Sectoral Development Multipliers*
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Francisco Buera
Washington University in St. Louis

Nicholas Trachter
Federal Reserve Bank of Richmond

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Abstract

How should industrial policies be directed to reduce distortions and modernize the structure of production? We study this question in a multi-sector model with technology adoption where the production of goods and modern technologies feature a rich network structure. We provide formulas for the policy multipliers, characterize the centrality of sectors in terms of both production and technology adoption, and provide insights regarding the power of alternative policy instruments. We devise a simple procedure to estimate the model parameters and the distribution of technologies across sectors, which we apply to Indian data. We find that technology adoption changes the ranking of priority sectors for industrial policy and that adoption subsidies are a more efficient way to promote development than revenue subsidies.

*Buera: fjbuera@wustl.edu. Trachter: trachter@gmail.com. ADD. The views expressed herein are those of the authors and do not necessarily represent the views of the Federal Reserve Bank of Richmond or the Federal Reserve System.

1 Introduction

There is consensus that the lack of economic development is the result of distortions that imply an inefficient organization of production and low incentives to modernize the production techniques of firms (Rosenstein-Rodan, 1943; Hsieh and Klenow, 2009; Restuccia and Rogerson, 2008). When designing industrial policies to overcome these inefficiencies, to the extent that sectors in the economy are asymmetric and interconnected, development calls for sectoral policies. The role of industrial policies as a instrument to improve the organization efficiency of production has recently been analyzed in the literature (Baqaee and Farhi, 2020; Liu, 2019). But the study of industrial policies in settings where these inefficiencies interplay with technology adoption in environments with a rich input-output structure is largely missing.[1] This paper fills this gap.

We begin by laying out a model where monopolistically competitive establishments across different sectors produce intermediate inputs that are then combined to produce the final consumption bundle, intermediate input bundles used by other establishments as a production input, and to produce adoption goods. Establishments can operate a traditional technology or pay for the adoption goods and modernize their production techniques. Sectors are interconnected as intermediate input bundles used by different sectors combine intermediate inputs in different ways, and because the adoption good used by different sectors also combine intermediate inputs in different ways. We provide approximated analytical expressions for sectoral development multipliers, which are the elasticity of aggregate consumption to a subsidy in a particular sector divided by the cost of running the policy. We relate to the results in Acemoglu et al. (2012) and show how influence in production and in technology adoption differ, and how influence in adoption encompasses two Leontieff Inverses rather than the standard one inverse. We also provide novel insights regarding the power of multipliers, and we show that this crucially relates to whether the key inputs are in fixed or variable supply.

We then apply the theory to the India economy. We devise a simple yet powerful estimation procedure to estimate the model parameters. With the estimated model at hand, we study quantitatively the extent by which technology adoption changes our understanding of which sectors are most relevant, in terms of revenue-based development multipliers (Baqaee and Farhi, 2020). We find that the adoption margin is highly relevant, amplifying the multiplier effects in some sectors, and reducing the multiplier effect in other sectors. Perhaps surprisingly, we find that the sectors with the largest multipliers are not those with the highest influence in production and/or adoption. Rather, the sectors with high revenue-based multipliers are small sectors with intermediate influence measures. Because the sectors are small, the gains from the policies dwarf its cost.

Revenue-based policies are useful as promote economic development by directly affecting inefficiencies and motivating firms to adopt modern techniques. However, they are an ineffective vehicle for economic development as they require a high cost to run. We show that adoption subsides, while more modest in their objectives, provide more 'bang for the buck' in the sense that the adoption-based subsidy elasticity

[1] Buera et al. (2021) evaluates an industrial policy in a fully symmetric environment where both margins are present.

gains are much larger than the cost of running the industrial policy.

2 Framework

We consider a multisector economy where establishments produce differentiated varieties within a sector to be used in the production of intermediate inputs and the final consumption aggregate. The economy is populated by a measure L of individuals who supply a unit of labor inelastically and a measure 1 of potential entrants in each sector $s \in S$.[2]

Within each sector s, each establishment has the ability to produce a differentiated product j. Production of the differentiated goods y_{js} is aggregated to the sector level output Y_s using a Dixit-Stiglitz aggregator,

$$Y_s = \left(\int y_{js}^{\frac{\eta-1}{\eta}} \, dj \right)^{\frac{\eta}{\eta-1}} \ , \ \eta > 1 \ ,$$

where η denotes the elasticity of substitution across the differentiated varieties produced within sector s. The output of all sectors is then combined to create aggregate consumption and the intermediate input aggregate,

$$C = \prod_s (C_s)^{\gamma_s} \ , \ X_s = \prod_{s'} (X_{ss'})^{\omega_{ss'}} \ ,$$

with $\gamma_s \in [0, 1]$, $\mathbf{\Gamma} = [\gamma_s]_{S\times 1}$ denoting a column vector collecting all γ_s, with $\mathbf{1}_{1\times S}\mathbf{\Gamma} = 1$. Likewise, let $\mathbf{\Omega} = [\omega_{ss'}]_{S\times S}$ denote a matrix collecting all $\omega_{ss'}$.

Potential entrants in sector s are endowed with productivity z, where $F(z)$ denotes the cumulative distribution of z within each sector and $f(z) \equiv \partial F(z)/\partial z$ denotes its density. We further assume that $F(z)$ is Pareto, with parameter $\zeta > 0$. Establishments can be either inactive or active. Inactive establishments do not operate and remain out of the market. Active establishments face an extra productivity shock e^{ε}, where we assume that ε is normally distributed with mean μ and standard deviation χ. Further, we set $\mu = -(\eta - 1)\chi^2/2)$.[3] An active establishment in sector s, endowed with productivity duple $\{z, \varepsilon\}$ and using technology $i \in \{t, m\}$, produces output y_s by combining labor l and the intermediate aggregate x according to the following Cobb-Douglas production function

$$y_{js} = \frac{zA_i e^{\varepsilon} x^{\nu} l^{1-\nu}}{\nu^{\nu} (1-\nu)^{(1-\nu)}} \ , \ \nu \in [0, 1] \ ,$$

where ν denotes the intermediate aggregate input elasticity, and where A_i is a technology shifter specific to i, with $A_t < A_m$. We refer to $i = t$ as the *traditional* technology and to $i = m$ as the *modern* technology.

[2] The framework is a multisector extension of model in Buera et al. (2021).

[3] This normalization for the mean of the ex-post distribution provides that ex-post heterogeneity does not affect equilibrium aggregates in the economy. Under this normalization, $E_{\varepsilon}[e^{(\eta-1)\varepsilon}] = 1$. Ex-post heterogeneity will be important for matching the size distribution of establishments implied by the model with that one observed in the data.

An active establishment in sector s operating the traditional technology must incur in an entry cost κ_{ts}. An active establishment in sector s operating the modern technology must incur in the entry cost κ_{ts} and modern technology adoption cost $P_{ms}\kappa_{ms}$, where P_{ms} denotes the price of the intermediate aggregate used to produce the adoption good used by sector s. The adoption good in sector s is produced by a representative competitive establishment that combines the output of the different sectors using the technology flow matrix $\Lambda = [\lambda_{ss'}]$. Likewise, the entry cost in sector s is also produced by a representative establishment, converting labor one-to-one into the entry cost.

Finally, we consider a variety of different instruments aimed at improving industrial outcomes: sector revenue subsidies r_s, sector labor cost subsidies r_s^l, sector intermediate input cost subsidies r_s^x, and sector technology adoption subsidies r_s^a. We stack all revenue subsidies in a column vector, $\boldsymbol{r} = [r_s]_{S\times 1}$. We stack all other subsidies in similar vectors. The government levies lump-sum taxes from consumers and runs a balanced budget.

Demand and Price Indexes. The economy has two relevant layers. In the first layer, the inner layer, the output of differentiated establishments is aggregated to produce sector output Y_s. Let P_s denote the sector price index corresponding to this output. Straightforward calculations provide that the demand for the output of a establishment producing variety j is given by $y_{js} = \left(\frac{p_{js}}{P_s}\right)^{-\eta} Y_s$, and the price index satisfies $P_s = (\int p_{js}^{1-\eta} dj)^{1/(1-\eta)}$. In the second layer, the outer layer, the output of the different sectors is aggregated to produce the consumption bundle C and the intermediate aggregate of each sector X_s, $s = 1, ..., S$. As before, it is immediate to obtain that final demand for the output of sector s satisfies $C_s = \gamma_s \frac{P_c}{P_s} C$, where the price index is defined as $P_c = \prod_s (P_s)^{\gamma_s}$. Likewise, we can obtain the intermediate demand by sector s for the output of sector s', $X_{ss'} = \omega_{ss'} \frac{P_{xs}}{P_{s'}} X_s$, where $P_{xs} = \prod_{s'} (P_{s'})^{\omega_{ss'}}$ and $P_{ms} = \prod_{s'} (P_{s'})^{\lambda_{ss'}}$.

The problem of a intermediate input producer. The operating profits of an active establishment using technology i in sector s are given by

$$
\begin{aligned}
\pi_{is}^o(z,\varepsilon) = & \max_{p,x,l} r_s p \left(\frac{p}{P_s}\right)^{-\eta} Y_s - \frac{P_{xs}}{r_s^x} x - \frac{1}{r_s^l} l \\
& \text{subject to } \frac{z A_i e^{\varepsilon} x^{\nu} l^{1-\nu}}{\nu^{\nu}(1-\nu)^{(1-\nu)}} \geq \left(\frac{p}{P_s}\right)^{-\eta} Y_s \ ,
\end{aligned}
$$

where the wage $w = 1$. That is, for a given productivity duple $\{z, \varepsilon\}$ and technology choice i, an active establishment in sector s chooses price p, intermediate inputs x and labor l in order to maximize profits, where the revenue of the establishment is affected by the gross subsidy r_s.

Using the first order conditions with respect to p, x and l we obtain the following expressions for prices and factor demands,

$$p_{is}(z,\varepsilon) = \frac{1}{r_s}\frac{\eta}{\eta-1}\left(\frac{1}{r_s^l}\right)^{1-\nu}\left(\frac{P_{xs}}{r_s^x}\right)^{\nu}\frac{1}{A_i e^{\varepsilon} z}\ , \tag{1}$$

$$l_{is}(z,\varepsilon) = (1-\nu)\left(\frac{1}{r_s}\frac{\eta}{\eta-1}\right)^{-\eta} r_s^l \frac{\left(A_i e^{\varepsilon} z\right)^{\eta-1} P_s^{\eta} Y_s}{\left(\left(\frac{1}{r_s^l}\right)^{1-\nu}\left(\frac{P_{xs}}{r_s^x}\right)^{\nu}\right)^{\eta-1}}\ , \tag{2}$$

$$x_{is}(z,\varepsilon) = \frac{\nu}{1-\nu}\frac{r_s^x}{P_{xs}}\frac{l_{is}(z,\varepsilon)}{r_s^l}\ . \tag{3}$$

Using the normalization of the distribution of ex-post heterogeneity, $\mu = (\eta-1)\chi^2/2$, expected operating profits for an active establishment in sector s, with ex-ante productivity z, operating technology $i = t, m$, are given by

$$\pi_{is}^o(z) \equiv \mathbb{E}_{\varepsilon}\left[\pi_{is}^o(z,\varepsilon)\right] = r_s^{\eta}\frac{1}{\eta}\left(\frac{\eta-1}{\eta}\right)^{\eta-1}\frac{\left(A_i z\right)^{\eta-1} P_s^{\eta} Y_s}{\left[\left(\frac{1}{r_s^l}\right)^{1-\nu}\left(\frac{P_{xs}}{r_s^x}\right)^{\nu}\right]^{\eta-1}}\ . \tag{4}$$

Operating profits are continuous in establishment productivity z. Also, given that $\eta > 1$, operating profits $\pi_{is}^o(z)$ are increasing in z. As a result, optimal entry and adoption decisions are given by threshold rules $\{z_{ts}, z_{ms}\}_{\forall s}$. A establishment is active and operates the traditional technology in sector s if and only if $z_{ts} \leq z < z_{ms}$, and is active and operates the modern technology if and only $z \geq z_{ms}$. For each sector s, the marginal entrant z_{ts} and marginal adopter z_{ms} satisfy

$$\pi_{ts}^o(z_{ts}) = \kappa_{ts} \text{ and,} \tag{5}$$

$$\pi_{ms}^o(z_{ms}) - \pi_{ts}^o(z_{ms}) = \frac{P_{ms}}{r_s^a}\kappa_{ms}\ . \tag{6}$$

The next lemma summarizes an important, albeit evident, relationship between the thresholds.

Lemma 1 *For all s, $z_{ms} \geq z_{ts}$. If $\kappa_{ms} > 0$, $z_{ms} > z_{ts}$.*

In other words, for establishments that decide to be active in sector s, i.e. those with $z \geq z_{ts}$, only a fraction of them operate the modern technology, $\frac{a_s}{e_s} = \frac{1-F(z_{ms})}{1-F(z_{ts})} = \left(\frac{z_{ts}}{z_{ms}}\right)^{\zeta} \leq 1$, where a_s and e_s denote the mass of adopters and entrants in sector s, respectively. And this fraction of adopters is selected: are those with the highest productivity within a sector. This will have important implications for the distribution of activity across establishments.

We now define a symmetric equilibrium,

Definition 1 *Given vector of subsidies $\boldsymbol{r}$, $\boldsymbol{r^l}$, $\boldsymbol{r^x}$ and $\boldsymbol{r^a}$, a symmetric equilibrium consists of thresholds $\{z_{ts}, z_{ms}\}_{s\,\in\,S}$, demand for labor and intermediate inputs by the different active establishments, prices*

$\{P_s, P_{xs}, P_{ms}\}_{s\,\in\,S}$ and P_c, and a level of aggregate consumption C, such that establishments maximize profits, markets clear,

$$Y_s = \gamma_s \frac{P_c}{P_s} C + \sum_{s'} \omega_{s's} \frac{P_{xs'}}{P_s} X_{s'} + \sum_{s'} \lambda_{s's} \frac{P_{ms'}}{P_s} a_{s'} \kappa_{ms'} \ , \tag{7}$$

$$L = \sum_{s} L_s + \sum_{s} e_s \kappa_{ts} \ , \tag{8}$$

where $X_s \equiv \int_{z_{ts}}^{z_{ms}} x_{ts}(z)dz + \int_{z_{ms}}^{\infty} x_{ms}(z)dz$ and $L_s \equiv \int_{z_{ts}}^{z_{ms}} l_{ts}(z)dz + \int_{z_{ms}}^{\infty} l_{ms}(z)dz$, with $x_{is}(z) = \mathbb{E}_\varepsilon[x_{is}(z,\varepsilon)]$ and $l_{is}(z) = \mathbb{E}_\varepsilon[l_{is}(z,\varepsilon)]$.

Adoption and Sectoral Productivity. Aggregating the output of establishments in sector s we obtain an expression for the sector output of sector s, Y_s, as a function of sectoral inputs, X_s and L_s,

$$Y_s = \frac{1}{\nu^\nu (1-\nu)^{1-\nu}} Z_s X_s^\nu L_s^{1-\nu} \ ,$$

where Z_s is a neutral Total Factor Productivity (TFP) shifter in sector s, defined as

$$\begin{aligned} Z_s &\equiv \left(A_t^{\eta-1} \int_{z_{ts}}^{z_{ms}} z^{\eta-1} f(z)dz + A_m^{\eta-1} \int_{z_{ms}}^{\infty} z^{\eta-1} f(z)dz \right)^{\frac{1}{\eta-1}} \\ &= \left[A_t^{\eta-1} e_s^{1-\frac{\eta-1}{\zeta}} + \left(A_m^{\eta-1} - A_t^{\eta-1} \right) a_s^{1-\frac{\eta-1}{\zeta}} \right]^{\frac{1}{\eta-1}} \ . \end{aligned} \tag{9}$$

We let $\boldsymbol{Z} = [Z_s]_{S\times 1}$ be a vector collecting all sector productivities.

Using (1) on the definition of the price index P_s, we can obtain a condition relating the sector price P_s, the price of the intermediate aggregate P_{xs}, and sector productivity Z_s,

$$P_s = \frac{1}{r_s} \frac{\eta}{\eta-1} \frac{\left(\frac{1}{r_s^l}\right)^{1-\nu} \left(\frac{P_{xs}}{r_s^x}\right)^{\nu}}{Z_s} . \tag{10}$$

A key feature of our economy is that, through adoption, sector productivity is endogenous. Higher adoption in sector s increases the share of value-added within the sector produced by modern establishments and, thus, sector productivity. In particular, the elasticity of sector TFP to a change in the mass of adopters is given by

$$\frac{\partial \ln Z_s}{\partial \ln a_s} = \frac{1}{\eta-1} \frac{\zeta + 1 - \eta}{\zeta} \frac{A_m^{\eta-1} - A_t^{\eta-1}}{A_m^{\eta-1}} M_s \equiv \beta M_s > 0 \ . \tag{11}$$

The value M_s represents the share of value-added in sector s attributed to modern establishments in the sector, and $\boldsymbol{M} = [M_s]_{S\times 1}$ collects all shares. The first expression shows how changes in adoption are converted into changes in sector TFP. Importantly, the elasticity is higher (i) the larger the productivity wedge among the two technologies, (i) the lower the elasticity of substitution and heterogeneity within a sector, as captured by η and $1/\zeta$, which are key determinants of complementarities across firms, (see Buera et al., 2021), and (iii) the higher is the share of value-added of modern establishments in the sector M_s. In addition, (11) implies that sectors with low M_s will have a muted effect of a given percent change in the adoption rate on sectoral aggregate productivity Z_s.

In turn, changes in sectoral productivity and policies affect sectoral and factor prices as these changes percolates through the production network. We combine (10) with the definition of the price index P_{xs} to obtain,

$$\ln \boldsymbol{P} = \mathbf{1}\frac{1}{1-\nu}\ln\frac{\eta}{\eta-1} - (\boldsymbol{I}-\nu\boldsymbol{\Omega})^{-1}\left(\ln \boldsymbol{Z} + \ln \boldsymbol{r} + (1-\nu)\ln \boldsymbol{r^l} + \nu \ln \boldsymbol{r^x}\right) \ . \tag{12}$$

where we used that $(\boldsymbol{I}-\nu\boldsymbol{\Omega})^{-1}\mathbf{1} = \mathbf{1}\frac{1}{1-\nu}$. Because of the IO structure of production, sector prices compound all sector aggregate TFP levels and markup distortions $\eta/(\eta-1)$ through the Leontieff inverse $(\boldsymbol{I}-\nu\boldsymbol{\Omega})^{-1} = \left(\boldsymbol{I}+\nu\boldsymbol{\Omega}+\nu^2\boldsymbol{\Omega}^2+\nu^3\boldsymbol{\Omega}^3+...\right)$.

