On the Investment Network and Development*

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Abstract

Capital accumulation and the systematic reallocation of economic activities across sectors are two of the most salient features of the process of economic development. These two processes are interconnected through the production of capital of various types and heterogeneous usage intensity across sectors, which is summarized by the investment network. Our paper introduces the first harmonized measures of the investment network across the development spectrum and documents novel empirical regularities. We then propose a simple theory linking disparities in this network and disparities in income per capita across countries. We show that Domar weights and the elasticity of output to sectorial productivity are non-trivial functions of the investment network and of the equilibrium sectorial investment rates along the Balanced Growth Path. For our sample of 58 countries, we show that 30% of the cross-country differences in steady state income per capita can be accounted for by disparities in the investment network. These differences in the "technology" for producing new capital are double what a standard development accounting exercise would predict for the role of capital in income disparities.

JEL Codes: E23; E21; O41.

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1 Introduction

Capital accumulation and the systematic reallocation of economic activities across sectors are two of the most salient features of the process of economic development. Sectors utilize different investment goods for production, which are either produced by other sectors of the economy or imported. As economic activity shifts across sectors, the economy's ability to produce new capital—or to export goods in exchange for these goods—changes. This facilitates further capital accumulation of various types and additional sectorial reallocation. Studying the nature of this continuous feedback is crucial for understanding the mechanics of economic development. It requires measures of sectorial links in both the production and use of new capital, that is, investment networks. This paper provides the first harmonized measures of investment networks across countries and at different stages of development. We document novel facts about how this network evolves as countries develop and construct a theory to evaluate its impact on the observed income differences across countries.

The importance of the nature of the investment network for economic development can be traced back to Hirschman (1958). He argued that a successful development strategy, along with the corresponding paths of capital accumulation, should emphasize sectors with strong forward and backward linkages to the rest of the economy. Our exercise formalizes these ideas while bringing empirical content to it. Recent studies have highlighted the changing sectorial composition of the production of (aggregate) investment in the economy Garcia-Santana, Pijoan-Mas and Villacorta (2021); Herrendorf, Rogerson and Valentinyi (2021), as well as disparities in the bundles of capital goods used for production Caunedo and Keller (2023). We propose a theory that rationalizes the full structure of production and uses of different capital types along the development spectrum. We then characterize the elasticity of aggregate output to changes in sectorial TFP as a function of the investment network and its interaction with the input-output structure, another important source of linkages in production. We show that these elasticities are non-linear functions of the nature of these networks, and that importantly, the role of a sector in boosting economic activity depends on its importance in the production of new investment, its relevance in the production of value added and the capital intensity of other sectors in the economy. Thus, while Hirschman's hypothesis about the role of investment linkages was correct, it was incomplete.

In our economy, welfare and GDP differ because investment is non-trivial. Interestingly, we show that the dynamics of capital accumulation enters into the equilibrium level of Domar weights, which drive the effect of sectorial productivity on Welfare. This is a novel result to the literature in production networks, which for the most part focuses on static economies.

We use our theory to inform measurement. We call the vector that summarizes aggregate output elasticities to sectorial TFP the "*influence vector*", following the now extensive literature that studies network properties of the economy, Acemoglu, Carvalho, Ozdaglar and Tahbaz-Salehi (2012). The influence vector summarizes the direct and indirect impact of changes in sectorial productivity and in the terms of trade for aggregate economic activity. Influence is a function of the input-output structure as well as of the investment network of the economy, through an augmented Leontief inverse. While measures of the input-output structure have become increasingly available across countries, estimates of the investment network are only available for the US (vom Lehn and Winberry, 2022) and a handful of years in OECD economies, see (Ding, 2023).[1] We advance previous measurement efforts by providing cross-country and time-series harmonized estimates of the investment network for 58 countries at different stages of development, i.e. income per capita between $428 and $81599 constant PPP dollars. Our dataset includes 9 countries from sub-Saharan Africa region, and, for many countries in the sample, noticeably South Korea, we provide time-series estimates of the investment network that go back to 1960s. In our analysis, capital is disaggregated into multiple equipment types, including ICT, Electronics, Machinery and Transportation; as well as structures, measured through Construction investment.[2]

To create our new harmonized measures of the investment network, we exploit a methodology similar to that of the Bureau of Economic Analysis (BEA) in the US. The BEA combines the occupational composition of each industry and an allocation rule for capital to workers, to estimate investment by capital type and sector. Unfortunately, the apportioning of stocks to workers is not publicly available. Hence, to assure replicability, we opt for an allocation of capital across sectors that follows Caunedo, Jaume and Keller (2023) for Equipment; and an allocation that follows intermediate inputs for Construction and other sectors with positive investment in final uses. While the allocation of investment may seem arbitrary, it is reassuring that our own estimates of capital-flow tables in the US follow closely those published by the BEA.

We start by documenting systematic disparities in homophily between the input-output and the investment network, which leads to differential roles for sectors as producers of (new) capital and intermediate goods for others. In other words, the diagonal of the input-output structure is heavier than that of the investment network. Hence, while the investment and the input-output networks are both sources of amplification of productivity shocks, their empirical nature is different, and warrant differential impact on aggregates.

A useful summary statistic to measure the relevance of sectors as providers of investment is the *outdegree* of a sector in the network, which corresponds to the row-sum of the entries in the network. Thus, outdegrees measure forward linkages which are strongly related to "upstreamness", as defined by Antras, Chor, Fally and Hillberry (2012). As countries develop, there is a notable decrease in the importance of the Construction and Machinery sectors as providers of investment, while the importance of ICT becomes 6 times higher. The outdegrees

[1] These measures are self-reported by country offices to the OECD Statistics office, and it is unclear whether measurement is comparable across countries. Ding (2023) exploits these investment flows to estimate capital services in each sector from different sectors and countries. To do so, he uses bilateral import flows to input cross-country linkages and estimates user costs along a BGP. In other words, he treats the investment network as a primitive, whereas we construct them in a harmonized way across countries.

[2] Our benchmark estimates include 8 sectors but estimates for as many as 19 sectors consistently defined across countries and time can be made readily available.

of Transportation and Electronics follow a hamp-shaped pattern with income. So, are sectors with high *outdegrees* in the investment network also sectors where changes in productivity have the strongest impact on aggregate activity? The answer is no. Our theory predicts that *influence* is the correct metric to answer this question.

We construct measures of influence using country-year variation in our estimates of the investment network, paired with measures of capital shares, measures of the input-output structure, and estimates of sectorial expenditure shares in value added. We then study the role of the investment network for persistent differences in income per capita through accounting exercises. We calibrate cross-country sectorial productivity differences to match disparities in value added across sectors within countries, and across countries for a sector. This way, our model matches exactly the variance in output per capita observed in the data. We then study the role of different channels in driving those disparities by first eliminating the investment network altogether, then eliminating the input-output structure altogether. Our main finding is that the investment network accounts for 30% of the observed disparities in income per worker. One interpretation of these results is that disparities in investment technology in each country can explain a non-negligible amount of income disparities. A standard development accounting exercise in our sample suggests that cross-country differences in aggregate capital-output ratios account for approximately 13% of observed income differences. Hence, taking into account roundabout effects in the technology for producing capital more than doubles its role.

We also explore the role of trade in driving these resources. Two findings are stark: first, despite trade in equipment being substantial across the world, we find little difference in imported expenditure shares across countries at different stages of development for broad equipment categories; second, we find that neglecting trade can lower steady state income per capita across countries by 44% on average, but that this decline is for the most part uniform across the development spectrum. In other words, trade contributes relatively little to observed income disparities across countries.

Disparities in the investment network reflect differences in the technology used for production, possibly as a consequence of distortions that shift relative prices or as a consequence of disparities in comparative advantage. Our theory rationalizes the choice of different investment networks as coming from differences in the technological frontier that each country faces. We explore the role of systematic disparities in the network along the development spectrum through counterfactuals where we replace the observed investment network by the investment network in Korea in 1965, before the country embarks in a sustained period of economic growth and catch up to developed economies. We show that poorer economies benefit relatively more from producing with the investment network of Korea in 1965, considering their input-output structures, sectoral productivities, and patterns of final expenditure shares. In contrast, richer economies are negatively affected by employing the same technology.

Contribution to the literature. There is a growing literature studying the relevance of sectorial linkages for differences in income per capita across countries. The role of intermediate input linkages has been highlighted by Ciccone (2002); Jones (2011). This role has been quantified in Fadinger, Ghiglino and Teteryatnikova (2022), who employs cross-country measures of input-output linkages as measured from the World Input-Output Dataset (WIOD) to show that differences in the input-output structure across countries amplify the role of sectorial TFP for differences in income per capita. We show that the input-output and the investment networks are empirically different, leading to different conclusions in terms of the role of sectors in boosting aggregate economic activity. In addition, the investment network directly affects the rate of convergence in the economy to its balanced growth path, making it an interesting object of study on its own right. Like us, Buera and Trachter (2024) study the role of sectorial multipliers in the spirit of Hirschman (1958). In their theory, distortions that affect endogenous technology choices and therefore sectorial productivity are at the forefront of the magnitude of these multipliers. Our newly harmonized data could be used to discipline such a theory along the development spectrum.

We contribute to the literature by allowing for dynamics in the accumulation of capital across sectors. Unitary elasticities of substitution in sectorial investment aggregators, as well as in intermediate inputs allow us to handle the empirical heterogeneity in factor intensities across sectors and the dynamics of capital-accumulation, while being consistent with balanced growth. The economy is efficient, so when assessing welfare, Domar aggregation holds. That is, aggregate welfare is a weighted sum of productivity growth, weighted by Domar weights. A key novel finding of our analysis is that the equilibrium level of those weights is a non-trivial function of the sectorial investment rates along the BGP (and along the transition). As in Acemoglu *et al.* (2012) and Liu (2019) output elasticities to sectorial productivity are different than Domar weights. In our framework, this is the result of non-trivial dynamics in capital rather than distortions in production. A static version of our economy with full capital depreciation would eliminate this disparity.

The main empirical contribution of our paper is to construct harmonized estimates of the investment network for many countries across time. We believe this effort opens the door to studying a myriad of questions related to the link between structural transformation and investment (Garcia-Santana *et al.*, 2021; Herrendorf *et al.*, 2021), as well as linking the nature and timing of investment in particular goods to the overall path of development, a timely discussion for policy makers. Indeed, we show that the path of sectorial influence to GDP across equipment categories in the cross-country evidence follows closely that of the development path of Korea during the XXth century.

Highlighting the role of imported equipment for economic growth brings new relevance to the higher cost of investment relative to consumption in poorer countries, and these country's ability to generate resources to trade for these capital goods (Hsieh and Klenow (2007)). Gaggl, Gorry and vom Lehn (2023) and Foerster, Hornstein, Sarte and Watson (2022) study the properties of the investment network in the US within a closed economy framework. Foerster *et al.* (2022) abstract from feedback effects between imported capital, the stock of capital available in the economy and sectorial output by assuming that either the share of imported investment is small, or that there are no time-trends in the terms of trade. Neither of these assumptions is realistic for the economies that we study. Gaggl *et al.* (2023) run their quantitative analysis with a single capital good for production, i.e. investment aggregators are assumed identical.[3]

The paper is organized as follows: in Section 2 we present the model, in Section 3 the methodology for the construction of the investment networks and empirical regularities of the investment network as countries develop; in Section 4 we present the results from the income accounting exercises, and characterize the role of the influence vector across the development spectrum; in Section 5 we conclude.

2 A model of the investment network and economic development

We build a framework to study the impact of long-term shifts in the composition of imported investment across sectors, as well as TFP growth in sectors producing equipment and structures for aggregate GDP growth. We do this in a context where markets for inputs and output are complete, and therefore we can characterize allocations through the technologies available to a planner.

The economy consists of N sectors that combine capital, labor and intermediate inputs to produce output:

$$y_{nt} = \left(\frac{v_{nt}}{\gamma_{nt}}\right)^{\gamma_{nt}} \left(\frac{m_{nt}}{1-\gamma_{nt}}\right)^{1-\gamma_{nt}}, \qquad \text{for } \gamma_{nt} \in [0,1],$$

with a measure of value added $v_{nt} = \exp(z_{nt}) \left(\frac{k_{nt}}{\alpha_n}\right)^{\alpha_n} \left(\frac{l_{nt}}{1-\alpha_n}\right)^{1-\alpha_n}$ that depends on productivity z_{nt}, and capital and labor allocations, k_{nt}, l_{nt}; and a constant returns to scale intermediate input aggregator $m_{nt} = \prod_{i=1}^{N} \left(\frac{m_{int}}{\mu_{int}}\right)^{\mu_{int}}$ with $\sum_i \mu_{int} = 1$.[4] The amount of intermediate inputs from sector i used in sector n is m_{int}. This flow of intermediate inputs is summarized by an input-output matrix, M_t, with typical element μ_{int}. The rows of M_t add to the importance of a sector as an intermediate inputs provider to the rest of the economy, the columns summarize the input composition of the intermediate input bundle in a sector. It will also be convenient to define $\Gamma_t = \text{diag}\{\gamma_{nt}\}$, a matrix of value added shares in production, as well as a matrix of capital expenditure shares, $\alpha_t = \text{diag}\{\alpha_{nt}\}$.

The capital stock used in each sector evolves according to the following law of motion,

$$k_{nt+1} = x_{nt} + (1-\delta_n)k_{nt},$$

for a composite of investment from different sectors.

There is a continuum of firms that produce investment goods for each sector, who optimally choose the intensity of use of different equipment types given a menu of technologies

[3] Our model economy can accommodate arbitrary CRS investment aggregators, but the data needed to discipline its behavior, namely detailed prices of equipment types across countries at different stages of development, is not available.

[4] We normalize inputs by expenditure shares to simplify the algebra.

available at a point in time. Technologies are summarized by the height of the production possibility frontier for investment in a sector, B_n, and the shape of the frontier, summarized by ν_n and its loadings ξ_{int}, as in Caselli and Coleman (2006).[5] Firms maximize profits by simultaneously choosing the amount of investment in each equipment type, and its intensity of use:

$$\max_{\omega_{int},\chi_{int}} p^x_{nt} x_{nt} - \sum_i p_{it}\chi_{int}$$

subject to

$$x_{nt} = \prod_{i=1}^{N} \left(\frac{\chi_{int}}{\omega_{int}}\right)^{\omega_{int}}, \tag{1}$$

$$\sum_i \xi_{int}\omega_{int}^{\nu_n} = B_n \tag{2}$$

for $\sum_{i=1}^N \xi_{int}^{\frac{1}{1-\nu_n}} = 1$, and ω_{int} the expenditure share in investment from sector i in sector n. The flow of investment across sectors is summarized by the investment network, Ω_t, with typical element ω_{int}, and with the sum across elements of each column being one, $\sum_{i=1}^N \omega_{int} = 1$. The rows of the investment network describe the production of investment by each sector, while the columns represent the use of investment by each sector. We assume $\nu_n > 1$ which assures an interior solution to the technology choice problem. Finally, inputs from sector i into the production of investment in other sectors, χ_{it} can be domestically produced or imported, $\chi_{int} = (\frac{\chi^d_{int}}{1-\phi_i})^{1-\phi_i}(\frac{\chi^f_{int}}{\phi_i})^{\phi_i}$, where ϕ_i is the expenditure share in foreign inputs for capital type i.

Each sectors' output can be used for production of final goods, c, intermediate uses m, or domestic investment, χ^d:

$$y_{nt} = c_{nt} + \sum_i m_{nit} + \sum_i \chi^d_{nit}.$$

Sectorial output allocated to the production of final goods is combined with a homothetic aggregator, Y_t, and can be used for exports, ϵ, or for consumption of the representative household:

$$Y_t = \prod_{n=1}^{N} \left(\frac{c_{nt}}{\theta_n}\right)^{\theta_n}, \qquad \sum_{n=1}^{N} \theta_n = 1 \text{ and } \quad \theta_n > 0;$$

$$Y_t = C_t + \epsilon_t.\text{[6]}$$

The representative household derives utility $U(C_t)$ that satisfies usual regularity conditions, and discounts the future at rate β.

[5] A key difference to their environment is that firms choose across capital services produced within the economy, rather than endowment goods. Hence, there is potentially a non-trivial feedback between the nature of the investment network, input-output structure, and its expenditure shares, which determine relative prices and technology choices. Since we set the elasticity of substitution across equipment types in investment to 1, we abstract away from the role of relative prices. This is done partially for convenience, but importantly, because these equipment prices are not readily available across countries and sectors in a harmonized manner.

[6] Our findings are robust to having two different aggregators for exports and consumption. Results carry through except that the price of consumption should be defined in units of exports.

We define the value of net exports in the economy as the difference in the value of exports and imports:

$$NX_t = p_{Yt}\epsilon_t - p_{\epsilon^f t}\epsilon_t^f.$$

The value of imports is the product between the price index of imports and a composite import value $\epsilon_t^f = \prod_{i=1}^{N} \frac{x_{it}^{f}}{\phi_i^f}^{\phi_i^f}$, as in Basu, Fernald, Fisher and Kimball (2005).[7] The terms of trade are given by the ratio between the price of exports and the price of imports $\tau \equiv \frac{p_{Yt}}{p_{\epsilon^f t}}$, where the price of imported goods is a CRS aggregator of the (exogenous) prices of imported investment for production. Given the assumption that the production technology for exports goods is the same as that of the consumption good, the price of exports pins down the price of the consumption good in the economy.

2.1 Balanced Growth Path

Definition *A Balanced Growth Path (BGP) is an allocation such that sectorial output, consumption, investment and capital grow at a constant (possibly different) rate.*

Proposition .1. *There exists a BGP of this economy where the vector of gross output and consumption growth in each sector satisfies*

$$g^y = g^c = g^m = g^x = b_z^y \gamma_z + b_\tau^y \gamma_\tau.$$

where b_z^y, b_τ^y are parameters that depend on technology, namely, the investment network, the input-output network, the capital expenditure and value added shares in gross output.

The growth rate of final output is a consumption share weighted average of the growth rates of sectorial gross output;

$$g^Y = \theta' g^y$$

and the vector of capital and total investment growth in each sector satisfies

$$g^k = g^x = b_z^k \gamma_z + b_\tau^k \gamma_\tau$$

where b_z^k, b_τ^k are also functions of the technology in the economy.

The proof to this proposition can be found in Appendix A.1.1.

Absent trends in the terms of trade, the BGP of the economy is simply a function of the sectorial productivity trends. We use Proposition .1 to detrend the economy and characterize equilibrium allocations in the steady state and along its transition. The pass-through between productivity growth and sectorial output and investment depends on the characteristics of the networks in the economy, which we revisit when characterizing the equilibrium of the detrended economy.

[7] Any unitary elasticity aggregator preserves the balanced growth path properties discussed in the Appendix.

