BANKRUNS, FRAGILITY & REGULATION

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* The views are not the views of the Minneapolis Fed or the Federal Reserve system.

IN THIS PAPER

- General equilibrium model of bankruns (a la Cole-Kehoe) where banks have liquid assets.
- Runs are possible → Require eqm profits.

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- General equilibrium model of bankruns (a la Cole-Kehoe) where banks have liquid assets.
- Runs are possible → Require eqm profits.
- In the absence of runs:
  - Competitive Equilibria are constrained efficient
- But, in the presence of runs,
  - Banks are <u>over-leveraged</u>

ENVIRONMENT

- 3 periods, $t = 0, 1, 2$.
- One final consumption good and one factor (capital)
- K units of capital in fixed supply in all periods.
- SOE, interest rate R

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BANKS

- Aggregate borrowing and production decisions
- Start $t=0$ with same outstanding debt, $b_0 = B_0$; and same holdings of capital, $k_0 = K_0$

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DOMESTIC AGENTS
BANKS

- Aggregate borrowing and production decisions
- Start $t=0$ with same outstanding debt, $b_0 = B_0$; and same holdings of capital, $k_0 = K_0$
- Produce final good using linear technology:

$$z \cdot k$$

idiosyncratic   capital used

- Can default at time $t = 1, 2$

PREFERENCES

$$u(c_0) + \beta \mathbb{E}\, u(C_1) + \beta^2 \mathbb{E}\, u(C_2) \quad \text{with } u = \log$$

This gives us lots of tractability (won't show today)

PREFERENCES

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Budget constraints <u>under repayment</u>:

$$c_0 = \underbrace{z k_0}_{\text{production}} - R \underset{\substack{\text{initial} \\ \text{debt}}}{b_0} + \underset{\substack{\text{price of} \\ \text{debt}}}{q_0(b_1, k_1)} \underset{\substack{\text{new} \\ \text{borrowing}}}{b_1} + \overset{\text{capital price}}{p_0} (k_0 - \underset{\text{new } k}{k_1})$$

PREFERENCES

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capital price

$$c_0 = \underbrace{z\, k_0}_{\text{production}} - R\, b_0 + q_0(b_1, k_1)\, b_1 + p_0\,(k_0 - k_1)$$

initial debt

price of debt

new borrowing

new k

$$c_1 = z\, k_1 - R\, b_1 + q_1(b_2, k_2)\, b_2 + p_1\,(k_1 - k_2)$$

$$c_2 = z\, k_2 - R\, b_2$$

last period
no more borrowing

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$$c_0 = \underbrace{z k_0}_{\text{production}} - R \underset{\uparrow \\ \text{initial debt}}{b_0} + \underset{\uparrow \\ \text{price of debt}}{q_0(b_1, k_1)}\, \underset{\uparrow \\ \text{new borrowing}}{b_1} + p_0 (k_0 - \underset{\uparrow \\ \text{new } k}{k_1})$$

$$c_1 = z k_1 - R b_1 + q_1(b_2, k_2)\, b_2 + p_1 (k_1 - k_2)$$

$$c_2 = z k_2 - R b_2$$

last period
no more borrowing

$$n_t \equiv (z + p_t) k_t - R b_t \quad // \text{ Networth}$$

OUTSIDE OPTIONS

A Default triggers loss of productivity + exclusion

$t=2$: $\quad V_2^D(k) = u(z_2^D k)$

$t=1$: $\quad V_1^D(k, z_1^D) = u(z_1^D k) + \beta u(z_2^D k)$

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$\boxed{z_1^D \sim \text{iid across banks, cdf } F}$

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$z_1^D \sim$ iid across banks, cdf F

V_t^D is independent of prices and increasing in k.

$\to$ Endogenous borrowing limits as in Kehoe-Levine.

