Dynamic Adjustment to Trade Shocks

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Abstract

Global trade flows and supply chains adjust gradually. The empirical disparity of trade elasticity estimates between the short and long run suggests substantive adjustment frictions in trade. We develop a tractable framework that provides microfoundations for dynamic trade adjustment. The model features staggered sourcing decisions, nests the Eaton-Kortum model as the limiting long-run case, and provides a quantitative framework that rationalizes reduced-form estimation of horizon-specific trade elasticities. We calibrate the model with horizon-specific trade elasticities under industry-specific input-output relations and use it to quantify the welfare impact of the 2018 US-China trade war. Staggered sourcing decisions imply that the well-known static welfare formula based on observed domestic trade shares requires dynamic adjustment, so that predictions account for short-run distortions. Simulations suggest that the short-run welfare impact can be smaller than the long-run level for the United States but larger for China despite the same low short-run trade elasticity, while third countries such as Mexico and Vietnam may experience welfare losses in the short run but welfare gains in the long term.

Keywords: International trade; estimation of the elasticity of trade; dynamic trade adjustment; staggered sourcing decision; US-China trade war

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1 Introduction

Innovations and disruptions to global supply chains lead to gradual adjustments in international trade flows. It has long been recognized that the trade elasticity, a key parameter that captures the substitution between imported goods from different countries in response to trade costs, varies by time horizon (e.g. Dekle, Eaton and Kortum 2008). Boehm, Levchenko and Pandalai-Nayar (2023, henceforth BLP) use plausibly exogenous tariff changes to measure the trade elasticity by time horizon and find that the short-run trade elasticity is about half the size of the long-run elasticity. This differential implies substantial frictions in trade adjustment that a static trade model cannot account for. A dynamic framework is needed to provide a rigorous and plausible quantification of the transitory and lasting impacts of shocks to global supply chains.

This paper proposes a dynamic general-equilibrium model of trade with many countries and many industries, where staggered sourcing decisions give rise to horizon-specific trade elasticities. Under the Ricardian trade tenet, products are sourced from the least expensive global supplier. However, the opportunity to switch to a new supplier only arrives randomly following a Poisson process. As a consequence, only some buyers respond to a trade disruption by adjusting to optimal sourcing relations. Other buyers endure a suboptimal sourcing choice until they can adjust. In this framework, disruptions put the world economy through a sustained period of adjustment.

The model preserves the analytical tractability of a class of quantitative Ricardian models based on Eaton and Kortum (2002, henceforth EK). We characterize impulse responses in the model using the dynamic hat algebra method. We establish a closed-form expression for the horizon-specific trade elasticity, showing that our model rationalizes empirical estimates of the trade elasticity at different time horizons as a convex combination of short- and long-run elasticity parameters, linked by transitory weights that shift at a constant rate of decay. Furthermore, we derive a novel characterization of the horizon-specific gains from trade that sheds light on the importance of sourcing frictions. Our model shows how the original static welfare formula based on Arkolakis, Costinot and Rodríguez-Clare (2012) can be augmented to account for dynamic adjustment so it delivers welfare predictions at any time horizon under a time-varying trade elasticity.

Specifically, we assume that intermediate goods are produced using constant returns-to-scale technologies and producers differ by productivity drawn from a country-industry specific Fréchet distribution. Trade is subject to iceberg trade costs. An assembler of an industry's final good at a destination d seeks to buy from the least expensive global supplier but may not be able to instantaneously switch from one supplier to another. The assembler's sourcing decision is governed by a binary random process: an assembler is either in a position to choose the least expensive global supplier of an intermediate good from any source-industry, or the assembler has to continue purchasing from the same producer as in the preceding period. We can therefore characterize equilibrium as a set of measurable partitions of the space of intermediate goods for each supplier, and then derive the equilibrium distributions. An intermediate good's price at a moment in time equals the initial destination price adjusted for the cumulative changes in marginal costs since the supplier was last selected. We show that a destination country's expenditure shares by source country across intermediate goods take an analytic form as in EK and similar Ricardian frameworks that are consistent with the gravity equation of trade.

For legacy varieties that are imported from the same supplier as in the preceding period, the expenditure shares in the augmented gravity equation encode the price that a buyer paid at the time of the last supplier change. Through this unmoved component, the short-run trade elasticity governs cross-price effects of substitution at the intensive margin while buyer-supplier relationships last, similar to an Armington (1969) model. When supplier-buyer relationships are reset optimally, the gravity expression simplifies to the common gravity equation in an EK framework, so that the long-run trade elasticity prevails for newly sources varieties. With the equilibrium relationships at hand, we compute impulse responses recursively, and we analytically derive the trade elasticity ε_i^h for each time horizon h after a shock to the global supply network at time $t = 0$:

$$\varepsilon_i^h \equiv \frac{\partial \log \lambda_{sdi,h}}{\partial \log \tau_{sdi,0}} = -\theta_i \left[1 - (1-\zeta_i)^{h+1}\right] - (\sigma_i - 1)(1-\zeta_i)^{h+1},$$

where $\lambda_{sdi,h}$ is destination country d's expenditure share falling on intermediate goods from source country s in industry i in the hth period after the shock, $\tau_{sdi,0}$ is the trade cost component that is shocked at time $t = 0$, θ_i is the long-term trade elasticity as in EK, $\sigma_i - 1$ is the short-term trade elasticity as in Armington. The frequency at which buyers of intermediate goods from industry i can switch suppliers is $\zeta_i \in (0, 1)$, which we call the *supplier adjustment probability*. The prevailing trade elasticity ε_i^h increases over time in absolute value from the short-run to the long-run level (for the common parametrization $\theta_i > \sigma_i - 1$).

In the long-run, the trade elasticity converges to the familiar Fréchet parameter θ_i as in EK. The rate of convergence depends on the frequency at which buyers can establish a new sourcing relationship ζ_i. The key parameters of our model are therefore identifiable from reduced-form estimates of the trade elasticity at varying time horizons as in BLP. This characterization of the horizon-specific trade elasticity also implies a horizon-specific welfare formula, which we derive in closed form. The horizon-specific welfare formula features a dynamic adjustment component, which fades over time, and nests the well-known formula from Arkolakis, Costinot and Rodríguez-Clare (2012) as the limiting case in the long run.

We show how the above results can be used to derive a set of estimation equations for the relevant parameters governing short and long-run trade elasticities, document how existing results from BLP can be employed, and quantify our trade model. With the tractability of our model and data on input-output relations, we consider a model world economy consisting of 32 industries across 77 regions. We apply the model to the US-China trade war started in 2018 and show that rich industry-level dynamics can result, with consequential changes in welfare implications. First, despite the low trade elasticity in absolute value in the short-run, the United States main suffer a smaller welfare loss over the short run relative to the long-run outcome when sourcing frictions are no longer relevant. China, on the other hand, may suffer a short-run welfare loss that exceeds the long-run loss. The effect of a low short-run trade elasticity for short-run welfare depends on the trade balance. Second, a direct application of the static welfare formula from Arkolakis, Costinot and Rodríguez-Clare (2012), using realized domestic trade shares, can result in qualitatively misleading predictions over finite time horizons. The reason is that sourcing frictions, and the resulting time-varying trade elasticities, can induce substantive and shifting deviations from the long-term welfare outcome. Third, gains from trade can differ between the short and the long run in both sign and magnitude. In the short-run, price disruptions caused by the US-China trade war propagate through the network of existing supply relationships, leading to a global reduction in economic welfare. Those short-run losses, in part, reflect the limited scope for third-party countries to gain from the trade dispute by forming new supply relationships with the United States or China immediately. Gains for third countries may materialize in the medium to long term, however. As a consequence, countries whose previous trade linkages leave them most exposed to the US-China trade war, such as Mexico and Vietnam, experience initial welfare losses in the short-run, but marked increases in welfare in the long-run.

The wide discrepancy between a low (short-run) trade elasticity in absolute value in international macroeconomics and a high (long-run) trade elasticity in absolute value in international trade has been documented in, for example, Ruhl (2008, who calls the discrepancy an "international elasticity puzzle") and Fontagné, Martin and Orefice (2018). Fontagné, Guimbard and Orefice (2022), BLP and Anderson and Yotov (2022) offer estimation procedures to separately identify short- and long-run trade elasticities. de Souza et al. (2024) obtain horizon-specific trade elasticity estimates in a difference-in-differences design for anti-dumping tariff changes. Anderson and Yotov (2022) rationalize their estimation procedure with firm heterogeneity in lag times from recognition to action in the spirit of Lucas and Prescott (1971). In an alternative approach from a macroeconomic perspective, Yilmazkuday (2019) proposes a framework with nested CES models and derives the trade elasticity as the weighted average of macro elasticities. Our general equilibrium model offers a rationalization for the existing estimation methods with a mixture of the Armington and EK elasticities. Beyond Ricardian trade, Boehm et al. (2024) show for a family of firm-level trade models that the short-run trade elasticity can co-determine the steady-state gains from trade even in the long term.

The importance of staggered contracts for trade and exchange rate dynamics has been recognized since at least Kollintzas and Zhou (1992) and shares features with staggered pricing (Calvo 1983). We generalize deterministic contract ages to supplier relationships that end stochastically. In a related approach, Arkolakis, Eaton and Kortum (2011) embed a consumer with no knowledge of the identity of source countries into an EK model. The consumer can switch to the lowest-cost supplier at random intervals but cannot act strategically because the supplier is unknown. In comparison, we rationalize consumer behavior by introducing assemblers that operate similar to a wholesale or retail firm in that they source bundles of goods at lowest cost while the consumer cannot unbundle the assembled final good. An assembler, in turn, cannot incur losses in imperfect capital markets and entrants cannot evade existing supplier relationships, so all assemblers act as Ricardian price takers in our model. Our model allows us to derive a stationary equilibrium distribution of supplier prices by age of contract beyond a binary characterization in Arkolakis, Eaton and Kortum (2011).[1] Based on the mixture of the stationary equilibrium distributions of prices by contract age, we can fully characterize steady states as well as transition dynamics. As a result, we obtain the original EK model as the limit of the equilibria along the transition path. Our welfare formula therefore endogenously inherits the long-run elasticity as a special case when all supplier contracts are optimally set.

The remainder of the paper is organized as follows. We present the model in Section 2, with details on mathematical

[1] The underlying stochastic process shares features with the so-called Sisyphos Process (Montero and Villarroel 2016).

derivations relegated to the Appendix. In Section 3 we turn to the dynamic analysis of the model. Estimation of the key parameters follows in Section 4. To illuminate the novel dynamic features of the model for economic activity during the adjustment path and the welfare consequences, we present a case study of the US-China trade war in Section 5. Section 6 concludes.

2 Model

2.1 Fundamentals

Consider a world economy with N destination countries $d \in \mathcal{D} := \{1, 2, \cdots, N\}$, $s \in \mathcal{D}$ source countries of trade flows, and I industries $i, j \in \mathcal{I} := \{0, 1, 2, \cdots, I\}$. Time t is discrete. Subscripts sdi, t denote a trade flow from source region s to destination d in industry i at time t. Households inelastically supply a single production factor (labor) to domestic firms, and markets are perfectly competitive.

Households. In each period t, a mass of L_d infinitely-lived households in country d inelastically supplies one unit of the production factor to domestic firms at a competitive wage $w_{d,t}$. Household utility in country d at time t is given by $u(C_{d,t})$, where $C_{d,t}$ is the final good: a Cobb-Douglas aggregate over the composite goods $C_{di,t}$ from each industry with

$$C_{d,t} = \prod_{i \in \mathcal{I}} \left(C_{di,t}\right)^{\eta_{di}}. \tag{1}$$

The coefficient η_{di} is the consumption expenditure share of industry $i's$ composite good, with $\sum_{i \in \mathcal{I}} \eta_{di} = 1$. Let $P_{di,t}$ denote the price index of the industry i good in d at time t. Country $d's$ consumer price index is then given by $P_{d,t} = \prod_{i \in \mathcal{I}} \left(P_{di,t}/\eta_{d,i}\right)^{\eta_{di}}$. We assume that households consume their income in every period and discount future utility flows at rate $\beta \in (0, 1)$.

Intermediate Goods. Every industry i consists of a continuum of producers of intermediate goods $\omega \in [0, 1]$. For each intermediate good, there is a large set of potential producers in each country with different technologies to produce the good. In each industry, producers of an intermediate good ω have an individual productivity z and operate a constant-returns-to-scale technology to produce the good using domestic labor ℓ and composite goods M_{ji} sourced from other industries:

$$y_i(\omega) = z\, (\ell)^{\alpha_{di}} \prod_{j \in \mathcal{I}} (M_{ji})^{\alpha_{dji}}, \tag{2}$$

where $y_i(\omega)$ is the output of good ω. The coefficient α_{di} is the value-added share of industry i and the parameters $\alpha_{dij} \geq 0$ are such that $\alpha_{di} = 1 - \sum_{j \in \mathcal{J}} \alpha_{dji}$.

We assume that intermediate goods can be traded across countries subject to an iceberg transportation cost, which implies that shipping one unit of a good in industry i from country s to country d at time t requires producing $d_{sdi,t} \geq 1$ units in s, where $d_{ddi,t} = 1$ for all d. Moreover, goods imported by d from s at t may be subject to an ad-valorem tariff $\bar{\tau}_{sdi,t}$. We combine both trade costs into one parameter $\tau_{sdi,t} \equiv d_{sdi,t}\bar{\tau}_{sdi,t}$.

Given this formulation of trade costs and technologies, there is a *common unit cost component* at destination d for all intermediate goods produced in country s, which we denote with

$$c_{sdi,t} \equiv \Theta_{si} \tau_{sdi,t} \left(w_{s,t}\right)^{\alpha_{si}} \prod_{j \in \mathcal{J}} \left(P_{sj,t}\right)^{\alpha_{sji}}, \tag{3}$$

where Θ_{si} is a collection of Cobb-Douglas coefficients. The resulting unit cost of good ω at destination d produced in country s with a productivity $z(\omega)$ is given by $c_{sdi,t}/z(\omega)$.

Production technologies for intermediate goods arrive stochastically and independently at a rate that varies by country and industry. Following EK, the mass of intermediate goods ω in country s's industry i that can be produced with a productivity higher than z is distributed Poisson with mean $A_{si} z^{-\theta_i}$.

Assembly of Composite Goods. In each industry, assemblers bundle intermediate goods into a composite good for consumption or production. An assembler procures intermediate goods at the lowest possible price and aggregates the sourced intermediates into $Y_{di,t}$ units of industry i's composite good using the technology

$$Y_{di,t} = \left(\int_{[0,1]} y_{di,t}(\omega)^{(\sigma_i - 1)/\sigma_i} \mathrm{d}\omega \right)^{\frac{\sigma_i}{\sigma_i - 1}}, \tag{4}$$

where $y_{di,t}(\omega)$ is the quantity purchased of an intermediate good ω by an assembler in country d, and σ_i is the elasticity of substitution between intermediate goods in industry i. We let $p_{di,t}(\omega)$ denote the lowest possible price at which an intermediate good ω can be purchased in destination d. We explain the exact price at which this intermediate good is available in detail below. As we elaborate in Appendix A.1, cost minimization given (4) implies that the price of industry i's composite good at destination d satisfies

$$P_{di,t} = \left(\int_{[0,1]} p_{di,t}(d\omega)^{-(\sigma_i - 1)} \mathrm{d}\omega \right)^{-\frac{1}{\sigma_i - 1}}. \tag{5}$$

2.2 Sourcing decisions and trade flows

Under the Ricardian trade tenet, assemblers seek to source an intermediate good from the least expensive global supplier. However, an assembler may not have the opportunity to adjust its choice of suppliers at any given time due to a sourcing friction, which we describe now. For every intermediate good ω, there is a continuum of producers in every country. Under perfect competition, an assembler optimally sources any given intermediate good ω from only one source country when given the choice.

The assembler's choice of source country for any given intermediate good ω is governed by an i.i.d. random variable $x_{i,t}(\omega) \in \{0, 1\}$ for each industry. If $x_{i,t}(\omega) = 1$, that is if the global draw for an intermediate good ω from industry i gives all assemblers worldwide the green light to switch to their preferred source country, then the assemblers in industry i optimally choose to purchase from the least costly source country for good ω in industry i

at time t. Between assemblers in different countries the optimal source country can vary because of different trade costs. Else, if $x_{i,t}(\omega) = 0$, that is if the global draw for intermediate ω does not turn to green for the assemblers in industry i worldwide, then all assemblers must purchase their intermediate goods ω in industry i from the same producer as in the preceding period $t-1$. While the identity of the source country does not change, the quantity procured and the price that the assembler pays can differ from the preceding period if the factory gate price moves (because of changing factor costs) or the currently prevailing trade cost moves (because of a shock).

This formulation of sourcing frictions captures search costs and other types of impediments that prevent the optimal rematch of supply relationships at a moment in time. An implication of the sourcing friction is that price elasticities of demand will differ across intermediate goods according to when their suppliers were last chosen. Let $\Omega^k_{j,t}$ denote the set of industry j goods whose supplier at time t was last chosen k periods ago:

$$\Omega^k_{i,t} = \left\{\omega : x_{di,t-k}(\omega) = 1,\ \prod_{\varsigma=t-k+1}^{t} x_{di,\varsigma}(\omega) = 0\right\}, \tag{6}$$

where $\cup_k \Omega^k_{j,t} = [0,1]$. We call the goods that were last sourced optimally $k > 0$ periods ago the legacy varieties. The sets $\Omega^k_{i,t}$ mutually exclusively and exhaustively partition the unit interval of intermediate goods for each industry i.

2.2.1 Demand for intermediate goods with newly formed supply relationships

We now describe the global demand for intermediate goods in each of these sets, beginning with those that are concurrently formed, $\omega \in \Omega^0_{dj,t}$.

If country s is chosen by an assembler in destination d to supply industry i's intermediate good ω at time t, the combination of the producer's productivity ω, factor cost in source country s and the trade cost between s and d in industry i must make the intermediate good the least expensive.