2.2 Equilibrium characterization, detrended economy

Most choices in this problem are standard, except perhaps for the choice of technology, which we describe first.

Technology choice. Optimality in the choice of technologies requires,

$$\frac{\omega_{i'nt}}{\omega_{int}} = \left(\frac{\xi_{i'nt}}{\xi_{int}}\right)^{\frac{1}{1-\nu_n}},$$

and a relative demand for investment goods that follows the relative intensities.[8] Hence, the optimal (relative) intensity of use of each equipment category reflects the shape of the production possibility frontier, and through it, the menu of technologies available in each country for a given sector.

The level of the intensity is pin down by the height of the productivity possibility frontier, given a normalization of the shape parameters, i.e. $\xi_{int} = 1$.

$$\omega_{int} = B_n^{\frac{1}{\nu_n}}.$$

The remainder of the analysis focuses on optimal factor demand and expenses allocation, describing equilibrium aggregate GDP and welfare in our economy. We emphasize how these key variables depend on the features of the investment network. Because markets are complete, the envelope theorem dictates that welfare is indeed a Domar-weighted average of the sectorial productivities. However, value added and welfare differ in our economy because investment is non-trivial.

The main analysis focuses on a small open economy that exports final goods in exchange for capital goods of different types, similarly to the set up in Jones (2011) for intermediate inputs.[9]

Domar weights. Let the Domar weight of sector n be $\eta_n \equiv \frac{p_n y_n}{pv}$, let the share of value added allocated to the production of final goods be $\zeta_n \equiv \frac{p_n c_n}{pv}$ and the value added share of each sector be $\tilde{\zeta}_n \equiv \zeta_n + \frac{p_n \chi_n^d}{pv}$. Finally, let the adjusted rate of depreciation for the detrended economy be $\hat{\delta}_i \equiv 1 - \frac{1-\delta}{1+g_i^k}$.

Proposition .2. *The equilibrium Domar weights along the BGP of the economy satisfy*

$$\left[I - \tilde{\beta}^{-1}\Gamma\boldsymbol{\alpha}(1-\boldsymbol{\phi})\Omega - (1-\Gamma)M\right]^{-1}\boldsymbol{\zeta} \equiv \boldsymbol{\eta} \tag{3}$$

for $\tilde{\beta}_i \equiv \frac{\frac{1}{\beta}-(1-\hat{\delta}_i)}{\hat{\delta}_i}$.

In vector form

$$\eta_n = \zeta_n + \sum_{i=1}^{N} \alpha_i \gamma_i \omega_{ni}(1-\phi_i)\eta_i + \sum_{i=1}^{N}(1-\gamma_i)\mu_{ni}\eta_i.$$

[8] Appendix A.2.1 presents a more general version of this problem with an arbitrary CRS aggregator for investment, while Appendix A.2.2 presents a version with wedges in the cost of capital.

[9] We present closed economy versions of these results, where $\phi_i = 0$, or $\chi_{it}^f = 0$ in all sectors i, and there are no exports $\epsilon_t = 0$ in Appendix A.1.3.

Along the transition to the steady-state, Domar weights are functions of their full equilibrium path:

$$\left[I - \tilde{\beta}_{t+1}^{-1}\Gamma\boldsymbol{\alpha}(1-\boldsymbol{\phi})\Omega\frac{x_{t+1}}{k_{t+1}}\frac{g_{\eta_{t+1}}}{g_{x_{t+1}}} - (1-\Gamma)M\right]^{-1}\boldsymbol{\zeta}_t \equiv \boldsymbol{\eta}_t, \tag{4}$$

for $\tilde{\beta}_{it+1} \equiv \frac{1}{R_t} - (1-\hat{\delta}_{it})\frac{p^x_{it+1}}{p^x_{it}}$.

Notice that the role of the investment network for the Domar weight scales with the importance of domestic investment across sectors, $(1-\phi) \in (0,1)$. The lower the importance of domestic investment, the less relevant the investment network is for equilibrium Domar weights.[10]

In our economy, equilibrium Domar weights are non-trivial functions of the investment rates along the BGP. That is, the dynamics of the system affects these weights. Indeed, along the transition path to the BGP, Domar weights are functions of the entire path of future Domar weights. These dynamics is introduced into the problem through the durable nature of capital.

Welfare. We start by describing how aggregate welfare in the economy depends on the investment network. Welfare is defined as a function of final consumption:

Proposition .3. *Along the equilibrium path, welfare satisfies*

$$\ln(C) \approx \boldsymbol{\eta}\Gamma z,$$

where $\boldsymbol{\eta}$ *are the equilibrium Domar weights.*

The proof to this result follows from the envelope theorem, and is analogous to the extensive literature in production networks studying the implications of round-about effects on the aggregate economy. Value added shares scale productivity levels because of the way productivity has been defined within the production technology. Proposition .3 is consistent with results in vom Lehn and Winberry (2022) for short-run fluctuations: Domar weights are scaled by the ratio between the value of GDP and final consumption. Along the BGP, this ratio is a constant. In the transition, the welfare effect of sectorial productivity shocks can be amplified or dampened depending on the relative allocation of value added between to consumption and investment uses.

Aggregate GDP. In an economy with investment, aggregate consumption and GDP differ. Next, we study the effect of the investment network for the aggregate level of GDP.

Proposition .4. *The equilibrium level of value added in the economy satisfies*

$$\ln(\nu) = \Phi\tilde{\boldsymbol{\eta}}'\Gamma(\boldsymbol{z} + \alpha\boldsymbol{\phi}\Omega'\tau) + \boldsymbol{\epsilon},$$

where $\tilde{\boldsymbol{\eta}}$ *is the vector of sectorial influence;* $\Phi \equiv (I - \tilde{\boldsymbol{\eta}}\Gamma\alpha\boldsymbol{\phi}'\boldsymbol{\Omega}')^{-1}$ *is an adjustment factor for the tradable nature of investment, and* $\boldsymbol{\epsilon}$*, an adjustment factor that depends on the equilibrium Domar*

[10] A salient feature is that the role of sectors as exporters do not show up in the equilibrium expression of the Domar weight. This is due to the assumption that exports stem from the composite final goods. If each sector is allowed to allocated output to exports, these flows show in the Domar weight of the sector.

weights.[11] *Sectorial influence is the product between sectorial value added shares, $\tilde{\boldsymbol{\zeta}}'$, and an adjusted Leontief inverse $\Xi \equiv (I - \tilde{\beta}^{-1}\Gamma\alpha(1-\phi)\Omega - (1-\Gamma)M)^{-1}$, i.e. $\tilde{\boldsymbol{\eta}} \equiv \tilde{\boldsymbol{\zeta}}'\Xi$.*

In vector form, GDP can be described as

$$\begin{aligned}\ln(\nu)(1 - \sum_n \tilde{\eta}_n \gamma_n \alpha_n \sum_i \omega_{in}\phi_i) &= \sum_n \tilde{\eta}_n \gamma_n z_n + \sum_n \tilde{\eta}_n \gamma_n \alpha_n \sum_i \omega_{in}\phi_i \ln(\tau) - \\ &\quad \ln(\sum_n \gamma_n (1-\alpha_n)\eta_n) \sum_n \tilde{\eta}_n \gamma_n (1-\alpha_n).\end{aligned}$$

Value added is therefore a function of sectorial productivities, $\boldsymbol{z}$, the terms of trade, τ and a constant $\boldsymbol{\epsilon}$. The first term showcases the impact of productivity on value added, and the vector of sectorial influence $\tilde{\eta}$, similarly to Acemoglu *et al.* (2012). [12] This effect gets augmented through the tradable nature of investment $\Phi \equiv (I - \tilde{\boldsymbol{\eta}}\Gamma\alpha\boldsymbol{\phi}'\boldsymbol{\Omega}')^{-1}$ as in Jones (2013) for tradable intermediate inputs. The reason is that when productivity increases within the economy, so does its export capacity, and due to trade balance, that implies higher imports of investment. The strongest the dependence on imported equipment, ϕ, and the intensity of use of capital in gross output, $\Gamma\alpha$, the strongest this amplification channel is.

The influence vector differs from Domar weights because in an economy with investment, the GDP deflator is not necessarily the deflator for consumption. Indeed, welfare in our economy is characterized through Domar weights as we described above. The main difference between influence on Welfare and influence on GDP is whether sectors are loaded by their relevance in consumption ζ, or their relevance as producers of value added $\tilde{\zeta}$.

Second, the terms of trade enter as a channel directly affecting value added in the economy. Once adjusted for the role of imported investment, the capital share and the investment network, the terms of trade affect the economy similarly to a TFP shock. Notice that as $\phi \to 0$ the economy losses its dependence on tradable investment, and Proposition .2 and .4 boil down to their closed economy which we describe in detail in Appendix A.1.3.

Our quantitative analysis will focus on assessing the role of the investment network in driving disparities across countries in GDP per capita along the BGP. However, the model economy is rich enough to have implications for the distribution of capital, consumption and output across sectors. It also has implications for the speed of convergence to the BGP. Whereas a full analysis of the transition dynamics is outside the scope of the current paper, we can already hint at features of the allocations that are interesting for future work, which we present next.

2.3 Implications of the investment network for steady state allocations

To highlight the implications of the investment network for steady state allocations, we work with a simplified closed economy with only two sectors that produce for consumption and

[11] This term, $\boldsymbol{\epsilon} \equiv -\Phi\tilde{\boldsymbol{\eta}}'\Gamma(1-\alpha)\ln(\Gamma(1-\alpha)\boldsymbol{\eta})$, maps into the equilibrium distribution of employment and is quantitatively small, so for most of the analysis, it can be omitted.

[12] Value added shares also mediate this effect because productivity enters into the value added expression. If modeled into gross output, the factor Γ drops out.

investment; no labor or intermediate inputs. Details are described in Appendix A.2.3.

Table 1: Comparative statics: Two sectors-two capital economy

Baseline

	k_1	k_2	C	κ_{11}	κ_{22}	convergence
$\omega_{11} = 0.15$	0.60	3.42	0.73	0.06	0.75	0.61
$\omega_{11} = 0.5$	1.56	1.56	0.92	0.50	0.50	1.00
$\omega_{11} = 0.85$	2.26	1.63	0.98	0.64	0.76	1.01

$z = [1.1\ \ 1]$

	k_1	k_2	C	κ_{11}	κ_{22}	convergence
$\omega_{11} = 0.15$	0.54	3.94	0.69	0.05	0.78	0.65
$\omega_{11} = 0.5$	2.41	1.35	1.04	0.64	0.35	0.96
$\omega_{11} = 0.85$	3.44	0.81	1.12	0.90	0.62	0.91

$\alpha = [0.2\ \ 0.2]$

	k_1	k_2	C	κ_{11}	κ_{22}	convergence
$\omega_{11} = 0.15$	2.66	0.58	0.81	0.45	0.17	0.41
$\omega_{11} = 0.5$	0.89	0.89	0.85	0.50	0.50	0.88
$\omega_{11} = 0.85$	1.24	0.91	0.88	0.63	0.77	1.06

Notes: The baseline economy is parameterized with identical sectors, $\omega_{ii} = 0.5, \alpha = 0.3, z = 1, \delta = 0.7$. The rate of convergence is computed as the average half-time of the system using the largest eigenvalue of the Jacobian to bound this speed. Convergence rates are relative to the Baseline economy with symmetric investment network, $\omega_{11} = 0.5$.

Table 1 shows alternative parameterizations of the model economy and its implications for steady state levels of capital of each type, a measure of aggregate consumption and the share of gross output from each sector devoted to investment in the capital used within the sector, $\kappa_{ii} = \frac{\chi_{ii}}{\chi_{ii}+\chi_{ij}}$. The dynamics of this economy are not only characterized by the capital used in each sector, but also by the share of gross-output allocated to the investment of new capital goods of each type. The dependence on the allocation of output across sectors is reminiscent of results in multisector economies with heterogeneous capital intensity (albeit with a unique capital good as in Acemoglu and Guerrieri, 2008). We also report the speed of convergence of the system to the steady state, computed as the average half-time of the process using the largest eigenvalue of the Jacobian to bound this speed. Baseline levels are normalized to 1 so that convergence is a measure of relative size of the largest eigenvalue across specifications.

Our baseline computation sets an homogeneous investment network with loadings equal to 0.5 for each investment type in each sector. An effective depreciation rate of 7% (accounting for capital obsolescence); and a discount factor of 4%. Preferences are assumed to put equal loadings on each type of consumption. Finally, the production technology is parameterized with a unit productivity z and a capital expenditure share of 0.3.

The row labelled $\omega_{11} = 0.5$ top panel of Table 1 shows results for this baseline parameterization. Given that technologies are identical across sectors, the allocation of investment across sectors is endogenously identically, $\kappa_{ii} = 0.5$. We then run comparative statics around the dependence of sector 1 in the production of investment from this same sector. When the dependence increases $\omega_{11} = 0.85$ the stock of capital in that sector in steady state increases (it actually increases in both sectors). These movement imply a higher marginal product of investment in that sector. Aggregate consumption also raises as well as the speed of convergence. Differently, when its dependence falls, the capital in steady state in that sector falls while capital in the second sector raises. This higher steady state capital in the second sector implies that the share of investment allocated to own production in sector 1 falls dramatically. Importantly, aggregate consumption falls by slightly more than 20%, and the speed of convergence is 61% of its baseline level. In other words, the strength of the diagonal term in the investment network (related to homophily in models of networks) has implications for steady state allocations as well as the speed of convergence of the system.

The second panel of Table 1 runs identical exercises but with a technology that, at baseline, is 10% more productive in sector 1 than in sector 2. This slight difference in productivity implies that the steady state level of capital in the productive sector almost doubles relative to the unproductive one. The speed of convergence of this system is 4% slower than its level when the investment network is homogeneous $\omega_{ii} = 0.5$ and productivities are the same across sectors. Interestingly, increasing the weight of the diagonal in the sector that is more productive can slow down the rate of convergence, despite sustaining higher aggregate consumption in steady state.[13]

The third panel of Table 1 analyzes an economy as in Panel 1 but setting the capital share in production to 0.2 instead of 0.3, i.e. reducing the marginal product of capital. It is not surprising that the speed of convergence in this economy is 12% slower than in the benchmark, the steady state level of capital and aggregate consumption is lower. A noticeable difference to the economy of Panel 1, is that when the capital shares are lower (and identical across sectors), less weight on the diagonal term for sector one increases the stock of capital in steady state. In our first exercise, this same movement in the investment network leads to a lower stock of capital. These non-linear responses are associated to relative factor intensities, which are not only characterized by the output elasticity to capital, α_i, but also the investment elasticity to each investment type. Relative output prices depend on the size of the output elasticity to capital.

3 Investment network

We are now ready to outline the methodology that we have created to provide estimates of the investment network across countries. We describe data sources, explain our methodology and finally characterize the properties of the investment network at different stages of development.

[13]Even this simple framework has therefore interest.

We group sectors into eight categories: four equipment types—Information and Communication Technology (ICT) [14] for our correspondence., Electronics, Machinery, and Transportation Equipment—along with Construction, Agriculture, Manufacturing (excluding equipment), and Services (see Table 11 for details).

3.1 Methodology

Table 2: The Investment Network

Investment Production	Agri	Const	Elec	ICT	Mach	Manuf	Serv	Transp
				Investment Expenditures				
Agriculture								
Construction						$\omega_{Cons,Man}$		
Electronics			$\omega_{Elec,Elec}$					
ICT				$\omega_{ICT,ICT}$			$\omega_{ICT,Ser}$	
Machinery	$\omega_{Mach,Agri}$							
Manufacturing								
Services								
Transportation					$\omega_{Tra,Mach}$			$\omega_{Tra,Tra}$

Table 2 illustrates an investment network table. Each entry (i, i') in the table indicates the total investment expenditures by column-sector i' purchased from row-sector i. Summing across columns yields the total production of investment by each sector, while summing across rows yields the total investment expenditures for each sector. For instance, each element of the ICT row indicates how significant ICT is as a provider of investment for each sector i', whereas each element of the Agriculture column represents how much Agriculture purchases investment from each other sector i. To express the investment network in terms of expenditure shares ω_{ij}, we simply divide each entry of column-sector i' by total expenditures in that sector, so that the sum across rows for each column is equal to one, $\sum_i \omega_{ij} = 1$.

Estimates of investment produced by each sector are readily available from *Use tables*, which record the uses of sectoral output between intermediate and final uses, including consumption and investment. Our contribution is to estimate how much of the investment produced (or imported) by each sector is purchased by other sectors of the economy. To do so, we follow two assignment rules that depend on whether the sector produces equipment or other capital goods, as we describe next.

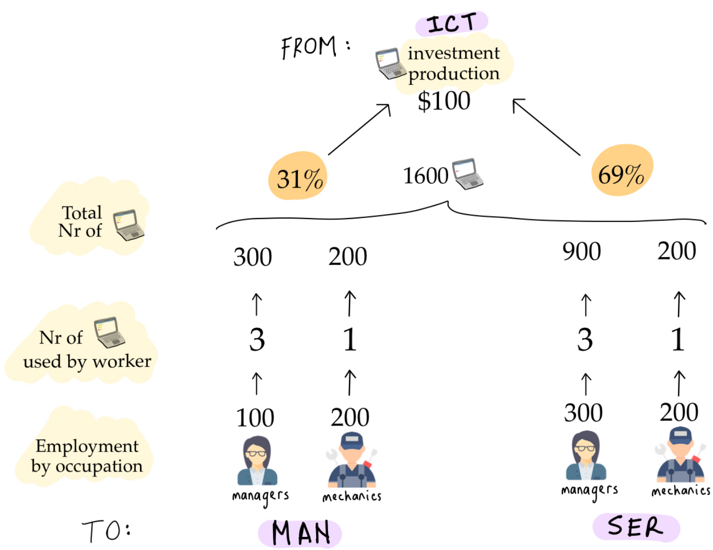
Allocation of equipment investment flows. Equipment-producing sectors include Electronics, ICT, Machinery and Transportation. We allocate their investment flows following the methodology of the Bureau of Economic Analysis (BEA) for the investment network in the US.

[14] Software is included under ICT equipment, which is produced by the Information and Communication sector. Data for years previous to 2000 and for countries in sub-Saharan Africa region include Professional Services together with ICT, therefore we include them under the ICT category to maintain cross country and across time consistency, see Table 11.

