Partial Homeownership: A Quantitative Analysis

Eirik Eylands Brandsaas and Jens Soerlie Kvaerner *

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Abstract

Partial Ownership (PO), which allows households to buy a fraction of a home and rent the remainder, is increasing in many countries with housing affordability challenges. We incorporate an existing for-profit PO contract into a life-cycle model to quantify its impact on homeownership, households’ welfare, and its implications for financial stability. We have the following results: 1) PO increases homeownership rates. 2) Willingness to pay increases with housing unaffordability and is highest among low-income and renting households. 3) PO increases aggregate debt as renters become partial owners but also reduces the average leverage ratios as indebted homeowners become partial owners.

Keywords: Partial Homeownership, Housing Affordability Crisis, Financial Innovation, Financial Stability

*Brandsaas: Federal Reserve Board of Governors; eebrandsaas@gmail.com. Kvaerner: Tilburg University; jkverner@gmail.com. We thank OBOS for sharing data that made this project possible and Ari Gelbard for excellent research assistance. Special thanks to the participants at BI Norwegian Business School, Federal Reserve Board, Tilburg University, Icelandic Economists Abroad conference, UW-Madison Alumni Conference, SFS Cavalcade, Indiana Kelley-Chicago Fed Conference on Housing Affordability, AREUEA, and Philadelphia Fed Consumer Finance Conference for their helpful comments. We benefited from discussions with Bas Werker, Joost Driessen, Jón Steinsson, Patrick Moran, John Driscoll, Ingjerd Gilhus, Espen Henriksen, Frank de Jong, Jac Kragt, Paul Lengerman, Pascal Maenhout, João Cocco, Søren Secher, Lu Liu, Nuno Clara (discussant), Carlos Garrica (discussant), and Dayin Zhang (discussant). Disclaimer: The views expressed in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of anyone else associated with the Federal Reserve System
1 Introduction

One of households’ most important financial choices is deciding whether to rent or buy a home. Today, many large cities struggle with high house prices and face housing affordability concerns (Favilukis, Mabille and Van Nieuwerburgh, 2022). Hsieh and Moretti (2019) predict that the lack of affordable housing options prevents cities from reaching their full growth potential. Housing and mortgage choices early in life are among the strongest predictors of where households end up in the wealth distribution at retirement (Bach, Calvet and Sodini, 2020).

Without financial innovations to promote homeownership, many future households will have few housing choices, which can reduce economic growth and perpetuate wealth inequality. While there is much work on housing affordability (see e.g., Favilukis and Nieuwerburgh, 2021, Garriga, Gete and Tsouderou, 2023, Molloy, Nathanson and Paciorek, 2022) and an increasing interest in optimal mortgage design (Campbell, Clara and Cocco, 2021, Guren, Krishnamurthy and McQuade, 2021), the effects of alternative homeownership contracts that bridge the gap between renting and owning remains an open question. In this paper, we study a contract that combines renting and owning—partial ownership (PO)—and study its effects on household welfare and financial fragility. The contract is now commonly used in Norway, Sweden, England, Australia, and China.

Our study is the first to incorporate a for-profit PO contract in a life-cycle model standard in the housing literature (see, e.g., Cocco, 2005, Attanasio, Bottazzi, Low, Nesheim and Wakefield, 2012, Davis and Van Nieuwerburgh, 2015). A PO contract allows households to buy a fraction of a home and rent the remainder. PO contracts are offered by for-profit homebuilders, financial intermediaries, and private-public partnerships.\(^1\) It has received substantial attention in the popular press since its introduction a few years ago. A recent survey in Norway revealed that 37% of all households and 70% of renters will consider PO in their next housing transaction.\(^2\)

In our model, households choose between renting, homeownership, and PO. The PO option includes buying between 50% and 90% of a house and renting the remainder. Home-

\(^1\)For example, coo.no, a fintech company, offers PO contracts for new and existing homes. OsloBolig, a joint venture between Oslo Municipality and private companies buys new apartments on the open market and offers PO contracts. Many large builders, such as OBOS, Selvaag and JM, also offer PO.

\(^2\)Opinion: Morgendagens Boformer (en: Ownership Options in the Future).
owners and partial owners can sell their entire housing investment but not a fraction. Partial owners can increase their ownership share at any time. Households differ in wealth and education and face uninsurable idiosyncratic income risk and uncertainty about future house prices. We estimate the model using simulated method of moments (SMM) on the administrative data from Norway and proprietary data on partial ownership.

Our first application is to understand how PO affects aggregate homeownership rates. In the short run, PO leads to a considerable reduction of households that rent as they switch to PO. Among 35-year-olds (the average age of PO users in the data), about 20% renter, which drops to 10% shortly after PO becomes available. In one year, PO has little impact on regular homeownership. In the long run, PO also decreases traditional homeownership. In the model, 20% of young households are partial homeowners, matching the previously mentioned survey evidence on the hypothetical demand for PO.

Our next application quantifies the willingness-to-pay (WTP) for PO. The mean welfare gain from having access to PO for households between 25 to 45 years—the primary users—is between 23% to 5% of disposable income. The estimated welfare gains exceed that of reverse mortgages (see Nakajima and Telyukova, 2017) and are comparable to optimizing financial investments (see, e.g., Cocco, Gomes and Maenhout, 2005), yet naturally smaller than the estimated lifetime gains of insurance (see Kojien, Nieuwerburgh and Yogo, 2016).

The model allows us to understand the heterogeneous demand for PO. As expected, renters have higher WTP for PO than owners. For example, a 35-year-old renter is willing to pay 33% of disposable income, compared to 6% for owners. There are two main reasons for the difference in WTP. First, PO allows current renters to obtain most of the utility benefits associated with 100% homeownership. Second, PO relaxes borrowing constraints that are more binding for renters than owners. Moreover, WTP is higher for households for whom housing is unaffordable: low-income, low-education, or low-wealth households, and households facing high house prices.

While PO has a high potential to increase welfare for many households, policymakers and regulators have financial stability concerns. The concerns of the Norwegian Financial

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3Several academic papers support their concerns. For example, Karapetyan, Kvaerner and Rohrer (2023) find households have high WTP for unregulated debt suitable to bypass mortgage regulation. Braggion, Manconi and Zhu (2022) find that online borrowing in China increases more for borrowers in areas subject to tighter mortgage regulation. Aastveit, Juelsrud and Wold (2022) find that tighter regulation decreases households’ liquidity buffers and increases financial fragility for affected borrowers.
Stability Authority and the Central Bank became evident in their reluctance to a recent policy proposal to relax regulations that allow builders to offer more PO contracts. Motivated by their skepticism, we study how PO affects two key household financial variables: debt-to-income (DTI) and homes entering liquidation.

We first calculate debt-to-income (DTI) ratios in many scenarios and for several household types. As expected, as many traditional renters borrow to become partial owners, the DTI ratio in the population rises. More interestingly, many households that are just wealthy enough to satisfy the regulatory constraints necessary for regular homeownership prefer PO and borrow less. Second, we study downsizing along the extensive and intensive margin to shed light on whether PO increases the number of financially vulnerable households. We find that PO has little impact on downsizing along the extensive margin but leads to a 50% decrease in the housing value lost among downsizers. From a financial stability viewpoint, our findings are remarkable: PO reduces the right tail of the DTI distribution and lowers the value of involuntary downsizing in bad times.

The finding that many renters and some traditional homeowners prefer partial ownership has several potential implications. First, the high take-up rates among young households suggest that PO has the potential to revert the drop in young homeownership after the Great Recession documented by Mabille (2022). D’Acunto and Rossi (2021) show that mortgage lending to low-income households declined in the U.S. immediately following the stricter regulation in 2010. Partial ownership offers an alternative homeownership method requiring less initial equity and borrowing.

Second, Cocco (2005) shows that due to investment in housing, younger and less fortunate households have limited financial wealth to invest in stocks, which reduces the benefits of equity market participation. He concludes that house price risk crowds out stockholdings. Because PO reduces the size of the house investment, it can mitigate the potential crowding-out effect housing investment can have on equities.

By the same logic, PO could also reduce several other documented adverse effects of regular homeownership (see e.g., Oswald, 2019, Kaplan and Violante, 2022, Kermani and Wong, 2021, Bond and Eriksen, 2021, Diamond, Guren and Tan, 2020, Campbell and Cocco, 2007). A common cause of these effects is the size of the housing investment. For example, PO could increase geographic mobility because it reduces the financial costs of moving. By the same token, PO relaxes the borrowing constraint and reduces the loan amount. That has
several potential implications, such as the distribution of “hand-to-mouth” households, the importance of parental wealth, and the impact of house price fluctuations on consumption.

There are many possible ways to “convexify” the rent or own decision. The perhaps best-known alternative is a shared equity loan (SEM), in which the lender funds part of the buyer’s downpayment in exchange for an equity share (see Benetton, Bracke, Cocco and Garbarino, 2021). Instead of interest payments, they receive their share of the home’s sales price. The main difference between SEM and PO lies in the ownership structure versus the financing arrangement. SEM users cover all expenses and cannot change ownership shares. PO involves joint ownership, where the ownership rights and responsibilities are divided among the co-owners according to a pre-specified agreement. A share appreciation mortgage (SAM) is another related product. It refers to a type of mortgage arrangement where the lender provides funds to a homeowner in exchange for a share of the future home price appreciation (see e.g., Greenwald, Landvoigt and Nieuwerburgh, 2021). For data availability reasons, our focus is on Scandinavia’s most common for-profit PO contract.

From a practical viewpoint, PO or similar equity-type instruments may have some advantages compared to the corresponding mortgage products, for example, regarding institutional barriers. In the US, government-sponsored enterprises such as Fannie Mae may impede the implementation of new mortgage products. In contrast, as Norway and Sweden have shown, PO is implementable without policy interventions. In the US, the Fintech company Quarter Inc. has offered partial ownership contracts since 2023.

To our knowledge, we are the first to incorporate a traded for-profit PO contract into a life cycle model calibrated and estimated using comprehensive microdata on wealth and homeownership and novel data on partial ownership. The paper closest to ours—developed coincidentally—is Koch (2023). While some of the analysis and results overlap with ours, the main focuses of the papers are different: We use our data to estimate preferences for for-profit partial ownership and use the estimated model to make predictions about the potential development of a PO market and to understand its implications for financial stability. She uses her model to understand portfolio choices and entry and exits in the housing market over the life cycle. Barras and Betermier (2020) study a theoretical asset allocation problem
with safe assets, equity, and housing but do not consider borrowing constraints. Their central insight is that households end up with smaller houses and relatively more of their wealth in housing than what they would if they could buy a fraction of a home. Partial ownership thus improves welfare in their model by allowing households to hold a combination of a better-diversified portfolio and living in a house that better matches their desired housing consumption.

In addition to previously cited papers, our paper is related to the long literature that uses structural models to understand life cycle patterns and quantify the cost of suboptimal choices or frictions and the benefits of new financial products resolving these issues, especially in the context of homeownership (see e.g., Ameriks, Caplin, Laufer and Nieuwerburgh, 2011, Nakajima and Telyukova, 2017, Kovacs and Moran, 2021, Karlman, Kinnerud and Kragh-Sørensen, 2021) Our contribution to this literature is to extend the standard housing model with PO. Apart from being necessary to answer our research question, the extension forces us to introduce a new utility parameter, the ownership-elasticity. We develop an identification strategy for the ownership-elasticity and find that households derive most of the utility benefits of ownership with modest ownership shares. This parameter is a necessary input to future papers that study partial ownership. For example, in a general equilibrium analysis of the housing market with PO, the ownership-elasticity will affect how the composition of PO users changes with the fees financial intermediaries charge for PO: A low ownership-elasticity implies less adverse selection.

2 Institutional Setting and Data

We explain the institutional setting, provide facts about who uses PO, and then describe our data sources.

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5The ownership utility premium measures the extra utility from housing services that are owned rather than rented (Davis and Van Nieuwerburgh, 2015). The ownership-elasticity measures how much of the utility premium partial owners obtain relative to their ownership share. We find that household owning 50% of a house receives 80% of the utility premium. We estimate the ownership-elasticity using granular data on PO.
2.1 The Norwegian Housing Market

We begin with a brief overview of the Norwegian housing market.\(^\text{6}\) Norway is characterized by high homeownership. Approximately 80\% of the Norwegians own their home, and 20\% rent. The homeownership rate in Norway exceeds those in the US and Australia (each at 66\%) and the European Union average (70\%), though it remains below China’s (90\%).

Households, corporations, and the government own these properties. Private landlords (e.g., households with two housing units) dominate the rental market with a market share of about 80\%. Corporations own about 75\% of the remaining units, and the government owns the rest (Sandlie and Sørvell, 2017; Stamsø, 2023). In Norway, there are two types of owner-occupation: “traditional” or through a co-op/housing association. Since the 1980s these two types are essentially identical.