Let $z_{si}(\omega)$ denote the highest realized productivity by any producer in country-industry si. Similar to EK, our distributional assumptions imply that z_{si} has a country-industry specific Fréchet distribution given by[2]

$$\Pr\left[z_{si}(\omega) \le z | A_{si}, \theta_i\right] = \exp\left\{-A_{si} z^{-\theta_i}\right\}. \tag{7}$$

For an assembler in destination d the price of an intermediate good ω from the least expensive available source country at time t is

$$p_{di,t}(\omega) = \min_{s \in \mathcal{D}} \left\{\frac{c_{sdi,t}}{z_{si}(\omega)}\right\} \tag{8}$$

for the common unit cost component $c_{sdi,t}$ given by (3) and the producer with the highest realized productivity $z_{si}(\omega)$ in country-industry si.

[2]Our model could also accommodate productivity change over time with a country-industry-time specific Fréchet distribution and resulting $z_{si,t}(\omega)$ realizations that vary over time. To focus most sharply on adjustment to trade shocks, we do not specify time-varying productivity shocks.

As in EK, the distribution of paid prices across intermediate goods in the set $\Omega^0_{i,t}$ in destination d at time t satisfies

$$G^0_{di,t}\left[p_{di,t}(\omega) \leq p\right] \equiv \text{Pr}\left[p_{di,t}(\omega) \leq p \middle| x_{i,t}(\omega) = 1\right] = 1 - \exp\left\{-\Phi^0_{di,t} p^{-\theta_i}\right\}, \tag{9}$$

where

$$\Phi^0_{di,t} \equiv \sum_{n \in \mathcal{N}} A_{ni}[c_{ndi,t}]^{-\theta_i} \tag{10}$$

is a measure of destination d's market access for intermediate goods $\omega \in \Omega^0_{i,t}$, given trade cost and factor prices behind the common unit cost component $c_{ndi,t}$ by (3). We relegate the derivation of these results to Appendix A.2. To guarantee that the distribution of paid prices has a finite mean, we impose the standard parametric restriction that $\theta_i > \sigma_i - 1$ for all $i \in \mathcal{I}$.

The properties of the Fréchet distribution imply that $G^0_{di,t}$ also equals the distribution of prices for intermediate goods $\omega \in \Omega^0_{i,t}$ sourced from any source country s. As a result, country d's expenditure share for each potential source country s across intermediate goods $\omega \in \Omega^0_{i,t}$ must equal the probability that this source country offers the lowest global price:

$$\lambda^0_{sdi,t} = \frac{A_{sj}[c_{sdi,t}]^{-\theta^i}}{\Phi^0_{di,t}}. \tag{11}$$

with the common unit cost component $c_{sdi,t}$ given by (3).

Within the set of intermediate goods that are sourced through concurrently and optimally formed supply relationships, the partial equilibrium elasticity of trade flows with respect to trade cost is governed by the familiar Fréchet parameter:

$$\left.\frac{\partial \log \lambda^0_{sdi,t}}{\partial \log \tau_{sdi,t}}\right|_{\Phi^0_{di,t}} = -\theta_j.$$

2.2.2 Demand for intermediate goods with legacy supply relationships

The legacy intermediate goods $\omega \in \Omega^k_{j,t}$ with $k > 0$ are purchased from a supplier that was chosen at time $t - k$. To characterize prices and expenditure allocations across these intermediate goods at time t, we denote changes over time for a variable x_t succinctly by $\hat{x}_t \equiv x_t / x_{t-1}$.

Suppose an assembler in d first sourced an intermediate good ω from s at time $t - k$ under the unit input cost $c_{sdi,t-k}/z_{si}(\omega)$, which depends on equilibrium factor prices and parameters by the common unit cost component (3). If the intermediate good is still sourced from the same producer at time t, its price will then equal[3]

$$p^k_{sdj,t}(\omega) = \frac{c_{sdi,t}}{z_{si}(\omega)} = \frac{c_{sdi,t-k} \prod_{\varsigma=t-k+1}^{t} \hat{c}_{sid,\varsigma}}{z_{si}(\omega)}, \tag{12}$$

[3]Note that $x_t = x_{t-k} \frac{x_{t-k+1}}{x_{t-k}} \cdots \frac{x_t}{x_{t-1}} \equiv x_{t-k} \hat{x}_{t-k+1} \cdots \hat{x}_t$. For a composite variable such as $c_{sdi,t} = \tau_{sdi,t}\, w_{s,t}$, the change over time is $\hat{c}_{sdi,t} = \hat{\tau}_{sdi,t}\, \hat{w}_{s,t}$.

which is the initial destination price adjusted for the cumulative changes in iceberg trade costs and factor cost since $t-k$.

We show in Appendix A.3 that country d's expenditure share by source country across intermediate goods $\omega \in \Omega^k_{i,t}$ equals

$$\lambda^k_{sdi,t} = \frac{\lambda^0_{sdi,t-k}\left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{sid,\varsigma}\right)^{-(\sigma_i-1)}}{\Phi^k_{di,t}}, \tag{13}$$

where

$$\Phi^k_{di,t} \equiv \sum_{n\in\mathcal{N}} \lambda^0_{ndi,t-k}\left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{nid,\varsigma}\right)^{-(\sigma_i-1)} \tag{14}$$

reflects the mean price that a buyer pays for the set of intermediate goods $\Omega^k_{i,t}$ at time $t-k$ through the trade shares $\left\{\lambda^0_{nid,t-k}\right\}_{n\in\mathcal{N}}$.

Comparing equations (13) and (11) shows how cross-price effects differ across intermediate goods depending on when a supply relationship was formed. If assemblers can optimally source from the least expensive global supplier of an intermediate good at time t, cross-price demand effects are governed by the Fréchet parameter θ_i and trade flows follow comparative advantage. Conversely, if an assembler is unable to switch suppliers, then the extensive margin is inoperative. The only margin of adjustment is the intensive margin, which is captured by the terms that collect the product of changes in unit input costs. Effectively, over those partitions of legacy varieties, trade happens as if the varieties were differentiated across countries, where the measure of varieties of each source is defined by the last period of adjustment in period $t-k$ for partition $\Omega^k_{i,t}$.

For each legacy partition $\Omega^k_{i,t}$ with $k>0$ Armington forces therefore determine trade flows. Intuitively, the price elasticity of demand is governed by the elasticity of substitution $\sigma_i - 1$, which captures Armington trade:

$$\left.\frac{\partial \log \lambda^k_{sdi,t}}{\partial \log \tau_{sdi,\varsigma}}\right|_{\Phi^k_{di,t}} = -\left(\sigma_i - 1\right) \quad \text{for } t-k<\varsigma<t.$$

To close the model, we now show how aggregate global demand for industry i's composite good follows from aggregating the trade shares in (11) and (13).

2.3 Aggregation

To find aggregate demand, we leverage the homotheticity of assembly. The partial price index for the composite of legacy intermediate goods purchased at time t from suppliers chosen $t-k$ periods ago satisfies $(P^k_{di,t})^{1-\sigma_j} = \int_{\omega\in\Omega^k_{i,t}} p(\omega)^{1-\sigma_j}_{di,t}\mathrm{d}\omega$. The sets $\left\{\Omega^k_{i,t}\right\}_{k=0}^{\infty}$ partition industry i's product space, so we can obtain country d's price index for industry i goods at time t by aggregating these partial price indices over all partitions and find $P^{1-\sigma_j}_{di,t} = \sum_{k=0}^{\infty}\left(P^k_{di,t}\right)^{1-\sigma_j}$.

We establish in Appendix A.2 that the partial price index for the set of intermediate goods whose suppliers are being chosen optimally at time t takes the familiar EK form

$$P^0_{di,t} = \gamma_i \, \mu_{i,t}(0)^{-1/(\sigma_i-1)} \left(\Phi^0_{di,t}\right)^{-\frac{1}{\theta_i}}, \tag{15}$$

where $\gamma_i \equiv \Gamma\left([\theta_i - \sigma_i + 1]/\theta_i\right)^{-(\sigma_i-1)}$ is a constant, $\Phi^0_{di,t}$ is given by (10), and $\mu_{i,t}(0)$ denotes the measure of the set $\Omega^0_{i,t}$. Following the previous discussion, the endogenous market access term $\Phi^0_{di,t}$ represents the mean price of intermediate goods whose suppliers are chosen optimally at time t. The measure $\mu_{i,t}(0)$ accounts for gains from variety. This measure recursively evolves over time according to the stochastic process that governs sourcing decisions, given by

$$\mu_{i,t}(k) = \begin{cases} \zeta_i, & k = 0 \\ (1-\zeta_i)\mu_{i,t-1}(k-1), & k > 0. \end{cases} \tag{16}$$

The parameter $\zeta_i \in (0,1)$ is the supplier adjustment probability: the frequency at which assemblers from industry i can switch suppliers.

We show in Appendix A.3 that the partial price index across legacy intermediate goods, whose suppliers were last chosen at time $t-k$, is given by

$$P^k_{di,t} = P^0_{di,t-k} \left(\frac{\mu_{i,t}(k)}{\mu_{i,t-k}(0)} \Phi^k_{di,t}\right)^{1/(1-\sigma_i)} \qquad \text{for } k > 0, \tag{17}$$

which is the period $t-k$ price index of the basket of intermediate goods Ω^0_{t-k} adjusted for the subsequent change in variety composition, captured by $\mu_{i,t}(k)/\mu_{i,t-k}(0)$, and prices, captured by $\Phi^k_{di,t}$.

Given equations (15) and (17), we can solve for the composite price index of industry i goods in country d at time t:

$$P_{di,t} = \gamma_i \left(\Phi^0_{di,t}\right)^{-\frac{1}{\theta_i}} \left[\mu_{i,t}(0) + \sum_{k=1}^{\infty} \mu_{i,t}(k) \left(\frac{\Phi^0_{di,t}}{\Phi^0_{di,t-k}}\right)^{-\frac{\sigma_i-1}{\theta_i}} \Phi^k_{di,t}\right]^{-\frac{1}{\sigma_i-1}}. \tag{18}$$

The term $\gamma_i \left(\Phi^0_{di,t}\right)^{-1/\theta_i}$ on the right-hand-side of (18) captures the prices paid under flexible supplier choice. The term in brackets quantifies the extent to which current aggregate demand is affected by the stickiness of supply relationships. The market access variable $\Phi^k_{di,t}$ reflects differences in demand across legacy intermediate goods driven by the varying age of their supply relationships and measures their impact on aggregate demand at time t. The market access ratio $(\Phi^0_{di,t}/\Phi^0_{di,t-k})^{-(\sigma_i-1)/\theta_i}$ measures the current demand of an assembler whose supplier relationship from k periods ago differs from that of an assembler who just updated its supplier.

Using the above price indices, we can derive country d's expenditure share on industry i goods sourced from country s:

$$\lambda_{sdi,t} = \sum_{k=0}^{\infty} \lambda_{sdi,t}^{k} \left(\frac{P_{di,t}^{k}}{P_{di,t}} \right)^{-(\sigma_i - 1)}, \tag{19}$$

where $\lambda_{sdi,t}^{k}$ is given by (11) if $k = 0$ and (13) if $k > 0$.

The set of trade shares $\{\lambda_{sdi,t}\}_{s,d \in \mathcal{N}, i \in \mathcal{I}}$ fully characterizes demand in the world economy at time t. To close the model, we now describe the conditions for market clearing and define general equilibrium.

2.4 Equilibrium

We denote the total revenue of an industry i in a source country s at time t with $X_{si,t}$. To define equilibrium, we express each industry's revenue in terms of trade shares, given by (19), total expenditures on consumption $E_{d,t}$ and sales anywhere in the world:

$$X_{si,t} = \sum_{d \in \mathcal{N}} \lambda_{sdi,t} \left[\eta_{di} E_{d,t} + \sum_{j \in \mathcal{I}} \alpha_{dij} X_{dj,t} \right]. \tag{20}$$

A country's national consumption spending is the sum of its factor income and trade deficit, $E_{d,t} = w_{d,t} L_{d,t} + D_{d,t}$, with $\sum_{d \in \mathcal{N}} D_{d,t} = 0$. We follow the conventional approach in the international trade literature and treat aggregate trade deficits as exogenous. To clear the factor market, wages then adjust to ensure that expenditures equal disposable income,

$$w_{d,t} L_{d,t} = \sum_{i \in \mathcal{I}} (1 - \alpha_{di}) X_{di,t}, \tag{21}$$

and goods market clearing is guaranteed by Walras' law.

We are now ready to define a dynamic general equilibrium and a steady state.

Definition 1. *An economy is described by a set of time-invariant parameters summarizing technologies, preferences and factor endowments,* $\mathbf{\Theta} = \left\{\theta_i, \sigma_i, \{\alpha_{dji}\}_{j \in \mathcal{I}}, \varphi_{di}, A_{di}, \eta_{di}, L_d\}_{d \in \mathcal{N}}\right\}_{i \in \mathcal{I}}$*, sourcing frictions* $\boldsymbol{\zeta} = \{\zeta_i\}_{i \in \mathcal{I}}$*, as well as a measure* $\boldsymbol{\mu}_{t_0} = \{\mu_{t_0}(k)\}_{k \in \{0,1,\cdots\}}$ *for some* t_0*. Given histories of trade costs* $\boldsymbol{\tau}_{t-1} \equiv \{\tau_\varsigma\}_{\varsigma < t} = \left\{\tau_{sid,\varsigma}\right\}_{s,d \in \mathcal{N}, i \in \mathcal{I}, \varsigma < t}$ *and their changes* $\hat{\tau}_t \equiv \{\hat{\tau}_{sdi,t}\}_{s,d \in \mathcal{N}, i \in \mathcal{I}}$ *as well as nominal wages* $\boldsymbol{w}_{t-1} = \{w_\varsigma\}_{\varsigma < t} = \{w_{d,\varsigma}\}_{d \in \mathcal{N}, \varsigma < t}$*:*

1. *A static equilibrium at time t is a vector of wages* $w(\hat{\tau}_t \times \tau_{t-1} \cup \boldsymbol{\tau}_{t-1}, \boldsymbol{w}_{t-1}, \boldsymbol{\zeta}, \mathbf{\Theta}) = w_t$ *that jointly solves equations (19), (20) and (21) for all* $s, d \in \mathcal{N}$ *and* $i \in \mathcal{I}$*.*

2. *A dynamic equilibrium at time t is a history of wages* $\boldsymbol{w}_t$ *so that, for all* $w_\varsigma \in \boldsymbol{w}_t$, $w_\varsigma = w(\hat{\tau}_{\varsigma-1} \times \tau_{\varsigma-1} \cup \boldsymbol{\tau}_{\varsigma-1}, w_{\varsigma-1} \cup \boldsymbol{w}_{\varsigma-2}, \boldsymbol{\zeta}, \mathbf{\Theta})$*.*

3. *A dynamic equilibrium at time t is a steady state if* $w(\mathbf{1}_{N \times N \times I} \times \tau_t \cup \boldsymbol{\tau}_{t-1}, w_t \cup \boldsymbol{w}_{t-1}, \boldsymbol{\zeta}, \mathbf{\Theta}) = w_t$*.*

2.5 Steady-state properties

We now show that our model preserves the properties of quantitative trade models based on EK in the limit when the economy is in steady state—irrespective of the magnitude of the frictions underlying imperfect supplier adjustment. Intuitively, the concrete supplier adjustment probability $\zeta_i \in (0,1)$ regulates the speed of adjustment to the long-term limit but does not affect the limit itself. The transitory effects of trade disruptions that arise in our model reflect how opportunities for finding new suppliers are limited in the short-run. As assemblers get to adjust all supply relationships in the long-run, we obtain the EK-model as the limit of the equilibria along the transition path.

Formally, let $w^{EK}(\hat{\tau}_t \times \tau_{t-1} \cup \boldsymbol{\tau}_{t-1}, \boldsymbol{w}_{t-1}, \mathbf{1}, \boldsymbol{\Theta})$ represent the equilibrium allocation in an economy in which suppliers can be flexibly adjusted for all goods, $\zeta_i = 1$ for all i. We can then establish

Proposition 1. *If w_{t^*} is a steady state equilibrium, then*

1. *For any $\boldsymbol{\zeta}$, $w_{t^*} = w(\mathbf{1}_{N\times N\times I} \times \tau_{t^*} \cup \boldsymbol{\tau}_{t^*-1}, w_{t^*} \cup \boldsymbol{w}_{t^*-1}, \boldsymbol{\zeta}, \boldsymbol{\Theta}) = w^{EK}(\mathbf{1}_{N\times N\times I} \times \tau_{t^*} \cup \boldsymbol{\tau}_{t^*-1}, w_{t^*} \cup \boldsymbol{w}_{t^*-1}, \mathbf{1}, \boldsymbol{\Theta})$.*
2. *For all $k \in \{0,1,...\}$, the measure of goods $\omega \in \Omega^k_{i,t}$ equals $\mu_{i,t^*}(k) = \zeta_i(1-\zeta_i)^k$, and trade flows are given by $\lambda^k_{sdi,t^*} = \lambda_{sdi,t} = \lambda^{EK}_{sid}$ where λ^{EK}_{sid} denotes the trade shares in the frictionless economy.*

Proof. See Appendix A.6.1. □

Proposition 1 shows the relationship of our model with sourcing frictions to existing models and the age distribution of legacy variables. The first statement clarifies that the solution tools partitioning product space, developed in the existing literature for equilibrium properties of static quantitative trade models, extend to the existence and uniqueness of steady state in our model. The second part of Proposition 1 highlights properties of the steady states that we can later leverage to quantify the model. In particular, the second statement shows that the process governing the evolution of the age distribution of supply relationships over time has a simple geometric stationary distribution. Moreover, the second part shows that steady state expenditure allocations are equalized across goods within an industry, irrespective of when their supplier was last chosen optimally.

