# Private Equity for Pension Plans? Evaluating Private Equity Performance from an Investor's Perspective*

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**Abstract**

We propose a methodology to evaluate private equity (PE) investments by using investor-specific stochastic discount factors. The methodology allows a direct way of decomposing an investor's PE return into a risk-compensation and an "alpha". It also helps determine whether a given investor could benefit from investing (more) in PE. Applying our metrics to U.S. public pension plans, our key results are that: a) during our sample period, pension plan allocations to PE funds were optimal overall, although the average plan was underexposed to buyout; b) plans invest in PE funds that have higher risk-adjusted performance, but this is because of some pension plans' superior access to successful PE managers; c) the higher PE returns obtained by some pension plans appear to be the result of a higher willingness to take risk without earning superior risk-adjusted returns, broadly consistent with agency problems within pension plans.

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The performance evaluation of investments is one of the most actively researched topics in finance. The typical approach is to compare historical rates of return against the return of similarly risky investments as predicted by various asset pricing models. In illiquid asset classes like private equity (PE), returns are not observed at regular intervals, and valuations are stale and potentially biased, complicating this task (see Korteweg, 2023, for a review of the literature). The most commonly used performance measures in PE, such as the internal rate of return (IRR) and cash multiples, therefore rely primarily on fund cash flows (and do so exclusively in the case of fully liquidated funds). However, a high IRR or cash multiple could simply indicate a very risky investment, rather than an investment that raises a portfolio's Sharpe ratio upon inclusion. These limitations led to the development of performance metrics that use a stochastic discount factor (SDF) to discount cash flows. The key feature is the use of the cumulative rate of return of some portfolio (or a "levered" version of that portfolio) for discounting. For instance, the Kaplan and Schoar (2005) public market equivalent (PME) metric uses the public stock market return to discount cash flows back to the fund's inception date.

The SDF approach to performance evaluation has several theoretical advantages, as we discuss in detail in section 1.1. For example, if the cash flows of the PE fund can be replicated by some dynamic trading strategy in publicly traded assets, then the present value of the private equity fund's cash flows has a value of zero for all SDFs that price these same public assets. However, in the more realistic case where such a replication is not possible, there could be multiple SDFs that price all the publicly traded assets but assign different values to the "unspanned" risks of private equity. This problem becomes particularly important if investors hold different optimal portfolios (e.g., due to differences in risk-aversion, or to non-participation in certain markets), as they are likely to assign very different values to these unspanned risks.

In this paper we propose a pragmatic approach to determining the SDF that is to be used for discounting. The key idea is to use a given investor's own portfolio return to form the stochastic discount factor. We provide several theoretical arguments why using the investor's own return to form the SDF has some appealing properties, even if the financial market is incomplete. Specifically, we propose two variants of our measure. The first measure, the "investor portfolio equivalent" (IPE), is essentially the same measure as the PME, except that we use the investor's own portfolio return rather than the return of the market portfolio when discounting the cash flows of the private equity fund. We show that when this measure has a positive value, it indicates that an investor could raise the (logarithmic) growth rate of her investment portfolio by allocating a marginal dollar towards PE. While simple to compute and easy to interpret, the IPE measure has the disadvantage that it could depend on different risk attitudes, which are reflected in an investor's optimal mix of stocks and bonds. For this reason, we also consider a generalized version (GIPE), which takes into account different investor risk aversions. In effect, the GIPE uses an investor-specific, levered version of the investor's portfolio to form the SDF. An attractive property of the GIPE is that it is zero if a PE investment is just a levered version of the return on public equity. Both the IPE and GIPE measure allow the computation of an annualized "alpha". This alpha can be interpreted as the component of the internal rate of return of the investment that is not due to risk, but rather due to a meaningful expansion of the investment opportunity set for a specific investor.

We also develop diagnostics to determine whether simple, *long-only* public market strategies can produce the same gains as a given PE investment (e.g., a value strategy for buyout funds). The comparison to long-only investment alternatives is important, because many large investors in private equity are constrained or altogether prohibited from shorting.

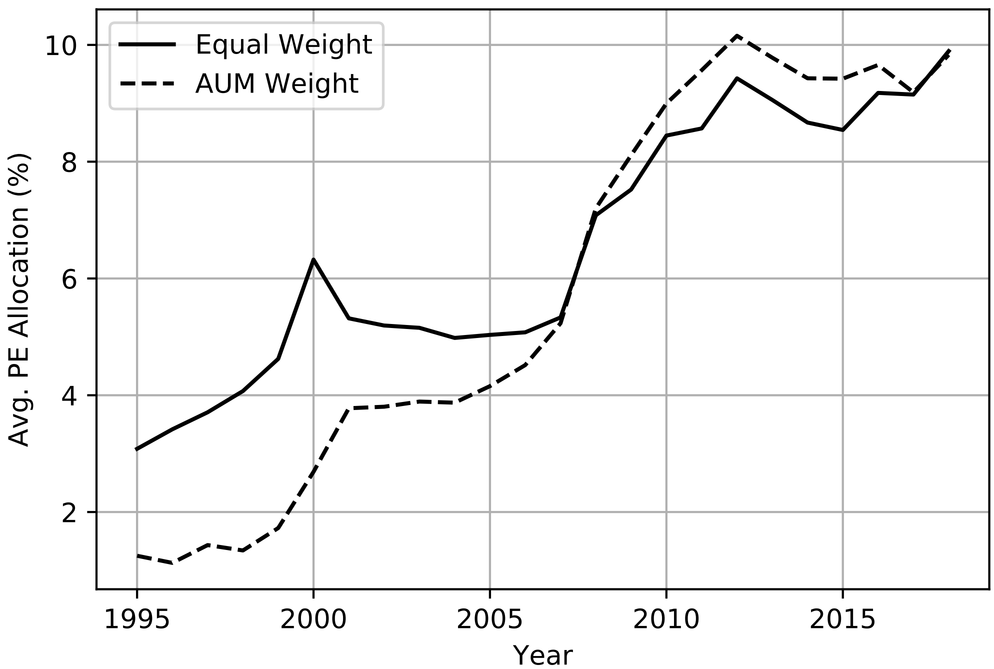
To illustrate our approach, we compute the IPE and GIPE of several PE strategies from the perspective of U.S. public pension plans.[1] We focus on public pension plans for several reasons. First, as Figure 1 illustrates, public pension plans have been very active investors in private equity, and many plans now have double-digit percentage allocations to this asset class. Second, underfunding and corporate governance concerns about pension plans make it especially important to risk-adjust their investments. Third, there is large cross-sectional heterogeneity in portfolios. For example, in 2018, the Public School Employees Retirement System of Pennsylvania had an allocation to public equities of 20.9%, compared to 61.9% for the Employees' Retirement System of Georgia. The cross-sectional standard deviation across all pension plans for the year was 10.9 percentage points (the average was 47.6%). Dispersion in portfolio holdings was similar in other years. Fourth, data on pension plan investment returns are readily available, which helps in forming our SDFs.

Our results can be summarized as follows. First, for our sample period that spans from 1995 to 2018, pension plans appear to have allocated their portfolios to long-only PE optimally, in the sense

[1] For disambiguation, we use the nomenclature pension *plans* (rather than pension funds) to distinguish them from private equity *funds*. Also, when we write private equity we mean all forms of PE, including but not limited to venture capital, buyout, and real estate funds.

Figure 1: Average Public Pension Plan Allocation to Private Equity

This figure shows the time series of average portfolio allocations to private equity (PE) by 179 U.S. public pension plans from 1995 to 2018. The solid line shows the equal-weighted average allocation across plans. The dashed line is the weighted average using pension plans' assets under management (AUM) as weights. Source: Comprehensive Annual Financial Report (CAFR) data from the Center for Retirement Research at Boston College, the Center for State and Local Government Excellence, and the National Association of State Retirement Administrators (available at `https://publicplansdata.org`), and authors' own work.




that there was no benefit to the average pension plan of investing more or less in a representative PE fund. Phrased more succinctly, the GIPE measure (averaged across pension plan and PE fund combinations) is approximately zero. The main exception is the performance of buyout funds, which has a positive GIPE over the sample period. However, this positive GIPE is at least in part a result from buyout having a value exposure, and public value equities performed well over the sample period. For venture capital (VC), we find that the average fund underperformed, but this is driven by the poor performance of small-growth firms overall (including publicly traded firms).

Second, we do not find evidence that pension plans had market timing skill. We do find that the PE funds that were selected by public pension plans outperform the average PE fund of the same vintage year, but this does not seem to be due to a genuine ability to select the better-performing PE investments. Rather, it appears that the PE funds select their investors and give them preferential access to the better PE investments. Indeed, when we we confine attention to the universe of private equity investments that are continuations of ongoing relationships with a given pension plan, or when we consider first time funds (which are likely to be less selective about their clients), the "selectivity" skill disappears.

Third, we decompose the IRR of a pension plan's private equity investments into a risk-compensation and an alpha component and examine whether plan characteristics that correlate with a high IRR do so because of the riskiness of these PE investments, or because of a meaningful expansion of the investment opportunity set (alpha). We find that underfunded pension plans, and plans that have a larger fraction of state officials who serve ex-officio and members of the public appointed by a government official, take more risk but earn lower risk-adjusted returns in their PE investments. Investments in PE funds that are located in the pension plan's state do not differ in risk from out-of-state funds, but they earn lower risk-adjusted returns. These results are suggestive of agency problems, such as gambling for resurrection and political influence, playing an important role in the selection of PE investments by pension plans.

The paper is related to several strands of literature. First, our SDF-based metrics are related to the public market equivalent measure of Kaplan and Schoar (2005) and, especially, the generalized PME (GPME) of Korteweg and Nagel (2016). The key difference is that we use investor-specific SDFs, which allows for the possibility of heterogeneous investors with different risk assessments of the same investment opportunity. Compared to GPME, we show that our GIPE measure produces a performance distribution that is more stable over time, consistent with relatively constant investment skills. In other related work, Augustin et al. (2024) find that the choice of PE investment benchmarks by pension plans is subject to agency problems with respect to investment consultants. GIPE-type metrics provide a more objective way of assessing PE performance for an individual investor.

The second related strand is the literature on limited partner performance in private equity. Prior work has shown that different types of limited partners experience different performance (e.g., Lerner et al., 2007; Sensoy et al., 2014; Cavagnaro et al., 2019; Goyal et al., 2022). Korteweg and Westerfield (2022) survey this literature. For pension plans specifically, several papers consider the gambling for resurrection (also known as risk-shifting) concern for underfunded plans (e.g., Rauh, 2009; Pennacchi and Rastad, 2011; Mohan and Zhang, 2014; Bradley et al., 2016; Andonov et al., 2017; Myers, 2022). This literature studies whether underfunding changes the share of their portfolios that pension plans allocate to risky assets, and how it affects overall plan performance and the total return on their PE investments. Our contribution is to separate risk and excess return within PE investments, which allows us to examine whether the difference in PE performance is simply due to a difference in risk-taking. Mittal (2022) shows that underfunded plans tend to invest in PE managers whose portfolio firms reduce labor productivity, providing one possible channel to explain our result that underfunded plans experience lower risk-adjusted returns. Similarly, the literature that considers home bias (e.g., Lerner et al., 2007; Hochberg and Rauh, 2013; Bradley et al., 2016; Andonov et al., 2018) and board structure (Andonov et al., 2017, 2018) in public pension investing only considers broad plan performance (not specific to PE), or only total, not risk-adjusted PE performance.

The paper is organized as follows. Section 1 develops the theory behind our performance measures. Section 2 describes the pension plan and private equity fund data. Sections 3 and 4 present the empirical results on PE fund performance and the heterogeneity in performance across pension plans, respectively. Section 6 concludes.

# 1 Theoretical framework

Private equity funds are structured as limited partnerships. Investors, such as pension funds, are the limited partners (LPs) of the fund. The LPs are pure capital providers and have no control over which deals are invested or exited. Capital is committed at fundraising but not immediately transferred to the fund. Instead, the fund manager (general partner, or GP) searches for deals, and "calls" capital from the LPs when they have identified an investment. Money from the sale of investments is distributed to the LPs, after fees to the GP. The fund has a limited lifetime to invest its committed capital and realize exits (typically 10 years, with limited extension options in case of unexited investments). When all portfolio investments have been sold, the fund is liquidated. For an in-depth description of the PE industry, see, for example, Korteweg and Westerfield (2022).

From an LP's perspective, committing to a PE fund may be viewed as producing a sequence of future net-of-fee fund cash flows $C = \{C_{t_0}, .., C_{t_K}\}$ of random magnitude, timing, and number. Capital calls are negative flows for the LP ($C_{t_k} < 0$) whereas distributions are positive ($C_{t_k} > 0$).

Typically, the first cash flows are capital calls, with distributions occurring later in the fund's life, but we place no restriction on the sign of each flow.

The fund's "net asset position", $A_t$, is the economic value of the assets that help finance the cash flow series $C$. At $t_0$, the time of fund inception, $A_{t_0} = 0$. The asset value increases when a capital call occurs and decreases upon a distribution, that is, $A_{t_k^+} = A_{t_k} - C_{t_k}$. After the final cash flow (i.e., at fund liquidation), the asset value is zero again: $A_{t_K^+} = 0$. Between capital calls and distributions, $A_t$ evolves randomly according to some diffusion process.

We assume throughout that an econometrician cannot observe the true $A_t$. This is a realistic assumption in PE, because quarterly fund net asset values (NAVs) reported by GPs are subject to staleness and manipulation.[2] Therefore, as is typical in the literature and in practice, our performance measures rely on the observed cash flows, only using the final reported NAV as a pseudo-distribution in cases where the fund is not yet liquidated at the end of the sample period.

In addition to the cash flow sequence $C$, investors have access to $N$ other risky investments (public equities, long term bonds, commodities, etc.) and a risk-free security yielding the instantaneous interest rate $r_t$. The vector of risky security returns follows a diffusion process

$$\underbrace{dR_t}_{N\times 1} = \underbrace{\mu_t}_{N\times 1} dt + \underbrace{\sigma_t}_{N\times d}\underbrace{dB_t}_{d\times 1}, \tag{1}$$

where $\mu_t$ is the vector of expected returns and $\sigma_t$ is an $N \times d-$dimensional matrix of exposures to the $d-$dimensional Brownian motion, $dB_t$. We assume that $d \geq N$ to allow for the possibility that the market is incomplete.

## 1.1 SDF-based performance evaluation

A common approach to evaluating whether an investment in the cash-flow stream $C$ is an attractive investment opportunity is to test whether its net present value is zero, using a stochastic discount factor (SDF) to discount the sequence of payments in $C$.

