# Models of Balance of Payments Crises with Capital Controls *


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**Abstract**

We study techniques to delay the collapse of an unsustainable fixed exchange rate regime where the rate of growth of domestic credit exceeds the rate of growth of the exchange rate.

Capital controls (with free trade) delay the collapse. In the transition, there is a trade deficit and a consumption boom, real interest rates are above international ones, and when reserves are depleted, there is an anticipated devaluation. Delaying monetization, keeping fiscal policy constant, extends the period of low inflation at the expense of higher inflation later.

Adding import restrictions that balance the current account to the capital controls makes the fixed exchange rate regime sustainable at the cost of misallocating resources. Binding import quotas create a wedge between domestic and international prices valued at the official exchange rate, which is an implicit export tax. The continuous excessive expansion of the money supply increases this wedge and leads to a steady state with no exports. In the transition path, inflation is lower than the rate of growth of domestic credit, aggregate consumption decreases as the economy transitions to autarky, real interest rates are below international ones, and the financial exchange rate is above the price level.



***Keywords***: dual exchange rates, capital controls, liberalization

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Governments that finance deficits by printing money would like to avoid their inevitable inflationary consequence. When the natural remedy of a fiscal adjustment is politically unfeasible, many governments fall to the temptation of using the exchange rate as a nominal anchor, fixing it at a level inconsistent with the money supply. Krugman (1979) showed that, with free capital mobility, this policy inexorably leads to a speculative attack on the central bank's reserves that forces the monetary authority to let the exchange rate float. After that, inflation rises to a level consistent with the deficit monetization. This story, with a smooth regime transition, however, rarely occurs. Before letting the low inflation fixed exchange rate regime go, governments often impose capital controls to delay the onset of higher inflation. When this is not enough, they impose import restrictions to balance the current account and protect their reserves. This paper models balance of payments crises under capital controls and other regime switch delaying techniques.

We study a small economy with an unsustainable fixed exchange rate regime due to excessive expansion of the monetary base's domestic credit component relative to the exchange rate's rate of crawl. We assume that the economy switches to a floating exchange rate regime whenever the central bank's reserves are depleted[1]. We compare the equilibria for three economic environments: an economy with free trade and free capital mobility—a Krugman (1979) economy, an economy with free trade and no capital mobility, and an economy with import restrictions and no capital mobility. In all the cases we compare the equilibrium paths of inflation, domestic interest rates, the current account, the shadow market exchange rate, and the timing of the collapse of the fixed exchange rate. We do so by unexpectedly imposing capital controls and import restrictions in a Krugman economy at date zero. In this context, we consider temporary contractions in the rate of growth of domestic credit, keeping fiscal policy constant, i.e., a Sargent and Wallace (1981) experiment.

We model capital controls as a quantitative restriction on the private sector's accumulation of foreign assets (zero for simplicity), which precludes agents from Krugman-style portfolio shifts. Restricting private capital mobility has several important implications. First, the current account becomes identical to the official accumulation of net foreign assets. Second, there is no arbitrage between domestic and international interest rates. At the equilibrium domestic interest rates, the government-imposed international portfolio flows are the private sector's optimal choice.

Imposing capital controls in a free trade environment with an unsustainable fixed exchange rate does not prevent the regime's collapse. Quite the contrary, it leads to a gradual depletion of reserves that ends with a fully anticipated devaluation. The excess supply of money created by the constant expansion of domestic credit has to clear through the drainage of official net foreign assets (reserves from now on), that is, through current account deficits. In the endowment model

[1] We assume there are no fundamental incentives for private capital flows as the domestic discount rate equals the international interest rate. The sole private incentive for accumulating foreign assets is to game the exchange rate regime, and the government imposes capital controls.

economy, this occurs through higher consumption. As the real money supply is a state variable, the jump in consumption increases velocity, which requires high nominal interest rates, leading to even higher consumption, velocity, and interest rates. Along this path, real money balances fall, exacerbating the excess supply of money. When reserves are finally depleted, the exchange rate floats, consumption falls to a new steady state, and nominal interest rates jump to a new steady state with high inflation. The money demand falls, and the exchange rate jumps. The return on domestic assets when the regime changes is negative (nominal assets are inflated away). This capital loss is fully anticipated and consistent with standard asset pricing theory: the marginal rate of substitution between consumption just before and right after the devaluation is smaller than one due to the fall in consumption when the regime changes. All along, the shadow exchange rate is increasing, and it is equal to the new equilibrium exchange rate at the regime switch time. The capital control is successful at extending the peg's survival. The government can prolong the low inflation regime even more, without changing its fiscal policy, by issuing debt and delaying the deficit monetization. This policy extends the duration of the fixed exchange rate regime at the cost of more inflation and a larger devaluation when it collapses.

Imposing import restrictions, in addition to capital controls, makes the fixed exchange rate regime sustainable and keeps inflation low for a long time. If import restrictions are set so that the current account balance is zero, the central bank's reserves are constant, and the regime becomes sustainable. This comes at the cost of misallocating resources. Import quotas break the arbitrage between the domestic price of tradeable goods and their import parity level. Capital controls force exporters to surrender their export proceeds at the official exchange rate. When domestic credit grows faster than the exchange rate, import constraints bind, domestic prices rise above their import parity, and there is a shadow exchange rate premium. The wedge between the official exchange rate and domestic prices is an implicit tax on exports that discourages production. As time goes by, the ratio between domestic prices and the official exchange rate diverges until the implicit export tax is 100%. At this point, the country is in a new steady state with autarky[2]. In this context, we compute the shadow price level that would prevail if authorities unexpectedly let the exchange rate float and remove import restrictions. Keeping fiscal and monetary constant, exchange rate unification would induce a jump in the price level. If a credible fiscal adjustment is announced at the time of liberalization, the higher money demand might induce deflation or

[2]Along this path, policymakers and commentators often talk about the country's foreign exchange constraint and the need for an export strategy when all that is happening is that the exchange rate is used as an implausible nominal anchor to an overly expansionary monetary policy.

allow authorities to buy reserves if they impose an exchange rate floor.

**Relevance**.

**Collapse of the Bretton Woods system**. The narrative of the central bank losing reserves in a Krugman economy, imposing capital controls when a speculative attack is imminent, and then devaluing and letting the exchange rate float seems to capture the end of the Bretton Woods system in which currencies were pegged to the dollar and the dollar, in turn, to gold. The Gold pool suffered a speculative attack in December 1967; in March 1968, the gold pool ended, and capital controls were introduced on gold transactions. Appendix D contains graphs of the Fed's Balance sheet and the price of gold consistent with this story. See Garber (1993) and Bordo et al. (2019).

**Emerging markets**. The model with import restrictions, in addition to capital controls, seems to be a good interpretation of how dual exchange regimes worked in Latin America in the 1980s (Kiguel et al., 1997) and of how they work today in some countries. Table 1 contains a list of countries with capital controls and dual exchange rates. Most have import restrictions and monetize fiscal deficits. Some examples are Ethiopia and Argentina.

**Literature**. The models in this paper rest on macroeconomic models developed in the 1970s and 1980s. The basic building block is Krugman (1979)'s model of speculative attacks, as developed in Calvo (1987), which incorporates Krugman's logic into a dynamic general equilibrium model with perfect foresight. Modeling equilibrium dynamics as the solution of forward-looking differential equations builds on Sargent and Wallace (1973) while the analysis of temporary policies builds on Sargent and Wallace (1981), Calvo (1986), and Calvo (1989). Modeling capital controls and dual exchange rates by finding the interest rates that support an equilibrium that satisfies the restrictions on international capital flows follows Frenkel and Razin (1989). Knowing the interest rate differential we value the shadow exchange rate as the ratio of the same Lucas (1978) tree traded onshore and offshore. Our treatment of the balance of payments blends the intertemporal approach to the current account (Obstfeld and Rogoff, 1995) with the monetary approach to the balance of payments (Frenkel and Johnson, 1976).

The closest paper to our model of balance of payments crisis and capital controls with free trade is Park and Sachs (1987), which introduces capital controls in Calvo (1987)'s model. This paper has the key insight of the Euler condition at the time of the anticipated devaluation in equation (17c). Our main contribution is to solve the model with an interest-elastic velocity in the micro-founded money demand specification. This is important because the demand for money

Table 1: Countries with Capital Controls and Dual Exchange Rates - March 2023

| | Exchange rates on March 31, 2023 | | |
|---|---|---|---|
| Country | Official | Parallel | Premium |
| Lebanon | 15,000 | 107,500 | 616.7 |
| Yemen (Sana'a vs. Aden) | 250 | 1,230 | 392.0 |
| Syria | 3,015 | 7,550 | 150.4 |
| Islamic Republic of Iran | 42,000<br>285,000 | 544,000 | 1195.2<br>1900 |
| Argentina | 209 | 391 | 87.1 |
| Ethiopia | 54.4 | 100.2 | 84.2 |
| Zimbabwe | 930 | 1,600 | 72.0 |
| Burundi (as of 12/31 /2022) | 2,061 | 3,359 | 63.0 |
| Nigeria | 461 | 745 | 61.6 |
| Algeria | 136 | 209 | 53.7 |
| Malawi | 1,028 | 1,495 | 45.4 |
| Myanmar | 2,100 | 2,857 | 36.0 |
| Congo, Democratic Rep. | 2,036 | 2,323 | 14.1 |
| Angola (as of 01 /27/2023) | 504 | 560 | 11.1 |
| Bangladesh | 106 | 113.3 | 6.9 |
| Lao PDR (as of 02/28/2023) | 16,221 | 17,327 | 6.8 |
| Ghana | 11.01 | 11.75 | 6.7 |
| Libya | 4.79 | 5.09 | 6.3 |
| Mozambique | 64.5 | 67.4 | 4.5 |
| Ukraine | 36.6 | 37.7 | 3.0 |
| Sri Lanka | 327 | 337 | 3.1 |
| Sudan | 590 | 605 | 2.5 |
| Venezuela | 24.5 | 24.7 | 0.8 |
| South Sudan | 851 | 850 | -0.1 |

Source: Malpass (2023)

Note: Iran has two official exchange rates: the baseline official exchange rate reported in the IFS database maintained by the IMF and the "NIMA" rate. NIMA is an online currency system launched by the central bank where exporters can sell foreign currency. Each is shown separately in this table in relation to the parallel market rate.

leading to the devaluation in our model is falling, while with a cash-in-advance constraint, the high consumption resulting from the excess supply of money and the monetary approach to the balance of payments increases the money demand before the anticipated devaluation. Our model also has a richer dynamic for real interest rates, and we compute the shadow exchange rate. We also add to Park and Sachs (1987) by analyzing a temporary reduction in the rate of growth of domestic credit in the spirit of Sargent and Wallace (1981) and Calvo (1986).

To the best of our knowledge, the model with import restrictions and capital controls is new. It shares common elements with Schmitt-Grohé and Uribe (2023) who study exchange rate controls as fiscal instruments. They also emphasize the misallocation caused by the implicit trade taxes embodied in the multiple exchange rate regime and find that a Ramsey planner would prefer to raise seignorage than trade taxes.

# 1 The Policy Environment

## 1.1 Capital Controls

We consider a regime in which foreign exchange transactions between residents and non-residents are treated differently depending on whether they reflect real transactions recorded in the current account of the balance of payments or financial transactions recorded in the capital account. Current account transactions are settled at a commercial (official) exchange rate exchange rate, and capital account transactions are settled at a financial (parallel) exchange rate. Furthermore, the monetary authority fixes the official exchange rate while the financial one floats.

Current account transactions, exports, and imports of goods and services, as well as the returns on foreign assets, are settled at the official exchange rate. In the presence of a positive parallel market premium, this amounts to taxing foreign exchange receipts and subsidizing payments. Proceeds from exports and net foreign asset income are compulsorily settled at the official exchange rate, typically below the floating financial exchange rate. On the other hand, importers and net debtors can buy foreign exchange at the official exchange rate. This institutional setup implies that the change in the Central Bank's international reserves equals the current account. Moreover, the fact that the change in reserves equals the current account implies that aggregate private international capital flows are zero. This is illustrated in the balance of payments identity in equation equation (1), where $f_t$ are the central bank´s net foreign assets (reserves).

$$\Delta f_t + \Delta \text{Private Net Foreign Assets}_t \equiv \text{Current Account} \tag{1}$$

If the counterpart to all current account transactions is the central bank so that $\Delta f_t = \text{Current Account}$, it must be the case that $\Delta\text{Private Net Foreign Assets}_t = 0$.

**Equivalent capital flow management taxes.** We consider an economy with quantitative restrictions on international capital flows that create a wedge between onshore and offshore interest rates. The capital flow management literature often considers taxes on international capital flows as the policy instrument, letting capital flows be endogenous. In our setup, the policy instrument is the quantities of international private capital flows, and the implicit taxes on international capital flows are the *equilibrium* wedge between offshore and onshore interest rates.

### 1.2 Multiple exchange rates and onshore interest rates

In spite of the fact that aggregate net international capital flows are zero, there might be non-zero gross international private capital flows that aggregate to zero. We capture this by allowing domestic trade in a Lucas (1978) that pays a constant dividend equal to the international interest rate, $r$, in units of the foreign good. The ratio between the tree's onshore domestic currency price and its offshore price is the shadow exchange rate. Its offshore price (in foreign currency) is one. This bond's local currency price is $Q_t$, which also represents the financial exchange rate. The parallel market premium, $q_t$, is the ratio between the tree´s price and the official exchange rate, $q_t \equiv Q_t/E_t$.

It is useful to define the real and the nominal onshore return on this bond and to express its price as the local present discounted value of its cash flow.

We assume that there are no anticipated jumps in the nominal price of the foreign bond so that $Q_t$ is a continuous function of time. Even though $Q_t$ is continuous, its derivative $\dot{Q}_t$ may be discontinuous, with $\dot{Q}_t \equiv \lim_{\delta\downarrow 0} \frac{Q_{t+\delta}-Q_t}{Q_t}$.

**Assumption 1 (No anticipated jumps in nominal asset prices)** *$Q_t$ is continuous in* $[0, \infty)$*.*

We use the following notational conventions when there is a jump in variable $x$ at time $T$: $x_T^+ = lim_{\delta\to 0} x(T+\delta)$, $x_T^- = lim_{\delta\to 0} x(T-\delta)$, $T^+ = lim_{\delta\to 0} T+\delta$ and $T^- = lim_{\delta\to 0} T-\delta$.

In the presence of capital controls and fixed exchange rates, there might be anticipated jumps in the official exchange rate. Let $T$ be the time of an expected devaluation of the official exchange rate. If at $T$ the exchange rate starts to float, $Q(T) = E(T^+)$, as there cannot be two prices for the same object. This implies the exchange rate premium the instant before the devaluation is $q_T^- \equiv \frac{Q_T}{E_T^-} = \frac{E_T^+}{E_T^-}$,

The nominal price of the bond must satisfy the following no-arbitrage condition

$$Q_t i_t = E_t r_t + \dot{Q}_t, \tag{2}$$

where $i_t$ is the nominal onshore interest rate. The opportunity cost of investing $Q_t$ units of local currency in the bond must equal its return, which consists of the coupon $r$ plus the capital gain $\dot{Q}_t$.