3 Dynamic Adjustment to Trade Shocks

We now characterize the model economy's dynamic response to trade disruptions. We derive a new structural estimating equation for the trade elasticity at different time horizons along with a new formula for the horizon-specific gains from trade. We show that transitional dynamics can be characterized using dynamic hat algebra.

3.1 Trade elasticities by time horizon

We begin by showing how the trade elasticity (the elasticity of trade flows with respect to transport cost) varies over time in our model. The trade elasticity $\varepsilon^h_{sdi,t-1}$ at time horizon h is defined as

$$\varepsilon^h_{sdi,t-1} \equiv \left. \frac{\partial \log X_{sdi,t+h}}{\partial \log \tau_{sdi,t}} \right|_{\{\Phi^k_{di,t+\varsigma}\}_{t \leq \varsigma \leq h,k}}, \tag{22}$$

for trade flows in industry i from country s to d at time $t+h$ after a trade cost change at time t. The elasticity measures the proportional change in trade flows $X_{sdi,t+h}/X_{sdi,t-1}$, compared to the pre-shock period, with respect a to change in trade costs at t: $\mathrm{d}\log \tau_{sdi,t} = \log \hat{\tau}_{sdi,t}$. The elasticity definition holding fixed the general equilibrium terms that summarize changes in market access for industry i goods at destination d. Proposition 2 provides an analytical characterization of this elasticity.

Proposition 2. *Suppose the economy is in steady state at $t = -1$. Then, up to first order, the elasticity of the horizon-h trade flows with respect to a shock to trade cost at time $t = 0$ is*

$$\varepsilon^h_i = -\theta_i \left[1 - (1-\zeta_i)^{h+1}\right] - (\sigma_i - 1)(1-\zeta_i)^{h+1}. \tag{23}$$

If $\zeta_i \in (0,1)$, then $\lim_{h\to\infty} \varepsilon^h_i = -\theta_i$, and the rate of convergence equals

$$\lim_{h\to\infty} \frac{\varepsilon^{h+1}_i + \theta_i}{\varepsilon^h_i + \theta_i} = \log(1-\zeta_i).$$

Proof. See Appendix A.6.2. □

By (23) the trade elasticity $\varepsilon^h_i > -\theta_i$ strictly decreases towards the long-run limit of $-\theta_i$ (strictly increases over time in absolute value) if $\theta_i > \sigma_i - 1$. In the long-run, the trade elasticity equals the Fréchet parameter θ_i in absolute value. The rate of convergence to the industry's long-run elasticity depends on the industry-specific supplier adjustment probability ζ_i, the frequency at which assemblers can establish a new sourcing relationship.

The definition of the horizon-specific trade elasticity in (22) is consistent with reduced-form estimates at varying time horizons as in BLP. For estimation, we will leverage this equivalence to identify the structural parameters governing the trade elasticity. The horizon-specific formulation of the trade elasticity implied by our model also gives rise to a horizon-specific welfare formula, which we derive next.

3.2 Welfare gains from trade by time horizon

When supply relationships are slow to adjust to shocks, temporary trade disruptions can put the world economy through sustained periods of adjustment. To illuminate the economic forces underlying this adjustment, we now show that our model admits an analytical characterization of the horizon-specific welfare gains from trade.

Proposition 3. *Suppose the economy is in steady state at $t = -1$. Then, the change in real wages $\hat{W}_d^h = C_{d,h}/C_{d,-1}$ in country d at time horizons $h = \{0, 1, ...\}$ after a set of arbitrary shocks to trade cost at time at $t = 0$ is given by*

$$\hat{W}_d^h = \prod_{j\in\mathcal{I}} \left[\left(\frac{\lambda_{ddj,h}}{\lambda_{ddj,-1}} \right)^{-\frac{1}{\theta_j}} \left(\Xi_{dj,h}\right)^{\frac{1}{\sigma_j - 1}} \right]^{\sum_{i\in\mathcal{I}} \bar{a}_{dji}\eta_i}, \tag{24}$$

where

$$\Xi_{dj,h} \equiv (1-\zeta_j)^{h+1} \left(\frac{\lambda_{ddj,h}}{\lambda_{ddj,-1}} \right)^{-\frac{\theta_j - (\sigma_j - 1)}{\theta_j}} + \sum_{\varsigma=0}^{h} \zeta_j (1-\zeta_j)^{\varsigma} \left(\frac{\lambda_{ddj,h}}{\lambda^0_{ddj,h-\varsigma}} \right)^{-\frac{\theta_j - (\sigma_j - 1)}{\theta_j}} \tag{25}$$

and $\bar{a}_{dji}$ is the (j,i)-th element of the Leontief inverse $(\mathbf{I} - \boldsymbol{A}_d)^{-1}$, with the elements of $\boldsymbol{A}_d$ given by α_{dji}.

If $\zeta_i \in (0,1)$, then $\lim_{h\to\infty} \hat{W}_d^h = \lim_{h\to\infty} \prod_{j\in\mathcal{I}} \left(\lambda_{ddj,t+h}/\lambda_{ddj,-1}\right)^{-\sum_{i\in\mathcal{I}} \bar{a}_{dji}\eta_i/\theta_j}$.

Proof. See Appendix A.6.3. □

Following (24), welfare analysis can be conducted using sufficient ex-post statistics.

The aggregate change in a country's real wage incorporates shifts in its comparative advantage as measured by the terms $(\lambda_{ddj,h}/\lambda_{ddj,-1})^{-1/\theta_j}$ on the right-hand-side of (24). The Fréchet parameter θ_j corresponds to the price elasticity of demand for newly sourced goods, so the terms measure the change in an industry's consumer price index as if all varieties were being sourced optimally. For finite horizons $h < \infty$, the home expenditure shares $\lambda_{ddj,h}$ are distorted under the sourcing frictions. All goods are sourced optimally when $h \to \infty$, only then do the home expenditure shares capture the entire real wage response in the long-run as in EK.

The exact welfare distortion at any finite horizon $h < \infty$ depends on the terms $\Xi_{dj,h}$ in (25). These welfare distortion factors $(\Xi_{dj,h})^{1/(\sigma_j - 1)}$ capture how the price distortions under the sourcing frictions contribute to the overall change in a country's terms of trade. There are two aspects to the distortions. First, for the legacy varieties that have not yet been optimally sourced between $t = 0$ and $t = h$ the current home expenditure shares $\lambda_{ddj,h}$ in each industry j deviate from the initial steady state at $t = -1$. Second, the home expenditure shares $\lambda_{ddj,h}$ of the legacy varieties that were at least once optimally sourced between the shock at $t = 0$ and the current horizon $t = h$ deviate from the optimal level $\lambda^0_{ddj,h-\varsigma}$. Those latter terms vary across legacy varieties, depending on when their current supplier was last optimally chosen between $t = 0$ and $t = h$.

As an implication of Proposition 3, the trade elasticity relevant for welfare analysis varies over time. To further illustrate this point, it is useful to consider the following first-order approximation around the initial steady state:

$$\log \left(\frac{\lambda_{ddj,h}}{\lambda_{ddj,-1}} \right)^{-\frac{1}{\theta_j}} \left(\Xi_{dj,h}\right)^{\frac{1}{\sigma_j - 1}} \approx -\frac{1}{\theta_j}[1-(1-\zeta_j)^{h+1}] \log \frac{\lambda^0_{ddj,h}}{\lambda_{ddj,-1}} - \frac{1}{\sigma_j - 1}(1-\zeta_j)^{h+1} \log \frac{\lambda^{k=h+1}_{ddj,h}}{\lambda_{ddj,-1}} - \mathcal{E}^h_{dj}, \tag{26}$$

where $\mathcal{E}_{dj}^{h}=\sum_{\varsigma=1}^{h+1}\zeta(1-\zeta)^{\varsigma}\left[-\frac{1}{\theta_j}\log\frac{\lambda^0_{ddj,h+1-\varsigma}}{\lambda^0_{ddj,-1}}-\frac{1}{\sigma_j-1}\log\frac{\lambda^0_{ddj,h-\varsigma+1}}{\lambda^{k=\varsigma}_{ddj,h}}\right]$.

The first term on the right side of (26) incorporates the partial-equilibrium price response for varieties whose supplier was reset at least once since the shock arrived and the second term the partial-equilibrium price response for varieties whose suppliers have not reset. The aggregate importance of these two effects intuitively varies over time, in tandem with the structural trade elasticity. The final term $\mathcal{E}_{dj}^{h}$ captures the extent to which past sourcing decisions distort present-day prices at horizon h through general-equilibrium effects.

Due to the dynamic interaction of sourcing decisions and factor prices, the trade elasticity relevant for welfare analysis thus differs from the structural elasticity in (23). Consequently, the welfare effects of trade shocks may vary both quantitatively and qualitatively over time, even conditional on the structural parameters governing time variation in the trade elasticity. Proposition 3 allows us to summarize these dynamic effects in terms of a small set of key statistics. As we will now describe, these statistics also enable us to deploy familiar tools from the international trade literature to solve exactly for the equilibrium response of prices and wages to shocks.

3.3 Impulse responses

We now show that solving for the responses of trade and production to shocks does not require knowledge of the economy's structural fundamentals (productivities and trade costs). As an implication, the so-called "hat algebra" of Dekle, Eaton and Kortum (2007) can be deployed to characterize impulse responses in our model.

Absent between-industry linkages, trade flows at time t can be expressed in terms of succinct changes in trade costs and wages, as well as past changes in trade flows for optimally sourced goods, trade costs and wages:

$$\lambda_{sdi,t}=\frac{\left[1+\left(\hat{\tau}_{sdi,t}\hat{w}_{s,t}/\hat{w}_{d,t}\right)^{\theta_i-(\sigma_i-1)}\kappa_{sdi,t-1}\right]\lambda^0_{sdi,t-1}\left(\hat{\tau}_{sdi,t}\hat{w}_{s,t}\right)^{-\theta_i}}{\sum_{s'\in\mathcal{N}}\left[1+\left(\hat{\tau}_{s'id,t}\hat{w}_{s',t}/\hat{w}_{d,t}\right)^{\theta_i-(\sigma_i-1)}\kappa_{s'id,t-1}\right]\lambda^0_{s'id,t-1}\left(\hat{\tau}_{s'id,t}\hat{w}_{s',t}\right)^{-\theta_i}},$$

where the wedges

$$\kappa_{sdi,t-1}\equiv\frac{\mu_{i,t}(1)}{\mu_{i,t}(0)}+\sum_{k'=2}^{\infty}\frac{\mu_{i,t}(k')}{\mu_{i,t}(0)}\left(\frac{\lambda^0_{ddi,t-1}}{\lambda^0_{ddi,t-k'}}\right)^{\frac{\sigma_i-1}{\theta_i}}\frac{\lambda^{k=k'}_{sdi,t-1}}{\lambda^0_{sdi,t-1}}\prod_{\varsigma=t-k'+1}^{t-1}\left(\hat{\tau}_{sid,\varsigma}\frac{\hat{w}_{s,t}}{\hat{w}_{d,\varsigma}}\right)^{-(\sigma_i-1)},$$

summarize how prior distortions in factor prices continue to impact trade flows at time t by distorting the terms of trade.

Now suppose that the economy was in steady state at some time prior to t. Then—given bilateral country-industry trade flows, industry-level consumption and intermediate good expenditure shares as well as per-capita GDP—the only additional industry-level parameters that are required to recursively compute changes in trade flows at increasing time horizons are given by $\{\zeta_i,\theta_i,\sigma_i\}$. Given this recursive formulation for trade flows, we can express the market clearing conditions (21) in terms of changes in trade costs and factor prices, as in Dekle, Eaton and Kortum (2007) and hence solve for the period-by-period change in wages associated with (a sequence of) trade shocks.

4 Estimation

We turn to the quantitative implications of our theory for the responses of production and welfare to trade shocks. In this section, we outline and implement our approach to estimating the structural parameters that govern time variation of the trade elasticity. In the next section, we will use these estimates for a quantitative re-evaluation of how the 2018 US-China trade war impacted trade, production and welfare.

4.1 Minimum distance estimation

To obtain the structural parameters governing the horizon-specific trade elasticity, we apply the explicit analytical characterization in equation (2) to existing reduced-form estimates of the trade elasticity. Intuitively, the parameter σ_i governs the empirical behavior of the trade elasticity in the short-run, while θ_i pins down its long-run level. The rate at which the trade elasticity changes from the short-run to the long-run level informs the structural parameter ζ_i, governing the stickiness of supply relationships in our model. Below, we formally lay out a general minimum distance estimator for the vector of structural parameters $\Theta_i \equiv \{\theta_i, \sigma_i, \zeta_i\}$.

From (2), we can define a continuous mapping f_i^h from the vector of structural parameters $\mathbf{\Theta}$ into the reduced-form trade elasticity at horizon h:

$$f_i^h(\Theta_i) \equiv \varepsilon_i^h = \frac{\partial \log X_{sdi,t+h}}{\partial \log \tau_{sdi,t}} = -\theta_i \left[1 - (1-\zeta_i)^{h+1}\right] + (1-\sigma_i)(1-\zeta_i)^{h+1}.$$

Now suppose we have access to a consistent estimate of $\hat{\varepsilon}_i \equiv \{\hat{\varepsilon}_i^h\}_{h=0}^H \in \mathbb{R}^{H+1}$ for the vector of reduced-form parameters $\varepsilon_i \equiv \{\varepsilon_i^h\}_{h=0}^H \in \mathbb{R}^{H+1}$, up to some $H \geq 2$, that satisfies

$$\sqrt{n}(\hat{\varepsilon}_i - \varepsilon_i) \xrightarrow{d} \mathcal{N}(0, \mathbf{V})$$

for a $(H+1) \times (H+1)$ variance-covariance matrix $\mathbf{V}$. Then a minimum distance estimator of Θ_i is given by

$$\hat{\Theta}_i = \arg\min_{\Theta_i} \left(\hat{\varepsilon}_i - f_i(\Theta_i)\right)' \hat{\mathbf{W}} \left(\hat{\varepsilon}_i - f_i(\Theta_i)\right), \tag{27}$$

where $f_i = \{f_i^h\}_{h=0}^H \in \mathbb{R}^{H+1}$ and $\hat{\mathbf{W}}$ is a $(H+1) \times (H+1)$ symmetric positive semi-definite matrix satisfying $\hat{\mathbf{W}} \xrightarrow{p} \mathbf{W}$. Under standard regularity conditions

$$\sqrt{n}(\hat{\Theta}_i - \Theta_i) \xrightarrow{d} \mathcal{N}\left(0, (\mathbf{G}'\mathbf{W}\mathbf{G})^{-1}\mathbf{G}'\mathbf{W}\mathbf{V}\mathbf{W}\mathbf{G}(\mathbf{G}'\mathbf{W}\mathbf{G})^{-1}\right), \tag{28}$$

where $\mathbf{G} \equiv \nabla_{\Theta_i} f_i(\Theta_i) \in \mathbb{R}^{3\times(H+1)}$. The above assumptions allow inference of the structural parameters estimates based on asymptotic approximation.

4.2 Implementation and results

For our baseline estimates of the structural parameters Θ_i, we follow BLP to identify the reduced-form trade elasticity at varying time horizons. For simplicity, we assume that $\Theta_i = \Theta$ for all i and hence there is no heterogeneity of the trade elasticity by industry. This restriction can be relaxed by repeating the estimation with industry-specific estimates for each industry i.

The BLP estimates are appealing for mainly two reasons. First, our structural model provides a microfoundation for the estimating equation that BLP specify to estimate the trade elasticity at different time horizons. BLP use tariff changes in third countries as a novel instrument for trade policy, conceptually consistent with the notion of shocks in our framework. Specifically, BLP instrument tariff changes with changes in most favored nation (MFN) tariffs for non-major trade partners to obtain plausibly exogenous variation in trade policy. The resulting empirical tariff changes can arguably be viewed as unanticipated permanent shocks and thus proxies for structural shocks in our model environment. Second, the BLP approach yields a comprehensive set of estimates of the trade elasticity over 11 consecutive year-horizons, providing considerable variation and over-identification for the minimum distance estimator to recover our three parameters of interest.[4]

The BLP estimates come with two important caveats. First, while the estimates inform the tariff-exclusive elasticity of trade flows, the relevant trade elasticity in our model is based on tariff-inclusive trade flows. This discrepancy can be resolved in our model by simply adding 1 to each of the reduced-form estimates.[5] Second, it is well recognized that the yearly horizons after a tariff change do not exactly coincide with multiples of calendar years, since tariff changes may in practice come into effect at any time within a calendar year. In our model, we adopt the timing convention that tariff changes happen at the beginning of period 0. To address the resulting issue of time aggregation, which could affect our estimates of Θ, we reinterpret a model period as half a year for the sake of estimation. We let ζ_H be the half-year counterpart of ζ such that

$$\zeta = 1 - (1 - \zeta_H)^2. \tag{29}$$

We then interpret the estimates from BLP associated with horizons $0, \ldots, 10$ as corresponding to the hth half-year for $h = 1, 3, \ldots, 21$. Estimates for ζ_H are then transformed back to estimates for ζ using (29).

Table 1 presents our estimates of the structural parameters governing the trade elasticity based on BLP estimates. Estimates in the first column are those to be used for the quantification exercise in Section 5. Here, we include

[4] The minimum distance approach requires estimates for at least three different time horizons to be feasible. In practice, however, we find that the reduced-form targets used in estimation must also cover a sufficiently wide time window. For instance, we attempted an alternative implementation based on de Souza and Li's 2020 estimates of the response of trade flows to changes in anti-dumping duties, identifying the trade elasticity up to a four-year horizon. Under that short time horizon, the resulting estimates for θ and ζ were comparable to our baseline, but the estimate for σ hit the theoretical lower bound of zero. When we naively applied our minimum-distance estimation approach to Anderson and Yotov's 2022 estimates, which cover a time window of 18 years, we obtained $\hat{\theta} = 7.32$, $\hat{\sigma} - 1 = -.62$, $\hat{\zeta} = .04$. Anderson and Yotov's estimates are based on a gravity approach with no direct mapping into the horizon-specific trade elasticity in our model or BLP.