From the definition of the consumption price index P_c and technology adoption price index P_{ms}, we obtain the following relationships,

$$d\ln P_c = \boldsymbol{\Gamma}' d\ln \boldsymbol{P}, \text{ and } d\ln \boldsymbol{P_m} = \boldsymbol{\Lambda} d\ln \boldsymbol{P} \ , \tag{13}$$

relating changes in sector prices to changes in the consumption bundle and technology adoption price indexes.

3 Sectoral Development Multipliers

In this section we study the mechanism by which technology adoption affects industrial policy. To keep things as simple as possible, our benchmark analysis follows Baqaee and Farhi (2020) and only considers revenue subsidies $\boldsymbol{r}$ (that is, $\ln \boldsymbol{r^l} = \ln \boldsymbol{r^x} = \ln \boldsymbol{r^a} = \mathbf{0}$). Later we show how the different subsidies operate through similar channels. Finally, in Section 4.5 we study quantitatively which subsidy provides the highest multipliers and thus can be considered as the most effective instrument for industrial policy.

To maintain things as simple as possible in this section, we abstract from the entry margin, thus treating e_s as an exogenous object across all s and abstract from the entry costs. We do this to provide a more transparent analysis of the adoption margin, which is the main focus of the paper. The entry margin is relevant for matching the data, but it is somewhat orthogonal to endogenous technology adoption. In fact, in our quantitative analysis, as shown in Table III, we show that the entry margin accounts for a very small fraction of the development multipliers, and thus little is lost by not accounting for this margin.[4] Still, we emphasize that we do take the entry margin into account when we quantify the multipliers using data.

[4]Follows from comparing the columns for ϵ and ϵ^e.

We follow Liu (2019) and define the sectoral development multiplier of a revenue subsidy in sector s as

$$\epsilon_{r,s} \equiv \overbrace{\left.\frac{d\ln C}{d\ln r_s}\right|_{\boldsymbol{r}=\mathbf{1}}}^{\text{subsidy elasticity}} /(\text{Fiscal cost of policy}) \ . \tag{14}$$

Basically, the multiplier accounts for the elasticity of aggregate consumption C to a revenue subsidy in sector s—the subsidy elasticity, r_s, around the equilibrium with no subsidies, $\ln \boldsymbol{r} = \mathbf{0}$, relative to the fiscal cost of the policy. A multiplier ϵ_s above one implies that an industrial policy in sector s increases aggregate consumption by more than the fiscal cost, a multiplier equal to one is neutral, a positive multiplier lower than one implies that the increase in aggregate consumption is lower than the fiscal costs, and a negative multiplier implies that aggregate consumption falls as result of the policy.[5]

3.1 Subsidy and Productivity Elasticity of Aggregate Consumption

Let $\delta_s = \frac{P_{ms} a_s \kappa_{ms}}{P_c C}$ denote adoption share of GDP in sector s, with $\boldsymbol{\Delta} = [\delta_s]_{S\times 1}$ a column vector collecting these shares. We define the Domar and Forward Domar weights in the economy as

$$\boldsymbol{\Psi_p}' = \left(\boldsymbol{\Gamma}' + \boldsymbol{\Delta}'\boldsymbol{\Lambda}\right)\left(\boldsymbol{I} - \nu\frac{\eta-1}{\eta}\boldsymbol{\Omega}\right)^{-1} \ , \ \tilde{\boldsymbol{\Psi}}_{\boldsymbol{p}}' = \left(\boldsymbol{\Gamma}' + \boldsymbol{\Delta}'\boldsymbol{\Lambda}\right)\left(\boldsymbol{I} - \nu\boldsymbol{\Omega}\right)^{-1} \ . \tag{15}$$

The Forward Domar weight is usually referred in the literature as the Cost Domar weight. Our choice of language will be clear in the discussion that follows.

Proposition 1 *Consider independent vectors of changes in revenue subsidies $d\log \mathbf{r}$ and changes in sector TFP induced by changes in adoption $d\log \mathbf{Z} = \beta\ \mathtt{diag}\left(\boldsymbol{M}\right) d\ln \boldsymbol{a}$, then*

$$d\ln C = \overbrace{\left(\tilde{\boldsymbol{\Psi}}_{\boldsymbol{p}}' - \boldsymbol{\Psi_p}'\right)}^{\textit{production channel}} d\ln \boldsymbol{r} + \overbrace{\left(\tilde{\boldsymbol{\Psi}}_{\boldsymbol{p}}' - \frac{\eta-1}{\eta}\boldsymbol{\Psi_p}'\right)}^{\textit{TFP channel}} d\ln \boldsymbol{Z} \ . \tag{16}$$

The term $\tilde{\boldsymbol{\Psi}}_{\boldsymbol{p}}' - \boldsymbol{\Psi_p}'$ gives the aggregate consumption gains that follow from changes in the allocation of resources across sectors. Ignoring the resources used, the change in the policy induces changes in the quantity produced by the different sectors. The first round effects of these changes are proportional to the final demand elasticities, which are measured by the consumption elasticities vector $\boldsymbol{\Gamma}'$ and the adoption elasticities $\boldsymbol{\Delta}'\boldsymbol{\Lambda}$. The effects from the subsequent rounds are proportional to the intermediate input elasticities, which are measured by the matrix $\nu\boldsymbol{\Omega}$, and its subsequent powers, which themselves give the increase in the output of all sectors as a results of the greater availability of inputs from the promoted industries. The

[5] While in our setting lump-sum taxation would imply that any positive subsidy elasticity would suffice to motivate the use of the instrument, one could consider that there are alternative uses for the income taxed away from consumers. In such a setting, taking into account the fiscal cost of the policy is appropriate.

impact of these subsequent rounds are ultimately also proportional to the aggregate demand elasticities, thus $(\mathbf{\Gamma}' + \mathbf{\Delta}'\mathbf{\Lambda})\left(\nu\mathbf{\Omega} + (\nu\mathbf{\Omega})^2 + ...\right)$. The Forward Domar weight $\tilde{\mathbf{\Psi}}_{\boldsymbol{p}}$ accounts for all these cumulative effects.[6] Increasing production requires reallocating labor across sectors. The cost of the reallocation is measured by the Domar weight $\mathbf{\Psi}_{\boldsymbol{p}}$, which accounts for the labor used by the sector directly and, indirectly, by all sectors supplying inputs to this sector. This cumulates to the labor or Gross Output share of the sector. Because of markup distortions, gross labor shares and cumulative final demand elasticities differ, and thus $\tilde{\mathbf{\Psi}}_{\boldsymbol{p}}{}' - \mathbf{\Psi}_{\boldsymbol{p}}{}'$ can be positive. The larger is the markup, the larger the divergence between these vectors, and larger the gains. When the markup approaches zero, i.e. $\eta/(\eta-1) \to 1$, forward and backward Domar weights are equal and thus the production channel of the subsidy elasticity is zero. Overall, gains are large when a sector has large (forward) cumulative final demand elasticities and small (backward) cumulative resource shares.

Endogenous technology adoption affects the subsidy elasticity by inducing a change in sector productivities $\boldsymbol{Z}$, the TFP channel. The term $\tilde{\mathbf{\Psi}}_{\boldsymbol{p}}{}' - \frac{\eta-1}{\eta}\mathbf{\Psi}_{\boldsymbol{p}}{}'$ regulates this margin. Notice that the intuition that we discussed for the production channel also applies here, with the only difference being that the vector of Domar weights, which accounts for the cost of reallocating labor, is now *discounted* by $(\eta-1)/\eta$. This occurs as the total resource of implementing a given increase in TFP combines a direct labor reallocation cost, $\mathbf{\Psi}_{\boldsymbol{p}}{}'$, with the way this labor is transformed into sector productivity through adoption. The reciprocal of the adoption elasticity of TFP, $(\partial \ln Z_s / \partial \ln a_s)^{-1}$, accounts for how adoption is transformed into productivity, and it is proportional to $(\eta - 1)$. Generating this increase in adoption requires changing the identity of the marginal adopter. Thus, the cost per unit of adoption are given by the profits of the marginal adopter, which are proportional to $1/\eta$. The ratio $(\eta-1)/\eta$ combines the marginal rate of transformation from adoption to sector productivity with its marginal cost. As the elasticity of substitution declines, a lower increase in adoption is required to implement a given change in TFP, relative to the cost per unit of adoption. Therefore, as the elasticity of substitution declines, fewer resources are needed to increase sectoral productivity.

The entries of the vectors $(\tilde{\mathbf{\Psi}}_{\boldsymbol{p}}{}' - \mathbf{\Psi}_{\boldsymbol{p}}{}')$ and $(\tilde{\mathbf{\Psi}}_{\boldsymbol{p}}{}' - \frac{\eta-1}{\eta}\mathbf{\Psi}_{\boldsymbol{p}}{}')$ in (16) give the differential effects of sector-specific subsidies and TFP changes, respectively. The relative magnitude of each entry depends critically on the details of the production network, given by the matrix $\mathbf{\Omega}$. Aggregating these entries results in the effect of a uniform and unitary policy or change in TFP. The following remark gives a simple expression for the elasticity of aggregate consumption to an uniform unitary change in revenue subsidies and TFP.

Remark 1 *Consider a uniform unitary changes in revenue subsidies and TFP in all sectors,* $d\ln \boldsymbol{r} = \mathbf{1}$ *and* $d\ln \boldsymbol{Z} = \mathbf{1}$*, then the expression in (16) reduces to*

$$d\ln C = \left(1+\bar{\delta}\right)\left(\frac{\nu}{1-\nu} + \frac{1}{1-\nu}\right)\frac{1}{\eta}\frac{1}{1-\nu\frac{\eta-1}{\eta}} \ ,$$

[6] In deriving these expression it is convenient to work with the dual, and trace out the fall in sector and final consumption prices induced by the change in policies—see (12) and (13).

where $\bar{\delta} = \sum_s \delta_s$.

A notable feature about this result is that the expression shows that, under uniform policies, the complexities of the network structure of production do not play any role. Indeed, the gains accrued by the uniform policy and TFP gains are exactly those obtained in a one-sector economy with roundabout production.

3.2 Subsidy Elasticity of Adoption and Development Multiplier

To find an expression relating how changes in the subsidy affect sector productivities, We need to find an expression for $d\ln \boldsymbol{a}$ from the marginal adopter's condition. In this case, it is convenient to consider both revenue and adoption subsidies, $d\ln \boldsymbol{r}$ and $d\ln \boldsymbol{r^a}$.

Log-differentiating the vector of conditions for the marginal adopter in (6), and using (10), we obtain a condition linking adoption, sectoral productivity, gross output, and the price of the adoption good, with the vector of revenue and adoption subsidies,

$$d\ln \boldsymbol{r} - (\eta - 1) d\ln \boldsymbol{Z} + d\ln (\boldsymbol{P} \circ \boldsymbol{Y}) - \frac{\eta - 1}{\zeta} d\ln \boldsymbol{a} = -d\ln \boldsymbol{r^a} + d\ln \boldsymbol{P_m} \;, \tag{17}$$

where the symbol $\circ$ denotes the Hadamard (element-wise) product. Notice how revenue $\boldsymbol{r}$ and adoption $\boldsymbol{r^a}$ subsidies have the same direct effect on adoption. Revenue subsidies also affect adoption indirectly through its effect on sector prices, which affect the aggregate demand channel and, importantly, the vector of prices of the adoption good $\boldsymbol{P_m}$.

A simple case to study is when an adoption subsidy is applied to a sector where the modern share is zero, i.e. $M_s = 0$. The next Remark describes the result.

Remark 2 *Assume that* $M_{\hat{s}} = 0$ *for some sector* $\hat{s}$. *Then,* $\frac{d\ln a_{\hat{s}}}{d\ln r^a_{\hat{s}}} = \frac{\zeta}{\eta - 1}$, *and* $\frac{d\ln a_s}{d\ln r^a_{\hat{s}}} = 0$ *for all* $s \neq \hat{s}$.

The Remark follows immediately after noticing that (11), given that $M_{\hat{s}} = 0$, provides that the elasticity of the sector's TFP to adoption is also zero, and thus $d\ln \boldsymbol{Z} = d\ln (\boldsymbol{P} \circ \boldsymbol{Y}) = d\ln \boldsymbol{P_m} = \boldsymbol{0}$. Adoption increases in sector $\hat{s}$ because of the direct effect of the policy: η measures the elasticity of the distribution of ex-ante heterogeneity, which is normalized by the curvature of the profit function with respect to this productivity, $\eta - 1$. But the percentage increase in adoption in this sector does not translate into a percentage increase in the sector's TFP. As a result, the sector's price index does not vary, and the same occurs to the price of adopting a modern technology in other sectors, and thus adoption in other sectors is unaffected by the policy. A notable implication of Remark 2 is that the modern share, through its effect on the elasticity of TFP to adoption, is a crucial ingredient for understanding how subsidies in one sector percolate to other sectors through adoption.

We now follow to solve for $d\ln \boldsymbol{a}$ from (17), together with the price feedbacks in (12) and (13). Combining the relationships between prices, policies and TFP in (12), (13) and (17), we obtain an expression for the elasticity of adoption $d\ln \boldsymbol{a}$ with respect to an adoption subsidy $d\log \boldsymbol{r^a}$. The next Proposition presents this result.

Proposition 2 *The elasticity adoption with respect to a adoption subsidy is given by*

$$d\ln \boldsymbol{a} = \underbrace{\overbrace{\frac{\zeta}{\eta-1}}^{\textit{direct effect}} \overbrace{\left\{\boldsymbol{I} - \frac{\zeta}{\eta-1}\left[\boldsymbol{\Lambda}\left(\boldsymbol{I}-\nu\boldsymbol{\Omega}\right)^{-1} - (\eta-1)\boldsymbol{I}\right]\beta\texttt{diag}\left(\boldsymbol{M}\right) - \frac{\zeta}{\eta-1}\boldsymbol{\nabla}_{\boldsymbol{PY},\boldsymbol{a}}\right\}^{-1}}^{\textit{amplification}}}_{\equiv \boldsymbol{\nabla}_{\boldsymbol{a},\boldsymbol{r^a}}} d\ln \boldsymbol{r^a} \ . \tag{18}$$

Details on the derivation are available in Appendix A.3. The operator $\texttt{diag}(\cdot)$ converts a column vector into a diagonal matrix, and $\boldsymbol{\nabla}_{\boldsymbol{PY},\boldsymbol{a}}$ denotes the elasticity of $\boldsymbol{P} \circ \boldsymbol{Y}$ with respect to $\boldsymbol{a}$, which is presented in closed-form in the appendix. To keep things simple, we will provide intuition abstracting from this last channel. The elasticity of adoption with respect to an adoption subsidy combines the direct effect of the policy, measured by the term $\zeta/(\eta-1)$, with the amplification effect that follows from the sector interactions. The degree of amplification depends on the structure of the production ($\boldsymbol{\Omega}$) and investment ($\boldsymbol{\Lambda}$) networks, as the effect of policies percolates through them. A subsidy to a sector promotes adoption and TFP in that sector, which in turns lower the sector price, lowering the cost of adoption in all sectors. This leads to a further increase in adoption in other sectors, resulting in further feedback rounds. The magnitude of these multiplier effects depends on parameters governing the elasticity of productivity with respect to adoption given in (11), which are subsumed in β and the vector of modern shares $\boldsymbol{M}$ and, for given values of these quantities, the effects are increasing in the elasticity of substitution and heterogeneity within a sector, as captured by η and $1/\zeta$, which are key determinants of complementarities across firms (see Buera et al., 2021).

In virtually all research exploring the relevance of input-output structures for economic aggregates, the key object is the Leontieff Inverse $(\boldsymbol{I} - \nu\boldsymbol{\Omega})^{-1}$, or linear combinations of it. This occurs as heterogeneity in the way sectors combine sectoral output to use as a intermediate aggregate in production results in heterogenenity in sector price indices. Because the intermediate aggregate used by any sector aggregates the output of all other sectors through $\boldsymbol{\Omega}$, so does sector prices $\boldsymbol{P}$. Absent technology adoption, aggregate effects of policies are the result of comparisons of objects that directly map to the Leontieff Inverse. Technology adoption adds a double inverse to the loop. Adoption increases TFP, which lowers sector prices and thus the cost of adoption. This is the inner inverse. Also, adoption in a sector reduces the marginal cost of production in all sectors, thus increasing the profits accrued from adoption by the marginal adopter. This feedback accounts for the outer loop of the double inverse.

To further understand the role of individual parameters and of the value-added share of modern establishments in shaping the double Leontieff inverse, the next remark characterizes the impact of a uniform unitary change in adoption subsidies on the elasticity of adoption, under the additional restriction that there is no heterogeneity in the modern share of value-added across sectors. This expression is the same one would obtain in a one-sector economy with roundabout production.

Remark 3 *Assume $\boldsymbol{M} = m\boldsymbol{I}$ with $m \geq 0$, and $d\ln(\boldsymbol{P} \circ \boldsymbol{Y}) = 0$. Consider a uniform unitary change in* *adoption subsidies in all sectors,* $d\ln \boldsymbol{r}^{\boldsymbol{a}} = \mathbf{1}$*, then the expression (18) reduces to*

$$d\ln \boldsymbol{a} = \frac{1}{1-\left[\frac{1}{1-\nu}-(\eta-1)\right]\beta m \frac{\zeta}{\eta-1}} \frac{\zeta}{\eta-1} \ . \tag{19}$$

As briefly discussed earlier, the subsidy elasticity of adoption features two nested multipliers. Subsidies promote adoption which enhance productivity, mediated by the elasticity $\beta m\zeta/(\eta-1)$, which lowers sector prices and, therefore, the price of adoption. These effects get amplified as they percolate through the input-output structure. The inner multiplier $1/(1-\nu)$, which is the standard multiplier of a roundabout economy, encodes these effects. The increase in the productivity of competing producers, captured by the term $\eta-1$, partially dampens this effect. The outer multiplier captures the infinite rounds of adoption lowering the marginal cost of production, further enhancing adoption. Finally, $\zeta/(\eta-1)$ is the direct elasticity of adoption with respect to the subsidy, which we described earlier.

The expression in (19) also allows us to see ingredients that, through the adoption channel, can greatly amplify the effect of policies even under uniform policies. While in production economy the multiplier becomes unbounded as ν approaches one, with adoption the multiplier becomes unbounded when ν approaches $1-\frac{1}{\eta-1}\frac{\beta m\zeta}{1+\beta M\zeta}$. Under our calibration later on, this upper bound is always between zero and one. This implies that, through the adoption channel, the multiplier becomes unbounded for lower values of the intermediate input elasticity ν. In other words, through the adoption channel, even the effect of uniform policies can be greatly amplified, and it is possible to generate larger amplification than through the production channel for a wider set of parameter values. In our calibrated economy this is not the case, i.e. amplification through the adoption channel is the result of heterogeneity, which is consistent with the findings in Buera et al. (2021) for a symmetric economy with roundabout production.

