Expectations and the UIP Puzzles when Foresight is Limited*

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Abstract

This paper investigates exchange rate dynamics and its forecast errors by incorporating bounded rationality in a small open-economy New Keynesian model. Decision-makers possess limited foresight, capable of planning only up to a finite distance into the future. This yields dynamic overshooting of forecast errors in the real exchange rate across different time horizons. It also distinguishes between short- and long-term expectation formations, where the Law of Iterated Expectations breaks. This framework provides a micro-foundation for understanding time- and forecast-horizon variability in uncovered interest parity (UIP) puzzles. Our model predictions on these UIP violations align both qualitatively and quantitatively with empirical estimates.

Keywords: Finite planning horizon; value function learning; small open economy; exchange rate; UIP violations

JEL codes: E43; E70; F31; F41

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1 Introduction

The uncovered interest rate parity (UIP) condition is the rational expectations (RE)-based asset pricing condition in the currency market and is a cornerstone in models of international macroeconomics and finance. It states that countries with higher interest rates should experience future currency depreciation, thereby preventing arbitrage profits solely from interest rate differentials. The RE-UIP condition implies unpredictable excess currency returns across different time horizons and also predicts identical responses of the exchange rate to anticipated future interest rate differentials across different forecast horizons.

However, these theoretical foundations have encountered substantial empirical challenges, known as the UIP puzzles. On the one hand, the literature documents predictable short-run excess returns for higher interest rate currency bonds (e.g., Fama, 1984; Eichenbaum and Evans, 1995), with the reversal of the predictability over longer time horizons (e.g., Bacchetta and van Wincoop, 2010; Engel, 2016; Valchev, 2020). On the other hand, exchange rates asymmetrically respond to expected short- and long-term interest rate differentials (Galí, 2020). Recent empirical literature further suggests that the main driver of these puzzles for advanced economies is subjective expectations, deviating from the common assumption of RE (e.g., Kalemli-Özcan and Varela, 2022; Candian and De Leo, 2023). Despite this empirical evidence, a comprehensive theory incorporating subjective expectations that can uniformly address these puzzles remains missing.

This paper fills this gap by developing a theory with bounded rationality, addressing the time- and forecast-horizon aspects of the UIP puzzles in a unified manner. We propose a small open-economy New Keynesian (SOE-NK) model where decision-makers optimize and form expectations under a finite planning horizon (FH). This modeling approach, first introduced by Woodford (2019), has sound empirical support from both survey evidence and macroeconomic aggregates.[1]

Specifically, we assume decision-makers can plan for only a finite distance into the future and use a coarse value function learned from past experiences to evaluate potential situations at the end of their planning horizons. This feature of limited foresight leads to dynamic overshooting of forecast errors in the real exchange rate (RER) across different time horizons, while also distinguishing the term structure between short- and long-term expectations.

The value function used by decision-makers to approximate continuation values and its

[1] Coibion et al. (2023) present survey evidence suggesting that household planning horizons in the U.S. are no more than two years and they may well be capable of planning forward about three or four quarters. Gust, Herbst and López-Salio (2022), utilizing Bayesian estimation with aggregate U.S. data, estimate several behavioral models and establishes the superiority of limited foresight in terms of fitting aggregate dynamics in a closed economy and its ability to deliver aggregate persistence without resorting to habits or price indexation.

associated updating behavior are pivotal elements in determining equilibrium dynamics. First, we assume the value function does not contain all state variables of the economy—it only includes individual state variables, excluding aggregate state variables such as aggregate stochastic shocks. As a consequence, when envisioning the future beyond their planning horizons, decision-makers do not factor in the evolution of these aggregate state variables. Second, the construction of the coarse value function is backward-looking, extended over time through updating based on past experiences, following a constant-gain learning process.

When the fundamentals of the economy change due to aggregate shocks, the long-run beliefs of decision-makers, captured by their coarse value function, display different patterns over time. Initially, these beliefs underreact to the shocks due to the coarseness of the value function and the finite planning horizon. However, over time, learning in the value function leads to over-extrapolation, causing overreaction in beliefs. This initial underreaction, followed by overreaction, generates dynamic overshooting of the forecast error for the RER in our model. This mechanism is crucial for understanding the predictability of excess return on currency investment and its sign reversal across time horizons.

Furthermore, the behavior of value function learning results in the breakdown of the Law of Iterated Expectations (LIE). The reason is that expectations formed at any time are contingent on the value functions at that same time. Thus, expectations of future endogenous variables could differ from the expectations of expected future endogenous variables, as long as decision-makers' value functions are not time-invariant. This feature breaks the relationship in an RE model that the RER is equal to the sum of the expected future real interest rate differentials, a result coming from the LIE. This mechanism is crucial for understanding the asymmetric response of the RER to anticipated interest rate differentials across different forecast horizons.

The rest of the model remains conventional and it includes the domestic productivity shock and the foreign interest rate shock as aggregate shocks. We calibrate the FH model by disciplining structural parameters to match key macro-international aggregates in Canada, in conjunction with the U.S. data. Comparing the impulse responses of equilibrium dynamics between the FH model and its RE counterpart reveals that, the learning behavior of the value function in the FH model leads to more persistent and hump-shaped movements in aggregate variables and a dynamic overshooting of forecast errors.

To evaluate the external validity of the FH model in addressing the UIP puzzles, we use model-simulated data to run the same regressions commonly used in the empirical literature. Our model aligns well with empirical patterns, matching the UIP violations using Canada as an example. The model produces predictable excess currency returns over the short-run time horizon and predictable returns with an opposite sign across long-run horizons. It also generates asymmetric reactions of the real exchange rate to the forecasts of future short- and long-term interest rate differentials. Notably, despite the fact that the UIP violations are not targeted in model calibration, the FH model successfully matches the empirical pattern both qualitatively and quantitatively. We also observe that the foreign interest rate shock is the primary driver for the model's quantitative success.

Related Literature. This paper contributes to the recent literature that challenges the full-information RE assumption by considering behavioral biases to assess their policy implications and empirical relevance.[2] In this context, our focus is on exploring the consequences of limited foresight in an open-economy setting. Compared with other popular behavioral variants, the FH approach naturally distinguishes between expectation biases arising from finite planning horizons and those originating from the approximating behavior of the value function. The feature of initial underreaction to new information, followed by overreaction due to value function learning, distinctly sets our model apart from others that emphasize only one direction.[3] Compared to those that also discuss underreaction followed by overreaction (e.g., Angeletos, Huo and Sastry, 2021), the distinct characteristic of our FH model is that the behavioral bias and its associated subjective expectations originate from imperfect *optimization*, akin to the concept described by Ilut and Valchev (2022), rather than arising from imperfect expectations, which distort the perceived true probability of states.

Our paper directly contributes to the literature studying the UIP puzzles by incorporating bounded rationality. As previously noted, a vast body of research highlights the importance of forecast errors derived from survey-based expectation data in explaining the UIP violations in advanced countries (e.g., Froot and Frankel, 1989; Chinn and Frankel, 2019; Kalemli-Özcan and Varela, 2022; Candian and De Leo, 2023). These findings suggest that the expectation channel can be the main driver of the UIP puzzles associated with ex-post realization data, on top of alternative explanations such as risk premia or financial frictions.[4] Our FH model offers a unified theoretical framework capable of explaining both the time- and forecast-horizon aspects of the UIP violations. Furthermore, by focusing on

[2] The literature has developed several approaches to model bounded rationality that address the forward guidance puzzle, such as cognitive discounting (Gabaix, 2020), lack of common knowledge (Angeletos and Lian, 2018), level-k thinking (e.g., García-Schmidt and Woodford, 2019; Farhi and Werning, 2019), and finite planning horizons (Woodford, 2019).

[3] Models featuring underreaction to new information include cognitive discounting, lack of common knowledge, level-k thinking, imperfect common knowledge (Woodford, 2002), sticky information (Mankiw and Reis, 2002), and rational inattention (Sims, 2003). For modeling overreaction, one leading approach is diagnostic expectations (e.g., Bordalo, Gennaioli and Shleifer, 2018; Bordalo et al., 2020).

[4] The literature has also developed various approaches in the RE framework to address some parts of these UIP puzzles, such as by considering time-varying risk premia (Verdelhan, 2010), infrequent portfolio adjustments (Bacchetta and van Wincoop, 2010), or the convenience yield (Valchev, 2020).

the expectation channel, our model underscores a distinct perspective on the source of UIP wedges compared to Itskhoki and Mukhin (2021). They consider a segmented financial market with noise traders and risk-averse intermediaries, where limits-to-arbitrage results in a wedge in the RE-UIP condition. In contrast, our model attributes endogenous deviations from the RE-UIP condition to decision-makers' behavioral responses to aggregate shocks.

Our paper also has distinct features compared to existing studies that attempt to explain the UIP puzzles through expectation channels. Several studies employ distorted beliefs and shock misperception to explain the time-horizon aspect of the UIP puzzles. Gourinchas and Tornell (2004) consider investors with distorted beliefs, resulting in misperceptions about the relative weight of persistent versus transitory interest rate shocks. Candian and De Leo (2023) extend this work by incorporating investors' extrapolation of underlying shocks. Valente, Vasudevan and Wu (2021) similarly model decision-makers receiving noisy signals and extrapolating an exogenous interest-rate process.

In contrast, decision-makers in our model do not misperceive underlying shocks; instead, expectation biases stem from imperfect optimization that is inherent in limited foresight. Our approach is also related to Molavi, Tahbaz-Salehi and Vedolin (2023), which employs a model with constraints on the complexity of agents' beliefs when addressing the UIP puzzles. However, individuals' subjective expectations satisfy the LIE in their model and hence it does not explain the forecast-horizon aspect of the UIP puzzles. In addition, our approach differs from that of Kolasa, Ravgotra and Zabczyk (2022), which tackles the UIP puzzles by considering cognitive discounting.

Our paper also expands the scope of FH models by applying to an open-economy context. Woodford and Xie (2019, 2022) emphasize the policy implications of limited foresight, particularly under the zero lower bound. Xie (2020) posits that this method facilitates the examination of equilibrium dynamics without having to tackle equilibrium selection issues. Dupraz, Bihan and Matheron (2022) reconsider the effects of make-up policies by reconciling small inflation response with large response in asset prices. Our paper underlines the implications of finite planning horizon for the dynamics of exchange rates and its forecasts. Methodology-wise, the model in Woodford (2019) considers only the state variables associated with agents' continuation value functions. We introduce a method in this paper to incorporate a larger set of endogenous state variables into his framework, which can also be applied to extend the FH model to medium- or large-scale DSGE models.

This paper proceeds as follows. Section 2 illustrates an SOE-NK model in which decision-makers are subject to limited foresight. Section 3 summarizes the full equilibrium conditions when decision-makers share a homogeneous planning horizon and discusses the model solution method. Section 4 analyzes the equilibrium dynamics of the FH model. Section 5 applies the model to address the UIP puzzles. Section 6 discusses the robustness of the main results by considering alternative setups of the model, including extending to heterogenous planning horizons. Section 7 concludes.

2 A Small Open-Economy New Keynesian Model under Limited Foresight

We develop an SOE-NK model in which decision-makers have bounded rationality. The small open economy is subject to nominal price rigidities and incomplete international financial market. The economy is populated by households and producers who optimize based on finite-horizon planning à la Woodford (2019). The household consumption basket includes both domestically produced goods and imported foreign goods. Producers of domestic goods have market power and can set prices in the domestic currency (i.e., producer currency pricing). We assume nominal rigidity in goods prices whereby firms have limited ability to reset their prices à la Calvo (1983). There are two types of bonds: one is a domestic currency bond that is only traded domestically, and the other is an international bond (denominated in foreign currency) that is traded with the rest of the world (RoW). We consider two types of shocks in the model: shocks to domestic productivity and the foreign interest rate.

2.1 Households

Let us begin with a description of the households' forward planning problem. The small open economy consists of infinitely many households indexed in the unit interval $[0, 1]$. At any time t, household i seeks to maximize

$$\hat{\mathbb{E}}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[u\left(C_\tau^i\right) - \varpi\left(N_\tau^i\right) \right], \tag{2.1}$$

where C_τ^i is the consumption composite at date τ and N_τ^i is the labor supply of household i. Function $u(\cdot)$ denotes a periodic utility, which is strictly increasing and concave, while function $\varpi(\cdot)$ denotes a periodic disutility of labor supply, which is strictly increasing and convex. Parameter $0 < \beta < 1$ is the subjective discount factor. Operator $\hat{\mathbb{E}}_t$ represents the subjective expectation of household i at time t. We will specify this expectation operator later which features a finite planning horizon.

The consumption basket C_τ^i is an aggregate of home goods, $C_{H,\tau}^i$, and imported foreign goods, $C^i_{F,\tau}$. The aggregation follows a constant elasticity of substitution (CES):

$$C^i_\tau = \left[(1-\alpha)^{\frac{1}{\eta}}(C^i_{H,\tau})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}(C^i_{F,\tau})^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}},$$

where $1-\alpha$ governs the degree of home bias and η represents the elasticity of substitution between home and foreign goods. $C^i_{H,\tau}$ and $C^i_{F,\tau}$ are the Dixit-Stiglitz aggregates of the home and foreign varieties, respectively; that is,

$$C^i_{H,\tau} = \left(\int_0^1 C^i_{H,\tau}(j)^{\frac{\epsilon-1}{\epsilon}}dj\right)^{\frac{\epsilon}{\epsilon-1}}, \qquad C^i_{F,\tau} = \left(\int_0^1 C^i_{F,\tau}(j)^{\frac{\epsilon-1}{\epsilon}}dj\right)^{\frac{\epsilon}{\epsilon-1}},$$

where ϵ represents the elasticity of substitution between within-country varieties.

The household faces the following sequential budget constraint:

$$P_\tau C^i_\tau + \frac{B^i_{\tau+1}}{1+i_\tau} + \frac{\mathcal{E}_\tau B^{*,i}_{\tau+1}}{1+i^*_\tau} = B^i_\tau + \mathcal{E}_\tau B^{*,i}_\tau + W_\tau N^i_\tau + \Phi_\tau, \tag{2.2}$$

where P_τ is the consumer price index (CPI), $B^i_{\tau+1}$ is the nominal payoff in period $\tau+1$ of the household's domestic bond holdings at the end of period τ, and $1+i_\tau$ is the short-term riskless nominal interest rate of the domestic bond. Variable $\mathcal{E}_\tau$ is the effective nominal exchange rate between the home country and the rest of the world, representing the price of foreign currency in units of domestic currency. Variable $B^{*,i}_{\tau+1}$ is the nominal payoff of a foreign bond in foreign currency and $1+i^*_\tau$ is its associated nominal foreign interest rate. Variable W_τ is the nominal wage and Φ_τ is the nominal dividends from firms that household i receives.[5]

The household's static cost minimization problem for consumption expenditure yields the following demand functions for each variety:

$$C^i_{H,\tau}(j) = \left(\frac{P_{H,\tau}(j)}{P_{H,\tau}}\right)^{-\epsilon} C^i_{H,\tau}, \qquad C^i_{H,\tau} = (1-\alpha)\left(\frac{P_{H,\tau}}{P_\tau}\right)^{-\eta} C^i_\tau,$$
$$C^i_{F,\tau}(j) = \left(\frac{P_{F,\tau}(j)}{P_{F,\tau}}\right)^{-\epsilon} C^i_{F,\tau}, \qquad C^i_{F,\tau} = \alpha\left(\frac{P_{F,\tau}}{P_\tau}\right)^{-\eta} C^i_\tau,$$

where the price indices for the two consumption bundles are

$$P_{H,\tau} = \left(\int_0^1 P_{H,\tau}(j)^{1-\epsilon}dj\right)^{\frac{1}{1-\epsilon}}, \qquad P_{F,\tau} = \left(\int_0^1 P_{F,\tau}(j)^{1-\epsilon}dj\right)^{\frac{1}{1-\epsilon}}.$$

[5]Households own the domestic firms, but the dividends transfer is beyond household i's control. Thus, Φ_τ is not indexed by i.

The aggregate price level (CPI) is

$$P_\tau = \left[(1-\alpha)P_{H,\tau}^{1-\eta} + \alpha P_{F,\tau}^{1-\eta}\right]^{\frac{1}{1-\eta}}.$$

For the sake of parsimony, we follow the assumption on the labor market in Woodford (2019). That is, the country has a labor organization in which each household is asked by firms to supply its share of the aggregate domestic labor demand N_τ. Thus, the expected path of $N_\tau^i = N_\tau$ is beyond household i's control. This implies the expected path of equilibrium income (in domestic currency), that is,

$$W_\tau N_\tau + \Phi_\tau = P_{H,\tau} Y_\tau, \tag{2.3}$$

is exogenous to individual household i.

Now, we characterize the intertemporal decision-making of household i to maximize (2.1) under limited foresight. Suppose in each period, household i engages in explicit forward planning for finite h periods ahead; that is, household i has a planning horizon h. We assume that h is exogenous and time-invariant. At time t, the household chooses state-contingent plans $\{C_\tau^i(\boldsymbol{z}_\tau)\}$ for all possible states $\boldsymbol{z}_\tau$ within periods $t \leq \tau \leq t+h$. The household chooses the finite-horizon plans to maximize the objective

$$\mathbb{E}_t^h \left[\sum_{\tau=t}^{t+h} \beta^{\tau-t} u(C_\tau^i) + \beta^{h+1} v\left(\mathcal{B}_{t+h+1}^i, \mathcal{B}_{t+h+1}^{*,i}\right)\right], \tag{2.4}$$

where $v(\cdot)$ is the value function that the household uses at time t to approximate continuation values if the asset portfolio it holds at the end of the planning horizon is $\mathcal{B}_{t+h+1}^i(\boldsymbol{z}_{t+h})$ and $\mathcal{B}_{t+h+1}^{*,i}(\boldsymbol{z}_{t+h})$. It is a *coarse* value function such that it is contingent only on individual state variables instead of the complete state-contingent structure as under the RE case. Here $\mathcal{B}_\tau \equiv B_\tau / P_{\tau-1}$ and $\mathcal{B}_\tau^* \equiv B_\tau^* / P_{\tau-1}^*$ denote the nominal value of bonds maturing in period τ deflated by last-period price indices, and thus $\mathcal{B}_\tau$ and $\mathcal{B}_\tau^*$ are real variables that are purely predetermined in period $\tau - 1$. Operator $\mathbb{E}_t^h(\cdot)$ is the subjective expectation $\hat{\mathbb{E}}_t(\cdot)$ in expression (2.1), representing the expectations at time t of a decision maker that plans h periods ahead.

By combining (2.2) and (2.3), household i's budget constraint can be expressed in real terms as follows:

$$C_\tau^i + \frac{\mathcal{B}_{\tau+1}^i}{1+i_\tau} + \frac{Q_\tau \mathcal{B}_{\tau+1}^{*,i}}{1+i_\tau^*} = \frac{\mathcal{B}_\tau^i}{\Pi_\tau} + \frac{Q_\tau \mathcal{B}_\tau^{*,i}}{\Pi_\tau^*} + \mathcal{S}_\tau Y_\tau, \tag{2.5}$$

where $\Pi_{\tau+1} \equiv P_{\tau+1}/P_\tau$ and $\Pi_{\tau+1}^* \equiv P_{\tau+1}^*/P_\tau^*$ are the domestic and foreign inflation rates,

$Q_\tau \equiv \mathcal{E}_\tau P^*_\tau / P_\tau$ is the real exchange rate, and $\mathcal{S}_\tau Y_\tau$ is the household income from wages and dividends (beyond the household's control). $\mathcal{S}_\tau \equiv P_{H,\tau}/P_\tau$ is the ratio between the price index for domestically produced goods and the price index for the aggregate consumption basket.

Note that in the household's finite-horizon forward planning problem (2.4), if the household's subjective expectation operator $\mathbb{E}^h_t[\cdot]$ is the model-consistent expectation and the value function $v(\cdot)$ is the accurate model-consistent value function with a complete state-contingent structure (as in standard dynamic programming under infinite planning horizons), the household's optimization problem replicates the conventional intertemporal optimization problem. That is, in such a case, the household makes the optimal infinite-horizon contingent plans under RE.

However, the decision-making under limited foresight features optimal plans and expectation formations that deviate from the infinite-horizon RE benchmark. At date t, the household constructs a contingent plan for the subsequent h forward dates but implements the plan only for the current date t. When the following date $t+1$ arrives, the household reconstructs the contingent plans for future h dates, which are not necessarily identical to those made at the *previous* date t. The household implements the new plans only for the current date $t+1$. In terms of expectation formation, at each date t, the h-horizon household makes a contingent plan up to date $t+h$. At each date τ within the planning horizon $t \leq \tau \leq t+h$, the household is assumed to plan forward for the remaining $t+h-\tau$ dates. In addition, the household assumes that spending and pricing decisions made by other households and firms at any date τ within its planning horizon are made with the same remaining planning horizon $t+h-\tau$.

We now define how the expectation operator for the h-horizon household is linked to the model-consistent expectation. For any endogenous variable X_τ determined at future date τ within the planning horizon ($t \leq \tau \leq t+h$), the household's expectation conditional on state $\boldsymbol{z}_t$ at date t is assumed to satisfy

$$\mathbb{E}^h_t[X_\tau | \boldsymbol{z}_t] = \mathbb{E}_t\left[X^{t+h-\tau}_\tau\right],$$

where operator $\mathbb{E}^h_t[\cdot]$ represents the expectation of the decision maker made at period t that plans h-periods ahead. $\mathbb{E}_t[\cdot]$ is the standard model-consistent expectation operator conditional on being in state $\boldsymbol{z}_t$ and the superscript $t+h-\tau$ indexes the (remaining) planning horizon when the household forecasts variable X_τ at date τ.[6] The household's

[6] We follow the notation in Woodford (2019) for the definition of expectation operator with finite planning horizon. The superscript $t+h-\tau$ here is needed because decision-makers could have different planning horizons, hence their forecasted X_τ is contingent on their remaining planning horizon at date τ. We consider

expectation for X_τ conditional on future state $\boldsymbol{z}_\tau$ in its period-t planning exercise is given by

$$\mathbb{E}_t^h[X_\tau|\boldsymbol{z}_\tau] = \mathbb{E}_\tau\left[X_\tau^{t+h-\tau}\right].$$

Finally, the household's expectation for $X_{\tau+1}$ conditional on the same information structure above is given by

$$\mathbb{E}_t^h[X_{\tau+1}|\boldsymbol{z}_\tau] = \mathbb{E}_\tau\left[X_{\tau+1}^{t+h-\tau-1}\right].$$

By imposing the expectation operator transformation, the first-order conditions to maximize the objective (2.4) with respect to C_τ^i, $\mathcal{B}_{\tau+1}^i$, and $\mathcal{B}_{\tau+1}^{*,i}$, with horizon $h \geq 1$ at any date $t \leq \tau \leq t+h-1$, yield

$$u'(C_\tau^{t+h-\tau}) = \beta\mathbb{E}_\tau\left[(1+i_\tau^{t+h-\tau})\frac{u'(C_{\tau+1}^{t+h-\tau-1})}{\Pi_{\tau+1}^{t+h-\tau-1}}\right], \tag{2.6}$$

$$u'(C_\tau^{t+h-\tau}) = \beta\mathbb{E}_\tau\left[(1+i_\tau^{*,t+h-\tau})\frac{u'(C_{\tau+1}^{t+h-\tau-1})}{\Pi_{\tau+1}^{*,t+h-\tau-1}}\frac{Q_{\tau+1}^{t+h-\tau-1}}{Q_\tau^{t+h-\tau}}\right]. \tag{2.7}$$

In terminal period $\tau = t+h$ where the forward planning is truncated, or in the case of $h=0$, the first-order conditions related to the value function are

$$u'(C_{t+h}^0) = \beta(1+i_{t+h}^0)v_1(\mathcal{B}_{t+h+1}^0, \mathcal{B}_{t+h+1}^{*,0}), \tag{2.8}$$

$$u'(C_{t+h}^0) = \beta(1+i_{t+h}^{*,0})v_2(\mathcal{B}_{t+h+1}^0, \mathcal{B}_{t+h+1}^{*,0})/Q_{t+h}^0. \tag{2.9}$$

In the nonstochastic steady state, we have that $1+\bar{i} = \beta^{-1}\bar{\Pi}$ and $1+\bar{i}^* = \beta^{-1}\bar{\Pi}^*$, and the (time-invariant) value function in the steady state is given by

$$v(\mathcal{B}, \mathcal{B}^*) = (1-\beta)^{-1}u\left(\frac{(1-\beta)\mathcal{B}}{\bar{\Pi}} + \frac{(1-\beta)\bar{Q}\mathcal{B}^*}{\bar{\Pi}^*} + \bar{\mathcal{S}}\bar{Y}\right). \tag{2.10}$$

Details of deriving (2.10) can be found in Appendix A.

We define the domestic variables after log-linear approximation as

$$\hat{c}_t \equiv \log\left(\frac{C_t}{\bar{C}}\right), \qquad \hat{y}_t \equiv \log\left(\frac{Y_t}{\bar{Y}}\right), \qquad \hat{\imath}_t \equiv \log\left(\frac{1+i_t}{1+\bar{i}}\right),$$

$$\hat{b}_t \equiv \frac{\mathcal{B}_t - \bar{\mathcal{B}}}{\bar{\Pi}\bar{C}}, \qquad \hat{q}_t \equiv \log\left(\frac{Q_t}{\bar{Q}}\right), \qquad \pi_t \equiv \log\left(\frac{\Pi_t}{\bar{\Pi}}\right),$$

an extension to heterogeneous planning horizons in Section 6.

and for the foreign variables,

$$\hat{\imath}_t^* \equiv \log\left(\frac{1+i_t^*}{1+\bar{i}^*}\right), \qquad \hat{b}_t^* \equiv \frac{\bar{Q}(\mathcal{B}_t^* - \bar{\mathcal{B}}^*)}{\bar{\Pi}^*\bar{C}}, \qquad \pi_t^* \equiv \log\left(\frac{\Pi_t^*}{\bar{\Pi}^*}\right),$$

Throughout the paper, we use lowercase to denote variables after taking logs unless otherwise stated, and further use hats to denote log-deviation from the steady state.

Log-linearizing equations (2.6) and (2.7) yield

$$\hat{c}_\tau^{t+h-\tau} = \mathbb{E}_\tau[\hat{c}_{\tau+1}^{t+h-\tau-1}] - \sigma^{-1}\left[\hat{\imath}_\tau^{t+h-\tau} - \mathbb{E}_\tau \pi_{\tau+1}^{t+h-\tau-1}\right], \tag{2.11}$$

$$\hat{c}_\tau^{t+h-\tau} = \mathbb{E}_\tau[\hat{c}_{\tau+1}^{t+h-\tau-1}] - \sigma^{-1}\left[\hat{\imath}_\tau^{*,t+h-\tau} + \mathbb{E}_\tau(\hat{q}_{\tau+1}^{t+h-\tau-1} - \hat{q}_\tau^{t+h-\tau} - \pi_{\tau+1}^{*,t+h-\tau-1})\right], \tag{2.12}$$

for any date $t \leq \tau \leq t+h-1$ with horizon $h \geq 1$, where $\sigma^{-1} \equiv -u'(\bar{C})/(u''(\bar{C})\bar{C})$ is the elasticity of intertemporal substitution of households.

Under the assumption that households always adopt the steady-state value function in their forward-planning exercise, log-linearizing equations (2.8) and (2.9) yields

$$\hat{c}_\tau^0 = -\sigma^{-1}\hat{\imath}_\tau^0 + (1-\beta)\hat{b}_{\tau+1}^0 + (1-\beta)\hat{b}_{\tau+1}^{*,0}, \tag{2.13}$$

$$\hat{c}_\tau^0 = -\sigma^{-1}\hat{\imath}_\tau^{*,0} + (1-\beta)\hat{b}_{\tau+1}^0 + (1-\beta)\hat{b}_{\tau+1}^{*,0} + \sigma^{-1}\hat{q}_\tau^0. \tag{2.14}$$

Details of the derivation can be found in Appendix B. In Section 2.5, we also introduce a learning process for updating the value function over time by averaging past experiences.

2.2 Firms

A set of continuum producers $f \in [0,1]$ in the economy produce a variety of differentiated intermediate goods as inputs for the domestically produced final goods. The intermediate goods market is monopolistically competitive, and the producers of each intermediate good can be price-setters in domestic currency but face staggered pricing, as in the style of Calvo (1983) and Yun (1996). Specifically, we assume that at each time, fraction $1-\theta$ of firms are randomly selected to be able to reoptimize their prices. A producer j that belongs to the remaining fraction θ cannot reset its price, and we assume that its price satisfies $P_{H,t}(j) = P_{H,t-1}(j)\bar{\Pi}_H$, where $\bar{\Pi}_H$ is the inflation rate of the domestic goods in the nonstochastic steady state. This implies that the prices are automatically revised by considering the long-run inflation rate for domestically produced goods. This assumption is an open-economy variation of Woodford (2019), which implies that all equilibrium relative prices among the varieties of domestic goods are the same as those under flexible prices in the steady state.

