A Currency Premium Puzzle[*]

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Abstract

Canonical long-run risk and habit models reconcile high equity premia with smooth risk-free rates by inducing an inverse functional relationship between the variance and the mean of the stochastic discount factor. We show this highly successful resolution to closed-economy asset pricing puzzles is fundamentally problematic when applied to open economies with complete markets: It requires that differences in currency returns arise almost exclusively from predictable appreciations, not from interest rate differentials. In the data, by contrast, exchange rates are largely unpredictable, and currency returns differ because interest rates differ widely across currencies. We show currency risk premia arising in canonical long-run risk and habit preferences cannot match this fact. We argue this tension between canonical asset pricing and international macroeconomic models is a key reason researchers have struggled to reconcile the observed behavior of exchange rates, interest rates, and capital flows across countries. The lack of such a unifying model is a major impediment to understanding the effect of risk premia on international markets.

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Introduction

Interest in the role of risk premia in shaping international capital markets has surged in recent years. Numerous studies have examined the ramifications of shocks to both global and country-specific risk premia, investigating their impact on exchange rates, interest rates, capital flows, and financial stability. This research addresses critical phenomena such as the violation of uncovered interest parity (di Giovanni et al., 2022), contagion (Forbes, 2013), the global financial cycle (Miranda-Agrippino and Rey, 2020),[1] and events such as flight to safety, capital retrenchments (Forbes and Warnock, 2021; Chari, Dilts Stedman, and Lundblad, 2020), and sudden stops (Mendoza, 2010), which are of paramount concern for policymakers.

Despite the intense interest in these topics, constructing quantitative models that can reconcile the observed behavior of exchange rates with large and persistent differences in interest rates across countries has proven difficult. For example, to our knowledge, no quantitative model has been able to reproduce the fact that New Zealand on average has had a risk-free interest rate five percentage points above that of Japan for multiple decades, or that the cost of borrowing in Switzerland is persistently lower than that in Norway. The lack of such a unifying model is a major impediment to understanding the effect of risk premia on the allocation of capital across countries.

In this paper, we show these challenges are rooted in a fundamental tension between the canonical asset pricing models, which have been highly successful in addressing closed-economy asset pricing puzzles (Bansal and Yaron, 2004; Campbell and Cochrane, 1999), and the empirically observed behavior of exchange rates: strong forces in these models require that any large differences in risk premia across countries manifest not as persistent differences in interest rates, but as highly predictable exchange rates. In the data, we see large and persistent differences in interest rates across countries and exchange rates that are notoriously hard to predict (Meese and Rogoff, 1983), putting this class of model at odds with the data. We term this tension a "currency premium puzzle" and argue it is a fundamental reason the literature has struggled to construct a quantitative model that can match both asset prices and macroeconomic quantities across countries.

The classical quantitative challenge in asset pricing is to reconcile a high equity premium (Mehra and Prescott, 1985) with a low and stable risk-free rate (Weil, 1989). The former requires the stochastic discount factor (SDF) to be highly volatile, whereas the latter requires the sum of the mean and variance of the log SDF to be low and stable. Canonical long-run risk and habit models resolve these puzzles jointly by introducing a negative, functional relationship between the mean and the variance of the log SDF, so that whatever

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[1]Also see, e.g., Rey (2015), Bai et al. (2023), Morelli, Ottonello, and Perez (2022).

increases the SDF's variance also decreases its mean. This negative functional relationship keeps the risk-free rate low and stable while allowing for a large equity premium.

This "trick"—hardwiring a functional relationship between the mean and the variance of the log SDF—has been highly successful in closed-economy settings, allowing the construction of representative agent models that match both asset prices and macroeconomic quantities, such as aggregate investment and consumption. *We argue the same trick is also the fundamental reason why these models struggle in an open economy setting.*

Specifically, in an open economy with complete markets and log-normal SDFs, the means of log SDFs govern expected changes in exchange rates, while the variances of the log SDFs govern the expected return on the currency (Backus, Foresi, and Telmer, 2001). Data on exchange rates and currency premia thus puts bounds on how these moments should differ across countries. In particular, a large literature documents that (1) exchange rates are largely unpredictable, which requires the means of log SDFs to be similar across countries, and (2) currency premia are quantitatively large, which requires the variances of log SDFs to differ significantly across countries (Hassan and Mano, 2019). Together, these prior findings mean countries' log SDFs should differ widely in their variances but not in their means. This restriction from the international data, however, is difficult to fit with canonical long-run risk and habit preferences, which hardwire an almost one-for-one negative functional relationship between the two moments.

In this sense, large currency risk premia pose a fundamentally different quantitative challenge to these models than the classical closed-economy asset pricing puzzles, and the literature has not widely recognized this challenge.

To illustrate this point, we consider a conventional long-run risk model with complete markets, two representative agents (one in the home country and one in the foreign country), and identical Epstein-Zin (EZ) preferences with parameters equal to those in Bansal and Shaliastovich (2013). Suppose the two countries differ in their risk characteristics (the variance of their log SDFs) for *any* reason, so that one country's currency is riskier than the other. We show that, in this model, 94% of any difference in currency premia between the two countries must manifest as a predictable depreciation of the exchange rate, with only the remaining 6% coming from interest rate differentials. Because of the hard-wired functional relationship between the mean and variance of the log SDFs, this 94:6 split is fully determined by the parameters of the EZ utility function and independent of the features of the economic environment.

In other words, we demonstrate a theorem showing currency risk premia arising in models with these canonical preferences (risk aversion around 10 and elasticity of intertemporal substitution around one) cannot

fit the international data.

The same holds for canonical habit preferences (Campbell and Cochrane, 1999), where—again—the link between the mean and variance of the log SDF forces the vast majority of cross-country differences in currency premia to manifest as predictable depreciations, not persistent interest rate differentials.

This finding is all the more problematic because leading theories of why risk premia and interest rates might differ persistently across countries typically point to differences in the economic environment, such as differences in country size (Martin, 2011; Hassan, 2013), their role in global trade (Richmond, 2019), resource endowments (Ready, Roussanov, and Ward, 2017), their level of indebtedness (Wiriadinata, 2021), or the volatility of shocks (Menkhoff et al., 2012; Colacito et al., 2018a). All of these features of the economic environment are irrelevant for the results above: with complete markets, canonical long-run risk and habit preferences force all of these drivers of currency risk to manifest as large predictable depreciations, not persistent interest rate differentials.

To corroborate our theoretical results, we simulate widely used versions of long-run risk and habit models and show the currency premium puzzle manifests in each of them.

We show the same issue also arises in models that combine disaster risk with Epstein and Zin (1989) preferences (Gourio, 2012; Gourio, Siemer, and Verdelhan, 2013) and in other canonical applications of the rare disaster paradigm (Farhi and Gabaix, 2016).

Moreover, our analytical results show stochastic volatility as in Bansal and Yaron (2004), and departures from log-normality do not substantially alleviate the problem. At a deep level, any complete-markets model in which representative agents in two countries have identical habit or EZ preferences with a preference for early resolution of uncertainty appears to feature the currency premium puzzle.

In sum, the currency premium puzzle we highlight in this paper applies to the most advanced quantitative models that have attempted a synthesis between international asset prices and quantities. We do not see a straightforward resolution, which, of course, is our motivation for writing this paper.

One possibility is that the patterns we observe in the data could be rationalized with systematic differences in preferences across countries. However, in the absence of direct evidence of such differences—such as systematic demonstrations of higher risk aversion among households in New Zealand versus Japan, or in Norway relative to Switzerland—we are unable to evaluate this possibility. Ultimately, any observed economic behavior can be explained by appealing to sufficiently heterogeneous preferences, which is why economists are traditionally skeptical of purely preference-based explanations, favoring instead interpretations grounded

in empirical evidence.

In the paper's final two sections, we examine two additional avenues for further developing the literature. First, it is possible that frictions induced by market incompleteness may take a form that solves the tension between large interest rate differentials and unpredictable exchange rates. Because the vast majority of the existing literature relies on the complete-markets assumption, assessing the extent to which this might be possible in general is difficult. However, we are able to study one specific type of incompleteness that arises under incomplete spanning (Lustig and Verdelhan, 2019): when the representative agent in one country cannot trade all assets available in other countries.

Here again we find significant challenges: although incomplete spanning loosens the relationship between marginal utility and exchange rates, it does so in a limited way. In particular, we show that every percentage point that incomplete spanning deducts from exchange rate predictability is also eliminated from that currency's expected return. That is, although it may be possible to construct an (offsetting) pattern of incomplete spanning wedges that eliminates exchange-rate predictability from the models mentioned above, that same friction will then again compress cross-sectional differences in currency returns toward zero, and thus to levels similar to those one would obtain with conventional (constant relative risk aversion) preferences. As a result, incomplete spanning may help eliminate exchange rate predictability, but those models would then likely again fail to generate the large differences in interest rates and currency returns we see in the data.[2]

Finally, we explore synthesizing risk-based models, which have been the focus of recent literature, with the traditional macroeconomic view of cyclical interest rate differentials. In risk-based models, persistent differences in currency returns arise because investing in some countries is inherently riskier than in others. In contrast, the traditional macroeconomic view attributes short-term interest rate differences to temporary variations in expected growth rates and inflation, which are reversed by predictable appreciations (uncovered interest parity holds). If these temporary differences were instead long-lasting, one could construct a synthesis where countries with high currency risk premia (and thus expected appreciations) also had persistently higher growth rates or inflation (expected depreciations). Differences in the means and variances of the countries' SDF could then potentially offset each other, transforming predicted appreciations into persistent interest

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[2]This result dovetails with several recent papers (Lustig and Verdelhan, 2019; Jiang, 2023; Jiang et al., 2022; Jiang, Krishnamurthy, and Lustig, 2023; Sandulescu, Trojani, and Vedolin, 2021; Chernov, Haddad, and Itskhoki, 2023) that have consistently found various forms of incomplete markets do not necessarily help resolve many of the well-known puzzles in international finance, including the volatility puzzle of Brandt, Cochrane, and Santa-Clara (2006), the Backus and Smith (1993) puzzle, and the exchange rate disconnect puzzle (Meese and Rogoff, 1983).

rate differentials. Though this approach has its challenges, we highlight promising work in this area.

Our paper contributes to a large and growing literature at the intersection of international macroeconomics and asset pricing. One important strand of this literature studies in reduced-form the effects of time variation in global and country-specific risk premia, where shocks to global and local risk-bearing capacity motivate variation in global and local borrowing costs, retrenchments, capital flight, and the global financial cycle (Mendoza, 2010; Forbes, 2013; Rey, 2015; Miranda-Agrippino and Rey, 2020; Chari, Dilts Stedman, and Lundblad, 2020; di Giovanni et al., 2022; Forbes and Warnock, 2021; Morelli, Ottonello, and Perez, 2022; Bai et al., 2023; Colacito et al., 2018b).

Several important papers have attempted to microfound such time variation in global risk premia by combining conventional international macro models with long-run risk, habits, and other advances from the asset pricing literature (e.g., Colacito and Croce, 2011, 2013; Colacito et al., 2018a; Verdelhan, 2010; Heyerdahl-Larsen, 2014; Stathopoulos, 2017; Gourio, Siemer, and Verdelhan, 2013). While most of these models are designed to address a variety of quantitative puzzles, ranging from the forward premium puzzle to exchange rate disconnect, none of them are able to match the empirical fact that currency premia are large, whereas exchange rates are largely unpredictable. We contribute to this literature by highlighting this tension as a major obstacle to its further development.

Our paper is also tightly linked to the classic approaches to resolving the equity premium and risk-free rate puzzles. In particular, we show that models that are highly successful in resolving these well-known puzzles (Campbell and Cochrane, 1999; Bansal and Yaron, 2004) face new challenges when confronted with international data.

Our paper also speaks to the growing international macroeconomics literature that has documented large and persistent differences in currency risk, currency returns, and interest rates across countries (Lustig and Verdelhan, 2007; Lustig, Roussanov, and Verdelhan, 2011; Hassan and Mano, 2019).[3] Various papers have related these persistent differences in currency risk to features of the economic environment, such as differences in country size (Hassan, 2013; Martin, 2011), trade centrality (Richmond, 2019), commodity trade (Ready, Roussanov, and Ward, 2017), indebtedness (Wiriadinata, 2021), fiscal conditions (Jiang, 2021), and others. We contribute to this literature by showing all of these forces, when evaluated through the lens of a quantitative model with long-run risk or habit preferences, will result in large predicted depreciations, but not the large interest rate differentials we observe in the data.

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[3]See Hassan and Zhang (2021) for a survey of this literature.

Lastly, our paper also relates to a growing body of papers that highlights the importance of incompleteness of international asset markets. Since the seminal work of Backus, Foresi, and Telmer (2001), complete markets have been a popular assumption in the international finance literature for the assumption's simplicity. But recent works by Sandulescu, Trojani, and Vedolin (2021), Lustig and Verdelhan (2019), Jiang (2023), Jiang et al. (2022), Jiang, Krishnamurthy, and Lustig (2023), and Chernov, Haddad, and Itskhoki (2023), to highlight a few prominent contributions, have established in a reduced-form way that market incompleteness, or even segmentations and Euler equation wedges, help match salient features of currency markets. We show incomplete spanning is also subject to the currency premium puzzle. Although other forms of market incompleteness might help resolve the puzzle, we are not aware of any work that has successfully done so.[4]

The remainder of this paper is organized as follows. Section 1 lays out the basic concepts and framework for analysis. Section 2 discusses the long-run risk model, and section 3 discusses the habit model. Sections 4, 5, and 6 expand the argument by allowing for departures from log-normal distributions, incomplete markets, and heterogeneous growth rates. Section 7 concludes.

1   Basic Framework and Data

This section introduces a simple graphical representation of the currency premium puzzle. We show how data on exchange rates and currency returns put restrictions on the choice of SDFs across countries. For simplicity, we first focus the discussion on log-normally distributed SDFs and complete markets. We show in sections 4 and 5 that the currency premium puzzle also arises in more complex settings with incomplete markets and non-normal shocks.

1.1   Closed Economy Asset Pricing Puzzles

Our starting point is a complete-markets setup where we assume a log-normal distribution for the SDF M_{t+1}. Absence of arbitrage requires

$$\mathbb{E}_t(M_{t+1} R^a_{t+1}) = 1,$$

where $\mathbb{E}_t$ denotes the mathematical expectation conditional on the information set at time t and R^a_{t+1} denotes the return on an arbitrary asset. Applied to the risk-free rate with log return r_t, this equation relates the mean

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[4]Recently, Fang (2021) proposes a model with financial intermediaries, which takes heterogeneous risk-free rates as exogenous and generates cross-country variations in currency premia. In his framework, risk-free rates are the causes instead of the consequences of currency riskiness. However, the drivers of the large cross-country variations in risk-free rates, and how these drivers would interact with currency premia, remain unclear.

and variance of the SDF via

$$\frac{1}{2} \text{var}_t(m_{t+1}) = -\mathbb{E}_t(m_{t+1}) - r_t. \tag{1}$$

Throughout the paper, lowercase letters denote the logarithm of a variable such that $x = \log(X)$.

Resolving the equity premium puzzle of Mehra and Prescott (1985) requires the variance of the logarithm of the SDF (log SDF), $\text{var}_t(m_{t+1})$, to be large (Hansen and Jagannathan, 1991). Equation (1) shows this restriction has an immediate implication for the risk-free rate. In the data, risk-free interest rates tend to be low and stable, as is known from the risk-free rate puzzle of Weil (1989).

The joint resolution of the risk-free rate and equity premium puzzles thus requires a mean of the log SDF that moves to accommodate a high, and potentially changing, variance of the SDF to maintain low and stable risk-free rates. As we show in later sections, standard resolutions of the puzzles such as long-run risks and habit utility achieve this feature by imposing a negative functional relationship between the two moments of the log SDF.

Figure 1 represents equation (1) graphically. It plots the (conditional) mean of the log SDF on the horizontal axis and the scaled conditional variance on the vertical axis. Each point in this "SDF space" determines a log SDF represented by its first two moments. By equation (1), the first two moments of each SDF pin down the risk-free interest rate. In fact, equation (1) implies any SDF that lies on the same negatively sloping 45-degree line produces the same risk-free rate. We refer to these lines as "iso-risk-free rate (or iso-rf) lines."

Figure 1 shows two examples of iso-rf lines. The intercept of the line with the x-axis represents the (negative of the) risk-free rate, and lower lines represent higher risk-free rates. We thus have a graphical representation of (1) for different risk-free rates. The equity premium puzzle and the risk-free rate puzzle require a high variance with $(\mathbb{E}_t(m_{t+1}), \frac{1}{2} \text{var}_t(m_{t+1}))$ pairs on or around a particular iso-rf line to maintain a stable risk-free rate.

1.2 Exchange Rates and the SDF

Next, we show the logic of the previous section puts additional restrictions on the log SDFs when applied to open economies. Under complete markets, Backus, Foresi, and Telmer (2001) show the expected change in exchange rates, quoted as units of foreign currency per home currency, is given by[5]

$$\mathbb{E}(\Delta s_{t+1}) = \mathbb{E}(m_{t+1}) - \mathbb{E}(m_{t+1}^{\star}), \tag{2}$$

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[5]All equations in this subsection also hold conditionally. We show unconditional equations here because we can only estimate the unconditional moments given data on exchange rates, and we emphasize the cross-country variations of unconditional moments.

Figure 1: The SDF Space and Iso-rf Lines
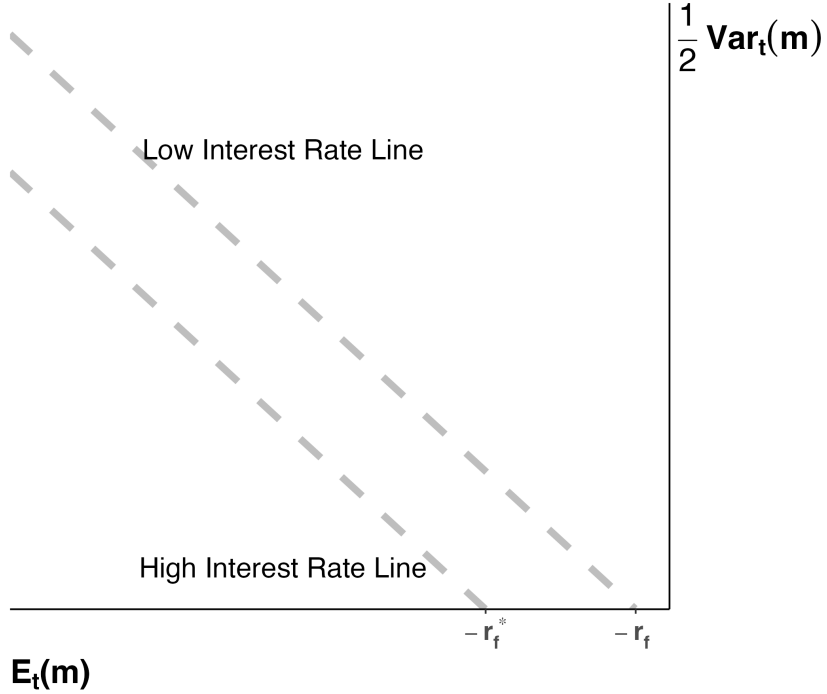
This figure plots two iso-rf lines (grey dashed line) in the SDF space. The lower line represents a higher risk-free rate. The intercept of each line with the x-axis represents the negative of the corresponding risk-free rate.

where Δs_{t+1} is the change in log exchange rates (depreciation rate of the foreign currency), and we use $\star$ to denote the foreign country.