Putting together the results from Propositions 1 and 2, we obtain the following expression for the elasticity of aggregate consumption with respect to independent revenue and an adoption subsidies:

$$\begin{aligned} d\ln C = & \left(\tilde{\boldsymbol{\Psi}}_{\boldsymbol{p}}{}' - \boldsymbol{\Psi}_{\boldsymbol{p}}{}'\right) d\ln \boldsymbol{r} & (20)\\ & + \left(\tilde{\boldsymbol{\Psi}}_{\boldsymbol{p}}{}' - \frac{\eta-1}{\eta}\boldsymbol{\Psi}_{\boldsymbol{p}}{}'\right)\beta \,\texttt{diag}\left(\boldsymbol{M}\right)\boldsymbol{\nabla}_{\boldsymbol{a},\boldsymbol{r}^{\boldsymbol{a}}}\left[\boldsymbol{I}+\boldsymbol{\nabla}_{\boldsymbol{PY},\boldsymbol{r}}+\boldsymbol{\Lambda}\left(\boldsymbol{I}-\nu\boldsymbol{\Omega}\right)^{-1}\right] d\ln\boldsymbol{r} & \\ & + \left(\tilde{\boldsymbol{\Psi}}_{\boldsymbol{p}}{}' - \frac{\eta-1}{\eta}\boldsymbol{\Psi}_{\boldsymbol{p}}{}'\right)\beta \,\texttt{diag}\left(\boldsymbol{M}\right)\boldsymbol{\nabla}_{\boldsymbol{a},\boldsymbol{r}^{\boldsymbol{a}}} d\ln \boldsymbol{r}^{\boldsymbol{a}} \ , & (21) \end{aligned}$$

where $\boldsymbol{\nabla}_{\boldsymbol{PY},\boldsymbol{r}}$ denotes the elasticity of $\boldsymbol{P}\circ\boldsymbol{Y}$ with respect to $\boldsymbol{r}$, which is provided in close-form in Appendix A.3.

We note the relevance of the double inverse, present in the key object $\boldsymbol{\nabla}_{\boldsymbol{PY},\boldsymbol{r}}$, as showcased in Proposition 18. This matrix converts subsidies into adoption, which in turn is converted to TFP through $d\ln\boldsymbol{Z} = \beta\texttt{diag}\left(\boldsymbol{M}\right)d\ln\boldsymbol{a}$. Key determinants of this conversion rate is the technology gap $A_m^{\eta-1} - A_m^{\eta-1}$, as discussed in (11). Then, the change in TFP is converted to aggregate consumption through the elasticity

$\left(\tilde{\boldsymbol{\Psi}}_{\boldsymbol{p}}{}' - \frac{\eta-1}{\eta}\boldsymbol{\Psi}_{\boldsymbol{p}}{}'\right)$. We provided a discussion of this term in Proposition 1.

In settings with exogenous productivity, Acemoglu et al. (2012) noted that $\tilde{\boldsymbol{\Psi}}_{\boldsymbol{p}}$ accounts for the Influence matrix. That is, the entry s of the Influence matrix provides the change in aggregate consumption resulting from a shock to exogenous sector productivity Z_s.[7] With endogenous sector TFP, we note that $\tilde{\boldsymbol{\Psi}}_{\boldsymbol{p}}$ accounts for Influence in production, while $\boldsymbol{\nabla}_{\boldsymbol{a},\boldsymbol{r}^a}{}'\beta\texttt{diag}\left(\boldsymbol{M}\right)\tilde{\boldsymbol{\Psi}}_{\boldsymbol{p}}$ accounts for influence in adoption. That is, the entry s of this object accounts for the resulting change in aggregate consumption following from an exogenous shock to the adoption technology.

With elasticities at hand, we need expressions for the fiscal cost of the different policies to be able to produce the multipliers. As in Liu (2019), the fiscal cost of the revenue subsidy is given by the Domar weight $\boldsymbol{\Psi}_{\boldsymbol{p}}$, as this weight provides the size of each sector, and thus, around the equilibrium with no subsidies, the fiscal cost of the subsidy. Therefore, obtaining an expression for ϵ_r for sector s requires dividing the s entry of (20) by the s entry of vector of Domar weights $\boldsymbol{\Psi}_{\boldsymbol{p}}$. Similarly, the fiscal cost of the adoption subsidy in sector s is given by the adoption share of the sector, the s entry of $\boldsymbol{\Delta}$, the fiscal cost of a intermediate input subsidy in sector s is given by the s entry of $\nu\boldsymbol{\Psi}_{\boldsymbol{p}}$, and the fiscal cost of a labor subsidy in sector s is given by the s entry of $(1-\nu)\boldsymbol{\Psi}_{\boldsymbol{p}}$.

The following remark provide expressions for the development multipliers associated with uniform and unitary revenue and adoption subsidies,

Remark 4 *Suppose that $\boldsymbol{M} = m\boldsymbol{I}$ and $d\ln\boldsymbol{P}\circ\boldsymbol{Y} = \boldsymbol{0}$ in the marginal adopter's problem in each sector. Then, under uniform and unitary revenue and adoption subsidies, i.e. $d\ln\boldsymbol{r} = d\ln\boldsymbol{r}^{\boldsymbol{a}}$, we have that*

$$\epsilon_r^u = \frac{\nu}{1-\nu}\frac{1}{\eta} + \frac{\eta-1}{\eta}\beta m\epsilon_{r^a}^u\left(1+\frac{1}{1-\nu}\right),$$

where

$$\epsilon_{r^a}^u = \frac{1}{\eta-1}\frac{1}{1-\nu}\frac{1}{1-\left[\frac{1}{1-\nu}-(\eta-1)\right]\beta m\frac{\zeta}{\eta-1}}\frac{\zeta}{\eta-1}.$$

These expressions highlight the counteracting forces driving the relative importance of the development multiplier associated with revenue and adoption subsidies. On the one hand, revenue subsidies have a direct effect enhancing production efficiency, as captured by the first term of ϵ_r^u. In addition, revenue subsidies affects the adoption incentive directly and indirectly, through their effect on the price of the adoption good, as capture by the last bracketed term. On the other hand, adoption subsidies have an advantage given their lower fiscal cost as the adoption share is smaller than the Domar weight, i.e. $\delta_s = ((\eta-1)/\eta)\beta M_s\Psi_{ps} < \Psi_{ps}$.

[7] Exogenous sectoral productivity implies no adoption, and thus $\boldsymbol{\Delta} = \boldsymbol{0}$. Then, under the restriction that $\boldsymbol{\Gamma}' = \boldsymbol{1}$, we get that $\tilde{\boldsymbol{\Psi}}_{\boldsymbol{p}} = \boldsymbol{\Gamma}'\left(\boldsymbol{I}-\nu\boldsymbol{\Omega}\right)^{-1}$ which is the influence measure described in Acemoglu et al. (2012).

3.3 Additional Results

In this section we present two additional analysis. First, we study the sectoral development multipliers associated with input subsidies, and compare them to the multipliers associated with revenue subsidies. To simplify the analysis, we study them in the case with no adoption. This section sheds light of the relevance of different policy instruments as an engine of development, and will aid on our quantitative analysis contrasting the power of different instruments. Second, we consider a version of the model where the adoption good is produced with labor only. This section sheds light on the importance of using fixed or variable inputs in the production of the adoption good.

3.3.1 Alternative Policy Instruments

Consider the case of revenue $\boldsymbol{r}$, intermediate inputs $\boldsymbol{r^x}$ and labor subsidies $\boldsymbol{r^l}$ in the economy with no adoption. The results in this section will shed light on (i) the differential relevance of the adoption margin depending on whether the adoption good is produced with intermediate inputs or labor, and (ii) the potential of the different policy instruments as the engine of industrial modernization.

In this case, the subsidy elasticity of aggregate consumption is given by

$$d\ln C = \left(\tilde{\boldsymbol{\Psi}}_{\boldsymbol{p}}{}' - \boldsymbol{\Psi}_{\boldsymbol{p}}{}'\right) d\ln \boldsymbol{r} + \left(\tilde{\boldsymbol{\Psi}}_{\boldsymbol{p}}{}' - \frac{\eta-1}{\eta}\boldsymbol{\Psi}_{\boldsymbol{p}}{}'\right) \nu d\ln \boldsymbol{r^x} + \left((1-\nu)\tilde{\boldsymbol{\Psi}}_{\boldsymbol{p}}{}' - \left(1-\nu\frac{\eta-1}{\eta}\right)\boldsymbol{\Psi}_{\boldsymbol{p}}{}'\right) d\ln \boldsymbol{r^w} \ .$$

Naturally, the contribution of the revenue subsidy $(\tilde{\boldsymbol{\Psi}}_{\boldsymbol{p}}{}' - \boldsymbol{\Psi}_{\boldsymbol{p}}{}')$ is the sum of the contributions of the intermediate input and labor subsidies. When considering intermediate input subsidies, the Forward Domar weight $\tilde{\boldsymbol{\Psi}}_{\boldsymbol{p}}{}'$ is weighted by the elasticity of intermediate inputs in production, ν, and the Backward Domar weight $\boldsymbol{\Psi}_{\boldsymbol{p}}{}'$ is weighted by the intermediate input share, $\nu(\eta-1)/\eta$. A similar argument applies for labor subsidies.

To further gain insights about the power of the different policy instruments we restrict to uniform and unitary subsidies, $d\ln \boldsymbol{r} = d\ln \boldsymbol{r^x} = d\ln \boldsymbol{r^l}$, and we contrast the resulting multipliers. Under the uniform policy,

$$\epsilon^u_{r^l} = 0 < \epsilon^u_{r} = \frac{\nu}{1-\nu}\frac{1}{\eta} < \epsilon^u_{r^x} = \frac{1}{1-\nu}\frac{1}{\eta} \ .$$

While a labor subsidy to an individual sector has generically an effect on aggregate consumption, either positive or negative, these individual effects cancel out when aggregated up, implying that the labor subsidy multiplier is zero. Key to this result is that labor is a fixed input. When aggregating the Backward Domar weight we obtain the gross output share of consumption, $1/(1-\nu(\eta-1)/\eta)$, which results from accumulating infinite rounds of the (variable) intermediate inputs share, $\left(\boldsymbol{I} - \nu\frac{\eta-1}{\eta}\boldsymbol{\Omega}\right)^{-1}\boldsymbol{1} = \left(\sum_{n=0}^{\infty}\left(\nu\frac{\eta-1}{\eta}\boldsymbol{\Omega}\right)^n\right)\boldsymbol{1}$. This share is then multiplied by the share of the (fixed) labor input, one minus the share of the variable input. A similar argument applies to $(1-\nu)\tilde{\boldsymbol{\Psi}}_{\boldsymbol{p}}{}'\boldsymbol{1}$, resulting in zero consumption effect of a labor uniform subsidy.

Intuitively, in the aggregate, nothing can be gained by promoting uniformly the use of a fixed input. This is reminiscent of the results in Liu (2019).

The results of the uniform labor policy are useful as an input to understand the values of the uniform revenue and intermediate input multipliers. In fact, the revenue and intermediate input elasticities are the same,

$$\left.\frac{d\ln C}{d\ln \boldsymbol{r}}\right|_{\boldsymbol{r}=\mathbf{1}} = \left.\frac{d\ln C}{d\ln \boldsymbol{r}^{\boldsymbol{x}}}\right|_{\boldsymbol{r}^{\boldsymbol{x}}=\mathbf{1}} = (1+\bar{\delta})\frac{\nu}{1-\nu}\frac{1}{\eta}\frac{1}{1-\nu\frac{\eta-1}{\eta}} \ .$$

They are equal because a uniform subsidy provides no gains in terms of labor reallocation and all gains follow from the intermediate input channel. This is why ν appears in the numerator. But, although the two uniform policies share the same value for the elasticity, they differ in the cost of the policy. The revenue uniform policy is 'paying' for labor reallocation with no gains, while the intermediate inputs uniform policy circumnavigates paying for the added cost. Therefore, while both policies have the same subsidy elasticities, the intermediate input policy cost is only ν for every unit of cost in the revenue policy. As a result, $\epsilon^u_r/\epsilon^u_{r_x} = \nu < 1$.

While the intermediate input subsidy dominates the revenue subsidy in the economy with no adoption, this is not necessary the case in the model with adoption. This occurs as, while both the revenue and intermediate input subsidies affect the marginal adopter's condition indirectly through their effect on sector prices, the revenue subsidy also adds a direct effect on the marginal adopter's profit. As a result, we resort to our quantitative analysis of multipliers to gauge the relative importance of the different subsidies as an engine of development.

3.3.2 Adoption goods produced with labor only

While we see technologies in our model as embedded in intermediate inputs (closer to capital in a dynamic version of the model), it is useful to evaluate the extent to which sectoral multipliers are sensitive to this assumption. In this section we study the polar opposite case, where the adoption good is produced solely with labor instead of solely with goods.

When labor is the sole input of production for the adoption good, the subsidy elasticity expression in (16) is given by

$$d\ln C = \left(\tilde{\boldsymbol{\Psi}}_{\boldsymbol{p}}{}' - \boldsymbol{\Psi}_{\boldsymbol{p}}{}'\right) d\ln \boldsymbol{r} + \frac{1}{1-\nu}\left[(1-\nu)\tilde{\boldsymbol{\Psi}}_{\boldsymbol{p}}{}' - \left(1-\nu\frac{\eta-1}{\eta}\right)\boldsymbol{\Psi}_{\boldsymbol{p}}{}'\right] d\ln \boldsymbol{Z} \ ,$$

where, as before, $\tilde{\boldsymbol{\Psi}}_{\boldsymbol{p}}{}' = \boldsymbol{\Gamma}'\left(\boldsymbol{I}-\nu\boldsymbol{\Omega}\right)^{-1}$, $\boldsymbol{\Psi}_{\boldsymbol{p}}{}' = \boldsymbol{\Gamma}'\left(\boldsymbol{I}-\nu\frac{\eta-1}{\eta}\boldsymbol{\Omega}\right)^{-1}$, and $d\ln \boldsymbol{Z} = \beta\texttt{diag}(\boldsymbol{M})d\ln \boldsymbol{a}$.

First, notice that When $\nu = 0$ we have that $d\ln C = \mathbf{0}'d\ln \boldsymbol{r} + \mathbf{0}'d\ln \boldsymbol{Z}$. While there are markup distortions in the economy, they do not manifest in creating a wedge between labor shares and final demand elasticities, thus rendering industrial policy useless.

Second, consider the effect of uniform subsidies in this economy. As before, we set $d\ln \boldsymbol{r} = d\ln \boldsymbol{Z} = \mathbf{1}$. The contribution of the production margin is analogous to the uniform policy in the baseline economy, $\frac{\nu}{1-\nu}\frac{1}{\eta}\frac{1}{1-\nu\frac{\eta-1}{\eta}}$. However, this analogy does not follow through for the contribution from productivity,

$$\left[(1-\nu)\tilde{\boldsymbol{\Psi}}_{\boldsymbol{p}}{}' - \left(1-\nu\frac{\eta-1}{\eta}\right)\tilde{\boldsymbol{\Psi}}'\right]\mathbf{1} = \boldsymbol{\Gamma}'\left[(1-\nu)\left(\boldsymbol{I}-\nu\boldsymbol{\Omega}\right)^{-1} - \left(1-\nu\frac{\eta-1}{\eta}\right)\left(\boldsymbol{I}-\nu\frac{\eta-1}{\eta}\boldsymbol{\Omega}\right)^{-1}\right]\mathbf{1} = 0\ . \tag{22}$$

This result is similar to the case of a labor subsidy in the economy without adoption. While the cumulative effect of the policy must be zero, aggregate consumption can increase or decrease by reallocating adoption labor across sectors. In fact, if a subsidy in one sector has a positive effect on aggregate consumption, there must be another sector with a negative effect on aggregate consumption.

Finally, in a symmetric economy with roundabout production in each sector, i.e. $\boldsymbol{\Omega} = \boldsymbol{I}$, we have that the vector of TFP contributions satisfy

$$\Gamma'\left[(1-\nu)\left(1-\nu\right)^{-1}\boldsymbol{I} - \left(1-\nu\frac{\eta-1}{\eta}\right)\left(1-\nu\frac{\eta-1}{\eta}\right)^{-1}\boldsymbol{I}\right] = \mathbf{0}'\ .$$

That is, each sector's contribution through TFP must be zero, and thus there are no gains from reallocating adoption. This last result is reminiscent of that one in Atkeson and Burstein (2010). As a result, adoption reallocation can have relevant effects for aggregate multipliers if there is substantial heterogeneity in the network structure of production.

4 Estimation and Calibration

We begin this section by presenting a simple identification strategy that allow us to structurally estimate the model. Our strategy consists of three stages, to be described, and combines structural estimation and calibration methods. After describing our strategy, we introduce the data that we use and present the estimates we obtain by following our procedure. Finally, we perform a validation exercise of the structural estimates using independent proxies of technology adoption.

4.1 Identification

While the model is able to capture complex sectoral production and adoption complementarities, it is parameterized in a relatively parsimonious fashion. We need to assign values to the following set of parameters (scalars, vectors and matrices): $A_t, A_m, \zeta, \chi, \eta, \nu, \{\kappa_{ts}, \kappa_{ms}\}_{\text{for all } s}, \boldsymbol{\Gamma}, \boldsymbol{\Omega}, \boldsymbol{\Lambda}$. To do this, we follow a three-stage procedure. In the first stage, we exploit the implication of the theory for the size of an establishment within a sector, and estimate a reduced-form representation of the parameters governing the distribution of technologies in the model, i.e. combination of the deep parameters of the model. In the second stage, we combine direct measurements from the data and estimates from other sources to calibrate the production and adoption networks, i.e. $\boldsymbol{\Omega}$ and $\boldsymbol{\Lambda}$, and the elasticity of substitution across varieties within a sector η. Finally, in the third stage, we use the equilibrium construct of the model to unbundle the reduced-form estimates obtained in the first stage and recover the deep parameters of the model governing technology adoption. We also show that this procedure provides a unique mapping from the reduced-form and deep parameters estimates. Further, following the working assumptions for multipliers, we assume no subsidies in the starting equilibirum. That is, $\ln \boldsymbol{r} = \ln \boldsymbol{r^l} = \ln \boldsymbol{r^x} = \ln \boldsymbol{r^a} = \mathbf{0}$.

Brute force estimation of the multisector economy is a challenging task. For example, given that entry and adoption in a sector depend on entry and adoption in other sectors, it is unclear how to operationalize the procedure for the estimation of entry and adoption costs. This problem gets exponentially more convoluted the more sectors are considered. Our insight to address these complications is that to explain variation in the size of establishments within a sector, a common estimation target in the literature, one does not need to understand how sectors are connected, nor the level of aggregate prices, nor entry and adoption rates in other sectors. This partial information procedure allows us to provide reduced-form estimates for entry and adoption thresholds for each sector, productivity parameters, and the distribution of ex-ante and ex-post heterogeneity.