At time t, similar to the objective function of households, firm f with a k-period planning horizon that can reset its price chooses $P^f_{H,t}$ to maximize

$$\max_{P^f_{H,t}} \mathbb{E}^k_t \left[\sum_{\tau=t}^{t+k} (\beta\theta)^{\tau-t} \lambda_\tau H\left(r^f_\tau; \mathcal{S}_\tau, \boldsymbol{Z}_\tau\right) + \tilde{v}\left(r^f_{t+k}\right) \right], \tag{2.15}$$

where $\lambda_\tau \equiv \int u_c(C^i_\tau)di$ is the average marginal utility of household consumption and $r^f_\tau \equiv \frac{P^f_{H,t}}{P_{H,\tau}} \bar{\Pi}^{\tau-t}_H$ denotes the relative price between firm f's goods price and domestically produced final goods. $H\left(r^f_\tau\right)$ represents the real profits of the firm at date τ, where $\boldsymbol{Z}_\tau$ is the vector of real state variables that are beyond firm f's control. A detailed expression of the functional form $H(\cdot)$ can be found in Appendix C. The last term $\tilde{v}(r^f_{t+k})$ in (2.15) is the firm's value function at the end of the planning horizon that is used to approximate the value of discounted future real profits from date $t+k+1$ onward.

We begin with the assumption that the firm's value function $\tilde{v}(\cdot)$ is the one learned from the nonstochastic steady-state equilibrium; that is, we now consider the following steady-state firm value function:

$$\tilde{v}\left(r^f\right) = (1-\theta\beta)^{-1} \bar{\lambda} H\left(r^f; \bar{\mathcal{S}}, \bar{\boldsymbol{Z}}\right),$$

where $\bar{\lambda} = u_c(\bar{C})$ is the constant value of λ_τ in the steady state. In Section 2.5, we relax this assumption by incorporating a learning process into $\tilde{v}_t(\cdot)$.

The firm's expectation formation $\mathbb{E}^k_t[\cdot]$ is isomorphic to that of the household. That is, the firm with planning horizon k assumes that the endogenous variables determined at any date τ in $t \le \tau \le t+k$ are based on the decisions of all agents in the economy with the remaining planning horizon $t+k-\tau$. Therefore, the firm's subjective expectation operator for endogenous variables is represented by the model-consistent expectation in the same fashion as in equations (2.1)-(2.1), where h is now replaced with k.

Then, with the notation $p^f_{H,t} \equiv \log[P^f_{H,t}/(P_{H,t-1}\bar{\Pi}_H)]$, any firm f that reoptimizes its price at time t with a k-period planning horizon sets $p^f_{H,t} = p^k_{H,t}$, which is given by

$$p^k_{H,t} = \mathbb{E}_t \sum_{\tau=t}^{t+k} (\beta\theta)^{\tau-t} \left[\pi^{t+k-\tau}_{H,\tau} + (1-\beta\theta)\widehat{mc}^{t+k-\tau}_\tau \right], \tag{2.16}$$

where $\widehat{mc}_\tau \equiv \log\left(\mathcal{MC}_\tau/\overline{\mathcal{MC}}\right)$ is the log-deviation of the real marginal cost, $\mathcal{MC}_\tau \equiv \frac{MC_\tau}{P_{H,\tau}}$, around its steady state at date τ. In particular, it satisfies $\widehat{mc}_t = -H'(1;1,\boldsymbol{Z}_t)/H''(1;1,\bar{\boldsymbol{Z}})$. The details of deriving (2.16) and $\widehat{mc}_t$ can be found in Appendix C.

The evolution of the aggregate domestic price index $P_{H,t}$ satisfies

$$P_{H,t}^{1-\epsilon} = \theta(P_{H,t-1}\bar{\Pi}_H)^{1-\epsilon} + (1-\theta)(P_{H,t}^f)^{1-\epsilon},$$

and its log-linear approximation around the steady state yields $\pi_{H,t}^k = (1-\theta)p_{H,t}^k$.

Thus, equation (2.16) becomes

$$\pi_{H,t}^k = (1-\theta)\mathbb{E}_t \sum_{\tau=t}^{t+k} (\beta\theta)^{\tau-t} \left[\pi_{H,\tau}^{t+k-\tau} + (1-\beta\theta)\widehat{mc}_\tau^{t+k-\tau} \right].$$

The isomorphic form of the equation holds if we replace k with any horizon $j \geq 0$. That is, $\{\pi_{H,t}^j\}$ for any horizon $j \geq 1$ satisfies the following recursive form:

$$\pi_{H,t}^j = \kappa \widehat{mc}_t^j + \beta \mathbb{E}_t \pi_{H,t+1}^{j-1}, \tag{2.17}$$

where $\kappa \equiv (1-\theta)(1-\beta\theta)/\theta$, and when $j = 0$, we have

$$\pi_{H,t}^0 = \kappa \widehat{mc}_t^0. \tag{2.18}$$

2.3 Labor Market and Real Marginal Cost

The wage is determined following the approach of Woodford (2019), which abstracts labor supply decision-making from any individual household while maintaining the aggregate labor-supply curve as in the canonical New Keynesian models. As mentioned in Section 2.1, the labor market organization has representatives who bargain for wages on behalf of households. Henceforth, we drop the superscripts on the planning horizon for the sake of parsimony when they are redundant for explicitly understanding the equilibrium relationships. We formally state the full equilibrium conditions with the FH in Section 3.

A representative determines the number of working hours provided by households for any given wage, and households must supply that number of hours and receive the same wage. There are many such representatives, and no representative has any market power. Then, the representatives choose the number of hours N_t to maximize the average utility of the households in the economy, which yields the following labor supply:

$$\varpi_N(N_t) = \lambda_t \frac{W_t}{P_t}.$$

We assume the standard disutility function of the labor supply in the form of

$$\varpi(N_t) = \frac{N_t^{1+\varphi}}{1+\varphi},$$

where φ is the inverse of the Frisch elasticity of labor supply. Then, the labor supply equation after taking log becomes

$$\varphi n_t = -\sigma c_t + w_t - p_t. \tag{2.19}$$

Each firm $j \in [0,1]$ has a linear technology represented by the production function $Y_t(j) = A_t N_t(j)$, hence the marginal cost is common across domestic firms. The real marginal cost in terms of domestic prices is then given by

$$\mathcal{MC}_t = \frac{W_t}{P_{H,t} A_t},$$

and thus the log of the real marginal cost becomes:

$$mc_t = w_t - p_{H,t} - a_t. \tag{2.20}$$

2.4 Closing the Economy

Exchange Rate and the Terms of Trade. From the definition of the real exchange rate, we have the following accounting relationship between the log of the nominal exchange rate, real exchange rate, and domestic and foreign price indices:

$$q_t = e_t + p_t^* - p_t, \tag{2.21}$$

which yields the following log-linearized equation

$$\hat{\varepsilon}_t = \hat{q}_t - \hat{q}_{t-1} + \pi_t - \pi_t^*. \tag{2.22}$$

Here, $\hat{\varepsilon}_t \equiv \log(\frac{\mathcal{E}_t}{\bar{\varepsilon}\mathcal{E}_{t-1}})$ is the log-deviation of the nominal depreciation rate from its steady-state value $\bar{\varepsilon}$.

Without loss of generality, the price of foreign composite goods in the foreign currency is normalized to one; that is, $p_t^* = 1$. We further assume that the law of one price (LOP) always holds (for each variety of goods j), hence $e_t = p_{F,t}$. Then, taking into account the linearized CPI index

$$p_t = (1-\alpha)p_{H,t} + \alpha p_{F,t}, \tag{2.23}$$

we have

$$\pi_t = (1-\alpha)\pi_{H,t} + \alpha\hat{\varepsilon}_t. \tag{2.24}$$

The terms of trade between the domestic country and the rest of the world is defined as $S_t \equiv P_{F,t}/P_{H,t}$. Taking the log of this expression yields

$$s_t = p_{F,t} - p_{H,t}. \tag{2.25}$$

By further combining (2.21) and (2.23), the terms of trade in the first-order approximation satisfies

$$\hat{s}_t = \frac{\hat{q}_t}{(1-\alpha)}. \tag{2.26}$$

International Goods Market Clearing. The international market clearing condition for domestically produced goods is

$$Y_t = C_{H,t} + C^*_{H,t}, \tag{2.27}$$

where $C^*_{H,t}$ is the foreign demand from the RoW. Following the literature (e.g., Davis and Presno, 2017), we assume an elastic foreign demand for the home country:

$$C^*_{H,t} = \left(\frac{P^*_{H,t}}{P^*_t}\right)^{-\gamma} C^*_t, \tag{2.28}$$

where $\gamma > 0$ is the demand elasticity of the RoW for domestically produced goods and C^*_t represents the RoW consumption. We abstract from the foreign demand shock and thus C^*_t is time-invariant.

Log-linearizing (2.27) and (2.28) yields the following goods market clearing condition:

$$\hat{y}_t = \vartheta_{yc}\hat{c}_t + \vartheta_{ys}\hat{s}_t, \tag{2.29}$$

where ϑ_{yc} and ϑ_{ys} are constant. They are functions of structural parameters and steady-state equilibrium values. Details of (2.29) can be found in Appendix D.

Utilizing the log-linearized production function $\hat{y}_t = \hat{a}_t + \hat{n}_t$ and the labor supply function (2.19), along with (2.23), (2.25), and (2.29), the log-linearized real marginal cost (2.20) is given by

$$\widehat{mc}_t = (\sigma + \varphi\vartheta_{yc})\hat{c}_t + (\alpha + \varphi\vartheta_{ys})\hat{s}_t - (1+\varphi)\hat{a}_t. \tag{2.30}$$

Without loss of generality, we assume that the steady-state level of domestic productivity is $\bar{A} = 1$, which implies $\bar{a} = 0$ and $\hat{a}_t = a_t$.

Domestic Monetary Policy. For the domestic interest rate, we consider a monetary policy rule intended to stabilize the domestic CPI inflation rate following a standard Taylor-type form; that is,

$$\hat{\imath}_t = \phi_\pi \pi_t, \tag{2.31}$$

where $\phi_\pi > 1$ is a constant parameter.

Evolution of Foreign Bond Holdings. Without loss of generality, we assume that the foreign inflation rate is always one; that is, $\Pi_t^* = \bar{\Pi}^* = 1$ and $\pi_t^* = 0$ for any t. Then, the foreign nominal interest rate $\hat{\imath}_t^*$ is equal to $\hat{r}_t^*$ and the steady-state relationship satisfies $1 + \bar{r}^* = \beta^{-1}$. In addition, given that the domestic bonds are not internationally tradable and are cleared domestically in equilibrium, we assume the net supply of domestic bonds in this small open economy is always zero ($\mathcal{B}_t = 0$ for any t).

In equilibrium, the resource constraint (2.5) of the domestic economy now becomes:

$$\frac{Q_t \mathcal{B}_{t+1}^*}{1 + i_t^*} = \frac{Q_t \mathcal{B}_t^*}{\Pi_t^*} + \mathcal{S}_t Y_t - C_t.$$

After log-linearization around the steady state and by noticing $\hat{\mathcal{S}}_t = -\alpha \hat{s}_t$, we have

$$\hat{b}_{t+1}^* = \beta^{-1}(\hat{b}_t^* + \vartheta_1 \hat{q}_t - \vartheta_2 \alpha \hat{s}_t + \vartheta_2 \hat{y}_t - \hat{c}_t) - \vartheta_1 \hat{q}_t + \vartheta_1 \hat{r}_t^*, \tag{2.32}$$

where $\vartheta_1 \equiv \frac{\bar{\mathcal{B}}^* \bar{Q}}{\bar{C}}$ and $\vartheta_2 \equiv \frac{\bar{\mathcal{S}} \bar{Y}}{\bar{C}}$. Equation (2.32) governs the evolution of the foreign bond holdings (net foreign asset position) $\hat{b}_t^*$ in the equilibrium.

Foreign Interest Rate. We assume that the foreign interest rate faced by the domestic country endogenously responds to its level of net foreign asset position:

$$\hat{r}_t^* = \phi_b \tilde{b}_{t+1}^* + \mu_t, \tag{2.33}$$

where $\tilde{b}_{t+1}^*$ is the cross-sectional average of the foreign bond holdings across households (which is beyond household i's control), $\phi_b < 0$ is a constant, and μ_t is the random foreign interest rate shock. In equilibrium, we have $\tilde{b}_{t+1}^* = \hat{b}_{t+1}^*$. Here $\phi_b < 0$ indicates that as the country borrows more from the rest of the world ($\tilde{b}_{t+1}^* < 0$), it is charged with a higher interest rate by the international market. This specification follows Schmitt-Grohé and Uribe (2003) through an external debt-elastic foreign interest rate, a common device to induce stationarity in linearized small open-economy models under RE with incomplete asset markets.

As discussed in Woodford (2019) and Xie (2020), a model with limited foresight by design always guarantees a unique equilibrium solution in the closed-economy setting, regardless of the restrictions on the monetary/fiscal policy reaction function. This feature of equilibrium determinacy also holds in our small open-economy model under limited foresight.[7] However, we still impose the same assumption (2.33) in the model of limited foresight to facilitate its comparison with the counterpart model under RE, so that we can isolate the role of limited foresight on the different equilibrium performances.

2.5 Decision-makers' Learning in Value Functions

Thus far, we have assumed that the value functions of households $v\left(\mathcal{B}, \mathcal{B}^{*}\right)$ and of firms $\tilde{v}\left(r^{f}\right)$ are the fixed ones learned from the nonstochastic stationary environment. Starting from this section, we relax this assumption such that decision-makers update their value functions over time based on their past experiences. Similar to Woodford (2019), we assume that the learning behaviors of households and firms follow a constant-gain process:

$$v_{t+1}(\mathcal{B}, \mathcal{B}^{*}) = \gamma_{v} v_{t}^{est}(\mathcal{B}, \mathcal{B}^{*}) + (1-\gamma_{v}) v_{t}(\mathcal{B}, \mathcal{B}^{*}),$$

$$\tilde{v}_{t+1}\left(r^{f}\right) = \gamma_{\tilde{v}} \tilde{v}_{t}^{est}\left(r^{f}\right) + (1-\gamma_{\tilde{v}}) \tilde{v}_{t}\left(r^{f}\right),$$

where $\gamma_{v}, \gamma_{\tilde{v}} \in [0,1]$ are learning gain parameters. That is, decision-makers *extrapolate* their priors of future value functions (which will be used in the planning exercise at time $t+1$) using their priors of the value functions at time t and estimates of the value function obtained as a result of their planning exercises at time t. Therefore, the value functions that describe decision-makers' perceptions of the future beyond their planning horizons reflect past and estimated value functions.

We now consider a local approximation of the dynamics implied by the constant-gain learning rule through a perturbation of the steady-state solution. We parameterize a log-linear approximation of $v_1(\mathcal{B}, \mathcal{B}^{*})$ in the household's optimal finite-horizon plan with respect to the domestic bond as

$$\log(v_{1,t}(\mathcal{B}, \mathcal{B}^{*})/v_{1}^{*}(\bar{\mathcal{B}}, \bar{\mathcal{B}}^{*})) = -\sigma\left[\nu_{t} + \chi_{t}\hat{b} + \zeta_{t}\hat{b}^{*}\right].$$

Here we use $v^{*}(\cdot)$ to denote the steady-state value function. Using this approximation, we can compute a log-linear approximation of the solution to the household's optimal finite-horizon plan in period t.

[7] When there is no learning in agents' value function, the equilibrium under limited foresight is stationary even with an exogenous path of $\hat{r}_{t}^{*}$. However, when agents learn and update their value function, a device similar to (2.33) is necessary to guarantee the stationarity of equilibrium.

Let $C_t^i(\mathcal{B}, \mathcal{B}^*)$ denote the optimal expenditure plan of household i under the counterfactual assumption $\mathcal{B}^i = \mathcal{B}$. Then, the derivative of the estimated value function is equal to

$$v_{1,t}^{est}(\mathcal{B}, \mathcal{B}^*) = \mathbb{E}_t^i[u_C(C_t^i(\mathcal{B}, \mathcal{B}^*)/\Pi_t].$$

By log-linearization, we then have

$$\log(v_{1,t}^{est}(\mathcal{B}, \mathcal{B}^*)/v_1^*(\bar{\mathcal{B}}, \bar{\mathcal{B}}^*)) = -\sigma \hat{c}_t^h(\hat{b}, \hat{b}^*) - \pi_t^h, \tag{2.34}$$

where h is the planning horizon of household i. Our log-linear approximation of the optimal household plan satisfies $\hat{c}_t^h(\hat{b}, \hat{b}^*) = \hat{c}_t^h(\bar{b}, \bar{b}^*) + \hat{c}_{1,t}^h \hat{b} + \hat{c}_{2,t}^h \hat{b}^*$. Approximating the left-hand side as $-\sigma\left[\nu_t^{est} + \chi_t^{est}\hat{b} + \zeta_t^{est}\hat{b}^*\right]$ of equation (2.34) directly implies

$$\nu_t^{est} = \hat{c}_t^h + \sigma^{-1}\pi_t^h - \zeta_t^{est}\hat{b}_t^*,$$

where we have used the condition $\hat{b}_t = 0$ for all t. Equating the coefficients on both sides by substituting the expression of $\hat{c}_t^h(\hat{b}, \hat{b}^*)$ yields

$$\chi_t^{est} = \hat{c}_{1,t}^h, \qquad \zeta_t^{est} = \hat{c}_{2,t}^h.$$

The intercept term of the estimated marginal value of the domestic bond, ν_t^{est}, depends on current consumption, the CPI inflation, and the predetermined net foreign asset position. ν_t^{est} increases with a decline in both current consumption and inflation. The intuition is that an increase in the marginal utility of consumption raises the marginal value of the real domestic bond via the standard wealth effect, while inflation reduces the domestic bond's real value.

Together with the constant-gain learning rule, we have

$$\begin{aligned}
\nu_{t+1} &= \gamma_v \nu_t^{est} + (1-\gamma_v)\nu_t, \\
\chi_{t+1} &= \gamma_v \chi_t^{est} + (1-\gamma_v)\chi_t, \\
\zeta_{t+1} &= \gamma_v \zeta_t^{est} + (1-\gamma_v)\zeta_t.
\end{aligned}$$

We can show that χ_t and ζ_t are univariately mean-reverting to a constant $1-\beta$, and thus we assume that these two variables have converged. Details can be found in Appendix E.

Similarly, we parameterize a log-linear approximation of $v_2(\mathcal{B}, \mathcal{B}^*)$ in the households' optimal finite-horizon plan with respect to the foreign bond as

$$\log(v_{2,t}(\mathcal{B},\mathcal{B}^*)/v_2^*(\bar{\mathcal{B}},\bar{\mathcal{B}}^*)) = -\sigma\left[\nu_t^* + \chi_t'\hat{b} + \zeta_t'\hat{b}^*\right].$$

Using this approximation, we can compute a log-linear approximation of the solution to the household's optimal finite-horizon plan in period t.

Let $C_t^i(\mathcal{B},\mathcal{B}^*)$ denote the optimal expenditure plan of household i under the counterfactual assumption $\mathcal{B}^{*i} = \mathcal{B}^*$. Then, the derivative of the estimated value function will be equal to

$$v_{2,t}^{est}(\mathcal{B},\mathcal{B}^*) = \hat{\mathbb{E}}_t^i[u_C(C_t^i(\mathcal{B},\mathcal{B}^*))Q_t/\Pi_t^*].$$

Note we have assumed that $\Pi_t^* = 1$ for any t. By log-linearization, we then have

$$\log(v_{2,t}^{est}(\mathcal{B},\mathcal{B}^*)/v_2^*(\bar{\mathcal{B}},\bar{\mathcal{B}}^*)) = -\sigma\hat{c}_t^h(\hat{b},\hat{b}^*) + \hat{q}_t^h. \tag{2.35}$$

Our log-linear approximation of the optimal household plan satisfies $\hat{c}_t^h(\hat{b},\hat{b}^*) = \hat{c}_t^h(\bar{b},\bar{b}^*) + \hat{c}_{1,t}^h\hat{b} + \hat{c}_{2,t}^h\hat{b}^*$. Approximating the left-hand side as $-\sigma[\nu_t^{*,est} + \chi_t^{\prime,est}\hat{b} + \zeta_t^{\prime,est}\hat{b}^*]$ and equating coefficients yield

$$\nu_t^{*,est} = \hat{c}_t^h - \sigma^{-1}\hat{q}_t^h - (1-\beta)\hat{b}_t^*,$$

$$\chi_t^{\prime,est} = \hat{c}_{1,t}^h = \chi_t^{est},$$

$$\zeta_t^{\prime,est} = \hat{c}_{2,t}^h = \zeta_t^{est},$$

where we have utilized the fact that $\zeta_t^{est} = 1-\beta$ as shown in Appendix E. Thus, the estimated marginal value of the foreign bond, $\nu_t^{*,est}$, depends on the current real exchange rate, the CPI inflation, and the predetermined net foreign asset position. $\nu_t^{*,est}$ decreases as current consumption falls and the real exchange rate depreciates. The intuition is that an increase in the marginal utility of consumption raises the marginal value of the real foreign bond via the standard wealth effect, and a depreciation of the real exchange rate increases the foreign bond's real value (in domestic currency).

Note further that the constant-gain learning rule yields

$$\nu_{t+1}^* = \gamma_v\nu_t^{*,est} + (1-\gamma_v)\nu_t^*.$$

Therefore, we have characterized the learning process of the household value function.

For the firm (with planning horizon k), we similarly have

$$\tilde{\nu}_t^{est} = (1-\theta)^{-1}\pi_{H,t}^k,$$

and

$$\tilde{\nu}_{t+1} = \gamma_{\tilde{v}} \tilde{\nu}_t^{est} + (1 - \gamma_{\tilde{v}}) \tilde{\nu}_t.$$

3 Equilibrium Characterization with Homogeneous Planning Horizons across Agents

In this section, we focus on the steps to pin down the equilibrium path with the assumption that all the agents share the same planning horizon h. Therefore, the equilibrium dynamics of aggregate variables satisfy $\hat{y}_t = \hat{y}_t^h$, $\pi_t = \pi_t^h$, etc., and also $\hat{b}_{t+1}^* = \hat{b}_{t+1}^{*,h} = \tilde{b}_{t+1}^*$. Focusing on the case of homogeneous agents allows us to abstract from the aggregation problem across the population, while we can still analyze how the equilibrium dynamics change with respect to the degree of foresight (i.e., the common planning horizon h). In Section 6, we extend the analyses to heterogeneous agents with different planning horizons and discuss the robustness of the main results.[8]

First, given state variables $\{\hat{q}_{t-1}, \hat{b}_t^*, \nu_t, \nu_t^*, \tilde{\nu}_t\}$ and exogenous shocks $\{a_t, \mu_t\}$ that follow AR(1) processes, we solve the problem of the finite planning exercise in period t. Let $\hat{y}_{\tau|t}^j$ be the expected value of $\hat{y}_\tau$ at date τ, as a result of aggregation of decisions made by agents with (counterfactual) planning horizon j in that period. Here $|t$ in the subscript indexes the date at which the finite planning takes place. It matters because different value functions are used in finite planning in different periods. $\hat{y}_{\tau|t}^j$ is a function of the state $\{\hat{q}_{t-1}, \hat{b}_t^*, \nu_t, \nu_t^*, \tilde{\nu}_t\}$ and $\{a_t, \mu_t\}$ in period t. Then, we have the actual aggregate output in period t given by $\hat{y}_t = \hat{y}_{t|t}^h$. Similarly, we can define other variables in the finite planning exercise with the same notation.

The equilibrium conditions for the finite planning exercise in period t are given in Appendix F. The system consists of a finite number of equations as a function of state variables $\{\hat{q}_{t-1}, \hat{b}_t^*, \nu_t, \nu_t^*, \tilde{\nu}_t\}$ and exogenous shocks $\{a_t, \mu_t\}$. Thus, we can solve for all endogenous variables $\{\hat{c}_{\tau|t}^j, \hat{y}_{\tau|t}^j, \hat{\imath}_{\tau|t}^j, \hat{r}_{\tau|t}^{*,j}, \pi_{H,\tau|t}^j, \pi_{\tau|t}^j, \hat{q}_{\tau|t}^j, \hat{s}_{\tau|t}^j, \hat{\varepsilon}_{\tau|t}^j, \hat{b}_{\tau+1|t}^{*,j}\}_{\tau=t}^{t+h}$ with a unique solution, where $j = h + t - \tau$. The actual aggregate variables in period t are then given by

$$\begin{aligned} &\hat{c}_t = \hat{c}_{t|t}^h, \qquad \hat{y}_t = \hat{y}_{t|t}^h, \qquad \hat{\imath}_t = \hat{\imath}_{t|t}^h, \qquad \hat{r}_t^* = \hat{r}_{t|t}^{*,h}, \qquad \pi_{H,t} = \pi_{H,t|t}^h, \\ &\pi_t = \pi_{t|t}^h, \qquad \hat{q}_t = \hat{q}_{t|t}^h, \qquad \hat{s}_t = \hat{s}_{t|t}^h, \qquad \hat{\varepsilon}_t = \hat{\varepsilon}_{t|t}^h, \qquad \hat{b}_{t+1}^* = \hat{b}_{t+1|t}^{*,h}. \end{aligned} \tag{3.1}$$

[8] Our conclusions are also robust when extending the set of exogenous shocks to include domestic demand shock and domestic interest rate shock, in addition to the productivity shock and foreign interest rate shock.

From period t to period $t+1$, the value functions evolve over time; that is,

$$V_{t+1} = \Gamma V_t^{est} + (I - \Gamma) V_t, \tag{3.2}$$

where $V_t \equiv [\nu_t \ \tilde{\nu}_t \ \nu_t^*]'$, $V_t^{est} \equiv [\nu_t^{est} \ \tilde{\nu}_t^{est} \ \nu_t^{*,est}]'$, $\Gamma \equiv \mathrm{diag}(\gamma_v, \gamma_v, \gamma_{\tilde{v}})$, and I is an identity matrix. The details expression of V_t^{est} can be found in Appendix F.

Now, we describe how to solve the planned solution at date τ calculated in period t, which is characterized by the equilibrium conditions of the finite planning exercise as shown in Appendix F. We can write the solution to any endogenous variable $x_{\tau|t}^j$ except $\hat{b}_{\tau+1|t}^{*j}$ in forward planning as a function of the state variables and exogenous shocks; that is,

$$x_{\tau|t}^j = \psi_{x,q}^j \hat{q}_{\tau-1|t}^{j+1} + \psi_{x,b}^j \hat{b}_{\tau|t}^{*,j+1} + \psi_{x,a}^j a_\tau + \psi_{x,\mu}^j \mu_\tau + \psi_{x,\nu}^j \nu_t + \psi_{x,\tilde{\nu}}^j \tilde{\nu}_t + \psi_{x,\nu^*}^j \nu_t^*, \tag{3.3}$$

for any (counterfactual) $j \geq 0$, and similarly,

$$\hat{b}_{\tau+1|t}^{*j} = \psi_{b,q}^j \hat{q}_{\tau-1|t}^{j+1} + \psi_{b,b}^j \hat{b}_{\tau|t}^{*,j+1} + \psi_{b,a}^j a_\tau + \psi_{b,\mu}^j \mu_\tau + \psi_{b,\nu}^j \nu_t + \psi_{b,\tilde{\nu}}^j \tilde{\nu}_t + \psi_{b,\nu^*}^j \nu_t^*. \tag{3.4}$$

Then, one can solve the undetermined coefficients via the equilibrium conditions of the finite planning exercise for any j in the following steps: (i) utilizing the equilibrium conditions for $j = 0$ and equating the coefficients yields the coefficients for $j = 0$; (ii) solving the undetermined coefficients for any (counterfactual) j by forward induction. That is, given the coefficients for $j - 1$, the undetermined coefficients for j are uniquely given by the equilibrium conditions for j. See Appendix G for the detailed procedure.

Thus far, we have derived the solution of the entire forward planning calculated in period t. Then, one can easily solve for the equilibrium path (3.1) with the evolution of the state variables (F.23)-(F.24) together with exogenous shocks.

4 Equilibrium Analyses

This section investigates the equilibrium features of the FH model. We first explore the characteristics of finite planning horizons by focusing on the Law of Iterated Expectation. We then calibrate the model parameters to match key moments of macro-international aggregates of Canada, in conjunction with the U.S. Following this calibration, we analyze the equilibrium dynamics of the model by examining impulse responses. Lastly, we discuss how the model with limited foresight generates systematic forecast errors in the real exchange rate.

4.1 The Breakdown of the Law of Iterated Expectation

The LIE holds true in a model with RE. However, it no longer holds in the FH model when the learning gain parameter, utilized in updating agents' value functions, is non-zero.

Proposition 1 *The Law of Iterated Expectation (LIE) does not hold when the economy is not always at the steady state and the learning gain of value function is not zero ($\gamma_v, \gamma_{\tilde{v}} \neq 0$), that is,*

$$\hat{\mathbb{E}}_t x_{t+2} \neq \hat{\mathbb{E}}_t \hat{\mathbb{E}}_{t+1} x_{t+2}.$$

Proof: see Appendix H.

