}} \left[\Sigma_{(k-1),n,\phi} - \Theta_{(k-1),n,\phi}\right]
\end{aligned}
$$

Proof. From Lemma 1, note that the discounting parameter on $E_t \hat{S}^f_{(k+1),t+1}$ and $\hat{S}^l_{(k+1),t-1}$ is always ß. Therefore, we can find similar results for $\hat{S}^f_{a,t}$ and $\hat{S}^l_{a,t}$:

$$
\begin{aligned}
\hat{S}^f_{k,a,t} \equiv & \sum_{n=1}^{N} \mathfrak{w}_{kn} \hat{S}^f_{k,n,t} = \sum_{n=1}^{N} \mathfrak{w}_{kn} \left[\left(\Sigma_{k,n,\phi} - \Theta_{k,n,\phi}\right) E_t \hat{\varkappa}^a_{n,t+1} + \Theta_{k,n,\phi} \hat{\varkappa}^b_{n,t} + \text{ß} E_t \hat{S}^f_{(k+1),n,t+1}\right] \\
= & \sum_{n=1}^{N} \mathfrak{w}_{kn} \left(\Sigma_{k,n,\phi} - \Theta_{k,n,\phi}\right) E_t \hat{\varkappa}^a_{n,t+1} + \sum_{n=1}^{N} \mathfrak{w}_{kn} \Theta_{k,n,\phi} \hat{\varkappa}^b_{n,t} + \tfrac{\mathfrak{w}_k \text{ß}}{\mathfrak{w}_{(k+1)}} E_t \sum_{n=1}^{N} \mathfrak{w}_{(k+1)n} \hat{S}^f_{(k+1),n,t+1} \\
= & \sum_{n=1}^{N} \mathfrak{w}_{kn} \left(\Sigma_{k,n,\phi} - \Theta_{k,n,\phi}\right) E_t \hat{\varkappa}^a_{n,t+1} + \sum_{n=1}^{N} \mathfrak{w}_{kn} \Theta_{k,n,\phi} \hat{\varkappa}^b_{n,t} + \tfrac{\mathfrak{w}_k \text{ß}}{\mathfrak{w}_{(k+1)}} E_t \hat{S}^f_{(k+1),a,t+1}
\end{aligned}
$$

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\begin{aligned}
\hat{S}^l_{k,a,t} \equiv & \sum_{n=1}^{N} \mathfrak{w}_{kn} \hat{S}^l_{k,n,t} = \sum_{n=1}^{N} \mathfrak{w}_{kn} \left[\left(\Sigma_{k,n,\phi} - \Theta_{k,n,\phi}\right) \hat{\varkappa}^a_{n,t} + \Theta_{k,n,\phi} \hat{\varkappa}^b_{n,t} + \text{ß} \hat{S}^l_{(k+1),n,t-1}\right] \\
= & \sum_{n=1}^{N} \mathfrak{w}_{kn} \left(\Sigma_{k,n,\phi} - \Theta_{k,n,\phi}\right) \hat{\varkappa}^a_{n,t} + \sum_{n=1}^{N} \mathfrak{w}_{kn} \Theta_{k,n,\phi} \hat{\varkappa}^b_{n,t} + \tfrac{\mathfrak{w}_k \text{ß}}{\mathfrak{w}_{(k+1)}} \sum_{n=1}^{N} \mathfrak{w}_{(k+1)n} \hat{S}^l_{(k+1),n,t-1} \\
= & \sum_{n=1}^{N} \mathfrak{w}_{kn} \left(\Sigma_{k,n,\phi} - \Theta_{k,n,\phi}\right) \hat{\varkappa}^a_{n,t} + \sum_{n=1}^{N} \mathfrak{w}_{kn} \Theta_{k,n,\phi} \hat{\varkappa}^b_{n,t} + \tfrac{\mathfrak{w}_k \text{ß}}{\mathfrak{w}_{(k+1)}} \hat{S}^l_{(k+1),a,t-1}
\end{aligned}
$$

■

Therefore, using Lemma 1, and using the fact that $\mu_2 = (\delta\bar{\wp}_s)\frac{\theta}{(\theta_2-1)}\frac{\Sigma_{\delta 2}}{\Sigma_{s2}}$, we simplify the first six equations of our system into:

$$\begin{aligned}
\hat{Y}_{s,t} &= \hat{Y}_t - \hat{\wp}_{s,t} \\
\hat{\wp}_{\delta,t} &= \frac{1}{\theta}\hat{\wp}_{s,t} \\
\hat{\wp}_{s,t} &= -\bar{\alpha}_1(\bar{\wp}^*)\frac{\Sigma_{1,s1}\Omega_2}{\Sigma_{s2}\Omega_1}\left(\hat{\pi}_t - \hat{\pi}_t^{ind}\right) + \bar{\alpha}_1(\bar{\wp}^*)\frac{1}{\Sigma_{s2}\Omega_1}\left[\Theta_{0,\delta 1}\hat{S}_{1,s1,t-1} - \Theta_{0,s1}\hat{S}_{1,\delta 1,t-1}\right] \\
\hat{\wp}_t^* &= \bar{\alpha}_1\frac{\Omega_3}{\Omega_1}\left(\hat{\pi}_t - \hat{\pi}_t^{ind}\right) + \bar{\alpha}_1\frac{1}{\Omega_1}\left[\frac{\left(1-\bar{\theta}_2\right)}{\theta}\hat{S}_{1,s1,t-1} + (\delta\bar{\wp}_s)\hat{S}_{1,\delta 1,t-1}\right] \\
\hat{S}_{k,s1,t} &= \bar{\alpha}_1\Sigma_{(k+1),s1}\left(\hat{\pi}_t - \hat{\pi}_t^{ind}\right) - \Theta_{k,s1}\hat{\wp}_t^* + \bar{\alpha}_1\hat{S}_{(k+1),s1,t-1} \\
\hat{S}_{k,\delta 1,t} &= \bar{\alpha}_1\Sigma_{(k+1),\delta 1}\left(\hat{\pi}_t - \hat{\pi}_t^{ind}\right) - \Theta_{k,\delta 1}\hat{\wp}_t^* + \bar{\alpha}_1\hat{S}_{(k+1),\delta 1,t-1}
\end{aligned}$$

for $k = \{0,1,2,...,\infty\}$.

We make a scale transformation in the ancillary variables: $\hat{S}_{k,s,t} \equiv \frac{\hat{S}_{k,s1,t}}{\Sigma_{(k+1),s1}}$ and $\hat{S}_{k,\delta,t} \equiv \frac{\hat{S}_{k,\delta 1,t}}{\Sigma_{(k+1),\delta 1}}$.And so, we obtain:

$$\begin{aligned}
\hat{Y}_{s,t} &= \hat{Y}_t - \hat{\wp}_{s,t} \\
\hat{\wp}_{\delta,t} &= \frac{1}{\theta}\hat{\wp}_{s,t} \\
\hat{\wp}_{s,t} &= -\bar{\alpha}_1(\bar{\wp}^*)\frac{\Sigma_{1,s1}\Omega_2}{\Sigma_{s2}\Omega_1}\left(\hat{\pi}_t - \hat{\pi}_t^{ind}\right) + \bar{\alpha}_1(\bar{\wp}^*)\frac{\Theta_{0,\delta 1}\Sigma_{2,s1}}{\Sigma_{s2}\Omega_1}\left[\hat{S}_{1,s,t-1} - \frac{\Theta_{0,s1}\Sigma_{2,\delta 1}}{\Theta_{0,\delta 1}\Sigma_{2,s1}}\hat{S}_{1,\delta,t-1}\right] \\
\hat{\wp}_t^* &= \bar{\alpha}_1\frac{\Omega_3}{\Omega_1}\left(\hat{\pi}_t - \hat{\pi}_t^{ind}\right) + \bar{\alpha}_1\frac{\Sigma_{2,s1}}{\Omega_1}\left[\frac{\left(1-\bar{\theta}_2\right)}{\theta}\hat{S}_{1,s,t-1} + (\delta\bar{\wp}_s)\frac{\Sigma_{2,\delta 1}}{\Sigma_{2,s1}}\hat{S}_{1,\delta,t-1}\right] \\
\hat{S}_{k,s,t} &= \bar{\alpha}_1\left(\hat{\pi}_t - \hat{\pi}_t^{ind}\right) - \frac{\Theta_{k,s1}}{\Sigma_{(k+1),s1}}\hat{\wp}_t^* + \bar{\alpha}_1\frac{\Sigma_{(k+2),s1}}{\Sigma_{(k+1),s1}}\hat{S}_{(k+1),s,t-1} \\
\hat{S}_{k,\delta,t} &= \bar{\alpha}_1\left(\hat{\pi}_t - \hat{\pi}_t^{ind}\right) - \frac{\Theta_{k,\delta 1}}{\Sigma_{(k+1),\delta 1}}\hat{\wp}_t^* + \bar{\alpha}_1\frac{\Sigma_{(k+2),\delta 1}}{\Sigma_{(k+1),\delta 1}}\hat{S}_{(k+1),\delta,t-1}
\end{aligned} \tag{22}$$