The justification for the SDF-approach is a replication argument. Specifically, suppose that

[2] As of 2007, Accounting Standards Code (ASC) Topic 820 (formerly known as FAS 157) requires the disclosure of fair values. However, there is no market to mark PE investments to. GPs and auditors usually rely on the pricing of recently traded comparable assets, but this is a subjective process. For empirical evidence of staleness and manipulation of reported NAVs, see Phalippou and Gottschalg (2009); Jenkinson et al. (2013); Barber and Yasuda (2017); Chakraborty and Ewens (2018); Brown et al. (2019); Jenkinson et al. (2020).

there exists a portfolio $\phi_t$ of the $N$ risky assets such that the fund's net asset position evolves as

$$\frac{dA_t}{A_t} = r_t dt + \phi_t' \left(dR_t - r_t 1_N dt\right), \tag{2}$$

where $1_N$ is a column vector of ones. The position in the risk-free asset is $1 - 1_N'\phi_t$, ensuring that the portfolio is self-financing. An implication of (2) is that the cash flows $C$ can be exactly replicated with the dynamic, self-financing strategy $\phi_t$ in existing assets. Letting $H_t$ denote any SDF that prices the $N$ risky assets, a standard no-arbitrage argument implies that the net present value (NPV) of $C$ should be zero:

$$0 = A_{t_0} = E_{t_0} \sum_{k=0}^{K} \left(\frac{H_{t_k}}{H_{t_0}}\right) C_{t_k}. \tag{3}$$

Matters become more complicated in the more realistic situation where $C$ cannot be replicated by some dynamic trading strategy. In this case,

$$\frac{dA_t}{A_t} = r_t dt + \phi_t' \left(dR_t - r_t 1_N dt\right) + d\widetilde{R}_t, \tag{4}$$

where $d\widetilde{R}_t$ is a residual; equivalently, $d\widetilde{R}_t$ can be viewed as the excess return of a fictitious asset that is orthogonal to all $N$ risky asset excess returns.[3] Now, choosing an SDF that prices the $N$ existing risky assets no longer implies that the zero-NPV equation (3) holds. The sign and magnitude of the NPV will depend on the choice of SDF, since the usual no-arbitrage replication argument cannot provide a unique and unambiguous way of pricing the unreplicable return component $d\widetilde{R}_t$.[4]

[3] To obtain (4), regress $\frac{dA_t}{A_t} - r_t dt$ on a constant and the excess return vector $dR_t - r_t 1_N$:

$$\frac{dA_t}{A_t} - r_t dt = a dt + \phi_t' \left(dR_t - r_t 1_N dt\right) + d\eta_t,$$

and define $d\widetilde{R}_t \equiv d\eta_t + a dt$. Note that the residual $d\eta_t$ is orthogonal to the excess returns on all $N$ risky assets.

[4] For example, there could be one or more PE-specific (risk) factors in the unspanned component $d\widetilde{R}_t$. But, even if $d\widetilde{R}_t$ were independent across PE funds, the usual APT-like asymptotic arbitrage argument that a well-diversified PE portfolio would be spanned by the $N$ risky factors is unlikely to hold. Unlike in publicly traded securities, diversification in PE is difficult and expensive. Funds are not traded publicly and the market in LP stakes has low liquidity and high transaction costs (e.g., Nadauld et al. (2019)). Moreover, GPs require minimum commitments, and negotiations between GPs and LPs are time-consuming. Indeed, Gredil et al. (2023) document that most LPs invest in only one or two PE funds per year. The low correlation of returns of alternative-asset positions across investors, which we document below, further strengthens this argument: if it was simple to create a well-diversified portfolio, these returns should be highly correlated, as investors should only retain the factor risk(s), but not idiosyncratic risk.

## 1.2 Investor Portfolio Equivalent

To account for the fact that in incomplete markets different investors may value the unspanned return components differently, we propose a relatively simple modification to the NPV criterion that incorporates investor-specific SDFs. To fix ideas, and in order to relate our performance measures to the ones conventionally used in the PE literature, we first consider an investor who is interested in maximizing the expected logarithmic growth rate of their investments:

$$V \equiv E_{t_0} \log W_T - \log W_{t_0}, \tag{5}$$

where $W_t$ is the value of their portfolio at time $t$, and $T > t_K$ is some distant time. Absent PE, the investor maximizes (5) over the $N \times 1$ vector of portfolio weights in the existing risky assets, $w_t \in \mathcal{W}$, with the weight on the risk-free asset being $1 - 1_N' w_t$. The set $\mathcal{W}$ captures any constraints placed on the investor's portfolio (position limits, shorting constraints, borrowing constraints, etc.).

We next present the investor with the opportunity to invest a small amount $\varepsilon > 0$ in the PE cash flow sequence $C$, and ask whether this investment improves her objective function. The following proposition is an implication of the envelope theorem (all proofs are in appendix A).

**Proposition 1** *Suppose that an investor maximizes (5) over $w_t \in \mathcal{W}$ . Let $\varepsilon C$ denote an investment of $\varepsilon > 0$ in the cash flow process $C$, and define the Investor Portfolio Equivalent (IPE)*

$$IPE \equiv E_{t_0} \sum_{k=0}^{K} \left( \frac{W_{t_0}}{W_{t_k}} \right) C_{t_k}, \tag{6}$$

*where $\frac{W_{t_0}}{W_{t_k}}$ is the inverse of the cumulative return on the investor's portfolio. Then $\frac{dV}{d\varepsilon} = V_W \left(W_{t_0}\right) \times IPE$, where $V_W \left(W_{t_0}\right)$ is the marginal value of wealth at time $t_0$.*

The IPE resembles the PME as defined by Korteweg and Nagel (2016), except that the return on the investor's portfolio is used in place of the stock market return. Thus, instead of a "one-size-fits-all" pricing assumption, the IPE uses each investor's own investment return to price the unspanned return components. Its theoretical advantage is its direct connection to the investor's objective: Per Proposition 1, the change in the investor's objective for an $\varepsilon$ commitment equals the IPE times the investor's marginal value of wealth at time $t_0$, $V_W \left(W_{t_0}\right)$. This means that the investor is indifferent between committing to the cash flow sequence $C$ or receiving a lump sum of $IPE$ dollars at time $t_0$. As such, the investor finds it worthwhile to commit to the fund if and only if $IPE > 0$.

**Remark 1** *Under the assumptions that (a) the investor's objective function* (5) *is correctly specified and (b) portfolio constraints are not binding, $H_t = W_t^{-1}$ is also a valid stochastic discount factor for all assets that comprise the investor's portfolio.*[5] *Per the result of the previous section, this implies that if the cash flow sequence $C$ can be replicated with a trading strategy using the assets that the investor is already invested in, then the IPE will be zero and $C$ does not present a meaningful expansion of the investor's investment opportunity set.*

It turns out that, even if portfolio constraints prevent investors (endowed with the objective function (5)) from maximizing (5), IPE can still be a useful metric:

**Corollary 1** *Suppose that the investor does not maximize (5) but instead assume that the addition of the cash flow $C$ does not change the investor's portfolio $w_t$. Then the conclusion of Proposition 1 remains valid, that is, $\frac{dV}{d\varepsilon} = V_W(t_0, W_{t_0}) \times IPE$.*

Thus, as long as the introduction of $C$ does not change the portfolio $w_t$, then a positive IPE implies that a commitment to $C$ increases the expected logarithmic growth rate of the investor's wealth.

### 1.3 Generalized IPE

In this section we generalize the IPE by considering a power utility investor with a constant relative risk aversion (CRRA) coefficient $\gamma > 0$, whose objective is to maximize

$$V^{(\gamma)} = E_{t_0} \frac{(W_T)^{1-\gamma}}{1-\gamma}. \tag{7}$$

The log-wealth objective (5) in the previous section is a special case when $\gamma = 1$.

A straightforward adaptation of Proposition (1) yields a similar result, namely a generalized IPE (GIPE) metric that represents the marginal improvement in wealth from investing in PE:

[5] The fact that the return of the expected-logarithmic-growth-maximizing portfolio is a valid SDF is a well-known result (see Long (1990) and papers cited therein).

**Proposition 2** *Consider the same assumptions as in Proposition 1, except that the investor maximizes (7). Also assume constant* $\mu_t = \mu, \sigma_t = \sigma, r_t = r$*, and define*

$$GIPE \equiv E_{t_0} \sum_{k=0}^{K} e^{-r(t_k - t_0)} \frac{\left(\frac{W_{t_k}}{W_{t_0}}\right)^{-\gamma}}{E_0\left(\frac{W_{t_k}}{W_{t_0}}\right)^{-\gamma}} C_{t_k}. \tag{8}$$

*Then* $\frac{dV^{(\gamma)}}{d\varepsilon} = GIPE.$

Appendix A includes a generalization of Proposition 2 to time-varying $\mu_t$, $\sigma_t$, and $r_t$, that shows that, under certain conditions, GIPE and $\frac{dV^{(\gamma)}}{d\varepsilon}$ have the same sign, so that GIPE is the correct metric to determine whether the investment in $C$ should be undertaken.

As before, under the additional assumption that the investor's portfolio choice is unconstrained, $H_t = e^{-rt} \frac{(W_t)^{-\gamma}}{E_0(W_t)^{-\gamma}}$ is a valid SDF for the assets in the investor's portfolio. Therefore, if $C$ can be replicated by some self-financing trading strategy in these assets, then GIPE equals zero.

The GIPE is closely related to the familiar beta-based approach to performance evaluation:

**Proposition 3** *Assume that the investor maximizes (7) and that no portfolio constraint is binding. Suppose the return process* $\frac{dA_t}{A_t} = \mu_A dt + \sigma_A dB_t$ *is observed, where* $\mu_A$ *is the expected return of* $\frac{dA_t}{A_t}$ *and* $\sigma_A$ *is a row vector of exposures to the* $d-$*dimensional Brownian motion of equation* (1)*. Let* $\beta_t$ *denote the regression coefficient from regressing* $\frac{dA_t}{A_t}$ *on* $\frac{dW_t}{W_t}$*, and let* $\mu^W - r \equiv w' (\mu - r)$ *denote the expected excess return on the investor's portfolio. Then the GIPE is zero if*

$$\mu_A - r = 0 + \beta_t \left(\mu^W - r\right). \tag{9}$$

Equation (9) is in the form of a familiar "Jensen's alpha" regression. The left-hand side is the expected excess return of investing in the asset with return process $A_t$. The right-hand side is the sum of a "Jensen's alpha" of zero and a "risk compensation component" which is the product of the expected excess return on the investor's overall portfolio times the "beta" from a regression of the returns of $A_t$ on the investor's overall portfolio return. Equation (9) shows that a zero GIPE is essentially the same as a zero "Jensen's alpha" in a regression of $\frac{dA_t}{A_t}$ on the investor's portfolio return. The advantage of the GIPE is that it does not require $A_t$ to be observed. Instead, it can be computed from the cash flow sequence $C$ alone.

Viewing the GIPE through the lens of equation (9) implies that an investor's level of risk aversion does not affect the sign of GIPE, as long as the investor faces no binding portfolio constraints.[6] That is, risk aversion does not determine whether a PE investment is attractive in the first place. (It can, however, change *how* attractive it is. Put differently, $\gamma$ changes the magnitude of GIPE, but not its sign.) It is also evident from equation (9) that the sign and magnitude of GIPE depends on the investor's portfolio composition, and therefore the sign and magnitude of the GIPE of a given private equity investment can differ across investors . For instance, adding $C$ to the portfolio of an investor who is already heavily invested in private equity will (in most practical situations) result in a higher required risk compensation, $\beta_t \left(\mu^W - r\right)$, compared to an investor who has no alternative investments; therefore the GIPE may be negative for the first investor and positive for the second. This is a "feature, not a bug": Intuitively, the same risk is more "diversifiable" in the second investor's portfolio.

## 1.4 Portfolio constraints and mimicking funds

We have shown that the sign of any cash flow's (G)IPE corresponds to the directional change in the investor's objective function resulting from a marginal investment in that cash flow process. If investment constraints are nonbinding (i.e., if optimal portfolio weights are an interior solution), then the (G)IPE of the *existing* assets in the investor's portfolio is zero. In practice, constraints may bind because of restrictions on leverage and mandates to invest, or refrain from investing, in certain asset classes. Such restrictions are commonplace, especially for pension plans. In fact, this is one possible reason why portfolio allocations differ across investors.

A consequence of binding portfolio constraints is that even the (G)IPE of the cash flows produced by some publicly tradeable assets could be nonzero. It is then possible that the investor is better off allocating a marginal dollar to these assets (by loosening constraints) than to the PE fund, even if the latter's (G)IPE is positive. As a corollary, the (G)IPE of a PE fund could be

[6] To see this, note that neither side of equation (9) depends on $\gamma$. The left-hand side follows by inspection. For the right-hand side, note that an investor's optimal portfolio is proportional to $\frac{1}{\gamma}$,

$$w = \frac{1}{\gamma} \left(\sigma\sigma'\right)^{-1} (\mu - r) .$$

Accordingly,

$$\beta_t \left(\mu^W - r\right) = \frac{(w'\sigma\sigma_A') \left(w' (\mu - r)\right)}{w'\sigma\sigma' w}$$

is independent of $\gamma$.

positive even if $C$ is replicable by some portfolio of existing assets. In effect, the PE fund could provide a "backdoor" way to increase exposure to risk factors that the investor cannot obtain directly, similar to the argument that certain financial instruments (such as options and leveraged ETFs) may alleviate borrowing constraints for investors (see Frazzini and Pedersen, 2022).

It is therefore useful to know whether a positive (G)IPE is due to the attractiveness of PE's unreplicable return component, or whether it reflects an investor's portfolio constraints. To this end, we construct a cash-flow process, $\widehat{C}$, which mimics the cash-flows of a PE fund by using existing assets, with cash flows occurring at the same times $t_k$ as $C$. Specifically, capital calls are equal to those of $C$,:

$$\widehat{C}_{t_k} \equiv C_{t_k}, \text{ for } C_{t_k} < 0. \tag{10}$$

The capital calls are invested in a benchmark portfolio of the existing assets $1, ..., N$. The benchmark portfolio earns an (instantaneous) return $dR_t^b$. We define $G_t$ to be the cumulative return on investing a dollar in the benchmark, so that $G_{t_0} = 1$, $\frac{dG_t}{G_t} = dR_t^b$.

Whenever the PE fund has a distribution, the mimicking fund distributes[7]

$$\widehat{C}_{t_k} = \widehat{V}_0 \omega_{t_k} G_{t_k}, \tag{11}$$

where

$$\widehat{V}_0 \equiv \sum_{k=0..K} \frac{|C_{t_k}|}{G_{t_k}} 1_{\{C_{t_k}<0\}} \qquad\qquad \omega_{t_k} \equiv \frac{\frac{C_{t_k} 1_{\{C_{t_k}>0\}}}{G_{t_k}}}{\sum_{k=0..K} \frac{C_{t_k} 1_{\{C_{t_k}>0\}}}{G_{t_k}}}.$$

The quantity $\widehat{V}_0$ is the present value of all capital commitments, and the ratio $\omega_{t_k}$ is the present value of the distribution at time $t_k$ as a fraction of the present value of all distributions, with the benchmark return serving as the discount rate.

The mimicking fund has desirable characteristics, as established in the following proposition. The most important one is that $C$ and $\widehat{C}$ coincide when the return on the PE fund's net asset position, $\frac{dA_t}{A_t}$, is the same as the return on the benchmark portfolio.

[7] The construction of $\widehat{C}$ is similar to the modified PME proposed by Cambridge Associates. Both approaches consider a mimicking fund that invests the capital calls in the benchmark portfolio. In both approaches the distributions are positive. But our approach enforces that under the null hypothesis (12) the sequences $C$ and $\widehat{C}$ coincide. Moreover, our approach does not rely on GP-provided NAVs, which may be problematic, as discussed above.

**Proposition 4** *a) The distributions (11) are non-negative. b) The cash flows $\widehat{C}_{t_k}$ given by (10) and (11) can be financed by the trading strategy of investing the capital calls at the benchmark return $R_t^b$ and making distributions equal to (11). c) Under the null hypothesis that the net asset position associated with the cash flow $C$ grows at the benchmark return,*

$$\frac{dA_t}{A_t} = dR_t^b, \tag{12}$$

*the cash flows of the mimicking fund $\widehat{C}$ are identically equal to the cash flows of the fund $C$.*

The mimicking fund $\widehat{C}$ allows us to perform an exercise similar in spirit to the popular "style" analysis that Sharpe (1988, 1992) introduced for mutual funds. Specifically, we choose a benchmark portfolio, construct the mimicking cash-flow sequence $\widehat{C}$, and examine its (G)IPE, as well as that of the differential cash flows $C - \widehat{C}$.[8] As a practical matter, we are particularly interested whether the PE cash flows can be replicated by simple, long-only strategies in publicly-traded benchmark portfolios (e.g., a value portfolio return could be used as a benchmark for buyout funds, or a small-growth portfolio return could be used as a benchmark for venture capital funds).

