Energy Shocks as Keynesian Supply shocks: Implications for Fiscal Policy*

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Abstract

This paper analyses the economic impact of and the optimal policy response to energy supply shocks in a flexible price model with heterogeneous households. We introduce energy as a consumption good on the demand side and as an input to production on the supply side. A distinguishing feature is that, in line with empirical evidence, we allow households' energy demand to be non-homothetic. The model provides three main insights. First, (negative) energy supply shocks act as a (negative) demand shock, or Keynesian supply shock, when two conditions are met: On the demand side household income inequality needs to be large, while on the supply side, the price elasticity of consumption goods needs to be high. Second, the social planner can implement the first-best allocation by subsidising firms and poor households while taxing rich households. Energy shocks then act as standard supply shocks. Last, issuing public debt can help implement the first best allocation when energy shocks are large and/or the economy's overall energy intensity is low.

Keywords: Energy shocks, non-homothetic demand, heterogeneous households, fiscal policy, public debt.

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1 Introduction

The recent energy crisis has elicited much debate about its distributional and macroeconomic consequences and the appropriate fiscal (and monetary) policy response. One feature that has been highlighted is the negative impact of higher energy prices on the distribution of income and consumption. Poorer households typically spend a larger share of their income on energy (see Figure 1).[1] As a result, they experience a stronger decline in their real disposable income following negative energy shocks.

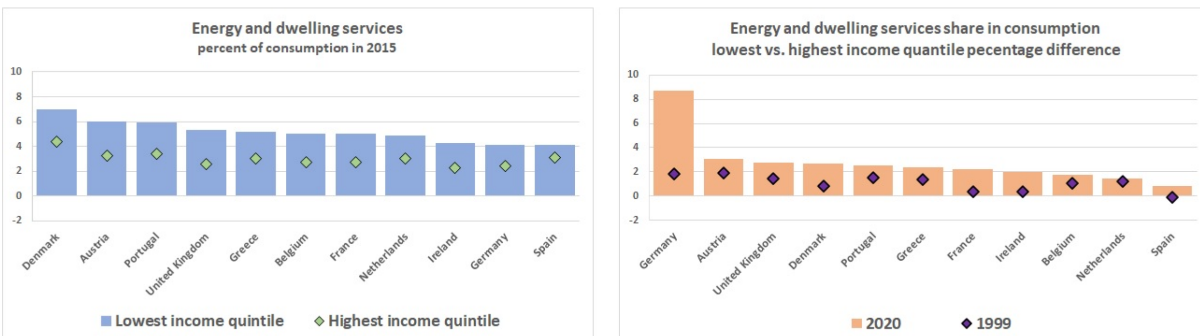

Figure 1: Poor households spend more on energy.

Moreover, as these households often have low saving buffers and are typically credit constrained, they tend to adjust their non-energy consumption much more than richer households in response to an adverse energy shock. Accordingly, the fiscal policy debate has focused on the need to target income support (in the form of subsidies or tax rebates) to the most vulnerable households. This can alleviate the negative distributional consequences and support aggregate demand, while limiting the fiscal cost.[2]

In this paper we develop a flexible price model with heterogeneous households and non-homothetic demand for energy to investigate the economic impact of and the optimal fiscal policy response to energy supply shocks. Like previous papers in the literature, we introduce energy as a consumption good on the demand side and as an input to production on the supply side.[3] In addition, our framework carries two distinguishing features. One

[1]This evidence, based on Eurostat Household Budget Survey, also shows that the difference between between the rich and the poor in the energy share in consumption has been growing over time.

[2]See, for instance, Ari et al. (2022) for a review of policy initiatives meant to alleviate the fallout of the energy crisis.

[3]see Kim and Loungani (1992), Hunt (2005), Bodenstein et al. (2008) or Dhawan and Jeske (2008) for a similar modelling.

is that households need to consume a minimum amount of energy.[4] As a result, as income rises, the energy share in total expenditure falls. Second, households are heterogeneous in their income as well as in their ability to tap credit markets. As a consequence, the minimum energy consumption barely matters for high-income households, but may become binding for low-income households, especially when negative income shocks hit.

In this environment, a negative energy supply shock typically has a larger impact on the ability of low-income, credit-constrained households to consume non-energy goods.[5] We trace out the implications of this feature for the aggregate economy and the optimal policy response investigating three main questions. First, what are the implications of a negative energy supply shock? Second, which distortions, if any, does the market allocation suffer from? Third, what tools can the social planner use to correct these distortions, and in particular, is there a role for public debt?

First, (negative) energy supply shocks can act as (negative) demand shocks under two conditions. On the one hand, household income inequality needs to be large. This requires the income gap between rich and poor households **and** the number of poor, credit-constrained, households, to be large. On the other hand, firms producing consumption goods should charge low markups, i.e. substituting between non-energy consumption goods should be easy. A negative energy supply shock makes energy more scarce and the economy as a whole, poorer. As such it should raise the equilibrium interest rate, as households want to borrow to smooth out the shock. However, low-income, credit-constrained households cannot borrow and have to cut on consumption. Moreover, households' demand for energy being non-homothetic, low-income households have to cut on their demand for consumption goods, because some of their demand for energy is sticky. As a result, aggregate demand for consumption goods falls and the relative price of consumption goods drops. The initial negative energy supply shock can then lead to a shortage of aggregate demand, turning the original shock into a so-called "Keynesian supply shock".[6]

[4]We follow here Geary (1950) and Stone (1954) in assuming that household derive utility from energy consumption only when it exceeds a certain minimum amount.

[5]Motivated by the recent European experience, Gornemann et al. (2023) also consider energy shocks as reductions in the quantity of energy, and investigate how such shocks can create self-fulfilling fluctuations.

[6]Generally speaking, Keynesian supply shocks are shocks to aggregate supply that lead to a shock to aggregate demand that is even larger than the original supply shock.

This happens when the household income gap and the fraction of low-income, credit-constrained households are large, as this implies a larger drop in the price of consumption goods. In addition, the price of consumption goods needs to be sufficiently responsive to the fall in demand, i.e. the price elasticity of demand for non-energy goods needs to be large.[7]

Second, the decentralised equilibrium suffers two distortions. First, because of monopolistic competition, firms do not consume enough energy. Second, because households are heterogeneous, high-income households consume too much while low-income households consume too little. The social planner can therefore replicate the first-best allocation by subsidising firms and low-income households while taxing high-income households. Under the first-best allocation, energy shocks always act as standard supply shocks. By equalising incomes across households, the social planner prevents the price externality due to low-income households disproportionately cutting on non-energy consumption. In addition, negative energy shocks require *proportionally* larger transfers to low-income households. This is because such shocks hurt low-income households twice: once as labour income falls *and* twice as a larger fraction of income needs to be devoted to energy consumption. Implementing the first-best therefore requires to tax a larger fraction of rich households' income, the larger the negative energy shock.

Finally, there is a role for public debt in implementing the social optimum, when the economy faces a large shock, and/or when the economy' overall energy intensity is low. In principle, there should not be any. Under the first-best allocation, there is no shortage of demand, as negative energy supply shocks always raise the equilibrium interest rate. However, under balanced budget, implementing the first-best allocation is possible, only insofar as tax rates can be set arbitrarily high. If there is an upper bound on tax rates, as is likely in practise, large negative energy shocks may require taxing high-income households beyond what is feasible. Issuing public debt can then bypass this problem. That said, the social planner must still ensure that future tax revenues —which are bounded by maximum

[7] Hence unlike other theories of Keynesian supply shocks looking at asymmetric shocks in the context of low elasticity of substitution between consumption goods (Guerrieri et al. (2022)), we highlight the role of inelastic demand for some goods like energy. Such inelastic demand can lead to a disproportionate reduction in the demand for other goods, paving the way for Keynesian supply shocks.

feasible taxation— are enough to service the debt *and* implement the first-best allocation once the economy is back to the steady state. Summarising these constraints, we derive a ”fiscal space” statistic, which computes the largest (negative) energy shock for which the social planner can still implement the first-best allocation, while ensuring public debt is sustainable. We show that ”fiscal space” is typically larger, when the share of energy in output and consumption is lower, and/or when household inter-temporal elasticity of substitution is higher.[8]

The literature on the economic effects of energy shocks is large (see for instance Kilian (2008) for a survey). The distributional consequences and the optimal policy response are, however, less well explored. Our paper contributes to a recent literature focusing on this angle. First, several empirical papers document the distributional consequences of energy price shocks. Battistini et al. (2022), Gelman et al. (2023) and Känzig (2021) show that low-income and/or liquidity-constrained households adjust their non-energy expenditures to energy price shocks to a larger extent. Similarly, Peersman and Wauters (2022) show, using Belgian survey data, that non-energy consumption is more sensitive to energy price increases than to energy price decreases. Marginal propensities to consume (MPCs) are also significantly larger for low-income and low-saving buffer households, while non-energy consumption declines more strongly for households who report higher uncertainty on their future financial situation. Our paper embeds these empirical findings in a two-agent macroeconomic model.