[5] Our model abstracts from variety entry. Given the unit mass of varieties, the tariff-inclusive elasticity of trade flows and the tariff-exclusive elasticity of trade flows differ by 1 both in steady state as well as along short-run transition paths.

Table 1: Parameter Estimates for Trade Elasticity

Parameter	Targeted Moments: Baseline BLP	Exclude horizons 5–6	Exclude horizons 4–8
Long-run Trade Elasticity θ	2.54	2.17	1.51
	(2.06)	(1.29)	(.63)
Short-run Trade Elasticity $(\sigma - 1)$	–0.81	–0.84	–0.89
	(.09)	(.10)	(.11)
Supplier adjustment probability ζ	0.08	0.10	0.14
	(.06)	(.06)	(.06)

Notes: Horizon-h trade elasticities by Boehm, Levchenko and Pandalai-Nayar (2023, BLP), using tariff-inclusive trade values for all time horizons. "Baseline BLP" refers to the baseline estimates from BLP. "Exclude horizons 5–6" and "Exclude horizons 4–8" refer to dropping BLP estimates for horizons 5–6 and 4–8 respectively. Parameters θ, $\sigma-1$, ζ from minimum distance estimator based on trade elasticity formula. Standard errors in parenthesis from (28), using standard error estimates by BLP to a obtain diagonal variance-covariance matrix. Model-based trade elasticity at impact: $\varepsilon^0 < -(\sigma-1)$ for $\theta > \sigma - 1$ based on tariff-exclusive trade value in BLP of $-.76$ and transformed to trade elasticity at impact for tariff-inclusive trade value of .24.

estimates for all 11 horizons from BLP as targets for our minimum distance estimator, using the standard errors associated with these estimates to construct the optimal diagonal weight matrix $\hat{\mathbf{W}}$. For the other two columns, we alter the set of targeted reduced-form parameters to exclude BLP estimates of the trade elasticity at medium-term horizons.

Our estimate for the Fréchet parameter implies that the long-run tariff-inclusive elasticity of trade flows to exogenous tariff changes is between -2.54 and -1.51.[6] The estimates for $(\sigma - 1)$ imply for the elasticity of substitution σ that it lies between zero and one so that existing intermediate-goods varieties are net complements from the perspective of assemblers, suggesting limited scope for expenditure switching when adjustment happens at the intensive margin. Finally, we find that the adjustment rate for supplier relationships ζ ranges between about 8% and 14% per annum, indicating substantial stickiness in supplier relationships.

Figure 1 plots the horizon-specific trade elasticity from the baseline parameter estimates in the first column of Table 1 alongside the reduced-form estimates of BLP, stated in tariff-inclusive terms. The model counterpart matches the empirical counterpart.[7]

4.3 Alternative estimation approach

We close this section by outlining an alternative approach to recovering our structural parameters of interest. Using Proposition 2, we can state the following estimating equation for the h-horizon elasticity of tariff-exclusive

[6] These estimates are significantly smaller in absolute value than the conventionally considered range of -5 to -10 from studies that combine empirical diff-in-diffs designs with disaggregated data to account for tariff endogeneity. See BLP for a detailed discussion.

[7] The model elasticity is computed with ζ_H for $h = 1, 3, \ldots, 21$ so as to address the time aggregation issue using equation (29).

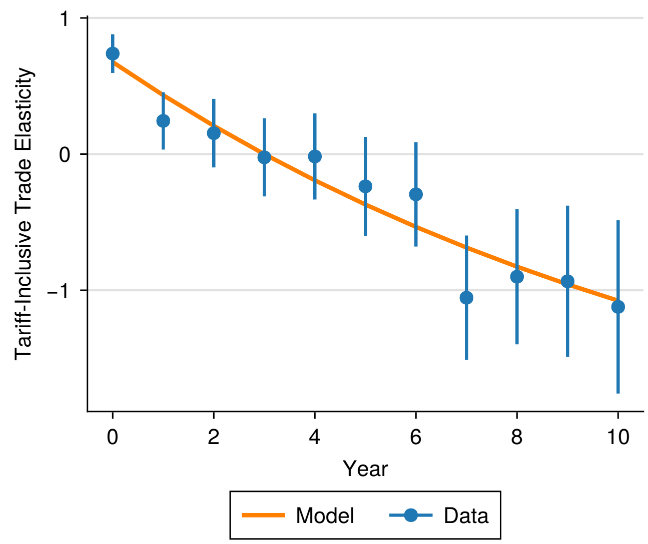

Figure 1: Horizon-Specific Trade Elasticity

Note: Model elasticity computed from the baseline estimates for structural parameters in Table 1, first column. For tariff-inclusive trade elasticities, the empirical counterpart corresponds to one plus the baseline estimates of BLP.

bilateral exports $X_{sdi,t+h}$ to a change in tariffs $\ln \bar{\tau}_{sdi,t}/\bar{\tau}_{sdi,t-1}$ at year t, similar to BLP:

$$\ln\left(\frac{X_{sdi,t+h}}{X_{sdi,t-1}}\right) = (\epsilon_i^h + 1)\ln\left(\frac{\bar{\tau}_{sdi,t}}{\bar{\tau}_{sdi,t-1}}\right) + \delta_{si,t+h} + \delta_{di,t+h} + u_{sdi,t+h}, \tag{30}$$

where ϵ_i^h is the time varying trade elasticity, δ denotes fixed effects and $u_{sdi,t+h}$ is a residual.

Using the findings from Proposition 2 to substitute for ϵ_i^h in (30), we propose to recover the structural parameters in two ways: either by instrumenting tariff changes and using non-linear OLS[8] or by pooling all horizons available in the data from $h = 0$ to $h = H$, in a long panel of bilateral trade flows and estimate

$$\ln\left(\frac{X_{sdi,t+h}}{X_{sdi,t-1}}\right) = (\bar{\gamma}_i + \gamma_i^h)\ln\left(\frac{\bar{\tau}_{sdi,t}}{\bar{\tau}_{sdi,t-1}}\right) + \delta_{si,t+h} + \delta_{di,t+h} + u_{sdi,t+h}, \tag{31}$$

with tariff changes instrumented as in BLP. Here, $\bar{\gamma}_i$ is an estimator for $-\theta_i + 1$, restricted to not vary by time horizon. The coefficient γ_i^h is an estimator for $\left[\theta_i - (\sigma_i - 1)\right](1 - \zeta_i)^{h+1}$. By the functional form of (23), we recover estimates for $-\theta_i + 1$ directly, for $\sigma_i - 1$ from the coefficient estimate $\hat{\gamma}_i^h$ at any available time horizon

[8]For implementation, we can capitalize on the fact that the estimating equation is linear in independent variables. Consequently, the Frisch-Waugh logic continues to apply and we can residualize the model before running standard non-linear OLS.

Table 2: Model Parameters and Variable Levels for Initial Steady State

Parameters or Initial Levels	Notation	Level of Variation
Matching Input-Output Data Exactly		
Producer expenditure shares across inputs	α_{sij}, α_{sj}	Producer region-industry
Initial import shares by source region	λ_{sdi}	User region
Initial level of bilateral trade flows	X_{sdi}	Industry-specific bilateral pair
Derived from Model Equilibrium		
Household expenditure shares across industries	η_{dj}	User region
Initial aggregate labor income	$w_s L_s$	Producer region
Deficit (difference between expenditure and income)	D_d	User region

and for ζ_i from average changes in the estimated coefficients $\hat{\gamma}_i^h$ in h, for all available horizons:

$$\begin{aligned}
\hat{\theta}_i =& 1 - \hat{\bar{\gamma}}_i \\
(\hat{\sigma}_i - 1) =& -\frac{\hat{\gamma}_i^h}{(1-\zeta_i)^{h+1}} - \hat{\bar{\gamma}}_i, \text{ for any } h, \\
1 - \hat{\zeta}_i =& (1/H) \sum_{h=0}^{H-1} \exp\left\{ \frac{\hat{\gamma}_i^{h+1}}{\hat{\gamma}_i^h} \right\}
\end{aligned}$$

For finite time horizons $H < \infty$ in the data, the estimator is subject to approximation error that decays at a faster rate than ζ_i. The estimation error can be assessed by varying the time horizons used in estimation up to H. Standard errors can be estimated using the Delta method. Implementation of both approaches is currently in progress.[9]

5 Quantitative Application: The 2018 US-China Trade War

We now use our model to examine the general equilibrium responses of trade and production to the 2018 US-China trade war.

5.1 Calibration of the initial steady state

We assume that the world economy is in steady state prior to the announcement of the trade-war tariff changes. The remaining model parameters and initial production and trade levels are calibrated so that the model equilibrium matches the data in 2017 in the absence of any shock. For this purpose, we utilize the 2017 table from the 2023 edition of the OECD Inter-Country Input-Output (ICIO) tables (OECD 2023). Table 2 summarizes the parameters and initial levels for our calibration.

[9] In a preliminary implementation of the pooled regression approach using the same data and instrumental variable as Boehm, Levchenko and Pandalai-Nayar (2023), we find that the long-run trade elasticity is around -7.

The ICIO database covers 45 industries in 76 economies along with a constructed rest of the world (ROW). In the model, we allow 77 economies corresponding to those in the data. For China (mainland) and Mexico, the data separately record the input-output relations for a subset of manufacturing activities only intended for export. To take advantage of this additional detail for China and Mexico in the model, we implement these two economies as consisting of two types of producers for each industry. Concretely, for each industry-specific good in these two economies, there is a set of regular producers delivering output for both domestic and foreign use; an additional set of producers produce special varieties that are only delivered abroad.The technological parameters including those governing trade shares are allowed to be different across these two types of producers. However, the value added generated from all these producers is pooled for computation of aggregate income. Labor inputs are assumed to be perfectly mobile across the two types of producers. The producers therefore face possibly different prices for intermediate inputs but identical wages.

From the 45 ICIO industries, we exclude three that are primarily for public expenditure or services that are hard to classify. We further aggregate the remaining 42 industries into 32 industries by combining non-manufacturing industries. After aggregating the ICIO data, the remaining trade and production values no longer necessarily satisfy restrictions under accounting identities.[10] To reestablish identities, we choose some variables that we target exactly and use model equilibrium conditions to derive those relationships that we cannot target simultaneously.

We set the technological parameters $\{\{\alpha_{sij}\}_i, \alpha_{sj}\}_{sj}$ so that the expenditure shares across production inputs match those in the data exactly.[11] We also match the initial import shares $\{\lambda_{sdi}\}_{sdi}$ exactly. These parameters and variables already determine a complete input requirement matrix for the world economy. However, we still need to determine the relative levels of output across all region-industry pairs, and there are alternative approaches. Each economy's exposure to the trade war depends on the initial levels of bilateral trade flows, so we choose to target the levels of bilateral trade flows exactly, which include home expenditures. From these bilateral trade flows, we directly obtain the levels of output from each region-industry pair and the total expenditure for every industry-level good in each region. From output levels and technological parameters we obtain total expenditure for every intermediate input in each region. Subtracting the intermediate uses from total expenditures yields the final uses for every industry in each region, ensuring that all accounting identities hold. We set the household expenditure shares across goods from different industries based on these derived final uses. Finally, with the region-industry specific value added shares at hand, we compute the initial levels of aggregate labor income. The discrepancy between total expenditure and total labor income in each country is treated as an exogenous trade imbalance that our model does not address.

5.2 Measuring the tariff changes

Tariff changes during the US-Chinas trade war are from Fajgelbaum et al. (2020). Tariffs are reported at a detailed Harmonized System (HS) code level. We compute weighted averages of these tariff changes within each of the

[10] The output from a region-industry pair must be identical to the sum of intermediate and final uses of the region-industry good around the world.

[11] The ICIO tables account for taxes and subsidies. We treat taxes and subsidies as special expenditures that are not contributing to any part of disposable income. For this reason, the sum of expenditure shares across inputs is smaller than one.

Table 3: US Tariff Increases on Imports from China

	2017 Imports in Total (%)		Cumulative Increases in Tariffs (%)		
Affected Industry in Model	OECD ICIO	US Census	2018	2019	2020–
Agriculture, forestry and fishing	0.5	0.6	2.5	14.7	20.6
Mining and quarrying	0.0	0.1	1.0	5.6	7.4
Food products, beverages and tobacco	1.9	0.8	2.6	15.5	22.3
Textiles, textile products, leather and footwear	17.7	12.8	0.6	6.6	13.8
Wood and products of wood and cork	1.3	0.8	2.9	16.4	22.1
Paper products and printing	1.2	1.3	2.1	11.0	15.8
Coke and refined petroleum products	0.2	0.1	2.4	14.2	20.5
Chemical and chemical products	3.2	3.1	2.7	12.7	17.7
Pharmaceuticals, medicinal and botanical products	1.3	0.5	0.0	0.1	0.1
Rubber and plastics products	3.7	3.6	2.2	10.9	15.1
Other non-metallic mineral products	2.4	1.7	2.1	12.3	17.4
Basic metals	1.0	0.9	8.8	22.4	24.5
Fabricated metal products	3.8	4.1	3.4	15.0	20.0
Computer, electronic and optical equipment	29.0	36.3	2.0	8.1	11.2
Electrical equipment	9.1	8.9	3.9	14.9	18.8
Machinery and equipment, nec	6.9	7.3	6.1	18.3	22.3
Motor vehicles, trailers and semi-trailers	4.2	3.2	4.7	19.3	24.7
Other transport equipment	0.8	0.7	7.2	20.5	24.0
Furniture and other manufacturing	11.7	13.3	1.1	7.1	11.0

Notes: "Imports in Total" are the shares of industry-specific US imports in total imports from China. "OECD ICIO" refers to the input-output table used for calibrating the model. "US Census" refers to the HS-level bilateral trade data accessed via USA Trade Online. The tariff changes are aggregated based on weights derived from the US Census data. Tariff changes from Fajgelbaum et al. (2020).

model industries over time. The tariff changes are aggregated both across different HS codes and over the months when they take effect. For aggregation across product categories, we determine the most relevant model industry based on the associated industry classification and use the annual bilateral trade volume of each product in 2017 as weight. Fajgelbaum et al. (2020) offer aggregates over months, using the shares of months within a year for which the tariff changes are in effect as weights. Table 3 collects the aggregated tariff changes on US imports from China along with the industry composition of US imports from China in 2017. The Table 4 also present the aggregate retaliatory tariff changes on US exports to China. For model calibration, we rely on the OECD ICIO data, which reconcile trade data with national accounts. To aggregate tariff changes, we use the 10-digit HS-code level data from the US Census. Given discrepancies in industry classifications between OECD ICIO and the US Census, there are discrepancies in import and export volumes. Many products were targeted by trade-war tariffs only during the second half of 2018, so the aggregate changes at the annual level in 2018 are smaller than those in 2019. By the end of 2020, all tariff changes associated with the trade war were in place.

5.3 General equilibrium impact of the trade war

We use our model to infer the response of trade flows, production and welfare to the 2018 US-China trade war. We compute the general-equilibrium response of the world economy to the bilateral changes in tariffs between China and the United States as described in Section 5.2, assuming that all other fundamentals remain unchanged at their

Table 4: Retaliatory Tariff Increases on US Exports to China

	2017 Exports in Total (%)		Cumulative Increases in Tariffs (%)		
Affected Sector in Model	OECD ICIO	US Census	2018	2019	2020–
Agriculture, forestry and fishing	14.7	15.1	11.9	31.1	31.3
Mining and quarrying	7.7	7.0	3.5	11.2	14.0
Food products, beverages and tobacco	4.2	2.8	10.4	19.9	21.0
Textiles, textile products, leather and footwear	0.4	0.9	2.5	12.1	15.3
Wood and products of wood and cork	1.5	1.5	2.7	12.9	16.3
Paper products and printing	2.0	2.5	2.1	7.7	8.8
Coke and refined petroleum products	2.6	1.0	10.2	26.0	26.0
Chemical and chemical products	10.6	9.6	3.7	12.5	14.3
Pharmaceuticals, medicinal and botanical products	2.5	2.8	0.3	1.6	2.7
Rubber and plastics products	1.4	1.3	2.3	10.0	12.4
Other non-metallic mineral products	0.6	0.8	4.0	13.7	15.8
Basic metals	10.7	1.9	4.1	15.4	18.9
Fabricated metal products	1.2	1.4	2.5	10.9	13.3
Computer, electronic and optical equipment	10.4	13.8	2.4	9.3	11.2
Electrical equipment	1.4	2.5	3.8	15.8	19.4
Machinery and equipment, nec	6.0	8.0	2.1	9.1	11.1
Motor vehicles, trailers and semi-trailers	8.6	11.0	10.5	21.5	21.7
Other transport equipment	12.4	13.3	0.0	0.1	0.1
Furniture and other manufacturing	1.1	2.8	4.1	12.8	14.2

Notes: "Exports in Total" are the shares of industry-specific US exports in total exports to China. "OECD ICIO" refers to the input-output table used for calibrating the model. "US Census" refers to the HS-level bilateral trade data accessed via USA Trade Online. The tariff changes are aggregated based on weights derived from the US Census data. Tariff changes from Fajgelbaum et al. (2020).

initial 2017 value.

Trade Flows. We begin with a description of how the trade war impacted trade flows between the United States and China. Figure 2 depicts the counterfactual response of imports, in tariff-inclusive terms and aggregated across industries. Import values increase during the initial years following the tariff escalation, reflecting the low degree of substitutability among suppliers implied by the short-run structural trade elasticity. However, as the trade elasticity increases over time, trade flows start to decline sharply soon after the final round of retaliatory tariff increases is implemented in 2020. In the long-run, US imports from China settle at level that is 10 percent lower than in 2017, while Chinese imports from the US decrease by over 15 pecent. As we will describe in further detail below, this discrepancy in the magnitude of trade adjustment in part reflects differences in how shifts in world factor prices brought about by the trade war affect production and trade.