This methodology exploits the occupational composition of the labor force in each sector and the types of capital that these occupations likely use.[15]

Figure 1 illustrates with an example how the production of ICT equipment investment is allocated across purchasing sectors following our methodology. Suppose there are three sectors in the economy: ICT, Manufacturing and Services, and that ICT sector produces \$100 worth of new capital goods, i.e. computers. Our goal is to determine how much of the \$100 of ICT investment has been purchased by the Manufacturing sector, and how much by the Services sector. As we mentioned above, we leverage the occupational composition of workers in the purchasing sectors and the type of capital that these occupations are more likely to use. In the example, both Manufacturing and Services employ 200 mechanics wach, along with 100 managers in the Manufacturing sector and 300 managers in Services sector. We normalize the use of computers by mechanics in any industry to 1 and, using data from the allocation of tools to workers (Caunedo *et al.*, 2023), assign three times as many computers to each manager in any industry. Hence, the total demand of computers (in units of the normalized usage for mechanics) in the Manufacturing sector is 500, with 300 of them being used by managers and 200 used by mechanics. The total demand of computers in the services sector is 1100 computers, with 900 of them used by managers and 200 used by mechanics. Out of 1600 computers used in the economy, 31% are used in the Manufacturing sector and 69% are used in the Services sector. Accordingly, of the \$100 worth of computers produced by the ICT sector, 31% are purchased by the Manufacturing sector, and 69% are purchased by the Services sector.

Figure 1: Example: Allocation of ICT Investment Flows



Our assignment follows the tools utilized in each occupation in the US, as described by O*NET. We implement the methodology introduced by Caunedo *et al.* (2023) to assign equip-

[15]BEA's allocation is as outlined in their publicly available documentation, but details of the exact assignment to workers and sector are not available.

ment investment (and therefore stocks) to workers of different occupations.[16] Following the example in Figure 1, the underlying identification assumption is that the number of computers used by a mechanic relative to those used by a manager is the same across countries, and equal to the one in US.[17] The amount of investment assigned to each purchasing sector still differs across countries because the aggregate investment flow of ICT is different across countries, and because the number of mechanics and managers that work in Manufacturing and Services is different across countries, i.e. the occupational composition of the industry varies with development.

Formally, we first compute the share of total production of equipment capital type j purchased by industry i in country c at time t, $\tilde{\omega}^c_{ijt}$, as:

$$\tilde{\omega}^c_{ijt} = \sum_o \frac{\tau_j^{o,US} n_{it}^{oc}}{\sum_{o,i} \tau_j^{oUS} n_{it}^{oc}}, \tag{5}$$

where n_{it}^{oc} is the number of workers in occupation o and industry i in country c at time t, and τ_j^{oUS} is the number of tools of capital type j used by a worker in occupation o in the US.

Since $\tilde{\omega}^c_{ijt}$ represent shares of investment goods allocated to different sectors in the economy, they sum up to 1.

Next, we compute the product between production of investment of capital type j by sector i' in country c at time t, $x^c_{j^{i'}t}$, and $\tilde{\omega}^c_{ijt}$, to obtain the dollar value assigned to each industry i, $x^c_{ij^{i'}t}$.

$$x^c_{ij^{i'}t} = \tilde{\omega}^c_{ijt} x^c_{j^{i'}t} \quad \text{if } j \in \text{equipment type}.$$

From sectors to equipment. One nuance to the assignment of flows to different equipment types is that the mapping is not one to one. In other words, a sector may produce multiple equipment types, and an equipment type may be produced by different industries. This information is encoded in "bridge tables" which underlie national accounts. For example, using an average bridge table between 2000 and 2018 in the US, one can see that 78% of the investment in computers is produced by the Electronics sector, while 22% of it is produced by the ICT sector. The flip side of this figure is that 28% of the output produced by the Electronic sector is computers' production, 34% of it is communication equipment, and the rest are other equipment categories. To the best of our knowledge, bridge tables are not available across countries. Hence, we use the average allocation of sectorial production to equipment types from the US

[16] The methodology in Caunedo *et al.* (2023) cross-walks equipment categories to the tools used within each SOC occupation.We use Dingel and Neiman (2020)'s crosswalk between SOC and ISCO to map these tools to harmonized cross-country occupational definitions.

[17] This identification restriction can be relaxed projecting tool usage in each occupation to the tasks performed on the job. Then cross-country variation in tasks for the same occupation, as the one documented in Caunedo, Keller and Shin (2021) can be used to predict tool usage for the same occupation across countries at different stages of development. The task projection is available in slightly more than half of our sample, and mostly for middle and high income countries.

Bridge tables between 2000 and 2018.[18] Formally, we construct first the total investment from a producing sector i' to equipment type j and we assign this flow following the imputation above. Then, for each demanding sector i, we add across all equipment type investment flows that are relevant to the producing sector i', which generates the relevant investment flows,

$$x^c_{ii't} = \sum_{j \in i'} x^c_{ij^{i'}t}.$$

We can then renormalize this investment flows by the total demand of investment in a sector, to generate the loadings of the investment network, $\omega_{ii't}$.[19]

Allocation of Construction and Other Sectors' Investment Flows. There exist no information on worker's usage of capital goods produced by the construction (i.e. structures); Agriculture, Manufacturing (except equipment) and Services sectors. Hence, we use the input-output structure of each country and assign the flows of investment from these sectors proportionally to their role as intermediate goods providers of other sectors in the economy.

Denote by $\tilde{\mu}^c_{ii't}$ the share of total intermediate inputs produced by sector i' that are purchased by sector i. For the non-equipment sectors, we compute the dollar value of each entry in the investment table as:

$$x^c_{ii't} = \tilde{\mu}^c_{ii't} x^c_{i't} \quad \text{if } i' \in \text{Non-equipment sector.}$$

The Investment Network. Lastly, to express the investment network in terms of expenditure shares $\omega^c_{ii't}$—such that the columns of the matrix add up to 1— we simply divide each dollar-value entry by their respective column-sum, i.e. the total expenses in new capital for any given sector:

$$\omega^c_{ii't} = \frac{x^c_{ii't}}{\sum_{i'} x^c_{ii't}}.$$

3.2 Data Description

Five pieces of data are necessary to construct the investment network: production of investment goods by each sector x^c_{it}, the number of tools by worker in each occupation τ_j^{oUS}, the employment distribution by occupation and sector n^{oc}_{it}, the Bridge table to construct equipment-sector flows $x_{ji'}$ and the Input-Output structure $\tilde{\mu}^c_{ii't}$.Table 3 summarizes the data sources, and a detailed description of the data sources for each country in our sample can be found in Table 10.

[18] The total production of a sector allocated to equipment types does not include replacement of used goods nor trade margins. Hence, when we impute investment flows from a sector to equipment in other countries (which may include these margins), we are effectively distributing these margins equally across equipment types.

[19] Our results are robust to constructing a cross walk between sectors and equipment types that assigns the total flow from a sector to its most common use. Results available upon request.

Table 3: Investment Network: Data Sources

Data Description	Source
Investment production by sector	Mensah and de Vries (2023),WIOD, OECD
Sector-commodity bridge	US Bridge tables (BEA)
Nr of tools by worker in each occupation	Caunedo *et al.* (2023)
Employment by occupation and sector	IPUMS, ILOSTAT, PIAAC
IO structure	Mensah and de Vries (2023), WIOD, OECD

Investment production by sector and Input-Output Tables. We obtain production of Gross Fixed Capital Formation (GFCF) by sector from the *Use Tables* that underlie the measurement of the Input-Output matrix. For the 9 countries from sub-Saharan Africa region in our sample, we use data provided by Mensah and de Vries (2023). For the remaining countries, we source this information from the World Input Output Dataset (WIOD) and OECD input-output tables.[20]

Employment by occupation and sector. We use the estimates of employment by occupation and sector from the PIAAC's survey, IPUMS International and ILOSTAT. For those countries with data available from all sources, we favor PIAAC over IPUMS International and ILOSTAT because the level of occupational disaggregation is higher.[21]

Country Coverage. Our dataset covers 58 countries at different stages of development, with income levels ranging from \$428 and \$81599 GDP per capita (PPP). For 20 of these countries, we construct time-series of investment networks from 1965-2014, and for the 9 countries in the sub-Saharan Africa region we construct time-series of investment networks from 1990-2019. For the remaining 29 countries, the investment network time series covers the period 2000-2014. See Table 10 in the Appendix for a full description.

3.3 Comparison with US Investment Networks

In this section, we compare our estimates of the investment network for the United States to the investment networks constructed by vom Lehn and Winberry (2022) ("*VLW*"), which are based on the capital flows tables from the Bureau of Economic Analysis (BEA). We use 2012 as the

[20]GFCF flows are reported in nominal currency, which we deflate using the output PPP prices from Penn World Tables. To abstract from business cycle fluctuations, we hp-filter sectoral GFCF flows.

[21]PIAAC measurement aggregated at the 1-digit level correlates strongly with IPUMS data, Caunedo *et al.* (2021). In IPUMS International, the industry classification does not include disaggregation of equipment sectors within Manufacturing. However, detailed industry classifications are available prior to their harmonization procedure. For each country for which we source data from IPUMS, we manually construct cross-walks between the disaggregated (not harmonized) industries and our 8 sectors. In PIAAC and ILOSTAT, sectors are classified according to ISIC Rev.4 or ISIC Rev.3 which we also cross-walk to our 8 sector dataset.

primary reference year, as this is the year for which data on employment distribution by sector and occupation were collected by the PIAAC survey.[22] To control for potential disparities in the sectoral Gross Fixed Capital Formation (GFCF) flows from BEA and WIOD, we apply our methodology using the same estimates of sectorial production of investment as vom Lehn and Winberry (2022). We aggregate their estimates of the investment network for 41 sectors to our 8 sectors, consistently with the ISIC Rev.4 cross-walk presented in Table 11.

In Table 4 we compare estimates of (a) the sectoral outdegrees of the investment network, a measure of each sector's relevance as an investment provider to other sectors in the economy, and (b) the homophily of the investment network, a measure of each sector's relevance as a provider of investment for its own sector. The *outdegree* is calculated as the row sum of the entries in the investment network, and the *homophily* is calculated as the diagonal elements of the matrix. Comparing the estimates in the second and third columns of Table 4 panel (a) and (b), we find that our methodology aligns very well with the estimates from vom Lehn and Winberry (2022), especially considering that the only common input is the estimates of the sectoral investment flows. The exceptions are the Machinery and Manufacturing sectors, where our estimates are below and above their estimates, respectively. This pattern suggests that some components of the Machinery sector might be included in our Manufacturing estimates.[23]

For further comparison, we regress the elements of our investment network estimates against those from VLW and report the Mean Squared Errors (MSE) as a measure of prediction accuracy. The MSE values are 0.005 for 2012, 0.008 for 1992, and 0.017 for 1972, indicating relatively small differences between our estimates and theirs.

Table 4: Comparison with VLW

(a) Outdegrees

Sector	This Paper	VLW
Agriculture	0.01	0.00
Construction	1.11	1.05
Electronics	0.74	0.58
ICT	2.5	2.85
Machinery	1.11	1.51
Manufacturing	0.57	0.18
Services	0.82	0.87
Transportation	1.15	0.94

(b) Homophily

Sector	This Paper	VLW
Agriculture	0.00	0.00
Construction	0.01	0.03
Electronics	0.12	0.07
ICT	0.52	0.60
Machinery	0.19	0.26
Manufacturing	0.17	0.04
Services	0.11	0.12
Transportation	0.22	0.27

[22] Table 12 and Table 13 in the Appendix show that the patterns observed in 2012 are largely consistent with those from the years 1972 and 1992. In fairness, since the occupational composition has changed through time, it is surprising that the assignment works relatively well 20 and 40 years ago.

[23] These disparities are likely due to differences in sector definitions and the method used to apportion intermediate input uses in the Manufacturing sector.

3.4 The Investment Network in the Development Spectrum

We now characterize the investment network for countries at various stages of development, using a handful of statistics that we borrow from the networks literature.

3.4.1 Investment Network Outdegrees

We begin by documenting the *outdegree* of each sector. In Table 5, we divide the sample countries into three groups based on their income per capita and report the median sectoral investment network outdegrees for each group.[24] As countries develop, there is a notable decrease in the importance of the Construction and Machinery sectors as providers of investment. The outdegree for low-income countries in these sectors is 22% and 31% higher, respectively, than in high-income countries.

Additionally, the outdegrees of the Transportation and Electronics sectors follow a hump-shaped pattern with respect to income, with Transportation outdegrees being more than 40% higher than those of Electronics. In contrast, the outdegree of the ICT sector increases significantly with development, being 6 times higher in high-income countries compared to low-income ones. Similarly, the outdegree of the Manufacturing sector is 15% higher in high-income countries.

Finally, the role of other service sectors as providers of investment exhibits a mild U-shape with income per capita, though the magnitudes remain relatively similar across income levels.

Table 5: Investment network outdegrees

	Low Income	Medium Income	High Income
Agriculture	0.17	0.11	0.06
Construction	2.92	2.73	2.39
Electronics	0.61	0.73	0.56
ICT	0.16	0.65	0.97
Machinery	1.17	1.09	0.89
Manufacturing	0.62	0.58	0.71
Services	0.99	0.97	0.99
Transportation	0.89	1.09	0.97

Notes: Data for 2005; outdegrees represent sectoral row-sum of the elements of the investment network. Average per capita GDP (PPP): Low Income $5144, Medium Income $21554, High Income $42995.

[24]For ease of exposition, we present empirical descriptives forr reference year 2005, which is the year for which we later conduct income accounting.

3.4.2 Investment Network vs Input-Output Network

In Table 6 we document substantial empirical differences between the Investment Network and the Input-Output Network, as measured by the median outdegrees for these two networks across three income levels. Several notable patterns emerge. Agriculture is a significant provider of intermediate inputs, with its importance declining as development progresses. However, and perhaps not surprisingly, it plays a very minor role as a provider of investment. Construction has the highest investment network outdegree in all country groups, yet it is consistently among the lowest Input-Output outdegrees. In low-income countries, the investment outdegree is up to 30 times higher than the intermediate inputs outdegree. Furthermore, the development pattern in the Construction sector varies: investment outdegrees decrease with income, while intermediate input outdegrees increase.

In the Electronics sector, both investment and Input-Output outdegrees follow a hump-shaped pattern with development, with investment outdegrees being slightly higher. ICT exhibits similar qualitative patterns in both investment and Input-Output linkages, although the magnitudes of the Input-Output outdegrees are up to 4 times higher for low-income countries and 1.5 times higher for high-income countries. A similar situation is observed with Services, where qualitative patterns are alike, but Input-Output outdegrees are twice as high in magnitude.

For Machinery, investment outdegrees decrease with development, but Input-Output outdegrees remain at similar (and significantly lower) levels across income groups. In the Manufacturing sector, we observe opposite qualitative patterns: the investment network outdegree increases with income, while the Input-Output outdegree decreases. The magnitudes of the Input-Output outdegrees are 4.5, 4.3, and 2.9 times higher for low, medium, and high-income countries, respectively. Lastly, for Transportation sector, there is a hump-shape pattern with development for investment linkages, but an increasing outdegree for intermediate linkages.

Figure 2 panel (a) shows a scatter plot illustrating these significant differences between the investment and the Input-Output outdegrees for countries at different income levels.

Table 6: Investment Network vs Input-Output Network Outdegrees

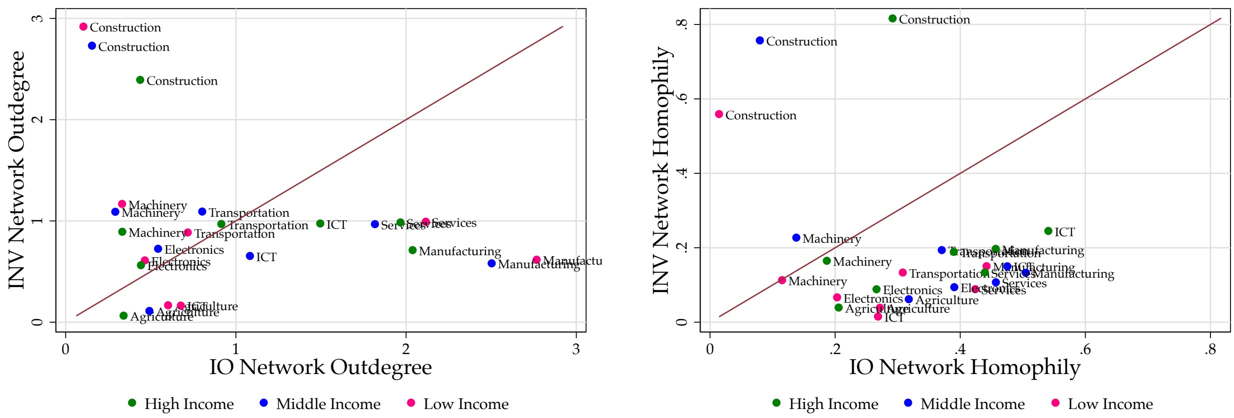
	Low Income		Medium Income		High Income	
	INV	IO	INV	IO	INV	IO
Agriculture	0.17	0.60	0.11	0.49	0.06	0.34
Construction	2.92	0.10	2.73	0.15	2.39	0.44
Electronics	0.61	0.47	0.73	0.54	0.56	0.44
ICT	0.16	0.68	0.65	1.08	0.97	1.50
Machinery	1.17	0.33	1.09	0.29	0.89	0.33
Manufacturing	0.62	2.77	0.58	2.50	0.71	2.04
Services	0.99	2.12	0.97	1.82	0.99	1.97
Transportation	0.89	0.72	1.09	0.80	0.97	0.91

Notes: Data for 2005; outdegrees represent sectoral row-sum of the elements of the investment network. Average per capita GDP (PPP): Low Income \$5144, Medium Income \$21554, High Income \$42995.

Figure 2: Investment Network vs Input-Output Network

(a) Outdegrees

(b) Homophily



Next, we compare the homophily of the investment network relative to the input output network, by examining the values of the diagonal entries of each respective matrix. As shown in Figure 2 panel (b), sectors are significant providers of intermediate inputs for themselves but depend more on other sectors for investment goods. An exception is the Construction sector, which is a major provider of investment goods for itself but not of intermediate inputs.

Differences in investment network across countries through time or income levels could prima facie reflect systematic disparities in technologies for production, either as a result of distortions or comparative advantage. The study of the sources of these disparities exceeds the scope of the current analysis but are nevertheless of key importance to understand the process of development. As a first step to highlight the implications of these newly uncovered patterns for income differences across countries, we now combine the structural predictions of the model with our newly constructed measures of the investment network to conduct an income accounting exercise.

4 Income Accounting

With our newly constructed measures of the investment network, we quantify the sectorial influence, which summarizes output elasticities to sectorial productivity growth and to changes in the terms of trade, see Proposition .4.

4.1 Data description

To estimate sectorial influence as described in Proposition .4 we need data on sectoral value added shares in gross output ($\boldsymbol{\Gamma}$), sectoral value added shares ($\tilde{\boldsymbol{\zeta}}$), capital shares in value added ($\boldsymbol{\alpha}$), sectoral imported share of investment ($\boldsymbol{\phi}$), sectoral depreciation rates ($\hat{\delta}$), and estimates of sectoral TFP (z) and terms of trade (τ) for each country in the sample.