Proposition 1 is a result of the updating behavior of the agents' value function. Intuitively, $\hat{\mathbb{E}}_t x_{t+2}$ is a function of the time-t value function v_t, whereas $\hat{\mathbb{E}}_{t+1} x_{t+2}$ is a function of the time-$(t+1)$ value function v_{t+1}. Since the evolution from v_t to v_{t+1} follows constant-gain learning, it implies $v_t \neq \hat{\mathbb{E}}_t v_{t+1}$ as long as $\gamma_v, \gamma_{\tilde{v}} \neq 0$. Instead, if $\gamma_v, \gamma_{\tilde{v}} = 0$, $v_{t+1} = v_t$ at any time t and then the LIE holds. One should note that the breakdown of the LIE in the FH model comes from the assumption that the value function in agents' finite forward planning exercise is *coarse*; if the value function is accurate enough, being the same as the one under RE, v_t becomes time-invariant and thus the LIE holds. We demonstrate in Section 5 that this feature of the FH model provides a natural micro-foundation for the puzzling aspect of the RE-UIP condition across the forecast horizons, as documented by Galí (2020).

4.2 Calibrated Parameters

We calibrate the FH model to a quarterly frequency; see Table 1. Following the common practice in the open-economy macro literature, we assume a symmetric steady state across the domestic country and the rest of the world, with $\bar{\mathcal{B}}^* = 0$, $1 + \bar{r}^* = 1/\beta$, and $\bar{Q} = 1$. It directly implies $\bar{\mathcal{S}} = \bar{S} = 1$ and $\bar{C} = \bar{Y}$.

The following parameters are standard in the literature. We set the subjective discount factor $\beta = 0.99$, the inverse of intertemporal elasticity of substitution $\sigma = 2$, and the inverse of the Frisch elasticity of labor supply $\varphi = 1$. Parameter α, which governs the home bias $(1-\alpha)$, is set to 0.2, a common value in the literature (e.g., Valchev, 2020). The Calvo-Yun price stickiness parameter is set to $\theta = 0.75$, implying an average duration of four quarters between two consecutive price adjustments. We set the parameter of policy reaction in the Taylor rule $\phi_\pi = 2.15$, following Clarida, Galí and Gertler (1999), and the parameters of trade elasticity γ and η to 1.5, following Backus, Kehoe and Kydland (1994) and Chari, Kehoe and McGrattan (2002). The sensitivity of the foreign interest rate to foreign bond holdings is set to $\phi_b = -0.01$, following Benigno (2009) and Justiniano and Preston (2010).

Table 1: Calibrated Parameters

Parameter	Value	Description
h	8	Length of Planning Horizon (quarter)
β	0.99	Subjective Discount Factor
σ	2	Inverse of Intertemporal Elasticity of Substitution
α	0.2	1 - Home Bias
φ	1	Inverse of the Frisch Elasticity of Labor Supply
θ	0.75	Calvo-Yun Sticky Price Parameter
γ	1.5	Foreign Demand Elasticity for Home Goods
η	1.5	Elasticity of Substitution between Goods of Home and Foreign
ϕ_π	2.15	Monetary Policy Reaction Coefficient to the CPI Inflation Rate
ϕ_b	-0.01	External Bond Sensitivity of the Foreign Interest Rate
ρ_x	0.9	Persistence of TFP and Foreign Interest Rate Shocks
σ_a/σ_μ	2.55	Relative Std. Deviations of Shocks
γ_v	0.14	Household's Learning Gain

Notes: We normalize the standard deviation of the foreign interest rate shock to one without loss of generality, as we do not target the absolute standard deviation of variables.

Together with the steady-state values, we calculate the coefficients ϑ_1, ϑ_2, ϑ_{yc}, and ϑ_{ys} in (2.30) and (2.32) by their definitions.

Following Woodford and Xie (2022), we set the length of the planning horizon $h = 8$ (that is, eight quarters). This is a *conservative* value because empirical findings suggest an even shorter planning horizon; for instance, Gust, Herbst and López-Salio (2022) estimate an average planning horizon as being one-quarter of the U.S. economy.[9] If the planning horizon is shorter, our conclusions are strengthened as they deviate more from the case under RE.

We set the persistence of the TFP shock to $\rho_a = 0.9$, following Candian and De Leo (2023). We also set the persistence of the foreign interest rate shock, ρ_μ, to 0.9, the same as that of the TFP shock. This is in line with the commonly assumed interest rate inertia coefficient for the U.S. (e.g., Valchev, 2020). We then calibrate the two remaining parameters: the relative standard deviations between the TFP shock and the foreign interest rate shock, σ_a/σ_μ, and the household's learning gain parameter, γ_v.[10] These two parameters are calibrated to match the following two moments, utilizing Canadian data from 1970:Q3 to 2007:Q4: (i) the relative standard deviation of consumption growth to output growth, $\sigma(\Delta c)/\sigma(\Delta y)$; and (ii) the persistence of the real exchange rate growth $\rho(\Delta q)$.[11] We obtain

[9]The survey evidence in Coibion et al. (2023) suggests that household planning horizons in the U.S. are no more than two years and they may well be capable of planning forward about three or four quarters.

[10]In our benchmark numerical analysis, we nullify the firm's learning behavior on its value function by setting $\gamma_{\tilde{v}} = 0$ for parsimony. Incorporating the firm's learning behavior has negligible effects on our findings in Sections 4 and 5. Thus, we leave the case of incorporating the firm's learning behavior in the robustness checks; see Appendix L.

[11]Appendix I presents the data source and variable construction. The exchange rate we use in calibration

Table 2: Data and Model-Implied Second-Order Moments

	Data (1)	Calibrated FH (2)	Non-calibrated RE (3)	Calibrated RE (4)
Targeted Moments				
$\sigma(\Delta c)/\sigma(\Delta y)$	0.94	0.94	1.15	0.94
$\rho(\Delta q)$	0.30	0.30	0.19	0.24
Non-Targeted Moments				
$\rho(q)$	0.98	0.97	0.90	0.96
$\rho(\varepsilon)$	0.34	0.40	0.21	0.25
$\rho(\varepsilon, \Delta q)$	0.94	0.97	0.97	0.99
$\sigma(\varepsilon)/\sigma(\Delta y)$	2.41	2.05	2.06	0.88
$\sigma(\Delta q)/\sigma(\Delta y)$	2.44	1.56	1.68	0.75
$\rho(\Delta y, \Delta c)$	0.51	0.30	0.30	0.86
$\rho(\Delta c, \Delta q)$	-0.11	-0.42	-0.54	0.21

Notes: Column (1) shows the interested moments in the data. Column (2) shows the moments from the FH model with calibrated parameters in Table 1. Column (3) shows the moments from the RE model with the same parameters as in Column (2). Column (4) shows the moments from the RE model by re-calibrating the shock persistence and relative standard deviations to match the same targeted moments. $\sigma(\Delta c)/\sigma(\Delta y)$ represents the relative standard deviation between consumption growth and output growth; $\rho(\Delta q)$ represents the persistence of the real exchange rate growth; and $\rho(\Delta y, \Delta c)$ represents the cross-correlation between output growth and consumption growth. Similar notations apply to the other moments. For model-implied second-order moments, each entry is the median of the moments derived from 10,000 simulations, each spanning 150 quarters. The length of quarters matches that of the Canadian data we utilize.

a relative standard deviation of $\sigma_a/\sigma_\mu = 2.55$ and a learning gain of $\gamma_v = 0.14$. The calibrated learning gain parameter is also close to the benchmark estimate in Gust, Herbst and López-Salio (2022) using the U.S. data, which is around 0.14.

Table 2 reports a number of moments for the macroeconomic aggregates, comparing both the data and the model-generated moments with the calibrated parameters. The FH model matches the overall moments reasonably well, including the two targeted moments and other non-targeted ones. Among the non-targeted moments, the FH model matches the data well for the autocorrelations of real exchange rate level and nominal exchange rate growth, as well as the high correlation close to one between nominal and real exchange rate growth. The FH model also successfully predicts significantly higher exchange rate growth volatility

is the one between Canada and the U.S. The sample starts from 1970:Q3 because the US-Canada exchange rate was pegged prior to this date and stops at 2007:Q4 to exclude the period of the Great Recession. In addition, we target the growth rate of the real exchange rate to ensure consistency among the targeted moments, where the first moment includes the growth rates of consumption and output. Nonetheless, our results are robust even if we target the persistence of the real exchange rate level rather than growth (see the first row of non-targeted moments in Table 1).

(both real and nominal) compared to output volatility, though the ratio is slightly lower than in the data. It also captures the negative correlation between consumption growth and real exchange rate growth, albeit with a higher magnitude than in the data.

Table 2 also shows that the FH model fits the aggregate data better than the RE models. Given that the parameter values in Table 1 are calibrated to fit the FH model to the data, it is not surprising that the RE counterpart with the same parameters does not perform better than the FH model (shown in Column (3)). However, even when the RE model is recalibrated to match the same targeted moments (shown in Column (4)), it does not match the data as well as the FH model does.[12] The recalibrated RE model cannot accurately match the targeted persistence of real exchange rate growth. Additionally, it fails to match the non-targeted moments in several dimensions, including the higher volatility of exchange rate growth compared to output growth volatility, and the negative correlation between consumption growth and real exchange rate growth. It also predicts a positive correlation between output growth and consumption growth, which is opposite to the data.

4.3 Impulse Responses

We now analyze the equilibrium dynamics by comparing the impulse response functions between the FH model and its RE counterpart (Column (3) in Table 2). To isolate the role of value function learning, we also compare with the FH model in which the learning gain in value function is zero (that is, $\gamma_v = 0$, labeled as FH-NG).

Figure 1 illustrates the impulse responses of the variables of interest to the two structural shocks, respectively, with a size of one standard deviation. In essence, across all three models, the foreign interest rate and domestic productivity shocks (shown in panels (a) and (b), respectively) conform to their roles in price and quantity determination. The former shock causes real exchange rate depreciation, decreases consumption, and elevates CPI inflation via exchange rate pass-through. The latter shock depreciates the real exchange rate, raises consumption, and triggers a small temporary rise in CPI inflation due to the exchange rate pass-through, followed by a decline in inflation induced by the positive supply shock.

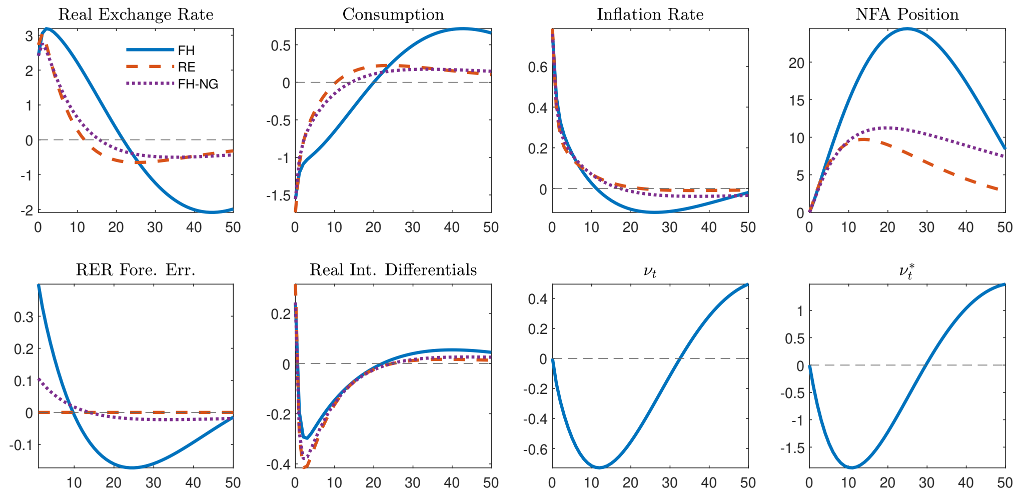
Both shocks prompt households to boost savings, resulting in a gradual rise in net foreign asset positions $\hat{b}_t^*$ in subsequent periods. Defined by $\hat{r}_t - \hat{r}_t^*$ (with $\hat{r}_t$ being the nominal interest rate minus expected one-period ahead inflation), the real interest rate differential between domestic and foreign bonds experiences an initial surge but later adjusts downwards, indicating higher subsequent real returns from foreign bonds.[13]

[12] In this case, the persistence of the shocks in the RE model has to increase to 0.99 and the relative standard deviation σ_a/σ_μ increases to 16.63.

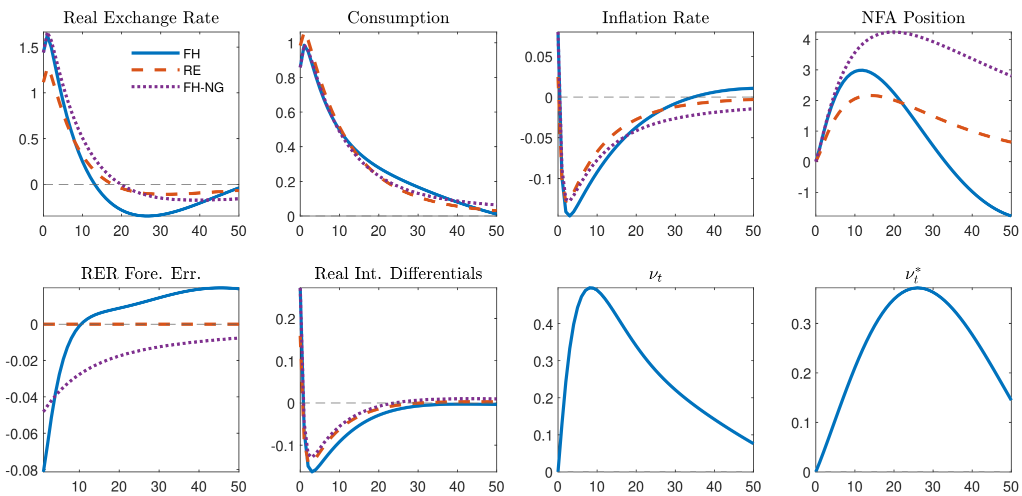
[13] Figure 1 shows an initial surge in the real interest rate differential in both the FH and RE models. Thus,

Figure 1: Impulse Responses to Exogenous Shocks

(a) Foreign Interest Rate Shock



(b) Domestic Productivity Shock



Notes: This figure shows the impulse responses of selected variables in three models, subject to one standard deviation foreign interest rate shock (panel (a)) and to one standard deviation domestic productivity shock (panel (b)). "FH" refers to the benchmark calibrated finite planning horizon model, "RE" refers to the rational expectation model, and "FH-NG" refers to the finite planning horizon model with no learning gain ($\gamma_v = 0$). "RER Fore. Err." represents the forecast error of the real exchange rate (RER), defined as $\hat{q}_{t+1} - \hat{\mathbb{E}}_t \hat{q}_{t+1}$; and "Real Int. Differential" represents the real interest rate differential between domestic and foreign bonds, defined as $\hat{r}_t - \hat{r}_t^*$. The x-axis is time in quarters.

this surge is not specific to the FH model but is attributed to the substantial persistence of shocks, set at 0.9.

These shared patterns of impulse responses across the FH and RE models suggest that the FH model need not compromise its ability to match aggregate moments compared to the RE model while offering a genuine expectation formation. This expectation formation of the FH model, as discussed in more detail in Section 5, contributes to explaining the RE-UIP violations. Therefore, the FH model, without sacrificing matching aggregate moments and, if anything, potentially improving upon them, provides a more realistic depiction of expectation formation on exchange rate dynamics compared to the RE model.

Despite those common patterns, we also observe notable differences across the models. Particularly, in contrast to the RE model, the FH model typically demonstrates more persistent, hump-shaped movements of the aggregate variables. Furthermore, the FH model generates non-trivial dynamics of the forecast errors, a distinctive feature absent in the RE model. We now examine these differences in detail.

Dynamics of the Value Functions. Figure 1 shows a hump-shaped dynamics of ν_t and ν_t^*. As detailed in Section 2.5, ν_t and ν_t^* capture the log-linear approximations of the derivatives of the value function of households in the FH model to domestic and foreign bonds, $v_{1,t}(\mathcal{B}, \mathcal{B}^*)$ and $v_{2,t}(\mathcal{B}, \mathcal{B}^*)$, respectively. Similarly, ν_t^{est} and $\nu_t^{*,est}$ capture the log-linear approximations of the derivatives of the estimated value function of households to domestic and foreign bonds, $v_{1,t}^{est}(\mathcal{B}, \mathcal{B}^*)$ and $v_{2,t}^{est}(\mathcal{B}, \mathcal{B}^*)$, respectively.

When shocks occur at $t = 0$, households use the steady-state evaluated priors ν_0 and ν_0^* for planning. Subsequently, they observe the realized macroeconomic outcomes and estimate ν_0^{est} and $\nu_0^{*,est}$.[14] As outlined in equation (3.2), households then update ν_1 and ν_1^* for planning exercise at $t = 1$ by averaging the estimates with the previous priors ν_0 and ν_0^*. After observing the realized macroeconomic outcomes at the end of $t = 1$, they obtain new estimates ν_1^{est} and $\nu_1^{*,est}$ and update ν_2 and ν_2^* similarly for planning at $t = 2$. This process is repeated thereafter.

Since the household value function shapes their long-run beliefs of the future beyond their planning horizons, their updating behavior on the value function leads to dynamic adjustments of their long-run beliefs. Initially, households behave based on the long-run beliefs derived from the steady-state value function, resulting in an underreaction due to the coarseness of the value function and the finite planning horizon. Over time, households gradually update their value function via constant gain (with $\gamma_v = 0.14$), which eventually leads to excess extrapolation of long-run beliefs and subsequent overreaction. This sequence of initial underreaction followed by gradual overreactions results in the hump-shaped dynamics of ν_t and ν_t^* shown in Figure 1.

[14]See the expressions of ν_0^{est} and $\nu_0^{*,est}$ as a function of realized macroeconomic outcomes in Appendix F.

It is worth noting that because the macroeconomic aggregates respond to the two shocks in different directions, the hump-shaped dynamics of the value functions via learning also move in different directions. As shown in Section 2.5, given the predetermined net foreign asset position $\hat{b}_t^*$, ν_t^{est} increases with a decline in both consumption and inflation; on the other hand, $\nu_t^{*,est}$ decreases as consumption falls and the real exchange rate depreciates.

With the calibrated parameters, $\{\nu_t^{est}, \nu_t^{*,est}\}$ follow: $\nu_t^{est} = \hat{c}_t + 0.5\pi_t - 0.01\hat{b}_t^*$ and $\nu_t^{*,est} = \hat{c}_t - 0.5\hat{q}_t - 0.01\hat{b}_t^*$.[15] The values of these coefficients suggest that consumption, inflation, and the real exchange rate are the primary drivers of ν_t^{est} and $\nu_t^{*,est}$ quantitatively. Given that Figure 1 shows the two shocks affect these endogenous macroeconomic variables differently, both in direction and magnitude, ν_t and ν_t^* follow U-shaped dynamics in response to the foreign interest rate shock and inverse U-shaped dynamics in response to the domestic productivity shock.

This adjustment process in long-run beliefs plays a crucial role in shaping the equilibrium response of macroeconomic outcomes. Simultaneously, the realized macroeconomic outcomes reciprocally impact the process of belief updating. In equilibrium, this interactive feedback leads to distinctive hump-shaped dynamics of macroeconomic aggregates in the FH model, distinguishing it from the corresponding dynamics in the RE model.

Dynamics of RER Forecast Errors. Another crucial feature of the FH model is that it generates dynamic overshooting of forecast errors, characterized by a *sign reversal* over time. Here, we focus on the dynamics of the one-period ahead forecast errors of the real exchange rate, $\hat{q}_{t+1} - \hat{\mathbb{E}}_t\hat{q}_{t+1}$. Panel (a) in Figure 1 shows that when subject to the foreign interest rate shock, the FH model initially exhibits a positive forecast error in the short-run and then reverses to negative in the subsequent periods. That is, in the FH model, households initially underestimate the depreciation in response to a foreign interest rate shock, but overestimate it across the time horizon. The dynamic overshooting of the forecast error also applies to the domestic productivity shock but in the opposite direction. Nonetheless, one can also observe that the magnitude of the forecast errors subject to the foreign interest rate shock are substantially larger than those under the productivity shock.

Although the FH-NG model also displays dynamic forecast errors, the magnitudes and dynamic reversals are significantly less pronounced compared to the FH model, due to the absence of the value function learning. To see the quantitative significance of value function learning, we decompose the forecast error of the one-period ahead real exchange rate in the FH and FH-NG models as follows:

[15] See Appendix F for the details of these two expressions.

$$\hat{q}_{t+1} - \hat{\mathbb{E}}_t \hat{q}_{t+1} = \underbrace{\underbrace{\left(\psi^8_{q,q} - \psi^7_{q,q}\right)\hat{q}_t}_{q \text{ part}} + \underbrace{\left(\psi^8_{q,b} - \psi^7_{q,b}\right)\hat{b}^*_{t+1}}_{b^* \text{ part}} + \underbrace{\left(\psi^8_{q,\mu} - \psi^7_{q,\mu}\right)\rho_\mu \mu_t}_{\mu \text{ part}} + \underbrace{\left(\psi^8_{q,a} - \psi^7_{q,a}\right)\rho_a a_t}_{a \text{ part}}}_{\text{forecast errors from finite forward planning}}$$
$$+ \underbrace{\underbrace{\left(\psi^8_{q,\nu}\nu_{t+1} - \psi^7_{q,\nu}\nu_t\right)}_{\nu \text{ part}} + \underbrace{\left(\psi^8_{q,\nu^*}\nu^*_{t+1} - \psi^7_{q,\nu^*}\nu^*_t\right)}_{\nu^* \text{ part}}}_{\text{forecast errors from value function extrapolation}}. \tag{4.1}$$

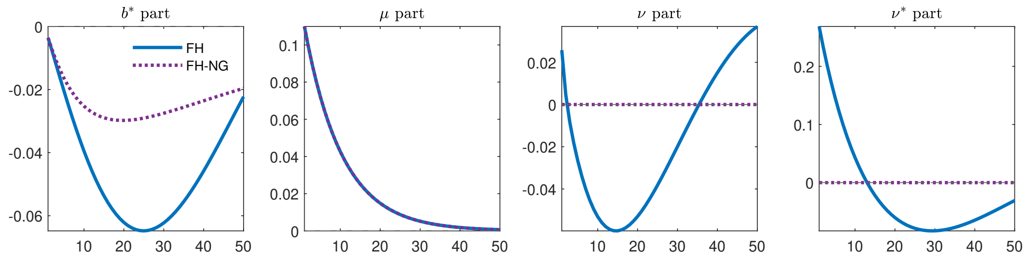
The decomposition (4.1) indicates that the forecast error of the real exchange rate in the FH model can be decomposed into two broad categories stemming from: (i) finite forward planning (q, b^*, μ, a), and (ii) gradual excess extrapolation through value function learning (ν, ν^*). The FH-NG model only contains the first category. Figure 2 shows the decomposition of the forecast errors of the real exchange rate shown in Figure 1. One can observe that the value function extrapolation of households in the FH model, from both ν_t and ν^*_t, plays a major role in contributing to the dynamic overshooting of the forecast error.

Equation (4.1) also sheds light on why the forecast errors of real exchange rates exhibit different signs in response to the two types of shocks in this quantitative exercise. Based on the calibration in Table 1, the coefficients of the μ and a parts are $\psi^8_{q,\mu} - \psi^7_{q,\mu} = 0.11$ and $\psi^8_{q,a} - \psi^7_{q,a} = -0.02$, respectively. The different signs of the coefficients indicate opposite initial responses of the forecast errors to the shocks when the value functions ν and ν^*_t have not moved much yet. The values of these coefficients also account for the different magnitudes of the forecast errors between the two panels in Figure 1. In the subsequent periods, the dynamics of the value functions dominate the movement of forecast errors, hence the forecast errors exhibit an overshooting.

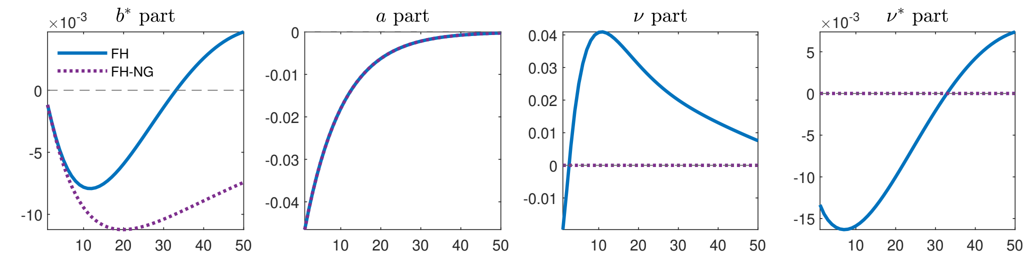
Despite the real exchange rate forecast errors showing opposite impulse responses to the two shocks in Figure 1, we find that this feature depends on the length of the planning horizon. Whereas our benchmark analyses adopt a conservative calibration for the planning horizon, the coefficient of the μ part in equation (4.1) becomes more positive and the coefficient in the a part becomes less negative as the planning horizon shortens. When the horizon is less than four quarters ($h \leq 4$), the latter even turns positive. In this scenario, the impulse responses of the forecast errors move in the same direction subject to a positive innovation in the two shocks.

Figure 2: Decomposition of the Forecast Errors of the Real Exchange Rate

(a) Foreign Interest Rate Shock



(b) Domestic Productivity Shock



Notes: This figure shows the selected components of the impulse response of the one-period ahead forecast errors of the real exchange rate in the two models, subject to one standard deviation foreign interest rate shock (panel (a)) and one standard deviation domestic productivity shock (panel (b)). "FH" refers to the benchmark finite planning horizon model and "FH-NG" refers to the finite planning horizon model with $\gamma_v = 0$. The x-axis is time in quarters. The q part is omitted because it is quantitatively negligible.

5 Addressing the UIP Puzzles

In this section, we show that the FH model adeptly addresses several puzzling characteristics concerning the time- and forecast-horizon aspects of the UIP puzzles. Recognizing that the calibrated parameters in Table 1 do not specifically target any empirical moments associated with UIP violations, our analyses thus provide external validity. This underscores the capability of the calibrated model to explain both the qualitative and quantitative aspects of expectation formations related to the UIP puzzles.

We begin by formally describing the RE-UIP condition and its implications.

5.1 RE-UIP Condition and Its Implications

The asset pricing equations of the real domestic currency bond and the real foreign currency bond in the RE framework (corresponding to (2.6)-(2.9) in the FH model) are:

$$u'(C_t) = \beta \mathbb{E}_t \left[(1+i_t)\, u'(C_{t+1})/\Pi_{t+1} \right], \tag{5.1}$$

$$u'(C_t) = \beta \mathbb{E}_t \left[(1+i_t^*)\, u'(C_{t+1})(Q_{t+1}/Q_t)(1/\Pi_{t+1}^*) \right]. \tag{5.2}$$

Combining (5.1) and (5.2) yields

$$\mathbb{E}_t\left[\frac{u'(C_{t+1})}{u'(C_t)}\left(\frac{1+i_t}{\Pi_{t+1}}-\frac{1+i_t^*}{\Pi_{t+1}^*}\frac{Q_{t+1}}{Q_t}\right)\right]=0. \tag{5.3}$$

The log-linear approximation of equation (5.3) around the nonstochastic steady state gives

$$\mathbb{E}_t\hat{q}_{t+1}-\hat{q}_t=\hat{r}_t-\hat{r}_t^*, \tag{5.4}$$

where $\hat{r}_t \equiv \hat{\imath}_t - \mathbb{E}_t\pi_{t+1}$ and $\hat{r}_t^* \equiv \hat{\imath}_t^* - \mathbb{E}_t\pi_{t+1}^*$. Equation (5.4) is the RE-UIP condition in real form. This implies that when the real interest rate differential between domestic and foreign currency bonds is positive (that is, $\hat{r}_t - \hat{r}_t^* > 0$), future real depreciation $\mathbb{E}_t\hat{q}_{t+1}-\hat{q}_t > 0$ should result of the same magnitude.[16]

We define the ex-post real excess return on foreign currency bonds from period t to $t+1$ as follows:

$$\Delta_{t+1} \equiv \hat{q}_{t+1}-\hat{q}_t+\hat{r}_t^*-\hat{r}_t. \tag{5.5}$$

Then, the UIP condition (5.4) under RE implies

$$\mathbb{E}_t\Delta_{t+1}=0, \tag{5.6}$$

indicating that the ex-post excess return Δ_{t+1} should be unpredictable with the information set at time t.

Furthermore, by extending equation (5.6) to time $t+k$ and applying the law of iterated expectation (LIE), one can obtain a corollary of the unpredictability result:

$$\mathbb{E}_t\Delta_{t+k}=0, \tag{5.7}$$

where $\Delta_{t+k} \equiv \hat{q}_{t+k}-\hat{q}_{t+k-1}+\hat{r}_{t+k-1}^*-\hat{r}_{t+k-1}$ is the ex-post one-period excess return between time $t+k-1$ and $t+k$. Thus, the RE-UIP condition implies that the ex-post excess return for any future time horizon $t+k$ is unpredictable based on the information set at time t. We regard this feature of unpredictability as the time-horizon aspect of the RE-UIP condition.