Prices. Figure B.1 in the appendix depicts the industry-level price effects of the trade war in China and the United States. In the US, the rise in trade costs brings about an increase in prices across all industries and time horizons. Due to sluggish adjustment of demand, these price increases are particularly pronounced in the short-ru,. Some industries—notably textiles, basic metals, and electrical equipment—see prices rise by over 4 percent once all retaliatory tariffs are in place. As sourcing decisions gradually adjust to the initial rise in trade cost, prices partially decline but remain high. In contrast to the substantial and uneven price hikes in the United States,

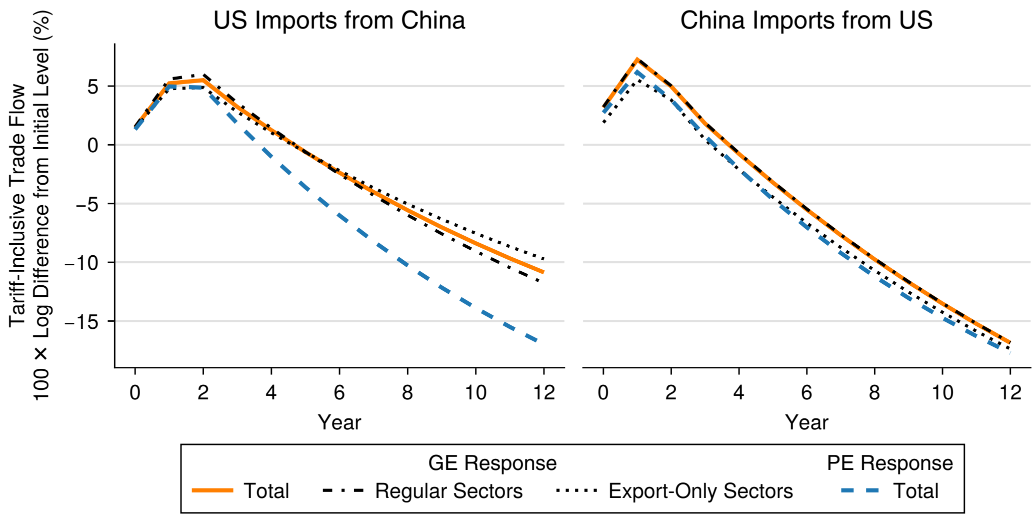

Figure 2: Changes in Tariff-Inclusive Trade Flows

Note: Tariff changes implemented gradually over the first two years. The model determines changes in trade shares at the industry level. The country-level outcomes are based on aggregate trade flows summed across industries. "GE Response" and "PE Response" refer to results generated from the full model involving factor price changes and results only based on the PE trade elasticity respectively. "Regular Sectors" and "Export-Only Sectors" are only relevant to China and Mexico for according ICIO tables, as explained in Section 5.1.

domestic prices in China decline across all industries. Similarly to the US, price changes are smaller in absolute terms in the short-run, reflecting the limited scope for demand reallocation in the short-run.

Figure B.2 in the appendix shows that the price responses of industries not directly exposed to tariff changes follow the same qualitative patterns as those described above. This outcome reflects the impact of the trade war on domestic factor prices, which we discuss next.

Real Wages. Figure 3 traces the counterfactual response of real wages, as well as nominal wages and consumer prices in the United States and China. In the long run, the trade war reduces the real wage in both countries. The short-run impact, however, differs substantially across the two countries. In the United States, the real wage responds gradually, with a moderate decline within the first two years of the trade war (by -0.1 percent) that corresponds to about 50 percent of the overall effect (of -0.22 percent). In contrast, while long-run income losses in China are similar to those in the United States, China experiences a substantially larger decline in real income over the first two years of the trade war (by -0.34 percent).

In Figure 4, we compare the transitory dynamics of real wages in our baseline economy to the equilibrium response of real wages in a variant of the model with equivalent fundamentals but instantaneous trade adjustment ($\zeta = 1$).[12] This exercise points to an important asymmetry in how the real income of households in the United States and China is affected by the trade war over time. While households in the US experience no noticeable change in real wages in the initial two years of the trade war, households in China immediately see their real wage decline.

[12]From Proposition 1 we know that shocks have identical long-run effects in both economies but cause transitory dynamics in the baseline economy. The fact that the impact of the trade war on the friction-less economy is smaller during the first two years reflects the fact that the tariff changes are not fully implemented until the end of 2019.

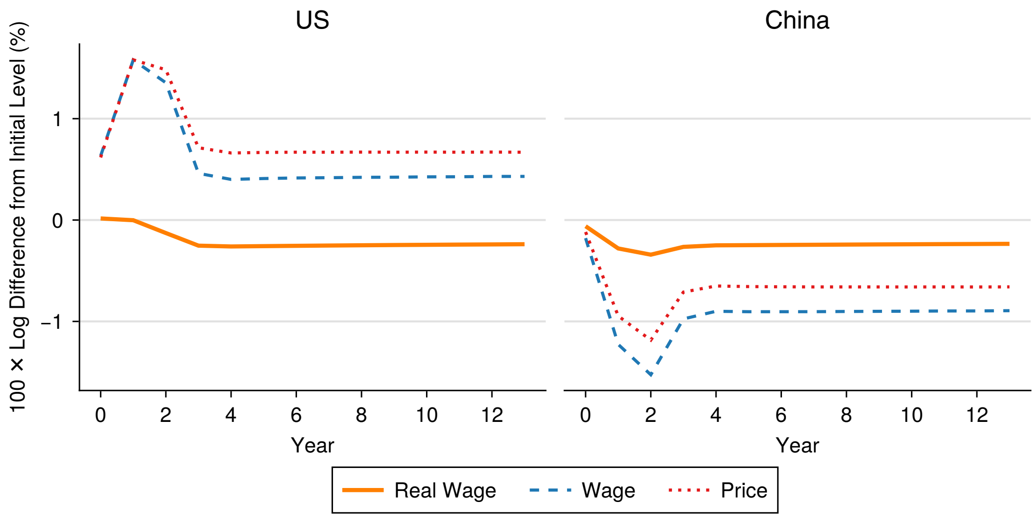

Figure 3: Changes in Real Wages, Wages and Consumer Prices

Note: Tariff changes implemented gradually over the first two years. "Real Wage" in year t refers to real wage changes between t and the initial steady state generated by the full model. "Wage" refers to the corresponding change in the nominal wage. "Price" is the change in the aggregate consumer price index.

Subsequently, this pattern reverses as real wages in China start to increase, sharply at first and then steadily until the new steady state is reached. Meanwhile the real income of US households decreases substantially in years 3 and 4, to then increase slowly until the new steady state is reached. The reason for the differential outcomes is the trade imbalance. In the United States, a delay in trade adjustments generates tariff revenues from Chinese imports while supply relationships last, and the tariff revenues are distributed to US households in the model—partly offsetting losses from trade distortions. In China, in contrast, the delay in trade adjustments prevents Chinese exports from penetrating new markets instead of the United States but the small volume of US exports to China fails to generate substantive tariff revenues for China.

A crucial insight from the simulations is that a lower short-run trade elasticity does not mechanically imply that the welfare costs of trade disruptions will be larger in the short-run compared to the long-run. Depending on the direction of trade imbalances, sourcing frictions can both mitigate or amplify the costs of trade disruptions in the short run. As we saw in Figure 3, in the US prices and wages increase by a similar magnitude in the initial years following the onset of the trade war. In contrast, wages in China decline sharply and by a substantially larger amount than consumer prices. This discrepancy stems from the sluggish trade adjustment, which helps smooth the transition for the United States in two ways—by limiting importers' scope to switch expenditures and thereby creating additional tariff revenue that bolsters domestic demand, and by preventing export demand from fully adjusting to the rise in export prices. In contrast, adjustment frictions bring about larger short-run costs for Chinese households: the adjustment frictions prevent Chinese firms from leveraging the fall in domestic wages to reallocate exports to third-party countries. US imports to China are relatively low, so the additional tariff revenue from short-run frictions does little in the way of strengthening domestic demand in China.

In Figure B.3 in the appendix we use Proposition 3 to further elucidate how adjustment frictions contribute to the

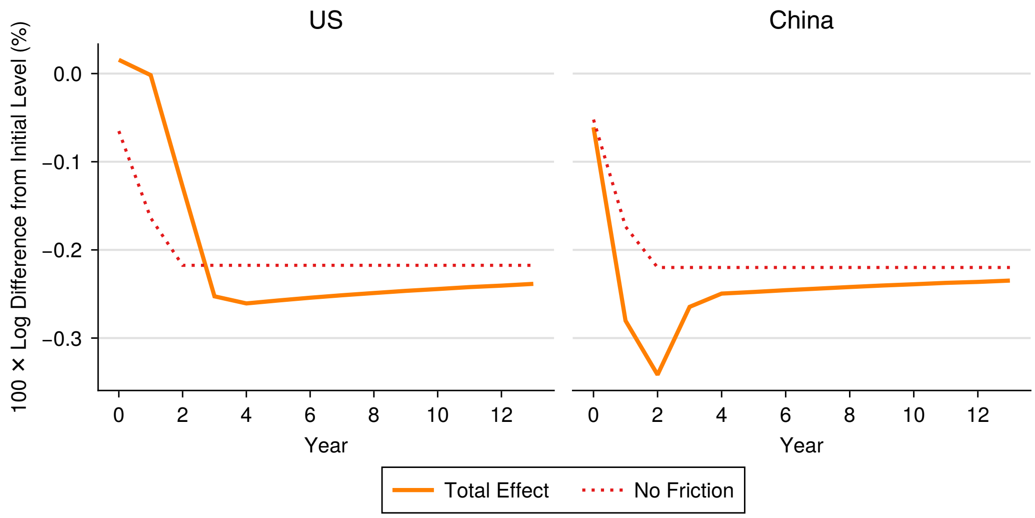

Figure 4: Horizon-Specific Impact on Real Wages

Note: Tariff changes implemented gradually over the first two years. "Total Effect" in year t refers to the real wage change between t and the initial steady state generated by the full model. "No Friction" refers to the change in real wages under the assumption that the economy reaches the long-run outcomes instantly ($\zeta = 1$).

transitory dynamics of real wages. The distortion term Ξ accounts for most of the equilibrium response. Consistent with the reasoning above, we see that price distortions improve the US terms-of-trade in the short run. In the long run, as trade flows gradually realign and start to be driven increasingly by comparative advantage, price distortions ultimately fully reflect the welfare effects of the trade war for US households, while ameliorating the shot-term excess impact on real wages in China.

Effects on Third-Party Countries. We conclude the discussion of the simulations by highlighting the importance of short-run adjustment frictions for assessing the welfare effects of the US-China trade war in third-party countries. In Figure 5, we display the counterfactual real income responses in Mexico and Vietnam, with Figure B.4 in the appendix depicting the underlying aggregate price and wage responses. While both countries ultimately stand to benefit from the US-China trade war, they bear income losses in the short-run. These initial income losses are substantial and, in the case of Mexico, the largest among all third-party countries. Over time Vietnam's real wage increases by 0.15 percent between steady states—a magnitude that is comparable in absolute terms to the corresponding losses borne by households in China and the United States.

The cases of Mexico and Vietnam also illustrate a broader point: the welfare effects of trade disruptions can change sign over time when adjustment is subject to frictions. Adjustment frictions imply that third-party countries stand to gain little from bilateral trade disruptions in the short-run when supply relationships are sticky and respond little to shocks. In the long-run, however, some countries stand to benefit from the realignment of supply relationships. In the case of Mexico and Vietnam, this realignment leads to sustained increases in domestic factor prices, reflecting both the reallocation of US and Chinese demand as well as their favorable positions in the international production network.

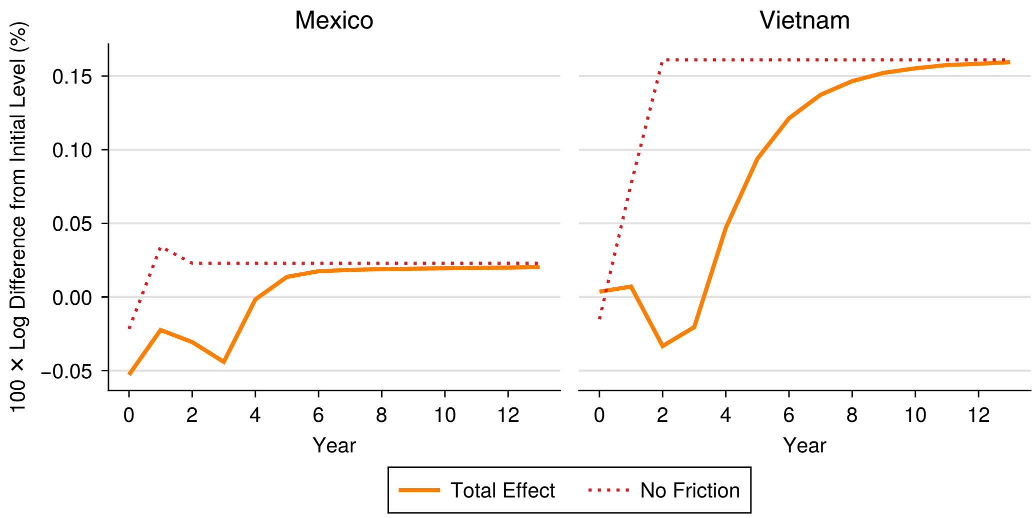

Figure 5: Welfare Impact in Mexico and Vietnam

Note: Tariff changes implemented gradually over the first two years. "Total Effect" in year t refers to the real wage change between t and the initial steady state generated by the full model. "No Friction" refers to the change in real wages under the assumption that the economy reaches the long-run outcomes instantly ($\zeta = 1$).

6 Concluding Remarks

To account for imperfect adjustment to global supply chain shocks, we develop a Ricardian trade framework with frictions that result from staggered decisions of producers to change global suppliers. We obtain novel formulas for accounting welfare changes to trade openness and trade shocks over time, derive novel estimation equations for a horizon-specific trade elasticity, and quantify the model. Counterfactual experiments of the US-China trade war suggest that rich sectoral dynamics ensue, resulting in considerable short-term reallocations and substantive welfare fluctuations at odds with long-term welfare predictions for economies with no sourcing frictions.

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Appendix A Additional Details on Model and Proofs

A.1 Ideal price indexes and generic trade shares

The composite good in industry j is

$$Y_{dj,t} \equiv \left(\int_{[0,1]} y_{dj,t}(\bar{\omega})^{\frac{\sigma_j - 1}{\sigma_j}} \mathrm{d}\bar{\omega} \right)^{\frac{\sigma_j}{\sigma_j - 1}}.$$

Product space $\Omega_j = [0,1]$ can be partitioned into disjoint sets with $\Omega_j = \bigcup_{k=0}^{\infty} \Omega_{j,t}^k$, so we can rewrite the composite good as

$$Y_{dj,t} \equiv \left(\sum_{k=0}^{\infty} \int_{\Omega_{j,t}^k} y_{dj,t}(\bar{\omega})^{\frac{\sigma_j - 1}{\sigma_j}} \mathrm{d}\bar{\omega} \right)^{\frac{\sigma_j}{\sigma_j - 1}}. \tag{A.1}$$

The assembler's associated cost minimization problem is

$$\begin{aligned}
\min_{\{y_{dj,t}(\bar{\omega})\}_{\bar{\omega}\in\Omega_{j,t}}, \{Y_{dj,t}^k\}} P_{dj,t} Y_{dj,t} &= \sum_{k=0}^{\infty} P_{dj,t}^k Y_{dj,t}^k \\
s.t. \qquad Y_{dj,t} &= \left(\sum_{k=0}^{\infty} \left(Y_{dj,t}^k\right)^{\frac{\sigma_j - 1}{\sigma_j}} \right)^{\frac{\sigma_j}{\sigma_j - 1}}, Y_{dj,t}^k \equiv \left(\int_{\Omega_{j,t}^k} y_{dj,t}(\bar{\omega})^{\frac{\sigma_j - 1}{\sigma_j}} \mathrm{d}\bar{\omega} \right)^{\frac{\sigma_j}{\sigma_j - 1}}, \\
P_{dj,t}^k Y_{dj,t}^k &= \int_{\Omega_{j,t}^k} p_{dj,t}(\bar{\omega}) y_{dj,t}(\bar{\omega}) \mathrm{d}\bar{\omega},
\end{aligned}$$

where we define the partial composite good $Y_{dj,t}^k \equiv \left(\int_{\Omega_{j,t}^k} y_{dj,t}(\bar{\omega})^{\frac{\sigma_j - 1}{\sigma_j}} \mathrm{d}\bar{\omega} \right)^{\frac{\sigma_j}{\sigma_j - 1}}$ for each partition k as a helpful construct for derivations and implicity define the associated partial ideal price index $P_{dj,t}^k$ that satisfies $P_{dj,t}^k Y_{dj,t}^k = \int_{\Omega_{j,t}^k} p_{dj,t}(\bar{\omega}) y_{dj,t}(\bar{\omega}) \mathrm{d}\bar{\omega}$.