Taking logs to (2), and setting $r_s = r_s^l = r_s^x = 1$, provides that

$$\ln l_{is}(z, \varepsilon) = \ln \tilde{A}_i + \ln \tilde{z}_s + \ln \tilde{\varepsilon} \ , \tag{23}$$

where $\tilde{A}_i \equiv A_i^{\eta-1}$, $\tilde{z}_s \equiv (1-\nu)\frac{\eta-1}{\eta}\frac{P_s Y_s}{Z_s^{\eta-1}} z^{\eta-1}$, and $\tilde{\varepsilon} \equiv \varepsilon^{\eta-1}$. In this representation, the size of an establishment within sector s combines (i) a technology-specific component $\tilde{A}_i$, (ii) a mix of a sector component and ex-ante heterogeneity of the establishment, and (iii) an idiosyncratic ex-post component. Notice that the sector component cannot be identified by observing heterogeneity in size of establishments within a sector. Unbundling the sector and the ex-ante heterogeneity components is the main objective of the third stage in our devised procedure.

Within a sector, heterogeneity in technology use manifests sector through its impact on the employment-size distribution. Let $H_s(l) = \Pr(l_s(\tilde{z}, \tilde{\varepsilon}) \leq l)$, with density $h_s(l) \equiv \frac{\partial H_s(l)}{\partial l}$. The distribution $H_s(l)$ provides the employment size distribution of establishments within sector s.

$$\begin{aligned} h_s(l) = & l^{-\tilde{\zeta}-1} \frac{\tilde{\zeta}}{\tilde{z}_{ts}^{-\tilde{\zeta}}} e^{\tilde{\mu}\tilde{\zeta} + \frac{\tilde{\chi}^2}{2}\tilde{\zeta}^2} \left\{ \tilde{A}_t^{\tilde{\zeta}} \left[\Phi\left(\frac{\ln \tilde{z}_{ms} - \ln l + \tilde{\mu} + \tilde{\chi}^2\tilde{\zeta}}{\tilde{\chi}} \right) - \right.\right. \\ & \left.\left. \Phi\left(\frac{\ln \tilde{z}_{ts} - \ln l + \tilde{\mu} + \tilde{\chi}^2\tilde{\zeta}}{\tilde{\chi}} \right) \right] + \tilde{A}_m^{\tilde{\zeta}} \left[1 - \Phi\left(\frac{\ln \tilde{z}_{ms} - \ln l + \tilde{\mu} + \tilde{\chi}^2\tilde{\zeta}}{\tilde{\chi}} \right) \right] \right\} \ , \end{aligned} \tag{24}$$

where $\tilde{\zeta} \equiv \frac{\zeta}{\eta-1}$, $\tilde{\chi} \equiv (\eta-1)\chi$, $\tilde{\mu} \equiv (\eta-1)\mu = -\tilde{\chi}^2/2$, and where $\tilde{z}_{ts}$ and $\tilde{z}_{ms}$ are the entry and adoption thresholds under the reduced-form representation. Details on the derivation are available in Appendix A.4. If one were to shut down technology heterogeneity and the ex-post shock ε, i.e. $\tilde{A}_t = \tilde{A}_m$ and $\tilde{\chi} \to 0$, $h_s(l)$ is Pareto. Departures from Pareto are the result of heterogeneity in technology adoption and dispersion following from $\tilde{\varepsilon}$. Similarly, we can define the employment weighted size distribution across establishments $G_s(l)$, where $g_s(l) \equiv \partial G_s(l)/\partial l = l h_s(l) / \int \hat{l} h_s\left(\hat{l}\right) d\hat{l}$.

Inference using $h_s(l)$ or $g_s(l)$ in the reduced-form model provides estimates for $\tilde{A}_t$, $\tilde{A}_m$, $\tilde{\chi}$, $\tilde{\zeta}$, and for the reduced-form entry and adoption thresholds $\{\tilde{z}_{ts},\ \tilde{z}_{ms}\}_{\text{for all } s}$. The next two remarks show how the shape of $h_s(l)$ and $g_s(l)$ is informative about the extent of heterogeneity in technology use within sector s, which is encoded in the technology parameters $\tilde{A}_t$ and $\tilde{A}_m$, and in the entry and adoption thresholds $\tilde{z}_{ts}$ and $\tilde{z}_{ms}$. We explore the informativeness of the distributions regarding technology adoption by characterizing two extreme cases. In the first remark we show that if there is no variation in technology use within a sector the resulting distributions are unimodal. In the second remark we show that with no ex-post heterogeneity, heterogeneity in technology use results in bimodal distributions.

Remark 5 *Suppose only one technology is used in sector s. Then, the distributions $H_s(l)$ and $G_s(l)$ are unimodal.*

The proof is available in Appendix A.5. The proposition states that a combination of Pareto and Log-Normal shocks, both unimodal distributions, imply equilibrium distributions that are also unimodal. Because the ex-post shock $\tilde{\varepsilon}$ is iid, it shifts mass in such a way that does not generate two modes.

Remark 6 *Let $\chi \to 0$. Then, the distributions $H_s(l)$ and $G_s(l)$ have two modes if and only if $\tilde{A}_m > \tilde{A}_t$. In this case, the two modes are at $l = \tilde{z}_{ts}$ and $l = \tilde{z}_{ms}$. If $\tilde{A}_m = \tilde{A}_t$, both distributions have one mode, located at $l = \tilde{z}_{ts}$.*

The proof is available in Appendix A.6. The proposition establishes that, by shifting mass from the middle of the distribution to the right of the distribution, technology adoption generates bimodality.

Under the reduced-form representation the endogenous choices of establishments regarding entry and technology adoption are summarized by the thresholds $\tilde{z}_{ts}$ and $\tilde{z}_{ms}$ for all s. In order to compute development multipliers and perform counterfactual analysis we need to unbundle these thresholds and uncover the entry and adoption costs, i.e. κ_{ts} and κ_{ms} for all s, that correspond to these thresholds. The next proposition uses the equilibrium conditions of the model and establishes that there is a unique mapping between the reduced-form thresholds and the underlying entry and adoption costs.

Proposition 3 *Given values for η, ν, $\mathbf{\Gamma}$, $\mathbf{\Omega}$, $\mathbf{\Lambda}$, and the reduced-form objects $\tilde{A}_t$, $\tilde{A}_m$, $\{\tilde{z}_{ts}, \tilde{z}_{ms}\}_{for\ all\ s}$, then the vectors of entry and adoption costs, $\{\kappa_{ts}, \kappa_{ms}\}_{for\ all\ s}$, are uniquely identified.*

The proof of this proposition follows from the fact that the equilibrium of the model can be described by a system of linear equations given the reduced-form thresholds, $\tilde{z}_{ts}$ and $\tilde{z}_{ms}$ for all s.

4.2 Data and calibration

We focus our quantitative analysis on the India economy and combine three sources of data: (i) Indian Economic Census, (ii) the World Input-Output Database (WIOD) and, (iii) data on the investment network from vom Lehn and Winberry (2022).

The Indian Economic Census is a complete count of all establishments/units located within the geographical boundaries, engaged in production or distribution of goods or services other than for the sole purpose of own consumption, crop production and plantation, public administration and defence, or activities of households as employers of domestic personnel. We further drop establishments in animal production, forestry and logging, and fishing and aquaculture. The sectoral data is given at the 3-digit level as per the National Industrial Classification (NIC 2004 and 2008, for the 2005 and 2012-13 Economic Censuses, respectively). We use the Sixth Indian Economic Census from 2012-13, which is our main source for data on the size distribution of establishment by sector, which is the data used to estimate the reduced-form representation of technology adoption by sector. We measure employment in an establishment as the sum of non-hired and hired workers, including family workers and owner.[8] We also partially rely on the Fifth Indian Economic Census as it provides information about the type of power used by individual establishments, a proxy for the technology used at the establishment level. This data is used in the validation exercise at the end of this Section. The final dataset for the Sixth (Fifth) Indian Economic Census includes 45363786 (35171881) establishments employing 108411367 (84134947) individuals.

The WIOD (Timmer et al., 2015) contains international comparable information on the input-output linkages and sectoral intermediate and final uses, covering 43 countries, for the period 2000-2014. Data for 56 sectors are classified according to the International Standard Industrial Classification revision 4 (ISIC Rev. 4). We use the table for India in 2010. This data gives us information to directly calibrate $\mathbf{\Gamma}$ and $\mathbf{\Omega}$.

There is no data available for India that can be easily mapped to the network of investment $\mathbf{\Lambda}$. We overcome this limitation by using the network of investment produced in vom Lehn and Winberry (2022) for the United States in 2010. In particular, the investment network records the share of new tangible and intangible investment expenditures of sector s that were purchased from sector s' for each pair of sectors. They construct the investment network using the BEA Fixed Assets and Input-Output databases for a sample of 37 private non-farm sectors from 1947-2018. We consider this as the best available proxy for the production network of *modern* technologies in India.

After harmonizing the sectors across the three datasets, the India economy that we consider is composed of 30 2-digit sectors.[9] Also, we follow Broda and Weinstein (2006) and Hsieh and Klenow (2009) and we set $\eta = 3$. Given the value of η, we set $\nu = 0.75$ so that the intermediate input share of gross output equals 0.49, the value in the WIOD for India in 2010.

[8] The question ask for the number of persons found working comprising, hired, non- hired (including family members; unpaid apprentice and owner), on the last working day in the establishment. Regular wage/salaried workers, owner/other family workers, who are temporarily absent on the last working day are also counted.

[9] Details are available upon request.

4.3 Structural Estimation

In this section we structurally estimate the parameters of the model. We use the insights of previous sections and estimate reduced-form parameters for each sector within the Indian economy. Then, we use the system of equilibrium equations in the model to back out the fundamental parameters that make the partial information estimates that we obtained for each sector consistent across sectors.

We assume that all sectors in the economy share the same distribution of (normalized) ex-ante and ex-post heterogeneity, $\{\tilde{\zeta},\ \tilde{\varepsilon}\}$, and gains from adoption $\tilde{A}_m$. Without loss of generality, we set $\tilde{A}_t = 1$. Thus, for each sector we estimate the (normalized) entry and adoption thresholds $\{\tilde{z}_{ts},\ \tilde{z}_{ms}\}_{s\ in\ S}$. While our approach may seem restrictive, we show that this parsimonious approach is able to provide a good fit of the data. Operationally, we estimate the parameters of the model by matching the binned employment share distribution for each sector—we use 21 bins per sector, where for the common parameters we weight equally all sectors. We want to notice that, unlike for other parameters, we estimate the tail parameter $\tilde{\zeta}$ by using direct information from the tail of the size-distribution of establishments for the whole Indian economy.

Table I presents the implied share of modern establishments, a_s/e_s, and the their value-added share, M_s, for the 30 sectors of the Indian economy, as well as the estimated entry cost κ_{es} and the entry cost relative to the cost of adoption, $\kappa_{es}/(P_{ms}\kappa_{as})$. The table shows large heterogeneity across sectors. For example, the estimation provides that modern establishments account for 60% of value-added share in the Mining sector, 0.4% in the Wood and Cork sector, and 87% in the Motor Vehicle sector. Heterogeneity in these shares is rationalized by heterogeneity in the relative cost of adoption: in Mining the entry cost is 2.3% of the added cost of adoption, in Wood and Cork it is basically negligible (in other words, the cost of adoption is very large), and in Motor Vehicles is 10% of the added cost.

Figure 1: Employment share distributions, 3 sectors

1 - Mining | 4 - Wood and Cork | 16 - Motor Vehicles

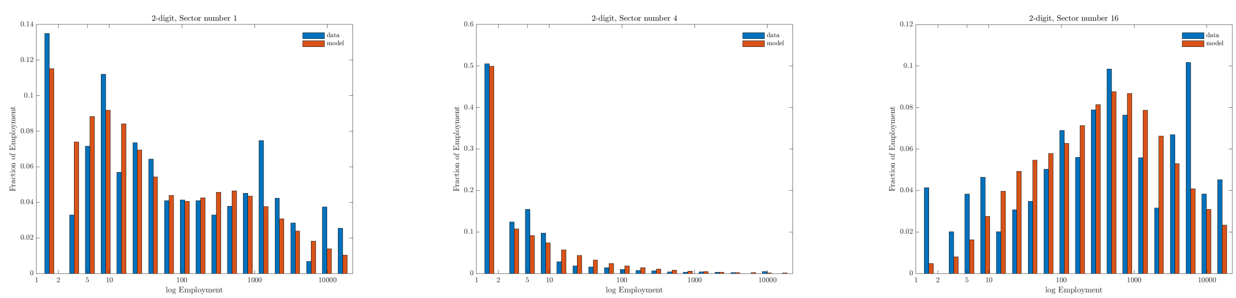


Figure 1 presents the empirical and the model implied employment share distributions for these three sectors. The model closely replicates the distributions at each sector, even though the only degrees of freedom at the sector level are the entry and adoption thresholds. The Mining sector presents a evident bimodal employment share distribution, implying that both small and large establishments are highly relevant. As a result, the estimation procedure matches this by having 60% of value-added share accounted for by modern

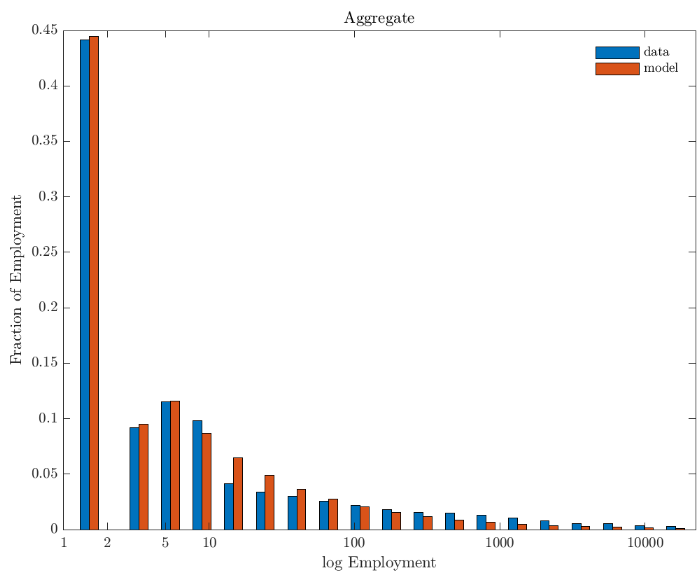
Table I: Modern firms in India

Division	a_s/e_s	M_s	κ_{es}	$\frac{\kappa_{es}}{P_{ms}\kappa_{as}}$
1 - Mining	0.021	0.606	1.387	0.023
2 - Food, beverages and tobacco	0.002	0.358	0.235	0.005
3 - Textiles, wearing and leather	0.001	0.305	0.337	0.004
4 - Wood and cork, except furniture	0.000	0.004	0.262	0.000
5 - Paper and paper products	0.534	0.948	0.366	0.179
6 - Printing and reproduction of media	0.000	0.033	0.819	0.000
7 - Coke and petroleum products	0.031	0.648	0.607	0.029
8 - Chemicals and pharma. products	0.149	0.824	1.745	0.080
9 - Rubber and plastic products	0.761	0.978	0.536	0.224
10 - Other non-metallic products	0.098	0.780	0.938	0.061
11 - Basic metals	0.054	0.711	1.078	0.042
12 - Metal products, except machinery and equip.	0.002	0.340	0.810	0.005
13 - Computer, electronic and optical products	0.070	0.742	1.083	0.049
14 - Electrical equipment	0.074	0.748	0.990	0.051
15 - Machinery and equipment	0.011	0.532	0.624	0.016
16 - Motor vehicles, trailers and semi-trailers	0.227	0.868	8.340	0.104
17 - Other transport equipment	0.100	0.782	50.892	0.062
18 - Furniture and other	0.000	0.000	0.422	0.000
19 - Utilities	0.024	0.622	0.640	0.025
20 - Construction	0.001	0.277	0.369	0.003
21 - Trade	0.000	0.003	0.156	0.000
22 - Transportation	0.000	0.209	0.125	0.002
23 - Accommodation and food service activities	0.000	0.005	0.391	0.000
24 - Information	0.000	0.000	0.585	0.000
25 - Professional services	0.015	0.568	0.577	0.019
26 - Finance and insurance	0.617	0.961	0.185	0.196
27 - Real estate and related support	0.001	0.313	0.316	0.004
28 - Education	0.994	0.999	0.282	0.265
29 - Human health and social work activities	0.000	0.000	0.614	0.000
30 - Repair and other services	0.000	0.002	0.125	0.000

establishments. Consistent with this, Table I shows that the cost of adoption in the sector, $P_{ms}\kappa_{ms}$, is large relative to the entry cost, κ_{es}, but not extreme. Highly productive establishments end up adopting and, while accounting for a small share of establishments in the sector, they account for a relatively large share of value-added. In the Wood and Cork sector, the empirical distribution resembles a Pareto distribution and thus the estimation procedure considers the sector to have almost no establishments adopting modern technologies, with a value-added share of modern establishments close to zero, which, as shown in Table I, is obtained in the model by setting large adoption costs relative to entry costs.[10] For the Motor Vehicles sector, the empirical distribution shows that most of employment is concentrated in large establishments, which the model matches with a high adoption rate. In fact, as previously discussed, the value-added share of modern establishments in the sector is almost 0.87. To generate high adoption rates in this sector, the estimation provides that the adoption cost accounts for around 90% of total effective cost of adoption.

While we estimate parameters to match individual sector employment share distributions, the model provides a employment share distribution for the whole India economy which we can contrast with the empirical one. This can be found in Figure 2. The model-implied distribution closely resembles the distribution observed in the data, although over-predicting the employment share of mid-size establishments, and under-predicting the employment share of the very large ones.

Figure 2: Employment share distribution in the aggregate Indian economy



Given that we calibrated the elasticity of substitution to be equal to 3, it is immediate to back out our estimates for ζ, χ and A_m from the reduced-form estimates. We also note that we provide 95% confidence intervals for the different objects of interest, constructed following a Bootstrapping technique. Table II presents the estimates and confidence intervals for the common parameters.