Table 1 summarizes the interpretation of the (G)IPE of $\widehat{C}$ and that of $C - \widehat{C}$. If the (G)IPE of $\widehat{C}$ is zero, then the replicable component of PE cash flows does not improve the investor's objective function, suggesting that portfolio constraints are not binding. In this case, the (G)IPE of $C$ is equal to the (G)IPE of $C - \widehat{C}$, and its sign indicates whether the non-replicable component of the PE fund improves the investor's objective function. If instead the (G)IPE of $\widehat{C}$ is positive (resp. negative), then even a marginal increase (decrease) in the allocation to the publicly-traded benchmark portfolio would improve the investor's objective, indicating a portfolio mis-allocation to publictly traded assets. In those situations, the (G)IPE of $C - \widehat{C}$ reveals whether the non-replicable component of the PE cash-flows raises the investor's objective function relative to investing in (or shorting) the benchmark portfolio.

**Remark 2** *One other (not mutually exclusive) reason why the (G)IPE of the mimicking fund $\widehat{C}$ could deviate from zero is market timing ability of the PE manager in the benchmark asset. For example, suppose that the manager simply invests in the public stock market index. Suppose also*

[8] Since (G)IPE is linear in cash flows, the (G)IPE of $C - \widehat{C}$ is equal to the (G)IPE of $C$ minus that of $\widehat{C}$.

Table 1: Interpretation of Performance with Private Equity-mimicking Funds

The first column of this table explains the interpretation of the (G)IPE of the cash flows of PE-mimicking funds, $\widehat{C}$. These funds have identical capital calls to the PE funds, invest in a chosen benchmark portfolio, and distribute cash at the same times as the PE funds (see the main text for a complete description of their construction). The second column describes the interpretation of the cash flow component of the PE funds that is not replicable by investing in the benchmark, i.e., the difference between the PE fund cash flows, $C$, and the mimicking flows $\widehat{C}$. The columns are to be read independently, for example, $\widehat{C}$ could have a (G)IPE of zero while $C - \widehat{C}$ has a positive (G)IPE.

| (G)IPE of | $\widehat{C}$ | $C - \widehat{C}$ |
|---|---|---|
| $< 0$ | Portfolio constraints prevent reducing/shorting exposure to existing (non-PE) assets and/or the PE manager has negative timing ability. | The non-replicable component of the PE payoff has negative value for the investor. |
| $= 0$ | The replicable component of PE neither adds nor destroys value for the investor. | The non-replicable component of the PE payoff neither raises nor lowers the investor's objective function. |
| $> 0$ | PE relaxes binding (long) portfolio constraints and/or the PE manager has positive timing ability. | The non-replicable component of the PE payoff has positive value for the investor. |

*that the expected return of the public stock market, $\mu_t$, can have one of two values, $\mu^H > \mu^L$, that switch according to some regime-switching process. The manager has the ability to predict regime switches, which allows her to collect a capital call at the beginning of regime $H$, invest it in the stock market, and liquidate it immediately before the regime is about to switch to L. By contrast, the investor (LP) does not know which regime the economy is in and holds a constant portfolio in the stock market. In this example, the cash flow stream $C$ has a positive GIPE but the GIPE of $C - \widehat{C}$ is zero, since assumption (12) holds by construction. This example shows that the (G)IPE of $C - \widehat{C}$ is tailored to identify whether the non-replicable component of the PE investment is valuable for the investor above and beyond any timing ability the manager may posses.*

# 2 Data

We use data on pension plans and private equity funds to compute (G)IPE metrics. We describe each data source in turn.

## 2.1 Pension plans

We use pension plan data collected from Comprehensive Annual Financial Reports (CAFRs) of U.S. defined benefit public pension plans. These reports contain balance sheet and income statement information, returns, valuations, actuarial data, and other key statistics, audited to conform to Government Accounting Standards Board (GASB) reporting requirements.

We start with a CAFR data set developed and maintained by a collaboration of the Center for Retirement Research at Boston College, the Center for State and Local Government Excellence, and the National Association of State Retirement Administrators.[9] The data include 179 state and local pension plans for the years 2001 to 2018, and cover 95% of U.S. public pension membership and assets. We extend coverage by hand-collecting additional CAFRs going back to 1995, when most start to be available online.

We consolidate pension plans whose assets are jointly owned and managed, since their reported returns are virtually identical. For example, Maine's local employee plan that covers participating districts and its state employee and teacher plan are both managed by the Maine Public Employees' Retirement System (PERS), even though they are treated separately for accounting purposes and publish separate CAFRs. Maine PERS is also the investor listed in the private equity fund commitment data described below. After consolidation, this leaves 142 plans.

Next, we drop four pension plans because they lack a return observation for one or more years. We need a continuous time series of returns to discount private equity fund cash flows and calculate the (G)IPE performance measures. This leaves us with a final sample of 138 pension plans.

Panel A of Table 2 reports descriptive statistics of the pension plans. The first four columns show characteristics across all 138 plans. The most common plan covers state and/or local employees (96 plans, or 70% of the sample), followed by teachers (41 plans, 30%) and police or fire personnel (29 plans, 21%). Note that these categories are not mutually exclusive due to the merging of some jointly managed plans that cover multiple employee types. Most plans are administered at the state level (78 plans, 57%), with the remainder administered locally (either by a county, city, or school district). About two thirds cover multiple employers, the vast majority with a cost-sharing agreement that pools the employers' pension assets and obligations.

The size distribution of pension plans is highly skewed. Across all plan-years, the median assets under management (AUM) is $7.81 billion. The average, $20.45 billion, is pulled upwards by a few very large plans, such as CalPERS, which had $354 billion in AUM in 2018. Most plans are underfunded on an actuarial basis, with the average (median) funding rate across plan-years equal

[9] This data set is freely downloadable from: `https://publicplansdata.org`.

## Table 2: Descriptive Statistics

Panel A shows descriptive statistics for public pension plans between 1995 and 2018. The first four columns show statistics for the full sample of 138 pension plans, the next four columns show the statistics for the sample of 103 plans for which we observe at least one private equity (PE) fund commitment in the Preqin data, and the column labeled "Diff. test $p$" shows the $p$-value for the test of equal proportions or means across the 103-plan subsample with commitment data and the 35-plan subsample without commitment data (for panel variables, *Assets*, *Funded ratio*, and *Annual return*, the tests include year fixed effects and double-cluster residuals by year and pension plan state). Numbers in the "%" column refer to the percentage of plans that belong to the category described in that row. The covered employee categories (*State/local employees*, *Teachers*, *Police/fire*) are not mutually exclusive. *State-administered plan* refers to a plan administered at the state level (rather than at the county, city, or school level). A *Multiple employer plan* covers employees for more than one employer, and *Cost sharing across employers* is the percentage of multiple employer plans in which employers are jointly responsible for each others' pension liabilities. *Assets* is the market value of pension assets in billions of U.S. dollars. *Funded ratio* is the ratio of actuarial assets to liabilities under GASB 25 accounting standards. *Annual return* is the one-year return on the pension plan (in %). Statistics for these three panel variables are calculated across all plan-years. *PE fund commitments per plan* is the plan's number of observed commitments to PE funds in the Preqin data over the sample. Panel B shows descriptive statistics for 1,303 North American private equity funds raised between 1995 and 2013, by strategy (venture capital (VC), buyout, and real estate). *Number of GPs* is the number of unique fund managers. *Fund size* is the fund's committed capital in millions of U.S. dollars. *Percentage of funds liquidated* is the percentage of funds that are fully liquidated (*100% liquidated*) or that have less than 5% of committed capital left in residual net asset value (*95% liquidated*). *Fund effective years* is the number of years between the first and the last observed cash flow for the fund, where cash flows are observed until the end of June 2018. *IRR* is the internal rate of return, *TVPI* the total value to paid-in capital, and *PME(KS)* and *PME(KN)* are the Public Market Equivalent of Kaplan and Schoar (2005) and Korteweg and Nagel (2016). *Size-weighted* is the NAV-weighted average performance metric. *Funds with matched LP data* is the number of funds for which we observe at least one public pension plan commitment in our data set. *Number of matched LPs / fund* is the number of observed pension plans commitments in a given fund in our data.

*Panel A: Pension plans.*

| | Full Sample (N = 139) | | | | Commitment Sample (N = 104) | | | | Diff. |
|---|---|---|---|---|---|---|---|---|---|
| | % | Mean | Median | St.Dev. | % | Mean | Median | St.Dev. | Test $p$ |
| Plans covering | | | | | | | | | |
| State/local employees | 69.57 | | | | 69.90 | | | | 0.882 |
| Teachers | 29.71 | | | | 29.13 | | | | 0.797 |
| Police/fire | 21.01 | | | | 18.45 | | | | 0.204 |
| State-administered plan | 56.52 | | | | 61.17 | | | | 0.059 |
| Inception year | | 1943.79 | 1944 | 18.35 | | 1942.70 | 1942 | 18.29 | 0.223 |
| Multiple employer plan, of which: | 64.49 | | | | 68.93 | | | | 0.062 |
| Cost sharing across employers | 93.26 | | | | 91.55 | | | | 0.133 |
| Assets (market value, $b) | | 19.91 | 7.56 | 33.51 | | 18.87 | 8.31 | 32.16 | 0.462 |
| Funded ratio (actuarial) | | 0.81 | 0.82 | 0.18 | | 0.80 | 0.81 | 0.18 | 0.062 |
| Annual return (%) | | 8.15 | 10.64 | 10.51 | | 8.17 | 10.80 | 10.38 | 0.194 |
| PE fund commitments per plan | | | | | | 49.88 | 26.00 | 59.5 | N/A |

*Panel B: Private equity funds.*

| | VC funds (N = 527) | | | Buyout funds (N = 527) | | | Real Estate funds (N = 249) | | |
|---|---|---|---|---|---|---|---|---|---|
| | Mean | Median | St.Dev. | Mean | Median | St.Dev. | Mean | Median | St.Dev. |
| Number of GPs | 261 | | | 256 | | | 134 | | |
| Funds per GP | 2.02 | 2.00 | 1.37 | 2.06 | 2.00 | 1.22 | 1.86 | 1.00 | 1.33 |
| Fund size ($m) | 348.19 | 250.00 | 356.60 | 1,588.85 | 700.00 | 2,634.58 | 929.05 | 535.00 | 1,429.91 |
| Percentage of funds liquidated: | | | | | | | | | |
| 100% liquidated | 36.81 | | | 34.72 | | | 27.71 | | |
| 95% liquidated | 43.64 | | | 42.88 | | | 36.14 | | |
| Fund effective years | 12.22 | 12.26 | 4.30 | 11.13 | 11.13 | 4.14 | 8.33 | 7.51 | 3.55 |
| IRR (%) | 8.27 | 4.48 | 36.74 | 13.66 | 13.39 | 14.89 | 12.46 | 12.26 | 11.55 |
| Size-weighted | 7.36 | | 28.06 | 14.58 | | 11.46 | 12.80 | | 12.57 |
| TVPI | 1.54 | 1.22 | 1.85 | 1.67 | 1.63 | 0.65 | 1.45 | 1.44 | 0.40 |
| Size-weighted | 1.45 | | 1.37 | 1.69 | | 0.47 | 1.46 | | 0.44 |
| PME(KS) | 1.00 | 0.79 | 1.22 | 1.18 | 1.13 | 0.49 | 0.98 | 1.01 | 0.31 |
| Size-weighted | 0.94 | | 0.92 | 1.17 | | 0.37 | 0.99 | | 0.33 |
| PME(KN) | -0.03 | -0.17 | 1.01 | 0.16 | 0.11 | 0.42 | -0.03 | 0.00 | 0.30 |
| Size-weighted | -0.07 | | 0.78 | 0.15 | | 0.34 | -0.02 | | 0.33 |
| Funds with matched LP data | 465 | | | 488 | | | 211 | | |
| Number of matched LPs / fund | 3.43 | 2.00 | 2.90 | 5.90 | 4.00 | 5.88 | 4.45 | 3.00 | 4.25 |

to 79% (80%).

The average (median) annual reported return across plan-years is 8.11% (10.60%), with a standard deviation of 10.54%. Most of the variance is coming from the time-series, but there is also an economically meaningful degree of cross-sectional dispersion. This can be seen in Panel A of Figure 2, which shows the time series of the average return across plans, as well as the 10th and 90th percentiles of plan returns.

For a subset of plans we have data on their private equity fund commitments (described in detail in the next section). Panel A of Table 2 shows the descriptive statistics for the 103 pension plans that we can match to at least one investment in a PE fund (we call this the "commitment sample"). The commitment sample is statistically indistinguishable from the subsample of 35 plans without commitment data on most dimensions, as shown by the $p$-values of the difference in proportions and means tests in the final column of Panel A. The exceptions are that the commitment sample has a higher proportion of state-administered and multiple-employer plans, and its mean annual return is different (all three are significant at the 10% level). The latter result appears surprising, given that Table 2 shows the same average return for the full sample and the commitment sample, when computed across all plan-years. The explanation is that the test controls for year fixed effects, revealing a difference in mean returns when measured *within the same year*.[10] However, the difference is economically small: the commitment sample has only a 30 basis point lower average return (regression not reported for brevity). The small magnitude of the difference is confirmed visually in Panel B of Figure 2, which plots the average return for both the commitment sample and the subsample of plans without commitment data.

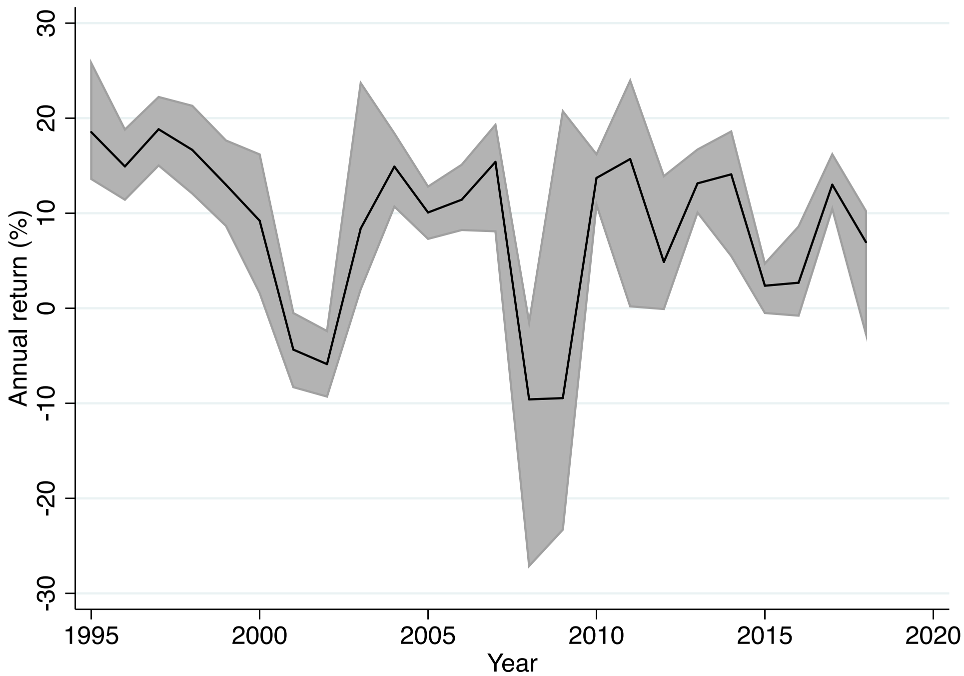
Over the period from 1995 to 2013, the commitment sample plans made an average (median) of 52.51 (27) commitments to PE funds in our data set (we drop PE funds with vintages post-2013 for reasons explained below). Similar to the size distribution, the number of commitments is highly skewed with a long right tail.