As for remaining 12 equations of the optimal pricing system, after using Lemma 1 applied to discounted sums, we aggregate ancillary variables with discounting $\bar{\alpha}_1\beta$ or $\bar{\alpha}_2\beta$ into two synthetic ancillary variables, $\hat{S}_{k,\alpha 1,t}$ and $\hat{S}_{k,\alpha 2,t}$, defined as follows:

$$\begin{aligned}
\hat{S}_{k,\alpha 1,t} &\equiv \frac{\left[\frac{\mu_\theta}{\mu}\kappa_\delta\hat{S}_{k,N1,t} - \hat{S}_{k,D1,t} + \delta\bar{\wp}_s\left(\mu_\theta\hat{S}_{k,N2,t} - \frac{\mu_\theta}{\mu}\left(1+\bar{\theta}_1\right)\kappa_\delta\hat{S}_{k,N5,t} - \delta\bar{\wp}_s\mu_\theta(1+\theta)\hat{S}_{k,N6,t} + \theta\hat{S}_{k,D3,t}\right)\right]}{\Phi_{\Sigma 1,(k+1),1}} \\
\hat{S}_{k,\alpha 2,t} &\equiv \frac{\left[\frac{\mu_\theta}{\mu}\kappa_\delta\left(1+\bar{\theta}_1\right)\hat{S}_{k,N3,t} - \theta\hat{S}_{k,D2,t} + \delta\bar{\wp}_s\mu_\theta(1+\theta)\hat{S}_{k,N4,t}\right]}{\Phi_{\Sigma,(k+1),3}}
\end{aligned}$$

Therefore, using Corollary 2, we obtain the following simplified system for $k = \{0,1,2,...,\infty\}$:

$$\begin{aligned}
\hat{\wp}_t^* &= \bar{\alpha}_2\beta\frac{\Phi_{\Sigma,1,3}}{\Phi_{\Sigma,0,3}}E_t\left(\hat{\pi}_{t+1} - \hat{\pi}_{t+1}^{ind}\right) + \frac{1}{(\bar{\wp}^*)}\left[\frac{\mu_\theta}{\mu}\kappa_\delta(\sigma+\omega)\frac{\Theta_{0,N1}}{\Phi_{\Sigma,0,3}}\hat{x}_t + \bar{\alpha}_1\beta\frac{\Phi_{\Sigma,1,1}}{\Phi_{\Sigma,0,3}}\hat{\ell}_t - (\delta\bar{\wp}_s)\frac{\Phi_{\Theta,0,4}}{\Phi_{\Sigma,0,3}}\hat{\wp}_{s,t}\right] \\
&+ \frac{1}{(\bar{\wp}^*)}\bar{\alpha}_1\beta\frac{\Phi_{\Sigma 1,2,1}}{\Phi_{\Sigma,0,3}}E_t\hat{S}_{1,\alpha 1,t+1} - \bar{\alpha}_2\beta\frac{\Phi_{\Sigma,2,3}}{\Phi_{\Sigma,0,3}}E_t\hat{S}_{1,\alpha 2,t+1} \\
\hat{S}_{k,\alpha 1,t} &= \frac{\mu_\theta\kappa_\delta(\sigma+\omega)}{\mu}\frac{\Theta_{k,N1}}{\Phi_{\Sigma 1,(k+1),1}}\hat{x}_t + \bar{\alpha}_1\beta\frac{\Phi_{\Sigma,(k+1),1}}{\Phi_{\Sigma 1,(k+1),1}}\hat{\ell}_t - (\delta\bar{\wp}_s)\frac{\Phi_{\Theta,k,4}}{\Phi_{\Sigma 1,(k+1),1}}\hat{\wp}_{s,t} + \bar{\alpha}_1\beta\frac{\Phi_{\Sigma 1,(k+2),1}}{\Phi_{\Sigma 1,(k+1),1}}E_t\hat{S}_{(k+1),\alpha 1,t+1} \\
\hat{S}_{k,\alpha 2,t} &= -\bar{\alpha}_2\beta E_t\left(\hat{\pi}_{t+1} - \hat{\pi}_{t+1}^{ind}\right) + \bar{\alpha}_2\beta\frac{\Phi_{\Sigma,(k+2),3}}{\Phi_{\Sigma,(k+1),3}}E_t\hat{S}_{(k+1),\alpha 2,t+1}
\end{aligned}$$

where $\hat{\ell}_t \equiv E_t\left(\hat{q}_{t+1} + \hat{\mathfrak{g}}_{t+1} + \hat{\pi}_{t+1}^{ind}\right)$ and $\hat{\xi}_t$ is the aggregate shock:

$$\begin{aligned}
\hat{\ell}_t &= -E_t\left(\hat{\pi}_{t+1} - \hat{\pi}_{t+1}^{ind}\right) + (1-\sigma)E_t\left(\hat{x}_{t+1} - \hat{x}_t\right) + \hat{\xi}_t \\
\hat{\xi}_t &= \frac{(1+\omega)}{(\sigma+\omega)}E_t\left[\left(\hat{\epsilon}_{t+1} - \hat{\epsilon}_t\right) + (1-\sigma)\left(\widehat{\mathcal{A}}_{t+1} - \widehat{\mathcal{A}}_t\right)\right]
\end{aligned}$$