PERIOD 2

Simple deterministic problem:

$$V_2(b_2, k_2) = \max_{d_2 \in \{0,1\}} \left\{ (1-d_2)\, u(z k_2 - R b_2) + \right.$$
$$\left. + d_2\, u(z_2^D k_2) \right\}$$

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$$\Rightarrow \qquad d_2(b_2, k_2) = \begin{cases} 1 & \text{if } R b_2 > \phi k_2 \\ 0 & \text{otherwise} \end{cases}$$

$$\phi = z - z_2^D > 0$$

Borrowing limit at $t=1$ $\Rightarrow$ $R b_2 \le \phi k_2$

PERIOD 1: TWO VALUE FUNCTIONS

WITHOUT RUNS

$$V_1^R(n_1) = \max_{c_1, k_2, b_2} \left\{ u(c_1) + \beta u(z_2 - R b_2) \right\}$$

$$\text{s.t.} \quad c_1 = n_1 + b_2 - p_1 k_2 \geq 0$$

$$R b_2 \leq \phi k_2$$

$$k_2 \geq 0$$

Period 1 : Two value functions

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WITH RUNS

$$V_1^{Run}(n_1) = \max_{c_1, k_2, b_2} \left\{ u(c_1) + \beta u(z_2 - R b_2) \right\}$$

$$\text{s.t.} \quad c_1 = n_1 + b_2 - p_1 k_2 \geq 0$$

$$b_2 \leq 0$$

$$k_2 \geq 0$$

Can save, but can't borrow

Period 1: Two value functions

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$$V_1^R(n_1) = \max_{c_1, k_2, b_2} \left\{ u(c_1) + \beta u(z_2 - Rb_2) \right\}$$

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WITH RUNS

$$V_1^{Run}(n_1) = \max_{c_1, k_2, b_2} \left\{ u(c_1) + \beta u(z_2 - Rb_2) \right\}$$

$$\text{s.t.} \quad c_1 = n_1 + b_2 - p_1 k_2 \geq 0$$

$$b_2 \leq 0$$

$$k_2 \geq 0$$

Can save, but can't borrow

These functions are indexed by p_1

Period 1: Default Thresholds

given (n_1, k_1) and p_1, define thresholds

Fundamental: $V_1^R(n_1) = V_1^D(k_1, \hat{z}^F)$

Run: $V_1^{Run}(n_1) = V_1^D(k, \hat{z}^{Run})$

PERIOD 1: DEFAULT THRESHOLDS

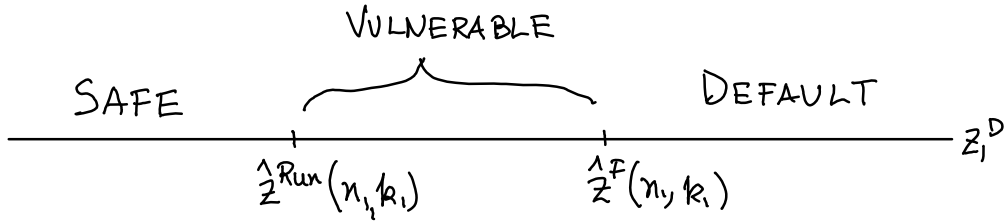
given (n_1, k_1) and p_1, define thresholds

Fundamental: $V_1^R(n_1) = V_1^D(k_1, \hat{z}^F)$

Run: $V_1^{Run}(n_1) = V_1^D(k_1, \hat{z}^{Run})$

$$\boxed{\begin{array}{l}\underline{\text{Result}}:\ \hat{z}^F(n_1,k_1) \geq \hat{z}^{Run}(n_1, k_1). \\ \hline \text{If } \quad p_1 < z/R \quad \text{then} \\ \qquad \hat{z}^F(n_1,k_1) > \hat{z}^{Run}(n_1,k_1)\end{array}}$$

