

Sectoral Development Multipliers

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Motivation

▶ Economic development

- Idea hinges on the **adoption** of modern, complementary, technologies
- Distortions hinder this process → underdevelopment
- Industrial policies targeting key sectors are called for alleviating these distortions and promoting the investment in modern technologies (Hirschman, 1958)

▶ Recent developments in multisector with IO networks

- Relevance of sectoral distortions (Baqaee and Fahri, 2020; Bigio and La'O, 2020)
- Relevance of sector policies as effective tools to alleviate distortions (Liu, 2019)

▶ Open questions

- How to think about industrial policy for economic development?
- Which sectors/ policy instruments to foster development?

⇒ **OUR PAPER!**

Our paper

- ▶ Quantitative study of technology adoption in a multisector economy with complementarities
(Buera, Hopenhayn, Shin and Trachter, 2021)
- ▶ Ingredients: tech. adoption, distortions, rich IO structure
- ▶ Use size distr. to estimate distribution of technologies by sector
- ▶ Laboratory for sectoral industrial policy, local analysis
 - ▶ Which sectors, policies instruments to foster development?

⇒ **Sectoral Development Multipliers**

Related literature

1. Sectoral shocks and distortions in settings with sectoral linkages
(Acemoglu et al., 2012; Baqae and Fahri, 2020; Bigio and La'O, 2020; Baqae and Fahri, 2021; Caliendo et al., 2022)
2. Analysis of sectoral policies
(Liu, 2019; Liu and Ma, 2021; Bartelme et al., 2019)
3. Complementarities in development/tech. adoption
(Buera et al., 2021; Alvarez et al., 2023; Boehm and Oberfield, 2023; Crouzet et al., 2023; Demir et al., 2024)
4. Investment networks and the propagation of sectoral productivity shocks
(Foerster et al., 2022; vom Lehm and Winberry, 2022; Casal and Caunedo, 2023)

Plan for today

1. Theory

Multisector model of technology adoption; multipliers; sources of amplification; alternative instruments

2. Parameterization

Identification; GMM partial information estimation; empirical fit/ model validation; measure adoption

3. Sectoral revenue multipliers

Amplification through adoption; decomposition

4. Alternative policy instruments

most effective instrument; decomposition of most effective instrument

5. Concluding remarks

next steps

THEORY

Model

- CES consumption aggregator across sectors s , $C = \prod_s (C_s)^{\gamma_s}$, $\mathbf{\Gamma} = [\gamma_s]$
- Monopolistic competition within each sector, $Y_s = \left(\int y_{js}^{\frac{\eta-1}{\eta}} dj \right)^{\frac{\eta}{\eta-1}}$
- Firm heterog.: Ex-ante, Pareto with tail ζ ; Ex-post, $\varepsilon \sim N\left(-\frac{(\eta-1)\chi^2}{2}, \chi\right)$
- Produce using labor and intermediate inputs w/ **rich IO architecture**
 - ▶ Intermediate input aggregate is $X_s = \prod_{s'} (X_{ss'})^{\omega_{ss'}}$, $\mathbf{\Omega} = [\omega_{ss'}]$
- (Cobb-Douglas) Production, with **traditional** or **modern** tech
 - ▶ Traditional: tech A_t , cost in labor κ_{ts} ($w = 1$)
 - ▶ Modern: tech A_m , cost in labor κ_{ts} and in goods $P_{ms}\kappa_{ms}$
 - ▶ ν : share of intermediate aggregate in production
- Capital-embodied tech. adoption: compet. sector combines inputs, $\mathbf{\Lambda} = [\lambda_{ss'}]$
- Subsidies: **revenue**, \mathbf{r} , labor, \mathbf{r}^l , intermediate inputs, \mathbf{r}^x , and adoption, \mathbf{r}^a

Profits, entry and adoption thresholds

(Ex-post) Operating profit of a intermediate input producer in sector s ,

$$\pi_{is}^o(z, \varepsilon) = \max_{p, x, l} r_s p \left(\frac{p}{P_s} \right)^{-\eta} Y_s - \frac{P_{xs}}{r_s^x} x - \frac{1}{r_s^l} l, \text{ s.t. } z A_i e^\varepsilon x^\nu l^{1-\nu} \geq \left(\frac{p}{P_s} \right)^{-\eta} Y_s$$

Ex-ante problem, entry and adoption thresholds

- ▶ Entry with traditional tech: $\pi_{ts}^o(z_{ts}) \equiv \mathbb{E}_\varepsilon [\pi_{ts}^o(z_{ts}, \varepsilon)] = \kappa_{ts}$
- ▶ Entry with modern tech: $\pi_{ms}^o(z_{ms}) - \pi_{ts}^o(z_{ms}) = \frac{P_{ms}}{r_s^a} \kappa_{ms}$

Price indexes: $\mathbf{P} = [P_s]$, $\mathbf{P}_x = [P_{xs}]$, $\mathbf{P}_m = [P_{ms}]$

