Monetary Policy and Inflation Scares*

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Abstract

A salient feature of the post-COVID inflation surge is that economic activity has remained resilient despite unfavorable supply-side developments. We develop a macroeconomic model with nonlinear price and wage Phillips curves, endogenous intrinsic indexation and an unobserved components representation of a cost-push shock that is consistent with these observations. In our model, a persistent large adverse supply shock can lead to a persistent inflation surge while output expands if the central bank follows an inflation forecast-based policy rule and thus abstains from hiking policy rates for some time as it (erroneously) expects inflationary pressures to dissipate quickly. A standard linearized formulation of our model cannot account for these observations under identical assumptions. Our nonlinear framework implies that the standard prescription of "looking through" supply shocks is a good policy for small shocks when inflation is near the central bank's target, but that such a policy may be quite risky when economic activity is strong and large shocks drive inflation well above target. Moreover, our model implies that the economic costs of "going the last mile" – i.e. a tight stance aimed at returning inflation quickly to target – can be substantial.

JEL Classification: E1, E3, E5.

Keywords: Inflation dynamics, New Keynesian model, inflation risk, monetary policy, linearized model, nonlinear model, real rigidities.

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1. Introduction

Following several decades of low and stable inflation in advanced economies, the post-COVID inflation surge caught central banks, economic forecasters, and academic economists by surprise. A vibrant debate has ensued about the causes of the surge, on why models failed to predict it, and on how to refine policy strategies.[1]

Supply shocks were widely viewed as generating much of the initial spike in inflation after the pandemic, including from COVID-related supply disruptions and escalating energy prices. However, while the shocks were clearly very large and drove headline inflation to levels not seen in decades, both the shocks and their effects on underlying inflation were forecast to be transient. The view that "second round" effects were likely small - implying weak endogenous propagation of supply shocks - was supported by a wide body of empirical evidence, and attributed to well-anchored inflation expectations as well as to structural features such as relatively flexible labor markets (Blanchard and Gali, 2007). Accordingly, advanced economy central banks choose to largely "look through" these temporary shocks and maintain highly accommodative policies even as realized inflation ran far above their targets.

The growing perception headline inflation pressures were reverberating to wages and core inflation eventually led to a rapid tightening in monetary policy that has helped markedly slow inflation. But the experience has left many open questions. Why did such a broad range of forecasters and forecasting models predict that high inflation would rapidly dissipate? What modeling features could help account for why second-round effects have generally been small since the start of the Great Moderation, but much higher in the recent surge? Are large shocks different in their effects? And, if transmission is more state-dependent than recognized, what are the implications for policy strategy, including for "looking through" supply shocks and how policy responds to inflation forecasts?

Our paper develops a modeling framework that aims to help address these questions. Following Harding, Lindé and Trabandt (2022, HLT), we start by embedding a nonlinear Phillips Curve into a fairly standard DSGE model that arises from a quasi-kinked demand schedule. The Phillips curve is flat when inflationary pressures are subdued and steepens as inflationary pressures rise. As shown by HLT (2023), these features imply that large shocks can have an outsized effect on inflation when inflation is already running above target.

We extend the analysis in HLT along several important dimensions. First, we introduce gradual

[1] See e.g. Federal Reserve Chair J. Powell speech at the 2021 Jackson Hole conference as well as the debate between L. Summers and P. Krugman that took place since early 2021. See also Gopinath (2022).

learning about the nature of the cost push shocks that are assumed to (mainly) drive inflation. Specifically, agents can't tell if the shocks are transitory or persistent (only observing the sum of the components), and must solve a signal extraction problem to estimate the underlying shock. The initial misperception of the shock as transitory - when it in fact turns about to be much more persistent - is one factor helping our model account for the large inflation forecast errors observed during the early phases of the inflation surge.

Second, we allow for endogenous indexation of prices and wages by firms and households. In particular, the fraction of prices and wages set in a backward-looking manner is higher when inflation runs persistently above target and by a greater magnitude. Thus, the stock of past inflation forecast errors has important consequences for inflation persistence in the Phillips Curve. Our calibration implies that this "intrinsic" persistence is very low if inflation has been reasonably close to target in recent years, consistent with estimates during the Great Moderation period; but implies much higher intrinsic persistence if inflation rises significantly above target for some time, as during the recent inflation surge.

Third, we consider the implications of a forecast-based targeting rule in which the central bank responds to an inflation forecast one or two years ahead, and compare this to a standard instrument rule with contemporaneous inflation. The forecast-based rule aims to capture how inflation-targeting central banks typically respond to medium-term forecasts of inflation and thus "look through" transient supply shocks (consistent with a wide literature on forecast-based Taylor rules).

Interactions of these three key features – information about the nature of the adverse cost-push shock, endogenous indexation, and the policy reaction function - allow our model account for key facets of the persistent post-COVID inflation surge and the monetary policy response. Given that the shock is initially perceived as transitory and intrinsic persistence is viewed as low, inflation is expected to quickly revert to baseline, and the central bank under the forecast-based rule keeps policy rates nearly unchanged. When inflationary pressures turn out to be more pervasive than initially projected the central bank starts to hike rates materially, but at this stage the "inflation ghost" is already out of the bottle. In particular, the state-dependent sensitivity of inflation to economic activity (from the nonlinear Phillips Curve) interacts with endogenous indexation mechanism to produce a full "inflation cycle" in our nonlinear model.

Importantly, the inflation cycle is associated with an expansion in economic activity followed by a subsequent contraction later in the inflation cycle when real interest rates rise. Because nominal rates depend on the medium-term forecast of inflation and hence only rise gradually, short-term real interest rates actually fall for some time (since inflation in the very near-term rises more). This stimulus contrasts with a standard full-information model in which an adverse supply shock triggers higher inflation along with a sizeable contraction in output. Thus, our model can account for some of the observed resilience in economic activity that accompanied the initial inflation surge even without demand shocks playing a material role.

We highlight how the implications of our nonlinear model contrast sharply with the those of a linearized formulation embedding otherwise identical assumptions. The linearized variant implies only a modest rise in inflation rather than large inflation boom as in the nonlinear model. The stark difference between the linearized and nonlinear formulations of the model is driven by the size of the underlying supply shock. When the supply disturbance is notably smaller, the stark difference between the nonlinear and linear model dissipates. We also show that our nonlinear model – when fed with a combination of large adverse supply shocks and moderate demand shocks – does a good job of accounting for the overall contours of U.S. data in 2021-2023. Our nonlinear model also accounts well for the forecast revisions of professional forecasters. By contrast, the linearized model fails to account for the data and forecasts.

From a policy perspective, our analysis suggests that the standard monetary policy prescription of "looking through" adverse supply shocks should be applied cautiously. While there is a strong case for the merits of such a policy for small adverse cost-push shocks which does not drive inflation far away from its target, our nonlinear model provides an example that such a policy may prove very costly when the economy is hit with a rare and large adverse cost-push shock which drives inflation well above the central bank's target.[2]

Our model can be used to assess the costs (and benefits) of the forecast-based policy framework assuming that the economy is initially at the steady state before the adverse supply shock hits. In this case, a central bank that use a policy rule based on actual instead of forecasted inflation moderates the surge in inflation somewhat, since the central bank from the onset acts more aggressively to battle higher inflation. However, a tighter monetary policy stance to curb inflationary pressures comes at a cost, namely that output falls more. Even so, the expected loss – measured as the sum of the squared deviations of annualized inflation from target plus the sum of squared output gaps for the first 5 years – falls by about 30 percent from a loss value of about 255 under the forecast-based rule to a loss of about 174 under a rule which responds to actual inflation.[3]

[2] Our calibration of the unobserverd components representation of the cost-push shock places a predominant role on transitory realizations of cost-push shocks and hence rationalizes that the central bank uses a forecast-based policy rule in normal times.

[3] The improvement in the loss function in the rule with actual inflation is not contingent on the weight of the output gap in the loss function; the loss improves for any weight on the output gap between 0 and 2 in the loss function. Debortoli, Kim, Linde, and Nunes (2019) shows that an inflation-output gap loss function approximates

An additional important policy implication from our nonlinear model is that the central bank faces a more severe tradeoff between inflation and output stabilization if it strives to push inflation all the way back to target aggressively when inflation has peaked and is gradually receding. While the Phillips curve appears to have steepened and the sensitivity of inflation and inflation expectations becomes elevated when the adverse supply shock hits the economy, the steepening is temporary and it flattens again when the Phillips curve shifts out. This feature of our model makes it costly for the central bank to aggressively attempt to push inflation all the way back to target quickly unless a sizeable part of the underlying adverse supply shock reverses and helps to push inflation back to target. Put differently, we find that the economic costs of "going the last mile" and bringing inflation quickly back to target can be sizeable.

Our findings rest importantly on the interaction between the steeper portion of the Phillips curve and on the relevance of inflation indexation mechanisms when inflation exceeds the central banks' inflation target. Taken together, these mechanisms imply that all shocks in the model transmit stronger to inflation when inflation rises materially above its steady state level. In particular, cost-push shocks generate conditional heteroskedasticity in inflation and inflation risk in our nonlinear model, consistent with the seminal paper by Engle (1982) and the more recent work by López-Salido and Loria (2020). Since these shocks are commonly believed to have played an important role during the post-COVID period, we argue that our model can account better for inflation dynamics during this period than a standard linearized macroeconomic model. Regression analysis supports the view that cost-push-type shocks have a larger impact on inflation if inflation is high to begin with (see e.g., Gelos and Ustyugova 2017; Forbes, Gagnon and Collins 2021a; Forbes, Gagnon and Collins 2021b; and Ball, Leigh and Mishra 2022).

We establish our main results in a nonlinear variant of the benchmark Erceg, Henderson and Levin (2000, henceforth EHL) model with sticky wages and prices. The EHL model shares most of the model features in Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2007, henceforth SW), except that it excludes endogenous capital accumulation. To parameterize the model, we follow HLT (2023) and allow for a more prominent role for Kimball (1995) quasi-kinked demand in goods markets. HLT shows that the more prominent role for quasi-kinked demand increases the marginal data density in the basic SW model provided that the average markup aligns with micro- and macroeconomic empirical evidence. Recent work by Dupraz (2017) and Ilut et al. (2022) provides a microfounded theory of kinked demand.

household welfare policy closely in the standard Erceg, Henderson and Levin (2000) and Smets-Wouters (2007) sticky price-wage models.

The remainder of the paper is organized as follows. Section 2 presents and discusses cross-country data for inflation and policy rates as well as the projections for the survey of professional forecaster forecasts for the U.S. economy. Section 3 presents the quantitative macroeconomic model with real rigidities in a dynamic stochastic general equilibrium framework with nominal price and wage stickiness. Section 4 discusses our results. Section 5 discusses the related literature. Finally, Section 6 provides some concluding remarks.

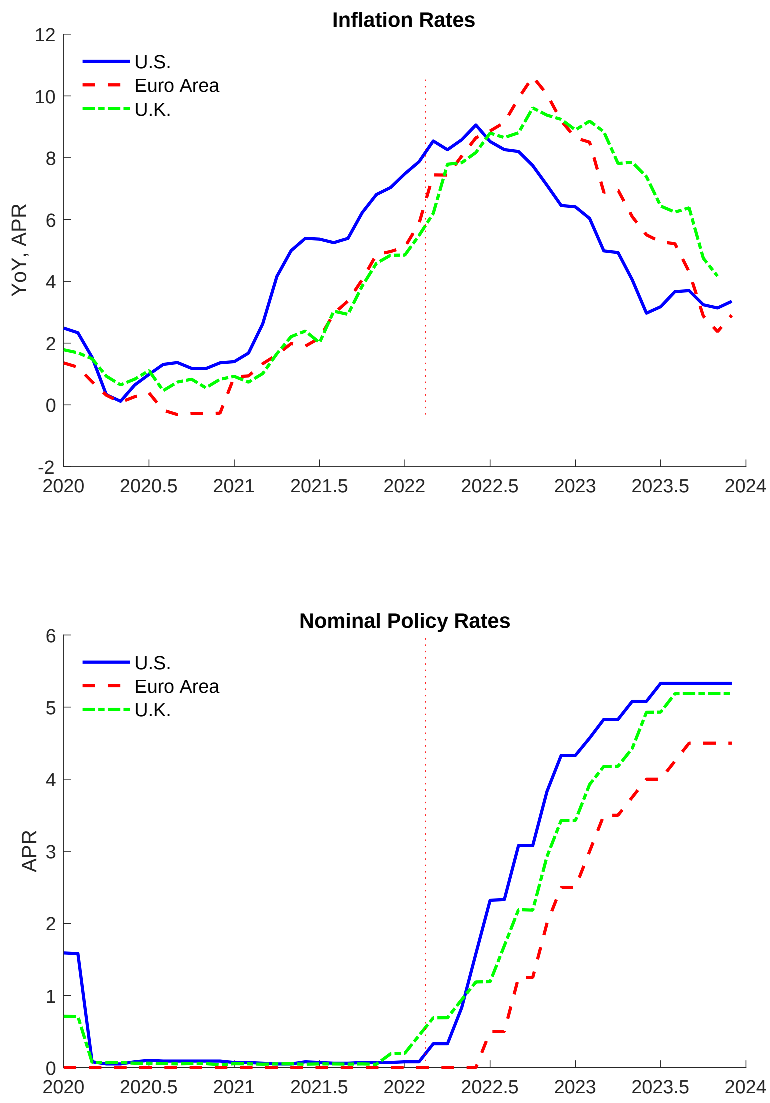

Figure 1: Inflation rates and nominal policy rates in the U.S., Euro Area, and United Kingdom.

2. Data

Figure 1 depicts monthly data for inflation (12-month change) and nominal policy rates for the U.S., the Euro Area and the U.K. Several facts emerge from the figures. First, inflation has surged in all these major economies, with the US about half-year ahead. Second, the central banks in these economies – US Federal Reserve, ECB and the Bank of England – kept policy rates unchanged for about 1 to 1.5 years after inflation rose above their 2 percent targets. In 2022 (the vertical dotted line in the figure), the central banks pivoted and hiked rates materially. Figure 2 shows quarterly U.S. data for PCE inflation, PCE growth and the three-months T-bill rate. In addition, the figure contains projections by survey of professional forecasters. Several facts emerge from Figure 2. First, professional forecasters underestimated both the size and persistence of the increase in U.S. inflation from early 2021. Second, professional forecasters underestimated the resilience of economic activity initially. Third and finally, professional forecasters also underestimated how much the Fed would need to raise policy rates and 3-months T-bills to cool the economy and bring inflation back to target. We seek to develop a model that can account for these facts and then use it to garner policy lessons.

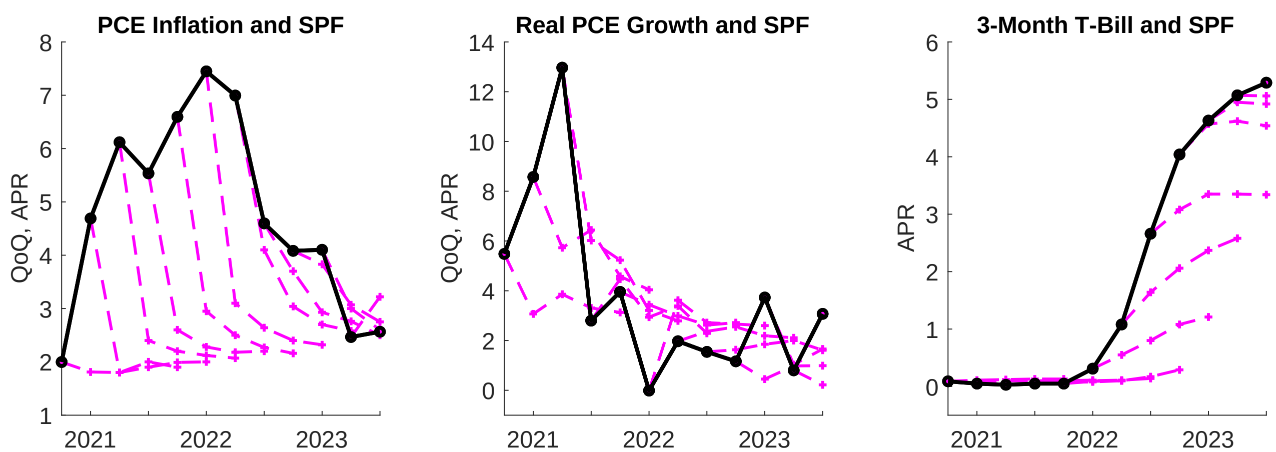

Figure 2: U.S. PCE inflation, PCE growth and 3-months T-Bill rate (solid: data; dashed: survey of professional forecasters (SPF) data.)

3. Quantitative Model

The model developed below modifies and extends the model in Lindé and Trabandt (2018) and Harding, Lindé and Trabandt (2022, 2023).

3.1. Households

There is a continuum of households $j \in [0,1]$ in the economy. Each household supplies a specialized type of labor j to the labor market. The j^{th} household is the monopoly supplier of the j^{th} type of labor service. The j^{th} household maximizes

$$\max_{c_t, n_t, b_t} E_0 \sum_{t=0}^{\infty} \beta^t \varsigma_t \left\{ \ln\left(c_t - hC_{t-1}\right) - \frac{1}{1+\chi} n_{j,t}^{1+\chi} \right\} \tag{1}$$

subject to

$$P_t c_t + B_t = W_{j,t} n_{j,t} + R_{t-1} B_{t-1} - T_t + \Gamma_t + a_{j,t}$$

where the choice variables of the j^{th} household are consumption c_t and risk-free bonds B_t. Bonds are in zero net supply. The j^{th} household also chooses the wage W_j subject to Calvo sticky prices as in Erceg, Henderson and Levin (2000, EHL). The household understands that when choosing W_j that it must supply the amount of labor n_j demanded by a labor contractor according to equation (2).

In principle, the presence of wage setting frictions implies that households have idiosyncratic levels of wealth and, hence, consumption. However, we follow EHL in supposing that each household has access to perfect consumption insurance. Because of the additive separability of the family utility function, perfect consumption insurance at the level of households implies equal consumption across households. Given this, we have simplified our notation and not include a subscript, j, on the j^{th} family's consumption (and bond holdings). Note that even though consumption is equal across households, consumption in response to shocks is not constant over time across households. P_t denotes the aggregate price level, and R_t the gross nominal interest rate on bonds purchased in period $t-1$ which pay off in period t. T_t are lump-sum taxes net of transfers and Γ_t denotes the share of profits that the household receives. Finally, $a_{j,t}$ denotes the payments and receipts associated with the insurance associated with wage stickiness. E_0 denotes the conditional expectation operator. Further, $0 \leq \beta < 1$ and $0 \leq h < 1$ are parameters. Finally, the variable ς_t is an exogenous shock to the discount factor. We assume that $\delta_t = \frac{\varsigma_{t+1}}{\varsigma_t}$ is exogenous and follows an AR(1) process:

$$\delta_t - \delta = \rho_\delta \left(\delta_{t-1} - \delta\right) + \varepsilon_{\delta,t},$$

with $\delta = 1$ in steady state.

3.2. Labor Contractors

Competitive labor contractors aggregate specialized labor inputs $n_{t,j}$ supplied by households into homogenous labor n_t which is hired by intermediate good producers. Labor contractors maximize profits

$$\max_{n_{t,j},n_t} W_t n_t - \int W_{t,j} n_{t,j} dj, \text{ or } \max_{n_{t,j}/n_t} 1 - \int \frac{W_{t,j}}{W_t} \frac{n_{t,j}}{n_t} dj$$

where $W_{t,j}$ is the wage paid by the labor contractor to households for supplying type j labor. W_t denotes the wage paid to the labor contractor for homogenous labor. Maximization of profits is subject to

$$\int G_w \left(\frac{n_{t,j}}{n_t} \right) dj = 1$$

where

$$G_w \left(\frac{n_{t,j}}{n_t} \right) = \frac{\omega_w}{1+\psi_w} \left[(1+\psi_w) \frac{n_{t,j}}{n_t} - \psi_w \right]^{\frac{1}{\omega_w}} - \frac{\omega_w}{1+\psi_w} + 1$$

is the Kimball aggregator specification as used in Dotsey and King (1995) or Levin, Lopez-Salido and Yun (2007) adapted for the labor market. Note that $\omega_w = \frac{(1+\psi_w)\phi_w}{1+\phi_w\psi_w}$ and $\phi_w = 1 + \theta_w$ where $\theta_w \geq 0$ denotes the net wage markup, $\phi_w \geq 1$ denotes the gross wage markup and $\psi_w \leq 0$ is the Kimball parameter that controls the degree of complementarities in wage setting. Let ϑ_t^w denote the multiplier on the labor contractor's constraint. The appendix contains detailed derivations that result in the following equations:

$$\frac{n_{t,j}}{n_t} = \frac{1}{1+\psi_w} \left(\left[\frac{W_{t,j}}{W_t} \right]^{-\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} [\vartheta_t^w]^{\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} + \psi_w \right) \tag{2}$$

$$W_t \vartheta_t^w = \left[\int W_{t,j}^{-\frac{1+\psi_w+\theta_w\psi_w}{\theta_w}} dj \right]^{-\frac{\theta_w}{1+\psi_w+\theta_w\psi_w}} \tag{3}$$

$$\vartheta_t^w = 1 + \psi_w - \psi_w \int \frac{W_{t,j}}{W_t} dj \tag{4}$$

Where equation (2) denotes the demand for labor, equation (3) is the aggregate wage index and equation (4) is the zero profit condition for labor contractors. Note that for $\psi_w = 0$ we get the standard Dixit-Stiglitz expressions $\frac{n_{t,j}}{n_t} = \left[\frac{W_{t,j}}{W_t} \right]^{-\frac{(1+\theta_w)}{\theta_w}}$, $W_t = \left[\int W_{t,j}^{-\frac{1}{\theta_w}} dj \right]^{-\theta_w}$, and $\vartheta_t^w = 1$.

3.3. Wage Setting

The household faces a standard monopoly problem of selecting $W_{j,t}$ to maximize the welfare, (1) subject to the demand for labor (2). Following EHL, we assume that the household experiences Calvo-style frictions in its choice of $W_{j,t}$. In particular, with probability $1 - \xi_w$ the j^{th} family has the opportunity to re-optimize its wage rate. With the complementary probability, the family must set its wage rate according to the following rule:

$$W_{j,t} = \tilde{\Pi}_t^w W_{j,t-1} \tag{5}$$

where $\tilde{\Pi}_t^w$ is an indexation factor whose determinants we discuss in Section 3.6 below.

Let Λ_t denote the Lagrange multiplier on the household budget constraint. To compute the optimal choice for $\tilde{W}_{j,t}$, the household seeks to maximize:

$$\max_{\tilde{W}_{j,t}} E_t \sum_{i=0}^{\infty} (\beta\xi_w)^i \varsigma_{t+i} \left\{ -\frac{1}{1+\chi} n_{j,t+i}^{1+\chi} + \Lambda_{t+i} \tilde{W}_{j,t} \left(\tilde{\Pi}_{t+i}^w \times ... \times \tilde{\Pi}_{t+1}^w \right) n_{j,t+i} \right\}$$

subject to labor demand:

$$n_{t+i,j} = \frac{1}{1+\psi_w} \left(\left[\frac{\tilde{W}_{j,t} \left(\tilde{\Pi}_{t+i}^w \times ... \times \tilde{\Pi}_{t+1}^w \right)}{W_{t+i}} \right]^{-\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} \left[\vartheta_{t+i}^w \right]^{\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} + \psi_w \right) n_{t+i}$$

In what follows, we assume that $\chi = 0$ for simplicity.[4] The appendix provides detailed derivations for optimal wage setting.