[10]We run the same test (with year fixed effects) for all panel variables in Table 2: AUM, funded ratio, and annual returns. All other variables are measured only in the cross-section.

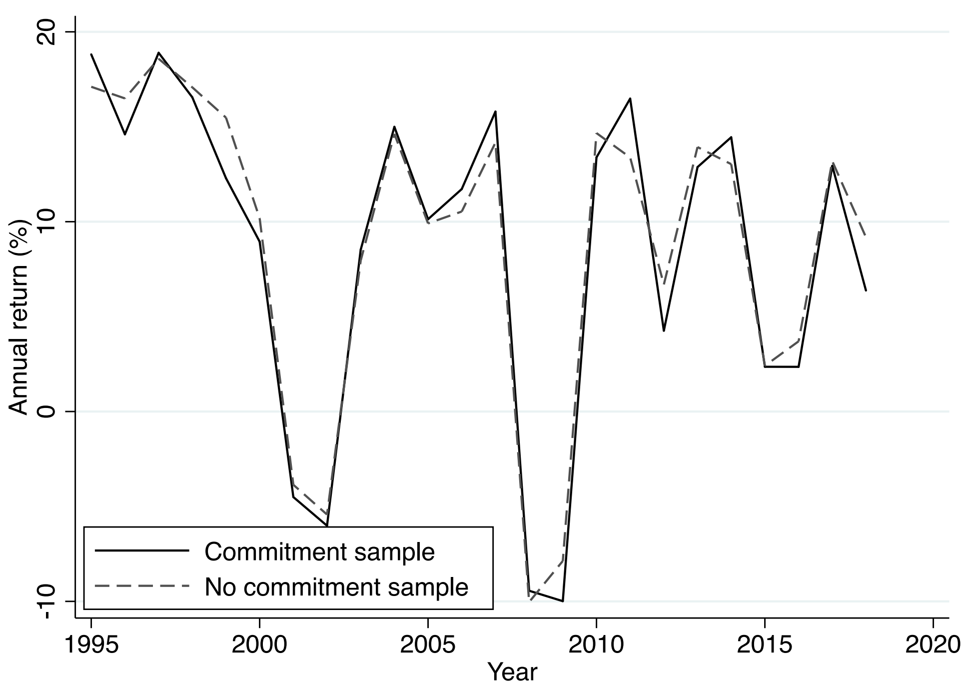
Figure 2: Pension Plan Returns

Panel A shows the time series of the average one-year return across pension plans, for the full sample of 138 plans described in Table 2. The shaded area represents the region between the 10th and the 90th percentile of plan returns. Panel B graphs the time series of average one-year plan returns for the subsample of 103 plans with at least one observed commitment to a PE fund (the solid line; this is the "commitment sample" described in Table 2) and the subsample of 35 plans without observed commitment data (the dashed line, labeled "No commitment sample").

*Panel A: Pension plan returns for the full sample.*




*Panel B: Average pension plan returns for subsamples with and without PE fund commitment data.*



## 2.2 Private equity funds

Our private equity data is sourced from Preqin, and contains fund capital calls and distributions net of fees paid to the GPs, as well as quarterly net asset values (NAVs) for a large number of PE funds. Following the literature, we limit the sample to North American funds with at least $5 million in committed capital. We focus on the three main strategies in PE; venture capital (VC), buyout, and real estate. Given the data limitations for pension plans described above, we only include funds raised since 1995, and we use all cash flows until the end of June 2018.Our final filter drops fund vintages after 2013, so we observe at least 5 years of cash flow data for each fund. The final sample contains 1,303 funds.

Panel B of Table 2 reports descriptive statistics by strategy. We observe 527 VC funds managed by 261 unique GPs, with the median GP raising two funds during 1995 to 2013 period. The median VC fund has $250 million in committed capital, whereas the average is higher at $348 million due to a few very large funds. For buyout, the number of funds and GPs are similar to VC (527 funds by 256 GPs), but funds are substantially larger, at an average (median) size of $1,589 million ($700 million). There are fewer real estate funds (249 funds by 134 GPs, with the median GP raising just one fund). The average (median) real estate fund size of $929 million ($535 million) is between VC and buyout fund sizes.

For VC and buyout, just over a third of funds have been fully liquidated by the end of the sample period (June 2018). The liquidation rate increases to roughly 43% for both strategies if we include funds that have less than 5% of committed capital in remaining NAV, as it is not uncommon for funds to be extended after their original 10-year life if any un-exited portfolio companies remain. As Table 2 shows, the time between the first and final observed cash flow for the median VC (buyout) fund is 12.3 (11.1) years. The proportion of liquidated real estate funds is lower, with only 28% fully liquidated, and the time between first and final cash flow is shorter (7.5 years for the median fund), in large part because PE real estate is a younger strategy with a higher proportion of funds raised in more recent times.

With respect to performance, we compute the standard metrics in the literature; total value to paid-in capital (TVPI), which is a cash multiple of total fund distributions to date divided by total capital calls, internal rate of return (IRR), and public market equivalent (PME). We compute two versions of the PME. The first is the Kaplan and Schoar (2005) defined as the sum of fund distributions discounted to fund inception at the public equity market rate of return, divided by the similarly discounted sum of capital calls. The second version is defined by Korteweg and Nagel (2016) as the sum (not the ratio) of discounted net cash flows (distributions minus capital calls, normalized by the fund's committed capital). Interpreting the PME as a benchmark against the public equity market, a PE fund has outperformed public equities if the Kaplan-Schoar (Korteweg-Nagel) PME is above one (zero). For funds that are not yet liquidated by the end of June 2018, we follow standard practice and include their final reported net asset value (NAV) as a pseudo-distribution in all return metrics. Panel B shows that VC funds had the worst performance during the sample period by most measures, and buyout funds experienced the best performance. VC also has by far the highest variance in fund outcomes (and real estate is the least variable), and its performance is the most skewed, as indicated by the difference between mean and median metrics.

Finally, our Preqin data includes commitments to PE funds by LPs, which we match to our pension plan data. For roughly 9 out of 10 PE funds we observe at least one investment by a pension plan in our data, depending on the strategy (465 out of 527 VC funds, 488 out of 527 buyout funds, and 211 out of 249 real estate funds). For the median VC fund we see 2 pension plan investments, 4 for the median buyout fund, and 3 for the median real estate fund. The averages are higher (3.43, 5.90, and 4.45, respectively) due to a number of large funds for which we see many commitments.

# 3 Private equity fund performance

## 3.1 IPE

We start our examination of PE fund performance by computing the IPE for each private equity fund and pension plan combination in our data, regardless of whether these combinations represent actual investments. Specifically, following equation (6), $IPE_{i,j}$ is the sum of the net cash flows (distributions minus capital calls) of private equity fund $i$, discounted to fund inception by the compounded cumulative return of pension plan $j$. For non-liquidated funds we include the final reported NAV at the end of the sample period as a pseudo-distribution, similarly discounted. Since pension plan returns are only available at the annual frequency, we discount a cash flow on day $d$

of year $\tau$ by the factor $\left(\prod_{t=1..\tau-1} R_{j,t}\right)^{-1} e^{-(\log R_{j,\tau})\frac{d}{365}}$, where $R_{j,t}$ is plan $j$'s return in year $t$.

The first row of Table 3 Panel A shows that the average IPE is 0.128, which is statistically significant at the 1% level. We double-cluster standard errors by pension plan and vintage year to account for the fact that each fund and each plan may be represented in multiple observations (since a given pension plan can price multiple funds, and a given fund can be evaluated by multiple plans), and to allow for other sources of cross-correlation within vintage year, such as correlated idiosyncratic shocks. The positive IPE implies that a larger allocation to an average PE fund would have increased the logarithmic growth rate of assets of a typical pension plan over the sample period. With the additional assumption of log-utility, an IPE of 0.128 represents a net present value of 12.8 cents generated over the fund's lifetime on a marginal one dollar commitment, for the average pairing of a pension plan and PE fund.

The average IPE is higher than the average Korteweg and Nagel (2016) PME of 0.036. The 0.091 difference between these two averages, which is statistically significant at the 1% level, is solely due to the difference in SDFs. Whereas PME assesses the potential added value from adding PE exposure to a portfolio that is invested exclusively in public equities, IPE considers the added value to an investor's actual portfolio, which, for pension plans, includes not only public equities but also fixed income and other asset classes (including existing PE investments). In comparing the Korteweg-Nagel PME and IPE it is important to remember that the difference in these two metrics does not merely reflect the difference in expected returns, but also the *riskiness* of PE as perceived by investors with different portfolios. To see this, consider the value of a cash flow at time $t_k$,

$$E_{t_0}\left(\frac{H_{t_k}}{H_{t_0}} C_{t_k}\right) = E_{t_0}\left(\frac{H_{t_k}}{H_{t_0}}\right) E_{t_0}\left(C_{t_k}\right) + Cov_{t_0}\left(\frac{H_{t_k}}{H_{t_0}}, C_{t_k}\right). \tag{13}$$

As discussed above, for a log-utility investor $H_{t_k}/H_{t_0}$, is proportional to the reciprocal of their return on wealth. Thus, the first term on the right-hand side of equation (13) discounts expected cash flows by the expected return on the wealth portfolio (approximately, as this ignores a Jensen's Inequality term). The second term adjusts for risk. An investor who is fully in invested in, say, Treasuries likely finds an average PE fund attractive because discounting the fund's cash flows at the risk-free rate yields a high value, and the covariance between PE cash flows and Treasuries is probably not very strong. In contrast, an investor who already has a 99% allocation to PE may well find a negative IPE value from adding more PE for two distinct reasons: first, discounting a fund's expected cash flows at the rate of return typically produced by PE investments likely yields a number close to zero making the first term of (13) small , and second, the covariance of PE cash flows and PE returns is very strong (such that the covariance term in (13) is strongly negative). Te combination of these two effects will tend to make the IPE value small, and potentially negative. This is sensible, since an investor who allocates 99% to PE could probably benefit (that is, raise the expected logarithmic growth rate of her portfolio) by diversifying way from PE.

To facilitate comparison with IRR and traditional performance metrics in public equities, Table 3 also shows performance expressed in "alpha", the additional annualized return (not due to a risk premium) that is required to make the $(G)IPE$ equal to zero.[11] The overall IPE alpha across all strategies is 3.5% per year, which, like its IPE counterpart, is significant at the 1% level. The average difference between IRR and alpha of 7.6% can be interpreted as the investment's risk premium, indicating that compensation for risk is a large component of a private equity fund's $IRR$. We employ this measure below, when we consider risk-taking behavior by pension plans.

Figure 3 shows the time series of average IPE across PE fund vintage years. The shaded area represents the 10th-90th percentile range across plan-fund combinations. The top-left plot shows that IPE is high initially (around 1995) but quickly drops to hover around zero from the late 1990s vintages until the mid 2000s, after which it is mildly positive until the end of our sample. The time series of alpha follows the same pattern, and is not shown here for brevity.

With respect to individual PE strategies, Table 3 shows that the average IPE (alpha) for buyout funds is 0.223 (6.2%), which is statistically significant at the 1% level, compared to 0.052 (0.4%) and 0.087 (4.3%) for VC and real estate. The VC metrics and real estate IPE are statistically not significantly different from zero, but the real estate alpha is significant at the 5% level. The time series pattern also varies by PE strategy, as shown in Figure 3. The average buyout IPE is positive throughout the sample period but does not exhibit the high initial IPE seen in overall PE. Those high early numbers are driven by venture capital, which generally follows the overall IPE pattern but in a more exaggerated manner. The average real estate IPE is negative for the mid-to-late 2000s vintages, whose investments span the global financial crisis.

[11] Mathematically, we define $\alpha$ as the number that makes $\sum e^{-\alpha t} H_{j,t} C_{i,t} = 0$ where $H_{j,t}$ is the SDF used in $(G)IPE$. The relation between alpha and $(G)IPE$ is analogous to the relation between direct alpha and PME as described in Gredil et al. (2022).

Table 3: Private Equity Fund Performance Metrics

This table reports performance results for private equity funds. The first column shows the average performance across all possible pairs of pension plans and PE funds. The second column shows the standard error (s.e.) for the test of the null hypothesis that the mean is zero, double-clustered by pension plan and vintage year. The second, third, and fourth sets of two columns shows the average performance and its standard error separated by PE strategy: buyout, venture capital, and real estate funds. Panel A shows IPE-type metrics. The first row is the *IPE* of equation (6). The second row, *PME(KN)*, shows the PME of Korteweg and Nagel (2016), for comparison. The *IPE(market-repl.)* is the IPE of PE-mimicking funds with the same capital calls as the PE fund, and the same timing of distributions, but invested in the CRSP value-weighted market portfolio. Similarly, *IPE(value-repl.)* and *IPE(growth-repl.)* refer to mimicking funds that invest in the top quintile of value stocks, and those in the intersection of the lowest size and book-to-market quintiles. Panel B shows similar results for GIPE-type metrics, where *GIPE* is as defined in equation (21), and *GPME* is the Generalized PME of Korteweg and Nagel (2016). The row labeled $\alpha$ shows the annualized excess return that makes the (G)IPE equal to zero, computed as described in the text. An $\alpha$ of 0.01 represents an excess return of 1% per year. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% level, respectively.

| | All funds | | Buyout | | VC | | Real Estate | |
|---|---|---|---|---|---|---|---|---|
| | Mean | s.e. | Mean | s.e. | Mean | s.e. | Mean | s.e. |
| *Panel A: IPE-type metrics.* | | | | | | | | |
| IPE | 0.128 | 0.036*** | 0.223 | 0.025*** | 0.052 | 0.083 | 0.087 | 0.054 |
| PME (KN) | 0.036 | 0.028 | 0.156 | 0.037*** | -0.049 | 0.068 | -0.035 | 0.053 |
| IPE (mkt-repl.) | 0.087 | 0.025*** | 0.073 | 0.024*** | 0.079 | 0.031** | 0.132 | 0.016*** |
| IPE (value-repl.) | 0.187 | 0.027*** | 0.182 | 0.033*** | 0.211 | 0.03*** | 0.147 | 0.017*** |
| IPE (growth-repl.) | -0.106 | 0.027*** | -0.1 | 0.027*** | -0.127 | 0.032*** | -0.074 | 0.023*** |
| $\alpha$(IPE) | 3.515 | 1.128*** | 6.233 | 0.905*** | 0.398 | 2.111 | 4.269 | 1.76** |
| | | | | | | | | |
| IPE - PME(KN) | 0.091 | 0.024*** | 0.067 | 0.031** | 0.101 | 0.024*** | 0.122 | 0.015*** |
| IPE - IPE(mkt-repl.) | 0.041 | 0.032 | 0.149 | 0.029*** | -0.027 | 0.069 | -0.045 | 0.062 |
| IPE - IPE(value-repl.) | -0.059 | 0.049 | 0.041 | 0.046 | -0.158 | 0.099 | -0.06 | 0.062 |
| IPE - IPE(growth-repl.) | 0.234 | 0.025*** | 0.323 | 0.033*** | 0.18 | 0.063*** | 0.161 | 0.037*** |
| IRR - $\alpha$(IPE) | 7.59 | 0.345*** | 7.868 | 0.33*** | 6.98 | 0.554*** | 8.244 | 0.369*** |
| | | | | | | | | |
| *Panel B: GIPE-type metrics.* | | | | | | | | |
| GIPE | -0.035 | 0.032 | 0.106 | 0.037*** | -0.138 | 0.063** | -0.112 | 0.074 |
| GPME | -0.115 | 0.134 | 0.178 | 0.21 | -0.285 | 0.11*** | -0.366 | 0.148** |
| GIPE(mkt-repl.) | -0.049 | 0.021** | -0.045 | 0.024* | -0.054 | 0.022** | -0.045 | 0.021** |
| GIPE(value-repl.) | 0.066 | 0.057 | 0.067 | 0.053 | 0.11 | 0.082 | -0.023 | 0.024 |
| GIPE(growth-repl.) | -0.241 | 0.041*** | -0.218 | 0.041*** | -0.263 | 0.039*** | -0.24 | 0.051*** |
| $\alpha$(GIPE) | -1.8 | 0.958* | 1.635 | 1.038 | -4.874 | 1.464*** | -2.576 | 1.59 |
| | | | | | | | | |
| GIPE-GPME | 0.08 | 0.127 | -0.072 | 0.185 | 0.147 | 0.104 | 0.254 | 0.085*** |
| GIPE - GIPE (mkt-repl.) | 0.014 | 0.027 | 0.151 | 0.034*** | -0.084 | 0.066 | -0.067 | 0.07 |
| GIPE - GIPE(value-repl.) | -0.101 | 0.063 | 0.039 | 0.046 | -0.248 | 0.128* | -0.09 | 0.07 |
| GIPE - GIPE(growth-repl.) | 0.206 | 0.024*** | 0.324 | 0.047*** | 0.126 | 0.057** | 0.128 | 0.03*** |
| IRR - $\alpha$(GIPE) | 12.913 | 1.229*** | 12.472 | 1.316*** | 12.268 | 1.591*** | 15.089 | 1.266*** |
| | | | | | | | | |
| N | 160499 | | 64611 | | 64010 | | 31878 | |