Under homotheticity of the assembler's production, this problem can be solved in two steps. First, the assembler decides which share of cost it allocates to each partial composite good $Y_{dj,t}^k$. Given those choices, the assembler then decides the optimal cost for each intermediate good $y_{dj,t}(\bar{\omega})$. Optimal demand satisfies

$$Y_{dj,t}^k = \left(\frac{P_{dj,t}^k}{P_{dj,t}} \right)^{-\sigma_j} Y_{dj,t} \quad \text{and} \tag{A.2}$$

$$y_{dj,t}^k(\bar{\omega}) = \left(\frac{p_{dj,t}(\bar{\omega})}{P_{dj,t}^k} \right)^{-\sigma_j} Y_{dj,t}^k = \left(\frac{p_{dj,t}(\bar{\omega})}{P_{dj,t}} \right)^{-\sigma_j} Y_{dj,t} \quad \text{for each } \bar{\omega} \in \Omega_{j,t}^k, \tag{A.3}$$

where the last equality also shows that the partitioned solution equals the standard solution under a constant elasticity of substitution. Replacing the demand functions above in the definition of the budget constraint results in the expressions for the ideal price indices:

$$P_{dj,t} = \left(\int_{[0,1]} p_{dj,t}(\bar{\omega})^{1-\sigma_j} \mathrm{d}\bar{\omega} \right)^{\frac{1}{1-\sigma_j}}, \qquad P^k_{dj,t} = \left(\int_{\Omega^k_{j,t}} p_{dj,t}(\bar{\omega})^{1-\sigma_j} \mathrm{d}\bar{\omega} \right)^{\frac{1}{1-\sigma_j}}. \tag{A.4}$$

We have now established that partitioning the product space into disjoint sets results in well-behaved demand functions such that, given optimal choices within each set, we can analyze demand for each intermediate good independently and then aggregate. In subsequent derivations, expenditure shares within each partition k will play a crucial role, so we state a general definition here:

$$\begin{aligned} \lambda^k_{sdj,t} \equiv \frac{X^k_{sdj,t}}{X^k_{dj,t}} &\equiv \frac{\int_{\Omega^k_{j,t}} \mathbf{1}\left\{s \text{ is } \omega\text{'s source country}\right\} p_{dj,t}(\omega) y_{dj,t}(\omega)\, \mathrm{d}\omega}{\int_{\Omega^k_{j,t}} p_{dj,t}(\omega) y_{dj,t}(\omega)\, \mathrm{d}\omega} \\ &= \frac{\int_{\Omega^k_{j,t}} \mathbf{1}\left\{s \text{ is } \omega\text{'s source country}\right\} p_{dj,t}(\omega) y_{dj,t}(\omega)\, \mathrm{d}\omega}{\sum_n \int_{\Omega^k_{j,t}} \mathbf{1}\left\{n \text{ is } \omega\text{'s source country}\right\} p_{dj,t}(\omega) y_{dj,t}(\omega)\, \mathrm{d}\omega}. \end{aligned} \tag{A.5}$$

A.2 Trade shares when firms are sourcing optimally ($k = 0$)

Under perfect competition, the destination price for intermediate good $\omega \in \Omega^0_{j,t}$ offered by country s to country d is $p_{sdj,t}(\omega) = c_{sdj,t}/z_{sj}(\omega)$ for the common unit cost component $c_{sdj,t}$ by (3) and supplier ω's productivity $z_{si}(\omega)$. Under the EK assumptions, the cumulative distribution function of prices is therefore

$$\tilde{F}_{sdj,t}(p) = \mathbb{P}\left[p_{sdj,t}(\omega) < p\right] = 1 - F_{sj}\left(\frac{c_{sdj,t}}{p}\right) = 1 - \exp\left\{-A_{sj}(c_{sdj,t})^{-\theta_j} p^{\theta_j}\right\}. \tag{A.6}$$

The resulting probability that country d sources an intermediate good $\omega \in \Omega^0_{j,t}$ from country s is

$$\mathbb{P}\left[s = \arg\min_n \left\{p_{ndj,t}(\omega)\right\}\right] = \int_0^\infty \prod_{n \neq s} \left[1 - \tilde{F}_{ndj,t}(p)\right] \mathrm{d}\tilde{F}_{sdj,t}(p) = \frac{A_{sj}(c_{sdj,t})^{-\theta_j}}{\Phi_{dj,t}}, \tag{A.7}$$

where $\Phi_{dj,t} \equiv \sum_n A_{sj}(c_{sdj,t})^{-\theta_j}$.

For products in $\Omega^0_{j,t}$, the distribution of prices $G^0_{sdj,t}(p)$ paid in country d on products sourced from country s equals the overall distribution of prices paid in country d: $G^0_{dj,t}(p)$. For any given source country s:

$$G^0_{sdj,t}(p) = \mathbb{P}\left[p_{dj,t}(\omega) \leq p \,\middle|\, s = \arg\min_n \left\{p_{ndj,t}(\omega)\right\}\right] = 1 - \exp\left\{-\Phi_{dj,t} p^{\theta_j}\right\}.$$

The unconditional distribution is the same as the distribution conditional on each source country, so

$$\begin{aligned} G^0_{dj,t}(p) &= \sum_s \mathbb{P}\left[p_{dj,t}(\omega) \le p \middle| s = \arg\min_n \left\{p_{ndj,t}(\omega)\right\}\right] \mathbb{P}\left[s = \arg\min_n \left\{p_{ndj,t}(\omega)\right\}\right] \\ &= \sum_s \left(1 - \exp\left\{-\Phi_{dj,t} p^{\theta_j}\right\}\right) \lambda^0_{sdj,t} = 1 - \exp\left\{-\Phi_{dj,t} p^{\theta_j}\right\}, \end{aligned} \tag{A.8}$$

where the last equality follows from the fact that $\sum_s \lambda^0_{sdj,t} = 1$.

Putting these results together, we can now solve for the expenditure share within partition 0. Starting from the definition of expenditure shares,

$$\begin{aligned} \lambda^0_{sdj,t} &\equiv \frac{\int_{\Omega^0_{j,t}} \mathbf{1}\left\{s = \arg\min_m \left\{p_{mdj,t}(\omega)\right\}\right\} \left(p_{sdj,t}(\omega)\right)^{1-\sigma_j} \mathrm{d}\omega}{\sum_n \int_{\Omega^0_{j,t}} \mathbf{1}\left\{n = \arg\min_m \left\{p_{mdj,t}(\omega)\right\}\right\} \left(p_{ndj,t}(\omega)\right)^{1-\sigma_j} \mathrm{d}\omega} \\ &= \frac{\int_{\Omega^0_{j,t}} \mathbf{1}\left\{s = \arg\min_m \left\{p_{mdj,t}(\omega)\right\}\right\} \int_0^\infty (p)^{1-\sigma_j} \mathrm{d}G_{sdj,t} \mathrm{d}\omega}{\sum_n \int_{\Omega^0_{j,t}} \mathbf{1}\left\{n = \arg\min_m \left\{p_{mdj,t}(\omega)\right\}\right\} \int_0^\infty (p)^{1-\sigma_j} \mathrm{d}G_{ndj,t} \mathrm{d}\omega} \\ &= \frac{\int_{\Omega^0_{j,t}} \mathbf{1}\left\{s = \arg\min_m \left\{p_{mdj,t}(\omega)\right\}\right\} \mathrm{d}\omega \int_0^\infty (p)^{1-\sigma_j} \mathrm{d}G_{dj,t}}{\sum_n \int_{\Omega^0_{j,t}} \mathbf{1}\left\{n = \arg\min_m \left\{p_{mdj,t}(\omega)\right\}\right\} \mathrm{d}\omega \int_0^\infty (p)^{1-\sigma_j} \mathrm{d}G_{dj,t}} \\ &= \frac{\int_{\Omega^0_{j,t}} \mathbf{1}\left\{s = \arg\min_m \left\{p_{mdj,t}(\omega)\right\}\right\} \mathrm{d}\omega}{\int_{[0,1]} \mathbf{1}\left\{\omega \in \Omega^0_{j,t}\right\} \mathrm{d}\omega} \\ &= \frac{\mu_{j,t}(0) \mathbb{P}\left[s = \arg\min_m \left\{p_{mdj,t}(\omega)\right\}\right]}{\mu_{j,t}(0)} \\ &= \frac{A_{sj}(c_{sdj,t})^{-\theta_j}}{\Phi_{dj,t}}, \end{aligned} \tag{A.9}$$

where $\mu_{i,t}(0)$ is the measure of the set $\Omega^0_{i,t}$. The third line uses the fact again that the distribution of prices conditional on the source country is the same as the unconditional distribution of prices, and the last equality uses the probability that a given source country hosts the lowest-cost supplier.

We can derive the corresponding ideal price indices using

$$\begin{aligned} \left(P^0_{dj,t}\right)^{1-\sigma_j} &= \int_{\Omega^0_{j,t}} p_{dj,t}(\bar{\omega})^{1-\sigma_j} \mathrm{d}\bar{\omega} = \int_{\Omega^*_{j,t}0} \int_0^\infty (p)^{1-\sigma_j} \mathrm{d}G_{dj,t} \, \mathrm{d}\bar{\omega} \\ &= \int_{\Omega^0_{j,t}} \int_0^\infty (p)^{1-\sigma_j} \theta_j \Phi_{dj,t} p^{\theta_j - 1} \exp\left\{-\Phi_{dj,t} p^{\theta_j}\right\} \mathrm{d}p \, \mathrm{d}\bar{\omega}. \end{aligned}$$

For a change of variables, define $x \equiv p^{\theta_j} \Phi_{dj,t}$, which implies that $\mathrm{d}x = \theta_j \Phi_{dj,t} p^{\theta_j - 1} \mathrm{d}p$ and $p = \left(x/\Phi_{dj,t}\right)^{1/\theta_j}$.

Denoting $\gamma_j \equiv \Gamma\left([\theta_j + 1 - \sigma_j]/\theta_j\right)$, we can then rewrite the integral above as

$$\left(P^0_{dj,t}\right)^{1-\sigma_j} = \int_{\Omega^0_{j,t}} \int_0^\infty \left(\frac{x}{\Phi_{dj,t}}\right)^{\frac{1-\sigma_j}{\theta_j}} \exp\{-x\}\, \mathrm{d}x\, \mathrm{d}\bar{\omega} = \gamma_j\, \mu_{j,t}(0) \left(\Phi_{dj,t}\right)^{-\frac{1-\sigma_j}{\theta_j}}, \tag{A.10}$$

$\mu_{j,t}(0)$ denotes the measure of the set $\Omega^0_{j,t}$. The results show that, when firms are adjusting, trade shares operate as in the frictionless economy of EK.

Using standard hat algebra for changes in the common unit cost component $\hat{c}_{sdj,t} \equiv c_{sdj,t}/c_{sdj,t-1}$, we can express trade shares and price levels within partition $k = 0$ as:

$$\lambda^0_{sdj,t} = \frac{\lambda^0_{sdj,t-1}\hat{c}^{-\theta_j}_{sdj,t}}{\sum_n \lambda^0_{ndj,t-1}(\hat{c}_{ndj,t})^{-\theta_j}} \tag{A.11}$$

$$P^0_{dj,t} = P^0_{dj,t-1}\left[\sum_s \lambda^0_{sdj,t-1}(\hat{c}_{sdj,t})^{-\theta_j}\right]^{-\frac{1}{\theta_j}}. \tag{A.12}$$

We next derive an analogous result for partitions $k > 0$ when firms are not adjusting their extensive margin of suppliers.

A.3 Legacy trade shares when firms are not adjusting ($k > 0$)

For intermediate goods $\omega \in \Omega^k_{j,t}$, assemblers last adjusted the least-cost supplier $t - k$ periods ago. In order to account for changes in trade shares and price levels, we therefore need to recall optimal sourcing choices at period $t - k$ and trace changes in parameters and prices since $t - k$.

Suppose that in period $t - k$ intermediate good ω was optimally sourced from country s to country d in industry j. Then the destination price in period t for this intermediate good will be:

$$p_{sdj,t}(\omega) = \frac{c_{sdj,t}}{z_{sj}(\omega)} = \frac{\prod_{\varsigma=t-k+1}^{t} c_{sdj,t-k}\,\hat{c}_{sdj,\varsigma}}{z_{sj}(\omega)} = p_{sdj,t-k}(\omega) \prod_{\varsigma=t-k+1}^{t} \left(\hat{c}_{sdj,\varsigma}\right), \tag{A.13}$$

which is the initial destination price adjusted for the cumulative changes in trade costs and factor costs. Using this result, we can derive country d's expenditure share by source country across intermediate goods $\omega \in \Omega^k_{j,t}$

$$
\begin{aligned}
\lambda^k_{sdj,t} &\equiv \frac{\int_{\Omega^k_{j,t}} \mathbf{1}\left\{s = \arg\min_m \left\{p_{mdj,t-k}(\omega)\right\}\right\} \left(p_{sdj,t-k}(\omega) \prod_{\varsigma=t-k+1}^{t} \hat{c}_{sdj,\varsigma}\right)^{1-\sigma_j} \mathrm{d}\omega}{\sum_n \int_{\Omega^k_{j,t}} \mathbf{1}\left\{n = \arg\min_m \left\{p_{mdj,t-k}(\omega)\right\}\right\} \left(p_{ndj,t-k}(\omega) \prod_{\varsigma=t-k+1}^{t} \hat{c}_{ndj,\varsigma}\right)^{1-\sigma_j} \mathrm{d}\omega} \\
&= \frac{\int_{\Omega^k_{j,t}} \mathbf{1}\left\{s = \arg\min_m \left\{p_{mdj,t-k}(\omega)\right\}\right\} \int_0^\infty (p)^{1-\sigma_j} \,\mathrm{d}G_{sdj,t-k}\mathrm{d}\omega \left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{sdj,\varsigma}\right)^{1-\sigma_j}}{\sum_n \int_{\Omega^k_{j,t}} \mathbf{1}\left\{n = \arg\min_m \left\{p_{mdj,t-k}(\omega)\right\}\right\} \int_0^\infty (p)^{1-\sigma_j} \,\mathrm{d}G_{ndj,t-k}\mathrm{d}\omega \left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{ndj,\varsigma}\right)^{1-\sigma_j}} \\
&= \frac{\int_{\Omega^k_{j,t}} \mathbf{1}\left\{s = \arg\min_m \left\{p_{mdj,t-k}(\omega)\right\}\right\} \mathrm{d}\omega \int_0^\infty (p)^{1-\sigma_j} \,\mathrm{d}G_{dj,t-k} \left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{sdj,\varsigma}\right)^{1-\sigma_j}}{\sum_n \int_{\Omega^k_{j,t}} \mathbf{1}\left\{n = \arg\min_m \left\{p_{mdj,t-k}(\omega)\right\}\right\} \mathrm{d}\omega \int_0^\infty (p)^{1-\sigma_j} \,\mathrm{d}G_{dj,t-k} \left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{ndj,\varsigma}\right)^{1-\sigma_j}} \\
&= \frac{\int_{\Omega^k_{j,t}} \mathbf{1}\left\{s = \arg\min_m \left\{p_{mdj,t-k}(\omega)\right\}\right\} \mathrm{d}\omega \left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{sdj,\varsigma}\right)^{1-\sigma_j}}{\sum_n \int_{\Omega^k_{j,t}} \mathbf{1}\left\{n = \arg\min_m \left\{p_{mdj,t-k}(\omega)\right\}\right\} \mathrm{d}\omega \left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{ndj,\varsigma}\right)^{1-\sigma_j}} \\
&= \frac{\mu_{j,t}(k)\lambda_{sdj,t-k} \left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{sdj,\varsigma}\right)^{1-\sigma_j}}{\sum_n \mu_{j,t}(k)\lambda_{ndj,t-k} \left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{ndj,\varsigma}\right)^{1-\sigma_j}} \\
&= \frac{\lambda^0_{sdj,t-k} \left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{sdj,\varsigma}\right)^{1-\sigma_j}}{\sum_n \lambda^0_{ndj,t-k} \left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{ndj,\varsigma}\right)^{1-\sigma_j}},
\end{aligned}
\tag{A.14}
$$

where $\mu_{i,t}(k)$ is the measure of the set $\Omega^k_{i,t}$. The third line again uses the fact that, at $t-k$, the distribution of prices conditional on the source is the same as the unconditional distribution; and the last line uses the result from the previous section that $\lambda^0_{sdj,t-k} = \mathbb{P}\left[s = \arg\min_s \left\{p_{sdj,t-k}(\omega)\right\}\right]$.

We can derive the corresponding ideal price indices using

$$
\begin{aligned}
\left(P^k_{dj,t}\right)^{1-\sigma_j} &= \int_{\Omega^k_{j,t}} p_{dj,t}(\bar{\omega})^{1-\sigma_j} \mathrm{d}\bar{\omega} \\
&= \sum_s \int_{\Omega^k_{j,t}} \mathbf{1}\left\{s = \arg\min_m \left\{p_{mdj,t-k}(\omega)\right\}\right\} \left(p_{sdj,t-k}(\omega) \prod_{\varsigma=t-k+1}^{t} \hat{c}_{sdj,\varsigma}\right)^{1-\sigma_j} \mathrm{d}\omega \\
&= \sum_s \int_{\Omega^k_{j,t}} \mathbf{1}\left\{s = \arg\min_m \left\{p_{mdj,t-k}(\omega)\right\}\right\} \int_0^\infty (p)^{1-\sigma_j} \,\mathrm{d}G_{sdj,t-k}\mathrm{d}\omega \left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{sdj,\varsigma}\right)^{1-\sigma_j} \\
&= \int_0^\infty (p)^{1-\sigma_j} \,\mathrm{d}G_{dj,t-k} \sum_s \int_{\Omega^k_{j,t}} \mathbf{1}\left\{s = \arg\min_m \left\{p_{mdj,t-k}(\omega)\right\}\right\} \mathrm{d}\omega \left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{sdj,\varsigma}\right)^{1-\sigma_j} \\
&= \frac{\mu_{j,t}(k)}{\mu_{j,t-k}(0)} \left(P^0_{dj,t-k}\right)^{1-\sigma_j} \sum_s \lambda^0_{sdj,t-k} \left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{sdj,\varsigma}\right)^{1-\sigma_j}
\end{aligned}
\tag{A.15}
$$

The price level change in partition 0 satisfies $P^0_{dj,t} = P^0_{dj,t-1}\left[\sum_s \lambda^0_{sdj,t-1}(\hat{c}_{sdj,t})^{-\theta_j}\right]^{-\frac{1}{\theta_j}}$ by (A.10), so we can rewrite the ideal price for composite goods with the last supplier selection k periods ago

$$\left(P^k_{dj,t}\right)^{1-\sigma_j} = \frac{\mu_{j,t}(k)}{\mu_{j,t-k}(0)}\left(P^0_{dj,t-k-1}\right)^{1-\sigma_j}\left[\sum_n \lambda^0_{ndj,t-k-1}\hat{c}^{-\theta_j}_{ndj,t-k}\right]^{-\frac{1-\sigma_j}{\theta_j}}\sum_s \lambda^0_{sdj,t-k}\left(\prod_{\varsigma=t-k+1}^{t}\hat{c}_{sdj,\varsigma}\right)^{1-\sigma_j}.$$

Denoting $\gamma_j \equiv \Gamma\left([\theta_j+1-\sigma_j]/\theta_j\right)$ and using the fact that $\left(P^0_{dj,t}\right)^{1-\sigma_j} = \mu_{j,t}(0)\left(\Phi_{dj,t}\right)^{-\frac{1-\sigma_j}{\theta_j}}\gamma_j$, we can rewrite the expression above as:

$$\left(P^k_{dj,t}\right)^{1-\sigma_j} = \gamma_j\mu_{j,t}(k)\left(\Phi_{dj,t-k}\right)^{-\frac{1-\sigma_j}{\theta_j}}\sum_s\left[\lambda^0_{sdj,t-k-1}\hat{c}^{-\theta_j}_{sdj,t-k}\right]^{-\frac{1-\sigma_j}{\theta_j}}\left(\prod_{\varsigma=t-k+1}^{t}\hat{c}_{sdj,\varsigma}\right)^{1-\sigma_j} \tag{A.16}$$

after expressing $\lambda^0_{sdj,t-k}$ recursively.

