Time-Limited Subsidies: Optimal Taxation with Implications for Renewable Energy Subsidies

Michael David Ricks* ⓡ Owen Kay†

February 3, 2024

Abstract

Pigouvian subsidies are efficient, but subsidies with uncertain or limited durations are not Pigouvian. We show that optimal "time-limited" policies subsidize output *and* investment with investment subsidies separably correcting for the limited duration. Because the change in production after the subsidy ends is a sufficient statistic for the optimal duration, we estimate this statistic using the US Renewable Energy Production Tax Credit for wind energy. Wind facilities reduce generation by 5-10% after the ten-year subsidy ends, demonstrating that time limits distort production even in inelastic industries. We provide additional evidence that correctly addressing time-limited policies may improve other industrial, tax, and energy policies.

*University of Nebraska. Email: mricks4@unl.edu

†University of Michigan. Email: okay@umich.edu

This research is the product of generous feedback from many helpful people including Catie Hausman, Jim Hines, Ash Craig, Joe Aldy, Tatyana Deryugina, Matt Kotchen, Rich Sweeney, Danny Schaffa, Alecia Cassidy, Yeliz Kacamak, Justin Wolfers, Matt White, Sam Stolper, Sarah Johnston, Ying Fan, Zach Brown, Rich Patterson, Michael Moore, Andrew Simon, and from seminar and conference participants at the University of Michigan, Brigham Young University, Yale University, the Harvard Kennedy School, Kansas State University, the National Tax Association, IIPF, AERE, the Michigan Tax Invitational, OSWEET, and ASSA Annual Meetings.

1. Introduction

As growing subsidy programs across the world usher in a new era in industrial and energy policy, a key question is how to efficiently correct economic externalities. The theoretical answer is simple—whether the externality is innovation, offsetting emissions, guaranteeing supply chain resilience, or maintaining a strong working class. In each case, the optimal "Pigouvian" correction is to directly subsidize every externality generating unit by its marginal external benefit. In practice, however, output subsidies typically have finite or uncertain durations, so they do *not* subsidize all externality-generating units. We call these subsidies "time-limited" output subsidies and study them in this paper.

Time-limited subsidies are prevalent across the world. For example, in the United States, the Advanced Manufacturing Production Tax Credit lasts for seven years (White House, 2022), and the Renewable Energy Production Tax Credit and Clean Vehicle Credit both create ten-year subsidies (TREAS, 2021). In Germany, feed-in-tariffs for renewable energy last for twenty years (OECD, 2022) while similar Chinese tax cuts last for six (Nyberg et al., 2020). Many agricultural policies have short subsidy durations, including annual Chinese subsidies for oilseeds (Mcdonald, 2022) and market price supports for dairy in Canada and the United States (CRS, 2014). Even taxes and subsidies without explicit time limits can have uncertain durations due to changes in the political environment as demonstrated by successively proposed and repealed alcohol taxes in the US (CRS, 1999; Blanchette et al., 2020); carbon taxes in Australia and Alberta, Canada (Dayton, 2014; Raymond, 2020); and soda, sugar, or fat taxes in Denmark and many US cities and states (Schmacker and Smed, 2023; The Economist, 2017; Urban Institute, 2023).[1] Despite the ubiquity of time-limited policies, we know nothing about how time limits affect our optimal policy considerations.

We show that whenever time limits are shorter than the capital life, they change firms' incentives and optimal policy. First, because time-limited polices only subsidize output produced during a limited "subsidy period,"[2] firms have incentives to invest less up front and to reduce production after the subsidy period. Second, time limits hamper policymakers' ability to target corrective policies well because they do not affect all externality-generating units. This paper characterizes how subsidy duration affects optimal subsidy rates and the optimal choice of subsidy instruments.

By developing an optimal tax framework for time-limited output subsidies, we demonstrate that time limits affect which policies a social planner should use. Rather than only subsidizing output as the canonical Pigouvian policy does, the optimal policy combines out-

[1] See Appendix Table A.1 for more examples of corrective policies with uncertain or limited durations.

[2] We use this generalization of the phrase "credit period" used for tax credits (e.g., TREAS, 2021).

put and investment subsidies. This result diverges from production-efficiency intuition about only subsidizing output (see Diamond and Mirrlees, 1971) because time-limited output subsidies create an incentive to under-invest, and investment subsidies efficiently counteract this incentive. The optimal subsidy for both investment and output is strictly positive whenever the (expected) output subsidy duration is less than the life of the fixed inputs. As such, policymakers may want to consider output and investment subsidies as complements rather than substitutes for corrective policy.

Although the optimal subsidies we characterize are larger than the externality if only subsidizing output, we show that also subsidizing investment returns the optimal output subsidy rate to the externality value—*no matter* the subsidy duration. This is because investment subsidies can influence production after the subsidy period more effectively than large output-only subsidies. Interestingly, the two subsidies are fully separable: The optimal output subsidy equals the marginal externality, and only the investment subsidy changes with the (expected) duration of the output subsidy. Policymakers should therefore subsidize investment more when output subsidies have shorter durations, all else equal, and set output subsidies equal to the externality regardless of the subsidy duration.

After defining the best subsidy rate for any given duration, we characterize a sufficient statistic for the optimal duration given administrative, compliance, or political frictions associated with a longer duration. In this framework, the efficient subsidy duration trades off the marginal external value of increased production during a longer subsidy period against its marginal administrative or political cost—as in the optimal tax system literature (Dharmapala et al., 2011; Keen and Slemrod, 2017) and politically feasible optimal tax literature (Scheuer and Wolitzky, 2016; Bierbrauer et al., 2021). Changes in production after the subsidy period are a sufficient statistic for the optimal subsidy duration because they capture the marginal social benefit of a longer duration (see similar result in Costa and Gerard (2021) for evaluating temporary corrective policies in the presence of hysteresis). This means that, all else equal, policymakers should establish longer subsidy periods in industries with larger expected changes in production.

Our main empirical application focuses on the US wind industry and the Renewable Energy Production Tax Credit (PTC). The PTC is one of the largest output subsidies in the world, but its subsidy period is less than half of a wind turbine's lifespan.[3] Furthermore, wind energy is a theoretically interesting setting. Given its production technology, changes in production should be relatively small because turbines are essential, wind is free, and after the subsidy period there are relatively few margins for response (e.g., improved maintenance,

[3] The PTC subsidy period lasts for 10 years, and wind turbines last for 20-30 years(Wiser and Bolinger, 2021).

forecasting, optimization, etc.). Because time-limited subsidies are only optimal when the change in production is small, the wind industry provides a limiting case to test the model. If firm behavior changes after the subsidy period in the wind industry, time limits will cause larger distortions in more elastic industries. Of course, the wind industry is also of policy interest because of its role in the global energy transition.

We estimate the change in electricity generation after the PTC's ten-year subsidy period, showing that wind facilities reduce their output by 5-10%. This response may seem surprising given the production technology, but it highlights the importance of subsidy duration when considering optimal policy. This response also has broader market implications. Each month, PTC ineligibility results in over 500 GWh (Gigawatt hours) of forgone production and externality benefits, amounts that will increase as additional turbines age out of subsidization.

In addition to our primary empirical application, we illustrate the role of time limits in two other settings. We recast economic evaluations of Danish sin taxes on sugary drinks (as documented by Schmacker and Smed, 2020, 2023) and US industrial policy for electric-vehicle manufacturing (as documented by Lohawala, 2023) into our framework, illustrating the general relevance of our model. Because time limits affect many industries, these exercises underscore the need to consider their broader market implications.

Our paper makes four main contributions. First, our results highlight additional theoretical justifications for taxing and subsidizing inputs. Although production efficiency suggests it is only efficient to subsidize output (Diamond and Mirrlees, 1971; Parish and McLaren, 1982; Ganapati et al., 2020), we show this is no longer true when output subsidies have time limits. This reversal relates to findings in the behavioral optimal tax and tax systems literatures where production efficiency must be weighed against behavioral biases (Farhi and Gabaix, 2019) or differential evasion opportunities (Emran and Stiglitz, 2005; Best et al., 2015). In addition to our main results about time limits, subsidizing both investment and output can also be an optimal response to policy uncertainty, firm heterogeneity, budget concerns, and network externalities. Given the quantitative magnitude of these concerns,[4] investment subsidies may be critical in many optimal policy conversations.

Second, we document a core complementarity between output and investment subsidies. Empirical research has generally considered output and investment subsidies as substitutes. Case studies have found that while investment subsidies do distort production efficiency (e.g., Burr, 2016; Aldy et al., 2019), other frictions and *cost effectiveness* can justify their

[4]For example, Farrell and Klemperer (2007), Seto et al. (2016), and Acemoglu et al. (2023) consider network effects and lock-in—particularly in the case of carbon intensive technologies—and Kellogg (2014), Baker et al. (2016) Handley and Li (2020), Chen (2023) and Wang et al. (2023) document large effects of uncertainty, especially on investment decisions.

use (see Parish and McLaren, 1982; Dunne et al., 2013; De Groote and Verboven, 2019; Yi et al., 2018). Our *efficiency* justification for investment subsidies reveals that output and investment subsidies may work better when combined than when compared. It is well known that subsidizing both output and investment is also efficient when investment subsidies directly correct a second externality (as in Acemoglu et al., 2012, 2023), but in our setting investment subsidies are used to correct a production externality.

Third, our results expand our understanding of optimal policy under imperfect externality targeting. According to the targeting principle, whenever externality-generating commodities are taxable, the optimal policy is separable between a Pigouvian correction and any other taxes (Sandmo, 1975; Kopczuk, 2003).[5] Although this logic is often used to calibrate output subsidies, time limits disrupt targeting in the real world. We show that using an investment subsidy restores a targeting-like result—even when not all units are targeted. In more general settings, the efficient policy may even choose to target fewer units in order to avoid administrative or fiscal costs. These results build on renewed interest in corrective taxation with imperfect targeting. But whereas most settings feature an inability to tax the externality-generating margin (e.g., Rothschild and Scheuer, 2016; Griffith et al., 2019; Dubois et al., 2020; Jacobsen et al., 2020)[6], our setting features an inability to tax all externality-generating units.[7]

Finally, our empirical results extend conversations about renewable energy subsidies by showing how time limits affect production. Of the many papers studying subsidies for wind energy,[8] two document differences in production and intermittency between firms that receive output versus investment subsidies (Petersen et al., 2022; Aldy et al., 2023), and only Hamilton et al. (2020) consider the time-limited nature of the PTC. Although mainly focused on turbine degradation over time, Hamilton et al. (2020) also document an immediate drop in output after the PTC subsidy period. Our empirical approach builds on these results to estimate long-run (rather than contemporaneous) impacts and does so using an empirical approach robust to both confounding intertemporal policy changes and cross-cohort differences in effects (Sun and Abraham, 2021)—resulting in effects that are roughly twice

[5]As shown in case studies of commodity taxation (Sandmo, 1975), international tax policy (Dixit, 1985), public good provision (Bovenberg and van der Ploeg, 1994), and joint income and commodity taxation (Cremer et al., 1998), all generalized by Kopczuk (2003).

[6]Empirical examples include taxing fuel-efficiency not emissions (Langer et al., 2017; Jacobsen et al., 2020), beverage volume rather than sugar or alcohol content (e.g., Grummon et al., 2019; Dubois et al., 2020; Miravete et al., 2020; O'Connell and Smith, 2021), or using attribute based regulation (Ito and Sallee, 2018; Kellogg, 2020).

[7]In addition to time limits, other empirical examples could include taxing formal markets but not informal markets and only having corrective taxes in some geographical jurisdictions.

[8]See for example Schmalensee (2012); Johnston (2019); Abrell et al. (2019); Helm and Mier (2021).

as large.[9]

Our empirical results also have implications for larger discussions about optimal industrial and energy policy. In the PTC context, production responses after the subsidy period will lead the current fleet of wind turbines to under-produce over 190,000 GWh over the next two decades: enough renewable energy to power every household in the United States for over 18 months. If subsidies are part of a proposed energy transition, accounting for the effect of time limits on production is critical for designing optimal policy. Similarly, industries with expensive inputs like agriculture and manufacturing may have even more elastic changes in production, suggesting that choosing subsidy periods (and investment subsidies) well may have even larger welfare implications for industrial policy.

The remainder of the paper is organized as follows: Section 2 presents a simplified model of time-limited subsidies with efficiency results; Section 3 generalizes the model and discusses changes and extensions; Section 4 contains our empirical application to the wind industry; Section 5 discusses additional applications to sin taxation and industrial policy; and Section 6 concludes.

2. Optimal Time-Limited Subsidies

This section builds time-limited subsidies into an intuitive optimal tax framework. We present the optimal subsidy rates using only one instrument (either output or investment) or combining both. We then derive the optimal subsidy duration given institutional frictions. Throughout we will use subsidy-oriented language, noting that corresponding arguments hold for taxes as well.

2.1 Model Setup and Intuition

We begin by considering a simple two-period model where the duration of output subsidies is (weakly) shorter than the lifetime of the capital. As show in in Appendix Table A.1, subsidies with limited durations are ubiquitous, and these limits are often much shorter than the expected capital life. Time limits may arise from a variety of factors including direct costs of administration or compliance (e.g., Dharmapala et al., 2011), policy features that encourage time limits,[10] or even incomplete contracting (e.g., Persson and Svensson, 1989; Alesina and Tabellini, 1990; Battaglini and Harstad, 2020).

[9] Quantitatively similar results can be derived from Hamilton et al. (2020) for some earlier cohorts by interpreting changes in slopes as part of the dynamic effect.

[10] For example, in the United States, only outlays meeting time-specific objectives can pass via reconciliation (see discussion in Wessel, 2021) and federal mandates in Germany require time limits on subsidies (German Federal Ministry of Finance, 2022).