From equation (1), the difference in risk-free rates between the two countries is given by

$$\mathbb{E}(r_t^\star - r_t) = \mathbb{E}(m_{t+1}) - \mathbb{E}(m_{t+1}^\star)$$
$$+ \frac{1}{2}\mathbb{E}(\text{var}_t(m_{t+1} - \text{var}_t(m_{t+1}^\star)). \tag{3}$$

Analogous to the equity premium, which is defined as the unconditional difference between returns on equity and the risk-free rate, we define the currency premium as the expected return a home investor would earn if she invests in the foreign risk-free bond versus the home bond.[6] She earns the difference in interest rates net of the depreciation rate of the foreign currency. Using (2) and (3), we obtain the currency premium as

$$\mathbb{E}(rx_{t+1}) = \mathbb{E}(r_t^\star - r_t) - \mathbb{E}(\Delta s_{t+1}) = \frac{1}{2}\mathbb{E}(\text{var}_t(m_{t+1}) - \text{var}_t(m_{t+1}^\star)), \tag{4}$$

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[6]In the literature, this return is sometimes referred to as "currency risk premia" or simply "currency returns." We call it currency premia, analogously to equity premium.

where rx_{t+1} denotes the currency return earned between t and $t + 1$. For ease of exposition, we assume the home country has a lower risk-free rate and that the investor is short the home country and long the foreign country unless specified otherwise.
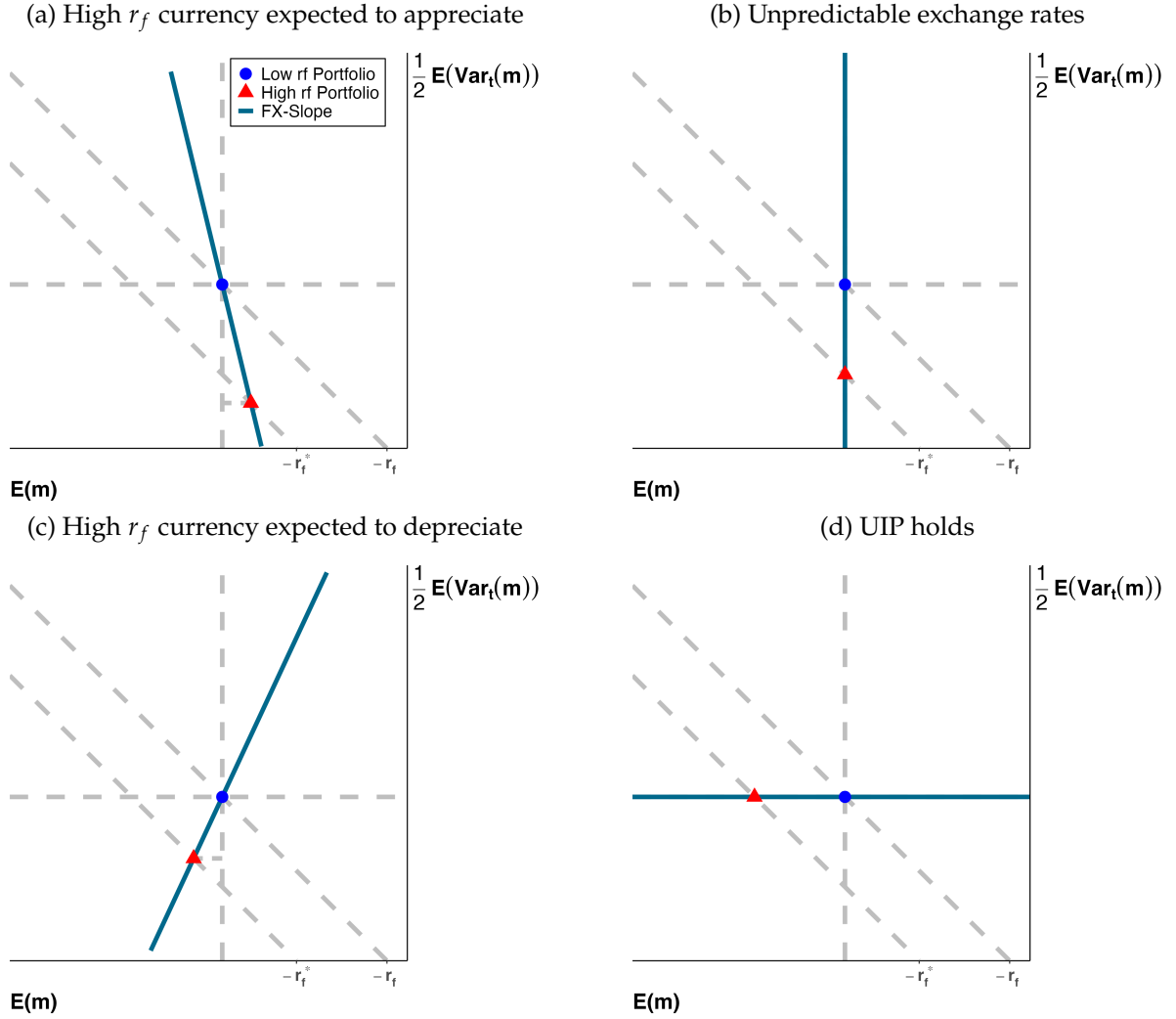
Note that we can decompose currency premia into an interest rate differential and an expected change in the exchange rate. The relative importance of these two terms is key to our analysis. To facilitate later discussion, we define the share of the expected change in the exchange rate in the currency premium, $\frac{-\mathbb{E}(\Delta s_{t+1})}{\mathbb{E}(rx_{t+1})}$, and label it "FX-share." The FX-share measures the fraction of the currency premium that comes from predicted changes in the exchange rate, rather than the difference in interest rates. In particular, a negative FX-share means investors lose money from expected depreciations; that is, the high interest rate currency depreciates on average.

Equations (2) and (4) show exchange rates and currency premia are tightly linked to means and variances of log SDFs. Each country's SDF is characterized by one mean-variance pair $(\mathbb{E}(m), \frac{1}{2}\mathbb{E}(\mathrm{var}_t(m_{t+1})))$, which we represent as a point in the (unconditional) SDF space. Data on exchange rates and currency premia between any two countries provide information on the relative position of their corresponding points. In particular, expected depreciations of the exchange rate determine the horizontal differences (equation (2)) and currency premia govern vertical differences (equation (3)).

Figure 2 visualizes properties of exchange rates and currency premia in SDF space under various configurations. Panel (a) shows the high-interest-rate currency appreciating on average relative to the low-interest-rate currency. The high-interest-rate foreign currency (red triangle) lies on a lower iso-rf line and is expected to appreciate $(\mathbb{E}_t(m_{t+1}) < \mathbb{E}_t(m_{t+1}^{\star}))$ relative to the low-interest-rate home currency (blue dot). The blue line connecting the two dots has a negative slope. In panel (b), exchange rates are unpredictable, namely, $\mathbb{E}(\Delta s) = 0$, and the means of the log SDFs are consequently equalized across countries, so that the blue line is vertical. Panel (c) shows the situation where the high-interest-rate currency is expected to depreciate – the blue line has a positive slope. If uncovered interest rate parity holds, interest-rate differences are exactly offset by expected exchange rates, and currency premia (differences in variances) vanish, so that the blue line is horizontal (panel (d)).

The main takeaway from Figure 2 is that the slope of the solid blue line that connects the two countries provides a useful statistic for their relative position in the SDF space. We call this statistic the "FX-slope."

Figure 2: Currency Premium in the SDF Space

(a) High r_f currency expected to appreciate

(b) Unpredictable exchange rates

(c) High r_f currency expected to depreciate

(d) UIP holds

This figure plots four scenarios of currency premia in the SDF space. In each panel, grey dashed lines represent iso-rf lines and vertical/horizontal lines. The blue dot represents the low-interest-rate country, and the red triangle represents the high-interest-rate country. Panel (a) represents the case when the high-interest-rate currency is expected to appreciate; panel (b) represents the case when exchange rates are unpredictable; panel (c) represents the case when the high-interest-rate currency is expected to depreciate; and panel (d) represents the case when uncovered interest rate parity holds.

Note the FX-slope is equal to the negative reciprocal of the FX-share

$$\text{FX-slope} = \frac{\frac{1}{2}\mathbb{E}(\text{var}_t(m_{t+1})) - \frac{1}{2}\mathbb{E}(\text{var}_t(m^{\star}_{t+1}))}{\mathbb{E}(m_{t+1}) - \mathbb{E}(m^{\star}_{t+1})}$$

$$= \frac{\mathbb{E}(rx)}{\mathbb{E}(\Delta s)} = -\frac{1}{\text{FX-share}}. \tag{5}$$

This FX-slope characterizes the composition of the currency premium: it pins down the split between expected

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depreciations and interest-rate differentials. As we discuss below, the currency-premium puzzle states that long-run risk and habit models generate an FX-slope that is at odds with the data, or a currency premium that is of the wrong composition (FX-share).

What do the data tell us about the FX-slope? A voluminous literature going back to Meese and Rogoff (1983) has documented that exchange rates are largely unpredictable. That is, we already know from this literature that the FX-slope must be close to vertical.

To estimate the FX-slope more directly, we build on a key result in Hassan and Mano (2019), who estimate the share of (conditional and unconditional) currency returns that are attributable to predictable depreciations. Table 1 summarizes their results for a baseline sample of countries. It shows the returns on a simple (unconditional) version of the carry trade that goes long currencies that have high interest rates and goes short currencies that have low interest rates. The portfolio weights each currency in proportion to its average interest differential with the US in the 15 years prior to 1995, without allowing any adjustments of weights thereafter.[7] Hassan and Mano (2019) refer to this strategy as the "Static Trade," because it mimics the famous carry trade, but without allowing for rebalancing of the portfolio in response to short-term fluctuations in the interest rate.

The table shows two main insights. First, unconditional differences in currency returns are large, with a mean return to this strategy of 3.46% annually and a Sharpe ratio of 0.39. Second, the vast majority of these returns are due to long-lasting differences in interest rates between countries (4.76 percentage points), whereas, if anything, high-interest-rate currencies depreciate on average, deducting 1.30 percentage points from the overall return. In other words, the FX-share is -0.37 with a 95% confidence interval ranging from -1.16 to 0.23.

Hassan and Mano (2019) show how to estimate this share more formally using regression analysis and how to construct appropriate standard errors. Moreover, they show both features of the data (large interest-rate differentials and a slightly positive FX-slope) are robust to a wide range of samples and estimation techniques. Appendix C gives details.[8]

On average, investing in high-interest-rate currencies makes money on the interest-rate differential and

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[7]The data range from 1983–2010. Our results are robust to including newer data. We keep our sample identical to Hassan and Mano (2019) to facilitate comparison. The table shows results from their "1 Rebalance" sample, which consists of 16 countries: Australia, Canada, Switzerland, Denmark, Hong Kong (China), Japan, Kuwait, Malaysia, Norway, New Zealand, Saudi Arabia, Sweden, Singapore, United Kingdom, South Africa, and the US. The empirical results below are robust to alternative means of portfolio construction, and the general pattern is confirmed in many related works (e.g., Lustig, Roussanov, and Verdelhan (2011)).

[8]Formally, and using the notation in their paper, the FX-slope in the unconditional SDF space is $\beta^{stat}/(1 - \beta^{stat})$, whereas the equivalent FX-slope in conditional SDF space is $\beta^{ct}/(1 - \beta^{ct})$.

Table 1: FX-Share and FX-Slope in the Data

	Return (%) $\mathbb{E}(rx)$	Change in FX (%) $-\mathbb{E}(\Delta s)$	Interest Rate Diff (%) $\mathbb{E}(r^\star - r)$	FX-share $\frac{-\mathbb{E}(\Delta s)}{\mathbb{E}(rx)}$	FX-slope $-\frac{1}{\text{FX-share}}$
Static Trade	3.46	-1.30	4.76	-0.37	2.67
	[1.18,5.54]	[-3.82,0.60]	[1.30,8.46]	[-1.16,0.23]	$[0.86, \infty) \cup (-\infty, -4.42]$

We use the 1-rebalance sample of Hassan and Mano (2019). All moments are annual. Static trade returns are calculated by first sorting the unconditional forward premia (interest rates), and then always shorting a weighted portfolio of the currencies with below-average unconditional forward premia, and longing a weighted portfolio of the rest. Details on the construction of portfolios can be found in Appendix C. Confidence intervals are reported in brackets and are obtained by bootstrapping over countries. The confidence interval of the FX-slope is obtained by choosing FX-slopes between which 95% of the bootstrapped points lie. Alternative regression-based estimations can be found in Appendix C and in Hassan and Mano (2019).

loses money on the exchange rate. The FX-share is negative (-37%). Consequently, the FX-slope is positive, as in panel (c) in Figure 2, with a confidence interval that includes a vertical relationship (unpredictable exchange rates as in panel (b)). However, the data clearly exclude the possibility that investors on average expect high-interest-rate currencies to appreciate (panel (a)) or that UIP might hold (panel (d)).

In sum, a complete-markets model that rationalizes the large and persistent differences in interest rates that we see in the data must satisfy two key properties:

Property 1. *A large difference in the variances of log SDFs*[9]

$$\mathbb{E}(var_t(m) - var_t(m^\star)) \geq 0.07$$

Property 2. *The FX-slope is positive or vertical*

$$\frac{\frac{1}{2}\mathbb{E}(var_t(m) - var_t(m^\star))}{\mathbb{E}(m_{t+1} - m^\star_{t+1})} \geq 0$$

Equivalently, the FX-share is negative, $\frac{-\mathbb{E}(\Delta s)}{\mathbb{E}(rx)} \leq 0$, and the high-interest-rate currency depreciates on average.

We incorporate these empirical results in the graphical representation of SDFs in Figure 3. In SDF space, the data require a weakly positive FX-slope.

These features of the data are not only of importance for our understanding of financial markets; they also affect the relative marginal product of capital across countries and direct capital flows. They thereby have first-order implications for the real economy, as we discuss further below (see Hassan, Mertens, and Zhang

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[9]To put things into perspective, the Hansen and Jagannathan (1991) bound implies $var_t(m) \geq 0.25$.

Figure 3: Static Trade in the SDF Space
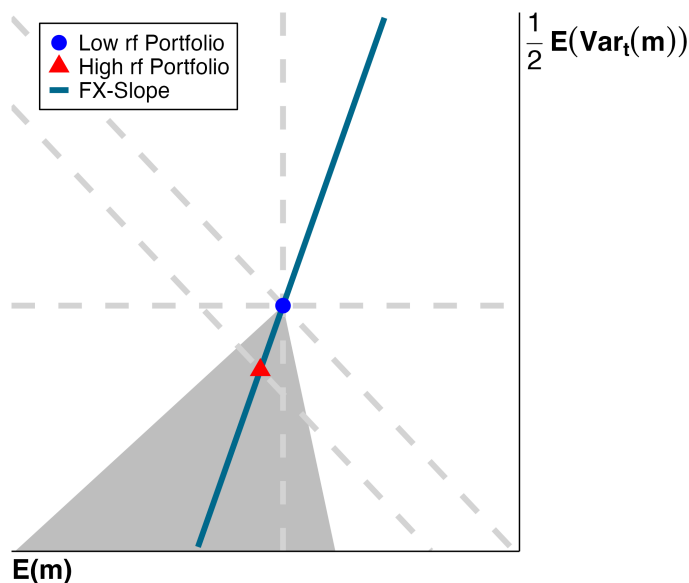
This figure plots the relative position of the low-interest-rate portfolio and the high-interest-rate portfolio implied by the static trade returns in the data, as well as the confidence intervals of the FX-slope. The position of the low-interest-rate portfolio is arbitrarily chosen because only the relative positions of these dots matter for our discussion. The position of the high-interest-rate portfolio is inferred from the data using equations (2) and (4). The shaded area represents confidence intervals for the FX-slope.

(2016, 2021) and Richers (2021)).

In the following sections, we show risk premia generated in models with canonical long-run risk and habit models cannot satisfy both properties simultaneously. They either generate unpredictable exchange rates with small currency returns (Property 2 but not Property 1) or generate large currency returns through predictable changes in exchange rates (Property 1 but not Property 2); that is, risk premia from both models are incompatible with large interest rate differentials that are the primary drivers of currency returns. We show theoretically that the fundamental reason these models fail to account for the two properties is that they introduce a negative functional relationship between means and variances of log SDFs.

2   Long-Run Risk Models

In this section, we set up and analyze a canonical long-run risk model under complete markets. We derive in closed form the relationship between the first two moments of the log SDFs and argue such a relationship is at odds with data from currency markets. We then numerically solve and simulate a number of prominent versions of the long-run risk model and show the currency premium puzzle quantitatively for each of them.

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A representative agent derives utility according to

$$U_t = \left((1-\delta)C_t^{1-\frac{1}{\psi}} + \delta \left\{ \mathbb{E}_t \left[U_{t+1}^{1-\gamma} \right] \right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right)^{\frac{1}{1-\frac{1}{\psi}}}. \tag{6}$$

U_t denotes utility at time t and C_t denotes consumption. δ is the subjective time discount factor, ψ governs the elasticity of intertemporal substitution, and γ scales risk aversion. Following the long-run risk approach, we further assume consumption follows[10]

$$\Delta c_{t+1} = \mu + z_t + \sigma \varepsilon_{t+1}$$

$$z_t = \rho z_{t-1} + \sigma_{LR} \varepsilon_{LR,t},$$

where μ is the mean growth rate of consumption, z_t is a long-run process that moves the mean of consumption growth, and ρ denotes its persistence. σ governs the volatility of short-run shocks and σ_{LR} governs the volatility of long-run shocks. ε_{t+1} and $\varepsilon_{LR,t}$ are short-run and long-run shocks, respectively. For ease of exposition, we set $\sigma = 0$ for our derivation in the main text. (Appendix A.1.2 shows the full solution including short-run shocks.)