Figure 3: Time Series of IPE and GIPE

This figure plots the average IPE (left column of graphs) and GIPE (right column of graphs) across pension plan-PE fund pairs, by fund vintage year. The first row depicts data for all funds, the second row for buyout funds only, the third row for venture capital funds and the fourth row for real estate funds. The horizontal axis is the vintage year of the funds. The solid line represents the mean, the shaded area corresponds to the 10th-90th percentile range of the observations. The mean and standard deviation of all observations across all vintages is displayed in the top right corner of each graph. Note that there is no North American real estate fund for the 2001 vintage year in the data, hence the gap for real estate funds.

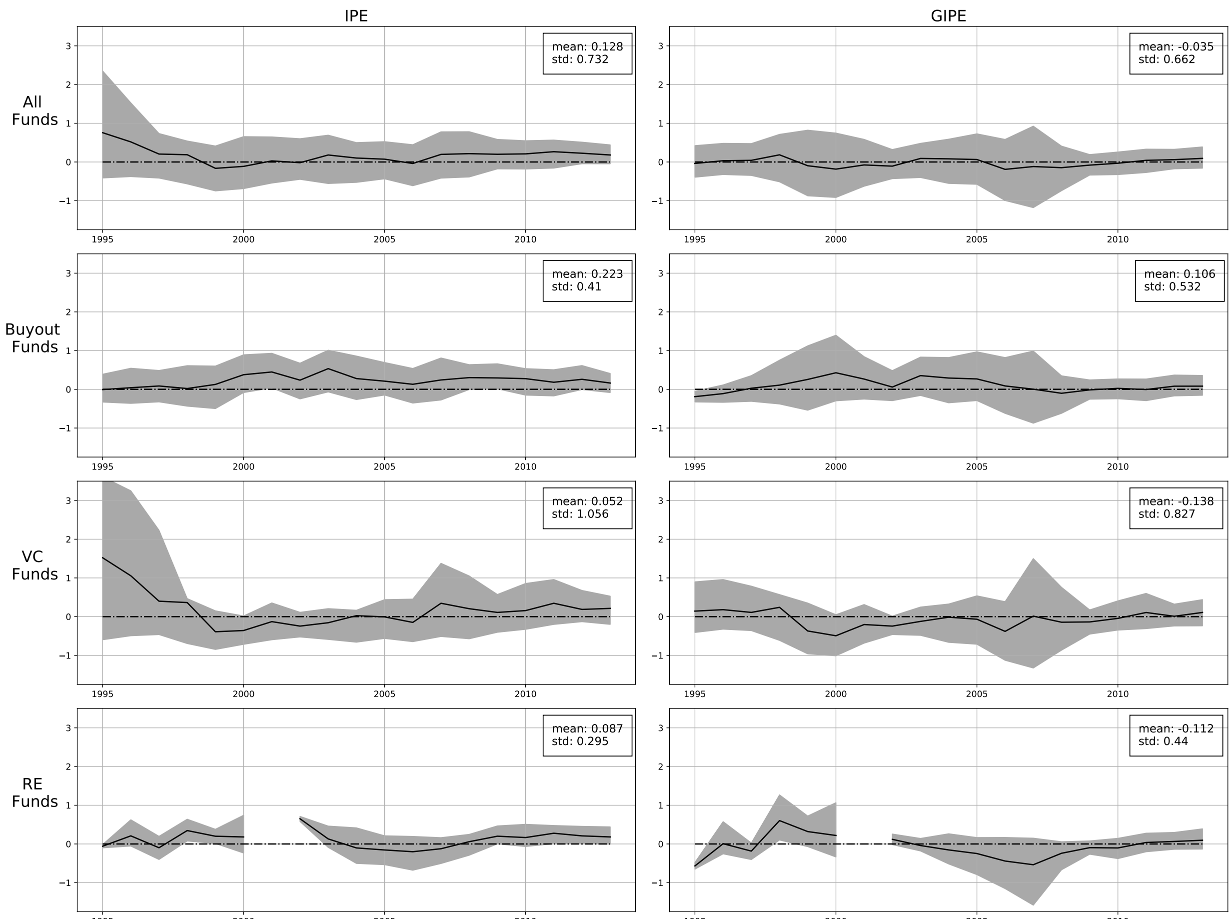
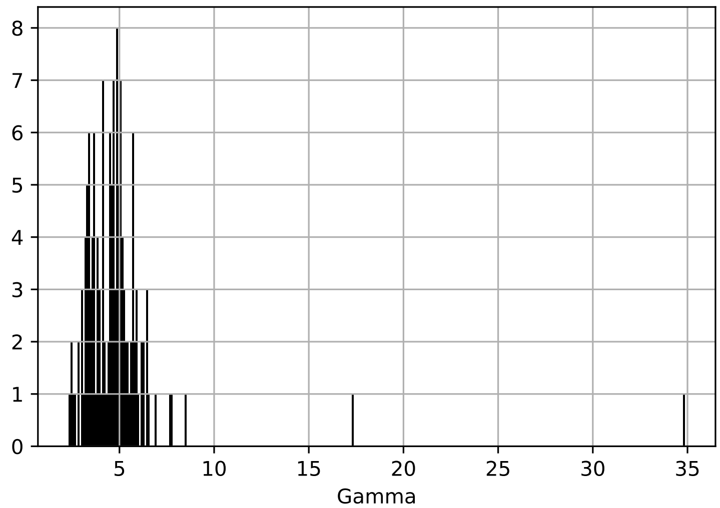


Figure 4: Histogram of Pension Plan Risk Aversion Coefficients

This figure shows the distribution of estimated risk aversion coefficients ($\gamma$) for U.S. public pension plans. Each pension plan has one risk aversion coefficient that is estimated such that the plan prices the excess return on the public stock market (i.e., it satisfies the Euler equation (14)), using all available annual returns for the plan.




### 3.2 GIPE

GIPE is a more general metric than IPE, allowing for a coefficient of relative risk aversion, $\gamma_j$, that may be different from one and may vary by pension plan. For each plan, we determine its $\gamma_j$ such that the plan correctly prices the excess return on the CRSP value-weighted market portfolio, $R_t^m - \left(1 + r_{t-1}^f\right)$. Specifically, we solve the Euler equation

$$\frac{1}{n_T - n_0 + 1} \sum_{t=n_0 \dots n_T} (R_{j,t})^{-\gamma_j} \left(R_t^m - \left(1 + r_{t-1}^f\right)\right) = 0, \tag{14}$$

using all available annual plan returns. The histogram in Figure 4 shows that the mass of the gamma distribution is centered around 5, and all but two plans have gammas below 10.[12]

Panel B of Table 3 shows that the GIPE results are quite different from IPE. The average GIPE across all strategies is economically small at $-0.035$, and statistically insignificant. The corresponding alpha of $-1.8\%$ per year is only marginally significant, at the 10% level. Across strategies, only buyout has a positive and significant average GIPE, equal to 0.106. However its

[12]The two outliers are the Boston Retirement System, with an estimated $\gamma$ of about 17, and the Texas Municipal Retirement System, with a $\gamma$ around 35. Excluding these two plans does not materially change the empirical results.

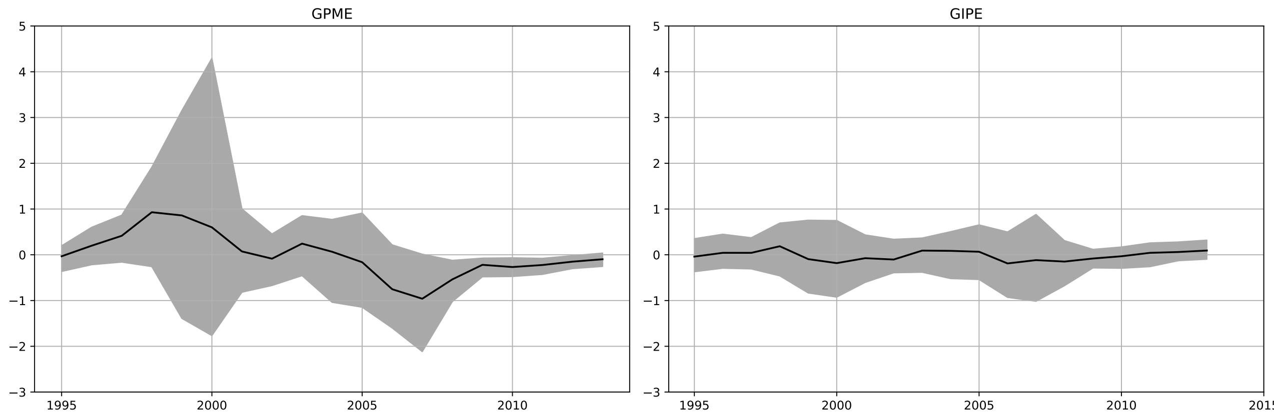
alpha of 1.6% is not statistically significant. Venture capital and real estate have a negative GIPE (alpha) of −0.138 (−4.9%) and −0.112 (−2.6%). The venture capital numbers are statistically significant, whereas those for real estate are not. The time series plots in the right column of Figure 3 show that GIPE tends to be closer to zero than IPE. Most striking is that the strong IPE performance of VC in the mid-1990s does not show up in GIPE. Unlike the results for IPE, the GIPE numbers suggest that a typical pension plan would not have benefited from an increased allocation to an average PE fund over the sample period. This underscores the importance of taking into account the level and heterogeneity of risk aversion.

It is useful to compare GIPE to the Korteweg and Nagel (2016) GPME, because both measures are constructed with the aim of assigning a zero value to a fund whose cash flows are replicable with some (possibly) levered trading strategy in stocks and bonds. Table 3 shows that, across all plan-fund combinations, the average GIPE-GPME of 0.080, while economically meaningful, is statistically insignificant. The only PE strategy for which the difference is significant is real estate, where GIPE is 0.254 higher than GPME. The table also shows that average GPME has a substantially larger standard error than GIPE. This is due to the fact that the cross-sectional standard deviation of GPME within a given vintage is much higher than for GIPE, as can be seen in Figure 5. The figure also shows that the cross-sectional dispersion of GPME changes dramatically from year to year. It is reassuring that the distribution of GIPE is more stable across vintage years, as one would expect that the cross-sectional distribution of risk-adjusted performance (e.g., from manager skill) does not change dramatically from one year to the next.

Next, we consider the breakdown of the GIPE between the component of PE payoffs that is replicable using publicly-traded securities versus the non-replicable component. We focus the discussion on the more general GIPE results, but Table 3 also reports the IPE results for completeness. The row labeled "GIPE(market-repl.)" reports results for mimicking funds that make capital calls that are identical to the PE funds but invest them in the CRSP value-weighted market portfolio instead, with distributions following equation (11). The average GIPE is quite consistent across strategies at around −0.050, and significant at the 5% level. The negative values imply that, if anything, PE fund managers have inferior market-timing ability. The difference in the GIPE of a PE fund and that of its market-replicating portfolio is informative in determining whether there is

Figure 5: Time Series of GPME and GIPE

This figure shows the distribution of GIPE and GPME by vintage year. The horizontal axis is the vintage year of the funds. The solid line represents the mean (For GIPE, equally weighted across all pension plan and PE fund combinations, irrespective of whether a commitment has been made) and the shaded area represents the 10th-90th percentile range of the observations within each vintage.




added value to investors in the PE-specific component of payoffs.[13] The row labeled "GIPE - GIPE (market-repl.)" shows that the only strategy for which this difference is statistically significant is buyout. The positive difference of 0.151 shows that – at least historically – buyout funds gave pension funds access to investments that on average dominated the alternative of investing the same capital calls in the stock market and making similar distributions to the buyout fund.

Buyout funds historically tended to invest in value stocks (Stafford, 2022) and venture capital in (small) growth firms. Given the strong performance of buyout, we are especially interested in examining whether buyout funds just represent "covert" value strategies. To that end, we construct mimicking funds that invest in publicly traded portfolios of value (the equally-weighted top quintile of book-to-market stocks) and small-growth stocks (those in the lowest size and book-to-market quintile, also equally weighted).[14] The row labeled "GIPE(value-repl.)" in Table 3 shows that the average value-mimicking buyout fund GIPE is 0.066. While positive, this number is statistically not different from zero, so one cannot reject the null hypothesis that the SDF of a typical pension plan can price a publicly traded value portfolio with cash flows that mimic those of an average buyout fund. The results do suggest, however, that buyout owes its success partly due to its value exposure.

[13]Note that the difference in $(G)IPE$ of the two cash flow streams is identical to the $(G)IPE$ of the difference in the two cash flow streams (i.e., the $(G)IPE$ of $C_{i,t} - \widehat{C}_{i,t}$), since these performance metrics are linear in cash flows.

[14]Size and value portfolios are downloaded from Kenneth French's website: `https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`

The difference between the GIPE of the actual cash flows of buyout funds and their mimicking cash flows using the value return as a benchmark is positive at 0.039, but not statistically significant (see the row labeled "GIPE - GIPE(value-repl.)"). In that sense, the historical outperformance of buyout funds is due to both: a) their value exposure and; b) their ability to select better investments. Combining a) and b) leads to a positive and significant GIPE, even though individually the two components are not significant.

It is useful to contrast the buyout value results with the GIPE of the mimicking funds that invest in small-growth stocks (see the row labeled "GIPE(growth-repl.)" in Table 3). The GIPEs are significantly negative for all PE strategies, ranging from $-0.218$ to $-0.263$. While we also found negative performance for the market-replicating funds, the numbers here are a factor four to five larger. This implies that the SDF of a given pension plan cannot "price" these strategies. The existence of these large and negative GIPEs may be due to shorting constraints or investment mandates that do not allow pension plans to reduce an existing exposure, preventing them from capitalizing on strategies with negative long-only excess returns. Interestingly, the poor performance of VC is explained away by the poor performance of small-growth companies. Relative to the public small-growth mimicking funds, VC performance was positive and significant.