Coupons are paid in goods and valued at $E_t$[3].

Re-arranging terms, we can express nominal interest rates as

$$i_t = \frac{rE_t}{Q_t} + \frac{\dot{Q}_t}{Q_t}. \tag{3}$$

When $Q_t = E_t$ equation (3) reduces to $i_t = r + \epsilon$.

Let the real interest rate be defined as

$$\rho_t \equiv i_t - \pi_t \quad \text{for } t \neq T.$$

Under free trade, the rate of inflation equals the rate of depreciation so that $\pi_t = \epsilon$[4]. Equation (3) and the definition of $q_t$ imply

$$\rho_t = \frac{r}{q_t} + \frac{\dot{q}_t}{q_t} \qquad \text{for } t \neq T. \tag{4}$$

Equation (4) states that the domestic real return on a dollar-denominated bond equals its coupon, $r$, plus the capital gains as a percentage of its purchase price. Typically, when the exchange rate premium differs from one, or when it is expected to change from zero, domestic and international rates will differ[5].

The forward solution to equation (4) is

$$q_t = q_T^- e^{-\int_t^T \rho_s ds} + r \int_t^T e^{-\int_t^s \rho_x dx} ds \tag{5}$$

The black market premium at $t$ is the present value of its left limit at $T$, $q_T^- = \frac{Q_t}{E_T^-} = E_T^+/E_T^-$, plus the present value of the perpetuity's coupon between $t$ and $T$. It depends on the expected devaluation, the time of the devaluation, and the path of domestic interest rates. It follows that the shadow nominal exchange rate is $Q_t = q_t E_t$. Equivalently, integrating equation (3) and using $Q_T = E_T^+$ we get an analogous expression for the nominal shadow exchange rate,

$$Q_t = E_T^+ e^{-\int_t^T i_s ds} + r \int_t^T E_s e^{-\int_t^s i_x dx} ds \tag{6}$$

The shadow exchange rate anticipates the market exchange rate after the devaluation as it is equal to its value discounted by the nominal interest rate plus the present value of the coupon payments (settled at the official exchange rate) up to the regime switch.

[3]The term $rE_t/Q_t$ can be interpreted as the coupon payment, $r$, corrected by the implicit tax on foreign currency income derived from forcing asset holders to sell the proceeds of the bond's foreign currency interest at the exchange rate $E_t$ when its market price is $Q_t$.

[4]The zero lower bound restricts asset prices and the domestic real interest rate as $i_t \geq 0 \iff \dot{Q}_t > -rE_t \iff \rho_t > -\epsilon$.

[5]$r = \rho_t \iff \dot{q}_t = r(q_t - 1)$

### 1.3 Monetary, Exchange Rate, and Fiscal Policy

In this section, we describe the behavior of the central bank's balance sheet and its implications for the government's budget constraint and fiscal policy. We pay particular attention to portfolio rebalancing and jumps in prices at the time of regime switch as well as to the wedge between international and domestic real interest rates. The policy environment is very simple and follows the literature on speculative attacks on the balance of payments.

We assume that the central bank's credit to the government grows at a constant exogenous rate $\theta$. Under the fixed exchange rate regime, the exchange rate grows at the exogenous rate $\epsilon$. We study the case in which $\theta > \epsilon$.

We start with an analysis of the central bank's balance sheet as it is the root of the balance of payments crises. It is a record of the composition of the central bank's assets and liabilities which can be written as $E_t f_t + D_t = M_t$, where $f_t$ is the stock of international reserves, which are valued at the official exchange rate $E_t$, and $D_t$ denotes the central bank's net domestic assets or domestic credit[6] and $M_t$ is the money supply. It is convenient to denominate the CB's balance sheet in foreign currency,

$$f_t + d_t = m_t, \tag{7}$$

where $d_t \equiv D_t/E_t$ and $m_t \equiv M_t/E_t$.

We now relate the central bank's balance sheet to the government's budget. The consolidated public sector's budget constraint is

$$\dot{f}_t = (\tau_t - g) + r f_t - \frac{i_t B_t^g}{E_t} + \frac{\dot{M}_t + \dot{B}_t^g}{E_t} \qquad \text{for } t \neq T \tag{8a}$$

$$Q_T \left( f_T^+ - f_T^- \right) = M_T^+ + B_T^{+g} - M_T^- - B_T^{-g} \qquad \text{for } T, \tag{8b}$$

where $\tau_t$ are lump sum taxes, $g$ government expenditures, and $B_t^g$ is nominal government debt. International reserves are invested in a Lucas (1978) tree with a constant dividend $r$ per unit, and its foreign currency price is 1. For $t \neq T$ there are neither discrete portfolio shifts nor jumps in $E_t$ so that the derivatives in equation (8a) and $\dot{E}_t$ exist. Equation (8b) is a portfolio reallocation constraint restricting the value of the portfolio before and after a trade to be the same at the transacted price. This constraint has no flows as the time elapsed at $T$ is zero. We can write the revenue from money

[6] In the data $D_t$ would be the central bank's claims on government plus the claims on banks and its net worth, net of its sterilization liabilities.

creation as $\dot{M}_t/E_t = \dot{f}_t + f_t\epsilon + \dot{D}_t/E_t$. That is, the change in the money supply can be allocated to extending credit to the government or to accumulating foreign exchange reserves. This expression combined with equation (8a) implies that unfunded government deficits are financed with central bank credit[7];

$$\frac{\dot{D}_t}{E_t} = \left(i_t - \frac{\dot{B}_t^g}{B_t}\right)\frac{B_t^g}{E_t} - (r + \epsilon_t)\, f_t + g - \tau_t. \tag{9}$$

In most of the paper, we assume $B_t^g = 0$ for all $t$ and $\dot{D}_t = \theta D_t$, $\dot{E}_t = \epsilon E_t$ given $D_0$ and $E_0$ and taxes $\tau_t$ are chosen so that equation (9) holds. In some sections of the paper, we study the equilibrium impact of a Sargent and Wallace (1981) exercise. That is, delaying monetization by keeping $D_t$ constant for a while and then expanding $D_t$ to monetize primary deficits plus the real interest on the debt accumulated by the delay in the deficit monetization.

**A note on the literature**. Let the unfunded deficit be $\Delta_t$ be the right-hand side of equation (9) so that $\dot{D}_t = \Delta_t E_t$. A frequent assumption in the literature, for example Krugman (1979) and Calvo (1987), is that the rate of growth of domestic credit is constant, $\dot{D}_t/D_t = \theta$, and that the government sets the rate of devaluation at a constant level, $\epsilon_t = \epsilon \geq 0$ for $t < T$. These policies are inconsistent with the sustainability of the peg when $\epsilon < \theta$. This assumption is convenient to obtain simple expressions for computing the timing of the collapse of an unsustainable fixed exchange rate regime. However, it has the unappealing feature that the unfunded deficit financed with seignorage, $\Delta_t = \theta D_0 e^{(\theta-\epsilon)t}$, grows exponentially while the government pursues the policy with $\epsilon < \theta$. Once the fixed exchange rate regime collapses and the exchange rate floats, $\epsilon = \theta$, and the deficit is constant at $\Delta_T = \theta D_0 e^{(\theta-\epsilon)T}$.

Perhaps a more natural assumption would be a constant unfunded deficit, $\Delta$, and a fixed exchange rate crawling at the rate $\epsilon$. In this case, $\dot{D}_t = \Delta E_t$ and $D_t = D_0 + \Delta E_0 e^{\epsilon t} t$. Real domestic credit becomes $\frac{D_t}{E_t} \equiv d_t = d_0 e^{-\epsilon t} + \Delta t$, which is a more complicated function of time (than the exponential) and increases if, and only if, $\epsilon < \theta_0 \equiv \frac{\Delta E_0}{D_0}$. The rate of growth of domestic credit, $\theta_t = \epsilon + \frac{\dot{d}_t}{d_t}$, is decreasing and converges to $\epsilon$ from above. We follow the literature by assuming a constant $\theta$.

[7]We allow for a budget constraint in which the government issues debt as we will analyze the impact of the government´s financial policy on the dynamics of the balance of payments crisis.

## 2 The Private Sector: Households

### Preferences

Consider an infinitely lived representative household with preferences described by the intertemporal utility function

$$\int_0^\infty u(c_t, m_t)e^{-rt}dt, \tag{10}$$

where we assumed that her discount factor is the international real interest rate and a constant endowment of $y$ units of the consumption good.

It is convenient to disentangle consumption dynamics from monetary dynamics by assuming $u(c_t, m_t)$ is homothetic and separable in $c$ and $m$. In the examples we compute, we assume the functional form

$$u(c_t, m_t) = \frac{c_t^{1-\sigma}}{1-\sigma} + \alpha\frac{m_t^{1-\sigma}}{1-\sigma}, \tag{11}$$

where $\sigma > 0$, $1/\sigma$ is the elasticity of intertemporal substitution of consumption as well as the elasticity of the money demand with respect to the nominal interest rate. Standard calibrations of the elasticity of intertemporal substitution in consumption set $\sigma \simeq 2$ (Crump et al., 2022) and Benati et al. (2021) estimate an elasticity of the money demand with $\sigma \simeq 2$.

### 2.1 Private budget constraints

The household's budget constraints are

$$\frac{\dot{M}_t + \dot{B}_t + Q_t\dot{b}_t^*}{E_t} = y - c_t - \tau_t + rb_t^* + \frac{i_tB_t}{E_t} \qquad \text{for } t \neq T \tag{12a}$$

$$0 = Q_T\left(b_T^{*+} - b_T^{*-}\right) + M_T^+ + B_T^+ - M_T^- - B_T^- \tag{12b}$$

Consumers accumulate wealth in the form of money balances, nominal domestic currency bonds, and foreign currency-denominated bonds. The constraint in equation (12b) restricts the value of portfolios at $T$ prices to be the same before and after a portfolio reallocation. Flows are ignored in equation (12b) as there is no accrual time.

Defining private wealth as $a_t \equiv m_t + b_t + q_tb_t^*$, and recalling $\rho_t \equiv r/q_t + \dot{q}_t/q_t$, we can write the representative agent's budget constraint as

$$\dot{a}_t = \rho_ta_t + y - c_t - \tau_t - (\rho_t + \epsilon_t)m_t \qquad \text{for } t \neq T \tag{13a}$$

$$a_T^+ - a_T^- = \left(\frac{1}{E_T^+} - \frac{1}{E_T^-}\right)\left(Q_T b_T^{*-} + M_T^- + B_T^-\right) = \left(\frac{E_T^-}{E_T^+} - 1\right) a_T^- \tag{13b}$$

Equation (13b) is the real capital gain of a portfolio reallocation at $T^-$, that satisfies constraint equation (12b) when the exchange rate is allowed to jump.

The household's intertemporal budget constraint is derived integrating equation (13a) considering equation (13b), assuming there is only one $T$, and imposing the no-Ponzi game condition $lim_{t\to\infty} a_t^g e^{-\int_0^t \rho_s ds} > 0$;

$$\int_0^\infty (c_t + i_t m_t)\, e^{-\int_0^t \rho_s ds}\, dt \le a_0 + \left(a_T^+ - a_T^-\right) e^{-\int_0^T \rho_s ds} + \int_0^\infty (y - \tau_t)\, e^{-\int_0^t \rho_s ds}\, dt, \tag{14}$$

As usual, the present value of the consumption plus the opportunity cost of holding real money balances has to be not greater than the present value of the consumer's wealth, which consists of her initial wealth, the present value of disposable income plus the present value of her capital gain at $T$. Observe that the capital gain at $T$, the term $a_T^+ - a_T^-$ in the intertemporal budget constraint, is a function of the household's choice of $a_T^-$ or, equivalently, her nominal wealth, $Q_T b_T^{*-} + M_T^- + B_T^-$, per equation (13b).

### 2.2 Households optimal choices

**Under free capital mobility**, with $\rho_t = r$ and $q_t = 1$ for all $t$, the household's problem is

$$\max_{\{c_t, m_t\}} \int_0^\infty u(c_t, m_t) e^{-rt} dt \text{ subject to } \begin{cases} \dot{a}_t = r a_t + y - c_t - \tau_t - i_t m_t \\ a_0 \text{ given and } \lim_{t\to\infty} a_t e^{-rt} \ge 0. \end{cases} \tag{15}$$

The solution to the household's problem is standard. For additively separable preferences, consumption is constant, $c$, and for homothetic preferences, there is a money demand function of the form $m = \ell(i_t)c$, with $\ell' < 0$.

Solving the household's problem **under capital controls** poses the challenge of the choice of nominal wealth at $T$ in the face of an anticipated devaluation. How should the household choose her nominal wealth the instant before she knows that prices will jump?

The consumer's problem is to maximize

$$\max_{c_t, m_t} \int_0^\infty u(c_t, m_t) e^{-rt} dt, \text{ subject to} \begin{cases} \dot{a}_t = \rho_t a_t + y - c_t - \tau_t - (\rho_t + \epsilon_t) m_t \text{ for } t \ne T \\ a_T^+ - a_T^- = \left(\frac{E_T^-}{E_T^+} - 1\right) a_T^- \\ a_0 \text{ given and } \lim_{t\to\infty} a_t e^{-\int_0^t \rho_s ds} \ge 0. \end{cases} \tag{16}$$

The key insight to solve the consumer's problem with an anticipated devaluation is to incorporate the discontinuity in asset dynamics in equation (13b) into the constraints of the household's problem. In Appendix A we show that the solution to this problem satisfies the following equations

$$\frac{u_m(c_t, m_t)}{u_c(c_t, m_t)} = i_t \qquad \text{for all } t \neq T, T^- \text{ and } T^+ \tag{17a}$$

$$\frac{\dot{c}_t}{c_t} = \frac{1}{-\frac{u_{cc}(c_t, m_t) c_t}{u_c(c_t, m_t)}} (\rho_t - r) \qquad \text{for all } t \neq T \tag{17b}$$

$$u_c\left(c_T^-, m_T^-\right) = u_c\left(c_T^+, m_T^+\right) \frac{E_T^-}{E_T^+} \tag{17c}$$

$$\int_0^\infty (c_t + i_t m_t) e^{-\int_0^\infty \rho_s ds} \, dt = a_0 + a_T^- \left(\frac{E_T^-}{E_T^+} - 1\right) + \int_0^\infty (y - \tau_t) e^{-\int_0^\infty \rho_s ds} \, dt. \tag{17d}$$

The first-order conditions to solve the household's problem for all $t \neq T$ are familiar. Equation (17a) yields a standard demand for money and equation (17b) is a standard Euler equation. As consumption and the nominal interest rate are discontinuous at $T$, the money demand at $T$ has to be evaluated for the interest rate and consumption at $T^-$ and $T^+$. For $t = T$ the Euler equation (17c) recognizes the non-differentiability of consumption and wealth. It states that the marginal rate of substitution between consumption at $T^-$ and $T^+$ is equal to the gross return on assets $\frac{E_T^-}{E_T^+}$. Agents arrive to period $T$ with nominal wealth $Q_T b_0^* + M_T^- + B_T^-$ knowing that they will suffer the capital loss $\frac{E_T^-}{E_T^+} - 1$. Nevertheless, they are willing to hold nominal assets because their value function from $T$ onward is an increasing function of their wealth at $T^+$, $a_T^- \frac{E_T^-}{E_T^+}$. The is illustrated by breaking the budget constraint in equation (17d) at $T$.

$$a_T^- = a_0 e^{\int_t^T \rho_s ds} + \int_0^T (y - c_t - i_t m_t - \tau_t) e^{\int_t^T \rho_s ds} \, dt \tag{18a}$$

$$\int_T^\infty (c_t + i_t m_t) e^{-\int_T^\infty \rho_s ds} \, dt = a_T^- \frac{E_T^-}{E_T^+} + \int_T^\infty (y - \tau_t) e^{-\int_T^\infty \rho_s ds} \, dt. \tag{18b}$$

The consumer's choices $\{c_t, m_t\}_{t=0}^T$ determine the wealth at $T^-$ through the budget constraint in equation (18a), which after the capital loss becomes the wealth with which the consumer starts her optimization problem at $T$ as equation (18b) illustrates. The marginal utility of consumption at $T^+$ is the shadow value of wealth at $T^+$, equation (18b)'s Lagrange multiplier for the consumer's problem starting at $T^+$.