Under these preferences and with complete markets, the log SDF is given by

$$\begin{aligned} m_{t+1} = \log(\delta) - \frac{1}{\psi} \Delta c_{t+1} \\ + \left(\frac{1}{\psi} - \gamma \right) \left(u_{t+1} - \frac{1}{1-\gamma} \log \left(\mathbb{E}_t [\exp((1-\gamma)u_{t+1})] \right) \right). \end{aligned} \tag{7}$$

Assuming u_{t+1} is normally distributed, we solve for the first and second moment of the log SDF as

$$\mathbb{E}(m_{t+1}) = \log(\delta) - \frac{1}{\psi}\mu - \frac{1}{2}(1-\gamma)\left(\frac{1}{\psi} - \gamma \right) \mathbb{E}(\mathrm{var}_t(u_{t+1})) \tag{8}$$

$$\frac{1}{2}\mathbb{E}(\mathrm{var}_t(m_{t+1})) = \frac{1}{2} \left(\frac{1}{\psi} - \gamma \right)^2 \mathbb{E}(\mathrm{var}_t(u_{t+1})). \tag{9}$$

These two equations show the first and second moments of the log SDF are tightly linked whenever utility is not time separable ($\gamma \neq \frac{1}{\psi}$). In fact, when we substitute out $\mathbb{E}(\mathrm{var}_t(u_{t+1}))$ from these conditions, we get a

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[10]For simplicity, we do not consider time-varying volatility here. In Appendix A.2, we show our results hold up under stochastic volatility.

functional relationship between the mean and variance:

$$\frac{1}{2}\mathbb{E}(\text{var}_t(m_{t+1})) = -\frac{\frac{1}{\psi} - \gamma}{1 - \gamma}\mathbb{E}(m_{t+1}) + \frac{\frac{1}{\psi} - \gamma}{1 - \gamma}\left(\log(\delta) - \frac{1}{\psi}\mu\right). \tag{10}$$

Standard calibrations assume a preference for early resolution of uncertainty, where $\gamma > 1$ and $\gamma > \frac{1}{\psi}$. In this case, The EZ utility function (8) implies a negative coefficient on the mean log SDF. As a result, any shocks that increase the variance lower the mean log SDF.

Importantly, all SDFs that satisfies the relationship in equation (10) lies on a line. We add this line to Figure 3 as the blue line with a common calibration of $\gamma = 6.5$ and $\psi = 1.6$ (see Figure 4). Note the blue line has the slope -1.07 and thus has a negative slope very similar to that of the iso risk-free lines (-1). For a given country, this feature of the long-run risk model helps resolve both the equity premium puzzle and the risk-free rate puzzle because it ensures a stable risk-free rate even when the variance of the SDF is large or changes over time.

Figure 4: The LRR Line in the SDF Space
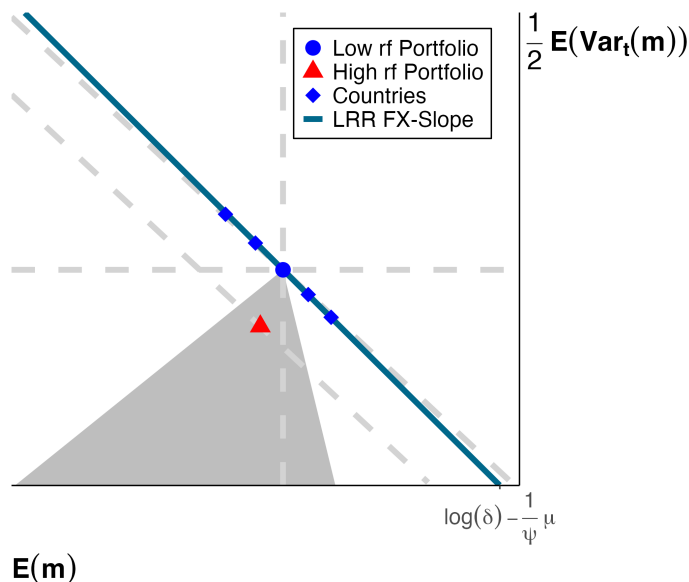
This figure plots the long-run risk FX Slope with $\gamma = 6.5$ and $\psi = 1.6$. The red triangle represents the high-interest-rate portfolio in the data. The blue dot represents the low-interest-rate portfolio in the data. The shaded area represents confidence intervals inferred from the data. Blue squares are example countries that differ in the variance of their log SDFs. These differences could be driven by heterogeneous loadings on a global shock, country size, trade centrality, or any other differences in the economic environment.

However, if all countries share the same preference parameters, all SDFs have to lie on the same blue line. This line thus represents the long-run risk model implied FX-slope (LRR FX-slope) across countries. Given

that the LRR FX-slope is close to -1, no specification of the model exists that would lead to a large difference in interest rates without generating substantial exchange rate predictability. Specifically, the model cannot generate a high-interest-rate portfolio with little or no exchange rate predictability, such as the red triangle in the figure. Put differently, the LRR FX-slope is clearly rejected by the data (outside of the admissible grey cone).

To illustrate this point, in Figure 4, we plot five representative countries (blue rhomboids) that differ in the variances of their log SDFs. These differences in variances could be driven by various forms of cross-country heterogeneities in risk characteristics proposed in the literature (different loadings on a global shock, country size, trade centrality, etc.). As long as all countries share the same preference parameters, they lie on the same line and no linear combination of them would match the empirical pattern established in section 1.2 (the red triangle).[11]

The long-run risk model thus pins down the slope of the relationship between means and variances of log SDFs via equation (10) whenever $\psi \neq 1/\gamma$.

Proposition 1. *Let γ, ψ, μ, and δ be identical across countries and assume absence of short-run risk ($\sigma = 0$). Further assume u_{t+1} is normally distributed. Then, for any two countries, the long-run risk model implies*

$$\text{FX-slope} = \frac{\mathbb{E}(rx_{t+1})}{\mathbb{E}(\Delta ex_{t+1})} = -\frac{\frac{1}{\psi} - \gamma}{1 - \gamma}. \tag{11}$$

If agents prefer early resolution of uncertainty so that $\gamma > 1/\psi$, and we assume $\gamma > 1$, the FX-slope is negative and the model cannot satisfy Property 2. In particular, if $\gamma > 2 - \frac{1}{\psi}$, FX-share $= \frac{-\mathbb{E}(\Delta ex_{t+1})}{\mathbb{E}(rx_{t+1})} = \frac{1-\gamma}{\frac{1}{\psi} - \gamma} > \frac{1}{2}$, the expected change in the exchange rate $\mathbb{E}(\Delta ex_{t+1})$ accounts for more than 50% of the currency premium.

Proof. Implied by equations (2), (4), and (10). □

Equation (11) shows the LRR FX-slope is determined purely by preference parameters. It is thus deeply connected with the assumption of recursive preferences: when agents prefer an early (or late) resolution of uncertainty ($\gamma \neq \frac{1}{\psi}$), the expected continuation utility enters non-linearly in (6), so that its higher-order moments (in the case of log-normal shocks, its second moment) move not just the variance but also the

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[11]For example, we confirm that in the calibrated long-run risk model of Colacito et al. (2018a) (a heterogeneous country general equilibrium model where countries differ in their loading on a global endowment shock), all countries indeed lie on the same LRR line. We confirm this by solving their model using risk-adjusted-affine approximation (log-linearization with risk-adjustments. See, e.g., Chen and Palomino (2019), Malkhozov (2014) and Lopez, Lopez-Salido, and Vazquez-Grande (2015)). We use the calibration in Table II (in particular, $\gamma = 6.5$, $\psi = 1.6$), and the heterogeneous loadings on the global shock are evenly spaced between [-0.65,0.65], as in the paper's calibration on page 18. We then numerically solve for the unconditional variances and means of each country's SDF. The Mathematica file for the solution is available upon request.

conditional mean of the log SDF. The FX-share, and thus the composition of currency premia, directly follows from the choice of preferences and is, in this sense, independent of the economic environment. As we show in Appendix A.2 and section 4, this logic is very general, so this relationship is robust to adding volatility shocks and relaxations of the log-normality assumption.

Table 2: Static Trade Returns under LRR Models

	Return (%) $\mathbb{E}(rx^{st})$	Change in FX (%) $-\mathbb{E}(\Delta s^{st})$	Interest Rate Diff (%) $\mathbb{E}(r^{\star,st} - r^{st})$	FX-share $\frac{-\mathbb{E}(\Delta s^{st})}{\mathbb{E}(rx^{st})}$	FX-slope $-\frac{1}{\text{FX-share}}$	P1	P2
Data	3.46 [1.18,5.54]	-1.30 [-3.82,0.60]	4.76 [1.30,8.46]	-0.37 [-1.16,0.23]	2.67 $[0.86, \infty) \cup (-\infty, -4.42]$	-	-
Colacito, Croce, Gavazzoni and Ready (2018) JF	7.10	5.98	1.12	0.93	-1.07	Yes	No
Colacito, Croce, Ho and Howard (2018) AER	0.00	0.00	0.00	0.98	-1.02	No	No
Bansal and Shaliastovich (2013) RFS	0.00	0.00	0.00	0.96	-1.04	No	No
Colacito and Croce (2013) JF	0.00	0.00	0.00	0.95	-1.05	No	No
Bansal and Yaron (2004) JF	-	-	-	0.94	-1.06	No	No

This table shows the simulation results of four long-run risk models. For Colacito et al. (2018a), we simulate the model in their section II, with calibrations in their Table II (exactly the same model as the heterogeneous EZ model in their Table III). For Colacito et al. (2018b), we simualte the model in their section II, with calibrations in their Table 2 (the same model as EZ-BKK in their Table 3). For Bansal and Shaliastovich (2013), we simulate the model in their Section 2, with calibrations in their Table 4 and section 4.4 (the same model as Table 7), we report the simulation results for the real model to be consistent with other models. For Colacito and Croce (2013), we simulate the model in their sections II and III, with calibrations in their Table II, Panel A (the same model as model (1) in their Table II, Panel B). For Colacito et al. (2018a), we follow their approach and simulate 100 economies, each with 320 periods and 100 burn-in periods (discarded). For all the other models, we simulate 100 economies of 10,000 periods. Static trade returns are computed using the same method in our empirical analysis. All moments are averaged across periods and simulations. Because Colacito et al. (2018b), Bansal and Shaliastovich (2013) and Colacito and Croce (2013) feature symmetric countries and currency premia are close to 0, we show the theoretical FX-slopes for these models using Proposition 1 instead. We also show the theoretical FX-slope for the calibration of Bansal and Yaron (2004) for comparison (with $\gamma = 10$, $\psi = 1.5$). The last two columns summarize whether the simulated results can match our empirical Properties 1 and 2, respectively.

To show the implications of the theoretical analysis, we solve and simulate five state-of-the-art long-run risk models in the literature and summarize the results in Table 2. These models are replications of the seminal works in Colacito et al. (2018a), Colacito et al. (2018b), Bansal and Shaliastovich (2013), and Colacito and Croce (2013), as well as the original long-run risk model in Bansal and Yaron (2004).[12] The most recent of the five models (Colacito et al. (2018a)) allows countries to differ in their loadings on global productivity

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[12]We thank Ric Colacito, Max Croce, Federico Gavazzoni, and Robert Ready for providing their code.

shocks, so that some countries (currencies) are permanently riskier than others in equilibrium. Through this channel, the model is able to generate sizable currency premia, matching our empirical Property 1. However, consistent with the currency premium puzzle, more than 90% of these currency premia arise from predictable appreciation of the high-interest-rate currencies, implying an FX-slope close to -1 and contradicting our empirical Property 2. All of the remaining models do not explicitly model asymmetries across countries. (In this sense, they fail to match Property 1.) However, the table shows they would suffer from the currency premium puzzle even if they did. In each of the four models, going back to the seminal work of Bansal and Yaron (2004), the FX-slope is very close to -1 (the slope of the iso-rf line), suggesting FX-shares would again make up well over 90% of any currency returns the models could produce. This feature, while useful in keeping the risk-free rate low and stable, leads to a currency premium with a counterfactual composition of interest rate differences and predictable changes in exchange rates. To be clear, the composition of currency premia was not the object of any of the papers, which are each highly successful on matching the data in a number of other dimensions. They are the state-of-the art in terms of matching both prices and quantities in open economies. However, our analysis reveals that the standard long-run risk setup struggles to match this additional empirical fact.

Table 3: Carry Trade Returns under LRR Models

	Return (%) $\mathbb{E}(rx^{ct})$	Change in FX (%) $-\mathbb{E}(\Delta s^{ct})$	Interest Rate Diff (%) $\mathbb{E}(r^{\star,ct} - r^{ct})$	FX-share $\frac{-\mathbb{E}(\Delta s^{ct})}{\mathbb{E}(rx^{ct})}$	FX-slope $-\frac{1}{\text{FX-share}}$	P1	P2
Data	4.95 [1.50,8.34]	-2.15 [-4.98,0.49]	7.11 [2.22,13.22]	-0.43 [-1.10,0.15]	2.30 [0.90, ∞)∪(−∞, -6.56]	-	-
Colacito, Croce, Gavazzoni and Ready (2018) JF	4.47	2.76	1.71	0.62	-1.62	Yes	No
Colacito, Croce, Ho and Howard (2018) AER	-0.09	-0.52	0.44	6.11	-0.16	No	No
Bansal and Shaliastovich (2013) RFS	-0.03	-0.26	0.23	9.54	-0.10	No	No
Colacito and Croce (2013) JF	0.05	-0.35	0.41	-7.02	0.14	No	No

This table shows the simulation results for the four long-run risk models of Table 2. Instead of the static trade, this table focuses on the carry trade return. The last two columns summarize whether the simulated results can match our empirical Properties 1 and 2, respectively.

Aside from the simple static trade introduced in Table 2, an alternative popular trading strategy, the carry

trade, aims at exploiting time-varying differences in interest rates across countries. The carry trade goes long high-interest-rate currencies and short low-interest-rate currencies, just like the static trade. It differs, however, in that the carry trade compares risk-free rates period by period and rebalances the portfolio accordingly. Because of period-by-period re-balancing, carry trade returns also contain conditional information, which is missing in our analysis of unconditional moments so far. Therefore, we report carry trade returns in Table 3.

Table 3 displays a similar pattern to the static trade. The data again suggest large currency premia that are mostly accounted for by risk-free rate differences.[13] Symmetric country models with long-run risk fail to generate the size of currency premia. And all four models imply counterfactually large predictale appreciations in exchange rates.

To summarize, in this section, we derived a closed-form functional relationship between the mean and the variance of the log SDFs under long-run risk models with (recursive) EZ preferences. We show that under standard long-run risk calibrations (Bansal and Yaron, 2004), where agents prefer early resolution of uncertainty, the model's risk premia can only generate a negative FX-slope and cannot satisfy Property 2. Most of the generated currency returns are therefore accounted for by an appreciation of the high-interest-rate currency, in contrast to the data.

3   External Habit Models

In this section, we show external habit models display similar difficulties in matching the data as long-run risk models. We illustrate these challenges using the pioneering work in Verdelhan (2010). This model has the advantage of being a direct extension of the classical habit model in Campbell and Cochrane (1999) and can be solved in closed form. We study more involved variations of the habit model numerically below.

Agents feature habit utilities of the form

$$\mathbb{E} \sum_{t=0}^{\infty} \delta^t \frac{(C_t - H_t)^{1-\gamma} - 1}{1 - \gamma},$$

where H_t is an externally given habit level. δ denotes the time discount factor and γ governs risk aversion as before. With the surplus consumption ratio

$$X_t \equiv \frac{C_t - H_t}{C_t},$$

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[13]The FX-slope now measures the relative position of the portfolio that we long each period and the portfolio that we short each period.

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the pricing kernel becomes

$$M_{t+1} = \delta \left(\frac{X_{t+1}}{X_t} \frac{C_{t+1}}{C_t} \right)^{-\gamma}.$$

Log consumption follows a random walk with drift given by

$$\Delta c_{t+1} = \mu + \sigma \varepsilon_{t+1},$$

where μ is the mean growth rate and σ is the conditional volatility. ε_{t+1} is an i.i.d. normal shock. Moreover, Campbell and Cochrane (1999) and Verdelhan (2010) assume the following process for the log surplus consumption ratio:

$$x_{t+1} = (1 - \phi)\bar{x} + \phi x_t + \lambda(x_t)(\Delta c_{t+1} - \mu),$$

where ϕ governs persistence and the sensitivity function $\lambda(x_t)$ is specified as

$$\lambda(x_t) = \begin{cases} \frac{1}{\bar{X}}\sqrt{1 - 2(x_t - \bar{x})} - 1 & \text{when } x < x_{max} \\ 0 & \text{elsewhere} \end{cases}$$

and the logarithm of the upper bound x_{max} is given by

$$x_{max} = \bar{x} + \frac{1 - (\bar{X})^2}{2}.$$

The steady state of the surplus consumption ratio is

$$\bar{X} = \sigma \sqrt{\frac{\gamma}{1 - \phi - B/\gamma}}.$$

Note $\gamma(1 - \phi) - B > 0$ by construction.[14] Using the process for the log surplus consumption ratio x_t, one can derive the log SDF as

$$m_{t+1} = \log(\delta) - \gamma(\Delta c_{t+1} + \Delta x_{t+1}).$$

_____

[14] The specification of the $\lambda()$ function strictly follows Campbell and Cochrane (1999). As they point out, the specific functional form is designed to keep the risk-free rates stable. Parameter B nests different SDFs from the literature. $B < 0$ in Verdelhan (2010), $B = 0$ in Campbell and Cochrane (1999), and $B > 0$ in Wachter (2006).

This specification implies the first two moments of the log SDF are

$$\mathbb{E}_t(m_{t+1}) = \log(\delta) - \gamma\mu + \gamma(1 - \phi)(x_t - \bar{x}) \tag{12}$$

$$\frac{1}{2}\text{var}_t(m_{t+1}) = \frac{1}{2}\gamma^2(1 + \lambda(x_t))^2\sigma^2$$

$$= \frac{1}{2}(\gamma(1 - \phi) - B) - (\gamma(1 - \phi) - B)(x_t - \bar{x}). \tag{13}$$

Note the mean and the variance of the log SDF depend on the term $x_t - \bar{x}$. Substituting it out unveils a strong relationship between the first two moments, which are now conditioned on time t information:

$$\frac{1}{2}\text{var}_t(m_{t+1}) = -\frac{\gamma(1 - \phi) - B}{\gamma(1 - \phi)}\mathbb{E}_t(m_{t+1})$$

$$+ \frac{\gamma(1 - \phi) - B}{\gamma(1 - \phi)}(\log(\delta) - \gamma\mu) + \frac{1}{2}(\gamma(1 - \phi) - B). \tag{14}$$

Equation (14) shows that, similar to long-run risk models, habit models also imply a negative functional relationship between the first two moments of log SDFs. Here, again, this feature is useful for reconciling a large equity premium with low and stable risk-free rates.

Equation (14) again implies a line in the—now conditional—SDF space. We plot this line of the habit model in Figure 5 with a calibration of $\gamma = 2$, $\phi = 0.995$, and $B = -0.01$, taken directly from Verdelhan (2010).

Figure 5 shows all SDFs implied by this external habit model (blue line) lie close to the iso-rf line, effectively immobilizing the risk-free rate while allowing for high risk premium (high variance of log SDF).

However, same as long-run risk models, when agents share the same preference, they end up on exactly the same line and the slope of this line determines the FX-slope implied by habit models (Habit FX-slope). As a result, the currency premium puzzle also applies to the habit model when taken to the international data. We summarize the habit model's FX-Slope in terms of the two properties that the data call for, in the following proposition.