To summarize and conclude the replication exercises, the key takeaway is that some mimicking funds exhibit negative and statistically significant GIPEs, but none of the positive GIPEs are significant. Thus, while the plan's portfolio decisions may be constrained with respect to shorting (or reducing exposures), they do not appear constrained or inconsistent with our proposed SDF with respect to long-only strategies.

## 4 Pension plan heterogeneity and PE performance

In this section we dig deeper into the drivers of PE performance across pension plans. First, we test whether (risk-adjusted) returns are different for funds that pension plans actually invested in. Table 4 shows estimates from regressions with either the IRR or the GIPE-implied alpha of all plan-fund combinations as the dependent variable. These two performance metrics are useful to compare, since the former measures a fund's total annualized return and the latter its risk-adjusted annualized return (we omit the IPE-implied alpha for brevity). Both IRR and alpha are measured in percentage points per year (for example, an IRR of 1.0 means a return of 1 percent per year). Panel A shows specifications with vintage fixed effects, and Panel B adds pension plan fixed effects. All regressions in this section double-cluster standard errors by vintage and by pension plan.

The regressions in Table 4 include covariates that are intended to capture performance heterogeneity due to pension plans' PE investment decisions. We first consider the coefficient on the indicator variable *Active*, which measures if a pension plan was an active investor in PE in a given year: It equals one if the plan made a commitment to any PE fund for the year, and zero otherwise. For IRR, the estimated coefficient is negative in specifications with vintage year fixed effects only (Panel A). However, the estimates are economically small across the board and, for the individual PE strategies, not statistically significant. When adding pension plan fixed effects in Panel B, the economic and statistical magnitudes become negligible, on the order of a few basis points. This means that pension plans do not tend to enter PE when IRRs are expected to be higher, or to get out of PE when future returns are lower.

Results are different for alpha. Without plan fixed effects, plans that invest in PE have average alphas of 1 to 1.4 percentage points higher (across strategies) than plans that do not. This could indicate that some plans make a rational decision to stay out of PE investments, given the low alphas that they expect to earn. Indeed, the result is due to heterogeneity across plans: with plan fixed effects, the *Active* coefficient is close to zero and insignificant across the board, suggesting that pension plans do not have skill in timing their entry or exit from PE investing altogether.

To test whether plans experience better performance in the PE funds that they actually invest in, we construct the *Commit* indicator, which equals one only if, in a given plan-fund combination, the pension plan made a commitment to that specific PE fund. In the regressions that use all PE funds, the coefficients are similar in magnitude - around 2 to 2.5 percentage points - and statistically significant for both IRR and alpha and both with and without pension plan fixed effects. There is some heterogeneity across PE strategies, but all estimates are positive. This improved performance could come from pension plans having selection skill or differential access to better PE funds. To disentangle these two channels, we consider the performance of funds of GPs with which the pension plan has invested before, measured by the indicator *Relationship*. This indicator equals one only if the pension plan has made a commitment to a prior fund of the same GP. Prior investors are usually given the option to reinvest in follow-on funds (Lerner et al., 2007), such that access is not a concern in the performance of reinvestment decisions. The large, positive coefficient on *Relationship* shows that such funds tend to perform well, both in terms of IRR and alpha. This result could be due to the fact that these PE managers have survived to raise follow-on funds. What is more important for our purpose, is the interaction between *Relationship* and *Commit*. The coefficient is negative and almost of equal magnitude of *Commit*. This means that, conditional on a pre-existing relationship with a GP, a commitment is not associated with higher performance. This suggests that the unconditional higher performance from committed funds is driven by access, not skill.

The literature also considers reinvestments and first-time funds to distinguish access and skill (Lerner et al., 2007; Sensoy et al., 2014; Andonov et al., 2018; Cavagnaro et al., 2019). The empirical evidence is mixed. For public pension plans, Lerner et al. (2007) and Cavagnaro et al. (2019) conclude that performance is due to more than just access (but not as much as for endowments), whereas Sensoy et al. (2014) find no evidence of skill after controlling for access, especially post-1999. Given that competition in PE has increased, and performance persistence has weakened for some strategies (e.g., Harris et al. (2022)), the fact that our sample period is more recent than these prior studies may help explain why we find little evidence of skill.

Next, we introduce additional pension plan-specific covariates to further explore performance differences between plans. We are especially interested in agency problems that may arise depending on the funding status of a plan, its governance structure, and the location of the PE fund that the plan invests in. We focus on the subsample of plan-fund combinations for which the plan actually made an investment in the PE fund. Furthermore, since many of the explanatory variables, such as the structure of the board, do not vary much (or at all) over time, we do not include pension plan fixed effects. Instead, we use plan-state fixed effects (in addition to vintage fixed effects), so that we are effectively comparing the outcome of investments made by different pension plans within the same state in the same year.

Table 5 reports the estimated coefficients of regressions of IRR and alpha on plan characteristics. To determine the effect of geography we include an indicator variable, *Home State*, that equals one if the PE fund has the same state of domicile as the pension plan, and zero otherwise. A negative coefficient means that local investments underperform relative to out-of-state investments. This could come about, for example, as a result of political pressure to invest in the home state (Hochberg and Rauh, 2013). Conversely, a positive coefficient suggests that pension plans may have superior

Table 4: **Pension Plan Commitments and Performance**

Each column shows the coefficients of a regression of private equity fund performance on two indicator variables plus controls. The observations are all possible combinations of pension plans and PE funds. The indicator *Active* equals one if the pension plan made a commitment to any PE fund in that vintage year. and zero otherwise. *Commit* equals one when the pension plan committed to this particular PE fund, and zero otherwise. *Relationship* equals one for all plan-fund combinations where a prior commitment has been made between the pension plan and PE fund manager. The first two columns consider all PE funds. Columns (3) and (4) only include buyout funds, columns (5) and (6) only venture capital funds, and columns (7) and (8) only real estate funds. The odd-numbered columns use IRR as the performance metric, and the even-numbered columns use the GIPE-implied $\alpha$ (both measured in percentage points per year, i.e., an IRR of 1 means one percent per year). All regressions include vintage year fixed effects. The regressions in Panel B also include pension plan fixed effects. Standard errors are double clustered by vintage year and pension plan, and are shown in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% level

*Panel A: Without pension plan fixed effects*

| | All Funds | | Buyout Funds | | Venture Capital Funds | | Real Estate Funds | |
|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| | IRR | $\alpha(GIPE)$ | IRR | $\alpha(GIPE)$ | IRR | $\alpha(GIPE)$ | IRR | $\alpha(GIPE)$ |
| Active=1 | -0.180*** | 1.162 | -0.113 | 1.365* | -0.149 | 1.022 | -0.0515 | 1.334 |
| | (0.0589) | (0.689) | (0.0685) | (0.711) | (0.101) | (0.619) | (0.0775) | (0.878) |
| Commit=1 | 2.318*** | 2.292*** | 0.828 | 1.142** | 3.291 | 2.375* | 0.272 | 0.443 |
| | (0.737) | (0.498) | (0.618) | (0.543) | (2.292) | (1.183) | (0.679) | (0.632) |
| Relationship=1 | 2.860*** | 3.697*** | 2.280 | 2.999** | 4.471** | 5.635*** | 0.900 | 1.699 |
| | (0.871) | (0.928) | (1.336) | (1.296) | (1.727) | (1.527) | (1.326) | (1.492) |
| Relationship x Commit | -2.781* | -2.758* | -1.881 | -1.807 | -5.609 | -5.194 | -0.314 | -1.223 |
| | (1.520) | (1.354) | (1.157) | (1.303) | (4.359) | (3.010) | (1.086) | (1.067) |
| $N$ | 159660 | 158490 | 64215 | 63689 | 63567 | 63172 | 31878 | 31629 |
| adj. $R^2$ | 0.059 | 0.032 | 0.136 | 0.090 | 0.119 | 0.059 | 0.320 | 0.170 |
| Vintage FE | Y | Y | Y | Y | Y | Y | Y | Y |
| Pension Plan FE | N | N | N | N | N | N | N | N |

*Panel B: With pension plan fixed effects*

| | All Funds | | Buyout Funds | | Venture Capital Funds | | Real Estate Funds | |
|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| | IRR | $\alpha(GIPE)$ | IRR | $\alpha(GIPE)$ | IRR | $\alpha(GIPE)$ | IRR | $\alpha(GIPE)$ |
| Active=1 | -0.0218 | -0.158 | -0.0275 | -0.0805 | 0.0459 | -0.136 | 0.0335 | -0.0518 |
| | (0.0482) | (0.314) | (0.0309) | (0.334) | (0.102) | (0.321) | (0.0394) | (0.206) |
| Commit=1 | 2.435*** | 1.969*** | 0.898 | 0.856 | 3.601 | 1.780 | 0.294 | 0.507 |
| | (0.724) | (0.519) | (0.641) | (0.521) | (2.309) | (1.373) | (0.705) | (0.666) |
| Relationship=1 | 2.999*** | 2.637*** | 2.366 | 1.939 | 4.796** | 4.699*** | 0.939 | 0.679 |
| | (0.889) | (0.833) | (1.382) | (1.226) | (1.784) | (1.588) | (1.369) | (1.372) |
| Relationship x Commit | -2.889* | -2.129 | -1.940 | -1.359 | -5.929 | -4.453 | -0.329 | -0.377 |
| | (1.523) | (1.358) | (1.181) | (1.274) | (4.403) | (3.163) | (1.103) | (0.971) |
| $N$ | 159660 | 158490 | 64215 | 63688 | 63567 | 63172 | 31878 | 31629 |
| adj. $R^2$ | 0.059 | 0.109 | 0.134 | 0.224 | 0.117 | 0.098 | 0.317 | 0.440 |
| Vintage FE | Y | Y | Y | Y | Y | Y | Y | Y |
| Pension Plan FE | Y | Y | Y | Y | Y | Y | Y | Y |

information about funds in their home state. Across all plan-fund investments, we find a negative coefficient on IRR, consistent with Hochberg and Rauh (2013), that is marginally statistically significant (at the 10% level). The coefficient is of similar magnitude and statistical significance when we use alpha as the dependent variable. This result complements the literature by showing that local underperformance is due to poorer investments in a risk-adjusted sense, and not only due to investments being potentially less risky. The coefficients are also negative for VC and real estate strategies, and significant in the case of real estate IRRs. For buyout, however, the coefficients are positive, and significant (at the 10% level) for IRR, suggesting that for this strategy, pension funds may have an information advantage when it comes to investing in local funds.

Turning to the funding status of the plan, we find that better funded plans, measured as the ratio of actuarial assets to actuarial liabilities, have lower IRR but higher alpha when we consider all plan-fund investments. The coefficients are insignificant, though economically meaningful: a one standard deviation increase in funded ratio lowers the expected annual IRR by 0.79 percentage points, and raises the expected alpha by 0.59 percentage points. Across all strategies, the coefficient estimates imply that IRR and alpha are closer together for higher funding ratios, an observation that we will return to below when we consider risk-taking.

To analyze the impact of governance structure on performance, we follow Andonov et al. (2018) and use the fraction of trustees on the pension plan's board that are state officials, plan participants, or members of the public, and whether they were elected by plan members, appointed by a government official, or serving ex-officio. Across these 9 categories, the most common ones are elected plan participants (27% of the average board), state officials, such as the state treasurer, serving ex-officio (25%), appointed members of the public (25%), appointed plan participants (12%), and appointed state officials (8%). The other four categories are less than 2% of the average board each, and we lump them together as *Other trustees*. Following Andonov et al. (2018), the omitted category is appointed plan participants. Despite covering a different sample period, and a different regression specification, our IRR results are qualitatively in line with Andonov et al.: plans with a higher fraction of state-appointed and ex-officio trustees, member-elected, or appointed members of the public (relative to appointed plan participants), experience lower IRRs on their PE investments. The effect is largest for the two categories of state official trustees. Unlike Andonov et al., we can also consider the effect on risk-adjusted returns. With alpha as the dependent variable, we

Table 5: **Pension Plan Characteristics and Performance**

Each column shows the coefficients of a regression of private equity fund performance on pension plan characteristics plus controls. The observations are all plan-fund combinations for which the pension plan invested in the PE fund. The first two columns consider all PE funds. Columns (3) and (4) only include buyout funds, columns (5) and (6) only venture capital funds, and columns (7) and (8) only real estate funds. The odd-numbered columns use IRR as the performance metric, and the even-numbered columns use the GPME-implied $\alpha$. *Home state = 1* is an indicator variable that equals one if the PE fund and pension plan are located in the same state, and zero otherwise. *Funded ratio* is the ratio of actuarial assets to actuarial liabilities. *Public Equity Wt* is the fraction of the plan's portfolio allocated to public equity. *State Appointed*, *State Ex-officio*, *Member Elected*, and *Public appointed* are the fraction of the plan's board members who are state officials appointed by government official, state officials serving ex-officio, plan members elected by their peers, and members of the public appointed by a government official, respectively. *Other Trustees* is the fraction of the board who were installed by other means, with the omitted category being plan members who were appointed by an official. *Board Size* is the number of pension plan board members. *Log(AUM)* is the natural logarithm of the pension plan's assets under management, and *Log(commitment %)* is the natural logarithm of the commitment of the plan to the fund, as a percentage of the plan's assets under management. All regressions include vintage year and plan-state fixed effects. Standard errors are double clustered by vintage year and pension plan, and are shown in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% level

| | All Funds | | Buyout Funds | | Venture Capital Funds | | Real Estate Funds | |
|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| | IRR | $\alpha(GIPE)$ | IRR | $\alpha(GIPE)$ | IRR | $\alpha(GIPE)$ | IRR | $\alpha(GIPE)$ |
| Home State=1 | -1.746* | -1.487* | 1.582* | 1.324 | -1.304 | -1.377 | -3.119** | -0.955 |
| | (0.947) | (0.754) | (0.873) | (1.146) | (1.888) | (1.643) | (1.075) | (1.131) |
| Funded Ratio | -4.364 | 3.874 | -2.159 | 3.895 | -9.893 | 1.154 | 1.636 | 10.19** |
| | (3.924) | (3.071) | (3.713) | (3.108) | (10.79) | (6.144) | (2.817) | (4.256) |
| Public Equity Wt | -15.10** | -23.18*** | -4.978 | -13.69*** | -18.61 | -21.36* | -9.405** | -20.47** |
| | (6.748) | (5.128) | (4.351) | (3.269) | (15.75) | (10.97) | (4.050) | (7.255) |
| State Appointed | -5.034 | -6.200 | -4.466 | -5.968 | 3.154 | 6.341 | 1.273 | 1.349 |
| | (4.505) | (4.824) | (5.564) | (6.177) | (9.801) | (7.746) | (4.479) | (4.742) |
| State ex-officio | -5.748 | -8.896** | -5.578 | -12.62*** | -7.367 | -4.865 | 6.628* | 4.026 |
| | (3.455) | (3.705) | (3.434) | (3.750) | (7.307) | (5.159) | (3.417) | (4.894) |
| Member Elected | -1.637 | -1.691 | -0.992 | -2.289 | -3.001 | -4.498 | 3.028 | 4.581 |
| | (2.032) | (2.239) | (1.718) | (2.248) | (3.510) | (2.752) | (2.963) | (5.518) |
| Public Appointed | -0.910 | -4.787 | -2.687 | -6.882* | -1.029 | -5.288 | 1.311 | 2.475 |
| | (2.032) | (3.017) | (1.964) | (3.559) | (4.458) | (3.973) | (2.612) | (3.455) |
| Other Trustees | 7.280** | 1.114 | 4.312 | -1.322 | 9.250 | 1.848 | 0.0102 | -2.086 |
| | (3.287) | (3.420) | (3.821) | (4.616) | (7.613) | (4.598) | (2.673) | (4.772) |
| Board Size | -0.0254 | 0.190*** | -0.0806 | 0.140** | 0.311 | 0.374*** | -0.0431 | 0.224* |
| | (0.0590) | (0.0542) | (0.0528) | (0.0589) | (0.181) | (0.120) | (0.0391) | (0.116) |
| Log(AUM) | 0.598 | 0.950 | 0.769** | 1.411*** | -1.837 | -1.611 | -0.355 | -0.643 |
| | (0.580) | (0.563) | (0.364) | (0.435) | (1.531) | (1.023) | (0.539) | (0.499) |
| Log (Commitment %) | 1.275* | 1.213** | 1.006** | 0.928* | -1.067 | -1.084 | -0.751 | 0.0875 |
| | (0.700) | (0.541) | (0.453) | (0.506) | (0.978) | (0.645) | (0.832) | (0.806) |
| *N* | 3957 | 3957 | 2149 | 2149 | 1082 | 1082 | 716 | 716 |
| adj. $R^2$ | 0.102 | 0.096 | 0.235 | 0.207 | 0.134 | 0.074 | 0.366 | 0.271 |
| Vintage FE | Y | Y | Y | Y | Y | Y | Y | Y |
| Plan State FE | Y | Y | Y | Y | Y | Y | Y | Y |

find even larger (i.e., more negative) coefficients. However, the only statistically significant results are for state ex-officio trustees. Economically, a 10 percentage point increase in the proportion of state ex-officio (appointed public) trustees lowers the annual alpha by 0.96 (0.55) percentage points. For the individual PE strategies, buyout looks similar to PE overall. VC looks similar with the exception of state-appointed trustees, who have a positive (though insignificant) coefficient. The coefficient signs are all reversed in real estate, though none are statistically significant, so it's difficult to draw strong conclusions for this strategy.