A.4 Aggregation over partitions

The aggregate ideal price level of the final good can be rewritten as a combination of the price levels of the partial price indices for the composites of intermediate goods purchased at time t from suppliers chosen $t-k$ periods ago:

$$\left(P_{dj,t}\right)^{1-\sigma_j} = \int_{[0,1]} p_{dj,t}(\bar{\omega})^{1-\sigma_j}\mathrm{d}\bar{\omega} = \sum_{k=0}^{\infty}\int_{\Omega^k_{j,t}} p_{dj,t}(\bar{\omega})^{1-\sigma_j}\mathrm{d}\bar{\omega} = \sum_{k=0}^{\infty}\left(P^k_{dj,t}\right)^{1-\sigma_j}.$$

Using the price index expressions (A.10) and (A.16) from the preceding subsections yields

$$\begin{aligned}\left(P_{dj,t}\right)^{1-\sigma_j} &= \gamma_j\sum_{k=0}^{\infty}\mu_{j,t}(k)\left(\Phi_{dj,t-k}\right)^{-\frac{1-\sigma_j}{\theta_j}}\sum_s\left[\lambda^0_{sdj,t-k-1}\,\hat{c}^{-\theta_j}_{sdj,t-k}\right]^{-\frac{1-\sigma_j}{\theta_j}}\\ &\qquad\times\exp\left\{\mathbf{1}\{k>0\}\log\left(\prod_{\varsigma=t-k+1}^{t}\hat{c}_{sdj,\varsigma}\right)^{1-\sigma_j}\right\}\\ &= \sum_{k=0}^{\infty}\frac{\mu_{j,t}(k)}{\mu_{j,t-k}(0)}\left(P^0_{dj,t-k-1}\right)^{1-\sigma_j}\sum_n\left[\lambda^0_{ndj,t-k-1}\hat{c}^{-\theta_j}_{ndj,t-k}\right]^{-\frac{1-\sigma_j}{\theta_j}}\\ &\qquad\times\exp\left\{\mathbf{1}\{k>0\}\log\left[\sum_s\lambda^0_{sdj,t-k}\left(\prod_{\varsigma=t-k+1}^{t}\hat{c}_{sdj,\varsigma}\right)^{1-\sigma_j}\right]\right\}.\end{aligned} \tag{A.17}$$

Recall that, by optimal demand, expenditure shares of each partition relative to total expenditures are

$$\frac{P^k_{dj,t}Y^k_{dj,t}}{P_{dj,t}Y_{dj,t}} = \left(\frac{P^k_{dj,t}}{P_{dj,t}}\right)^{1-\sigma_j}$$

Total expenditure shares are therefore simply the weighted average of trade shares across partitions

$$\lambda_{sdj,t} \equiv \sum_{k=0}^{\infty} \frac{P_{dj,t}^{k} Y_{dj,t}^{k}}{P_{dj,t} Y_{dj,t}} \lambda_{sdj,t}^{k} = \sum_{k=0}^{\infty} \left(\frac{P_{dj,t}^{k}}{P_{dj,t}} \right)^{1-\sigma_j} \lambda_{sdj,t}^{k}, \tag{A.18}$$

which can also be stated as

$$\lambda_{sdj,t} = \left(\frac{P_{dj,t}^{0}}{P_{dj,t}} \right)^{1-\sigma_j} \frac{\lambda_{sdj,t-1}^{0} \hat{c}_{sdj,t}^{-\theta_j}}{\sum_n \lambda_{ndj,t-1}^{0} \hat{c}_{ndj,t}^{-\theta_j}} + \sum_{k=1}^{\infty} \left(\frac{P_{dj,t}^{k}}{P_{dj,t}} \right)^{1-\sigma_j} \frac{\lambda_{sdj,t-k}^{0} \left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{sdj,\varsigma} \right)^{1-\sigma_j}}{\sum_n \lambda_{ndj,t-k}^{0} \left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{ndj,\varsigma} \right)^{1-\sigma_j}}.$$

Writing $\lambda_{sdj,t-k}^{0}$ and $\lambda_{ndj,t-k}^{0}$ recursively, we can express trade shares compactly as

$$\lambda_{sdj,t} = \sum_{k=0}^{\infty} \left(\frac{P_{dj,t}^{k}}{P_{dj,t}} \right)^{1-\sigma_j} \frac{\lambda_{sdj,t-k-1}^{0} \hat{c}_{sdj,t-k}^{-\theta_j} \exp\left\{ \mathbf{1}\{k>0\} \log \left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{sdj,\varsigma} \right)^{1-\sigma_j} \right\}}{\sum_n \lambda_{ndj,t-k-1}^{0} \hat{c}_{ndj,t-k}^{-\theta_j} \exp\left\{ \mathbf{1}\{k>0\} \log \left(\prod_{\varsigma=t-k+1}^{t} \hat{c}_{ndj,\varsigma} \right)^{1-\sigma_j} \right\}}. \tag{A.19}$$

A.5 Convergence

Results in the preceding subsection imply that trade shares can be expressed a sum over infinitely many partitions. We now establish regularity conditions for convergence.

Lemma 1 (Convergence)**.** *If cumulative changes in trade costs are finite-valued*

$$\lim_{k\to\infty} \left| \prod_{\varsigma=t-k+1}^{t} \hat{c}_{ndj,\varsigma} \right| < \infty,$$

then price levels $P_{dj,t}^{k} < \infty$ *and trade shares* $0 < \lambda_{dj,t} < 1$ *are finite-valued.*

Proof. Note that $\left(\Phi_{dj,t-k}\right)^{(\sigma_j-1)/\theta_j} < \infty$ and $\sum_s \left[\lambda_{sdj,t-k-1}^{0} A_{sj} \hat{c}_{sdj,t-k}^{-\theta_j}\right]^{(\sigma_j-1)/\theta_j} < \infty$ are both finite-valued, because they are equilibrium objects of a static equilibrium of the model. Also note that, for any $k > m$, if $|\prod_{\varsigma=t-k+1}^{t} \hat{c}_{ndj,\varsigma}| < \infty$, then $|\prod_{\varsigma=t-m+1}^{t} \hat{c}_{ndj,\varsigma}| < \infty$, since the product up to k includes every term in the product up to m. Therefore, if $\lim_{k\to\infty} |\prod_{\varsigma=t-k+1}^{t} \hat{c}_{ndj,\varsigma}| < \infty$, then, for every $k < \infty$, the product will also be finite. It follows that $P_{dj,t}^{k} < \infty$ is finite valued for every k. Given that $\lim_{k\to\infty} \mu_{j,t}(k) = \lim_{k\to\infty}(1-\zeta_j)^k \zeta_j = 0$. These findings also guarantee that $P_{dj,t} < \infty$. $\square$

A.6 Proofs

A.6.1 Proof of Proposition 1.

When the economy is in steady state, then for any $t <$ changes must satisfy $\hat{\mathbf{F}}_t = \hat{\mathbf{F}}_{\not k}$ and $\hat{\mathbf{w}}_t = \hat{\mathbf{w}}_{\not k}$ so that $\hat{c}_{s,t} = 1$ for all $s \in \mathcal{D}$. For the firms that are adjusting at t ($k = 0$), evaluating equation (19) at those values, $\lambda^0_{sdj,t} = \lambda^0_{sdj,t-1} = \cdots = \lambda^0_{sdj,0}$ for all t. For the firms that are not adjusting at t ($k > 0$), we have $t - k > 0$ in equilibrium as long as the partition exists and can evaluate (19) using the same logic as above: $\lambda^k_{sdj,t} = \lambda^0_{sdj,t-k} = \lambda^0_{sdj,0}$ for all t. From (19), it is easy to see that $\lambda_{sdj,t} = \lambda^0_{sdj,t}$, which shows that $\boldsymbol{\lambda}_t = \boldsymbol{\lambda}^{EK}$ in steady state.

To derive the stationary distribution of contract lengths, begin by noting that the case $k = 0$ is trivial, since $\mu(0) = \mathbb{P}[K_t = 0] = \zeta_j$ does not vary. Now consider the case $k > 0$. Note that:

$$\begin{aligned}
\mathbb{P}[K_t = k, k > 0] &= \sum_{l=0}^{\infty} \mathbb{P}[K_t = k, k > 0 | K_{t-1} = l]\, \mathbb{P}[K_{t-1} = l] \\
&= (1 - \zeta_j)\mathbb{P}[K_{t-1} = k - 1]
\end{aligned}$$

The remaining proof for $k > 0$ then follows by induction. For $K_t = 1$, $\mathbb{P}[K_t = 1] = (1 - \zeta_j)\zeta_j$, and for $K_t = 2$, $\mathbb{P}[K_t = 2] = (1 - \zeta_j)\mathbb{P}[K_{t-1} = 1] = (1 - \zeta_j)^2\zeta_j$, and so forth recursively, for an arbitrary $K_t = k$ we must have $\mathbb{P}[K_t = k] = (1 - \zeta_j)^k\zeta_j$. This is the probability density function of a geometric distribution with mean $(1 - \zeta_j)/\zeta_j$ and standard deviation $\sqrt{1 - \zeta_j}/\zeta_j$.

Finally, using the definition of the measure μ, $\mu_{j,t}(k) = \mathbb{P}[K_t = k]$ for $t \geq k$. Given the Markov property of K_t, the following distribution will be stationary for all $k \in \mathbb{N}_0$:

A.6.2 Proof of Proposition 2.

For ease of notation, we suppress industry subscripts throughout the derivations. Consider a one-time permanent change in trade costs such that $\hat{\tau}_{sd,t} \neq 1$ and $\hat{\tau}_{sd,t+h} = 1\ \forall h > 0$. To characterize the partial trade elasticity at horizon h, we first characterize the elasticity for trade shares of each partition, then aggregate them up using the consumption shares derived from the CES preferences over partitions. The change in expenditure shares on intermediate goods in the kth partition in period $t + h$, relative to period $t - 1$ is given by

$$\log \frac{\lambda^k_{sd,t+h}}{\lambda^k_{sd,t-1}} = \begin{cases}
-(\sigma - 1)\log \hat{\tau}_{sd,t} + \log \frac{\lambda^0_{sd,t+h-k}}{\lambda^k_{sd,t-1}} \left(\frac{(c_{s,t+h}/P^k_{d,t+h})}{(c_{s,t+h-k}/P^k_{d,t+h-k})}\right)^{1-\sigma} & , k \geq h \\
\log \frac{\lambda^0_{sd,t+h-k}}{\lambda^k_{sd,t-1}} \left(\frac{(c_{s,t+h}/P^k_{d,t+h})}{(c_{s,t+h-k}/P^k_{d,t+h-k})}\right)^{1-\sigma} & , 1 \leq k < h \\
\log \frac{\lambda^0_{sd,t+h-1}}{\lambda^k_{sd,t-1}} \left(\frac{(c_{s,t+h}/P^0_{d,t+h})}{(c_{s,t-1}/P^0_{d,t-1})}\right)^{\theta} & , k = 0
\end{cases}$$

The first line denotes intermediate goods that have not updated suppliers since the shock arrived. For such intermediate goods, changes in expenditure shares still explicitly depend on the shock to trade costs. The remaining intermediate goods have updated at least once, and a "new" optimal sourcing share $\lambda^0_{sd,t+h-k}$ from a time period between t and $t+h$ encodes the "initial price index" relative to which changes in expenditure shares are updated as well as the effect of the shock in trade costs. Unit costs are the relevant GE variables.

Denote

$$\Delta \boldsymbol{G}^{EK}_{sd,t,t+h} = -\theta \log \prod_{k=1}^{h} \frac{\hat{c}_{sd,t+k}}{\hat{P}^0_{sd,t+k}}$$

and

$$\Delta \boldsymbol{G}^{k}_{sd,\varsigma,t+h} = (1-\sigma) \log \prod_{\varsigma'=\varsigma+1}^{t+h} \frac{\hat{c}_{sd,\varsigma'}}{\hat{P}^k_{sd,\varsigma'}}$$

Then we can solve backwards to express all changes in trade shares above in terms of $\lambda^0_{sd,t-1}$, if possible:

$$\log \frac{\lambda^k_{sd,t+h}}{\lambda^k_{sd,t-1}} = \begin{cases} -(\sigma-1)\log \hat{\tau}_{sd,t} + \log \frac{\lambda^0_{sd,t+h-k}}{\lambda^k_{sd,t-1}} + \Delta \boldsymbol{G}^k_{sd,t,t+h} & , k \geq h \\ -\theta \log \hat{\tau}_{sd,t} + \log \frac{\lambda^0_{sd,t-1}}{\lambda^k_{sd,t-1}} + \Delta \boldsymbol{G}^{EK}_{sd,t,t+h-k} + \Delta \boldsymbol{G}^k_{sd,t+h-k,t+h} & , 1 \leq k < h \\ -\theta \log \hat{\tau}_{sd,t} + \Delta \boldsymbol{G}^{EK}_{sd,t,t+h} & , k = 0 \end{cases}$$

Use the fact that outcomes determined at t and earlier do not respond to the change in trade costs. Hence, the elasticity of $\lambda^k_{sd,t+h}$ with respect to a change in trade costs at t, is hence given by,

$$\frac{\mathrm{d}\log(\lambda^k_{sd,t+h}/\lambda^k_{sd,t})}{\mathrm{d}\log \tau_{sd,t}} = \begin{cases} -(\sigma-1) + \frac{\mathrm{d}\Delta \boldsymbol{G}^k_{sd,t,t+h}}{\mathrm{d}\log \tau_{sd,t}} & , k \geq h \\ -\theta + \frac{\mathrm{d}\Delta \boldsymbol{G}^{EK}_{sd,t,t+h-k}}{\mathrm{d}\log \tau_{sd,t}} + \frac{\mathrm{d}\Delta \boldsymbol{G}^k_{sd,t+h-k,t+h}}{\mathrm{d}\log \tau_{sd,t}} & , 1 \leq k < h \\ -\theta + \frac{\mathrm{d}\Delta \boldsymbol{G}^{EK}_{sd,t,t+h}}{\mathrm{d}\log \tau_{sd,t}} & , k = 0 \end{cases}$$

To a first order, the change in overall expenditures at time $t+h$ caused by a one-time permanent shock to trade costs at t is given by

$$
\begin{aligned}
\frac{\mathrm{d}\log(\lambda_{sd,t+h}/\lambda_{sd,t})}{\mathrm{d}\log\tau_{sd,t}} &= \sum_{k=0}^{\infty}\omega_k\left\{\frac{\mathrm{d}\log\lambda^k_{sd,t+h}/\lambda^k_{sd,t}}{\mathrm{d}\log\tau_{sd,t}} + (1-\sigma)\frac{\mathrm{d}\log\frac{P^k_{sd,t+h}P_{sd,t}}{(P^k_{sd,t}P_{sd,t+h})}}{\mathrm{d}\log\tau_{sd,t}}\right\} \\
&= \sum_{k=0}^{h-1}\omega_k\left\{-\theta + \frac{\mathrm{d}\Delta\boldsymbol{G}^{EK}_{sd,t,t+h}}{\mathrm{d}\log\tau_{sd,t}} + \frac{\mathrm{d}\Delta\boldsymbol{G}^{k}_{sd,t+h-k,t+h}}{\mathrm{d}\log\tau_{sd,t}} + (1-\sigma)\frac{\mathrm{d}\log\frac{P^k_{sd,t+h}P_{sd,t}}{(P^k_{sd,t}P_{sd,t+h})}}{\mathrm{d}\log\tau_{sd,t}}\right\} \\
&\quad + \sum_{k=h}^{\infty}\omega_k\left\{(1-\sigma) + \frac{\mathrm{d}\Delta\boldsymbol{G}^{k}_{sd,t,t+h}}{\mathrm{d}\log\tau_{sd,t}} + (1-\sigma)\frac{\mathrm{d}\log\frac{P^k_{sd,t+h}P_{sd,t}}{(P^k_{sd,t}P_{sd,t+h})}}{\mathrm{d}\log\tau_{sd,t}}\right\} \\
&= -\theta\sum_{k=0}^{h-1}\omega_k + (1-\sigma)\sum_{k=h}^{\infty}\omega_k \\
&\quad + \sum_{k=0}^{h-1}\omega_k\frac{\mathrm{d}\Delta\boldsymbol{G}^{EK}_{sd,t,t+h}}{\mathrm{d}\log\tau_{sd,t}} \\
&\quad + \sum_{k=0}^{h-1}\omega_k(1-\sigma)\left\{\frac{\sum_{\varsigma=t+h-k+1}^{t+h}\mathrm{d}\log c_{sd,\varsigma}}{\mathrm{d}\log\tau_{sd,t}} + \frac{\sum_{\varsigma=t}^{t+h-k}\mathrm{d}\log P^k_{sd,\varsigma}}{\mathrm{d}\log\tau_{sd,t}}\right\} \\
&\quad + \sum_{k=h}^{\infty}\omega_k(1-\sigma)\left\{\frac{\sum_{i=0}^{t+h}\mathrm{d}\log c_{sd,t+i}}{\mathrm{d}\log\tau_{sd,t}}\right\} \\
&\quad - (1-\sigma)\frac{\sum_{i=0}^{h}\mathrm{d}\log P_{sd,t+i}}{\mathrm{d}\log\tau_{sd,t}},
\end{aligned}
$$

where $\omega_k \equiv \frac{\left(\frac{P^k_{dj,t}}{P_{dj,t}}\right)^{1-\sigma}\lambda^k_{sdj,t}}{\sum_k\left(\frac{P^k_{dj,t}}{P_{dj,t}}\right)^{1-\sigma}\lambda^k_{sdj,t}} = \frac{\mu_t(k)\lambda^k_{sdj,t}}{\sum_k\mu_t(k)\lambda^k_{sdj,t}}$. If t was a steady state, then $\omega_k = \mu(k)$, and the partial horizon-h trade elasticity equals:

$$
\varepsilon^{t+h}_{sd} \equiv \frac{\partial\log\lambda_{sdj,t+h}}{\partial\log\tau_{sd,t}} = -\theta\sum_{k=0}^{h-1}\mu(k) + (1-\sigma)\sum_{k=h}^{\infty}\mu(k).
$$

Using the stationary distribution of $\mu_t(k)$ to substitute for $\mu(k)$, we obtain the expression stated in the main text.