3.4. Final Goods Producers

Competitive final goods producers maximize profits:

$$\max_{y_{t,i}/y_t} 1 - \int \frac{P_{t,i}}{P_t} \frac{y_{t,i}}{y_t} di$$

subject to

$$\int G\left(\frac{y_{t,i}}{y_t}\right) di = 1$$

where

$$G\left(\frac{y_{t,i}}{y_t}\right) = \frac{\omega_p}{1+\psi_p} \left[(1+\psi_p) \frac{y_{t,i}}{y_t} - \psi_p \right]^{\frac{1}{\omega_p}} - \frac{\omega_p}{1+\psi_p} + 1$$

is the Kimball (1995) aggregator specification used by Dotsey and King (1995) and Levin, Lopez-Salido and Yun (2007). Note that $\omega_p = \frac{(1+\psi_p)\phi_p}{1+\phi_p\psi_p}$ and $\phi_p = 1 + \theta_p$ where $\theta_p \geq 0$ denotes the net price markup, $\phi_p \geq 1$ denotes the gross price markup and $\psi_p \leq 0$ is the Kimball parameter that controls the degree of complementarities in firm's pricing decisions. Let ϑ_t denote the multiplier on the constraint. The appendix contains detailed derivations that result in the following equations:

$$\begin{aligned}
\frac{y_{t,i}}{y_t} &= \frac{1}{1+\psi_p} \left(\left[\frac{P_{t,i}}{P_t} \right]^{-\frac{1+\theta_p}{\theta_p}(1+\psi_p)} \vartheta_t^{\frac{1+\theta_p}{\theta_p}(1+\psi_p)} + \psi_p \right) \\
P_t \vartheta_t &= \left[\int P_{t,i}^{-\frac{1+\psi_p+\psi_p\theta_p}{\theta_p}} di \right]^{-\frac{\theta_p}{1+\psi_p+\psi_p\theta_p}} \\
\vartheta_t &= 1 + \psi_p - \psi_p \int \frac{P_{t,i}}{P_t} di
\end{aligned}$$

[4] In a future version of this paper, we might consider allowing for $\chi > 0$.

where $\phi_p = 1 + \theta_p$ and $\varepsilon_p = \frac{\phi_p(1+\psi_p)}{1-\phi_p}$. Note that for $\psi_p = 0$ we get the standard Dixit-Stiglitz expressions $\frac{y_{t,i}}{y_t} = \left[\frac{P_{t,i}}{P_t}\right]^{-\frac{1+\theta_p}{\theta_p}}$, $\vartheta_t = 1$ and $P_t = \left[\int P_{t,i}^{-\frac{1}{\theta_p}} di\right]^{-\theta_p}$.

3.5. Intermediate Goods Producers

Intermediate goods firms have the following production function:

$$y_{t,i} = n_{t,i}$$

Total costs for the firm are:

$$TC_{t,i} = \tau_t^{1/\kappa} W_t n_{t,i}$$

where $\tau_t^{1/\kappa}$ is an exogenous shifter of firms' total costs. $\tau_t^{1/\kappa}$ is a stand in for e.g. a tax shock (with lump-sum redistribution to households) or any other shock to firms' total cost. In what follows, we refer to a shock to τ_t as a cost-push shock (which we use interchangeably as a markup shock, too). Note that the shock to marginal cost is scaled by the inverse slope of the price Phillips curves once the model is log-linearized, i.e. $\kappa_p = \frac{1}{1+\beta\varkappa}\frac{(1-\beta\xi_p)(1-\xi_p)}{\xi_p}\frac{1}{1-(1+\theta_p)\psi_p}$. This way, after log-linearization, the shock enters the Phillips curve additively separable with a unit coefficient as in Smets and Wouters (2007) and other estimated DSGE models. An additional attractive feature with this scaling is that small shocks to τ_t propagate identically in the nonlinear and linearized formulation of the model. τ_t has a mean of unity so that the scaling does not affect the steady state. Consistent with the literature, e.g. Smets and Wouters (2007) and Galí, Smets and Wouters (2012), we assume that the cost-push shock only affects the actual economy but not potential output, i.e. the cost-push shock is inefficient.[5]

Firms minimize costs (or maximize negative cost) subject to production:

$$\max_{n_{t,i}} -\tau_t^{1/\kappa} W_t n_{t,i} + MC_{t,i}\left[n_{t,i} - y_{t,i}\right]$$

$MC_{t,i}$ denotes the lagrange multiplier and coincides with firm marginal cost. Since all firms face the same marginal costs, the first order condition reads as:

$$MC_{t,i} \equiv MC_t = \tau_t^{1/\kappa} W_t$$

Profit maximization:

$$\max_{\tilde{P}_{t,i}} \; E_t \sum_{j=0}^{\infty} (\beta\xi_p)^j \varsigma_{t+j}\Lambda_{t+j}\left[\left(\tilde{\Pi}_{t+j} \times ... \times \tilde{\Pi}_{t+1}\right)\tilde{P}_{t,i} y_{t+j,i} - MC_{t+j} y_{t+j,i}\right]$$

[5] It would be interesting to consider other types of adverse supply shocks (such as e.g. adverse total factor productivity shocks reflecting e.g. supply chain disruptions or energy price hikes) that affect the potential economy with flexible prices and wages.

subject to

$$y_{t+j,i} = \frac{1}{1+\psi_p}\left(\left[\frac{\left(\tilde{\Pi}_{t+j} \times ... \times \tilde{\Pi}_{t+1}\right)\tilde{P}_{t,i}}{P_{t+j}}\right]^{-\frac{1+\theta_p}{\theta_p}(1+\psi_p)} \vartheta_{t+j}^{\frac{1+\theta_p}{\theta_p}(1+\psi_p)} + \psi_p\right) y_{t+j}$$

where $\tilde{\Pi}_t$ is an indexation factor whose determinants we discuss in Section 3.6 below. The appendix provides a detailed set of derivations of optimal price setting.

3.6. Endogenous Price and Wage Indexation

Define the gross inflation rate $\Pi_t = P_t/P_{t-1}$. As noted earlier, $\tilde{\Pi}_t$ and $\tilde{\Pi}_t^w$ are the price- and wage-setting indexation inflation rates for non-optimizing firms and labor unions. We consider the following state-dependent price and wage indexation specification:

$$\tilde{\Pi}_t = \tilde{\Pi}_t^w = \bar{\Pi}^{1-\varkappa_t}\Pi_{t-1}^{\varkappa_t} \tag{6}$$

where

$$\varkappa_t = e^{-\frac{\varrho}{\max(\Pi_t^* - \bar{\Pi},\, 0.0001)}} - e^{-\frac{\varrho}{0.0001}} \tag{7}$$

and

$$\Pi_t^* = \left(\Pi_{t-1}^*\right)^{\omega}\left(\Pi_{t-1}\right)^{1-\omega} \tag{8}$$

with $0 \leq \omega < 1$ and $\varrho \geq 0$. $\bar{\Pi}$ denotes steady state inflation. Note that prices and wages are indexed by the same factor which is a function of a geometric lag of past inflation rates, provided $\omega > 0$.[6] We will henceforth use the term 'endogenous' indexation when $\varkappa_t > 0$ since it depends on the aggregate rate of inflation which is an endogenous variable in our model.[7]

To illustrate the indexation specification, consider the following parameters. We set $\varrho = 0.002$ and $\bar{\Pi} = 1.005$. The number 0.0001 inside the max operator is set for numerical stability. Note that $\varkappa_t$ is zero in steady state. Figure 3 provides a graphical illustration of the endogenous indexation feature of our model. Specifically, varying Π_t^* on a grid and calculating the resulting indexation factor $\varkappa_t$ results in the relationship between Π_t^* and $\varkappa_t$ displayed in Figure 3.

[6] It would be interesting to study the implications of making indexation dependent on annual lagged inflation $\Pi_{t-1}^a = (\Pi_{t-1}\Pi_{t-2}\Pi_{t-3}\Pi_{t-4})^{1/4}$ and compare the results to our our baseline specifiation with quarterly lagged inflation, Π_{t-1}.

[7] Note that our indexation scheme offers one way of rationalizing state dependence. An alternative mechanism could be that firms re-optimize prices and wages more frequently in high-inflation environments, i.e. state-dependent Calvo probabilities. This would be consistent with many S-s type models. We favour the endogenous indexation feature of our model since it implies that inflation may remain persistently high in response to inflation surges. In other words, the endogenous inflation indexation feature generates endogenous inflation persistence. In addition, our inflation indexation feature implies that disinflations are more costly the higher inflation is to begin with. By contrast, in models where prices and wages become more flexible as a consequence of high inflation, disinflations become less costly in terms of economic activity the higher the rate of inflation.

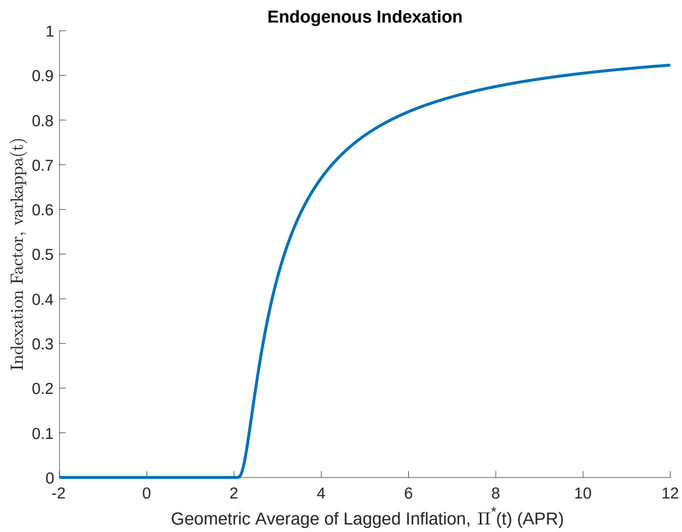

Figure 3: Endogenous Indexation

Note that when taking logs of the indexation equation $\tilde{\Pi}_t = \bar{\Pi}^{1-\varkappa_t}\Pi_{t-1}^{\varkappa_t}$ we get

$$\ln \frac{\tilde{\Pi}_t}{\bar{\Pi}} = \widehat{\tilde{\Pi}}_t = \varkappa_t \ln \frac{\Pi_{t-1}}{\bar{\Pi}} = \varkappa_t \hat{\Pi}_{t-1}$$

Taking a first order Taylor series approximation gives:

$$\widehat{\tilde{\Pi}}_t = \varkappa \hat{\Pi}_{t-1} \tag{9}$$

that is, the state-dependency feature of endogenous indexation disappears in the log-linearized model variant. The reason for eq. (9) is that $\bar{\Pi} = \Pi_{t-1}$ in eq. (6) in the steady state, which is the point of approximation. This implies no dynamic indexation in the linearized model, since $\varkappa = 0$. By contrast, in the nonlinear model, indexation to past inflation is state-dependent. Consistent with the empirical evidence in Smets and Wouters (2007) and Fernandez-Villaverde and Rubio-Ramirez (2008), there is little or no dynamic indexation when inflation is close or below the steady state, but dynamic indexation arises endogenously when inflation runs up well above the central banks' inflation target.[8]

[8] As as alternative to the indexation scheme described above, we have also examined the implications when adopting the following indexation rule: $\tilde{\Pi}_t = \left(\frac{\Pi_{t-1}}{\bar{\Pi}}\right)^{\varkappa_t}$. This indexation rule has similar implications in terms of dynamic indexation as the rule in the main text, i.e. it implies no dynamic indexation after log-linearization and endogenous dynamic indexation in the nonlinear model. In addition to these properties, the indexation rule also implies no indexation in the steady state, i.e. the model then features price dispersion in the steady state. Our qualitative and quantitative results are very little affected with this alternative indexation scheme, although it implies that the average time that is takes for firms to change their prices becomes endogenous in our nonlinear model. When inflation is close or below the steady state, firms change their prices once every three quarters. But in response to large persistent shocks that drives inflation well above the central banks' inflation target, firms change their prices more often. This model feature is consistent with recent micro evidence documented in Cavallo, Lippi, and Miyahara (2023, Figure 2).

3.7. Aggregate Resources

The aggregate resource constraint can be written as:

$$c_t = y_t = (p_t^*)^{-1} (w_t^*)^{-1} l_t$$

where p_t^* and w_t^* are measures of price and wage dispersion. See the appendix for detailed derivations and expressions for these variables.

Note that the payments and receipts associated with the insurance associated with wage stickiness are in zero net supply, i.e.

$$\int a_{j,t} dj = 0$$

and bonds are in zero net supply

$$B_t = 0.$$

3.8. Monetary Policy

Consider the following monetary policy rule which defines the so-called notional interest rate:

$$\frac{R_t^{not}}{R} = \left(\frac{R_{t-1}^{not}}{R}\right)^{\rho} \left(\frac{\mathrm{E}_t \Pi_{t+4}}{\bar{\Pi}}\right)^{(1-\rho)\gamma_\pi} \left(\frac{Y_t}{Y} / \frac{Y_t^{pot}}{Y^{pot}}\right)^{(1-\rho)\gamma_x} e^{\varepsilon_{R,t}} \tag{10}$$

where the monetary policy shock $\varepsilon_{R,t}$ is assumed to be i.i.d. zero mean with positive variance.[9]

Monetary policy is subject to the zero lower bound on interest rates, i.e. the actual nominal interest rate is

$$R_t = \max(0, R_t^{not})$$

Regarding the fiscal authority we assume that net lump-sum taxes adjust to balance the government budget. Because of Ricardian equivalence we don't spell out the government budget and fiscal rule for lump sum transfers.

3.9. Learning about the Cost-Push Shock

We adopt the following unobserved components representation for the cost-push shock. First, define: $a_t \equiv \tau_t - 1$, i.e. a_t is the deviation of τ_t from its steady state. Now suppose that the stochastic process a_t consists of a transitory part $a_{T,t}$ and a persistent part $a_{P,t}$. Agents can observe a_t but not $a_{T,t}$ or $a_{P,t}$. Below we set up the signal-extraction problem that households and firms are solving

[9] Our results are robust to replacing $\frac{E_t \Pi_{t+4}}{\bar{\Pi}}$ in the monetary policy rule with $\frac{E_t \Pi_{t+4}^{1yoy}}{\bar{\Pi}}$ where $\Pi_t^{1yoy} = (\Pi_{t+4}\Pi_{t+3}\Pi_{t+2}\Pi_{t+1})^{1/4}$ denotes the annual rate of change (at quarterly rate) of the price level. The results based on this alternative monetary policy rule are available upon request.

following Erceg and Levin (2003), who used this approach to solve a signal-extraction problem about transitory and persistent shocks to the central bank's inflation target. Edge, Laubach and Williams (2007) apply the same technique on a signal extraction problem for productivity.

Following e.g. Erceg and Levin (2003), Hamilton (1994), Ljungqvist and Sargent (2018), or Canova (2007), we set up the following state space system:

$$a_t = \underbrace{\begin{bmatrix} 1 & 1 \end{bmatrix}}_{H'} \begin{bmatrix} a_t^P \\ a_t^T \end{bmatrix}$$

$$\begin{bmatrix} a_{t+1}^P \\ a_{t+1}^T \end{bmatrix} = \underbrace{\begin{bmatrix} \rho_P & 0 \\ 0 & \rho_T \end{bmatrix}}_{F} \begin{bmatrix} a_t^P \\ a_t^T \end{bmatrix} + \underbrace{\begin{bmatrix} \sigma_P & 0 \\ 0 & \sigma_T \end{bmatrix}}_{Q} \begin{bmatrix} \varepsilon_{t+1}^P \\ \varepsilon_{t+1}^T \end{bmatrix}$$

Then, using the Kalman filter, the estimate of state variables is (state update):

$$\begin{bmatrix} \hat{a}_{t|t}^P \\ \hat{a}_{t|t}^T \end{bmatrix} = \begin{bmatrix} \hat{a}_{t|t-1}^P \\ \hat{a}_{t|t-1}^T \end{bmatrix} + L_t \left(a_t - H' \begin{bmatrix} \hat{a}_{t|t-1}^P \\ \hat{a}_{t|t-1}^T \end{bmatrix} \right)$$

where

$$\begin{aligned} L_t &= P_{t|t-1} H (H^{'} P_{t|t-1} H)^{-1} \\ P_{t|t} &= P_{t|t-1} - L_t H^{'} P_{t|t-1} \end{aligned}$$

The optimal forecast of state variables is (state forecast):

$$\begin{bmatrix} \hat{a}_{t+1|t}^P \\ \hat{a}_{t+1|t}^T \end{bmatrix} = F \begin{bmatrix} \hat{a}_{t|t-1}^P \\ \hat{a}_{t|t-1}^T \end{bmatrix} + K_t \left(a_t - H' \begin{bmatrix} \hat{a}_{t|t-1}^P \\ \hat{a}_{t|t-1}^T \end{bmatrix} \right)$$

where

$$\begin{aligned} K_t &= F P_{t|t-1} H (H^{'} P_{t|t-1} H)^{-1} \\ P_{t+1|t} &= \left(F - K_t H^{'} \right) P_{t|t-1} \left(F^{'} - H K_t^{'} \right) + Q Q^{'}. \end{aligned}$$

Note that:

$$\begin{bmatrix} \hat{a}_{t+j|t}^P \\ \hat{a}_{t+j|t}^T \end{bmatrix} = F^j \begin{bmatrix} \hat{a}_{t|t}^P \\ \hat{a}_{t|t}^T \end{bmatrix} \text{ for } j = 1, ...\infty$$

The forecast of e.g. a_{t+1} is straightforward:

$$\hat{a}_{t+j|t} = H^{'} \begin{bmatrix} \hat{a}_{t+j|t}^P \\ \hat{a}_{t+j|t}^T \end{bmatrix} \text{ for } j = 1, ...\infty$$

We assume that the agents have an infinite amount of past data available so that they run the recursions of K_t and $P_{t+1|t}$ until convergence. In other words, we solve for the fixed point $P_{t+1|t} = P_{t|t-1} = P$.

Table 1: Model parameter values

Parameter	Value	Description
$\bar{\Pi}$	1.005	Steady state gross inflation rate
θ_p	0.1	Net price markup in steady state
ξ_p	2/3	Calvo price stickiness parameter
ψ_p	-12	Parameter Kimball aggregator prices
ϱ	0.002	Curvature parameter endogenous indexation
ω	0.8	Parameter in endogenous. indexation
$\varkappa$	0	Inflation indexation parameter in linear model
θ_w	0.1	Net wage markup in steady state
ξ_w	0.75	Calvo wage stickiness parameter
ψ_w	-6	Parameter Kimball aggregator wages
ρ	0.85	Taylor rule: interest rate smoothing
γ_π	1.5	Taylor rule: coef. on expected inflation
γ_x	0.125	Taylor rule: coef. on output gap
β	0.995	Household discount factor
h	0.7	Household consumption habit
χ	0	Inverse Frisch elasticity of labor supply
ρ_P	0.9	AR(1) persistent markup shock
ρ_T	0	AR(1) transitory markup shock
σ_P	1	Standard deviation persistent markup shock
σ_T	10	Standard deviation transitory markup shock
ρ_δ	0.9	AR(1) discount factor shock

3.10. Equilibrium, Solution and Parameters

The appendix provides full sets of equilibrium equations for the nonlinear and linear model. The appendix also provides the steady state as well as details about how we solve the model.

Table 1 contains the parameter values that we use in the analysis. Most of the values are taken from HLT (2022, 2023) and considered standard in the literature. But parameters related to the learning of the cost-push shock and dynamic indexation features of the model necessitate further discussion. Beginning with the parameters related to the unobserved components representation (ρ_P , ρ_T, σ_P and σ_T) for the cost-push shock, we calibrate them such that the observed cost-push shock process $a_t = a_t^T + a_t^P$ follows closely the estimated ARMA(1,1) process in Smets and Wouters (2007).[10] This implies setting $\rho_P = 0.9$, $\rho_T = 0$, and $\sigma_P/\sigma_T = 1/10$. This parameterization also enables the model to account well for the projections and forecast errors made by the survey of professional forecasters (see Figure 4 that we discuss in the next section). The indexation parameters ω and ϱ are chosen such that the nonlinear model captures well the hump-shaped dynamics of inflation in the data.

Notice that dynamic price and wage indexation is absent (i.e. $\varkappa = 0$ in eq. 6) in steady state in both the nonlinear and linear models and also absent in the linearized model away from the steady state. This is consistent with many studies showing that there is not little (if at all) evidence in favor dynamic indexation in micro and macro data when inflation is close to target. However, once inflation surges endogenous indexation kicks in our nonlinear model, whereas this model feature is absent in the linearized model.

4. Results

In this section, we report our results. We start out in the next subsection by showing that our model can capture the key features of realized and forecasted inflation, output growth and federal funds rate paths during the post-COVID inflation surge. Section 4.2 discusses the key elements in the model that enables it to explain the key contours of the data. Next, Section 4.3 provides additional details on the state-dependent amplification nature of cost-push shocks in our model, while the final Section 4.4 discusses the state-dependent costs of disinflating the economy by comparing the paths for different variables to a monetary policy tightening that reduces inflation with 1 percent in both the nonlinear and linearized formulations of the model.

4.1. U.S. Data-Model Comparison

Figure 4 provides a U.S. data-model comparison for the nonlinear model. The data is actual realizations (red solid with dots) and projections (green dashed with dots) made by survey of professional forecasters (SPF). In the model simulation underlying Figure 4, we use both an adverse cost-push shock and an expansionary demand (discount factor) shock. Both shocks are phased-in over two to three quarters amid a baseline in which the central bank is assumed to be at the ELB for an extended period absent any shocks. The data are expressed in changes relative to 2020Q4.[11] Model variables are expressed in changes relative to steady state. The nonlinear model does a good

[10] Smets and Wouters (2007) estimate the following cost-push shock process: $a_t = .9a_{t-1} + \varepsilon_{a,t} - 0.75\varepsilon_{a,t-1}$.

[11] We choose 2020Q4 as a reference date since inflation – the key object of interest in our paper – was close to its target of two percent.

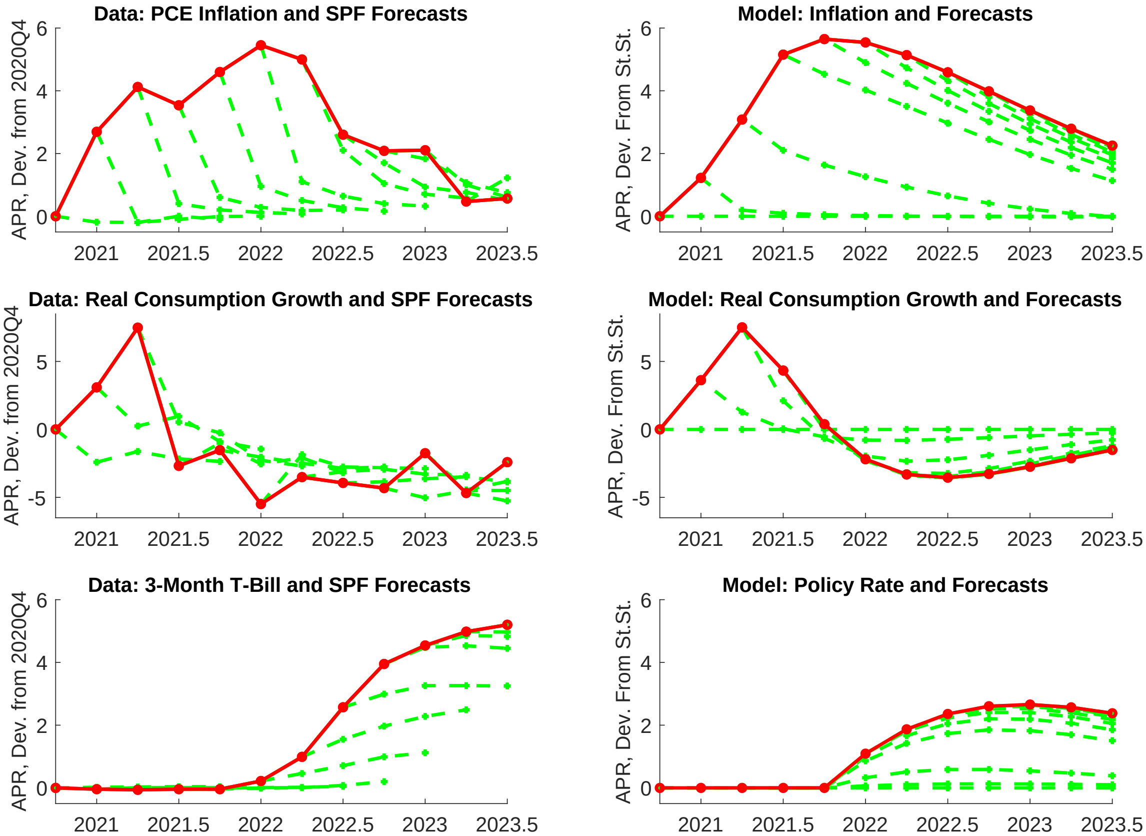

Figure 4: Comparison of U.S. data vs. nonlinear model.

job of accounting for the overall contours of U.S. data. In addition, the model also accounts well for the forecast revisions of the survey of professional forecasters (SPF). The nonlinearities in the model are key behind the good fit. In Appendix B show that the corresponding figure (Figure B.1) for the linearized formulation of the model provides a significantly worse adaptation to the data: the linearized model cannot replicate the inflation boom for the same set of shocks. The following subsection unpack the key drivers of the simulation results depicted in Figure 4 into its components, focusing on how adverse cost-push shocks can have expansionary short-run output effects while triggering an inflation cycle.