Proposition 2. *If preferences are symmetric across countries, the FX-slope is given by*

$$FX\text{-}slope = \frac{\mathbb{E}_t(rx_{t+1})}{\mathbb{E}_t(\Delta s_{t+1})} = -\frac{\gamma(1 - \phi) - B}{\gamma(1 - \phi)}.$$

Because $\gamma(1 - \phi) - B > 0$ is required by stationarity, the FX-slope is always negative and the model cannot satisfy Property 2. Furthermore, if $\gamma(1 - \phi) > -B$, FX-share $= \frac{-\mathbb{E}(\Delta s_{t+1})}{\mathbb{E}(rx_{t+1})} = \frac{\gamma(1-\phi)}{\gamma(1-\phi)-B} > \frac{1}{2}$. Thus, an appreciation of the

Figure 5: The Habit Line in the SDF Space
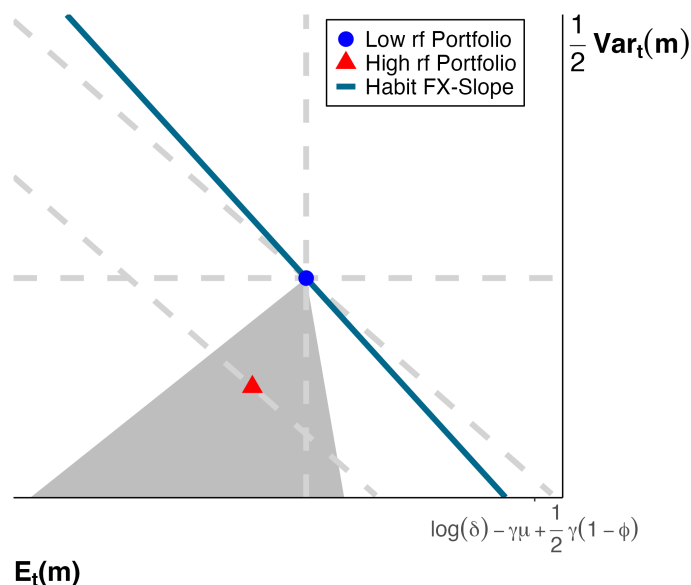
This figure plots the FX-Slope of the habit model with $\gamma = 2$, $\phi = 0.995$, and $B = -0.01$. The red triangle represents the high-interest-rate portfolio in the data. The blue dot represents the low-interest-rate portfolio in the data. The shaded area shows the confidence intervals inferred from the data.

high-interest-rate currency accounts for more than 50% of the currency premium.

Proof. Implied by equations (2), (4), and (14). □

We focus our numerical comparison of various models with the data on carry trade returns. An analysis of static trades would not be suitable, because all major models in the literature feature fully symmetric countries with no unconditional differences in interest rates or expected currency returns across countries. To show that the currency premium puzzle applies to a wider set of habit models, we again numerically solve and simulate several prominent and state-of-the-art models in the literature.

We summarize our results in Table 4. The habit model of Verdelhan (2010) closely matches the carry trade returns in the data (Property 1). Almost half of these returns, however, arise from expected appreciations of the high-interest-rate currency, contradicting Property 2. The FX-slope is strongly negative at -2.07 in this model, whereas it is positive in the data (2.30).

In the continuous-time habit model of Stathopoulos (2017), the high-interest-rate currency does depreciate on average.[15] But it depreciates so much that the expected depreciation exceeds the interest rate differential, and the resulting currency premium is negative, at odds with the data and contradicting both our empirical

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[15]We thank Andreas Stathopoulos for providing us with his code.

Table 4: Carry Trade Returns under Habit Models

	Return (%) $\mathbb{E}(rx^{ct})$	Change in FX (%) $-\mathbb{E}(\Delta s^{ct})$	Interest Rate Diff (%) $\mathbb{E}(r^{\star,ct} - r^{ct})$	FX-share $\frac{-\mathbb{E}(\Delta s^{ct})}{\mathbb{E}(rx^{ct})}$	FX-slope $-\frac{1}{\text{FX-share}}$	P1	P2
Data	4.95 [1.50,8.34]	-2.15 [-4.98,0.49]	7.11 [2.22,13.22]	-0.43 [-1.10,0.15]	2.30 $[0.90, \infty) \cup (-\infty, -6.56]$	-	-
Verdelhan (2010) JF	4.54	2.19	2.35	0.48	-2.07	Yes	No
Stathopoulos (2017) RFS	-1.23	-2.40	1.17	1.95	-0.51	No	No
Heyerdahl-Larsen (2014) RFS	3.48	3.05	0.43	0.88	-1.14	Yes	No
Campbell and Cochrane (1999) JPE	-	-	-	1.00	-1.00	No	No

This table shows the simulation results of three habit models. For Verdelhan (2010), we simulate the model in section I, with calibrations in Table II. For Stathopoulos (2017), we simulate the model in section 1, with calibrations in Table 1. For Verdelhan (2010) and Heyerdahl-Larsen (2014), we simulate 100 economies of 10,000 periods. All moments are averaged across periods and simulations. Results for Heyerdahl-Larsen (2014) are obtained by using the same simulated samples that generated his Table 10. Details on carry trade return construction can be found in Appendix B, and the same method is used for both the empirical and the simulation results. We also show the theoretical FX-slope for the calibration of Campbell and Cochrane (1999) for comparison. The last two columns summarize whether the simulated results can match our empirical properties 1 and 2, respectively.

properties. The deep habit model of Heyerdahl-Larsen (2014) features an FX-share of 87.67% and and FX-slope of -1.14 and thus also fails to match Property 2.[16] We further report the FX-slope for Campbell and Cochrane (1999), which is -1 exactly. In fact, the authors explicitly parameterize the model to fully immobilize the risk-free rate, so that it also allows for no variation in interest rates across countries.

As for the literature on long-run risks, it is worth noting that none of these models aimed to fit the composition of currency premia, and instead focused on other salient features of the data. Our analysis shows that the persistent differences in interest rates combined with unpredictable exchange rates pose a tough challenge for a wide range of models in international finance.

We should note that unlike in long-run risk models, no clear separation exists between preference parameters and the economic environment (endowment process) in habit models, so that they may allow for more degrees of freedom. By construction, habit models allow changes in the endowment process to directly affect agents' risk aversion (preferences). One possible path forward might be to allow for cross-country variation in σ, which governs the volatility of the surplus consumption ratio. However, this parameter has no effect on interest rates in the standard formulation of the model. One might also consider allowing B, and thus the long-run habit level, to differ across countries to generate unconditional differences in variance of log SDFs

[16]We thank Christian Heyerdahl-Larsen for providing us with his simulated samples.

(see equations (12) and (13)), though one might argue doing so might amount again to simply modifying preferences in arbitrary ways to fit the data. More research is needed to establish how such variations could be disciplined by the data.[17]

To summarize, in this section, we derived a closed-form functional relationship between the conditional mean and variance of the log SDFs under external habit models. We showed that under standard calibrations, standard models in this literature generate a negative FX-slope and cannot satisfy Property 2, thereby running into the currency premium puzzle.

4 Relaxing Log-Normality

In addition to long-run risk and habit models, a large number of authors have considered rare disasters and other departures from log-normal shocks as possible explanations for the equity premium puzzle and other closed-economy asset pricing phenomena. In this section, we show that, perhaps surprisingly, the currency premium puzzle also applies in this broader class of models.

We first generalize our basic framework from section 1 to a non-normal distribution and show the tension between the equity premium puzzle, the risk-free rate, and exchange rate predictability exists regardless of log-normality. We then examine two existing prominent international asset pricing models that feature disaster risk and find both are subject to the currency premium puzzle.

For general distributions, the risk-free rate can be written as (see Backus, Foresi, and Telmer (2001))

$$r_t = -\log(\mathbb{E}_t M_{t+1}) = -\mathbb{E}_t(m_{t+1}) - \left[\log(\mathbb{E}_t(M_{t+1})) - \mathbb{E}_t(m_{t+1})\right]$$
$$= -\mathbb{E}_t(m_{t+1}) - \Xi_t(m_{t+1}),$$

where $\Xi_t(m_{t+1}) = \log(\mathbb{E}_t(M_{t+1})) - \mathbb{E}_t(m_{t+1})$ denotes the entropy of the SDF.

We investigate whether the composition of currency premia also puts restrictions on models with higher moments in the distribution. In this generalized case, the expected change in exchange rates and currency

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[17]In fact, studies have tied B to slopes of term structure (Wachter (2006), Verdelhan (2010)), but to our knowledge, no study has tied B to a specific economic source.

premia are now given by

$$\mathbb{E}_t(\Delta s_{t+1}) = \mathbb{E}_t(m_{t+1}) - \mathbb{E}_t(m_{t+1}^\star)$$

$$\mathbb{E}_t(rx_{t+1}) = \Xi_t(m_{t+1}) - \Xi_t(m_{t+1}^\star).$$

The expressions are identical to the ones in section 1 except that the entropy $\Xi_t(m_{t+1})$ takes the place of what used to be the variance of the log SDF, $\frac{1}{2}\operatorname{var}_t(m_{t+1})$. The relationship between risk-free rates, exchange rates, currency premia, and the mean, and now the entropy of the SDFs is, however, preserved. As a result, the discussion in section 1 still applies. The log-normal model emerges as a special case, and the tension between a high equity premium, a low and stable risk-free rate, and the composition of currency premium extends to the generalized setup.

As an illustration, we first consider the framework of Gourio, Siemer, and Verdelhan (2013), which focuses on disaster risk and thus higher moments of the distribution of the SDF. Agents feature EZ preferences[18] as in (6), and their log SDF is given by (7). The first moment and the entropy are then given by[19]

$$\mathbb{E}_t(m_{t+1}) = \log(\delta) - \frac{1}{\psi}\mathbb{E}_t(\Delta c_{t+1})$$

$$+ \frac{\frac{1}{\psi} - \gamma}{1 - \gamma}\mathbb{E}_t\left((1-\gamma)u_{t+1}\right) - \frac{\frac{1}{\psi} - \gamma}{1 - \gamma}\log\left(\mathbb{E}_t[U_{t+1}^{1-\gamma}]\right)$$

$$\Xi_t(m_{t+1}) = \log\mathbb{E}_t(M_{t+1}) - \mathbb{E}_t(m_{t+1})$$

$$= \Xi_t(-\frac{1}{\psi}\Delta c_{t+1}) + \log\left(\operatorname{cov}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}, \left(\frac{U_{t+1}^{1-\gamma}}{\mathbb{E}_t\left[U_{t+1}^{1-\gamma}\right]}\right)^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}}\right]\right)$$

$$- \mathbb{E}_t\left(\left(\frac{1}{\psi} - \gamma\right)u_{t+1}\right) + \log\left(\mathbb{E}_t\left(U_{t+1}^{\frac{1}{\psi}-\gamma}\cdot\right)\right)$$

To simplify the exposition, we assume the entropy of next period's consumption growth $\Xi_t(-\frac{1}{\psi}\Delta c_{t+1}) \approx 0$; that is, all higher moments of consumption growth are approximately equal to 0.[20] We relax this assumption in our quantitative exercise and show simulation results are consistent with our theoretical predictions. Note

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[18]The model in Gourio, Siemer, and Verdelhan (2013) features labor supply. Thus, leisure shows up in the utility function. We abstract away from this setup for simplicity. We also suppress the production side of the economy because all our results do not depend on the specific setup of the economic environment. We confirm this with numerical exercises.

[19]For a detailed derivation, see Appendix B.

[20]Entropy is a function of all the higher moments. Thus, this is similar to the "no-short-run-shock" assumption in the log-normal case of section 2. Empirically, aggregate consumption growth is quite smooth.

the covariance term in the second equation above also equals 0 under this assumption, and the unconditional mean and the entropy of the SDF are given by

$$\mathbb{E}(m_{t+1}) = \log(\delta) - \frac{1}{\psi}\,\mathbb{E}(\Delta c_{t+1}) + \frac{\frac{1}{\psi} - \gamma}{1 - \gamma}\,\mathbb{E}\left[((1-\gamma)u_{t+1}) - \log\left(\mathbb{E}_t[U_{t+1}^{1-\gamma}]\right)\right] \tag{15}$$

$$\mathbb{E}(\Xi_t(m_{t+1})) = -\mathbb{E}\left[\left(\frac{1}{\psi} - \gamma\right)u_{t+1} - \log\left(\mathbb{E}_t\left(U_{t+1}^{\frac{1}{\psi}-\gamma}\right)\right)\right]. \tag{16}$$

An immediate observation is that the mean and the entropy are again tightly linked, because both depend on continuation utility (the terms in square brackets). Once again, this result follows directly from the structure of EZ preferences.[21]

We can make further headway in two special cases. We first consider the special case where the elasticity of intertemporal substitution ψ is equal to 1. In this case, we obtain a straightforward functional link between the mean of the log SDF and its entropy:

$$\mathbb{E}(\Xi_t(m_{t+1})) = -\mathbb{E}(m_{t+1}) + \log(\delta) - \mathbb{E}(\Delta c_{t+1}).$$

This linear functional relationship shows an FX-Slope of -1 in a generalized SDF space, where the y-axis is now entropy instead of variance of SDF. That is, countries with identical preference parameters and $\psi = 1$ must always have the same interest rate. All of these countries are on the same iso-rf line, so the model does not allow for risk premia to induce any cross-country variation in risk-free rates. As a result, the FX-share is one implying currency premia exclusively induce predictable variation in exchange rates, regardless of the configuration of higher-order moments of shocks. Models of risk premia with EZ preferences, complete markets, and an intertemporal elasticity of substitution of 1 thus generally fail to satisfy the two empirical properties required by the data, independently of any differences in the configuration of disaster risk across countries.[22]

More broadly, as we saw for long-run risk models, EZ preferences with an intertemporal elasticity of substitution near one help in keeping the risk-free rate stable. But this feature also prevents the risk premia from generating large differences in interest rates across countries.

Looking beyond the case of a unit elasticity of intertemporal substitution, we can make further progress

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[21]Although the defining feature of disaster models is the existence of disaster risk, EZ preferences are commonly used. See Barro (2009), Gourio (2012), for example.

[22]One path forward might be to combine heterogeneity in disaster risk to an off-setting source of variation in the conditional mean of the log SDF. We discuss this possibility below.

by limiting countries to differ in only one higher-order moment of the distribution of their shocks. For concreteness, we may capture the key ingredient in the disaster risk literature by allowing for country-specific skewness in the distribution of consumption. To highlight the role of higher moments, we exploit the recursiveness of preferences to recover the relationship between the first moment and the entropy of the SDF. From equations (15) and (16), using cumulant generating functions (Backus, Foresi, and Telmer (2001)), we obtain

$$\mathbb{E}(m_{t+1}) = \left(\log(\delta) - \frac{1}{\psi}\mathbb{E}(\Delta c_{t+1})\right) - \frac{1}{2}(1-\gamma)\left(\frac{1}{\psi}-\gamma\right)\mathbb{E}(\kappa_{2,t}(u_{t+1})) - \frac{1}{6}(1-\gamma)^2\left(\frac{1}{\psi}-\gamma\right)\mathbb{E}(\kappa_{3,t}(u_{t+1})) + \ldots,$$

where $\kappa_{i,t}(u_{t+1})$ is the ith cumulant of u_{t+1}.[23] A similar expansion applies to the entropy

$$\mathbb{E}(\Xi_t(m_{t+1})) = \frac{1}{2}\left(\frac{1}{\psi}-\gamma\right)^2\mathbb{E}(\kappa_{2,t}(u_{t+1})) + \frac{1}{6}\left(\frac{1}{\psi}-\gamma\right)^3\mathbb{E}(\kappa_{3,t}(u_{t+1})) + \ldots$$

With skewness ($\kappa_{3,t}(u_{t+1})$) differing across countries and setting all other cumulants to be identical, we get

$$\mathbb{E}(\Xi_t(m_{t+1})) = -\left(\frac{\frac{1}{\psi}-\gamma}{1-\gamma}\right)^2\mathbb{E}(m_{t+1}) + \text{constant}. \tag{17}$$

We again see a tight relationship between the entropy and the first moment. In particular, (17) implies

$$\text{FX-slope} = -\frac{1}{\text{FX-share}} = -\left(\frac{\frac{1}{\psi}-\gamma}{1-\gamma}\right)^2,$$

which is always negative and close to -1 under standard calibrations ($\gamma > \frac{1}{\psi}$, $\gamma > 1$ and $\psi > 1$). For example, with $\gamma = 8.5$ and $\psi = 2$, we have a FX-slope $= -1.13$.

Both special cases thus strongly suggest departures from log-normality do not fundamentally change the challenge of addressing the currency premium puzzle, as long as (symmetric) EZ preferences are involved.

We next show quantitatively that this intuition holds across some of the most influential models in the existing literature. To this end, we consider two highly successful contributions from the international finance literature, as well as a special case of the closed-economy model by Gourio (2012), and compare them to the data.[24] Closely related to our theoretical results, Gourio, Siemer, and Verdelhan (2013) features a model with

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[23]Cumulants are functions of the usual central moments.

[24]Barro (2009) is another closed-economy disaster model with EZ preferences. Given that our theoretical results only rely on the use of these preferences, his setup is also likely to be subject to the currency premium puzzle.

both EZ utility and heterogeneous exposures to global disaster risk.

Table 5: Carry Trade Returns under Disaster Models

	Return (%) $\mathbb{E}(rx^{ct})$	Change in FX (%) $-\mathbb{E}(\Delta s^{ct})$	Interest Rate Diff (%) $\mathbb{E}(r^{\star,ct} - r^{ct})$	FX-share $\frac{-\mathbb{E}(\Delta s^{ct})}{\mathbb{E}(rx^{ct})}$	FX-slope $-\frac{1}{\text{FX-share}}$	P1	P2
Data	4.95 [1.50,8.34]	-2.15 [-4.98,0.49]	7.11 [2.22,13.22]	-0.43 [-1.10,0.15]	2.30 [0.90, ∞)∪(−∞, -6.56]	-	-
Gourio, Siemer and Verdelhan (2013) JIE	2.36	1.81	0.55	0.77	-1.31	Yes	No
Gourio (2012) AER	-	-	-	1.00	-1.00	No	No
Farhi and Gabaix (2016) QJE (UN)	4.9	3.39	1.51	0.69	-1.44	Yes	No
Farhi and Gabaix (2016) QJE (ND)	-	-	-	0.75	-1.33	Yes	No

The return, FX-share, and FX-slope for Gourio, Siemer and Verdelhan (2013) are calculated from their Tables 2 and 4; return for Farhi and Gabaix (2016) is from their Table III, FX-shares and FX-slopes are calculated using their calibrations in Tables I and II, and their equations (24) and (25). UN stands for unconditional moments, and ND stands for moments conditional on no disaster in the sample. We also show the theoretical FX-share and FX-slope for a special case of the closed-economy model of Gourio (2012) for comparison, assuming an IES of 1 ($\psi = 1$). Alternatively, if we use Gourio's preferred calibration ($\psi = 2$, $\gamma = 3.8$) and use the approximation in equation (17), we end up with FX-share = 0.72 and FX-slope = -1.39. The last two columns summarize if the simulated results can match our empirical properties 1 and 2, respectively.