In addition, the RE-UIP condition also exhibits another feature from the perspective of term structure. Iterating the expectation term of real exchange rate $\mathbb{E}_t\hat{q}_{t+1}$ forward in

[16] The literature also often uses the nominal version of the RE-UIP condition. Choosing either nominal or real version is not consequential for our results. We use the real version simply to facilitate the discussion with Section 4. Also, note that the RE-UIP condition (5.4) is a simple specification and it can be extended to more complex forms, for example, by considering a stationary trend in the real exchange rate dynamics. For the sake of parsimony, we use the simplest specification to emphasize the role of behavioral biases from the FH models in addressing the UIP puzzles that we focus on in an essential modeling environment.

equation (5.4) by the LIE yields

$$\hat{q}_t = \sum_{k=0}^{\infty} \mathbb{E}_t[\hat{r}^*_{t+k} - \hat{r}_{t+k}] + \lim_{T\to\infty} \mathbb{E}_t \hat{q}_{t+T}.$$

Following Galí (2020), we decompose the sum of expectations into the short-term and long-term:

$$\hat{q}_t = D^S_t(M) + D^L_t(M) + \lim_{T\to\infty} \mathbb{E}_t \hat{q}_{t+T}, \tag{5.8}$$

where M is the threshold period for the short-term and the long-term summations, and

$$D^S_t(M) \equiv \sum_{k=0}^{M-1} \mathbb{E}_t[\hat{r}^*_{t+k} - \hat{r}_{t+k}], \qquad D^L_t(M) \equiv \sum_{k=M}^{\infty} \mathbb{E}_t[\hat{r}^*_{t+k} - \hat{r}_{t+k}],$$

are defined as the sum of expectations on the short- and long-term real interest rate differential, respectively.[17] Since the real exchange rate is a stationary variable, one can assume that $\lim_{T\to\infty} \mathbb{E}_t \hat{q}_{t+T} = 0$. Then, (5.8) indicates that the RE-UIP condition predicts the horizon invariance for the impact of the forecast of the real interest rate differential on the real exchange rate. That is, the forecast of the short-term interest rate differential $D^S_t(M)$ and that of the long-term interest rate differential $D^L_t(M)$ have identical effects on the current real exchange rate with the same weight of one. We regard this feature of horizon invariance as the forecast-horizon aspect of the RE-UIP condition.

5.2 Excess Return Predictability and the Predictability Reversal

A challenge to the RE-UIP condition (5.4) is a predictable excess return observed in the data. Early studies, such as Fama (1984) and Eichenbaum and Evans (1995), show a short-run positive predictable excess return of currency bonds that bear higher interest rates. Recent studies (e.g., Bacchetta and van Wincoop, 2010; Engel, 2016; Valchev, 2020) further document that the movements of the predictable excess return are more complicated over the time horizon: the excess return is positive in the short run, whereas it reverses to negative in the medium to long run. Thus, the UIP violations have time horizon variability.

Within both the FH and RE models, the ex-post one-period excess return on foreign currency bonds between time t and $t+1$ is, by construction, equivalent to the one-period-

[17] The definition of the real interest rate differential in Galí (2020) is represented as $\hat{r}^* - \hat{r}$. Thus, in the discussion regarding the forecast horizon invariance, we follow Galí (2020)'s definition to maintain consistency, whereas in the rest of the paper, we adopt the common practice by defining the real interest rate differential as $\hat{r} - \hat{r}^*$.

ahead forecast error of the real exchange rate:

$$\Delta_{t+1} \equiv \hat{q}_{t+1} - \hat{q}_t + \hat{r}^*_t - \hat{r}_t = \hat{q}_{t+1} - \hat{\mathbb{E}}_t q_{t+1},$$

where the symbol $\hat{\mathbb{E}}$ is intended to encompass both FH and RE expectation operators. This formulation intentionally disregards other potential factors in excess return predictability driven by time-varying risk or liquidity premia. Thus, our model isolates the expectation channel from other factors and this formulation is also consistent with the empirical finding that emphasizes the importance of subjective forecast errors in explaining the UIP deviations in advanced countries (e.g., Froot and Frankel, 1989; Chinn and Frankel, 2019; Kalemli-Özcan and Varela, 2022; Candian and De Leo, 2023).

Following the empirical specification in the literature, we run the regression model using the simulated data from the RE and FH models as follows:

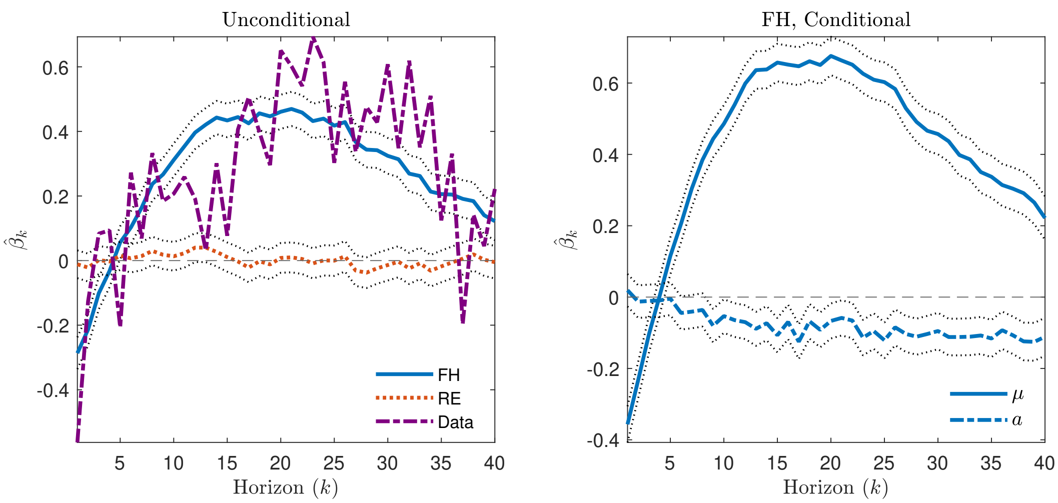
$$\Delta_{t+k} = \beta_0 + \beta_k(\hat{r}_t - \hat{r}^*_t) + \xi_{t+k}, \tag{5.9}$$

where the coefficient β_k captures the predictable excess return at time t for future horizon k. A negative β_k suggests that the foreign currency bonds yield a positive real excess return when the foreign bonds carry a higher real interest rate ($\hat{r}^*_t > \hat{r}_t$). Conversely, a positive β_k implies the opposite.

The left panel in Figure 3 illustrates the estimates of excess return coefficients, $\hat{\beta}_k$, from the two models, together with the empirical estimates obtained from the actual data.[18] In the RE model, the confidence intervals for the estimates nearly always contain zero, highlighting the unpredictability of excess returns as implied by the RE-UIP condition. In contrast, the FH model demonstrates that excess returns are predictable. It initially exhibits a negative $\hat{\beta}_k$ for the first five horizons, which then reverses to positive, peaks at time horizon $k = 21$, and diminishes thereafter. Thus, the FH model predicts that the foreign currency bond yields a short-run positive real excess return when the foreign bond bears a higher real interest rate. Meanwhile, it predicts a negative excess return in the medium- and long-run time horizons. Furthermore, one can observe that the estimates from the simulated FH model match the overall dynamics of the empirical estimates reasonably well, both qualitatively and quantitatively. Considering that we do not specifically target the UIP deviation in calibrating the FH model, its successful matching serves as evidence of external validity.

[18] The empirical estimates are conducted using real exchange rates and real interest rate differentials between Canada and the U.S. To construct the real interest rates in the data, we consider ex-ante real interest rates based on AR(1) fitted forecasted inflation. We also consider alternative approaches of constructing real interest rates based on current and ex-post inflation and find that the results are robust. For details on the construction of the data and further results, see Appendix J.1.

Figure 3: Excess Return Predictability across Time Horizons: Regression Coefficients



Notes: This figure presents the estimates of excess return coefficients, $\hat{\beta}_k$, across time horizon k. The bold lines represent point estimates and the surrounding thin dotted lines represent 95% confidence intervals. In the left panel, "FH" refers to the benchmark finite planning horizon model and "RE" refers to the rational expectation model. "Data" refers to the point estimates from the regressions using actual data, in which the ex-ante real interest rates are calculated based on AR(1) fitted forecasted inflation. The models in the left panel use the series generated by the two shocks. The right panel presents the regression coefficients in the FH model, using simulated data series conditional on the foreign interest rate shock (μ) and the domestic productivity shock (a), respectively. All estimates from the models are obtained using samples from 100 simulations with each spanning 150 quarters (15,000 observations in total).

Whereas the left panel of Figure 3 shows the *unconditional* profile of excess return predictability, the right panel of Figure 3 displays the $\hat{\beta}_k$ estimates, each conditioned on a distinct shock. Consistent with Figure 1 and the discussion in Section 4.3, the real exchange rate forecast errors in the FH model respond in qualitatively opposite ways to the two shocks, mirroring the behavior of the estimated $\hat{\beta}_k$. Despite both shocks indicating opposite excess return profiles in the FH model, the foreign interest rate shock dominates the unconditional profile of excess return.[19]

The sign reversal of forecast errors in the FH model, induced by the finite planning horizon and value function learning, introduces a novel explanation for the reversal of excess return predictability. It enriches existing theoretical explanations in the literature that attribute this phenomenon to infrequent portfolio decisions (Bacchetta and van Wincoop, 2010), convenience yields (Valchev, 2020), or over-extrapolation on misperceived shocks (Candian and

[19]The opposite conditional profiles depend on the length of the planning horizon. As the planning horizon becomes shorter ($h \leq 4$), whereas the conditional profile of $\hat{\beta}_k$ for the foreign interest rate shock remains qualitatively similar, the conditional profile for the domestic productivity shock flips its sign. Furthermore, in this case, the foreign interest rate shock becomes more quantitatively dominating compared with the productivity shock. So, recognizing that we adopt a conservative calibration of the length of the planning horizon, our findings are even stronger if the horizon is shorter.

De Leo, 2023).

5.3 Breakdown of the Forecast Horizon Invariance

Another challenge to the RE-UIP condition is the empirical breakdown of the forecast horizon invariance. Consider the following regression specification based on equation (5.8):

$$\hat{q}_t = \gamma_0 + \gamma_S D_t^s(M) + \gamma_L D_t^L(M) + \zeta_t, \tag{5.10}$$

where γ_0, γ_S, and γ_L are regression coefficients and ζ_t is an orthogonal error term. If the forecast horizon invariance from the RE-UIP condition holds, the regression model should yield estimates of $\hat{\gamma}_S = 1$ and $\hat{\gamma}_L = 1$.

Galí (2020) tests specification (5.10) using the data of government zero-coupon bond yields and inflation swaps at different maturities from the U.K. and Germany, paired with the U.S. He finds robust results on $\hat{\gamma}_S > 1$ and $\hat{\gamma}_L < 1$ for those countries.[20] The empirical findings imply that the current real exchange rate *overreacts* to the forecast of the short-term interest rate differential but *underreacts* to the forecast of the long-term interest rate differential. Thus, the UIP violations exhibit forecast horizon variability, and Galí (2020) refers to this phenomenon as the "forward guidance exchange rate puzzle."

We conduct the same exercise using the series generated by the FH model and show that the FH model can address the short-term overreaction and the long-term underreaction of the real exchange rate. For a given threshold horizon M between the short-term and the long-term, together with the given planning horizon h, we construct the expected real interest rate differentials under limited foresight as

$$D_t^S(M) = \sum_{k=0}^{M-1} \mathbb{E}_t^h[\hat{r}^*_{t+k} - \hat{r}_{t+k}], \qquad D_t^L(M) = \sum_{k=M}^{h} \mathbb{E}_t^h[\hat{r}^*_{t+k} - \hat{r}_{t+k}], \tag{5.11}$$

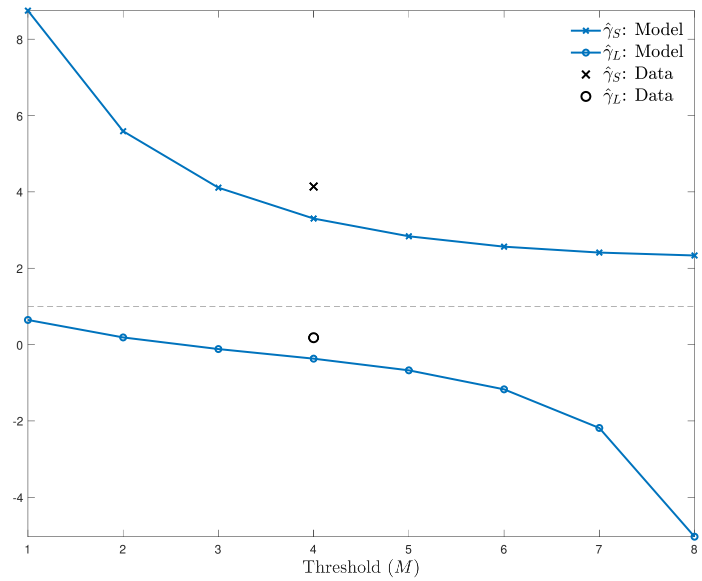
where $0 < M \leq h$; see the detailed construction procedure in Appendix K.

We run the regression for the empirical specification (5.10), using the constructed variables from the FH model. Figure 4 shows the estimates of the reaction coefficients $\hat{\gamma}_S$ and $\hat{\gamma}_L$ under the threshold $1 \leq M \leq 8$.[21] We have also labeled the corresponding empirical estimates of Canadian data (paired with the U.S. data) at the threshold $M = 4$ in Figure 4

[20] Galí (2020) documents that this phenomenon is empirically robust after controlling a time trend, or taking first-order differences, or controlling term premia.

[21] Figure K.9 in Appendix K shows the conditional estimates of the reaction coefficients $\hat{\gamma}_S$ and $\hat{\gamma}_L$, each conditioned on a distinct shock. The foreign interest rate shock primarily drives the breakdown of the forecast horizon invariance in the unconditional profile shown in Figure 4, whereas the domestic productivity shock plays a lesser role.

Figure 4: Reaction of Real Exchange Rate to Expected Interest Rate Differentials



Notes: This figure shows the (unconditional) estimates of $\hat{\gamma}_S$ and $\hat{\gamma}_L$ in specification (5.10), representing the reaction of the real exchange rate to the forecasts of short- and long-term real interest rate differentials in the benchmark FH model. The simulated data series are generated by the two shocks and the estimates are obtained using samples from 100 simulations with each spanning 150 quarters (15,000 observations in total). The black cross- and circle-markers represent the empirically estimated $\hat{\gamma}_S$ and $\hat{\gamma}_L$ using actual data, respectively.

for illustration purposes.[22]

Figure 4 shows that, across all the threshold M, $\hat{\gamma}_S > 1$ and $\hat{\gamma}_L < 1$. At the threshold $M = 4$, the estimates from the model simulations are close to the empirical estimates from the actual data, even though they are not targeted. As the threshold M increases, the reaction coefficients decrease monotonically, which is also consistent with the empirical estimates shown in Appendix J.2. Thus, the FH model predicts the breakdown of the forecast horizon invariance reasonably well with the actual data, which also aligns with the empirical findings for other countries documented in Galí (2020).

In contrast to the FH model, the simulated series consistently yields $\hat{\gamma}_S = \hat{\gamma}_L = 1$ in both the RE and FH-NG models. Hence, the value function learning in the FH model plays a pivotal role in the breakdown of the forecast horizon invariance in Figure 4. What are the mechanisms at play here? First, as shown in Proposition 1, the LIE does not apply in the FH model with value function learning. This variation disrupts the forecast horizon invariance (5.8), a result based on the LIE. Indeed, an alternative interpretation of the empirical finding in Galí (2020) is that it rejects the LIE.

[22] As considered in Galí (2020), we also obtain empirical estimates after controlling a time trend or taking first-order differences. The results show a similar pattern. For more details, see Appendix J.2.

Second, the intuition of why $\hat{\gamma}_S > \hat{\gamma}_L$ comes from the channel of increasing marginal impact of value function along the planning horizons. Decision-makers form expectations at time t for future period $t+k$ ($k \leq h$) by assuming that the aggregate conditions are determined by agents with a *remaining* planning horizon of $h-k$. As the date approaches the end of planning horizon (that is, k increases), the marginal impact of the value function on agents' forecasted variables becomes stronger. Thus, its marginal impact is greater on D_t^L than on D_t^S and is the smallest on $\hat{q}_t$. As a result, the response of $\hat{q}_t$ to D_t^S is larger than its response to D_t^L, that is, $\hat{\gamma}_S > \hat{\gamma}_L$.

To see the marginal impact of value function explicitly, consider the decision-makers' perception of the counterfactual interest rate differential at the end of their planning horizon ($h = 8$). The real interest rate differential at date $t+8$ is governed by

$$\hat{r}^{*,0}_{t+8|t} - \hat{r}^{0}_{t+8|t} = \hat{q}^{0}_{t+8|t} + \sigma(\nu_t^* - \nu_t), \tag{5.12}$$

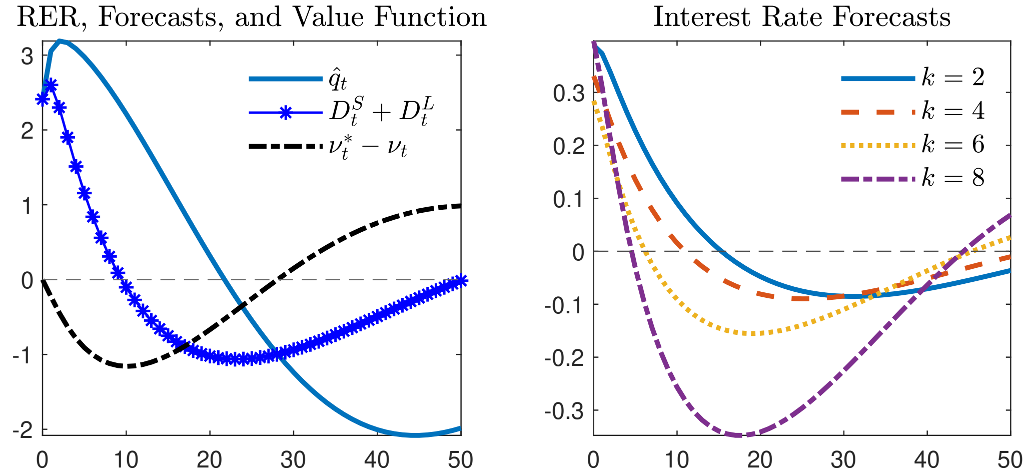
where the value function term $\nu_t^* - \nu_t$ directly influences the counterfactual real interest rate differential at that date.[23] Intuitively, when decision-makers perceive a higher relative marginal value of holding foreign bonds over domestic bonds (that is, a drop in $\nu_t^* - \nu_t$), it leads to an increase in relative foreign bond prices and thus a corresponding decrease in the counterfactual interest rate differential $\hat{r}^{*,0}_{t+8|t} - \hat{r}^{0}_{t+8|t}$. As agents make forecasts for more recent dates, the marginal impact of the value function becomes weaker.

Figure 5 visualizes this insight by showing the impulse response of the related variables in the FH model to the two structural shocks. Regardless of the type of shock, the figure suggests the following common patterns. The left plot of each panel shows that $\hat{q}_t \neq D_t^S + D_t^L$, reflecting the breakdown of LIE. It is also worth noting that even though ν_t and ν_t^* show opposite dynamics for each shock, as displayed in Figure 1, their gaps $\nu_t^* - \nu_t$ display a movement of similar shapes. The right plot shows the forecasts of real interest rate differentials $\hat{r}^{*,h-k}_{t+k|t} - \hat{r}^{h-k}_{t+k|t}$ at different forecast horizons $k = 2, 4, 6$, and 8, respectively. One can observe that as k increases, the dynamics of the forecasts become more similar to that of $\nu_t^* - \nu_t$. This is especially pronounced when $k = 8$, in line with equation (5.12). On the other hand, the forecasts with smaller k show more similarity to that of $\hat{q}_t$, being further away from $\nu_t^* - \nu_t$. Thus, the marginal impact of value function $\nu_t^* - \nu_t$ is larger on D_t^L than D_t^S and is the smallest on $\hat{q}_t$, causing the asymmetric reaction coefficients $\hat{\gamma}_S > \hat{\gamma}_L$.

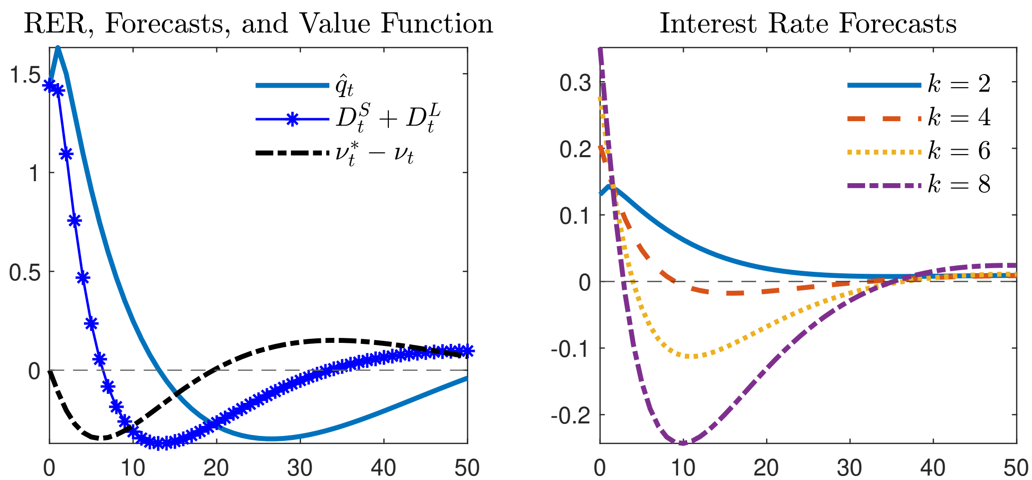
[23] See Appendix F for the derivation of (5.12).

Figure 5: Breakdown of the Forecast Horizon Invariance: Intuition

(a) Foreign Interest Rate Shock



(b) Domestic Productivity Shock



Notes: This figure shows impulse responses to a one standard deviation foreign interest rate shock (panel (a)) and a one standard deviation domestic productivity shock (panel (b)). In each panel, the left plot presents the real exchange rate ($\hat{q}_t$), the sum of the forecasts for the short- and long-term real interest rate differentials ($D_t^S + D_t^L$), and the difference between the marginal values of holding foreign and domestic bonds in value function ($\nu_t^* - \nu_t$). The right plot presents the forecast of the real interest rate differential, $\hat{r}^{*,h-k}_{t+k|t} - \hat{r}^{h-k}_{t+k|t}$, at different horizons $k \in \{2, 4, 6, 8\}$. The planning horizon h is set to 8 quarters and the x-axis is time in quarters.

6 Robustness

We evaluate the robustness of the main results in Section 5 by taking into account alternative lengths of planning horizon h and by extending the model to incorporate agents with heterogeneous planning horizons. In these robustness checks, we set all structural parameters the same as in Section 4.3 except for the parameter of interest. The main results remain broadly robust. In addition, whereas the firm's value function learning is muted in Sections 4 and 5 (that is, $\gamma_{\tilde{\nu}} = 0$), we also vary the firm's learning gain parameter $\gamma_{\tilde{v}}$ and confirm that the firm's learning behavior has negligible effects on our findings. This is consistent with intuition in the sense that the asset-pricing condition for the exchange rate and interest rate differentials originates from households' optimizations rather than those of firms. Thus, households' learning parameter plays an important role in explaining those UIP violations, whereas firms' learning parameter does not. To save space, we leave the details to Appendix L.

7 Concluding Remarks

In this paper, we reconsider the conclusions from the RE assumption in a standard SOE-NK model by assuming that decision-makers are subject to limited foresight when making decisions. Our analysis indicates that the dynamics of the model's equilibrium are significantly influenced by both the degree of decision-makers' foresight and how they update their value functions. The FH model generates dynamic overshooting of the forecast errors of the real exchange rate across time horizons, along with inherent differences in the formation of short-term and long-term expectations.

Our model provides an intrinsic and comprehensive micro-foundation for those renowned UIP puzzles that feature time- and forecast-horizon variability. We show that our model can explain (i) the time-varying excess return predictability and its reversal of sign over longer time horizons and (ii) the breakdown of the forecast horizon invariance, marked by the diverse responses of the real exchange rate to the term structure of expected real interest rate differentials. Our model's predictions are both qualitatively and quantitatively consistent with empirical estimates.

While our paper concentrates on the model's capability to address the UIP puzzles, the standard general equilibrium feature of our model leaves room for extension and application in more general aspects. Natural extensions could involve the incorporation of additional frictions and wedges, along with more endogenous variables such as capital and investment. It may also be beneficial to estimate the quantitative model in line with dynamic moments from a more comprehensive data set, including financial and expectation-related elements. We leave these possibilities for future research.

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Online Appendix: Not for Publication

Expectations and the UIP Puzzles when Foresight is Limited

Seunghoon Na and Yinxi Xie

A Steady-State Value Function of Households

The steady-state value function solves the following Bellman equation:

$$v(\mathcal{B}, \mathcal{B}^*) = \max_{C, \mathcal{B}', \mathcal{B}^{*\prime}} \{u(C) + v(\mathcal{B}', \mathcal{B}^{*\prime})\}$$

s.t.

$$\beta \mathcal{B}' + \beta \frac{\bar{Q}\mathcal{B}^{*\prime}\bar{\Pi}}{\bar{\Pi}^*} = \mathcal{B} + \frac{\bar{Q}\mathcal{B}^*\bar{\Pi}}{\bar{\Pi}^*} + (\bar{\mathcal{S}}\bar{Y} - C)\bar{\Pi}.$$

The first-order conditions yield

$$v_1(\mathcal{B}', \mathcal{B}^{*\prime}) = \frac{\beta u'(C)}{\bar{\Pi}}, \qquad v_2(\mathcal{B}', \mathcal{B}^{*'}) = \frac{\beta \bar{Q} u'(C)}{\bar{\Pi}^*},$$

$$v_1(\mathcal{B}, \mathcal{B}^*) = \frac{u'(C)}{\bar{\Pi}}, \qquad v_2(\mathcal{B}, \mathcal{B}^*) = \frac{\bar{Q} u'(C)}{\bar{\Pi}^*},$$

where the last two equations come from the envelope theorem.

It can be easily verified that the following solution satisfies the above system of first-order conditions, which is given by

$$v(\mathcal{B}, \mathcal{B}^*) = (1-\beta)^{-1} u \left(\frac{(1-\beta)\mathcal{B}}{\bar{\Pi}} + \frac{(1-\beta)\bar{Q}\mathcal{B}^*}{\bar{\Pi}^*} + \bar{\mathcal{S}}\bar{Y} \right).$$

B Log-Linearization of the F.O.C.s of Households in the Ending Period of Forward Planning

We now show the steps of log-linearizing equations (2.8) and (2.9) under the assumption that households use a steady-state value function in their forward planning. First, taking logs of (2.8) and conducting first-order Taylor expansion at the steady state with notation $\tau = t + h$ yields

$$\ln u'(C_\tau^0) = \ln \beta + \ln(1 + i_\tau^0) + \ln v_1(\mathcal{B}_{\tau+1}^0, \mathcal{B}_{\tau+1}^{*,0})$$

$$\Rightarrow \frac{u''(\bar{C})}{u'(\bar{C})}(C_\tau^0 - \bar{C}) = \frac{1+i_\tau^0 - (1+\bar{i})}{1+\bar{i}} + \frac{v_{1,1}(\bar{\mathcal{B}},\bar{\mathcal{B}}^*)}{v_1(\bar{\mathcal{B}},\bar{\mathcal{B}}^*)}(\mathcal{B}_{\tau+1}^0 - \bar{\mathcal{B}}) + \frac{v_{1,2}(\bar{\mathcal{B}},\bar{\mathcal{B}}^*)}{v_1(\bar{\mathcal{B}},\bar{\mathcal{B}}^*)}(\mathcal{B}_{\tau+1}^{*,0} - \bar{\mathcal{B}}^*)$$

$$\Rightarrow \frac{u''(\bar{C})\bar{C}}{u'(\bar{C})}\hat{c}_\tau^0 = \hat{i}_\tau^0 + \frac{u''(\bar{C})\bar{C}}{u'(\bar{C})}\frac{(1-\beta)}{\bar{\Pi}}\frac{\mathcal{B}_{\tau+1}^0 - \bar{\mathcal{B}}}{\bar{C}} + \frac{u''(\bar{C})\bar{C}}{u'(\bar{C})}\frac{(1-\beta)\bar{Q}}{\bar{\Pi}^*}\frac{\mathcal{B}_{\tau+1}^{*,0} - \bar{\mathcal{B}}^*}{\bar{C}}. \tag{B.1}$$

Given the definition of $\sigma^{-1} \equiv -\frac{u'(\bar{C})}{u''(\bar{C})\bar{C}}$, (B.1) can be rewritten as

$$\hat{c}_\tau^0 = -\sigma^{-1}\hat{i}_\tau^0 + (1-\beta)\hat{b}_{\tau+1}^0 + (1-\beta)\hat{b}_{\tau+1}^{*,0},$$

which gives (2.13).