Government policy encourages homeownership, and owner-occupied housing is treated favorably in the tax code, possibly contributing to the high homeownership rate. Households can deduct mortgage interest payments from income tax, and capital gains on the primary residence are tax-exempt. There is a progressive wealth tax with a maximum tax of 1.1\%. Only 25\% of the market value of the primary residence is subject to wealth tax. The municipality determines property taxation, and approximately 80\% of municipalities use it. In most cases, property tax is paid as part of other municipal taxes and makes up a tiny portion of the user cost of housing. To buy a house in Norway, one typically obtains a pre-qualification letter (“finansieringsbevins”) from a lender that verifies the borrower’s income, performs in-house risk assessments, and ensures compliance with loan-to-value (LTV) and debt-to-income (DTI) requirements (discussed below). Mortgages are generally floating rates with a 20-30-year payment plan. Most existing houses are sold anonymously at an ascending English auction, while most new homes sell at a fixed price.

The Norwegian rental market is deregulated, with little government intervention, and landlords can decide on the rent without restrictions, unlike Sweden and Denmark (e.g., Sodini, Van Nieuwerburgh, Vestman and von Lilienfeld-Toal, 2023). As in most countries, regulations provide tenant protection, contracts are largely standardized, and landlords have limited rights to terminate leases. Renters and landlords can annually re-adjust the rent in

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\(^\text{6}\)We discuss the ownership structure in the Norwegian housing market in Section I.A of the Online Appendix. For a discussion of financial regulation, we refer to Aastveit et al. (2022) and for more details on the rental market, we refer to Bø (2021).
line with the inflation index. This contributes to a stable and efficient rental market.

For landlords, there are differences between renting a part of a primary residence and a separate unit: Landlords who lease a portion of their primary residence or, in extenuating circumstances, can offer contracts for only one year. In contrast, all other lessees typically offer a three-year tenancy agreement. The consumer rights are stronger in the latter case. Moreover, landlords who rent out parts of their primary residence do not pay taxes on rental income. These factors contribute to the low share of commercial landlords.

Following the financial crisis, Norway, like many others, implemented stricter mortgage regulations, focusing on controlling loan-to-value (LTV) ratios for home purchases. In March 2010, regulators introduced guidelines capping mortgages at 90 percent of the house’s market value, with a subsequent reduction to 85 percent in December 2011, and formalized in 2015. Further amendments related to interest-only loans, payment capacity evaluations, and a debt-to-income (DTI) limit of five times gross annual income were introduced in in December 2016.

2.2 Facts, Myths, and Predictions about Partial Ownership

2.2.1 Partial Ownership Around the World

There are many types of PO contracts offered around the world. On one side of the spectrum are the PO contracts offered by for-profit companies, for instance, in Norway and Sweden, with no link to government-sponsored housing programs. The other side of the spectrum is the PO contract targeted at low-income families, offered, for example, in England and Australia. In China, pilot programs of a PO variation (“gong you chan quan fang”) were launched in 2007 in Huai’an and Shanghai and in Beijing in 2018 (Li, Qin and Wu, 2020). Some countries have both types of PO. While the English Government Shared Ownership Scheme offers PO mainly to low-income families, some private companies, such as Wayhome and Swan Housing, offer similar products to everyone.

For data reasons, we focus on Norway’s oldest and most common for-profit PO contract (“deleie”) offered by OBOS. OBOS is one of the biggest residential builders in Scandinavia. This contract allows households to buy a minimum ownership share of 50%. The household

\[\text{Whitehead and Yates (2010) provide a historical overview of the evolution of shared equity and ownership programs, focusing on Australia and England.}\]
can then later increase ownership in 10 percentage point increments. When buying larger shares, the price equals the maximum of the initial and current market price (the current market price equals the initial price times the local house price index). Both LTI and DTI requirements apply to PO. For example, if purchasing a 50 percent share of a home valued at 4 million NOK, the minimum downpayment would be 300,000 NOK, representing 15 percent of the purchase price of the household’s share. The household pays rent on the share not owned, with rent indexed to inflation just as for standard rental contracts. The household can list and sell the apartment at any time; at this point, proceeds are divided according to the ownership share, with sales costs divided proportionally. The household is responsible for in-unit maintenance (e.g., painting walls and maintaining appliances) while “shared fees” (e.g., maintenance of common areas and property taxes) are split according to the ownership share. OBOS guarantees the contract for ten years, after which they can list the property for sale. The income from the sale is shared according to the ownership shares. Because the contract is so new, no one has used the contract for ten years. Another possibility is that OBOS could extend the contract for ten more years.

Partial ownership has received much attention in the media since its introduction in Norway in 2020. A recent survey of adults in Oslo quantified the high interest in the PO contract among consumers: 37% of all households and 70% of renters consider PO in their next housing transaction and interest for new contracts was most prominent among low income households.\footnote{Opinion: Morgendagens Boformer (en: Ownership Options in the Future).} Indeed, the strong demand for these properties prompted other homebuilders (e.g., JM and Selvaag) to offer PO contracts, motivated public-private joint partnerships (e.g., Oslo Bolig), and led financial intermediaries to offer PO contracts (e.g., Coo) on both new and existing homes.

### 2.2.2 PO, Housing Frictions and PO in the Long-Run

PO reduces three frictions in rental contracts, which has clear implications for the development of the PO market. First, it mitigates the “fundamental rental externality”—resulting from moral hazard—that leads to higher maintenance costs on rental units since tenants have fewer incentives to reduce wear and tear (see, e.g., Henderson and Ioannides, 1983). With PO, the households have some ownership, which aligns incentives better. In addition,
in the Norwegian contract, the household is responsible for all in-unit maintenance. Second, PO also reduces costs associated with adverse selection in the rental market (e.g., households with volatile incomes are more likely to have problems paying rent). Landlords typically use security deposits or other screening mechanisms such as credit reports to learn about the quality of the tenant (see, e.g., Miceli, 1989). PO owners have, prior to the purchaser, often been through similar screening by the mortgage provider, and hence, less effort is necessary from the owner of the remaining share of the house. Third, vacancies are expensive for landlords. For example, in the U.S., the rental vacancy rate is typically between 5-10%. With PO, there is no vacancy: Either the household becomes a traditional homeowner after some time or the unit is sold. PO also reduces inefficiencies in the owner-occupied market. For example, a common concern with new homes is that builders take shortcuts to save costs that eventually lead to higher maintenance. When the house builder is also the owner of the fraction of the house not owned by the household (as is typical with PO contracts), it will share maintenance costs and, therefore, have little less incentive to take shortcuts in the construction process. In summary, the fact that competitive rental and housing markets have existed for hundreds of years; a simple convex combination of the two that, in addition, reduces traditional inefficiencies in the two markets also to be sustained.

2.2.3 The Supply Side

While there are good reasons why a PO market is sustainable, it does not answer why developers—such as OBOS in Norway and Sweden—have begun to offer PO in addition to traditional homeownership. For developers, one motivation is to sell a higher quantity of houses in a world with tight mortgage regulations and increasing development costs without lowering prices. For financial intermediaries, such as “Coo.no,” the most obvious motivation is that households’ high willingness to pay for PO is large enough to make PO a positive NPV project. To examine the profitability of PO, we have used the model to compare the present value of cash flows from selling a unit, renting it out as a traditional landlord, and offering PO. In expectations, renting it out gives the highest present value (due to house price appreciation). In contrast, PO gives the highest risk-adjusted present value (i.e., the average present value from the simulation scaled by the volatility of present values). Thus,
our simulations provide a simple financial rationale for offering PO.\(^9\)

### 2.2.4 Do people understand what they pay for with PO?

While we think people in our sample understand the PO contract, we acknowledge that financial illiteracy, behavioral biases, or mistakes can also affect the demand for PO. Nevertheless, the contract appears transparent and easy to understand.\(^10\) The only choice the buyer makes is the ownership share, and there are no hidden fees or cross-subsidies across consumers. All PO providers have websites with contact details and frequently asked questions. Moreover, in all OBOS sale listings that allow PO, households can adjust their desired ownership share and see how it adjusts the purchase amount, adjustment costs, maintenance costs, and rent payments (see Figure A1 for an illustration). This allows potential users to visualize all expenses associated with PO, which improves contract transparency.\(^11\)

### 2.2.5 Who Use Partial Ownership?

We now briefly discuss, based on data provided to us by OBOS, information on which households buy homes using PO contracts, reported in Figure 1.

The left panel in Figure 1 shows high growth in the share of new homes sold with PO contracts. In personal communication with OBOS, they shared that the share of homes sold with PO would be larger without legal barriers limiting the number of PO contracts per apartment building. Under current regulations, a building can have PO on up to 20% of the

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\(^9\) An exciting avenue for future research is to understand, from a general equilibrium risk-sharing viewpoint, what agents should supply the PO contract. For example, developers are already positively exposed to property markets through their business model; holding more assets that correlate positively with their regular activities is not necessarily optimal from a societal perspective.

\(^10\) A rationale for contract transparency would be that the long-term gains from development in terms of being able to sell more property in a market with high house prices and multiple borrower-based mortgage regulations exceed the short-term profits, which has been the primary objective of several financial products (e.g., Hirshleifer, 2015).

\(^11\) We refer to Appendix A.2 for further details.
Figure 1: Summary Statistics from PO Contracts. The data is provided by OBOS and is based on PO transactions. The left panel plots the share of sales in PO. The middle panel plots the mean and standard deviation of the buyer’s age. The right panel plots the initial ownership share.

The center panel in Figure 1 displays the age of PO users. The average buyer is just under 40, with a relatively large variation. The low age suggests that PO is most attractive to first-time homebuyers, who tend to be young. One reason the contract is relatively more appealing for younger households is the 10-year contract length, making PO a poor option for older households who want to downsize or tap into home equity using reverse mortgages. In addition, the characteristics of the average PO units suit young households: it has 2.67 rooms (in addition to a kitchen and bathroom), is 59 square meters, and is in a housing association with 117 units. The right panel of Figure 1 plots the distribution of initial ownership shares. We see that 59% of PO contracts start with the smallest possible ownership share of 50%. As of September, 31% of partial owners have increased their ownership shares at least once. According to OBOS, a change in ownership share tends to exceed the minimum share.

For example, if OBOS builds a five-unit building and sells one unit with PO, where they own 10% and the household the remaining 90%, they are counted as a full owner and so “own” 20% of the apartment building, though they only own 2%. A recent government public policy proposal suggested changing the limit to 50% and counting the ownership share accurately. If passed, the proposal would relax the constraint limiting OBOS and other PO providers to meet the current demand for PO. The proposal has gathered broad support from builders, regional governments, and the financial industry. The Norwegian Central Bank and the Financial Supervisory Authority have objected to the proposal on financial stability grounds and concerns about households’ financial vulnerabilities. These concerns motivate our study of how PO affects household fragility in Section 5.3.

This reflects, of course, in part that the current contract is made to fit the preferences of young households.
of 10%. In sum, the typical PO buyers are relatively young, use PO on small apartments in apartment buildings in the largest urban areas in Norway, and most choose initial ownership shares of 50%. We reproduce a standard PO unit for sale listing in Figure A1.

2.3 Other Data Sources

We use multiple additional data sources to estimate the model. Information on wealth, residential choices, income, and education comes from the Norwegian Tax Registry (NTR) and Statistics Norway (SSB). NTR is responsible for collecting income and wealth taxes in Norway. By law, employers, banks, and public agencies must submit personal information on income, total assets, and transfers to the NTR every year. Individuals are accountable for the accuracy of the information in their tax returns, and the submission of inaccurate information is punishable by law. We have data on PO contracts from OBOS, the largest homebuilder in Scandinavia and the largest supplier of PO. Eiendomsverdi AS provides transaction data on housing. EV estimates the market value of the Norwegian residential real estate market. Section A.1 of the Appendix explains how we construct our sample, calculate the statistics we use to estimate the model. Section 1.2 of the Online Appendix explain how we estimate the parameters for the income process. Together, these statistics are sufficient to replicate all the results in the paper.

3 Model

We now present the model, which nests a standard life-cycle homeownership model. Our innovation is the introduction of the PO contract.

3.1 Setup

The unit of analysis is a household $i$ of age $a$. Each period $t$ corresponds to one year. The household enters the model at age 24, works for $K$ years, and spends $T - K$ years in retirement. During this period, the household maximizes utility by choosing consumption, $C_{i,a}$ and housing services $H_{i,a}$, including the ownership share $S$. 
3.1.1 Preferences, Choices, and the Life Cycle

Omitting subscript \(i\), households choose consumption, housing, and ownership to maximize the discounted sum of lifetime utility:

\[
\max_{C_a, H_a, S_a} \mathbb{E}_{24} \sum_{a=24}^{T} \beta^{a-24} \frac{(C_a^{1-\eta} H_a^\eta \chi(S_a))^{1-\gamma}}{1-\gamma},
\]

where \(\beta < 1\) is the discount factor, \(\gamma\) is the coefficient of relative risk aversion, and \(\eta\) measures the relative importance of housing services.