Then, fraction of establishments operating modern tech. is

$$\frac{a_s}{e_s} = \frac{1 - F(z_{ms})}{1 - F(z_{ts})} = \left(\frac{z_{ts}}{z_{ms}} \right)^\zeta \leq 1, \mathbf{a} = [a_s], \mathbf{e} = [e_s]$$

Equilibrium

Given subsidies r , r^x , r^l and r^a , a symmetric eq. consists of

- ▶ thresholds $\{z_{ts}, z_{ms}\}_{s \in S}$,
- ▶ demands for labor and intermediate inputs and aggregate demand C ,
- ▶ and prices P_C , P , P_x and P_m ,

such that establishments maximize profits and markets clear,

$$Y_s = \gamma_s \frac{P_C}{P_s} C + \sum_{s'} \omega_{s's} \frac{P_{x_{s'}}}{P_s} X_{s'} + \sum_{s'} \lambda_{s's} \frac{P_{m_{s'}}}{P_s} a_{s'} \kappa_{m_{s'}} \quad \text{for all } s,$$

$$L = \sum_s L_s + \sum_s e_s \kappa_{ts}$$

with,

$$X_s \equiv \int_{z_{ts}}^{z_{ms}} x_{ts}(z) df(z) + \int_{z_{ms}}^{\infty} x_{ms}(z) df(z), \quad x_{is}(z) = \mathbb{E}_{\varepsilon} [x_{is}(z, \varepsilon)]$$

$$L_s \equiv \int_{z_{ts}}^{z_{ms}} l_{ts}(z) df(z) + \int_{z_{ms}}^{\infty} l_{ms}(z) df(z), \quad l_{is}(z) = \mathbb{E}_{\varepsilon} [l_{is}(z, \varepsilon)]$$

Aggregates

- Aggregate output in s is $Y_s = \overbrace{Z_s}^{\text{TFP}} X_s^\nu L_s^{1-\nu}$, where

$$Z_s = \left(A_t^{\eta-1} \int_{z_{ts}}^{z_{ms}} z^{\eta-1} f(z) dz + A_m^{\eta-1} \int_{z_{ms}}^{\infty} z^{\eta-1} f(z) dz \right)^{\frac{1}{\eta-1}}$$

- Sector prices

$$\ln \mathbf{P} = \left(\frac{1}{1-\nu} \ln \frac{\eta}{\eta-1} \right) \mathbf{I} - (\mathbf{I} - \nu \mathbf{\Omega})^{-1} (\ln \mathbf{Z} + \ln \mathbf{r} + (1-\nu) \ln \mathbf{r}^l + \nu \ln \mathbf{r}^x)$$

$$\Rightarrow d \ln \mathbf{P}_c = \mathbf{\Gamma}' d \ln \mathbf{P}, \Rightarrow d \ln \mathbf{P}_x = \mathbf{\Omega}' d \ln \mathbf{P}, \Rightarrow d \ln \mathbf{P}_m = \mathbf{\Lambda} d \ln \mathbf{P}$$

- Adoption and TFP

$$d \ln \mathbf{Z} = \frac{1}{\eta-1} \overbrace{\frac{\zeta + 1 - \eta}{\zeta} \frac{A_m^{\eta-1} - A_t^{\eta-1}}{A_m^{\eta-1}}}^{\equiv \beta > 0} \overbrace{\text{diag}(M)}^{M_s \geq 0} d \ln \mathbf{a}$$

More adoption \rightarrow Higher \mathbf{Z} \rightarrow Lower \mathbf{P} \rightarrow Lower MC and adoption costs \rightarrow More adoption
 \rightarrow Interactions generate complementarities across plant investments

Development multipliers

Let ϵ_{rs} denote the sectoral revenue development multiplier in sector s ,

$$\epsilon_{rs} \equiv \underbrace{\frac{d \ln C}{d \ln r_s} \Big|_{r=1}}_{\text{subsidy elasticity}} / (\text{Fiscal cost of policy})$$

(local, first-order, analysis)

Markup distortions. Effect of revenue subsidy

- ▶ Production. Subsidy affects demands for l and $x \rightarrow$ effect on C
- ▶ Adoption.
 1. Subsidy affects incentives to adopt.
 2. Adoption is converted to TFP through $d \ln \mathbf{Z} = \beta \text{diag}(M) d \ln \mathbf{a}$
 3. Changes in TFP are converted to changes in C

Subsidy and TFP elasticity of Aggregate Consumption

Consider independent changes to $d \ln \mathbf{r}$ and $d \ln \mathbf{Z} = \beta \text{diag}(\mathbf{M}) d \ln \mathbf{a}$,

$$d \ln C = \overbrace{\left(\tilde{\Psi}' - \Psi' \right)}^{\text{production channel}} d \ln \mathbf{r} + \overbrace{\left(\tilde{\Psi}' - \frac{\eta - 1}{\eta} \Psi' \right)}^{\text{TFP channel}} d \ln \mathbf{Z}$$

Production channel (Baqaee-Fahri)

- ▶ Forward Domar weight, $\tilde{\Psi}' = (\Gamma' + \Delta' \Lambda) (I - \nu \Omega)^{-1}$, $\Delta = \frac{P_m \circ \alpha \circ \kappa \circ m}{P_C C}$
(output cumulative effects)