Period 1: Runs with Liquid Assets



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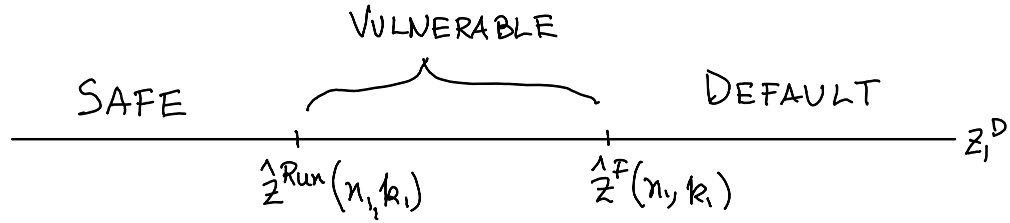
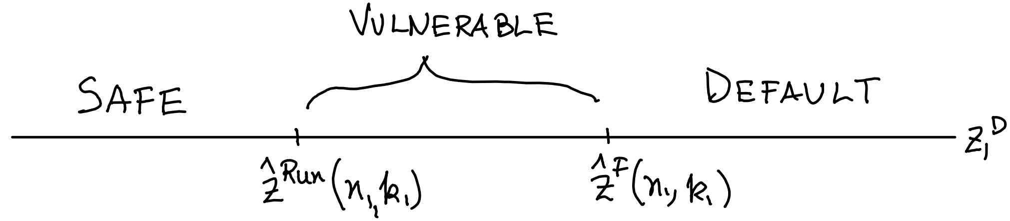


Vulnerability is possible even though assets are liquid

Period 1: Runs with Liquid Assets



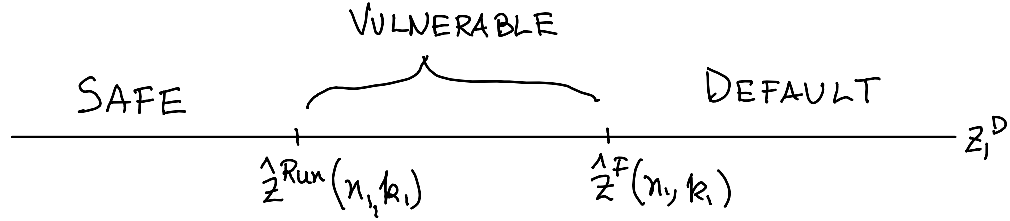
Vulnerability is possible even though assets are liquid

$$\to \quad p_1 < \frac{z}{R} \iff \boxed{R < \frac{z}{p_1} \equiv R^K}$$

Borrowing Cost (R)

Return to capital ($\frac{z}{p_1}$)

Period 1: Runs with Liquid Assets



Vulnerability is possible even though assets are liquid!

$\hookrightarrow \quad p_1 < \dfrac{z}{R} \iff$

$$R < \frac{z}{p_1} \equiv R^K$$

Borrowing Cost

Return to ...

If $R^K > R$; there are PROFITS, but PROFITS dissappear if you can't borrow.

↳ This is the "illiquidity" that facilitates runs.

If $\dfrac{z}{p_1} > R$:

$$V_1^{\text{Run}}(n_1) = A + (1+\beta)\log n_1 + \beta \log\left(\frac{z}{p_1}\right)$$

cannot borrow

If $\frac{z}{p_1} > R$:

excess return from leverage

$$V_1^R(n_1) = A + (1+\beta)\log n_1 + \beta \log\left(\frac{z - \emptyset}{p_1 - \emptyset/R}\right)$$

$$V_1^{Run}(n_1) = A + (1+\beta)\log n_1 + \beta \log\left(\frac{z}{p_1}\right)$$

cannot borrow

$$\Rightarrow \quad V_1^{Run}(n_1) < V_1^R(n_1)$$

Period 1: Default Decisions

Follow Cole-Kehoe $\longrightarrow$

Repaying under a Run is off-equilibrium

A vulnerable bank defaults with λ prob.

PERIOD 1: DEFAULT DECISIONS

Follow Cole-Kehoe $\longrightarrow$

Repaying under a Run is off-equilibrium

A vulnerable bank defaults with λ prob.

$$d_1(n_1, k_1, z_1^D) = \begin{cases} 0 & \text{if } z_1^D \leq \hat{z}^{Run} \\ \lambda & \text{if } \hat{z}^{Run} < z_1^D \leq \hat{z}^F \\ 1 & \text{if } z_1^D > \hat{z}^F \end{cases}$$

PERIOD 0

$$V_0(n_0) = \log c_0 + \beta \int_{\underline{z}}^{\bar{z}} \left[d_1 \cdot V_1^D(k_1, \tilde{z}) + (1-d_1) V_1^R(n_1, \tilde{z}) \right] dF(\tilde{z})$$

$$c_0 = n_0 + q_0(n_1, k_1) b_1 - p_0 k_1$$

$$n_1 = (z + p_1) k_1 - R b_1$$

PERIOD 0

From before

$$V_0(n_0) = \log c_0 + \beta \int_{\underline{z}}^{\bar{z}} \left[d_1 \cdot V_1^D(k_1, \tilde{z}) + (1-d_1) V_1^R(n_1, \tilde{z}) \right] dF(\tilde{z})$$