The utility from housing services depends on ownership status \(S_{i,a}\) through a premium capturing any enjoyment agents derive from owning rather than renting their home, \(\chi(S) = 1 + \chi S^\alpha\), where \(\alpha\) is the ownership-elasticity. Without PO, we have \(S = 0\) for renters and \(S = 1\) for owners, and \(\alpha\) is redundant. With PO, households with ownership status \(S \in (0, 1)\) are partial owners, and \(\alpha\) is a free parameter. As \(\alpha \to 0\), households receive the full premium regardless of the ownership share. As \(\alpha \to \infty\), sole ownership is necessary to receive the full premium. If \(\alpha = 1\), the utility shift is linear in the ownership share.

Since the average age of PO users is 35 years we abstract away from features important only for older and retired households, such as stochastic mortality, bequest motives (Ameriks et al., 2011, Lockwood, 2018, Kvaerner, 2022), and “aging-at-home” preferences (Cocco and Lopes, 2019) to simplify the problem.

3.1.2 The Labor Income Process

Households enter the model at age \(a\) with an education level \(e\), which affects their income stream. Before retirement, the labor income, \(Y_{a,e}\), is exogenously given by:

\[
\ln(Y_{a,e}) = f(a, e) + \nu_{a,e} + \epsilon_{a,e}, \tag{2}
\]

where \(f(a, e)\) is a deterministic function of age \((a)\) and education \((e)\). The stochastic component governs the sum of a transitory shock \(\epsilon_{a,e} \sim N(0, \sigma^2)\) and a persistent shock:

\[
\nu_{e,a} = \rho_e \nu_{e,a-1} + u_{e,a}. \tag{3}
\]
where \( u_{a,e} \sim N(0, \sigma_u^2) \). Following Fagereng, Gottlieb and Guiso (2017), the parameters \( \rho_e, \sigma_u, \sigma_\epsilon \) depend on education. After retirement \((a > K)\), income is a constant proportion \( \phi \) of income at retirement age \( K \). We provide variable definitions and explain the estimation procedure for the income process in Appendix C.2.

### 3.1.3 Housing

To understand the demand for PO today, it is crucial to have a realistic representation of house prices, which has a direct effect on the marginal households’ demand for PO. While in the long-term, widespread use of PO can also affect households’ demand functions indirectly through a potential impact on house prices and rental contracts, in the short to medium term, this effect is negligible.\(^\text{14}\) Therefore, we follow the standard practice in the literature using finite life cycle models with housing to understand household choices and assume one stochastic house price index. Let \( P_t^H \) denote the date \( t \) real housing price, and let \( p_t^H \equiv \ln(P_t^H) \). We model the log real house price as a random walk with drift as in Vestman (2019), Cocco and Lopes (2019) Thus, the growth in real house prices is:

\[
\Delta p_t^H = \mu + \sigma_h Z, \tag{4}
\]

where \( Z \) is i.i.d. \( N(0, 1) \). As PO is primarily offered in densely populated areas where the supply of housing stock is restricted, we omit an endogenous construction sector as in, for example, Murphy (2018).

The rent-to-price ratio, denoted by \( \kappa \), is constant. As is standard, a subset of housing sizes is available for rent and another for owner-occupation (see, e.g., Kaplan, Mitman and Violante, 2020). We denote the housing choice sets by \( \mathcal{H}(S) \). Transacting in the real estate market is costly. First, homeowners pay depreciation proportional to the market value, which is the sum of property maintenance \( \delta \) and interior house maintenance \( \tau \). For owners, depreciation and capital gains are due for in the following year: \( S_{a+1}(P_{t+1}H_{a+1}(1 - \tau - \delta)) \), where \( H_{a+1} \) is the house the household chooses to live in this period. In contrast, rent

\(^{14}\)Consequently, our model cannot speak to whether PO impacts house prices in specific housing market segments. In our data, there is no evidence that apartment types that offer PO sell at prices different from those without PO. Yet, as mentioned, it is possible that widespread use of PO in the long term affects the cross-sectional distribution of house and rental prices and thus indirectly impacts choices through this channel. We leave such a general equilibrium analysis for future research.
\( \kappa P_t H_{a+1} \) is due in the current year. Second, to buy (sell) an owner-occupied unit, households pay adjustment costs \( m_b \) (\( m_s \)) proportional to the market value.

Since house prices follow a random walk and the rent-to-price ratio is constant while income is stationary, prices can, in theory, reach a level where households cannot afford the rent for the smallest unit. We include a welfare system that provides a price-dependent minimum wage \( y(P) \), indexed to the market rent of the smallest unit plus a consumption floor \( c \).

### 3.1.4 Wealth

Household wealth, \( W \), equals a cash account plus housing wealth. The cash account pays an interest rate of \( r_f \). The household can take out a mortgage. Mortgages are available at the interest rate \( r_f + \theta \), where \( \theta \) denotes the mortgage premium.

All mortgages are one-period instruments rolled over every year. Households can costlessly adjust the size of the mortgage. As a result, households with mortgages do not have cash. Depending on context, we refer to the net position as liquid wealth, \( LW \), or debt \( D \). The return on liquid wealth is:

\[
r(LW) = \begin{cases} 
  r_f + \theta, & \text{if } LW < 0 \\
  r_f, & \text{otherwise.}
\end{cases}
\]  

(5)

Borrowing is subject to a liquid wealth-to-value (LTV) and a debt-to-income (DTI) constraint.

### 3.1.5 Partial Ownership

We now discuss how we model the most common PO contract in Norway, as described in Section 2.2. To reduce the computational cost, we only allow partial ownership shares of 50% and 75%, i.e., \( S \in \{0.0, 0.5, 0.75, 1.0\} \). Once a household achieves 100% ownership, the contract is terminated, and they become a traditional homeowner. A partial owner can increase ownership but not decrease it without selling the entire share.
Buying, selling, and changing ownership shares entails fixed costs: $l_b, l_s, l_c$. When changing ownership shares, the purchase price depends on the house price ($P_0^H$) at the start of the contract and the current one ($P_t^H$). If today’s price exceeds the price at the contract’s start, the issuer charges today’s price. Otherwise, it charges the starting price. The cost of increasing the ownership from $S_a$ to $S_{a+1}$ is therefore:

$$P_t H_a \Delta S_{a+1} (1 + m_b) + l_c.$$

The first term in Eq. 6 is the payment for buying a house fraction $\Delta S_{a+1}$, $l_c$ is a fee, and $P_t = \max\{P_t^H, P_0^H\}$. Note, perhaps surprisingly, that the option element of the contract is not particularly important for our analysis; removing it increases WTP by only 1-2 percentage points. The main reason for its low value is that households can, if prices fall, “reset” the contract by selling the unit, receiving their share, and then entering into a new PO contract. Once the price level falls substantially below $P_0$ this becomes cheaper. As a result, $P_0^H$ is only relevant when households experience small price drops from the initial purchase price. In addition, real estate prices and labor income are uncorrelated in the model, which means that few households experience a large drop in labor income and a rise in the value of the option they are short at the same time.

There are three recurring costs associated with PO. First, the partial owner must pay market rent on the share of the property not owned, i.e., $P_t H_{a+1} \kappa (1 - S_{a+1})$. Second, the household pays all interior house maintenance $\tau$, irrespective of their ownership share. Third, maintenance expenses $\delta$ (e.g., outdoor painting) are divided between the owners according to ownership shares.

As our focus is on non-retirees, we simplify the risks and preferences necessary to match the behavior of retirees. Specifically, we omit stochastic mortality and bequest motives, health risks, and preferences for aging-in-place. This mechanically leads old households to choose the instruments in the model, which leads to a rapid decumulation of wealth. In our model, this means that essentially all old households would use PO, which is inconsistent.

---

Footnotes:

15For example, a household who owns 75% of its home and wants to own 50% would have to sell its home, receive 50% of the value, then buy a new home with a 75% share, and pay their share of the sales cost $m_s$ (75%), their share of the buying cost $m_b$ (50%), and 100% of associated legal fees $l_b$ and $l_s$ (one for selling and for buying).

16We provide a numerical example to illustrate this point in Section B.2 of the Appendix.
with the data and the intention of the PO contract.\textsuperscript{17} To prevent retirees from dominating the PO market, we impose a cost that is gradually rising after age 55.

\[ c(a, \varpi) = \varpi \max\{0, a - 55\}. \]  

(7)

The function is a parsimonious way of smoothly modeling details in the actual PO contract, which makes it unattractive for senior households (see Appendix A.2), mainly that the contract is only guaranteed to last for 10 years. We choose this ad hoc function for simplicity, instead of adding more features as described above or including a state variable for length of ownership. For young households, the function does not impact the demand for PO.

3.2 Recursive Formulation and Decision Problems

We now state the recursive formulation and explain the decision problem. We omit subscripts and use prime superscript for next-period values to save on notation.

The timing is as follows: The house price and the household-specific income shocks appear at the start of a period. After observing these values, the household chooses consumption and housing to maximize indirect utility. The utility of today’s choices is realized in the current period. For example, a household that enters age \( a \) as a renter (\( S = 0 \)) but buys a house (\( S' = 1 \)) receives the utility kick at age \( a \).

3.2.1 Budget Equations

Households choose consumption \( C \), housing \( H' \), ownership status \( S' \), and liquid wealth \( LW' \). Renters pay rent, while homeowners keep the house on the balance sheet. We introduce the function \( ac(P, S, H, S', H') \) that calculates the adjustment costs as a function of house price, choices, and size. The budget equation for a household with wealth \( W \) and income of \( Y \) is:

\[ W + Y = C + LW' + ac(P, S, H, S', H') + (1 - S')\kappa PH' + S'PH'. \]  

(8)

A partial homeowner faces the same budget equation, except for the possibility of chang-\textsuperscript{17}This result is similar to the “puzzle” of why older households have such high homeownership rates (Cocco and Lopes, 2019, Nakajima and Telyukova, 2017), and points to an unexplored market for PO that we leave to future research.
ing the ownership share:

\[
W + Y = C + LW' + ac(P, S, H, S', H') + (1 - S')\kappa PH' + S'PH' + 1_{\Delta S' \neq 0, \Delta H' = 0}\Delta S' \bar{P}_t H'. \tag{9}
\]

The evolution of wealth is determined by liquid wealth \((LW')\) and the market value of the ownership share in housing net of depreciation:

\[
W' = LW'(1 + r(LW')) + 1_{S' > 0}P'H'(S'(1 - \delta) - \tau) \tag{10}
\]

### 3.2.2 Decision Problems

All households make the same choices regardless of homeownership. The state variables for renters and regular homeowners are: \(\Xi \equiv \{W, H, S, \nu, P, a\}\) while partial owners have an extra state variable: the house price level when they first became partial owners, \(P_0^H\), since the price of increasing ownership shares depends on the initial price.

Because the decision problems of retired households are identical except for the removal of permanent income shocks, we only explain the case of working-age households. The Bellman equation for renters and regular homeowners is:

\[
V(\Xi) = \max_{C, H', LW', S'} \left\{ u(C, H') + \beta E[V(\Xi)] \right\}, \tag{11}
\]

subject to

\[
\begin{align*}
C &> 0, \tag{12} \\
S' &\in \{0, 0.5, 0.75, 1\}, \tag{13} \\
H' &\in \mathcal{H}(S'), \tag{14} \\
D' &\geq LTV \times PH'S', \tag{15} \\
D' &\geq DTI \times YS', \tag{16}
\end{align*}
\]

the age constraint (Eq. 7), the budget equation (Eq. 10), and the law of motion in Eq. 8. The constraints have simple interpretations: Eq. 13 shows that a household must choose to rent or own. Eq. 14 shows that the housing choice set depends on ownership status. Eqs.
and 16 show that borrowing is only available for home purchases and must satisfy both LTV and DTI requirements.

The analog decision problem of a partial owner is identical, except that the initial price becomes an additional state variable:

$$V(\Xi; P^0_H) = \max_{C, H', B', S'} \left\{ u(C, H') + \beta E \left[ V(\Xi'; P^H_0) \right] \right\},$$

subject to Eq.: 7, 10, 9, and Eq. 12-16.