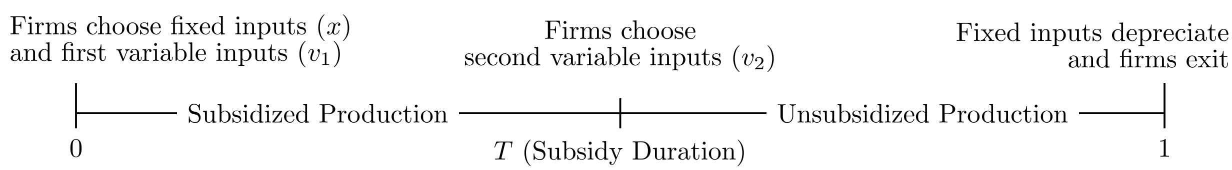
2.1.1 Firm Problem

Firms choose fixed and variable inputs given a set of subsidy policies, θ; market prices; and a production technology, $q()$. We write the following firm profit maximization problem:

$$\max_{x,v_1,v_2} \pi(x, v_1, v_2; \theta) = T[(p + \tau^o)q(x, v_1) - mv_1] + (1 - T)[pq(x, v_2) - mv_2] - x(c - \tau^i) \quad (1)$$

The policy vector $\theta = (\tau^i, \tau^o, T)$ includes the investment subsidy rate for the fixed input, τ^i; the output subsidy rate, τ^o; and the duration of the output subsidy T (as a fraction of the capital life). As illustrated in Figure 1, T partitions the life of the fixed input into two portions where output is either subsidized or unsubsidized. In response to these policies, firms choose fixed inputs x (used to produce both subsidized and unsubsidized units) and different levels of variable inputs v_1 and v_2 (corresponding to production during and after the subsidy period). Fixed and variable inputs are purchased at input prices c and m. Output is produced with a production technology $q(x, v)$ and is sold at price p. Because of the subsidies, firm revenue in the subsidized period is $(p + \tau^o)$ per unit, and investment only costs $(c - \tau^i)$. At the end of the capital life, capital fully depreciates.

Figure 1: Example of Time-Limited Subsidy Structure



Note: This figure displays the economic problem presented when output subsidies have time limits. Firms make investment and input decisions with the recognition that only units produced during the subsidy period (before T), will receive the output subsidy.

Time limits create two problematic incentives for firms. We call the first problem *under-utilization*. That is, conditional on production capacity, production will be inefficiently low after the subsidy period. The second problem is *under-investment* in the fixed input. Because the same fixed inputs produce both subsidized and unsubsidized units, firms will invest less than is socially optimal. Each problem results from not targeting units produced after the subsidy period.

2.1.2 Social Planner Problem

The social planner wants to design a subsidy system to maximize welfare given the firm's response to policy, (x^f, v_1^f, v_2^f); the value of the externality, γ; and the costs imposed by current institutional features. We write the following maximization problem:

$$\max_{\tau^i,\tau^o,T} \mathcal{W}(\tau^i, \tau^o, T) \equiv \max_{\tau^i,\tau^o,T} \Pi + \gamma Q - \lambda TC - \phi(T) \tag{2}$$

Social welfare, $\mathcal{W}$, consists of four terms. The first represents firm profits, $\Pi = \pi(x^f, v_1^f, v_2^f; \theta)$. The second term is the external benefit of total production where $Q = Tq(x^f, v_1^f) + (1 - T)q(x^f, v_2^f)$. The third term is the social cost of funding the subsidy: the marginal cost of public funds, λ, multiplied by the total tax expenditures, $TC = T\tau^o q(x^f, v_1^f) + \tau^i x^f$. Finally, we include a social cost associated with the subsidy duration, $\phi(T)$. This term captures the administrative costs and institutional frictions of a subsidy with duration T. Note that there is no consumer surplus term because demand is perfectly elastic.

Consider both the first-best "Pigouvian" subsidy and a time-limited subsidy in this framework. A Pigouvian policy subsidizes each unit of the externality-generating good by the marginal external value (Pigou, 1920). By setting $T = 1$, $\tau^o = \gamma$, and $\tau^i = 0$, it is consistent with the targeting principle and the production efficiency principle. However, revealed preference suggests that there are social costs, $\phi(T)$, that make Pigouvian subsidies impractical. From a welfare perspective, time-limited subsidies trade off these social costs against the costs of imperfect targeting and production inefficiency that shorter subsidy durations create.

2.1.3 A Note on Simplifications

This setup features a representative firms who does not discount, has perfect foresight, faces perfectly elastic demand, experiences one-hoss shay depreciation, and produces a constant "atmospheric" externality. Section 3 shows that the intuition gained from this model is unaffected by relaxing these assumptions and also microfounds $\phi(T)$. Although other aspects of this model are simplified in this section, the production technology is already quite general. We only assume there are decreasing returns to scale, that inputs produce only one output, that there is no avoidance or evasion, and that a zero-profit condition is attained through costs on the margin of potential entry.

2.2 Optimal Subsidy Policies

To build intuition, this section solves four optimal policy problems in order of increasing complexity. First, we describe the optimal investment-only subsidy and the optimal output-only subsidy. After considering these policies separately, we then describe the optimal combined subsidy. Finally, we characterize the optimal subsidy duration given the social costs that generate time limits and derive a sufficient statistic for subsidy duration.

We solve these optimal tax problems given one main assumption. Assumption 1, stated formally in Appendix B, assumes that $\lambda = 1$ and standard regularity conditions hold.[11] We make this assumption to focus our attention on correcting the production externality rather than correcting the fiscal externality (a common argument in optimal corrective taxation, e.g., Griffith et al., 2019). This assumption would be met if the tax and redistribution system is optimally calibrated (Jacobs, 2018) or if revenue is raised using non-distortionary lump-sum taxes.[12]

2.2.1 Optimal Investment-Only Subsidy

A social planner trying to maximize welfare with only an investment subsidy will face a tradeoff between increasing the quantity of externality-generating units and raising costs by distorting production efficiency. Proposition 1 characterizes the optimal subsidy.

Proposition 1. Optimal Investment-Only Subsidy. Under Assumption 1, if $\tau^o = 0$, then

$$\tau^{i*} = \gamma \frac{\mathrm{d}q(x^f, v_2^f)}{\mathrm{d}x^f}$$

Proof in Appendix B.

Proposition 1 illustrates the use of the investment subsidy to correct for the externality. Because all production is captured in the $q(x^f, v_2^f)$ term, $\gamma \frac{\mathrm{d}q(x^f, v_2^f)}{\mathrm{d}x^f}$ captures the marginal external benefit of increasing investment—just as a Pigouvian subsidy rate captures the marginal external benefit of increasing production. Note that this is a total derivative, not a partial derivative, so the change in quantity includes the direct effect of additional fixed inputs, x, as well as any endogenous change in v_2 caused by increasing x.

[11] These conditions guarantee that $q(x, v)$ generates a unique solution given prices and subsidies and ensures that the conditions of the Implicit Function Theorem are met. The solution to the firm's problem under these conditions is also in Appendix B.

[12] In the case that $\gamma < 0$ and some optimal policies are taxes, this also means that the government will not try to reduce the fiscal externality by raising additional taxes on the externality generating industry.

This characterization also captures the production-efficiency trade-off created by subsidizing investment. To see this, note that $\frac{\mathrm{d}q(x^f,v_2^f)}{\mathrm{d}x^f}$ can be rewritten as $\frac{\frac{\mathrm{d}q(x^f,v_2^f)}{\mathrm{d}\tau^i}}{\frac{\partial x^f}{\partial \tau^i}}$. Therefore, the optimal investment subsidy will be larger in settings with large externalities or production responses ($\frac{\mathrm{d}q(x^f,v_2^f)}{\mathrm{d}\tau^i}$), all else equal. The optimal investment subsidy will be smaller in settings with larger marginal investment distortions ($\frac{\partial x^f}{\partial \tau^i}$), all else equal.[13]

2.2.2 Optimal Output-Only Subsidy with a Given Duration

On the other extreme, a social planner trying to maximize welfare with only an time-limited output subsidy will face a tradeoff between increasing the quantity of externality-generating units and generating over-utilization during the subsidized period. Proposition 2 characterizes the optimal subsidy.

Proposition 2. Optimal Output-Only Subsidy. Under Assumption 1, if $\tau^i = 0$ and T is fixed, then

$$\tau^{o*} = \gamma + \gamma \frac{1-T}{T} \frac{\frac{\mathrm{d}q_(x^f,v_2^f)}{\mathrm{d}\tau^o}}{\frac{\mathrm{d}q_(x^f,v_1^f)}{\mathrm{d}\tau^o}}$$

Proof in Appendix B.

This characterization of τ^o as a function of the subsidy duration captures the trade-offs time limits induce. The first term reflects the base subsidy rate, targeted to the marginal external value of production. The second term adjusts the subsidy rate up to compensate for under-investment. If $T = 1$, all units are subsidized, and policy simplifies to the Pigouvian first best, $\tau^o = \gamma$. However, as T approaches 0, τ^o diverges to infinity. Intuitively, this is because τ^o can only change production after the subsidy period by incentivizing investment, and only a larger subsidy rate can change investment if the subsidy period is shorter.

The extent to which τ^o should be adjusted in response to a time limit depends on how effectively the subsidy affects production after the subsidy period. This efficacy is described by the ratio $\frac{\frac{\mathrm{d}q_2}{\mathrm{d}\tau^o}}{\frac{\mathrm{d}q_1}{\mathrm{d}\tau^o}}$.[14] This ratio will be small if investment is unresponsive to the output subsidy, if the marginal product of capital is small, or if fixed and variable inputs are more substitutable. In this case, τ^{o*} remains close to γ for any T, but when the ratio is close to 1,

[13] Multiplying both sides by $\frac{\partial x^f}{\partial \tau^i}$ reveals that the marginal external benefits from a change in τ^i will be set equal to the marginal distortion in investment costs at the optimum.

[14] This ratio is related to the under-utilization incentive discussed at the outset as it depicts the ratio between a subsidy's effect on production after the subsidy period relative to the effect during the subsidy period. In this context, however, the ratio will technically reflect over-utilization during the subsidy period whenever τ^o is raised above the marginal externality.

τ^{o*} may be quite large with shorter durations. Intuitively, it's not worth raising the subsidy rate above γ if investment in unresponsive or will not increase production after the subsidy period.

One special, policy-relevant case is when production is based entirely on fixed inputs, so the time-limited output-only subsidy and investment-only subsidy both attain the first best for any time limit. In this case, the ratio $\frac{\frac{\mathrm{d}q_2}{\mathrm{d}\tau^o}}{\frac{\mathrm{d}q_1}{\mathrm{d}\tau^o}} = 1$, and the optimal subsidy will be $\frac{\gamma}{T}$. Because variable inputs are irrelevant, the resulting allocation is welfare-equivalent to both the Pigouvian subsidy (with no time limit) and the investment subsidy.[15] This insight may reflect the policy intuition behind time-limited subsidies in industries with large fixed costs and relatively small variable costs.

2.2.3 Optimal Combined Subsidy with a Given Duration

Given the shortcomings of investment-only and time-limited output-only subsidies, we now assess the benefits of combining both policy instruments. Whether there are gains from having multiple instruments is not *ex ante* obvious. For example, without time limits ($T = 1$), it is well known that it is optimal to only subsidize output even when an investment subsidy is available (Diamond and Mirrlees, 1971). At the same time, there are intuitive arguments for gains from using both instruments. Investment subsidies can target x but cannot directly affect q_1 or q_2 (resulting in the breakdown of production efficiency) whereas output-only subsidies can target q_1, but cannot directly affect x or q_2 (resulting in under-investment and under-utilization). Proposition 3 shows that there are gains from targeting both x and q_1.

Proposition 3. Optimal Combined Subsidy. Under Assumption 1, if T is fixed, then

$$\begin{aligned}\tau^{i*} =& (1-T)\gamma\frac{\mathrm{d}q(x^f, v_2^f)}{\mathrm{d}x^f}\\ \tau^{o*} =& \gamma\end{aligned}$$

Proof in Appendix B.

This result has two major implications. First, it shows that an investment subsidy can correct the under-investment problem created by time limits. When $T = 0$, and all output is unsubsidized, τ^{i*} takes the same form as in Proposition 1. When $T = 1$, $\tau^{i*} = 0$ because there is no need to subsidize investment (Diamond and Mirrlees, 1971). In all other cases the investment subsidy is positive, despite creating production inefficiency, because it addresses

[15] Although these policies create the same amount of production, the output subsidies transfers more money to firms (see Parish and McLaren, 1982), but with $\lambda = 1$, this is welfare-irrelevant.

under-investment and promotes production after the subsidy period more effectively than increasing the output subsidy.

Second, combining subsidy instruments restores a targeting-like calibration for the output subsidy even though targeting is imperfect. Whereas τ^{o*} increased above the marginal externality whenever $T < 1$ in Proposition 2, it now remains constant. The optimal output subsidy is $\tau^{o*} = \gamma$ for all values of T—whether targeting includes all or almost none of the production. The optimal response to changes in subsidy duration is captured in τ^{i*} and is fully separable from τ^{o*}. This "separability" is reminiscent of many other results in the targeting literature (e.g., Sandmo, 1975; Dixit, 1985; Bovenberg and van der Ploeg, 1994; Cremer et al., 1998; Kopczuk, 2003), but in our setting the appropriate tax instruments can restore a targeting-like result *even without perfect targeting.*[16]

Figure 2 depicts a comparison of the optimal policies presented in Propositions 1-3. It depicts the investment subsidy rate and the output subsidy rate as functions of T for cases where the social planner is restricted to only one of the instruments or has both available. Although the degree of curvature will depend on the production technology $q(\cdot)$, the intercepts and limits reflect the optimal policies in general.

2.2.4 Optimal Subsidy Duration Choice

Now consider the optimal subsidy duration. It is well known that the first best policy chooses $T = 1$ and targets perfectly, but revealed preference suggests that real constraints make this infeasible. Recall that $\phi(T)$ reflects these constraints, administrative costs, and institutional frictions.