4.2. How Adverse Cost-push Shocks can be Expansionary

Figure 5 provides the impulse responses in the nonlinear model to an adverse cost-push shock. Specifically, we assume $\varepsilon_0^P = 0.0025$ and $\varepsilon_0^T = 0$ so that the realized cost-push shock is driven by

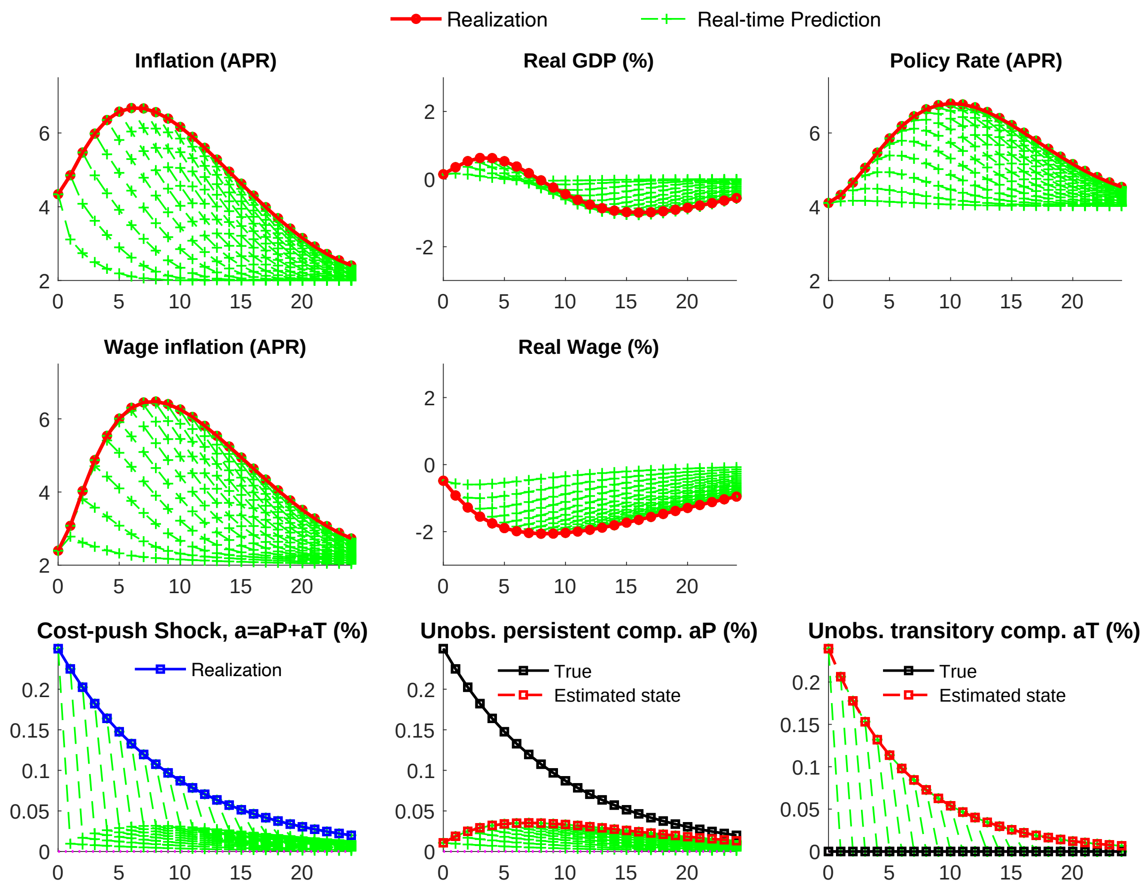

Figure 5: Impulse responses to an adverse cost-push shock in the baseline nonlinear model.

the persistent component of the unobserved components representation. The central bank follows the inflation-forecast based Taylor rule in equation (10) and can only observe the sum of the persistent and transitory component but has to filter in periods $t = 0, 1, 2, ...$ which of the two components that drives the cost-push shock. Figure 5 shows the realized impulse responses as well as real-time forecasts of households, firms and the central bank at each point in time. In line with the SPF forecast patterns following the COVID pandemic discussed in Figure 4, the figure shows that it takes considerable time for households, firms and the central bank before they recognize out that the persistent component of the cost-push shock process is the underlying driver of the realized cost-push shock path and economic dynamics.

Interestingly, output increases in the short run in response to the adverse cost-push shock, which causes a hump-shaped inflation boom. Two key forces in the model are responsible for this result.

First, agents are not sure which component of the cost-push shock process is realized. Given that the transitory component is calibrated to be the most important driver of cost-push shocks, agents forecast it to recede soon and hence that inflation and output will return quickly to the steady state. Second, the central bank responds to 4-quarter ahead inflation in the rule (10) and thus reacts very little to the rise in the cost-push shock as is expects inflationary pressure to dissipate quickly. And by keeping the policy rate nearly unchanged in the short run triggers a fall in the real interest rate when expected inflation rises somewhat. So, while the cost-push shock itself puts downward pressure on GDP, the fall in the real interest rate induces by the slow policy response provides a countervailing upward force for GDP. On net, GDP expands in the wake of the cost-push shock since the real rate effect is stronger than the cost-push effect. In short, the adverse cost-push shock is expansionary with respect to GDP – a novel result that we are not aware of has been shown previously in the literature.

Figure 5 shows that our quantitative model captures many of the salient features of the post-Covid episode depicted in Figure 4 with just cost-push shocks: inflation surges, output remains strong, policy rates rise, wage inflation jumps but less so than price inflation and thus the real wage falls. Hence, in principle, we do not need demand-type shocks to capture salient features of the data, at least not initially.[12] This has important consequences for assessing how much of the inflation scare was driven by cost-push vs. demand shocks. According to our model, there is in principle no need to rely on demand shocks to obtain co-movement between inflation and output when inflation took off in early 2021 – a result that is worthwhile to revisit in empirical analysis.

Figure 6 provides the impulse responses of the nonlinear model to a cost-push shock when the central bank reacts to the current inflation gap ($\Pi_t/\bar{\Pi}$) rather than to 4-quarter-ahead inflation ($\mathrm{E}_t\Pi_{t+4}/\bar{\Pi}$) in the Taylor rule (10). When the central bank reacts to actual inflation, it hikes the policy rate significantly faster and thus the real rate effect is less pronounced. Consequently, the cost-push shock is contractionary – a well-known finding in the literature. So, the conduct of monetary policy is crucial for the transmission of large adverse persistent cost-push development that households, firms and the central bank for some time incorrectly perceives to be transient.

Figure 7 provides a comparison of the impulse responses of the nonlinear model to a cost-push shock when the central bank reacts to current inflation gap ($\Pi_t/\bar{\Pi}$), one-year ahead expected inflation (baseline rule) and two-years ahead expected inflation ($\mathrm{E}_t\Pi_{t+8}/\bar{\Pi}$) in the Taylor rule (10). When the central bank reacts to one-year ahead and even more starkly with two-year ahead

[12] We include demand shocks in the analysis to allow for an expansionary fiscal policy stance operating in the economy during our sample.

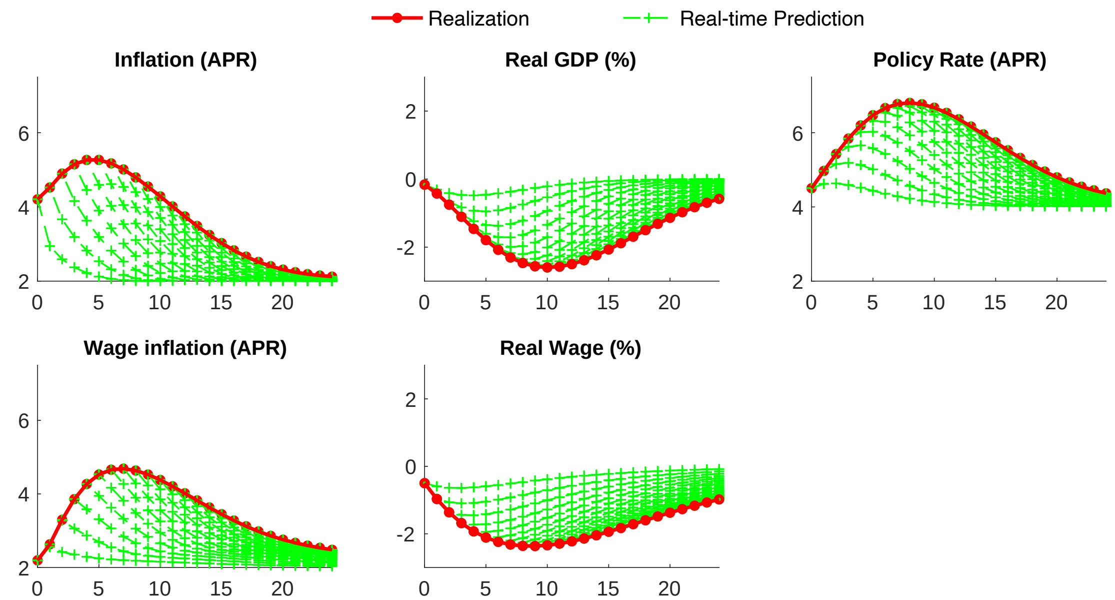

Figure 6: Impulse responses to a cost-push shock in the nonlinear model when the central bank reacts to current inflation (rather than 4-quarter ahead inflation) in the Taylor rule.

expected inflation, the markup shock generates a boom in output at the same time as inflation is booming. That is, output and inflation move in the same direction, i.e. feature a demand-like pattern although the driving force in this simulation is an adverse cost-push shock. Now, if the central bank reacts to contemporaneous inflation, output tanks while inflation surges as reported in e.g. Clarida, Gali and Gertler (1999) and many others. Our key result has several implications. First, it questions a routine empirical identification assumption that cost-push shocks drive output and inflation in opposite directions. But it also calls into question the standard empirical identification assumption that demand shocks is the only driver of positive co-movement of output and inflation in the neat term. Our key result casts doubt on both these identification assumptions. Second, it has implications for monetary policy. Specifically, our key result provides caution against on the common practice on relying on inflation projections at long forecast horizons (say 2-3 years) ahead in the formulation of monetary policy when actual near-term inflation is well above the central banks target. Many central banks seemed to have relied on a forecast-based approach to rationalize a muted policy rate response to intensifying inflation pressures in 2021 into the beginning of 2022. They argued in essence that policy rates should remain low in 2021 because they forecasted that inflation would return run to target 2-3 years ahead. These forecasts eventually became untenable and many central banks around the globe had to reset their policy stance quickly. Third, co-movement of inflation and output in the data across many countries

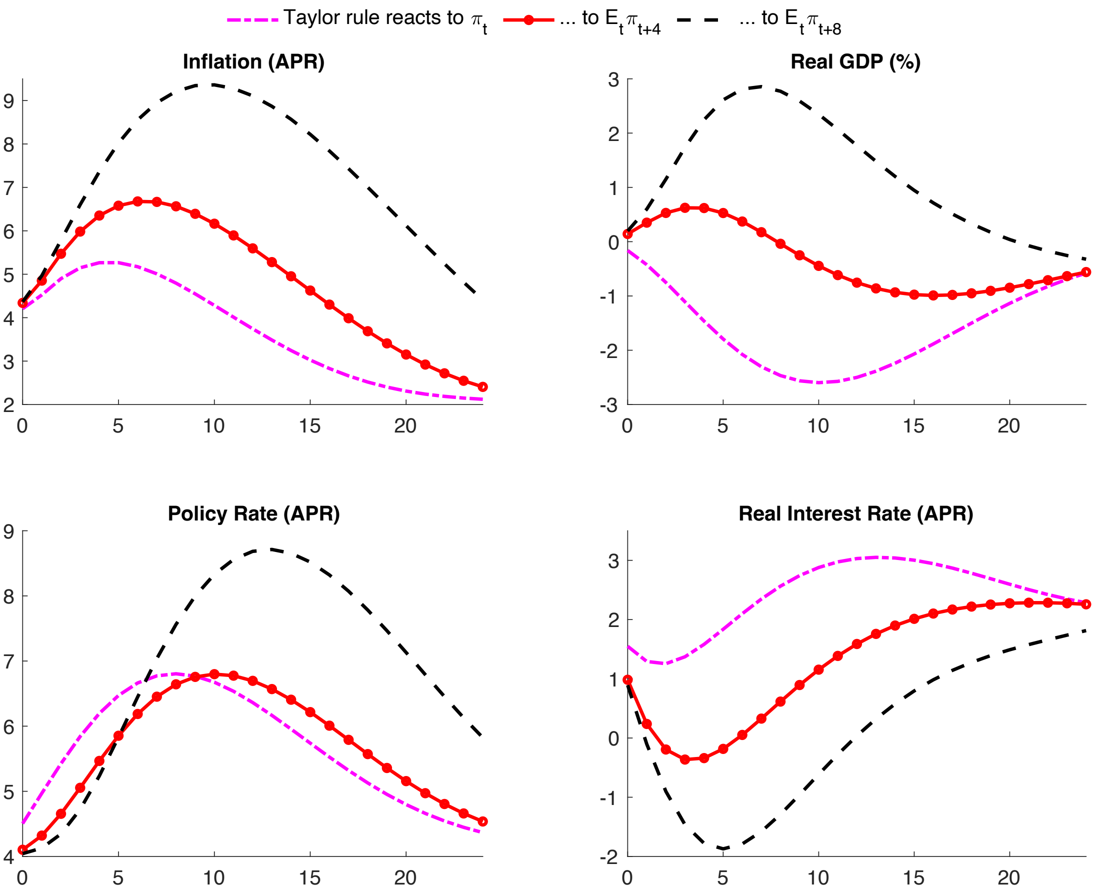

Figure 7: Impulse responses to a cost-push shock in the nonlinear model when the central bank reacts to current inflation, one-year ahead expected inflation, or two-year ahead expected inflation in the Taylor rule.

– especially the softening of inflation figures coupled with relatively strong economic activity – came as a surprise to many economists. Our model offers an explanation to these facts. From a more normative perspective, since the underlying cost-push shocks are inefficient and do not affect potential output, real GDP and core inflation should move in opposite directions. Hence, although the tighter monetary policy stance to curb inflationary pressures implies that real GDP falls more, the expected loss – measured as the sum of the squared deviations of annualized inflation from target plus the sum of squared output gaps for the first 5 years – falls by about 30 percent from a loss value of about 255 under the forecast-based rule to a loss of about 174 under a rule which responds to actual inflation.[13]

[13] The improvement in the loss function in the rule with actual inflation is not contingent on the weight of the output gap in the loss function; the loss improves for any weight on the output gap between 0 and 2 in the loss

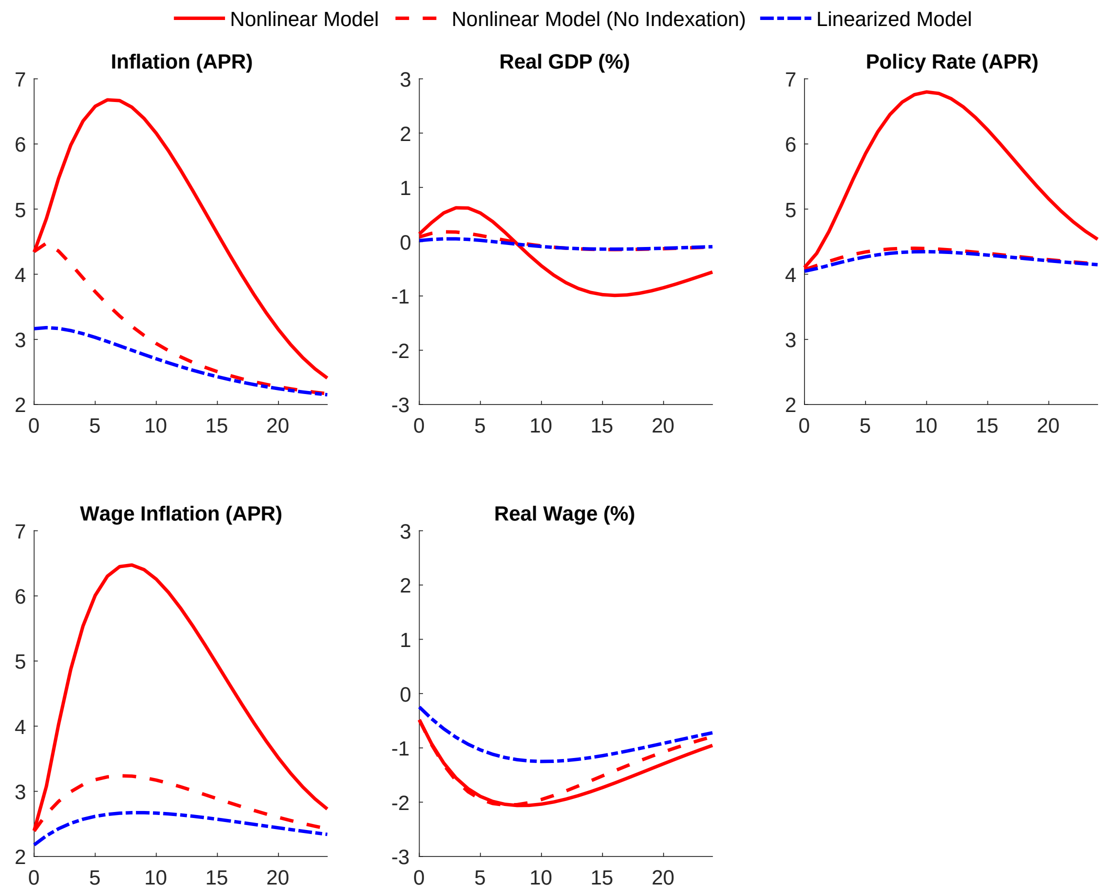

Figure 8: Impulse responses to a cost-push shock in the nonlinear model with and without indexation, and in the linearized formulation of the model.

Figure 8 provides a comparison of the impulse responses of the nonlinear and linearized models to a cost-push shock. Clearly, nonlinearities also plays a key role for our model to provide explain this episode: the nonlinear model generates a much stronger and much more persistent jump in inflation than the linearized model. There are two reasons for this result. First, in line with HLT (2023), the Kimball aggregator is one important driving force that triggers a stronger inflation response to cost-push shocks (compare blue dash-dotted with red dashed line). Second, the endogenous indexation feature of our model (allowing $\varkappa_t > 0$ in eq. 6) is another important driving force that accounts for an important part of the differences between the nonlinear and linearized model as

function. Debortoli, Kim, Linde, and Nunes (2019) shows that an inflation-output gap loss function approximates household welfare policy closely in the standard Erceg, Henderson and Levin (2000) and Smets-Wouters (2007) sticky price-wage models.

can be teased by difference between the red dashed and solid lines.[14]

To further illuminate the role of endogenous indexation in Figure 8, Figure 9 provides the impulse responses of the endogenous indexation variables $\tilde{\Pi}_t$ and Π_t^* to a cost-push shock in the nonlinear and linearized model. These results show that when inflation is low, there is very little (if any) indexation. But once inflation rises well above the central bank's inflation target, indexation starts to kick in. The differences between the nonlinear and linearized model responses for $\tilde{\Pi}_t$ and Π_t^* have important quantitative implications for assessing post-Covid inflation dynamics.

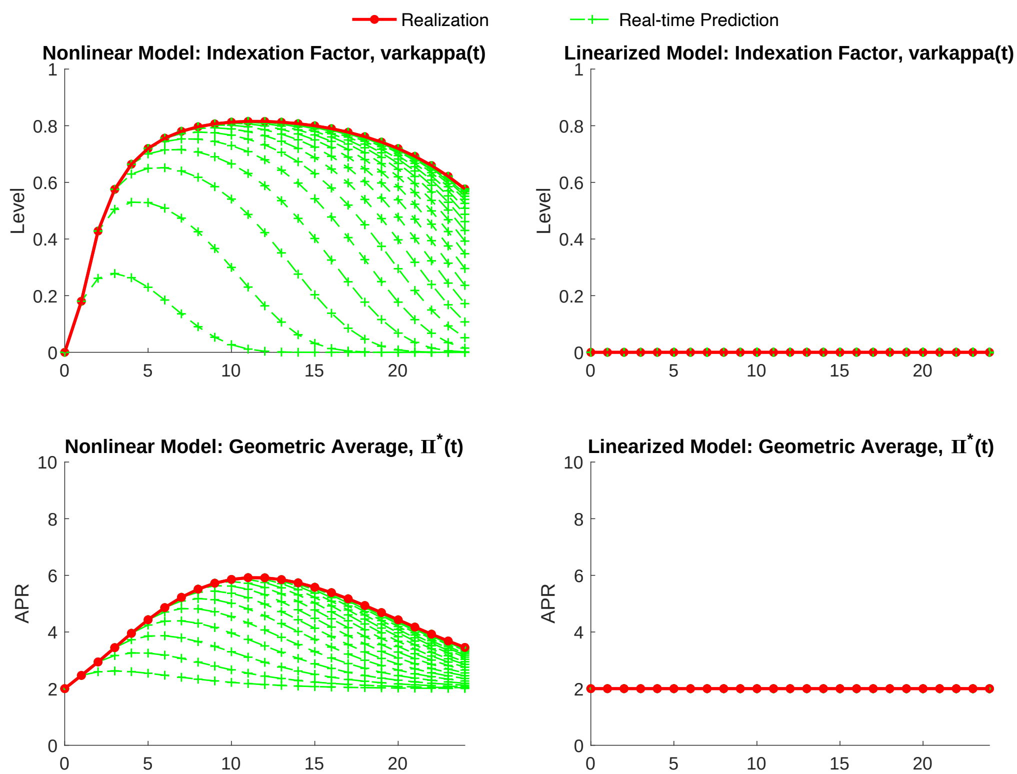

Figure 9: Impulse responses of indexation variables to the adverse cost-push shock in the baseline nonlinear and linearized models.