With respect to size, we find that larger pension plans (measured by the natural logarithm of assets under management) have better performance. The relation is stronger for alpha compared to IRR, but neither are statistically significant. This result is driven by buyout, where coefficients are significant. The signs reverse in VC and real estate investments (but the coefficients are insignificant). This result is consistent with Dyck and Pomorski (2015), who find that plans that have more dollars invested in PE, realize better plan returns. Possible explanations are that larger LPs have access to better GPs (Lerner et al., 2007), and a wider scope of due diligence, monitoring, and other related activities (Da Rin and Phalippou, 2017). Interestingly, we find a positive and significant effect of board size on alpha, even after conditioning on plan size. Adding another member to the board increase the annual alpha by 0.20 percentage points.

Finally, we control for the fraction of the plan's portfolio that is allocated to public equity. Plans with a higher allocation to public equity, indicative of lower risk aversion, have worse performance in private equity, both in terms of IRR and alpha, for all strategies. This echoes the results in Andonov et al. (2017), who find that pension plans with a higher allocation to risky assets have lower benchmark-adjusted plan returns.

The relation between performance and the funded ratio, as well as the equity share, suggest that risk-taking by pension plans may play an important role. A unique advantage of our approach is that we can measure the degree of risk-taking by pension plans more directly, at the individual investment level, as measured by the difference between PE fund IRR and alpha. Table 6 reports regression results with $IRR - \alpha$ as the dependent variable.[15]

A key result is that better-funded pension plans take less risk in their private equity investments,

[15]Note that the coefficients in the last four columns of Table 6 are equal to the difference in the corresponding coefficients of the IRR and alpha regressions in Table 5.

Table 6: **Pension Plan Risk-taking**

This table shows results for regressions of the difference between IRR and the GIPE-implied $\alpha$, a measure of compensation for risk, on pension plan characteristics plus controls. The observations are all plan-fund combinations for which the pension plan invested in the PE fund. Columns (1) and (5) include all PE funds. Columns (2) and (6) only include buyout funds, columns (3) and (7) only venture capital (VC) funds, and columns (4) and (8) only real estate (RE) funds. All explanatory variables are as described in Table 5. All regressions include vintage year and plan-state fixed effects. Standard errors are double clustered by vintage year and pension plan, and are shown in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% level.

| | (1)<br>All | (2)<br>Buyout | (3)<br>VC | (4)<br>RE | (5)<br>All | (6)<br>Buyout | (7)<br>VC | (8)<br>RE |
|---|---|---|---|---|---|---|---|---|
| Home State=1 | -0.330 | 0.358 | -0.225 | -2.020* | -0.259 | 0.258 | 0.0730 | -2.164** |
| | (0.622) | (0.751) | (0.493) | (1.085) | (0.579) | (0.742) | (0.542) | (0.988) |
| Funded Ratio | -7.618*** | -7.607*** | -8.640** | -7.534** | -8.238*** | -6.054** | -11.05* | -8.550** |
| | (2.264) | (2.584) | (3.460) | (2.998) | (2.303) | (2.783) | (5.294) | (3.753) |
| Public Equity Wt | 8.801** | 9.632*** | 3.198 | 11.00* | 8.080** | 8.709** | 2.747 | 11.06* |
| | (3.751) | (3.229) | (7.559) | (5.643) | (3.702) | (3.535) | (7.821) | (5.728) |
| Log(AUM) | -0.682 | -0.778** | -0.922** | 0.548 | -0.352 | -0.642 | -0.227 | 0.289 |
| | (0.411) | (0.337) | (0.382) | (0.480) | (0.453) | (0.498) | (0.738) | (0.447) |
| State Appointed | | | | | 1.166 | 1.502 | -3.186 | -0.0766 |
| | | | | | (4.651) | (4.834) | (6.473) | (4.759) |
| State ex-officio | | | | | 3.149 | 7.042** | -2.502 | 2.602 |
| | | | | | (2.445) | (2.630) | (4.516) | (4.912) |
| Member Elected | | | | | 0.0537 | 1.297 | 1.496 | -1.553 |
| | | | | | (2.061) | (1.835) | (1.861) | (3.409) |
| Public Appointed | | | | | 3.877 | 4.196* | 4.260 | -1.164 |
| | | | | | (2.608) | (2.003) | (3.027) | (3.468) |
| Board Size | | | | | -0.216*** | -0.221*** | -0.0628 | -0.268* |
| | | | | | (0.0569) | (0.0590) | (0.115) | (0.139) |
| Log (Commitment %) | | | | | 0.0620 | 0.0785 | 0.0170 | -0.839 |
| | | | | | (0.337) | (0.395) | (0.436) | (0.674) |
| Other Trustees | | | | | 6.165** | 5.634* | 7.402 | 2.096 |
| | | | | | (2.901) | (3.114) | (4.762) | (3.924) |
| $N$ | 3957 | 2149 | 1082 | 716 | 3957 | 2149 | 1082 | 716 |
| adj. $R^2$ | 0.341 | 0.440 | 0.336 | 0.300 | 0.345 | 0.446 | 0.337 | 0.303 |
| Vintage FE | Y | Y | Y | Y | Y | Y | Y | Y |
| Plan State FE | Y | Y | Y | Y | Y | Y | Y | Y |

Standard errors in parentheses
$^{*}$ $p < 0.10$, $^{**}$ $p < 0.05$, $^{***}$ $p < 0.01$

consistent with gambling for resurrection by underfunded plans (Rauh, 2009; Pennacchi and Rastad, 2011; Mohan and Zhang, 2014; Bradley et al., 2016; Myers, 2022). At the same time, plans with a higher public equity allocation take more risk in PE funds, consistent with these plans having lower risk aversion. As to governance and risk-taking, plans with a higher fraction of trustees who are appointed members of the public take more risk, especially in buyout and VC. Boards with a higher fraction of state officials who serve ex-officio also take more risk, especially in buyout. Plan size, measured by assets under management, is not significantly related to risk-taking, whereas having a larger board (conditional on plan AUM) has a small negative, but statistically significant, relation with risk-taking. Regarding location, home-state investments are not related to risk-taking, except for real estate, where local investments tend to be less risky.

To summarize, the main results in this section are as follows. Pension plans experience better risk-adjusted performance in PE funds that they invest in compared to those in which they do not, but this appears to be driven primarily by access to high-quality PE managers. Across all PE investments, underfunded plans take more risk, which translates into higher IRRs but lower alphas. Pension plan boards composed of a higher fraction of (ex-officio) state officials and appointed members of the public tend to invest in more risky PE funds, but earn lower alphas on these investments. Finally, PE funds in a pension plan's home state tend to under-perform, and this is not due to these investments being less risky.

# 5 Robustness

As a robustness check, we also calculate pension plan returns following the methodology outlined in Andonov and Rauh (2021). Specifically, annual returns are computed as the total net investment income divided by the beginning-of-the-year assets. This alternate measure of annual returns may be influenced by the timing of contributions and pension benefit payments throughout the year, as well as the valuation methods used for unrealized stakes in private equity and other illiquid assets. Figure 6 plots a time series of the average one-year reported return and alternate return. The alternate returns data is available only from 2001 onwards. The difference between the reported and alternate returns is +30bps and significant at the 1% level, likely due to the reported return being a gross-of-fee number. The average correlation between the two returns is 97% (equally weighted across plans).

Table 7 shows PE performance metrics computed using reported and alternate returns. For each of the performance metric (IPE, GIPE, and their alphas), there are three values computed – *Full*, *Alt*, and *Match*. The *Full* measure is computed using the full sample of available reported returns for all pension plan and PE fund combinations (regardless of commitment) and is identical to the performance measures reported in Table 3. The *Alt* measure is computed using the alternate returns. Since alternate returns data is available only from 2001 onwards, for each pension plan, we restrict the reported data to the same sample length as the alternate returns. Performance measures computed using this matched sample length are labeled as *Match*.

In Panel A of Table 7, the IPE (*Alt*) is higher than IPE (*Full*) because the alternate returns, likely a net-of-fee measure, are lower than the reported returns. A similar pattern is observed for the alphas, where $\alpha$-IPE (*Alt*) is 21 basis points higher than $\alpha$-IPE (*Match*) and significant at the 1% level. In Panel B, $\alpha$-GIPE (*Alt*) is 1.6% lower than $\alpha$-GIPE (*Full*) and significant at the 1% level. Note, that for computing the GIPE measures, gammas are estimated separately for each of the returns sample (*Full*, *Alt*, *Match*) such that the Euler equation (14) is satisfied [16]. The difference in alphas between the *Full* and *Alt* samples arises predominantly due to the difference in gammas, as the difference in alphas between the *Alt* and *Match* samples is statistically indistinguishable from zero.

As an additional robustness check, the two outliers in Figure 4 are the Boston Retirement System, with an estimated $\gamma$ of about 17, and the Texas Municipal Retirement System, with a $\gamma$ around 35. Excluding these two plans does not materially change any of our empirical results.

## 6 Conclusion

Evaluating performance and the distinction between risk-taking and informed trading in observed returns are fundamental issues in the theory and practice of investments. A key step in addressing such questions is to separate the compensation for bearing risk from excess returns. With heterogeneous agents, the assessed riskiness of the same investment opportunity can vary across

[16] The average gammas for the *Full*,*Alt*, *Match* samples are 4.8, 3.9, and 4 respectively. The difference in gammas between the *Alt* and *Match* sample is statistically not different from 0. The difference between the *Full* and *Alt* sample gammas is 0.4 and significant at the 1% level.

investors. The measures developed in this paper extend existing metrics to allow for the estimation of investor-specific risk-adjusted performance in long-dated investments, without relying on stale or potentially biased valuations.

We apply our method to U.S. public pension plan investments in private equity funds. We find that for the 1995 to 2018 sample period, pension plans appear to have pursued an optimal allocation to private equity investments overall, in the sense that there was no significant benefit in changing the investment in a representative PE fund. Looking at individual PE strategies, the average plan could have benefited from a higher allocation to buyout funds, although this is at least in part due to buyout's value exposure. While there is no evidence of market timing skill, pension plans did realize higher risk-adjusted returns in the funds they chose to invest in, compared to an average PE fund of the same vintage. However, this appears to be due to differences in access rather than skill in picking outperforming funds.

We find systematic differences across plans: Underfunded plans take more risk, which yields higher total returns, but lower risk-adjusted returns. Similarly, pension plan boards that have a higher fraction of (ex-officio) state officials and appointed members of the public tend to invest in riskier funds, but earn lower alpha. Home-state PE investments also earn lower alpha, but do not differ in risk compensation, so that total returns are lower. These results are broadly consistent with agency problems within pension plans, such as gambling for resurrection and political influence, playing an important role in investment decisions. Our findings augment the prior literature that considers total, but not risk-adjusted, plan returns and papers that only consider pension returns at a highly aggregated level.

The paper does not explore the mechanisms by which pension plans identify attractive PE funds, or the underlying reasons why certain types of board members have a preference for riskier investments. We also note a large degree of cross-pension plan dispersion in alternative asset returns, but we do not pursue pension plans' diversification strategies, and whether they price idiosyncratic risk. These and other questions we leave for future research.

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# A Proofs

**Proof.** [Proof of Proposition 1]Applying Ito's Lemma to compute the dynamics of $d\log W_t$ and taking expectations, implies that for any $t_k$ the value function $V(W_{t_k}, t_k)$ satisfies

$$\begin{aligned} V(W_{t_k}, t_k) &= E_{t_k} \log W_T = \\ & \log(W_{t_k}) + E_{t_k} \int_{t_k}^{T} \max_{w_u} \left( r_u + w_u' \left( \mu_u - r_u 1_{N_j} \right) - \frac{1}{2} w_u' \sigma_u \sigma_u' w_u \right) du. \end{aligned} \tag{15}$$

Accordingly, an implication of the envelope theorem is

$$\frac{d\left(E_{t_0} \log W_T\right)}{d\varepsilon} = \sum_{k=0}^{K} V_W\left(W_{t_k}, t_k\right) C_{t_k} = \sum_{k=0}^{K} \frac{C_{t_k}}{W_{t_k}}. \tag{16}$$

Therefore

$$\frac{d\left(E_{t_0} \log W_T\right)}{d\varepsilon} = \frac{1}{W_{t_0}} \times IPE = V_W\left(t_0, W_{t_0}\right) \times IPE.$$

■

**Proof.** [Proof of Corollary 1]Under the additional assumption that $\frac{dw_t}{d\varepsilon} = 0$, all steps of the proof of proposition 1 are unchanged, except that the maximization in equation (15) is no longer needed. Hence, equation (16) follows without invoking the Envelope theorem. ■

**Proof.** [Proof of Proposition 2] For any $t_k$ we have that that the value function $V(W_{t_k}, t_k)$ satisfies

$$V\left(W_{t_k}, t_k\right) = \frac{E_{t_k} W_T^{1-\gamma}}{1-\gamma} = \frac{W_{t_k}^{1-\gamma}}{1-\gamma} f\left(t_k\right), \tag{17}$$

where

$$f\left(t_k\right) = E_{t_k} \exp\left\{ \begin{array}{c} (1-\gamma)\left(r + w'\left(\mu - r1_{N_j}\right) - \frac{1}{2} w'\sigma\sigma' w\right)(T - t_k) \\ + (1-\gamma) w'\sigma\left(B_T - B_{t_k}\right) \end{array} \right\}$$

and the optimal portfolio is $w = \frac{1}{\gamma}\left(\sigma\sigma'\right)^{-1}\left(\mu - r1_{N_j}\right)$. Accordingly, an implication of the envelope theorem is

$$\frac{d\left(\frac{E_{t_0} W_T^{1-\gamma}}{1-\gamma}\right)}{d\varepsilon} = E_{t_0} \sum_{k=0..K} V_W\left(W_{t_k}, t_k\right) C_{t_k} = E_{t_0} \sum_{k=0..K} W_{t_k}^{-\gamma} f\left(t_k\right) C_{t_k}. \tag{18}$$

The Euler equation for bonds states that

$$e^{r(t_k - t_0)} E_{t_0} \frac{V_W (W_{t_k}, t_k)}{V_W (W_{t_0}, t_0)} = 1. \tag{19}$$

Using (17) inside (19) leads to

$$e^{r(t_k - t_0)} \frac{f(t_k)}{f(t_0)} E_{t_0} \left( \frac{W_{t_k}}{W_{t_0}} \right)^{-\gamma} = 1, \tag{20}$$

and using (20) inside (18) gives

$$\frac{d\left(\frac{E_{t_0} W_T^{1-\gamma}}{1-\gamma}\right)}{d\varepsilon} = f(t_0)\, W_{t_0}^{-\gamma} \times GIPE = V_W(t_0, W_{t_0}) \times GIPE.$$

■

**Generalization of Proposition 2** Assume that $\mu_t, r_t, \sigma_t$ are functions of some vector of state variables $X_t$ that follow some diffusion $dX_t = \mu_X dt + \sigma_X dB_t$. Assume also that $\sigma_X \sigma_A' = 0$, i.e., innovations to $dX_t$ and $\frac{dA_t}{A_t}$ are independent. Assume that $\mu_{A,t} - r_t + \frac{\langle dW_t^{-\gamma}; dA_t \rangle}{W_t^{-\gamma} A_t}$ has the same sign for all $t$ and define

$$GIPE^{(\gamma)} \equiv E_{t_0} \sum_{k=t_0,t_1 \ldots t_K} e^{-\int_{t_0}^{t_k} r_u du} \frac{(W_{t_k})^{-\gamma}}{E_0 (W_{t_k})^{-\gamma}} C_{t_k}. \tag{21}$$

Then $sign\left(\frac{dV^\gamma}{d\varepsilon}\right) = sign\left(GIPE^{(\gamma)}\right) = sign\left(\mu_{A,t} - r_t + \frac{\langle dW_t^{-\gamma}; dA_t \rangle}{W_t^{-\gamma} A_t}\right)$.