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PERIOD 0

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$$c_0 = n_0 + q_0(n_1, k_1) b_1 - p_0 k_1$$

$$n_1 = (z + p_1) k_1 - R b_1$$

Bond price schedule:

$$q_0(n_1, k_1) = (1-\lambda) F(\hat{z}(n_1, k_1)) + \lambda F(\hat{z}^{Run}(n_1, k_1))$$

EQUILIBRIUM

Usual optimization conditions + symmetry

+

Aggregate demand for capital is K at $t \in \{0, 1\}$

EQUILIBRIUM

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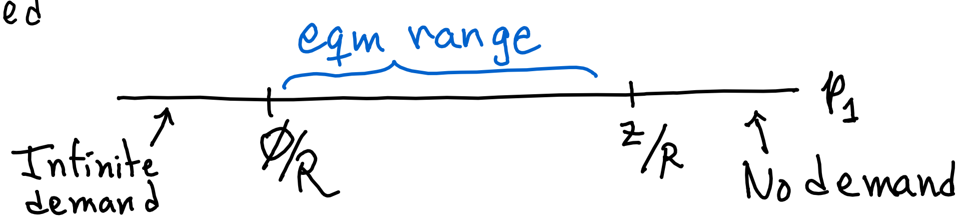
Aggregate demand for capital is K at $t \in \{0, 1\}$

<u>IN A C.E.:</u> $\quad q_0(N_0, K_1) > 0$

(Not certain default)

and

price of capital is bounded $\rightarrow$ $\emptyset/R < p_1 \leq z/R$



LEVERAGE

Define $l_1 = \dfrac{b_1}{k_1}$

The redefine $\hat{z}^F(l_1 | p_1)$ and $\hat{z}^{Run}(l_1 | p_1)$ and $q_0(l_1 | p_1)$.

LEVERAGE

Define $\ell_1 = \dfrac{b_1}{k_1}$

The redefine $\hat{z}^F(\ell_1 | p_1)$ and $\hat{z}^{Run}(\ell_1 | p_1)$ and $q_0(\ell_1 | p_1)$.

$$\boxed{\begin{array}{l}\text{In a competitive equilibrium} \\ \nexists\, \ell_1 \text{ s.t. } q_0(\ell_1 | p_1)\,\ell_1 \geq p_0\end{array}} \Rightarrow n_0 > 0.$$

cannot start from nothing

AGGREGATE LEVERAGE

Relationship between L_1 and p_1

$$p_1 = \mathcal{P}_1(L_1) = \begin{cases} Z/R & \text{for } L_1 \leq \hat{L} \\ \beta Z + (1+\beta)\frac{\emptyset}{R} - \beta R L_1 & \text{for } L_1 \in (\hat{L}, \bar{L}) \end{cases}$$

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$$K_2 = \frac{\beta}{(1+\beta)(p_1 - \phi/R)} N_1 = K$$

(capital demand = K)

AGGREGATE LEVERAGE

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Higher L_1 leads to a lower N_1 and a reduction in the demand for capital in period $t=1$.

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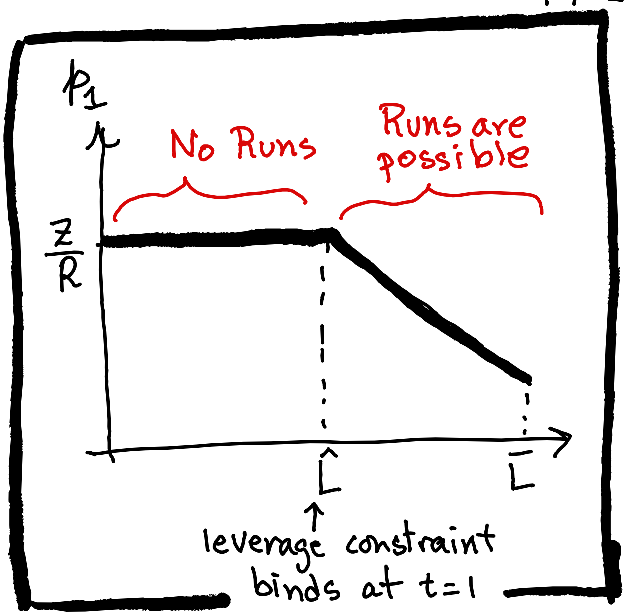

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CONSTRAINED EFFICIENCY

L_1 characterizes the rest of the equilibrium objects.