Second, a burgeoning literature uses Heterogeneous Agents New Keynesian (HANK) models to explore the distributional and macroeconomic implications of energy price shocks. Chan et al. (2022) develop a small, open-economy Two Agents New Keynesian (TANK) model, where labour and energy complement each other. Higher energy prices then reduce the labour share in total income, which depresses aggregate demand, because of borrowing constraints. In turn, price flexibility insures firm profits against adverse energy price shocks and further depresses labour income and demand.

Pieroni (2023) and Auclert et al. (2023) develop a full-scale HANK model to address

[8]Conversely ”fiscal space” is smaller when household income inequality is larger, and/or where public debt is higher to start with, these correlations being magnified in low energy-intensive economies.

similar questions. Auclert et al. (2023) focuses on the open economy implications of alternative monetary and fiscal policy responses. They show that an increase in imported energy prices can cause a recession by pushing down real wages and consumer spending, provided the elasticity of substitution between energy and domestic goods is realistically low. They also analyse the cross-border impact of alternative monetary and fiscal policies. Our paper also emphasises the possible negative demand effects of an energy supply shock, but our mechanism does not rely on a limited substitutability on the supply side. Instead we emphasise the empirically relevant non-homothetic energy demand on the household side and the implications for MPCs of credit-constrained households. We also characterise optimal fiscal policy in this environment.

The rest of the paper proceeds as follows. The next section lays out the main building blocks of the model. Section 3 derives the decentralised equilibrium and its main properties. The social optimum, its implementation and the implications for public debt issuance, are analysed in section 4. Finally conclusions are drawn in section 5.

2 The model

The model consists of households and firms. Households are heterogeneous. They supply labour to firms and consume goods that firms produce under monopolistic competition.[9] Moreover, households consume energy (in addition to consumption goods) while firms use energy as an input to production (in addition to labour). Importantly, the demand for energy on the household side is non-homothetic. Let us now describe the model's main assumptions more systematically, starting with households and following with firms.

[9] We restrict heterogeneity to the household side, while firms are assumed to be symmetric. Introducing heterogeneity on the firm side would add further mechanisms in the model, not least in terms of reallocation across firms and sectors, that we leave out of this paper.

2.1 The demand side

2.1.1 Household preferences

Households live infinitely. Each period, they consume energy E and a composite consumption good C. In addition they are endowed each period with some quantity of energy and some quantity of labour —the latter is normalised to one for simplicity. Households' preferences U write as:

$$U = \sum_{t \geq 0} \beta^t \frac{[u(E_t; C_t)]^{1-\frac{1}{\gamma}} - 1}{1 - \frac{1}{\gamma}} \tag{1}$$

with

$$u(E_t; C_t) = \left[\delta_h^{\frac{1}{\sigma_h}} (E_t - e_h)^{1-\frac{1}{\sigma_h}} + (1-\delta_h)^{\frac{1}{\sigma_h}} (C_t)^{1-\frac{1}{\sigma_h}} \right]^{\frac{1}{1-\frac{1}{\sigma_h}}} \text{ if } E_t \geq e_h \tag{2}$$

Here γ denotes households' inter-temporal elasticity of substitution, σ_h denotes households' (intra-temporal) elasticity of substitution between energy and consumption goods and β is the rate at which households discount the future. Following Geary (1950) and Stone (1954), we introduce a minimum (subsistence) consumption level —denoted e_h— for energy, which acts as a lower bound on households' demand for energy. Households therefore devote their income to energy consumption up to $E_t = e_h$ and then split whatever is left between energy and consumption goods, the share of energy in household expenditures being then δ_h.[10] The composite consumption good C, is in turn, a CES aggregation of consumption goods produced across the different sectors in the economy:

$$C_t = \left[\int_0^1 [c_{st}]^{1-\frac{1}{\eta}} ds \right]^{\frac{1}{1-\frac{1}{\eta}}} \tag{3}$$

Here $\eta > 1$ is the elasticity of substitution between the different consumption goods. Denoting p_{et} the price of energy at time t and p_{st} the price of consumption good s at time t, the price of the composite consumption good p_{ct} and the general price level P_t —assuming energy consumption E_t exceeds the minimal level e_h— satisfy the usual expressions:

$$p_{ct}^{1-\eta} = \int_0^1 p_{st}^{1-\eta} ds \quad \text{and} \quad P_t^{1-\sigma_h} = \delta_h p_{et}^{1-\sigma_h} + (1-\delta_h) p_{ct}^{1-\sigma_h} \tag{4}$$

[10]Household demands for energy and consumption goods are formally derived in the next sections.

2.1.2 Household heterogeneity

There are two types of households. A first group of households —called *unconstrained* households— is endowed with a quantity of energy $(en)_u$ that exceeds the minimum energy consumption e_h: $(en)_u > e_h$; owns the firms producing consumption goods; and can freely lend and borrow. A second group of households —called *constrained* households— is endowed with a quantity of energy $(en)_c$ that falls short of the minimum energy consumption e_h : $(en)_c < e_h$; has no ownership rights over firms producing consumption goods; and cannot save nor borrow. A fraction ϕ of households are constrained.

2.1.3 Household income and demand for energy and consumption goods

Let e_c denote constrained households' net energy needs, i.e. $e_c = e_h - (en)_c > 0$. Then constrained households' expenditures R_t^c are simply the difference between labour income w_t and the cost $p_{et}e_c$ of net minimum energy needs:

$$R_t^c \equiv [w_t - p_{et}e_c]^+ \tag{5}$$

Turning now to the case of unconstrained households, and denoting e_u their energy endowment net of the minimum energy consumption, i.e. $e_u = (en)_u - e_h > 0$, expenditures R_t^u writes as:

$$R_t^u \equiv w_t + p_{et}e_u + \frac{\pi_t}{1-\phi} + (1 + i_{t-1})\, s_{t-1} - s_t \tag{6}$$

Here s_t denotes savings at time t, π_t the firms' profits at time t, and i_t is the nominal interest rate on savings s_t. Based on these expressions, and denoting R_t household aggregate net expenditures, i.e. $R_t = \phi R_t^u + (1-\phi)R_t^c$, demands by households for energy and for the consumption goods respectively write as:

$$E_t = e_h + \delta_h \left[\frac{p_{et}}{P_t}\right]^{-\sigma_h} \frac{R_t}{P_t} \quad \text{and} \quad C_t = (1-\delta_h)\left[\frac{p_{ct}}{P_t}\right]^{-\sigma_h} \frac{R_t}{P_t} \quad \text{and} \quad C_{st} = \left[\frac{p_{st}}{p_{ct}}\right]^{-\eta} C_t \tag{7}$$

As Figure 2 below shows, households' marginal propensity to consume energy (energy MPC) is constant equal to δ_h but the *average* propensity to consume energy (energy APC) gradually decreases with income from 1 when net income R_t is zero to δ_h when net income

R_t becomes very large, consistent with energy accounting for a larger fraction of lower-income households' consumption basket.

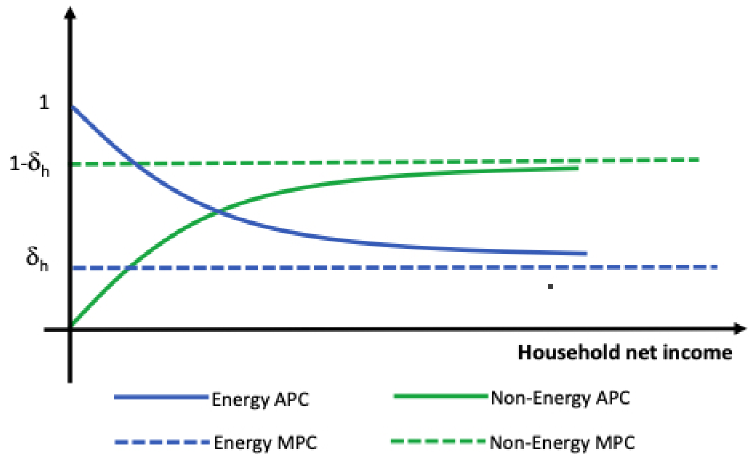

Figure 2: Households' average and marginal propensities to consume.