As Table 5 shows, the currency premium puzzle is also evident in these disaster models. The conclusions are the same as for long-run risk and habit models: with FX-shares above 50% and FX-slopes close to -1, the models do not match the composition of currency premia.

Perhaps more surprisingly, virtually identical results hold for Farhi and Gabaix (2016), who build a disaster model with constant relative risk aversion and tradeable and nontradeable goods. We list the unconditional results (UN) as well as the results conditional on no disaster happening in the sample (ND). Although the time-separable preferences in this model do not force a negative functional relationship between the mean and higher-order moments of the log SDF, this relationship appears to assert itself nevertheless when calibrating the model to reflect low and stable risk-free rates.

To summarize, the inability of standard models to simultaneously generate a high equity premium, a low and stable risk-free rate, and largely unpredictable exchange rates extends beyond the case of log-normality. In particular, existing disaster models are also subject to the currency premium puzzle.[25] That said, allowing multiple higher cumulants or moments to differ across countries at the same time may offer more degrees of freedom. How such a model should be constructed and what would drive these differences in cumulants is not obvious and requires further research.

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[25]Relatedly, Jurek (2014) and Farhi et al. (2009) find that even when disaster risk is hedged using options, carry trade returns remain large, suggesting disasters are not the sole explanation for systematic variation in currency returns.

5 Incomplete Spanning

Can market incompleteness help resolve the currency premium puzzle? In this section, we relax the assumption of complete markets and explore incomplete spanning as a potential avenue for matching the empirical patterns in interest rates and exchange rates. We find that this form of market incompleteness does not offer a clear-cut resolution to the currency premium puzzle.

Specifically, we consider a scenario where agents do not have full access to foreign financial markets: agents can buy and sell their own country's (domestic) risk-free assets, but cannot trade financial assets across borders. In this case, the expected change in the exchange rate is no longer fully determined by the ratio of expected SDFs across countries. In other words, (2) no longer holds. Following Lustig and Verdelhan (2019), we summarize incomplete spanning in international financial markets by a wedge, η_{t+1}, between changes in exchange rates and log SDFs so that [26]

$$\mathbb{E}(\Delta s_{t+1}) = \mathbb{E}(m_{t+1}) - \mathbb{E}(m^*_{t+1}) - \mathbb{E}(\eta_{t+1}). \tag{18}$$

(As before, all unconditional equations in this section also hold conditionally.)

$\mathbb{E}(m^\star_{t+1}) + \mathbb{E}(\eta_{t+1})$ is then the expectation of a foreign log SDF adjusted for the incomplete-market wedge. Plugging (1) and (18) into equation (4) yields the currency premium

$$\mathbb{E}(rx_{t+1}) = \frac{1}{2}\mathbb{E}(\mathrm{var}_t(m_{t+1})) - \left(\frac{1}{2}\mathrm{var}_t(m^*_{t+1})) - \mathbb{E}(\eta_{t+1})\right). \tag{19}$$

Equations (18) and (19) show the expected incomplete market wedge, $\mathbb{E}(\eta_{t+1})$, provides an extra degree of freedom. The question is whether this degree of freedom can help resolve the currency premium puzzle.

The answer is it likely cannot. To see why, recall that as long as agents can freely buy and sell their own country's risk-free asset, equation (1) holds, so that the wedge $\mathbb{E}(\eta_{t+1})$ affects currency returns and predicted depreciations in (18) and (19), but not the size of the interest rate differential. Even when spanning is incomplete, the interest differential thus depends only on the mean and variance of each country's log SDF,

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[26]Lustig and Verdelhan (2019) assume agents can trade all the risk-free assets across borders. For what follows, we only need the weaker assumption that each agent have access to her own domestic risk-free asset.

as before:

$$\mathbb{E}(r_t^\star - r_t) = \mathbb{E}(m_{t+1}) - \mathbb{E}(m_{t+1}^\star)$$
$$- \frac{1}{2} \mathbb{E}(\mathrm{var}_t(m_{t+1}^\star) - \mathrm{var}_t(m_{t+1})).$$

In this sense, incomplete spanning cannot address the core problem that all the canonical models we examined in the sections above force FX-slopes close to -1, and thus interest rate differentials that are far too small relative to the data.

Another way of seeing the same point is that modulating $\mathbb{E}(\eta_{t+1})$ for a given country pair allows movement away from the now familiar blue lines in SDF space, but only along the (negatively sloped) 45-degree line.[27] Figure 6, panel (a) shows this effect graphically, using as an example the same LRR model and parameters as in Figure 4 with $\gamma = 6.5$ and $\psi = 1.6$. As in our prior example, each dot on the blue line represents a country—indicating the mean and variance of its log SDF. The blue dots depict the situation under complete markets, where differences in countries' risk characteristics determine where on the blue line each country lies.

To the extent that the FX-Slope in the long run risk model is different from -1, that is, if $\psi \neq 1$, the wedge induced by incomplete spanning now allows movement off the blue line, but again only along a given iso-interest-rate line. That is, by altering a country i's expected wedge $\mathbb{E}(\eta_{t+1}^i)$, we can increase or decrease its currency's expected rate of depreciation, and the part of the currency return determined by it, but not the interest differential. Panel (a) of Figure 6 illustrates these possible movements along the (negative) 45 degree line intersecting at each point along the blue line.

In the best possible case, one might come up with a mechanism that generates an off-setting wedge for each country that is exactly large enough to render its exchange rate unpredictable, that is, $\mathbb{E}(\Delta s_{t+1}) = 0$. We plot this special case as red squares in Figure 6, panel (b). However, even if one had such a mechanism, as long as the FX-Slope under complete markets is close one (as in the canonical long-run risk and habit models), the same wedge would then again wipe out the vast majority of the expected currency return, leaving us with the same (small) interest rate differentials we obtained in the complete-markets model—the vertical distances between the red squares.

Two observations from this analysis stand out. First, as shown in Figure 6, panel (b), the incomplete

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[27]We are slightly abusing the notation in the sense that we fix the home country point $(\mathbb{E}(m), \mathbb{E}(\mathrm{var}_t(m)))$, and treat the foreign point as $(\mathbb{E}(m^\star) + \mathbb{E}(\eta), \mathbb{E}(\mathrm{var}_t(m^\star) - \mathbb{E}(\eta)))$, as implied by (18) and (19).

Figure 6: Incomplete Markets vs. Complete Markets
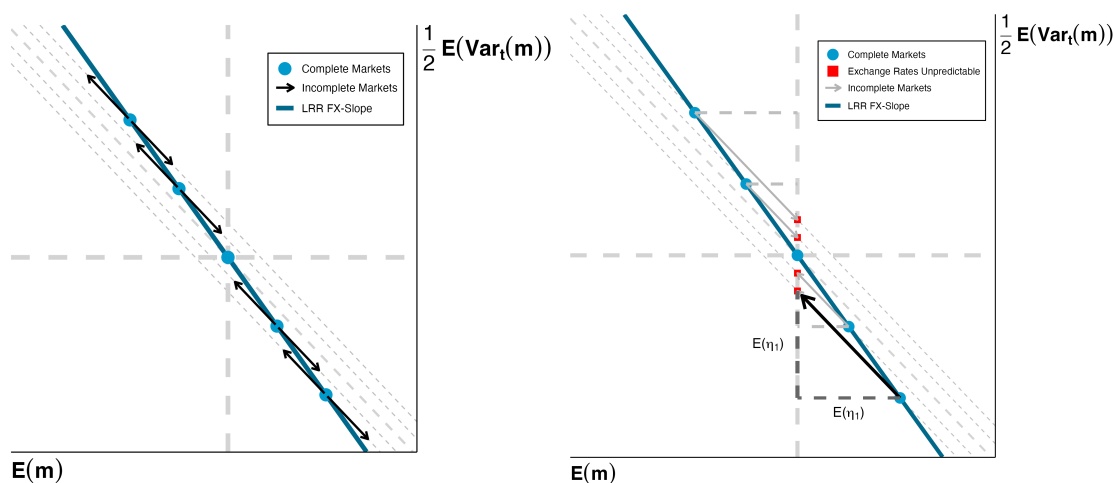
(a) Effect of Incomplete-market Wedge (b) Achieving Unpredictable Exchange Rates

This figure plots a long-run risk FX-slope (blue line) with five countries. The five countries must lie on the same LRR line under complete markets (blue dots). Panel (a) shows how the incomplete market wedge could move each country around in the SDF space; panel (b) shows what the incomplete market wedge of each country needs to do to achieve unpredictable exchange rates (red squares).

market wedges $\mathbb{E}(\eta^i)$ have to differ significantly across countries and would have to line up in an off-setting pattern to achieve exchange rate unpredictability across country pairs. What type of mechanism would bring about such a constellation of wedges so as to exactly render each country's exchange rate unpredictable is unclear.

Second, even if one were to achieve such an off-setting pattern, the expected wedge lowers exchange rate predictability and currency premia by the same amount (as implied by equations (18) and (19)). Consequently, incomplete spanning, in combination with wedges that reduce exchange rate predictability, make it more difficult for any model to satisfy empirical Property 1, which states that currency premia need to be large.

To illustrate these two observations quantitatively, Table 6 shows currency premia and exchange rate appreciations for five representative countries under both complete and incomplete markets, using country 3 (the middle country) as the base currency. The currency premium puzzle is again evident across all country pairs, with counterfactually small interest rate differences and large currency premia driven mostly by expected appreciations. As in Figure 6, the incomplete market wedge can be utilized to achieve unpredictable exchange rates as in the data. In particular, by setting $\mathbb{E}(\Delta s_{t+1})$ in (18) to 0, we can back out the required wedges to eliminate predictability from exchange rates for each of the country pairs. We report these implied wedges in the last column. Setting the incomplete wedge to these values improves the model's performance

Table 6: Implied Wedges in Canonical LRR Model

Country	Return (%)		Change in FX (%)		Interest Rate Diff (%)	Implied Wedge
	Complete	Incomplete	Complete	Incomplete		
1	2.91	0.16	-2.75	0.00	0.16	-2.75
2	1.52	0.08	-1.44	0.00	0.08	-1.44
3	0.00	0.00	0.00	0.00	0.00	0.00
4	-1.64	-0.07	1.57	0.00	-0.07	1.57
5	-3.41	-0.12	3.28	0.00	-0.12	3.28

This table shows currency premia, expected depreciation of the high-interest-rate currency, and interest rate differences for the five representative countries under both complete and incomplete markets. The currency of the middle, country 3, is used as the base currency. The implied incomplete-market wedge is set to equal differences in the means of log SDFs to match the fact that exchange rates are unpredictable.

in matching the composition of currency premia (Property 2), because all currency premia arise entirely from interest rate differences. The implied wedges strongly differ across country pairs.

As an example, country 1 (perhaps New Zealand) produces a currency premia of 2.91% relative to country 3 under complete markets. If we set the incomplete market wedge to -2.75% so that exchange rates are unpredictable, we end up with a currency premium of merely 0.16%.[28] In other words, whereas variation in the wedge across countries can ensure Property 2 holds, Property 1, which says currency premia are large, will fail in those cases.[29]

This argument applies more broadly. Recall that in all models we have considered, changes in exchange rates account for more than 50%—in many cases, for more than 90%—of currency premia. Fixing the composition of currency premia in these models using the incomplete spanning wedge implies a significant decrease in currency premia, weakening their ability to match the empirically large currency premia. This fundamental issue is again embedded in preferences chosen to keep risk-free rate stable by imposing a tight functional relationship between means and variances of log SDFs.

[28]Similarly, consider our simulation results in Table 2: the simulated difference in variances of log SDFs is 7.10%, whereas the difference in means is -5.98%. If we believe exchange rates are unpredictable as the data suggest and set $\mathbb{E}(\eta_{t+1}) = 5.98\%$, we can indeed generate a $\mathbb{E}(\Delta s_{t+1}) = 0$ and $\mathbb{E}(rx_{t+1}) = 1.12\%$. Now, all currency premia are accounted for by differences in risk-free rates: incomplete markets helps the long-run risk model of Colacito et al. (2018a) to match Property 2, that is, to get the right composition of currency risk premia. But it does so by shrinking the currency premia by more than 90%, significantly weakening the model's ability to match Property 1.

[29]In principle, one could design a long-run risk (or habit) model with even larger differences in the variance of SDFs and the right incomplete market wedge to match the data, but how such extreme heterogeneity can be justified, and how such strong market frictions can prevent exchange rates to move in such models, remain unclear.

6 Off-setting Differences in $\mathbb{E}(m)$

Finally, we explore a deeper synthesis between risk-based models, which have been the focus of the recent literature, and the traditional macroeconomic view of cyclical interest rate differentials.

The premise of risk-based models is that countries differ in their risk characteristics (var(m) in equation (4)) so that some countries' currencies (such as the US dollar and the Euro) are safer than others and pay lower returns to foreign investors. This risk-based view of currency returns has become the dominant paradigm in the literature based on two key empirical observations: First, interest rate differentials between countries are large and long-lasting, so that uncovered interest parity fails and investors can make money by borrowing in the low interest rate country and lending in the high interest rate country (Fama, 1984; Hassan and Mano, 2019). Second, there is clear evidence of risk factors in currency markets, where currencies with low interest rates tend to appreciate at times of stress in the global economy (Lustig and Verdelhan, 2007; Campbell, Serfaty-De Medeiros, and Viceira, 2010; Lustig, Roussanov, and Verdelhan, 2011; Lettau, Maggiori, and Weber, 2014).

As we have shown above, however, under canonical long-run risk and habit preferences, because of the hardwired functional relationship between $\mathbb{E}(m)$ and var(m), heterogeneity in risk characteristics forces risk premia to manifest as predictable appreciations (equation (2)), which contradicts the data.

One possible avenue to addressing the currency premium puzzle might thus be adding an additional source of heterogeneity that moves the conditional means of the log SDFs to off-set these predictable exchange rate movements.

In fact, a voluminous macroeconomic literature has focused on cyclical variation in interest rates based on short-run differences in the means of countries log SDFs ($\mathbb{E}(m)$'s are not equalized in equation (3)) (Backus, Kehoe, and Kydland, 1992; Galí and Monacelli, 2005). In this view, countries with temporarily high expected consumption growth or inflation have a temporarily high interest rate.

However, these models have their own problems – they require that any cyclical interest rate differentials are reversed by predictable depreciations of the high-interest-rate currency, so that uncovered interest parity holds. To see this, note that differences in $\mathbb{E}(m)$ affect interest rates in equation (3) but not expected returns (equation (4)).

Thus, heterogeneity in risk characteristics can rationalize differences in expected returns, whereas heterogeneity in $\mathbb{E}(m)$ cannot. Panel (a) of Figure 7 shows this contrast graphically. Differences in $\mathbb{E}(m)$ across countries temporarily move them along the horizontal arrows in the figure. While these movements induce

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Figure 7: Heterogeneous Growth vs Homogeneous Growth
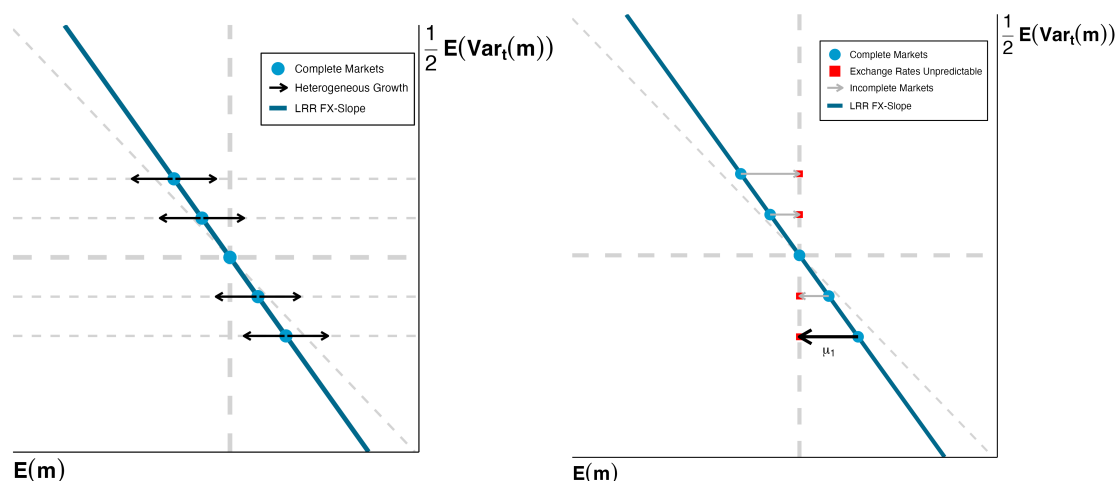
(a) Effect of Heterogeneous Growth   (b) Achieving Unpredictable Exchange Rates

This figure plots the FX-Slope from a long-run risk model (blue line) with five countries. The five countries must lie on the same blue line under complete markets (blue dots). Panel (a) shows how an additional source of heterogeneity in $\mathbb{E}(m)$ can move each country horizontally in the SDF space; panel (b) shows what the specific level of $\mathbb{E}(m)$ of each country needs to do to achieve unpredictable exchange rates (red squares).

additional variation in interest rates, this additional variation is fully reversed by predictable depreciations of the exchange rate.

Although the traditional macroeconomic forces driving differences in $\mathbb{E}(m)$, of course, operate in all of the quantitative models surveyed above, they tend to operate only at cyclical frequencies and do not affect currency premia so that they typically play only a minor role in rationalizing the size of long-term differences in currency returns.

One avenue for constructing a deeper synthesis between the two approaches may be to explore whether long-lasting differences in countries' risk characteristics might be offset by *long-lasting* (rather than merely cyclical) differences in the means of their log SDFs. If that were the case, one could construct models where heterogeneity in risk characteristics produces large differences in expected returns that transmit themselves through large expected appreciations of high-interest-rate currencies (as required by the canonical long-run risk and habit models), but are then transformed into large interest rate differentials by off-setting differences in the means of countries log SDFs: Countries with high var(m) would then also have persistently high $\mathbb{E}(m)$'s, so that the expected appreciation of the riskier country is offset by an off-setting expected depreciation due to its high $\mathbb{E}(m)$, as shown in Panel (b) of Figure 7.

Of course, one would have to propose some theory of why and how heterogeneity in the two moments

of the log SDF might be linked. Doing so might be challenging because long-run differences in the expected growth rate of consumption, and other forces controlling $\mathbb{E}(m)$, could then lead to issues with stationarity (e.g. one country "disappearing" in the long-run).