$\hookrightarrow p_1 = p_1(L_1)$

CONSTRAINED EFFICIENCY

L_1 characterizes the rest of the equilibrium objects.

Planner chooses L_1

$$\max_{c_0, L_1} \; u(c_0) + \beta \int \Big[d_1(L_1, \tilde{z} \mid p_1) V_1^D(K, \tilde{z}) + \\ + \big(1 - d_1(L_1, \tilde{z} \mid p_1)\big) V_1^R(n_1 \mid p_1) \Big] dF(\tilde{z})$$

optimally chosen at $t=1$ (subject to runs.)

Depends on n_1 and p_1

CONSTRAINED EFFICIENCY

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subject to:

$$c_0 = z \cdot K - RB_0 + q_0(L_1 \mid p_1) L_1 K_1$$

$$n_1 = (z + p_1)K - R L_1 K$$

$$p_1 = P_1(L_1)$$

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$$p_1 = \mathcal{P}_1(L_1) \longleftarrow \text{GE}$$

EFFICIENCY WITH NO RUNS

PROPOSITION

> If $\lambda = 0$; then any competitive equilibrium is constrained efficient

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PROPOSITION

If $\lambda = 0$; then any competitive equilibrium is constrained efficient

Preliminary:

Consider any L_1 and $p_1 = \mathcal{P}_1(L_1)$ then

(i) $V_1^R\left((z+p_1)K - RL_1 \mid p_1\right) \leq$

$$V_1^R\left((z+\hat{p}_1)K - RL_1 \mid \hat{p}_1\right)$$

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$$V_1^R\big((z+\hat{p}_1)K - RL_1 \,|\, \hat{p}_1\big)$$

(ii) $q_0(L_1 | p_1) \leq q_0(L_1 | \hat{p}_1)$

where first inequality strict if $p_1 \neq \hat{p}_1$

WHY?

$p_1 = \mathcal{P}_1(L_1)$ means that at $t=1$,

demand for capital must equal K

$\Rightarrow$ Banks are neither net <u>buyers</u> or <u>sellers</u> of capital

$$c_1 = n_1 + b_2 - p_1 k_2$$
$$= (z + p_1) k_1 - R b_1 + b_2 - p_1 k_2$$
$$= z \cdot k_1 - R b_1 + b_2 - p_1 \underbrace{(k_2 - k_1)}$$

for $p_1 = \mathcal{P}(L_1) \Rightarrow k_2 = k_1 = K$.

WHY?

$p_1 = P_1(L_1)$, means that at $t=1$,
demand for capital must equal K

$\Rightarrow$ Banks are neither net buyers or sellers of capital

$\hat{p}_1 \neq P_1(L_1)$,

$\Rightarrow$ Banks face a different price at $t=1$.
than $p_1 = P_1(L_1)$,
can now buy/sell capital and
and strictly increase value.

PROOF OF EFFICIENCY (NO RUNS)

L_1^E, $p_1^E(L_1^E)$: Equilibrium

L_1^P, $p_1^P(L_1^P)$: Solution to planner's (this exists)

Proof of Efficiency (No Runs)

$L_1^E,\ p_1^E(L_1^E)$: Equilibrium

$L_1^P,\ p_1^P(L_1^P)$: Solution to planner's (this exists)

In C.E.: L_1^E is preferred to L_1^P when $p_1 = p_1^E$

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In C.E.: L_1^E is preferred to L_1^P when $p_1 = p_1^E$

$$u(zK - RB_0 + q_0(L_1^E | p_1^E) L_1^E K) + \beta\, \mathbb{E}\, V_1(L_1^E, K | p_1^E)$$

$$\geq u(zK - RB_0 + q_0(L_1^P | p_1^E) L_1^P K) + \beta\, \mathbb{E}\, V_1(L_1^P, K | p_1^E)$$