Value added shares in production (Γ) and sectorial value-added shares ($\tilde{\zeta}$). We compute sectoral value added shares in gross output and sectoral value added shares using the same data sources as the Input-Output tables for each respective country: Mensah and de Vries (2023), WIOD, and OECD.

Capital share in value added (α). We exploit data from Penn World Tables version 10.01 to compute labor expenditure share. We estimate capital shares as residuals from labor expenditure shares, under the assumption of Constant Returns to Scale value-added production technologies. The capital expenditure share is computed at the aggregate level, and therefore country-specific but common across sectors.[25]

Sectoral imported investment shares ($\boldsymbol{\phi}$). For each sector and country, we compute the share of sectoral investment that is source from abroad using the *Use Tables* described in Section 3.2, which contain information on imported and domestic sectoral investment.

Sectoral depreciation rates ($\hat{\delta}$). Estimates of depreciation rates by sector are not available across countries. Given this data limitation, we compute sectoral depreciation rates for US using data from Fixed Assets Tables from BEA. We first compute, for each sector, the associated depreciation rates ($\hat{\delta}_i$) of equipment, structures and intellectual property, as the ratio of depreciation over net stock of each capital type. We then construct a sectoral-level depreciation rate as as a weighted average of the sectoral depreciation rate of each capital type, weighted by the share of each type in the total capital stock of the sector. We impose the same depreciation rates for a given sector across countries.

"Productivity-like" shifters. As we discussed in the model economy, the terms of trade work similarly to a TFP shock across sectors. Both of these can be treated as shocks to these

[25]Some high and medium income countries have data on sectoral capital shares in value added from WIOD. But this information more than half of the countries in our sample. Hence, for consistency we use country-specific aggregate capital shares.

sectors, and inferred as a residual using the structural restrictions of the model. Let this residual be $\boldsymbol{a} \equiv (\mathbf{z} + \alpha\boldsymbol{\phi}\Omega'\tau)$.[26] Then,

$$\boldsymbol{a} = \left(I - \Gamma\alpha\boldsymbol{\phi}'\boldsymbol{\Omega}'\right)^{-1} \Xi\Gamma \ln(\boldsymbol{\nu}), \tag{6}$$

with $\ln(\boldsymbol{\nu})$ being a vector of log of sectorial value added. We use this identity to infer relative productivities across sectors, and then discipline the level of productivity in an economy (i.e. for a sector) $\boldsymbol{a}$ to match the observed level of income per capita in each country.

To ensure comparability across countries, we rely on data from WIOD in the year 2005, as it is the only year with available Purchasing Power Parity (PPP) sectoral prices that we use to convert nominal values into real units. For those countries not included in the WIOD sample, we use GDP PPP price deflators from Penn World Tables (PWT).

4.2 Accounting

Equipped with estimates of sectoral influence and the implied GDP in each country, we run income accounting to address the following question: How important are cross-country differences in the investment network in explaining variations in income per capita?

Then, we construct counterfactual estimates of GDP for alternative measures of sectorial influence, driven by alternative assumptions on the investment and intermediate input networks. To ease the exposition, we repeat the main expression for GDP:

$$\boldsymbol{\eta}^{\text{GDP}}\boldsymbol{a} \equiv \underbrace{\Phi}_{\substack{\text{trade} \\ \text{amplification}}} \underbrace{\tilde{\zeta}'}_{\text{exp share in VA}} \underbrace{(I - \tilde{\beta}^{-1}\Gamma\alpha(1-\phi)\Omega - (1-\Gamma)M)^{-1}}_{\substack{\text{augmented} \\ \text{Leontief inverse}}} \Gamma\boldsymbol{a}, \tag{7}$$

with trade amplification term Φ as defined in Proposition .4, $\tilde{\beta}_i \equiv \frac{\frac{1}{\beta} - (1-\hat{\delta}_i)}{\hat{\delta}_i}$, and $\boldsymbol{a}$ defined in equation 6. We set the discount factor to $\beta = 0.96$.

To assess the role of the investment network, and given the non-linear nature of its effect on GDP per capita, we estimate model-based income using equation 7 in two counterfactual scenarios: One where we only include the investment network, and another one where we remove the investment network altogether (including its effect through trade amplification Φ). Intuitively, we compute the change in the variance of income to the investment network at two different points. The role of the investment network in explaining income variances can be assessed as an average across these two "orderings" of the counterfactual exercises.

Table 7 presents the results. We define the *Baseline* scenario as the model-based income when all elements in equation 7 are included, which matches with the observed levels of income per capita. We normalize the *Baseline* cross-country income variance to 1. When we only

[26] The next version of this paper will split this residuals between TFP and the terms of trade effect. This last term depends on the shape of the investment network and is potentially important when running counterfactuals.

include investment links (second row in Table 7), the model predicts 36% of the observed income disparities. When we eliminate the investment network altogether (third row in Table 7), we find that the investment network can account for 25% of the observed disparities. We conclude from these exercises that the investment network accounts for 30% of the observed income differences in our sample.

Table 7: Development Accounting

	Income Variance	Contribution of Ω
Baseline	1	
Only Investment Links	0.36	36%
Only Intermediate Inputs Links	0.75	25%

The role of trade. Given that capital goods (in particular equipment) are produced in a handful of countries (Eaton and Kortum (2001)), it is natural to ask whether trade is a driver of the role of the investment network for income disparities. We would expect some role if the incidence of trade differs across countries along development and across equipment types.

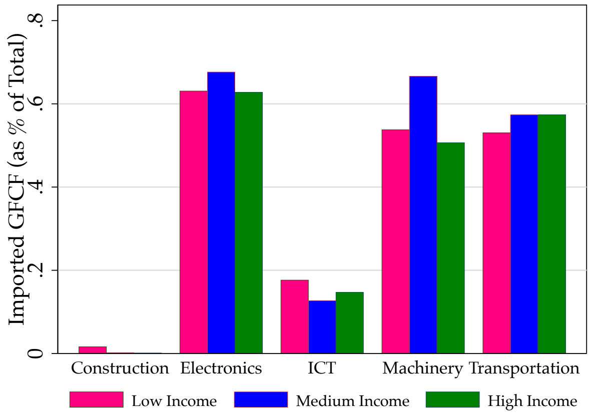
Quantitatively, we answer this question by constructing an economy where there is (a) no multiplier Φ from exports that can induce further equipment imports; and (b) the investment network is recomputed only considering sectoral domestic investment flows. Perhaps surprisingly, we find almost no effect of trade in capital for explaining income disparities across countries (see Table 8). The reason behind this result is that import shares of equipment across the income spectrum do not vary much, as we show in Figure 3 panel (a).

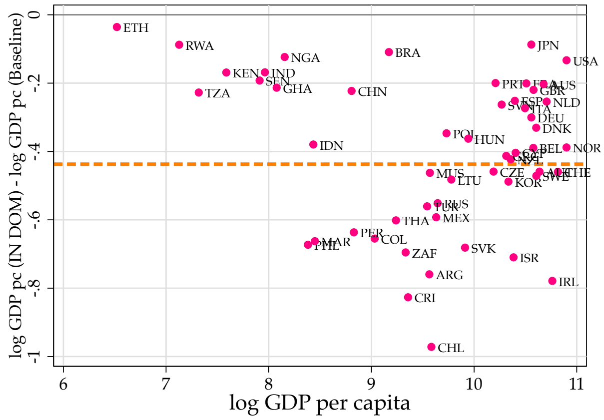
Does this mean that trade in capital is not relevant in determining income per capita? No! Indeed, when we shut down trade in equipment, average income per capita falls by 44% in our sample, see Figure 3 panel (b).

Table 8: Impact of Trade

	Income Variance	Contribution of Ω
Baseline	1	
Only Investment Links	0.36	36%
Only Domestic Investment Links	0.36	36%

Figure 3: Impact of Trade





Notes: *Panel (a):* Average across countries within each income group, period 2000-2015. *Panel (b)*: Changes in income per capita in an economy with only domestic investment links and total intermediate input links, relative to baseline. Countries with outlier income changes were excluded from the graph: Zambia (log GDP per capita 7.4, change of 12%), Malaysia (log GDP per capita 9.8, change of-168%), and Singapore (log GDP per capita 11.1, change of -148%).

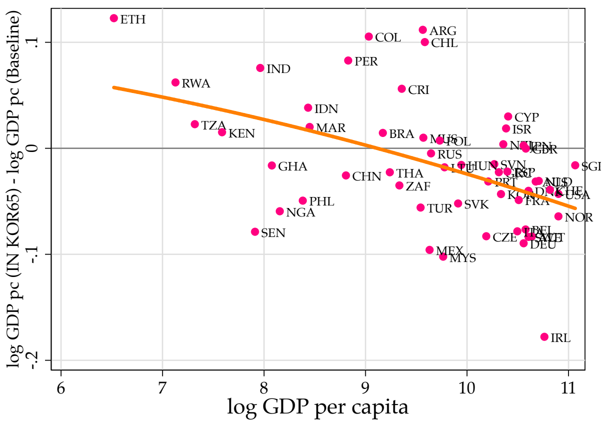
Alternative investment networks. As we mentioned before, investment networks can be interpreted as technologies for the production of new capital goods in the economy. As discussed in section 2.2, these technologies are the outcome of some endogenous choice of production, given endowments, comparative advantage, and (possibly) distortions. One interpretation of the documented disparities in investment network across countries is that countries differ in the technology frontier over which they choose how to produce investment, i.e different B_n and ζ_{in} per sector n. Appendix Figure 6 shows the network estimates for Korea in 1965 and in 2014. The network becomes more diversified in recent years, but the non-diagonal terms of the network are still important. One can interpret this shifts through the availability of a technology with higher height, B_n, because ICT shares rise throughout; as well as shifts in the relative intensity of equipment, with movements away from the Construction, Machinery and Services sectors which are prevalent in 1965.

So what would happen to our economies if we introduce a technology for investment production that resembles that of Korea at the beginning of its development process in 1965, when its income per capita was \$1450 PPP, more than 30 times lower than it currently is? In terms of the investment network, this exercise is equivalent to giving countries a technological frontier with lower B_n in all sectors, and a schedule of ζ_{in} that is relatively concentrated in Construction, Machinery and Services.

Figure 4 shows the differences in income levels relative to baseline if each country had the same investment network as Korea in 1965: a positive value indicates an improvement in GDP per capita relative to the observed one, while a negative number indicates a deterioration in GDP per capita relative to the observed one. We find that poorer countries would benefit relatively more from producing with the investment network of Korea in 1965, but the vast majority of economies would suffer from this technology. This finding suggests that economies are shifting investment technologies as they develop, perhaps optimally. Our next section studies the plausibility of these systematic shifts along the development spectrum.

We also run robustness exercises where we impose the composition of domestic and imported shares of capital in Korea in 1965. In that case, the gradient with development is even more negative (see Figure 7 in the Appendix) suggesting that poorer countries are relatively closed and that rich countries' shifts in the investment network is also consistent with their opennes in terms of equipment trade.

Figure 4: KOR Investment Network in 1965



Notes: Changes in income per capita in an economy with investment network of Korea in 1965, relative to baseline. Countries with outlier income changes were excluded from the graph: Zambia (log GDP per capita 7.4, change of 74%).

4.3 The role of the Influence Vector

Table 9 reports the magnitudes of the influence vector across countries at different stages of development. The most salient features are a steady decline in the influence of Agriculture and Manufacturing (except equipment) and a steady increase in the influence of ICT and Services. Transportation, Electronics and Machinery display a hump-shape in sectorial influence across income groups (although last two with milder levels).

Figure 5 shows the full pattern across the development spectrum. Each gray entry in the graph is a year-country observation, whereas those highlighted in orange correspond to the development path of South Korea from 1965 with a GDP per capita of \$1450 PPP to 2014 with a GDP per capita of \$35525 PPP. It is surprising to see how well the time-series of the path of Korea lines up with the fitted average across the sample. This finding suggests that there might be systematic patterns in the nature of the shifts of the investment network across development that, albeit outside of the scope of this paper, deserve further attention.

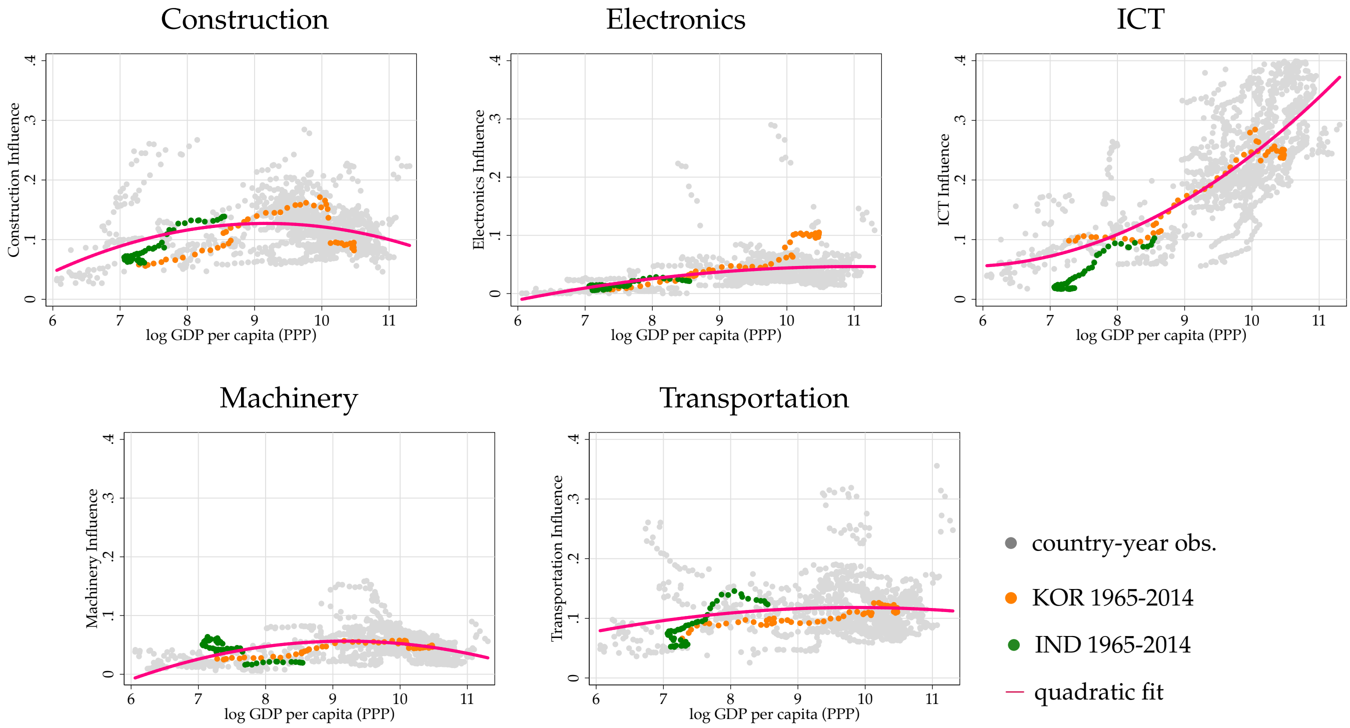
Prima facie, these patterns could be driven entirely by the sectorial shares of value added, $\tilde{\zeta}$. Hence, we separately report the outdegrees of the augmented Leontief inverse, Ξ (see Table 14 in the Appendix). Comparing these magnitudes to those of the influence vector, it can be seen that the dynamics of influence for Services are mostly driven by sectorial value-added shares. The reason is that the outdegrees of Leontief inverse for Services are relatively stable along the income spectrum at levels of 2.6. For the remaining sectors, the qualitative patterns of influence correlate with the dynamics of the outdegrees of the Leontief-inverse, although the relative magnitudes change across sectors.

Table 9: Influence Vectors, average

	Low Income	Medium Income	High Income
Agriculture	0.26	0.07	0.03
Construction	0.14	0.15	0.11
Electronics	0.02	0.05	0.04
ICT	0.12	0.22	0.29
Machinery	0.03	0.05	0.03
Manufacturing	0.34	0.26	0.15
Services	0.49	0.62	0.64
Transportation	0.12	0.16	0.12

Notes: Low Income countries have an average per capita GDP (PPP) of 5030, Medium Income countries an average per capita GDP (PPP) of 44472, and High Income countries an average per capita GDP (PPP) of 84671 in 2005.

Figure 5: Influence across countries



One takeaway from this analysis is that the influence of ICT and Services increases with development, and that those are mostly driven by an increase in importance as providers of investment and intermediate inputs to the rest of the economy. To explore their role as potentially high forward-linkage sectors to the rest of the economy, we can refer again to the outdegrees of the investment network across income levels in Table 5. The outdegrees of ICT equipment indeed increase with income levels but the one for Services is stable. In other words, forward linkages from ICT are relatively low at low-stages of development, but become more important as economies develop. Perhaps surprisingly, the role of Machinery and Construction as providers of investment to the rest of the economy declines with development.

5 Final Remarks

We have constructed novel measures of the investment network for 58 countries across the development spectrum and time series estimates that cover years 1965 to 2014. Our analysis reveals systematic disparities in the sectors' roles as providers of investment goods as economies progress. Notably, ICT emerges as increasingly pivotal in investment provision as countries develop. We also document significant empirical disparities between the investment network and the input output network for countries at different income levels.

Leveraging our estimates of the investment network across countries at different development stages, we conduct an income accounting exercise, finding that disparities in the investment network can account for 30% of observed differences in income per capita across countries, almost double the effect of capital in standard income accounting exercise. Additionally, we find that poorer economies could increase output from adopting an investment network similar to that of Korea in 1965, during the onset of its growth miracle, suggesting that economies are shifting investment technologies as they develop.

How is this shift coming to place? To what extent do comparative advantages, distortions, or variations in human capital endowments explain adoption decisions? What are the quantitative implications of optimal investment network choices for transitioning to development? We defer exploration of these questions to future research, and hope this is the groundwork to study those critical questions to the process of economic development.

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A Appendix

A.1 Proofs & Derivations

A.1.1 Balanced growth path

Proof of Proposition .1. Let us start by defining GDP in the economy, ν as the value of consumption and investment expenses plus net exports, $p_Y C + \sum p_n x_n + NX = \nu$, in units of consumption.