## 3 Equilibrium Balance of Payments Crises

We now define and characterize equilibria with inconsistent monetary and exchange rate policies, $\theta > \epsilon$, under different economic environments. First, we consider Krugman's classic model in which there is free trade and free capital mobility as a benchmark case (Krugman, 1979). Then, we consider a model with capital controls and free trade and, finally, one with capital controls and import restrictions.

### 3.1 Aggregate Budget Constraints

It is usefull to know the aggregate budget constraint to characterize equilibria. Aggregating the flow public and private budget constraints, equation (8a) and equation (13a) for all $t \neq T$, we get the country's budget constraint

$$\dot{\bar{a}} \equiv \dot{f}_t + \dot{b}^*_t = r\left(f_t + b^*_t\right) + y - c_t - g \qquad \text{for all } t \tag{19}$$

The aggregate intertemporal budget constraint is continuous in $t$, and net foreign assets accrue the international interest rate. Portfolio reallocation between $f_t$ and $b^*_t$ or redistributions between the private and the public sectors due to changes in the value of money do not affect aggregate wealth.

Under capital controls, $\dot{b}^*_t = 0$. In this case, the monetary approach to the balance of payments links the current account of the balance of payments to the central bank's balance sheet by exploiting the fact that under capital controls, the balance of payments reduces to $\dot{\bar{a}} = \dot{f}_t$. This accounting identity implies that the current account balance is equal to the difference between the change in the money demand, $\dot{m}_t$, and domestic credit creation $\dot{d}_t$, that is, $\dot{f}_t = \dot{m}_t - \dot{d}_t$. The monetary approach to the balance of payments (Frenkel and Johnson (1976)) is captured by the equation

$$\dot{m}_t - \dot{d}_t = r\left(f_t + b^*_t\right) + y - c_t - g \qquad \text{for all } t, \tag{20}$$

which will play an important role in the determination of equilibrium under capital controls.

### 3.2 Perfect Capital Mobility and Free Trade

This section introduces Krugman (1979)'s model of a speculative attack on the central bank's reserves under perfect capital mobility as a benchmark.

**Assumption 2 (Krugman)** *Monetary and fiscal policy*

1. *Monetary and exchange rate policy*

   (a) *The rate of growth of domestic credit,* $\dot{D}_t/D_t = \theta$,

   (b) *The rate of crawl of the "fixed" exchange rate,* $\epsilon_t$, *is constant for* $t \leq T$.

   (c) *The fixed exchange rate regime is unsustainable:* $\theta > \epsilon$

2. *Fiscal dominance*

   (a) *Government's deficit is financed with central bank credit,* $\dot{D}_t = [g - \tau_t - (r + \epsilon_t) f_t] E_t$

   (b) *No access to credit,* $\forall t : B_t = 0$.

3. *Regime change: at time T when reserves are zero the exchange rate regime switches to a floating one.*

**Assumption 3** *Free capital mobility and positive interest rates* $r_t = \rho_t > 0$.

Under assumption 2 there is fiscal dominance in the sense that the central bank finances the treasury. But, as the rate of growth of domestic credit is exogenous, in a sense, there is monetary dominance: taxes adjust to the stock of domestic credit and to the exchange rate so that $\tau_t = g - rf_t - \theta D_0/E_0 \, e^{(\theta-\epsilon)t}$ while the fixed exchange rate regime is in place and $\tau_t = g - \theta m_t$ for $t \geq T$ once the fixed exchange rate regime collapses. This implies the government's budget constraint is satisfied.

**Definition 1 (Krugman Equilibrium)** *A* Krugman equilibrium with free capital mobility *is a regime switch time,* $T$, *a sequence of allocations* $\left\{c_t, m_t, b_t^*, f_t, \tau_t\right\}_{t=0}^{\infty}$, *and floating exchange rates* $\{E_t\}_{t=T}^{\infty}$ *such that given initial conditions* $\{D_0, E_0, a_0, a_0^g\}$, *international interest rates,* $r$, *assumption 2, assumption 3, and a sequence of government expenditures* $g$ *the following conditions hold.*

1. *Nominal interest rates satisfy the no-arbitrage condition equation (3).*

2. *Households solve the problem equation (15) given* $a_0$ *and the sequence of prices* $r$ *and* $\{i_t\}_{t=0}^{\infty}$.

3. $T$ *is the smallest* $t$ *such that* $f_t = 0$.

4. *The government's intertemporal budget constraint is satisfied.*

**Proposition 1** *The* Krugman Equilibrium *is characterized by equations (21)*

$$c = r(f_0 + b_0^*) + y - g \tag{21a}$$

$$i_t = \begin{cases} r + \epsilon & \text{for } t < T \\ r + \theta & \text{for } t \geq T \end{cases} \tag{21b}$$

$$m_t = \begin{cases} \ell(r + \epsilon)c & \text{for } t < T \\ \ell(r + \theta)c & \text{for } t \geq T \end{cases} \tag{21c}$$

$$f_t = \begin{cases} \ell(r + \epsilon)c - d_0 e^{(\theta - \epsilon)t} & \text{for } t < T \\ 0 & \text{for } t \geq T \end{cases} \tag{21d}$$

$$E_t = \begin{cases} E_0 e^{\epsilon t} & \text{for } t < T \\ E_0 e^{\epsilon T} e^{\theta(t-T)} & \text{for } t \geq T \end{cases} \tag{21e}$$

$$b_t^* = \begin{cases} a_0 - \ell(r + \epsilon)c + d_0 \left(e^{(\theta - \epsilon)t} - 1\right) & \text{for } t < T \\ a_0 - \ell(r + \theta)c & \text{for } t \geq T \end{cases} \tag{21f}$$

$$\tau_t = \begin{cases} g - r f_t - \theta D_0 / E_0 \, e^{(\theta - \epsilon)t} & \text{for } t < T \\ g - \theta m_T & \text{for } t \geq T \end{cases} \tag{21g}$$

$$T = \frac{\ln\left(\ell(r + \theta)c\right) - \ln d_0}{\theta - \epsilon} \tag{21h}$$

In order to characterize a Krugman equilibrium, it is useful to start with the central bank's balance sheet. Differentiating equation (7) we obtain

$$\dot{f} = \dot{m} - (\theta - \epsilon_t)\, d_t. \tag{22}$$

Equation (22) is an accounting identity that describes the evolution of reserves. Interpreting $\dot{m}$ as the change in the demand for real money balances and $(\theta - \epsilon_t)\, d_t$ as the real value of money creation, equation (22) states that reserves dynamics mirror the dynamics of the excess demand for money. In a Krugman equilibrium, reserves are zero for $t > T$, implying that the rate of growth of the nominal money supply is constant at $\theta$ and therefore $\epsilon_t = \theta$ for $t \geq T$ and real money balances are constant. For $t < T$, $\epsilon$ is also constant so $\dot{m}$ is constant as well. It follows that under the assumption $\theta > \epsilon$, $\dot{f} = -(\theta - \epsilon_t)\, d_t < 0$ for $t < T$ and the regime is unsustainable.

Equilibrium consumption in equation (21a) is obtained from the optimality condition equation (17b) for $\rho_t = r$ for all $t$, the separability of money balances and consumption in preferences (equation (10)), and the aggregation of the government's and the household's intertemporal budget constraints, into equation (19) with $\dot{\bar{a}}_t = 0$. Equation (21c) implies that when interest rates increase from $r + \epsilon$ to $r + \theta$ at time $T$ the money demand drops by

$$m_{t>T} - m_{t<T} = c\ell\left(r + \theta\right) - c\ell\left(r + \epsilon\right) = f_{t>T} - f_{t<T} < 0.$$

This drop in the money demand when inflation jumps is Krugman's speculative attack on the balance of payments. The time of speculative attack, $T$, is also the one for which there is no jump in the exchange rate at $T$. That is, at $T$,

$$E_T = \frac{M_T^-}{\ell(r+\epsilon)c} = \frac{M_T^+}{\ell(r+\theta)c}.$$

### 3.3 Capital Controls and Free Trade

This section characterizes equilibrium prices and allocations under an unsustainable fixed exchange rate regime with capital controls and free trade.

Let $a_0^- \equiv m_0 + b_0^*$ be the private sector's assets the instant before capital controls are implemented.

As discussed in section 4, the aggregate net foreign assets of the private sector are fixed under capital controls. We also assume that the initial level of reserves when capital controls are implemented is an equilibrium in the Krugman model with perfect capital mobility.

**Assumption 4** *Capital Controls*

1. $b_t^* = b_0^*$ *for all* $t$.
2. $m_0$, $b_0^*$ *and* $f_0$ *are a Krugman equilibrium given initial private wealth* $a_0^-$ *under assumption 2 and assumption 3.*

**Definition 2** *A* Krugman equilibrium with capital controls *is a regime switch time, $T$, a sequence of allocations* $\left\{c_t, m_t, b_t^*, f_t, \tau_t\right\}_{t=0}^{\infty}$, *shadow prices for the exchange rate* $\{Q_t\}_{t=0}^{\infty}$, *and floating exchange rates* $\{E_t\}_{t=T}^{\infty}$ *such that given initial conditions* $\{D_0, E_0, a_0^-, a_0^g\}$, *international interest rates, $r$, a sequence of government expenditures and endowments, $g$, $y$, assumption 2 and assumption 4 the following conditions hold.*

1. *Nominal interest rates satisfy the no-arbitrage condition equation (3).*
2. *Households solve the problem equation (16) given the sequence of prices $r$ and* $\{i_t, E_t, Q_t\}_{t=0}^{\infty}$.

3. *T is the smallest t such that* $f_t = 0$.

4. *The government's intertemporal budget constraint is satisfied.*

In definition 2, the exchange rate premium $q_t \equiv Qt/E_t$ and the onshore real return $\rho \equiv r/q_t + \dot{q}_t/q_t$ are convenient constructs for the formulation of the budget constraint in the consumer's problem.

### Characterization of Equilibrium

We characterize the equilibrium by breaking the problem in two (using the principle of optimality). First, we solve for the equilibrium after $T$, and then we solve for the equilibrium before $T$ and for the timing of the regime switch $T$.

**Equilibrium for t ≥ T.** The exchange rate floats after $T$, so $Q_T = E_T$, and $\rho_t = r$. This implies the equilibrium after $T$ has constant consumption, interest rates, and real money balances.

$$c_t = rb_0^* + y - g \qquad \text{for all } t \geq T \tag{23a}$$

$$m_t = \ell(r + \theta)c_t \qquad \text{for all } t \geq T \tag{23b}$$

$$i_t = r + \theta \qquad \text{for all } t \geq T \tag{23c}$$

$$E_t = \frac{D_0 e^{\theta t}}{\ell(r + \theta)c_t} \qquad \text{for all } t \geq T \tag{23d}$$

Under our assumptions, agents have no incentives to change their asset position so the presence of capital controls after $T$ is immaterial.

**Equilibrium for t < T.** The equilibrium before $T$ is characterized by the following dynamical system.

$$\begin{cases} \dot{m}_t = r(m_t + b_0^*) + y - g - c_t + (\theta - \epsilon - r)\, d_0 e^{(\theta-\epsilon)t} & \text{(24a)} \\ \dot{c}_t = \dfrac{c_t}{\sigma}\left( \underbrace{\overbrace{\dfrac{u_m(c_t, m_t)}{u_c(c_t, m_t)}}^{i_t} - \epsilon}_{\rho_t} - r \right) & \text{(24b)} \\ \dot{f}_t = r f_t + y - g - c_t & \text{(24c)} \end{cases}$$

Equation (24a) represents the monetary approach to the balance of payments (Frenkel and Johnson (1976)) in equation (20)[8], which captures the restriction on private capital mobility, the

[8] In equation (24a) the terms $f_t$ and $\dot{f}_t$ in equation (20) is substituted using $f_t = m_t - d_t$.

country's aggregate budget constraint, and the monetary and exchange rate policy. Equation (24b) represents the consumer's optimal choices. It combines the Euler equation (17b), the money demand equation (17a), and the definition of the real interest rate $\rho_t = i_t - \epsilon$. A path for $\{c_t, m_t\}$ that solves equations (24a)-(24b) solves the household's optimization problem and clears the money market for the equilibrium interest rates $i_t = \frac{u_m(c_t,m_t)}{u_c(c_t,m_t)}$. Finally, given the equilibrium path for consumption equation (24c) tracks the equilibrium evolution of reserves.

In order to determine the equilibrium, we need boundary conditions for each of the differential equations (24) plus a condition to find the time at which reserves are depleted, $T$. These are;

$$\begin{cases} m_0 = \ell(r+\epsilon)\left[r\left(f_0 + b_0\right) + y - g\right] & \text{(25a)} \\ u_c\left(c_T^-, m_T^-\right) = u_c\left(c_T^+, \ell(r+\theta)c_T^+\right)\dfrac{E_T^-}{E_T^+} & \text{(25b)} \\ f_0 = m_0 - d_0 & \text{(25c)} \\ f_T = 0, & \text{(25d)} \end{cases}$$

where $c_T^+ = rb_0^* + y - g$ and $\frac{E_T^-}{E_T^+} = \frac{\ell(r+\theta)c_T^+}{d_0 e^{(\theta-\epsilon)t}}$. The initial conditions for $f_0$ and $m_0$ in equations (25a) and (25c) ensure that given the policy parameters $D_0, E_0$, and $\epsilon$, the initial level of real money balances and reserves are a Krugman equilibrium. The Euler condition at $T^-$, equation (25b), acts as a terminal condition for consumption at $T$. Its right-hand side is given by the equilibrium at $T^+$ characterized in equation (23). If there is a devaluation at $T$ [9] the marginal utility at $T^-$ needs to be lower than at $T^+$. As there is a discrete drop in real money balances at $T$, this implies that for $u_{cm} \geq 0$ consumption before the regime switch is higher than afterward. In other words, there is a current account deficit just before $T$. Finally, the initial condition $f_0$ and the differential equation (24c) evaluated at the values of $\{c_t\}$ that solve equation (24a) and (24b) yield the value of $T$ for which $f_T = 0$.