Similarly, log-linearizing equation (2.9) yields

$$\ln u'(C_\tau^0) = \ln\beta + \ln(1+i_\tau^{*,0}) + \ln v_2(\mathcal{B}_{\tau+1}^0, \mathcal{B}_{\tau+1}^{*,0}) - \ln Q_\tau^0$$

$$\Rightarrow \frac{u''(\bar{C})}{u'(\bar{C})}(C_\tau^0 - \bar{C}) = \frac{1+i_\tau^{*,0} - (1+\bar{i}^*)}{1+\bar{i}^*} + \frac{v_{2,1}(\bar{\mathcal{B}},\bar{\mathcal{B}}^*)}{v_2(\bar{\mathcal{B}},\bar{\mathcal{B}}^*)}(\mathcal{B}_{\tau+1}^0 - \bar{\mathcal{B}}) + \frac{v_{2,2}(\bar{\mathcal{B}},\bar{\mathcal{B}}^*)}{v_2(\bar{\mathcal{B}},\bar{\mathcal{B}}^*)}(\mathcal{B}_{\tau+1}^{*,0} - \bar{\mathcal{B}}^*) - \frac{Q_\tau^0 - \bar{Q}}{\bar{Q}}$$

$$\Rightarrow \frac{u''(\bar{C})\bar{C}}{u'(\bar{C})}\hat{c}_\tau^0 = \hat{i}_\tau^0 + \frac{u''(\bar{C})\bar{C}}{u'(\bar{C})}\frac{(1-\beta)}{\bar{\Pi}}\frac{\mathcal{B}_{\tau+1}^0 - \bar{\mathcal{B}}}{\bar{C}} + \frac{u''(\bar{C})\bar{C}}{u'(\bar{C})}\frac{(1-\beta)\bar{Q}}{\bar{\Pi}^*}\frac{\mathcal{B}_{\tau+1}^{*,0} - \bar{\mathcal{B}}^*}{\bar{C}} - q_\tau^0.$$

After plugging the definition of σ, we have

$$\hat{c}_\tau^0 = -\sigma^{-1}\hat{i}_\tau^{*,0} + (1-\beta)\hat{b}_{\tau+1}^0 + (1-\beta)\hat{b}_{\tau+1}^{*,0} + \sigma^{-1}\hat{q}_\tau^0,$$

which gives (2.14).

C Firm Profit Function and Optimal Pricing Solution

In this section, we first show the explicit expression for profit function $H(\cdot)$ of the firms in each period and then derive the firms' optimal pricing decision given by (2.16). The period profit function of firms is the same regardless of whether they are infinitely forward-looking or have limited foresight. We therefore derive the profit function by considering the case of the standard RE framework with infinite planning horizons.

In the RE framework, a firm f that is able to reoptimize its goods price sets $P_{H,t}^f$ to maximize

$$\max_{P_{H,t}^f} \mathbb{E}_t \sum_{k=0}^{\infty}(\beta\theta)^k \left[\left(\frac{C_{t+k}}{C_t}\right)^{-\sigma}\left(\frac{P_t}{P_{t+k}}\right) Y_{t+k}(j)\left(P_{H,t}^f \bar{\Pi}_H^k - MC_{t+k}\right)\right], \tag{C.1}$$

subject to the demand constraint

$$Y_{t+k}(j) \leq \left(\frac{P^f_{H,t}\bar{\Pi}^k_H}{P_{H,t+k}}\right)^{-\epsilon} \underbrace{\left(C_{H,t+k} + \int_0^1 C^l_{H,t+k} dl\right)}_{\equiv Y_{t+k}}.$$

We rewrite the firm's problem (C.1) as follows:

$$\max_{P^f_{H,t}} \mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta)^k \left[C^{-\sigma}_{t+k} \left(\frac{P^f_{H,t}\bar{\Pi}^k_H}{P_{H,t+k}}\right)^{-\epsilon} Y_{t+k} \frac{P_{H,t+k}}{P_{t+k}} \left(\frac{P^f_{H,t}\bar{\Pi}^k_H}{P_{H,t+k}} - \frac{MC_{t+k}}{P_{H,t+k}}\right)\right], \tag{C.2}$$

where we have applied the demand constraint and dropped C_t and P_t (note that they are taken as given by firm f at time t, and dropping them does not change the solution to the optimality problem).

Now, let us define

$$\lambda_{t+k} \equiv C^{-\sigma}_{t+k}, \qquad r^f_{H,t+k} \equiv \frac{P^f_{H,t}\bar{\Pi}^k_H}{P_{H,t+k}},$$
$$\mathcal{S}_{t+k} \equiv \frac{P_{H,t+k}}{P_{t+k}}, \qquad \mathcal{MC}_{t+k} \equiv \frac{MC_{t+k}}{P_{H,t+k}}.$$

Then, the optimality problem (C.2) can be summarized as follows:

$$\max_{P^f_{H,t}} \mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta)^k \left[\lambda_{t+k} H(r^f_{H,t+k}; \mathcal{S}_{t+k}, \boldsymbol{Z}_{t+k})\right], \tag{C.3}$$

where $H(r^f_{H,t+k}; \mathcal{S}_{t+k}, \boldsymbol{Z}_{t+k})$ is the function of real profit in period $t+k$; that is,

$$H(r^f_{H,t+k}; \mathcal{S}_{t+k}, \boldsymbol{Z}_{t+k}) = \left(r^f_{H,t+k}\right)^{-\epsilon} Y_{t+k}\mathcal{S}_{t+k} \left(r^f_{H,t+k} - \mathcal{MC}_{t+k}\right), \tag{C.4}$$

and $\boldsymbol{Z}_{t+k}$ is the vector of all real state variables at time $t+k$. In the steady state with $r^f_H = \mathcal{S} = 1$, the derivative of function $H(\cdot)$ becomes

$$H'(1; 1, \bar{\boldsymbol{Z}}) = \bar{Y}(1 - \epsilon + \epsilon \cdot \overline{\mathcal{MC}}) = 0 \tag{C.5}$$

by noting that the real marginal cost in the steady state is $\overline{\mathcal{MC}} = (\epsilon - 1)/\epsilon$.

Now, we show that the firms' optimal pricing decision is given by (2.16). We assume that the firms use a value function to approximate discounted future profits beyond its planning horizon that is learned from the nonstochastic steady state given by (2.2). Then, the first-order condition of maximizing the firm's objective function (2.15) is

$$\mathbb{E}_t^f\left[\sum_{\tau=t}^{t+k}(\beta\theta)^{\tau-t}\lambda_\tau H_1\left(\frac{P_{H,t}^f\bar{\Pi}_H^{\tau-t}}{P_{H,\tau}};\mathcal{S}_\tau,\boldsymbol{Z}_\tau\right)\frac{P_{H,t}\bar{\Pi}_H^{\tau-t}}{P_{H,\tau}}+\frac{(\beta\theta)^{k+1}}{1-\beta\theta}\bar{\lambda}H_1\left(\frac{P_{H,t}^f\bar{\Pi}_H^k}{P_{H,t+k}};\bar{\mathcal{S}},\bar{\boldsymbol{Z}}\right)\frac{P_{H,t}\bar{\Pi}_H^k}{P_{H,t+k}}\right]=0. \tag{C.6}$$

Log-linearizing (C.6) around the steady state yields

$$\mathbb{E}_t^f\left\{\sum_{\tau=t}^{t+k}(\beta\theta)^{\tau-t}\left[p_{H,t}^f-\sum_{s=t}^{\tau}\pi_{H,s}-m_\tau\right]+\frac{(\beta\theta)^{k+1}}{1-\beta\theta}\left[p_{H,t}^f-\sum_{s=t}^{t+k}\pi_{H,s}\right]\right\}=0, \tag{C.7}$$

where

$$p_{H,t}^f\equiv\log\left(\frac{P_{H,t}(f)}{P_{H,t-1}\bar{\Pi}_H}\right),\qquad \pi_{H,t}\equiv\log\left(\frac{\Pi_{H,t}}{\bar{\Pi}_H}\right),$$

and

$$m_t\equiv-\frac{H'(1;1,\boldsymbol{Z}_t)}{H''(1;1,\bar{\boldsymbol{Z}})}=\frac{Y_t}{\bar{Y}}\left(\frac{\epsilon}{\epsilon-1}mc_t-1\right). \tag{C.8}$$

We define

$$\hat{m}_t\equiv m_t-\bar{m},\qquad \widehat{mc}_t\equiv\log\left(\frac{\mathcal{MC}_t}{\overline{\mathcal{MC}}}\right),$$

where $\bar{m}$ is the value of m_t in the nonstochastic steady state.

By noting that $\bar{m}=\frac{\bar{Y}}{\bar{Y}}\left(\frac{\epsilon}{\epsilon-1}\overline{\mathcal{MC}}-1\right)=0$, we have $\hat{m}_t=m_t$. Then, the log-linear approximation of (C.8) yields

$$m_t=\widehat{mc}_t.$$

Thus, by replacing m_t with $\widehat{mc}_t$ in (C.6) and reorganizing its expression, we have the firms' optimal pricing $p_{H,t}^f$ characterized by (2.16).

D International Goods-Market Clearing Condition

The law of one price holds, implying $\mathcal{E}_tP_{H,t}^*=P_{H,t}$. With the assumption that $P_t^*=1$, combining equations (2.27) and (2.28) yields

$$\begin{aligned}Y_t &= (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta}C_t+\left(\frac{P_{H,t}}{\mathcal{E}_t}\right)^{-\gamma}C_t^*\\ &= (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta}C_t+\left(\frac{P_{H,t}}{P_t}\right)^{-\gamma}\left(\frac{P_t}{\mathcal{E}_t}\right)^{-\gamma}C_t^*\\ &= (1-\alpha)\mathcal{S}_t^{-\eta}C_t+\mathcal{S}_t^{-\gamma}Q_t^{\gamma}C_t^*. \end{aligned} \tag{D.1}$$

Log-linearizing (D.1) gives

$$\bar{Y}\hat{y}_t = (1-\alpha)\bar{\mathcal{S}}^{-\eta}\bar{C}\left(\hat{c}_t + \alpha\eta\hat{s}_t\right) + \bar{\mathcal{S}}^{-\gamma}\bar{Q}^{\gamma}\bar{C}^*\left(\alpha\gamma\hat{s}_t + \gamma\hat{q}_t + \hat{c}_t^*\right), \tag{D.2}$$

where we have utilized $\hat{\mathcal{S}}_t = -\alpha\hat{s}_t$. By further utilizing the relation $\hat{s}_t = \hat{q}_t/(1-\alpha)$ and assuming $\hat{c}_t^* = 0$ (no foreign demand shock), (D.2) can be rewritten into

$$\hat{y}_t = \vartheta_{yc}\hat{c}_t + \vartheta_{ys}\hat{s}_t,$$

where $\vartheta_{yc} \equiv (1-\alpha)\mathcal{S}^{-\eta}\bar{C}/\bar{Y}$ and $\vartheta_{ys} \equiv [\alpha\eta(1-\alpha)\bar{\mathcal{S}}^{-\eta}\bar{C} + \gamma\bar{\mathcal{S}}^{-\gamma}\bar{Q}^{\gamma}\bar{C}^*]/\bar{Y}$. This gives equation (2.29).

Note, in Section 4.2, we assume a symmetric steady state between the home country and the rest of the world with balanced trade, which implies $\bar{Y} = \bar{C}$, $\bar{\mathcal{S}} = \bar{Q} = 1$, and $\bar{C}^* = \alpha\bar{C}$. In this case, $\vartheta_{yc} = (1-\alpha)$ and $\vartheta_{ys} = \alpha[\eta(1-\alpha)+\gamma]$.

E Proof of the Mean-Reverting Processes of χ_t and ζ_t

First, we log-linearize the resource constraint (2.5) as follows:

$$\hat{c}_t + \beta(\hat{b}_{t+1} + \hat{b}_{t+1}^*) - \beta\vartheta_1'\hat{\imath}_t + \beta\vartheta_1(\hat{q}_t - \hat{r}_t^*) = \hat{b}_t - \vartheta_1'\pi_t + \hat{b}_t^* + \vartheta_1\hat{q}_t + \vartheta_2(\hat{y}_t - \alpha\hat{s}_t),$$

where $\vartheta_1' \equiv \frac{\bar{\mathcal{B}}}{\bar{\Pi}\bar{C}}$. Here we have used the relation $\hat{\mathcal{S}}_t = -\alpha\hat{s}_t$, $\pi_t^* = 0$, hence $\hat{\imath}_t^* = \hat{r}_t^*$, together with the steady-state relationship $\bar{\Pi}^* = 1$ and $\beta^{-1} = (1+\bar{\imath})/\bar{\Pi} = (1+\bar{\imath}^*)/\bar{\Pi}^*$. It can be rewritten as

$$\hat{b}_t + \hat{b}_t^* = \beta(\hat{b}_{t+1} + \hat{b}_{t+1}^*) + \left[\hat{c}_t - \beta\vartheta_1'\hat{\imath}_t + \beta\vartheta_1(\hat{q}_t - \hat{r}_t^*) + \vartheta_1'\pi_t - \vartheta_1\hat{q}_t - \vartheta_2(\hat{y}_t - \alpha\hat{s}_t)\right]. \tag{E.1}$$

In the FH model, we can rewrite equation (E.1) at any date τ as the version of interest:

$$\hat{b}_\tau^{j+1} + \hat{b}_\tau^{*j+1} = \beta(\hat{b}_{\tau+1}^j + \hat{b}_{\tau+1}^{*,j}) + \left[\hat{c}_\tau^j - \beta\vartheta_1'\hat{\imath}_\tau^j + \beta\vartheta_1(\hat{q}_\tau^j - \hat{r}_\tau^{*,j}) + \vartheta_1'\pi_\tau^j - \vartheta_1\hat{q}_\tau^j - \vartheta_2(\hat{y}_\tau^j - \alpha\hat{s}_\tau^j)\right], \tag{E.2}$$

where j is the (counterfactual) planning horizon at date τ.

Let time t be the point at which forward planning occurs. Then, iterating (E.2) forward to the end of the planning horizon yields

$$
\begin{aligned}
&\hat{b}_t^{h+1} + \hat{b}_t^{*,h+1} \\
&= \mathbb{E}_t \sum_{j=0}^{h} \beta^j \left[\hat{c}_{t+j}^{h-j} - \beta\vartheta_1' \hat{\imath}_{t+j}^{h-j} + \beta\vartheta_1 (\hat{q}_{t+j}^{h-j} - \hat{r}_{t+j}^{*,h-j}) + \vartheta_1' \pi_{t+j}^{h-j} - \vartheta_1 \hat{q}_{t+j}^{h-j} - \vartheta_2 (\hat{y}_{t+j}^{h-j} - \alpha \hat{s}_{t+j}^{h-j}) \right] \\
&+ \beta^{h+1} \mathbb{E}_t (\hat{b}_{t+h+1}^0 + \hat{b}_{t+h+1}^{*,0}), \qquad \text{(E.3)}
\end{aligned}
$$

where $\hat{b}_t^{h+1}$ and $\hat{b}_t^{*,h+1}$ are the household's initial financial position in period t.

We parameterize the log-linear approximations of $v_1(\mathcal{B}, \mathcal{B}^*)$ and $v_2(\mathcal{B}, \mathcal{B}^*)$ by

$$
\begin{aligned}
\log(v_{1,t}(\mathcal{B}, \mathcal{B}^*)/v_1^*(\bar{\mathcal{B}}, \bar{\mathcal{B}}^*)) &= -\sigma(\nu_t + \chi_t \hat{b} + \xi_t \hat{b}^*), \\
\log(v_{2,t}(\mathcal{B}, \mathcal{B}^*)/v_2^*(\bar{\mathcal{B}}, \bar{\mathcal{B}}^*)) &= -\sigma(\nu_t^* + \chi_t' \hat{b} + \xi_t' \hat{b}^*).
\end{aligned}
$$

Then the first-order conditions of a household's optimality problem at the end of its planning horizon (2.8) and (2.9) can be log-linearized as

$$
\begin{aligned}
\hat{c}_{t+k}^0 &= -\sigma^{-1} \hat{\imath}_{t+k}^0 + \nu_t + \chi_t \hat{b}_{t+k+1}^0 + \xi_t \hat{b}_{t+k+1}^{*,0}, \\
\hat{c}_{t+k}^0 &= -\sigma^{-1} \hat{\imath}_{t+k}^{*,0} + \nu_t^* + \chi_t' \hat{b}_{t+k+1}^0 + \xi_t' \hat{b}_{t+k+1}^{*,0} + \sigma^{-1} \hat{q}_{t+k}^0,
\end{aligned}
$$

which implies $\chi_t = \chi_t'$ and $\xi_t = \xi_t'$.

In the standard dynamic programming problem under the RE assumption in the benchmark model, the household's holdings of domestic bonds $\hat{b}$ and foreign bonds $\hat{b}^*$ (in terms of the domestic currency) are perfect substitutes in their value functions. Since the bond holdings $\hat{b}$ and $\hat{b}^*$ are also perfect substitutes in the budget constraint of the household in the finite planning problem, we have $\chi_t = \xi_t$. Thus, the Euler equation at the end of the planning horizon reduces to

$$
\tilde{b}_{t+h+1}^0 \equiv \hat{b}_{t+h+1}^0 + \hat{b}_{t+h+1}^{*,0} = \chi_t^{-1} (\hat{c}_{t+h}^0 - \nu_t + \sigma^{-1} \hat{\imath}_{t+h}^0), \qquad \text{(E.4)}
$$

where $\tilde{b}$ represents the total holdings of bond positions.

Similar to Woodford (2019), by the household's optimal expenditure conditions (2.11) and (2.13), together with (E.3), we have

$$
\hat{c}_t^h = g_k(\chi_t) \tilde{b}_t^{h+1} + rest,
$$

where "rest" indicates the terms not including total asset position $\tilde{b}_t^{h+1}$ and

$$g_k(\chi_t) \equiv \frac{\chi_t}{\beta^{k+1} + \left(\frac{1-\beta^{h+1}}{1-\beta}\right)\chi_t}.$$

Thus, we have $\chi_t^{est} = g_k(\chi_t)$. Because the evolution process of χ_t follows a constant-gain learning rule; that is,

$$\chi_{t+1} = \gamma g_k(\chi_t) + (1-\gamma)\chi_t,$$

χ_t monotonically converges to the fixed point $1-\beta$. Similarly, since $\chi_t = \xi_t$ for any t, the same is true for the evolution process of ξ_t.

F Equilibrium Conditions of the Forward Planning

This section summarizes the equilibrium conditions in the finite planning exercise calculated in period t. Let $y_{\tau|t}^j$ be the value of $\hat{y}_\tau$ that is predicted at date τ as a result of aggregation of decisions made by agents with (counterfactual) planning horizon $j = h + t - \tau$, which is calculated at date t by agents with planning horizon h. It is a function of the state $\{\hat{q}_{t-1}, \hat{b}_t^*, \nu_t, \nu_t^*, \tilde{\nu}_t\}$ and exogenous shocks $\{a_t, \mu_t\}$ in period t. Then, the actual aggregate output in period t is given by $\hat{y}_t = \hat{y}_{t|t}^h$. Similarly, we define other variables in the finite planning exercise with the same notation. The additional subscript $|t$ matters because different value functions are used in finite planning in different periods. All the exogenous shocks are assumed to follow an AR(1) process.

In the forward planning exercise by the agents in period t with planning horizon h, at any date $t \le \tau < t+h-1$, we have

$$\hat{c}^{h+t-\tau}_{\tau|t} = \mathbb{E}_\tau \hat{c}^{h+t-\tau-1}_{\tau+1|t} - \frac{1}{\sigma}(\hat{\imath}^{h+t-\tau}_{\tau|t} - \mathbb{E}_\tau \pi^{h+t-\tau-1}_{\tau+1|t}), \tag{F.1}$$

$$\hat{q}^{h+t-\tau}_{\tau|t} = \mathbb{E}_\tau \hat{q}^{h+t-\tau-1}_{\tau+1|t} + \hat{r}^{*,h+t-\tau}_{\tau|t} - (\hat{\imath}^{h+t-\tau}_{\tau|t} - \mathbb{E}_\tau \pi^{h+t-\tau-1}_{\tau+1|t}), \tag{F.2}$$

$$\hat{\varepsilon}^{h+t-\tau}_{\tau|t} = \hat{q}^{h+t-\tau}_{\tau|t} - \hat{q}^{h+t-\tau+1}_{\tau-1|t} + \pi^{h+t-\tau}_{\tau|t}, \tag{F.3}$$

$$\hat{s}^{h+t-\tau}_{\tau|t} = \frac{\hat{q}^{h+t-\tau}_{\tau|t}}{1-\alpha}, \tag{F.4}$$

$$\pi^{h+t-\tau}_{H,\tau|t} = \kappa \widehat{mc}^{h+t-\tau}_{\tau|t} + \beta \mathbb{E}_\tau \pi^{h+t-\tau-1}_{H,\tau+1|t}, \tag{F.5}$$

$$\widehat{mc}^{h+t-\tau}_{\tau|t} = (\sigma + \varphi\vartheta_{yc})\hat{c}^{h+t-\tau}_{\tau|t} + (\alpha + \varphi\vartheta_{ys})\hat{s}^{h+t-\tau}_{\tau|t} - (1+\varphi)a_\tau, \tag{F.6}$$

$$\hat{\imath}^{h+t-\tau}_{\tau|t} = \phi_\pi \pi^{h+t-\tau}_{\tau|t}, \tag{F.7}$$

$$\hat{y}^{h+t-\tau}_{\tau|t} = \vartheta_{ys}\hat{s}^{h+t-\tau}_{\tau|t} + \vartheta_{yc}\hat{c}^{h+t-\tau}_{\tau|t}, \tag{F.8}$$

$$\pi^{h+t-\tau}_{\tau|t} = (1-\alpha)\pi^{h+t-\tau}_{H,\tau|t} + \alpha\hat{\varepsilon}^{h+t-\tau}_{\tau|t}, \tag{F.9}$$

$$\hat{r}^{*,h+t-\tau}_{\tau|t} = \phi_b \hat{b}^{h+t-\tau}_{\tau+1|t} + \mu_\tau, \tag{F.10}$$

$$\begin{aligned}\hat{b}^{*,h+t-\tau}_{\tau+1|t} &= \beta^{-1}(\hat{b}^{*,h+t+1-\tau}_{\tau|t} + \vartheta_1\hat{q}^{h+t-\tau}_{\tau|t} - \vartheta_2\alpha\hat{s}^{h+t-\tau}_{\tau|t} + \vartheta_2\hat{y}^{h+t-\tau}_{\tau|t} - \hat{c}^{h+t-\tau}_{\tau|t}) \\ &\quad - \vartheta_1\hat{q}^{h+t-\tau}_{\tau|t} + \vartheta_1\hat{r}^{*,h+t-\tau}_{\tau|t},\end{aligned} \tag{F.11}$$

where $\hat{q}^{h+1}_{t-1|t}$ is simply a notational simplification defined by $\hat{q}^{h+1}_{t-1|t} \equiv \hat{q}_{t-1}$ and similarly $\hat{b}^{h+1}_{t|t} \equiv \hat{b}^*_t$. Here $\vartheta_{yc} = (1-\alpha)\frac{\bar{S}^{-\eta}\bar{C}}{\bar{Y}}$, $\vartheta_{ys} = \alpha[\gamma + \eta(1-\alpha)]\frac{\bar{S}^{-\eta}\bar{C}}{\bar{Y}}$, $\vartheta_1 = \frac{\bar{\mathcal{B}}^*\bar{Q}}{\bar{C}}$, and $\vartheta_2 = \frac{\bar{S}\bar{Y}}{\bar{C}}$.

At the end of finite planning date $\tau = t+h$, we have

$$\hat{c}^0_{\tau|t} = -\frac{1}{\sigma}\hat{\imath}^0_{\tau|t} + (1-\beta)\hat{b}^{*,0}_{\tau+1|t} + \nu_t, \tag{F.12}$$

$$\hat{q}^0_{\tau|t} = \hat{r}^{*,0}_{\tau|t} - \hat{\imath}^0_{\tau|t} + \sigma(\nu_t - \nu^*_t), \tag{F.13}$$

$$\hat{\varepsilon}^0_{\tau|t} = \hat{q}^0_{\tau|t} - \hat{q}^1_{\tau-1|t} + \pi^0_{\tau|t}, \tag{F.14}$$

$$\hat{s}^0_{\tau|t} = \frac{\hat{q}^0_{\tau|t}}{1-\alpha}, \tag{F.15}$$

$$\pi^0_{H,\tau|t} = \kappa\widehat{mc}^0_{\tau|t} + (1-\theta)\beta\tilde{\nu}_t, \tag{F.16}$$

$$\widehat{mc}^0_{\tau|t} = (\sigma + \varphi\vartheta_{yc})\hat{c}^0_{\tau|t} + (\alpha + \varphi\vartheta_{ys})\hat{s}^0_{\tau|t} - (1+\varphi)a_\tau, \tag{F.17}$$

$$\hat{\imath}^0_{\tau|t} = \phi_\pi\pi^0_{\tau|t}, \tag{F.18}$$

$$\hat{y}^0_{\tau|t} = \vartheta_{ys}\hat{s}^0_{\tau|t} + \vartheta_{yc}\hat{c}^0_{\tau|t}, \tag{F.19}$$

$$\pi^0_{\tau|t} = (1-\alpha)\pi^0_{H,\tau|t} + \alpha\hat{\varepsilon}^0_{\tau|t}, \tag{F.20}$$

$$\hat{r}^{*,0}_{\tau|t} = \phi_b\hat{b}^0_{\tau+1|t} + \mu_\tau, \tag{F.21}$$

$$\hat{b}^{*,0}_{\tau+1|t} = \beta^{-1}(\hat{b}^{*,1}_{\tau|t} + \vartheta_1\hat{q}^0_{\tau|t} - \vartheta_2\alpha\hat{s}^0_{\tau|t} + \vartheta_2\hat{y}^0_{\tau|t} - \hat{c}^0_{\tau|t}) - \vartheta_1\hat{q}^0_{\tau|t} + \vartheta_1\hat{r}^{*,0}_{\tau|t}. \tag{F.22}$$

The above system of equations consists of a finite number of equations as a function of state variables $\{\hat{q}_{t-1}, \hat{b}^*_t, \nu_t, \nu^*_t, \tilde{\nu}_t\}$ and exogenous shocks $\{a_t, \mu_t\}$. Thus, we can solve for all endogenous variables $\{\hat{c}^{h+t-\tau}_{\tau|t}, \hat{y}^{h+t-\tau}_{\tau|t}, \hat{\imath}^{h+t-\tau}_{\tau|t}, \hat{r}^{*,h+t-\tau}_{\tau|t}, \pi^{h+t-\tau}_{H,\tau|t}, \pi^{h+t-\tau}_{\tau|t}, \hat{q}^{h+t-\tau}_{\tau|t}, \hat{s}^{h+t-\tau}_{\tau|t}, \hat{\varepsilon}^{h+t-\tau}_{\tau|t}, \hat{b}^{*,h+t-\tau}_{\tau+1|t}\}^{t+h}_{\tau=t}$ with a unique solution. See Appendix G for the detailed solution method.

From period t to period $t+1$, the value functions evolve over time; that is,

$$\nu_{t+1} = \gamma_v \nu^{est}_t + (1-\gamma_v)\nu_t, \tag{F.23}$$

$$\tilde{\nu}_{t+1} = \gamma_{\tilde{v}} \tilde{\nu}^{est}_t + (1-\gamma_{\tilde{v}})\tilde{\nu}_t, \tag{F.24}$$

$$\nu^*_{t+1} = \gamma_v \nu^{*,est}_t + (1-\gamma_v)\nu^*_t, \tag{F.25}$$

where

$$\nu^{est}_t = \hat{c}_t + \sigma^{-1}\pi_t - (1-\beta)\hat{b}^*_t, \tag{F.26}$$

$$\tilde{\nu}^{est}_t = (1-\theta)^{-1}\pi_{H,t}, \tag{F.27}$$

$$\nu^{*,est}_t = \nu^{est}_t - \sigma^{-1}(\hat{q}_t + \pi_t). \tag{F.28}$$

G Solution to Policy Functions

We show the solution to the policy functions for the equilibrium characterized in Section 3 and Appendix F. Similar to expressions (3.3)-(3.4), we can write the solution to any endogenous variable $x^j_{\tau|t}$ except for $\hat{b}^{*j}_{\tau+1|t}$ in agents' forward planning as a function of the state variables with exogenous shocks; that is,

$$x^j_{\tau|t} = \psi^j_{x,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{x,b}\hat{b}^{*,j+1}_{\tau|t} + \psi^j_{x,a}a_\tau + \psi^j_{x,\mu}\mu_\tau + \psi^j_{x,\nu}\nu_t + \psi^j_{x,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{x,\nu^*}\nu^*_t, \tag{G.1}$$

for any (counterfactual) $j \geq 0$, and

$$\hat{b}^{*j}_{\tau+1|t} = \psi^j_{b,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{b,b}\hat{b}^{*,j+1}_{\tau|t} + \psi^j_{b,a}a_\tau + \psi^j_{b,\mu}\mu_\tau + \psi^j_{b,\nu}\nu_t + \psi^j_{b,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{b,\nu^*}\nu^*_t. \tag{G.2}$$

First, we aim to pin down the coefficients for $j=0$. From (F.12)-(F.22), one can easily eliminate $\{\hat{\varepsilon}^0_{\tau|t}, \hat{s}^0_{\tau|t}, \widehat{mc}^0_{\tau|t}, \hat{\imath}^0_{\tau|t}, \hat{r}^{*,0}_{\tau|t}\}$. Additionally, note that since $y^0_{\tau|t}$ only enters (F.19), we only need to solve $\{\hat{c}^0_{\tau|t}, \hat{q}^0_{\tau|t}, \pi^0_{\tau|t}, \pi^0_{H,\tau|t}, \hat{b}^{*,0}_{\tau+1|t}\}$, and then $\hat{y}^0_{\tau|t}$ is uniquely pinned down by (F.19).