The results in Figure 9 implies that endogenous indexation plays a key role for our findings in Figure 8. Without this feature, inflation still jumps by more than in the linearized model, but less so and less persistently as in the baseline nonlinear model which allows for $\varkappa_t > 0$. The remaining differences between the nonlinear model without indexation and the linearized model are due to the

[14] Note that due to endogenous indexation, medium-term inflation expectations become more elevated in the nonlinear than the linearized model. Our nonlinear framework implies that inflation expectations become more sensitive to realized inflation rates above target due to endogenous indexation ($\varkappa_t > 0$). Thus, our nonlinear model implies more 'de-anchoring' of medium-term inflation expectations than the linearized model.

nonlinearities embedded in the Kimball aggregator as discussed in further detail in HLT (2023).

We also want to underscore the importance the size of the cost-push shocks for these dynamics to materialize. Figure 10 illustrates this by reporting impulse responses of inflation in the nonlinear and linearized model for different sizes of the cost-push shock. For the baseline cost-push shock, the differences between the nonlinear and linearized model are very large. Figure 10 also shows that the differences disappear for sufficiently small shocks. Thus, if cost-push shocks are small, a

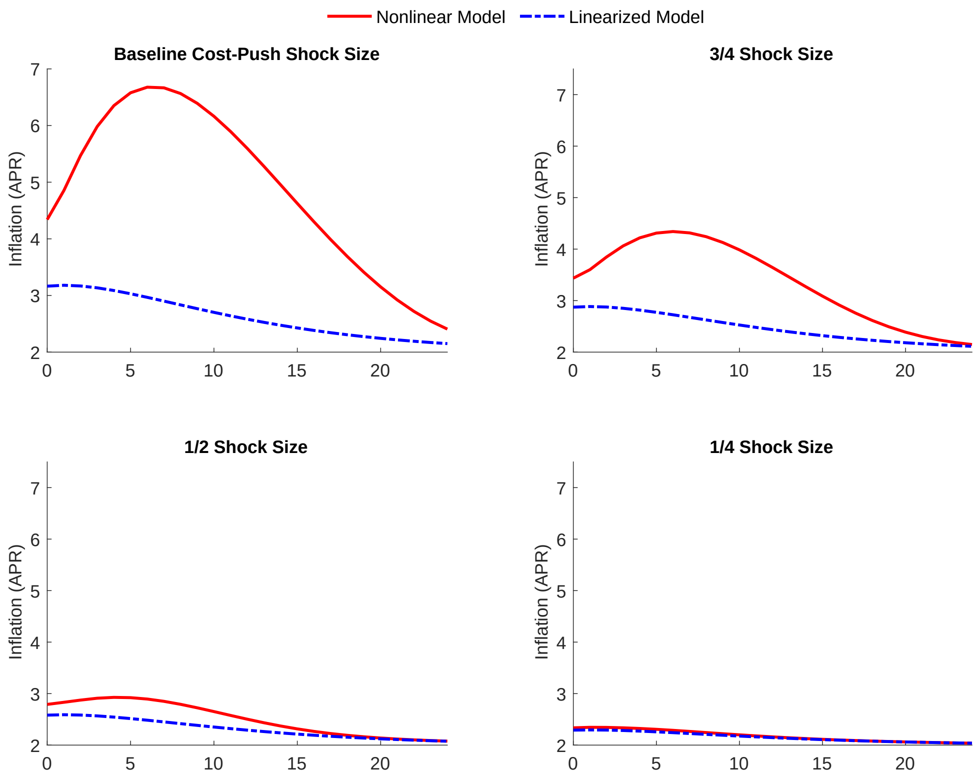

Figure 10: Impulse responses of inflation in the nonlinear and linearized model for different sizes of the cost-push shock.

linearized variant captures the transmission of the shock well and there is no need to work with the nonlinear model. But, if the cost-push shock is large, it is crucial to work with the nonlinear model to capture the dynamic effects of cost-push shocks quantitatively.

In Appendices D and E we study the ratio of standard deviations (σ_P/σ_T) of the persistent and transitory components of the unobserved components representation of the cost push shock in the nonlinear model. Figure D.3 in Appendix D provides the impulse responses of the nonlinear model to a cost-push shock when the latter is driven by a pure transitory (iid) shock. This generates a large transient spike in inflation but output and policy rates are unaffected. Next, Figure E.4 in Appendix E shows the effects of assuming alternative values for the ratio of standard deviations (σ_P/σ_T) of the persistent and transitory components of the unobserved components representation of the cost push shock in the nonlinear model. If agents are more rapidly able to filter out that the cost-push shocks is driven by the persistent component, firms would change prices more, the central bank would recognize more persistent upward price pressures and tighten their policy stance more, and as a result output would decline notably more. All told, both figures provide further support and intuition for the mechanisms in our model and underscore the importance of the unobserved components representation (and more generally the importance of learning by households, firms and the central bank) for our results.

We close this section by noting that a forecast-based targeting rule is a good policy for normal recurring transient cost-push shocks that do not drive inflation persistently away from the central banks' inflation target. Figure 11 shows that a transient cost-push shock a_t^T does not move real GDP much under a forecast-based policy rule, whereas the same shock lowers output notably under a rule which responds to actual inflation. Since the inflation paths are largely unchanged, the monetary policy rule that looks-through the transient cost-push shock yields more favorable macroeconomic outcomes.[15]

4.3. State-dependent Amplification of Cost-push Shocks

In this subsection, we illustrate that cost-push shocks affect inflation in a state-dependent manner in the nonlinear model. Put differently, we show that the same cost-push shock results in a stronger surge in inflation in an environment with strong demand and inflation is (somewhat) elevated to begin with.

To do this exercise we first generate a baseline scenario for differently-sized positive demand shocks with inflation peaks above the 2 percent steady state rate and output above its potential value. The first column in Table 2 shows the output (gap) peak, whereas the impact on peak inflation in each of the baselines are shown in the second column. Following Christiano, Eichenbaum and Rebelo (2011), we capture variations in expansionary demand conditions with discount factor shocks as these shocks affect the potential real rate but leaves potential output unaffected.[16] The first row show the smallest (zero) positive demand shock for which inflation simply remains at its steady state value and output remains at potential, whereas the last row shows the biggest demand

[15] Without the sizeable degree of interest rate smoothing that we assume in the interest rate rule ($\rho_R = 0.85$ in eq. 10), the difference between the rules would be further amplified.
[16] Appendix C illustrates the impulse responses to a discount factor shock in our model. We assume the path of this shock is known and does not have to filtered by agents in the model.

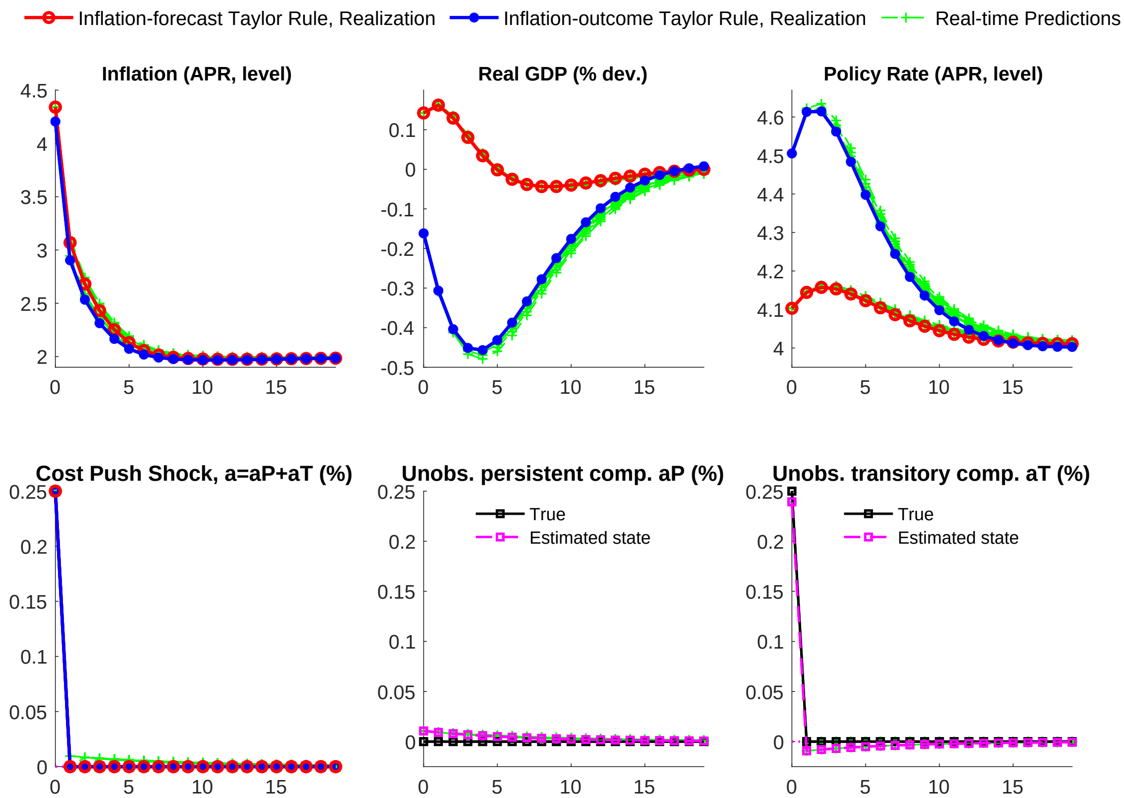

Figure 11: Impulse responses to a transient cost-push shock under alternative monetary policy rules in the nonlinear model.

shock that drives up peak inflation to 3.2 percent and the output (gap) to 5.4 percent.[17] Although the increment of the underlying demand shock is the same in the second to last row in the table, the peak impact on demand and inflation in the first two columns in Table 2 increases with the size of the shocks in our nonlinear model.

Amid the baseline simulations, we add a same-sized (i.e. identical) adverse cost-push shock to each of the baseline simulation. As a result, inflation surges even more in the scenario with the cost-push shock – reported in the third column – compared to the baseline simulation. More importantly, the last column of Table 2 calculates the differences in terms of the inflation peaks between the scenario with the added cost-push shock and the baseline simulation with the stronger demand shock only. This column shows the key result that the amplification of cost-push shocks is

[17] In the table, the underlying discount factor shock varies from 0 (no shock), -0.5, -1, -1.5 and -2.0; i.e. the increment of the underlying demand shock is constant. A fall in the discount factor implies a rise in demand.

state dependent in the following sense. When inflation is close to target, the same sized cost-push shock pushes inflation up by less compared to the case when demand is strong and inflation is elevated to begin with. With such initial conditions, the same-sized cost-push shock triggers a much more vigorous jump in inflation. In other words, if inflation is high to begin with, a cost-push shock triggers a larger jump in inflation compared to the case when inflation is – say – at target. This feature of our model has important implications for the conduct of monetary policy, which we will discuss in the next section.

Table 2: Amplification of Cost-Push Shocks in Nonlinear Model

Baseline: Discount Factor Shock		**Scenario:** Baseline+Same-Sized Cost-Push Shock	State-dependent Effects of Cost-Push Shock
Output Peak	Inflation Peak	Inflation Peak	Δ Inflation Peak **(Scenario-Baseline)**
0.0%	2% (Steady State)	6.7%	4.7%
1.4%	2.2%	8.0%	5.8%
2.7%	2.4%	9.4%	7.0%
4.1%	2.7%	10.8%	8.1%
5.4%	3.2%	12.2%	9.0%

4.4. Effects of Monetary Tightening

When inflation exceeds the central banks' inflation target as in Figure 5, it is natural that the central bank reconsiders the pros and cons of bringing inflation faster back towards target. In this subsection, we there study the effects of more monetary tightening than embedded in the systematic response of the central bank to the cost-push shock. Specifically, we let the monetary policy shock $\varepsilon_{R,t}$ in the Taylor rule (10) follow an AR(1) process with a persistence coefficient of 0.75. We size the monetary policy shock in both the nonlinear and linearized models such that inflation is reduced with one percentage point (APR) below its baseline path, which is constructed using the cost-push shock in Figure 5 plus a one percent discount factor shock. The monetary policy intervention is assumed to start when inflation attains its peak in the nonlinear model.[18] In addition, we consider

[18] In Appendix F, we consider the implications when the central bank becomes more aggressive at different points in time. Specifically, Figure F.5 shows the simulation results for more aggressive monetary policy in the nonlinear model for different start dates of the monetary intervention. The key takeaway is that the earlier the central bank intervenes, the larger the reduction in inflation for a given hike in the policy rate. Put differently, monetary policy becomes more effective the higher inflation is to begin with. In this sense, the efficacy of monetary tightening and the associated sacrifice ratio are state-dependent in our model.

the case that in the linear model, the slope of the price and wage Phillips curves is twice as large as in steady state (reflecting empirical evidence as well as the fact that in the nonlinear model, the slopes of price and wage Phillips curves steepen endogenously by about factor two).[19]

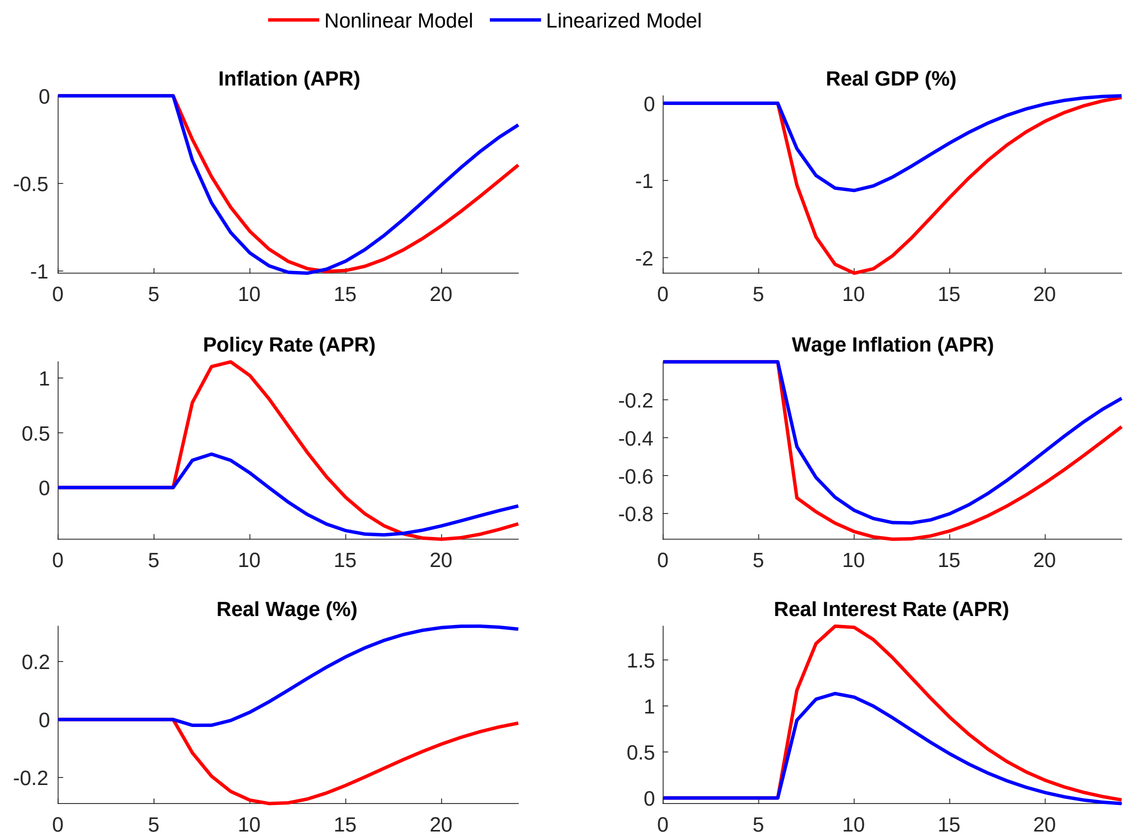

Figure 12: Effects of more aggressive monetary policy (deviation from baseline) when inflation peaks in nonlinear and linearized models.

Figure 12 shows the deviations of model variables from the baseline due to the additional dose of monetary tightening. The figure shows that to attain a one percentage point (APR) lower trajectory for inflation, the nonlinear model implies that the nominal policy rate has to be tightened notably more, and that this will result in notably lower output relative to the output path in the linearized model. Put differently, if the central bank needs to disinflate faster than implied by the baseline in

[19] Finally, to put the nonlinear and linearized model on a more equal footing, we make the following additional assumption. We allow for endogenous indexation in the linear model too, i.e. we work with what we call a pseudo-linearized model in which indexation is nonlinear (and endogenous) but all other model equations are linearized in the linear model.

order to preserve credibility for the inflation target, our nonlinear model implies that the resulting output costs are notably larger than implied by a standard linearized model with Phillips curve intended to match the dynamics of inflation when it surged above the inflation target.

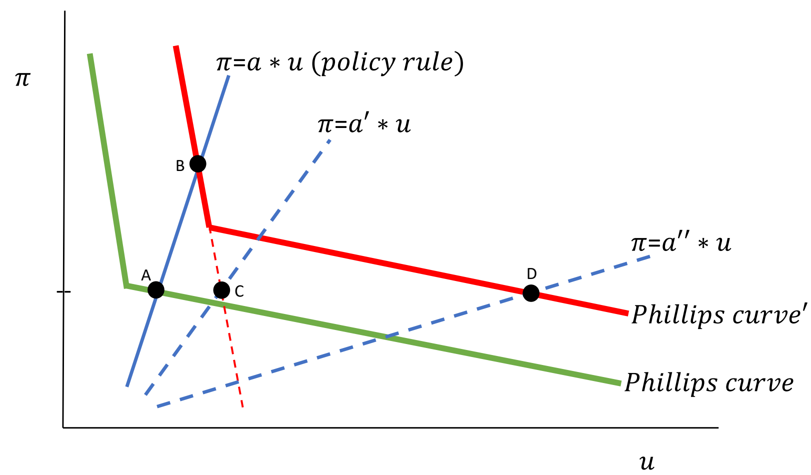

Figure 13: Intuition for transition dynamics and economic costs of more monetary tightening than embedded in the baseline.

Figure 13 provides a graphical illustration of the intuition underlying our quantitative result that additional monetary tightening goes hand in hand with larger economic costs in an environment with nonlinear (kinked) Phillips curves. In Figure 13, the economy is initially in point A, i.e. at the intersection of a Phillips curve and a monetary policy rule. In the figure π stands for inflation and u stands for the unemployment rate which is assumed to be proportional to the negative output gap, say $-x$. For simplicity, we assume a monetary policy rule as in Clarida, Gali, and Gertler (1999) in which the central bank conducts optimal monetary policy under discretion in response to a cost-push shock, i.e. $\pi = a * u$ or, after substituting out for the unemployment rate, $\pi = -a * x$. Now, an adverse cost-push shock shifts the Phillips curve up persistently to point B.[20] If the central bank pursues its historical policy rule then the economy travels slowly back from point B to point A, i.e. the baseline dynamics. However, if the central bank seeks to bring the economy back to target inflation faster than implied by the baseline, the central bank can adapt a more aggressive policy stance. Point C illustrates the outcome if the central bank (erroneously)

[20] Note that in the graphical illustration in Figure 13, the cost-push shock affects the levels of unemployment and inflation at which the Phillips curve kinks. We have chosen to do so for expositional purposes and with an eye toward the quantitative model. There, the slope of the Phillips curve steepens with the level of inflation which at some point results in a shift of the kink. An alternative way to draw the Phillips curve after the cost-push shock would be to maintan the location of the kink unchanged in term of inflation. However, the same qualitative conclusions would arise as in our baseline illustration. Finally, there are surely alternative ways to draw the graphical illustration such that they would contradict the key result in our quantitative model. Given that we want to convey intution about our quantitative model, we have restricted our illustration to a case that allows to shed light on the inner-workings of our quantitative model.

extrapolates the steep slope of the Phillips curve implying only relatively small economic costs, i.e. the conclusions drawn with applying the insights from the linearized model analysis in Figure 12. By contrast, point D illustrates the implications in the true underlying nonlinear model in which a much stronger monetary tightening is required to bring inflation back to target faster. The resulting economic costs can – depending on the slope of the nonlinear Phillips curve – become very substantial as shown in Figure 12. All told, Figure 13 illustrates that the costs of additional monetary tightening than embedded in the baseline can be substantial in a nonlinear framework while a linearized framework may suggest notably lower costs.

The left column of Figure 14 shows the implications for output and inflation when allowing for stochastic cost-push shocks and discount factor shocks around the baseline. The resulting densities shown in the figure are constructed as follows. Starting at the baseline path at $t = 8$, the economy is hit by random unexpected cost-push and discount factor shocks in each period $t >= 8$.[21] The cost-push shocks follow the unobserved components specification embedded in the model, i.e. have realizations of transitory and persistent shocks. The variances of cost-push and discount factor shocks are chosen such that the model generates roughly the unconditional standard deviation of core PCE inflation, the unconditional standard deviation of real consumption per capita growth, and the correlation between consumption growth and inflation in post-war/pre-Covid U.S. data.

The density plots are then constructed by using 500 random sequences of these shocks, simulating the model with each sequence separately and such that each period agents are surprised by new realizations of cost-push and discount factor shocks. The density plots in Figure 14 show the $\{2.5, 10, 20, \ldots, 90, 97.5\}$ percentiles and the median.

Strikingly, according to the left column of Figure 14, the density plots are asymmetric for inflation. There are more realizations of high inflation than low inflation in the stochastic simulations. The reason for this result is due to the amplification effects of Kimball aggregation and endogenous indexation. That is, to the extend that the economy is hit by cost-push and discount factor shocks and inflation already runs above the central bank's inflation target, our results indicate that the economy will see bursts of inflation more often than inflation declines.

The right column of Figure 14 illustrates the effects when a central bank adopts a more aggressive stance toward inflation surges. Specifically, starting in period $t = 8$, the central bank increases the weight of inflation in the Taylor rule by factor three and reduces the weight of the output gap by factor three.[22] With this change to the systematic parts of the central bank's interest rate feedback

[21] An alternative simulation setup would be to consider that the supply shock up to the time of the intervention is actually driven by observationally equivalent draws of the transitory component. This would allow for a more rapid post-intervention decay in inflation.