The optimal subsidy duration for a given policy weighs the welfare from better targeting against the social costs of a longer duration. We use a second-best interpretation of the results that follow, where the Pigouvian first-best policy is impeded by institutional frictions and the social planner optimizes accordingly.[17]

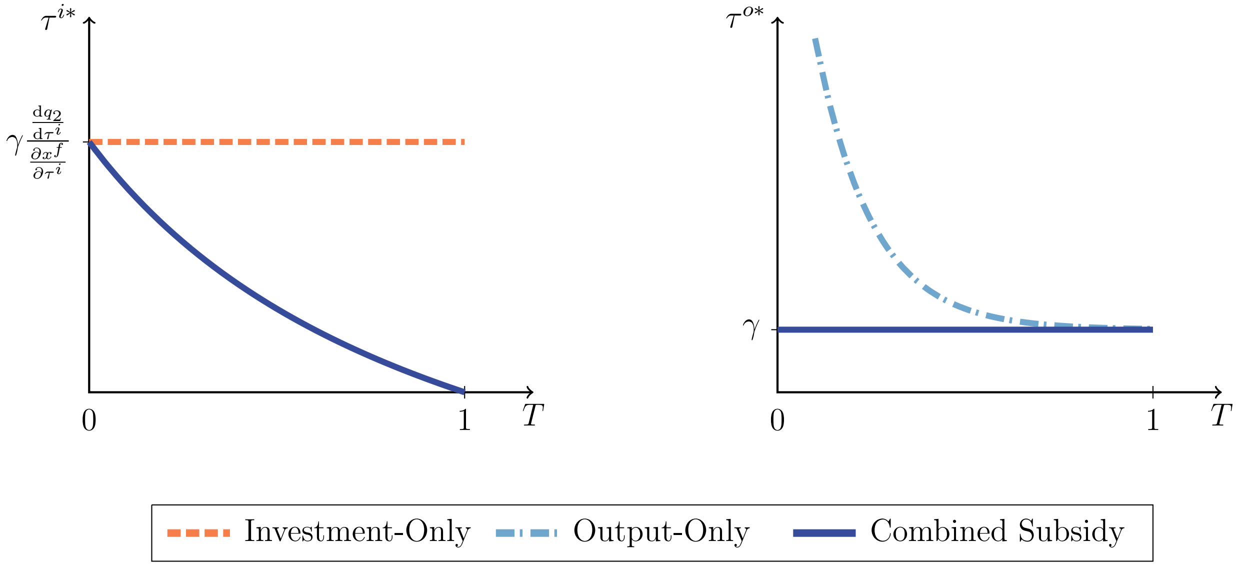
Proposition 4. Optimal Subsidy Duration. Under Assumption 1, a first-order Taylor approximation where $\Delta v = v_2 - v_1$ is small and q_v is locally linear, and a positive, convex, and twice differentiable $\phi(T)$, the optimal subsidy duration is unique and satisfies the following at interior solutions:

$$\phi'(T^*) = -\gamma\big[q(x^f, v_2^f) - q(x^f, v_1^f)\big] \equiv -\gamma \Delta q(\theta^*)$$

[16] This restoration of targeting through the application of additional (if not directly related) tax instruments reflects similar insights about the power of multiple instruments from elsewhere in the optimal income tax literature (e.g., Rothschild and Scheuer, 2016; Scheuer and Werning, 2016, etc.).

[17] The choice of subsidy duration is technically a "first-best" problem if $\phi(T)$ is considered to be a real social cost as in the optimal tax systems literature (e.g. Keen and Slemrod, 2017).

Figure 2: Comparing Optimal Time-Limited Subsidies



Note: This figure shows the optimal rates for τ^i and τ^o as functions of the subsidy duration, T. Three policies are represented; the optimal investment-only subsidy, the optimal output-only subsidy, and the optimal combined subsidy. For the combined subsidy, note that whereas the total value of the investment subsidy is decreasing in T, the total value of the output subsidy is increasing—even though the per-unit rate is constant.

with corner solutions characterized by

$$\begin{aligned} T^* &= 1 \quad \text{if } \phi'(1) \leq -\gamma \Delta q(\theta^*|_{T=1}) \\ T^* &= 0 \quad \text{if } \phi'(0) \geq \gamma \Delta q(\theta^*|_{T=0}) \end{aligned}$$

Proof in Appendix B.

This characterization shows that the optimal T trades off the costs and benefits of a longer subsidy duration. A longer subsidy period will increase the amount of the externality good produced. At the same time, it will also cost more to implement.

Proposition 4 reveals a sufficient statistic for the optimal subsidy duration. For a social planner who values production at γ but pays $\phi(T)$ to implement the subsidy, the change in production after the subsidy period (Δq) is a sufficient statistic for T^*. In corner solutions the externality benefits are greater than all costs ($T^* = 1$) or less than any cost ($T^* = 0$). This implies an elasticity rule for optimal subsidy durations: policymakers should choose a longer subsidy duration in industries with more elastic changes in production (and, thus, externality generation) and a shorter subsidy duration in markets with less elastic production.

The fact that this change in quantity depends on the price elasticity may be particularly useful to policymakers. The dependence on primitives means that engineering-based estimates of supply and model-based estimates of demand can inform the optimal policy—before implementing a policy experiment to identify the sufficient statistic. This approach may be particularly useful since firms who know they are participating in a policy experiment will have an incentive to feign a greater elasticity in order to secure a longer duration and subsidy transfer.

3. Generalized Optimal Time-Limited Subsidies

With intuition established, this section extends our analysis to a more general setting. After generalizing the model from Section 2, we present extensions to additional economic environments. In all cases the main conclusions from Section 2 carry through: time limits create a complimentary relationship between output and investment subsidies where output subsidies are used to target the externality during the subsidy period and investment subsidies are used to target the externality after the subsidy period.

3.1 Model Setup

3.1.1 Firm Problem

Consider a set of heterogeneous firms, indexed by j, that make a single investment decision and a series of production decisions over the lifetime of their initial capital. The timing of investment decisions identifies firms' cohorts, s_j. Let firms choose their initial fixed input X_j and set of variable inputs, $v_{j,t}$ ($t \geq s_j$), given subsidies and prices. The firm problem is below (suppressing j subscripts)

$$\begin{aligned}\max_{X,\{v_t\}} \pi &= \max_{X,\{v_t\}} \int_{s_j}^{\infty} e^{-\beta t}\pi_t \,\mathrm{d}\,t \\ &= \max_{X,\{v_t\}} \int_{s}^{T+\kappa} e^{-\beta t}\left[(p_t+\tau^o)q(x_t,v_t)-m_t v_t\right]\mathrm{d}\,t + \int_{T+\kappa}^{\infty} e^{-\beta t}\left[p_t q(x_t,v_t)-m_t v_t\right]\mathrm{d}\,t - X(c_s-\tau^i)\end{aligned}$$

This setup generalizes a number of features from Section 2. First, each time period has an endogenous output price and changing (exogenous) prices for inputs. Second, producers exponentially discount future profits at rate β. Third, firms have heterogeneous production functions $q_j(x,v)$. Fourth, fixed inputs depreciate, such that $x_{j,t} = X_j \cdot \delta(t-s)$ with $\delta(\cdot) \in [0,1]$ and $\delta'(\cdot) \leq 0$. Finally, the new term $\kappa \in \{0, s\}$ reflects whether the subsidy period ends at the same time for all firms, $\kappa = 0$, (often called policy "sunsetting"—e.g., Tax Cuts and Jobs Act provisions that expire in 2025) or has the same duration for firms in every cohort, $\kappa = s$, (e.g., the 10-year PTC).

3.1.2 Social Planner Problem

The social planner chooses $\theta = (\tau^o, \tau^i, T)$ to maximize the sum of five terms: consumer utility, firm profits, the production externality, the total fiscal costs of the subsidy, and the duration costs. Welfare is given by

$$\max_{\tau^o,\tau^i,T} \mathcal{W}(\tau^o, \tau^i, T) = \max_{\tau^o,\tau^i,T} \int_0^\infty e^{-\beta t} \left[U_t + \int_{\mathcal{J}_t} \pi_{j,t} + \gamma_{j,t} q_{j,t} + \lambda TC_{j,t} \, \mathrm{d}\, F(j) \right] \mathrm{d}\, t - \phi(T)$$

where the set of firms producing in period t is $\mathcal{J}_t$; where $F(j)$ is the distribution of firms;[18] where the fiscal costs of subsidizing firm j at time t is $TC_{j,t}$;[19] and where the consumer utility, U_t, is a quasi-linear function of the total quantity produced at time t.

This welfare function enriches the model from Section 2 in three notable ways. First, consumer utility is now welfare relevant. Second, we address the possibility that $\lambda \geq 1$. Third, and most importantly, the marginal externality, $\gamma_{j,t}$, is allowed to vary over firms and across time. This nuance seems particularly relevant because time-limited subsidies might have intuitive appeal when an externality is decreasing over time. For example, in the energy transition application, $\gamma_{j,t}$ likely vary over time and space—especially as the share of energy generated by renewables increases.

This welfare function also generates a micro-foundation for $\phi(T)$. Appendix C.2 shows that if the mass of firms present at each time period is increasing quickly enough, then $\phi(T)$ will be convex if each firm generates a fixed cost for administration (such as an administrative or audit staff as in Dharmapala et al., 2011; Keen and Slemrod, 2017) or compliance (such as hiring accountants to file subsidy applications) in each period.

3.2 Optimal Time-Limited Subsidies

To continue building intuition we consider three optimal policy questions with increasing orders of complexity. First, we demonstrate how the optimal subsidies are changed by dynamics (i.e., discounting, depreciation, and price changes), then we add heterogeneity in technology and marginal externalities as well as revenue costs of taxation—all taking the time limit as given. We then consider optimal time limits in the general case.

[18] Without loss of generality, this notation requires that each cohort produces unique "types" of firms. As such, changes in entry over time are captured by the changing measure of $\mathcal{J}_t$.

[19] The total fiscal cost includes the cost of the investment subsidy in s_j and the output subsidy for each $t \in [s_j, T + \kappa]$.

3.2.1 Optimal Combined Subsidy with Dynamics

First, to focus on firm dynamics, consider one cohort of homogeneous firms. The same trade-offs characterize this optimal policy problem but with greater analytical complexity.

Proposition 5. If Assumption 1 holds for a set of homogeneous firms that all enter at $s_j = 0$, and the marginal externality is constant, then for a given subsidy duration, T, then

$$\tau^{i*} = \frac{(1 - \widetilde{T}_0)}{\beta} \gamma \ \mathbb{E}_2 \left[\frac{\mathrm{d}q(x_t^f, v_t^f)}{\mathrm{d}X_j^f} \right]$$

$$\tau^{o*} = \gamma$$

where $\widetilde{T}_0 = 1 - e^{-\beta T}$ and where $\mathbb{E}_2[\cdot]$ returns the present-value average after the subsidy period. Proof in Appendix B.

The key terms from Proposition 3 remain readily recognizable in these new formulas. In fact, many extensions have no impact on the analytical expressions at all. For example, consumer surplus, endogenous output prices, and changing input prices do not affect the expressions (although these considerations will affect the investment subsidy rate through $\frac{\mathrm{d}q_j(x_t^f, v_t^f)}{\mathrm{d}X^f}$).

Although depreciation and discounting change the investment subsidy rate, they do not affect separability. This is seen in two places. First, $\frac{(1-\widetilde{T}_0)}{\beta}$ captures the relative importance of production after the subsidy period and is determined by the discount rate. For any T, steeper discounting reduces the social value of production after the subsidy period, reducing the optimal investment subsidy. Second, in the expectation, depreciation affects $\mathbb{E}_2\left[\frac{\mathrm{d}q(x_t^f, v_t^f)}{\mathrm{d}X^f}\right]$ by attenuating the production response in each period from additional initial fixed inputs X_j, therefore reducing the optimal investment subsidy.[20] Note, discounting also impacts the *present-value weighted* expectation by weighting earlier production responses more.

3.2.2 Optimal Combined Subsidy with Dynamics and Heterogeneity

We now extend the analysis to allow for multiple cohorts of heterogeneous firms and for heterogeneous externalities. We assume (Assumption 2) that the regularity conditions from

[20] Mathematically this is because the depreciation enters multiplicatively in the derivative

$$\frac{\mathrm{d}q(x_t^f, v_t^f)}{\mathrm{d}X_j^f} = \delta(t) \left(\frac{\partial q_t}{\partial X_j} + \frac{\frac{\partial^2 q_t}{\partial X, v_t}}{\frac{\partial^2 q_t}{\partial, v_t, v_t}} \frac{\partial q_t}{\partial v_t} \right)$$

Assumption 1 hold for all firms j and that $\lambda \geq 1$. Note that λ can be interpreted generally as capturing the social value of \$1 of government revenue relative to \$1 of profits.

Proposition 6. Under Assumption 2, if $T+\kappa$ is fixed, then

$$
\begin{aligned}
\tau^{i*} &= \frac{(1-\widetilde{T}_\kappa)}{\beta} \frac{\overline{\gamma_2} \ \ \mathbb{E}_2\left[\frac{\mathrm{d}q_j(x_t^f, v_t^f)}{\mathrm{d}X_j^f}\right]}{\lambda} \\
&\qquad + \frac{\widetilde{T}_\kappa}{\beta}\Omega_{\frac{\gamma}{\lambda}, \frac{\partial q_1}{\partial \tau^i}} + \frac{(1-\widetilde{T}_\kappa)}{\beta}\Omega_{\frac{\gamma}{\lambda}\frac{\mathrm{d}q_2}{\mathrm{d}X}, \frac{\partial X}{\partial \tau^i}} + \frac{(1-\lambda)}{\lambda}\Psi_{\tau^i} \\
\tau^{o*} &= \frac{\overline{\gamma_1}}{\lambda} \qquad + \Omega_{\frac{\gamma}{\lambda}, \frac{\partial q_1}{\partial \tau^o}} + \frac{(1-\widetilde{T}_\kappa)}{\widetilde{T}_\kappa}\Omega_{\frac{\gamma}{\lambda}\frac{\mathrm{d}q_2}{\mathrm{d}X}, \frac{\partial X}{\partial \tau^o}} + \frac{(1-\lambda)}{\lambda}\Psi_{\tau^o}
\end{aligned}
$$

where $\overline{\gamma_1}$ and $\overline{\gamma_2}$ are the average externalities during and after the subsidy period, where $\widetilde{T}_\kappa = 1 - \mathbb{E}_0[e^{-\beta(T+\kappa-s_j))}]$, where $\mathbb{E}_0$ is a present-value firm-level average, and where the Ω's are adjusted covariance terms. Proof in Appendix B.