Nevertheless, two papers suggest avenues for doing so. Andrews et al. (2024) recently propose a partial equilibrium long-run risk model where risky countries (exogenously) have higher consumption growth rates and inflation for long periods of time. Although again their focus is on other features of the data, this link between long-lasting heterogeneity in the first and second moments of log SDFs reduces the FX-share of currency returns. For example, one version of their model with long-lasting differences in consumption growth and inflation generates returns to the static trade of 3.87%, 1.64 percentage points of which are from interest rate differentials, and an FX Share of 58%. Although this FX Share is still too high relative to the data, it is much closer than the 90+% in Table 2 (see Appendix Table 7 for details).[30]

Another approach might be to make differences in the expected log SDF unobservable in-sample, for example by postulating differences in countries' resilience to disasters. This possibility is hinted at in Farhi and Gabaix (2016). Again, more work is needed to determine the potential of this approach.

7   Conclusion

In this paper, we highlight a fundamental tension between canonical asset pricing models, which have been notably successful in explaining closed-economy puzzles, and the empirical observations in open economies. This tension, which we term the 'currency premium puzzle', manifests in the inability of these models to reconcile large and persistent interest rate differentials with the unpredictability of exchange rates.

We have demonstrated that in the context of an open economy with complete markets, canonical long-run risk and habit models generate currency risk premia that predominantly manifest as predictable exchange rate movements, rather than interest rate differentials. This theoretical prediction sharply contrasts with empirical observations, where exchange rates are notoriously difficult to predict and interest rate differentials are the primary source of differences in currency returns.

This failure is not mitigated by introducing stochastic volatility, by deviating from log-normal distributions, or by modifying the economic environment in other ways. Instead, it is inherent to the preference structures in these models: the same preference structures that reconcile large equity premia with low and stable risk-free rates in the context of a closed economy also require that the vast majority of any differences in

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[30]See Appendix A.3 for a detailed derivation and results for carry trade.

currency premia across countries must transmit themselves through predicted depreciations of the exchange rate—as long as markets are complete.

This leaves loosening the complete markets paradigm as an important avenue to explore. Predominantly, the existing literature in this area presupposes the completeness of financial markets, so that more research is needed to assess to what extent deviations from this standard could contribute to resolving the currency premium puzzle in general.

Our own exploration has focused on one specific type of market incompleteness—incomplete spanning. We showed this form of market imperfection does not offer a straightforward solution. Incomplete spanning tends to simultaneously diminish the predictability of exchange rates and the magnitude of currency returns, thus falling short in explaining the significant interest rate differentials observed in the data. This finding underscores that although moving away from the assumption of complete markets is a logical direction for future research, it is far from being a straightforward solution.

The implications of our findings are two-fold. First, they underscore a significant limitation of the current generation of asset pricing models when applied to open economies. Second, the currency premium puzzle highlights the necessity for new models that can simultaneously account for large interest rate differentials and the unpredictability of exchange rates. This need is not merely academic; it is a crucial step in building models that can assess the effect of variation in global and local risk premia on capital flows and the allocation of capital across countries.

By pinpointing a crucial inconsistency between canonical asset pricing and international macroeconomic models, we hope to spur more work on the broader challenge of integrating these two areas. In our view, this integration is essential for developing a more comprehensive understanding of critical phenomena, including the violation of uncovered interest parity, contagion, the global financial cycle, flights to safety, capital retrenchments, and sudden stops. All these phenomena ultimately result from the interplay of international financial markets, risk premia, and allocations. They are critical to understand. The currency premium puzzle, as we have defined it, calls for innovative approaches to address this challenge.

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A   Details on the LRR Models

A.1   Derivation of moments of log SDF

Start from EZ preferences.

$$
U_t = \left((1-\delta)C_t^{1-\frac{1}{\psi}} + \delta \left\{ \mathbb{E}_t \left[U_{t+1}^{1-\gamma} \right] \right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right)^{\frac{1}{1-\frac{1}{\psi}}}. \tag{A1}
$$

SDF is given by

$$
\begin{aligned}
M_{t+1} &= \frac{\frac{\partial U_t}{\partial C_{t+1}}}{\frac{\partial U_t}{\partial C_t}} \\
&= \frac{\frac{\partial U_t}{\partial U_{t+1}} \frac{\partial U_{t+1}}{\partial C_{t+1}}}{\frac{\partial U_t}{\partial C_t}}.
\end{aligned}
$$

Note

$$
\begin{aligned}
\frac{\partial U_t}{\partial U_{t+1}} &= \delta \left((1-\delta)C_t^{1-\frac{1}{\psi}} + \delta \left\{ \mathbb{E}_t \left[U_{t+1}^{1-\gamma} \right] \right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right)^{\frac{\frac{1}{\psi}}{1-\frac{1}{\psi}}} \left\{ \mathbb{E}_t \left[U_{t+1}^{1-\gamma} \right] \right\}^{\frac{\gamma-\frac{1}{\psi}}{1-\gamma}} U_{t+1}^{-\gamma} \\
&= \delta U_t^{\frac{1}{\psi}} \left\{ \mathbb{E}_t \left[U_{t+1}^{1-\gamma} \right] \right\}^{\frac{\gamma-\frac{1}{\psi}}{1-\gamma}} U_{t+1}^{-\gamma} \\
\frac{\partial U_t}{\partial C_t} &= (1-\delta) \left((1-\delta)C_t^{1-\frac{1}{\psi}} + \delta \left\{ \mathbb{E}_t \left[U_{t+1}^{1-\gamma} \right] \right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right)^{\frac{\frac{1}{\psi}}{1-\frac{1}{\psi}}} C_t^{-\frac{1}{\psi}} \\
&= (1-\delta) U_t^{\frac{1}{\psi}} C_t^{-\frac{1}{\psi}}.
\end{aligned}
$$

We can then easily see that

$$
M_{t+1} = \delta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \left(\frac{U_{t+1}}{\left\{ \mathbb{E}_t \left[U_{t+1}^{1-\gamma} \right] \right\}^{\frac{1}{1-\gamma}}} \right)^{\frac{1}{\psi}-\gamma}.
$$

Taking logs on both side, we have the log SDF as

$$m_{t+1} = \log(\delta) - \frac{1}{\psi}\Delta c_{t+1} + \left(\frac{1}{\psi} - \gamma\right)\left(u_{t+1} - \frac{1}{1-\gamma}\log(\mathbb{E}_t\left[\exp((1-\gamma)u_{t+1})\right])\right).$$

A.1.1 Model without short-run shocks

Now assume

$$\Delta c_{t+1} = \mu + z_t \tag{A2}$$

$$z_t = \rho z_{t-1} + \sigma\varepsilon_t. \tag{A3}$$

Assumption 1. *Assume u_{t+1} is normal.*

We can easily see that now

$$\mathbb{E}_t(m_{t+1}) = \log(\delta) - \frac{1}{\psi}\mu - \frac{1}{\psi}z_t - \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)(1-\gamma)\operatorname{var}_t(u_{t+1})$$

$$\frac{1}{2}\operatorname{var}_t(m_{t+1}) = \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)^2\operatorname{var}_t(u_{t+1}).$$

In particular, if we look at unconditional moments,

$$\mathbb{E}(m_{t+1}) = \log(\delta) - \frac{1}{\psi}\mu - \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)(1-\gamma)\mathbb{E}(\operatorname{var}_t(u_{t+1})) \tag{A4}$$

$$\frac{1}{2}\mathbb{E}(\operatorname{var}_t(m_{t+1})) = \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)^2\mathbb{E}(\operatorname{var}_t(u_{t+1})). \tag{A5}$$

A.1.2 Model with short-run shocks

If we instead assume

$$\Delta c_{t+1} = \mu + z_t + \varepsilon_{SR,t+1},$$

$$z_t = \rho z_{t-1} + \sigma\varepsilon_t.$$

Now calculating the second moment is a bit tricky. To simplify things, we define $V_t = \frac{U_t}{C_t}$. Now we have

$$V_t = \left((1-\delta) + \delta \left\{ \mathbb{E}_t \left[\left(\frac{U_{t+1}}{C_t} \right)^{1-\gamma} \right] \right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right)^{\frac{1}{1-\frac{1}{\psi}}}$$

$$= \left((1-\delta) + \delta \left\{ \mathbb{E}_t \left[\left(V_{t+1} \frac{C_{t+1}}{C_t} \right)^{1-\gamma} \right] \right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right)^{\frac{1}{1-\frac{1}{\psi}}}.$$

The nice thing about this transformation is that the conditional covariance of v_{t+1} and Δc_{t+1} is now 0. To see this, suppose after some terminal date T, all shocks are 0 and $z_T = 0$. Then, we have

$$v_T = \frac{1}{1-\frac{1}{\psi}} \log \left((1-\delta) + \delta \left\{ \mathbb{E}_T \left[\exp(1-\gamma)(v_{T+1} + \Delta c_{T+1}) \right] \right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right)$$

$$= \frac{1}{1-\frac{1}{\psi}} \log \left((1-\delta) + \delta \left\{ \mathbb{E}_T \left[\exp(1-\gamma)(v_{T+1} + \mu) \right] \right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right).$$

Obviously, $v_T = v_{T+1}$, so we can solve for v_T as a constant. At $T-1$, we have

$$v_{T-1} = \frac{1}{1-\frac{1}{\psi}} \log \left((1-\delta) + \delta \left\{ \mathbb{E}_{T-1} \left[\exp(1-\gamma)(v_T + \Delta c_T) \right] \right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right)$$

$$= \frac{1}{1-\frac{1}{\psi}} \log \left((1-\delta) + \delta \left\{ \mathbb{E}_{T-1} \left[\exp(1-\gamma)(v_T + \mu + z_{T-1} + \varepsilon_{SR,T}) \right] \right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right).$$

Note $\mathbb{E}_{T-1}\left[\exp(1-\gamma)(v_T + \mu + z_{T-1} + \varepsilon_{SR,T}) \right]$ is a function of z_{T-1} only. So, v_{T-1} is a function of z_{T-1} only. Similarly,

$$v_{T-2} = \frac{1}{1-\frac{1}{\psi}} \log \left((1-\delta) + \delta \left\{ \mathbb{E}_{T-2} \left[\exp(1-\gamma)(v_{T-1} + \Delta c_{T-1}) \right] \right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right)$$

$$= \frac{1}{1-\frac{1}{\psi}} \log \left((1-\delta) + \delta \left\{ \mathbb{E}_{T-2} \left[\exp(1-\gamma)(v_{T-1} + \mu + z_{T-2} + \varepsilon_{SR,T-1}) \right] \right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right).$$

One can see v_{T-2} is also a function of z_{T-2} only. Using backward induction, we can conclude that for any t, v_{t+1} is a function of z_{t+1} only, and thus,

$$\text{cov}_t(v_{t+1}, \Delta c_{t+1}) = 0.$$

The reason is simple, v_{t+1} depends on $z_{t+1} = \rho z_t + \sigma \varepsilon_{t+1}$ only and $\Delta c_{t+1} = \mu + z_t + \varepsilon_{SR,t+1}$. Given z_t, because $\text{cov}_t(\varepsilon_{t+1}, \varepsilon_{SR,t+1}) = 0$, obviously, $\text{cov}_t(v_{t+1}, \Delta c_{t+1}) = 0$.

Now let's revisit our SDF. Our SDF has the following form:

$$
\begin{aligned}
M_{t+1} &= \delta \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}} \left(\frac{U_{t+1}}{\left\{\mathbb{E}_t\left[U_{t+1}^{1-\gamma}\right]\right\}^{\frac{1}{1-\gamma}}}\right)^{\frac{1}{\psi}-\gamma} \\
&= \delta \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}} \left(\frac{V_{t+1}\frac{C_{t+1}}{C_t}}{\left\{\mathbb{E}_t\left[\left(V_{t+1}\frac{C_{t+1}}{C_t}\right)^{1-\gamma}\right]\right\}^{\frac{1}{1-\gamma}}}\right)^{\frac{1}{\psi}-\gamma}.
\end{aligned}
$$

Taking logs, we have

$$m_{t+1} = \log(\delta) - \frac{1}{\psi}\Delta c_{t+1} + \left(\frac{1}{\psi} - \gamma\right)\left(v_{t+1} + \Delta c_{t+1} - \frac{1}{1-\gamma}\log\mathbb{E}_t\left[\exp((1-\gamma)(v_{t+1} + \Delta c_{t+1}))\right]\right).$$

Now, assuming v_{t+1} is log-normal, we have

$$
\begin{aligned}
\mathbb{E}_t(m_{t+1}) &= \log(\delta) - \frac{1}{\psi}\mathbb{E}_t(\Delta c_{t+1}) \\
&\quad + \left(\frac{1}{\psi} - \gamma\right)\left(\mathbb{E}_t(v_{t+1}) + \mathbb{E}_t(\Delta c_{t+1}) - \mathbb{E}_t(v_{t+1}) - \mathbb{E}_t(\Delta c_{t+1}) - \frac{1}{2}(1-\gamma)\text{var}_t(v_{t+1} + \Delta c_{t+1})\right) \\
&= \log(\delta) - \frac{1}{\psi}\mathbb{E}_t(\Delta c_{t+1}) - \frac{1}{2}(1-\gamma)\left(\frac{1}{\psi} - \gamma\right)(\text{var}_t(v_{t+1} + \Delta c_{t+1}).)
\end{aligned}
$$

And the conditional variance of the log SDF is given by

$$\frac{1}{2}\operatorname{var}_t(m_{t+1}) = \frac{1}{2}\left(\frac{1}{\psi}\right)^2 \operatorname{var}_t(\Delta c_{t+1}) + \frac{1}{2}\left(\frac{1}{\psi}-\gamma\right)^2 \operatorname{var}_t(v_{t+1}+\Delta c_{t+1}) - \frac{1}{\psi}\left(\frac{1}{\psi}-\gamma\right) cov_t(\Delta c_{t+1}, v_{t+1}+\Delta c_{t+1})$$

$$= \frac{1}{2}\left(\frac{1}{\psi}\right)^2 \operatorname{var}_t(\Delta c_{t+1}) + \frac{1}{2}\left(\frac{1}{\psi}-\gamma\right)^2 \operatorname{var}_t(v_{t+1}+\Delta c_{t+1}) - \frac{1}{\psi}\left(\frac{1}{\psi}-\gamma\right) \operatorname{var}_t(\Delta c_{t+1}).$$

The second equality uses the fact that $cov_t(v_{t+1}, \Delta c_{t+1}) = 0$. Now, substituting in our processes, we have

$$\mathbb{E}_t(m_{t+1}) = \log(\delta) - \frac{1}{\psi}\mathbb{E}_t(\mu + z_t) - \frac{1}{2}(1-\gamma)\left(\frac{1}{\psi}-\gamma\right)(\operatorname{var}_t(v_{t+1}+\Delta c_{t+1}))$$

$$\frac{1}{2}\operatorname{var}_t(m_{t+1}) = \frac{1}{2}\left(\frac{1}{\psi}\right)^2 \sigma_{SR}^2 + \frac{1}{2}\left(\frac{1}{\psi}-\gamma\right)^2 \operatorname{var}_t(v_{t+1}+\Delta c_{t+1}) - \frac{1}{\psi}\left(\frac{1}{\psi}-\gamma\right)\sigma_{SR}^2.$$

Unconditionally,[31]

$$\mathbb{E}(m_{t+1}) = \log(\delta) - \frac{1}{\psi}\mu - \frac{1}{2}(1-\gamma)\left(\frac{1}{\psi}-\gamma\right)\mathbb{E}(\operatorname{var}_t(v_{t+1}+\Delta c_{t+1}))$$

$$\frac{1}{2}\mathbb{E}(\operatorname{var}_t(m_{t+1})) = \frac{1}{2}\left(\frac{1}{\psi}\right)^2 \sigma_{SR}^2 + \frac{1}{2}\left(\frac{1}{\psi}-\gamma\right)^2 \mathbb{E}(\operatorname{var}_t(v_{t+1}+\Delta c_{t+1})) - \frac{1}{\psi}\left(\frac{1}{\psi}-\gamma\right)\sigma_{SR}^2.$$

All short-run shocks do is that it adds a constant. If we don't allow short-run shock volatilities to differ across countries, nothing changes; if we do allow them to differ across countries, the currency premia they can generate is given by

$$\frac{1}{\psi}\left(-\frac{1}{2}\frac{1}{\psi}+\gamma\right)(\sigma_{SR}^2 - (\sigma_{SR}^\star)^2).$$

Consumption growth volatility is low, as is the difference across countries, so this term is quantitatively not important.

A.2 Long-run risk model with time-varying volatility

A.2.1 Main results

In this section, we explore if adding time variation in the second moments might help in mitigating our currency premium puzzle. We abstracted from this mechanism in our discussion in section 2. Following

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[31]Note we have left $\operatorname{var}_t(v_{t+1} + \Delta c_{t+1})$ as a whole although further simplification could be made. The reason is to highlight the fact that this term as a whole is the same in the mean and the variance of the EZ SDFs and thus can be cancelled when calculating the EZ line.

Bansal and Yaron (2004), long-run risk models often feature stochastic volatility to generate time-varying risk premia. We find the puzzle is deeply linked to the structure of EZ preferences and changing the endowment process to incorporate second moment shocks matters little for the FX-share or the FX-slope.

With time-varying volatility, the endowment process for the long-run risk model is given by

$$\Delta c_{t+1} = \mu + z_t + \sigma \varepsilon_{SR,t+1}$$

$$z_t = \rho z_{t-1} + w_{t-1} \varepsilon_{LR,t}$$

$$w_t^2 = (1 - \phi) w_0^2 + \phi w_{t-1}^2 + \sigma_w \varepsilon_{w,t},$$

where w_t^2 is the time-varying volatility and ϕ governs its persistence and σ_w its volatility. The remaining setup is identical to the model in section 2. That is, we continue to abstract from short-run shocks so that $\sigma = 0$.

When including stochastic volatility, a closed-form solution is only available for approximations of the model. Using a log-linearization with risk adjustments to solve the model, we show [32]

$$\mathbb{E}(\hat{m}_{t+1}) = -\frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)(1 - \gamma)(A_{vw}^2 \sigma_w^2 + A_{vz}^2 w_0^2) \tag{A6}$$

$$\frac{1}{2}\mathbb{E}(\text{var}_t(\hat{m}_{t+1})) = \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)^2 (A_{vw}^2 \sigma_w^2 + A_{vz}^2 w_0^2). \tag{A7}$$

The various coefficients labeled A are constants and the variables with a hat denote deviations from the deterministic steady state. Substituting out the right-hand side again shows the tight link between first and second moments of the log SDFs:

$$\frac{1}{2}\mathbb{E}(\text{var}_t(\hat{m}_{t+1})) = -\frac{\frac{1}{\psi} - \gamma}{1 - \gamma}\mathbb{E}(\hat{m}_{t+1}).$$

Adding stochastic volatility to the model does not help resolve the currency premium puzzle. In fact, this link between the moments is identical to the case without time-varying volatility (see equation (10)). The reason is that the relationship between the first and second moments of the log SDF is deeply embedded in the EZ preferences and determined only by preference parameters. Changing the structure of the endowment process then matters little in terms of breaking the link.

---

[32]See a detailed proof in section A.2.2.

A.2.2 Detailed derivation

Now we solve the model using risk-adjusted affine approximation.