PROOF OF EFFICIENCY (NO RUNS)

$L_1^E,\ p_1^E(L_1^E)$: Equilibrium

$L_1^P,\ p_1^P(L_1^P)$: Solution to planner's (this exists)

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from Preliminary

PROOF OF EFFICIENCY (NO RUNS)

$L_1^E,\ p_1^E(L_1^E)$: Equilibrium

$L_1^P,\ p_1^P(L_1^P)$: Solution to planner's (this exists)

In C.E.: L_1^E is preferred to L_1^P when $p_1 = p_1^E$

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$$\geq u(zK - RB_0 + q_0(L_1^P | p_1^P) L_1^P K) + \beta \mathbb{E} V_1(L_1^P, K | p_1^P)$$

from Preliminary

$\Rightarrow$ Banks utility in C.E. higher than planner

$\Rightarrow$ L_1^E must also solve the planner's problem

UNIQUENESS + EXISTENCE (NO RUNS)

Uniqueness

Proposition. Suppose that (i) there is a unique solution to planner's problem; or (ii) there exists a C.E. with $L_1 > \hat{L}$ then there is <u>at most</u> one C.E.

UNIQUENESS + EXISTENCE (NO RUNS)

Uniqueness

Proposition. Suppose that (i) there is a unique solution to planner's problem; or (ii) there exists a C.E. with $L_1 > \hat{L}$ then there is at most one C.E.

Existence is harder (just as in G.E.)

↳ Paper provides conditions

EFFICIENCY (NO RUNS)

<u>With no runs</u> ($\lambda = 0$)

$\Rightarrow$ C.E. constrained efficient

No room for policy affecting choice of leverage.

Not totally surprising

$\Rightarrow$ No moral hazard

No deposit insurance

<u>What about runs?</u>

Runs

off equilibrium

A bank that faces a run and <u>repays</u> is a net seller of Capital.

RUNS

off equilibrium

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$$\frac{\partial \hat{z}^{Run}(L_1 | p_1)}{\partial p_1} = \frac{\hat{z}^{Run} \quad (1+\beta)\phi}{R(z+p_1-RL_1)p_1} > 0$$

Assumes $p_1 < \frac{z}{R}$ so runs are possible

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The effect on $\hat{z}^F$ is zero order.

INEFFICIENCY WITH RUNS

FOC for planner:

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Privately efficient choice of L_1 given p_1.

INEFFICIENCY WITH RUNS

FOC for planner:

$$\frac{1}{C_0} - \frac{\beta R}{C_1} = \text{Effect of } L_1 \text{ choice on default} + \text{and thresholds}$$

$$+ \frac{\lambda f(\hat{z}^{Run})}{q_0 K} \times \frac{\partial \hat{z}^{Run}}{\partial p_1} P_1'(L_1) \times$$

GE TERM

$$\times \left[\frac{1}{C_0} L_1 K + \beta \underbrace{\left(V_1^R(n_1 | p_1) - V_1^D(K, \hat{z}^{Run}) \right)}_{\text{Cole-Kehoe: Default is "forced" at the margin}} \right]$$

mg utility

Lender's loss in Default

+ Cole-Kehoe: Default is "forced" at the margin = Total loss at the margin

INEFFICIENCY WITH RUNS

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$$\times \left[\underbrace{\frac{1}{C_0} L_1 K + \beta \left(V_1^R(n_1 | p_1) - V_1^D(K, \hat{z}^{Run}) \right)}_{>0} \right]$$

In a C.E., there is too much borrowing.

INEFFICIENCY WITH RUNS

- Paper provides conditions for inefficiency

- Implementing a constrained efficient level of leverage requires a tax on borrowing

  (tax is effectively the last term in the F.O.C.)

INEFFICIENCY WITH RUNS

- Paper provides conditions for inefficiency
- Implementing a constrained efficient level of leverage requires a <u>tax</u> on borrowing

  (tax is effectively the last term in the F.O.C.)
- There are other implementations (i.e. capital requirements)
- Numerical illustrations in the paper

THE END