Definition: *A balanced growth path is an allocation where output, consumption, investment and capital in each sector grow at a constant, possibly different, growth rate.*

Along the BGP

$$g^{\nu} = g^{p_Y} + g^{c} = g^{p^x} + g^{x} = g^{NX},$$

The growth rate of net exports is

$$g^{NX} = g^{p_Y} + g^{\epsilon} = g^{p^f} + g^{\chi^f}.$$

It follows that the growth rate of the terms of trade (considered exogenous) determines the relative growth of real exports and imports whenever trade is balanced.

$$g^{\tau} \equiv g^{p_Y} - g^{p^f} = g^{\chi^f} - g^{\epsilon}. \tag{8}$$

Define g^y as the vector collecting the growth rates of gross output across sectors $g^y = (g^{y_1}, \ldots.., g^{y_N})$. We define g^{ν}, g^{m}, g^{k} and g^{x} analogously. The growth rate of output in each sector grows at a constant rate equal to growth rate of its uses, including consumption, investment and intermediate goods. Feasibility then implies that $g^{m_{in}} = g^{y_i}$, and therefore, given the aggregator of intermediate inputs in sector n, $g^{m_n} = \sum_{i=0}^{N} \mu_{in} g^{y_i}$. In other words, $g^{m_n} = M' g^y$.

Along the BGP, the law of motion for capital requires $g^x = g^k$, where investment includes domestically and foreign sourced investment. Hence,

$$g^k = g^x = (1-\phi)\Omega' g^{\chi^d} + \phi\Omega' g^{\chi^f}$$

$$g^k = g^x = (1-\phi)\Omega' g^{\chi^d} + \phi\Omega' g^{\epsilon} + \phi\Omega' g^{\tau}$$

Note that because of trade balance the amount of exports in equilibrium equals the amount of imported equipment.

Finally, the production technology implies $g^y = \Gamma g^{\nu} + (1-\Gamma) g^{m}$, and by definition, $g^{\nu} = g^{z} + \alpha g^{k} + (1-\alpha) g^{l}$. But aggregate labor supply is fixed and along a BGP the share of labor allocated to each sector is constant (because relative sectorial output is constant). Using the growth rate of capital and collecting the terms with the growth rate of gross output yields $g^y = \Gamma g^z + \Gamma\alpha(1-\phi)\Omega' g^y + \Gamma\alpha\phi\Omega' g^{\nu} + \Gamma\alpha\phi\Omega' g^{\tau}) + (1-\Gamma) M' g^y$. The third term in the RHS of this expression corresponds to the growth rate of exports.

$$g^y = \Xi'\Gamma(g^z + \alpha\phi\Omega' g^{\underline{\tau}} + \alpha\phi\Omega' g^{\nu}).$$

Hence, the growth rate of gross output in the economy depends on the productivity growth in each sector, the terms of trade and the growth rate of exports, proportional to value added, with a multiplier $\Xi' \equiv (I - \Gamma\alpha(1-\phi)\Omega' - (1-\Gamma)M')^{-1}$. The matrix Ξ is the generalized Leontief inverse.

$$g^{v} = (I - \alpha\phi\Omega'(I + \alpha(1-\phi)\Omega'\Xi'\Gamma))^{-1}(I + \alpha(1-\phi)\Omega'\Xi'\Gamma)[g^{z} + \alpha\phi g^{\tau}]$$

Hence, the growth rate of value added follows from the factor structure of the economy, as in Long and Plosser (1983).

A.1.2 Equilibrium outcomes, open economy

Proof Proposition (open ec) .2. Use the optimality conditions of the firm, to rewrite the expenses in different intermediate and investment goods as a function of gross output, i.e.

$$\mu_{ni}(1-\gamma_i)p_{it}y_{it} = p_{nt}m_{nit}$$

$$\alpha_i\gamma_i p_{it}y_{it} = r_{it}k_{it}$$

$$(1-\phi_{jt})\omega_{ji}p^{x}_{it}x_{it} = p_{jt}\chi^{d}_{jit}$$

By no arbitrage, the user cost of capital satisfies,

$$r_{it} = p^{x}_{it-1}\left[\frac{1}{R_t} - (1-\hat{\delta}_i)\frac{p^{x}_{it}}{p^{x}_{it-1}}\right]$$

where $1-\hat{\delta}_i$ corresponds to the adjusted undepreciated value of a unit of capital adjusted along the BGP, i.e. $1-\hat{\delta}_i \equiv \frac{1-\delta_i}{1+g^{k}_{i}}$; and $R_t = \beta\frac{U'(c_t)}{U'(c_{t-1})}$ is the interest rate in the economy.

Combining the optimality conditions for capital and investment, as well as the steady-state level of capital

$$\alpha_i\gamma_i p_{it}y_{it} = \left[\frac{1}{R_t} - (1-\hat{\delta}_i)\frac{p^{x}_{it}}{p^{x}_{it-1}}\right]\frac{p_{jt-1}\chi^{d}_{jit-1}}{(1-\phi_{jt})\omega_{ji}}\frac{x_{it}}{x_{it-1}}\frac{k_{it}}{x_{it}},$$

which we can use to write the feasibility constraint in each sector n,

$$p_{nt}y_{nt} = p_{nt}c_{nt} + \sum_i p_{nt}\chi^{d}_{nit} + \sum_j p_{nt}m_{njt}.$$

Then

$$\zeta_{nt}\frac{y_{nt}}{c_{nt}} = \zeta_{nt} + \sum_i \frac{\alpha_i\gamma_i(1-\phi_{nt})\omega_{ni}}{\frac{1}{R_t} - (1-\hat{\delta}_i)\frac{p^{x}_{it+1}}{p^{x}_{it}}}\frac{x_{it+1}}{k_{it+1}}\frac{x_{it}}{x_{it+1}}\frac{p_{it+1}y_{it+1}}{p_{it}y_{it}}\zeta_{it}\frac{y_{it}}{c_{it}} + \sum_j(1-\gamma_j)\mu_{njt}\zeta_{jt}\frac{y_{jt}}{c_{jt}}$$

This is a system of equations across sectors that can be solved for the Domar weights $\eta_n \equiv \zeta_n\frac{y_n}{c_n}$. Along the BGP, the solution satisfies,

$$\left[I - \tilde{\beta}^{-1}\Gamma\alpha\Omega(1-\phi) - (1-\Gamma)M\right]^{-1}\zeta \equiv \boldsymbol{\eta} \tag{9}$$

where $\tilde{\beta}$ is an adjustment factor due to the dynamic nature of investment with typical element $\tilde{\beta}_i \equiv \frac{\frac{1}{\beta}-(1-\hat{\delta}_i)}{\hat{\delta}_i}$. ■

Proof Proposition (open ec) .3. The planner's problem associated to our economy is,

$$W \equiv \max_{C_t, Y_t, \omega_{int}, \chi_{int}, x_{nt}, k_{nt+1}, m_{int}, \epsilon_t, \epsilon_t^f} \sum_{t=0}^{\infty} \beta^t \ln(C_t)$$

subject to

$$y_{nt} = \left(\frac{\tilde{z}_{nt} \left(\frac{k_{nt}}{\alpha_n}\right)^{\alpha_n} \left(\frac{l_{nt}}{1-\alpha_n}\right)^{1-\alpha_n}}{\gamma_{nt}} \right)^{\gamma_{nt}} \left(\frac{m_{nt}}{1-\gamma_{nt}} \right)^{1-\gamma_{nt}}, \qquad \text{for } \gamma_{nt} \in [0,1],$$

$$k_{nt+1} = x_{nt} + (1-\delta_n) k_{nt},$$

$$x_{nt} = \prod_{i=1}^{N} \left(\frac{\chi_{int}}{\omega_{int}} \right)^{\omega_{int}}, \quad \sum_{in} \xi_{in} \omega_{int}^{\nu_n} = B_n,$$

$$y_{nt} = c_{nt} + \sum_i m_{nit} + \sum_i \chi_{nit}^d,$$

$$Y_t = \prod_{n=1}^{N} \left(\frac{c_{nt}}{\theta_n} \right)^{\theta_n}, \qquad \sum_{n=1}^{N} \theta_n = 1 \text{ and } \quad \theta_n > 0;$$

$$Y_t = C_t + \epsilon_t, \qquad \epsilon_t - \frac{\epsilon_t^f}{\tau} = 0$$

$$\epsilon_t^f = \prod_{i=1}^{N} \frac{\chi_{it}^{f\ \phi_i^f}}{\phi_i^f} \qquad \chi_{it}^f = \sum_n \chi_{int}^f,$$

$$\chi_{int} = (\frac{\chi_{int}^d}{1-\phi_i})^{1-\phi_i} (\frac{\chi_{int}^f}{\phi_i})^{\phi_i}.$$

where we have defined $\tilde{z} \equiv \exp z$ for notational convenience.

The envelope condition then yields that

$$\frac{\partial C}{\partial \tilde{z}_{nt}} \tilde{z}_{nt} = \lambda_{nt} y_{nt} \frac{\partial y_{nt}}{\partial \tilde{z}_{nt}} \frac{\tilde{z}_{nt}}{y_{nt}}$$

where λ_n is the lagrange multiplier associated to the feasibility constraint for good n and the last term in the above equation is simply the elasticity of gross output to productivity, i.e. γ_n. We can rewrite this in terms of the change in Welfare which is proportional to $d\ln(C_t)$ because utility is separable in time.

$$\frac{\partial C_t}{\partial \tilde{z}_{nt}} \frac{\tilde{z}_{nt}}{C_t} = \frac{\nu_t}{C_t} \frac{\lambda_{nt} y_{nt}}{\nu_t} \gamma_n$$

Along the BGP, aggregate consumption is a constant fraction of GDP, ν, and by definition $\eta_n = \frac{\lambda_{nt} y_{nt}}{\nu_t}$, i.e the Domar weight.

$$\frac{d \ln C_t}{d \ln \tilde{z}_{nt}} = \frac{\nu_t}{C_t} \eta_{nt} \gamma_n.$$

■

Proof Proposition .4. Use the solution and the definition of ζ_i to solve for relative prices, given investment rates.

$$\frac{p_i}{p_j} = \frac{c_j}{c_i}\frac{\zeta_i}{\zeta_j} = \frac{\eta_i}{\eta_j}\frac{y_j}{y_i}$$

These relative prices are useful to define the demand for intermediate inputs, investment and labor, as a function of the vector of sectorial gross output. The demand for intermediate inputs follows $(1-\gamma_i)\frac{\eta_i}{\eta_n}y_n = m_{ni}$, while the demand for domestic investment goods is $\frac{1}{\tilde{\beta}_i\hat{\delta}}\frac{x_i}{k_i}(1-\phi_j)\omega_{ji}\alpha_i\gamma_i\frac{\eta_i}{\eta_j}y_j = \chi_{ji}$. The demand for imported investment satisfies

$$\frac{1}{\tilde{\beta}_i\hat{\delta}}\frac{x_i}{k_i}(\phi_j)\omega_{ji}\alpha_i\gamma_i\frac{\eta_i}{p_j^f}\nu = \chi_{ji}^f.$$

Total investment in sector i defines the level of the stock of capital as

$$x_i = \prod_j\left(\frac{1}{\tilde{\beta}_i\hat{\delta}}\left(\frac{x_i}{k_i}\alpha_i\gamma_i\frac{\eta_i}{\eta_j}y_j\right)^{1-\phi_j}\left(\frac{x_i}{k_i}\alpha_i\gamma_i\frac{\eta_i}{p_j^f}\nu\right)^{\phi_j}\right)^{\omega_{ji}},$$

or what is the same $k_i = \prod_j\left(\frac{1}{\tilde{\beta}_i\hat{\delta}}\left(\alpha_i\gamma_i\frac{\eta_i}{\eta_j}y_j\right)^{1-\phi_j}\left(\alpha_i\gamma_i\frac{\eta_i}{p_j^f}\nu\right)^{\phi_j}\right)^{\omega_{ji}}$.

Assume that the supply of labor is inelastic at 1, so the fraction of labor allocated to each sector follows Domar weights adjusted by the sectorial labor expenditure shares in gross output,

$$l_i^\star = \frac{(1-\alpha_i)\gamma_i p_i y_i}{\sum_i(1-\alpha_i)\gamma_i p_i y_i} = \frac{(1-\alpha_i)\gamma_i\eta_i}{\sum_i(1-\alpha_i)\gamma_i\eta_i}.$$

For the purpose of describing final demand, it would be useful to define $\tilde{l}_i = \frac{l_i^\star}{\gamma_i(1-\alpha_i)}$.

Final output in each sector is then

$$y_n = \left[\exp(z_n)\left(\prod_i\left(\frac{1}{\tilde{\beta}_i\hat{\delta}}\left(\frac{\eta_n}{\eta_i}y_i\right)^{1-\phi_i}\left(\frac{\eta_n}{p_i^f}\nu\right)^{\phi_i}\right)^{\omega_{in}}\right)^{\alpha_n}(\tilde{l}_i)^{1-\alpha_n}\right]^{\gamma_n}\left[\prod_i\left(\frac{\eta_n}{\eta_i}y_i\right)^{\mu_{in}}\right]^{1-\gamma_n}$$

Taking logs and writing output in matrix form we obtain

$$\ln(\mathbf{y}) = \Gamma\boldsymbol{z} + \boldsymbol{\iota} + \Gamma\alpha\boldsymbol{\phi}'\boldsymbol{\Omega}' ln(\nu) + \Gamma\alpha(1-\boldsymbol{\phi})'\Omega'\ln(\mathbf{y}) + (1-\Gamma)M'\ln(\mathbf{y})$$

where each element of the vector $\boldsymbol{\iota}$ can be described as $\iota_n \equiv \gamma_n(1-\alpha_n)\ln(\tilde{l_n}) + \gamma_n\alpha_n\sum_i(1-\phi_i)\omega_{in}\ln(\frac{\eta_n}{\eta_i}) + \gamma_n\alpha_n\sum_i\phi_i\omega_{in}\ln(\frac{\eta_n}{p_i^f}) - \gamma_n\alpha_n\sum_i\omega_{in}\ln(\tilde{\beta}_i\hat{\delta}_i) + (1-\gamma_n)\sum_i\mu_{in}\ln(\frac{\eta_n}{\eta_i})$.

The solution for gross output is then,

$$\ln(\mathbf{y}) = \Xi\Gamma\boldsymbol{z} + \Xi\boldsymbol{\iota} + \Xi\Gamma\alpha\boldsymbol{\phi}'\boldsymbol{\Omega}'\ln(\nu) \tag{10}$$

where the multiplier on sectorial productivity is $\Xi \equiv (I-\Gamma\alpha(I-\boldsymbol{\phi})'\Omega' - (1-\Gamma)M')^{-1}$. Let the price level of the economy be normalized to $p=1$, then aggregate value added is $\nu = \frac{p_n y_n}{\eta_n}$ for any n. We can compute a geometric average of each of the terms using the expenditure shares consumption and investment $\tilde{\zeta}_n \equiv \frac{p_n \hat{c}_n}{v} + \frac{p_n x_n^d}{v}$ as weights. Note that $p_n \hat{c}_n$ is the value of final uses from sector n that are allocated to aggregate consumption (these values can be split due to the constant returns aggregator for final uses). Hence, weights add up to 1 since trade is balanced.

$$\ln(\nu) = \sum_n \tilde{\zeta}_n \ln(p_n) + \sum_n \tilde{\zeta}_n \ln(y_n) - \sum_n \tilde{\zeta}_n \ln(\eta_n).$$

Given a CRS aggregator of sectorial output, the price index for final goods satisfies, $\ln(p) = \sum_n \tilde{\zeta}_n \ln(p_n)$. Because final output is the numeraire, the log of the price index equals zero, and therefore the first term in the expression for value added drops out. The weighting of the terms in the sum also include investment shares in value added. Investment shares are proportional to consumption shares in value added whenever sectorial value added shares are proportional to consumption shares across sectors. This is by construction the assumption in canonical models of input-output linkages without capital and we assume that feature here.[27]

We have already characterized the solution to each of the last two terms, in equations 9 and 10.

$$\ln(\nu) = \tilde{\boldsymbol{\zeta}}' \Xi(\Gamma \boldsymbol{z} + \boldsymbol{\iota} + \Gamma \alpha \boldsymbol{\phi}' \boldsymbol{\Omega}' \ln(\nu)) - \sum_n \tilde{\zeta}_n \ln(\eta_n) \tag{11}$$

where we can define the elasticity of value to sectorial TFP as $\tilde{\eta} \equiv \tilde{\boldsymbol{\zeta}}' \Xi$. Unlike the Domar weight, these elasticities are not adjusted by the investment rate.

Because of the presence of tradable investment goods we obtain an additional amplification (as in Jones (2011) for tradable intermediate inputs). The reason is that as productivity increases within the economy, the export capacity improves, and due to trade balance that implies higher imports of investment. The strongest the dependence on imported equipment and the intensity of use of capital, the strongest is this amplification channel.

$$\ln(\nu) = \left(I - \tilde{\boldsymbol{\zeta}}' \Xi \Gamma \alpha \boldsymbol{\phi}' \boldsymbol{\Omega}'\right)^{-1} \left[\tilde{\boldsymbol{\zeta}}' \Xi(\Gamma \boldsymbol{z} + \boldsymbol{\iota}) - \sum_n \tilde{\zeta}_n \ln(\eta_n)\right] \tag{12}$$

Unpacking the vectors, $\tilde{\zeta}_n = \tilde{\eta}_n - \sum_j \gamma_j \alpha_j (1 - \phi_j) \omega_{nj} \tilde{\eta}_j - \sum_j (1 - \gamma_j) \mu_{nj} \tilde{\eta}_j$

$$\sum_n \tilde{\zeta}_n ln(\mu_n) = \sum_n \tilde{\eta}_n \ln(\eta_n) - \sum_n \sum_j \gamma_n \alpha_n (1 - \phi_n) \omega_{nj} \tilde{\eta}_j \ln(\mu_n) - \sum_n \sum_j (1 - \gamma_n) \mu_{nj} \tilde{\eta}_j \ln(\mu_n)$$

Now consider the term, $\tilde{\eta}\iota$

$$\begin{aligned} \sum_n \tilde{\eta}_n \iota_n &= \sum_n (\tilde{\eta}_n \gamma_n (1 - \alpha_n) \ln(\tilde{l}_n) + \tilde{\eta}_n \gamma_n \alpha_n \sum_j (1 - \phi_j) \omega_{jn} \ln(\frac{\eta_n}{\eta_j}) + \gamma_n \alpha_n \sum_j \phi_j \omega_{jn} \ln(\frac{\eta_n}{p_j^f}) \\ &\quad + \tilde{\eta}_n (1 - \gamma_n) \sum_j \mu_{jn} \ln(\frac{\eta_n}{\eta_j})) - \gamma_n \alpha_n \sum_i \omega_{in} \ln(\tilde{\beta}_i \hat{\delta}_i) \end{aligned}$$

[27] Alternatively, one can set up the economy so that investment in different capital types is produced through the final good. This economy would also allow us to define the price of value added as a function of sectorial prices in a way that they drop out from the expression above, while allowing for investment shares that need not be proportional to consumption shares. The undesirable feature of this economy is that sector producing for final production and intermediate inputs are decoupled from those producing investment.

which can be rewritten as

$$\begin{aligned}\sum_n \tilde{\eta}_n \iota_n &= \sum_n \tilde{\eta}_n \gamma_n (1-\alpha_n) \ln(\tilde{l}_n) + \sum_n \tilde{\eta}_n (\gamma_n \alpha_n + 1 - \gamma_n) \ln(\eta_n) - \gamma_n \alpha_n \sum_i \omega_{in} \ln(\tilde{\beta}_i \hat{\delta}_i) \\ &- \sum_n \tilde{\eta}_n \gamma_n \alpha_n \sum_j \omega_{jn} \left((1-\phi_j) \ln(\eta_j) + \phi_j \ln(p_j^f) \right) - \sum_n \tilde{\eta}_n (1-\gamma_n) \sum_j \mu_{jn} \ln(\eta_j)\end{aligned}$$

Therefore the difference in the last two terms of the expression for value added are

$$\begin{aligned}\sum_n \tilde{\eta}_n \iota_n - \sum_n \tilde{\zeta}_n \ln(\eta_n) &= \sum_n \tilde{\eta}_n \gamma_n (1-\alpha_n)(\ln(\tilde{l}_n) - \ln(\eta_n)) - \sum_n \tilde{\eta}_n \gamma_n \alpha_n \sum_j \omega_{jn} \phi_j \ln(p_j^f) \\ &-\gamma_n \alpha_n \sum_i \omega_{in} \ln(\tilde{\beta}_i \hat{\delta}_i)\end{aligned}$$

The last term can be written as a function of the terms of trade for imported equipment j, $\ln(\tau_j) = \ln(p) - \ln(p_j^f)$. Because the final good is the numeraire, p=1. Hence,

$$\begin{aligned}\sum_n \tilde{\eta}_n \iota_n - \sum_n \tilde{\zeta}_n \ln(\eta_n) &= \sum_n \tilde{\eta}_n \gamma_n (1-\alpha_n)(\ln(\tilde{l}_n) - \ln(\eta_n)) + \sum_n \tilde{\eta}_n \gamma_n \alpha_n \sum_j \omega_{jn} \phi_j \ln(\tau_j) \\ &-\gamma_n \alpha_n \sum_i \omega_{in} \ln(\tilde{\beta}_i \hat{\delta}_i)\end{aligned}$$

which proves our result. ∎

A.1.3 Closed economy.