The core expressions in Proposition 6 remain largely unchanged, but the optimal policies now must account for heterogeneity and budget considerations. Analytically similar terms now account for more nuance. For example, the present-value, firm-weighted averages reflect the responses of all types of firms, no matter the period they enter in. As such, economic primitives like the nature of technological progress will now matter for the optimal policy considerations.

Consider three implications of these formulas for understanding time limited subsidies. First, the new externality terms $\overline{\gamma_1}$ and $\overline{\gamma_2}$ reinforce the core separability result from Section 2. When allowing for heterogeneous marginal externalities, the optimal output subsidy is equal to the average externality during the subsidy period, $\overline{\gamma_1}$, and the optimal investment subsidy is related to the average externality after the subsidy period $\overline{\gamma_2}$.[21] This differentiation underlies the economic intuition behind separability: the optimal policy uses the output subsidy to correct for externality generation during the subsidy period and uses the investment subsidy to correct for externality generation after the subsidy period (or equivalently to correct for the suboptimal subsidy duration).

Second, the new Ω terms are adjustments based on the covariance between the marginal externality and the marginal effects of subsidies, similar to those in Diamond (1973). Intuitively, the ideal policy accounts for how much behavioral change each tax instrument induces among high-externality versus low-externality firms and time periods. Because we

[21]Note these averages are not present-value averages, just as they would not be in the Pigouvian case.

have two policy instruments, the optimal policy also accounts for both cross-policy effects[22] (analogous to those in Griffith et al. (2019)[23]) and for the possibility for improved targeting among heterogeneous firms.[24] All Ω terms will be zero if the marginal externality is constant or if it varies for reasons unrelated to the production function.

One striking takeaway about targeting from this result is that (except in knife-edge cases), non-zero subsidies for both investment and output are almost always optimal—even as the time limit goes to infinity. To see this analytically, note that when there is no time limit, the optimal investment subsidy is not equal to zero, but equal to $\frac{1}{\beta}\Omega_{\frac{\gamma}{\lambda},\frac{\partial q_1}{\partial \tau^i}}$. The implication is that when firms are heterogeneous, a social planner using a uniform output subsidy can increase welfare by either also subsidizing or taxing investment—with the welfare gains arising through improved externality targeting. As such, it may be optimal to subsidize or tax both investment and output under a quite general array of settings.

Third, now that $\lambda \geq 1$, changes from revenue costs extend standard separability results. The optimal subsidies now include Ψ_{τ^i} and Ψ_{τ^o} to capture the (cross-policy adjusted) costs of transferring money to firms. They also rescale the marginal production externality, $\gamma_{j,t}$, by λ to account for the welfare costs of raising additional revenue. Two interesting points arise from these changes. First, whenever Ψ_{τ^i} is non-zero, it is efficient to recoup revenue by taxing investment. Second, these formulas demonstrate that the production externality, the time limit, and fiscal externalities can all be corrected separably—extending the results of Kopczuk (2003) to settings with imperfect targeting. As the literature on optimal corrective taxation often relies on such separability to focus on production externalities, it is useful to know such separability can hold even when targeting does not.

3.2.3 Optimal Time Limit in General

We now consider the optimal subsidy duration, which trades-off the economic benefits from targeting with the administrative or institutional costs captured by $\phi(T)$.

Proposition 7. Let Δq_j and Δv_j denote the instantaneous change in firm j's output and the variable input at the end of the subsidy period ($t = T + \kappa$). Then under Assumption 2, a first-order Taylor approximation of q_{v_j} in Δv_j, and increasing and convex $\phi(T)$, the optimal

[22]At an optimum, increasing either subsidy rate will reduce the optimal rate of the other subsidy. The Ω terms reflect the welfare gains from the increase in the subsidy rate *minus* welfare costs from the reduction in the other rate that each arise through the correlation of behavioral responses with $\gamma_{j,t}$.

[23]Our cross-policy effects depend on the responsiveness of production and investment to each policy instruments whereas those in Griffith et al. (2019) depend on cross-price elasticities. Appendix B contains analytical details and additional discussion.

[24]It now matters whether firms with large investment responses also tend to have higher or lower marginal products of fixed inputs, as this will affect the relative efficiency of output versus investment subsidies.

subsidy duration satisfies

$$\begin{aligned}\phi'(T^*) = & - \mathbb{E}_{T^*+\kappa}[\Delta q_j \gamma_j] + \\ & + \Omega_{\gamma, \frac{\partial q_1}{\partial T}} + \Omega_{\gamma \frac{\mathrm{d}q_2}{\mathrm{d}X}, \frac{\partial X}{\partial T}} + (1-\lambda)\Psi_T.\end{aligned}$$

Proof in Appendix B.

As with the other results from the general model, this formula reflects the same intuition from Section 2 but accounts for additional complications. Most model extensions have very similar effects as they do on the other subsidies: discounting reduces the present-value benefits of a longer duration, and greater depreciation reduces the change in production after the subsidy period if fixed and variable inputs are complements. While the average change in externality generation features prominently, cross-policy adjusted covariance terms and budget considerations also matter. We consider three points about how the model generalizations affect the optimal time limit.

First, the change in production after the subsidy period is still key, and remains economically interpretable even with a demand side of the market. Because consumers no longer have perfectly elastic demand, the average Δq_j reflects the elasticity of both supply and the demand. Despite relying on both sets of market primitives, this reduced-form change in quantity is still readily observable from market outcomes and remains fully sufficient for the optimal policy in cases where $\Omega = 0$ and $\lambda = 1$. Note that the covariance of Δq and γ will now matter because it will make the expectation of the product larger (or smaller).

Second, budget considerations directly affect T^*. The Ψ_T terms are now adjusted for cross-policy effects of both τ^o and τ^i,[25] but the key intuition is that as long as the main transfer term ($\mathbb{E}_{T+\kappa}[\tau^o q 1_j]$) is larger than the cross-policy adjustments, larger revenue costs imply a shorter optimal duration. This builds on the intuition that when there are decreasing returns to scale it is more cost effective to subsidize inputs as they affect production more on average than at the margin (Parish and McLaren, 1982).

When considered in the context of targeting and optimal taxation, these budget considerations mean that a social planner may choose policies that reduce their ability to target all units in order to also reduce their budgetary costs. This insight complements many research insights about optimal tax by considering the extent of targeting as a choice variable rather than considering ideal policy taking perfect targeting as given (e.g., Diamond and Mirrlees, 1971; Sandmo, 1975; Kopczuk, 2003) or examining ways to quantify the welfare losses from imperfect targeting (e.g., Rothschild and Scheuer, 2016; Griffith et al., 2019; Jacobsen et

[25] See Appendix B for details. The Ω terms are similarly adjusted for both cross-policy effect, but do not add much economic intuition.

al., 2020; Dubois et al., 2020). In this sense, the optimal extent of targeting is a policy instrument to be weighed against other costs similar to the tax base in Keen and Slemrod (2017) and constituents tax knowledge in Craig and Slemrod (2022).

Finally, note that $\phi'(T)$ in Proposition 7 can alternatively be interpreted as the social benefit of relaxing an arbitrarily imposed time limit. In the case that the subsidy duration was not optimally determined or $\mathcal{W}$ is not globally convex, we can no longer characterize an optimal duration; however, even in this case, the formula in Proposition 7 reflects the shadow value on extending T. This interpretation requires no assumptions on the shape or source of $\phi(T)$, but it may still be informative regarding which industries may benefit more from having longer time-limits.

3.3 Additional Extensions

In this section we consider three extensions: uncertain subsidy durations, network externalities, and expanding the policy space beyond two uniform subsidies. Rather than carry the analytical and notational baggage from the general tax formulas, we present each example in the simplest possible case to focus on the core economic results.

3.3.1 Extension I: Policy Uncertainty

In the real world, many tax and subsidy policies are repealed rather than ending at a predetermined time limit. The possibility of repeal introduces uncertainty that shapes firms' investment and production decisions even in the absence of statutory time limits. In this extension, we show that the optimal output and investment subsidies in response to uncertain durations are *ex ante* equivalent to statutory time limits on duration.

To illustrate this connection, consider a firm making an investment decision in the face of uncertainty. At the moment of investment, an output and an investment subsidy are both in place, but there is a constant hazard p that the output subsidy will be repealed by $t = 1$. If the subsidy is repealed, the firm will be able to adjust their variable input, v_t, but not the fixed input, x.

Corollary 1. If Assumption 1 holds for a representative firm with one-hoss shay depreciation, and the social planner is infinitely patient but faces policy uncertainty p, then

$$\tau^i = p\frac{\gamma\frac{\partial q(x^f, v_2^f)}{\partial \tau^i}}{\frac{\partial x^f}{\partial \tau^i}}$$

$$\tau^o = \gamma$$

Proof in Appendix B.

Corollary 1 shows that both uncertain and time-limited subsidies feature the same economic incentives. As such, any output subsidy (or tax) with positive probability of repeal should be accompanied by an investment subsidy. Because uncertainty reduces the expected returns to investment in the same way as a statutory time limit, the (*ex ante*) optimal subsidy corrects this under-investment with an investment subsidy. The higher the probability, p, the larger the optimal investment subsidy.

3.3.2 Extension II: Network Externalities

Another motivation for a time limited subsidy might be to make a "big push" toward a better equilibrium—allowing the subsidy period to end once the transition is guaranteed. Implicit in this logic is the presence of a network externality or learning spillover that makes competitive markets stay in a bad equilibrium for too long. In this extension, we show that policy makers can separably correct such a network externality by subsidizing investment.

Since most examples of network externalities are based on the stock of constructed capital, we consider a model where input costs, technology, demand, or the marginal externalities depend on prior capital investments. Defining the total prior investment in period t as $\mathcal{X}_t = \int_0^t \int_{\mathcal{J}_t} X_j \, \mathrm{d}\, F(j)$, we include these totals as determinants of investment costs $c_t(\mathcal{X}_t)$, production technology $q_j(\cdot; \mathcal{X}_t)$, utility of consumption $U_t(\mathcal{X}_t)$, and marginal externalities $\gamma_{j,t}(\mathcal{X}_t)$. This addition to the model allows there to be a "big push" such that substantial investments could move the market to a new equilibrium.

Corollary 2. If Assumption 1 holds for all firms, and cost, production, and externalities are allowed to be functions of $\mathcal{X}_t$, then

$$\tau^{i*} = \tau_6^i + \omega_{\mathcal{X}}^i \overline{\gamma_{\mathcal{X}}} + \Omega_{\gamma_{\mathcal{X}}, \frac{\partial \mathcal{X}-}{\partial \tau^i}}$$
$$\tau^{o*} = \tau_6^o + \omega_{\mathcal{X}}^o \overline{\gamma_{\mathcal{X}}} + \Omega_{\gamma_{\mathcal{X}}, \frac{\partial \mathcal{X}}{\partial \tau^o}}$$

where τ_6 are analytically identical to the optimal subsidies from Proposition 6, and $\overline{\gamma_{\mathcal{X}}}$ is the average network externality, and $\omega_{\mathcal{X}}^i$ and $\omega_{\mathcal{X}}^o$ characterize the relative effectiveness of increasing the capital stock in early periods with investment and output subsidies. Proof in Appendix B.

Corollary 2 provides an additional argument for the importance of subsidizing both output and investment. If the network externality is non-zero, then it is critical to subsidize investment even when there is no time-limit or uncertainty surrounding the output subsidy.

This argument follows similar intuition to Acemoglu et al. (2012), where optimal policy involves both carbon taxes to correct the production externality and research subsidies to account for endogenous technological change. Furthermore, this result reveals that if the main externality is a "big push" rather than a direct production externality, the social planner should subsidize the margin of the network effect—likely at the margin of investment.

3.3.3 Extension III: Expanding the Policy Space

Although the set of possible time-limited subsidies is much more expansive than traditional comparisons between either the optimal uniform investment subsidy or the optimal uniform output subsidy, we also consider two extensions to this policy space.

Adding Variable Input Subsidies. Given the optimality of combining output and investment subsidies, we consider the efficiency of also subsidizing variable inputs. Corollary 3 in Appendix B demonstrates that at the optima characterized by Proposition 3, the optimal variable input subsidy is zero. Therefore, the exclusion of a variable input subsidy in the baseline model is without loss of generality. Of course, this intuition would be complicated if output and variable input subsidies had different time limits, if firms are heterogeneous (leading to gains from targeting), or if there are cost effectiveness concerns (as in Parish and McLaren, 1982).

Allowing Subsidies to Vary Between Firms and Over Time. Since uniform subsidization does not effectively target firm-level marginal externalities (and as a result introduces the adjusted covariance terms in Propositions 6 and 7), we consider non-linear output subsidies differentiated among firms and time periods. Corollary 4 in Appendix B reveals that subsidies targeting marginal firm externalities satisfy the necessary conditions for optimality and eliminate the adjusted covariance terms during the subsidy period. This is unsurprising as a differentiated subsidy is a more efficient solution than a uniform one, but varying the subsidy amount by firm and over time may be infeasible in many situations.

3.4 Market Implications from the Generalized Model

With our theoretical results defined, we consider their practical implication in preparation for the empirical examples in Sections 4 and 5. We consider the connections between our model generalizations and real-world markets then discuss the limitations of the model to certain contexts.