- First round effects prop. to final demand elast., Γ' and $\Delta' \Lambda$
- All subsequent rounds prop. to intermediate input elast., $\nu \Omega$

$$(\Gamma' + \Delta' \Lambda) (I + \nu \Omega + (\nu \Omega)^2 + \dots) = (\Gamma' + \Delta' \Lambda) (I - \nu \Omega)^{-1}$$

- ▶ (Backward) Domar weight, $\Psi' = (\Gamma' + \Delta' \Lambda) \left(I - \nu \frac{\eta - 1}{\eta} \Omega \right)^{-1}$
(labor or gross output)
- direct: labor used by sector
- indirect: labor used by all sectors supplying to the sector

Markup distortions \rightarrow gross labor shares and final demand elast. differ!

Subsidy and TFP elasticity of Aggregate Consumption

Consider independent changes to $d \ln \mathbf{r}$ and $d \ln \mathbf{Z} = \beta \text{diag}(\mathbf{M}) d \ln \mathbf{a}$,

$$d \ln C = \overbrace{\left(\tilde{\Psi}' - \Psi' \right)}^{\text{production channel}} d \ln \mathbf{r} + \overbrace{\left(\tilde{\Psi}' - \frac{\eta - 1}{\eta} \Psi' \right)}^{\text{TFP channel}} d \ln \mathbf{Z}$$

TFP channel:

- ▶ Same logic that in production channel, but cost *discounted* by inverse of markups
- ▶ resource cost is the product of the cost per unit of adoption \times reciprocal of adoption elast. of TFP, $(\partial \ln Z_s / \partial \ln a_s)^{-1}$
 - Cost per unit of adoption is given by profits of marginal adopter, prop. to $1/\eta$
 - Increase in adoption required to implement a given change in TFP is prop. to $\eta - 1$
 - Fall in η , lower change in adoption required for a given change in TFP

→ lower η implies that fewer resources are needed to increase sector TFP!

Subsidy elasticity of adoption

- Log-differentiating the marginal adopter's condition,

$$d \ln \mathbf{r} - (\eta - 1) d \ln \mathbf{Z} + d \ln (\mathbf{P} \circ \mathbf{Y}) - \frac{\eta - 1}{\zeta} d \ln \mathbf{a} = -d \ln \mathbf{r}^{\mathbf{a}} + d \ln \mathbf{P}_m$$

- Subsidy elasticity of adoption,

$$d \ln \mathbf{a} = \underbrace{\frac{\zeta}{\eta - 1}}_{\text{direct effect}} \underbrace{\left\{ \mathbf{I} - \frac{\zeta}{\eta - 1} \left[\mathbf{\Lambda} (\mathbf{I} - \nu \mathbf{\Omega})^{-1} - (\eta - 1) \mathbf{I} \right] \beta \text{diag}(\mathbf{M}) - \frac{\zeta}{\eta - 1} \nabla_{\mathbf{P}\mathbf{Y}, \mathbf{a}} \right\}^{-1}}_{\text{amplification}} d \ln \mathbf{r}^{\mathbf{a}}$$

$\equiv \nabla_{\mathbf{a}, \mathbf{r}^{\mathbf{a}}}$

Double-Leontieff inverse!

- Inner: adoption raises TFP, lowering sector prices and thus cost of adoption ... Leontieff Inverse
 - Outer: adoption raises TFP, lowering MC of production in other sectors, fostering adoption in other sectors ... Leontieff Inverse
- Simple case, $\mathbf{M} = m\mathbf{I}$ & $\nabla_{\mathbf{P}\mathbf{Y}, \mathbf{a}} = \mathbf{0}$. Let $d \ln \mathbf{r}^{\mathbf{a}} = \mathbf{1}$,

$$d \ln \mathbf{a} = \frac{\zeta}{\eta - 1} \frac{1}{1 - \left[\frac{1}{1 - \nu} - (\eta - 1) \right] \beta m \frac{\zeta}{\eta - 1}}$$

Multiplier, $\epsilon_s \equiv \left. \frac{d \ln C}{d \ln r_s} \right|_{r=1}$ / (Fiscal cost of policy)

Consider revenue r and adoption subsidies r^a ,

$$\begin{aligned} d \ln C &= (\tilde{\Psi}' - \Psi') d \ln r \\ &+ \left(\tilde{\Psi}' - \frac{\eta - 1}{\eta} \Psi' \right) \beta \text{diag}(M) \nabla_{\alpha, r^a} [I + \nabla_{PY, r} + \Lambda (I - \nu \Omega)^{-1}] d \ln r \\ &+ \left(\tilde{\Psi}' - \frac{\eta - 1}{\eta} \Psi' \right) \beta \text{diag}(M) \nabla_{\alpha, r^a} d \ln r^a \end{aligned}$$

- $\tilde{\Psi}$: influence in production - Acemoglu et al.