### An Example

In this section, we illustrate the equilibrium through an example with the preferences defined in equation (11), $u = \frac{c^{1-\sigma}}{1-\sigma} + \alpha\frac{m^{1-\sigma}}{1-\sigma}$. We set $\sigma = 2$, which is consistent with the elasticity of intertemporal substitution in Crump et al. (2022) and with the elasticity of the money demand with respect to the nominal interest rates in Benati et al. (2021).

[9] A sufficient condition for a devaluation is for $T$ to be larger than the regime switch-time under the Krugman equilibrium with free capital mobility.

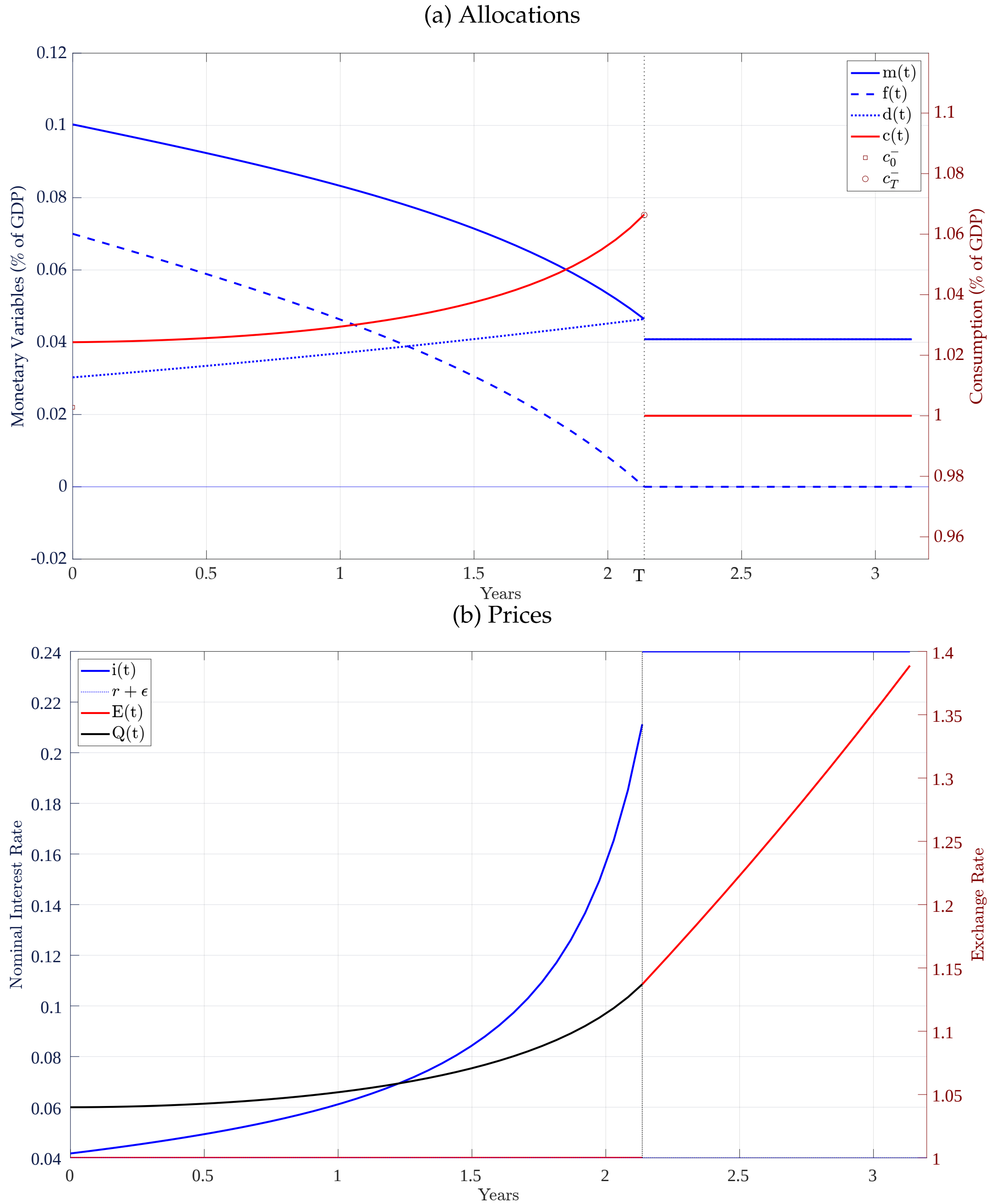
In this example, the optimization condition in equation (24b) becomes $\frac{\dot{c}_t}{c_t} = \frac{1}{\sigma}\left(\alpha\left(\frac{c}{m}\right)^{\sigma} - \epsilon - r\right)$ and the boundary condition in equation (25b) becomes $c_T^- = \left(rb_0^* + y - g\right)^{1-\frac{1}{\sigma}}\left(\frac{\alpha}{r+\theta}\right)^{\frac{1}{\sigma^2}} d_0^{-\frac{1}{\sigma}} e^{\frac{\theta-\epsilon}{\sigma}\mathbf{T}}$.

We compute the example through the following shooting algorithm. For each $c_0$, the initial conditions $\{c_0, m_0, f_0\}$ and the dynamic system governed by equation (24) yields terminal values $\{c_T^-, m_T^-, f_T^-\} = \{c_T^-, d_T, 0\}$. Thus, the terminal condition $f_T = 0$ defines a $T$ for each $c_0$, $T(c_0)$. Given $m_0$ and $f_0$, the shooting algorithm finds the value of $c_0$ for which $c_T^-(T(c_0), c_0) = c_T^-$, where $c_T^-(T(c_0), c_0)$ is the system of differential equations (24)'s solution for consumption at time $T(c_0)$ given initial conditions $\{c_0, m_0, f_0\}$, and $c_T^-$ stems from the boundary condition equation (25b) evaluated at $T(c_0)$.

Figure 1 shows the equilibrium allocations and prices in this example. We assume that before the imposition of capital controls, the economy was in a Krugman equilibrium with free capital mobility. At $t = 0$, agents know that the regime is unsustainable and that a devaluation will take place at $T$. The vertical dotted line at $T$ represents the endogenous time of the regime switch. The dotted blue line represents the exogenous path of the central bank's domestic credit, $d_0 e^{(\theta-\epsilon)t}$. At $t = 0$, when capital controls are imposed, the central bank's monetary expansion clears through current account deficits as predicted by the monetary approach to the balance of payments. In our endowment economy, consumption has to jump to induce a trade deficit. Given initial real money balances, the jump in consumption forces a jump in nominal interest rates, $m/c$ falls, to clear the money market. The higher nominal and real interest rates are shown in the solid blue line in figure 1b. They are the same since inflation is zero. As $i_t = \rho_t > r = r + \epsilon$ for $t < T$, consumption, represented by the red line in figure 1a, grows even more for $t > 0$. The level of consumption at $t = 0$ is the one for which consumption at $T^-$ is equal to $C_T^-$ from consumption's terminal condition in equation (25b). This terminal condition is represented by a red circle at T[10]. Another red circle at $t = 0$ represents consumption in the Krugman equilibrium, $rf_0 + y - g$. As time evolves, real money balances fall, and consumption grows at increasing rates, with nominal interest rates growing so that $i_t = \alpha(m_t/c_t)^{-\sigma}$. International reserves, $f_t = m_t - d_t$, are represented by the blue dashed lines in figure 1a. As nominal interest rates rise and the money demand falls while domestic credit rises steadily, reserves fall through current account deficits. The shadow exchange rate, $Q_t$, is depicted by the black line in figure 1b and obeys equation (6). At $t = 0$, it jumps above the official exchange rate depicted by the red line and grows until it reaches the value of the floating exchange rate at $T$. The shadow exchange rate grows at the rate $\dot{Q}_t/Q_t = i_t - \frac{r}{q_t}$. At the time of the regime switch, there is a perfectly anticipated devaluation. The exchange rate jumps almost 15%. At $t = T$, as

[10]The difference between the red line and the red circle at $T$ is due to the numerical error in the shooting algorithm.

Figure 1: Balance of Payments Crisis with Capital Controls and Free Trade




Note: $u = \frac{c^{1-\sigma}}{1-\sigma} + \alpha\frac{m^{1-\sigma}}{1-\sigma}$; $\sigma = 2$; $r = 0.04$; $\epsilon = 0$; $\theta = 0.2$; $y = 1$; $f_0 = 0.07$; $c_{ss} = rf_0 + y$; $m_0 = 0.1c_{ss}$; $d_0 = m_0 - f_0$ ; $\alpha = 0.1^\sigma(r+\epsilon)$;

reserves are zero, the money supply is $M_T = D_0 e^{\theta T}$. The jump in the exchange rate reduces real money balances while the interest rate jumps from $i_T^- = \frac{r}{q_T^-} + \frac{\dot{Q}_T^-}{Q_T^-}$ to $i_T^+ = r + \theta$. Consumption falls according to equation (25b) to the level $c_T^+ = y - g$, which is $rf_0$ lower than the one in the Krugman equilibrium with free capital mobility.

Figure 2 compares the dynamics of reserves in the models with and without capital controls. In the model with free capital mobility, while the exchange rate remains fixed, the nominal interest rate is constant at $i = r + \epsilon$, and real money balances are constant. The fall in reserves mirrors the increase in domestic credit with $\dot{f}_t = -\dot{d}_t$. The interest rate under the floating exchange rates regime becomes $i = r + \theta$, and at the time of the regime switch, the jump in interest rates reduces the money demand, and agents purchase the central bank's reserves with the excess money, $\Delta f_t = (rf_0 + y - g)\left[\ell(r + \theta) - \ell(r + \epsilon)\right]$. Along the transition path, agents build their holdings of foreign assets purchasing reserves from the central bank, i.e., $\dot{b}_t^* = -\dot{f}_t = \dot{d}_t$, and consumption is constant. In the regime with capital controls, reserves fall faster as the money demand falls along the transition path. In the example in figure 2, capital controls delay the collapse of the fixed exchange rate.

**Delayed Monetization: Unpleasant Monetarist Arithmetic**

In this section, we ask whether delaying monetization by issuing debt and monetizing deficits later, Sargent and Wallace (1981)'s exercise, can delay the fixed exchange rate's collapse in an economy with capital controls.

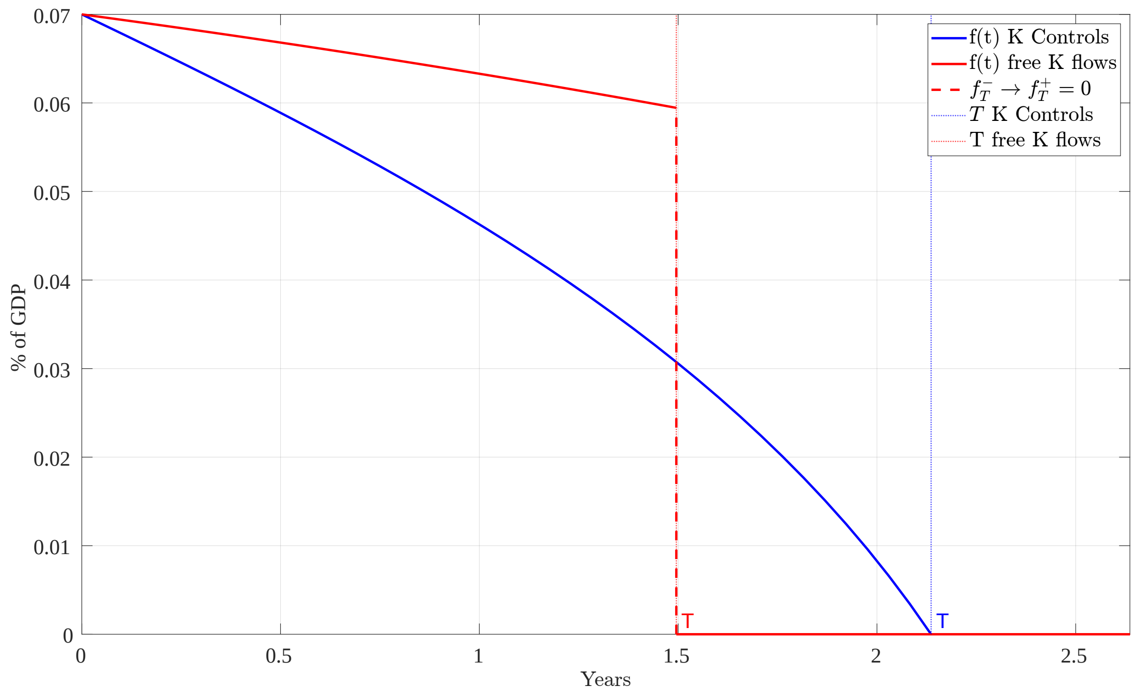
Appendix C shows that in the Krugman (1979)-Calvo (1987) model with perfect capital mobility, a Wallace neutrality proposition (Wallace, 1981) holds. Delaying monetization does not affect the timing of the collapse or consumption allocations. This is no longer the case in the presence of capital controls.

Consider a policy where the path of primary deficits is constant, and the government delays its monetization by issuing debt up to time $\bar{T}$ and keeping domestic credit constant. At date $\bar{T}$ it stops issuing debt, and it finances the primary deficit plus the service of the new debt by issuing money.

The debt accumulated between 0 and $\bar{T}$ equals the sum of the debt-financed deficits and their accrued interest.

$$B_{\bar{T}} = \int_0^{\bar{T}} \underbrace{\theta D_0 e^{\theta t}}_{\text{Deficit}_t} \underbrace{e^{\int_t^{\bar{T}} i_s ds}}_{\text{Interest}} dt \tag{26}$$

Figure 2: Balance of Payments Crisis with and without Capital Controls




Note: $u = \frac{c^{1-\sigma}}{1-\sigma} + \alpha\frac{m^{1-\sigma}}{1-\sigma}$; $\sigma = 2$; $r = 0.04$; $\epsilon = 0$; $\theta = 0.2$; $c_T^+ = y = 1$; $f_0 = 0.07$; $c_{ss} = rf_0 + y$; $m_0 = 0.1c_{ss}$; $d_0 = m_0 - f_0$ ; $\alpha = 0.1^\sigma(r+\epsilon)$; $g = 0$.