Dividing both sides of the EZ preference by C_t and defining $V_t = U_t/C_t$, we have

$$\exp\left(\left(1 - \frac{1}{\psi}\right)v_t\right) = 1 - \delta + \delta \exp\left(\left(1 - \frac{1}{\psi}\right)q_t\right)$$

$$q_t = \frac{1}{1 - \gamma} \log\left\{\mathbb{E}_t\left[(\exp(v_{t+1} + \Delta c_{t+1}))^{1-\gamma}\right]\right\}$$

$$m_{t+1} = \log(\delta) - \frac{1}{\psi}\Delta c_{t+1} + \left(\frac{1}{\psi} - \gamma\right)(v_{t+1} - q_t + \Delta c_{t+1}).$$

Log-linearizing the system around the deterministic steady state, we have

$$\Delta \hat{c}_{t+1} = z_t + \sigma_{SR}\varepsilon_{SR,t+1}$$

$$z_t = \rho z_{t-1} + w_{t-1}\varepsilon_{LR,t}$$

$$w_t^2 = (1 - \phi)w_0^2 + \phi w_{t-1}^2 + \sigma_w \varepsilon_{w,t}$$

$$\hat{v}_t = \delta \exp\left(\left(1 - \frac{1}{\psi}\right)\mu\right)\hat{q}_t$$

$$\hat{m}_{t+1} = -\frac{1}{\psi}\Delta \hat{c}_{t+1} + \left(\frac{1}{\psi} - \gamma\right)(\hat{v}_{t+1} - \hat{q}_t + \Delta \hat{c}_{t+1}.)$$

We leave the equation with an expectation operator as it is:

$$\hat{q}_t = \frac{1}{1 - \gamma} \log\left\{\mathbb{E}_t\left[(\exp(\hat{v}_{t+1} + \Delta \hat{c}_{t+1}))^{1-\gamma}\right]\right\}.$$

Now guess

$$\hat{v}_t = A_{v0} + A_{vz}z_t + A_{vw}w_t^2.$$

We plug it into the equation immediately above and obtain

$$\frac{1}{\delta \exp\left(\left(1 - \frac{1}{\psi}\right)\mu\right)}\left(A_{v0} + A_{vz}z_t + A_{vw}\hat{w}_t^2\right) = \frac{1}{1-\gamma}\log\left\{\mathbb{E}_t\left[\left(\exp(A_{v0} + A_{vz}z_{t+1} + A_{vw}\hat{w}_{t+1}^2 + z_t + \sigma_{SR}\varepsilon_{SR,t+1})\right)^{1-\gamma}\right]\right\}$$

$$= \frac{1}{1-\gamma}\log\left\{\mathbb{E}_t\left[\left(\exp(A_{v0} + A_{vz}\rho z_t + A_{vz}\hat{w}_t\varepsilon_{LR,t+1})\right)^{1-\gamma}\right]\right\}$$

$$+ \frac{1}{1-\gamma}\log\left\{\mathbb{E}_t\left[\left(\exp(A_{vw}(1-\phi)w_0^2 + A_{vw}\phi w_t^2 + A_{vw}\sigma_w\varepsilon_{w,t+1})\right)^{1-\gamma}\right]\right\}$$

$$+ \frac{1}{1-\gamma}\log\left\{\mathbb{E}_t\left[\left(\exp(z_t + \sigma_{SR}\varepsilon_{SR,t+1})\right)^{1-\gamma}\right]\right\}$$

$$= A_{v0} + A_{vz}\rho z_t + \frac{1}{2}(1-\gamma)A_{vz}^2 w_t^2$$

$$+ A_{vw}(1-\phi)w_0^2 + A_{vw}\phi w_t^2 + \frac{1}{2}(1-\gamma)A_{vw}^2\sigma_w^2$$

$$+ z_t + \frac{1}{2}(1-\gamma)\sigma_{SR}^2$$

$$= A_{v0} + A_{vw}(1-\phi)w_0^2 + \frac{1}{2}(1-\gamma)A_{vw}^2\sigma_w^2 + \frac{1}{2}(1-\gamma)\sigma_{SR}^2$$

$$+ (1 + A_{vz}\rho)z_t + \left(\frac{1}{2}(1-\gamma)A_{vz}^2 + A_{vw}\phi\right)w_t^2.$$

Matching coefficients, we have

$$\frac{1}{\delta \exp\left(\left(1 - \frac{1}{\psi}\right)\mu\right)}A_{v0} = A_{v0} + A_{vw}(1-\phi)w_0^2 + \frac{1}{2}(1-\gamma)A_{vw}^2\sigma_w^2 + \frac{1}{2}(1-\gamma)\sigma_{SR}^2$$

$$\frac{1}{\delta \exp\left(\left(1 - \frac{1}{\psi}\right)\mu\right)}A_{vz} = 1 + A_{vz}\rho$$

$$\frac{1}{\delta \exp\left(\left(1 - \frac{1}{\psi}\right)\mu\right)}A_{vw} = \frac{1}{2}(1-\gamma)A_{vz}^2 + A_{vw}\phi.$$

We can easily solve for all three coefficients:

$$A_{vz} = \frac{1}{\frac{1}{\delta \exp\left(\left(1-\frac{1}{\psi}\right)\mu\right)} - \rho}$$

$$A_{vw} = \frac{\frac{1}{2}(1-\gamma)A_{vz}^2}{\frac{1}{\delta \exp\left(\left(1-\frac{1}{\psi}\right)\mu\right)} - \phi}$$

$$A_{v0} = \frac{A_{vw}(1-\phi)w_0^2 + \frac{1}{2}(1-\gamma)A_{vw}^2\sigma_w^2 + \frac{1}{2}(1-\gamma)\sigma_{SR}^2}{\frac{1}{\delta \exp\left(\left(1-\frac{1}{\psi}\right)\mu\right)} - 1}.$$

We can then solve for the pricing kernel:

$$
\hat{m}_{t+1} = -\frac{1}{\psi}\Delta\hat{c}_{t+1} + \left(\frac{1}{\psi} - \gamma\right)\left(\hat{v}_{t+1} - \hat{q}_t + \Delta\hat{c}_{t+1}\right)
$$

$$
= -\gamma\Delta\hat{c}_{t+1} + \left(\frac{1}{\psi} - \gamma\right)\left(\hat{v}_{t+1} - \hat{q}_t\right)
$$

$$
= -\gamma(z_t + \sigma_{SR}\varepsilon_{SR,t+1}) + \left(\frac{1}{\psi} - \gamma\right)\left(\hat{v}_{t+1} - \frac{1}{\delta\exp\left(\left(1 - \frac{1}{\psi}\right)\mu\right)}\hat{v}_t\right)
$$

$$
= -\gamma(z_t + \sigma_{SR}\varepsilon_{SR,t+1}) + \left(\frac{1}{\psi} - \gamma\right)\left(A_{v0} + A_{vz}(\rho z_t + w_t\varepsilon_{LR,t+1}) + A_{vw}((1-\phi)w_0^2 + \phi w_t^2 + \sigma_w\varepsilon_{w,t+1})\right)
$$

$$
- \left(\frac{1}{\psi} - \gamma\right)\left(\frac{1}{\delta\exp\left(\left(1 - \frac{1}{\psi}\right)\mu\right)}(A_{v0} + A_{vz}z_t + A_{vw}w_t^2)\right)
$$

$$
= \left(\frac{1}{\psi} - \gamma\right)\left(A_{v0} + A_{vw}(1-\phi)w_0^2 - \frac{1}{\delta\exp\left(\left(1 - \frac{1}{\psi}\right)\mu\right)}A_{v0}\right)
$$

$$
+ \left(-\gamma + \left(\frac{1}{\psi} - \gamma\right)\left(A_{vz}\rho - \frac{1}{\delta\exp\left(\left(1 - \frac{1}{\psi}\right)\mu\right)}A_{vz}\right)\right)z_t
$$

$$
+ \left(\frac{1}{\psi} - \gamma\right)\left(A_{vw}\phi - \frac{1}{\delta\exp\left(\left(1 - \frac{1}{\psi}\right)\mu\right)}A_{vw}\right)w_t^2 + \left(\frac{1}{\psi} - \gamma\right)A_{vz}w_t\varepsilon_{LR,t+1}
$$

$$
+ \left(-\gamma\sigma_{SR}\varepsilon_{SR,t+1} + \left(\frac{1}{\psi} - \gamma\right)(A_{vw}\sigma_w\varepsilon_{w,t+1})\right)
$$

$$
= -\left(\frac{1}{\psi} - \gamma\right)\left(\frac{1}{2}(1-\gamma)A_{vw}^2\sigma_w^2 + \frac{1}{2}(1-\gamma)\sigma_{SR}^2\right)
$$

$$
- \frac{1}{\psi}z_t + \left(\frac{1}{\psi} - \gamma\right)\left(-\frac{1}{2}(1-\gamma)A_{vz}^2\right)w_t^2 + \left(\frac{1}{\psi} - \gamma\right)A_{vz}w_t\varepsilon_{LR,t+1}
$$

$$
+ \left(-\gamma\sigma_{SR}\varepsilon_{SR,t+1} + \left(\frac{1}{\psi} - \gamma\right)(A_{vw}\sigma_w\varepsilon_{w,t+1})\right).
$$

And we have the conditional mean of the log SDF given by

$$
\mathbb{E}_t(\hat{m}_{t+1}) = -\frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)(1-\gamma)A_{vw}^2\sigma_w^2 - \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)(1-\gamma)\sigma_{SR}^2 - \frac{1}{\psi}z_t - \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)(1-\gamma)A_{vz}^2 w_t^2.
$$

And the conditional variance of the log SDF given by

$$\frac{1}{2}\operatorname{var}_t(\hat{m}_{t+1}) = \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)^2 A_{vz}^2 w_t^2 + \frac{1}{2}\gamma^2\sigma_{SR}^2 + \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)^2 A_{vw}^2\sigma_w^2.$$

Unconditionally,

$$\mathbb{E}(\hat{m}_{t+1}) = -\frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)(1-\gamma)A_{vw}^2\sigma_w^2 - \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)(1-\gamma)\sigma_{SR}^2 - \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)(1-\gamma)A_{vz}^2 w_0^2$$

$$\frac{1}{2}\mathbb{E}(\operatorname{var}_t(\hat{m}_{t+1})) = \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)^2 A_{vz}^2 w_0^2 + \frac{1}{2}\gamma^2\sigma_{SR}^2 + \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)^2 A_{vw}^2\sigma_w^2$$

$$= \frac{1}{\psi}\left(-\frac{1}{2}\frac{1}{\psi} + \gamma\right)\sigma_{SR}^2 - \frac{\frac{1}{\psi} - \gamma}{1 - \gamma}\mathbb{E}(\hat{m}_{t+1}).$$

Setting $\sigma_{SR} = 0$ yields (A6) and (A7).

A.3 Long-run Risk Models with Heterogeneous Growth and Inflation

In this subsection, we solve and simulate the model of Andrews et al. (2024), where countries differ in two ways: they feature heterogeneous loadings on global growth news as well as loadings on global inflation news. Moreover, in their framework, they assume that country-specific growth rates and inflation are mechanically linked to these loadings to match the empirical fact that high-loading countries feature low interest rates and high inflation.

Agents feature the following preference

$$U_{i,t} = (1 - \delta)\log C_{i,t} + \delta\theta\log\mathbb{E}_t\, e^{\frac{U_{i,t+1}}{\theta}}$$

where $\theta = \frac{1}{1-\gamma}$. The elasticity of substitution $\psi = 1$. Absence of any differences in growth rates or inflation, this preference alone means all countries would then share the same unconditional risk-free rates (See the theoretical results in Section 2.

Table 7: ACCG Results: Static Trade

	Return (%) $\mathbb{E}(rx^{st})$	Change in FX (%) $-\mathbb{E}(\Delta s^{st})$	Interest Rate Diff (%) $\mathbb{E}(r^{\star,st} - r^{st})$	FX-share $\frac{-\mathbb{E}(\Delta s^{st})}{\mathbb{E}(rx^{st})}$	FX-slope $-\frac{1}{\text{FX-share}}$	P1	P2
Data	3.46 [1.18,5.54]	-1.30 [-3.82,0.60]	4.76 [1.30,8.46]	-0.37 [-1.16,0.23]	2.67 [0.86, ∞)∪ (−∞, -4.42]	-	-
Real Simulation	5.38	4.50	0.88	0.84	-1.20	Yes	No
Nominal Simulation	3.64	2.38	1.25	0.66	-1.53	Yes	No
Real: Theory	4.38	3.46	0.92	0.79	-1.27	Yes	No
Nominal: Theory	3.87	2.23	1.64	0.58	-1.73	Yes	No
Nominal Sim: EW	2.75	1.54	1.21	0.65	-1.78	Yes	No

This table shows the static trade returns for the model in Andrews et al. (2024), using calibrations in their Tables 6 and F.1. For the simulation results, shocks are chosen so that $x_{c,0} = x_{\pi,0} = 0.13\%$ and $x_{c,t^\star} = x_{\pi,t^\star} = -0.54\%$, as in the paper. We simulate 1000 different samples with 100 periods, with $t^\star = 51$. Real Simulation shows the results for real variables, Nominal Simulation for nominal variables. We also report the real and nominal theoretical returns. Portfolios are constructed in the same way as our data (see Appendix C). In the last row, we report the return on the equal-weighted portfolio (top 3 minus bottom 3).

There's a long-run component in both consumption and inflation, and we have

$$\begin{bmatrix} x_{\pi,t} \\ x_{c,t} \end{bmatrix} = \begin{bmatrix} \rho_\pi & 0 \\ \rho_{c\pi} & \rho_c \end{bmatrix} \cdot \begin{bmatrix} x_{\pi,t-1} \\ x_{c,t-1} \end{bmatrix} + \begin{bmatrix} \sigma_{x,\pi} & 0 \\ 0 & \sigma_{x,c} \end{bmatrix} \begin{bmatrix} \varepsilon_{\pi_t} \\ \varepsilon_{c,t} \end{bmatrix}$$

Consumption and inflation follow

$$\Delta c_{i,t+1} = \mu_c^i + \beta_i^c x_{c,t} + \sigma_c \eta_{i,t+1}^c$$

$$\pi_{i,t+1} = \mu_\pi^i + \beta_i^\pi x_{\pi,t} + \sigma_\pi \eta_{i,t+1}^\pi$$

All shocks are i.i.d. normal. Country-specific mean growth rates and inflation rates

$$\mu_c^i = \bar{\mu}_c + \bar{\mu}_c(1 - \beta_c^i)$$

$$\mu_\pi^i = \bar{\mu}_\pi - \bar{\mu}_\pi(1 - \beta_\pi^i)$$

Table 8: ACCG Results: Carry Trade

	Return (%) $\mathbb{E}(rx^{ct})$	Change in FX (%) $-\mathbb{E}(\Delta s^{ct})$	Interest Rate Diff (%) $\mathbb{E}(r^{\star,ct}-r^{ct})$	FX-share $\frac{-\mathbb{E}(\Delta s^{ct})}{\mathbb{E}(rx^{ct})}$	FX-slope $-\frac{1}{\text{FX-share}}$	P1	P2
Data	4.95 [1.50,8.34]	-2.15 [-4.98,0.49]	7.11 [2.22,13.22]	-0.43 [-1.10,0.15]	2.30 [0.90, ∞)∪ (−∞, -6.56]	-	-
Real Simulation	5.29	4.41	0.89	0.83	-1.20	Yes	No
Nominal Simulation	3.53	1.68	1.85	0.48	-2.10	Yes	No
Nominal Sim: EW	2.98	1.14	1.85	0.38	-2.62	Yes	No

This table shows the carry trade returns for the model in Andrews et al. (2024), using calibrations in their Tables 6 and F.1. For the simulation results, shocks are chosen so that $x_{c,0} = x_{\pi,0} = 0.13\%$ and $x_{c,t^\star} = x_{\pi,t^\star} = -0.54\%$, as in the paper. We simulate 1000 different samples with 100 periods, with $t^\star = 51$. Real Simulation shows the results for real variables, Nominal Simulation for nominal variables. Portfolios are constructed in the same way as our data (see Appendix C). In the last row, we report the return on the equal-weighted portfolio (top 3 minus bottom 3).

As we can clearly see, growth rates are hardwired to be linked to the loadings.

Real SDF is then given by

$$m_{i,t+1} = \bar{m}_i^{real} - \beta_c^i x_{c,t} - k_{\varepsilon c}^i \sigma_{x,c} \varepsilon_{c,t+1} + k_{\varepsilon \pi}^i \sigma x, \pi \varepsilon_{\pi,t+1} - \gamma \sigma_c \eta_{i,t+1}^c$$

Unconditional expectation of the real SDF is given by

$$\mathbb{E}(m_{i,t+1}^{real}) = \log(\delta) - \frac{1}{2}(1-\gamma)^2 \sigma_c^2 - \mu_c^i - \frac{1}{2}\left[(k_{\varepsilon c}^i \sigma_{x,c})^2 + (k_{\varepsilon \pi}^i \sigma_{x,\pi})^2\right]$$

where

$$k_{\varepsilon c}^i = (\gamma - 1)\beta_c^i \left(\frac{\delta}{1 - \delta\rho_c}\right)$$

$$k_\varepsilon \pi^i = -\rho_{c\pi} k_{\varepsilon c}^i \left(\frac{\delta}{1 - \delta\rho_\pi}\right)$$

Unconditional expectation of the nominal SDF is given by

$$\mathbb{E}(m_{i,t+1}^{nominal}) = \mathbb{E}(m_{i,t+1}^{real}) - \mu_\pi^i$$

Unconditional expectation of real risk-free rates is given by

$$\mathbb{E}(r^{i,real}) = \mu_c^i - \log(\delta) - \left(\frac{1}{2} - \frac{1}{\theta}\right)\sigma_c^2$$

Unconditional expectation of nominal risk-free rates is given by

$$\mathbb{E}(r^{i,nominal}) = \mu_c^i + \mu_\pi^i - \log(\delta) - \left(\frac{1}{2} - \frac{1}{\theta}\right)\sigma_c^2 - \frac{1}{2}\sigma_\pi^2$$

Currency premium (note that σ_π are the same across countries) is given by

$$\mathbb{E}(rx) = r^{i,real} - r^{US,real} + m^{i,real} - m^{US,real}$$

$$= r^{i,nominal} - r^{US,nominal} + m^{i,nominal} - m^{US,nominal}$$

$$= \frac{1}{2}\left[(k_{\varepsilon c}^{US}\sigma_{x,c})^2 + (k_{\varepsilon\pi}^{US}\sigma_{x,\pi})^2\right] - \frac{1}{2}\left[(k_{\varepsilon c}^i\sigma_{x,c})^2 + (k_{\varepsilon\pi}^i\sigma_{x,\pi})^2\right]$$

Change in real exchange rates is given by

$$\mathbb{E}(\Delta s) = \mathbb{E}(m_{US}^{real}) - \mathbb{E}(m_i^{real})$$

$$= \mu_c^{US} - \mu_c^i + \frac{1}{2}\left[(k_{\varepsilon c}^{US}\sigma_{x,c})^2 + (k_{\varepsilon\pi}^{US}\sigma_{x,\pi})^2\right] - \frac{1}{2}\left[(k_{\varepsilon c}^i\sigma_{x,c})^2 + (k_{\varepsilon\pi}^i\sigma_{x,\pi})^2\right]$$

Change in nominal exchange rates is given by

$$\mathbb{E}(\Delta s) = \mathbb{E}(m_{US}^{nominal}) - \mathbb{E}(m_i^{nominal})$$

$$= \mu_c^{US} - \mu_c^i + \mu_\pi^{US} - \mu_\pi^i + \frac{1}{2}\left[(k_{\varepsilon c}^{US}\sigma_{x,c})^2 + (k_{\varepsilon\pi}^{US}\sigma_{x,\pi})^2\right] - \frac{1}{2}\left[(k_{\varepsilon c}^i\sigma_{x,c})^2 + (k_{\varepsilon\pi}^i\sigma_{x,\pi})^2\right]$$

Real Interest rate diffs is given by

$$r^{i,nominal} - r^{US,nominal} = \mu_c^i - \mu_c^{US}$$

Nominal Interest rate diffs is given by

$$r^{i,nominal} - r^{US,nominal} = \mu_c^i - \mu_c^{US} + \mu_\pi^i - \mu_\pi^{US}$$

We can again see that both nominal rates and real rates are identical when growth rates and inflation are the same across countries, confirming our results in Section 2.