Let the Domar weight of sector n be $\eta_n \equiv \frac{p_n y_n}{pv}$, let the share of value added allocated to the production of final goods be $\zeta_n \equiv \frac{p_n c_n}{pv}$ and the value added share of each sector be $\tilde{\zeta}_n \equiv \zeta_n + \frac{p_n \chi_n^d}{pv}$.

Proposition .5. *The equilibrium Domar weights are functions of sectorial investment rate.*

$$[I - \Gamma \alpha \Omega - (1-\Gamma) M]^{-1} \boldsymbol{\zeta} \equiv \boldsymbol{\eta} \tag{13}$$

or in vector form

$$\eta_n = \zeta_n + \sum_{i=1}^{N} \alpha_i \gamma_i \omega_{ni} \eta_i + \sum_{j=1}^{N} (1-\gamma_j) \mu_{nj} \eta_j.$$

Proof Proposition .5. Use the optimality conditions of the firm, to rewrite the expenses in different intermediate and investment goods as a function of gross output, i.e.

$$\mu_{ni}(1-\gamma_i) p_{it} y_{it} = p_{nt} m_{nit}$$

$$\alpha_i \gamma_i p_{it} y_{it} = r_{it} k_{it}$$

$$\omega_{ji} p_{it}^x x_{it} = p_{jt} \chi_{jit}$$

By no arbitrage, the user cost of capital satisfies,

$$r_{it} = p_{it-1}^x \left[\frac{1}{R_t} - (1-\hat{\delta}_i) \frac{p_{it}^x}{p_{it-1}^x} \right]$$

where $1-\hat{\delta}_i$ corresponds to the adjusted undepreciated value of a unit of capital adjusted along the BGP, i.e. $1-\hat{\delta}_i \equiv \frac{1-\delta_i}{1+g_i^k}$.

Combining the optimality conditions for capital and investment

$$\alpha_i \gamma_i p_{it} y_{it} = \left[\frac{1}{R_t} - (1-\hat{\delta}_i)\frac{p_{it}^x}{p_{it-1}^x}\right] \frac{p_{jt-1}\chi_{jit-1}}{\omega_{ji}} \frac{x_{it}}{x_{it-1}} \frac{k_{it}}{x_{it}},$$

which we can use to write the feasibility constraint in each sector n,

$$p_{nt} y_{nt} = p_{nt} c_{nt} + \sum_i p_{nt} \chi_{nit} + \sum_j p_{nt} m_{njt}.$$

We can rewrite this condition as

$$\zeta_{nt} \frac{y_{nt}}{c_{nt}} = \zeta_{nt} + \sum_i \frac{\alpha_i \gamma_i \omega_{ni}}{\frac{1}{\beta} - (1-\hat{\delta}_i)\frac{p_{it+1}^x}{p_{it}^x}} \frac{x_{it+1}}{k_{it+1}} \frac{x_{it}}{x_{it+1}} \frac{p_{it+1} y_{it+1}}{p_{it} y_{it}} \zeta_{it} \frac{y_{it}}{c_{it}} + \sum_j (1-\gamma_j) \mu_{njt} \zeta_{jt} \frac{y_{jt}}{c_{jt}}$$

The above define a system of equations across sectors that can be solved for the Domar weights $\eta_{nt} \equiv \zeta_{nt} \frac{y_{nt}}{c_{nt}}$, given investment rates in each sector $\frac{x_{it+1}}{k_{it+1}}$ and the growth rates of nominal gross output, $g_{p_i y_i}$ and investment, g_{x_i}. Note that the equilibrium Domar weight depends on the full path of output by sector, as well as the investment rates.[28]

That is, the solution to the equilibrium Domar weight depends directly on the path of the growth rates of the Domar weights and the investment growth rates.

$$\left[I - \frac{1}{\frac{1}{R_t} - \frac{1-\delta_i}{1+g^k}\frac{p_{it+1}^x}{p_{it}^x}} \Gamma \alpha \Omega \frac{\mathbf{x_{t+1}}}{\mathbf{k_{t+1}}} \frac{\mathbf{g}_{\eta_{t+1}}}{\mathbf{g}_{\mathbf{x_{t+1}}}} - (1-\Gamma) M_t\right]^{-1} \boldsymbol{\zeta_t} \equiv \boldsymbol{\eta_t} \tag{14}$$

Along an BGP the Domar weight is a constant and the investment rate is proportional to the discount factor.[29] The adjustment factor along the BGP is then $\tilde{\beta} = \frac{\frac{1}{\beta} - (1-\hat{\delta}_i)}{\hat{\delta}_i}$.

■

Proposition .6. *The BGP level of value added in the economy satisfies*

$$\ln(\nu) = \tilde{\boldsymbol{\eta}}' \Gamma \boldsymbol{z} - \tilde{\boldsymbol{\eta}}' \Gamma (1-\alpha) \ln(\Gamma(1-\alpha)\boldsymbol{\eta}),$$

or in vector form

$$\ln(\nu) = \sum_n \tilde{\eta}_n \gamma_n z_n - \ln(\sum_n \gamma_n (1-\alpha_n) \eta_n) \sum_n \tilde{\eta}_n \gamma_n (1-\alpha_n).$$

[28] Alternatively, describe this as

$$\zeta_{nt} \frac{y_{nt}}{c_{nt}} = \zeta_{nt} + \sum_i \frac{\alpha_i \gamma_i \omega_{ni}}{\frac{1}{R_t} - (1-\hat{\delta}_i)\frac{p_{it+1}^x}{p_{it}^x}} \frac{x_{it+1}}{k_{it+1}} \frac{x_{it}}{x_{it+1}} \zeta_{it+1} \frac{y_{it+1}}{c_{it+1}} + \sum_j (1-\gamma_j) \mu_{njt} \zeta_{jt} \frac{y_{jt}}{c_{jt}}$$

Which shows that the equilibrium Domar weights solve a first order equation in differences.

[29] Prices of sectorial output move inversely proportional to consumption, which in turn grows at the same rate as final output.

Proof Proposition .6. Use the solution and the definition of ζ_{it} to solve for relative prices, given investment rates.

$$\frac{p_{it}}{p_{jt}} = \frac{c_{jt}}{c_{it}}\frac{\zeta_{it}}{\zeta_{jt}} = \frac{\eta_{it}}{\eta_{jt}}\frac{y_{jt}}{y_{it}}$$

These relative prices are useful to define the demand for intermediate inputs, investment and labor, as a function of the vector of sectorial gross output. The demand for intermediate inputs follows $(1-\gamma_i)\frac{\eta_{it}}{\eta_{nt}}y_{nt} = m_{nit}$, while the demand for investment goods is

$$\frac{x_{it+1}}{k_{it+1}}\frac{x_{it}}{x_{it+1}}\frac{\omega_{ji}\alpha_i\gamma_i}{\frac{1}{R_t}-(1-\delta_i)\frac{p^x_{it+1}}{p^x_{it}}}\frac{\eta_{it+1}}{\eta_{j_t}}y_{j_t} = \chi_{jit}.$$

Total investment in sector i defines the level of the stock of capital as

$$x_{it} = \prod_j\left(\frac{x_{it}}{k_{it+1}}\frac{\alpha_i\gamma_i}{\frac{1}{R_t}-(1-\delta_i)\frac{p^x_{it+1}}{p^x_{it}}}\frac{\eta_{it+1}}{\eta_{jt}}y_{jt}\right)^{\omega_{ji}} \quad \text{or} \quad k_{it+1} = \prod_j\left(\frac{\alpha_i\gamma_i}{\frac{1}{R_t}-(1-\delta_i)\frac{p^x_{it+1}}{p^x_{it}}}\frac{\eta_{it+1}}{\eta_{jt}}y_{jt}\right)^{\omega_{jit}}.$$

Along the BGP, the rate of adjustment is $\tilde{\beta}_i$ as defined before and

$$x_{it} = \prod_j\left(\frac{\alpha_i\gamma_i}{\tilde{\beta}_i}\frac{\eta_{it+1}}{\eta_{jt}}y_{jt}\right)^{\omega_{ji}} \quad \text{or} \quad k_{it+1} = \prod_j\left(\frac{\alpha_i\gamma_i}{\tilde{\beta}_i\hat{\delta}_i}\frac{\eta_{it+1}}{\eta_{jt}}y_{jt}\right)^{\omega_{jit}}.$$

Assume that the supply of labor is inelastic at 1, so the fraction of labor allocated to each sector follows Domar weights adjusted by the sectorial labor expenditure shares in gross output,

$$l^\star_i = \frac{(1-\alpha_i)\gamma_i p_i y_i}{\sum_i(1-\alpha_i)\gamma_i p_i y_i} = \frac{(1-\alpha_i)\gamma_i\eta_i}{\sum_i(1-\alpha_i)\gamma_i\eta_i}.$$

For the purpose of describing final demand, it would be useful to define $\tilde{l}_i = \frac{l^\star_i}{\gamma_i(1-\alpha_i)}$. Also note that the employment allocation is constant along the BGP, because Domar weights are constant.

Final output in each sector is then

$$y_{nt} = \left[z_{nt}\left(\prod_i\left(\frac{\eta_{nt}}{\eta_{it-1}}y_{it-1}\right)^{\omega_{in}}\right)^{\alpha_n}(\tilde{l_{nt}})^{1-\alpha_n}\right]^{\gamma_n}\left[\prod_i\left(\frac{\eta_{nt}}{\eta_{it}}y_{it}\right)^{\mu_{in}}\right]^{1-\gamma_n}$$

Taking logs and writing output in matrix form we obtain

$$\ln(\mathbf{y}_t) = \Gamma\boldsymbol{\lambda_t} + \boldsymbol{\iota_t} + \Gamma\alpha\Omega'\ln(\mathbf{y}_{t-1}) + (1-\Gamma)M'\ln(\mathbf{y}_t)$$

where each element of the vector $\boldsymbol{\iota}$ can be described as $\iota_{nt} \equiv \gamma_n(1-\alpha_n)\ln(\tilde{l_{nt}}) + \gamma_n\alpha_n\sum_i\omega_{in}\ln(\frac{\eta_{nt}}{\eta_{it}}) + \gamma_n\alpha_n\sum_i\omega_{in}(\ln(\frac{y_{it-1}}{y_{it}}) + \ln(\frac{\eta_{it}}{\eta_{it-1}}) - \ln(\tilde{\beta}_i\delta_i)) + (1-\gamma_n)\sum_i\mu_{in}\ln(\frac{\eta_{nt}}{\eta_{it}})$.

Notice that the growth rate of gross output adjusted by the growth rate of the Domar weights is nothing else than the growth rate of sectorial prices. Along the steady state of the detrended economy, these are constant and therefore $\ln(\frac{y_{it-1}}{y_{it}}) + \ln(\frac{\eta_{it}}{\eta_{it-1}})$ is simply zero.

The solution for gross output is then,

$$\ln(\mathbf{y}) = \Xi_t\Gamma\boldsymbol{z} + \Xi_t\boldsymbol{\iota} \tag{15}$$

where the elasticity of output to sectorial productivity is proportional to $\Xi_t \equiv (I - \Gamma\alpha_d\Omega' - (1-\Gamma)M')^{-1}$. Unlike the Domar weight, these elasticities are not adjusted by the investment rate. Let the price level of the economy be normalized to $p = 1$, then aggregate value added is $\nu = \frac{p_n y_n}{\eta_n}$ for any n. We can compute a geometric average of each of the terms using the expenditure shares of consumption and investment $\tilde{\zeta}_n \equiv \zeta_n + \frac{p_n \sum_i x_{ni}}{\nu}$ as weights (since these weights add up to 1).

$$\ln(\nu) = \sum_n \tilde{\zeta}_n \ln(p_n) + \sum_n \tilde{\zeta}_n \ln(y_n) - \sum_n \tilde{\zeta}_n \ln(\eta_n)$$

Given a CRS aggregator of sectorial output, the price index for final goods satisfies, $\ln(p) = \sum_n \zeta_n \ln(p_n)$. Because final output is the numeraire, the log of the price index equals zero, and therefore the first term in the expression for value added drops up. The weighting of the terms in the sum also include investment shares in value added. Investment shares are proportional to consumption shares in value added whenever sectorial value added shares are proportional to consumption shares across sectors. This is by construction the assumption in canonical models of input-output linkages without capital and we assume that feature here.[30]

We have already characterized the solution to each of the last two terms, in equations 13 and 15.

$$\ln(\nu_t) = \tilde{\boldsymbol{\zeta}}_t{}'\Xi_t(\Gamma\boldsymbol{\lambda}_t + \boldsymbol{\iota}_t) - \tilde{\boldsymbol{\zeta}}_t{}'\ln(\boldsymbol{\eta}_t).$$

where we can define the elasticity of value to sectorial TFP as $\tilde{\eta} \equiv \tilde{\boldsymbol{\zeta}}'\Xi$.

Unpacking this expression in vector form, $\tilde{\zeta}_{jt} = \tilde{\eta}_{jt} - \sum_n \gamma_n\alpha_n\omega_{jn}\tilde{\eta}_{nt} - \sum_n (1-\gamma_n)\mu_{jn}\tilde{\eta}_{nt}$

$$\sum_j \tilde{\zeta}_{jt}\ln(\eta_{jt}) = \sum_j \tilde{\eta}_{jt}\ln(\eta_{jt}) - \sum_j\sum_i \gamma_n\alpha_n\omega_{ji}\tilde{\eta}_{it}\ln(\eta_{jt}) - \sum_j\sum_i (1-\gamma_n)\mu_{ji}\tilde{\eta}_{it}\ln(\eta_{jt})$$

Now consider the term, $\tilde{\eta}_t\iota_t$

$$\begin{aligned}\sum_n \tilde{\eta}_{nt}\iota_{nt} &= \sum_n \tilde{\eta}_n\gamma_n(1-\alpha_n)\ln(\tilde{l_n}) + \tilde{\eta}_{nt}\gamma_n\alpha_n\sum_j \omega_{jn}\ln(\frac{\eta_{nt}}{\eta_{jt}}) \\ &+\tilde{\eta}_{nt}(1-\gamma_n)\sum_j \mu_{jn}\ln(\frac{\eta_{nt}}{\eta_{jt}})) - \tilde{\eta}_{nt}\gamma_n\alpha_n\sum_i \omega_{in}\ln(\tilde{\beta}_i\hat{\delta}_i)\end{aligned}$$

which can be rewritten as

$$\begin{aligned}\sum_n \tilde{\eta}_n\iota_n &= \sum_n \tilde{\eta}_n\gamma_n(1-\alpha_n)\ln(\tilde{l_{nt}}) + \sum_n \tilde{\eta}_{nt}(\gamma_n\alpha_n + 1 - \gamma_n)\ln(\eta_{nt}) \\ &- \sum_n \tilde{\eta}_{nt}\gamma_n\alpha_n\sum_j \omega_{jn}\ln(\eta_{jt}) - \sum_n \tilde{\eta}_n(1-\gamma_n)\sum_j \mu_{jn}\ln(\eta_j) \\ &- \tilde{\eta}_{nt}\gamma_n\alpha_n\sum_i \omega_{in}\ln(\tilde{\beta}_i\delta_i).\end{aligned}$$

[30] Alternatively, one can set up the economy so that investment in different capital types is produced through the final good. This economy would also allow us to define the price of value added as a function of sectorial prices in a way that they drop out from the expression above, while allowing for investment shares that need not be proportional to consumption shares. The undesirable feature of this economy is that sector producing for final production and intermediate inputs are decoupled from those producing investment.