4 Parameterization

We solve the model with standard numerical methods, with details in Section 3 of the Online Appendix. The parameterization of the model contains three stages. First-stage parameters are from other papers, or estimated directly in our data. In the second-stage, we estimate the discount factor $\beta$ and the utility shifter for homeownership $\chi$. We estimate these parameters by matching wealth levels and homeownership rates. In the third-stage we estimate the ownership-share-elasticity $\alpha$ by matching the average ownership share of new partial owners. Table 1 summarizes all parameters.

4.1 First-stage Parameters

We first discuss the calibration of parameters common for the model with and without PO and then discuss parameters only relevant to PO.

4.1.1 Common Parameters

Following Campbell and Cocco (2015), we set the coefficient of relative risk aversion $\gamma$ to 2.0. We set the preference weight on housing $\eta$ to 0.3, the average for households aged 27-45 in Yao, Fagereng and Natvik (2015), which is calibrated to Norwegian data.

We estimate the labor income process using administrative data for the Norwegian population from 1993 to 2018. As Campbell and Cocco (2015), we rely on a broad measure of household income, the sum of gross salary income and pension plus net capital income and total government transfers minus tax. Retirement income is a fraction of the last income
before retirement. We set the pension-to-income ratio to match the estimate in Fagereng et al. (2017). Details of the estimation of the deterministic part of labor income and the variances of the transitory and permanent shocks are in Appendix C.2.

Our model relies on several assumptions and parameters related to housing. First, we start with the nominal home price index to estimate mean house price growth ($\mu$) and its standard deviation ($\sigma_h$). We deflate this index by median after-tax household income since income is stationary in the model. Figure 2a displays the evolution of nominal, real, and income-deflated house prices. Expected log house price growth equals the time-series mean, $\mu = 0.023$. Because prices of individual homes are about twice as volatile as price indices (Landvoigt, Piazzesi and Schneider, 2015, Case and Shiller, 1989), we double the volatility and set $\sigma_h = 0.0564$.19

Second, we include three house sizes: 44, 77, and 100 square meters. These sizes correspond to the 5th, 25th, and 50th percentile of residential units. We omit large houses because wealthy households with large homes are not in the partial ownership market. We use the smallest unit as a numeraire. Only the two smallest units are available for renting and the two largest for owning. Only the middle unit is available for PO.20

Third, we need a rent-to-price ratio $\kappa$, which we calculate as follows. We start with yearly rent statistics, which differ by size, number of rooms, and type (single-family, small multifamily, and multifamily). We divide the rent per square meter for units with five rooms by the single-family square meter price. Similarly, we divide the 4-room rental price by the small multifamily price and the 3 and 2-room prices by the multifamily price. The normalization ensures that the rent-to-price ratios are functions of both square meters and housing type. The ratios are relatively stable in the years we have data, 2012-2022.21 Our rent-to-price ratio in the model is equal to the average ratio of these four series, $\kappa = 0.044$, which is close to the commonly used American estimate of 0.05 by Davis, Lehnert and Martin (2008) (see Figure A4e for illustration).

---

18All data are publicly available at Statistics Norway: Income (Table 04751), CPI (Table 03014), and existing home price index (Table 07230).
19As a reference point, the ratio average arithmetic real house price growth to volatility is 0.44, comparable to a value-weighted Norwegian stock index over the same period.
20In Appendix B.1, we show that our overall conclusions regarding demand for PO and WTP are insensitive to reasonable changes to the size of the PO unit.
21All data are publicly available at Statistics Norway: Square meter prices (Table 06035) and square meter rent (Table 09895).
Fourth, there are costs associated with adjusting the housing choice. We set these costs to match the institutional setting. First, buyers pay a transaction tax of 2.5% of the purchase price, so we set $m_b = 0.025$. Second, the real estate agent charges an average of 2% in sales commissions, and households often pay additional costs such as staging (Yao et al., 2015). Hence, we set $m_s = 0.025$.

The risk-free rate $r_f = 0.0143$ as in Fagereng et al. (2017). We set total depreciation on housing to $\tau + \delta = 2.5\%$ as in Yao et al. (2015). Following Norwegian law, the maximum LTV and DTI are 0.85 and 5.0. Since 2000, the mortgage tax deduction has ranged from 22-28%, and the average mortgage premium has been 2.6%. Thus, we set the mortgage premium $\theta = 0.016$ to account for the tax deduction on the mortgage rate (given by the risk-free rate plus the premium). We set the consumption floor to $\underline{c} = 100,000$ NOK, comparable to the financial subsistence benefits of a single household.

When simulating the households, we calibrate the joint distribution of initial prices, wealth, and productivity as discussed in Section 2 of the Online Appendix.

---

22 The maximum DTI ratio of 5 are on total income before taxes. The average income tax rate on 35 year old households is about 20%, implying a DTI of 6 on after-tax income. However, unlike the LTI, it also includes all other debt, such as student loans, car loans, and unsecured credit, which many young households have. Moreover, regulation requires that borrowers must be able to manage a mortgage rate increase of 5 percentage points (3 percentage points from 2022). This ‘stress test’ primarily results in lower income with uncertain income having lower DTIs. Taking all these factors into account, we set the after-tax DTI limit 5.0.

23 Table 08175, Statistics Norway.
4.1.2 Partial Ownership Parameters

Entering a PO contract costs \( l_b = 2,128 \text{ NOK} \) (filing fees). The cost of changing ownership shares \( l_c \) is NOK 8,701. This value matches what OBOS charges, plus various government fees.\(^{24}\) Selling the contract is free \( l_s = 0.0 \). PO is available for medium-sized houses, which are the closest to the average size of units sold with PO (66 square meters in 2023). Restricting PO in the model to the most common size sold with PO in the data ensures consistency between PO parameters and speeds up the model.

The relative share of depreciation due to property \( \delta \) and interior maintenance \( \tau \) matter for PO, since only the latter is scaled by the ownership share. Since PO is offered only on new construction, which requires little work beyond regular property maintenance, we set \( \delta = 0.02 \) and \( \tau = 0.005 \).

4.2 Second-Stage: Wealth and Homeownership over Age

We use SMM to estimate the discount factor \( \beta \) and the utility shifter for homeownership \( \chi \). We need these parameters to solve the model without PO. The point is to match a set of stylized facts before we introduce PO.

The empirical moments we target are the average net worth and the homeownership rate of households aged between 30 and 50 years. Each age contains two moments, resulting in an overidentified system with two parameters and 40 moments. The identification is straightforward. A higher discount factor \( (\beta) \) increases wealth accumulation. A higher ownership preference \( (\chi) \) increases homeownership.

Although our model simulation starts at 24, we do not target the first ages in the estimation. The reason is to reduce the impact on initial conditions and the high homeownership rates among the youngest households in the data—typically due to reasons not in the model (e.g., parental support)—on the estimates.

The SMM estimator is defined as follows. Let \( \hat{m} \) denote the vector of empirical moments we target in the estimation. The parameter vector of interest is \( \omega \equiv \{ \beta, \chi \} \). Given a candidate parameter vector \( \omega \), we solve the model and calculate the equivalent simulated

\(^{24}\)OBOS charges six times the standardized inflation-indexed legal fee, the so-called “rettsgebyr” set to NOK 1,243 in 2023. To include other smaller fees charged by the public sector, we set the total change cost to seven times the “rettsgebyr”. 
### Panel A: Externally Calibrated Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preference</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Risk aversion</td>
<td>$\gamma$</td>
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</tr>
<tr>
<td>Exp. Share Housing</td>
<td>$\eta$</td>
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<tr>
<td>Non-Housing Markets</td>
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<td></td>
</tr>
<tr>
<td>Risk-free rate</td>
<td>$r_f$</td>
<td>0.0143</td>
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<tr>
<td>Mortgage premium</td>
<td>$\theta$</td>
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<tr>
<td>Wage profiles</td>
<td>$f(a)$</td>
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</tr>
<tr>
<td>Permanent income shock</td>
<td>$\sigma_v$</td>
<td>Tab. A2</td>
</tr>
<tr>
<td>Transitory shock var.</td>
<td>$\sigma_\epsilon$</td>
<td>Tab. A2</td>
</tr>
<tr>
<td>Retirement Income Drop</td>
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<tr>
<td>Housing</td>
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<td></td>
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<tr>
<td>Rent-to-price ratio</td>
<td>$\kappa$</td>
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</tr>
<tr>
<td>Sales cost</td>
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</tr>
<tr>
<td>Purchase cost</td>
<td>$m_b$</td>
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<td>Property maintenance</td>
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<td>In-unit maintenace</td>
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<td>PO change cost</td>
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<td>PO sales cost</td>
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<tr>
<td>Loan-to-Value</td>
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<tr>
<td>Debt-to-Income</td>
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</tr>
<tr>
<td>Price growth</td>
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</tr>
<tr>
<td>Price volatility</td>
<td>$\sigma^2$</td>
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<td>Rental sizes</td>
<td>$\mathcal{H}(0)$</td>
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<tr>
<td>Owner-occupied sizes</td>
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<tr>
<td>PO sizes</td>
<td>$\mathcal{H}((0, 1))$</td>
<td>[1.75]</td>
</tr>
<tr>
<td>Other</td>
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<tr>
<td>Starting age</td>
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<tr>
<td>Retirement age</td>
<td>$K$</td>
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<tr>
<td>Final age</td>
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<tr>
<td>Consumption Floor</td>
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<td>Initial Distribution of wealth</td>
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<td>Fig. A4b†</td>
</tr>
<tr>
<td>Initial Distribution of prices</td>
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<td>Fig. A4d†</td>
</tr>
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</table>

### Panel B: Internally Estimated Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount Factor</td>
<td>$\beta$</td>
<td>0.961</td>
</tr>
<tr>
<td>Homeownership utility</td>
<td>$\chi$</td>
<td>0.30</td>
</tr>
<tr>
<td>Ownership share Elasticity</td>
<td>$\alpha$</td>
<td>24</td>
</tr>
</tbody>
</table>

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
</table>

Table 1 Model Parameters This table reports the value of all parameters used in the model. Superscript † denotes variables in NOK10,000.
Figure 3 Model Fit. The figure compares the predicted average net worth and the homeownership rate of households aged between 30 and 50 years using the parameter vector that solves Eq. 18 with the empirical counterparts.

\[
\omega^*(\Omega) = \arg \min_{\omega} \{[\hat{m}(\omega) - \hat{m}]' \Omega [\hat{m}(\omega) - \hat{m}] \}.
\]  \hspace{1cm} (18)

Here \( \Omega \) is a diagonal weighting matrix with elements equal to the inverse of the empirical moments, \( 1/\hat{m} \), so that the moment conditions are expressed as percentage deviations from their targets. This normalization prevents some moment conditions from receiving a high weight because of their units.

Figure 3 presents the model fit. We estimate the discount factor \( \beta \) of 0.961 (standard error = 0.002) and the homeownership preference parameter \( \chi \) of 0.30 (standard error = 0.054). We obtain standard errors using a bootstrap procedure, as we explain in detail in Section 2 of the Online Appendix.

4.3 Third-Stage: Introducing Partial Ownership

Due to our new data on PO users, we can identify two preference parameters: the preference for ownership and the ownership elasticity in the case of partial ownership. We now explain how we identify the ownership elasticity and what the preference parameter tells us about
the preference for homeownership.

The mechanics of the identification are as follows. We augment the above model with the preference parameter for PO, the ownership-elasticity \( \alpha \) in the ownership utility premium \( \chi(S) = 1 + \chi S^\alpha \), which we identify as follows: We start with the model without PO. We then introduce PO as part of the households’ choice set and simulate one year. In the simulation, some households switch to PO. Upon choosing a PO, the ownership share \( S \in (0, 1) \) depends on the ownership-elasticity \( \alpha \). If \( \alpha \) is small, households will choose a low initial ownership share to “harvest” most utility benefits from homeownership. As we increase \( \alpha \), partial owners will increase their ownership share because the utility value of low ownership now is closer to renting than owning.\(^{25}\) In practical terms, we first solve the model for 11 values of \( \alpha \). Then, we linearly interpolate between the points to find the parameter value that equates to the simulated and empirical moment.

Figure 4 shows the fit. As expected, the targeted moment, the initial ownership share, monotonically increases the ownership-elasticity \( \alpha \). The estimated \( \alpha \) is 0.352 (standard error of 0.031).\(^{26}\) What does the estimated \( \alpha \) of 0.32 tell us? In Table 2, we see that the average initial PO share is 57%, and 72% of households choose to own 50% (the lower limit). To match this, we need people to get a large share of the homeownership utility with a low ownership share (low alpha). Overall, households generally buy relatively large ownership shares, and some have, despite our relatively short sample, already increased their ownership. Taken together, this suggests that PO is not just a cheap way to get the non-pecuniary benefit.