**Proof.** [With a (Markovian) time-varying opportunity set, the value function is multiplicatively separable in $W_{t_k}^{1-\gamma}$ and $X_{t_k}$ :

$$V(W_{t_k}, X_{t_k}, t_k) = \frac{E_{t_k} W_T^{1-\gamma}}{1-\gamma} = \frac{W_{t_k}^{1-\gamma}}{1-\gamma} f(X_{t_k}, t_k), \tag{22}$$

The marginal valuation of $C$ is now

$$\begin{aligned}\frac{d\left(\frac{E_{t_0}W_T^{1-\gamma}}{1-\gamma}\right)}{d\varepsilon} &= E_{t_0}\sum_{k=0..K} V_W\left(W_{t_k},t_k\right)C_{t_k} \qquad (23)\\ &= E_{t_0}\sum_{k=0..K} V_W\left(W_{t_k},t_k\right)\left(A_{t_k}-A_{t_k^+}\right)\\ &= E_{t_0}\sum_{k=0..K-1}\left(V_W\left(W_{t_{k+1}},t_{k+1}\right)A_{t_{k+1}}-V_W\left(W_{t_k},t_k\right)A_{t_k^+}\right)\\ &= E_{t_0}\sum_{k=0..K-1}\int_{t_k^+}^{t_{k+1}} d(V_W\left(W_t,t\right)A_t)\end{aligned}$$

Using Ito's Lemma gives

$$\begin{aligned}\frac{d(V_W\left(W_t,t_k\right)A_t)}{V_W\left(W_t,t_k\right)A_t} &= \frac{d(V_W\left(W_t,t_k\right))}{V_W\left(W_t,t_k\right)}+\frac{dA_t}{A_t}+\frac{\langle dV_W\left(W_t,t_k\right);dA_t\rangle}{V_W\left(W_t,t_k\right)A_t}\\ &= -r_t dt+\frac{dA_t}{A_t}+\frac{\left\langle dW_t^{-\gamma};dA_t\right\rangle}{W_t^{-\gamma}A_t}\end{aligned}$$

Since $V_W\left(W_t,t_k\right)A_t$ is positive, then $E_{t_0}\sum_{k=0..K-1}\int_{t_k^+}^{t_{k+1}} d(V_W\left(W_t,t_k\right)A_t)$ has the same sign as $\mu_{A,t}-r_t+\frac{\langle dW_t^{-\gamma};dA_t\rangle}{W_t^{-\gamma}A_t}$. Using $H_t=e^{-\int_{t_0}^t r_u du}\frac{W_t^{-\gamma}}{E_{t_0}\left(W_t^{-\gamma}\right)}$ as a stochastic discount factor and repeating the same steps as in (23) leads to

$$GIPE=E_{t_0}\sum_{k=0..K} H_{t_k}C_{t_k}=E_{t_0}\sum_{k=0..K-1}\int_{t_k^+}^{t_{k+1}} d(H_tA_t).$$

Applying Ito's Lemma to $d(H_tA_t)$, noting that $\frac{W_t^{-\gamma}}{E_{t_0}\left(W_t^{-\gamma}\right)}$ is a martingale and taking expectations shows that $GIPE$ has the same sign as $\mu_{A,t}-r_t+\frac{\langle dW_t^{-\gamma};dA_t\rangle}{W_t^{-\gamma}A_t}$. ■

**Proof of Proposition 3.** Let $H_t=e^{-rt}\frac{W_t^{-\gamma}}{E_{t_0}W_t^{-\gamma}}$. Noting that $\frac{W_t^{-\gamma}}{E_{t_0}W_t^{-\gamma}}$ is a martingale, an application of Ito's Lemma yields

$$\frac{dH_t}{H_t}=-rdt-\gamma w'\sigma dB_t$$

where the optimal portfolio $w=\frac{1}{\gamma}\left(\sigma\sigma'\right)^{-1}\left(\mu-r1_{N_j}\right)$. Applying Ito's Lemma again shows that

$$\frac{d\left(H_tA_t\right)}{H_tA_t}=\left(\mu_A-r-\gamma w'\sigma\sigma_A'\right)dt-\left(\gamma w'\sigma-\sigma_A\right)dB_t \qquad (24)$$

Next note that

$$\begin{aligned}\beta_t\left(\mu^W - r\right) &= \beta_t w'\left(\mu - r1_{N_j}\right) = \frac{w'\sigma\sigma'_A}{w'\sigma\sigma' w} w'\left(\mu - r1_{N_j}\right) \\ &= \frac{\gamma w'\sigma\sigma'_A}{\left(\mu - r1_{N_j}\right)'\left(\sigma\sigma'\right)^{-1}\left(\mu - r1_{N_j}\right)}\left(\mu - r1_{N_j}\right)'\left(\sigma\sigma'\right)^{-1}\left(\mu - r1_{N_j}\right) \\ &= \gamma w'\sigma\sigma'_A.\end{aligned}$$

Accordingly, assumption (9) implies that $\mu_A - r = \beta_t\left(\mu^W - r\right)$ is equivalent to $\mu_A - r - \gamma w'\sigma\sigma'_A = 0$. Accordingly, (24) implies that $H_t A_t$ is a martingale, and therefore

$$H_{t_k^+} A_{t_k^+} = E_{t_k}\left(H_{t_{k+1}} A_{t_{k+1}}\right) = E_{t_k}\left(H_{t_{k+1}} A_{t_{k+1}^+}\right) + E_{t_k}\left(H_{t_{k+1}} C_{t_{k+1}}\right).$$

Iterating forward, using the law of the iterated expetation and noting that $A_{t_0} = A_{t_K^+} = 0$ implies that $GIPE = 0$. ■

**Proof of proposition 4.** a) All terms in (11) are non-negative for all $t_k$. b) It suffices to show that the present value of the cash flows $\widehat{C}_{t_k}$ discounted at the benchmark rate of return is zero, $\sum_{k=t_0,t_1\ldots t_K} \frac{\widehat{C}_{t_k}}{G_{t_k}} = 0$. We have that

$$\sum_{k=0..K} \frac{\widehat{C}_{t_k}}{G_{t_k}} 1_{\{\widehat{C}_{t_k}>0\}} = \widehat{A}_0 \times \sum_{k=0..K} \omega_{t_k} = \widehat{A}_0. \tag{25}$$

Equation (25) implies $\sum_{k=0..K} \frac{\widehat{C}_{t_k}}{G_{t_k}} = 0$, which shows that the strategy can be financed by investing the funds in the benchmark portfolio. c) By construction, $C_{t_k} = \widehat{C}_{t_k}$ whenever $\widehat{C}_{t_k} < 0$. So we focus on $C_{t_k} > 0$ to obtain

$$\begin{aligned}\widehat{C}_{t_k} &= \widehat{A}_0 \omega_{t_k} G_{t_k} \\ &= \frac{\widehat{A}_0}{\sum_{k=0..K} \frac{C_{t_k} 1_{\{C_{t_k}>0\}}}{G_{t_k}}} C_{t_k} \\ &= C_{t_k},\end{aligned} \tag{26}$$

where the last equality follow from assumption (12) and (25). ■

# B Robustness

Figure 6: **Pension Plan Reported and Alternate Returns**

Figure shows the time series of the average one-year return across the full sample of 142 pension plans described in Table 2. The solid line represents the reported returns, while the dashed line illustrates an alternate returns measure calculated using the methodology outlined in Andonov and Rauh (2021). Specifically, these alternate returns are computed as the total net investment income divided by the beginning-of-the-year assets. This alternate measure of annual returns may be influenced by the timing of contributions and pension benefit payments throughout the year, as well as the valuation methods used for unrealized stakes in private equity and other illiquid assets.

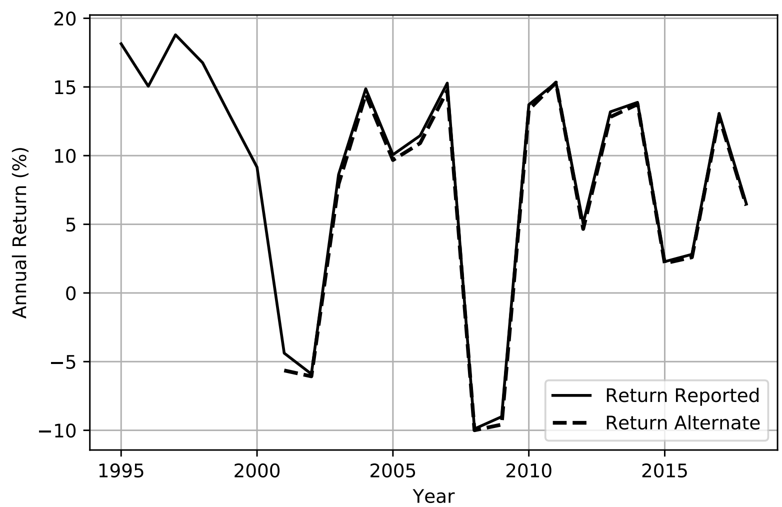

Table 7: PE Performance Metrics with Alternate Returns

This table reports performance results for private equity funds computed using reported returns and alternate returns (measure calculated using the methodology outlined in Andonov and Rauh (2021)). The first column shows the average performance across all possible pairs of pension plans and PE funds. The second column shows the standard error (s.e.) for the test of the null hypothesis that the mean is zero, double-clustered by pension plan and vintage year. The second, third, and fourth sets of two columns shows the average performance and its standard error separated by PE strategy: buyout, venture capital, and real estate funds. PE fund performance computed using all the available reported returns are denoted by *Full*. Performance computed using alternate return measures are denoted by *Alt.*. Performance computed using reported returns, but whose sample length matches the sample length of alternate returns are denoted by *Match*. Panel A shows IPE-type metrics. The first row is the *IPE* of equation (6) computed for all pension plan and PE fund combinations (regardless of commitment). Panel B shows similar results for GIPE-type metrics, where *GIPE* is as defined in equation (21). The row labeled $\alpha$ shows the annualized excess return that makes the (G)IPE equal to zero, computed as described in the text. An $\alpha$ of 0.01 represents an excess return of 1% per year. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% level, respectively.

| | All funds | | Buyout | | VC | | Real Estate | |
|---|---|---|---|---|---|---|---|---|
| | Mean | s.e. | Mean | s.e. | Mean | s.e. | Mean | s.e. |
| *Panel A: IPE-type metrics.* | | | | | | | | |
| IPE (Full) | 0.128 | 0.036*** | 0.223 | 0.025*** | 0.052 | 0.083 | 0.087 | 0.054 |
| IPE (Alt.) | 0.141 | 0.031*** | 0.26 | 0.027*** | 0.044 | 0.062 | 0.09 | 0.056 |
| IPE (Match) | 0.132 | 0.031*** | 0.249 | 0.026*** | 0.035 | 0.062 | 0.083 | 0.057 |
| IPE (Alt.) - IPE (Match) | 0.009 | 0.002*** | 0.011 | 0.002*** | 0.009 | 0.002*** | 0.007 | 0.001*** |
| | | | | | | | | |
| $\alpha$(IPE)(Full) | 3.52 | 1.13*** | 6.23 | 0.9*** | 0.4 | 2.11 | 4.27 | 1.76** |
| $\alpha$(IPE)(Alt.) | 3.69 | 1.12*** | 7.37 | 1.05*** | -0.85 | 1.53 | 4.46 | 1.84** |
| $\alpha$(IPE)(Match) | 3.48 | 1.13*** | 7.15 | 1.05*** | -1.06 | 1.54 | 4.26 | 1.85** |
| $\alpha$(IPE)(Alt.) - $\alpha$(IPE)(Match) | 0.21 | 0.04*** | 0.22 | 0.04*** | 0.21 | 0.03*** | 0.2 | 0.04*** |
| | | | | | | | | |
| *Panel B: GIPE-type metrics.* | | | | | | | | |
| GIPE (Full) | -0.035 | 0.032 | 0.106 | 0.037*** | -0.138 | 0.063** | -0.112 | 0.074 |
| GIPE (Alt.) | -0.097 | 0.034*** | 0.018 | 0.03 | -0.187 | 0.044*** | -0.153 | 0.072** |
| GIPE (Match) | -0.097 | 0.034*** | 0.016 | 0.029 | -0.185 | 0.044*** | -0.151 | 0.071** |
| GIPE (Alt.) - GIPE (Match) | 0.0003 | 0.003 | 0.002 | 0.004 | -0.002 | 0.002 | -0.001 | 0.002 |
| | | | | | | | | |
| $\alpha$(GIPE)(Full) | -1.8 | 0.96* | 1.63 | 1.04 | -4.87 | 1.46*** | -2.58 | 1.59 |
| $\alpha$(GIPE)(Alt.) | -3.4 | 0.93*** | 0.42 | 1.05 | -7.25 | 1.16*** | -3.89 | 1.56** |
| $\alpha$(GIPE)(Match) | -3.65 | 0.99*** | 0.13 | 1.08 | -7.5 | 1.21*** | -4.1 | 1.62** |
| $\alpha$(GIPE)(Alt.) - $\alpha$(GIPE)(Match) | 0.26 | 0.25 | 0.3 | 0.27 | 0.25 | 0.23 | 0.21 | 0.23 |
| | | | | | | | | |
| N (*Full*) | 160,499 | | 64,611 | | 64,010 | | 31,878 | |
| N (*Alt & Match*) | 133,317 | | 53,750 | | 50,104 | | 29,463 | |