Last, unconstrained households set their net savings $[(1 + i_{t-1})\, s_{t-1} - s_t]$ at time t, consistent with the usual Euler equation:

$$\frac{R_t^u}{P_t} = \left[\frac{\beta\,(1 + i_t)}{P_{t+1}/P_t}\right]^{-\gamma} \frac{R_{t+1}^u}{P_{t+1}} \tag{8}$$

2.2 The supply side

2.2.1 Technology

Firms produce consumption goods using labour which they hire from households, and energy which they purchase from unconstrained households. Specifically, output y_{st} at time t for the firm operating in sector s, writes as:

$$y_{st} = \left[\delta_f^{\frac{1}{\sigma_f}} E_{st}^{1-\frac{1}{\sigma_f}} + \left(1 - \delta_f\right)^{\frac{1}{\sigma_f}} L_{st}^{1-\frac{1}{\sigma_f}}\right]^{\frac{1}{1-\frac{1}{\sigma_f}}} \tag{9}$$

Here E_{st} denotes firm's s energy consumption at time t and L_{st} the amount of labour firm s hires at time t from households. The elasticity of substitution between energy and labour is σ_f while δ_f measures the energy intensity of output.

2.2.2 Firms optimal pricing and demand for inputs

Let mc_t denote the marginal cost to produce consumption goods at time t. Firm s then sets its price p_{st} to maximise profits, given the marginal cost mc_t and household demand for its good C^*_{st}:

$$\max_{p_{st}} \quad \pi_{st} = [p_{st} - mc_t]\, C_{st}$$
$$\text{s.t.} \begin{cases} C_{st} = \left[\frac{p_{st}}{p_{ct}}\right]^{-\eta} C_t \quad \text{and} \quad mc_t^{1-\sigma_f} = \delta_f p_{et}^{1-\sigma_f} + \left(1-\delta_f\right) w_t^{1-\sigma_f} \\ C_t = (1-\delta_h)\left[\frac{p_{ct}}{P_t}\right]^{-\sigma_h} \frac{R_t}{P_t} \quad \text{and} \quad R_t = \left(1-\phi\right) R_t^u + \phi R_t^c \end{cases} \tag{10}$$

Here the demand C_{st} for consumption good s depends on three terms. The first reflects the choice between different consumption goods, the second, the choice between energy and consumption goods and the last reflects the composition of aggregate expenditures R_t between constrained and unconstrained households.

The optimal price p_{st} of consumption good s at time t simply writes as a constant markup μ over the marginal cost, $p_{st} = \mu mc_t$ with $\mu = \frac{\eta}{\eta-1}$, this being also the expression for the composite price index for consumption goods: $p_{ct} = \mu mc_t$, as firms are symmetric. The equilibrium level of output in sector s is then:

$$y_{st} = \frac{1-\delta_h}{\mu}\left[\frac{p_{ct}}{P_t}\right]^{1-\sigma_h} \frac{R_t}{mc_t} \tag{11}$$

As is visible from (11), sectoral output y_{st} has standard properties: decreasing in the marginal cost of production mc_t as well as in the markup μ, but increasing in the share of expenditures $1-\delta_h$ spent on consumption goods, in addition to aggregate expenditures R_t. Then using (11), profits in sector s satisfy:

$$\pi_{st} = \frac{\mu - 1}{\mu}(1-\delta_h)\left[\frac{p_{ct}}{P_t}\right]^{1-\sigma_h} R_t \tag{12}$$

Under the assumption $\eta > \sigma_h \geq 1$, the comparative statics for profits π_{st} and output y_{st} are broadly similar: profits decrease in the marginal cost mc_t but increase with the share of income $1-\delta_h$ spent on consumption goods as well as with households' expenditures R_t.

Last, firms' demands for energy and labour are simply the sum of energy and labour demands across firms. Using the expression in (11) for firm-level output, aggregate energy and labour demand by firms respectively writes as:

$$E_f = \int_0^1 \delta_f \left[\frac{p_{et}}{mc_t}\right]^{-\sigma_f} y_{st} ds = \frac{1-\delta_h}{\mu}\left[\frac{p_{ct}}{P_t}\right]^{1-\sigma_h} \frac{\delta_f R_t}{p_{et}^{\sigma_f} mc_t^{1-\sigma_f}} \tag{13}$$

and

$$L_f = \int_0^1 \left(1-\delta_f\right)\left[\frac{w_t}{mc_t}\right]^{-\sigma_f} y_{st} ds = \frac{1-\delta_h}{\mu}\left[\frac{p_{ct}}{P_t}\right]^{1-\sigma_h} \frac{\left(1-\delta_f\right) R_t}{w_t^{\sigma_f} mc_t^{1-\sigma_f}} \tag{14}$$

Having determined households' demand for energy and consumption goods as well as firms' demand for energy and labour, we can now solve for the general equilibrium.

3 The decentralised equilibrium

In this economy, an equilibrium is a vector of prices and quantities such that:
(i) Household demands for energy and consumption goods maximise intra-temporal utility; household borrowing and savings maximise inter-temporal utility.
(ii) Firm' demands for energy and labour minimise total cost of output; consumption good prices maximise profits.
(iii) The price of consumption goods balance households' demand and firms' supply; the wage rate balances households' labour supply and firms' labour demand; the interest rate balances the market for lending and borrowing.

To derive the decentralised equilibrium, we first need to determine the equilibrium of the labour market. Then we can close the model and go through the main take-aways. Considering a symmetric equilibrium, setting the elasticities of substitution σ_h and σ_f to one, and denoting $1-\delta = \left(1-\delta_f\right)(1-\delta_h)$, we can derive the following result.

Proposition 1 *When constrained households' can cover their minimum energy consumption, the equilibrium wage rate $\omega = w_t/p_{et}$ satisfies*

$$\left(1-\phi\right)(\omega + e_u) + \phi\,(\omega - e_c) = \frac{\omega}{1-\delta}\left[1 + \delta_h(\mu - 1)\right] \tag{15}$$

Proof 1 *cf. Appendix A.1.*

As noted in the proposition, this equilibrium exists only insofar as constrained households can cover their minimum energy consumption e_c out of their labour income ω. The following lemma formally derives this necessary condition.

Lemma 2 *Denoting the economy's net energy endowment* $e = (1-\phi)e_u - \phi e_c$*, the equilibrium described in (15) exists, if and only if:*

$$\frac{e_c}{e} \leq \overline{\theta} \equiv \frac{1-\delta}{\delta + \delta_h(\mu - 1)} \tag{16}$$

In this case, households' aggregate real income is $R/P = \mu^{\delta_h} \left[\frac{\delta}{\delta+\delta_h(\mu-1)}\right]^{\delta} \frac{e^{\delta}}{\delta^{\delta}(1-\delta)^{1-\delta}}$

Proof 2 *cf. Appendix A.2.*

When condition (16) holds, constrained households' energy need e_c is sufficiently small that firms can then use a large volume of energy to produce consumption goods. As firms produce large volumes of output, they pay out high wages, which ensures that constrained households are able to cover their minimum energy consumption e_h at the equilibrium (see Figure 3 below). We will therefore assume from now on, that condition (16) holds.

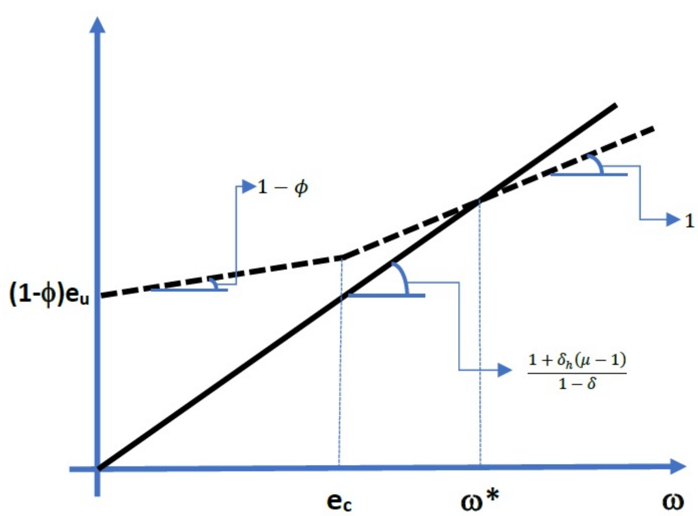

Figure 3: The decentralised equilibrium.

3.1 Demand and supply effects of energy shocks

How do output and real incomes vary in response to shocks affecting energy endowments? From the discussion above, one can easily conclude that constrained households are trivially worse-off when their energy endowment falls, i.e. when their energy needs e_c go up. But what about unconstrained households? Are they always worse-off when their energy endowment and that of households in general, shrinks? Or are there cases where they could end up better-off? The following proposition sheds some light on this question.