(with exogenous productivity: impact of productivity shock to sectoral output)

- $(\nabla_{\alpha, r^a})' \beta \text{diag}(M) \tilde{\Psi}$: influence in adoption - Paco and Nico

(impact of productivity shock to sectoral adoption technology)

Total fiscal cost of policy:

$$r_s : \psi_s, r_s^x : \nu \psi_s, r_s^l : (1 - \nu) \psi_s, r_s^a : \delta_s = \frac{\eta - 1}{\eta} \beta M_s \psi_s < \psi_s$$

PARAMETERIZATION OF MODEL

Roadmap

0. Data (2013 Indian Economic Census, 30 2-digit sectors)

1. Partial information (GMM) estimation

From problem of estab., $\ln l = \ln \tilde{A}_i + \ln \tilde{z}_s + \tilde{\varepsilon}$, $i \in \{t, m\}$

- ▶ reduced-form entry & adoption thresholds identification details on estimation
- ▶ technology gap, ex-ante, ex-post heterogeneity estimates fit: examples
- ▶ Validation w/ proxy measure (traditional vs. modern power) validation

2. Calibration

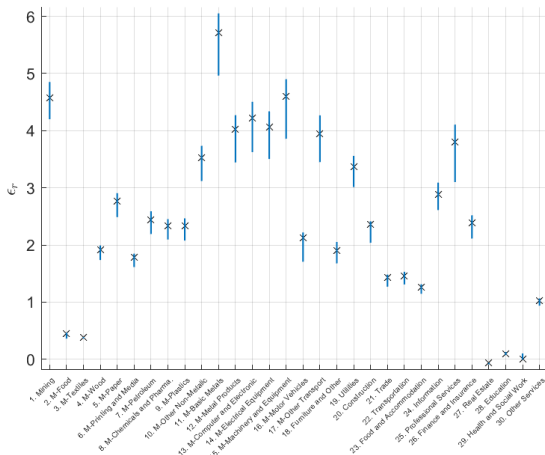
- ▶ Γ, Ω : WIOD, 2010; Λ : vom-Lehm and Winberry, from BEA
- ▶ $\eta = 3$: Broda and Weinstein, Hsieh and Klenow
- ▶ $\nu = 0.75$: match intermediate input share (0.49) in WIOD 2010

3. Full structural calibration estimates

- ▶ Use full model to identify structural costs, $\{\kappa_{ts}, \kappa_{ms}\}$ for all s
(unique mapping! 😊)

SECTORAL (REVENUE) MULTIPLIERS

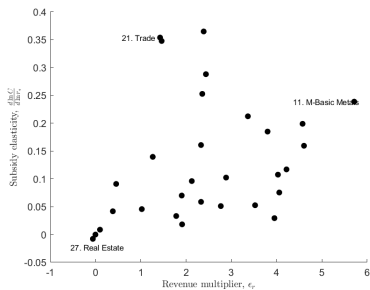
Sectoral (revenue) multipliers



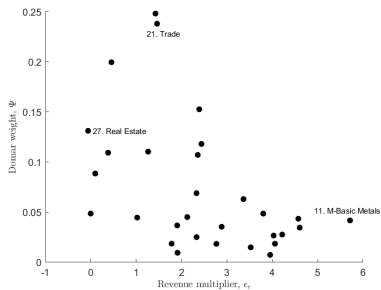
- Estimated multipliers are tight!
- Top engines of development: 11. M-Basic Metals, 15. M-Machinery and Equipment, 1. Mining, 13. M-Computer and Electronic, 14. M-Electrical Equipment