The deficit, financed issuing money after $\bar{T}$, is $\dot{D}_t = \theta D_0 e^{\theta t} + \rho_t B_{\bar{T}}$. We characterize monetary policy by the rate of growth of domestic credit[11], $\tilde{\theta}_t \equiv \frac{\dot{D}_t}{D_t}$;

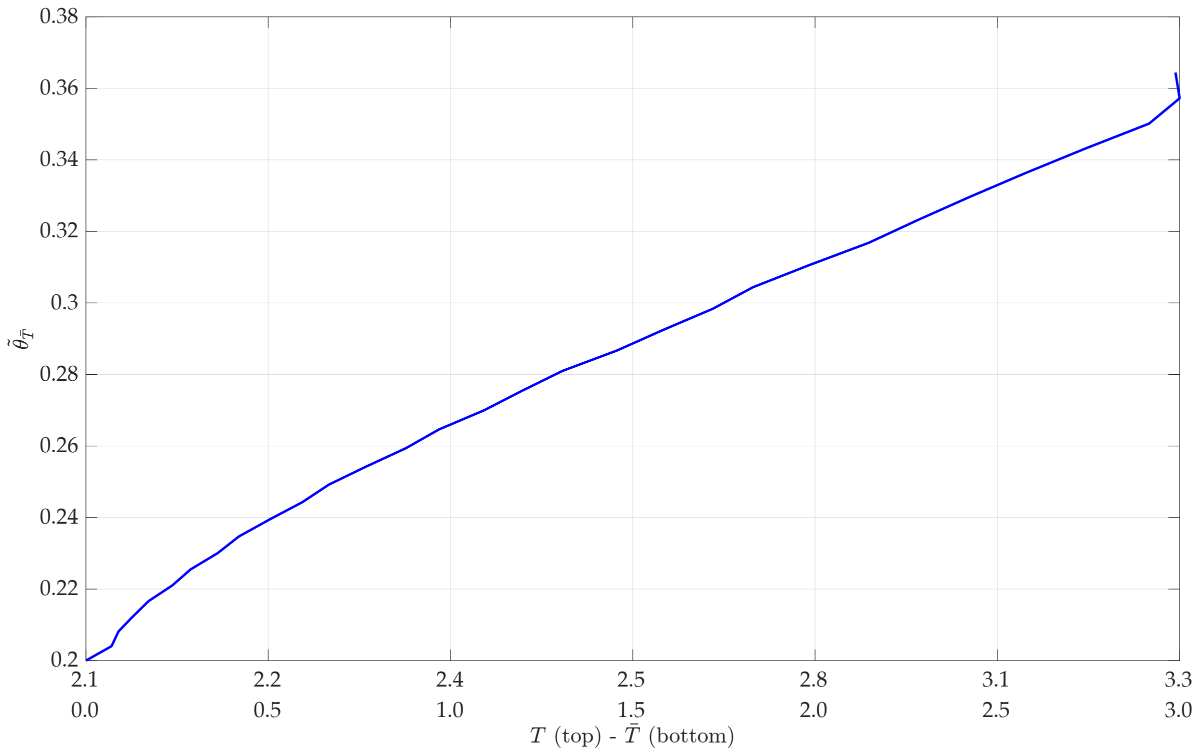
$$\tilde{\theta}_t = \begin{cases} 0 & \text{for } t \leq \bar{T} \\ \theta e^{\theta \bar{T}} + \rho_t \frac{B_{\bar{T}}}{D_0} e^{-\theta(t-\bar{T})} & \text{for } \bar{T} < t \end{cases} \quad (27)$$

Equation (27) captures the idea of unpleasant monetarist arithmetic. There are two sources of a higher growth rate of domestic credit after the government starts to monetize deficits at $\bar{T}$. First, the term $\theta e^{\theta\bar{T}}$ implies that the post-delay domestic credit growth is $e^{\theta\bar{T}}$ times the original one. This is because the post-delay need for monetizing the primary deficit $\dot{D}_t$ is the same, but the stock of domestic credit is smaller. The second source, the monetization of the new debt's service, is captured by the term $\rho_t \frac{B_{\bar{T}}}{D_0} e^{-\theta(t-\bar{T})}$. It is increasing in $\bar{T}$ as the longer the government borrows to service the debt, the higher the accumulated debt $B_{\bar{T}}$ is. The debt service also depends on the path of the domestic interest rate, which is very important as real rates before a balance of payments

[11] We can use the definition of $\tilde{\theta}_t$ to express domestic credit as $D_t = D_0 e^{\int_0^t \tilde{\theta}_s ds}$, with $\int_0^t \tilde{\theta}_s ds = \theta e^{\theta\bar{T}}(t-\bar{T}) + \frac{rB_{\bar{T}}}{\theta D_0}\left(1 - e^{\theta(\bar{T}-t)}\right)$.

crisis are higher than international rates. Finally, as the debt service is constant over time, under the assumption of constant credit growth to finance primary deficits, the debt service over GDP as a fraction of domestic credit converges to zero as $t \to \infty$.

Figure 3: Delayed monetization. Regime Switch and Inflation



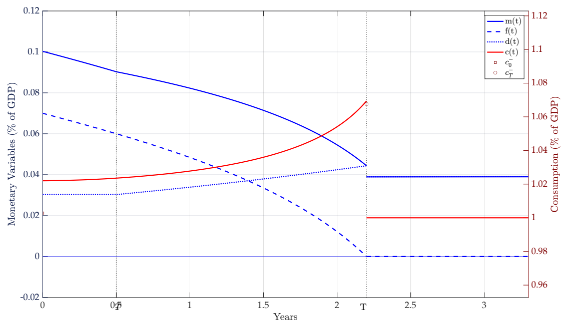

Note: $u = \frac{c^{1-\sigma}}{1-\sigma} + \alpha \frac{m^{1-\sigma}}{1-\sigma}$; $\sigma = 2$; $r = 0.04$; $\epsilon = 0$; $\theta = 0.2$; $y = 1$; $f_0 = 0.07$; $c_{ss} = rf_0 + y$; $m_0 = 0.1c_{ss}$; $d_0 = m_0 - f_0$ ; $\alpha = 0.1^{\sigma}(r + \epsilon)$;

Delaying monetization extends the period of low inflation while the fixed exchange rate is in place at the cost og higher inflation later. Figure 3 presents a frontier between the timing of the regime switch and inflation immediately after the regime switch. The two scales in the horizontal axis represent the time of the switch, $T$, and the delay, $\bar{T}$. Foe example, delaying monetization by three years extends the low inflation regime by tw years and two months at the cost of raising the post switch inflation from 20$ per year to 26%.

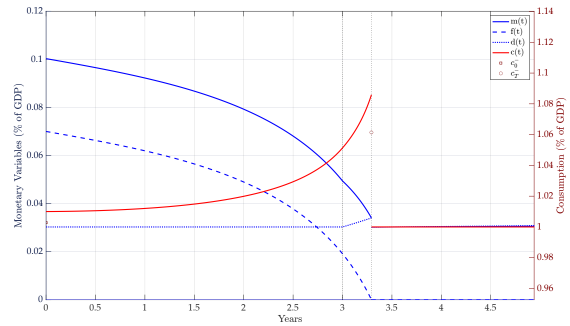
Figure 4 reports the equilibrium for the example in the previous section for values of $\bar{T}$ of 0.5 and 3 years. For each $\bar{T}$ we observe an equilibrium that follows the same logic as the one with $\bar{T} = 0$. The nominal interest rate shows the unpleasant monetarist arithmetic as delaying monetization

Figure 4: Balance of Payments Crisis with Delayed Monetization

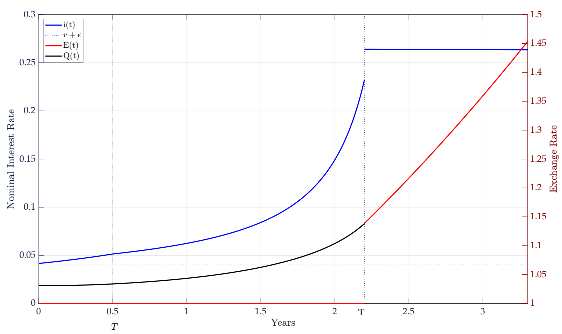
(a) Allocations



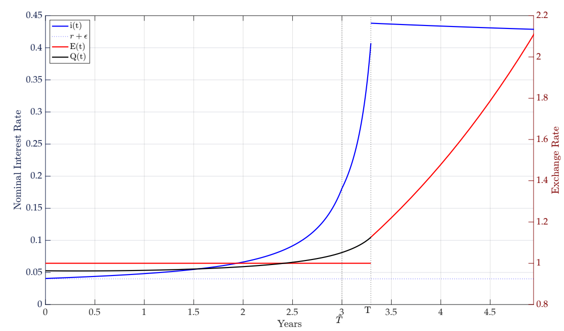

(b) Allocations



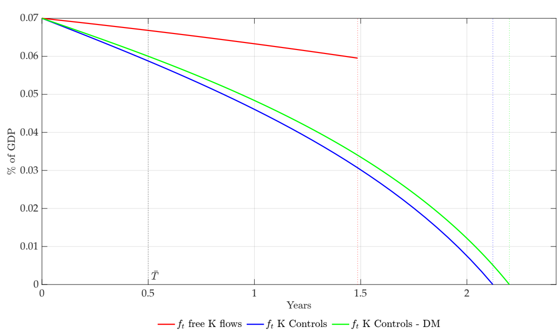

(c) Prices



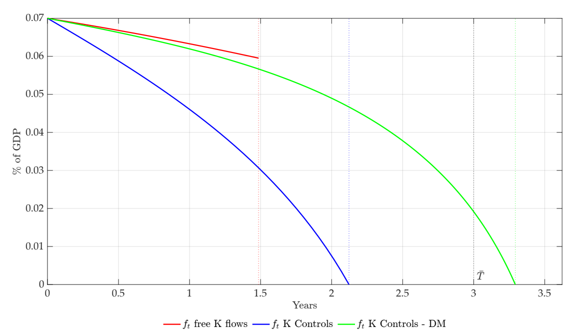

(d) Prices




(e) Reserves




(f) Reserves




Delayed monetization: $\bar{T} = 0.5$ years

Delayed monetization: $\bar{T} = 3$ years

Note: $u = \frac{c^{1-\sigma}}{1-\sigma} + \alpha\frac{m^{1-\sigma}}{1-\sigma}$; $\sigma = 2$; $r = 0.04$; $\epsilon = 0$; $\theta = 0.2$; $y = 1$; $f_0 = 0.07$; $c_{ss} = rf_0 + y$; $m_0 = 0.1c_{ss}$; $d_0 = m_0 - f_0$ ; $\alpha = 0.1^{\sigma}(r + \epsilon)$.

by three years increases the floating regime's inflation from 20% to somewhere around 40%.

## 3.4 Capital Controls and Import Restrictions

In this section we introduce import restrictions that ensure that the current account deficit is zero, reserves are never depleted, and the fixed exchange rate regime can last forever.

Quantitative import restrictions introduce a fundamental difference in the economy by breaking international arbitrage in the goods market. Very much like in the textbook case of an import quota, the domestic price of goods may exceed import parity prices at the official exchange rate. The wedge between the official exchange rate at which foreign trade transactions are settled implicitly creates an export tax and an import subsidy. Thus, the sustainability of the fixed exchange rate regime comes at the cost of introducing a distortion that misallocates resources.

In order to sketch the distortionary effect of import restrictions while keeping the simplicity of the endowment economy, we now assume that the economy produces two goods, an exportable good and a home good. Both are produced with labor, which is inelastically supplied. Under free trade all labor is allocated to the exportable good and the model reduces to the endowment economy in the previous section. When there is a wedge between domestic prices and the official exchange rate exporters receive, labor is misallocated to produce a home good.

### 3.4.1 Description of the Economy

Consider an economy that produces two goods: $T$ is internationally traded, and $H$ is not. Furthermore, assume that all the good $T$ produced at home is exported and all the consumption of good $T$ is imported. Goods $H$ and $T$ are perfect substitutes in consumption so that $c_t = c_{H,t} + c_{T,t}$.

**Assumption 5 (Perfect substitutability between $H$ and $T$ goods)** *Assume the goods, $H$ and $T$, are perfect substitutes in consumption;*

$$u\left(c_{H,t}, c_{T,t}, m_t\right) = u\left(c_t, m_t\right), \text{ with } c_t = c_{H,t} + c_{T,t}.$$

The price index, the cost of buying a unit of the utility aggregator $c_t$, is

$$P_t = \min\{P_{Tt}, P_{Ht}\},$$

where $P_{it}$ is the price of the respective good.

We maintain the previous assumption that $u$ is homothetic in $c$ and $m$ and $u_{cm} = 0$.

**Assumption 6 (Production technology)** *The production functions for H and T goods are;*

*i* $y_{T,t} = y(n_{T,t})$ *with* $y' > 0$, $y'' < 0$, *and* $y'(1) = 1$

*ii* $y_{H,t} = n_{H,t}$

*iii* $n_{H,t} + n_{T,t} = 1$

Assumption 5 together with assumption 6 imply that in an efficient allocation only traded goods are produced and consumed; $n_{T,t} = 1$, $y_{H,t} = n_{H,t} = 0$, $y_{T,t} = y(1)$ and the relative price between the two goods is one.

**Assumption 7 (Import restriction)**

$$c_{T,t} \leq \bar{c}_t \text{ for all } t \tag{28}$$

A consequence of assumption 7 is that when equation (28) binds the price of the consumption good may be higher than the import parity price, i.e., the official exchange rate.

We assume $\bar{c}$ is set so that the current account balance is zero and the government's international reserves are constant. This is the maximum level of imports consistent with the sustainability of the regime.

**Assumption 8 (Current account balance)** *Import restrictions,* $\bar{c}_t$, *are chosen so that for all* $t$,

$$\dot{f}_t = r\left(f_0 + b_0^*\right) + y_{T,t} - g - \bar{c}_t \geq 0$$

In the parameterization of interest, with $\theta > \epsilon$, the import restriction in equation (28) coupled with assumption 8 will always bind.

The government's budget constraint is

$$\dot{f}_t - \frac{\dot{M}_t + \dot{B}_t}{P_t} = r f_t - i_t \frac{B_t}{P_t} + \tau_t - g + \left(1 - \frac{E_t}{P_t}\right) y(n_{T,t}), \tag{29}$$

It is standard except for an implicit tax on export proceeds, $\left(1 - \frac{E_t}{P_t}\right) y_T$. In interpreting equation (29) one can think that the government buys exports from the private sector at a price $E_t$ and sells imported goods back to households at a price $P_t \geq E_t$. Recall the relative price between imports and exports is assumed to be one. The profit made by intermediating exports is $\left(1 - \frac{E_t}{P_t}\right) y_T$.