Proposition 3 *Denoting* $\lambda = (1 + \delta_h(\mu - 1) - \frac{\delta\mu}{\phi}) / (1 - \delta)^2$*, an unexpected and temporary shock that uniformly reduces energy endowments leaves unconstrained households better-off and acts as a negative aggregate demand shock when:*

$$\frac{e_c}{e} \geq \underline{\theta} \equiv [1 - \lambda] \frac{1 - \delta}{\delta + \delta_h (\mu - 1)} \tag{17}$$

Proof 3 *cf. Appendix A.3.*

Following a negative shock that equally cuts the energy endowment of constrained and unconstrained households, the equilibrium wage rate falls, which cuts both households' income and the price of consumption goods. As a result, both *constrained* and *unconstrained* households are poorer and reduce their demand for consumption goods. However, constrained households, as their income falls closer to the minimum energy consumption, have to cut their purchases of consumption goods disproportionately, given that a large fraction of their demand for energy is sticky and cannot be compressed. As constrained households cut their demand for consumption goods more than one-to-one, the price of consumption goods has to adjust downwards, and in some cases the drop in the price of consumption goods can actually be larger than the fall in unconstrained households' income. Expression (18) isolates these two opposite forces. Writing the equilibrium wage rate as $\omega^* = we$, unconstrained households' real expenditures write as

$$\frac{R^u}{P} = \underbrace{\left[\frac{e^\delta}{w^{1-\delta}}\right]}_{(-)} \underbrace{\left[1 + w - \left[\delta_h + \frac{1 - \delta_h}{\mu}\right] \phi \left[1 - w \frac{e_c}{e}\right]\right]}_{(+)} \tag{18}$$

On the one hand, a negative energy supply shock —a fall in e— reduces expenditures expressed in energy units. This is the first term in (18). On the other hand, a negative energy supply shock —an increase in e_c/e— creates a disproportionate fall in the demand for consumption goods, which cuts the relative price of consumption goods, expressed in energy units. This price effect is the second term of (18). The positive price effect then dominates the negative expenditure effect when condition (17) holds, i.e. when the energy need e_c of unconstrained households is large relative to the economy's total energy endowment e. In this case, constrained households have to make a significant cut to their demand for consumption goods when their income falls, which leaves unconstrained households better-off.

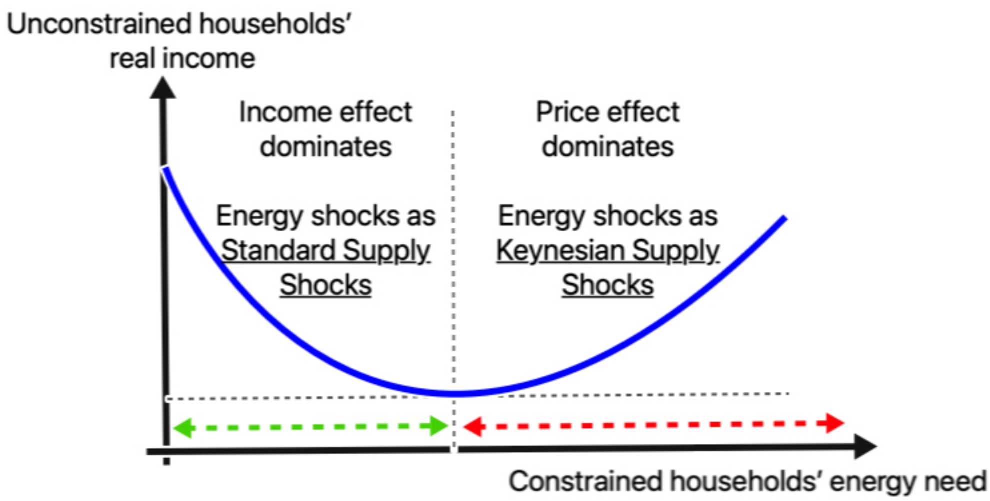

Figure 4: Negative energy supply shocks as negative demand shocks.

Figure 4 above plots unconstrained households' real income as a function of constrained households' energy need e_c. The figure shows that a negative energy shock —that raises constrained households' energy need relative to the economy's overall energy endowment— first reduces the real income of unconstrained households. In this (green) region, consumption patterns of constrained and unconstrained households are very similar, because constrained households' energy needs are relatively low. As a consequence, the relative price of consumption goods falls less quickly than unconstrained households' income following a negative energy shock. However, when constrained households' energy needs are larger (red region), the difference in consumption patterns between constrained and unconstrained households widens and a negative energy shock triggers a significant fall in constrained household demand for consumption goods. The fall in the relative price of consumption goods then dominates the fall in household income and, unconstrained households' real income goes up. Negative energy shocks therefore make unconstrained better-off when the dispersion in energy endowments between constrained and unconstrained households is sufficiently large. In this case, the equilibrium interest rate r_t that is governed by the Euler equation (8) has too fall, which is typical of a shortage of aggregate demand:

$$1 + r_t = \frac{1 + i_t}{P_{t+1}/P_t} = \frac{1}{\beta}\left[\frac{R^u_{t+1}/P_{t+1}}{R^u_t/P_t}\right]^{\frac{1}{\gamma}} \Rightarrow \frac{\partial\left(R^u_t/P_t\right)}{\partial e} \leq 0 \Leftrightarrow \frac{\partial r_t}{\partial e} \geq 0$$

Aggregate demand shortages, or Keynesian supply shocks, can happen only if the parameter λ is positive. This typically requires that the fraction ϕ of constrained households to be sufficiently large and the markup μ charged by firms producing consumption goods be sufficiently low. When ϕ is high and there are many constrained households, a negative energy supply shock implies a larger negative shock to the demand for consumption goods, hence a larger fall, everything else equal, in the relative price of consumption goods. In addition, the drop in the relative price of consumption goods depends not only the drop in aggregate demand for consumption goods, but also on the elasticity of the inverse demand function, and hence on the markup μ. When the markup is low, the fall in the demand for consumption goods produces a large drop in the relative price of consumption goods, so that a negative energy supply shock creates a larger drop in the price of consumption goods and is hence more likely to raise unconstrained households' real income.[11]

4 The social optimum

The social planner can improve on the decentralised equilibrium allocation in two ways. First, there are welfare gains in redistributing income between constrained and uncon-

[11]Note that if some prices were sticky, prices which are flexible would have to fall by more, requiring an even lower markup. The Keynesian supply shock property therefore rests on the assumption that there is a sufficient degree of price flexibility in the economy.

strained households as income differences imply differences in marginal utilities of consumption, and hence welfare losses. Second, households and firms compete in the decentralised equilibrium for energy and there can be welfare gains in redistributing energy consumption between households and firms. Let us first focus on this last issue, derive the socially optimal allocation and compare it to that of the decentralised equilibrium.

4.1 The socially optimal allocation of energy.

Setting aside household heterogeneity, at the social optimum, the planner allocates energy to households and firms such that the allocation maximises households' welfare under the constraints that (i) households' consumption (of consumption goods) is equal to firms' output; (ii) firms use all of households' labour supply; and (iii) the sum of energy consumption by households and firms cannot exceed the total energy endowment in the economy. Denoting E_h amd E_f the amount of energy allocated respectively to households and firms, the problem for the social planner therefore writes as:

$$\max_{E_h;E_f} \; u\left(E_h;C\right) = \left[\delta_h^{\frac{1}{\sigma_h}}\left(E_h - e_h\right)^{\frac{\sigma_h-1}{\sigma_h}} + \left(1-\delta_h\right)^{\frac{1}{\sigma_h}} C^{\frac{\sigma_h-1}{\sigma_h}}\right]^{\frac{\sigma_h}{\sigma_h-1}}$$

$$\begin{cases} C \leq \left[\delta^{f\,\frac{1}{\sigma}} E_f^{\frac{\sigma-1}{\sigma}} + \left(1-\delta_f\right)^{\frac{1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \\ E_f + E_h - e_h \leq e \end{cases} \tag{19}$$

As is clear, household labour supply is fully used at the social optimum, so that $L = 1$ and the upper bound on household consumption C only depends on the amount of energy E_f allocated to firms. We can then derive the following proposition.

Proposition 4 *Firms and constrained households consume too little energy in the decentralised equilibrium relative to the social optimum. Conversely, unconstrained households consume too much energy relative to the social optimum.*

Proof 4 *cf. Appendix A.4.*

The decentralised equilibrium features two types of inefficiencies. The first is the standard distortion due monopolistic competition. The second is the distortion due to the non-homotheticity in households' energy consumption. First, because of monopolistic competition, firms produce too little output. As a result, they use too little energy and the social planner corrects this inefficiency by allocating more energy to firms. Second, in the decentralised equilibrium, constrained households' consumption of both energy and purchases of consumption goods is inefficiently low, as their marginal utility of consumption is always higher than that of unconstrained households. This is why a social planner who wants to implement the first-best-allocation needs to address two issues: First it needs to neutralise the monopolistic competition distortion and provide firms with incentives to consume more energy. Second, it needs to equalise incomes across constrained and unconstrained households, which would eliminate welfare losses stemming from differences in marginal utilities of consumption.