We solve $\{\hat{c}^0_{\tau|t}, \hat{q}^0_{\tau|t}, \pi^0_{\tau|t}, \pi^0_{H,\tau|t}\}$ by equating coefficients. Note that from (F.12),

$$\hat{c}^0_{\tau|t} = -\frac{\phi_\pi}{\sigma}\pi^0_{\tau|t} + (1-\beta)\hat{b}^{*,0}_{\tau+1|t} + \nu_t,$$

and equating the coefficients yields

$$\psi^0_{c,q} = -\frac{\phi_\pi}{\sigma}\psi^0_{\pi,q} + (1-\beta)\psi^0_{b,q}, \qquad \psi^0_{c,b} = -\frac{\phi_\pi}{\sigma}\psi^0_{\pi,b} + (1-\beta)\psi^0_{b,b},$$
$$\psi^0_{c,a} = -\frac{\phi_\pi}{\sigma}\psi^0_{\pi,a} + (1-\beta)\psi^0_{b,a}, \qquad \psi^0_{c,\mu} = -\frac{\phi_\pi}{\sigma}\psi^0_{\pi,\mu} + (1-\beta)\psi^0_{b,\mu},$$
$$\psi^0_{c,\nu} = -\frac{\phi_\pi}{\sigma}\psi^0_{\pi,\nu} + (1-\beta)\psi^0_{b,\nu} + 1, \qquad \psi^0_{c,\tilde{\nu}} = -\frac{\phi_\pi}{\sigma}\psi^0_{\pi,\tilde{\nu}} + (1-\beta)\psi^0_{b,\tilde{\nu}},$$
$$\psi^0_{c,\nu^*} = -\frac{\phi_\pi}{\sigma}\psi^0_{\pi,\nu^*} + (1-\beta)\psi^0_{b,\nu^*}.$$

Similarly, from (F.13),

$$\hat{q}^0_{\tau|t} = \phi_b\hat{b}^0_{\tau+1|t} + \mu_\tau - \phi_\pi\pi^0_{\tau|t} + \sigma(\nu_t - \nu^*_t),$$

which yields

$$\psi^0_{q,q} = \phi_b\psi^0_{b,q} - \phi_\pi\psi^0_{\pi,q}, \qquad \psi^0_{q,b} = \phi_b\psi^0_{b,b} - \phi_\pi\psi^0_{\pi,b}$$
$$\psi^0_{q,a} = \phi_b\psi^0_{b,a} - \phi_\pi\psi^0_{\pi,a}, \qquad \psi^0_{q,\mu} = \phi_b\psi^0_{b,\mu} - \phi_\pi\psi^0_{\pi,\mu} + 1,$$
$$\psi^0_{q,\nu} = \phi_b\psi^0_{b,\nu} - \phi_\pi\psi^0_{\pi,\nu} + \sigma, \qquad \psi^0_{q,\tilde{\nu}} = \phi_b\psi^0_{b,\tilde{\nu}} - \phi_\pi\psi^0_{\pi,\tilde{\nu}},$$
$$\psi^0_{q,\nu^*} = \phi_b\psi^0_{b,\nu^*} - \phi_\pi\psi^0_{\pi,\nu^*} - \sigma.$$

Similarly, from (F.16)-(F.17),

$$\pi^0_{H,\tau|t} = \kappa(\sigma + \varphi\vartheta_{yc})\hat{c}^0_{\tau|t} + \frac{\kappa(\alpha + \varphi\vartheta_{ys})}{1-\alpha}\hat{q}^0_{\tau|t} - \kappa(1+\varphi)a_\tau + (1-\theta)\beta\tilde{\nu}_t,$$

which yields

$$\psi^0_{\pi_H,q} = \kappa(\sigma + \varphi\vartheta_{yc})\psi^0_{c,q} + \frac{\kappa(\alpha + \varphi\vartheta_{ys})}{1-\alpha}\psi^0_{q,q}, \qquad \psi^0_{\pi_H,b} = \kappa(\sigma + \varphi\vartheta_{yc})\psi^0_{c,b} + \frac{\kappa(\alpha + \varphi\vartheta_{ys})}{1-\alpha}\psi^0_{q,b},$$
$$\psi^0_{\pi_H,a} = \kappa(\sigma + \varphi\vartheta_{yc})\psi^0_{c,a} + \frac{\kappa(\alpha + \varphi\vartheta_{ys})}{1-\alpha}\psi^0_{q,a} - \kappa(1+\varphi), \qquad \psi^0_{\pi_H,\mu} = \kappa(\sigma + \varphi\vartheta_{yc})\psi^0_{c,\mu} + \frac{\kappa(\alpha + \varphi\vartheta_{ys})}{1-\alpha}\psi^0_{q,\mu},$$
$$\psi^0_{\pi_H,\nu} = \kappa(\sigma + \varphi\vartheta_{yc})\psi^0_{c,\nu} + \frac{\kappa(\alpha + \varphi\vartheta_{ys})}{1-\alpha}\psi^0_{q,\nu}, \qquad \psi^0_{\pi_H,\tilde{\nu}} = \kappa(\sigma + \varphi\vartheta_{yc})\psi^0_{c,\tilde{\nu}} + \frac{\kappa(\alpha + \varphi\vartheta_{ys})}{1-\alpha}\psi^0_{q,\tilde{\nu}} + (1-\theta)\beta,$$
$$\psi^0_{\pi_H,\nu^*} = \kappa(\sigma + \varphi\vartheta_{yc})\psi^0_{c,\nu^*} + \frac{\kappa(\alpha + \varphi\vartheta_{ys})}{1-\alpha}\psi^0_{q,\nu^*}.$$

Similarly, from (F.14) and (F.20),

$$\pi^0_{\tau|t} = (1-\alpha)\pi^0_{H,\tau|t} + \alpha(\hat{q}^0_{\tau|t} - \hat{q}^1_{\tau-1|t} + \pi^0_{\tau|t}),$$

which yields

$$
\begin{aligned}
(1-\alpha)\psi^0_{\pi,q} &= (1-\alpha)\psi^0_{\pi_H,q} + \alpha\psi^0_{q,q} - \alpha, & (1-\alpha)\psi^0_{\pi,b} &= (1-\alpha)\psi^0_{\pi_H,b} + \alpha\psi^0_{q,b},\\
(1-\alpha)\psi^0_{\pi,a} &= (1-\alpha)\psi^0_{\pi_H,a} + \alpha\psi^0_{q,a}, & (1-\alpha)\psi^0_{\pi,\mu} &= (1-\alpha)\psi^0_{\pi_H,\mu} + \alpha\psi^0_{q,\mu},\\
(1-\alpha)\psi^0_{\pi,\nu} &= (1-\alpha)\psi^0_{\pi_H,\nu} + \alpha\psi^0_{q,\nu}, & (1-\alpha)\psi^0_{\pi,\tilde{\nu}} &= (1-\alpha)\psi^0_{\pi_H,\tilde{\nu}} + \alpha\psi^0_{q,\tilde{\nu}},\\
(1-\alpha)\psi^0_{\pi,\nu^*} &= (1-\alpha)\psi^0_{\pi_H,\nu^*} + \alpha\psi^0_{q,\nu^*}.
\end{aligned}
$$

Similarly, from (F.19) and (F.22),

$$
\begin{aligned}
\hat{b}^{*,0}_{\tau+1|t} &= \beta^{-1}(\hat{b}^{*,1}_{\tau|t} + \vartheta_1\hat{q}^0_{\tau|t} - \vartheta_2\alpha\hat{s}^0_{\tau|t} + \vartheta_2\hat{y}^0_{\tau|t} - \hat{c}^0_{\tau|t}) - \vartheta_1\hat{q}^0_{\tau|t} + \vartheta_1\hat{r}^{*,0}_{\tau|t}\\
&= \beta^{-1}\left\{\hat{b}^{*,1}_{\tau|t} + \left[\vartheta_1 - \vartheta_2\frac{\alpha-\vartheta_{ys}}{(1-\alpha)}\right]\hat{q}^0_{\tau|t} - (1-\vartheta_2\vartheta_{yc})\hat{c}^0_{\tau|t}\right\} - \vartheta_1\hat{q}^0_{\tau|t} + \vartheta_1\hat{r}^{*,0}_{\tau|t},
\end{aligned}
$$

and it implies

$$
(1-\vartheta_1\phi_b)\hat{b}^{*,0}_{\tau+1|t} = \beta^{-1}\left\{\hat{b}^{*,1}_{\tau|t} + \left[\vartheta_1 - \vartheta_2\frac{\alpha-\vartheta_{ys}}{(1-\alpha)}\right]\hat{q}^0_{\tau|t} - (1-\vartheta_2\vartheta_{yc})\hat{c}^0_{\tau|t}\right\} - \vartheta_1\hat{q}^0_{\tau|t} + \vartheta_1\mu_\tau,
$$

which yields

$$
\begin{aligned}
(1-\vartheta_1\phi_b)\psi^0_{b,q} &= \beta^{-1}\left\{\left[\vartheta_1 - \vartheta_2\frac{\alpha-\vartheta_{ys}}{(1-\alpha)}\right]\psi^0_{q,q} - (1-\vartheta_2\vartheta_{yc})\psi^0_{c,q}\right\} - \vartheta_1\psi^0_{q,q},\\
(1-\vartheta_1\phi_b)\psi^0_{b,b} &= \beta^{-1}\left\{1 + \left[\vartheta_1 - \vartheta_2\frac{\alpha-\vartheta_{ys}}{(1-\alpha)}\right]\psi^0_{q,b} - (1-\vartheta_2\vartheta_{yc})\psi^0_{c,b}\right\} - \vartheta_1\psi^0_{q,b},\\
(1-\vartheta_1\phi_b)\psi^0_{b,a} &= \beta^{-1}\left\{\left[\vartheta_1 - \vartheta_2\frac{\alpha-\vartheta_{ys}}{(1-\alpha)}\right]\psi^0_{q,a} - (1-\vartheta_2\vartheta_{yc})\psi^0_{c,a}\right\} - \vartheta_1\psi^0_{q,a},\\
(1-\vartheta_1\phi_b)\psi^0_{b,\mu} &= \beta^{-1}\left\{\left[\vartheta_1 - \vartheta_2\frac{\alpha-\vartheta_{ys}}{(1-\alpha)}\right]\psi^0_{q,\mu} - (1-\vartheta_2\vartheta_{yc})\psi^0_{c,\mu}\right\} - \vartheta_1\psi^0_{q,\mu} + \vartheta_1,\\
(1-\vartheta_1\phi_b)\psi^0_{b,\nu} &= \beta^{-1}\left\{\left[\vartheta_1 - \vartheta_2\frac{\alpha-\vartheta_{ys}}{(1-\alpha)}\right]\psi^0_{q,\nu} - (1-\vartheta_2\vartheta_{yc})\psi^0_{c,\nu}\right\} - \vartheta_1\psi^0_{q,\nu},\\
(1-\vartheta_1\phi_b)\psi^0_{b,\tilde{\nu}} &= \beta^{-1}\left\{\left[\vartheta_1 - \vartheta_2\frac{\alpha-\vartheta_{ys}}{(1-\alpha)}\right]\psi^0_{q,\tilde{\nu}} - (1-\vartheta_2\vartheta_{yc})\psi^0_{c,\tilde{\nu}}\right\} - \vartheta_1\psi^0_{q,\tilde{\nu}},\\
(1-\vartheta_1\phi_b)\psi^0_{b,\nu^*} &= \beta^{-1}\left\{\left[\vartheta_1 - \vartheta_2\frac{\alpha-\vartheta_{ys}}{(1-\alpha)}\right]\psi^0_{q,\nu^*} - (1-\vartheta_2\vartheta_{yc})\psi^0_{c,\nu^*}\right\} - \vartheta_1\psi^0_{q,\nu^*}.
\end{aligned}
$$

Thus, in the case of $j=0$, we have 35 simple linear equations for 35 undetermined coefficients. By solving these linear equations, we obtain the solution of $\{\hat{c}^0_{\tau|t}, \hat{q}^0_{\tau|t}, \pi^0_{\tau|t}, \pi^0_{H,\tau|t}, \hat{b}^{*,0}_{\tau+1|t}\}$ at date τ calculated in period t. Then one can easily obtain all other endogenous variables at date τ calculated in period t.

Next, we solve the undermined coefficients for any (counterfactual) j as a function of the coefficients for $j-1$ through (F.1)-(F.11). Similarly, we can eliminate $\{\hat{\varepsilon}^j_{\tau|t}, \hat{s}^j_{\tau|t}, \widehat{mc}^j_{\tau|t}, \hat{\imath}^j_{\tau|t}, \hat{\imath}^{*,j}_{\tau|t}\}$

in (F.1)-(F.11). Additionally, since $\hat{y}^j_{\tau|t}$ only enters (F.8), we only need to solve $\{\hat{c}^j_{\tau|t}, \hat{q}^j_{\tau|t}, \pi^j_{\tau|t}, \pi^j_{H,\tau|t}, \hat{b}^{*,j}_{\tau+1|t}\}$, and $\hat{y}^j_{\tau|t}$ is pinned down by (F.8).

To solve $\{\hat{c}^j_{\tau|t}, \hat{q}^j_{\tau|t}, \pi^j_{\tau|t}, \pi^j_{H,\tau|t}, \hat{b}^{*,j}_{\tau+1|t}\}$, from (F.1), we have

$$\hat{c}^j_{\tau|t} = \mathbb{E}_\tau \hat{c}^{j-1}_{\tau+1|t} - \frac{1}{\sigma}(\phi_\pi \pi^j_{\tau|t} - \mathbb{E}_\tau \pi^{j-1}_{\tau+1|t}),$$

and substituting with the policy functions yields

$$\begin{aligned}
&\psi^j_{c,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{c,b}\hat{b}^{*,j+1}_{\tau|t} + \psi^j_{c,a}a_\tau + \psi^j_{c,\mu}\mu_\tau + \psi^j_{c,\nu}\nu_t + \psi^j_{c,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{c,\nu^*}\nu^*_t \\
&= \mathbb{E}_\tau\{\psi^{j-1}_{c,q}[\psi^j_{q,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{q,b}\hat{b}^{*,j+1}_{\tau|t} + \psi^j_{q,a}a_\tau + \psi^j_{q,\mu}\mu_\tau + \psi^j_{q,\nu}\nu_t + \psi^j_{q,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{q,\nu^*}\nu^*_t] \\
&+ \psi^{j-1}_{c,b}[\psi^j_{b,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{b,b}\hat{b}^{*,j+1}_{\tau|t} + \psi^j_{b,a}a_\tau + \psi^j_{b,\mu}\mu_\tau + \psi^j_{b,\nu}\nu_t + \psi^j_{b,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{b,\nu^*}\nu^*_t] \\
&+ \psi^{j-1}_{c,a}a_{\tau+1} + \psi^{j-1}_{c,\mu}\mu_{\tau+1} + \psi^{j-1}_{c,\nu}\nu_t + \psi^{j-1}_{c,\tilde{\nu}}\tilde{\nu}_t + \psi^{j-1}_{c,\nu^*}\nu^*_t\} \\
&- \frac{\phi_\pi}{\sigma}[\psi^j_{\pi,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{\pi,b}\hat{b}^{*,j+1}_{\tau|t} + \psi^j_{\pi,a}a_\tau + \psi^j_{\pi,\mu}\mu_\tau + \psi^j_{\pi,\nu}\nu_t + \psi^j_{\pi,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{\pi,\nu^*}\nu^*_t] \\
&+ \frac{1}{\sigma}\mathbb{E}_\tau\{\psi^{j-1}_{\pi,q}[\psi^j_{q,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{q,b}\hat{b}^{*,j+1}_{\tau|t} + \psi^j_{q,a}a_\tau + \psi^j_{q,\mu}\mu_\tau + \psi^j_{q,\nu}\nu_t + \psi^j_{q,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{q,\nu^*}\nu^*_t] \\
&+ \psi^{j-1}_{\pi,b}[\psi^j_{b,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{b,b}\hat{b}^{*,j+1}_{\tau|t} + \psi^j_{b,a}a_\tau + \psi^j_{b,\mu}\mu_\tau + \psi^j_{b,\nu}\nu_t + \psi^j_{b,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{b,\nu^*}\nu^*_t] \\
&+ \psi^{j-1}_{\pi,a}a_{\tau+1} + \psi^{j-1}_{\pi,\mu}\mu_{\tau+1} + \psi^{j-1}_{\pi,\nu}\nu_t + \psi^{j-1}_{\pi,\tilde{\nu}}\tilde{\nu}_t + \psi^{j-1}_{\pi,\nu^*}\nu^*_t\},
\end{aligned}$$

where we have substituted $\hat{q}^j_{\tau|t} = \psi^j_{q,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{q,b}\hat{b}^{*,j+1}_{\tau|t} + \psi^j_{q,a}a_\tau + \psi^j_{q,\mu}\mu_\tau + \psi^j_{q,\nu}\nu_t + \psi^j_{q,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{q,\nu^*}\nu^*_t$ and the similar expression of $\hat{b}^{*,j}_{\tau+1|t}$.

By equating the coefficients, we obtain

$$\begin{aligned}
\psi^j_{c,q} &= \psi^{j-1}_{c,q}\psi^j_{q,q} + \psi^{j-1}_{c,b}\psi^j_{b,q} - \frac{\phi_\pi}{\sigma}\psi^j_{\pi,q} + \frac{1}{\sigma}[\psi^{j-1}_{\pi,q}\psi^j_{q,q} + \psi^{j-1}_{\pi,b}\psi^j_{b,q}], \\
\psi^j_{c,b} &= \psi^{j-1}_{c,q}\psi^j_{q,b} + \psi^{j-1}_{c,b}\psi^j_{b,b} - \frac{\phi_\pi}{\sigma}\psi^j_{\pi,b} + \frac{1}{\sigma}[\psi^{j-1}_{\pi,q}\psi^j_{q,b} + \psi^{j-1}_{\pi,b}\psi^j_{b,b}], \\
\psi^j_{c,a} &= \psi^{j-1}_{c,q}\psi^j_{q,a} + \psi^{j-1}_{c,b}\psi^j_{b,a} + \rho_a\psi^{j-1}_{c,a} - \frac{\phi_\pi}{\sigma}\psi^j_{\pi,a} + \frac{1}{\sigma}[\psi^{j-1}_{\pi,q}\psi^j_{q,a} + \psi^{j-1}_{\pi,b}\psi^j_{b,a} + \rho_a\psi^{j-1}_{\pi,a}], \\
\psi^j_{c,\mu} &= \psi^{j-1}_{c,q}\psi^j_{q,\mu} + \psi^{j-1}_{c,b}\psi^j_{b,\mu} + \rho_\mu\psi^{j-1}_{c,\mu} - \frac{\phi_\pi}{\sigma}\psi^j_{\pi,\mu} + \frac{1}{\sigma}[\psi^{j-1}_{\pi,q}\psi^j_{q,\mu} + \psi^{j-1}_{\pi,b}\psi^j_{b,\mu} + \rho_\mu\psi^{j-1}_{\pi,\mu}], \\
\psi^j_{c,\nu} &= \psi^{j-1}_{c,q}\psi^j_{q,\nu} + \psi^{j-1}_{c,b}\psi^j_{b,\nu} + \psi^{j-1}_{c,\nu} - \frac{\phi_\pi}{\sigma}\psi^j_{\pi,\nu} + \frac{1}{\sigma}[\psi^{j-1}_{\pi,q}\psi^j_{q,\nu} + \psi^{j-1}_{\pi,b}\psi^j_{b,\nu} + \psi^{j-1}_{\pi,\nu}], \\
\psi^j_{c,\tilde{\nu}} &= \psi^{j-1}_{c,q}\psi^j_{q,\tilde{\nu}} + \psi^{j-1}_{c,b}\psi^j_{b,\tilde{\nu}} + \psi^{j-1}_{c,\tilde{\nu}} - \frac{\phi_\pi}{\sigma}\psi^j_{\pi,\tilde{\nu}} + \frac{1}{\sigma}[\psi^{j-1}_{\pi,q}\psi^j_{q,\tilde{\nu}} + \psi^{j-1}_{\pi,b}\psi^j_{b,\tilde{\nu}} + \psi^{j-1}_{\pi,\tilde{\nu}}], \\
\psi^j_{c,\nu^*} &= \psi^{j-1}_{c,q}\psi^j_{q,\nu^*} + \psi^{j-1}_{c,b}\psi^j_{q,\nu^*} + \psi^{j-1}_{c,\nu^*} - \frac{\phi_\pi}{\sigma}\psi^j_{\pi,\nu^*} + \frac{1}{\sigma}[\psi^{j-1}_{\pi,q}\psi^j_{q,\nu^*} + \psi^{j-1}_{\pi,b}\psi^j_{b,\nu^*} + \psi^{j-1}_{\pi,\nu^*}].
\end{aligned}$$

Similarly, from (F.2), we have

$$\hat{q}^j_{\tau|t} = \mathbb{E}_\tau\hat{q}^{j-1}_{\tau+1|t} + \phi_b\hat{b}^j_{\tau+1|t} + \mu_\tau - (\phi_\pi\pi^j_{\tau|t} - \mathbb{E}_\tau\pi^{j-1}_{\tau+1|t}),$$

and substituting with the policy functions yields

$$
\begin{aligned}
&\psi^j_{q,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{q,b}\hat{b}^{*,j+1}_{\tau|t} + \psi^j_{q,a}a_\tau + \psi^j_{q,\mu}\mu_\tau + \psi^j_{q,\nu}\nu_t + \psi^j_{q,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{q,\nu^*}\nu^*_t \\
&= \mathbb{E}_\tau\{\psi^{j-1}_{q,q}[\psi^j_{q,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{q,b}\hat{b}^{*,j+1}_{\tau|t} + \psi^j_{q,a}a_\tau + \psi^j_{q,\mu}\mu_\tau + \psi^j_{q,\nu}\nu_t + \psi^j_{q,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{q,\nu^*}\nu^*_t] \\
&+ \psi^{j-1}_{q,b}[\psi^j_{b,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{b,b}\hat{b}^{*,j+1}_{\tau|t} + \psi^j_{b,a}a_\tau + \psi^j_{b,\mu}\mu_\tau + \psi^j_{b,\nu}\nu_t + \psi^j_{b,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{b,\nu^*}\nu^*_t] \\
&+ \psi^{j-1}_{q,a}a_{\tau+1} + \psi^{j-1}_{q,\mu}\mu_{\tau+1} + \psi^{j-1}_{q,\nu}\nu_t + \psi^{j-1}_{q,\tilde{\nu}}\tilde{\nu}_t + \psi^{j-1}_{q,\nu^*}\nu^*_t\} \\
&+ \phi_b[\psi^j_{b,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{b,b}\hat{b}^{*,j+1}_{\tau|t} + \psi^j_{b,a}a_\tau + \psi^j_{q,\mu}\mu_\tau + \psi^j_{b,\nu}\nu_t + \psi^j_{b,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{b,\nu^*}\nu^*_t] \\
&+ \mu_\tau - \phi_\pi[\psi^j_{\pi,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{\pi,b}\hat{b}^{*,j+1}_{\tau|t} + \psi^j_{\pi,a}a_\tau + \psi^j_{\pi,\mu}\mu_\tau + \psi^j_{\pi,\nu}\nu_t + \psi^j_{\pi,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{\pi,\nu^*}\nu^*_t] \\
&+ \mathbb{E}_\tau\{\psi^{j-1}_{\pi,q}[\psi^j_{q,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{q,b}\hat{b}^{*,j+1}_{\tau|t} + \psi^j_{q,a}a_\tau + \psi^j_{q,\mu}\mu_\tau + \psi^j_{q,\nu}\nu_t + \psi^j_{q,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{q,\nu^*}\nu^*_t] \\
&+ \psi^{j-1}_{\pi,b}[\psi^j_{b,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{b,b}\hat{b}^{*,j+1}_{\tau|t} + \psi^j_{b,a}a_\tau + \psi^j_{b,\mu}\mu_\tau + \psi^j_{b,\nu}\nu_t + \psi^j_{b,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{b,\nu^*}\nu^*_t] \\
&+ \psi^{j-1}_{\pi,a}a_{\tau+1} + \psi^{j-1}_{\pi,\mu}\mu_{\tau+1} + \psi^{j-1}_{\pi,\nu}\nu_t + \psi^{j-1}_{\pi,\tilde{\nu}}\tilde{\nu}_t + \psi^{j-1}_{\pi,\nu^*}\nu^*_t\}.
\end{aligned}
$$

By equating the coefficients, we obtain

$$
\begin{aligned}
\psi^j_{q,q} &= \psi^{j-1}_{q,q}\psi^j_{q,q} + \psi^{j-1}_{q,b}\psi^j_{b,q} + \phi_b\psi^j_{b,q} - \phi_\pi\psi^j_{\pi,q} + \psi^{j-1}_{\pi,q}\psi^j_{q,q} + \psi^{j-1}_{\pi,b}\psi^j_{b,q}, \\
\psi^j_{q,b} &= \psi^{j-1}_{q,q}\psi^j_{q,b} + \psi^{j-1}_{q,b}\psi^j_{b,b} + \phi_b\psi^j_{b,b} - \phi_\pi\psi^j_{\pi,b} + \psi^{j-1}_{\pi,q}\psi^j_{q,b} + \psi^{j-1}_{\pi,b}\psi^j_{b,b}, \\
\psi^j_{q,a} &= \psi^{j-1}_{q,q}\psi^j_{q,a} + \psi^{j-1}_{q,b}\psi^j_{b,a} + \rho_a\psi^{j-1}_{q,a} + \phi_b\psi^j_{b,a} - \phi_\pi\psi^j_{\pi,a} + \psi^{j-1}_{\pi,q}\psi^j_{q,a} + \psi^{j-1}_{\pi,b}\psi^j_{b,a} + \rho_a\psi^{j-1}_{\pi,a}, \\
\psi^j_{q,\mu} &= \psi^{j-1}_{q,q}\psi^j_{q,\mu} + \psi^{j-1}_{q,b}\psi^j_{b,\mu} + \rho_\mu\psi^{j-1}_{q,\mu} + \phi_b\psi^j_{b,\mu} + 1 - \phi_\pi\psi^j_{\pi,\mu} + \psi^{j-1}_{\pi,q}\psi^j_{q,\mu} + \psi^{j-1}_{\pi,b}\psi^j_{b,\mu} + \rho_\mu\psi^{j-1}_{\pi,\mu}, \\
\psi^j_{q,\nu} &= \psi^{j-1}_{q,q}\psi^j_{q,\nu} + \psi^{j-1}_{q,b}\psi^j_{b,\nu} + \psi^{j-1}_{q,\nu} + \phi_b\psi^j_{b,\nu} - \phi_\pi\psi^j_{\pi,\nu} + \psi^{j-1}_{\pi,q}\psi^j_{q,\nu} + \psi^{j-1}_{\pi,b}\psi^j_{b,\nu} + \psi^{j-1}_{\pi,\nu}, \\
\psi^j_{q,\tilde{\nu}} &= \psi^{j-1}_{q,q}\psi^j_{q,\tilde{\nu}} + \psi^{j-1}_{q,b}\psi^j_{b,\tilde{\nu}} + \psi^{j-1}_{q,\tilde{\nu}} + \phi_b\psi^j_{b,\tilde{\nu}} - \phi_\pi\psi^j_{\pi,\tilde{\nu}} + \psi^{j-1}_{\pi,q}\psi^j_{q,\tilde{\nu}} + \psi^{j-1}_{\pi,b}\psi^j_{b,\tilde{\nu}} + \psi^{j-1}_{\pi,\tilde{\nu}}, \\
\psi^j_{q,\nu^*} &= \psi^{j-1}_{q,q}\psi^j_{q,\nu^*} + \psi^{j-1}_{q,b}\psi^j_{b,\nu^*} + \psi^{j-1}_{q,\nu^*} + \phi_b\psi^j_{b,\nu^*} - \phi_\pi\psi^j_{\pi,\nu^*} + \psi^{j-1}_{\pi,q}\psi^j_{q,\nu^*} + \psi^{j-1}_{\pi,b}\psi^j_{b,\nu^*} + \psi^{j-1}_{\pi,\nu^*}.
\end{aligned}
$$