Components of sectoral revenue multipliers

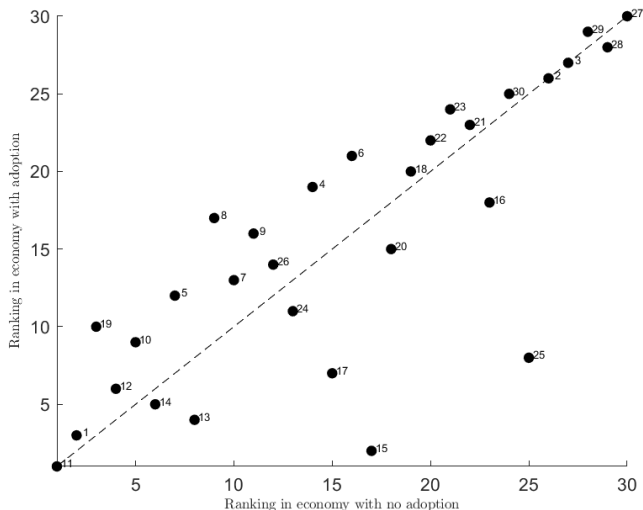
Subsidy elasticity of C



Domar weight, $\tilde{\Psi}$

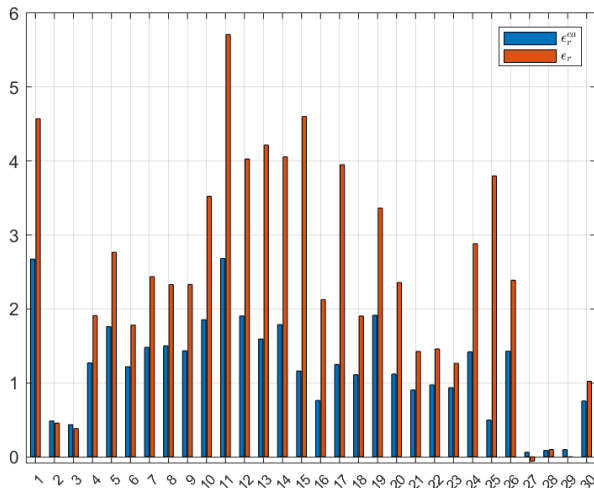


The ranking of multipliers changes with adoption



Top 5: 11, 1, 19, 12 and 10 → Top 5: 11, 15, 1, 13, and 14

Amplification through adoption



$$\mathcal{A} = \left\{ \left(\tilde{\Psi}' - \frac{\eta-1}{\eta} \Psi' \right) \beta \text{diag}(M) \nabla_{a,ra} \left[I + \nabla_{PY,r} + \Lambda (I - \nu\Omega)^{-1} \right] \right\} \otimes \Psi'$$

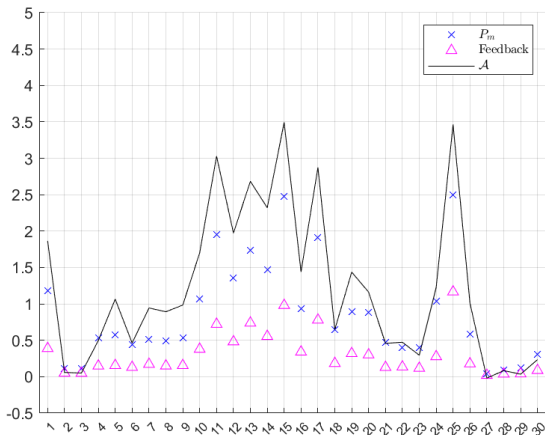
Decomposing amplification through adoption I

$$\begin{aligned}
 \mathcal{A} = & \overbrace{\left(\tilde{\Psi}' \circ \Psi' - \frac{\eta-1}{\eta} \mathbf{1} \right) \beta \text{diag}(M) \frac{\zeta}{\eta-1}}^{\text{direct incentive (SMALL)}} \\
 & + \overbrace{\left[\left(\tilde{\Psi}' - \frac{\eta-1}{\eta} \Psi' \right) \beta \text{diag}(M) \frac{\zeta}{\eta-1} \Lambda (I - \nu \Omega)^{-1} \right] \circ \Psi'}^{\text{price of adoption, } P_m} \\
 & + \overbrace{\left\{ \left(\tilde{\Psi}' - \frac{\eta-1}{\eta} \Psi' \right) \beta \text{diag}(M) [I + \Lambda (I - \nu \Omega)^{-1}] \left(\nabla_{a,r^a} - \frac{\zeta}{\eta-1} I \right) \right\} \circ \Psi'}^{\text{feedback through adoption}} \\
 & + \overbrace{\left[\left(\tilde{\Psi}' - \frac{\eta-1}{\eta} \Psi' \right) \beta \text{diag}(M) \nabla_{a,r^a} \nabla_{PY,a} \right] \circ \Psi'}^{\text{aggregate demand (SMALL)}} ,
 \end{aligned}$$

- ▶ Direct incentive - closely related to Liu's. For 'average' sector,

$$\text{direct incentive} = \overbrace{\epsilon_{rs}^{ea}}^{<3} \overbrace{\beta \bar{m} \frac{\zeta}{\eta-1}}^{=0.115} + \overbrace{\beta \bar{m} \frac{\zeta}{\eta(\eta-1)}}^{=0.038} \in [0.038, 0.383]$$

Decomposing amplification through adoption II

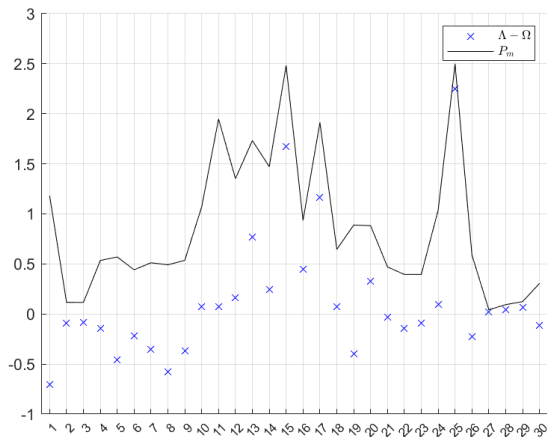


- ▶ Price of adoption channel is crucial!
- ▶ Feedback effect accounts for 20 to 30% in top sectors
(relevance of the Double-Leontieff Inverse)