[22] In this experiment, we have chosen to model monetary tightening with a more aggressive response coefficient to

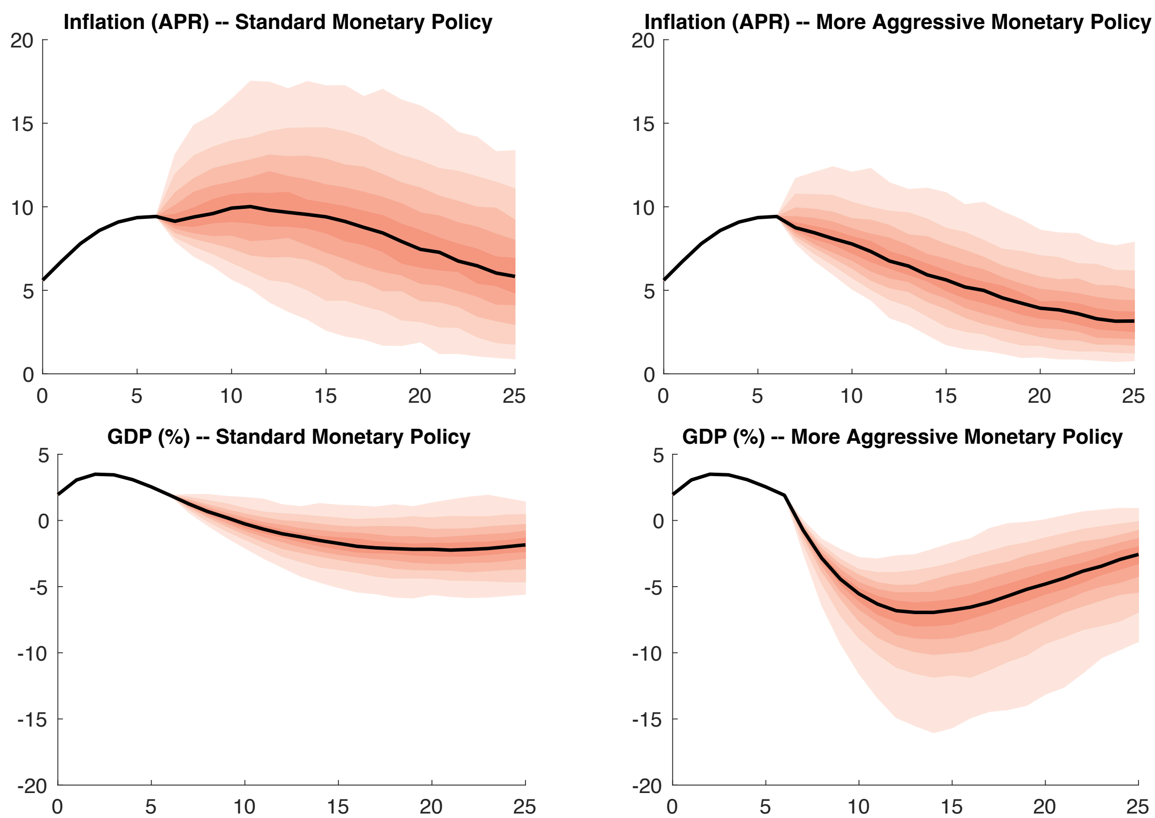

Figure 14: Distributions of inflation and output (dev from SS) with stochastic shocks for alternative monetary policy rules.

rule, the right column of Figure 14 suggests two important implications. First, the inflation is lower on average than with the standard Taylor rule specification. Second, these is almost no asymmetry in the distribution for inflation. Note, however, that the improved stabilization of inflation (i.e. lower mean and less upward asymmetric inflation risks) comes with substantial economic costs as illustrated by the bottom right subplot for output in Figure 14. The output panel displays a significant lower mean and downside risks relative to the left panel with the historical policy rule. This highlights the costs of disinflating late in an economic cycle once inflation and higher inflation expectations have become entrenched.

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expected inflation in the central bank's reaction function. An alternative approach would be to switch from forecasted inflation to current (observed) inflation in the policy rule whenever actual inflation exceeds a threshold inflation level. That is, whenever the observed rate of inflation becomes too high, the central bank abandons its inflation forecast-based policy rule in favor of a policy rule with actual inflaion. Given that actual inflation runs higher than the inflation forecast in Figure 14 the results of this alternative experiment should be similar to our approach to model more aggressive tightening.

5. Related Literature

In addition to the literature discussed in the previous sections, our paper is also related to the following body of work.

Many scholars are seeking to understand the causes of the recent inflation surge and the dynamics in the labor market. Bernanke and Blanchard (2023) develop a linear model of wage-price dynamics for understanding inflation dynamics to decompose the sources of the pandemic-era inflation. They show that supply and energy shocks were the main drivers behind the runup in inflation, whereas labor market conditions accounted only for a small share of the inflation spike at an early stage. However, according to their analysis, the influence of the product market shocks will fade, while the tight labor market and associated persistent nominal wage increases will become the main factors behind wage and price inflation going forward.

Gagliardone and Gertler (2023) develop a New Keynesian model that aims to account for the inflation surge with emphasis on the role of oil price shocks and accommodative monetary policy. An important feature of this model, which includes non-linearities, is that oil is treated as a complementary good for households and as a complementary input for firms. With these model features, a upward oil price shock which declines the oil intensity in production reduces the marginal product of labor (given the strong complementarity between oil and labor) and thereby increases marginal cost, which increases inflation . This, together with monetary policy accommodation, helps to explain the inflation surge, even after allowing for demand and labor market tightness shocks.

Lorenzoni and Werning (2023) discuss the concept of a wage-price spiral, highlighting the conflict between workers and firms on relative prices of labor and goods as a proximate cause of inflation. Notably, the model incorporates a scarce non-labor input with low substitutability in production and both nominal and price wage rigidities. The paper explores how this conflict unfolds within a New Keynesian framework, with a specific focus on the trajectory of real wages in response to demand and supply shocks. Their findings demonstrate that both demand and supply shocks can exhibit a similar three-phase pattern of adjustment in nominal prices, characterized by stronger price inflation early on, followed by wage inflation catching up later on.

Ball, Leigh, and Mishra (2022) analyze the recent surge in U.S. inflation, with a special focus on core and headline inflation. They argue that core inflation is influenced by a tighter labor market and past shocks from headline inflation, particularly due to higher energy prices and supply chain disruptions. The paper also explores future inflation scenarios, mainly focusing on one where unemployment rises modestly as projected by the Federal Reserve. Their analysis suggests that achieving the Fed's inflation target hinges on optimistic assumptions about inflation expectations and the relationship between unemployment and job vacancies. If these assumptions are not met, inflation may remain above the Fed's 2 percent target unless unemployment increases more than currently projected by the Federal Reserve.

Amiti et al. (2024) analyze how much the supply-side disruption and tight labor market contributed to the recent inflation surge. The authors develop a two-sector New Keynesian model with multiple input factors such as labor, domestic and foreign intermediate inputs, shocks to imported intermediate input prices, foreign competition in domestic markets, and workers' willingness to work. When all shocks hit at the same time, firms' ability to substitute between inputs is diminished, therefore the total effect on inflation is amplified and raises inflation by more compared to the case when the shocks hit the economy in isolation.

Benigno and Eggertsson (2023, 2024) develop a model that features a non-linear Phillips curve. The nonlinearity arises from asymmetries in wage setting. The authors provide evidence that the Phillips curve has a higher slope coefficient when market tightness is exceptionally high, which usually defines a labor shortage. They conclude that the key reason why policymakers failed to foresee the large persistent inflation surge was because the Phillips curve was assumed to be flat. An exceptionally tight labor market, which moved the economy on the steep segment of the Phillips curve, is, according to Benigno and Eggertsson (2023, 2024) responsible for the increase in inflation in the early 2020s.

Ferrante et al. (2023), Gudmundsson et al. (2024), Guerrieri et al. (2022), Guerrieri et al. (2024) among others study the implications for inflation of switching expenditures from services to goods during the pandemic followed by a switch back from goods to services in the aftermath of the pandemic. These papers typically find that disruptions in one sector can be helpful to understand inflation dynamics during and after the pandemic.

Finally, Hakamada and Walsh (2024) study the implications of a cost-push shock in a linear New Keynesian model when the central bank is assumed to keep the policy rate unchanged for some time. The authors report that the accommodative stance of the central bank renders the model capable of accounting for a surge in inflation and an expansion of economic activity.

Relative to the body of work cited above, our model combines a unique set of features whose interplay allows us to account for the joint dynamics of inflation, output, interest rates and the real wage in the post-Covid episode. The features highlighted in this paper are: i) nonlinear price and wage Phillips curves, ii) an unobserved components representation for cost-push shocks, iii) endogenous intrinsic price and wage indexation, and iv) an inflation forecast-based Taylor rule. With these elements, we have shown that a steep surge in inflation, resilient economic activity, a slow central bank response, as well as a fall of the real wage emerge endogenously in our model. These features also imply that our model has novel implications for the amplification of cost-push shocks and the conduct and effects of monetary policy. Moreover, our model not only applies to the recent post-Covid episode but is also useful to understand deep recessions such as the Great Recession as well as 'normal' business cycles. Specifically, given the boomerang-shaped nonlinear price and wage Phillips curves embedded in our model, our framework can be used to resolve the missing deflation puzzle, see HLT (2022).

6. Conclusion

We use a macroeconomic model with nonlinear Price and Wage Phillips curves, endogenous intrinsic indexation and an unobserved components representation of a cost-push shock to explain the post-COVID inflation surge. The cost push shock can be driven by a transitory or persistent component but households, firms and the central bank can only observe the sum of both components, and hence must solve a signal extraction problem to deduce which component drives the observed markup shock. We consider the case when agents and the central bank expect cost-push shocks to be transitory most of the time, but the realized cost-push shock in fact stems from the persistent component. In this environment, when assuming a central bank which follows an inflation forecast-based policy rule to see through transient inflation movements, we show that a nonlinear formulation of our model can explain the persistent inflation surge along with an initial expansion in economic activity in response to an adverse cost-push shock. Put differently, in our model, an adverse cost-push shock is expansionary in the short run. Our finding stems from the central bank misjudging the persistence of the underlying inflationary pressures and abstains from hiking policy rates as it (erroneously) expects inflationary pressures to dissipate quickly. Under identical assumptions, a standard linearized formulation of our model does not generate an inflation cycle and the output gap remains closed.

There are two important monetary policy implications of our nonlinear framework. First, while "looking through" supply shocks may be good policy for small shocks when inflation is near the central banks target, it may be quite risky when economic activity is strong and large adverse shocks drive inflation well above target. Second, our model implies that the economic costs of "going the last mile" – i.e. with a notably tighter stance than normal behavior would prescribe attempt to returning inflation quickly to target – can be considerable.

We leave several interesting issues for future research. First, the degree of indexation to past inflation in price- and wage-setting evolves as a function of the aggregate rate of inflation which is an endogenous variable in our nonlinear model. It would be very interesting to consider a version of our model in which firms are allowed to choose a desired rate of indexation and compare the implications of this in a nonlinear vs. linear model. Second, future research might consider allowing to switch from an "intensive margin" interpretation of indexation to an "extensive margin" interpretation – i.e., rather than allowing all non-re-optimizers to partially index to past inflation, one could allow a state-dependent fraction of non-re-optimizers to (fully) index to past inflation. In this case, the indexation rule could be calibrated to match differences in the observed frequency of price adjustment across high- and low-inflation episodes, or to match more refined estimates of the empirical relationship between the frequency of price adjustment and the prevailing inflation rate. It might also be worthwhile to consider using separate indexation rules for prices and wages, in which case the parameterization of the wage rule could be disciplined to match the prevalence of cost-of-living-adjustment clauses observed during high-inflation episodes.

All told, our analysis suggests that the interaction of nonlinearities and unexpectedly persistent shocks are crucial to understand the 2021-23 post-COVID episode and are critical to formulate good policy.

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Appendix A. Derivations, Equilibrium Equations, and Additional Results

A.1. Households

Let Λ_t denote the Lagrange multiplier on the household budget constraint. The first order conditions for consumption and bonds can be written as (in scaled form):

$$\begin{aligned} c_t &: \quad \frac{1}{c_t - hC_{t-1}} = \lambda_t \\ B_t &: \quad \lambda_t = \beta\delta_t E_t \frac{R_t}{\Pi_{t+1}} \lambda_{t+1} \end{aligned}$$

where $\Pi_t = P_t/P_{t-1}$ and $\lambda_t = \Lambda_t P_t$. Note that in steady state $R = \Pi/\beta$ and that in equilibrium, $c_t = C_t$.

A.2. Labor Contractors

Optimization:

$$\frac{W_{t,j}}{W_t} = \vartheta_t^w \frac{dG_w\left(\frac{n_{t,j}}{n_t}\right)}{d\frac{n_{t,j}}{n_t}}$$

Calculate derivative and rearrange:

$$\begin{aligned}
\frac{W_{t,j}}{W_t} &= \vartheta_t^w \left[(1+\psi_w)\frac{n_{t,j}}{n_t} - \psi_w\right]^{\frac{1-\omega_w}{\omega_w}} \\
\frac{n_{t,j}}{n_t} &= \frac{1}{1+\psi_w}\left(\left[\frac{1}{\vartheta_t^w}\frac{W_{t,j}}{W_t}\right]^{\frac{\omega_w}{1-\omega_w}} + \psi_w\right) \\
\frac{n_{t,j}}{n_t} &= \frac{1}{1+\psi_w}\left(\left[\frac{W_{t,j}}{W_t}\right]^{-\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} \left[\vartheta_t^w\right]^{\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} + \psi_w\right)
\end{aligned}$$

Substitute into aggregator which gives the aggregate price index resp. definition of lagrange multiplier:

$$\begin{aligned}
1 &= \int G_w\left(\frac{n_{t,j}}{n_t}\right) dj \\
1 &= \int \left(\frac{\omega_w}{1+\psi_w}\left[(1+\psi_w)\frac{n_{t,j}}{n_t} - \psi_w\right]^{\frac{1}{\omega_w}} - \frac{\omega_w}{1+\psi_w} + 1\right) dj \\
1 &= \int \frac{\omega_w}{1+\psi_w}\left[(1+\psi_w)\frac{n_{t,j}}{n_t} - \psi_w\right]^{\frac{1}{\omega_w}} dj - \int \frac{\omega_w}{1+\psi_w} dj + \int 1 dj \\
1 &= \int \left[(1+\psi_w)\frac{n_{t,j}}{n_t} - \psi_w\right]^{\frac{1}{\omega_w}} dj \\
\vartheta_t^w &= \left[\int \left[\frac{W_{t,j}}{W_t}\right]^{-\frac{1+\psi_w+\theta_w\psi_w}{\theta_w}} dj\right]^{-\frac{\theta_w}{1+\psi_w+\theta_w\psi_w}}
\end{aligned}$$

Note that after imposing zero profits for labor contractors (free entry), we can write

$$\begin{aligned}
1 &= \frac{1}{1+\psi_w}\vartheta_t^w + \frac{\psi_w}{1+\psi_w}\int \frac{W_{t,j}}{W_t} dj \\
\vartheta_t^w &= 1 + \psi_w - \psi_w \int \frac{W_{t,j}}{W_t} dj
\end{aligned}$$

A.3. Wage Setting

Substituting labor demand into the objective and re-arranging gives:

$$\max_{\tilde{W}_{j,t}} E_t \sum_{i=0}^{\infty} (\beta\xi_w)^i \frac{\varsigma_{t+i} n_{t+i} \lambda_{t+i}}{1+\psi_w} \left\{ \begin{array}{c} \frac{W_{t+i}}{P_{t+i}} \vartheta_{w,t+i}^{\varepsilon} \left[\frac{\left(\tilde{\Pi}_{t+i}^w \times \ldots \times \tilde{\Pi}_{t+1}^w\right)}{W_{t+i}}\right]^{1-\varepsilon} \tilde{W}_{j,t}^{1-\varepsilon} + \psi_w \frac{\left(\tilde{\Pi}_{t+i}^w \times \ldots \times \tilde{\Pi}_{t+1}^w\right)}{P_{t+i}} \tilde{W}_{j,t} \\ -mrs_{t+i} \vartheta_{w,t+i}^{\varepsilon} \left[\frac{\left(\tilde{\Pi}_{t+i}^w \times \ldots \times \tilde{\Pi}_{t+1}^w\right)}{W_{t+i}}\right]^{-\varepsilon} \tilde{W}_{j,t}^{-\varepsilon} - \psi_w mrs_{t+i} \end{array} \right\}$$

where

$$\begin{aligned}
mrs_{t+i} &= \frac{1}{\Lambda_{t+i} P_{t+i}} = \frac{1}{\lambda_{t+i}} \\
\varepsilon &= \frac{(1+\theta_w)(1+\psi_w)}{\theta_w}
\end{aligned}$$

Differentiating:

$$E_t \sum_{i=0}^{\infty} (\beta\xi_w)^i \frac{\varsigma_{t+i} n_{t+i} \lambda_{t+i}}{1+\psi_w} \left\{ \begin{array}{c} (1-\varepsilon) \frac{W_{t+i}}{P_{t+i}} \vartheta_{w,t+i}^{\varepsilon} \left[\frac{\left(\tilde{\Pi}_{t+i}^w \times \ldots \times \tilde{\Pi}_{t+1}^w\right)}{W_{t+i}} \right]^{1-\varepsilon} \tilde{W}_{j,t}^{-\varepsilon} + \psi_w \frac{\left(\tilde{\Pi}_{t+i}^w \times \ldots \times \tilde{\Pi}_{t+1}^w\right)}{P_{t+i}} \\ +\varepsilon mrs_{t+i} \vartheta_{w,t+i}^{\varepsilon} \left[\frac{\left(\tilde{\Pi}_{t+i}^w \times \ldots \times \tilde{\Pi}_{t+1}^w\right)}{W_{t+i}} \right]^{-\varepsilon} \tilde{W}_{j,t}^{-\varepsilon-1} \end{array} \right\} = 0$$

All wage adjusters choose the same wage, i.e. $\tilde{W}_{j,t} = \tilde{W}_t$. Re-arranging:

$$E_t \sum_{i=0}^{\infty} (\beta\xi_w)^i \frac{\varsigma_{t+i} n_{t+i} \lambda_{t+i}}{1+\psi_w} \left\{ \begin{array}{c} (1-\varepsilon) w_{t+i} \vartheta_{w,t+i}^{\varepsilon} \left[\frac{\left(\tilde{\Pi}_{t+i}^w \times \ldots \times \tilde{\Pi}_{t+1}^w\right) W_t}{W_{t+i}} \right]^{1-\varepsilon} \tilde{w}_t^{1-\varepsilon} + \psi_w w_{t+i} \frac{\left(\tilde{\Pi}_{t+i}^w \times \ldots \times \tilde{\Pi}_{t+1}^w\right) W_t}{W_{t+i}} \tilde{w}_t \\ +\varepsilon mrs_{t+i} \vartheta_{w,t+i}^{\varepsilon} \left[\frac{\left(\tilde{\Pi}_{t+i}^w \times \ldots \times \tilde{\Pi}_{t+1}^w\right) W_t}{W_{t+i}} \right]^{-\varepsilon} \tilde{w}_t^{-\varepsilon} \end{array} \right\} = 0$$

where

$$\tilde{w}_t = \frac{\tilde{W}_t}{W_t}, w_t = \frac{W_t}{P_t}$$

Note that we can write the first-order condition as:

$$\begin{array}{c} 0 = E_t \sum_{i=0}^{\infty} (\beta\xi_w)^i \varsigma_{t+i} n_{t+i} \lambda_{t+i} w_{t+i} \vartheta_{w,t+i}^{\varepsilon} \left[\frac{\left(\tilde{\Pi}_{t+i}^w \times \ldots \times \tilde{\Pi}_{t+1}^w\right) W_t}{W_{t+i}} \right]^{1-\varepsilon} \tilde{w}_t \\ -\frac{\varepsilon}{\varepsilon-1} E_t \sum_{i=0}^{\infty} (\beta\xi_w)^i \varsigma_{t+i} n_{t+i} \lambda_{t+i} mrs_{t+i} \vartheta_{w,t+i}^{\varepsilon} \left[\frac{\left(\tilde{\Pi}_{t+i}^w \times \ldots \times \tilde{\Pi}_{t+1}^w\right) W_t}{W_{t+i}} \right]^{-\varepsilon} \\ -\frac{\psi_w}{\varepsilon-1} E_t \sum_{i=0}^{\infty} (\beta\xi_w)^i \varsigma_{t+i} n_{t+i} \lambda_{t+i} w_{t+i} \frac{\left(\tilde{\Pi}_{t+i}^w \times \ldots \times \tilde{\Pi}_{t+1}^w\right) W_t}{W_{t+i}} \tilde{w}_t^{1+\varepsilon} \end{array}$$

Or

$$\begin{array}{c} 0 = E_t \sum_{i=0}^{\infty} (\beta\xi_w)^i \varsigma_{t+i} n_{t+i} \lambda_{t+i} w_{t+i} \vartheta_{w,t+i}^{\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} \left[\frac{\left(\tilde{\Pi}_{t+i}^w \times \ldots \times \tilde{\Pi}_{t+1}^w\right) W_t}{W_{t+i}} \right]^{-\frac{1+\psi_w+\theta_w\psi_w}{\theta_w}} \tilde{w}_t \\ -\frac{(1+\psi_w)(1+\theta_w)}{1+\psi_w+\theta_w\psi_w} E_t \sum_{i=0}^{\infty} (\beta\xi_w)^i \varsigma_{t+i} n_{t+i} \lambda_{t+i} mrs_{t+i} \vartheta_{w,t+i}^{\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} \left[\frac{\left(\tilde{\Pi}_{t+i}^w \times \ldots \times \tilde{\Pi}_{t+1}^w\right) W_t}{W_{t+i}} \right]^{-\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} \\ -\frac{\theta_w\psi_w}{\psi_w+\theta_w\psi_w+1} E_t \sum_{i=0}^{\infty} (\beta\xi_w)^i \varsigma_{t+i} n_{t+i} \lambda_{t+i} w_{t+i} \frac{\left(\tilde{\Pi}_{t+i}^w \times \ldots \times \tilde{\Pi}_{t+1}^w\right) W_t}{W_{t+i}} \tilde{w}_t^{1+\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} \end{array}$$

Or

$$\begin{array}{rcl} S_t^w & = & F_t^w \tilde{w}_t - A_t^w \tilde{w}_t^{1+\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} \\ & & \text{or in scaled terms} \\ \frac{S_t^w}{\varsigma_t} & = & \frac{F_t^w}{\varsigma_t} \tilde{w}_t - \frac{A_t^w}{\varsigma_t} \tilde{w}_t^{1+\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} \\ s_t^w & = & f_t^w \tilde{w}_t - a_t^w \tilde{w}_t^{1+\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} \end{array}$$

where

$$
\begin{aligned}
F_t^w &= E_t \sum_{i=0}^{\infty} (\beta\xi_w)^i \varsigma_{t+i} n_{t+i} \lambda_{t+i} w_{t+i} \vartheta_{w,t+i}^{\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} \left[\frac{\left(\tilde{\Pi}_{t+i}^w \times ... \times \tilde{\Pi}_{t+1}^w\right) W_t}{W_{t+i}} \right]^{-\frac{1+\psi_w+\theta_w\psi_w}{\theta_w}} \tilde{w}_t \\
A_t^w &= \frac{\theta_w \psi_w}{\psi_w + \theta_w\psi_w + 1} E_t \sum_{i=0}^{\infty} (\beta\xi_w)^i \varsigma_{t+i} n_{t+i} \lambda_{t+i} w_{t+i} \frac{\left(\tilde{\Pi}_{t+i}^w \times ... \times \tilde{\Pi}_{t+1}^w\right) W_t}{W_{t+i}} \tilde{w}_t^{1+\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} \\
S_t^w &= \frac{(1+\psi_w)(1+\theta_w)}{1+\psi_w+\theta_w\psi_w} E_t \sum_{i=0}^{\infty} (\beta\xi_w)^i \varsigma_{t+i} n_{t+i} \lambda_{t+i} mrs_{t+i} \vartheta_{w,t+i}^{\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} \left[\frac{\left(\tilde{\Pi}_{t+i}^w \times ... \times \tilde{\Pi}_{t+1}^w\right) W_t}{W_{t+i}} \right]^{-\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}}
\end{aligned}
$$