An alternative institutional setup to the one implicit in equation (29) would be to assume the government sells imported goods to an intermediary at the "subsidized" price $E_t$ and that this intermediary resells the imported consumption goods to the public at the price $P_t$. In this case, the consumption subsidy, $\left(1-\frac{E_t}{P_t}\right)\bar{c}_t$, would be a transfer to the private intermediary, which would be an expenditure in the government's budget constraints equation (29). Households would still pay the price $P_t$ for imported consumption, and the consumption subsidy, $\left(1-\frac{E_t}{P_t}\right)\bar{c}_t$, would be akin to a lump-sum transfer to the private "intermediaries" with access to imports at a preferential price.

**Household behavior**

A representative household produces and consumes the two goods. Recall that we assume that all the production of the tradeable good is exported and all consumption is imported. Capital control regulations imply that exporters must sell their foreign currency export revenues at the official exchange rate. Under the assumption that the foreign price of the traded goods is 1, exporters receive $E_t$ units of domestic currency for each unit of exports. The domestic price of the traded good is $P_t$, which may be greater than $E_t$. To simplify notation and exposition, we assume $P_t = P_{Tt}$, which will be true in equilibrium.

The private sector's and the government's budget constraints for all $t$ are, respectively;

$$\frac{\dot{M}_t+\dot{B}_t}{P_t}+q_t\dot{b}_t^* = i_t\frac{B_t}{P_t}+rb_t^*+\frac{E_t}{P_t}y\left(n_{T,t}\right)-c_{T,t}+\frac{p_{Ht}}{P_t}\left(1-n_{T,t}-c_{H,t}\right)-\tau_t \tag{30}$$

where $q_t \equiv \frac{Q_t}{P_t}$.

The private sector budget constraints, equation (30), states that asset accumulation equals the sum of the interest from the nominal bond, the interest on the foreign bond paid in units of good $T$, export proceeds settled at the official exchange rate and converted to units of $T$, net of good $T$ consumption, and lumpsum taxes. The term $1-n_{T,t}-c_{H,t}$ represents the difference between good $H$'s output and consumption, taking into account the production function and the labor constraint.

The no-arbitrage condition on the Lucas tree is

$$i_tQ_t = rP_t+\frac{\dot{Q}_t}{Q_t}. \tag{31}$$

It is analogous to equation (2) with the price of the consumption good $P_t$ instead of $E_t$. It implies the real interest rate in the foreign currency bond $\rho_t \equiv i_t-\pi_t \equiv \frac{r}{q_t}+\frac{\dot{q}_t}{q_t}$, where $q_t = \frac{Q_t}{P_t}$. The solution of the differential equation in $q_t$ is equation (5).

Defining $a = m_t+b_t+q_tb_t^*$, as before, we can write equation (30) as

$$\dot{a}_t = \rho_ta_t+\frac{E_t}{P_t}y-c_{T,t}-\tau_t-i_tm_t+\frac{p_{Ht}}{P_t}\left(1-n_{T,t}-c_{H,t}\right) \tag{32}$$

The consumer's problem is to choose $\{c_{H,t}, c_{T,t}, n_{H,t}, n_{T,t}, m_t\}$ to solve the problem

$$\max_{c_{H,t},c_{T,t},n_{T,t},m_t} \int_0^\infty u\left(c_{H,t} + c_{T,t}, m_t\right) e^{-rt} dt \ \text{ subject to (32)} \tag{33}$$

given an initial value for $a_0$, a no-Ponzi-game condition, and the sequence of prices $\{\rho_t, E_t, P_t, p_{Ht}\}$. The solution to the household's problem, when the non-negativity constraints on each consumption good are not binding, is given by

$$\frac{P_t}{E_t} = y'\left(n_{T,t}\right) \tag{34a}$$

$$m_t = \ell(i_t)c_t \tag{34b}$$

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\sigma}(\rho_t - r) \tag{34c}$$

for preferences described by equation (11) as shown in Appendix B.

### 3.4.2 Definition and Characterization of Equilibrium

**Definition 3 (equilibrium with capital controls and import restrictions)** *An Equilibrium with capital controls and import restrictions is a sequence of prices $\{Q_t, P_t, p_{H,t}, \rho_t, i_t\}_{t=0}^{\infty}$ and allocations $\left\{c_{T,t}, c_{H,t}, n_{T,t}, n_{H,t}, m_t, b_t^*, f_t, \tau_t\right\}_{t=0}^{\infty}$ such that given (i) initial conditions $\{D_0, E_0, a_0, a_0^g\}$, (ii) international interest rates, $r$, (iii) a sequence of government expenditures, $g$, (iv) the production technology in assumption 6, (v) fiscal and monetary policies satisfying assumption 2, (vi) capital controls described by assumption 4, and (vii) import restrictions in assumption 7, the following conditions hold*

1. *$f_0$ satisfies the Krugman equilibrium defined in definition 1.*
2. *The import restriction in equation (28) i satisfied.*
3. *Nominal interest rates and the shadow exchange rate satisfy the no-arbitrage condition equation (31),*
4. *The government's intertemporal budget constraint, equation (29) with initial conditions $f_0$, $b_0$, $m_0$ and a no ponzi game condition is satisfied.*
5. *Households solve the problem equation (33) given $a_0$ and the sequence of prices $\{i_t, \rho_t E_t, P_t, p_{Ht}\}_{t=0}^{\infty}$,*
6. *The aggregate consistency condition $c_{H,t} = n_{H,t}$ for all $t$.*

We characterize equilibria with inconsistent monetary and exchange rate policies, $\theta > \epsilon$, that, in the absence of import restrictions, result in the depletion of reserves and the collapse of the fixed exchange rate regime. Hence, import restrictions will bind. We also assume that the nominal price of the home and the traded good are the same, so the non-negativity constraints in each of these consumption goods are not binding.

Let $e_t \equiv \frac{E_t}{P_t}$ and $\omega_t \equiv 1 - e_t$ be a wedge that distorts the optimal allocation that is attained when $e_t = 1$. When the import constraint binds, $P_t > E_t$ and $\omega_t > 0$[12]. This implies $0 \leq e \leq 1$. We characterize equilibrium allocations and interest rates as a function of $e_t$ (or the wedge $\omega_t$) in equation (35) bellow;

$$1 = e_t y'\left(n_{T,t}\right) \iff n_{Tt} = n(e_t) \tag{35a}$$

$$m_t = \ell(\rho_t + \pi_t) c_t \tag{35b}$$

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\sigma}(\rho_t - r) \tag{35c}$$

$$\bar{c}_t = r\left(f_0 + b_0^*\right) + y\left(n(e_t)\right) - g \tag{35d}$$

$$c_{Ht} = 1 - n(e_t) \tag{35e}$$

$$c_t = 1 - n(e_t) + r\left(f_0 + b_0^*\right) + y\left(n(e_t)\right) - g \tag{35f}$$

Equations (35a)-(35c) represent the household's optimization conditions. Equations (35a) determines the optimal allocation of labor between the traded and the home sector. It sets the "after-tax" marginal rate of transformation between $T$ and $H$ equal to the marginal rate of substitution between the two goods, $u_H/u_T = p_H/p_T = 1$. It implies that the demand for labor in the $T$ sector can be written as a function $e$, $n(e)$ with $n'(e) = -1/y'' > 0$. The other two conditions for household optimization are the money demand and the consumption Euler equation. Equation (35d) is the import restriction, which is also an increasing function of $e$. A higher $e$ induces more labor to be allocated to the export sector, allowing to import more goods while balancing the current account. This condition is sometimes referred to as a country's foreign exchange constraint. Equation (35e) combines the market clearing condition in the labor market and in the $H$ good market. It implies $c_H = y_h$ and $n_H + n_T = 1$. Finally, equation (35f) is the consumption aggregator that is governed by the Euler equation equation (35c). If equations (35d), (35e), and the government budget constraints are satisfied, then the private budget constraint is satisfied as well.

[12] If $P_t < E_t$ agents would like to export, the import constraint cannot be binding.

The properties of the production functions in assumption 6 imply that

$$e = 1 \Rightarrow \begin{cases} n_T = n(1) = 1 \\ y_H = n_H = c_H = 0 \\ c = c_T = r\left(f_0 + b_0^*\right) + y(1) - g \end{cases} \tag{36}$$

$$e \to 0 \Rightarrow \begin{cases} n_T = n(0) = y_T(n(0)) = 0 \\ y_H = n_H = c_H = 1 \\ c_T = r\left(f_0 + b_0^*\right) - g \\ c = 1 + r\left(f_0 + b_0^*\right) - g \end{cases} \tag{37}$$

The optimal allocation with $e = 1$ assigns all labor to the traded sector while all the labor is allocated to the home good for $e = 0$.

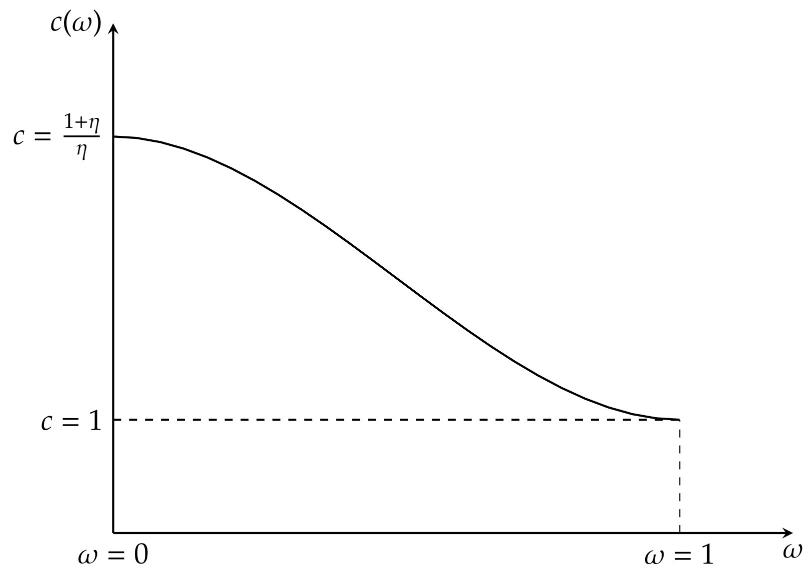



Figure 5: Aggregate output and the export wedge $\omega = 1 - e$

Note: $b_0 = f_0 = g = 0$ and $y_T = \frac{1+\eta}{\eta} n_T^{\frac{\eta}{1+\eta}}$ and $\eta = 2$. Consumption is $c(\omega) = \frac{1+\eta}{\eta}(1-\omega)^\eta + 1 - (1-\omega)^{1+\eta}$.

Figure 5 plots equilibrium consumption as a function of the wedge $\omega = 1 - e$. It shows that aggregate consumption, $c(\omega) = 1 - n(\omega_t) + r\left(f_0 + b_0^*\right) + y\left(n(\omega_t)\right) - g$ is a decreasing function of the wedge between the marginal product of labor in the $H$ and $T$ sectors, which is maximized when all labor is allocated to the $T$ sector. The figure assumes $b_0 = f_0 = g = 0$ and the technology $y_T = \frac{1+\eta}{\eta} n_T^{\frac{\eta}{1+\eta}}$. The consumer's optimal labor allocation is $n_T(e) = e^{1+\eta}$ and the tradeable supply function is $y_T(e) = \frac{1+\eta}{\eta} e^\eta$.

It is clear from equation (35) that is we find the equilibrium price level, given that $E$ is a policy variable, we can characterize the whole equilibrium. Moreover, the dynamics of the real equilibrium allocations will follow the dynamics of $e$, which is tantamount to those of $P$. The money market equilibrium implies $e_t = \frac{E_t}{P_t} = \frac{E_t m_t}{M_t}$ where $M_t$ is the money supply and $m_t$ is the money demand in equation (35b); that is,

$$e_t = \frac{\ell(i_t)c(e_t)}{f_0 + d_0 e^{(\theta-\epsilon)t}}$$

Using the money market clearing condition, the Euler equation (35c), and equation (35f) we can compute the equilibrium trajectory of $e_t$ solving the implicit differential equation (38a) with boundary condition equation (38b).

$$e_t = \frac{\ell\left(r + \epsilon + \left(\sigma\frac{(1-e_t)n'(e_t)}{c(e_t)} - 1\right)\frac{\dot{e}_t}{e_t}\right)c(e_t)}{f_0 + d_0 e^{(\theta-\epsilon)t}} \tag{38a}$$

$$\lim_{t\to\infty} e_t = 0. \tag{38b}$$

The differential equation (38a) is derived noticing that the Euler equation (35c) implies we can write the nominal interest rate as $i_t = r + \epsilon - \frac{\dot{e}_t}{e_t} + \sigma\frac{\dot{c}_t}{c_t}$ while equation (35f) implies $\dot{c}_t = \left(\frac{1}{e_t} - 1\right)n'(e_t)\dot{e}_t$.

The boundary condition $\lim_{t\to\infty} e_t = 0$ is the consequence of the fact that for $\theta > \epsilon$, $M_t/E_t = f_0 + d_0 e^{(\theta-\epsilon)t} \to \infty$ as $t \to \infty$, while the money demand, $\ell(i_t)c(e_t)$, is bounded above.

A corollary of equation (38b) is

$$\lim_{t\to\infty} \pi_t = \theta$$

Proof. In the limit as $\lim_{t\to\infty} \dot{e} = e(\epsilon - \pi) = 0$. Taking logs and differentiating the money market clearing condition with respect to time yields, $\epsilon - \pi = \ell'(i)\dot{i}_t + c'\dot{e}_t - (\theta - \epsilon)\frac{D_t}{M_t}$. In the limit, $\ell'(i)\dot{i}_t + c'\dot{e}_t = 0$ and $D_\infty = M_\infty$ implying that $\lim_{t\to\infty} \pi_t = \theta$. ∎

In contrast to the case with free trade, in which consumption is increasing over time and real interest rates are high relative to international ones, in the economy with import restrictions, if $e$ is monotonically decreasing, consumption, $c$, is monotonically decreasing, and real interest rates are below international rates.

We close this section by describing the behavior of the shadow exchange rate. Recall we defined it as $Q_t = q_t P_t$ and that $q_t$ is the solution of

$$\dot{q}_t = \rho_t q_t - r$$

for the values of $\rho_t$ that solve equation (35c). The forward solution is[13]

$$\begin{aligned} q_t =& r\int_t^{\infty} e^{-\int_t^s \rho_u du} ds \\ =& e^{-\int_t^{\mathrm{T}} \rho_u du} q_{\mathrm{T}} + r\int_t^{\mathrm{T}} e^{-\int_t^s \rho_u du} ds \end{aligned}$$

As $\lim_{t\to\infty}\rho_t = r$ there is a T such that $q_{\mathrm{T}} \simeq 1$. The ratio of the black market exchange rate to the price level at $t$ is the present value of the perpetuity's coupon discounted at the domestic real interest rate. It can be written as a weighted average of the excess returns of an offshore bond[14], i.e.

$$q_t = \int_t^{\infty} \frac{e^{-r(s-t)}}{\int_t^{\infty} e^{-r(s-t)}} e^{\int_t^s (r-\rho_u)du} ds.$$

In order to derive some intuition, consider the steady state in which $e_t$ and consumption fall over time until all labor is allocated to produce the home good $H$ and consumption is $c_t = 1$. In this equilibrium, $\rho_t \leq r$ and $\lim_{t\to\infty}\rho_t = r$. Let T be a date after which $\rho_t$ is very close to $r$—i.e., $\left|\rho_t - r\right| < \delta$ for some small $\delta > 0$ and for all $t \geq \mathrm{T}$. For $t \geq \mathrm{T}$, $q_t = 1$. For $t \leq \mathrm{T}$, the terms $e^{\int_t^{\mathrm{T}}(r-\rho_u)du} > 1$, implying that $q_t > 1$. Thus, the shadow exchange rate starts being larger than the price level, $Q_t > P_t$ for $t \leq \mathrm{T}$ and converges to the price level in the future $\lim_{t\to\infty} Q_t = P_t$.