The price of adoption channel

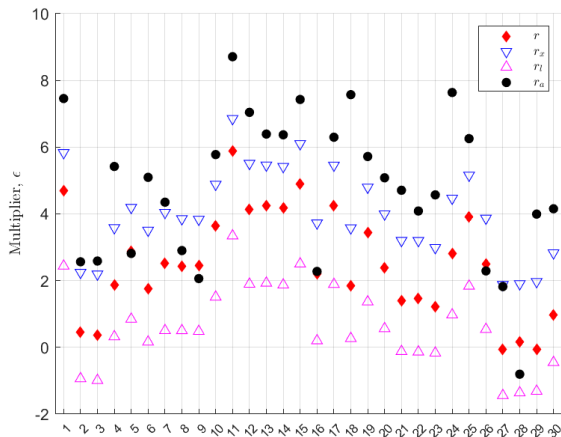
- ▶ How important is centrality in Ω vs Λ ?

$$\Lambda (I - \nu\Omega)^{-1} = \Omega (I - \nu\Omega)^{-1} + (\Lambda - \Omega) (I - \nu\Omega)^{-1}$$



ALTERNATIVE POLICY INSTRUMENTS

Alternative policy instruments



- ▶ Consistent with theory results with no adoption
- ▶ Top sectors always with adoption subsidies: 11, 1, 15, 12 and 13
- ▶ Why? Cost-effective! [details](#)

Concluding remarks

Build a lab. for the study of **industrial policy** in multisector economy

(technology adoption enhances complementarities, generating amplification)

Provide novel insights regarding amplification of policies through adoption

Estimate model of tech. adoption with rich IO architecture,

- ▶ Heterogeneity in technology use across and within sectors

(different shapes in the distributions)

Quantitative results regarding industrial policy,

- ▶ Great amplification through adoption
- ▶ Tech. adoption crucial for our understanding of important sectors
- ▶ Adoption subsidies are the most effective instrument

Related application: Policies to foster adoption of **clean technologies**

Uniform subsidies & Δ TFP, $d \ln \mathbf{r} = d \ln \mathbf{Z} = \mathbf{1}$

$$d \ln C = (1 + \bar{\delta}) \left(\overbrace{\frac{\nu}{1 - \nu}}^{\text{Production}} + \overbrace{\frac{1}{1 - \nu}}^{\text{TFP}} \right) \frac{1}{\eta} \frac{1}{1 - \nu \frac{\eta - 1}{\eta}}$$

- The complexities of the IO structure do not show up! (no Ω !)
- Same as in a one-sector economy with roundabout production
- Lower η or higher ν : higher elasticity
- Positive level effects through both channels

Revenue and adoption multipliers with uniform subsidies

- ▶ Simple case, $M = mI$ & $\nabla_{PY,a} = \nabla_{PY,r} = 0$. Let $d \ln r = d \ln r^a = 1$

$$\epsilon_r^u = \overbrace{\frac{\nu}{1-\nu} \frac{1}{\eta}}^{\text{production efficiency}} + \overbrace{\frac{\eta-1}{\eta} \beta m}_{\in[0,1)} \epsilon_{r^a}^u \overbrace{\left(1 + \frac{1}{1-\nu}\right)}^{>1}$$

- ▶ Revenue subsidy affects both production and TFP channels
- ▶ Revenue subsidy is more expensive than adoption subsidy ($\delta_s < \psi_s$)

→ Which instrument is more effective to promote development?

Alternative Uniform Subsidies with No Adoption

Contribution of r is the sum of contributions of r^x and r^l ,

$$d \ln C = \overbrace{(\tilde{\Psi}' - \Psi') \mathbf{1}}^{\text{revenue}} + \overbrace{\left(\nu \tilde{\Psi}' - \nu \frac{\eta - 1}{\eta} \Psi' \right) \mathbf{1}}^{\text{intermediate inputs}} + \overbrace{\left[(1 - \nu) \tilde{\Psi}' - \left(1 - \nu \frac{\eta - 1}{\eta} \right) \Psi' \right] \mathbf{1}}^{\text{labor}}$$

$$0 = \epsilon_{r,l}^u < \epsilon_{r,r}^u = \frac{\nu}{1 - \nu} \frac{1}{\eta} < \epsilon_{r,x}^u = \frac{1}{1 - \nu} \frac{1}{\eta} \rightarrow \frac{\epsilon_{r,r}^u}{\epsilon_{r,x}^u} = \nu < 1$$

- ▶ Labor: no gains from promoting uniformly the use of a fixed input
Gains are due to reallocating across sectors
(reminiscent of Liu)
- ▶ Intermediate inputs has larger multiplier than revenue
 - + With revenue, the labor effect is zero, and thus only effect goes through intermediate inputs, thus ν
 - + With r^x , only effect through the share of production ν
 - + Therefore, they have same elasticity but revenue has higher cost

Note: With adoption, this may not be true

Uniform Subsidies: Goods Only vs. Labor Only Adoption

- ▶ Uniform revenue subsidy & prod. impact $d \ln \mathbf{r} = \mathbf{1}$ and $d \ln \mathbf{Z} = \mathbf{1}$

$$d \ln C = (\tilde{\Psi}' - \Psi') \mathbf{1} + \left(\tilde{\Psi}' - \frac{\eta-1}{\eta} \Psi' \right) \mathbf{1}$$