Writing recursively:

$$
\begin{aligned}
F_t^w &= \varsigma_t n_t \lambda_t w_t \vartheta_{w,t}^{\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} + \beta\xi_w E_t \left[\frac{\tilde{\Pi}_{t+1}^w}{\Pi_{w,t+1}} \right]^{-\frac{1+\psi_w+\theta_w\psi_w}{\theta_w}} F_{t+1}^w \\
&\quad \text{or in scaled terms} \\
f_t^w &= n_t \lambda_t w_t \vartheta_{w,t}^{\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} + \beta\xi_w \delta_t E_t \left[\frac{\tilde{\Pi}_{t+1}^w}{\Pi_{w,t+1}} \right]^{-\frac{1+\psi_w+\theta_w\psi_w}{\theta_w}} f_{t+1}^w
\end{aligned}
$$

$$
\begin{aligned}
A_t^w &= \frac{\theta_w \psi_w}{\psi_w + \theta_w\psi_w + 1} \varsigma_t n_t \lambda_t w_t + \beta\xi_w E_t \frac{\tilde{\Pi}_{t+1}^w}{\Pi_{w,t+1}} A_{t+1}^w \\
&\quad \text{or in scaled terms} \\
\alpha_t^w &= \frac{\theta_w \psi_w}{\psi_w + \theta_w\psi_w + 1} n_t \lambda_t w_t + \beta\xi_w \delta_t E_t \frac{\tilde{\Pi}_{t+1}^w}{\Pi_{w,t+1}} \alpha_{t+1}^w
\end{aligned}
$$

$$
\begin{aligned}
S_t^w &= \frac{(1+\psi_w)(1+\theta_w)}{1+\psi_w+\theta_w\psi_w} \varsigma_t n_t \lambda_t mrs_t \vartheta_{w,t}^{\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} + \beta\xi_w E_t \left[\frac{\tilde{\Pi}_{t+1}^w}{\Pi_{w,t+1}} \right]^{-\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} S_{t+1}^w \\
&\quad \text{or in scaled terms} \\
s_t^w &= \frac{(1+\psi_w)(1+\theta_w)}{1+\psi_w+\theta_w\psi_w} n_t \lambda_t mrs_t \vartheta_{w,t}^{\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} + \beta\xi_w \delta_t E_t \left[\frac{\tilde{\Pi}_{t+1}^w}{\Pi_{w,t+1}} \right]^{-\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} s_{t+1}^w
\end{aligned}
$$

and the optimality condition in scaled terms

$$
s_t^w = f_t^w \tilde{w}_t - \alpha_t^w \tilde{w}_t^{1+\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}}
$$

A.4. Final Goods Producers

Optimization:

$$
\frac{P_{t,i}}{P_t} = \vartheta_t \frac{dG\left(\frac{y_{t,i}}{y_t}\right)}{d\frac{y_{t,i}}{y_t}}
$$

Calculate derivative and rearrange:

$$
\begin{aligned}
\frac{P_{t,i}}{P_t} &= \vartheta_t \left[\left(1+\psi_p\right) \frac{y_{t,i}}{y_t} - \psi_p \right]^{\frac{1-\omega_p}{\omega_p}} \\
\frac{y_{t,i}}{y_t} &= \frac{1}{1+\psi_p} \left(\left[\frac{P_{t,i}}{P_t} \vartheta_t^{-1} \right]^{\frac{\omega_p}{1-\omega_p}} + \psi_p \right) \\
\frac{y_{t,i}}{y_t} &= \frac{1}{1+\psi_p} \left(\left[\frac{P_{t,i}}{P_t} \right]^{-\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)} \vartheta_t^{\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)} + \psi_p \right)
\end{aligned}
$$

Substitute into aggregator which gives the aggregate price index resp. definition of lagrange multiplier:

$$
\begin{aligned}
1 &= \int \left(\frac{\omega_p}{1+\psi_p} \left[\left[\frac{P_{t,i}}{P_t} \right]^{-\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)} \vartheta_t^{\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)} \right]^{\frac{1}{\omega_p}} - \frac{\omega_p}{1+\psi_p} + 1 \right) di \\
\vartheta_t &= \left[\int \left[\frac{P_{t,i}}{P_t} \right]^{1-\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)} di \right]^{\frac{1}{1-\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)}}
\end{aligned}
$$

Note that after imposing zero profits, we can write

$$
\begin{aligned}
1 &= \frac{1}{1+\psi_p} \vartheta_t + \frac{\psi_p}{1+\psi_p} \int \frac{P_{t,i}}{P_t} di \\
\vartheta_t &= 1 + \psi_p - \psi_p \int \frac{P_{t,i}}{P_t} di
\end{aligned}
$$

A.5. Intermediate Goods Producers

Substituting the demand function into the profit function gives:

$$
\max_{\tilde{P}_{t,i}} \; E_t \sum_{j=0}^{\infty} \left(\beta \xi_p\right)^j \varsigma_{t+j} \Lambda_{t+j} \frac{y_{t+j}}{1+\psi_p} \left[\begin{array}{c}
P_{t+j} \left[\frac{\left(\tilde{\Pi}_{t+j} \times \ldots \times \tilde{\Pi}_{t+1}\right) \tilde{P}_{t,i}}{P_{t+j}} \right]^{-\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)+1} \vartheta_{t+j}^{\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)} \\
+ \left(\tilde{\Pi}_{t+j} \times \ldots \times \tilde{\Pi}_{t+1}\right) \tilde{P}_{t,i} \psi_p \\
-MC_{t+j} \left[\frac{\left(\tilde{\Pi}_{t+j} \times \ldots \times \tilde{\Pi}_{t+1}\right) \tilde{P}_{t,i}}{P_{t+j}} \right]^{-\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)} \vartheta_{t+j}^{\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)} - \psi_p MC_{t+j}
\end{array} \right]
$$

Differentiate

$$
\begin{aligned}
0 \quad = \quad & E_t \sum_{j=0}^{\infty} \left(\beta \xi_p\right)^j \varsigma_{t+j} \Lambda_{t+j} \frac{1+\psi_p+\theta_p \psi_p}{1+\psi_p} \left(\tilde{\Pi}_{t+j} \times ... \times \tilde{\Pi}_{t+1}\right) \tilde{P}_{t,i} \\
& \times \left[\frac{\left(\tilde{\Pi}_{t+j} \times ... \times \tilde{\Pi}_{t+1}\right) \tilde{P}_{t,i}}{P_{t+j}} \right]^{-\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)} \vartheta_{t+j}^{\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)} y_{t+j} \\
& - E_t \sum_{j=0}^{\infty} \left(\beta \xi_p\right)^j \varsigma_{t+j} \Lambda_{t+j} \frac{\psi_p}{1+\psi_p} \theta_p \left(\tilde{\Pi}_{t+j} \times ... \times \tilde{\Pi}_{t+1}\right) \tilde{P}_{t,i} y_{t+j} \\
& - E_t \sum_{j=0}^{\infty} \left(\beta \xi_p\right)^j \varsigma_{t+j} \Lambda_{t+j} \left(1+\theta_p\right) MC_{t+j} \\
& \times \left[\frac{\left(\tilde{\Pi}_{t+j} \times ... \times \tilde{\Pi}_{t+1}\right) \tilde{P}_{t,i}}{P_{t+j}} \right]^{-\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)} \vartheta_{t+j}^{\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)} y_{t+j}
\end{aligned}
$$

Define

$$
\tilde{p}_{t,i} = \frac{\tilde{P}_{t,i}}{P_t}, \; mc_{t+j} = \frac{MC_{t+j}}{P_{t+j}}
$$

Using the above definitions and after rearranging:

$$
\begin{aligned}
0 \quad = \quad & \underbrace{E_t \sum_{j=0}^{\infty} \left(\beta \xi_p\right)^j \varsigma_{t+j} \lambda_{t+j} y_{t+j} \left[\frac{\left(\tilde{\Pi}_{t+j} \times ... \times \tilde{\Pi}_{t+1}\right) P_t}{P_{t+j}} \right]^{-\frac{\psi_p+\theta_p\psi_p+1}{\theta_p}} \vartheta_{t+j}^{\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)} \tilde{p}_{t,i}}_{\equiv F_t} \\
& \underbrace{- E_t \sum_{j=0}^{\infty} \left(\beta \xi_p\right)^j \varsigma_{t+j} \lambda_{t+j} y_{t+j} \left[\frac{\left(\tilde{\Pi}_{t+j} \times ... \times \tilde{\Pi}_{t+1}\right) P_t}{P_{t+j}} \right]^{-\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)} \vartheta_{t+j}^{\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)} \frac{\left(1+\psi_p\right)\left(1+\theta_p\right)}{1+\psi_p+\theta_p\psi_p} mc_{t+j}}_{\equiv S_t} \\
& \underbrace{- \frac{\psi_p \theta_p}{1+\psi_p+\theta_p\psi_p} E_t \sum_{j=0}^{\infty} \left(\beta \xi_p\right)^j \varsigma_{t+j} \lambda_{t+j} y_{t+j} \left[\frac{\left(\tilde{\Pi}_{t+j} \times ... \times \tilde{\Pi}_{t+1}\right) P_t}{P_{t+j}} \right] \tilde{p}_{t,i}^{1+\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)}}_{\equiv A_t}
\end{aligned}
$$

Or

$$
S_t = F_t \tilde{p}_{t,i} - A_t \tilde{p}_{t,i}^{1+\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)}
$$

Note that, in each period, all firms that reset prices face the same problem and therefore set the same price, $\tilde{p}_{t,i} = \tilde{p}_t$,

$$
S_t = F_t \tilde{p}_t - A_t \tilde{p}_t^{1+\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)}
$$

It is convenient to scale the above equation by ς_t

$$\begin{aligned}\frac{S_t}{\varsigma_t} &= \frac{F_t}{\varsigma_t}\tilde{p}_t - \frac{A_t}{\varsigma_t}\tilde{p}_t^{1+\frac{1+\theta_p}{\theta_p}(1+\psi_p)} \\ s_t &= f_t\tilde{p}_t - \alpha_t\tilde{p}_t^{1+\frac{1+\theta_p}{\theta_p}(1+\psi_p)}\end{aligned}$$

Consider the expressions for S_t, F_t and A_t :

$$\begin{aligned}S_t &= E_t\sum_{j=0}^{\infty}\left(\beta\xi_p\right)^j \varsigma_{t+j}\lambda_{t+j}y_{t+j}\left[\frac{\left(\tilde{\Pi}_{t+j}\times...\times\tilde{\Pi}_{t+1}\right)P_t}{P_{t+j}}\right]^{-\frac{1+\theta_p}{\theta_p}(1+\psi_p)}\vartheta_{t+j}^{\frac{1+\theta_p}{\theta_p}(1+\psi_p)}\frac{\left(1+\psi_p\right)\left(1+\theta_p\right)}{1+\psi_p+\theta_p\psi_p}mc_{t+j} \\ F_t &= E_t\sum_{j=0}^{\infty}\left(\beta\xi_p\right)^j \varsigma_{t+j}\lambda_{t+j}y_{t+j}\left[\frac{\left(\tilde{\Pi}_{t+j}\times...\times\tilde{\Pi}_{t+1}\right)P_t}{P_{t+j}}\right]^{-\frac{\psi_p+\theta_p\psi_p+1}{\theta_p}}\vartheta_{t+j}^{\frac{1+\theta_p}{\theta_p}(1+\psi_p)} \\ A_t &= \frac{\psi_p\theta_p}{1+\psi_p+\theta_p\psi_p}E_t\sum_{j=0}^{\infty}\left(\beta\xi_p\right)^j \varsigma_{t+j}\lambda_{t+j}y_{t+j}\left[\frac{\left(\tilde{\Pi}_{t+j}\times...\times\tilde{\Pi}_{t+1}\right)P_t}{P_{t+j}}\right]\end{aligned}$$

Note that

$$\begin{aligned}S_t &= \varsigma_t\lambda_t y_t\vartheta_t^{\frac{1+\theta_p}{\theta_p}(1+\psi_p)}\frac{\left(1+\psi_p\right)\left(1+\theta_p\right)}{1+\psi_p+\theta_p\psi_p}mc_t \\ & E_t\sum_{j=1}^{\infty}\left(\beta\xi_p\right)^j \varsigma_{t+j}\lambda_{t+j}y_{t+j}\left[\frac{\left(\tilde{\Pi}_{t+j}\times...\times\tilde{\Pi}_{t+1}\right)P_t}{P_{t+j}}\right]^{-\frac{1+\theta_p}{\theta_p}(1+\psi_p)}\vartheta_{t+j}^{\frac{1+\theta_p}{\theta_p}(1+\psi_p)}\frac{\left(1+\psi_p\right)\left(1+\theta_p\right)}{1+\psi_p+\theta_p\psi_p}mc_{t+j}\end{aligned}$$

So that

$$\begin{aligned}S_t &= \frac{\left(1+\psi_p\right)\left(1+\theta_p\right)}{1+\psi_p+\theta_p\psi_p}\varsigma_t\lambda_t y_t\vartheta_t^{\frac{1+\theta_p}{\theta_p}(1+\psi_p)}mc_t+\beta\xi_p E_t\left(\tilde{\Pi}_{t+1}/\Pi_{t+1}\right)^{-\frac{1+\theta_p}{\theta_p}(1+\psi_p)}S_{t+1} \\ s_t &= \frac{\left(1+\psi_p\right)\left(1+\theta_p\right)}{1+\psi_p+\theta_p\psi_p}\lambda_t y_t\vartheta_t^{\frac{1+\theta_p}{\theta_p}(1+\psi_p)}mc_t+\beta\xi_p E_t\delta_{t+1}\left(\tilde{\Pi}_{t+1}/\Pi_{t+1}\right)^{-\frac{1+\theta_p}{\theta_p}(1+\psi_p)}s_{t+1}\end{aligned}$$

Similarly,

$$f_t = \lambda_t y_t\vartheta_t^{\frac{1+\theta_p}{\theta_p}(1+\psi_p)}+\beta\xi_p E_t\delta_{t+1}\left(\tilde{\Pi}_{t+1}/\Pi_{t+1}\right)^{-\frac{(1+\psi_p+\psi_p\theta_p)}{\theta_p}}f_{t+1}$$

Finally,

$$\alpha_t = \frac{\psi_p\theta_p}{1+\psi_p+\theta_p\psi_p}y_t\lambda_t+\beta\xi_p E_t\delta_{t+1}\left(\tilde{\Pi}_{t+1}/\Pi_{t+1}\right)\alpha_{t+1}$$

A.6. Aggregate Resources

$$\begin{aligned} y_t^{sum} &= \int y_{t,i} di \\ &= \int n_{t,i} di \\ y_t^{sum} &= n_t \end{aligned}$$

Also,

$$y_t^{sum} = y_t \int \left(\frac{1}{1+\psi_p} \left[\frac{P_{t,i}}{P_t} \right]^{-\frac{1+\theta_p}{\theta_p}(1+\psi_p)} \vartheta_t^{\frac{1+\theta_p}{\theta_p}(1+\psi_p)} + \frac{\psi_p}{1+\psi_p} \right) di$$

So that

$$\begin{aligned} y_t &= \frac{1}{\int \left(\frac{1}{1+\psi_p} \left[\frac{P_{t,i}}{P_t} \right]^{-\frac{1+\theta_p}{\theta_p}(1+\psi_p)} \vartheta_t^{\frac{1+\theta_p}{\theta_p}(1+\psi_p)} + \frac{\psi_p}{1+\psi_p} \right) di} n_t \\ y_t &= (p_t^*)^{-1} n_t \end{aligned}$$

where

$$\begin{aligned} p_t^* &= \frac{\vartheta_t^{\frac{1+\theta_p}{\theta_p}(1+\psi_p)}}{1+\psi_p} \Delta_{t,1}^{-\frac{1+\theta_p}{\theta_p}(1+\psi_p)} + \frac{\psi_p}{1+\psi_p} \\ \Delta_{t,1} &= \left[\int \left[\frac{P_{t,i}}{P_t} \right]^{-\frac{1+\theta_p}{\theta_p}(1+\psi_p)} di \right]^{-\frac{\theta_p}{(1+\theta_p)(1+\psi_p)}} \\ \Delta_{t,1} &= \left[\int \left[\frac{P_{t,i}}{P_t} \right]^{-\frac{1+\theta_p}{\theta_p}(1+\psi_p)} di \right]^{-\frac{\theta_p}{(1+\theta_p)(1+\psi_p)}} \\ \Delta_{t,1} &= \left[\left(1-\xi_p\right) \tilde{p}_t^{-\frac{(1+\theta_p)(1+\psi_p)}{\theta_p}} + \xi_p \left[\frac{\tilde{\Pi}_t}{\Pi_t} \Delta_{t-1,1} \right]^{-\frac{(1+\theta_p)(1+\psi_p)}{\theta_p}} \right]^{-\frac{\theta_p}{(1+\theta_p)(1+\psi_p)}} \end{aligned}$$

Denote aggregate hours worked by households by l_t. Then,

$$\begin{aligned} l_t &= \int n_{j,t} dj \\ &= \frac{1}{1+\psi_w} n_t \int \left(\left[\frac{W_{t,j}}{W_t} \right]^{-\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} [\vartheta_t^w]^{\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} + \psi_w \right) dj \\ &= n_t \int \left(\frac{1}{1+\psi_w} \left[\frac{W_{t,j}}{W_t} \right]^{-\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} [\vartheta_t^w]^{\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} + \frac{\psi_w}{1+\psi_w} \right) dj \end{aligned}$$

Or

$$n_t = (w_t^*)^{-1} l_t$$

where

$$\begin{aligned}
w_t^* &= \int \left(\frac{1}{1+\psi_w} \left[\frac{W_{t,j}}{W_t} \right]^{-\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} [\vartheta_t^w]^{\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} + \frac{\psi_w}{1+\psi_w} \right) dj \\
w_t^* &= \frac{[\vartheta_t^w]^{\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}}}{1+\psi_w} \left[\Delta_{t,1}^w\right]^{-\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} dj + \frac{\psi_w}{1+\psi_w} \\
\Delta_{t,1}^w &= \left[\int \left[\frac{W_{t,j}}{W_t} \right]^{-\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} dj \right]^{-\frac{\theta_w}{(1+\theta_w)(1+\psi_w)}} \\
\Delta_{t,1}^w &= \left[(1-\xi_w) \left[\tilde{w}_t\right]^{-\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} + \xi_w \left[\frac{\tilde{\Pi}_t^w}{\Pi_t^w} \Delta_{t-1,1}^w \right]^{-\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} dj \right]^{-\frac{\theta_w}{(1+\theta_w)(1+\psi_w)}}
\end{aligned}$$

So that the aggregate resource constraint reads as follows:

$$c_t = y_t = (p_t^*)^{-1} (w_t^*)^{-1} l_t$$

The zero profit condition for final goods producers can be written as:

$$\begin{aligned}
\vartheta_t &= 1 + \psi_p - \psi_p \int \frac{P_{t,i}}{P_t} di \\
\vartheta_t &= 1 + \psi_p - \psi_p \Delta_{t,2}
\end{aligned}$$

where

$$\begin{aligned}
\Delta_{t,2} &= \int \frac{P_{t,i}}{P_t} di \\
\Delta_{t,2} &= \left(1-\xi_p\right) \tilde{p}_t + \xi_p \left(\tilde{\Pi}_t / \Pi_t\right) \Delta_{t-1,2}
\end{aligned}$$

Further, the aggregate price index equation can be rewritten as follows:

$$\begin{aligned}
P_t \vartheta_t &= \left[\int P_{t,i}^{-\frac{1+\psi_p+\psi_p\theta_p}{\theta_p}} di \right]^{-\frac{\theta_p}{1+\psi_p+\psi_p\theta_p}} \\
\vartheta_t &= \Delta_{t,3} \\
\Delta_{t,3}^{-\frac{1+\psi_p+\psi_p\theta_p}{\theta_p}} &= (1-\xi_p) \tilde{p}_t^{-\frac{1+\psi_p+\psi_p\theta_p}{\theta_p}} + \xi_p \left(\left(\tilde{\Pi}_t / \Pi_t\right) \Delta_{t-1,3} \right)^{-\frac{1+\psi_p+\psi_p\theta_p}{\theta_p}}
\end{aligned}$$

The zero profit condition for labor contractors can be written as:

$$
\begin{aligned}
\vartheta_t^w &= 1+\psi_w-\psi_w\int\frac{W_{t,j}}{W_t}dj \\
\vartheta_t^w &= 1+\psi_w-\psi_w\Delta_{t,2}^w \\
\Delta_{t,2}^w &= \int\frac{W_{t,j}}{W_t}dj \\
\Delta_{t,2}^w &= (1-\xi_w)\,\tilde{w}_t+\xi_w\left(\tilde{\Pi}_t^w/\Pi_t^w\right)\Delta_{t-1,2}^w
\end{aligned}
$$

The aggregate wage index can be written as follows:

$$
\begin{aligned}
W_t\vartheta_t^w &= \left[\int W_{t,j}^{-\frac{1+\psi_w+\theta_w\psi_w}{\theta_w}}dj\right]^{-\frac{\theta_w}{1+\psi_w+\theta_w\psi_w}} \\
\vartheta_t^w &= \Delta_{t,3}^w \\
\left[\Delta_{t,3}^w\right]^{-\frac{1+\psi_w+\psi_w\theta_w}{\theta_w}} &= (1-\xi_w)\tilde{w}_t^{-\frac{1+\psi_w+\psi_w\theta_w}{\theta_w}}+\xi_w\left(\left(\tilde{\Pi}_t^w/\Pi_t^w\right)\Delta_{t-1,3}^w\right)^{-\frac{1+\psi_w+\psi_w\theta_w}{\theta_w}}
\end{aligned}
$$

Notice the following relation between the real wage, price inflation and wage inflation

$$
\begin{aligned}
\Pi_t^w &= \frac{W_t}{W_{t-1}} \\
\Pi_t^w &= \frac{P_t}{P_t}\frac{P_{t-1}}{P_{t-1}}\frac{W_t}{W_{t-1}} \\
\Pi_t^w &= \frac{P_t}{P_{t-1}}\frac{P_{t-1}}{W_{t-1}}\frac{W_t}{P_t} \\
\Pi_t^w &= \Pi_t\frac{w_t}{w_{t-1}}
\end{aligned}
$$

A.7. Solution and Implementation

We use the nonlinear ('simul') solver in Dynare to solve the model. Specifically, we will use the two-point boundary value solver that is implemented in dynare. In stylized form, the model can be written as follows:

$$
\begin{aligned}
f(y_{-1},y_0,y_1;a_0) &= 0 \text{ in period } t=0 \\
f(y_0,y_1,y_2;a_1) &= 0 \text{ in period } t=1 \\
f(y_1,y_2,y_3;a_2) &= 0 \text{ in period } t=2 \\
&\;\;\;\;.... \\
f(y_{T-1},y_T,y_{T+1};a_T) &= 0 \text{ in period } t=T
\end{aligned}
$$

Where y_0 denotes the vector of endogenous variables of the model. Dynare's <simul> command solves this set of equations for periods $t=0,...,T$ using a Newton algorithm.

y_{-1} and y_{T+1} are given, and most often equal the steady state of the model.

For each realization of shocks from their stochastic processes, we solve the above system of equations in which the agents form expectations using the Kalman filter.[A.1]

More precisely, say in period $t = 0$ a shock is observed. Then, we solve the system of equations from $t = 0$ to $t = T$ with agents forecasting future realizations of shocks from the Kalman filter. Then, we move one period forward, i.e. $t = 1$. There, a new shock is realized. We take the state y_0 from the previous simulation as an initial value and solve the system of equations from $t = 1$ to $t = T + 1$. And so on until no new shocks are realized.

[A.1] Note that we solve the model under certainty equivalence, i.e. in each simulation period, we solve for the deterministic solution of the model. Put differently, the solution method that we are using does not take possible interactions between non-linearities and uncertainty about future shocks into account, i.e. Jensen's inequality plays no role in shaping expectations. In future work, it might be worthwile to consider the effects of shock uncertainty.