### 4.4 Policy Functions

We present the policy functions for PO with comparative statics. We focus on how the policy functions or homeownership change with the ownership-elasticity \( \alpha \) and the ownership-utility premium \( \chi \). All examples show the optimal choices of a 25-year-old in the middle of the

\(^{25}\)A numerical example provides intuition for the identification. To simplify the math, assume \( \chi = 1 \) and define the log premium \( \ln(S^\alpha) \). The utility loss of switching from full to partial ownership is \( \alpha \ln(S) \) (because full ownership equals \( \alpha \ln(1) = 0 \)). With \( \alpha = 0 \), our starting value in the estimation, partial owners and regular homeowners receive the same non-monetary benefits from homeownership. As we increase the ownership-elasticity \( \alpha \), the utility loss from not having full homeownership increases. As the utility loss increases, households choose larger initial ownership shares.

\(^{26}\)We calculate standard errors using bootstrap. Specifically, we draw 100 bootstrapped samples of the initial ownership shares in the data and repeat the estimation procedure described above to find the standard error.
Figure 4 Moments from Third-Stage Estimation. The horizontal dashed red line is the empirical moment, while the orange solid vertical line is the estimated parameter value for $\alpha$. The gray lines denote the mean plus/minus the standard deviation of age in the data (dashed) and the model (solid).

<table>
<thead>
<tr>
<th>Moment</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average new initial ownership share</td>
<td>57.0%</td>
<td>57%</td>
</tr>
<tr>
<td>New owners owning 50%</td>
<td>72.0%</td>
<td>59%</td>
</tr>
<tr>
<td>Average Age</td>
<td>33.5</td>
<td>35.0</td>
</tr>
<tr>
<td>Std. Dev. Age</td>
<td>8.9</td>
<td>13.0</td>
</tr>
</tbody>
</table>

Table 2 Model Fit - Third Stage. The ownership-elasticity $\alpha$ is set to match the average new initial ownership share (first row). The following three rows report non-targeted moments related to PO. *Source:* OBOS.

income distribution who faces median house prices. \(^{27}\)

4.4.1 The Decision To Become a Partial Owner

Figure 5 shows how young households decide to rent or buy as a function of wealth. Many households prefer PO to rent and regular homeownership. There are two main reasons for this. First, PO allows “wealthy” renters to buy a fraction of a house. Second, PO allows households just wealthy enough to become regular homeowners to buy less than 100%. Taken together, this shows that PO increases household welfare by smoothing out the discrete real estate investment choice.

\(^{27}\)As a reference point, a wealth of 100 on the x-axis refers to 1 million NOK. The median 25-year-old’s net worth is zero, and the 75 percentile is 300,000 NOK.
Figure 5 Housing Choice Over The Wealth Distribution. The figures illustrate how young households decide to rent \((S = 0)\), partially own \((0 < S < 1)\), or own outright \((S = 1)\) as a function of wealth. The policy functions represent a 25-year-old high school graduate with median productivity who faces medium house prices. A wealth of 100 on the x-axis refers to 1 million NOK.

Figure 5 shows the housing choice for current renters (left), homeowners (center), and partial owners (right). Without PO, renters follow a threshold rule, only becoming homeowners when they are sufficiently wealthy. With PO, renters become partial owners at lower wealth levels, and as wealth increases, they gradually increase ownership. The threshold to own outright shifts to the right. The requirement that households must buy at least 50%, combined with mortgage borrowing constraints, limits the poorest households from becoming partial owners.

Without PO, current homeowners follow a threshold rule, with the threshold to the left of the one for current renters, due to sales cost. With PO, homeowners choose PO at modest wealth levels and rent only at the lowest wealth levels. Unlike for renters, few combinations of state variables result in a step function for homeowners. The reason is the considerable adjustment cost associated with going from, say, 100% to 75% ownership, which requires selling the home and buying 75% ownership. While introducing PO has less impact on owners, it matters for households close to the ownership threshold.

We plot two policy functions for partial owners based on different ownership shares, 50% and 75%. Without adjustment costs, these functions would overlap. With adjustment costs, households stick to their ownership share for longer. The slow adjustment to changes in wealth is most pronounced for reducing ownership shares. This is because the current contract prohibits reducing ownership shares on the same unit. As a result, it is only possible to lower the ownership share by selling the existing contract and entering a new one. The
total transaction costs are large enough to make partial owners reluctant to reduce ownership shares.

4.4.2 Drivers of the Demand for PO

We investigate how demand for PO is affected by the three preference parameters estimated internally: ownership-elasticity $\alpha$, the ownership-utility premium $\chi$, and the discount factor $\beta$ in a comparative statics exercise. Other preferences and state variables remain unchanged. The central finding is that the demand for PO is not only driven by preferences but also by what the contract offers.

Figure 6a shows how the housing choice of a renter depends on the ownership-elasticity $\alpha$. The dashed orange line shows the policy function with the estimated $\alpha$ of 0.352, and the solid red line shows the case with an $\alpha$ of 1 at which the ownership utility premium becomes $\chi(S) = 1 + \chi S$. Recall the lower $\alpha$ is, the smaller the necessary ownership share to receive a utility benefits ownership. With $\alpha$ equal to 1, demand for PO comes via channels other than the preference parameter $\alpha$.

The red line in Figure 6a shows that PO remains attractive with an $\alpha$ of 1. Thus, the demand for PO also comes from the properties of the contract. We present similar figures for other parameters in Appendix F, highlighting that part of the demand for PO comes through reducing frictions such as the indivisibility of housing and changing exposure to house price risk.
Figure 6b-6c shows how demand for PO depends on other preference parameters. In Figure 6b, we remove the ownership-utility premium $\chi = 0$. Without it, renting becomes, by definition, relatively more attractive, causing the demand function for homeownership to shift to the right. In Figure 6c, we increase the subjective discount factor $\beta$. Small changes in $\beta$ have little impact on housing choices.

5 Results

Our results contain three parts: First, we introduce PO into the calibrated model. This allows us to study take-up rates over the life cycle in the short and long term. Then, we quantify the willingness-to-pay (WTP) for PO across social strata and housing market conditions. Finally, we use the model to understand the effect of PO on debt and debt-to-income (DTI) ratios and housing downsizing.

5.1 Take-Up of PO in the Short and Long-run

We present aggregate outcomes after the introduction of PO in Figure 7. We separate take-up rates one year after the introduction of PO from long-run outcomes.

The top left plot shows the take-up rate of PO over the life cycle. Take-up declines with age and is around 20% at age 35, which is the average age of PO users. The top right plot shows the percentage of households that are renting. We observe that PO decreases rental rates, especially among the young, and this adjustment happens almost entirely within one year. The plot at the bottom left shows that PO only marginally decreases outright ownership initially. Over time, the outright ownership rate falls among young households who prefer PO. Hence, PO crowds out renting immediately and traditional ownership only in the long run.

The bottom right plot shows that the average ownership share held by households increases for all ages after the introduction of PO. Initially, as many renters become partial owners, the average household ownership share increases for all ages. Over time, household ownership falls again as the share of households owning outright falls. However, the average

\[28\] A simplified assumption in the model is that households can exit the rental contract at no cost. This introduces a small upward bias in the simulated one-year take-up rates.
ownership share remains above pre-PO levels because the shift from renters to partial owners is greater than the shift from regular to partial ownership.

5.1.1 **PO and Economic Outcomes Related to Homeownership**

We now discuss how PO relates to the large literature studying how homeownership affects other economic outcomes.

The high take-up rates of young households suggest that PO could reverse some of the fall in young homeownership after the Great Recession documented by Mabille (2022). He shows that a contraction in the availability of aggregate credit matches the observed fall in young homeownership rates. We show that PO is particularly popular among young households, who are generally more credit-constrained.\(^{29}\)

Bach et al. (2020) document that housing and mortgage choices early in life are among the most important predictors of where households end up in the wealth distribution at

\(^{29}\)There is empirical support for the hypothesis that many potential PO users are also likely to be credit-constrained. For example, D’Acunto and Rossi (2021) show that mortgage lending to low-income households declined in the U.S. immediately following the Dodd-Frank Act in 2010.
retirement. We show that many young households that would be renters without PO become partial owners and would now benefit from house price appreciation.

PO makes it possible to increase housing investments gradually over the life cycle. Cocco (2005) shows that because of investment in housing, younger and less fortunate households have limited financial wealth to invest in stocks, reducing the benefits of equity market participation. He concludes that house price risk crowds out stockholdings. We show that many households that would become homeowners without PO instead become partial owners with PO, which may mitigate some of the crowding-out effects housing investment can have on equities.

By reducing lock-in effects and barriers to buying a home, PO may dampen other adverse effects of homeownership, such as lowering geographic mobility (Oswald, 2019). Lock-in effects are smaller due to lower housing adjustment costs for partial homeowners, and barriers to buying are smaller due to lower initial investment in becoming a partial homeowner.

5.2 Welfare Effects of Partial Ownership

We measure the economic importance of PO as a one-time payment that makes a household’s indirect utility with PO the same as without PO. Specifically, we first calculate the value function without PO $V$. We define the WTP as the maximum one-time cost $c$ a household is willing to pay to obtain PO:

$$WTP(Ξ) = \{c \in \mathbb{R} : E[V(Ξ)] = E[\tilde{V}(W - c, H, S, ν, P, a)]\},$$

where $\tilde{V}$ denotes the value function with PO. The expectation operator is taken with respect to a particular group of households (e.g., low income households). The parameterization of the welfare cost calculations is based on the benchmark case (see Table 1 for parameters). This approach is widely used to quantify the importance of a particular choice, or a financial product (see, e.g., Cocco et al., 2005, Calvet, Campbell and Sodini, 2007, Koijen et al., 2016, Nakajima and Telyukova, 2017, Gomes, Michaelides and Zhang, 2022, among others).
Figure 8 Willingness to Pay (WTP) for PO. The figure plots the one-time WTP for PO as defined by Equation 19. The left plot shows WTP as a percent of annual household income. The plot on the right shows WTP measured in 10,000 NOK.

5.2.1 Average Welfare Gains from PO

Figure 8 presents the average and the median WTP as well as the 5th and the 95th percentile by age.

The WTP for PO among households aged 25 to 35—the primary users—is between 23% to 13% of after-tax income. In absolute terms, this is between 60,000 and 35,000 NOK. The WTP ranges from roughly 40% of after-tax income to zero. The median WTP is consistently below the mean.

The estimated welfare gains are high in absolute and relative terms. For example, Nakajima and Telyukova (2017) estimate that the WTP for a reverse mortgage option is between 0.84% and 5.13% of after-tax income at age 65. Cocco et al. (2005) calculate the welfare loss due to suboptimal portfolio choices. The most considerable losses are equivalent to a reduction in annual consumption between 1.5% and 2.0%. Calvet et al. (2007) estimate that the welfare cost of under-diversification is 0.5% of disposable income for the median Swedish household. Koijen et al. (2016) estimate that the typical lifetime welfare cost of market incompleteness and suboptimal insurance choice is 3.2% of total wealth. WTP for PO exceeds that of reverse mortgages and is comparable to having access to optimal portfolio advice but is, as expected, lower than the WTP for insurance.
5.2.2 Heterogeneity in WTP for PO

We use the model to understand the relative demand for PO. Our focus is on households that differ in terms of homeownership, income, wealth, and education—and on the affordability of housing. Figure 9 presents the results.

In the top left, we plot WTP by homeownership status before PO. The gains are greater for renters than for homeowners. The reason is that renters get much of the utility benefit from traditional homeownership with PO. Traditional homeowners benefit less from gradual exposure to real estate over the life cycle and smoother downsizing after adverse shocks. In the top middle, we break WTP down by the house size. We see that those who live in the smallest units have the highest WTP.

The bottom left plot shows that the PO WTP is highest for low-income households.
For example, at age 35, the WTP of low-income households is approximately five times higher than that of high-income households. The top right plot shows that households with less education—so those with a less favorable deterministic income path—benefit the most. Human capital explains both WTP differences and reflects the binding DTI constraint. The bottom middle plot displays variation in WTP across the wealth distribution. We observe that the difference in WTP between high and low-wealth households is smaller than for high and low-human capital households. The reason is that low income is a more common source of exclusion from the mortgage market than low wealth for households over 30. These results suggest that the variation in WTP in the population depends on the shadow cost of financing that gives access to the housing market.

The final plot at the bottom right shows how WTP changes with housing affordability. The higher the house prices are, the higher the WTP for PO at all ages. The impact of house price levels on WTP is particularly large for young households.