- ▶ If adoption good is produced only w/ labor

$$d \ln C = (\tilde{\Psi}' - \Psi') \mathbf{1} + \frac{1}{1-\nu} \underbrace{\left[(1-\nu) \tilde{\Psi}' - \left(1 - \nu \frac{\eta-1}{\eta} \right) \tilde{\Psi}' \right]}_{=0} \mathbf{1}$$

→ Aggregate effects result from reallocating adoption across sectors

- ▶ Symmetric economy with roundabout production, $\Omega = \mathbf{I}$,

$$\Gamma' \left[(1-\nu) (1-\nu)^{-1} \mathbf{I} - \left(1 - \nu \frac{\eta-1}{\eta} \right) \left(1 - \nu \frac{\eta-1}{\eta} \right)^{-1} \mathbf{I} \right] = \mathbf{0}'.$$

→ Each sector's contribution through TFP must be zero!

(reminiscent of Atkeson-Burstein)

→ No gains from reallocating adoption

Partial Information estimation I

From the problem of the firm, $i = t, m$,

$$\ln l = \ln \tilde{A}_i + \ln \tilde{z}_s + \tilde{\varepsilon}$$

(where $\tilde{A}_i \equiv A_i^{\eta-1}$, $\tilde{z}_s \equiv (1-\nu) \frac{\eta-1}{\eta} \frac{P_s Y_s}{Z_s^{\eta-1}} z^{\eta-1}$, and $\tilde{\varepsilon} = \varepsilon^{\eta-1}$)

1. Firm size proportional to GE object (encoded within \tilde{z}_s)
2. $\ln \tilde{z}_s + \tilde{\varepsilon}$: generates unimodal size distributions
3. Selection wrt. who operates A_t or A_m is a force towards two modes
4. Estimate reduced form parameters w/ GMM

Then, employment-size pdf,

$$h_s(l) = l^{-\tilde{\zeta}-1} \frac{\tilde{\zeta}}{\tilde{z}_{ts}^{-\tilde{\zeta}}} e^{\tilde{\mu}\tilde{\zeta} + \frac{\tilde{\chi}^2}{2}\tilde{\zeta}^2} \left\{ \tilde{A}_t^{\tilde{\zeta}} \left[\Phi \left(\frac{\ln \tilde{z}_{ms} - \ln l + \tilde{\mu} + \tilde{\chi}^2 \tilde{\zeta}}{\tilde{\chi}} \right) - \Phi \left(\frac{\ln \tilde{z}_{ts} - \ln l + \tilde{\mu} + \tilde{\chi}^2 \tilde{\zeta}}{\tilde{\chi}} \right) \right] + \tilde{A}_m^{\tilde{\zeta}} \left[1 - \Phi \left(\frac{\ln \tilde{z}_{ms} - \ln l + \tilde{\mu} + \tilde{\chi}^2 \tilde{\zeta}}{\tilde{\chi}} \right) \right] \right\}$$

(where $\tilde{\zeta} \equiv \frac{\zeta}{\eta-1}$, $\tilde{\chi} \equiv (\eta-1)\chi$, $\tilde{\mu} \equiv (\eta-1)\mu = -\tilde{\chi}^2/2$)

So that employment-share pdf,

$$g_s(l) = \frac{lh_s(l)}{\int \tilde{l}h_s(\tilde{l})d\tilde{l}}$$

Partial Information estimation II

- ▶ Assume $A_t = 1$.
- ▶ Estimate via GMM

$$\left(\begin{array}{c} \text{vary across sectors} \\ \{ \tilde{z}_{ts}, \tilde{z}_{ms} \}_{s \in S} \end{array} , \begin{array}{c} \text{common across sectors} \\ \tilde{A}_m, \tilde{\zeta}, \tilde{\chi} \end{array} \right)$$

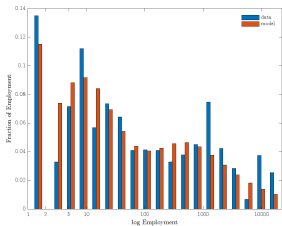
- ▶ Moments:
 - ▶ binned employment shares (40 bins for each of the 30 sectors)
 - ▶ tail coefficient (whole economy)