A.8. Nonlinear Equilibrium Equations

The nonlinear equilibrium equations can be written as:

$$
\begin{aligned}
\text{Marginal utility (n1)} &: \quad \frac{1}{c_t - hc_{t-1}} = \lambda_t \\
\text{MRS (n2)} &: \quad mrs_t = 1/\lambda_t \\
\text{Euler equation (n3)} &: \quad \lambda_t = \beta E_t \delta_{t+1} \frac{R_t}{\Pi_{t+1}} \lambda_{t+1} \\
\text{Resource Constraint (n4)} &: \quad c_t = y_t \\
\text{Production (n5)} &: \quad y_t = \left(p_t^*\right)^{-1} \left(w_t^*\right)^{-1} l_t \\
\text{Non.lin. pricing 1 (n6)} &: \quad s_t = \frac{\left(1+\psi_p\right)\left(1+\theta_p\right)}{1+\psi_p+\theta_p\psi_p} \lambda_t y_t \vartheta_t^{\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)} mc_t \\
&\quad +\beta \xi_p E_t \delta_{t+1} \left(\tilde{\Pi}_{t+1}/\Pi_{t+1}\right)^{-\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)} s_{t+1} \\
\text{Non.lin. pricing 2 (n7)} &: \quad f_t = \lambda_t y_t \vartheta_t^{\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)} + \beta \xi_p E_t \delta_{t+1} \left(\tilde{\Pi}_{t+1}/\Pi_{t+1}\right)^{-\frac{\left(1+\psi_p+\psi_p\theta_p\right)}{\theta_p}} f_{t+1} \\
\text{Non.lin. pricing 3 (n8)} &: \quad \alpha_t = \frac{\psi_p\theta_p}{1+\psi_p+\theta_p\psi_p} y_t \lambda_t + \beta \xi_p E_t \delta_{t+1} \left(\tilde{\Pi}_{t+1}/\Pi_{t+1}\right) \alpha_{t+1} \\
\text{Non.lin.pricing 4 (n9)} &: \quad s_t = f_t \tilde{p}_t - \alpha_t \tilde{p}_t^{1+\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)} \\
\text{Zero profit condition prices (n10)} &: \quad \vartheta_t = 1+\psi-\psi\Delta_{t,2} \\
\text{Aggregate price index (n11)} &: \quad \vartheta_t = \Delta_{t,3} \\
\text{Overall price dispersion (n12)} &: \quad p_t^* = \frac{\vartheta_t^{\frac{1+\theta_p}{\theta_p}(1+\psi)}}{1+\psi_p} \Delta_{t,1}^{-\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)} + \frac{\psi_p}{1+\psi_p} \\
\text{Price dispersion 1 (n13)} &: \quad \Delta_{t,1}^{-\frac{\left(1+\theta_p\right)\left(1+\psi_p\right)}{\theta_p}} = \left(1-\xi_p\right) \tilde{p}_t^{-\frac{\left(1+\theta_p\right)\left(1+\psi_p\right)}{\theta_p}} + \xi_p \left[\left(\tilde{\Pi}_t/\Pi_t\right) \Delta_{t-1,1}\right]^{-\frac{\left(1+\theta_p\right)\left(1+\psi_p\right)}{\theta_p}} \\
\text{Price dispersion 2 (n14)} &: \quad \Delta_{t,2} = \left(1-\xi_p\right) \tilde{p}_t + \xi_p \left(\tilde{\Pi}_t/\Pi_t\right) \Delta_{t-1,2} \\
\text{Price dispersion 3 (n15)} &: \quad \Delta_{t,3}^{-\frac{1+\psi_p+\psi_p\theta_p}{\theta_p}} = (1-\xi_p) \tilde{p}_t^{-\frac{1+\psi_p+\psi_p\theta_p}{\theta_p}} + \xi_p \left(\left(\tilde{\Pi}_t/\Pi_t\right) \Delta_{t-1,3}\right)^{-\frac{1+\psi_p+\psi_p\theta_p}{\theta_p}}
\end{aligned}
$$

$$\text{Wage inflation (n16)} \quad : \quad \Pi_t^w = \Pi_t \frac{w_t}{w_{t-1}}$$

$$\text{Non.lin. wage setting 1 (n17)} \quad : \quad s_t^w = f_t^w \tilde{w}_t - \alpha_t^w \tilde{w}_t^{1+\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}}$$

$$\text{Non.lin. wage setting 2 (n18)} \quad : \quad f_t^w = (w_t^*)^{-1} l_t \lambda_t w_t \vartheta_{w,t}^{\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} + \beta \xi_w \delta_t E_t \left[\frac{\tilde{\Pi}_{t+1}^w}{\Pi_{w,t+1}} \right]^{-\frac{1+\psi_w+\theta_w\psi_w}{\theta_w}} f_{t+1}^w$$

$$\text{Non.lin. wage setting 3 (n19)} \quad : \quad \alpha_t^w = \frac{\theta_w \psi_w}{\psi_w + \theta_w \psi_w + 1} (w_t^*)^{-1} l_t \lambda_t w_t + \beta \xi_w \delta_t E_t \frac{\tilde{\Pi}_{t+1}^w}{\Pi_{w,t+1}} \alpha_{t+1}^w$$

$$\text{Non.lin. wage setting 4 (n20)} \quad : \quad s_t^w = \frac{(1+\psi_w)(1+\theta_w)}{1+\psi_w+\theta_w\psi_w} (w_t^*)^{-1} l_t \lambda_t mrs_t \vartheta_{w,t}^{\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}}$$

$$+\beta \xi_w \delta_t E_t \left[\frac{\tilde{\Pi}_{t+1}^w}{\Pi_{w,t+1}} \right]^{-\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} s_{t+1}^w$$

$$\text{Zero profit condition wages (n21)} \quad : \quad \vartheta_t^w = 1 + \psi_w - \psi_w \Delta_{t,2}^w$$

$$\text{Agg. wage index (n22)} \quad : \quad \vartheta_t^w = \Delta_{t,3}^w$$

$$\text{Overall wage disperision (n23)} \quad : \quad w_t^* = \frac{\left[\vartheta_t^w\right]^{\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}}}{1+\psi_w} \left[\Delta_{t,1}^w\right]^{-\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} + \frac{\psi_w}{1+\psi_w}$$

$$\text{Wage dispersion 1 (n24)} \quad : \quad \left[\Delta_{t,1}^w\right]^{-\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} = (1-\xi_w)\left[\tilde{w}_t\right]^{-\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}}$$

$$+\xi_w \left[\frac{\tilde{\Pi}_t^w}{\Pi_t^w} \Delta_{t-1,1}^w \right]^{-\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}}$$

$$\text{Wage dispersion 2 (n25)} \quad : \quad \Delta_{t,2}^w = (1-\xi_w)\tilde{w}_t + \xi_w \left(\tilde{\Pi}_t^w / \Pi_t^w\right) \Delta_{t-1,2}^w$$

$$\text{Wage dispersion 3 (n26)} \quad : \quad \left[\Delta_{t,3}^w\right]^{-\frac{1+\psi_w+\psi_w\theta_w}{\theta_w}} = (1-\xi_w)\tilde{w}_t^{-\frac{1+\psi_w+\psi_w\theta_w}{\theta_w}}$$

$$+\xi_w \left(\left(\tilde{\Pi}_t^w / \Pi_t^w\right) \Delta_{t-1,3}^w \right)^{-\frac{1+\psi_w+\psi_w\theta_w}{\theta_w}}$$

$$\text{Indexation 1 (n27)} \quad : \quad \tilde{\Pi}_t = \bar{\Pi}^{1-\varkappa_t} \Pi_{t-1}^{\varkappa_t}$$

$$\text{Indexation 2 (n28):} \quad \tilde{\Pi}_t^w = \tilde{\Pi}_t$$

$$\text{Indexation 3 (n29)} \quad : \quad \varkappa_t = e^{-\frac{\varrho}{\max\left(\Pi_t^* - \Pi,\ 0.0001\right)}} - e^{-\frac{\varrho}{0.0001}}$$

$$\text{Indexation 4 (n30)} \quad : \quad \Pi_t^* = \left(\Pi_{t-1}^*\right)^{\omega} \left(\Pi_{t-1}\right)^{1-\omega}$$

$$\text{Marginal cost (n31)} \quad : \quad mc_t = \tau_t^{1/\kappa} w_t$$

$$\text{Taylor rule (n32)} \quad : \quad R_t^{not}/R = \left\{R_{t-1}^{not}/R\right\}^{\rho} \left\{E_t\left[\Pi_{t+4}/\Pi\right]\right\}^{(1-\rho)\gamma_\pi} \left\{ \frac{y_t}{y} / \frac{y_t^{pot}}{y^{pot}} \right\}^{(1-\rho)\gamma_x} e^{\varepsilon_{R,t}}$$

$$\text{ZLB (n33)} \quad : \quad R_t = \max(1, R_t^{not})$$

Flex-price-flex-wage (potential) economy: version of the model when prices and wages are flexible, i.e. $\xi_p = \xi_w = 0$. Also, we set the cost-push shock to zero in the potential economy. The potential economy can be summarized by the following two equations:

$$\text{Real potential rate, pot. econ (n34)} \quad : \quad \frac{1}{y_t^{pot} - hy_{t-1}^{pot}} = \beta E_t \delta_{t+1} rr_t^{pot} \frac{1}{y_{t+1}^{pot} - hy_t^{pot}}$$

$$\text{Potential output, pot. econ (n35)} \quad : \quad y_t^{pot} - hy_{t-1}^{pot} = \frac{1}{1+\theta_p} \frac{1}{1+\theta_w}$$

Note that potential output is constant and only the real potential rate moves in response to the discount factor shock.

We have 35 equations in the following 35 endogenous variables:

$$c_t \ \lambda_t \ w_t \ R_t \ R_t^{not} \ \Pi_t \ y_t \ p_t^* \ l_t \ s_t \ \vartheta_t \ \tilde{\Pi}_t \ mc_t$$

$$f_t \ \alpha_t \ \tilde{p}_t \ \Delta_{t,1} \ \Delta_{t,2} \ \Delta_{t,3} \ \Pi_t^* \ \varkappa_t \ \tilde{\Pi}_t^w \ w_t^* \ \Pi_t^w$$

$$\Delta_{t,1}^w \ \Delta_{t,2}^w \ \Delta_{t,3}^w \ \tilde{w}_t \ s_t^w \ f_t^w \ \alpha_t^w \ \vartheta_t^w \ mrs_t \ rr_t^{pot} \ y_t^{pot}$$

The variables δ_t, $\tau_t - 1 = a_t$ and $\varepsilon_{R,t}$ are exogenous.

A.9. Steady State

The following set of equations solve for the steady state of the model. Assume the central bank chooses a level of steady state inflation Π. Then:

$$(\text{n3}) : R = \frac{1}{\beta}\Pi$$

$$(\text{n27}) \quad : \quad \tilde{\Pi} = \Pi$$

$$\Pi^* \quad = \quad \Pi$$

Note that:

$$(\text{n10}) \quad : \quad \vartheta = 1 + \psi - \psi\Delta_2$$

$$(\text{n11}) \quad : \quad \Delta_3 = \vartheta$$

$$(\text{n12}) \quad : \quad p^* = \frac{\vartheta^{\frac{1+\theta_p}{\theta_p}(1+\psi)}}{1+\psi_p} \Delta_1^{-\frac{1+\theta_p}{\theta_p}(1+\psi_p)} + \frac{\psi_p}{1+\psi_p}$$

$$(\text{n13}) \quad : \quad \frac{\Delta_1}{\tilde{p}} = 1$$

$$(\text{n15}) \quad : \quad \frac{\Delta_3}{\tilde{p}} = 1$$

Then, using n10, n11, n14 and n15 we get:

$$\tilde{p} = 1$$

$$(\text{n14}) : \Delta_2 = \tilde{p}$$

$$(\text{n10}) : \vartheta = 1 + \psi - \psi \Delta_2$$

$$(\text{n11}) : \Delta_3 = \vartheta$$

$$(\text{n13}) : \Delta_1 = \tilde{p}$$

$$(\text{n12}) : p^* = \frac{\vartheta^{\frac{1+\theta_p}{\theta_p}(1+\psi)}}{1+\psi_p} \Delta_1^{-\frac{1+\theta_p}{\theta_p}(1+\psi_p)} + \frac{\psi_p}{1+\psi_p}$$

Use n6-n9:

$$mc = \left(\vartheta^{\frac{1+\theta_p}{\theta_p}(1+\psi_p)} \tilde{p} - \frac{\psi_p \theta_p}{1+\psi_p+\theta_p\psi_p} \tilde{p}^{1+\frac{1+\theta_p}{\theta_p}(1+\psi_p)} \right) \times \frac{1}{\frac{(1+\psi_p)(1+\theta_p)}{1+\psi_p+\theta_p\psi_p} \vartheta^{\frac{1+\theta_p}{\theta_p}(1+\psi_p)}}$$

$$(\text{n31}) : w = mc$$

Use n1, n2, n4, n5 and $mrs = 1/\lambda = w/(1+\theta_w)$ to get:

$$y = \frac{1}{1-h} \frac{w}{1+\theta_w}$$

$$(\text{n4}) : c = y$$

$$(\text{n1}) : \lambda = \frac{1}{(1-h)c}$$

$$\begin{aligned}
(\text{n6}) \quad &: \quad s = \frac{\frac{(1+\psi_p)(1+\theta_p)}{1+\psi_p+\theta_p\psi_p} \lambda y \vartheta^{\frac{1+\theta_p}{\theta_p}(1+\psi_p)}}{1-\beta\xi_p} mc \\
(\text{n7}) \quad &: \quad f = \frac{\lambda y \vartheta^{\frac{1+\theta_p}{\theta_p}(1+\psi_p)}}{1-\beta\xi_p} \\
(\text{n8}) \quad &: \quad \alpha = \frac{\frac{\psi_p\theta_p}{1+\psi_p+\theta_p\psi_p} y\lambda}{1-\beta\xi_p}
\end{aligned}$$

$$\begin{aligned}
w^* &= 1 \\
l &= n
\end{aligned}$$

$$
\begin{aligned}
\tilde{w} &= 1 \\
\Delta_1^w &= 1 \\
\Delta_2^w &= 1 \\
\Delta_3^w &= 1 \\
\vartheta^w &= 1 \\
\Pi^w &= \Pi
\end{aligned}
$$

$$
\begin{aligned}
mrs &= 1/\lambda \\
f^w &= \frac{1}{1-\beta\xi_w} n\lambda w \\
\alpha^w &= \frac{1}{1-\beta\xi_w} \frac{\theta_w\psi_w}{\psi_w+\theta_w\psi_w+1} n\lambda w \\
s^w &= \frac{1}{1-\beta\xi_w} \frac{(1+\psi_w)(1+\theta_w)}{1+\psi_w+\theta_w\psi_w} n\lambda mrs \\
s^w &= f^w - \alpha^w
\end{aligned}
$$

$$
\begin{aligned}
\tau &= 1 \\
\delta &= 1
\end{aligned}
$$

Flex-price flex-wage (potential) economy: version of the model when prices are flexible, i.e. $\xi_p = \xi_w = 0$.

$$
\text{(n34) } rr^{pot} = \frac{1}{\beta}
$$

$$
\text{(n35) } y^{pot} = \frac{1}{1-h} \frac{1}{1+\theta_p} \frac{1}{1+\theta_w}
$$

A.10. Log-Linearized Equilibrium Equations

Equations n10-15 can be expressed in log-linearized form as follows:

$$
\hat{\vartheta}_t = 0, \hat{p}_t^* = 0, \hat{\Delta}_{t,1} = 0, \hat{\Delta}_{t,2} = 0, \hat{\Delta}_{t,3} = 0, \hat{\tilde{p}}_t \; = \frac{\xi_p}{1-\xi_p}\left(\hat{\Pi}_t - \hat{\tilde{\Pi}}_t\right)
$$

After some tedious math, n6-n9 can be written as:

$$\begin{aligned}
\text{Non.lin. pricing 1 (n6)} &: \quad \hat{s}_t = \left(1-\beta\xi_p\right)\left[\hat{y}_t + \hat{\lambda}_t + \widehat{mc}_t\right] \\
&\quad +\beta\xi_p E_t\left[\hat{\delta}_{t+1} + \frac{\left(1+\theta_p\right)\left(1+\psi_p\right)}{\theta_p}\left(\hat{\Pi}_{t+1} - \hat{\tilde{\Pi}}_{t+1}\right) + \hat{s}_{t+1}\right] \\
\text{Non.lin. pricing 2 (n7)} &: \quad \hat{f}_t = \left(1-\beta\xi_p\right)\left(\hat{y}_t + \hat{\lambda}_t\right) + \beta\xi_p E_t\left[\hat{\delta}_{t+1} + \frac{\left(1+\psi_p+\psi_p\theta_p\right)}{\theta_p}\left(\hat{\Pi}_{t+1} - \hat{\tilde{\Pi}}_{t+1}\right) + \hat{f}_{t+1}\right] \\
\text{Non.lin. pricing 3 (n8)} &: \quad \hat{\alpha}_t = \left(1-\beta\xi_p\right)\left(\hat{y}_t + \hat{\lambda}_t\right) + \beta\xi_p E_t\left[\hat{\delta}_{t+1} - \left(\hat{\Pi}_{t+1} - \hat{\tilde{\Pi}}_{t+1}\right) + \hat{\alpha}_{t+1}\right] \\
\text{Non.lin.pricing 4 (n9)} &: \quad \hat{s}_t = \frac{1+\psi_p+\theta_p\psi_p}{1+\psi_p}\hat{f}_t - \frac{\psi_p\theta_p}{1+\psi_p}\hat{\alpha}_t + \frac{\xi_p\left(1-\psi_p-\psi_p\theta_p\right)}{1-\xi_p}\left(\hat{\Pi}_t - \hat{\tilde{\Pi}}_t\right)
\end{aligned}$$

Premultiply n7 and n8 by $\frac{1+\psi_p+\theta_p\psi_p}{1+\psi_p}$ and $\frac{\psi_p\theta_p}{1+\psi_p}$,respectively. Add n8 and substract n7 from n6:

$$\begin{aligned}
\hat{s}_t - \frac{1+\psi_p+\theta_p\psi_p}{1+\psi_p}\hat{f}_t + \frac{\psi_p\theta_p}{1+\psi_p}\hat{\alpha}_t &= \left(1-\beta\xi_p\right)\widehat{mc}_t \\
&\quad +\beta\xi_p E_t\left[\begin{array}{c} \hat{s}_{t+1} - \frac{1+\psi_p+\theta_p\psi_p}{1+\psi_p}\hat{f}_{t+1} + \frac{\psi_p\theta_p}{1+\psi_p}\hat{\alpha}_{t+1} \\ +\left(1-\theta_p\psi_p-\psi_p\right)\left(\hat{\Pi}_{t+1} - \hat{\tilde{\Pi}}_{t+1}\right) \end{array}\right]
\end{aligned}$$

Use equation n9 to get:

$$\begin{aligned}
\frac{\xi_p\left(1-\psi_p-\psi_p\theta_p\right)}{1-\xi_p}\left(\hat{\Pi}_t - \hat{\tilde{\Pi}}_t\right) &= \left(1-\beta\xi_p\right)\widehat{mc}_t \\
&\quad +\beta\xi_p E_t\left[\frac{\psi_p+\theta_p\psi_p-1}{\xi_p-1}\right]\left(\hat{\Pi}_{t+1} - \hat{\tilde{\Pi}}_{t+1}\right)
\end{aligned}$$

Or

$$\hat{\Pi}_t - \hat{\tilde{\Pi}}_t = \beta E_t\left(\hat{\Pi}_{t+1} - \hat{\tilde{\Pi}}_{t+1}\right) + \frac{\left(1-\beta\xi_p\right)\left(1-\xi_p\right)}{\xi_p}\frac{1}{1-\left(1+\theta_p\right)\psi_p}\widehat{mc}_t$$

The coefficient $\frac{1}{1-(1+\theta_p)\psi_p}$ is identical to the one in Levin, Lopez-Salido and Yun (2007).

Finally, $\tilde{\Pi}_t = \Pi^{1-\varkappa_t}\Pi_{t-1}^{\varkappa_t}$ can be log-linearized to yield:

$$\hat{\tilde{\Pi}}_t = \varkappa\hat{\Pi}_{t-1}.$$

So, the log-linearized New Keynesian Phillips curve reads:

$$\hat{\Pi}_t = \frac{\varkappa}{1+\beta\varkappa}\hat{\Pi}_{t-1} + \frac{\beta}{1+\beta\varkappa}E_t\hat{\Pi}_{t+1} + \kappa\widehat{mc}_t$$

where

$$\kappa = \frac{1}{1+\beta\varkappa}\frac{\left(1-\beta\xi_p\right)\left(1-\xi_p\right)}{\xi_p}\frac{1}{1-\left(1+\theta_p\right)\psi_p}$$

The nonlinear wage setting equations can be log-linearized to obtain:

$$
\begin{aligned}
\hat{f}_t^w &= (1-\beta\xi_w)\left(\hat{n}_t+\hat{\lambda}_t+\hat{w}_t\right)+\beta\xi_w E_t\left(\hat{\delta}_t+\frac{1+\psi_w+\theta_w\psi_w}{\theta_w}\left(\hat{\Pi}_{w,t+1}-\widehat{\tilde{\Pi}_{t+1}^w}\right)+\hat{f}_{t+1}^w\right)\\
\hat{\alpha}_t^w &= (1-\beta\xi_w)\left(\hat{n}_t+\hat{\lambda}_t+\hat{w}_t\right)+\beta\xi_w E_t\left(\hat{\delta}_t-\left(\hat{\Pi}_{w,t+1}-\widehat{\tilde{\Pi}_{t+1}^w}\right)+\hat{\alpha}_{t+1}^w\right)\\
\hat{s}_t^w &= (1-\beta\xi_w)\left(\hat{n}_t+\hat{\lambda}_t+\widehat{mrs}_t\right)+\beta\xi_w E_t\left(\hat{\delta}_t+\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}\left(\hat{\Pi}_{w,t+1}-\widehat{\tilde{\Pi}_{t+1}^w}\right)+\hat{s}_{t+1}^w\right)\\
\hat{s}_t^w &= \frac{1+\psi_w+\theta_w\psi_w}{1+\psi_w}f_t^w-\frac{\theta_w\psi_w}{1+\psi_w}\hat{\alpha}_t^w+(1-\psi_w-\theta_w\psi_w)\,\widehat{\tilde{w}}_t
\end{aligned}
$$

Also:

$$
\widehat{\tilde{w}}_t=\frac{\xi_w}{1-\xi_w}\left(\hat{\Pi}_{w,t}-\widehat{\tilde{\Pi}_t^w}\right)
$$

So that

$$
\begin{aligned}
\frac{1+\psi_w+\theta_w\psi_w}{1+\psi_w}\hat{f}_t^w &= \frac{1+\psi_w+\theta_w\psi_w}{1+\psi_w}(1-\beta\xi_w)\left(\hat{n}_t+\hat{\lambda}_t+\hat{w}_t\right)\\
&\quad+\frac{1+\psi_w+\theta_w\psi_w}{1+\psi_w}\beta\xi_w E_t\left(\hat{\delta}_t+\frac{1+\psi_w+\theta_w\psi_w}{\theta_w}\left(\hat{\Pi}_{w,t+1}-\widehat{\tilde{\Pi}_{t+1}^w}\right)+\hat{f}_{t+1}^w\right)\\
\frac{\theta_w\psi_w}{1+\psi_w}\hat{\alpha}_t^w &= \frac{\theta_w\psi_w}{1+\psi_w}(1-\beta\xi_w)\left(\hat{n}_t+\hat{\lambda}_t+\hat{w}_t\right)+\frac{\theta_w\psi_w}{1+\psi_w}\beta\xi_w E_t\left(\hat{\delta}_t-\left(\hat{\Pi}_{w,t+1}-\widehat{\tilde{\Pi}_{t+1}^w}\right)+\hat{\alpha}_{t+1}^w\right)\\
\hat{s}_t^w &= (1-\beta\xi_w)\left(\hat{n}_t+\hat{\lambda}_t+\widehat{mrs}_t\right)+\beta\xi_w E_t\left(\hat{\delta}_t+\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}\left(\hat{\Pi}_{w,t+1}-\widehat{\tilde{\Pi}_{t+1}^w}\right)+\hat{s}_{t+1}^w\right)\\
\hat{s}_t^w &= \frac{1+\psi_w+\theta_w\psi_w}{1+\psi_w}f_t^w-\frac{\theta_w\psi_w}{1+\psi_w}\hat{\alpha}_t^w+(1-\psi_w-\theta_w\psi_w)\frac{\xi_w}{1-\xi_w}\left(\hat{\Pi}_{w,t}-\widehat{\tilde{\Pi}_t^w}\right)
\end{aligned}
$$