We also provide an off-sample test of the validity of our estimates. We do so by contrasting adoption measures implied by the estimated model with adoption proxies that can be observed in the data. In particular, the Fifth Indian Economic Census provides the type of power (i.e. electricity, horsepower, etc.) that a establishment employs. We then classify each power source as either traditional (without power, fire wood, animal power, non conventional, and others) or modern (electricity, coal/sift coke, petrol/diesel/kerosen,

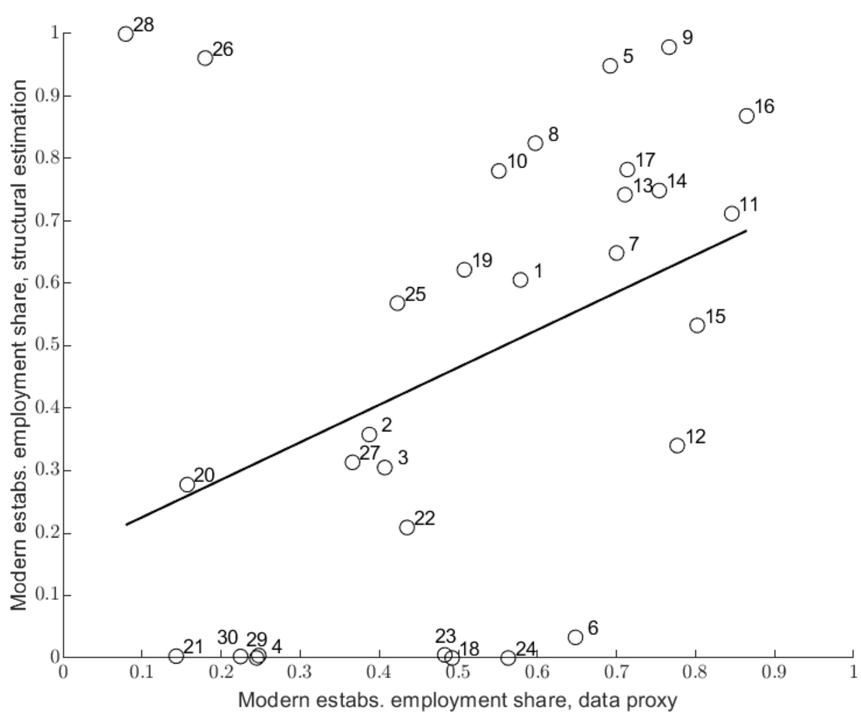
[10] It is also relevant that the Wood and Cork sector has relatively small establishments relative to the other sectors in the economy. Otherwise the estimation procedure would interpret the sector as having full adoption of modern technologies.

Table II: Common parameters, estimates

Ex-ante heterogeneity, ζ	Ex-post heterogeneity, χ	Technology, A_m
3.16	0.57	2.18
[3.03, 3.19]	[0.49, 0.60]	[2.01, 2.31]

and liquefied petroleum gas/natural gas). Figure 3 contrasts the employment share of modern establishments within a sector using this proxy (x-axis) with that one implied by the structural estimation of the model (y-axis). The figure shows a strong correlation between the two measures. In fact, the slope of a linear relationship among the two series is 0.6. However, it is worth noting that the relationship is far from perfect. For example, for some sectors the model predicts an employment share of modern establishments of nearly 1, while the proxy is close to zero.

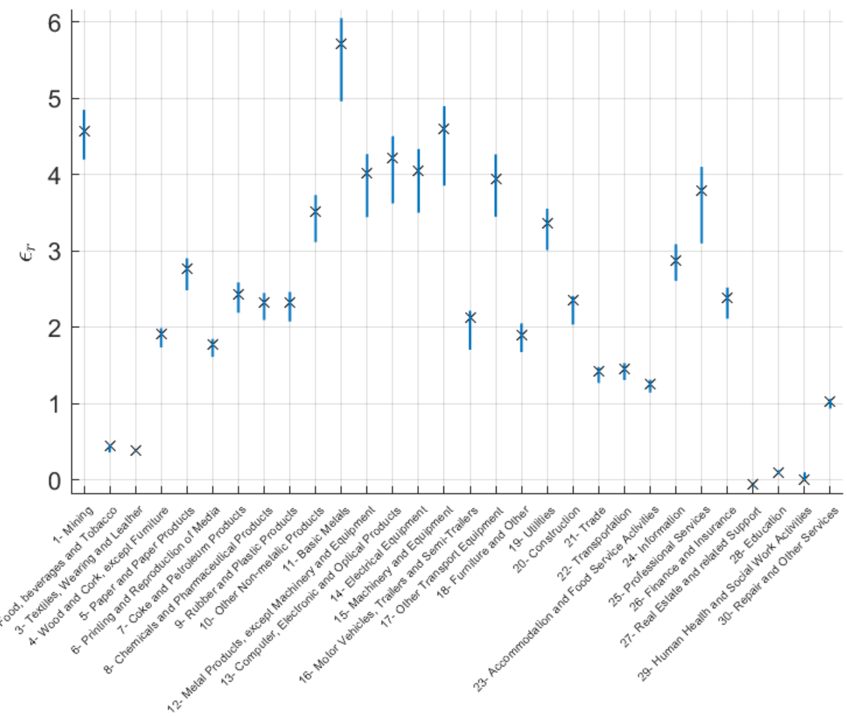
Figure 3: The employment share of modern establishments, proxy vs. estimated



4.4 Sectoral revenue multipliers

We begin our quantitative analysis of sectoral development multipliers by presenting estimates of revenue multipliers for all sectors, ϵ_r. Figure 4 presents our estimates, with 95% bootstrap confidence interval s. As seen in the figure, estimates are 'tight', in the sense that confidence intervals are small and they do not seem to affect the way one should interpret variation in multipliers. Regarding the estimates, the figure showcases large heterogeneity in multipliers across sectors. Some sectors are cost-effective, i.e. $\epsilon_r > 1$, some sectors are cost ineffective, i.e. $\epsilon_r < 1$, and even some sectors are such that it is counterproductive to subsidize them, i.e. $\epsilon_r < 0$. Among cost-effective sectors, there are a few that stand out as key engines for development, in decreasing order of relevance: 11- Basic Metals, 15- Machinery and Equipment, 1- Mining, 13- Computer, Electronic and Optical Products, and 14- Electrical Equipment.

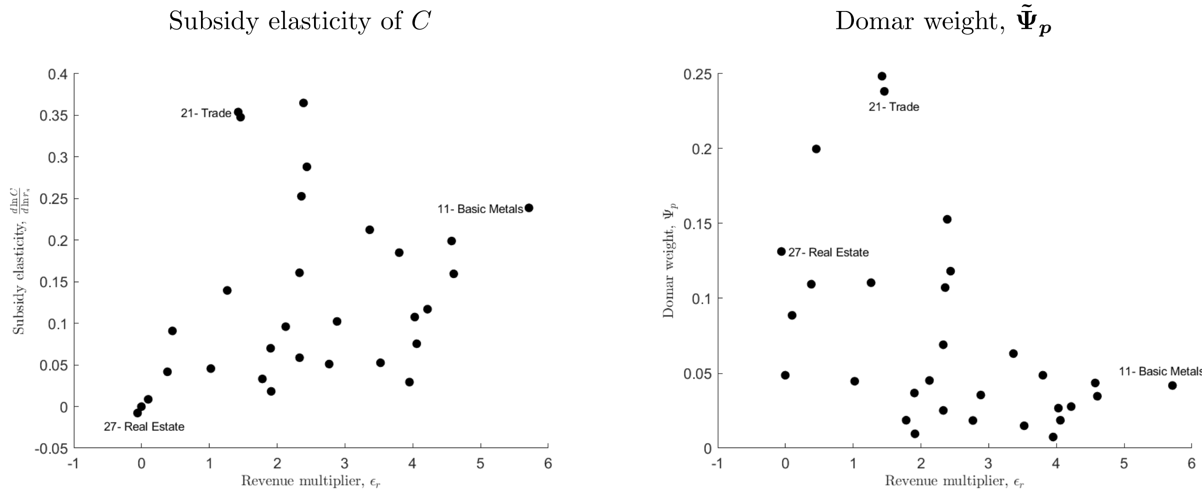
Figure 4: Sectoral revenue multipliers, ϵ_r



Multipliers are the result of normalizing the revenue subsidy elasticity of consumption by the cost of the subsidy, measured by the Domar weight. The left panel of Figure 5 provides a scatter plot with the subsidy elasticity in the y-axis and the revenue multiplier in the x-axis, while the right panel presents a scatter plot with the Domar weight in the y-axis and the revenue multiplier in the x-axis. As it is clear from the figure, there is a strong positive relationship between the subsidy elasticity and the multiplier, and a strong negative relationship between the Domar weight and the multiplier. The figure shows that sectors with high multipliers are those with intermediate-to-high values for the subsidy elasticity and low values for the

Domar weight. The Basic Metals sector falls in this category, exhibiting the highest multiplier, a subsidy elasticity that is high, but 30% lower than that one in the Trade sector, and a Domar weight that is very low, 85% lower than in the Trade sector. Overall, sectors with high multipliers exhibit intermediate values for consumption elasticity and, relative to sectors with large values for the elasticity, are disproportionately smaller, thus severely reducing the cost of the policy.

Figure 5: Component of sectoral revenue multipliers

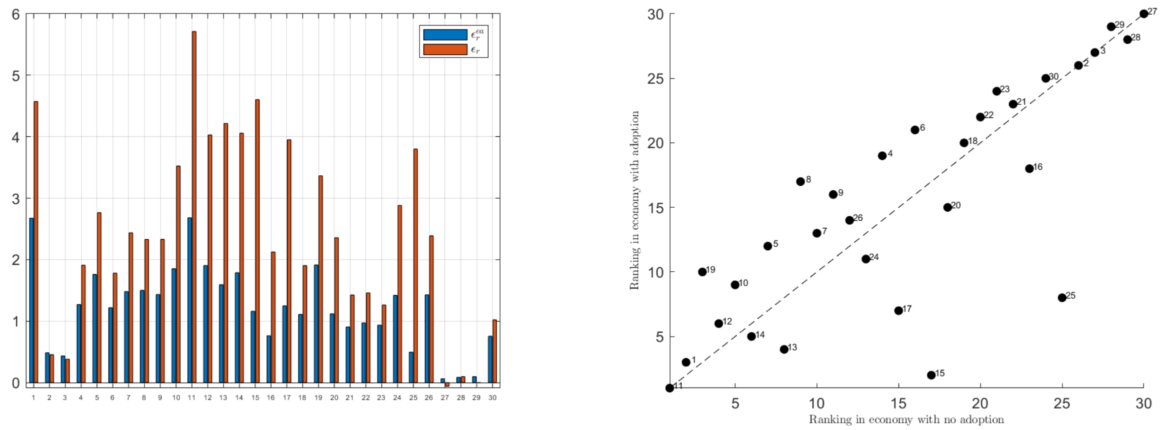


The relevance of adoption as a key ingredient of sectoral multipliers can be measured by comparing $\boldsymbol{\epsilon_r}$ with $\boldsymbol{\epsilon_r^{ea}}$, where this last one is the multiplier that would be obtained after shutting down entry and adoption margins. The left panel of Figure 6 presents the multipliers for these two cases. The effect of the adoption margin is highly heterogeneous across sectors. In some sectors, the adoption margin turns the multiplier negative. In other sectors, the adoption margin barely affects the level of the multiplier. And in some sectors, the adoption margin greatly amplifies the degree of the sectoral multiplier.

Heterogeneity of the effect of adoption for multipliers across sectors manifests on the way sectors are ranked in terms of multipliers with and without technology adoption. The right panel of Figure 6 presents in the x-axis the ranking of the sector revenue development multiplier in the economy with no entry and adoption, i.e. the ranking using ϵ^{ea}, and in the y-axis the ranking of the sector revenue development multiplier in the economy with entry and adoption, i.e. the ranking using ϵ. While there is little variation for those sectors considered less relevant in the economy with no adoption—i.e. those sectors with low multipliers, there is substantial variation in rankings among those exhibiting intermediate and high values for multipliers. For example, the top 5 sectors abstracting from adoption are, in descending order of relevance, 11- Basic Metals, 1- Mining, 19- Utilities, 12- Metal Products, and 10- Other Non-metallic Products, while the top 5 sectors in the economy with adoption are, in descending order of relevance, 11- Basic Metals, 15- Machinery and Equipment, 1- Mining, 13- Computer, Electronic and Optical Products, and 14- Electrical Equipment. With a finite amount of resources to be allocated to industrial policy, this result suggests that

Figure 6: The role of adoption

ϵ_r and ϵ_r^{ea} | Ranking



accounting for the adoption margin is crucial for the policy.

4.4.1 Determinants of amplification through adoption

Figure 6 showcases the relevance of adoption for multipliers: through adoption, there is substantial amplification, and this amplification is heterogeneous across sectors. In this section we study the determinants of amplification through adoption. To do so, we study the case with no entry as it is simpler and, as shown in Table III in the Appendix, multipliers barely change when the entry channel is considered. Using (20) together with the definition of a sectoral revenue multiplier, we define the contribution of adoption to the multiplier as

$$\boldsymbol{\mathcal{A}} = \left\{ \left(\tilde{\boldsymbol{\Psi}}_{\boldsymbol{p}}{}' - \frac{\eta - 1}{\eta} \boldsymbol{\Psi}_{\boldsymbol{p}}{}' \right) \beta \mathtt{diag}\left(\boldsymbol{M}\right) \boldsymbol{\nabla}_{\boldsymbol{a},\boldsymbol{r}^{\boldsymbol{a}}} \left[\boldsymbol{I} + \boldsymbol{\nabla}_{\boldsymbol{PY},\boldsymbol{r}} + \boldsymbol{\Lambda} \left(\boldsymbol{I} - \nu \boldsymbol{\Omega} \right)^{-1} \right] \right\} \oslash \boldsymbol{\Psi}_{\boldsymbol{p}}\boldsymbol{'} \ ,$$

where $\oslash$ denotes element-by-element division. We exploit the following decomposition of $\boldsymbol{\mathcal{A}}$ to study the determinants of amplification,

$$\boldsymbol{\mathcal{A}} = \overbrace{\left(\tilde{\boldsymbol{\Psi}}_{\boldsymbol{p}}{}' \oslash \boldsymbol{\Psi}_{\boldsymbol{p}}{}' - \frac{\eta-1}{\eta}\mathbf{1}'\right)\beta\bar{m}\frac{\zeta}{\eta-1}}^{\text{(1) direct incentive channel}}$$
$$+ \overbrace{\left\{\left(\tilde{\boldsymbol{\Psi}}_{\boldsymbol{p}}{}' - \frac{\eta-1}{\eta}\boldsymbol{\Psi}_{\boldsymbol{p}}{}'\right)\beta\frac{\zeta}{\eta-1}\left[\mathtt{diag}\left(\boldsymbol{M}\right) - \bar{m}\boldsymbol{I}\right]\right\} \oslash \boldsymbol{\Psi}_{\boldsymbol{p}}{}'}^{\text{(2) direct modern share channel}}$$
$$+ \overbrace{\left[\left(\tilde{\boldsymbol{\Psi}}_{\boldsymbol{p}}{}' - \frac{\eta-1}{\eta}\boldsymbol{\Psi}_{\boldsymbol{p}}{}'\right)\beta\mathtt{diag}\left(\boldsymbol{M}\right)\frac{\zeta}{\eta-1}\boldsymbol{\Omega}\left(\boldsymbol{I} - \nu\boldsymbol{\Omega}\right)^{-1}\right] \oslash \boldsymbol{\Psi}_{\boldsymbol{p}}{}'}^{\text{(3) direct price of adoption channel, } \boldsymbol{\Lambda} = \boldsymbol{\Omega}}$$
$$+ \overbrace{\left[\left(\tilde{\boldsymbol{\Psi}}_{\boldsymbol{p}}{}' - \frac{\eta-1}{\eta}\boldsymbol{\Psi}_{\boldsymbol{p}}{}'\right)\beta\mathtt{diag}\left(\boldsymbol{M}\right)\frac{\zeta}{\eta-1}\left(\boldsymbol{\Lambda} - \boldsymbol{\Omega}\right)\left(\boldsymbol{I} - \nu\boldsymbol{\Omega}\right)^{-1}\right] \oslash \boldsymbol{\Psi}_{\boldsymbol{p}}{}'}^{\text{(4) direct price of adoption channel, } \boldsymbol{\Lambda} \text{ effect}}$$
$$+ \overbrace{\left\{\left(\tilde{\boldsymbol{\Psi}}_{\boldsymbol{p}}{}' - \frac{\eta-1}{\eta}\boldsymbol{\Psi}_{\boldsymbol{p}}{}'\right)\beta\mathtt{diag}\left(\boldsymbol{M}\right)\left(\boldsymbol{\nabla}_{\boldsymbol{a},\boldsymbol{r}^{\boldsymbol{a}}} - \frac{\zeta}{\eta-1}\boldsymbol{I}\right)\left[\boldsymbol{I} + \boldsymbol{\Lambda}\left(\boldsymbol{I} - \nu\boldsymbol{\Omega}\right)^{-1}\right]\right\} \oslash \boldsymbol{\Psi}_{\boldsymbol{p}}{}'}^{\text{(5) feedback through adoption channel}}$$
$$+ \overbrace{\left[\left(\tilde{\boldsymbol{\Psi}}_{\boldsymbol{p}}{}' - \frac{\eta-1}{\eta}\boldsymbol{\Psi}_{\boldsymbol{p}}{}'\right)\beta\mathtt{diag}\left(\boldsymbol{M}\right)\boldsymbol{\nabla}_{\boldsymbol{a},\boldsymbol{r}^{\boldsymbol{a}}}\boldsymbol{\nabla}_{\boldsymbol{PY},\boldsymbol{a}}\right] \oslash \boldsymbol{\Psi}_{\boldsymbol{p}}{}'}^{\text{(6) aggregate demand channel}},$$

where $\bar{m}$ is equal to the average equally-weighted modern share across sectors.

The direct channel in $\boldsymbol{\mathcal{A}}$ relates to Remark 2, and considers the direct effect of a subsidy on adoption, as showcased in (18), abstracting from sectoral heterogeneity in $\boldsymbol{M}$ and $\boldsymbol{\nabla}_{\boldsymbol{a},\boldsymbol{r}^{\boldsymbol{a}}}$. This direct effect on adoption implies a change in sectoral TFP, $(d\ln Z/d\ln a)\,(d\ln a/d\ln r^a) = \beta\bar{m}\zeta/(\eta-1)$. For a particular sector s, the direct effect can be written as

$$\text{Direct incentive channel} = \epsilon^{ea}_{rs}\overbrace{\beta\bar{m}\frac{\zeta}{\eta-1}}^{=0.115} + \overbrace{\beta\bar{m}\frac{\zeta}{\eta(\eta-1)}}^{=0.038}\ . \tag{25}$$

From Figure 6, the multiplier ϵ^{ea}_{rs} is as small 0 and as large as 3. Thus, the direct channel of adoption can generate amplification raging from 0.038 to 0.383. Given that amplification is larger than 2 for many sectors, these simple calculations imply that the contribution of the direct margin is relatively small. Furthermore, in Table IV in the Appendix we show that the second term in the decomposition, i.e. adding heterogeneity in modern shares, does not alter this conclusion.[11]

The third term in the decomposition measures the importance of the revenue subsidy in lowering the

[11] Notice that the term $\epsilon^{ea}_{rs} = \frac{\tilde{\psi}_{rs}}{\psi_{rs}} - 1$ in (25) is the same as the key statistic in Liu (2019). Here, because of adoption, the key statistic is 'transformed' by the elasticity of TFP with respect to the subsidy.

price of the modern technology, under the counterfactual assumption that the adoption good combines sectoral output in the same proportions as the intermediate aggregate, i.e. $\mathbf{\Omega}$. This term captures the "direct" feedback effects of the revenue subsidy on the price of the adoption good through the production network, $(\boldsymbol{I} - \nu\mathbf{\Omega})^{-1}$, but ignore further feedback effects through adoption included in $\boldsymbol{\nabla}_{\boldsymbol{a},\boldsymbol{r^a}}$. The fourth term evaluates the relevance, for the production of the adoption good, of combining sectoral output with the investment network matrix $\mathbf{\Lambda}$. Large values for this term is a further manifestation that the sector has a higher importance in the investment network $\mathbf{\Lambda}$ than in the production network $\mathbf{\Omega}$.