Because the model is solved in closed form, we show both theoretical and simulation results of static trade in Tables 7. Table 8 shows the simulation results for carry trade. As we can clearly see, allowing growth rates and inflation to differ across countries helps the model to perform better. But how the link between loadings and mean growth rates/inflation could be micro-founded remain a challenge.

B   Details on the Disaster Models

The Gourio, Siemer, and Verdelhan (2013) model features EZ preferences, and thus, the SDF is given by

$$
M_{t+1} = \delta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \left(\frac{U_{t+1}}{\left\{ \mathbb{E}_t \left[U_{t+1}^{1-\gamma} \right] \right\}^{\frac{1}{1-\gamma}}} \right)^{\frac{1}{\psi} - \gamma}.
$$

We can easily get

$$
\mathbb{E}_t(m_{t+1}) = \log(\delta) - \frac{1}{\psi} \mathbb{E}_t(\Delta c_{t+1})
$$

$$
+ \mathbb{E}_t \left(\left(\frac{1}{\psi} - \gamma \right) u_{t+1} \right) - \frac{\frac{1}{\psi} - \gamma}{1 - \gamma} \log \left(\mathbb{E}_t[U_{t+1}^{1-\gamma}] \right)
$$

$$
= \log(\delta) - \frac{1}{\psi} \mathbb{E}_t(\Delta c_{t+1})
$$

$$
+ \frac{\frac{1}{\psi} - \gamma}{1 - \gamma} \mathbb{E}_t \left((1 - \gamma) u_{t+1} \right) - \frac{\frac{1}{\psi} - \gamma}{1 - \gamma} \log \left(\mathbb{E}_t[U_{t+1}^{1-\gamma}] \right), \tag{A8}
$$

To derive the entropy $\Xi_t(m_{t+1}) = \log \mathbb{E}_t(M_{t+1}) - \mathbb{E}_t(m_{t+1})$, we need $\log \mathbb{E}_t(M_{t+1})$. Note

$$
\log \mathbb{E}_t(M_{t+1}) = \log \mathbb{E}_t \left(\delta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \left(\frac{U_{t+1}^{1-\gamma}}{\mathbb{E}_t \left[U_{t+1}^{1-\gamma} \right]} \right)^{\frac{\frac{1}{\psi} - \gamma}{1 - \gamma}} \right)
$$

$$
= \log(\delta) + \log \left(\mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \right] \mathbb{E}_t \left[\left(\frac{U_{t+1}^{1-\gamma}}{\mathbb{E}_t \left[U_{t+1}^{1-\gamma} \right]} \right)^{\frac{\frac{1}{\psi} - \gamma}{1 - \gamma}} \right] + \mathrm{cov}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}}, \left(\frac{U_{t+1}^{1-\gamma}}{\mathbb{E}_t \left[U_{t+1}^{1-\gamma} \right]} \right)^{\frac{\frac{1}{\psi} - \gamma}{1 - \gamma}} \right] \right).
$$

To make things simpler, we consider the special case where $\frac{C_{t+1}}{C_t}$ is known at time t. That is, the entropy of consumption growth equals 0. Although this assumption seems strong at first sight, it is a direct extension of the "no-short-run-shocks" assumption in the long-run risk framework. We confirm the harmlessness of this assumption in our simulation results in Table 5. With this simplification, we have

$$
\log \mathbb{E}_t(M_{t+1}) = \log(\delta) + \log \mathbb{E}_t \left(\left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \right) + \log \mathbb{E}_t \left(\left(\frac{U_{t+1}^{1-\gamma}}{\mathbb{E}_t \left[U_{t+1}^{1-\gamma} \right]} \right)^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}} \right)
$$

$$
= \log(\delta) - \frac{1}{\psi}(\Delta c_{t+1}) + \left[\log \left(\mathbb{E}_t \left(U_{t+1}^{\frac{1}{\psi}-\gamma} \right) \right) - \frac{\frac{1}{\psi} - \gamma}{1 - \gamma} \log \left(\mathbb{E}_t \left[U_{t+1}^{1-\gamma} \right] \right) \right].
$$

Combining with (A8), we have the entropy

$$
\Xi_t(m_{t+1}) = \log \mathbb{E}_t(M_{t+1}) - \mathbb{E}_t(m_{t+1}) = -\mathbb{E}_t \left(\left(\frac{1}{\psi} - \gamma \right) u_{t+1} \right) + \log \left(\mathbb{E}_t \left(U_{t+1}^{\frac{1}{\psi}-\gamma} \right) \right). \tag{A9}
$$

By comparing the red terms in (A8) and (A9), we can already see a tight link between the two. In particular, if $\psi = 1$, we have

$$
\Xi_t(m_{t+1}) = -\mathbb{E}_t(m_{t+1}) + \log(\delta) - \mathbb{E}_t(\Delta c_{t+1}).
$$

Taking unconditional expectations, we have

$$
\mathbb{E}(\Xi_t(m_{t+1})) = -\mathbb{E}(m_{t+1}) + \log(\delta) - \mathbb{E}(\Delta c_{t+1}).
$$

This is a line with a slope of -1 in the generalized SDF space with entropy on the y-axis. This suggests that as long as all countries feature the same preference and consumption growth rate, they all lie on this "EZ-entropy line," which actually coincides with an iso-rf line. That is, when $\psi = 1$, all countries would feature the same risk-free rates.

In standard calibrations, ψ is typically close to but larger than 1. In this case, we can use cumulant

generating functions and obtain

$$\Xi_t(m_{t+1}) = \log\left(\mathbb{E}_t\left(U_{t+1}^{\frac{1}{\psi}-\gamma}\right)\right) - \mathbb{E}_t\left(\left(\frac{1}{\psi}-\gamma\right)u_{t+1}\right)$$

$$= \frac{1}{2}\left(\frac{1}{\psi}-\gamma\right)^2 \kappa_{2,t}(u_{t+1}) + \frac{1}{6}\left(\frac{1}{\psi}-\gamma\right)^3 \kappa_{3,t}(u_{t+1}) + \frac{1}{24}\left(\frac{1}{\psi}-\gamma\right)^4 \kappa_{4,t}(u_{t+1})\dots$$

$$\mathbb{E}_t(m_{t+1}) = \left(\log(\delta) - \frac{1}{\psi}\mathbb{E}_t(\Delta c_{t+1})\right) - \frac{\frac{1}{\psi}-\gamma}{1-\gamma}\left(\log\left(\mathbb{E}_t[U_{t+1}^{1-\gamma}]\right) - \mathbb{E}_t\left((1-\gamma)u_{t+1}\right)\right)$$

$$= \left(\log(\delta) - \frac{1}{\psi}\mathbb{E}_t(\Delta c_{t+1})\right)$$

$$- \frac{1}{2}(1-\gamma)\left(\frac{1}{\psi}-\gamma\right)\kappa_{2,t}(u_{t+1}) - \frac{1}{6}(1-\gamma)^2\left(\frac{1}{\psi}-\gamma\right)\kappa_{3,t}(u_{t+1}) - \frac{1}{24}(1-\gamma)^3\left(\frac{1}{\psi}-\gamma\right)\kappa_{4,t}(u_{t+1}) + \dots.$$

C   Portfolio Construction and Alternative Estimates

In this section, we discuss how our static trade and carry trade portfolios are constructed, as well as alternative estimates of the FX-slope. We follow the procedure and estimates in Hassan and Mano (2019). Our results are robust to alternative portfolio construction methods, for example, the equal-weighted method utilized in Lustig, Roussanov, and Verdelhan (2011).[33] We use the Hassan and Mano (2019) portfolios because of their close connection to regression-based results, on which we would base our alternative estimates of the FX-slope in this section. [34]

Static trade and carry trade returns are constructed by forming a portfolio of currencies weighted by their forward premium relative to the US (equivalently, their risk-free rates). Letting $f_{i,t}$ be the log one-period forward exchange rate of currency i at time t, and let $s_{i,t}$ be the spot rate, both quoted in units of currency i per US dollar, by covered interest-rate parity, we have

$$r_{i,t} - r_{US,t} = f_{i,t} - s_{i,t} = fp_{i,t}.$$

Sorting currencies by their forward premia ($fp_{i,t}$) is thus equivalent to sorting currencies on their risk-free rates. Letting $rx_{i,t+1} = f_{i,t} - s_{i,t+1} = r_{i,t} - r_{US,t} - \Delta s_{i,t+1}$ be the currency premium of currency i relative to the

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[33]The broad pattern that carry traders lose money on the exchange rate is also evident in Table 1 of Lustig, Roussanov, and Verdelhan (2011).

[34]We only provide essential information in this section. Interested readers should refer to Hassan and Mano (2019) for details

US dollar, our static trade return is then given by

$$\Sigma_{i,t}[rx_{i,t+1}(\overline{fp}_i^e - \overline{fp}^e)],$$

where $\overline{fp}_i^e$ denotes the estimated[35] forward premium of currency i over time and $\overline{fp}^e = \frac{1}{N}\Sigma_i\overline{fp}_i$. Intuitively, investors conducting the static trade would weight the currencies using their long-term forward premium, longing high-interest-rate currencies and shorting low-interest-rate ones. They fix their portfolio (the weights, $\overline{fp}_i^e - \overline{fp}^e$, do not change over time), thus conducting a "static" carry trade.

Our carry trade return is given by

$$\Sigma_{i,t}[rx_{i,t+1}(fp_{i,t} - \overline{fp}_t)],$$

where $\overline{fp}_t = \frac{1}{N}\Sigma_i fp_{i,t}$. The only difference from static trade is that the weights now change over time. In each period, investors would weight currencies based on their forward premium in that period.

For both of these trades, we can view the currency portfolio that investors long as one representative country, and the currency portfolio that investors short as another representative country. Using the static trade portfolio as an example, mathematically, the currency return for the representative high-interest-rate country relative to the US is given by

$$\Sigma_{i\in\{\forall i \text{ s.t. } \overline{fp}_i-\overline{fp}>0\},t}[rx_{i,t+1}(\overline{fp}_i - \overline{fp})].$$

The risk-free rate (or forward premium) and exchange rate of this representative country are defined in a similar manner, simply replacing $rx_{i,t+1}$ with $fp_{i,t+1}$ and $-\Delta s_{i,t+1}$, respectively. Using the unconditional moments of these returns, as well as their composition, we could infer FX-slopes for these representative countries as in the main text of this paper.

As shown in Hassan and Mano (2019), one major advantage of forming portfolios in this way is one can easily relate these portfolios to regression results. For example, the static trade is closely related to β^{static} in

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[35]From an investor's perspective, a currency's forward premium over the whole sample period is unknown. We assume investors simply expect $\overline{fp}_i$ to be equal to the mean of $\overline{fp}_{i,t}$ across all available data prior to the investment period. We do the same thing when running all our simulations in the paper.

Table 9: Estimation of FX-Slope

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Horizons (months)	1	1	6	12	1	1	6	12
Sample	1 Reblance				3 Reblance			
Static T: β^{stat}	0.47	0.37	0.56	0.60	0.26	0.18	0.26	0.25
	[0.31, 0.63]	[0.19, 0.55]	[0.36, 0.76]	[0.40, 0.80]	[0.16, 0.36]	[0.08, 0.28]	[0.18, 0.34]	[0.13, 0.37]
Static T: FX-slope	0.89	0.59	1.27	1.50	0.35	0.22	0.35	0.33
	[0.46, 1.68]	[0.24, 1.20]	[0.57, 3.10]	[0.68, 3.90]	[0.19, 0.56]	[0.09, 0.39]	[0.22, 0.51]	[0.15, 0.58]
Carry T: β^{ct}	0.68	0.55	0.62	0.71	0.57	0.45	0.42	0.43
	[0.15, 1.21]	[0.04, 1.06]	[0.05, 1.19]	[0.20, 1.22]	[0.20, 0.94]	[0.10, 0.80]	[0.01, 0.83]	[0.06, 0.80]
Carry T: FX-slope	2.13	1.22	1.63	2.45	1.33	0.82	0.72	0.75
	[0.18,+∞)∪(−∞,-5.78]	[0.04,+∞)∪(−∞,-17.78]	[0.05,+∞)∪(−∞,-6.31]	[0.25,+∞)∪(−∞,-5.55]	[0.25, 16.36]	[0.11, 4.07]	[0.01, 4.94]	[0.06, 4.06]
Sample	6 Reblance				12 Reblance			
Static T: β^{stat}	0.23	0.15	0.25	0.24	0.34	0.23	0.31	0.30
	[0.13, 0.33]	[0.05, 0.25]	[0.17, 0.33]	[0.14, 0.34]	[0.18, 0.50]	[0.05, 0.41]	[0.15, 0.47]	[0.14, 0.46]
Static T: FX-slope	0.30	0.18	0.33	0.32	0.52	0.30	0.45	0.43
	[0.15, 0.49]	[0.05, 0.33]	[0.21, 0.49]	[0.17, 0.51]	[0.22, 0.99]	[0.06, 0.68]	[0.18, 0.88]	[0.17, 0.84]
Carry T: β^{ct}	0.56	0.45	0.45	0.11	0.67	0.52	0.57	0.22
	[0.21, 0.91]	[0.12, 0.78]	[0.08, 0.82]	[-0.16, 0.38]	[0.36, 0.98]	[0.21, 0.83]	[0.26, 0.88]	[-0.11, 0.55]
Carry T: FX-slope	1.27	0.82	0.82	0.12	2.03	1.08	1.33	0.28
	[0.26,10.47]	[0.13,3.61]	[0.08,4.63]	[-0.14,0.62]	[0.55, 59.98]	[0.26, 5.01]	[0.34, 7.59]	[-0.10, 1.24]

Point estimates are taken from Table III in Hassan and Mano (2019). Confidence intervals are calculated using the corresponding standard errors.

the following regression:

$$rx_{i,t+1} - \overline{rx}_{t+1} = \beta^{static}(\overline{fp}_i^e - \overline{fp}^e) + \epsilon_{i,t+1}^{static}.$$

More importantly, estimates of β^{static} are closely related to the FX-slope:

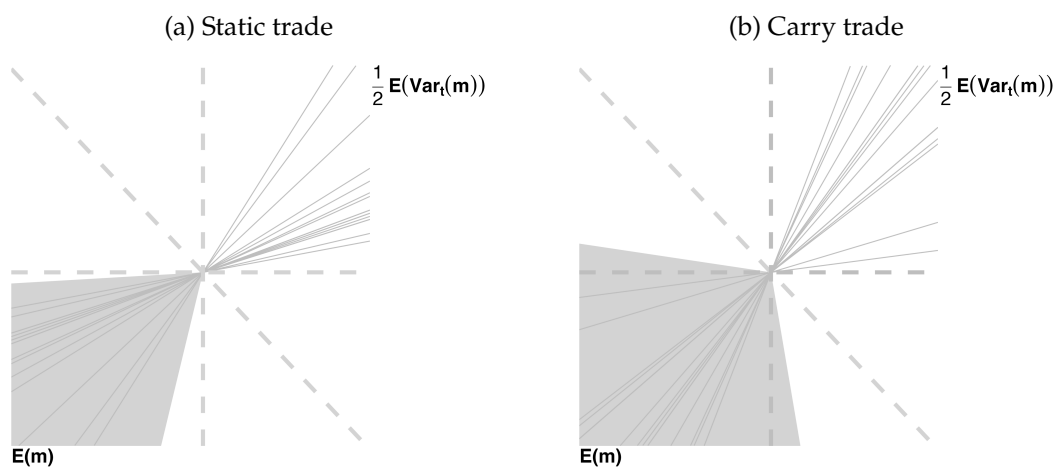$$\text{FX-slope} = \frac{\hat{\beta}^{static}}{1 - \hat{\beta}^{static}}.$$

To see this, note $\hat{\beta}^{static} = \frac{\Sigma_{i,t}[(rx_{i,t+1} - \overline{rx}_{t+1})(\overline{fp}_i^e - \overline{fp}^e)]}{\Sigma_{i,t}[(\overline{fp}_i^e - \overline{fp}^e)^2]} = \frac{\Sigma_{i,t}[rx_{i,t+1}(\overline{fp}_i^e - \overline{fp}^e)]}{\Sigma_{i,t}[(\overline{fp}_i^e - \overline{fp}^e)^2]}$, so we have

$$\frac{\hat{\beta}^{static}}{1 - \hat{\beta}^{static}} = \frac{\Sigma_{i,t}[rx_{i,t+1}(\overline{fp}_i^e - \overline{fp}^e)]}{\Sigma_{i,t}[\Delta s_{i,t+1}(\overline{fp}_i^e - \overline{fp}^e)]} = -\frac{1}{\text{FX-share}} = \text{FX-slope}.$$

A similar result can be obtained for carry-trade returns. We then use the estimates of β^{static} and β^{ct} in Hassan and Mano (2019) to construct alternative estimates of FX-slope.

Table 9 lists all the estimates using different samples. We illustrate all these estimates in our SDF space in Figure 8. Each of the lines represent an estimate of the FX-slope, with the shaded area showing the widest confidence interval. One can clearly see the results are consistent with our bootstrap-based estimates: high-interest-rate currency tends to depreciate, and the FX-slope tends to be positive or vertical at best.

Figure 8: Alternative Estimations of the FX-Slope

(a) Static trade

$\frac{1}{2}\,\mathbf{E(Var_t(m))}$

E(m)

(b) Carry trade

$\frac{1}{2}\,\mathbf{E(Var_t(m))}$

E(m)

This figure plots the point estimates and confidence intervals of the FX-slopes inferred from estimates of β^{static} and β^{carry}. Solid grey lines represent point estimates across different samples. The shaded grey area represents the widest confidence interval.