Substract first and add second equation to third equation and substitute last equation to get:

$$
\begin{aligned}
(1-\psi_w-\theta_w\psi_w)\frac{\xi_w}{1-\xi_w}\left(\hat{\Pi}_{w,t}-\widehat{\tilde{\Pi}_t^w}\right) &= (1-\beta\xi_w)\left(\widehat{mrs}_t-\hat{w}_t\right)\\
&\quad+\beta\xi_w E_t\left(\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}\left(\hat{\Pi}_{w,t+1}-\widehat{\tilde{\Pi}_{t+1}^w}\right)+\hat{s}_{t+1}^w\right)\\
&\quad+\frac{\theta_w\psi_w}{1+\psi_w}\beta\xi_w E_t\left(-\left(\hat{\Pi}_{w,t+1}-\widehat{\tilde{\Pi}_{t+1}^w}\right)+\hat{\alpha}_{t+1}^w\right)\\
&\quad-\frac{1+\psi_w+\theta_w\psi_w}{1+\psi_w}\beta\xi_w E_t\left(\frac{1+\psi_w+\theta_w\psi_w}{\theta_w}\left(\hat{\Pi}_{w,t+1}-\widehat{\tilde{\Pi}_{t+1}^w}\right)+\hat{f}_{t+1}^w\right)
\end{aligned}
$$

Or :

$$
\begin{aligned}
(1-\psi_w-\theta_w\psi_w)\frac{\xi_w}{1-\xi_w}\left(\hat{\Pi}_{w,t}-\widehat{\tilde{\Pi}_t^w}\right) &= (1-\beta\xi_w)\left(\widehat{mrs}_t-\hat{w}_t\right)\\
&\quad+\beta\xi_w(1-\psi_w-\theta_w\psi_w)\left(1+\frac{\xi_w}{1-\xi_w}\right)E_t\left(\hat{\Pi}_{w,t+1}-\widehat{\tilde{\Pi}_{t+1}^w}\right)
\end{aligned}
$$

Or:

$$
\left(\hat{\Pi}_{w,t}-\widehat{\tilde{\Pi}_t^w}\right)=\beta E_t\left(\hat{\Pi}_{w,t+1}-\widehat{\tilde{\Pi}_{t+1}^w}\right)+\frac{(1-\xi_w)(1-\beta\xi_w)}{\xi_w}\frac{1}{1-(1+\theta_w)\psi_w}\left(\widehat{mrs}_t-\hat{w}_t\right)
$$

where

$$\widehat{mrs}_t = -\hat{\lambda}_t$$

So, the set of log-linearized equilibrium equations can be written as:

$$\begin{aligned}
\text{Euler equation (l1)} &: \quad \hat{y}_t = \frac{1}{1+h} E_t \hat{y}_{t+1} + \frac{h}{1+h} \hat{y}_{t-1} - \frac{1-h}{1+h} E_t \left[\hat{R}_t - \hat{\Pi}_{t+1} + \hat{\delta}_{t+1} \right] \\
\text{Marginal cost (l2)} &: \quad \widehat{mc}_t = \frac{1}{\kappa} \hat{\tau}_t + \hat{w}_t \\
\text{Marg. rate of subst.(l3)} &: \quad \widehat{mrs}_t = \frac{1}{1-h} \left(\hat{y}_t - h \hat{y}_{t-1} \right) \\
\text{Taylor rule (l4)} &: \quad \hat{R}_t^{not} = \rho \hat{R}_{t-1} + (1-\rho) \left[\gamma_\pi E_t \hat{\Pi}_{t+4} + \gamma_x \left(\hat{y}_t - \hat{y}_t^{pot} \right) \right] + \varepsilon_{R,t} \\
\text{ZLB (l5)} &: \quad \hat{R}_t = \max(-\ln R, \hat{R}_t^{not}) \\
\text{Price Phillips Curve (l6)} &: \quad \hat{\Pi}_t - \hat{\bar{\Pi}}_t = \beta E_t \left(\hat{\Pi}_{t+1} - \hat{\bar{\Pi}}_t \right) + (1 + \beta \varkappa) \kappa \widehat{mc}_t \\
\text{Wage Phillips Curve (l7)} &: \quad \hat{\Pi}_{w,t} - \hat{\bar{\Pi}}_t = \beta E_t \left(\hat{\Pi}_{w,t+1} - \hat{\bar{\Pi}}_t \right) + \kappa^w \left(\widehat{mrs}_t - \hat{w}_t \right) \\
\text{Wage inflation (n8)} &: \quad \hat{\Pi}_{w,t} = \hat{\Pi}_t + \hat{w}_t - \hat{w}_{t-1} \\
\text{Real potential rate (l9)} &: \quad \widehat{rr}_t^{pot} = -E_t \hat{\delta}_{t+1} \\
\text{Potential output (l10)} &: \quad \hat{y}_t^{pot} = 0
\end{aligned}$$

Log-linearizing the indexation equations gives:

$$\text{Indexation (l11):} \hat{\bar{\Pi}}_t = 0$$

Finally, the slopes of the Phillips curves are defined as:

$$\begin{aligned}
\kappa &= \frac{\left(1 - \beta \xi_p\right) \left(1 - \xi_p\right)}{\xi_p} \frac{1}{1 - (1 + \theta_p) \psi_p} \frac{1}{1 + \beta \varkappa} \\
\kappa^w &= \frac{(1 - \xi_w)(1 - \beta \xi_w)}{\xi_w} \frac{1}{1 - (1 + \theta_w) \psi_w}
\end{aligned}$$

We have 13 equations in the following 13 endogenous variables:

$$\hat{R}_t \ \hat{R}_t^{not} \ \hat{\Pi}_t \ \hat{\bar{\Pi}}_t \ \hat{y}_t \ \hat{w}_t \ \widehat{mc}_t \ \widehat{mrs}_t \ \hat{\Pi}_{w,t} \ \widehat{rr}_t^{pot} \ \hat{y}_t^{pot}$$

The variables $\hat{\delta}_t$, $\hat{\tau}_t = \tau_t - 1 = a_t$ and $\varepsilon_{R,t}$ are exogenous.

Appendix B. Additional Results: Data-Model Comparison Linearized Model

Figure B.1 provides a comparison between the data and the linearized model.

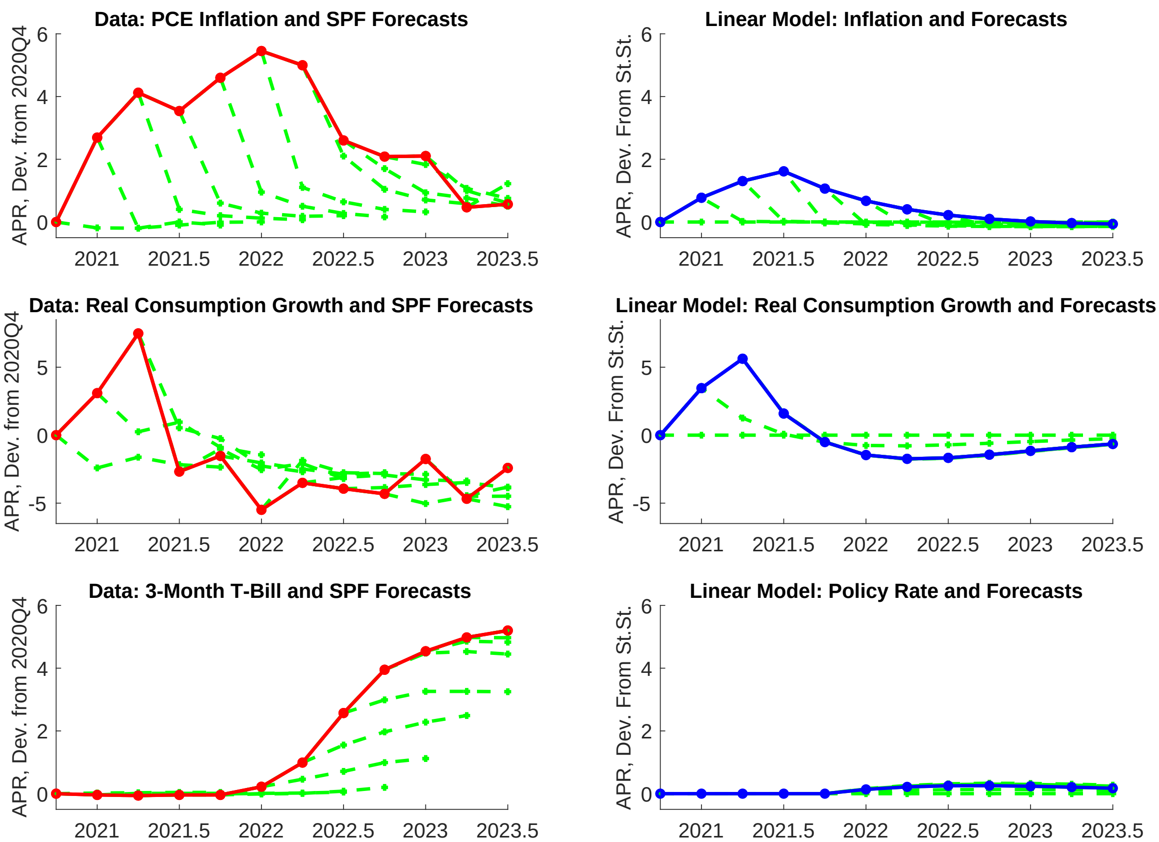

Figure B.1: Comparison of data vs. linear model.

Appendix C. Additional Results: Discount Factor Shock

Specification: discount factor shock of $\varepsilon_{\delta,0} = -0.01$, i.e. fall in discount factor of 1 percent (quarterly) or 4 percent (annualized). Fall in discount factor implies rise in demand (but no effects on potential output).

Figure C.2 provides the impulse responses of the nonlinear model to a discount factor shock.

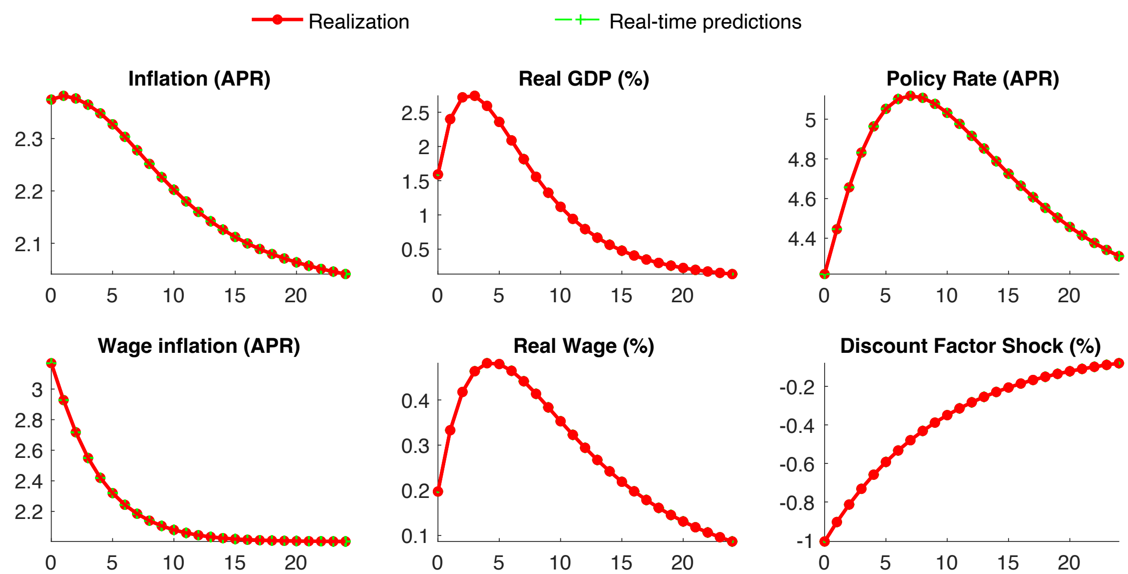

Figure C.2: Impulse responses to a discount factor shock in the nonlinear model.

Appendix D. Additional Results: Transitory Cost-push Shock

Here, we consider the case when the cost push shock is transitory, i.e. $\varepsilon_0^T = 0.0025$ shock to the transitory component of the cost push shock. I.e. 1/4 percent (quarterly) or 1 percent (annualized) cost-push shock. This analysis could be extended to show (or argue based on the the results below) that it is more optimal to 'look through' transitory cost push shocks.

Figure D.3 in the appendix provides the impulse responses of the nonlinear model to a cost-push shock when the latter is driven by the transitory (iid) component.

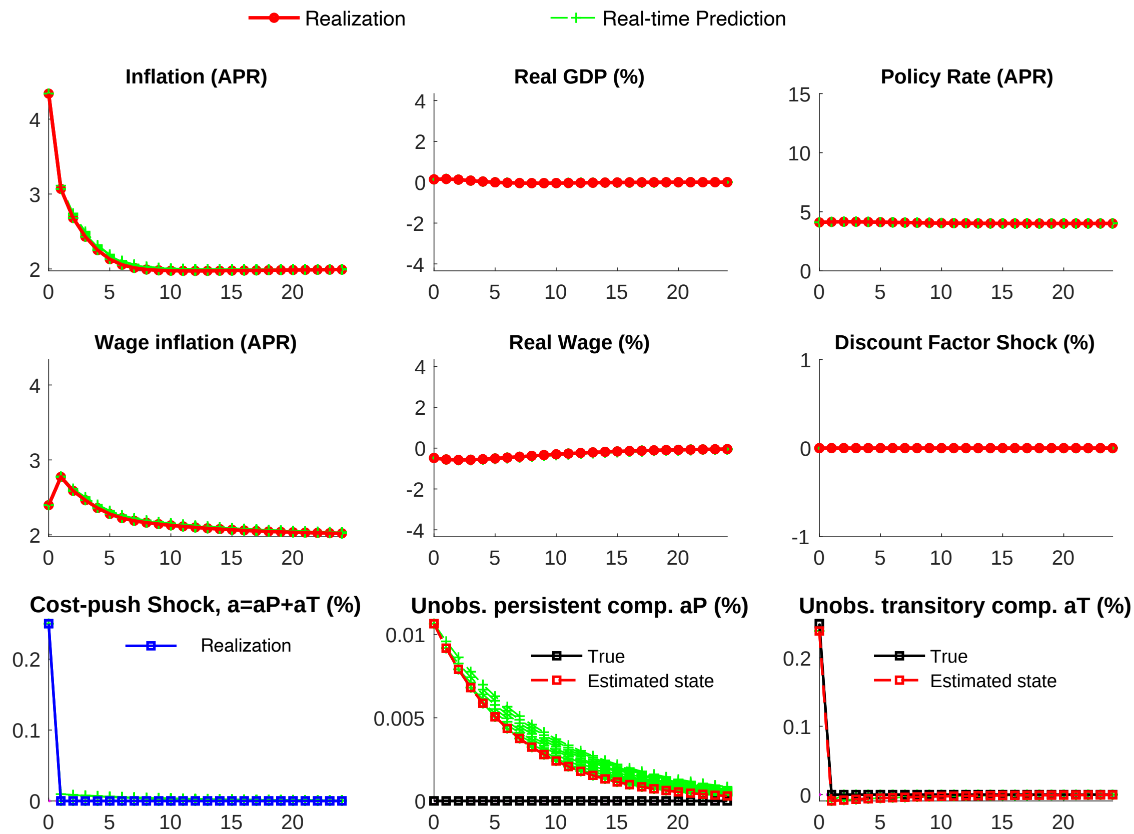

Figure D.3: Impulse responses to a transitory (iid) cost-push shock in the nonlinear model.

Appendix E. Additional Results: Standard Deviations of Components Representation

Figure E.4 shows the effects of assuming alternative values for the ratio of standard deviations (σ_P/σ_T) of the persistent and transitory components of the unobserved components representation of the cost push shock in the nonlinear model.

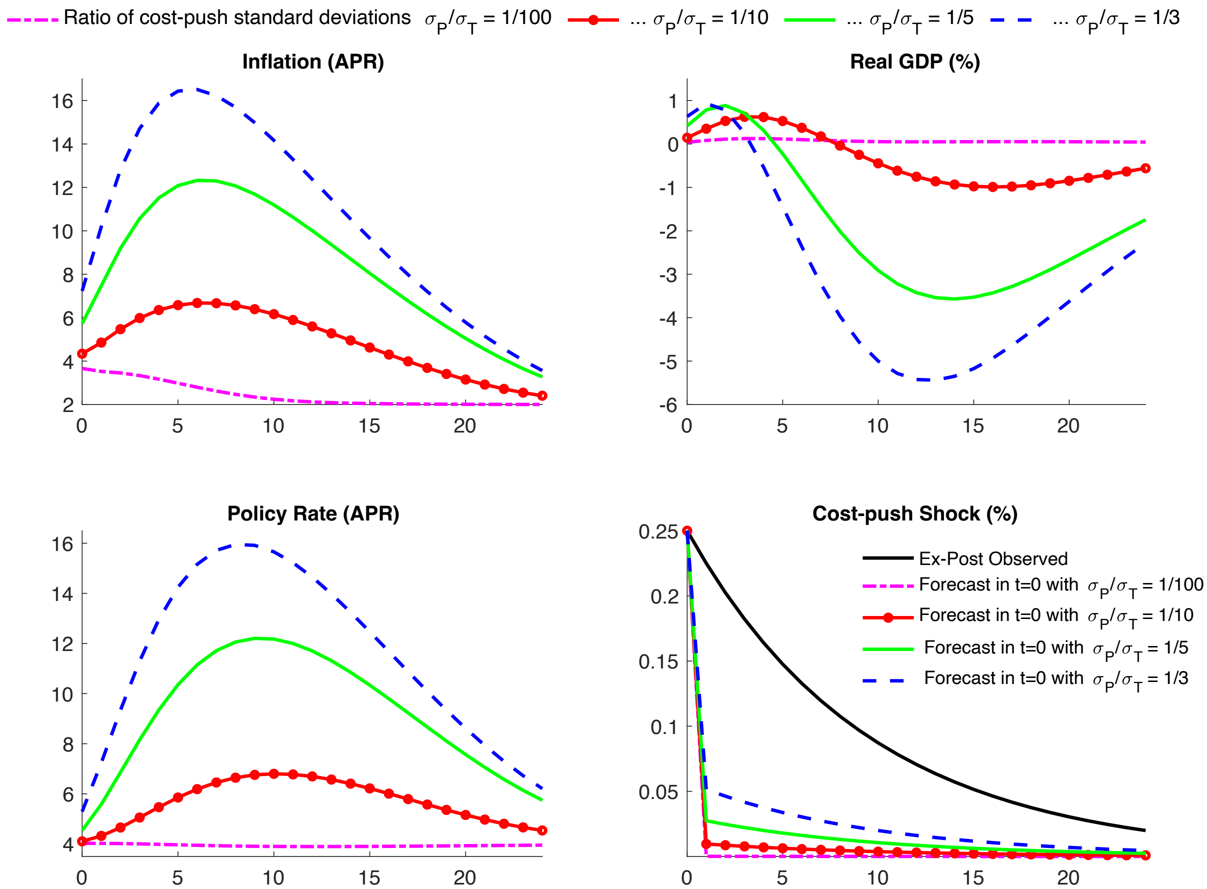

Figure E.4: Effects of alternative values for the ratio of standard deviations of the persistent and transitory components of the unobserved components representation of the cost push shock in the nonlinear model.

Appendix F. Additional Results: Timing of Monetary Policy Intervention

Here, we consider the implications when the central bank becomes more aggressive at different points in time. Specifically, Figure F.5 shows the simulation results for more aggressive monetary policy in nonlinear model for different start dates of the monetary policy intervention. All impulse responses are displayed in deviations from baseline. The key takeaway is that the earlier the central bank intervenes, the larger the reduction in inflation for a given hike in the policy rate. Put differently, monetary policy becomes less effective the higher inflation is to begin with. In this sense, the efficacy of monetary tightening and the sacrifice ratio are state-dependent in our model.

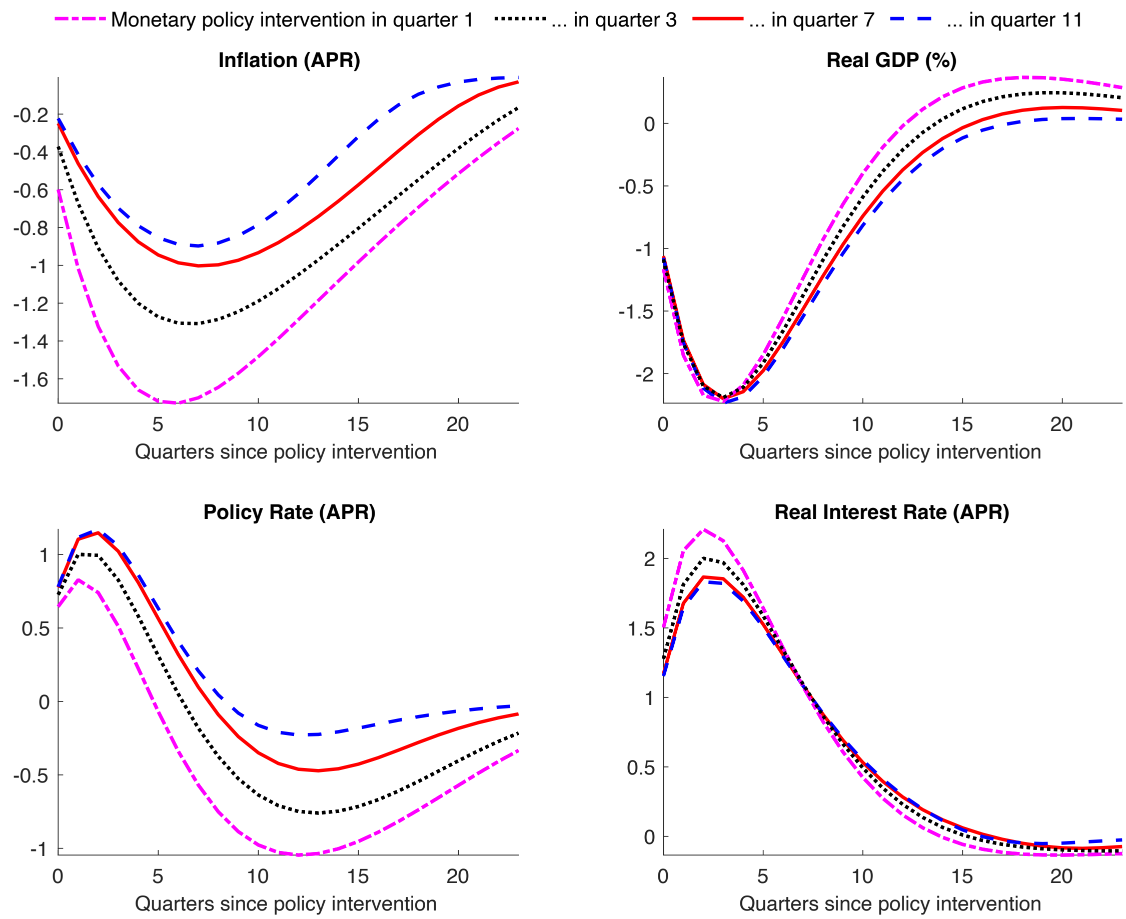

Figure F.5: Simulation results for more aggressive monetary policy in nonlinear model with different start dates for the monetary policy intervention. All responses in deviations from baseline.