Similarly, from (F.5)-(F.6), we have

$$
\pi^j_{H,\tau|t} = \kappa(\sigma + \varphi\vartheta_{yc})\hat{c}^j_{\tau|t} + \frac{\kappa(\alpha + \varphi\vartheta_{ys})}{1-\alpha}\hat{q}^j_{\tau|t} - \kappa(1+\varphi)a_\tau + \beta\mathbb{E}_\tau\pi^{j-1}_{H,\tau+1|t},
$$

and substituting with the policy functions yields

$$
\begin{aligned}
&\psi^j_{\pi_H,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{\pi_H,b}\hat{b}^{j+1}_{\tau|t} + \psi^j_{\pi_H,a}a_\tau + \psi^j_{\pi_H,\mu}\mu_\tau + \psi^j_{\pi_H,\nu}\nu_t + \psi^j_{\pi_H,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{\pi_H,\nu^*}\nu^*_t \\
&= \kappa(\sigma+\varphi\vartheta_{yc})[\psi^j_{c,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{c,b}\hat{b}^{j+1}_{\tau|t} + \psi^j_{c,a}a_\tau + \psi^j_{c,\mu}\mu_\tau + \psi^j_{c,\nu}\nu_t + \psi^j_{c,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{c,\nu^*}\nu^*_t] \\
&+ \frac{\kappa(\alpha+\varphi\vartheta_{ys})}{1-\alpha}[\psi^j_{q,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{q,b}\hat{b}^{j+1}_{\tau|t} + \psi^j_{q,a}a_\tau + \psi^j_{q,\mu}\mu_\tau + \psi^j_{q,\nu}\nu_t + \psi^j_{q,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{q,\nu^*}\nu^*_t] \\
&- \kappa(1+\varphi)a_\tau \\
&+ \beta\mathbb{E}_\tau\{\psi^{j-1}_{\pi_H,q}[\psi^j_{q,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{q,b}\hat{b}^{j+1}_{\tau|t} + \psi^j_{q,a}a_\tau + \psi^j_{q,\mu}\mu_\tau + \psi^j_{q,\nu}\nu_t + \psi^j_{q,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{q,\nu^*}\nu^*_t] \\
&+ \psi^{j-1}_{\pi_H,b}[\psi^j_{b,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{b,b}\hat{b}^{j+1}_{\tau|t} + \psi^j_{b,a}a_\tau + \psi^j_{b,\mu}\mu_\tau + \psi^j_{b,\nu}\nu_t + \psi^j_{b,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{b,\nu^*}\nu^*_t] \\
&+ \psi^{j-1}_{\pi_H,a}a_{\tau+1} + \psi^{j-1}_{\pi_H,\mu}\mu_{\tau+1} + \psi^{j-1}_{\pi_H,\nu}\nu_t + \psi^{j-1}_{\pi_H,\tilde{\nu}}\tilde{\nu}_t + \psi^{j-1}_{\pi_H,\nu^*}\nu^*_t\}.
\end{aligned}
$$

By equating the coefficients, we obtain

$$
\begin{aligned}
\psi^j_{\pi_H,q} &= \kappa(\sigma+\varphi\vartheta_{yc})\psi^j_{c,q} + \frac{\kappa(\alpha+\varphi\vartheta_{ys})}{1-\alpha}\psi^j_{q,q} + \beta[\psi^{j-1}_{\pi_H,q}\psi^j_{q,q} + \psi^{j-1}_{\pi_H,b}\psi^j_{b,q}], \\
\psi^j_{\pi_H,b} &= \kappa(\sigma+\varphi\vartheta_{yc})\psi^j_{c,b} + \frac{\kappa(\alpha+\varphi\vartheta_{ys})}{1-\alpha}\psi^j_{q,b} + \beta[\psi^{j-1}_{\pi_H,q}\psi^j_{q,b} + \psi^{j-1}_{\pi_H,b}\psi^j_{b,b}], \\
\psi^j_{\pi_H,a} &= \kappa(\sigma+\varphi\vartheta_{yc})\psi^j_{c,a} + \frac{\kappa(\alpha+\varphi\vartheta_{ys})}{1-\alpha}\psi^j_{q,a} - \kappa(1+\varphi) + \beta[\psi^{j-1}_{\pi_H,q}\psi^j_{q,a} + \psi^{j-1}_{\pi_H,b}\psi^j_{b,a} + \rho_a\psi^{j-1}_{\pi_H,a}], \\
\psi^j_{\pi_H,\mu} &= \kappa(\sigma+\varphi\vartheta_{yc})\psi^j_{c,\mu} + \frac{\kappa(\alpha+\varphi\vartheta_{ys})}{1-\alpha}\psi^j_{q,\mu} + \beta[\psi^{j-1}_{\pi_H,q}\psi^j_{q,\mu} + \psi^{j-1}_{\pi_H,b}\psi^j_{b,\mu} + \rho_\mu\psi^{j-1}_{\pi_H,\mu}], \\
\psi^j_{\pi_H,\nu} &= \kappa(\sigma+\varphi\vartheta_{yc})\psi^j_{c,\nu} + \frac{\kappa(\alpha+\varphi\vartheta_{ys})}{1-\alpha}\psi^j_{q,\nu} + \beta[\psi^{j-1}_{\pi_H,q}\psi^j_{q,\nu} + \psi^{j-1}_{\pi_H,b}\psi^j_{b,\nu} + \psi^{j-1}_{\pi_H,\nu}], \\
\psi^j_{\pi_H,\tilde{\nu}} &= \kappa(\sigma+\varphi\vartheta_{yc})\psi^j_{c,\tilde{\nu}} + \frac{\kappa(\alpha+\varphi\vartheta_{ys})}{1-\alpha}\psi^j_{q,\tilde{\nu}} + \beta[\psi^{j-1}_{\pi_H,q}\psi^j_{q,\tilde{\nu}} + \psi^{j-1}_{\pi_H,b}\psi^j_{b,\tilde{\nu}} + \psi^{j-1}_{\pi_H,\tilde{\nu}}], \\
\psi^j_{\pi_H,\nu^*} &= \kappa(\sigma+\varphi\vartheta_{yc})\psi^j_{c,\nu^*} + \frac{\kappa(\alpha+\varphi\vartheta_{ys})}{1-\alpha}\psi^j_{q,\nu^*} + \beta[\psi^{j-1}_{\pi_H,q}\psi^j_{q,\nu^*} + \psi^{j-1}_{\pi_H,b}\psi^j_{b,\nu^*} + \psi^{j-1}_{\pi_H,\nu^*}].
\end{aligned}
$$

Similarly, from (F.3) and (F.9), we have

$$
\pi^j_{\tau|t} = (1-\alpha)\pi^j_{H,\tau|t} + \alpha(\hat{q}^j_{\tau|t} - \hat{q}^{j+1}_{\tau-1|t} + \pi^j_{\tau|t}),
$$

and substituting with the policy functions yields

$$
\begin{aligned}
&(1-\alpha)[\psi^j_{\pi,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{\pi,b}\hat{b}^{j+1}_{\tau|t} + \psi^j_{\pi,a}a_\tau + \psi^j_{\pi,\mu}\mu_\tau + \psi^j_{\pi,\nu}\nu_t + \psi^j_{\pi,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{\pi,\nu^*}\nu^*_t] \\
&= (1-\alpha)[\psi^j_{\pi_H,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{\pi_H,b}\hat{b}^{j+1}_{\tau|t} + \psi^j_{\pi_H,a}a_\tau + \psi^j_{\pi_H,\mu}\mu_\tau + \psi^j_{\pi_H,\nu}\nu_t + \psi^j_{\pi_H,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{\pi_H,\nu^*}\nu^*_t] \\
&+ \alpha[\psi^j_{q,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{q,b}\hat{b}^{j+1}_{\tau|t} + \psi^j_{q,a}a_\tau + \psi^j_{q,\mu}\mu_\tau + \psi^j_{q,\nu}\nu_t + \psi^j_{q,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{q,\nu^*}\nu^*_t] - \alpha\hat{q}^{j+1}_{\tau-1|t}.
\end{aligned}
$$

By equating the coefficients, we obtain

$$
\begin{aligned}
(1-\alpha)\psi^j_{\pi,q} &= (1-\alpha)\psi^j_{\pi_H,q} + \alpha\psi^j_{q,q} - \alpha, & (1-\alpha)\psi^j_{\pi,b} &= (1-\alpha)\psi^j_{\pi_H,b} + \alpha\psi^j_{q,b},\\
(1-\alpha)\psi^j_{\pi,a} &= (1-\alpha)\psi^j_{\pi_H,a} + \alpha\psi^j_{q,a}, & (1-\alpha)\psi^j_{\pi,\mu} &= (1-\alpha)\psi^j_{\pi_H,\mu} + \alpha\psi^j_{q,\mu},\\
(1-\alpha)\psi^j_{\pi,\nu} &= (1-\alpha)\psi^j_{\pi_H,\nu} + \alpha\psi^j_{q,\nu}, & (1-\alpha)\psi^j_{\pi,\tilde{\nu}} &= (1-\alpha)\psi^j_{\pi_H,\tilde{\nu}} + \alpha\psi^j_{q,\tilde{\nu}},\\
(1-\alpha)\psi^j_{\pi,\nu^*} &= (1-\alpha)\psi^j_{\pi_H,\nu^*} + \alpha\psi^j_{q,\nu^*}.
\end{aligned}
$$

Finally, from (F.8) and (F.11), we have

$$
(1-\vartheta_1\phi_b)\hat{b}^{*,j}_{\tau+1|t} = \beta^{-1}\left\{\hat{b}^{*,j+1}_{\tau|t} + \left[\vartheta_1 - \vartheta_2\frac{\alpha-\vartheta_{ys}}{(1-\alpha)}\right]\hat{q}^j_{\tau|t} - (1-\vartheta_2\vartheta_{yc})\hat{c}^j_{\tau|t}\right\} - \vartheta_1\hat{q}^j_{\tau|t} + \vartheta_1\mu_\tau,
$$

and substituting with the policy functions yields

$$
\begin{aligned}
&(1-\vartheta_1\phi_b)[\psi^j_{b,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{b,b}\hat{b}^{*,j+1}_{\tau|t} + \psi^j_{b,a}a_\tau + \psi^j_{q,\mu}\mu_\tau + \psi^j_{b,\nu}\nu_t + \psi^j_{b,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{b,\nu^*}\nu^*_t]\\
&= \beta^{-1}\hat{b}^{*,j+1}_{\tau|t} + \vartheta_1\mu_\tau\\
&+ \left[(\beta^{-1}-1)\vartheta_1 - \beta^{-1}\vartheta_2\frac{\alpha-\vartheta_{ys}}{(1-\alpha)}\right][\psi^j_{q,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{q,b}\hat{b}^{j+1}_{\tau|t} + \psi^j_{q,a}a_\tau + \psi^j_{q,\mu}\mu_\tau + \psi^j_{q,\nu}\nu_t + \psi^j_{q,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{q,\nu^*}\nu^*_t]\\
&- \beta^{-1}(1-\vartheta_2\vartheta_{yc})[\psi^j_{c,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{c,b}\hat{b}^{j+1}_{\tau|t} + \psi^j_{c,a}a_\tau + \psi^j_{c,\mu}\mu_\tau + \psi^j_{c,\nu}\nu_t + \psi^j_{c,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{c,\nu^*}\nu^*_t]
\end{aligned}
$$

By equating the coefficients, we obtain

$$
\begin{aligned}
(1-\vartheta_1\phi_b)\psi^j_{b,q} &= \left[(\beta^{-1}-1)\vartheta_1 - \beta^{-1}\vartheta_2\frac{\alpha-\vartheta_{ys}}{(1-\alpha)}\right]\psi^j_{q,q} - \beta^{-1}(1-\vartheta_2\vartheta_{yc})\psi^j_{c,q}\\
(1-\vartheta_1\phi_b)\psi^j_{b,b} &= 1 + (\beta^{-1}-1) + \left[(\beta^{-1}-1)\vartheta_1 - \beta^{-1}\vartheta_2\frac{\alpha-\vartheta_{ys}}{(1-\alpha)}\right]\psi^j_{q,b} - \beta^{-1}(1-\vartheta_2\vartheta_{yc})\psi^j_{c,b}\\
(1-\vartheta_1\phi_b)\psi^j_{b,a} &= \left[(\beta^{-1}-1)\vartheta_1 - \beta^{-1}\vartheta_2\frac{\alpha-\vartheta_{ys}}{(1-\alpha)}\right]\psi^j_{q,a} - \beta^{-1}(1-\vartheta_2\vartheta_{yc})\psi^j_{c,a}\\
(1-\vartheta_1\phi_b)\psi^j_{b,\mu} &= \left[(\beta^{-1}-1)\vartheta_1 - \beta^{-1}\vartheta_2\frac{\alpha-\vartheta_{ys}}{(1-\alpha)}\right]\psi^j_{q,\mu} - \beta^{-1}(1-\vartheta_2\vartheta_{yc})\psi^j_{c,\mu} + \vartheta_1\\
(1-\vartheta_1\phi_b)\psi^j_{b,\nu} &= \left[(\beta^{-1}-1)\vartheta_1 - \beta^{-1}\vartheta_2\frac{\alpha-\vartheta_{ys}}{(1-\alpha)}\right]\psi^j_{q,\nu} - \beta^{-1}(1-\vartheta_2\vartheta_{yc})\psi^j_{c,\nu}\\
(1-\vartheta_1\phi_b)\psi^j_{b,\tilde{\nu}} &= \left[(\beta^{-1}-1)\vartheta_1 - \beta^{-1}\vartheta_2\frac{\alpha-\vartheta_{ys}}{(1-\alpha)}\right]\psi^j_{q,\tilde{\nu}} - \beta^{-1}(1-\vartheta_2\vartheta_{yc})\psi^j_{c,\tilde{\nu}}\\
(1-\vartheta_1\phi_b)\psi^j_{b,\nu^*} &= \left[(\beta^{-1}-1)\vartheta_1 - \beta^{-1}\vartheta_2\frac{\alpha-\vartheta_{ys}}{(1-\alpha)}\right]\psi^j_{q,\nu^*} - \beta^{-1}(1-\vartheta_2\vartheta_{yc})\psi^j_{c,\nu^*}.
\end{aligned}
$$

Thus, given the undetermined coefficients for $j-1$, we have 35 simple linear equations for the 35 undetermined coefficients for j, which yields a unique solution. Since we have derived the undetermined coefficients for the case of $j=0$, we can solve the expressions for $\{\hat{c}^j_{\tau|t}, \hat{q}^j_{\tau|t}, \pi^j_{\tau|t}, \pi^j_{H,\tau|t}, b^{*,j+1}_{\tau|t}\}$ by forward induction. All the other endogenous variables can be easily derived with the solutions of these four endogenous variables.

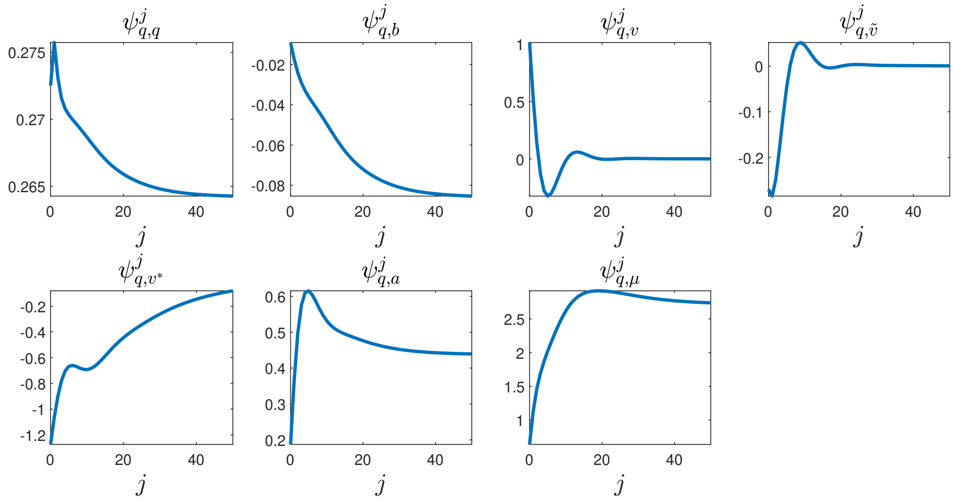
Thus far, we have derived the solution of the entire forward planning calculated in period t. Then, one can easily solve the equilibrium path solution (3.1) with the evolution of the state variables (F.23)-(F.24).

Policy Function Coefficients. With the calibrated parameters in Table 1, Figures G.6 and G.7 report the policy coefficients of the five variables $x \in \{q, b, c, \pi, \pi_H\}$ for different planning horizons $j \in [0, 50]$ quarters. As j increases, all coefficients converge to the unique RE equilibrium values, with household value functions $\{\psi^j_{x,\nu}, \psi^j_{x,\tilde{\nu}}, \psi^j_{x,\nu^*}\}$ becoming zero. As planning horizon j decreases, policy coefficients deviate from RE values, exhibiting non-monotonic, bumpy movements. Some coefficients, such as $\psi^j_{q,\nu}$, may even change signs with shorter horizons.

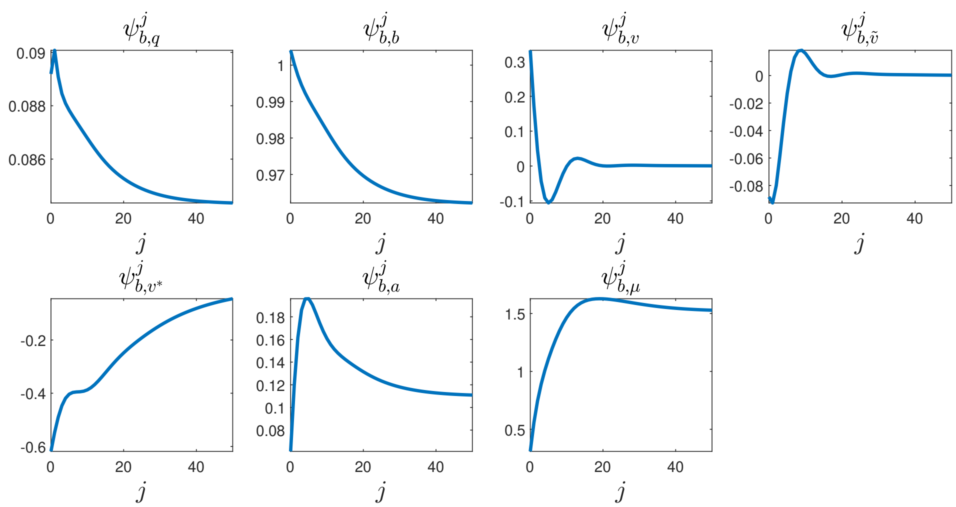
These policy coefficients entail several behavioral implications. Shorter planning horizons give rise to deviations from the RE equilibrium, attributed by limited foresight. Moreover, these biases do not consistently follow the same direction, as indicated by the non-monotonic policy coefficients. As a consequence, when decision-makers operate within a relatively short planning horizon, the resulting equilibrium can diverge in various ways from the RE equilibrium. Conversely, even with an extended planning horizon, expectations regarding distant future variables—assessed using policy coefficients for a truncated remaining planning horizon—may substantially deviate from RE.

Figure G.6: Policy Coefficients of the Five Variables with Planning Horizon $j \in [0, 50]$

(a) Real Exchange Rate



(b) Net Foreign Asset Position



(c) Consumption

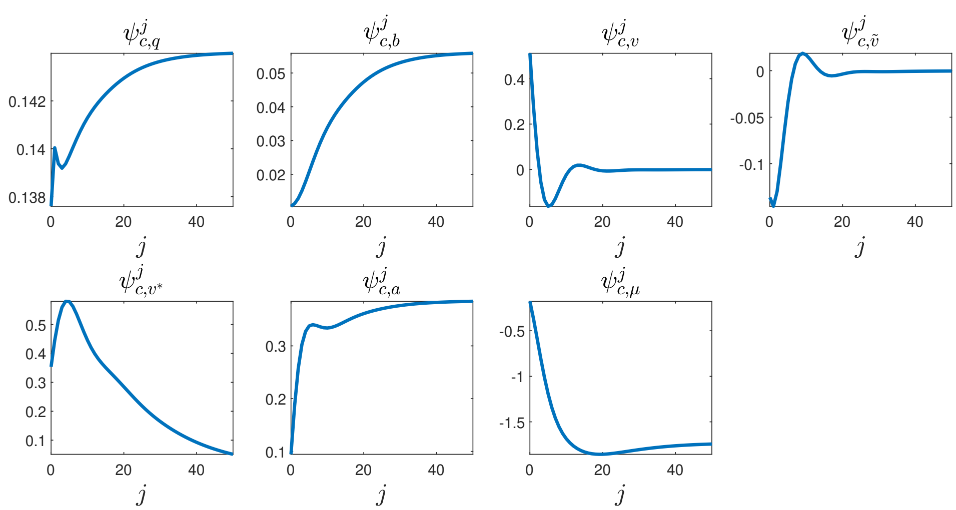
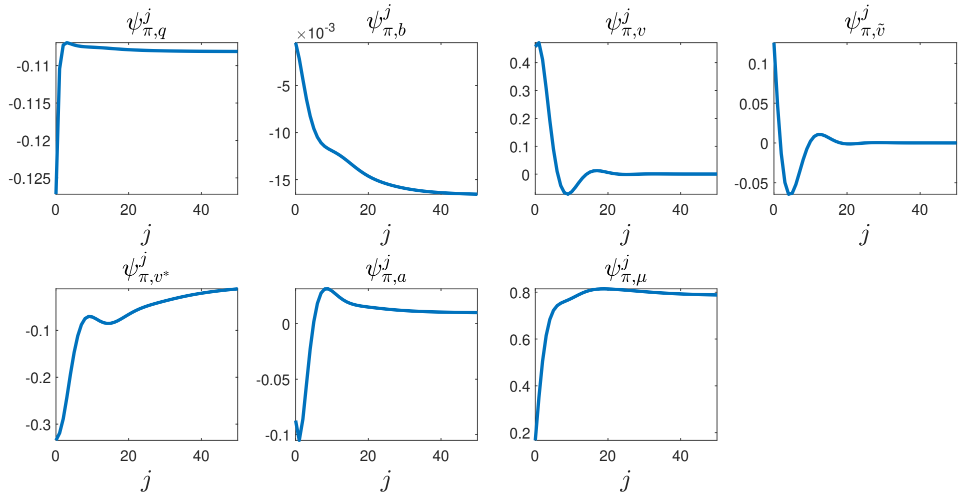
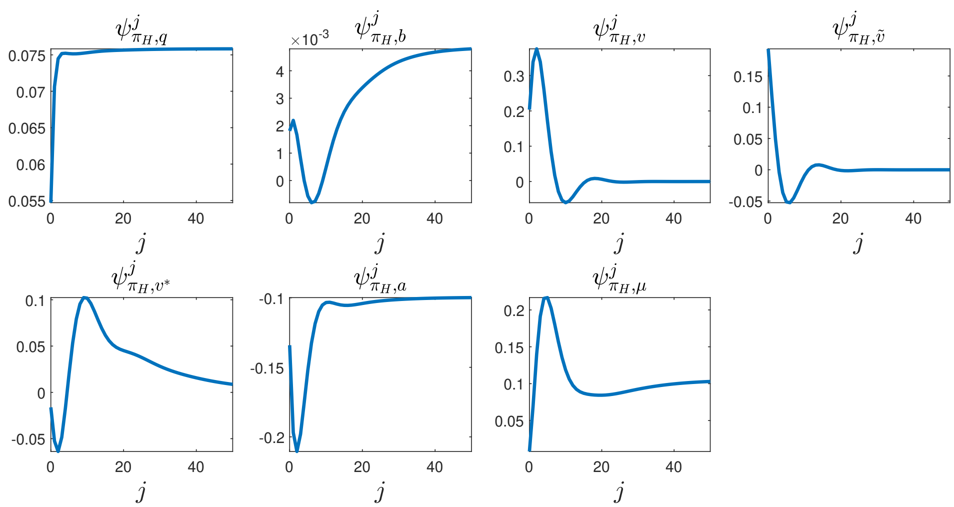


Figure G.7: Policy Coefficients of the Five Variables with Planning Horizon $j \in [0, 50]$ (continued)

(d) CPI Inflation



(e) Domestic Goods Inflation



H Proof of Proposition 1

We prove Proposition 1 by contradiction. Assume the LIE holds. It must imply that

$$\hat{\mathbb{E}}_t x^j_{t+2|t} = \hat{\mathbb{E}}_t \hat{\mathbb{E}}_{t+1} x^j_{t+2|t} \tag{H.1}$$

for any $j \geq 0$.

For any endogenous variable in the FH model, it can be written in the form

$$x^j_{\tau|t} = \Lambda_j Z^{j+1}_{\tau|t} + M_j U_\tau + N_j V_t,$$

where vector $Z^{j+1}_{\tau|t}$ denotes the vector of endogenous state variables at date τ in addition to those of value functions, U^j_τ denotes the vector of exogenous state variables at date τ, and V_t denotes the vector of value functions. Λ_j, M_j, and N_j are the matrices of coefficients. From expression (3.3), $Z^{j+1}_{\tau|t} = [\hat{q}^{j+1}_{\tau-1|t}\ \hat{b}^{*,j+1}_{\tau|t}]'$, $U_\tau = [a_\tau\ \mu_\tau]'$, and $V_t = [\nu_t\ \tilde{\nu}_t\ \nu^*_t]'$.

Thus, the left-hand side of equation (H.1) is

$$\hat{\mathbb{E}}_t x^j_{t+2|t} = \Lambda_j \hat{\mathbb{E}}_t Z^{j+1}_{t+2|t} + M_j \hat{\mathbb{E}}_t U_{t+2|t} + N_j V_t, \tag{H.2}$$

and its right-hand side is

$$\hat{\mathbb{E}}_t \hat{\mathbb{E}}_{t+1} x^j_{t+2|t} = \hat{\mathbb{E}}_t \left[\Lambda_j \hat{\mathbb{E}}_{t+1} Z^{j+1}_{t+2|t} + M_j \hat{\mathbb{E}}_t U_{t+2|t} + N_j V_{t+1} \right]. \tag{H.3}$$

Because the LIE holds, we have

$$\hat{\mathbb{E}}_t Z^{j+1}_{t+2|t} = \hat{\mathbb{E}}_t \hat{\mathbb{E}}_{t+1} Z^{j+1}_{t+2|t}, \qquad \hat{\mathbb{E}}_t U_{t+2|t} = \hat{\mathbb{E}}_t \hat{\mathbb{E}}_t U_{t+2|t}.$$

By equating (H.2) and (H.3), it yields

$$N_j V_t = N_j \hat{\mathbb{E}}_t V_{t+1} \tag{H.4}$$

for any j.

Meanwhile, the constant-gain learning rule implies

$$\hat{\mathbb{E}}_t V_{t+1} = \Gamma V^{est}_t + (I - \Gamma) V_t,$$

where $\Gamma \equiv \text{diag}(\gamma_v, \gamma_v, \gamma_{\tilde{v}})$ and I is an identity matrix. Plugging it into (H.4), we have

$$N_j \Gamma (V_t - V^{est}_t) = 0. \tag{H.5}$$

Since (H.5) has to hold for any j and N_j is not a zero matrix, it is equivalent to

$$\Gamma (V_t - V^{est}_t) = 0. \tag{H.6}$$

Condition (H.6) means that either $\Gamma = 0$ or $V_t = V^{est}_t$ for all t, that is, either the learning gain parameters are all zero, or the value functions are the fixed point of the constant-gain learning rule, which are the steady state value functions. Thus, once $\gamma_v, \gamma_{\tilde{v}} \neq 0$ and the economy is not always at its steady state, condition ($H.6$) does not hold, and the LIE breaks.

I Data and Variable Construction for Calibration

The data used in the calibration of Section 4.2 are constructed as follows, all at a quarterly frequency:

- *Canadian per capita real output growth and consumption growth.* We obtain the Canadian data of real GDP [V6E06896], household real final consumption expenditure [V6A89012], and population [V1] from Statistics Canada (StatsCan). The code in brackets represents StatsCan mnemonic. We divide the two data series by population to calculate the per capita real output and consumption and take the log difference over time to get the corresponding growth rates.

- *U.S. and Canada price levels.* We obtain the headline consumer price index (CPI) of the U.S. and Canada from Federal Reserve Economic Data [FRED: CPIAUCSL and CPALCY01CAQ661N]. The U.S. CPI data is seasonally adjusted, whereas the latter is not. Thus, we follow the U.S. Census Bureau model X-13ARIMA-SEATS to seasonally adjust the Canadian CPI data series. We further normalize the data series such that the average price level for both the U.S. and Canada in 2015 is 100.

- *U.S.-Canada real exchange rate.* We obtain the average of the daily nominal exchange rate between U.S. and Canadian dollars from Federal Reserve Economic Data [FRED: CCUSMA02CAM618N]. The data is in units of Canadian dollars. We calculate the real exchange rate by multiplying the nominal exchange rate with the U.S. price level and dividing it by the Canadian price level.

J Empirical Analyses with Actual Data

J.1 Excess Return Predictability Regression

The data is directly from the replication file of Valchev (2020). It contains forward and spot exchange rates of multiple countries (each paired with the U.S.) and inflation rates. The original data sources for forward and spot rates are Reuters/WMR and Barclays, and the original data source for inflation rates is the OECD. We select the exchange rate data for Canada (paired with the U.S.), as well as the inflation rate data for both Canada and the

U.S. The frequency of the exchange rates is daily, whereas the frequency of inflation rates is monthly. To align the frequencies, we use the end date of each month for the exchange rates data, same as in Engel (2016). The converted data starts from 1976Q1 and up until 2007Q4. We construct the real depreciation rate, $q_{t+1} - q_t$, which equals the nominal depreciation rate plus the U.S. inflation rate and minus the Canadian inflation rate.