3.4.1 Applying Theoretical Results

Consider seven connections between the results from the general model and real-world markets:

Depreciation and Discounting. The general model reveals some cases where naive Pigouvian subsidies[26] might be approximately optimal. Consider the limiting case: increasing investment will not affect welfare if firms' fixed inputs are depreciated by the end of the subsidy period or if the social planner does not care about the distant future. Although this insight is not likely to be relevant for wind energy (wind turbines often last for 25-30 years, much longer than the 10 year PTC time-limit), it could be applicable in industries with more rapid depreciation. For example, our results can explain why a government wanting to incentivize the production of cutting edge microchips or semiconductors might only subsidize output even in the presence of a relatively short time limit—the rapid depreciation of fixed inputs through obsolescence essentially eliminate the optimal investment subsidy.[27] In a sense the capital depreciates so fast that the time-limits no longer bind and the naive Pigouvian subsidy is actually the first best.

Heterogeneous Externalities. There are many settings where externality benefits are heterogeneous. If the externalities tend toward zero quickly enough, a naive Pigouvian subsidy without an investment subsidy could be optimal. For example, marginal wind and solar production are more likely to offset (dirty) coal in some times and places relative to (cleaner) natural gas in others (e.g., Cullen, 2013; Fell et al., 2021), and the amount of coal offset is changing over time (Holland et al., 2022; EIA, 2023). On the other hand, if externalities are growing, optimal investment subsidy rates may be dramatically higher. For example, because the external value of electric vehicle use depends on the energy mix of the local electricity markets (Holland et al., 2016, 2020), externalities will increase with wind and solar penetration.

Adjusted Covariances and Targeting. Although the adjusted covariance terms from the general model are not intrinsically related to time limits, whether they matter in practice depends on market characteristics. In industries like wind energy where technologies are fairly similar, the covariance terms will likely be small relative to the average externality, and these terms may not affect the optimal policy much. On the other hand, in settings like sin taxation with lots of heterogeneity, these terms may matter enormously. In setting where covariances are large, rather than altering a uniform subsidy to account for the heterogeneity, the social returns to differentiating subsidies may also be large (Griffith et al., 2019).

[26] i.e., a time limited subsidy equal to the average externality.

[27] Interestingly, in the US the main subsidies for these industries are investment-only subsidies (e.g., CHIPS and Science Act and FAB Act).

Policy Uncertainty. Many real-world subsidies and taxes have uncertain durations. As climate and energy policy become increasingly politicized, policies aimed at addressing climate change increasingly risk repeal as the tides of political power change. For example, the Australian carbon tax introduced in 2011 was repealed in 2014 (Dayton, 2014). In this context, a policymaker that wants to address climate change, but who is concerned that a carbon tax could be repealed, might also tax investments that extract or burn fossil fuels to prevent over-investment.

Network Effects. In many industries, total investments impact costs, technology, consumer demand, or externalities through network effects. For example, learning spillovers between wind turbine manufactures have generated significant cost reductions and have advanced the production technology available to the industry (Covert and Sweeney, 2022). Similarly, there is evidence that consumers who see neighbors' rooftop solar investments invest in more solar panels of their own (Bollinger et al., 2022). There may also be settings in industrial policy where the suitability of an equilibrium is itself defined by the production capacity, making the marginal externalities a function of the capital stock—as with arguments about supply chain security.

Subsidy Phase-Out. Our results also have implications for subsidy phase out. In practice, it is common to reduce subsidy rates over time. For example, the original subsidies for electric vehicles phased down from $7,500 to $3,750 and after 2016 each cohorts of wind facilities claiming the PTC received 20% smaller output subsidies.[28] Our results show that phase outs are not justified by discounting or depreciation, but can be optimal when the externality itself is changing over time.

Reducing Frictions. With the second-best nature of our results in mind, policymakers may want to explore options to facilitate first-best policies. In our analytical results, we take administrative and institutional costs as given, but systematic changes reducing these frictions would also generate welfare gains. Reducing or eliminating the barriers that necessitate time-limited subsidies could move policy toward the first-best allocation. Since the same institutional frictions could affect policy decisions in many markets, the benefits of reducing frictions (and thus implementing better policies) may be quite large.

3.4.2 Limitations of Modeling Choices

While the extended results cover a broad range of possible economic scenarios, the more fundamental aspects of the model setup still imply some restrictions worth discussing. This subsection addresses what we see as the most relevant points.

[28] Although some of the phaseout was retroactively reversed in 2020 and 2021.

First, in our model, inputs only create one output and both inputs and output can be directly measured and subsidized. This makes sense in a setting like the wind industry where turbines only have one purpose and where measuring energy produced is relatively straightforward, but the usefulness of investment subsidies may be undercut by evasion or shifting responses. For example, if automakers' capital can be used in producing multiple goods (with different externalities), output shifting may eliminate the external value of additional investment. Input shifting can likewise reduce the social value of subsidizing investment. For example, if wind turbines are subsidized but not the rental costs of land, turbines may produce less because of wake effects (over-crowding). These concerns might be exacerbated by evasion or imperfect compliance with one or more of the tax or subsidy instruments (as in Emran and Stiglitz, 2005; Best et al., 2015).

Second, in our model no potential entrant has an incentive to wait to enter. This setup is analogous to assuming "short lived" potential entrants that cannot strategically wait to enter later.[29] In this case, changes in policy do not impact which firms enter. Although the dynamic entry margins may be an important dimension for many policies, they seem tangential to how time limits affect optimal policy. Simplifying entry allows us to focus on the economic consequences of time limits rather than on the distortions created by incentives to delay (see, e.g., Langer and Lemoine, 2022).[30]

Third, there are no pricing complications from market power or price uncertainty. Assuming that no individual firm has market power in output markets allows us to focus on externality correction. Market power would add an additional source of under-production (as in O'Connell and Smith, 2021). Input markets are also competitive, with exogenous prices. Finally, although each price may vary, there is no price uncertainty. This simplification highlights our efficiency justifications for non-Pigouvian corrective policy even given perfect certainty.[31]

Finally, in the alternative energy setting, corrective subsidy policy is technically a second best solution. Because energy is generated by multiple sources and demanded inelastically, changes in energy production from one (clean) source offset production from another (dirtier) source.[32] As such, the first-best policy would be to tax dirty energy not subsidize clean

[29] Although the later assumption is used in IO methods (e.g., Doraszelski and Satterthwaite, 2010), the former could be micro-founded by competitive entry markets, where landowners capitalize marginal changes in overall profitability rather than wind facilities

[30] Note that under this assumptions (and perfect competition) the model applies to cases where firms make dynamic investment decisions and can invest in new capital in any period as their current capital stock depreciates.

[31] Other arguments to deviate from Pigouvian policy in response to price uncertainty do exist (if not for efficiency)—for example to improve cost effectiveness (as in Yi et al., 2018).

[32] Cullen (2013) and Fell et al. (2021) show that wind energy does, in fact, offset dirty energy in the status quo.

energy. In the energy market, the marginal externality, $\gamma_{j,t}$, could therefore be interpreted as the relative externality between $q_{j,t}$ and the (additively separable) numeraire good.

4. Empirical Application: Wind Energy and the 10-Year PTC

Given the theoretical results in Sections 2 and 3, the welfare implications of time-limited subsidies hinge on an empirical question: How big are changes in production after the subsidy period? Section 4 answers this question in the context of the US wind energy industry, describing the relevant features of the industry, estimating the change in production, then exploring the implications for energy markets.

4.1 A Brief Introduction to the US Wind Industry

4.1.1 Background and Motivation

Wind developers make investment and production decisions, deciding how many turbines to build and how to operate them. In the United States, investment costs average \$0.8-1.5 million per megawatt (MW) of capacity and are paid at the outset of the project (Wiser and Bolinger, 2021). These costs include turbine purchase and installation, interconnection costs, and balance of plant. For the years in our sample, the average ratio of production to capacity, called the capacity factor, was between 30-36% (Wiser and Bolinger, 2021). Wind farms sell electricity they produce for around \$40 per MWh on average and receive an additional \$25-40 per MWh in subsides. Production mainly depends on wind speed, but operation and management costs average \$7-10 per MWh (Wiser and Bolinger, 2021).[33] The five main margins affecting production, conditional on investment, are maintenance decisions, speed of repairs, subscribing to expensive forecasting and optimization programs, curtailment, and cut-in.

This capital-intensive production technology makes wind a theoretically interesting application for our model. The welfare losses from time-limits are proportional to the change in production after the subsidy period. Because wind energy generation is fixed-input intensive and wind is free, the change in production should be relatively small. In this sense, the wind industry is a limiting case in which to test the model. If wind facilities respond to the end of subsidization, the social burden of time-limited subsidies may be much more costly more elastic markets.

[33] Wiser and Bolinger (2021) report that in 2020 average costs are \$25 per kW-year—so a capacity factor between 0.3 and 0.4 implies average costs of \$7-\$9.5 per MWh. They note that these include both fixed and variable O&M costs like wages, materials for maintenance, and rent.

The wind industry has been widely subsidized as a central part of the worldwide energy transition. In the US (as in many other countries across the world) the industry receives many subsidies for both output and investment. The largest subsidy is the Renewable Energy Production Tax Credit (PTC), which since 1992 has awarded tax credits for every MWh of electricity a turbine produces from starting operation through a 10-year subsidy period. The credit is nonrefundable (Grobman and Carey, 2002; Johnston, 2019), indexed to inflation, and was $25 per MWh in 2020. Investment is typically subsidized by accelerated or bonus depreciation, worth roughly 10% of investment costs,[34] but from 2009 to 2012 new turbines could claim an investment grant (called a Section 1603 grant) worth an additional 30% of the investment costs in cash in lieu of the PTC.[35] Sub-national policies also subsidize both output and investment, such as Renewable Energy Credits in states with Renewable Portfolio Standards (see Lyon, 2016) and tax abatements on land and turbine sales.

4.1.2 Data and Sample Construction

We use administrative data about wind facilities and their investment, production, and subsidy receipt. Data on investment and production are available from a census of all utility-scale wind facilities in the United States through the Energy Information Administration (EIA). The annual EIA-860 form contains information on first date of operation, location, and investment information like the nameplate capacity (EIA, 2001-2021a). Realized production data come from the monthly EIA-923 form, which reports monthly net generation at the facility level. We calculate monthly capacity factors by dividing realized generation by the potential generation implied by capacity (EIA, 2001-2021b).[36]

Empirically we are interested in the change in production after the PTC subsidy period. Because the administrative data do not include tax filings, receiving the PTC is not directly observable. Instead we use the policy rule to determine eligibility. Specifically, we identify the first month each facility reports positive net-generation in the EIA-923 and impute subsidization from that first month until the end of the 10-year subsidy period (after the 120th month).

We also make four sample restrictions. First, we exclude firms who received the 1603 investment grant instead of the PTC from our baseline analysis (using the list from the replication data of Aldy et al., 2021). Second, because the EIA data cover production in 2001-2021, we keep firms who began producing in or after 2002 to observe their first month

[34] Although policies like accelerated and bonus depreciation do have large effects (Zwick and Mahon, 2017; Ohrn, 2018; Liu and Mao, 2019; Maffini et al., 2019), they are rarely discussed in terms of corrective policy.

[35] Aldy et al. (2023) describe the history and implications of this policy and compare it to the PTC.

[36] This drops 41 facilities with missing capacity information. We truncate all capacity factors above at 100, and impute 0 for periods with no generation data. Results are not sensitive to these specifications.

of production. Third, we drop facilities who renovated their turbines, called "repowering," during the sample period because we cannot determine their new capacity from the EIA data. To do this, we exclude firms that report repowering in the American Clean Power Association's CleanPowerIQ data (American Clean Power Association, 2020). Finally, we drop firm-month observations from the first 24 months of production because the staggered construction of turbines within a facility means that not all capacity is online in the first months of facility operation.

4.2 Measuring Production Responses after the PTC Subsidy Period

We now consider if and how much energy generation changes at wind facilities after the PTC subsidy period. In theory, reducing the after-tax revenue per MWh will incentivize less production, but it is an empirical question whether wind facilities actually respond to this incentive. On one hand, investment decisions are made only once, and firms have no control over how much the wind blows. On the other hand, firms may still be able to respond by optimizing, maintaining, or repairing their capital less effectively, by cutting in at lower speeds, by engaging in curtailment in the face of low (or negative) prices, or choosing to exit. In this subsection, we present our empirical strategy and demonstrate that facilities do decrease production after the PTC subsidy period. Appendix Table A.2 present evidence that the effect is not driven by exit or curtailment.

4.2.1 Event Study Design

To estimate the effect of output subsidies on net generation, we estimate an event study of production around the end of the PTC subsidy period. Our main outcome of interest is the capacity factor, but in the appendix, we show results with net generation, capacity, and exit. We estimate the following specification:

$$\begin{aligned} \text{Capacity Factor}_{j,t} = {} & \theta_j + \psi_{s_j,t} \\ & + \sum_{v' \in \mathcal{V}} \sum_{m' \in \mathcal{M}} \beta_{m',v'} \mathbb{1}[\text{First Month}_j \in v'] \mathbb{1}[m' = t - \text{First Month}_j] + \varepsilon_{j,t} \end{aligned} \tag{3}$$

where, capacity factor is indexed by firm j producing in state s_j during (monthly) time period t. The event study sums over vintage v (in which each v' is calendar year the facility started operation) and the month of operation m. The set $\mathcal{V}$ is partitioned into years, and the set of included event indicators is $\mathcal{M} = [\underline{m}, 60, 61, ..., 119, 121, ..., 180, \bar{m}]$. We exclude $m = 120$ (the last month of the subsidy period) as a reference period. Because there are no never-treated units we bin $m < 60$ and $m > 180$ together for a second normalization (see details in Sun and Abraham, 2021). We also include facility fixed effects θ_j and state-by-month-by-year fixed effects $\psi_{s_j,t}$. Note that the data are not a balanced panel because we only observe firms after their first month of production (i.e., $t >$ First Month$_j$).