5.2.3 Sensitivity Checks

We now perform some sensitivity checks. Its primary purpose is to examine whether the WTP estimates, essential output from our analysis, are sensitive to small changes in model parameters. We change parameters related to the house price process, LTV requirements, depreciation, and the rent-to-price ratio. The idea is not to see what the WTP would be in a different country with a different housing market but how each aspect of the housing market—as captured by these parameters—influences the demand for PO. We perform these calculations in the following way. We solve the model without PO after changing one parameter and then solve the same decision problems with PO. With the new policy functions and the simulated outcomes, we calculate WTP using Eq. 19. In all simulations, households receive the same shocks. Hence, the WTP estimate only reflects the parameter change. Table 3 presents the results.
Table 3 Housing Market Parameters and the Willingness-to-Pay for PO. The table reports the average WTP for PO, expressed as a percent of annual income, as we vary various housing-specific parameters. When we increase total depreciation, we keep the shares allocated to the two types of maintenance constant. The first panel reports the WTP by age groups while the second panel reports the average within the top 20% of the variable among households aged 25-55.

The WTP is increasing in price growth since the benefit of house price exposure—through PO—increases with expected price growth. Next, we increase the LTV. As the LTV increases, households need more wealth to become homeowners. As a result, PO, which relaxes borrowing constraints, becomes more attractive, especially for the youngest households. Next, we decrease depreciation on owner-occupied housing. This decrease in the user cost of owning increases the WTP for PO through the same mechanism as higher price growth; the expected net return on housing goes up. Finally, we decrease the rent-to-price ratio. Surprisingly, the WTP for PO still increases, even though rental prices are now lower. This happens for two reasons. First, part of the cost of being a partial owner is that the household still pays market rent on the share they do not own, which is now lower. All else equal, this increases the WTP for PO. Second, the drop in rental prices increases the region where households prefer PO to own outright, which increases the demand for PO. The WTP increases for all age groups since these benefits occur at all ages.
Table 3 also reports how the characteristics of the households with the highest WTPs change with changes in housing market conditions. A stricter LTV requirements lower the marginal PO user’s average age, wealth and educational level. A drop in the effective price of using PO modeled as a decline in house depreciation has the opposite effect. Small changes in expected house price growth, or the rent-to-price ratio, have a negligible impact on the composition of marginal PO users.

To conclude, the WTP estimates are relatively insensitive to small changes in housing parameters. The sensitivity checks provide suggestive evidence for why the PO contracts came early to Norway. The Norwegian real estate market has experienced high price growth, strict borrowing regulation, and low rent-to-price rates, all pushing up demand for PO.

5.3 Financial Stability Concerns

Past crises show that real estate markets are essential from a macroeconomic and financial stability perspective. A potential concern is that PO increases outstanding debt by inducing renters to become partial owners. Relatedly, introducing PO may transfer risk from various private property owners to a few commercial PO vendors and, ultimately, the banking sector and the real economy. Moreover, Norway’s Financial Stability Authority (FSA) released the following statement in October 2023: “The FSA cannot see that assessments have been made of the risk that new ownership models [PO] could lead to increased financial vulnerability in Norwegian Households.’’ Motivated by these concerns, we use counterfactual experiments to analyze PO’s impact on household debt and financial fragility.

Figure 10 presents the unconditional debt-to-income (DTI) ratios with and without PO in the short and long run. We report the results for the total population and households in the bottom and top 20% of age-specific income distribution.

The DTI ratio increases immediately after the introduction of PO due to renters becoming partial homeowners. The most considerable change in the demand for mortgages comes from young households, the primary PO users. In the long run, the increase is more modest, as some households use PO instead of traditional ownership.

The plot in the middle shows the same analysis but for households in the bottom 20% of
the age-specific income distribution. Without PO, most households under 40 in this group have no debt as they are unable, or unwilling, to borrow to become traditional homeowners. With PO, many switch from renting to partial ownership and use debt to pay for the ownership share. Again, the increase in debt is muted in the long-run.

The plot on the right shows that high-income households are almost unaffected by PO. The exception is the youngest households in the long-run (orange line), who borrow less when PO is available as they prefer a smaller mortgage and an ownership share below 100%.

Figure 11 presents the same graphs as Figure 10 but only includes households with debt. Quite startlingly, PO decreases the average debt level among households with debt. The decrease is large, most pronounced for the youngest households and those with the lowest incomes, and happens almost entirely within one year. These results hint that while PO increases total debt, it may decrease borrowers’ average debt, making borrowers less financially vulnerable.

5.3.1 A Difference-in-Differences Analysis of Housing Choice

The above analysis sheds light on the average effects of PO on DTI ratios. We now focus on three types of households that re-optimize their housing choices when PO becomes available.
Figure 11 Financial Stability: Debt to Income by Income Groups, Conditional on Having Debt. The figures plot the population average and the top and bottom 20% of age-specific income distribution. The solid red line shows the DTI ratio one year after the introduction of PO. The orange dashed line shows the long-term (steady state). The black line shows the same calibration without PO.

These households are renters who would remain renters but now switch to PO, owners who would remain owners but now switch to PO, and renters who would become owners but now instead use PO. Specifically, we take the simulated distribution before PO and simulate it for one year, both with (red line) and without PO (black line). Figure 12 presents the results.
Figure 12 Average Effects on Affected Households: Debt-to-Income (DTI) by Income Groups. The left plot shows households renting at time $t = 0$ who would be renting at $t = 1$ without PO but now use PO. The middle plot shows households owning in $t = 0$ who would be owning at $t = 1$ without PO but now use PO. The right plot shows households renting at time $t = 0$ who would be traditional homeowners at $t = 1$ without PO but now use PO.

The first plot shows the average DTI ratio for renters who switch to PO. They cannot borrow as they rent, but after PO is introduced, they take out modest mortgages to become partial owners. This is the main driver of the household sector’s increase in total borrowing. The plot in the middle shows the corresponding results for switchers from outright to partial ownership. Without PO, these households are close to the DTI constraint of 5.0. By selling their home and becoming partial owners, they become less indebted. The plot on the right shows the outcome of renters who would become traditional homeowners but now use PO. Without PO, these households would also take on substantial debt, resulting in a DTI ratio close to the limit. With PO, many choose partial ownership instead. The result is that the group, on average, has a substantially lower DTI ratio.

5.3.2 Household Downsizing

Downsizing in the housing market refers to buying a smaller or less expensive unit and is often motivated by financial considerations. For example, anticipating an increase in labor market uncertainty, downsizing would reduce costs as a smaller unit means lower interest payments and less maintenance. Lower fixed costs make the household better prepared for a
less prosperous future. Now, if changes in aggregate outcomes mainly drive downsizing, many households are likely to downsize simultaneously. Such collective downsizing can trigger a collapse of the financial system (Gabriel, Iacoviello and Lutz, 2020, Shleifer and Vishny, 2011, Corbae and Quintin, 2015). Regarding financial innovation in the housing market, the Norwegian Central Bank just released the following statement: “The new models [e.g., PO] make it possible to enter the housing market without meeting the usual financial requirements for buyers ...If the new models for house purchases become more widespread, they may have consequences for financial stability:...” In other words, the key concern is that PO turns too many renters into homeowners, which in turn increases the supply of housing in the case of collective downsizing. In the following, we use counterfactual experiments to quantify how PO impacts collective downsizing.

Figure 13 plots the share of households that downsize with and without PO. We observe that PO has little impact on the number of households that downsize. In the second panel, we plot the share of traditional owners who downsize. Fewer young households downsize, and with little change for older households. That drop in downsizing among young households is because fewer households are close to the regulatory borrowing constraints with PO. As a result, they can bear more adverse shocks without downsizing. The right panel shows that between 3% to 10% of partial owners downsize.

Figure 13b shows the average value of downsized housing \( (P(S'H' - SH)) \), conditional on downsizing. It measures the value of “fire sales” in housing. The left plot shows that the downsized amount is reduced by about 50%. Hence, the value of the total housing stock listed for sale due to downsizing is much smaller with PO. The main reason is that traditional homeowners can now downsize to partial ownership. The panel illustrates this mechanism. The last panel plots the value downsized among partial owners. This implies that most downsizing partial owners become renters.

To conclude this section, introducing PO can negatively and positively affect financial

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32 House prices volatility is one example of aggregate trigger of downsizing (see Banks, Blundell, Oldfield and Smith, 2007).

33 Original quote: “De nye modellene gjør det mulig å komme inn i boligmarkedet som eier, uten å oppfylle de vanlige økonomiske kravene til boligkjøper ...Ved økt utbredelse av de nye modellene for boligkjøp kan de ha konsekvenser for finansiell stabilitet:” Norges Bank 20.10.2023

34 The likelihood of downsizing increases non-linearly with age, partly due to how we model the cost with PO. This assumption does not impact the estimates for those below 45, which are the main users of PO in the data.
stability. A potential adverse effect is that more people borrow—as many switch from renting to partial ownership—and as a result, the average DTI ratio in the economy rises. On positive effects, we see that many households that are just wealthy enough to become traditional homeowners choose partial ownership instead and take out smaller loans. Adding up, PO leads to an increase in aggregate debt and a decrease in average debt among borrowers. In addition, PO leads to about 50% decrease in the housing value listed for sale along the intensive margin due to downsizing. From a household financial fragility viewpoint, our results indicate that PO leads to less extreme borrowing and smaller fire sales in the event of adverse aggregate shocks.

6 Conclusion

High house prices and strict borrowing regulation have triggered an affordable housing crisis. Responding to the affordability crisis, governments, real estate developers, and financial intermediaries have begun offering new financial housing contracts. Partial ownership (PO) is one example. It allows households to buy a fraction of a house and rent the rest. We show that PO contracts exists in multiple countries and that the number of new homes sold with PO is increasing. For example, in Norway, one year after its introduction, 10% of new homes sold had partial owners. China started their PO pilot programs in Huai’an in 2007 and in Shanghai and Beijing in 2018 (Li et al., 2020).

We are the first to incorporate a for-profit PO contract in a life-cycle model. Our analysis delivers predictions about a future PO market, including take-up rates, drivers of demand, willingness-to-pay (WTP), and its effect on household borrowing.

In the short run, PO reduces the share of households that rent with almost no reduction in regular homeownership. However, over time, PO decreases traditional homeownership because many households prefer gradually increase ownership shares. Hence, our findings suggest that PO improves household welfare by transforming housing investment from a discrete choice to an equity-like investment.

We measure the WTP for PO for households with multiple characteristics and under different housing market conditions. For example, households aged 25 to 45—the primary users—would pay between 23% to 5% of after-tax income to access PO. The welfare benefits of PO are particularly high for poorer households and those that are either just below the
regulatory constraints that exclude them from the mortgage market—or just above so that they become high-risk borrowers. Overall, our WTP estimates are high in absolute and relative terms, exceeding that of reverse mortgages and comparable to optimizing financial investments over the life cycle.

Our results have several potential implications. The high WTP and take-up rates among young households hints that PO has the potential to recover some of the lost potential young homeowners that have been renting after the Great Recession documented by Mabille (2022). A broader implication is that PO can mitigate the potential crowding-out effect housing investment can have on equities (Cocco, 2005).

While PO has a high potential to increase welfare for many households, we suspect that policymakers and regulators will have financial stability concerns. A potential concern is that PO increases the borrowing of financially fragile households. Because many of these households would rent without PO, introducing PO transfers risk from various landlords to these households and the commercial PO vendors and, ultimately, the banking sector and the real economy.

To shed light on the financial stability concern, we calculate debt-to-income ratios in many scenarios and for several household types. The results are both expected and unexpected. On the negative side, more people borrow—as many switch from renting to partial ownership—and as a result, the average debt-to-income ratio in the population rises. On the positive side, we find that many households that are just wealthy enough to become traditional homeowners choose partial ownership instead and take out smaller loans.

We complement the above analysis by looking at downsizing along the extensive and intensive margin. Our results show that PO has little impact on downsizing along the extensive margin but leads to a 50% decrease in the housing value listed for sale among downsizers. From a financial stability viewpoint, this finding and the above results for DTI are remarkable: With PO, we get a less skewed DTI distribution and a large drop in the housing wealth sold due to involuntary downsizing in bad times. Gabriel et al. (2020) find that more flexible mortgages induce less downsizing, while we show that more flexible ownership structures can accomplish the same.

There are several ways of extending our work. First, a general equilibrium analysis of PO would be interesting. On the one hand, by turning renters into partial owners, aggregate financial risk can increase. The reason is that even a fraction of a house would be the largest
investment for many households, particularly those that used to rent. Although less so than a
traditional house investment, partial ownership leads to a levered balance sheet tilted toward
housing, which increases households’ exposure to aggregate house price fluctuations. On the
other hand, by turning highly levered homeowners into partial owners, aggregate risk can
decrease. A general equilibrium analysis would trade off these two opposing effects, resulting
in an equilibrium price for PO. The equilibrium price would determine the composition of
PO users and, ultimately, whether a private market for PO is sustainable.