4.2 Decentralising the socially optimal allocation

Consider now a social planner who aims at implementing the first-best allocation, using taxes and subsidies. Starting with firms, given that they do not consume enough energy in the decentralised equilibrium, it makes sense for the social planner to subsidise firms' energy consumption as it would specifically address firms' energy consumption gap. Let us therefore assume the social planner grants a subsidy s_f to firms on energy consumption so that firms pay a price $\left(1 - s_f\right) p_e$ for energy instead of the market price p_e.[12] Turning now to households, given that unconstrained households consume too much energy and consumption goods, while constrained consume too little (of both energy and consumption goods), it is sensible for the social planner to raise a lump-sum tax T^u on the former and extend a lump-sum subsidy S^c to the latter. Given this tax/subsidy scheme, denoting $\omega(s_f, S^c, T^u)$ the wage rate —expressed in energy units—, constrained households' expenditures become $R^c/p_e = \omega(s_f, S^c, T^u) + S^c - e_c$, while unconstrained households' expenditures write as $R^u/p_e = \omega(s_f, S^c, T^u) - T^u + e_u + \pi(s_f, S^c, T^u)/(1 - \phi)$. In this last expression, $\pi(s_f, S^c, T^u)$ denotes firms' profits —expressed in energy units—, when the social planner

[12]The energy subsidy is by no means the sole possibility to correct for the monopolistic competition distortion, a production subsidy would also work. However, this would be more expensive as it would end up subsidising all production factors, including labour, which is unnecessary in this context.

extends a subsidy s_f to firms, a subsidy S^c to constrained households and imposes a tax T^u on unconstrained households. In this framework, the tax/subsidy policy implements the first-best allocation if and only if three conditions are satisfied:

- Firms and households should consume their first-best level of energy,
- Marginal utilities of consumption should be equalised across households,
- Tax revenues should cover for subsidy expenditures.

Considering the special case where elasticities of substitution σ_h and σ_f are both set to one, we can derive the following result.

Proposition 5 *The social planner can implement the first-best allocation, by paying a subsidy s_f to firms per unit of energy consumed, a subsidy S^c to constrained households, and raising a tax T^u on unconstrained households, such that:*

$$(1-s_f)\mu = 1 \quad \textit{and} \quad \frac{S^c}{p_e} = e_c + \frac{\mu-(1-\delta)}{\mu\delta}e \quad \textit{and} \quad \left(1-\phi\right)\frac{T^u}{p_e} = \phi S^c + \frac{\mu-1}{\mu}\frac{\delta-\delta_h}{\delta}e \tag{20}$$

In this case, the wage rate ω^o and households' real income R^o/P^o write as

$$\omega^o = \frac{1}{\mu}\frac{1-\delta}{\delta}e \quad \textit{and} \quad \frac{R^o}{P^o} = \frac{e^\delta}{(1-\delta)^{1-\delta}\,\delta^\delta} \tag{21}$$

Proof 5 *cf. Appendix A.5.*

The tax/subsidy scheme that implements the first-best allocation has four important properties. First, the subsidy extended to firms s_f is strictly positive and independent of the economy's energy endowment e. When the elasticity of substitution between energy and labour σ_f is one, firms' total energy consumption moves one-for-one with the wage rate ω which itself moves one-for-one with the economy's energy endowment e. As a consequence, the monopolistic competition distortion can be corrected with a constant subsidy per unit of energy consumed.

Second, the wage rate under the first-best allocation is lower than under the decentralised equilibrium, i.e. $\omega^o < \omega^*$. When firms get a subsidy for energy consumption, they increase their demand for energy, but they also reduce their demand for labour as energy and labour are partly substituable.[13] Lower labour demand then naturally translates into a lower equilibrium wage rate. The fall in the equilibrium wage rate in turn affects households' demand for consumption goods through the usual income and substitution effects. The former implies lower demand for both consumption goods and energy, while the latter leads households to substitute consumption goods for energy. When households' elasticity of substitution σ_h is one, these two effects cancel each other, leaving the demand for consumption goods unchanged. The demand for consumption goods being unaffected, the subsidy extended to firms ends up depressing the equilibrium wage rate.

Third, under the first-best allocation, households' real income R^o/P^o is increasing in the energy endowment e. As a result, negative energy supply shocks typically raise the equilibrium rate of interest, insofar as they reduce unconstrained household's real income. In other words, negative energy shocks always act as negative supply shocks under the first-best allocation.

Fourth and last, the subsidy extended to constrained households and hence the tax levied on unconstrained households, are both increasing in the economy's energy endowment e, i.e. $\partial S^c/\partial e > 0$ and $\partial T^u/\partial e > 0$. A negative energy supply shock, that cuts the economy's energy endowment e therefore reduces the subsidy S^c extended to constrained households, and the tax T^u imposed on unconstrained households both fall. However, the energy need e_c of constrained households being non-zero, the elasticity of taxes and subsidies to energy shocks is less than one:

$$\frac{\partial S^c}{\partial e}\frac{e}{S^c} < 1 \text{ and } \frac{\partial T^u}{\partial e}\frac{e}{T^u} < 1 \tag{22}$$

Based on this property, we can then derive the following lemma.

Lemma 6 *Implementing the first-best allocation requires taxing a larger fraction of unconstrained households' income when the economy faces a negative energy shock.*

Proof 6 *cf. Appendix A.6.*

[13] As is clear, the higher the elasticity of substitution σ_f, the larger the drop in labour demand

A negative energy shock that cuts households' energy endowments therefore requires to impose a larger tax rate on unconstrained households. Following negative energy shocks, constrained households take a double hit, one from the fall in the energy endowment —which leads to an increase in the minimum energy consumption e_c— *and* one from the fall in labour income ω. The marginal utility of consumption of constrained households therefore increases much more steeply than that of unconstrained households. Equalising incomes then requires transferring a larger fraction of unconstrained households' income to constrained households. In practise, this means that negative energy shocks, in spite of being uniformly distributed, act as a distributional shock, that hurt low-income, credit-constrained households, to a larger extent.

The comparative static results derived above have one obvious implication: Given that the social planner needs to impose a higher tax rate on unconstrained households in the face of negative energy shocks, i.e. at a time where incomes are lower, implementing the first-best allocation may become impossible if this implies imposing tax rates in excess of what is feasible in practise. The next proposition looks into this question.

Proposition 7 *Denoting $\overline{\tau}$ the maximum feasible tax rate, the social planner can implement the first-best allocation through subsidies to firm energy consumption and taxes/subsidies to households only if constrained households' net energy need satisfies $e_c/e \leq \theta_{fb}$ where*

$$\theta_{fb} = \left[\overline{\tau}\frac{\mu - \phi(1-\delta)}{\delta + \delta_h(\mu - 1)} - \phi\frac{\mu - (1-\delta)}{\mu\delta} - \frac{\mu - 1}{\mu}\frac{\delta - \delta_h}{\delta}\right]\frac{1}{(1-\overline{\tau})\phi} \tag{23}$$

Proof 7 *To replicate the first-best allocation, the social planner needs to extend taxes and subsidies in line with (20). Then assuming tax rates cannot exceed $\overline{\tau}$, implementing the first-best allocation through a set of taxes and subsidies is not feasible unless $T^u \leq \overline{\tau}R^u$, which simplifies as $e_c/e \leq \theta_{fb}$, with θ_{fb} satisfying (23).*

According to this proposition, the social planner is unable to implement the first best allocation —by relying solely on taxes and subsidies— when the energy need e_c of constrained households is large relative to the overall economy's energy endowment, which typically coincides with the condition under which energy shocks turn into Keynesian supply shocks and create a shortage of demand. Expression (23) also shows that the upper bound θ_{fb} is increasing in the maximum tax rate $\overline{\tau}$. In other words, unless $\overline{\tau}$ is sufficiently large, the planner may be unable to implement the first-best. There may even be cases where θ_{fb} turns negative implying that even in the case where constrained households' energy endowment covers for their minimum energy consumption, i.e. $e_c = 0$, tax revenues raised from unconstrained households would fall short of the subsidy extended to firms —to correct for the monopolistic competition distortion—, *and* the subsidy extended to constrained households —to equalise marginal utilities of consumption across households. Similarly, when θ_{fb} is positive, a larger fraction ϕ of constrained households tends to reduce the upper bound on constrained households' energy needs e_c below which the social planner can still implement the first-best allocation. In other words, a negative energy shock is then more likely to push the economy in the region where implementing the first-best allocation by relying solely on taxes and subsidies becomes impossible. In this case, the social planner may contemplate additional tools. Public debt could be one of them. We investigate this possibility in the next section.