There are four empirical considerations motivating this design. First, wind speeds vary across time and space. There are seasonal patterns (windy and slow months), annual patterns (windy and slow years), and geographic patterns (windy and slow locations). As these three dimensions of variation are correlated, naive time-period fixed effects or controls for seasonality will not capture the true heterogeneity and could leave spurious residual correlations between the event indicators and the error term. To account for this, we estimate the model with state-by-month-by-year fixed effects.[37]

Second, heterogeneity in the effects by vintage may bias the effects of a naive event study estimator. A rich literature on event study estimation has documented the importance of allowing for heterogeneous effects by treatment cohort (see Callaway and Sant'Anna, 2021; Sun and Abraham, 2021; Wooldridge, 2021). As wind production technology has been improving over time (Covert and Sweeney, 2022) and performance degradation varies by vintage (Hamilton et al., 2020), this is potentially a first-order concern. We use the estimator proposed by Sun and Abraham (2021) and estimate each event coefficient separately by vintage. Following Sun and Abraham (2021), we report vintage-weighted averages of the heterogeneous effects:

$$\beta_m = \sum_{v'} \omega_{m,v'} \beta_{m,v'}$$

where $\omega_{m,v}$ is the share of firms that entered in v' among those who produce for m months.

Third, PTC eligibility is not directly observed. PTC eligibility occurs at the turbine level, but our data are only available at the facility level, introducing "fuzziness" in the defined treatment. This could happen in two ways. Because of staggered construction, turbines that are completed after the first month of facility generation will still be subsidized after the 120th month of facility production. Furthermore, when tax filing dates do not line up with the month of first reported generation, some observations before the 120th observed month may be unsubsidized and some observations after may be subsidized.

Because each of these measurement imperfections will attenuate our estimated effects

[37] Using facility-by-month and month-by-year fixed effects produces similar results, but ignoring this heterogeneity produces noisier estimates.

near the end of the subsidy period, we report both short- and long-term effects:

$$\beta_{\text{short}} = \sum_{m'=121}^{144} \sum_{v'} \omega_{m',v'} \beta_{m,v'} \qquad \beta_{\text{long}} = \sum_{m'=145}^{180} \sum_{v'} \omega_{m',v'} \beta_{m',v'}$$

where the $\omega_{m,v}$ weights are now the unconditional share of firms with (m', v') among the short- or long-run period. Here β_{short} represents the average change in production in the two years immediately after the subsidy period. To the extent to which staggered turbine completion attenuates the effects, this will be limited to the short-run effect since all turbines seem to be completed by the end of year two. Our estimate of β_{long} will not be biased from the staggered turbine completion, but may still be attenuated if some facilities enter the unsubsidized period before their 120th month of reported production because of tax filing reasons.

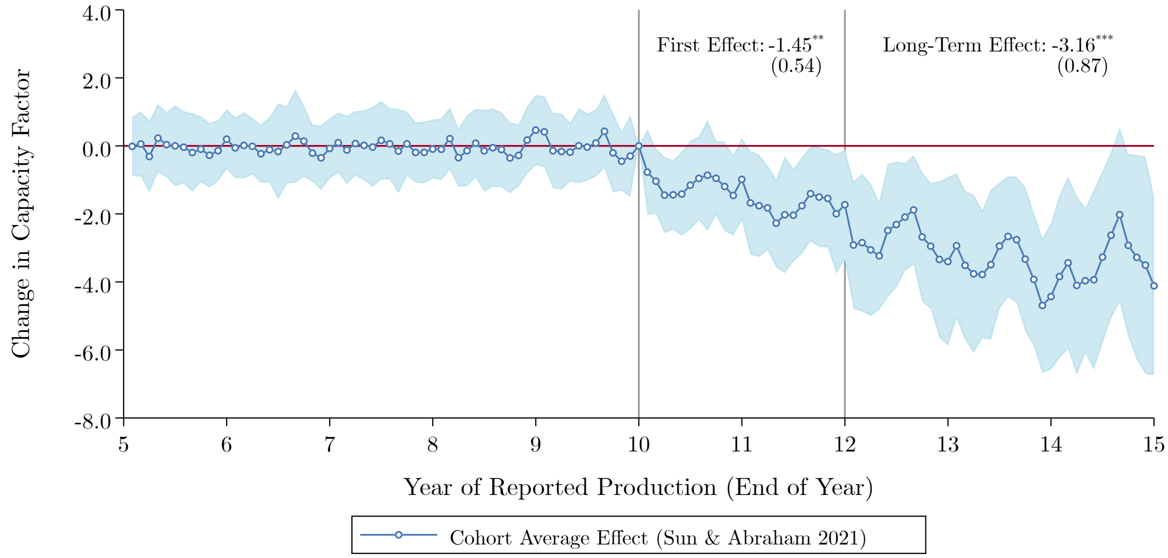
The fourth concern is that event study estimates will capture any acceleration in depreciation over time. Interestingly, other research has shown that the end of the PTC subsidy period is actually a determinate of heterogeneous degradation in wind turbine generation (Hamilton et al., 2020). In the presence of accelerating depreciation, we would expect to see a downward sloping pretrend that accelerates approaching the end of the subsidy period. Interestingly, whereas this pattern is visible in other countries that don't have the PTC, it is absent in the US (see discussion in Hamilton et al., 2020). This insight suggests that we should interpret changes in depreciation as part of the long-term treatment effect on production after the PTC subsidy period.[38]

4.2.2 Wind Facilities Reduce Production After the PTC Subsidy Period

Figure 3 presents our event study results. This figure presents the vintage-weighted average event estimates for each month of production, β_m, relative to the end of the subsidy period. The shaded area behind the series of estimates are two-way clustered, 95% pointwise confidence intervals computed using the delta method, clustering by facility and month-of-year. Looking at the patterns in these estimates, we see that the capacity factor essentially remains constant up through the end of the subsidy period. The average capacity factor in year 10 is 31.3%. After the subsidy period, production jumps down by 1 percentage point and begins sloping downward. In the first two years after the end of the PTC, the average decrease in the capacity factor is 1.5 percentage points (4.6%); this effect grows and stabilizes to 3.2 percentage points (10.1%) in the years thereafter.

[38]Technically, our results could still be driven by accelerating depreciation if the depreciation process happens to be nonlinear or discontinuous at the 10-year mark. This seems unlikely.

Figure 3: Production Decreases After When Production Tax Credit Subsidization Ends



Note: This graph shows event-study estimates of the change in production after the PTC subsidy period. The sample are 65,861 non-singleton, facility-month observations from 2002-2021, including 763 firms, 307 of which produced for more than 10 years. The series present the vintage-weighted average of the event coefficients from Equation 3, with the first and long-run effects reported as well. Standard errors and pointwise 95% confidence intervals are computed with the delta method with two-way clustering by facility and month-of-year. The average capacity factor in the tenth year of production is 31.3.

The first identifying assumption required to interpret these estimates as causal effects is parallel trends in baseline outcomes (Sun and Abraham, 2021). In our setting, this means subsidized production should evolve in parallel across firms over time absent the PTC time-limit. This assumption is met if the state-by-month-by-year fixed effects reflect the counterfactual changes in production had facilities still been subsidized. With the identifying variation for the event indicators 121-180 coming from relatively older-vintage facilities, the younger facilities act as “control” units to identify the geo-temporal fixed effects. The possibility of heterogeneous responses to seasonality by vintage is why we measure output in capacity factor and not MWh.[39] Although we cannot test this parallel trends assumption directly, Appendix Table A.2 presents additional confirmatory evidence from a placebo exercise showing that production also does not decrease in years 11-12 for 1603-Grant firms who did not receive the PTC.

The second identifying assumption is that there is no treatment effect in pre-treatment periods (Sun and Abraham, 2021). As discussed, tax filing issues could end subsidization for

[39]Furthermore, models with facility-by-month-of-year fixed effects are noisier but also suggest that this is not a problem. As discussed above, evidence suggests that subsidized production is indeed parallel despite the potential for degradation (Hamilton et al., 2020).

some turbines before month 120, violating this assumption. As the end of the subsidy period nears, firms that make strategic maintenance decisions or changes their capacity may also create anticipatory treatment effects.[40] Fortunately, the average level in the pre-treatment periods is very close to zero and does not drop until the subsidy period ends, suggesting that neither noise between tax filing and reported generation, nor anticipation significantly biases our effects.

In addition to concerns about identification and internal validity, consider three important points about external validity. First, production reductions after the subsidy period are an intensive- rather than an extensive-margin responses. Appendix Table A.2 shows that although both capacity factor and net generation decrease after the PTC subsidy period, the change in the probability of exit, measured by zero-generation, is almost zero and statistically insignificant [$p = 0.83$].

Second, our short-run effects are much smaller than the long-run effects. Because the event-study estimates are weighted by cohorts, the effects in periods farther from the end of the subsidy period (e.g., months 168-180) are identified off of firms from earlier vintages (e.g., 2002-2007).[41] In this case, the estimated effect in month 180 may not generalize to firms from later vintages, and the difference in long-term and short-term effects may be driven by composition rather than dynamics. To assess this concern, we estimate our event-study separately for three terciles of vintage: 2002-2006, 2007-2008, and 2009-2011. When we compare the short-term effects, the effects on the oldest and newest vintages are almost the same and there are no statistical differences between any group (see Appendix Table A.2).[42]

A third concern about external validity is that the reduction in production occurs because energy markets occasionally face negative prices. In very windy hours, turbines may generate more energy than needed but will want to still produce in order to capture the PTC, driving prices below zero. Although Aldy et al. (2023) document that curtailment accounts for at most one third of the difference in production between facilities that receive the PTC and 1603 investment grant, we also consider it in our data. We estimate our event-study separately for facilities that sold electricity above and below median average wholesale prices in their tenth year of operation to account for the possibility that firms selling to lower-price markets may be more likely to face negative prices. We find similar reductions in production for both groups (see Appendix Table A.2), suggesting that negative prices and curtailment

[40] Under the 80/20 rule investments that are updated at a cost of more than 80% of the original investment cost re-qualify for another 10 year of the PTC. This is another reason why we drop firms that report repowering.

[41] This is why the standard errors grow larger in Figure 3 the farther the series progresses to the right.

[42] The fact that the effect in the 2007-2008 tercile is smaller seems to be driven by a handful of firms that began production in 2008 actually producing more after the subsidy period. We conjecture that this is due to one or two repowering decision not observed in our data.

do not limit the interpretation of our results.

Considering this evidence, we conclude that wind facilities reduce production by 5-10% after the PTC subsidy period and that this represents a causal response to the marginal incentives to produce. Given the important role of fixed inputs like turbines, some readers may find it striking that there is any response at all. It is important to note that the end of the subsidy period reduces prices by 30%, so the implied elasticity is still quite small (about 0.1-0.25). Our event-study estimates are also smaller than the differences in Aldy et al. (2023) who show that in a subset of large facilities, those receiving the PTC produce 10-12% more than those receiving the 1603 investment grant. They point out and discuss the important margins of endogenous decisions about maintenance, repairs, forecasting, and optimization, concluding that effects even larger than ours could be very realistic.[43]

4.3 Implications for Energy Markets and Welfare

We now turn to the implications of the PTC time limit for energy markets. Wind facilities are a quickly growing feature of US energy markets, and by 2025 over 71,000 MW of wind capacity will have aged out of the PTC subsidy period. In this subsection we quantify how production changes from the PTC time limit will affect total wind energy production.

We quantify the dynamic production response attributable to PTC ineligibility with a simple extrapolation exercise using the event study estimates. For each month that firms produce after the subsidy period, we assume that their average net generation would have been lower by 733 MWh in the first two years after the subsidy period and by 1405 MWh in subsequent years. If anything, this will underestimate the total effect if the loss of the PTC leads to continued degradation beyond the 5-year window over which we estimate effects (as suggested by Hamilton et al., 2020) and because newer firms have larger name plate capacity (we are using MWh estimates rather than capacity factor estimates to be conservative). We estimate the cumulative effect to date and also project the effect on the existing fleet forward in time through the year 2045, assuming a capital life of 25 years.[44]

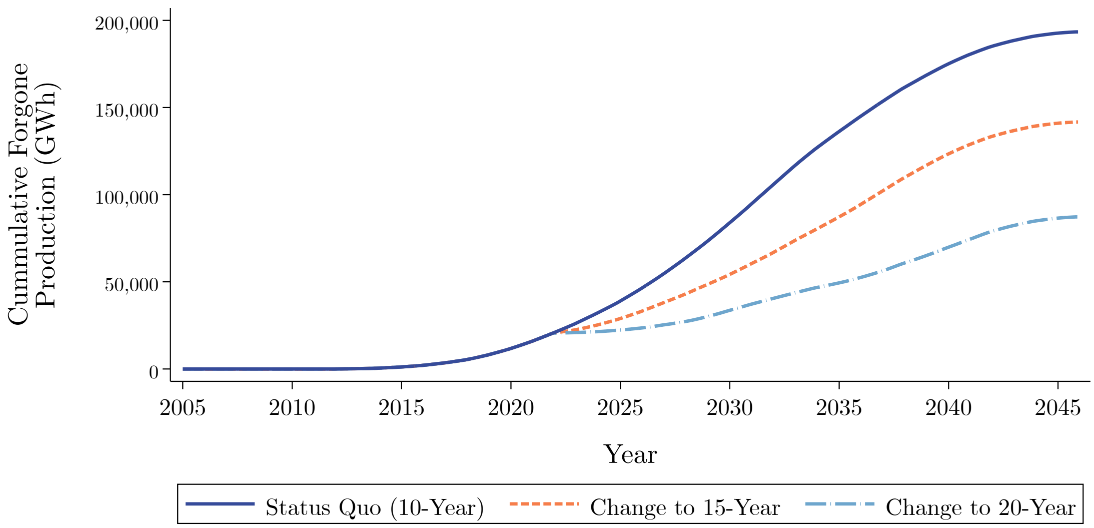
We compare the effects of the existing policy with two counterfactual policies extending the duration of the PTC subsidy period. For these counterfactuals, we estimate the energy production that would be forgone if in January 2022 the United States had extended the PTC to either 15 or 20 years. To be conservative, we assume that firms who "requalify" for the PTC after this policy return to full production and experience the short-term effects again when the policy expires rather than resuming where they had been in the dynamics.