A.6.3 Proof of Proposition 3.

We begin by rearranging equation (19) to express the prices of composite goods in terms of home expenditure shares

$$
\begin{aligned}
\lambda_{ddi,t} P_{di,t}^{-(\sigma_i-1)} &= \gamma_i \mu_i(0) \left(\Phi_{di,t}^0\right)^{-\frac{1-\sigma_i}{\theta_i}} \lambda_{ddi,t}^0 + \sum_{k\geq 1} \gamma_i \mu_i(k) \left(\Phi_{di,t-k}^0\right)^{-\frac{1-\sigma_i}{\theta_i}} \Phi_{di,t}^k \lambda_{ddi,t}^k \\
&= \gamma_i \mu_i(0) \left(\Phi_{di,t}^0\right)^{-\frac{1-\sigma_i}{\theta_i}} \lambda_{ddi,t}^0 + \sum_{k\geq 1} \gamma_i \mu_i(k) \left(\Phi_{di,t-k}^0\right)^{-\frac{1-\sigma_i}{\theta_i}} \lambda_{ddi,t-k}^0 \left(\frac{c_{dd,t}}{c_{dd,t-k}}\right)^{-(\sigma_i-1)} \\
&= \gamma_i \mu_i(0) \left(\frac{c_{dd,t}^{-\theta_i}}{\lambda_{ddi,t}^0}\right)^{-\frac{1-\sigma_i}{\theta_i}} \lambda_{ddi,t}^0 + \sum_{k\geq 1} \gamma_i \mu_i(k) \left(\frac{c_{dd,t-k}^{-\theta_i}}{\lambda_{ddi,t-k}^0}\right)^{-\frac{1-\sigma_i}{\theta_i}} \lambda_{ddi,t-k}^0 \left(\frac{c_{dd,t}}{c_{dd,t-k}}\right)^{-(\sigma_i-1)}.
\end{aligned}
$$

It follows that

$$
P_{di,t}^{-(\sigma_i-1)} = c_{dd,t}^{-(\sigma_i-1)} \left(\lambda_{ddi,t}^0\right)^{-\frac{\sigma_i-1}{\theta_i}} \frac{1}{\lambda_{ddi,t}} \gamma_i \left[\mu_i(0)\lambda_{ddi,t}^0 + \sum_{k\geq 1} \mu_i(k) \left(\frac{\lambda_{ddi,t}^0}{\lambda_{ddi,t-k}^0}\right)^{-\frac{1-\sigma_i}{\theta_i}} \lambda_{ddi,t-k}^0\right] \tag{A.20}
$$

where the price index is expressed in terms of unit cost and domestic trade shares. With the unit cost under Cobb-Douglas technology, the above equation can be rewritten as

$$
\frac{P_{di,t}}{w_{d,t}} = \left(\lambda_{ddi,t}^0\right)^{\frac{1}{\theta_i}} \left(\lambda_{ddi,t}\right)^{1/(\sigma_i-1)} \left(\gamma_i \xi_{di,t}\right)^{1/(1-\sigma_i)} \alpha_{di}^{-\alpha_{di}} \prod_j \left(\frac{P_{dj,t}}{\alpha_{dji} w_{d,t}}\right)^{\alpha_{dji}}
$$

where

$$
\xi_{di,t} \equiv \mu_i(0)\lambda_{ddi,t}^0 + \sum_{k\geq 1} \mu_i(k) \left(\frac{\lambda_{ddi,t}^0}{\lambda_{ddi,t-k}^0}\right)^{\frac{\sigma_i-1}{\theta_i}} \lambda_{ddi,t-k}^0.
$$

Taking logs yields

$$
\log \frac{P_{di,t}}{w_{d,t}} = \log B_{si,t} + \sum_j \alpha_{sji} \log \frac{P_{sj,t}}{w_{s,t}},
$$

where $B_{di,t} \equiv \alpha_{di}^{-\alpha_{di}} \left(\prod_j \alpha_{dji}^{-\alpha_{dji}}\right) \left(\lambda_{ddi,t}^0\right)^{\frac{1}{\theta_i}} \left(\lambda_{ddi,t}\right)^{1/(\sigma_i-1)} \left(\gamma_i \xi_{di,t}\right)^{1/(1-\sigma_i)}$. In matrix notation, this leads to

$$
(\mathbf{I} - A_d) \log \hat{\boldsymbol{P}}_{d,t} = \log \boldsymbol{B}_{d,t},
$$

where $A_d = \left\{\alpha_{dji}\right\}$ and $\log \hat{\boldsymbol{P}}_{d,t}$ and $\log \boldsymbol{B}_{d,t}$ are $I \times 1$ vectors. Inverting this system of equations, we obtain

$$
\frac{P_{di,t}}{w_{d,t}} = \prod_j B_{dj,t}^{\bar{a}_{dji}},
$$

where $\bar{a}_{dji}$ is the (j,i) entry of the Leontief matrix $(\mathbf{I}-A_d)^{-1}$. The consumer price index in country d can be written as

$$P_{d,t}=\prod_i \left(P_{di,t}\right)^{\eta_i}=w_{d,t}\prod_{i,j}B_{dj,t}^{\bar{a}_{dji}\eta_i}=w_{d,t}\prod_j B_{dj,t}^{\sum_i \bar{a}_{dji}\eta_i}$$

It follows that the real wage is

$$W_{d,t}\equiv\frac{w_{d,t}}{P_{d,t}}=\prod_j B_{dj,t}^{-\sum_i \bar{a}_{dji}\eta_i}.$$

Taking the ratio between real wages in $t-1$ and $t+h$ yields

$$\frac{W_{d,t+h}}{W_{d,t-1}}=\prod_j\left[\left(\frac{\lambda^0_{ddj,t+h}}{\lambda^0_{ddj,t-1}}\right)^{-\frac{1}{\theta_j}}\left(\frac{\lambda_{ddj,t+h}}{\lambda_{ddj,t-1}}\right)^{-\frac{1}{\sigma_j-1}}\left(\frac{\xi_{dj,t+h}}{\xi_{dj,t-1}}\right)^{\frac{1}{\sigma_j-1}}\right]^{\sum_i \bar{a}_{dji}\eta_i}.$$

If $t-1$ is a steady state, then $\lambda^k_{ddj,t-1}=\lambda_{ddj,t-1}$ for all $k\in\{0,1,2,...\}$ and the above expression simplifies to

$$\begin{aligned}
\frac{W_{d,t+h}}{W_{d,t-1}}&=\prod_j\left[\left(\frac{\lambda^0_{ddj,t+h}}{\lambda^0_{ddj,t-1}}\right)^{-\frac{1}{\theta_j}}\left(\frac{\lambda_{ddj,t+h}}{\xi_{dj,t+h}}\right)^{-\frac{1}{\sigma_j-1}}\right]^{\sum_i \bar{a}_{dji}\eta_i}\\
&=\prod_j\left[\left(\frac{\lambda_{ddj,t+h}}{\lambda_{ddj,t-1}}\right)^{-\frac{1}{\theta_j}}\left(\frac{\lambda^0_{ddj,t+h}}{\lambda_{ddj,t+h}}\right)^{-\frac{1}{\theta_j}}\left(\frac{\lambda_{ddj,t+h}}{\xi_{dj,t+h}}\right)^{-\frac{1}{\sigma_j-1}}\right]^{\sum_i \bar{a}_{dji}\eta_i}\\
&=\prod_j\left[\left(\frac{\lambda_{ddj,t+h}}{\lambda_{ddj,t-1}}\right)^{-\frac{1}{\theta_j}}\left(\Xi_{dj,h}\right)^{\frac{1}{\sigma_j-1}}\right]^{\sum_i \bar{a}_{dji}\eta_i},
\end{aligned}\tag{A.21}$$

where

$$\Xi_{dj,t}\equiv(1-\zeta_j)^{h+1}\left(\frac{\lambda_{ddi,t+h}}{\lambda_{ddj,t-1}}\right)^{-\frac{\theta_j-(\sigma_j-1)}{\theta_j}}+\zeta_j\left(\frac{\lambda_{ddj,t+h}}{\lambda^0_{ddj,t+h}}\right)^{-\frac{\theta_j-(\sigma_j-1)}{\theta_j}}+\sum_{\varsigma=1}^{h}\zeta_j(1-\zeta_j)^{\varsigma}\left(\frac{\lambda_{ddj,t+h}}{\lambda^0_{ddj,t+h-\varsigma}}\right)^{-\frac{\theta_j-(\sigma_j-1)}{\theta_j}}\tag{A.22}$$

follows from combining the last two factors in (A.21).

Appendix B Additional Figures

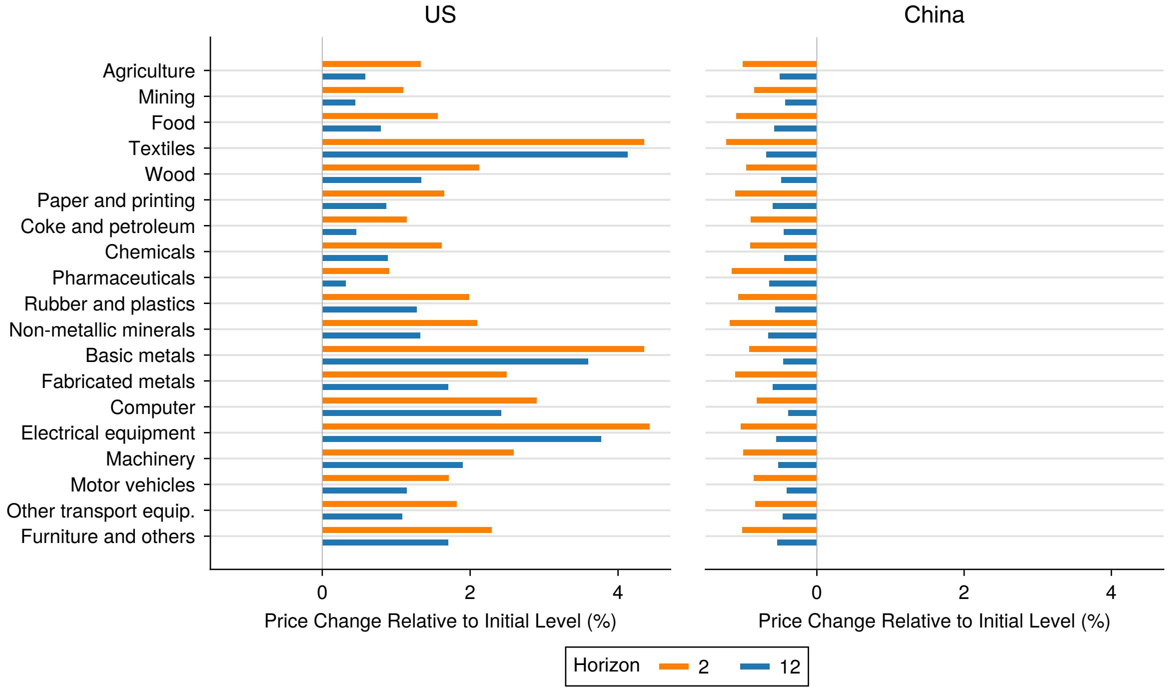

Figure B.1: Changes in Price Indices Among Directly Affected Industries

Note: Tariff changes implemented gradually over the first two years.

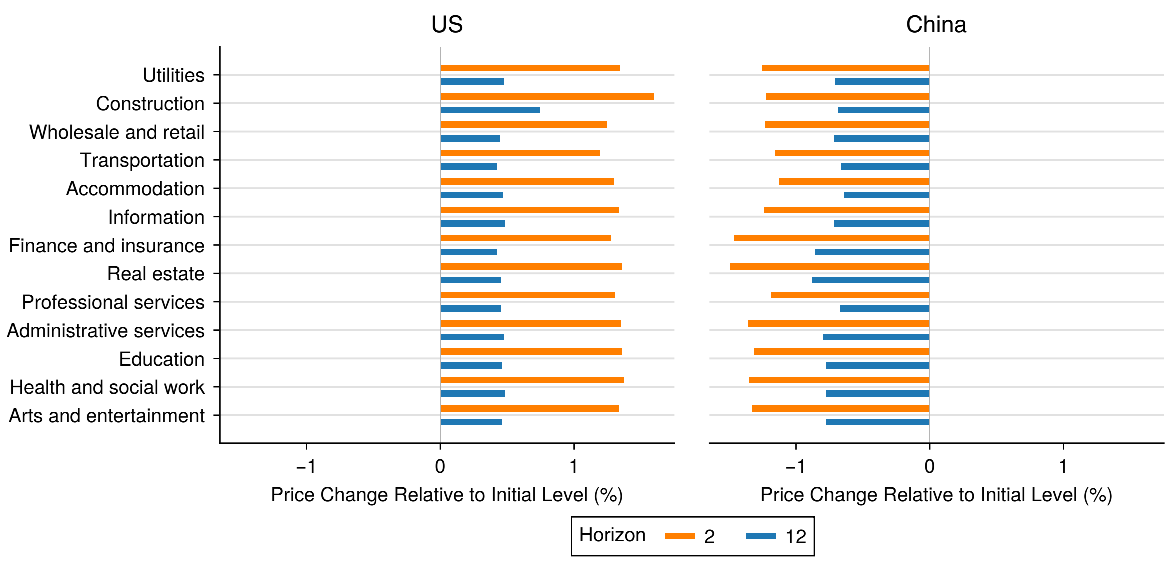

Figure B.2: Changes in Price Indices Among Industries Not Directly Affected

Note: Tariff changes implemented gradually over the first two years.

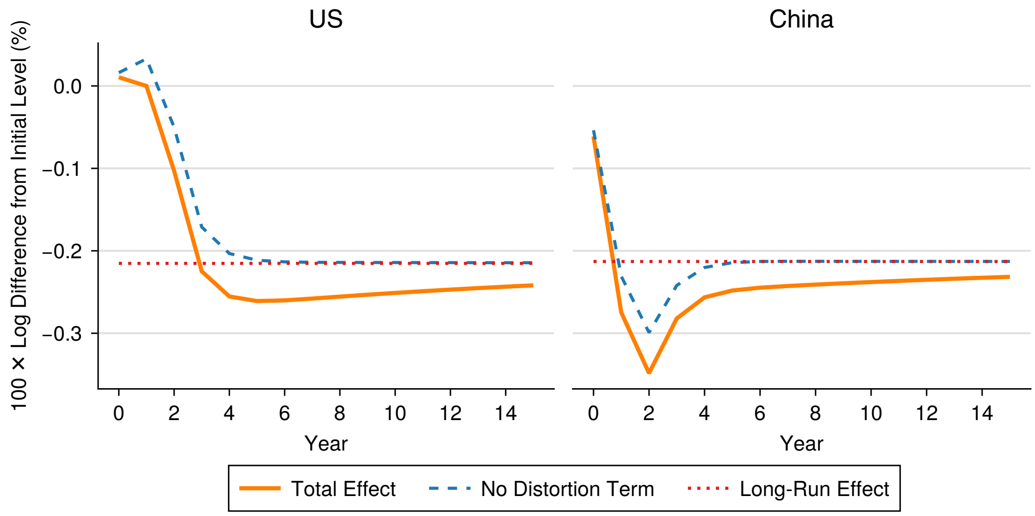

Figure B.3: Horizon-Specific Impact on Real Wages

Note: Tariff changes implemented gradually over the first two years. "Total Effect" in year t refers to the real wage changes between t and the initial steady state generated by the full model. "No Distortion" refers the change in real wages under the assumption that $\{\Xi_{dj,h}\} = 1$ for all d, j, h. "No Friction" refers to the change in real wages under the assumption that the economy reaches the long-run outcomes instantly ($\zeta = 1$).

Figure B.4: Changes in Prices and Wages in Mexico and Vietnam

Note: Tariff changes implemented gradually over the first two years. "Real Wage" in year t refers to real wage changes between t and the initial steady state generated by the full model. "Wage" refers to the corresponding change in the nominal wage. "Price" is the change in the aggregate consumer price index.