The fifth term in the decomposition of $\boldsymbol{\mathcal{A}}$ accounts for the relevance of feedback effects in adoption in generating large multipliers, as captured by $\boldsymbol{\nabla}_{\boldsymbol{a},\boldsymbol{r^a}} - \zeta/(\eta-1)\boldsymbol{I}$, taking the form of a double Leontieff Inverse as described in detail in (18). Large values for this term imply that the sector is central in the investment network.

Finally, the sixth term in the decomposition of $\boldsymbol{\mathcal{A}}$ accounts for the relevance of the feedback effects through the aggregate demand channel.

While we present all terms of the decomposition in Table IV in the Appendix, the main takeaways of the decomposition exercise are presented in the next two figures.

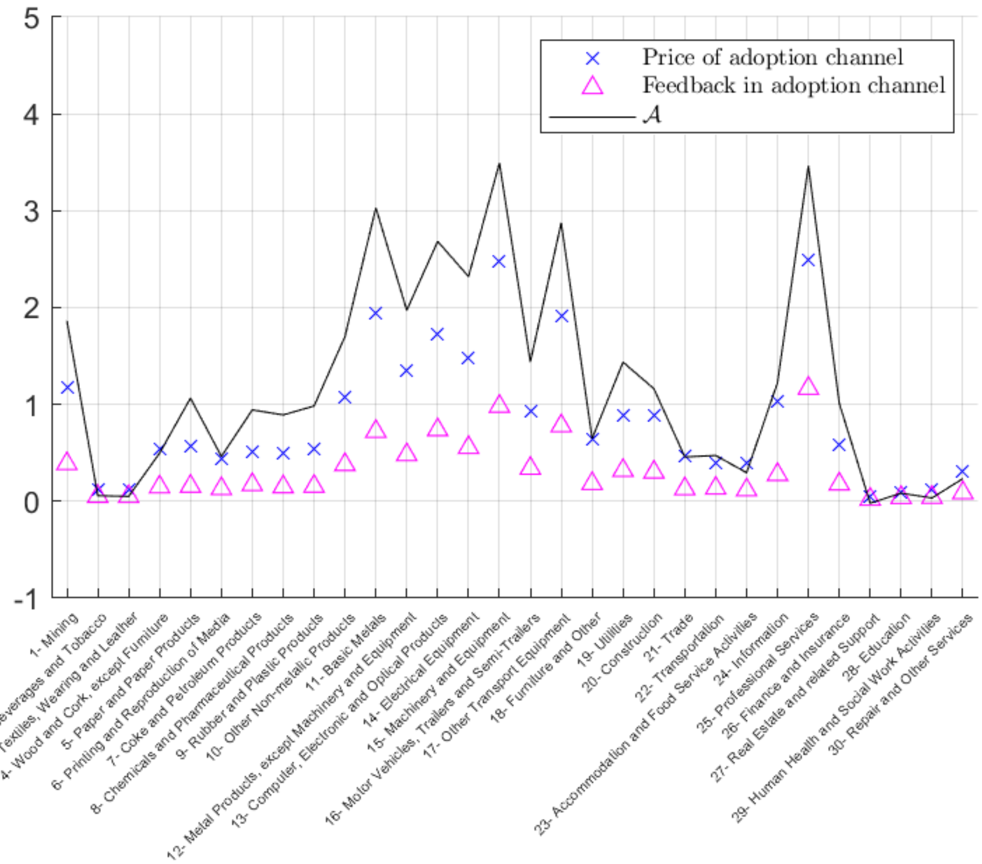
Figure 7 presents the contribution of the direct price of adoption channel (blue x, third and fourth terms in the decomposition of $\boldsymbol{\mathcal{A}}$), and the contribution of the feedback in adoption channel (pink triangle, fifth term in the decomposition of $\boldsymbol{\mathcal{A}}$). We also add $\boldsymbol{\mathcal{A}}$ to the figure to aid in the analysis. The other terms of the decomposition are not included as they are small relative to the level of amplification observed through adoption. The figure shows that the channel through the price of adoption $\boldsymbol{P_m}$ accounts for a large part of amplification through adoption $\boldsymbol{\mathcal{A}}$. While the price of adoption channel is relevant for all sectors, the feedback channel is only substantial for sectors exhibiting high values of $\boldsymbol{\mathcal{A}}$. For example, for the top 5 sectors in terms of multipliers, the feedback channel accounts for 20 to 30% of amplification through adoption.

Given the relevance of the channels through the price of adopting modern technologies $\boldsymbol{P_m}$, Figure 8 showcases the relevance of the two key ingredients shaping up the relevance of a sector as a central producer of inputs for adoption, $\mathbf{\Lambda}$ and $\mathbf{\Omega}$. A positive value for the Λ effect implies that the sector is more central in the investment network $\mathbf{\Lambda}$ than in the production network $\mathbf{\Omega}$, while a negative implies the opposite. For example, for sector 11- Basic Metals, amplification through this channel results from the fact that the sector is central in production, while for sector 15- Machinery and Equipment, amplification goes through the sector's centrality in the investment network. A particularly interesting sector is sector 13- Computer, Electronic and Optical Products, which belongs to the top 5 sectors in terms of multipliers, but with intermediate centrality in both production and adoption networks.

4.5 Alternative policy instruments

Which policy instrument is the most appropriate to promote economic development? The different instruments differ in (i) the way they improve production efficiency and promote the adoption of modern technologies, and (ii) the cost of implementation (for example, see Remark 4 in Section 3.2 and results in

Figure 7: Determinants of amplification



Section 3.3.1). As there are clear trade-offs among the different policy instruments, we resort to a quantitative analysis to understand which instrument is the most effective for development.

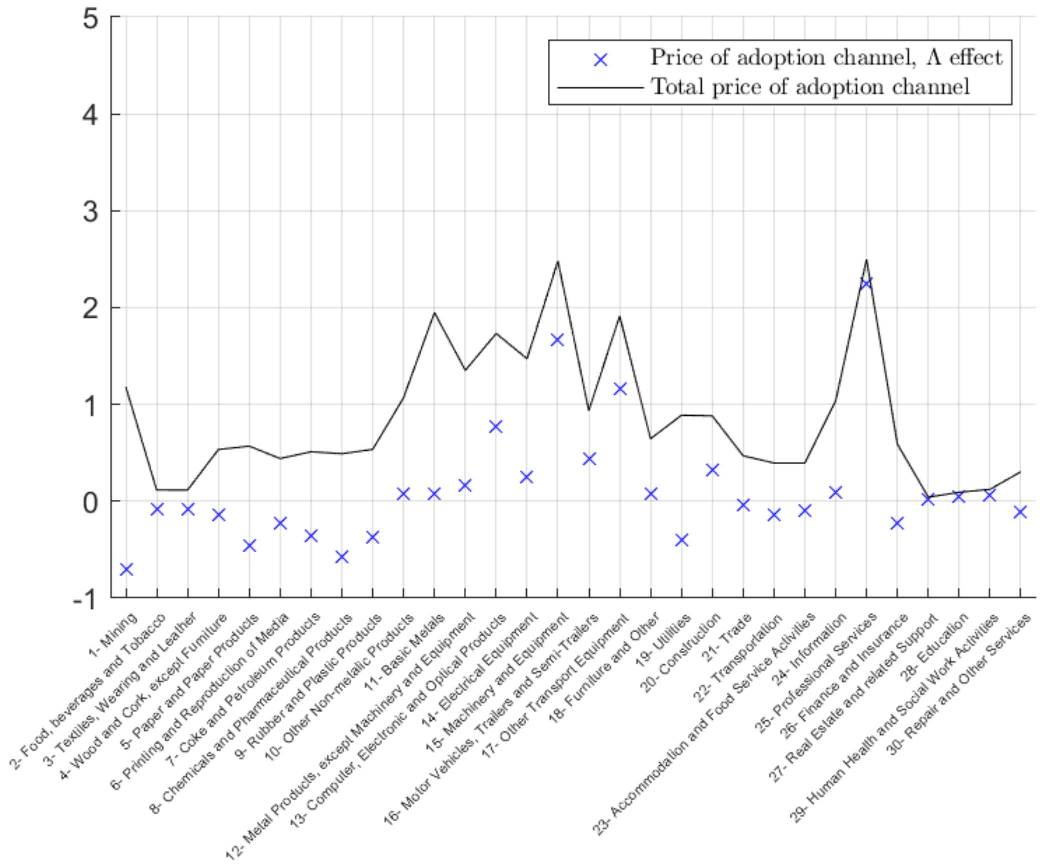
Figure 9 presents, for each sector, the resulting multiplier under the four alternative policy instruments: revenue subsidies, intermediate input subsidies, labor subsidies, and adoption subsidies. Next we discuss the main takeaways.

First, among all instruments, labor subsidies r_l are the most ineffective tool to promote development, as evident from the fact that, for all sectors, labor subsidies provide the lowest multipliers. This is expected, as labor is in fixed supply, and consistent with the results in Section 3.3.1.

Second, revenue multipliers ϵ_r are always above labor multipliers ϵ_{r_l} and below intermediate input multipliers ϵ_{r_x}. While consistent with the results in 3.3.1, it is surprising that there is no sector where mutlipliers under revenue subsidies are above multipliers under intermediate inputs, given that revenue subsidies have a direct effect on the marginal adopters' condition and intermediate input subsidies do not.

Third, and most importantly, adoption subsidies appear to be the most effective way to promote economic

Figure 8: Investment $\mathbf{\Lambda}$ or production $\mathbf{\Omega}$ networks?

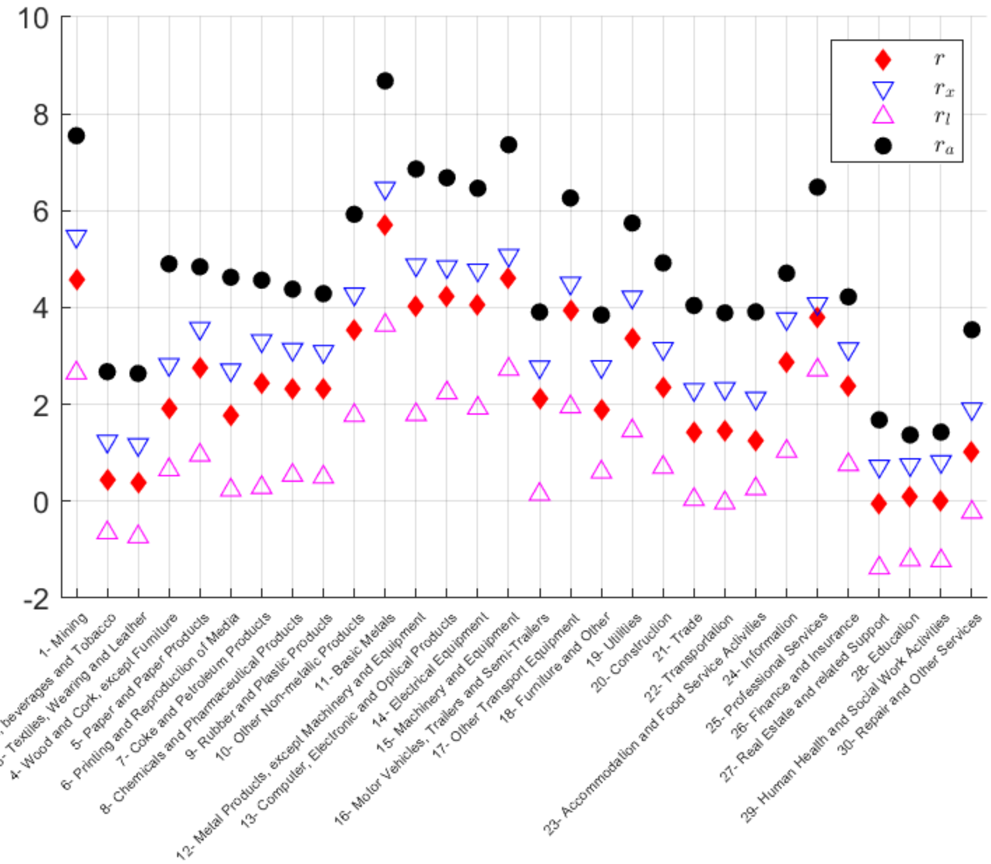


development, in spite from the fact that they do not have a direct effect on the production efficiency channel, as evident from (20).[12] Overall, among all sectors and among all policy instruments, the top 5 multipliers are obtained through implementing adoption subsidies in the following sectors (in order of relevance): 11- Basic Metals, 24- Information, 15- Machinery and Equipment, 1- Mining, and 12- Metal Products, except Machinery and Equipment.

Along the same lines as we did with the amplification term associated with a revenue subsidy, we can decompose the adoption multiplier can be decomposed into a direct effect and a feedback through the adoption channel:

[12]This explains why, for a small subset of sectors, intermediate input subsidies dominate adoption subsidies. This occurs for sector 10- Other Non-metallic products, sector 16- Motor Vehicles, Trailers and Semi-trailers, and sector 26- Finance and Insurance.

Figure 9: Multipliers under alternative policy instruments



$$\boldsymbol{\epsilon}_{\boldsymbol{r^a}} = \overbrace{\left(\tilde{\boldsymbol{\Psi}}_{\boldsymbol{p}}{}' \oslash \boldsymbol{\Psi}_{\boldsymbol{p}}{}' - \frac{\eta-1}{\eta}\mathbf{1}\right)\frac{\eta}{\eta-1}\frac{\zeta}{\eta-1}}^{\text{(1) direct effect}}$$

$$+\overbrace{\left[\left(\tilde{\boldsymbol{\Psi}}_{\boldsymbol{p}} - \frac{\eta-1}{\eta}\boldsymbol{\Psi}_{\boldsymbol{p}}{}'\right)'\beta\texttt{diag}\left(\boldsymbol{M}\right)\left(\boldsymbol{\nabla}_{\boldsymbol{a},\boldsymbol{r^a}} - \frac{\zeta}{\eta-1}\boldsymbol{I}\right)\right]\oslash\boldsymbol{\Delta}'}^{\text{(2) feedback through adoption channel}}.$$

5 Concluding remarks

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Appendix

A Proofs and derivations

A.1 Domar weights

Multiplying both sides of (7) by P_s and using that $P_sC_s = \gamma_s P_c C$ provides that

$$\gamma_s P_c C + \sum_{s'} \omega_{s's} P_{xs'} X_{s'} + \sum_{s'} \lambda_{s's} P_{ms'} a_{s'} \kappa_{ms'} = P_s Y_s \ ,$$

and by using the intermediate input demand in (3), the expression relating P_s and P_{xs} in (10), and the definition of sector productivity in (9) we obtain

$$\gamma_s P_c C + \nu \frac{\eta - 1}{\eta} \sum_{s'} \omega_{s's} r_{s'} P_{s'} Y_{s'} + \sum_{s'} \lambda_{s's} P_{ms'} a_{s'} \kappa_{ms'} = P_s Y_s \ . \tag{26}$$

When $r_s = 1$ for all s, and by dividing by $P_c C$ we obtain for each s that

$$\begin{aligned} \frac{P_s Y_s}{P_c C} - \nu \frac{\eta - 1}{\eta} \sum_{s'} \omega_{s's} \frac{P_{s'} Y_{s'}}{P_c C} &= \gamma_s + \sum_{s'} \lambda_{s's} \frac{P_{ms'} a_{s'} \kappa_{ms'}}{P_c C} \ , \\ \frac{P_s Y_s}{P_c C} - \nu \frac{\eta - 1}{\eta} \sum_{s'} \omega_{s's} \frac{P_{s'} Y_{s'}}{P_c C} &= \gamma_s + \sum_{s'} \lambda_{s's} \delta_{s'} \ . \end{aligned}$$

We can stack these equations and obtain an expression for the Domar weigths $\boldsymbol{\Psi_p}$,

$$\boldsymbol{\Psi_p} = \left(\boldsymbol{I} - \nu \frac{\eta - 1}{\eta} \boldsymbol{\Omega}' \right)^{-1} \left(\boldsymbol{\Gamma} + \boldsymbol{\Lambda}' \boldsymbol{\Delta} \right) \ .$$

Or, expressed in row vectors,

$$\boldsymbol{\Psi_p}' = \left(\boldsymbol{\Gamma}' + \boldsymbol{\Delta}' \boldsymbol{\Lambda} \right) \left(\boldsymbol{I} - \nu \frac{\eta - 1}{\eta} \boldsymbol{\Omega} \right)^{-1} \ .$$

A.2 Derivation of equation (16)

We begin by noting that we can combine labor input demand in (2), teh expression relating P_s and P_{xs} in (10), the definition of sector productivity in (9) with the labor market clearing condition in (8) to obtain the following expression,

$$L = (1 - \nu) \frac{\eta - 1}{\eta} \sum_s r_s P_s Y_s \ .$$

Log-differentiating this expression around $\boldsymbol{r}=\mathbf{1}$,

$$\sum_{s} P_s Y_s \left(d\ln r_s + d\ln P_s Y_s\right) = 0 \ . \tag{27}$$

Note how, by dividing by $P_c C$, this expression provides that $\boldsymbol{\Psi_p}' \left[d\ln \boldsymbol{r} + d\ln\left(\boldsymbol{P}\circ \boldsymbol{Y}\right)\right]=0$. That is, weighted by Domar weights, the total change in subsidies is offset by the change in gross product.