[43] Another endogenous mechanism could be strategically choosing cut-in speeds because, as one reader pointed out, wear and tear may be more closely related to hours of operation than to MWh produced.

[44] Which, if too short, would also lead us to understate the total forgone energy.

If there are persistent effects from the degradation that firms allow to occur after the end of subsidy eligibility, it will lead us to underestimate the social value of extending the subsidized period.

Figure 4: Forgone Clean Energy Production from PTC Ineligibility



Note: This figure shows the energy production that was lost from PTC ineligibility and projections for the amount of forgone energy resulting from different possible changes to the PTC for the existing fleet of wind facilities. To calculate these estimates, we apply the short- and long-term effects on net generation to each month and sum up the total effects. For the counter-factual policies we assume the same responses as estimated at the ten-year time limit, even though this is likely an underestimate of the true effect. Note that these estimates only capture the production lost along the intensive margin for the existing fleet, not for firm entry and investment decisions as new capacity comes online.

Figure 4 shows that the amount of forgone energy is increasing rapidly and will continue to do so. By December 2021, energy markets were forgoing over 420,000 MWh/month of energy produced by wind. This corresponds to the power used by over 470,000 homes [45] and a social externality value between \$14 and \$55 million per month[46]—figures that will more than double by 2030 under the current policy. By the end of 2045, when the last of the current fleet will retire, the energy market will have forgone over 190,000 GWh of clean energy from wind—enough energy to power every home in the US for over 18 months.

Figure 4 also shows how extending the PTC subsidy duration could reduce the amount of forgone energy. Lengthening the PTC subsidy duration would reduce the amount of forgone energy and would strongly reduce the rate at which that amount is increasing. Our estimates suggest that increasing the PTC time limit by 40% (20%) of the capital life could cut forgone production by more than 55% (25%).

[45]Using the EIA's estimate that the average household uses 0.893 MWh EIA (2022)

[46]See Appendix D for details on these calculations.

While these implications for markets are striking, we want to make two notes of caution about extrapolating from these production-based results to analyses of welfare. First, although the marginal externality per MWh is likely bigger than the current cost of the PTC, a longer subsidy duration would transfer surplus to firms because most of their production is inframarginal. If there are concerns about the marginal cost of public funds, then extending the PTC may be a poor use of tax revenue. Second, although we can quantify the externality today and the amount of production forgone in the next 20 years, we consider it unlikely that the external value of a MWh of wind will stay constant over this period. As the US transitions to cleaner energy, the pollution offset by wind energy should decrease, as will the social cost of forgone production.

4.4 Subsidies for the Energy Transitions

To conclude our main empirical application, we consider how our combined theoretical and empirical results relate to energy policy more broadly. Since countries across the world subsidize alternative energy with time-limited output subsidies, our results inform the design of energy transition policies. We show that optimal policies must account for production reductions after a subsidy period, and extend time limits (if feasible given frictions) or subsidize investment in addition to output (if extending time limits is infeasible).

With the complementarity of investment and output subsidies in mind, our theory also suggests that policies requiring firms to choose between output and investment subsidies may be less effective than allowing firms to claim both. Choosing between subsidies is common in energy settings. For example, producers of wind and geothermal energy have had to choose between the Production Tax Credit and an Investment Tax Credit. But in other industries both investment and output are simultaneously subsidized (e.g., low-income housing, healthcare, or research and development).

Finally, with regard to the PTC in particular, we conduct some inverse optimum exercises to consider the model primitives that would justify the current policy in our model context. These analyses are in Appendix D and show that the current subsidy regime is only optimal under three (somewhat restrictive) conditions. First, to justify the \$25 PTC, the social cost of carbon must be lower than estimates from recent research (40-70% lower). Second, for the 10-year time limit to be optimal, extending the PTC by one year must cost society at least \$350 million in administrative costs or other institutional frictions. Finally, to only subsidize investment with bonus depreciation, either the externality must have shrunk by a factor of 4 during the subsidy period or the total product of fixed inputs must be small. If these assumptions seem unlikely to be satisfied, there may be room to improve these policies.

5. Time Limits in Other Applications

This section discusses the relevance of our results for optimal policy in other contexts and empirical applications. Corrective subsidy instruments vary immensely both across and even within industries. For example, policies targeting production or consumption include sin and excise taxes; market price supports for agriculture, manufacturing, and energy; and tax credits such as the PTC and electric vehicle tax credit. Some policies subsidize variable inputs such as R&D, labor, and fuel, and even more policies target investment such as property and sales taxes or abatements; accelerated and bonus depreciation; loan guarantees and sub-market rates; and direct investment grants or tax credits (like the affordable housing, chip manufacturing, and investment tax credits in the US).

In this section, we consider two additional empirical examples. We consider policy uncertainty through a repeal of Danish sin taxes on sugary drinks (Schmacker and Smed, 2023) and subsidies in industrial policy using the US Electric Vehicle Tax Credit (replicating Lohawala, 2023). We then step back to discuss insights for corrective policy in general.

5.1 Application to Optimal Sin Taxes

Whereas the majority of this paper has focused on subsidies, the repeal of Danish soda taxes illustrates how the insights from our paper apply to taxes as well. For years Denmark taxed soft drinks, but the tax was repealed in 2013 and phased out by the end of the year (see discussion in Schmacker and Smed, 2020, 2023).[47] This example allows us to apply our model to negative externalities and to consider policies with uncertain durations. Given the need to influence both investment and output when the policy duration is limited or uncertain, this setting highlights the welfare implications of only taxing production (or consumption) when policy may change before the end of firms' capital life.

The changes in soda consumption demonstrate the policy relevance of using output and investment subsidies as complements. Panel (a) of Figure 5 shows that sales increase by about 25% after the tax is fully repealed,[48] implying a tax rate elasticity of roughly 1. Connecting this back to the theory, this response suggests that producers who expect a sin tax to be repealed will over-invest in "sin making" capital. As such, a social planner who can't fully commit to a permanent tax on a negative externality good should tax *both* investment and output.

[47] The tax rate was increased from 1.08 DKK to 1.58 DKK per liter at the beginning of 2012. Then the repeal cut the rate to 0.82 DKK in July 2013, completely eliminating it by January 2014.

[48] This figure is adapted from Figure 2 of Schmacker and Smed (2020) and Figure 2 (a) of Schmacker and Smed (2023), reporting percentage changes rather than levels for comparability.

Despite the causal rigor of the elasticities reported above, there are additional considerations that may complicate policy inferences surrounding optimal time-limited taxation. First, in a policy environment where the persistence of a production tax is uncertain, an investment tax may face a similar fate in practice. While this consideration is moot in the simple one-cohort version of the model in Section 2, it points out the important role continued taxation has in the more general model in Section 3. The soda repeal in Denmark suggests a similar pattern. The rate hikes in 2012 seem to have generated the equilibrium changes resulting in the tax's eventual repeal. In like fashion, a policymaker who optimally implements a carbon-intensive investment tax could spark political opposition against both the investment tax and the output tax precisely because the combined policies correct the negative externality more stringently.

5.2 Application to Industrial Policy

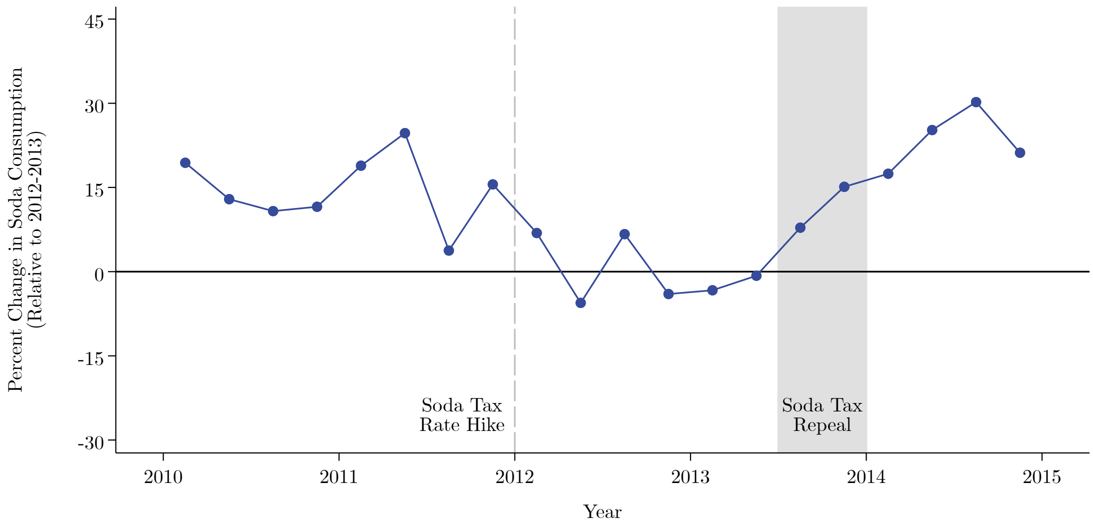
The United States Electric Vehicle Tax Credit is a large industrial policy subsidy. Prior to the Inflation Reduction Act, it remitted a $7,500 credit to buyers of new electric vehicles. This subsidy was time-limited, however. As discussed in detail in Lohawala (2023), each manufacturer faced a quota—after which the subsidy phased out. Time limits in this setting could have large welfare effects if auto manufacturers have elastic production responses. Additionally, this industry also represents one where there may be both an environmental and network externalities.

Sales of the Chevy Volt drop after the subsidy phases out, demonstrating the policy relevance of Propositions 4 and 7. Panel (b) of Figure 5 displays the 50% reduction in sales,[49] implying a large elasticity (greater than 2). In fact production of the Chevy Volt halted just months after the phase out began. According to our theoretical results, larger Δq requires larger subsidy duration (and smaller investment subsidies). As such, the responses in Panel (b) of Figure 5 suggest that the time-limited EV subsidies have real welfare costs.

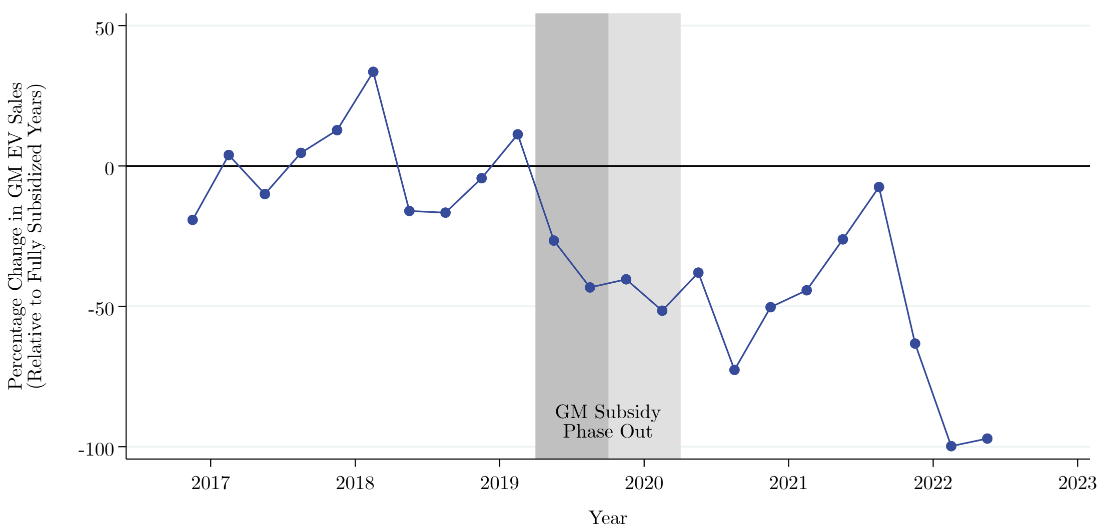
While these patterns are suggestive, they are less rigorously estimated than our main empirical example. In the case of auto manufacturing, Figure 5 may or may not reflect causal effects. Manufactures could strategically time sales during the subsidy period, and because cars are durable goods, changes in sales at the end of the subsidy period might also depress demand afterward. As such, the true elasticity may be smaller than 2; nevertheless, the reduction in sales and eventual exit of the Chevy Volt suggest that supply is quite elastic and that the subsidy period is a major determinant of sales quantities. These responses highlight the potential costs of subsidies with short durations in industrial policy settings.

[49]This figure is adapted directly from Appendix Figure 7 of Lohawala (2023), again reported in percent changes.

Figure 5: Markets Respond to Other Tax and Subsidy Time Limits



(a) Sweet Beverage Consumption in Denmark



(b) Electric Vehicle Production in United States

Notes: This figure shows the changes in production and consumption in two additional industries, recast in percent changes for comparability. Following Schmacker and Smed (2023) Panel (a) looks at the market for sweetened beverages in Denmark where sin taxes were hiked in 2012 and then gradually repealed in 2013. Percent changes in quarterly consumption are plotted relative to the 18 months after the rate hike (and before the phase out and repeal). Following Lohawala (2023) Panel (b) looks at the market for electric vehicles when the subsidy period for GM ended in 2019. Percent changes in monthly sales are plotted relative to the months before subsidization ended.

6. Conclusion

This paper characterizes the optimal policy implications of limited or uncertain subsidy duration in corrective taxation. We show the importance of investment subsidies when a ("Pigouvian") output subsidy with no time limit is infeasible and demonstrate that changes in production after the subsidy period inform the optimal subsidy duration. We also document a 5-10% decrease in wind energy production after the PTC subsidy period, quantify the implications of the PTC duration on energy markets, and discuss the implications of time-limited policy for energy transition goals, sin taxation, and industrial policy. We now conclude by considering our results in the context of future research.