To construct the real interest rate differential $r_t - r_t^*$, we take the following steps. First, we construct the Canada-U.S. nominal interest rate differential by computing the forward-to-spot ratio, employing the result from the covered interest rate parity (CIP), following the standard practice in the literature. Second, we subtract the respective inflation rates from the nominal interest rates of each country. We consider three approaches of constructing the real interest rate (symmetric specifications are applied for the U.S. real interest rate):

(1) Real interest rate based on current inflation: $r_t = i_t - \pi_t$.

(2) Real interest rate based on forecasted inflation: $r_t = i_t - \widehat{\mathbb{E}_t \pi_{t+1}}$, where $\widehat{\mathbb{E}_t \pi_{t+1}}$ is the inflation forecast based on AR(1) regression.

(3) Real interest rate based on ex post inflation: $r_t = i_t - \pi_{t+1}$.

We run the regression model (5.9) with the constructed data of the real depreciation rate and the real interest rate differential. The regression coefficients are then quarterly averaged to match the frequency of the model. Figure J.8 shows the estimates $\hat{\beta}_k$ under the three approaches of constructing the real interest rate. It also includes the estimated $\hat{\beta}_k$ from the FH model for comparison. Overall, the dynamic patterns of $\hat{\beta}_k$ across the time horizon k are both qualitatively and quantitatively robust across the three lines. All share close similarity to the estimates from the FH model. We have placed the estimates from the data construction method (2) into Figure 3 for illustration purposes.[24]

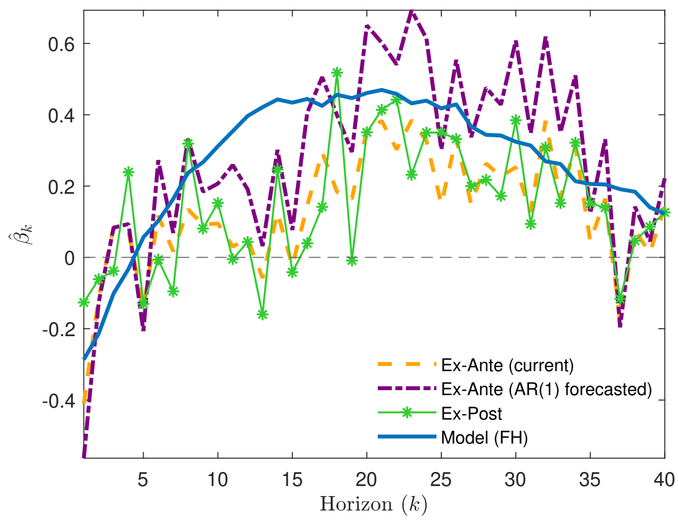
J.2 Forward Guidance Exchange Rate Puzzle

Below we first list the data sources and variable construction used in regression (5.10). Most of the variables are from monthly market prices or indices, following the data sources in Galí (2020). Unless otherwise noted, data represent the last price of each month.

- *U.S. inflation expectation.* We obtain the measure of U.S. inflation expectations through inflation swap rates. The inflation swap rates are collected from Bloomberg

[24]In either data construction method (1) or (2), the unpredictability feature of excess return under RE based on time-t information set is always rejected. We choose specification (2) as the benchmark since this method is closer to the conventional meaning of an ex-ante real interest rate and is also commonly used in the literature (see, e.g., Valchev, 2020).

Figure J.8: Excess Return Regression with Alternative Formulations of Real Interest Rates



Notes: This figure shows the estimates of excess return coefficients, $\hat{\beta}_k$, across time horizon k. The dashed line represents the point estimates by constructing the real interest rate via $r_t = i_t - \pi_t$. The dash-dotted line represents the point estimates by constructing the real interest rate via $r_t = i_t - \widehat{\mathbb{E}_t \pi_{t+1}}$, where $\widehat{\mathbb{E}_t \pi_{t+1}}$ is the inflation forecast based on AR(1) regression. The solid line with cycles represents the point estimates by constructing the real interest rate via $r_t = i_t - \pi_{t+1}$. Similar construction applies to the foreign real interest rate accordingly. The solid blue line represents the point estimates using FH model simulated data from Figure 3. The x-axis represents horizon k (in quarters).

under the mnemonics "USSWIT[M] Curncy," where [M] is the maturity for inflation swap rates (in years). We obtain the data series for $M = 1$, 2, 5, 10, and 30.

- *U.S. expected nominal interest rate.* We obtain the U.S. expected nominal interest rates through the zero-coupon yields of the U.S. Treasuries from Bloomberg under the mnemonics "I025[M]Y Index," where [M] is the maturity in years. We obtain the data series for $M = 1$, 2, 5, 10, and 30.

- *Canadian inflation expectation.* Since the inflation swap is not available for Canada, we obtain the survey-based Canadian inflation expectations for maturities $M = 1$, 2, 5, and 10 years from the Consensus Forecast. The data is available at a quarterly frequency since 2014Q3 but is only available semi-annually before that. To construct monthly data for the regressions in Galí (2020), we first interpolate the missing values with the average of two adjacent observations and assume that the inflation expectation is the same within the same quarter. We also assume inflation expectations at maturity $M = 30$ years is the same as those at $M = 10$ years. We then translate the data into the average inflation expectations over fixed horizons following the approach in Dovern, Fritsche and Slacalek (2012).

Table 3: Forward Guidance Exchange Rate Puzzle, U.S.-Canada

	(1) $M = 4$	(2) $M = 8$	(3) $M = 20$	(4) $M = 40$	(5) $M = 120$
Baseline					
γ_S	4.14***	3.46***	2.47***	2.03***	0.27**
	(1.51)	(1.16)	(0.48)	(0.26)	(0.12)
γ_L	0.18	0.12	-0.08	-0.37***	
	(0.11)	(0.12)	(0.13)	(0.14)	
Time trend					
γ_S	3.71***	3.14***	2.01***	1.50***	-0.09
	(0.82)	(0.72)	(0.27)	(0.15)	(0.14)
γ_L	-0.17	-0.25**	-0.40***	-0.61***	
	(0.14)	(0.12)	(0.09)	(0.09)	
First differences					
γ_S	1.93***	1.56***	0.99***	0.73***	0.14***
	(0.20)	(0.19)	(0.15)	(0.12)	(0.04)
γ_L	0.09***	0.07*	0.05	-0.03	
	(0.03)	(0.04)	(0.04)	(0.04)	

Notes: This table reports the estimated γ_S and γ_L in the three specifications in Galí (2020) for the case of U.S.-Canada: the original form of specification (5.10), the version with a time trend, and the version after taking first-order differences. Standard errors are reported in brackets, using the Newey-West adjustment with 12 lags. The sample period starts from July 2004 to December 2023. $^{***}p < 0.01, ^{**}p < 0.05, ^{*}p < 0.1$.

- *Canadian expected nominal interest rate.* We obtain the expected nominal interest rates through the zero-coupon yields of the Canadian government bonds from Bloomberg under the mnemonics "I007[M]Y Index," where [M] is the maturity in years. We obtain the data series for $M = 1$, 2, 5, 10, and 30.

- *U.S.-Canada real exchange rate.* We obtain the nominal exchange rate between U.S. and Canadian dollars from Bloomberg under mnemonics [CADUSD Curncy], the Canadian CPI price level under mnemonics [CACPI Index], and the U.S. CPI price level under mnemonics [CPURNSA Index]. Since the nominal exchange rate data is in units of US dollars, we calculate the real exchange rate in Canadian dollars by multiplying the inverse of the nominal exchange rate with the U.S. price level and dividing it by the Canadian price level.

Following Galí (2020), we construct the expected real interest differentials by calculating

$$D_t^S(M) \equiv \sum_{k=0}^{M-1} \mathbb{E}_t[\hat{\imath}^*_{t+k} - \hat{\pi}^*_{t+k} - (\hat{\imath}_{t+k} - \hat{\pi}_{t+k})],$$

where $M = 4$, 8, 20, 40, and 120 quarters. Then the expected long-term interest rate differentials are

$$D_t^L(M) \equiv D_t^S(120) - D_t^S(M)$$

for $M = 4$, 8, 20, and 40.

Table 3 reports the estimated γ_S and γ_L in the three specifications in Galí (2020) for the case of U.S.-Canada: the original form of specification (5.10), the version with a time trend, and the version after taking first-order differences. Because the U.S. inflation swap data has only been available since July 2004, the sample period in the regressions starts from July 2004 to December 2023. Table 3 suggests similar findings as in Galí (2020), in which the estimated γ_S is significantly larger than one in most cases and the estimated γ_L is significantly smaller than one. Same as in Figure 4 using model-generated data, the empirical estimated γ_S and γ_L become smaller as the threshold M increases.

K Construction of Short-Term and Long-Term Interest Rate Differentials

We construct the corresponding model-generated series via the following steps. First, to construct the simulated expected real interest rate differential under limited foresight, for each time t and for any horizon k, we need to pin down $\{\hat{q}^{h-k}_{t+k|t}, \hat{b}^{h-k}_{t+k+1|t}\}^h_{k=0}$ to back up $\{\hat{\pi}^{h-k}_{t+k|t}, \hat{\imath}^{h-k}_{t+k|t}, \hat{r}^{*,h-k}_{t+k|t}\}^h_{k=0}$. For each k, the solution to the forward planning exercise (3.3) yields

$$\mathbb{E}_t\hat{q}^{h-k}_{t+k|t} = \psi^{h-k}_{q,q}\mathbb{E}_t\hat{q}^{h-k+1}_{t+k-1|t} + \psi^{h-k}_{q,b}\mathbb{E}_t\hat{b}^{h-k+1}_{t+k|t} + \psi^{h-k}_{q,a}\rho^k_a a_t + \psi^{h-k}_{q,\mu}\rho^k_\mu \mu_t + \psi^{h-k}_{q,\nu}\nu_t + \psi^{h-k}_{q,\nu^*}\nu^*_t,$$

where $\mathbb{E}_t\hat{q}^{h+1}_{t-1|t} = \hat{q}_{t-1}$ and $\mathbb{E}_t\hat{b}^{h+1}_{t|t} = \hat{b}_t$ (that is, the equilibrium pre-determined real exchange rate and the net foreign asset position at time t). Then, the expected inflation under limited foresight satisfies

$$\mathbb{E}^h_t\pi_{t+k} \equiv \mathbb{E}_t\pi^{h-k}_{t+k|t} = \psi^{h-k}_{\pi,q}\mathbb{E}_t\hat{q}^{h-k+1}_{t+k-1|t} + \psi^{h-k}_{\pi,b}\mathbb{E}_t\hat{b}^{h-k+1}_{t+k|t} + \psi^{h-k}_{\pi,a}\rho^k_a a_t + \psi^{h-k}_{\pi,\mu}\rho^k_\mu \mu_t + \psi^{h-k}_{\pi,\nu}\nu_t + \psi^{h-k}_{\pi,\nu^*}\nu^*_t.$$

Since $\hat{\imath}_{\tau|t}^{h+t-\tau} = \phi_\pi \hat{\pi}_{\tau|t}^{h+t-\tau}$, we have the expected nominal interest rate given by

$$\mathbb{E}_t^h \hat{\imath}_{t+k} \equiv \mathbb{E}_t \hat{\imath}_{t+k|t}^{h-k} = \phi_\pi \mathbb{E}_t \pi_{t+k|t}^{h-k},$$

and then we construct the expected real interest rate under limited foresight as

$$\mathbb{E}_t^h \hat{r}_{t+k} \equiv \mathbb{E}_t \hat{\imath}_{t+k|t}^{h-k} - \mathbb{E}_t \pi_{t+k+1|t}^{h-k-1}, \tag{K.1}$$

for any $0 \leq k < h$, and for the case of $k = h$, we construct $\mathbb{E}_t^h \hat{r}_{t+h} \equiv \mathbb{E}_t \hat{\imath}_{t+h|t}^0$.

Second, for the expected foreign real interest rate under limited foresight, since (2.33) yields $\hat{r}_{\tau|t}^{*,h+t-\tau} = \phi_b \hat{b}_{\tau+1|t}^{h+t-\tau} + \mu_\tau$ for any $t \leq \tau \leq t+h$, we construct

$$\mathbb{E}_t^h \hat{r}_{t+k}^* \equiv \mathbb{E}_t \hat{r}_{t+k|t}^{h-k} = \phi_b \mathbb{E}_t \hat{b}_{t+k+1|t}^{h-k} + \rho_\mu^k \mu_t. \tag{K.2}$$

Therefore, for a given threshold horizon for the short-term and the long-term M and the given planning horizon h, we can construct the cumulative forecasts of real interest rate differentials under limited foresight as

$$D_t^S(M) = \sum_{k=0}^{M-1} \mathbb{E}_t^h [\hat{r}_{t+k}^* - \hat{r}_{t+k}], \qquad D_t^L(M) = \sum_{k=M}^{h} \mathbb{E}_t^h [\hat{r}_{t+k}^* - \hat{r}_{t+k}], \tag{K.3}$$

where $0 < M \leq h$.

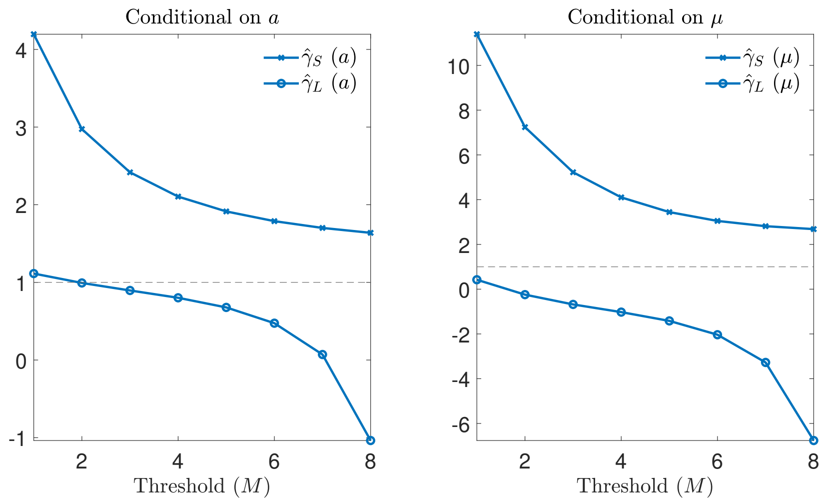
Using model-simulated data, whereas Figure 4 shows the conditional estimates of $\hat{\gamma}_S$ and $\hat{\gamma}_L$ in specification (5.10), Figure K.9 shows the conditional ones, each conditioned on a distinct shock.

L Robustness Checks

Planning Horizon h. First, we consider planning horizon $h \in \{2, 4, 40\}$ in the benchmark model with homogeneous agents. Subject to the two exogenous shocks, Figure L.10 illustrates the regression coefficients for both excess return predictability and the real exchange rate's response to short- and long-term real interest rate differentials with each planning horizon h. In summary, the attributes of the primary findings in Section 5 become stronger when planning horizons shorten and weaker as they lengthen, in comparison to the benchmark planning horizon $h = 8$.

Firm's Learning Gain $\gamma_{\tilde{v}}$. Next, we consider firm's learning behavior on its value function with various learning gain $\gamma_{\tilde{v}} \in \{0.1, 0.5, 0.99\}$. Figure L.11 illustrates the related regression

Figure K.9: Reaction of Real Exchange Rate to Expected Interest Rate Differentials (Conditional)



Notes: This figure shows the conditional estimates of $\hat{\gamma}_S$ and $\hat{\gamma}_L$ in specification (5.10), representing the reaction of the real exchange rate to the forecasts of short- and long-term real interest rate differentials in the benchmark FH model. The left panel uses the series from the domestic productivity shock only and the right panel uses the series from the foreign interest rate shock only. The estimates are obtained using samples from 100 simulations with each spanning 150 quarters (15,000 observations in total).

coefficients based on each learning gain $\gamma_{\tilde{v}}$. The outcomes for excess return predictability are not significantly impacted by variations in $\gamma_{\tilde{v}}$. Moreover, the results of the breakdown of the forecast horizon invariance are found to be unaffected by $\gamma_{\tilde{v}}$. This is consistent with intuition in the sense that the asset-pricing condition for the exchange rate and interest rate differentials comes from the F.O.C.s of households rather than those of firms. Thus, households' learning parameter plays a key role in explaining the UIP puzzles, whereas firms' learning parameter does not.

Heterogeneous Planning Horizons across Agents. Lastly, we extend the model to heterogeneous planning horizons across agents and examine its consequences for the aggregate variables. We assume that fraction $\omega_h \in (0,1)$ of households and fraction $\tilde{\omega}_h \in (0,1)$ of firms have planning horizon h. Following Woodford (2019), we further assume that those agents with horizon h make their decisions by assuming that all other agents have the same planning horizon of h. After the planning exercises in each period, the estimated value functions across households are aggregated by

$$\nu_t^{est} = \sum_h \omega_h \left[\hat{c}_t^h + \sigma^{-1}\pi_t^h - (1-\beta)\hat{b}_t^{*,h}\right], \tag{L.1}$$

$$\nu_t^{*,est} = \sum_h \omega_h \left[\nu_t^{est} - \sigma^{-1}(\hat{q}_t^h + \pi_t^h)\right], \tag{L.2}$$

and the estimated value functions across firms are aggregated by

$$\tilde{\nu}_t^{est} = \sum_h \tilde{\omega}_h (1-\theta)^{-1} \pi_{H,t}^h. \tag{L.3}$$

We consider a special case in which fractions ω_h and $\tilde{\omega}_h$ follow the same geometric distributions:

$$\omega_h = \tilde{\omega}_h = \rho^h (1-\rho)$$

for any $h \geq 0$, where $\rho \in (0,1)$. Then, the aggregate variables become the weighted average among the population. The current endogenous variables are aggregated as follows:

$$\hat{c}_t = \sum_h \omega_h \hat{c}_t^h, \qquad \hat{y}_t = \sum_h \omega_h \hat{y}_t^h, \qquad \hat{\imath}_t = \sum_h \omega_h \hat{\imath}_t^h, \qquad \hat{r}_t^* = \sum_h \omega_h \hat{r}_t^{*,h}, \qquad \pi_{H,t} = \sum_h \omega_h \pi_t^h,$$
$$\pi_t = \sum_h \omega_h \pi_t^h, \qquad \hat{q}_t = \sum_h \omega_h \hat{q}_t^h, \qquad \hat{s}_t = \sum_h \omega_h \hat{s}_t^h, \qquad \hat{\varepsilon}_t = \sum_h \omega_h \hat{\varepsilon}_t^h, \qquad \hat{b}_{t+1}^* = \sum_h \omega_h \hat{b}_{t+1}^h. \tag{L.4}$$

The k-period ahead expectations for CPI inflation are aggregated as follows:

$$\begin{aligned}
\mathbb{E}_t^{agg} \pi_{t+k} &\equiv \sum_h w_h \mathbb{E}_t^h \pi_{t+k} = \sum_h w_h \mathbb{E}_t \pi_{t+k|t}^{h-k} \\
&= \mathbb{E}_t \left[\rho^k (1-\rho) \pi_{t+k|t}^0 + \rho^{k+1} (1-\rho) \pi_{t+k|t}^1 + \rho^{k+2} (1-\rho) \pi_{t+k|t}^2 + \cdots \right] \\
&= \rho^k \mathbb{E}_t \left[\pi_{t+k|t} \right].
\end{aligned}$$

Similarly, we obtain

$$\mathbb{E}_t^{agg} \hat{q}_{t+k} = \rho^k \mathbb{E}_t \left[\hat{q}_{t+k|t} \right], \qquad \mathbb{E}_t^{agg} \hat{\imath}_{t+k} = \rho^k \mathbb{E}_t \left[\hat{\imath}_{t+k|t} \right], \qquad \mathbb{E}_t^{agg} r_{t+k}^* = \rho^k \mathbb{E}_t \left[r_{t+k|t}^* \right].$$

Thus, we have

$$\begin{aligned}
\mathbb{E}_t \left[\pi_{t+k|t} \right] &= \rho^{-k} \mathbb{E}_t^{agg} \pi_{t+k}, & \mathbb{E}_t \left[\hat{q}_{t+k|t} \right] &= \rho^{-k} \mathbb{E}_t^{agg} \hat{q}_{t+k}, \\
\mathbb{E}_t \left[\hat{\imath}_{t+k|t} \right] &= \rho^{-k} \mathbb{E}_t^{agg} \hat{\imath}_{t+k}, & \mathbb{E}_t \left[\hat{r}_{t+k|t}^* \right] &= \rho^{-k} \mathbb{E}_t^{agg} \hat{r}_{t+k}^*,
\end{aligned}$$

and the k-step ahead domestic real interest rate becomes

$$\mathbb{E}_t [\hat{r}_{t+k|t}] = \mathbb{E}_t \left[\hat{\imath}_{t+k|t} \right] - \mathbb{E}_t \left[\pi_{t+k+1|t} \right]. \tag{L.5}$$

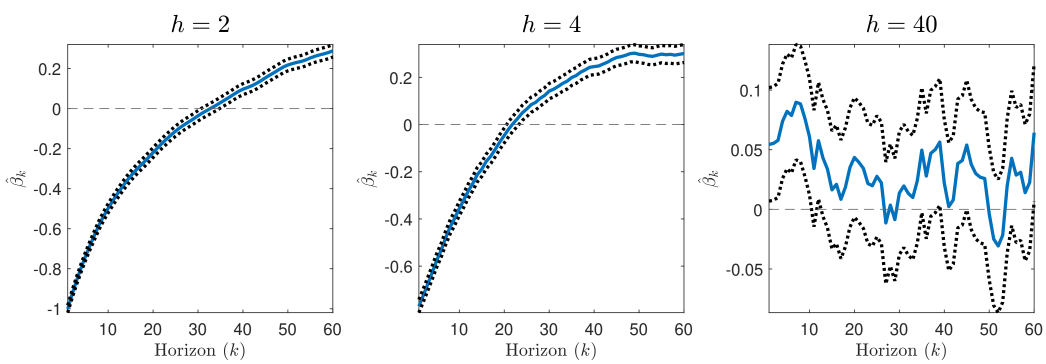
We can then construct $D_t^S(M) = \sum_{k=0}^{M-1} \mathbb{E}_t [\hat{r}_{t+k|t}^* - \hat{r}_{t+k|t}]$ and $D_t^L(M) = \sum_{k=M}^{\infty} \mathbb{E}_t [\hat{r}_{t+k|t}^* -$

$\hat{r}_{t+k|t}]$.

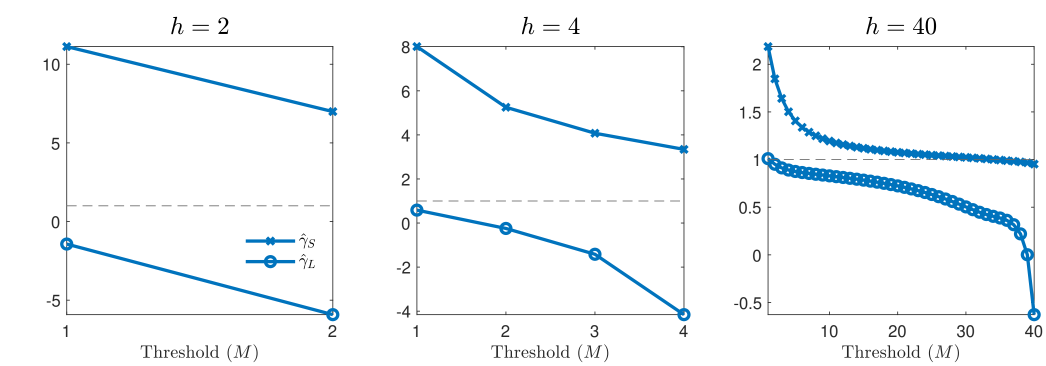
For the numerical exercise, we set $\rho = 8/9$ so that the average planning horizon in the economy is $\rho/(1-\rho) = 8$ quarters, the same as the planning horizon of the homogeneous agents in Section 4.[25] Figure L.12 illustrates the related regression coefficients; it shows that the coefficients for excess return predictability across various time horizons display a pattern similar to our benchmark results, as shown in Section 5. Additionally, there is a breakdown of the forecast horizon invariance, as shown in the middle and right panels. One observation is that the long-term reaction coefficient of the real exchange rate, $\hat{\gamma}_S$, exhibits quantitatively small variations across different thresholds M. This is because the long-term expected real interest rate differential $D_t^L(M)$ also shows relatively small variations across these thresholds, due to the aggregation effect of the value functions across heterogeneous planning horizons.

Figure L.10: Robustness Check with Alternative Planning Horizons $h \in \{2, 4, 40\}$

(a) Excess Return Predictability across Time Horizons



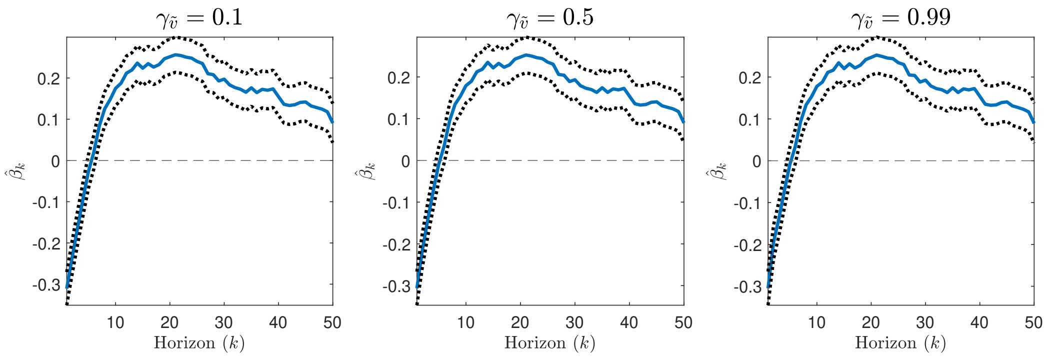
(b) Reaction Coefficients of Real Exchange Rate to Expected Real Interest Rate Differentials



[25]For numerical aggregation, we consider planning horizons ranging from 0 to 100. Also, we nullify the firms' learning in their value function.

Figure L.11: Robustness Check with Different Firm's Learning Gains $\gamma_{\tilde{v}} \in \{0.1, 0.5, 0.99\}$

(a) Excess Return Predictability across Time Horizons



(b) Reaction Coefficients of Real Exchange Rate to Expected Real Interest Rate Differentials

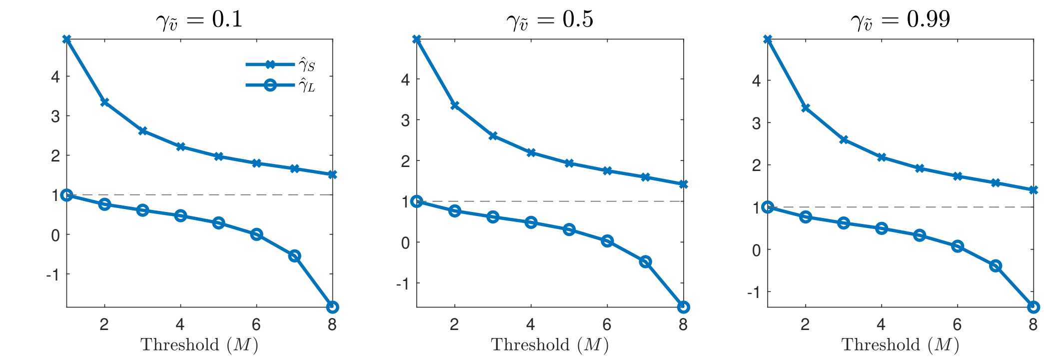
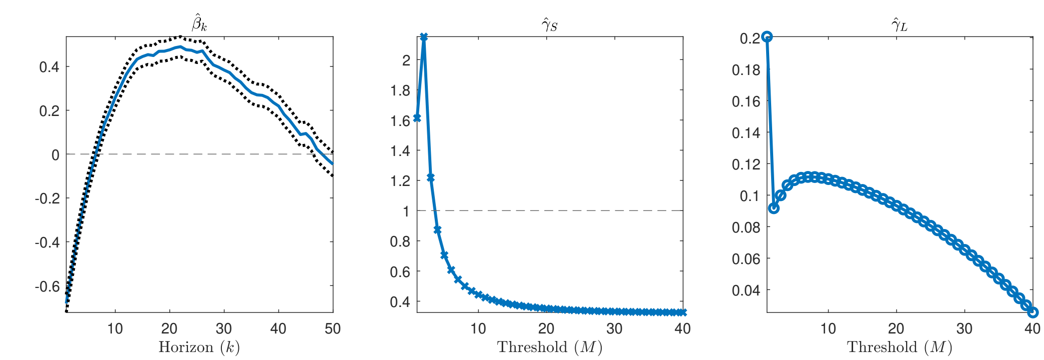


Figure L.12: The Case of Heterogeneous Planning Horizon across Agents



Notes: The average planning horizon of the population is set to be eight quarters ($\rho = 8/9$). The left panel shows the regression coefficients for the excess return predictability. The middle and right panels show the regression coefficients for the response of the real exchange rate to expected real interest rate differentials