4.3 Optimal public debt issuance.

Let us consider a steady state where the energy endowment e —and its distribution across constrained and unconstrained households— are such that the social planner can implement the first-best allocation, by taxing unconstrained households and subsidising firms' energy consumption and constrained households, so that the condition $e_c/e \leq \theta_{fb}$ holds. Or put differently, the tax rate τ_{ss} the social planner needs to impose on unconstrained households falls strictly below the maximum feasible tax rate $\overline{\tau}$. Then, consider an unexpected negative energy shock, say at date t, that cuts energy endowments uniformly across constrained and unconstrained households, so that the condition $e_c/e \leq \theta_{fb}$, stops holding and the social planner cannot anymore implement the first-best allocation, by solely relying on taxes and subsidies. We now ask if there is a role in this setting for public debt. More specifically, if the social planner cannot implement the first-best allocation because this would imply a tax rate that would exceed the maximum $\overline{\tau}$, could it be that issuing debt to raise the missing resources makes the economy as a whole better-off?

To answer these questions, let us describe the social planner's problem in more details.

Specifically the unexpected, temporary, shock that hits the economy at date t cuts the energy endowment from e down to $(1 - \varepsilon_t)e$, with $0 < \varepsilon_t < 1$. Unconstrained households' net energy endowment therefore drops from e_u to $e_u - \varepsilon_t e$ while constrained households' net energy need increases from e_c to $e_c + \varepsilon_t e$. Moreover, as indicated above, the negative energy shock ε_t is sufficiently large that the social planner cannot implement the first-best by relying solely on subsidies and taxes.[14] Finally from date $t + 1$ onward, the energy shock disappears, energy endowments come back to their pre-shock levels, and the social planner can implement again, the first-best allocation by relying on taxes and subsidies.

While the social planner cannot implement the first-best allocation at date t, as this would imply setting the tax rate on unconstrained households above the maximum possible tax rate $\overline{\tau}$, it can still set the tax rate at its maximum level $\overline{\tau}$ and make up for the missing revenues by issuing debt. The amount of debt issued then writes as:

$$d_t = T^u(\varepsilon_t) - \overline{\tau} R^u(\varepsilon_t) \equiv [\tau(\varepsilon_t) - \overline{\tau}] R^u(\varepsilon_t) \tag{24}$$

In this expression, $T^u(\varepsilon_t)$ represents the taxes the social planner needs to raise to pay out subsidies to constrained households and firms, when the economy is hit with a negative energy shock ε_t and $R^u(\varepsilon_t)$ stands for the income, unconstrained households devote to purchases of energy and consumption goods when the economy is hit with a negative energy shock ε_t. Finally $\tau(\varepsilon_t)$ is the tax rate that would need to be imposed on unconstrained households to implement the first-best allocation when the economy faces a negative energy shock ε_t.

Issuing debt, however, implies paying interest. How much interest the social planner has to pay at date $t+1$ for the debt issued at date t depends on unconstrained households' Euler equation, which in turn depends on unconstrained households' real incomes at date t and date $t+1$, respectively R_t^u/P_t and R_{t+1}^u/P_{t+1}. Given the expression for households' real income under the first-best allocation (21), the real interest rate at date t, i.e. at the time of

[14] Meanwhile, we restrict the energy shock ε_t to ensure the equilibrium still exists and constrained households are still able to consume a strictly positive amount of consumption goods.

the negative energy shock, satisfies:

$$1 + r_t = \frac{1}{\beta}\left[\frac{R^u_{t+1}/P_{t+1}}{R^u_t/P_t}\right]^{\frac{1}{\gamma}} = \frac{1}{\beta(1-\varepsilon_t)^{\frac{\delta}{\gamma}}} \tag{25}$$

Under the first-best allocation, the real interest rate increases when a negative energy shock hits, as unconstrained households who are temporarily poorer would like to borrow to smooth out the shock. Moreover, the increase in the real interest rate is larger when the energy intensity δ is higher. As would be expected, a negative energy shock raises the equilibrium interest rate disproportionately in economies which are more energy intensive.

Then from date $t+1$, when the economy is back to the steady-state, the social planner can simply implement the first-best allocation by raising taxes and paying out subsidies. It does however need to fund the debt issued at date t —to address the negative energy shock—, which requires raising some additional taxes, to make sure the debt does not blow up out of proportion. Given expression (25) for the interest rate r_t at the time of the energy shock, the expression for the date-$t+1$ value of the debt issued at date t simplifies as:

$$d_{t+1} = (1+r_t)(\tau(\varepsilon_t) - \overline{\tau})R^u(\varepsilon_t) = (1+r_t)\frac{\tau(\varepsilon_t) - \overline{\tau}}{1-\tau(\varepsilon_t)}R^o \tag{26}$$

where the second equality derives from the property that the tax rate $\tau(\varepsilon_t)$ that implements the first best allocation satsifies $(1-\tau(\varepsilon_t))R^u(\varepsilon_t) = R^o$.

Conversely, since the social planner is able to implement the first-best allocation from date $t+1$ onwards, the real interest satisfies $1 + r_{t+n} = 1 + r^o \equiv 1/\beta$ for any $n \geq 1$. The date-$t+1$ present value a_{t+1} of current and future tax revenues net of subsidies extended to constrained households and firms then writes as:

$$a_{t+1} = \sum_{n\geq 0} \frac{1}{(1+r^o)^n}\tau^a_{t+1+n}R^o = \sum_{n\geq 0}\beta^n \tau^a_{t+n+1}R^o \tag{27}$$

where $\left\{\tau^a_{t+n}\right\}_{n\geq 1}$ denotes the additional tax rates that the social planner imposes on unconstrained households to finance (part of) the debt coming due. Then considering the case where the social planner sets constant additional taxes, i.e. $\tau^a_{t+n} = \tau_a$ from date $t+n$

onwards (with $n \geq 1$), and given that additional taxes cannot exceed the planner's spare taxation capacity, i.e. $\tau_a \leq \overline{\tau} - \tau_{ss}$, we can derive the following proposition.

Proposition 8 *The social planner can implement the first-best allocation on the back of a negative energy supply shock, by issuing public debt, raising taxes and paying subsidies, as long as the energy shock ε_t satisfies*

$$1 + \frac{\beta}{1-\beta}\left[\overline{\tau} - \tau_{ss}\right]\left[1-\varepsilon_t\right]^{\frac{\delta}{\gamma}} \geq \frac{1-\overline{\tau}}{1-\tau\left(\varepsilon_t\right)} \tag{28}$$

Proof 8 *The planner can issue public debt to accommodate the negative energy shock only if the (present) value of liabilities does not exceed the present value of net revenues, i.e. $d_{t+1} \leq a_{t+1}$. Using expressions (26) and (27), debt liabilities do not exceed tax revenues if and only if condition (28) holds.*

Let us denote $\varepsilon\left(\overline{\tau}\right)$ the largest (negative) energy shock for which the social planner can implement the first-best without issuing any public debt and $\varepsilon_{\max}$ the shock for which (28) holds with equality. Then it is straightforward to note that as long as $\overline{\tau} - \tau_{ss} \geq 0$, i.e. as long as the social planner can implement the first-best allocation at the steady state without having to issue public debt, we always have $\varepsilon\left(\overline{\tau}\right) < \varepsilon_{\max}$. In other words, when the social planner can implement the first-best allocation at the steady state by setting a tax rate τ_{ss} below $\overline{\tau}$, issuing public debt allows the social planner to accommodate a larger set of negative energy shock.

The intuition for this result is fairly simple. When the economy faces a large negative energy shock, implementing the first-best allocation may imply taxing unconstrained households beyond what is practically possible. As a result, the social planner can issue public debt to make up for the missing resources and redistribute the proceeds to firms and constrained households. That said, the social planner has to commit to raise additional taxes in the future to ensure public debt does not spiral up. This means that the face value of public debt cannot exceed the present value of all future additional tax revenues that the social planner can raise, once the shock has dissipated. Hence, for the very same reason that public debt can be useful, i.e. to bypass the social planner's limited taxation power, public debt cannot grow indefinitely, as it has to be backed by future tax revenues, which are themselves limited by the social planner's maximum taxation power.