Another interesting GE effect is the impulse response function of the aggregate house
price following a demand shock due to an increase in households who become (partial)
owners. Such analysis requires estimates of housing supply elasticity in regions with many
PO users. The interaction between supply and demand will determine the evolution of
aggregate house prices after the introduction of PO. Given the infancy of the PO market, it
would be interesting to analyze different PO contracts. One twist on the existing contract
would be a continuous ownership contract allowing households to buy and sell any house
share. Such a contract would make housing investments similar to equity investments and
include a reverse mortgage element. Given the modest correlation between the return on
housing and equities, such a contract would increase household portfolios’ Sharpe ratios.
Given the high estimated WTP for reverse mortgages (see Nakajima and Telyukova, 2017,
Cocco and Lopes, 2019), we expect the welfare benefits of this alternative contract to exceed
the current PO contract—particularly for wealthy households with strong bequest motives
and preference for aging in their own home.

Finally, it is unclear how widespread use of PO would compare with an improved rental
market regarding comovement in aggregate house prices and consumption (Campbell and
Cocco, 2007). Kiyotaki, Michaelides and Nikolov (2023) use a macroeconomic model with
housing to examine the causes and consequences of volatility in the housing market and
evaluate alternative housing-related policies. They suggest that a well-functioning rental
market can protect young and less fortunate households from house price shocks. PO is a
potential substitute for improving the rental market but has an additional benefit. As rent-
ing, partial ownership reduces the sensitivity of household wealth to changes in house prices.
Unlike renting, PO allows households to benefit from the utility benefits of homeownership.
A welfare comparison of an improved rental market and PO would be interesting. We leave
these questions for future research.
Figure 13 PO and Downsizing. The upper plots look at downsizing along the extensive margin in a world with and without PO. The first plot presents the aggregate results, while the second and third plots show the results for traditional- and partial homeowners. The lower plots show the corresponding results along the intensive margin.
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A Appendix: Data and Institutional Details

A.1 Appendix: The Sample

For each individual, we observe the birth date (variable name: “foedsels_aar_mnd”) from the population database (In Norwegian: “Befolkning”). Educational level is based on the Norwegian standard for education grouping (“NUS”). NUS is a six-digit education code, where the first digit indicates the level of education. We report results for all educational levels, low (0-2), medium (3-5), and high (6-8). Low includes middle school, medium includes high school, and high starts with a bachelor’s degree.

The unit of analysis is the household. We distinguish between individuals living alone and individuals with a partner. We obtain the National identity number of the spouse/registered partner from the SSB’s population statistics. We observe the ID (anonymized) of the spouse (variable name: ‘ekt_fnr_aaaa”), or cohabitant (variable name: “sambo_snr_aaaa”). We use this information to classify an individual into a one-adult household (not registered ID for spouse or cohabitant) or more than one adult household. We refer to the oldest individual in the household as the household head. For tax purposes, the household can allocate wealth in a way that gives the lowest wealth tax. Thus, there are no incentives for tax-motivated asset allocation within the household. We restrict the sample to one-family households.

For each household, we define disposable income $Y$ (variable name: “wsaminnt”), as the sum of gross salary income and pension plus net capital income and total government transfers minus tax (“utskatt”). A broad measure of income implicitly allows for several ways of self-insurance against labor income risks (Campbell and Cocco, 2015). We define net worth, $W$, as total assets (variable name: “ber_brform”) minus debt (variable name: “gjeld”). We define a household as a homeowner ($S = 1$) if it does not rent (variable for renting: “eie_leie”). A household’s age and education are based on the household head. If two individuals have the same age, the man is the household head. All households are at least 25 years.

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35 That is, we require the first digit of the variable “regstat_hushtyp” to be one.
A.2 Appendix: The OBOS-contract

The paper uses a standard life-cycle homeownership model augmented with partial homeownership (PO). The calibration of the PO contract is based on the “OBOS-contract”. In what follows, we provide additional details about the “OBOS-contract”.\footnote{The main source is OBOS’s website: https://www.obos.no/ny-bolig/obos-deleie/?gclid=Cj0KCQjw1_SkBhDwARIsANbGpFuFSD2o4Pv0Xu8wuNQpDHz4ywhNDPvdWbn3WpJV7iEWoRrGGVBwUaAsMrEALw_wCB.} To be consistent with the main text, we refer to it also here as PO. Figure A1 provides an example from one of OBOS’ new housing projects in Oslo.

PO allows people to buy a fraction of a home yet use the home as a traditional homeowner. All new OBOS projects with housing cooperatives offer PO. The minimum ownership share is 50%. Above this threshold, one can choose any share in 10% increments. It is easy to increase ownership share later while living in the home. The equity requirement is the same for PO as traditional house investments. The equity requirement is 15% of the total purchase price of the share. Thus, if you buy 50% of a home for a total of NOK 4 million, the price for your share is NOK 2 million. For the fraction you do not own, you pay rent equivalent to the market rent but cover all maintenance. As explained in the main text, buying, upsizing, or selling partial housing incurs fixed legal fees. When buying larger shares, the market value is bounded from below by the initial price. All these features are in the model.

While we attempt to reproduce the contract as closely as possible, we deviate in some aspects. First, the PO contract ties rent to CPI, not the house price, which in the model would imply that the rental payment is given by the initial price $\kappa P^H_0$ instead of the market value $\kappa P^H_t$. However, this feature of the contract differs from standard rentals. We ignore this aspect to keep the contract valuation simple and the findings generalizable. Second, we omit the put option for the financial intermediary. That is, OBOS can unilaterally put the house for sale after ten years. If OBOS terminates the contract, the household has one chance to buy the remaining share as usual. If not, the unit is sold on the open market, with each party getting their share of the proceeds. Since the contract was introduced in 2020 we do not yet know whether the option will generally be used. In our simulations very few households hold the contract for more than 10 years. Finally, in practice, if the contract is terminated after 10 years, the household can simply become a partial owner again. Moreover, this contract feature is another reason for the age cost function (Eq. 7).
shorter guaranteed duration of the contract makes it less appealing for retired households, who want to gradually decrease ownership without moving, since contract termination forces a sale and a subsequent move. Third, there is a single price in the model, so there is no regional price index that is used to estimate the current market value.

Finally, we reiterate that other providers also offer PO contracts with slightly varying contract terms in Norway. Some providers are financial intermediaries offering contracts for both existing and new homes, with rental payments potentially linked to the intermediary’s interest payments. Other are provided by other home builders and even joint public-private partnerships.
Appendix Figure A1 PO Example: The Vollebekk Construction Project. We here show one typical example of a Obos project with PO. Figure a) shows the online ‘calculator’ where households can, by adjusting the different option, choose their desired ownership share and see what they pay for their unit as well as the various monthly payments. Newly built housing cooperatives are usually partially debt financed and each unit is associated their share of the debt, which is customary reflect in the purchase price. Some, such as this project, allows buying households to pay down all debt initially. Figure b) shows the location within Oslo with the Metro system overlaid. The nearest metro station is within a 5-10 minutes walk. The downtown area is approximately where ‘Oslo’ is labelled. Figure c) shows the scale of this big residential construction project, while figure d) shows one specific building (the example apartment in panel (a) is on the sixth floor of this building).
B Robustness Analysis

This section contains the robustness analysis referred to in the main text.

B.1 Robustness: House Sizes

A possible concern with our setup is that the demand for PO depends heavily on how we model house sizes, that is, the house size grids $H(0) = [1.0, 1.75]$, $H(1) = [1.75, 2.27]$ and $H((0, 1)) = [1.75]$. Note that the grids we use match the 5th, 25th, and 50th percentile of the empirical house size distribution.

One can ask two different but related questions: 1) if households could choose from a larger choice set of house sizes, how would that affect the demand for PO? 2) how important is the relative size of the PO unit for the results? We now show, using simulations, that our conclusions are insensitive to alternative specifications of the house size grids.

To address both concerns simultaneously, we vary the size of the medium unit from the smallest to the largest unit and calculate the main outcomes. Figure A2 plots, from left to right, the WTP/Income (summarizing Fig. 8 in the main text), the homeownership rate with and without PO (summarizing Fig. 7), and the share of households who are partial owners (same figure).

As expected, the WTP for PO is increasing in the size of the middle unit. As the medium
unit (the one available for rent, PO, and traditional homeownership) approaches the large house in size, the WTP flattens out somewhat and tops out at 20%. As the middle unit approaches the size of the smallest unit (which is only available for rent), the WTP remains at 5% of income. This result is striking: even in an economy where the homeownership is 100% without PO, some 5% of all households (and hence current owners) prefer to be partial owners and the average WTP for PO is still high at 2% of disposable income.

When the medium house is tiny (and thus cheap), most households become traditional homeowners and only a few switch to PO after its introduction. Unsurprisingly, as the risk of becoming indebted if house prices tank diminishes and the “housing ladder” stops being a “ladder” as many can afford their preferred house size early in the life cycle, the demand for PO declines. By the same logic, as we instead increase the size of the medium house, we observe fewer households becoming outright owners and more households using PO. To conclude, a realistic calibration of house sizes and valuations will generate demand for PO. Indeed, as our results show, even in a calibration so unrealistic that all households are outright owners when PO is not available, some (owning) households prefer to be partial owners, and hence the average WTP is positive.

B.2 The Robustness: The Option Value Element

As explained in the text, a reason why the option element of the PO contract has little impact on households’ WTP for PO is that households can, if prices fall, “reset” the contract by selling the unit, receiving their share, and then entering into a new PO contract. Concretely, the cost of buying a larger share using the contract is (including $l_c$, the cost of changing size)

$$\max\{P^H, P^H_0\} \times H \times (1 + m_b) \times (S' - S) + l_c. \quad (A1)$$

The cost of buying a larger share, by selling the current share and buying a new house is (including $l_s$, the cost of canceling a PO contract, and $l_b$ the cost of entering a new PO contract if they do not buy a new house outright):

$$P^H \times H[((1 + m_b)S' - (1 - m_s)S] + l_s + l_b \mathbf{1}_{s' < 1}. \quad (A2)$$

The household in the model chooses the cheapest alternative.
Now, consider a household with a medium house that bought half of it at a price of 100 per unit (price of entire unit is 1,750,000 NOK). In the next period, the price drops by half. If the household chooses to buy the remaining 50%, it pays $100H(1 + m_b)(S' - S) + l_c = 89.69875 + 0.87$ since the contract binds them to the initial price per unit of 100. If the household instead chooses to sell and buy it outright, it pays $50 \times H \times [(1 + m_b) - 0.5 \times (1 - m_s)] + l_s = 47.03125 + 0.0$. Thus, if $P_0^H$ is much higher than today’s market value, the household can reenter into a new contract at a low price. Figure A3 below illustrates the cost of increasing ownership to 100% or 75% from 50%, as we vary the initial price ($P_0^H$). The current market price is 100.

**Appendix Figure A3 Upsize costs, by choice of $S'$.** This figure plots the cost of increasing ownership shares as a function of the initial purchase price under various contracts. The dashed blue line plots the cost using the baseline contract in the model. The dashed red line plots the cost for a contract that does not include the initial price but is otherwise identical. The green line plots the cost of selling the unit and buying a new one. Finally, the black line is the lower envelope of the costs when the contract includes the initial price.

Suppose the initial price was below the current price (100), then the household always uses the contract; $P_0$ becomes irrelevant. If house prices have fallen but not by much, the cost of upsizing increases a little. If the prices tank, using the contract to increase ownership becomes very expensive, so the household uses the alternative strategy. On net, the option value element of the initial price has little impact.
Internet Appendix:
Partial Homeownership: A Quantitative Analysis

This Internet Appendix describes the details of data construction, the PO contract, and discusses the empirical methodology.
C Appendix to Section: Institutional Setting and Data

C.1 Homeownership in Norway

In Norway, the two dominant forms of homeownership is “traditional” ownership and through a co-op or a housing association (“borettslag”). In practice, these two types of ownership have been largely identical since 1980 and though co-ops originally served a social-civic minded purpose, they are today behaving like for-profit companies (Sørvoll and Bengtsson, 2018).

Strictly speaking, ‘owning’ a co-op apartment means owning a share in the co-op which includes the right to live in a specific unit, though in practice a Norwegian co-op is more like a condominium than a co-op in the North American setting. For more details on the history of co-ops we refer to Sørvoll and Bengtsson (2018). In Norway, housing cooperatives are mainly in cities. In Oslo, the shares of households living in the three types have been stable since 2015, with about 36% in ‘self-ownership’, about 32% in owned coops, and 32% in rental housing (Source: Statistics Norway Table 11084). For comparison, about 25% of American households live in some form of community managed developments (e.g., condominiums, homeowner association, or coops) and 80% in New York City. Coops mainly build new multifamily buildings in large cities, and OBOS is the largest home builder in Norway.