Log-differentiating the goods market clearing condition in (26) around $\boldsymbol{r}=\mathbf{1}$ provides,

$$\gamma_s P_c C \left(d\ln P_c + d\ln C\right) + \nu\frac{\eta-1}{\eta}\sum_{s'}\omega_{s's}P_{s'}Y_{s'}\left(d\ln r_{s'} + d\ln P_{s'}Y_{s'}\right)$$
$$+\sum_{s'}\lambda_{s's}P_{ms'}a_{s'}\kappa_{ms'}\left(d\ln P_{ms'} + d\ln a_{s'}\right) = P_s Y_s d\ln P_s Y_s \ .$$

Adding across all sectors s, using (27), and dividing by $P_c C$,

$$d\ln C = -\,d\ln P_c - \sum_{s'}\frac{P_{ms'}a_{s'}\kappa_{ms'}}{P_c C}\left(d\ln P_{ms'} + d\ln a_{s'}\right) - \sum_{s}\frac{P_s Y_s}{P_c C} d\ln r_s \ ,$$

or, in matrix form,

$$d\ln C = -\,d\ln P_c - \boldsymbol{\Delta}'\left(d\ln \boldsymbol{P_m} + d\ln \boldsymbol{a}\right) - \boldsymbol{\Psi_p}' d\ln \boldsymbol{r}$$

Using that from (12) we obtain that $d\ln \boldsymbol{P} = -\left(\boldsymbol{I}-\nu\boldsymbol{\Omega}\right)^{-1}\left(d\ln \boldsymbol{Z} + d\ln \boldsymbol{r}\right)$, and using (13),

$$d\ln C = \left[\boldsymbol{\Gamma}' + \boldsymbol{\Delta}'\boldsymbol{\Lambda}'\right]\left(\boldsymbol{I}-\nu\boldsymbol{\Omega}\right)^{-1}\left(d\ln \boldsymbol{Z} + d\ln \boldsymbol{r}\right) - \boldsymbol{\Delta}' d\ln \boldsymbol{a} - \boldsymbol{\Psi_p}' d\ln \boldsymbol{r} \ .$$

Finally, noting that $\tilde{\boldsymbol{\Psi}}_{\boldsymbol{p}}{}' = \left[\boldsymbol{\Gamma}' + \boldsymbol{\Delta}'\boldsymbol{\Lambda}'\right]\left(\boldsymbol{I}-\nu\boldsymbol{\Omega}\right)^{-1}$,

$$d\ln C = \left(\tilde{\boldsymbol{\Psi}}_{\boldsymbol{p}}{}' - \boldsymbol{\Psi_p}'\right) d\ln \boldsymbol{r} + \tilde{\boldsymbol{\Psi}}_{\boldsymbol{p}}{}' d\ln \boldsymbol{Z} - \boldsymbol{\Delta}' d\ln \boldsymbol{a} \ . \tag{28}$$

Combining (4), (6), and (10), and specializing to the case with only revenue subsidies, we obtain

$$\frac{r_s}{\eta}\frac{\left(A_m^{\eta-1} - A_t^{\eta-1}\right) a_s^{-\frac{\eta-1}{\zeta}}}{Z_s^{\eta-1}} P_s Y_s = P_{ms}\kappa_{ms}$$

or

$$r_s\frac{\eta-1}{\eta}\beta M_s P_s Y_s = P_{ms}\kappa_{ms}.$$

Finally, writing the conditions in vector form, dividing both sides of the expression by $P_c C$ and setting

$\boldsymbol{r} = \mathbf{1}$, there is a tight connection between the GDP share of a sector, or Domar weight, and its adoption share of GDP,

$$\frac{\eta-1}{\eta}\beta\,\texttt{diag}\left(\boldsymbol{M}\right)\boldsymbol{\Psi_p} = \boldsymbol{\Delta}\ .$$

Applying this expression to (28) completes the derivation.

A.3 The elasticity of adoption to revenue and adoption subsidies

We begin by computing an expression for $d\ln\left(\boldsymbol{P}\circ\boldsymbol{Y}\right)$. To this end, we first rewrite (26) in matrix form and log-differentiate to get,

$$\begin{aligned}\left(\boldsymbol{I}-\nu\frac{\eta-1}{\eta}\boldsymbol{\Omega}'\right)\texttt{diag}\left(\boldsymbol{\Psi_p}\right)d\ln\left(\boldsymbol{P}\circ\boldsymbol{Y}\right) =&\boldsymbol{\Gamma}\left(d\ln P_c + d\ln C\right)\\ &+\boldsymbol{\Lambda}'\texttt{diag}\left(\boldsymbol{\Delta}\right)\left(\frac{d\ln\boldsymbol{P_m}}{d\ln\boldsymbol{a}}+\boldsymbol{I}\right)d\ln\boldsymbol{a}\\ &+\left[\boldsymbol{\Lambda}'\texttt{diag}\left(\boldsymbol{\Delta}\right)\frac{d\ln\boldsymbol{P_m}}{d\ln\boldsymbol{r}}+\nu\frac{\eta-1}{\eta}\boldsymbol{\Omega}'\texttt{diag}\left(\boldsymbol{\Psi_p}\right)\right]d\ln\boldsymbol{r}\ .\end{aligned}$$

Also, we follow the same steps as in Appendix A.2 to obtain an expression for $d\ln P_c + d\ln C$,

$$d\ln P_c + d\ln C = -\boldsymbol{\Delta}'\left(\frac{d\ln\boldsymbol{P_m}}{d\ln\boldsymbol{a}}+\boldsymbol{I}\right)d\ln\boldsymbol{a} - \left(\boldsymbol{\Delta}'\frac{d\ln\boldsymbol{P_m}}{d\ln\boldsymbol{r}}+\boldsymbol{\Psi_p}\right)d\ln\boldsymbol{r}\ .$$

Combining these two expressions provide that

$$d\ln\left(\boldsymbol{P}\circ\boldsymbol{Y}\right) = \boldsymbol{\nabla_{PY,a}}d\ln\boldsymbol{a} + \boldsymbol{\nabla_{PY,r}}d\ln\boldsymbol{r}\ ,$$

where $\boldsymbol{\nabla_{PY,a}} = \left[\left(\boldsymbol{I}-\nu\frac{\eta-1}{\eta}\boldsymbol{\Omega}'\right)\texttt{diag}\left(\boldsymbol{\Psi_p}\right)\right]^{-1}\left[\boldsymbol{\Lambda}'\texttt{diag}\left(\boldsymbol{\Delta}\right)\left(\frac{d\ln\boldsymbol{P_m}}{d\ln\boldsymbol{a}}+\boldsymbol{I}\right)-\boldsymbol{\Gamma}\boldsymbol{\Delta}'\left(\frac{d\ln\boldsymbol{P_m}}{d\ln\boldsymbol{a}}+\boldsymbol{I}\right)\right]$, and $\boldsymbol{\nabla_{PY,r}} = \left[\left(\boldsymbol{I}-\nu\frac{\eta-1}{\eta}\boldsymbol{\Omega}'\right)\texttt{diag}\left(\boldsymbol{\Psi_p}\right)\right]^{-1}\left[\boldsymbol{\Lambda}'\texttt{diag}\left(\boldsymbol{\Delta}\right)\frac{d\ln\boldsymbol{P_m}}{d\ln\boldsymbol{r}}+\nu\frac{\eta-1}{\eta}\boldsymbol{\Omega}'\texttt{diag}\left(\boldsymbol{\Psi_p}\right)-\boldsymbol{\Gamma}\left(\boldsymbol{\Delta}'\frac{d\ln\boldsymbol{P_m}}{d\ln\boldsymbol{r}}+\boldsymbol{\Psi_p}\right)\right]$, and where, using (12) and (13), $\frac{d\ln\boldsymbol{P_m}}{d\ln\boldsymbol{a}} = -\boldsymbol{\Lambda}\left(\boldsymbol{I}-\nu\boldsymbol{\Omega}\right)^{-1}\frac{d\ln\boldsymbol{Z}}{d\ln\boldsymbol{a}}$, and $\frac{d\ln\boldsymbol{P_m}}{d\ln\boldsymbol{r}} = -\boldsymbol{\Lambda}\left(\boldsymbol{I}-\nu\boldsymbol{\Omega}\right)^{-1}$.

Having obtained an expression for $d\ln\left(\boldsymbol{P}\circ\boldsymbol{Y}\right)$, we can now solve for $d\ln\boldsymbol{a}$ using the marginal adopters' conditions,

$$d\ln\boldsymbol{r} - (\eta-1)d\ln\boldsymbol{Z} + d\ln\left(\boldsymbol{P}\circ\boldsymbol{Y}\right) - \frac{\eta-1}{\zeta}d\ln\boldsymbol{a} = -d\ln\boldsymbol{r^a} + d\ln\boldsymbol{P_m}\ .$$

Substituting the expression for $d\ln\left(\boldsymbol{P}\circ\boldsymbol{Y}\right)$ produced in (31) and rearranging,

$$\left\{\frac{\eta-1}{\zeta}\boldsymbol{I}+\left[(\eta-1)\boldsymbol{I}-\frac{d\ln\boldsymbol{P_m}}{d\ln\boldsymbol{Z}}\right]\frac{d\ln\boldsymbol{Z}}{d\ln\boldsymbol{a}}-\boldsymbol{\nabla_{PY,a}}\right\}d\ln\boldsymbol{a} = \left(\boldsymbol{I}+\boldsymbol{\nabla_{PY,r}}+\frac{d\ln\boldsymbol{P_m}}{d\ln\boldsymbol{r}}\right)d\ln\boldsymbol{r} + d\ln\boldsymbol{r^a}\ ,$$

so that

$$d\ln \boldsymbol{a} = \boldsymbol{\nabla}_{\boldsymbol{a},\boldsymbol{r^a}} \left[\boldsymbol{I} + \boldsymbol{\nabla}_{\boldsymbol{PY},\boldsymbol{r}} + \boldsymbol{\Lambda} \left(\boldsymbol{I} - \nu \boldsymbol{\Omega} \right)^{-1} \right] d\ln \boldsymbol{r} + \boldsymbol{\nabla}_{\boldsymbol{a},\boldsymbol{r^a}} d\ln \boldsymbol{r^a} \ ,$$

where

$$\begin{aligned}
\boldsymbol{\nabla}_{\boldsymbol{a},\boldsymbol{r^a}} &= \left\{ \frac{\eta-1}{\zeta} \boldsymbol{I} + \left[(\eta-1)\boldsymbol{I} - \boldsymbol{\Lambda} \left(\boldsymbol{I} - \nu \boldsymbol{\Omega} \right)^{-1} \right] \frac{d\ln \boldsymbol{Z}}{d\ln \boldsymbol{a}} - \boldsymbol{\nabla}_{\boldsymbol{PY},\boldsymbol{a}} \right\}^{-1} \ , \\
&= \frac{\zeta}{\eta-1} \left\{ \boldsymbol{I} - \left[\frac{\boldsymbol{\Lambda} \left(\boldsymbol{I} - \nu \boldsymbol{\Omega} \right)^{-1}}{\eta-1} - \boldsymbol{I} \right] \zeta \beta \mathtt{diag}\left(\boldsymbol{M} \right) - \frac{\zeta}{\eta-1} \boldsymbol{\nabla}_{\boldsymbol{PY},\boldsymbol{a}} \right\}^{-1} \ .
\end{aligned}$$

A.4 The density $h_s(l)$

We are interested in $H_s(l) = \Pr(l_{is}(\tilde{z},\tilde{\varepsilon}) < l) = \Pr(\ln l_{is}(\tilde{z},\tilde{\varepsilon}) < \ln l)$. Using (23) this reduces to

$$\begin{aligned}
H_s(l) &= \Pr\left(\ln \tilde{A}_i + \ln \tilde{z}_s + \tilde{\varepsilon} < \ln l \right) = \Pr\left(\tilde{\varepsilon} < \ln l - \ln \tilde{A}_i - \ln \tilde{z}_s \right) \ , \\
&= \frac{1}{\tilde{z}_{ts}^{-\tilde{\zeta}}} \int_{\tilde{z}_{ts}}^{\tilde{z}_{ms}} \Phi\left(\frac{\ln l - \ln \tilde{A}_t - \ln \tilde{z} - \tilde{\mu}}{\tilde{\chi}} \right) \tilde{\zeta} z^{-\tilde{\zeta}-1} d\tilde{z} + \frac{1}{\tilde{z}_{ts}^{-\tilde{\zeta}}} \int_{\tilde{z}_{ms}}^{\infty} \Phi\left(\frac{\ln l - \ln \tilde{A}_m - \ln \tilde{z} - \tilde{\mu}}{\tilde{\chi}} \right) \tilde{\zeta} z^{-\tilde{\zeta}-1} d\tilde{z} \ ,
\end{aligned}$$

where $\Phi(\cdot)$ denotes the CDF of a standard normal. To compute $h_s(l)$ recall that $h_s(l) = \frac{\partial H_s(l)}{\partial l}$. Then,

$$\begin{aligned}
h_s(l) =& \frac{1}{\tilde{\chi} l \tilde{z}_{ts}^{-\tilde{\zeta}}} \int_{\tilde{z}_{ts}}^{\tilde{z}_{ms}} \phi\left(\frac{\ln l - \ln \tilde{A}_t - \ln \tilde{z} - \tilde{\mu}}{\tilde{\chi}} \right) \tilde{\zeta} z^{-\tilde{\zeta}-1} d\tilde{z} \\
&+ \frac{1}{\tilde{\chi} l \tilde{z}_{ts}^{-\tilde{\zeta}}} \int_{\tilde{z}_{ms}}^{\infty} \phi\left(\frac{\ln l - \ln \tilde{A}_m - \ln \tilde{z} - \tilde{\mu}}{\tilde{\chi}} \right) \tilde{\zeta} z^{-\tilde{\zeta}-1} d\tilde{z} \ .
\end{aligned}$$

Integrating provides the expression in the text provided in (24).

A.5 Proof of Remark 5

Without loss of generality let $\tilde{A}_t = 0$. Because all entrants are adopters of the modern technology, $\tilde{z}_{ts} = \tilde{z}_{ms}$. Then (24) reduces to

$$h_s(l) = l^{-\tilde{\zeta}-1} \frac{\tilde{\zeta}}{\tilde{z}_{ms}^{-\tilde{\zeta}}} e^{\tilde{\mu}\tilde{\zeta} + \frac{\tilde{\chi}^2}{2}} \tilde{A}_m^{\tilde{\zeta}} \left[1 - \Phi\left(\frac{\ln \tilde{z}_{ms} - \ln l + \tilde{\mu} + \tilde{\chi}^2 \tilde{\zeta}}{\tilde{\chi}} \right) \right] \ .$$

Notice that

$$\frac{\partial h_s(l)}{\partial l} = \frac{h_s(l)}{l} \left[\frac{1}{\tilde{\chi}} haz \left(\frac{\ln \tilde{z}_{ms} - \ln l + \tilde{\mu} + \tilde{\chi}^2 \tilde{\zeta}}{\tilde{\chi}} \right) - \left(\tilde{\zeta} + 1 \right) \right] \ ,$$

where $haz(\cdot) \equiv \frac{\phi\left(\frac{\ln \tilde{z}_{ms} - \ln l + \tilde{\mu} + \tilde{\chi}^2 \tilde{\zeta}}{\tilde{\chi}}\right)}{1 - \Phi\left(\frac{\ln \tilde{z}_{ms} - \ln l + \tilde{\mu} + \tilde{\chi}^2 \tilde{\zeta}}{\tilde{\chi}}\right)}$ is the hazard rate of the normal distribution, where $haz' > 0$ and $haz'' > 0$.

Because $h_s(l) \geq 0$ and the hazard rate decreasing and convex in l, the distribution $H_s(l)$ has one mode. If $haz\left(\frac{\ln \tilde{z}_{ms} + \tilde{\mu} + \tilde{\chi}^2 \tilde{\zeta}}{\tilde{\chi}}\right) \leq \tilde{\chi}\left(\tilde{\zeta} + 1\right)$ then the mode is at $l = \tilde{z}_{ms}$. Otherwise, the mode is at some $l > \tilde{z}_{ms}$.

For the employment share distribution, recall that $g_s(l) = \frac{l h_s(l)}{\int \hat{l} h_s(\hat{l}) d\hat{l}}$, so that $\frac{\partial g_s(l)}{\partial l} \propto h_s(l) + l \frac{\partial h_s(l)}{\partial l}$. Then,

$$\frac{\partial g_s(l)}{\partial l} = \frac{g_s(l)}{l}\left[\frac{1}{\tilde{\chi}} haz\left(\frac{\ln \tilde{z}_{ms} - \ln l + \tilde{\mu} + \tilde{\chi}^2 \tilde{\zeta}}{\tilde{\chi}}\right) - \tilde{\zeta}\right].$$

As a result, again we have that if $haz\left(\frac{\ln \tilde{z}_{ms} + \tilde{\mu} + \tilde{\chi}^2 \tilde{\zeta}}{\tilde{\chi}}\right) \leq \tilde{\chi}\tilde{\zeta}$ then the mode is at $l = \tilde{z}_{ms}$. Otherwise, the mode is at some $l > \tilde{z}_{ms}$.

A.6 Proof of Remark 6

When $\chi \to 0$ we also have that $\tilde{\chi} \to 0$. Then, when $\chi \to 0$, the expression for $h_s(l)$ in (24) reduces to

$$h_s(l) = \begin{cases} \frac{l^{-\tilde{\zeta}-1}}{\tilde{z}_{ts}^{-\tilde{\zeta}}} \tilde{\zeta} \tilde{A}_t^{\tilde{\zeta}} & \text{if } \tilde{z}_{ts} \leq l \leq \tilde{z}_{ms}, \\ 0 & \text{if } \tilde{z}_{ms} \leq l < \tilde{A}_m \tilde{z}_{ms}, \\ \frac{l^{-\tilde{\zeta}-1}}{\tilde{z}_{ts}^{-\tilde{\zeta}}} \tilde{\zeta} \tilde{A}_m^{\tilde{\zeta}} & \text{if } l \geq \tilde{z}_{ms}. \end{cases} \tag{29}$$

Because $\tilde{\zeta} > 0$ and $\tilde{A}_m > \tilde{A}_t$, $H_s(l)$ has two modes: one at $l = \tilde{z}_{ts}$, and one at $l = \tilde{z}_{ms}$. Likewise, because $g_s(l) \propto l h_s(l)$, $G_s(l)$ has the same two modes. Further, if $\tilde{A}_m = \tilde{A}_t$ both distributions have a single mode, and the mode is at $l = \tilde{z}_{ts}$.

B Additional tables and figures

In this section we collect additional results.

B.1 Sectoral revenue multipliers

Table III produces the revenue development multipliers for all sectors. The first column provides the (backward) Domar weight or influence measure $\tilde{\boldsymbol{\Psi}}_{\boldsymbol{p}}$ and the second column provides the (forward) Domar weight $\boldsymbol{\Psi}_{\boldsymbol{p}}$—the size of the sector. In the spirit of Baqaee and Farhi (2020), the third column provides the development multipliers fixing entry and adoption, ϵ^{ea}. The fourth column presents the multiplier fixing only entry, ϵ^{e}, and the fifth column presents the multiplier with both entry and adoption being active. The fact that there are no substantial differences between column 4 and 5 provides reassurances of our analysis abstracting from the entry margin in Section 3.