Interestingly, issuing public debt, following a negative energy shock, can be useful for the social planner to implement the first-best allocation, even as the energy shock acts as a negative supply shock and raises the equilibrium rate of interest. There is hence a role for public debt in implementing the first-best allocation, not simply when the negative energy shock acts as negative demand shock, but also when it acts as a negative supply shock. In other words, public debt is useful even in the absence of demand shortages. That said, as noted above, issuing public debt is more likely to be help implementing the first best when energy shocks turn into Keynesian supply shocks and lead to a shortage of demand.

Looking at comparative statics, the social planner can accommodate a wider set of negative shocks by issuing public debt when the inter-temporal elasticity of substitution γ is higher (salmon + yellow regions in Figure 5).[15] A high inter-temporal elasticity of substitution γ indeed means that unconstrained agents can more easily substitute between current and future consumption. As a result, the negative energy shock raises the equilibrium real rate, but to a lesser extent when γ is higher. The cost of issuing public debt when the economy faces a negative energy shock is then lower and so is the additional taxation needed to ensure public debt does not balloon out of proportion.

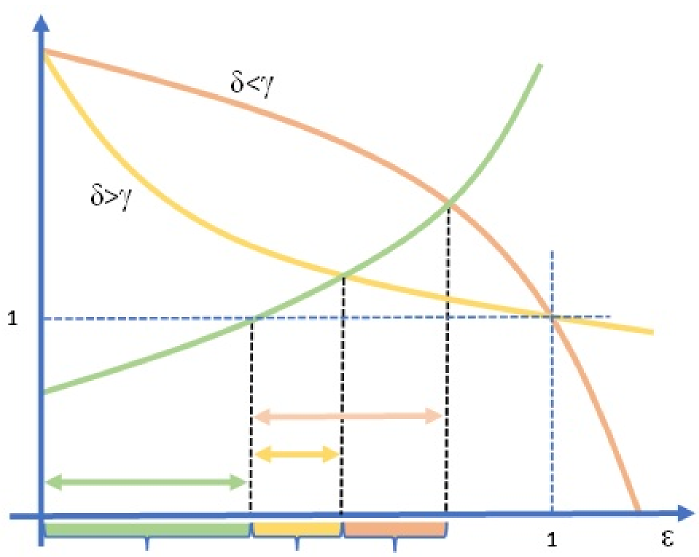

Figure 5: Negative energy shocks and public debt

[15]The term on the right-hand side of condition (28) is increasing in γ, implying that the shock ε_{max} for which condition (28) holds with equality also increases with the inter-temporal elasticity of substitution γ.

Similarly, the social planner can accommodate a larger set of negative energy shocks ε by issuing public debt when the energy intensity δ is lower as the term on the right-hand side of condition 28 is decreasing in δ, implying that the energy shock ε for which the conditions holds with equality is itself decreasing in the energy intensity δ. When consumption and/or production are less energy-intensive, public debt is more useful, in the sense that the social planner has more "fiscal space" and can accommodate a wider set of negative energy shocks. Conversely, and as would be expected, when the social planner's taxation power is more limited, i.e. when $\overline{\tau}$ is lower, the set of negative shocks ε under which the social planner can implement the first-best allocation using public debt is also more limited. Simply put, the higher the public debt to start with, the larger the tax revenues needed to fund pre-existing public debt. As a result, the ability to raise additional taxes is lower and additional public debt can accommodate a smaller set of shocks.

5 Conclusions

We have investigated the distributional and economic impact of energy supply shocks in a flexible price, two-agent model where energy is a necessity —there is a lower bound on energy consumption— and some households are poor and credit-constrained. In this framework, we showed that a negative energy supply shock can morph into a negative demand shock for consumption goods when income inequality amongst households is large — the income gap between constrained and unconstrained households is large and the economy comprises a large number of constrained households— and prices of consumption goods are flexible enough. In addition, we analyse optimal fiscal policy and show that a budget-neutral fiscal policy which subsidises firms' energy consumption and low-income credit-constrained households but taxes high-income unconstrained households, can replicate the first-best allocation. Yet, the planner needs to make larger transfers —relative to the size of the economy— to implement the first-best allocation, when the economy undergoes a larger negative shock to energy supply. As a result, in the presence of an upper limit on income tax rates, public debt can help implement the first-best outcome by spreading over time the budgetary cost of fiscal transfers.

Looking forward, this analysis could be extended in several directions. An obvious one would be to expand this model to include nominal rigidities, be it on the price or on the wage side. This would provide a natural framework to think about monetary and fiscal policy jointly. Another one would be to allow for non-homothetic demand for energy on the supply side in addition to the demand side. This would allow a deeper look into the supply-side effects of energy shocks.

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Appendix A.1: Proof of Proposition 1. Given firms' optimal pricing, the price of each consumption good writes as $p_{st}/p_{et} = \mu \left[\delta_f + \left(1 - \delta_f\right) \omega_t^{1-\sigma_f}\right]^{\frac{1}{1-\sigma_f}}$, where $\omega_t = w_t/p_{et}$. Applying this to the price of consumption goods p_c when $\sigma_f = 1$ yields $p_c = p_{ct}/p_{et} = \mu\omega^{1-\delta_f}$. Then aggregate net saving being zero in equilibrium, aggregate household net income devoted to consumption is $R_t/p_{et} = \left(1-\phi\right)(\omega_t + e_u) + \phi\,(\omega_t - e_c)^+ + \pi_t/p_{et}$. Then using expression (12) for equilibrium profits π_t, household aggregate income at date t writes as:

$$\frac{R_t}{p_{et}} = \frac{\left(1-\phi\right)(\omega_t + e_u) + \phi\,(\omega_t - e_c)^+}{1 - (1-\delta_h)\frac{\mu-1}{\mu}\left[\frac{p_{ct}}{P_t}\right]^{1-\sigma_h}} \tag{29}$$

Then using expression (14) for the demand for labour, and solving for the equilibrium of the labour market in the special case where $\sigma_h = 1$, the wage rate w_t that balances labour supply and demand satisfies $\mu w_t = (1-\delta)R_t$. Simplifying this equality using expression (29), the equilibrium wage rate ω_t satisfies (15).

Appendix A.2: Proof of Lemma 2. Assuming the equilibrium wage rate ω^* satisfies $\omega^* \geq e_c$, and denoting the economy's energy endowment $e = (1-\phi)e_u - \phi e_c$, then the labour market equilibrium implies that ω^* satisfies:

$$\omega^* = \frac{1-\delta}{\delta + \delta_h(\mu-1)}e \tag{30}$$

Based on this expression, the equilibrium wage rate ω^* satisfies the constraint $\omega^* \geq e_c$ when condition (16) holds. Then using (29), when the elasticities of substitution σ_f and σ_h are both equal to one, households' real income writes as

$$\frac{R}{P} = \frac{1}{\mu^{1-\delta_h}\left[\omega^*\right]^{1-\delta}}\frac{\omega^* + e}{1-(1-\delta_h)\frac{\mu-1}{\mu}} \tag{31}$$

And plugging in, expression (30) for the equilibrium wage rate ω^*, we end up with

$$\frac{R}{P} = \mu^{\delta_h}\frac{e^{\delta}}{\delta^{\delta}(1-\delta)^{1-\delta}}\left[\frac{\delta}{\delta + \delta_h(\mu-1)}\right]^{\delta} \tag{32}$$

Appendix A.3: Proof of Proposition 3.

Aggregate income for unconstrained households writes as $R^u = \left(1-\phi\right)(\omega + e_u)\,p_e + \pi$. Using expression (12) for equilibrium profits π, the expression for aggregate income R^u

simplifies as

$$\frac{R^u}{p_e} = \frac{(\omega + e_u) + (1-\delta_h)\frac{\mu-1}{\mu}\left[\frac{p_c}{P}\right]^{1-\sigma_h}\frac{\phi}{1-\phi}(\omega - e_c)^+}{1-(1-\delta_h)\frac{\mu-1}{\mu}\left[\frac{p_c}{P}\right]^{1-\sigma_h}} \tag{33}$$

When $\sigma_f = \sigma_h = 1$, then unconstrained households' *real* income writes, up to a positive multiplicative constant, as

$$\frac{R^u}{P} = \frac{\left(1-\phi\right)(\omega_t + e_u) + (1-\delta_h)\frac{\mu-1}{\mu}\phi\,(\omega_t - e_c)^+}{\mu^{1-\delta_h}\omega_t^{1-\delta}} \tag{34}$$

Given the expression for the equilibrium wage rate ω^*, unconstrained households' *real* income writes, up to a positive multiplicative constant, as:

$$(1-\phi)\frac{R^u}{P} = \mu^{\delta_h}\left[\frac{\mu-\phi(1-\delta)}{(1-\delta)\mu} + \frac{\phi}{\mu}\left[\frac{1-\delta_h+\delta_h\mu}{1-\delta} - 1\right]\frac{e_c}{e}\right]\left[\frac{1-\delta}{\delta+\delta_h(\mu-1)}e\right]^{\delta} \tag{35}$$

A negative energy shock that equally cuts the net energy endowments e_u and $-e_c$ then raises unconstrained households' real income R^u/P if and only if

$$\frac{e_c}{e} \geq \underline{\theta} \equiv (1-\lambda)\,\frac{1-\delta}{\delta+\delta_h\,(\mu-1)} \tag{36}$$

with $\lambda = (1-\delta_h+\delta_h\mu-\delta\mu/\phi)/\,(1-\delta)^2$. Given that the equilibrium requires $e_c/e \leq \overline{\theta}$, $\underline{\theta} \leq \overline{\theta}$, i.e. $\lambda \geq 0$, is a necessary condition for unconstrained households' real income to increase when the energy endowment e falls. This simplifies as $\phi \geq \frac{\delta\mu}{1+\delta_h(\mu-1)}$. Given that $\phi \leq 1$, this requires $\delta_f\mu \leq 1$.