### 3.4.3 Example

Figure 6 depicts the equilibrium values of the real exchange rate, $e$, consumption, interest rates, inflation, price levels, and labor. The behavior of these variables illustrates the patterns described in the characterization of equilibrium. This figures also presents the value of this endogenous variables before the implementation of the import restrictions.

We consider an economy with a yearly rate of growth of domestic credit $\theta = 0.2$ and a fixed exchange rate with $\epsilon = 0$. While it is in a Krugman equilibrium at $t = 0$, it suddenly imposes capital controls and import restrictions to balance the current account for all $t$.

The price level jumps on impact about 60%, reducing the real exchange rate, labor in the $T$ sector is reduced to 13%, and consumption falls 13%. The movement in relative prices induces the private sector to consume and produce less $T$ goods. Our specification of a frictionless reallocation of labor across sectors coupled with a linear technology in the $H$ sector exacerbates reallocation.

[13] $q_t = r\int_t^{\mathrm{T}} e^{-\int_t^s \rho_u du} ds + re^{-\int_t^{\mathrm{T}} \rho_u du} \int_{\mathrm{T}}^{\infty} e^{-\int_{\mathrm{T}}^s \rho_u du} ds = r\int_t^{\mathrm{T}} e^{-\int_t^s \rho_u du} ds + re^{-\int_t^{\mathrm{T}} \rho_u du} \frac{q_{\mathrm{T}}}{r}$.

[14] $\frac{1}{r} = \int_t^{\infty} e^{-r(s-t)}$

After the impact effect, the real exchange rate appreciates as a consequence of the policy of keeping $\theta > \epsilon$ as the intuition from looking at the money market clearing condition suggests. This movement in the real exchange rate is purely caused by nominal variables: the inconsistent monetary and exchange rate policy. There are no real shocks and no nominal rigidities in the model. As discussed in 5, the implicit export tax $\omega = 1 - e$ increase reduces aggregate consumption over time. Equilibrium real interest rates, therefore, are lower than international rates and, for over four years, negative.

The policy bundle of the low devaluation rate, capital controls, and import restrictions is successful at keeping inflation below the floating exchange rate benchmark $\pi = \theta$ for a long time.

Panel 6e shows the price level, the financial exchange rate, and the price level that will prevail if authorities would *unexpectedly* remove trade and capital flow restrictions at $t$, $P^*_{\theta,t}$. At the time capital and import controls are implemented, the price level jumps 60% because of a drastic fall in the money demand as interest rates jump from $i = 0.04$ to $i = 0.11$ and consumption falls 17%. As time goes by, it rises because the money supply is growing and rising interest and falling consumption reduce the money demand.

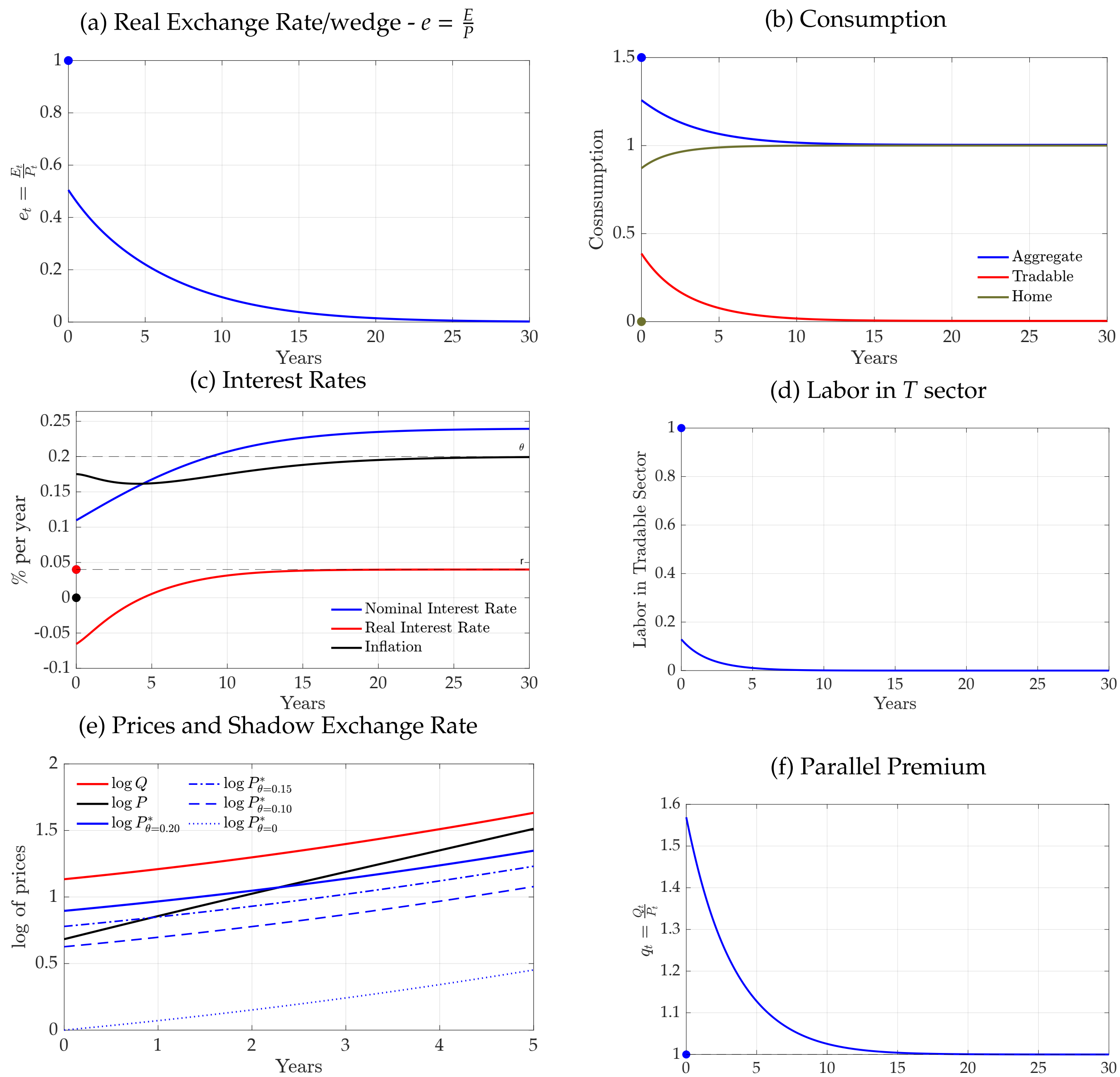
If liberalization occurs at $t = 0$ without a change in monetary and fiscal policy, the price level will jump at the time of liberalization due to the increase in the nominal interest rate to $i = r + \theta$ which will lower the demand for money. As time goes by and consumption falls in the distorted economy, the increase in consumption that occurs at the time of liberalization increases the money demand by more than the effect of higher nominal interest rates. Fiscal reforms that lower $\theta$ at the time of liberalization increase the money demand and result in a lower post-liberalization price level.

Two features of this equilibrium are of particular interest for future experiments.

The initial period with negative real interest rates might render the arithmetic of delayed monetization not so unpleasant. Issuing debt is a good deal when real interest rates are negative.

We could also compute the equilibrium path when agents anticipate the removal of capital and import controls under different post-liberalization fiscal policies summarized in $\theta$. This is relevant for countries that announce a future removal of capital controls, such as Argentina in 2015 and 2023.

Figure 6: Balance of Payments Crisis with Capital Controls and Import Restrictions




Note: $u = \frac{(c_T + c_H)^{1-\sigma}}{1-\sigma} + \alpha\frac{m^{1-\sigma}}{1-\sigma}$; $\sigma = 2$; $r = 0.04$; $\epsilon = 0$; $\theta = 0.2$; $y = 1$; $f_0 = 0.07$; $c_{ss} = rf_0 + y$; $m_0 = 0.1c_{ss}$; $d_0 = m_0 - f_0$ ; $\alpha = 0.1^{\sigma}(r + \epsilon)$; $y_T = \frac{1+\eta}{\eta} l_T^{\frac{\eta}{1+\eta}}$; $y_N = 1 - l_T$ ; $\eta = 2$.

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# Appendix

## A Consumer's Problem with an Anticipated Devaluation

The consumer's problem is to maximize

$$\max_{c_t, m_t} \int_0^\infty u(c_t, m_t) e^{-rt} dt, \text{ subject to} \begin{cases} \dot{a}_t = \rho_t a_t + y - c_t - \tau_t - (\rho_t + \epsilon_t) m_t \text{ for } t \neq T \\ a_T^+ - a_T^- = \left(\frac{E_T^-}{E_T^+} - 1\right) a_T^- \\ a_0 \text{ given and } \lim_{t\to\infty} a_t e^{-\int_0^t \rho_s ds} \geq 0. \end{cases} \tag{39}$$

The current value Hamiltonian for this problem is

$$\mathcal{H}_t = u\left(c_t, m_t\right) + \lambda_t \left(\rho_t a_t + y - c_t - \tau_t - (\rho_t + \epsilon_t) m_t\right) \text{ for } t \neq T \tag{40a}$$

$$\mathcal{H}_T = \lambda_T^+ \left(\frac{E_T^-}{E_T^+} - 1\right) a_T^-. \tag{40b}$$

The maximum principle implies the optimality conditions

$$\begin{aligned} u_c(c_t, m_t) &= \lambda_t && \text{for all } t \neq T, T^- \text{ and } T^+ \\ u_m(c_t, m_t) &= \lambda_t i_t && \text{for all } t \neq T, T^- \text{ and } T^+ \\ \dot{\lambda}_t &= r\lambda_t - \lambda_t \rho_t && \text{for all } t \neq T \\ \lambda_T^+ - \lambda_T^- &= -\lambda_T^+ \left(\frac{E_T^-}{E_T^+} - 1\right) \\ \lim_{t\to\infty} a_t e^{-rt} &= 0 \end{aligned}$$

Eliminating the Lagrange multipliers, integrating the budget constraints, and using the transversality condition we get

$$\frac{u_m\left(c_t, m_t\right)}{u_c\left(c_t, m_t\right)} = i_t \qquad \text{for all } t \neq T, T^- \text{ and } T^+ \tag{41a}$$

$$\frac{\dot{c}_t}{c_t} = \frac{1}{-\frac{u_{cc}(c_t, m_t) c_t}{u_c(c_t, m_t)}} \left(\rho_t - r\right) \qquad \text{for all } t \neq T \tag{41b}$$

$$u_c\left(c_T^-, m_T^-\right) = u_c\left(c_T^+, m_T^+\right) \frac{E_T^-}{E_T^+} \tag{41c}$$

$$\int_0^\infty \left(c_t + i_t m_t\right) e^{-\int_0^t \rho_s ds} dt = a_0 + \left(\frac{E_T^-}{E_T^+} - 1\right) a_T^- e^{-\int_0^T \rho_s ds} + \int_0^\infty \left(y - \tau_t\right) e^{-\int_0^t \rho_s ds} dt. \tag{41d}$$

# B Consumer´s Problem with Import Restrictions

The consumer's problem is to choose $\{c_{H,t}, c_{T,t}, n_{H,t}, n_{T,t}, m_t\}$ to solve the problem

$$\max_{c_{H,t}, c_{T,t}, n_{T,t}, m_t} \int_0^\infty u\left(c_{H,t} + c_{T,t}, m_t\right) e^{-rt} dt \text{ subject to}$$

$$\begin{aligned} \dot{a}_t =& \rho_t a_t + \frac{E_t}{P_t} y - c_{T,t} - \tau_t - i_t m_t + \frac{p_{Ht}}{P_t}\left(1 - n_{T,t} - c_{H,t}\right) \\ c_{Tt} \geq& 0, \quad c_{Ht} \geq 0 \end{aligned}$$

The current value Hamiltonian is

$$\begin{aligned} \mathcal{H} =& u\left(c_{H,t} + c_{T,t}, m_t\right) + \lambda_t \left(\rho_t a_t + \frac{E_t}{P_t} y(n_{Tt}) - c_{T,t} - \tau_t - i_t m_t + \frac{p_{Ht}}{P_t}\left(1 - n_{T,t} - c_{H,t}\right)\right) \\ &+ \gamma_{Ht} c_{Ht} + \gamma_{Tt} c_{Tt} \end{aligned}$$

The first order conditions for an interior solution with $\gamma_{Tt} = \gamma_{Ht} = 0$ are

$$\begin{aligned} c_T :& u_c = \lambda_t & c_{Tt} \geq 0 \\ c_H :& u_c = \lambda_t \frac{p_{Ht}}{P_t} & c_{Tt} \geq 0 \\ m :& u_m = i_t \lambda_t \\ n_T :& \frac{E_t}{P_t} y'\left(n_{T,t}\right) = \frac{p_{Ht}}{P_t} \\ a_t :& \dot{\lambda} = \lambda(r_t - \rho_t) \end{aligned}$$

The first order conditions for $c_T$ and $c_H$ imply that in an interior solution $P_t = P_{Ht}$. It follows that

$$\begin{aligned} \frac{u_m}{u_c} &= i_t \\ \frac{E_t}{P_t} y'\left(n_{T,t}\right) &= 1 \end{aligned}$$

The first order conditions for $a_t$ can be written as

$$\frac{\dot{c}_t}{c_t} = \frac{1}{-\frac{u_{cc}(c_t, m_t) c_t}{u_c(c_t, m_t)}} (\rho_t - r).$$

For preferences described by equation (11), $\frac{u_m}{u_c} = i_t$ implies a money demand function $m_t = \ell(i_t) c_t$ and the elasticity of intertemporal substitution $-\frac{u_{cc}(c_t, m_t) c_t}{u_c(c_t, m_t)} = \sigma$.

# C Unpleasant monetarist arithmetic with free capital mobility and free trade

In this section, we ask the question posed in Sargent and Wallace (1981), of what happens if the government delays the monetary financing of the deficit by issuing debt. Can this policy delay the collapse of a fixed exchange rate regime? How does it affect the dynamics for inflation, onshore real interest rates, the black market premium, and consumption allocations under capital controls?