Therefore the difference in the last two terms of the expression for value added are

$$\sum_n \tilde{\eta}_n \iota_n - \sum_n \tilde{\zeta}_n \ln(\eta_n) \quad = \quad \sum_n \tilde{\eta}_n \gamma_n (1-\alpha_n)(\ln(\tilde{l}_n) - \ln(\eta_n)) - \tilde{\eta}_{nt}\gamma_n \alpha_n \sum_i \omega_{in} \ln(\tilde{\beta}_i \delta_i)$$

We can rewrite the first two terms as a function of influence vectors by replacing the optimal labor demand,

$$\sum_n \tilde{\eta}_n \iota_n - \sum_n \tilde{\zeta}_n \ln(\eta_n) = \sum_n \tilde{\eta}_n \gamma_n (1-\alpha_n)(\ln(\eta_n) - \ln(\eta_n) - \ln(\sum_n \gamma_n (1-\alpha_n)\eta_n)$$

Hence,

$$\sum_n \tilde{\eta}_n \iota_n - \sum_n \tilde{\zeta}_n \ln(\eta_n) = -\ln(\sum_n \gamma_n (1-\alpha_n)\eta_n) \sum_n \tilde{\eta}_n \gamma_n (1-\alpha_n)$$

The equilibrium level of value added in the economy satisfies

$$\ln(\nu_t) = \tilde{\boldsymbol{\eta}}_t{}' \Gamma \boldsymbol{\lambda}_t - \tilde{\boldsymbol{\eta}}_t{}' \Gamma (1-\alpha) \ln(\Gamma(1-\alpha)\boldsymbol{\eta}_t) - \tilde{\boldsymbol{\eta}}_t{}' \Gamma \boldsymbol{\alpha} \Omega' \tilde{\beta}^{-1} \hat{\delta}$$

In vector form

$$\begin{aligned} \ln(\nu_t) \quad = \quad & \sum_n \tilde{\eta_{nt}} \gamma_n z_{nt} - \ln(\sum_n \gamma_n (1-\alpha_n)\eta_{nt}) \sum_n \tilde{\eta}_{nt} \gamma_n (1-\alpha_n) \\ & - \sum_n \tilde{\eta}_{nt} \gamma_n \alpha_n \sum_i \omega_{in} \ln(\tilde{\beta}_i \delta_i) \end{aligned}$$

■

A.2 Alternative features of the model economy

A.2.1 Technology choices, general set up.

We populate the economy by a continuum of firms that produce investment goods for each sector. These firms maximize profits by choosing the amount of investment in each equipment type, but also the intensity of use of each equipment for production following

$$\max_{\omega_{int}, \chi_{int}} r_{nt} x_{nt} - \sum_i p_{it} \chi_{int}$$

subject to

$$x_{nt} = \sum_{i=1}^{N} \left(\omega_{int} \chi_{int}^{\sigma_n} \right)^{\frac{1}{\sigma_n}}, \tag{16}$$

$$\sum_i \xi_{int} \omega_{int}^{\nu_n} = B_n \tag{17}$$

The production technology is a generalization of the investment aggregator described in equation 1.

The optimal (interior) choices of firms are characterized by two conditions

$$\left(\frac{\chi_{jnt}}{\chi_{int}} \right)^{1-\sigma_n} = \frac{\omega_{jnt}}{\omega_{int}} \frac{p_{it}}{p_{jt}} \tag{18}$$

$$\left(\frac{\chi_{int}}{\chi_{jnt}}\right)^{\sigma_n} = \frac{\xi_{int}}{\xi_{jnt}}\left(\frac{\omega_{int}}{\omega_{jnt}}\right)^{\nu_n - 1} \tag{19}$$

Replacing 18 into 19 we obtain

$$\frac{\chi_{int}}{\chi_{jnt}} = \left(\frac{\xi_{int}}{\xi_{jnt}}\left(\frac{p_{it}}{p_{jt}}\right)^{\nu_n - 1}\right)^{\frac{1}{\sigma_n \nu_n + 1 - \nu_n}} \tag{20}$$

as well as

$$\frac{\omega_{jnt}}{\omega_{int}} = \left(\frac{\xi_{jnt}}{\xi_{int}}\right)^{\frac{1-\sigma_n}{\sigma_n \nu_n + 1 - \nu_n}}\left(\frac{p_{jt}}{p_{it}}\right)^{\frac{\sigma_n}{\sigma_n \nu_n + 1 - \nu_n}} \tag{21}$$

Hence, if $\sigma_n \nu_n - (\nu_n - 1) < 0$ we obtain an interior solution. This is the same as requiring, $\nu_n > 1/(1 - \sigma_n)$. Such a condition requires more curvature in the technology choice than in the investment aggregator. As in Caselli and Coleman (2006), if $\sigma_n < 0$ firms choose to increase the efficiency of the relatively expensive factor, while if $\sigma_n > 0$, they increase the efficiency of the relatively cheap factor. At the same time, the relative demand for a particular investment type decreases in its price.

This economy reduces to our benchmark economy as we take the limit when $\sigma_n \to 0$. In that case, expenditure shares in the investment aggregators are simply the parameters characterizing the shape of the production possibility frontier in each economy $\omega_{int} \propto \xi_{int}^{\frac{1}{1-\nu_n}}$, and independent of relative prices.

A.2.2 The investment network in an economy with distortions.

Consider and economy with distortions, which introduce wedges in the price of investment,$(1 + \tau^i)$. These wedges could be policy distortions, or market power, etc. For the purpose of this analysis we are agnostic about the source of the gap between output prices and marginal costs

$$\max_{\omega_{int}, \chi_{int}} r_{nt} x_{nt} - \sum_i (1 + \tau_i) p_{it} \chi_{int}$$

subject to

$$x_{nt} = \prod_{i=1}^{N} \left(\chi_{int}^{\omega_{int}}\right),$$

$$\sum_i \xi_{int} \omega_{int}^{\nu_n} = B_n$$

Note that the choice of loadings into the production technology are as in the main paper, $\omega_{int} \approx \xi_{int}^{\frac{1}{1-\nu_n}}$. The quantities of investment however change,

$$\frac{\chi_{int}}{\chi_{jnt}} = \frac{\xi_{int}}{\xi_{jnt}}^{\frac{1}{1-\nu_n}}\left(\frac{p_{it}(1+\tau_i)}{p_{jt}(1+\tau_j)}\right) \tag{22}$$

In other words, with a Cobb-Douglas investment aggregator, distortions on the cost of investment affect the quantities demanded of each investment type, but not directly the loadings in the investment network.

It is only when allowing some substitutability or complementarity between investment types into the production of investment that relative prices affect the loadings of the network. From equation 21 (and with parameters that assure an interior solution) one can see that the loading of the investment aggregator in a sector is relatively lower for sectors with stronger distortions (higher τ).

A.2.3 A two sector, two capital example

We characterize the planner's problem of a two sector, two capital types economy with no labor and no trade. We present the detrended economy and work in continuous time for convenience.

$$\max_{c_n} \int_{t=0}^{\infty} exp(-\rho t) U(c_1(t), c_2(t))$$

subject to

$$\dot{k}_1(t) = x_1(t) - (\delta_1 + g_1^k) k(t)$$

$$\dot{k}_2(t) = x_2(t) - (\delta_2 + g_2^k) k(t)$$

$$k_1^{\alpha}(t) = c_1(t) + \sum_{i=1,2} \chi_{1i}(t)$$

$$k_2^{\alpha}(t) = c_2(t) + \sum_{i=1,2} \chi_{2i}(t)$$

$$x_1(t) = \prod_{j=1,2} \chi_{j1}(t)^{\omega_{j1}}$$

$$x_2(t) = \prod_{j=1,2} \chi_{j2}(t)^{\omega_{j2}}$$

Where g_i^k is the growth rate of capital along the BGP, which is in turn a combination of the growth rates of technological change of the investment bundle for each sectorial capital.

$$g_i^k = \sum_{j=1,2} \omega_{ji} g_j^y = \sum_{j=1,2} \omega_{ji} (g_j^z + \alpha_j g_j^k),$$

which can be solved as system of linear equation for the equilibrium growth rates of capital. If the growth rates of sectorial productivity are constant so are the growth rates of capital.

$$\boldsymbol{g}^k = (1 - \boldsymbol{\alpha})^{-1} \boldsymbol{\Omega}' \boldsymbol{g}^z$$

Optimality From the principle of optimality, we can solve for sufficient conditions for an optimum.

$$exp(-\rho t)\frac{\partial U(c_1(t), c_2(t))}{\partial c_i(t)} = \lambda_i$$
$$\lambda_i(t) = \mu_j(t)\omega_{ij}\frac{x_j(t)}{\chi_{ij}(t)}$$

$$\dot{\mu_i(t)} = -\left(\lambda_i(t)\frac{\partial F(k_i(t))}{\partial k_i(t)} - (\delta_i + g_i^k)\mu_i(t)\right)$$

We can use the optimality condition for investment to rewrite the dynamics in terms of the dynamics of the co-state λ

$$\frac{\dot{\mu}_i(t)}{\mu_i(t)} = \frac{\dot{\lambda}_j(t)}{\lambda_j(t)} - \frac{\dot{x_i(t)}}{x_i(t)} + \frac{\dot{\chi_{ji}(t)}}{\chi_{ji}(t)}$$

Totally differentiating the optimality condition with respect to consumption, we obtain $\dot{\lambda}_i(t) = -\sigma\frac{\dot{c_i(t)}}{c_i(t)} - \rho$. Hence,

$$\frac{\dot{\lambda}_j(t)}{\lambda_j(t)} - \frac{\dot{x_i(t)}}{x_i(t)} + \frac{\dot{\chi_{ji}(t)}}{\chi_{ji}(t)} = -\frac{\lambda_i(t)}{\mu_i(t)}\frac{\partial F(k_i(t))}{\partial k_i(t)} + (\delta_i + g_i^k)$$
$$\sigma\frac{\dot{c_j(t)}}{c_j(t)} = \frac{\lambda_i(t)}{\mu_i(t)}\frac{\partial F(k_i(t))}{\partial k_i(t)} - \frac{\dot{x_i(t)}}{x_i(t)} + \frac{\dot{\chi_{ji}(t)}}{\chi_{ji}(t)} - (\delta_i + g_i^k - \rho)$$

Which in terms of allocations is simply

$$\sigma\frac{\dot{c_i(t)}}{c_i(t)} = \omega_{ii}\frac{x_i(t)}{\chi_{ii}(t)}\frac{\partial F(k_i(t))}{\partial k_i(t)} - \frac{\dot{x_i(t)}}{x_i(t)} + \frac{\dot{\chi_{ji}(t)}}{\chi_{ji}(t)} - (\delta_i + g_i^k - \rho)$$

This shows already why the dynamics of the system will be government by the relative allocation of investment across sectors. The dynamics of sectorial investment can be written in terms of its composition as

$$\frac{\dot{x_i(t)}}{x_i(t)} = \sum_{j=1,2}\omega_{ji}\frac{\dot{\chi_{ji}(t)}}{\chi_{ji}(t)}$$

From the optimality condition for investment, we know that investment within the sector are inversely proportional to the shadow value of consumption, i.e.

$$\frac{\dot{\chi_{ji}(t)}}{\chi_{ji}(t)} = \frac{\dot{\chi_{ii}(t)}}{\chi_{ii}(t)} + \sigma\frac{\dot{c_i(t)}}{c_i(t)} - \sigma\frac{\dot{c_j(t)}}{c_j(t)}.$$

Therefore, the Euler equation is

$$\sigma\frac{\dot{c_i(t)}}{c_i(t)} = \omega_{ii}\frac{x_i(t)}{\chi_{ii}(t)}\frac{\partial F(k_i(t))}{\partial k_i(t)} - (\omega_{ii} - (1-\omega_{ji}))\frac{\dot{\chi_{ii}(t)}}{\chi_{ii}(t)} + (1-\omega_{ji})\sigma(\frac{\dot{c_i(t)}}{c_i(t)} - \frac{\dot{c_j(t)}}{c_j(t)}) - (\delta_i + g_i^k - \rho)$$

but because the investment aggregator is constant returns to scale, the second term in the RHS drops out and

$$\omega_{ji}\sigma\frac{\dot{c_i(t)}}{c_i(t)} = \omega_{ii}\frac{x_i(t)}{\chi_{ii}(t)}\frac{\partial F(k_i(t))}{\partial k_i(t)} - (1-\omega_{ji})\sigma\frac{\dot{c_j(t)}}{c_j(t)} - (\delta_i + g_i^k - \rho)$$

Using an analogous expression for consumption in sector j, we can solve for the Euler equation as a function of primitives

$$\omega_{ji}\sigma\frac{\dot{c_i(t)}}{c_i(t)} = \omega_{ii}\frac{x_i(t)}{\chi_{ii}(t)}\frac{\partial F(k_i(t))}{\partial k_i(t)} - (\delta_i + g_i^k - \rho) - (1-\omega_{ji})\left(\frac{\omega_{jj}}{\omega_{ij}}\frac{x_j(t)}{\chi_{jj}(t)}\frac{\partial F(k_j(t))}{\partial k_j(t)} - \frac{\delta_j + g_j^k - \rho}{\omega_{ij}}\right) \tag{23}$$

The optimal path is further characterized by

$$\dot{k}_i(t) = \prod_{j=1,2}\chi_{ji}(t)^{\omega_{ji}} - (\delta_i + g_i^k)k_i(t), \tag{24}$$

$$F(k_i(t)) = c_i(t) + \sum_{j=1,2}\chi_{ij}(t). \tag{25}$$

To understand the system dynamics when we need to keep track of consumption, capital stocks, investment paths and the path of relative allocations of investment across sectors. Let χ_i as the total investment coming from sector i. Then, we can combine 25 and 24 as follows,

$$\dot{k}_i(t) = \prod_{j=1,2}\chi_j(t)^{\omega_{ji}}\prod_{j=1,2}\kappa_{ji}^{\omega_{ji}} - (\delta_i + g_i^k)k_i(t),$$

where $\kappa_{ji} \equiv \frac{\chi_{ji}(t)}{\chi_j(t)}$ is the fraction of investment goods produced in sector j going to sector i.

We can then incorporate the feasibility constraint into the law of motion for capital as follows

$$\dot{k}_i(t) = \prod_{j=1,2}\left(F(k_j(t)) - c_j(t)\right)^{\omega_{ji}}\prod_{j=1,2}\kappa_{ji}(t)^{\omega_{ji}} - (\delta_i + g_i^k)k_i(t), \tag{26}$$

Hence, to complete the full dynamics, we need a dynamic equation for $\kappa_{ji}(t)$ which we obtain from the optimal allocation of investment across sectors. Consider investment goods from sector j used in i and i', optimality yields,

$$\frac{\kappa_{ji}(t)}{\kappa_{ji'}(t)} = \frac{\mu_i(t)}{\mu_{i'}(t)}\frac{\omega_{ij}}{\omega_{ji'}}\frac{x_i(t)}{x_{i'}(t)}$$

Totally differentiating,

$$\frac{\dot{\kappa}_{ji}(t)}{\kappa_{ji}(t)} - \frac{\dot{\kappa}_{ji'}(t)}{\kappa_{ji'}(t)} = \frac{\dot{\mu}_i(t)}{\mu_i(t)} - \frac{\dot{\mu}_{i'}(t)}{\mu_{i'}(t)} + \frac{\dot{x}_i(t)}{x_i(t)} - \frac{\dot{x}_{i'}(t)}{x_{i'}(t)},$$

and now replacing by the euler equation for the dynamic of the shadow price of capital

$$\frac{\dot{\kappa}_{ji}(t)}{\kappa_{ji}(t)} - \frac{\dot{\kappa}_{ji'}(t)}{\kappa_{ji'}(t)} = -(\omega_{ii}\frac{x_i(t)}{\chi_{ii}(t)}\frac{\partial F(k_i(t))}{\partial k_i(t)} - \omega_{i'i'}\frac{x_{i'}(t)}{\chi_{i'i'}(t)}\frac{\partial F(k_{i'}(t))}{\partial k_{i'}(t)}) + (\delta_i + g_i^k) - (\delta_{i'} + g_{i'}^k) + \frac{\dot{x}_i(t)}{x_i(t)} - \frac{\dot{x}_{i'}(t)}{x_{i'}(t)} \tag{27}$$

where we can use the definition for investment to write the last two terms as a function of capital, consumption in each sector and the sectorial allocation of investment. Given

$$x_i(t) = \prod_{j=1,2} \left(F(k_j(t)) - c_j(t)\right)^{\omega_{ji}} \prod_{j=1,2} \kappa_{ji}(t)^{\omega_{ji}}$$

then

$$\frac{\dot{x}_i(t)}{x_i(t)} - \frac{\dot{x}_j(t)}{x_j(t)} \approx \omega_ij \frac{\dot{\kappa}_{ji}(t)}{\kappa_{ji}(t)} - \omega_{jj}\frac{\dot{\kappa}_{jj}}{\kappa_{jj}} + \omega_{ii}\frac{\dot{\kappa}_{ii}(t)}{\kappa_{ii}(t)} - \omega_{ij}\frac{\dot{\kappa}_{ij}}{\kappa_{ij}}$$

By definition $\kappa_{ji}(t) = 1 - \kappa_{ji'}(t)$ and therefore

$$\frac{\dot{\kappa}_{ji}(t)}{\kappa_{ji}(t)} = -\frac{\kappa_{ji'}(t)}{1-\kappa_{ji'}(t)}\frac{\dot{\kappa}_{ji'}(t)}{\kappa_{ji'}(t)} \tag{28}$$

which implies,

$$\frac{\dot{x}_i(t)}{x_i(t)} - \frac{\dot{x}_j(t)}{x_j(t)} \approx \left(\omega_{ji} + \omega_{jj}\frac{\kappa_{jj}}{1-\kappa_{jj}}\right)\frac{\dot{\kappa}_{ji}(t)}{\kappa_{ji}(t)} - \left(\omega_{ij} + \omega_{ii}\frac{\kappa_{ii}}{1-\kappa_{ii}}\right)\frac{\dot{\kappa}_{ij}}{\kappa_{ij}}.$$

Hence, the dynamics of the allocation of capital 27

$$\begin{aligned} \frac{\dot{\kappa}_{ji}(t)}{\kappa_{ji}(t)}\left(\frac{1}{1-\kappa_{ji}} - (\omega_{ji} + \omega_{jj}\frac{\kappa_{jj}}{1-\kappa_{jj}})\right) &= (\omega_{jj}\frac{x_j(t)}{\chi_{jj}(t)}\frac{\partial F(k_j(t))}{\partial k_j(t)} - \omega_{ii}\frac{x_i(t)}{\chi_{ii}(t)}\frac{\partial F(k_i(t))}{\partial k_i(t)}) \\ &\quad +(\delta_i + g_i^k) - (\delta_{i'} + g_{i'}^k) - \left(\omega_{ij} + \omega_{ii}\frac{\kappa_{ii}}{1-\kappa_{ii}}\right)\frac{\dot{\kappa}_{ij}}{\kappa_{ij}} \end{aligned}$$

An analogous condition for $\frac{\dot{\kappa}_{ji}(t)}{\kappa_{ji}(t)}$ determines a system of linear equations that can be solved for the dynamics of the investment allocation. Let $\zeta_{ji} \equiv (\omega_{ji} + \omega_{jj}\frac{\kappa_{jj}}{1-\kappa_{jj}})$, then

$$\frac{\dot{\kappa}_{ji}(t)}{\kappa_{ji}(t)} = \frac{1}{(\frac{1}{1-\kappa_{ji}} - \zeta_{ji})(\frac{1}{1-\kappa_{ij}} - \zeta_{ij})}\left(\omega_{jj}\frac{x_j(t)}{\chi_{jj}(t)}\frac{\partial F(k_j(t))}{\partial k_j(t)} - \omega_{ii}\frac{x_i(t)}{\chi_{ii}(t)}\frac{\partial F(k_i(t))}{\partial k_i(t)} + \delta_i + g_i^k - \delta_j + g_j^k\right) \tag{29}$$

Finally,

$$\frac{x_i(t)}{\chi_{ii}(t)} = \prod_{j=1,2}\left(\frac{F(k_j(t)) - c_j(t)}{F(k_i(t)) - c_i(t)}\right)^{\omega_{ji}} \prod_{j=1,2}\frac{\kappa_{ji}(t)}{\kappa_{ii}(t)}^{\omega_{ji}} \tag{30}$$

Steady state. Along the steady state

$$exp(-\rho t)\frac{\partial U(c_1(t), c_2(t))}{\partial c_i(t)} = \lambda_i$$

$$\frac{\lambda_i(t)}{\mu_i(t)} = \frac{\partial F(k_i(t))}{\partial k_i(t)}^{-1}(\delta_i + g_i^k)$$

and therefore

$$\frac{x_j(t)}{\chi_{ij}(t)} = \left(\omega_{ij}\frac{\partial F(k_i(t))}{\partial k_i(t)}\right)^{-1}(\delta_i + g_i^k)$$

The Euler equation and the dynamic condition for the allocation of investment across sectors, implies that in a steady state

$$\omega_{ii}\frac{x_i(t)}{\chi_{ii}(t)}\frac{\partial F(k_i(t))}{\partial k_i(t)} = \delta_i + g_i^k - \rho.$$

Note that the one sector neoclassical growth model is a special case of this, where investment is fully specialized in one sector, $\omega_{ii} = 1$ and $\frac{x_i(t)}{\chi_{ii}(t)} = 1$.