Partial Information estimation III

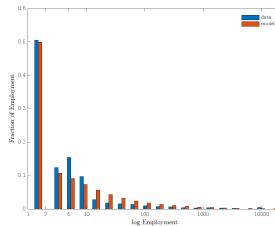
Division	a_s/e_s	M_s	κ_{es}	$\frac{\kappa_{es}}{P_{ms}\kappa_{as}}$
1. Mining	0.021	0.606	1.387	0.023
2. M-Food	0.002	0.358	0.235	0.005
3. M-Textiles	0.001	0.305	0.337	0.004
4. M-Wood	0.000	0.004	0.262	0.000
5. M-Paper	0.534	0.948	0.366	0.179
6. M-Printing and Media	0.000	0.033	0.819	0.000
7. M-Petroleum	0.031	0.648	0.607	0.029
8. M-Chemicals and Pharma.	0.149	0.824	1.745	0.080
9. M-Plastics	0.761	0.978	0.536	0.224
10. M-Other Non-Metallic	0.098	0.780	0.938	0.061
11. M-Basic Metals	0.054	0.711	1.078	0.042
12. M-Metal Products	0.002	0.340	0.810	0.005
13. M-Computer and Electronic	0.070	0.742	1.083	0.049
14. M-Electrical Equipment	0.074	0.748	0.990	0.051
15. M-Machinery and Equipment	0.011	0.532	0.624	0.016
16. M-Motor Vehicles	0.227	0.868	8.340	0.104
17. M-Other Transport	0.100	0.782	50.892	0.062
18. M-Furniture and Other	0.000	0.000	0.422	0.000
19. Utilities	0.024	0.622	0.640	0.025
20. Construction	0.001	0.277	0.369	0.003
21. Trade	0.000	0.003	0.156	0.000
22. Transportation	0.000	0.209	0.125	0.002
23. Food and Accommodation	0.000	0.005	0.391	0.000
24. Information	0.000	0.000	0.585	0.000
25. Professional Services	0.015	0.568	0.577	0.019
26. Finance and Insurance	0.617	0.961	0.185	0.196
27. Real Estate	0.001	0.313	0.316	0.004
28. Education	0.994	0.999	0.282	0.265
29. Health and Social Work	0.000	0.000	0.614	0.000
30. Other Services	0.000	0.002	0.125	0.000

Partial Information estimation IV: examples

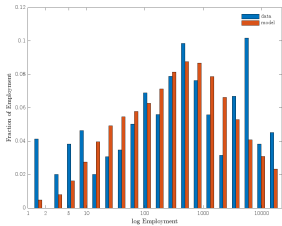
1. Mining (0.021)



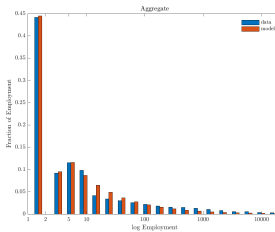
4. M-Wood (0.000)



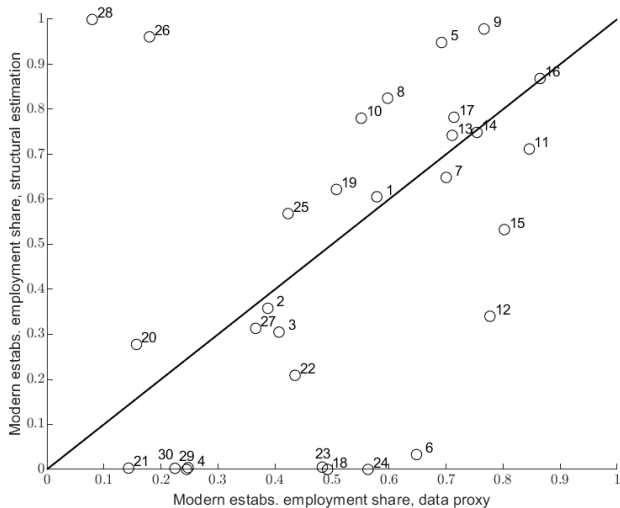
16. M-Motor vehicles (0.227)



Whole economy



Partial information estimation V: adoption proxy



Parameters of full model

- ▶ Given $\eta = 3$,

Ex-ante heterogeneity, ζ	Ex-post heterogeneity, χ	Technology, A_m
3.16 [3.03, 3.19]	0.57 [0.49, 0.60]	2.18 [2.01, 2.31]

(95% confidence intervals computed with bootstrap)

→ Given $\{\tilde{z}_{ts}, \tilde{z}_{ms}\}_{\text{for all } s}, \dots$, uniquely back out costs

$$\{k_{ts}, k_{ms}\}_{s \in \text{2-digit}}$$

Costs consistent with the *relative size of sectors* (through ν, Γ, Ω and Λ), and the *size distribution within sectors* (through $\{\tilde{z}_{ts}, \tilde{z}_{ms}\}_{\text{for all } s}, \tilde{A}_m, \tilde{\zeta}, \dots$), given η

Key insight: given all these other objects, linear set of equations on the structural costs

Decomposing adoption multipliers

$$\epsilon_{r,a} = \overbrace{\left(\tilde{\Psi}' \oslash \Psi' - \frac{\eta-1}{\eta} \mathbf{1} \right) \frac{\eta}{\eta-1} \frac{\zeta}{\eta-1}}^{\text{direct incentive}} + \overbrace{\left[\left(\tilde{\Psi} - \frac{\eta-1}{\eta} \Psi' \right)' \beta \text{diag}(M) \left(\nabla_{a,r^a} - \frac{\zeta}{\eta-1} I \right) \right] \oslash \Delta'}^{\text{feedback through adoption}}.$$

- ▶ Direct incentive is guaranteed to be large here,

$$\text{direct incentive, } r^a = \epsilon_{rs}^{ea} \overbrace{\frac{\eta}{\eta-1} \frac{\zeta}{\eta-1}}{=2.37} + \overbrace{\frac{\eta}{\eta-1} \frac{\zeta}{\eta(\eta-1)}}{=0.79}.$$

(only difference is size of fiscal cost!)

Decomposing adoption multipliers