Consider the case in which the government intends to make its monetary and exchange rate policy sustainable by lowering the domestic credit growth rate from $\theta$ to $\epsilon$, albeit without any fiscal reform. The financing gap from lowering the rate of growth of domestic credit from $\theta$ to $\epsilon$ is covered by issuing domestic currency debt, which is later paid by issuing money. On the other hand, as the real value of domestic credit is constant, there is no bleed of central bank reserves.

The debt issued to reduce the growth rate of domestic credit can be issued directly by the treasury, or the central bank can issue it to sterilize the excess money creation $(\theta - \epsilon)D_t$. Whether $B_t^g$ is treasury-issued domestic currency debt or sterilization debt issued by the central bank is immaterial.

We extend to this setup Wallace (1981)'s neutrality result, proving that the equilibrium allocations and the timing of the fixed exchange rate's collapse are independent of the government's open market operations.

If the central bank's transfers to the treasury are replaced by debt financing so that domestic credit is constant in real terms, its rate of growth will fall from $\theta$ to $\epsilon$ and debt will increase by $(\theta - \epsilon)D_t$ every period. At the same time, the government's equilibrium interest income will be higher due to the fact that it will earn income on $f_0$ instead of on the benchmark equilibrium's $f_t$ in equation (21d). These two effects result in the following law of motion for nominal domestic currency debt,

$$\dot{B}_t^g = i_t B_t^g + \underbrace{(\theta - \epsilon) D_t}_{\text{less domestic credit}} - \underbrace{(r + \epsilon) E_t \left( f_0 - \left( m_0 - d_0 e^{(\theta - \epsilon)t} \right) \right)}_{\text{more interest on reserves}} \qquad \text{for } t \neq T. \tag{42}$$

The dynamics follow from the government's flow budget constraint in equation (9) and the equilibrium taxes in equation (21g). As long as this regime is in place, consumption and real money balances are constant. Hence, as the real value of domestic credit is constant, and reserves are constant at their initial level, $f_0$.

Our first observation is that this policy is unsustainable for $\theta - \epsilon > r$. To see this, transform the nominal debt dynamics to real terms to get $\dot{b}^g_t = rb^g_t + (\theta - \epsilon - r)d_0 e^{(\theta-\epsilon)t} + rd_0$, which integrates to

$$b^g_t = b^g_0 + d_0\left(e^{(\theta-\epsilon)t} - 1\right).$$

It is clear that the effect of issuing debt to reduce domestic credit exceeds the benefits of earning more interest on reserves and that, for $\theta - \epsilon > r$, the real value of debt explodes in the sense that $\lim_{t\to\infty} b^g_t e^{-rt} = \infty$, which violates the government's budget constraint.

We propose an equilibrium in which the private sector is willing to hold this unsustainable debt as long as it is "backed" by the central bank's reserves. Once domestic debt loses its backing, the private sector redeems the debt, and buys all the reserves at the fixed exchange rate, the government has to return to fully finance its deficit by issuing money, and the rate of inflation jumps from $\epsilon$ to $\theta$, and money demand falls.

The timing of the collapse is when reserves (constant until the attack) are just enough to cover the debt plus the fall in the money demand, that is, $f_0 = b^g_t + m_0 - m_T$. Using the fact that $b_T = d_0\left(e^{(\theta-\epsilon)t} - 1\right)$, where without loss of generality we assumed $b^g_0 = 0$, the timing of the regime's collapse is given by

$$m_T = d_0 e^{(\theta-\epsilon)T},$$

which is exactly the same as in the Krugman equilibrium in equation (21h).

**Proposition 2 (Wallace neutrality in the Krugman Equilibrium)** *Consider a Krugman equilibrium with financial policy $\Pi = \{\dot{D}_t/D_t, B^g_t\}_{t=0}^{\infty} = \{\theta, 0\}_{t=0}^{\infty}$ and another one with financial policy $\Pi' = \{\hat{D}', B^{g\prime}_t\}_{t=0}^{\infty}$, where*

$$\hat{D}' = \begin{cases} \epsilon & \text{for } t < T \\ \theta & \text{for } t \geq T, \end{cases}$$

*and $B^{g\prime}_t$ is given by equation (42). Then:*

*i. $T$ and the allocations $\{c_t, m_t\}$ under the policies $\Pi$ and $\Pi'$ are the same;*

*ii. portfolio allocations under the two policies differ in that under policy $\Pi'$*

- *equation (21d) is $f_t = f_0$ for $t < T$, and*
- *equation (21f) is $b^*_t = \left(a_0 + a^g_0\right)e^{rt} + \left(e^{rt} - 1\right)(y - g - c_{t<T}) - f_0$,*

*and are the same for $t \geq T$.*

**Proof** Dot the i's and cross the t's. ∎

The question of how equilibrium prices and allocations change when the government changes its consolidated balance sheet, keeping its fiscal stance constant has been analyzed in a series of papers that derive Miller-Modigliani theorems for the government, starting with Wallace (1981)'s seminal contribution. Wallace neutrality refers to the fact that equilibrium prices and allocations are independent of open market operations.

Proposition 2 establishes that in a fixed exchange rate regime with perfect capital mobility and monetary, fiscal, and exchange rate policies satisfying assumption 2 the same type of neutrality proposition as in Wallace (1981) and Sargent and Smith (1987) holds. If the government conducts open market operations that sterilize the monetary financing of deficits aimed at keeping reserves constant in this setup, the timing of the collapse of the fixed exchange rate, as well as equilibrium allocations are unchanged.

The key insight is that in the Krugman setup in Proposition 1 prior to the fixed exchange rate's collapse, the government is indirectly financing its deficit by depleting its foreign exchange reserves. When the government sterilizes the monetary injections that finance its deficit with domestic currency debt it switches the financing of deficits from the depletion of foreign assets to the issuance of debt. As there is free capital mobility, these two schemes have the same cost. In the original scheme, the private sector accumulates foreign assets until reserves are equal to the fall in the money demand that will ensue when the fixed exchange rate is abandoned and the cost of holding money increases. When sterilization keeps the government's foreign exchange reserves constant prior to the collapse, the private sector accumulates government debt, which is unsustainable. When the domestic currency government debt held by the private sector is equivalent to the stock of reserves net of the fall in the money demand entailed by the abandonment of the exchange rate peg, the private sector stops buying government debt, redeems all the government debt it holds, and depletes the central bank's foreign exchange reserves. The run on reserves in this case is much more dramatic than in Krugman's case as all the central bank's initial reserves are sold in an instant, and there is a simultaneous rollover crisis on the local currency debt.

In spite of the fact that this policy experiment is similar to the one explored in Sargent and Wallace (1981)'s *unpleasant monetarist arithmetic*, in that the government is using open market operations to change the time profile of the monetary financing of deficits, the results are different.

The difference lies in the fact that under a fixed exchange rate regime, the inflation rate is pinned by the exchange rate peg and the money supply is endogenous. The open market operations that would reduce the money supply under floating exchange rates are "undone" by an increase in the central bank's foreign exchange rate reserves under a fixed exchange rate. Thus, the open market operations change the source of non-monetary financing of the deficit without altering either inflation or the money supply. As we shall see next, this is no longer the case under capital controls.

## D End of Bretton Woods?

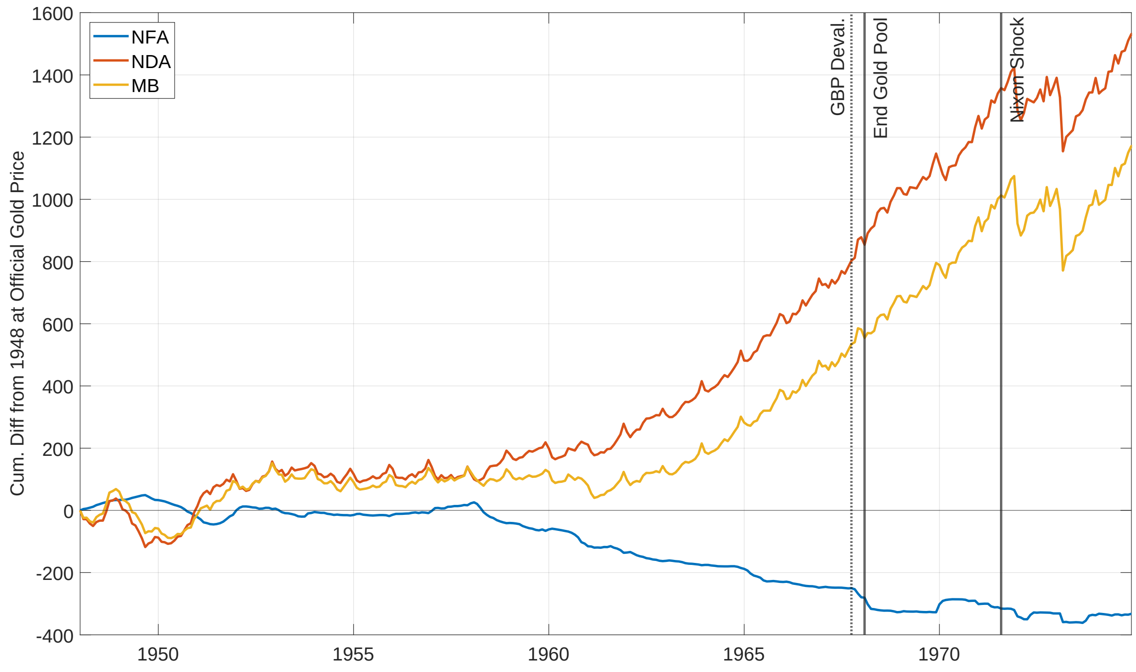


Figure 7: Fed Balance Sheet

Sources:

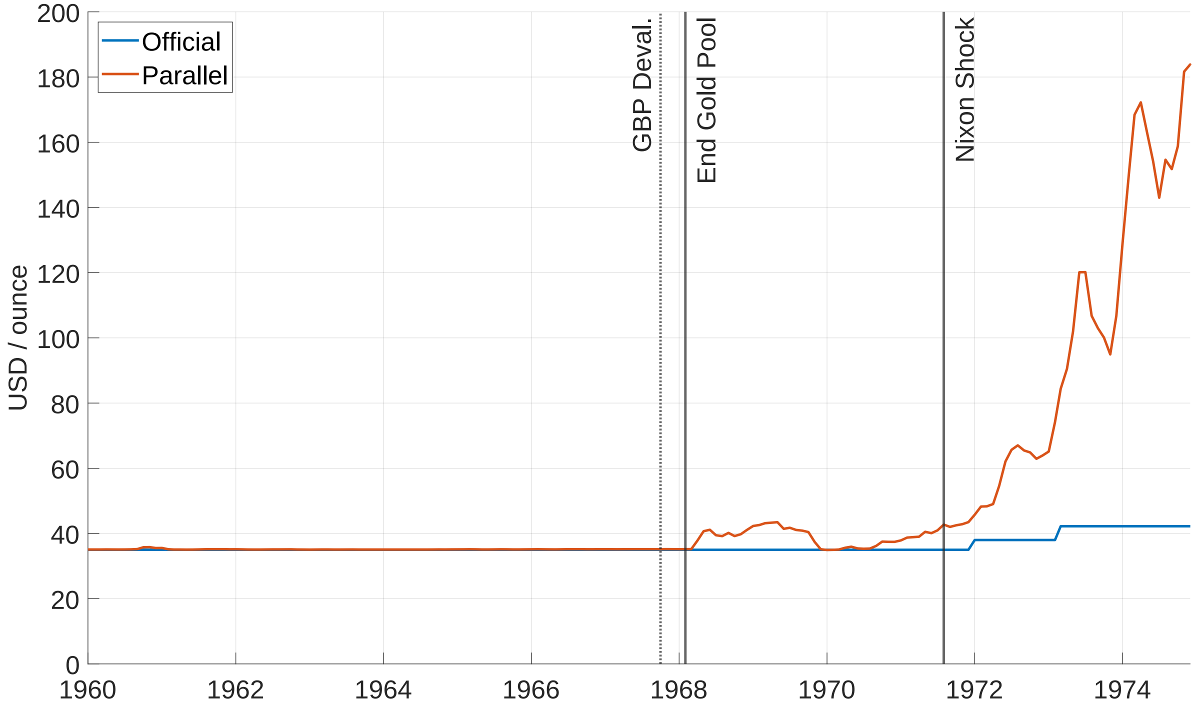

Figure 8: Gold Prices

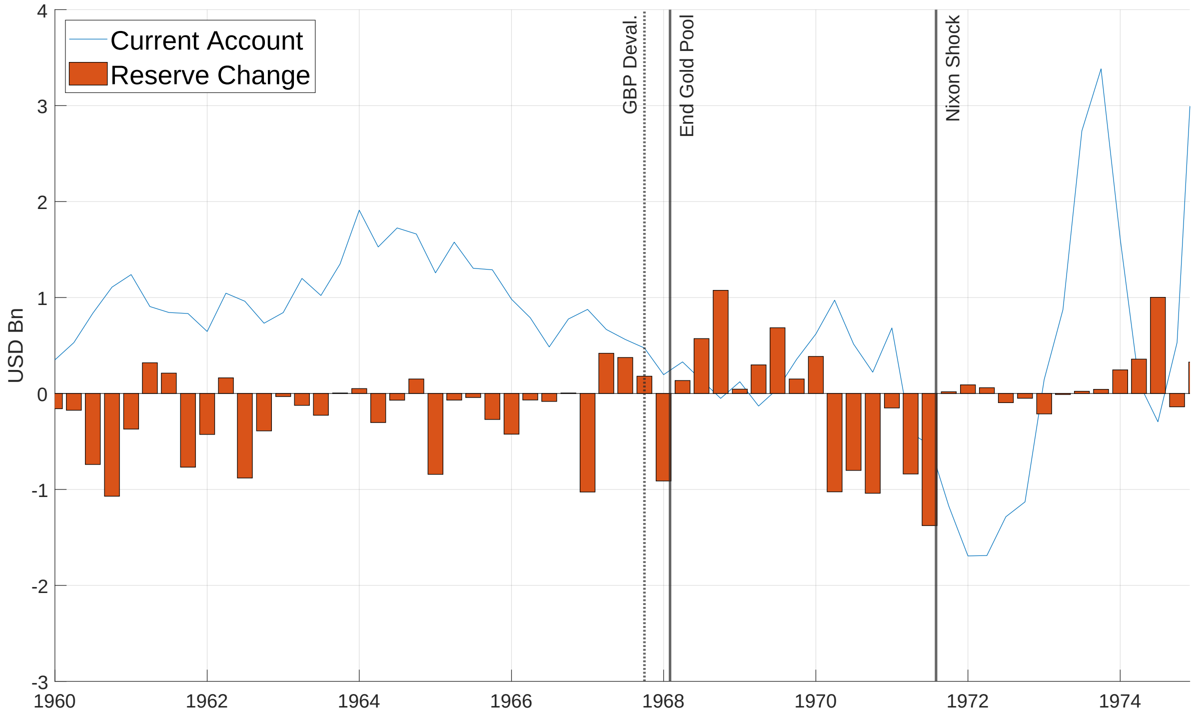


Figure 9: Reserves and BoP