Appendix A.4: Proof of Proposition 4. Based on the first-order condition for the planner's problem (19), the socially optimal allocation of energy between households and firms is such that firms' energy consumption E_f^o satisfies

$$\left[1 + \frac{\delta_h}{1-\delta_h}\left[\frac{1}{\delta_f}\right]^{\frac{\sigma_h}{\sigma_f}}\left[\delta_f^{\frac{1}{\sigma_f}} + \left(1-\delta_f\right)^{\frac{1}{\sigma_f}}\left[E_f^o\right]^{\frac{1-\sigma_f}{\sigma_f}}\right]^{\frac{\sigma_f-\sigma_h}{\sigma_f-1}}\right]E_f^o = e \tag{37}$$

In the special case where the elasticities of substitution are both set to one, ie. $\sigma_h = \sigma_f = 1$, simplifying this expression, the socially optimal allocation of energy satisfies:

$$E_f^o = \left[1 - \frac{\delta_h}{\delta}\right] e \quad \text{and} \quad E_h^o = e_h + \frac{\delta_h}{\delta} e \tag{38}$$

Conversely in the decentralised equilibrium, following on expressions (13) for firms' demand for energy and (29) for households' net income devoted to consumption, firms' energy use is $E_f = \frac{\delta_f}{1-\delta_f}\omega^{\sigma_f}$ while the equilibrium wage rate ω satisfies (15).
Hence, when condition (16) holds, the amount of energy E_f^* firms use in equilibrium, satisfies

$$\left[1 + \frac{\delta_h}{1-\delta_h}\left[\frac{1}{\delta_f}\right]^{\frac{\sigma_h}{\sigma_f}} [\mu]^{\sigma_h} \left[\delta_f^{\frac{1}{\sigma_f}} + \left(1-\delta_f\right)^{\frac{1}{\sigma_f}} \left[E_f^*\right]^{\frac{1-\sigma_f}{\sigma_f}}\right]^{\frac{\sigma_f - \sigma_h}{\sigma_f - 1}}\right] E_f^* = e \tag{39}$$

Here again, in the particular case where the elasticities of substitution are both set to one, ie. $\sigma_h = \sigma_f = 1$, the equilibrium allocation of energy simplifies as:

$$E_f^* = \left[1 - \frac{\mu\delta_h}{\delta + (\mu - 1)\delta_h}\right] e \quad \text{and} \quad E_h^* = e_h + \frac{\mu\delta_h}{\delta + (\mu-1)\delta_h} e \tag{40}$$

Expressions (38) and (40) show that firms consume too little energy in the decentralised equilibrium relative to the social optimum, while households as a group consume too much, i.e. $E_f^* \leq E_f^o$ but $E_h^* \geq E_h^o$. Moreover in the decentralised equilibrium, energy consumption by constrained households E_h^c is strictly lower than the socially optimal level, i.e. $E_h^c < E_h^o$. Applying expression (7) for households' demand for energy, E_h^c satisfies:

$$E_h^c = e_h + \delta_h \left[\frac{1-\delta}{\delta + \delta_h(\mu - 1)} e - e_c\right] < e_h + \frac{\delta_h}{\delta} e = E_h^o \tag{41}$$

Therefore while households as a group, consume too much energy in the decentralised equilibrium, constrained households actually consume too little.

Appendix A.5: Proof of Proposition 5.

Suppose the social planner extends a subsidy s_f to firms for each unit of energy consumed. Then, when $\sigma_h = \sigma_f = 1$, firms' demand for energy writes as:

$$E_f = \frac{\delta_f}{1-\delta_f} \frac{w_t}{\left(1-s_f\right) p_e} = \frac{\delta_f}{1-\delta_f} \frac{\omega_t}{1-s_f} \tag{42}$$

Moreover, when the social planner extends a subsidy S_c to constrained households and levies a tax T_u on unconstrained households, the equilibrium wage rate satisfies:

$$\left(1-\phi\right)\left[\omega + e_u - \frac{T^u}{p_e}\right] + \phi\left[\omega - e_c + \frac{S^c}{p_e}\right]^+ = \frac{\omega}{1-\delta_f}\left[1 + \frac{\delta_h \mu}{1-\delta_h}\right] \tag{43}$$

Re-writing expression (43), energy E_f consumed by firms at the equilibrium satisfies

$$\left[1 + \frac{\delta_h \mu}{\delta - \delta_h}(1-s_f)\right] E_f + \left(1-\phi\right) T^u = e + \phi S^c + s_f E_f \tag{44}$$

Moreover, at the social optimum, the planner should equalise incomes across constrained and unconstrained households. Denoting $\xi = (1-\delta_h)(\mu-1)/(1+\delta_h(\mu-1))$, the income equalisation condition writes as:

$$\omega_t + \frac{S^c}{p_e} - e_c = (1+\xi)\left[\omega_t - \frac{T^u}{p_e} + e_u\right] + \xi \frac{\phi}{1-\phi}\left[\omega_t + \frac{S^c}{p_e} - e_c\right] \tag{45}$$

Finally, tax revenues should cover for subsidy expenditures, i.e.

$$\left(1-\phi\right)\frac{T^u}{p_e} = \phi \frac{S^c}{p_e} + s_f E_f \tag{46}$$

Using (44) and (46), one can easily check that firms consume the first-best level of energy E_f^o when the subsidy s_f satisfies $\mu(1-s_f) = 1$. Then inverting firms' demand for energy (42), the equilibrium wage rate under the optimal policy writes as $\omega_t = \frac{1-\delta}{\delta}\frac{e}{\mu}$. Finally, given the expressions for the subsidy s_f, the wage rate ω_t, and firms' energy consumption E_f, the income equalisation condition (45) together with the balanced budget condition (46) imply that the subsidy S^c to constrained households and the tax T^u on unconstrained households should satisfy (20). Finally under the tax/subsidy scheme replicating the first-best allocation, household income R and the general price level P respectively satisfy:

$$R/p_e = \frac{e}{\delta} \quad \text{and} \quad P/p_e = \left[\frac{1-\delta}{\delta} e\right]^{1-\delta} \tag{47}$$

which yields expression (21) for household real income R/P under the first-best allocation.

Appendix A.6: Proof of Lemma 6.

The taxes T^u the social planner needs to raise on unconstrained households to implement the first-best allocation write as:

$$\left(1-\phi\right) T^u/p_e = \phi \left[e_c + \frac{\mu-(1-\delta)}{\delta\mu} e\right] + \frac{\mu-1}{\mu}\frac{\delta-\delta_h}{\delta} e \tag{48}$$

Moreover, in the decentralised equilibrium, i.e. absent any intervention from the social planner, unconstrained households' income R^u_t/p_{et} writes as:

$$\left(1-\phi\right) R^u/p_e = \left(1-\phi\right)(1+\xi)(\omega+e_u) + \xi\phi\left(\omega - e_c\right) \tag{49}$$

Hence, the tax rate for unconstrained households, i.e. taxes T^u as a ratio of income R^u writes as

$$\frac{T^u}{R^u} = \frac{\delta\mu\phi(e_c/e) + [\mu - \phi\,(1-\delta)] + [(\mu-1)(\delta-\delta_h) - (1-\phi)\mu]}{\delta\mu\phi(e_c/e) + \frac{\delta\mu}{\delta+(\mu-1)\delta_h}[\mu-\phi\,(1-\delta)]} \tag{50}$$

One can then easily check that the term on the RHS of (50) is increasing in the ratio e_c/e. As a consequence negative energy supply shocks that cut the energy endowment e and hence raise the ratio e_c/e typically require the social planner to levy a larger fraction of unconstrained households' income, to implement the first-best allocation.