The law of motion for capital, 24 implies,

$$x_i(t) = k_i(t)(\delta_i + g_i^k),$$

replacing back,

$$\omega_{ii}\frac{k_i(t)}{\chi_{ii}(t)}\frac{\partial F(k_i(t))}{\partial k_i(t)} = \hat{\rho}_i. \tag{31}$$

where $\hat{\rho}_i \equiv 1 - \frac{\rho}{\delta_i + g_i^k}$.

The optimal allocation of consumption under separable log-utility and using the feasibility constraint in each sector, 25 satisfies,

$$\lambda_i(t) = \frac{\theta_i}{F(k_i(t))(1 - \frac{\omega_{ii}\epsilon_i}{\hat{\rho}_i}) - \chi_{ij}(t)} \tag{32}$$

where $\epsilon_i \equiv \frac{k_i(t)}{F(k_i(t))}\frac{\partial F(k_i(t))}{\partial k_i(t)}$ is the output elasticity to capital.

Replacing 31 into the steady state law of motion for capital and using the definition of investment x_i we obtain

$$\chi_{ji}(t) = \left(k_i(t)(\delta_i + g_i^k)\right)^{\frac{1}{\omega_{ji}}} \left(\omega_{ii}\frac{\epsilon_i}{\hat{\rho}_i}F(k_i(t))\right)^{\frac{-\omega_{ii}}{\omega_{ji}}} \tag{33}$$

which we can replace back in the expression for the price λ_i, defining prices as a function of the stock of capital in each sector and parameters.

We can rewrite the steady state 24 as

$$\frac{k_i(t)}{\chi_{ii}(t)}(\delta_i + g_i^k) = \left(\frac{\chi_{ji}(t)}{\chi_{ii}(t)}\right)^{\omega_{ji}}$$

and using the optimal input demands,

$$\frac{k_i(t)}{\chi_{ii}(t)}(\delta_i + g_i^k) = \left(\frac{\lambda_i}{\lambda_j}\frac{\omega_{ji}}{\omega_{ii}}\right)^{\omega_{ji}}$$

Replacing back, 31 we can solve for

$$(\delta_i + g_i^k - \rho)\frac{k_i(t)}{\omega_{ii}\epsilon_i F(k_i)} = \left(\frac{\lambda_i}{\lambda_j}\frac{\omega_{ji}}{\omega_{ii}}\right)^{\omega_{ji}} \tag{34}$$

Therefore, equations 33, 32 and 34 solve for the capital stock in each sector.

Dynamics The optimality conditions of the problem, equation 23 to 25 and 30 yield the conditions describing the optimal dynamics of the system around a neighborhood of the BGP. We repeat them here to ease the exposition Using an analogous expression for consumption in sector j, we can solve for the Euler equation just a function of primitives

$$\omega_{ji}\sigma\frac{\dot{c_i(t)}}{c_i(t)}=\omega_{ii}\frac{x_i(t)}{\chi_{ii}(t)}\frac{\partial F(k_i(t))}{\partial k_i(t)}-(\delta_i+g_i^k-\rho)-(1-\omega_{ji})\left(\frac{\omega_{jj}}{\omega_{ij}}\frac{x_j(t)}{\chi_{jj}(t)}\frac{\partial F(k_j(t))}{\partial k_j(t)}-\frac{\delta_j+g_j^k-\rho}{\omega_{ij}}\right)$$

$$\dot{k}_i(t)=\prod_{j=1,2}\left(F(k_j(t))-c_j(t)\right)^{\omega_{ji}}\prod_{j=1,2}\kappa_{ji}(t)^{\omega_{ji}}-(\delta_i+g_i^k)k_i(t),$$

$$\frac{\dot{\kappa}_{ji}(t)}{\kappa_{ji}(t)}=\frac{1}{(\frac{1}{1-\kappa_{ji}}-\zeta_{ji})(\frac{1}{1-\kappa_{ij}}-\zeta_{ij})}\left(\omega_{jj}\frac{x_j(t)}{\chi_{jj}(t)}\frac{\partial F(k_j(t))}{\partial k_j(t)}-\omega_{ii}\frac{x_i(t)}{\chi_{ii}(t)}\frac{\partial F(k_i(t))}{\partial k_i(t)}+\delta_i+g_i^k-\delta_j+g_j^k\right)$$

where $\zeta_{ji}\equiv(\omega_{ji}+\omega_{jj}\frac{\kappa_{jj}}{1-\kappa_{jj}})$.

$$\dot{\kappa}_{jj}(t)=-\dot{\kappa}_{ji}(t)$$

$$\frac{x_i(t)}{\chi_{ii}(t)}=\prod_{j=1,2}\left(\frac{F(k_j(t))-c_j(t)}{F(k_i(t))-c_i(t)}\right)^{\omega_{ji}}\prod_{j=1,2}\frac{\kappa_{ji}(t)}{\kappa_{ii}(t)}^{\omega_{ji}}$$

The Jacobian of the system computed at the steady state would characterize the speed of convergence to the steady state, which is bounded by its largest eigenvalue (in absolute value). If the spectral radius of the system is below 1 the system is stable.

A.3 Data appendix

Table 10: Country Sample and Data Sources

	Country	GDP per capita 2005 (PPP)	Use-Tables; Input-Output Matrix VA Shares, GFCF imported share	Employment by Occup. and Sector	Investment Network
			Source	*Source*	*Available Years*
1	Ethiopia	679	MDV	ILOSTAT	1990-2019
2	Rwanda	1246	MDV	ILOSTAT	1990-2019
3	Tanzania	1507	MDV	ILOSTAT	1990-2019
4	Zambia	1710	MDV	IPUMS	1990-2019
5	Kenya	1972	MDV	ILOSTAT	1990-2019
6	Cambodia	2048	OECD	IPUMS	2005-2015
7	Senegal	2728	MDV	IPUMS	1990-2019
8	India	2872	WIOD	IPUMS	1965-2000; 2000-2014
9	Vietnam	3128	OECD	IPUMS	2005-2015
10	Ghana	3219	MDV	ILOSTAT	1990-2019
11	Nigeria	3481	MDV	IPUMS	1990-2019
12	Philippines	4366	OECD	IPUMS	2005-2015
13	Indonesia	4602	OECD	IPUMS	2005-2015
14	Morocco	4672	OECD	IPUMS	2005-2015
15	China	6681	OECD	IPUMS	2005-2015
16	Peru	6832	OECD	PIAAC	2005-2015
17	Colombia	8367	OECD	ILOSTAT	2005-2015
18	Tunisia	9353	OECD	ILOSTAT	2005-2015
19	Brazil	9610	WIOD	IPUMS	1965-2000; 2000-2014
20	Thailand	10293	OECD	IPUMS	2005-2015
21	South Africa	11311	OECD	IPUMS	2005-2015
22	Costa Rica	11580	OECD	IPUMS	2005-2015
23	Turkey	13941	WIOD	PIAAC	2000-2014
24	Argentina	14247	OECD	ILOSTAT	2005-2015
25	Mauritius	14325	MDV	IPUMS	1990-2019
26	Chile	14534	OECD	PIAAC	2005-2015
27	Mexico	15230	WIOD	PIAAC	1965-2000; 2000-2014
28	Russia	15450	WIOD	PIAAC	2000-2014
29	Poland	16838	WIOD	PIAAC	2000-2014
30	Malaysia	17412	OECD	IPUMS	2005-2015
31	Lithuania	17646	WIOD	ILOSTAT	2000-2014
32	Slovakia	20168	OECD	PIAAC	2000-2014
33	Hungary	20819	WIOD	PIAAC	2000-2014
34	Czechia	26624	WIOD	ILOSTAT	2000-2014
35	Portugal	27149	WIOD	IPUMS	1965-2000; 2000-2014
36	Slovenia	28821	OECD	PIAAC	2000-2014
37	Greece	30138	WIOD	PIAAC	1965-2000; 2000-2014
38	South Korea	30784	WIOD	PIAAC	1965-2000; 2000-2014
39	New Zealand	31485	OECD	PIAAC	2005-2015
40	Israel	32358	OECD	PIAAC	2005-2015
41	Spain	32769	WIOD	PIAAC	1965-2000; 2000-2014
42	Cyprus	33025	OECD	ILOSTAT	2005-2015
43	Italy	36167	WIOD	PIAAC	1965-2000; 2000-2014
44	France	36651	WIOD	PIAAC	1965-2000; 2000-2014
45	Japan	38466	WIOD	PIAAC	1965-2000; 2000-2014
46	Germany	38475	WIOD	PIAAC	1965-2000; 2000-2014
47	Belgium	39220	WIOD	PIAAC	1965-2000; 2000-2014
48	United Kingdom	39308	WIOD	PIAAC	1965-2000; 2000-2014
49	Denmark	40344	OECD	PIAAC	1965-2000; 2000-2014
50	Sweden	40381	WIOD	PIAAC	1965-2000; 2000-2014
51	Austria	41678	WIOD	ILOSTAT	1965-2000; 2000-2014
52	Australia	43333	WIOD	ILOSTAT	1965-2000; 2000-2014
53	Netherlands	44662	WIOD	PIAAC	1965-2000; 2000-2014
54	Ireland	47211	WIOD	PIAAC	1965-2000; 2000-2014
55	Switzerland	49859	OECD	IPUMS	2005-2015
56	Norway	54200	WIOD	PIAAC	2000-2014
57	United States	54210	WIOD	PIAAC	1965-2000; 2000-2014
58	Singapore	63949	OECD	PIAAC	2005-2015

Table 11: Aggregate Sectors Definition

ISIC Rev.4 Code	ISIC Rev.4 Description	Aggregate Sector
A01	Crop and animal production, hunting and related service activities	Agriculture
A02	Forestry and logging	
A03	Fishing and aquaculture	
F	Construction	Construction
C26	Manufacture of computer, electronic and optical products	Electronics
C27	Manufacture of electrical equipment	
C18	Printing and reproduction of recorded media	ICT
J58	Publishing activities	
J59_J60	Motion picture, video and television programme production, broadcasting activities	
J61	Telecommunications	
J62_J63	Computer programming, consultancy and related activities; information service activities	
M69_M70	Legal and accounting activities; activities of head offices; management consultancy activities	
M71	Architectural and engineering activities; technical testing and analysis	
M72	Scientific research and development	
M73	Advertising and market research	
M74_M75	Other professional, scientific and technical activities; veterinary activities	
N	Administrative and support service activities	
C28	Manufacture of machinery and equipment n.e.c.	Machinery
C33	Repair and installation of machinery and equipment	
C10-C12	Manufacture of food products, beverages and tobacco products	Manufacturing
C13-C15	Manufacture of textiles, wearing apparel and leather products	
C16	Manufacture of wood and of products of wood and cork, except furniture	
C17	Manufacture of paper and paper products	
C19	Manufacture of coke and refined petroleum products	
C20	Manufacture of chemicals and chemical products	
C21	Manufacture of basic pharmaceutical products and pharmaceutical preparations	
C22	Manufacture of rubber and plastic products	
C23	Manufacture of other non-metallic mineral products	
C24	Manufacture of basic metals	
C25	Manufacture of fabricated metal products, except machinery and equipment	
C31_C32	Manufacture of furniture; other manufacturing	
B	Mining and quarrying	Not included
D35	Electricity, gas, steam and air conditioning supply	Services
E36	Water collection, treatment and supply	
E37-E39	Sewerage; waste collection	
G46	Wholesale trade, except of motor vehicles and motorcycles	
G47	Retail trade, except of motor vehicles and motorcycles	
I	Accommodation and food service activities	
K64	Financial service activities, except insurance and pension funding	
K65	Insurance, reinsurance and pension funding, except compulsory social security	
K66	Activities auxiliary to financial services and insurance activities	
L68	Real estate activities	
O84	Public administration and defence; compulsory social security	
P85	Education	
Q	Human health and social work activities	
R_S	Other service activities	
T	Activities of households as employers	
U	Activities of extraterritorial organizations and bodies	
C29	Manufacture of motor vehicles, trailers and semi-trailers	Transportation
C30	Manufacture of other transport equipment	
G45	Wholesale and retail trade and repair of motor vehicles and motorcycles	
H49	Land transport and transport via pipelines	
H50	Water transport	
H51	Air transport	
H52	Warehousing and support activities for transportation	
H53	Postal and courier activities	

Table 12: Investment Network Outdegrees: Comparison with VLW

(a) 1972

Sector	This Paper	VLW
Agriculture	0.00	0.00
Construction	2.34	1.75
Electronics	0.48	0.61
ICT	0.79	1.76
Machinery	1.44	1.91
Manufacturing	1.00	0.24
Services	0.52	0.44
Transportation	1.37	1.29

(b) 1992

Sector	This Paper	VLW
Agriculture	0.00	0.00
Construction	1.77	1.29
Electronics	0.83	0.87
ICT	1.63	2.49
Machinery	1.13	1.45
Manufacturing	0.91	0.32
Services	0.62	0.57
Transportation	1.11	1.01

Table 13: Investment Network Homophily: Comparison with VLW

(a) 1972

Sector	This Paper	VLW
Agriculture	0.00	0.00
Construction	0.04	0.13
Electronics	0.1	0.07
ICT	0.08	0.36
Machinery	0.23	0.23
Manufacturing	0.28	0.07
Services	0.04	0.04
Transportation	0.30	0.45

(b) 1992

Sector	This Paper	VLW
Agriculture	0.00	0.00
Construction	0.03	0.14
Electronics	0.13	0.10
ICT	0.19	0.50
Machinery	0.18	0.17
Manufacturing	0.28	0.06
Services	0.05	0.06
Transportation	0.22	0.27

Table 14: Outdegrees: adjusted-Leontief inverse

	Low Income	Medium Income	High Income
Agriculture	1.62	0.89	0.71
Construction	1.11	1.09	0.95
Electronics	0.60	0.65	0.69
ICT	1.12	1.46	1.98
Machinery	0.62	0.70	0.63
Manufacturing	2.14	1.96	1.45
Services	2.55	2.57	2.75
Transportation	1.10	1.20	1.14

Notes: Low Income countries have an average per capita GDP (PPP) of 5030, Medium Income countries an average per capita GDP (PPP) of 44472, and High Income countries an average per capita GDP (PPP) of 84671 in 2005.

Figure 6: South Korea: Investment Network across time

(a) 1965 (b) 2014

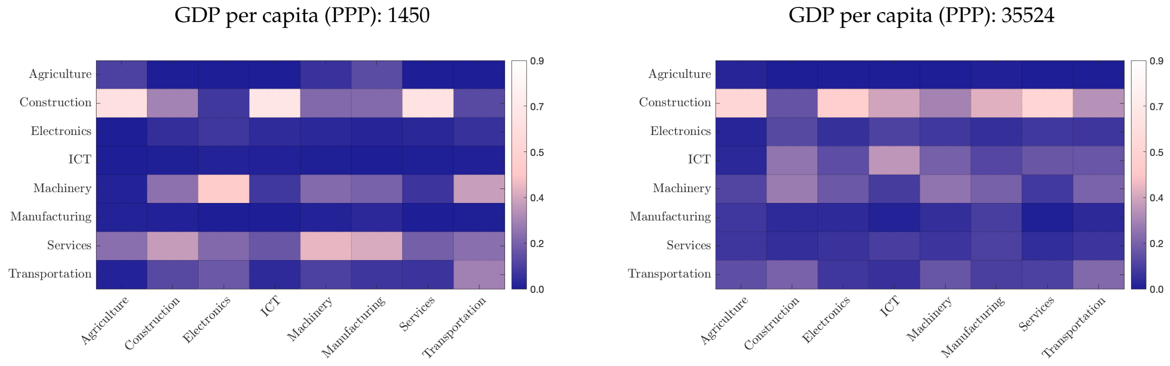
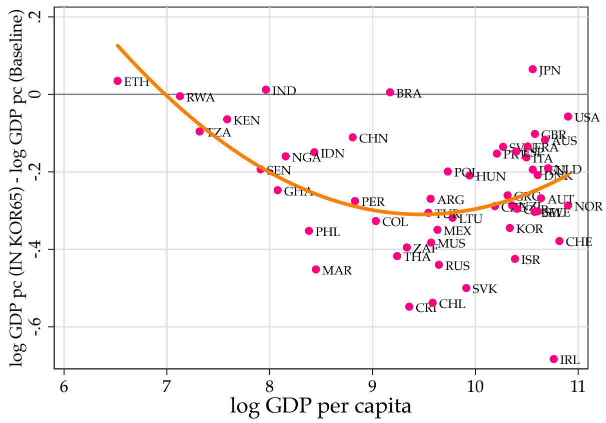


Figure 7: KOR Investment Network in 1965



Notes: Countries with outlier income changes were excluded from the graph: Zambia (log GDP per capita 7.4, change of 64%), Malaysia (log GDP per capita 9.8, change of-94%), and Singapore (log GDP per capita 11.1, change of -92%).