Buying a co-op unit is equivalent to acquiring a co-op share and a co-op association membership. With that, the co-op grants the holder the right to live in the unit indefinitely. To give such a right, the board must prepare a financial plan, including information on maintenance costs. The co-op charges monthly fees to its members, which must cover maintenance costs, the amortization of the co-op debt, and its interest expenses. Shares can be pledged as collateral against the home mortgage but do not entitle buyers to property rights over the unit. However, in practice, homeowners in Norwegian housing cooperatives have the same control over their homes as single-family owners.

C.2 Labor Income Calibration

We use Norwegian Microdata from Statistics Norway to estimate Equations 2 and 3. We scale nominal disposable income $Y$ by the consumer price index, and denote log real earnings by $y_{i,t} \equiv \ln(Y_{i,t}/ CPI_t)$. The base year is 2018. We require all households to have a minimum
Appendix Table A1 Labor Income Process: Age Polynomials. The table shows the coefficients of the third-order polynomial fitted to the estimated dummy variable coefficient in Equation A3.

<table>
<thead>
<tr>
<th></th>
<th>All</th>
<th>&lt; High School</th>
<th>High School</th>
<th>College</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>9.267</td>
<td>10.526</td>
<td>10.419</td>
<td>8.151</td>
</tr>
<tr>
<td></td>
<td>(0.170)</td>
<td>(0.059)</td>
<td>(0.112)</td>
<td>(0.258)</td>
</tr>
<tr>
<td>Age</td>
<td>0.218</td>
<td>0.129</td>
<td>0.148</td>
<td>0.284</td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(0.004)</td>
<td>(0.008)</td>
<td>(0.018)</td>
</tr>
<tr>
<td>Age2 × 100</td>
<td>-0.390</td>
<td>-0.218</td>
<td>-0.253</td>
<td>-0.511</td>
</tr>
<tr>
<td></td>
<td>(0.027)</td>
<td>(0.010)</td>
<td>(0.017)</td>
<td>(0.040)</td>
</tr>
<tr>
<td>Age3 × 1000</td>
<td>0.022</td>
<td>0.011</td>
<td>0.013</td>
<td>0.030</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Observations</td>
<td>43</td>
<td>43</td>
<td>43</td>
<td>43</td>
</tr>
<tr>
<td>R²</td>
<td>0.98</td>
<td>0.96</td>
<td>0.99</td>
<td>0.98</td>
</tr>
</tbody>
</table>

of 100,000 in disposable income Y and 5,000 NOK in financial wealth.

We use all one-family households and partition the sample into three education groups based on the educational attainment of the head of the household (low, medium, and high education, as defined in section in the main text).\(^{37}\) We estimate the following model for household i aged a at time t separately for each educational group

\[
y_{i,a,t} = \sum_{j=25}^{67} c_j 1(a_{i,a,t}=d_j) + \lambda_t + \varepsilon_{i,a,t},
\]

where \(1(a_{i,a,t}=d_j)\) takes the value of one if the age of household i at time t equals \(d_j\), \(\lambda_t\) denotes calendar year fixed effects, and \(\varepsilon_{i,a,t}\) denotes the regression residual. Following Cocco et al. (2005), we fit a third-order polynomial to the age coefficients, \(\hat{c}_{25}, \hat{c}_{26}, \ldots, \hat{c}_{67}\) to obtain the labor income profiles for the numerical solution. Table A1 presents the results.

To estimate the error structure of the labor income process, we use the full sample from 1993 to 2018. We impose the same requirement as in the estimation of the deterministic part of labor income, except for the one-family household criteria, which we only observe\(^{37}\) because information about whether the household is a one-family household starts in 2004, the estimation of the deterministic part of income is based on the period from 2004 to 2018.
from 2004 and onward.

We first define the $d$-year difference in labor income shock as

$$r_{a,d} \equiv (\nu_{a+d} + \epsilon_{a+d}) - (\nu_a + \epsilon_a) = (\nu_{a+d} - \nu_a) + (\epsilon_{a+d} - \epsilon_a) = \sum_{j=1}^d u_{a+j} + (\epsilon_{a+d} - \epsilon_a),$$

where the last equality follows from Equation (3). The variance of Equation (A4) is $\text{Var}(r_{a,d}) = d\sigma_u^2 + 2\sigma_\epsilon^2$. To estimate $\sigma_u^2$ and $\sigma_\epsilon^2$ we define the $d$-difference in prediction error from Equation A3 as:

$$\hat{r}_{i,d} \equiv \epsilon_{i,a,t+d} - \epsilon_{i,a,t}. \quad (A5)$$

With $h$ consecutive observations of income for household $i$, we get $h - 1$ estimates of $\hat{r}_{i,d}$. We calculate the variance of Equation (A5) by pooling together all individuals for each $d$. Following Campbell and Cocco (2015), we winsorize each $d$ sample at the 5% level top and bottom. Finally, we regress the empirical variances on $d$ and a constant. The coefficient in front of $d$ is the estimate of $\sigma_u^2$, and half of the intercept is the estimate of $\sigma_\epsilon^2$. Table A2 presents the results.

**Appendix Table A2 Labor Income Variance Decomposition.** This table reports the estimate of the volatility of permanent and transitory labor income shocks. The estimate is based on the decomposition in Equation A5.

<table>
<thead>
<tr>
<th></th>
<th>&lt; High School</th>
<th>High School</th>
<th>College</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>Education</td>
<td>0.176</td>
<td>0.408</td>
<td>0.416</td>
<td>1</td>
</tr>
<tr>
<td>Transitory</td>
<td>0.176</td>
<td>0.169</td>
<td>0.185</td>
<td>0.180</td>
</tr>
<tr>
<td>Permanent</td>
<td>0.062</td>
<td>0.064</td>
<td>0.066</td>
<td>0.067</td>
</tr>
<tr>
<td>Constant</td>
<td>0.062</td>
<td>0.0569</td>
<td>0.0684</td>
<td>0.0645</td>
</tr>
<tr>
<td>S.e</td>
<td>0.004</td>
<td>0.0037</td>
<td>0.0040</td>
<td>0.0041</td>
</tr>
<tr>
<td>d</td>
<td>0.004</td>
<td>0.0041</td>
<td>0.0043</td>
<td>0.0045</td>
</tr>
<tr>
<td>S.e</td>
<td>0.000</td>
<td>0.0002</td>
<td>0.0003</td>
<td>0.0003</td>
</tr>
</tbody>
</table>
D Appendix to Section 4:

We estimate the model in three steps. In this section, we explain the two first steps. The third step is explained in the main text. We first describe how we choose the initial conditions necessary to simulate the model and then outlay the estimation.

First, we assign net financial assets to each household by drawing from the empirical distribution. We estimate the empirical distribution by pooling households at age 25 into 10 financial wealth groups of equal size (see Figure A4b). In the first year, we randomly assign all households to a financial wealth bin and give everyone in the same bin identical initial values. Second, we draw the initial persistent income shock from the stationary distribution implied by Equation (3). Third, all households start as renters but may choose to become homeowners in the first period. Fourth, households are randomly allocated to an education group in line with our data, with the following PMF: 0.176, 0.408, and 0.416 in the less-than-high-school, high school, and college groups, respectively.

Households draw the initial house price $p_s$ from a five-binned discrete uniform distribution. We calibrate the mean of the initial price in the following way. We find the ratio of the average square meter price of owner-occupied housing (Table 06035) to median household income over time (Table 04751). We then multiply the square meter price-to-income ratio by 77, the size of the smallest owner-occupied house in our model, and take the average over 2002-2019 (the years we have data before PO). We find that the mean price to income of the unit is 3.9 (see Figure A4c). In our simulation, the average household income for households aged 24-45 is 45.3. Thus, to find the typical starting price, we take $3.9 \times 45.3 / 1.75 = 100.96$. We set each bin at ±10% increments (see Figure A4d).
Appendix Figure A4 Calibration. These figures present various moments used in the calibration, see Section 4 for details.
Appendix Figure A5 Structural Estimation: Optimization Space. This is the minimum value of the objective function (in logs) over our global search space, with darker colors indicating lower values (better fit). The white lines are level curves for the lowest 1, 2, and 5 percent. The crosses indicate the best 10 parameter vectors, with larger crosses meaning a better fit.

In the second stage estimation, we draw \( N = 4000 \) candidate parameter vectors \( \omega \), using Sobol sequencing. Figure A5 shows that the search space is big and surrounds the local minimum. As \( \beta \) is lower, we also need a higher \( \chi \) to match the data well. The crosses in Figure A5 mark the 10 best model fits, which are in a small area around \( \beta = 0.961 \) and \( \chi = 0.30 \).

The global optimization procedure lends itself to verifying identification, as we now show. After solving the model for the \( N \) parameter vectors and finding the simulated moments, we do the following procedure for all moments and parameters. First, pick a parameter, say \( \beta \), and divide it into 20 quantiles. Find the 25th, 50th, and 75th percentiles within each quantile for a moment. The remaining parameter is uniformly distributed within each...
quantile. We can then show how the moment depends on the parameter by plotting the percentiles within each quantile. One can think of this procedure as taking the partial derivative of the moment rate with respect to the parameter while keeping the distribution of the other parameter constant. We can then repeat this process for every moment.

A moment is informative for a parameter if the moment percentiles move as we move across quantiles while keeping the distribution of other parameters constant. The steeper the slope, the more informative the moment is for the parameter. A parameter is relatively more important when the distance between the 25th and 75th moment percentiles is smaller.

In Figure A6, we plot the results of this exercise, for homeownership and wealth at age 30. As expected, wealth and homeownership are both increasing in the two estimated parameters, \( \beta \) and \( \chi \). We observe that \( \chi \) allows us to pinpoint the homeownership rate for young households (Fig. A6a), which is key for identification. As households age, we see that \( \chi \) loses its importance for ownership as wealth becomes more important.

We get bootstrapped standard errors as a by-product since we repeat the procedure for 100 bootstrapped empirical moments.

E Appendix to Section: XX Numerical Details

The problem is solved backward by first solving the value function of a retiree in the final period when death is certain. For each discrete choice, we find optimal consumption (the choice which maximizes the current utility and the expected continuation value) using Brent’s root-finding algorithm. The optimal policy is then given by the discrete choice and its associated optimal consumption choice. This process is repeated backward until we reach the lowest age in the model. When evaluating continuation values, we perform linear interpolation over next-period wealth and house prices.

The persistent income process is discretized using the generalized Rouwenhorst algorithm (Fella, Gallipoli and Pan, 2019). The price shock and transitory income shocks are discretized on an equal probability basis. That is, for a grid with \( n \) points, the nodes are positioned at the midpoints between groups determined by the \( n − 1 \) quantiles, each having an equal probability of \( 1/n \). For instance, in a setup with three nodes, each node has a probability of \( 1/3 \), and the nodes would be positioned at the 16.66th, 50th, and 83.33rd percentiles (with the first tertile at the 33rd percentile and the second at the 66th).
Appendix Figure A6 Structural Estimation: Identification. Red dashed line is the empirical moment. The orange and black dots denote the 25th, 50th, and 75th percentiles.
The persistent income shock $\nu$ follows a 3-state Markov chain process, and the transitory income shock is discretized to 2 states, while the house price shock is discretized to 3 states. The net worth, price, and initial price grids are all unevenly spaced, with higher density for lower values with 71, 10, and 6 grid points, respectively. For the ownership grid we use 4 nodes, at $[0, 0.5, 0.75, 1]$ and the ownership share must be on the grid.

The model is solved in Julia 1.8.5, and in addition to standard packages we use `Interpolations.jl v0.14.7` and `Optim v1.7.5` for interpolation and optimization routines.
Appendix Figure A7 Comparative Statics on Housing Decisions. These figures are equivalent to the figures in Fig. 6, just with different parameters.
<table>
<thead>
<tr>
<th></th>
<th>Without PO</th>
<th>With PO</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Rent</td>
<td>PO</td>
</tr>
<tr>
<td>Owners</td>
<td>91.4%</td>
<td>0.0%</td>
</tr>
<tr>
<td>Renters</td>
<td>8.6%</td>
<td>10.2%</td>
</tr>
<tr>
<td>Totals</td>
<td>100.0%</td>
<td>0.9%</td>
</tr>
</tbody>
</table>

Appendix Table A3 Conditional Change in Housing Outcomes With PO at Age 45. This table reports the housing outcomes with PO, by housing outcomes without partial ownership. The leftmost column reports the share of households who were owning and renting at age 45. The three rightmost columns report the share of those households who would have been renting, partially owning, or full owners if PO had been available their entire life. For example, the row=Renters, column=PO cell, is the share of renters at age 45 without PO that would be partial owners had PO been available.

Appendix Figure A8 Financial Stability: Debt to Income Distribution. The figure plots debt to income (DTI) for households with debt. The legend includes the 95th percentile of the DTI distribution.