Table III: Sectoral (revenue) development multipliers

Division	$\tilde{\Psi}_p$	Ψ_p	ϵ_r^{ea}	ϵ_r^{e}	ϵ_r
1 - Mining	0.16	0.04	2.67	4.54	4.57
2 - Food, beverages and tobacco	0.30	0.20	0.48	0.54	0.45
3 - Textiles, wearing and leather	0.16	0.11	0.43	0.48	0.38
4 - Wood and cork, except furniture	0.02	0.01	1.27	1.77	1.91
5 - Paper and paper products	0.05	0.02	1.76	2.83	2.77
6 - Printing and reproduction of media	0.04	0.02	1.22	1.68	1.78
7 - Coke and petroleum products	0.29	0.12	1.48	2.43	2.44
8 - Chemicals and pharma. products	0.17	0.07	1.50	2.39	2.33
9 - Rubber and plastic products	0.06	0.03	1.43	2.42	2.33
10 - Other non-metallic products	0.04	0.01	1.85	3.55	3.52
11 - Basic metals	0.15	0.04	2.68	5.70	5.71
12 - Metal products, except machinery and equip.	0.08	0.03	1.91	3.87	4.03
13 - Computer, electronic and optical products	0.07	0.03	1.59	4.27	4.22
14 - Electrical equipment	0.05	0.02	1.79	4.11	4.06
15 - Machinery and equipment	0.07	0.03	1.16	4.63	4.60
16 - Motor vehicles, trailers and semi-trailers	0.08	0.05	0.76	2.20	2.13
17 - Other transport equipment	0.02	0.01	1.25	4.07	3.95
18 - Furniture and other	0.08	0.04	1.11	1.76	1.90
19 - Utilities	0.18	0.06	1.91	3.35	3.36
20 - Construction	0.23	0.11	1.12	2.28	2.36
21 - Trade	0.47	0.25	0.90	1.36	1.43
22 - Transportation	0.47	0.24	0.97	1.45	1.46
23 - Accommodation and food service activities	0.21	0.11	0.94	1.23	1.26
24 - Information	0.09	0.04	1.42	2.64	2.88
25 - Professional services	0.07	0.05	0.50	3.95	3.80
26 - Finance and insurance	0.37	0.15	1.43	2.44	2.39
27 - Real estate and related support	0.14	0.13	0.06	0.04	-0.06
28 - Education	0.10	0.09	0.09	0.17	0.10
29 - Human health and social work activities	0.05	0.05	0.10	0.13	-0.00
30 - Repair and other services	0.08	0.04	0.76	0.99	1.02

B.2 Decomposition of amplification under revenue multipliers

In this section we present the results of the decomposition of the amplification term, as described in Section 4.4.1. We perform the decomposition abstracting from the entry channel, as accounting for this channel is barely relevant for multipliers as evident in Table III. Each column of Table IV accounts for a term in the decomposition of $\boldsymbol{Amp}$.

Table IV: Decomposition

Division	(1)	(2)	(3)	(4)	(5)	(6)
1 - Mining	0.35	0.10	1.89	-0.71	0.39	-0.16
2 - Food, beverages and tobacco	0.09	-0.02	0.21	-0.09	0.05	-0.18
3 - Textiles, wearing and leather	0.09	-0.03	0.20	-0.09	0.05	-0.17
4 - Wood and cork, except furniture	0.18	-0.18	0.68	-0.14	0.15	-0.19
5 - Paper and paper products	0.24	0.23	1.03	-0.46	0.15	-0.13
6 - Printing and reproduction of media	0.18	-0.17	0.66	-0.22	0.13	-0.12
7 - Coke and petroleum products	0.21	0.08	0.87	-0.36	0.17	-0.03
8 - Chemicals and pharma. products	0.21	0.16	1.07	-0.57	0.15	-0.11
9 - Rubber and plastic products	0.20	0.20	0.91	-0.37	0.15	-0.11
10 - Other non-metallic products	0.25	0.16	1.00	0.07	0.38	-0.16
11 - Basic metals	0.35	0.18	1.87	0.07	0.72	-0.17
12 - Metal products, except machinery and equip.	0.26	-0.06	1.19	0.16	0.48	-0.06
13 - Computer, electronic and optical products	0.22	0.13	0.96	0.77	0.74	-0.14
14 - Electrical equipment	0.24	0.14	1.23	0.25	0.55	-0.09
15 - Machinery and equipment	0.17	0.03	0.81	1.67	0.98	-0.17
16 - Motor vehicles, trailers and semi-trailers	0.13	0.10	0.49	0.44	0.34	-0.06
17 - Other transport equipment	0.18	0.12	0.75	1.17	0.78	-0.12
18 - Furniture and other	0.17	-0.17	0.57	0.08	0.18	-0.18
19 - Utilities	0.26	0.09	1.28	-0.39	0.32	-0.12
20 - Construction	0.17	-0.06	0.55	0.33	0.30	-0.13
21 - Trade	0.14	-0.14	0.50	-0.03	0.13	-0.14
22 - Transportation	0.15	-0.08	0.54	-0.14	0.13	-0.13
23 - Accommodation and food service activities	0.15	-0.14	0.49	-0.09	0.12	-0.22
24 - Information	0.20	-0.20	0.94	0.10	0.28	-0.09
25 - Professional services	0.10	0.02	0.25	2.25	1.17	-0.32
26 - Finance and insurance	0.20	0.20	0.81	-0.23	0.18	-0.16
27 - Real estate and related support	0.05	-0.01	0.02	0.02	0.02	-0.11
28 - Education	0.05	0.05	0.05	0.04	0.04	-0.14
29 - Human health and social work activities	0.05	-0.05	0.06	0.06	0.04	-0.13
30 - Repair and other services	0.13	-0.12	0.42	-0.12	0.09	-0.16

C Online Appendix - Not for publication

C.1 The elasticity of aggregate consumption to revenue and adoption subsidies

In this Appendix we derive an expression for the elasticity of aggregate consumption with respect to revenue and adoption subsidies where both entry and technology adoption decisions are considered.

We begin by expanding the expression in (16),

$$d\ln C = \left(\tilde{\boldsymbol{\Psi}}_{\boldsymbol{p}}{}' - \boldsymbol{\Psi}_{\boldsymbol{p}}{}'\right) d\ln \boldsymbol{r} + \left(\tilde{\boldsymbol{\Psi}}_{\boldsymbol{p}}{}' - \frac{\eta-1}{\eta}\boldsymbol{\Psi}_{\boldsymbol{p}}{}'\right)\left(\frac{d\ln \boldsymbol{Z}}{d\ln \boldsymbol{e}} d\ln \boldsymbol{e} + \frac{d\ln \boldsymbol{Z}}{d\ln \boldsymbol{a}} d\ln \boldsymbol{a}\right) , \tag{30}$$

where $\frac{d\ln \boldsymbol{Z}}{d\ln \boldsymbol{a}} = \beta\texttt{diag}\left(\boldsymbol{M}\right)$, $\frac{d\ln \boldsymbol{Z}}{d\ln \boldsymbol{e}} = \beta_e \boldsymbol{I} - \beta\texttt{diag}\left(\boldsymbol{M}\right)$ and $\beta_e \equiv \frac{1}{\eta-1}\frac{\zeta-(\eta-1)}{\zeta}$. Obtaining an expression for the elasticity of aggregate consumption to revenue and adoption subsidies requires us to solve for $d\ln \boldsymbol{e}$ and $d\ln \boldsymbol{a}$.

We begin by computing an expression for $d\ln\left(\boldsymbol{P}\circ\boldsymbol{Y}\right)$. To this end, we first rewrite (26) in matrix form and log-differente to get,

$$\begin{aligned}
\left(\boldsymbol{I} - \nu\frac{\eta-1}{\eta}\boldsymbol{\Omega}'\right)\texttt{diag}\left(\boldsymbol{\Psi}_{\boldsymbol{p}}\right) d\ln\left(\boldsymbol{P}\circ\boldsymbol{Y}\right) =& \boldsymbol{\Gamma}\left(d\ln P_c + d\ln C\right) \\
&+ \boldsymbol{\Lambda}'\texttt{diag}\left(\boldsymbol{\Delta}\right)\left[\left(\frac{d\ln \boldsymbol{P}_{\boldsymbol{m}}}{d\ln \boldsymbol{a}} + \boldsymbol{I}\right) d\ln \boldsymbol{a}\right] \\
&+ \boldsymbol{\Lambda}'\texttt{diag}\left(\boldsymbol{\Delta}\right)\left(\frac{d\ln \boldsymbol{P}_{\boldsymbol{m}}}{d\ln \boldsymbol{e}} d\ln \boldsymbol{e}\right) \\
&+ \left[\boldsymbol{\Lambda}'\texttt{diag}\left(\boldsymbol{\Delta}\right)\frac{d\ln \boldsymbol{P}_{\boldsymbol{m}}}{d\ln \boldsymbol{r}} + \nu\frac{\eta-1}{\eta}\boldsymbol{\Omega}'\texttt{diag}\left(\boldsymbol{\Psi}_{\boldsymbol{p}}\right)\right] d\ln \boldsymbol{r} \ .
\end{aligned}$$

Also, we follow the same steps as in Appendix A.2 to obtain an expression for $d\ln P_c + d\ln C$,

$$\begin{aligned}
d\ln P_c + d\ln C = & -\boldsymbol{\Delta}'\left(\frac{d\ln \boldsymbol{P}_{\boldsymbol{m}}}{d\ln \boldsymbol{a}} + \boldsymbol{I}\right) d\ln \boldsymbol{a} \\
& - \left[\boldsymbol{\Delta}'\frac{d\ln \boldsymbol{P}_{\boldsymbol{m}}}{d\ln \boldsymbol{e}} + \frac{1-\nu\frac{\eta-1}{\eta}}{(1-\nu)\frac{\eta-1}{\eta}}\boldsymbol{\Delta}_{\boldsymbol{e}}{}'\right] d\ln \boldsymbol{e} \\
& - \left[\boldsymbol{\Delta}'\frac{d\ln \boldsymbol{P}_{\boldsymbol{m}}}{d\ln \boldsymbol{r}} + \boldsymbol{\Psi}_{\boldsymbol{p}}\right] d\ln \boldsymbol{r} \ ,
\end{aligned}$$

where $\boldsymbol{\Delta}_{\boldsymbol{e}} \equiv \boldsymbol{\kappa}_{\boldsymbol{t}}\circ\boldsymbol{e}/(P_c C)$. Also, using (12) and (13), $\frac{d\ln \boldsymbol{P}_{\boldsymbol{m}}}{d\ln \boldsymbol{e}} = -\boldsymbol{\Lambda}\left(\boldsymbol{I}-\nu\boldsymbol{\Omega}\right)^{-1}\frac{d\ln \boldsymbol{Z}}{d\ln \boldsymbol{e}}$, $\frac{d\ln \boldsymbol{P}_{\boldsymbol{m}}}{d\ln \boldsymbol{a}} = -\boldsymbol{\Lambda}\left(\boldsymbol{I}-\nu\boldsymbol{\Omega}\right)^{-1}\frac{d\ln \boldsymbol{Z}}{d\ln \boldsymbol{a}}$, and $\frac{d\ln \boldsymbol{P}_{\boldsymbol{m}}}{d\ln \boldsymbol{r}} = -\boldsymbol{\Lambda}\left(\boldsymbol{I}-\nu\boldsymbol{\Omega}\right)^{-1}$.

Combining these expressions provide

$$d\ln\left(\boldsymbol{P}\circ\boldsymbol{Y}\right) = \boldsymbol{\nabla}_{\boldsymbol{PY},\boldsymbol{a}} d\ln \boldsymbol{a} + \boldsymbol{\nabla}_{\boldsymbol{PY},\boldsymbol{e}} d\ln \boldsymbol{e} + \boldsymbol{\nabla}_{\boldsymbol{PY},\boldsymbol{r}} d\ln \boldsymbol{r} \ , \tag{31}$$

where

$$\boldsymbol{\nabla}_{\boldsymbol{PY},\boldsymbol{a}} = \left\{ \left[\boldsymbol{I} - \nu \frac{\eta - 1}{\eta} \boldsymbol{\Omega}' \right] \texttt{diag}\left(\boldsymbol{\Psi}_{\boldsymbol{p}}\right) \right\}^{-1} \left[\boldsymbol{\Lambda}' \texttt{diag}\left(\boldsymbol{\Delta}\right) - \boldsymbol{\Lambda}\boldsymbol{\Delta}' \right] \left(\frac{d \ln \boldsymbol{P}_{\boldsymbol{m}}}{d \ln \boldsymbol{a}} + \boldsymbol{I} \right) \ ,$$

$$\boldsymbol{\nabla}_{\boldsymbol{PY},\boldsymbol{e}} = \left\{ \left[\boldsymbol{I} - \nu \frac{\eta - 1}{\eta} \boldsymbol{\Omega}' \right] \texttt{diag}\left(\boldsymbol{\Psi}_{\boldsymbol{p}}\right) \right\}^{-1} \left[\left(\boldsymbol{\Lambda}' \texttt{diag}\left(\boldsymbol{\Delta}\right) - \boldsymbol{\Gamma} \right) \boldsymbol{\Delta}' \frac{d \ln \boldsymbol{P}_{\boldsymbol{m}}}{d \ln \boldsymbol{e}} - \boldsymbol{\Gamma} \frac{1 - \nu \frac{\eta-1}{\eta}}{(1-\nu)\frac{\eta-1}{\eta}} \boldsymbol{\Delta}_{\boldsymbol{e}}' \right] \ ,$$

$$\boldsymbol{\nabla}_{\boldsymbol{PY},\boldsymbol{r}} = \left\{ \left[\boldsymbol{I} - \nu \frac{\eta - 1}{\eta} \boldsymbol{\Omega}' \right] \texttt{diag}\left(\boldsymbol{\Psi}_{\boldsymbol{p}}\right) \right\}^{-1} \left[\left(\boldsymbol{\Lambda}' \texttt{diag}\left(\boldsymbol{\Delta}\right) - \boldsymbol{\Gamma}\boldsymbol{\Delta}' \right) \frac{d \ln \boldsymbol{P}_{\boldsymbol{m}}}{d \ln \boldsymbol{r}} + \nu \frac{\eta - 1}{\eta} \boldsymbol{\Omega}' \texttt{diag}\left(\boldsymbol{\Psi}_{\boldsymbol{p}}\right) - \boldsymbol{\Gamma} \boldsymbol{\Psi}_{\boldsymbol{p}}' \right] \ .$$

Having obtained an expression for $d \ln (\boldsymbol{P} \circ \boldsymbol{Y})$, we can now solve for $d \ln \boldsymbol{e}$ and $d \ln \boldsymbol{a}$ using the marginal entrant and marginal adopters' conditions,

$$d \ln \boldsymbol{r} - (\eta - 1) d \ln \boldsymbol{Z} + d \ln (\boldsymbol{P} \circ \boldsymbol{Y}) - \frac{\eta - 1}{\zeta} d \ln \boldsymbol{e} = 0 \ ,$$
$$d \ln \boldsymbol{r} - (\eta - 1) d \ln \boldsymbol{Z} + d \ln (\boldsymbol{P} \circ \boldsymbol{Y}) - \frac{\eta - 1}{\zeta} d \ln \boldsymbol{a} = -d \ln \boldsymbol{r}^{\boldsymbol{a}} + d \ln \boldsymbol{P}_{\boldsymbol{m}} \ .$$

Substituting the expression for $d \ln (\boldsymbol{P} \circ \boldsymbol{Y})$ produced in (31) and rearranging,

$$\overbrace{\left[\frac{\eta - 1}{\zeta} \boldsymbol{I} + (\eta - 1) \frac{d \ln \boldsymbol{Z}}{d \ln \boldsymbol{e}} - \boldsymbol{\nabla}_{\boldsymbol{PY},\boldsymbol{e}} \right]}^{\equiv \boldsymbol{\nabla}_{\boldsymbol{E},\boldsymbol{e}}} d \ln \boldsymbol{e} + \overbrace{\left[(\eta - 1) \frac{d \ln \boldsymbol{Z}}{d \ln \boldsymbol{a}} - \boldsymbol{\nabla}_{\boldsymbol{PY},\boldsymbol{a}} \right]}^{\equiv \boldsymbol{\nabla}_{\boldsymbol{E},\boldsymbol{a}}} d \ln \boldsymbol{a} = \overbrace{(\boldsymbol{I} + \boldsymbol{\nabla}_{\boldsymbol{PY},\boldsymbol{r}})}^{\equiv \boldsymbol{\nabla}_{\boldsymbol{E},\boldsymbol{r}}} d \ln \boldsymbol{r} \ ,$$
$$\overbrace{\left[(\eta - 1) \frac{d \ln \boldsymbol{Z}}{d \ln \boldsymbol{e}} - \boldsymbol{\nabla}_{\boldsymbol{PY},\boldsymbol{e}} \right]}^{\equiv \boldsymbol{\nabla}_{\boldsymbol{A},\boldsymbol{e}}} d \ln \boldsymbol{e} + \overbrace{\left[\frac{\eta - 1}{\zeta} \boldsymbol{I} + (\eta - 1) \frac{d \ln \boldsymbol{Z}}{d \ln \boldsymbol{a}} - \boldsymbol{\nabla}_{\boldsymbol{PY},\boldsymbol{a}} \right]}^{\equiv \boldsymbol{\nabla}_{\boldsymbol{A},\boldsymbol{a}}} = \overbrace{(\boldsymbol{I} + \boldsymbol{\nabla}_{\boldsymbol{PY},\boldsymbol{r}})}^{\equiv \boldsymbol{\nabla}_{\boldsymbol{A},\boldsymbol{r}}} d \ln \boldsymbol{r} + d \ln \boldsymbol{r}^{\boldsymbol{a}} \ .$$

Notice that these equations are linear in the vectors $d \ln \boldsymbol{e}$ and $d \ln \boldsymbol{a}$, and so there is a unique solution to the system. Stacking these equations in matrix form and solving for these two vectors provide,

$$\begin{pmatrix} d \ln \boldsymbol{e} \\ d \ln \boldsymbol{a} \end{pmatrix} = \begin{pmatrix} \boldsymbol{\nabla}_{\boldsymbol{E},\boldsymbol{e}} & \boldsymbol{\nabla}_{\boldsymbol{E},\boldsymbol{a}} \\ \boldsymbol{\nabla}_{\boldsymbol{A},\boldsymbol{e}} & \boldsymbol{\nabla}_{\boldsymbol{A},\boldsymbol{a}} \end{pmatrix}^{-1} \left[\begin{pmatrix} \boldsymbol{\nabla}_{\boldsymbol{E},\boldsymbol{r}} \\ \boldsymbol{\nabla}_{\boldsymbol{A},\boldsymbol{r}} \end{pmatrix} d \ln \boldsymbol{r} + \begin{pmatrix} \boldsymbol{0} \\ \boldsymbol{I} \end{pmatrix} d \ln \boldsymbol{r}^{\boldsymbol{a}} \right] \ .$$

Using these expressions in (C.1) imply that we fully specified the elasticity of aggregate consumption with respect to revenue, $\boldsymbol{r}$, and adoption subsidies, $\boldsymbol{r}^{\boldsymbol{a}}$.