Our research documents new complementarities between subsidizing investment and output that arise due to time limits. Because of the dynamic frictions binding time limits create, the efficient policy subsidizes output to correct the externality during the subsidy period and subsidizes investment to correct the externality afterward. We hope future research will continue to explore other settings where complementary policy instruments can be used to correct for unintended frictions created by features of commonly enacted policies. Furthermore, as policy uncertainty, network externalities, budget concerns, and firm heterogeneity can also justify the use of multiple subsidy instruments, we hope that future research will also continue to explore other complementarities between policy instruments for designing optimal policy in real-world, second-best settings.

Our results also reveal how separability and targeting-like results apply even under imperfect targeting. Even though time-limited output subsidies impede externality targeting, investment subsidies separably correct for the subsidy duration, restoring a Pigouvian-like output subsidy. Perhaps even more strikingly, administrative costs, firm heterogeneity, or budget concerns may make it optimal to choose a policy with less perfect targeting to address these other concerns. These insights could be extended to other settings such as multi-dimensional tagging, tax-systems aware income taxation, and behavioral public finance.

Finally, we demonstrate the quantitative importance of time-limited subsidization across a broad array of industrial and policy settings. Due to the ubiquity of time-limited and uncertain policies, we hope many more empirical papers explore the causal effects of subsidy ineligibility or repeal. Applying such insights can have major policy implications. For example, our results suggest that a full transition to clean energy would require at least 10% more capacity than expected because of production reductions after the subsidy period. It may also be fruitful to expand structural models of markets, dynamic climate and economic models, and optimal tax calibrations to include our insights about time limits and investment subsidies (or taxes) in response to time limits and policy uncertainty.

Although our results apply under a wide array of settings, other considerations could enrich them. For example, in addition to policy uncertainty, price uncertainty could generate additional complications (e.g., see Yi et al., 2018). Since a key benefit of price-controlling policies like feed in tariffs is eliminating price uncertainty, extending our analyses to non-traditional corrective policy could also be fascinating. Similarly, as real-world subsidies likely subsidize non-price-taking firms, the optimal policies might change when considered in tandem with correcting market power (e.g., see Dubois et al., 2020; O'Connell and Smith, 2021). Finally, research exploring the dynamic implications of subsidization for both investment and entry (as in Langer and Lemoine, 2022) may also be valuable.

While not related to future research, our results also rationalize many real-world policies. Although certain policies may seem inefficient from a naive Pigouvian perspective—like the presence of time-limited subsidies and the coexistence of output and investment subsidies in the same industry—we show time limits and investment subsidies are rational responses to institutional frictions, budget concerns, and policy uncertainty. In this light, our paper also suggests a hopeful message that policy makers may be making much more efficient decisions than a naive economic criticism would suggest.

On the whole, this paper underscores the real stakes of implementing time-limited subsidies. By connecting output and investment subsidies in a framework of time-limited subsidies, we articulate the crucial role investment subsidies play despite the intuition we inherit from the first-best. These insights can inform subsidy design in an era of increasing attention to industrial and energy policy and of increasing uncertainty about policy in general.

References

Abrell, Jan, Sebastian Rausch, and Clemens Streitberger, "The economics of renewable energy support," *Journal of Public Economics*, 2019, *176*, 94–117.

Acemoglu, Daron, Philippe Aghion, Leonardo Bursztyn, and David Hemous, "The environment and directed technical change," *American economic review*, 2012, *102* (1), 131–166.

_ , _ , Lint Barrage, and David Hémous, "Climate change, directed innovation, and energy transition: The long-run consequences of the shale gas revolution," Technical Report, National Bureau of Economic Research 2023.

Aldy, Joseph E, Todd D Gerarden, and Richard L Sweeney, "Investment versus output subsidies: Implications of alternative incentives for wind energy (NBER WP Version)," Technical Report, NBER Working Papers 2019.

_ , _ , and _ , "Investment versus output subsidies: Implications of alternative incentives for wind energy - Replication Data," Working Paper 2021.

_ , _ , and _ , "Investment versus output subsidies: Implications of alternative incentives for wind energy," *Journal of the Association of Environmental and Resource Economics*, 2023.

Alesina, Alberto and Guido Tabellini, "A positive theory of fiscal deficits and government debt," *The review of economic studies*, 1990, *57* (3), 403–414.

American Clean Power Association, "CleanPowerIQ," Technical Report 2020. https://cleanpoweriq.cleanpower.org/ (Accessed 2020).

Baker, Scott R, Nicholas Bloom, and Steven J Davis, "Measuring economic policy uncertainty," *The quarterly journal of economics*, 2016, *131* (4), 1593–1636.

Battaglini, Marco and Bård Harstad, "The political economy of weak treaties," *Journal of Political Economy*, 2020, *128* (2), 544–590.

Best, Michael Carlos, Anne Brockmeyer, Henrik Jacobsen Kleven, Johannes Spinnewijn, and Mazhar Waseem, "Production versus Revenue Efficiency with Limited Tax Capacity: Theory and Evidence from Pakistan," *Journal of Political Economy*, December 2015, *123* (6), 1311–1355.

Bierbrauer, Felix J, Pierre C Boyer, and Andreas Peichl, "Politically feasible reforms of nonlinear tax systems," *American Economic Review*, 2021, *111* (1), 153–91.

Blanchette, Jason G, Craig S Ross, and Timothy S Naimi, "The rise and fall of alcohol excise taxes in US states, 1933–2018," *Journal of studies on alcohol and drugs*, 2020, *81* (3), 331–338.

Bollinger, Bryan, Kenneth Gillingham, A Justin Kirkpatrick, and Steven Sexton, "Visibility and peer influence in durable good adoption," *Marketing Science*, 2022, *41* (3), 453–476.

Bovenberg, Ary Lans and Frederick van der Ploeg, "Environmental policy, public finance and the labour market in a second-best world," *Journal of Public Economics*, 1994, *55* (3), 349–390.

Burr, Chrystie, "Subsidies and Investments in the Solar Power Market," Working Paper November 2016.

Cai, Yongyang and Thomas S Lontzek, "The Social Cost of Carbon with Economic and Climate Risks," *Journal of Political Economy*, 2019, *127*, 2684–2734.

Callaway, Brantly and Pedro HC Sant'Anna, "Difference-in-differences with multiple time periods," *Journal of Econometrics*, 2021, *225* (2), 200–230.

Chen, Luming, "The Dynamic Efficiency of Policy Uncertainty: Evidence from Wind Industry," Working Paper 2023.

Costa, Francisco and François Gerard, "Hysteresis and the welfare effect of corrective policies: Theory and evidence from an energy-saving program," *Journal of Political Economy*, 2021, *129* (6), 1705–1743.

Covert, Thomas R and Richard L Sweeney, "Winds of change: Estimating learning by doing without cost or input data," Technical Report, Working paper 2022.

Craig, Ashley C and Joel Slemrod, "Tax knowledge and tax manipulation: A unifying model," Technical Report, National Bureau of Economic Research 2022.

Cremer, Helmuth, Firouz Gahvari, and Norbert Ladoux, "Externalities and optimal taxation," *Journal of Public Economics*, 1998, *70* (3), 343–364.

Cullen, Joseph, "Measuring the environmental benefits of wind-generated electricity," *American Economic Journal: Economic Policy*, 2013, *5* (4), 107–33.

Dayton, Leigh, "Australia scraps carbon tax: Becomes first country in the world to turn back on tool to curtail emissions," *Science: ScienceInsider*, July 2014.

De Groote, Olivier and Frank Verboven, "Subsidies and time discounting in new technology adoption: Evidence from solar photovoltaic systems," *American Economic Review*, 2019, *109* (6), 2137–72.

Dharmapala, Dhammika, Joel Slemrod, and John Douglas Wilson, "Tax policy and the missing middle: Optimal tax remittance with firm-level administrative costs," *Journal of Public Economics*, 2011, *95* (9-10), 1036–1047.

Diamond, Peter A, "Consumption externalities and corrective imperfect pricing," *The Bell Journal of Economics*, 1973, *4* (2), 526–538.

_ **and James A Mirrlees**, "Optimal taxation and public production I: Production efficiency," *American Economic Review*, 1971, *61* (1), 8–27.

Dixit, Avinash, "Tax policy in open economies," in "Handbook of Public Economics," Vol. 1, Elsevier, 1985, pp. 313–374.

Doraszelski, Ulrich and Mark Satterthwaite, "Computable Markov-perfect industry dynamics," *The RAND Journal of Economics*, 2010, *41* (2), 215–243.

Dubois, Pierre, Rachel Griffith, and Martin O'Connell, "How well targeted are soda taxes?," *American Economic Review*, 2020, *110* (11), 3661–3704.

Dunne, Timothy, Shawn D Klimek, Mark J Roberts, and Daniel Yi Xu, "Entry, exit, and the determinants of market structure," *The RAND Journal of Economics*, 2013, *44* (3), 462–487.

Emran, M. Shahe and Joseph E. Stiglitz, "On selective indirect tax reform in developing countries," *Journal of Public Economics*, April 2005, *89* (4), 599–623.

Farhi, Emmanuel and Xavier Gabaix, "Optimal Taxation with Behavioral Agents," Technical Report, National Bureau of Economic Research 2019.

Farrell, Joseph and Paul Klemperer, "Coordination and lock-in: Competition with switching costs and network effects," *Handbook of industrial organization*, 2007, *3*, 1967–2072.

Fell, Harrison, Daniel T Kaffine, and Kevin Novan, "Emissions, transmission, and the environmental value of renewable energy," *American Economic Journal: Economic Policy*, 2021, *13* (2), 241–72.

Ganapati, Sharat, Joseph S Shapiro, and Reed Walker, "Energy cost pass-through in US manufacturing: Estimates and implications for carbon taxes," *American Economic Journal: Applied Economics*, 2020, *12* (2), 303–42.

German Federal Ministry of Finance, "28th Subsidy Report: 2019–2022," Website 2022.

Griffith, Rachel, Martin O'Connell, and Kate Smith, "Tax design in the alcohol market," *Journal of Public Economics*, 2019, *172*, 20–35.

Grobman, Jeffrey H and Janis M Carey, "The effect of policy uncertainty on wind-power investment," *The Journal of Energy and Development*, 2002, *28* (1), 1–14.

Grummon, Anna H, Benjamin B Lockwood, Dmitry Taubinsky, and Hunt Allcott, "Designing better sugary drink taxes," *Science*, 2019, *365* (6457), 989–990.

Hamilton, Sofia D, Dev Millstein, Mark Bolinger, Ryan Wiser, and Seongeun Jeong, "How does wind project performance change with age in the United States?," *Joule*, 2020, *4* (5), 1004–1020.

Handley, Kyle and J Frank Li, "Measuring the effects of firm uncertainty on economic activity: New evidence from one million documents," Technical Report, National Bureau of Economic Research 2020.

Helm, Carsten and Mathias Mier, "Steering the energy transition in a world of intermittent electricity supply: Optimal subsidies and taxes for renewables and storage," *Journal of Environmental Economics and Management*, 2021, *109*, 102497.

Holland, Stephen P, Erin T Mansur, Nicholas Z Muller, and Andrew J Yates, "Are there environmental benefits from driving electric vehicles? The importance of local factors," *American Economic Review*, 2016, *106* (12), 3700–3729.

Holland, Stephen P., Erin T. Mansur, Nicholas Z. Muller, and Andrew J. Yates, "Decompositions and Policy Consequences of an Extraordinary Decline in Air Pollution from Electricity Generation," *American Economic Journal: Economic Policy*, November 2020, *12* (4), 244–274.

Holland, Stephen P, Matthew J Kotchen, Erin T Mansur, and Andrew J Yates, "Why marginal CO2 emissions are not decreasing for US electricity: Estimates and implications for climate policy," *Proceedings of the National Academy of Sciences*, 2022, *119* (8), e2116632119.

Hollingsworth, Alex and Ivan Rudik, "External impacts of local energy policy: The case of renewable portfolio standards," *Journal of the Association of Environmental and Resource Economists*, 2019, *6* (1), 187–213.

Ito, Koichiro and James M Sallee, "The economics of attribute-based regulation: Theory and evidence from fuel economy standards," *Review of Economics and Statistics*, 2018, *100* (2), 319–336.

Jacobs, Bas, "The marginal cost of public funds is one at the optimal tax system," *International Tax and Public Finance*, 2018, *25* (4), 883–912.

Jacobsen, Mark R, Christopher R Knittel, James M Sallee, and Arthur A Van Benthem, "The use of regression statistics to analyze imperfect pricing policies," *Journal of Political Economy*, 2020, *128* (5), 1826–1876.

Johnston, Sarah, "Nonrefundable tax credits versus grants: The impact of subsidy form on the effectiveness of subsidies for renewable energy," *Journal of the Association of Environmental and Resource Economists*, 2019, *6* (3), 433–460.

Keen, Michael and Joel Slemrod, "Optimal tax administration," *Journal of Public Economics*, 2017, *152*, 133–142.

Kellogg, Ryan, "The effect of uncertainty on investment: evidence from Texas oil drilling," *American Economic Review*, 2014, *104* (6), 1698–1734.

_ , "Output and attribute-based carbon regulation under uncertainty," *Journal of Public Economics*, 2020, *190*, 104246.

Kopczuk, Wojciech, "A note on optimal taxation in the presence of externalities," *Economics Letters*, 2003, *80* (1), 81–86.

Langer, Ashley and Derek Lemoine, "Designing dynamic subsidies to spur adoption of new technologies," *Journal of the Association of Environmental and Resource Economists*, 2022, *9* (6), 1197–1234.

— , **Vikram Maheshri, and Clifford Winston**, "From gallons to miles: A disaggregate analysis of automobile travel and externality taxes," *Journal of Public Economics*, 2017, *152*, 34–46.

Liu, Yongzheng and Jie Mao, "How do tax incentives affect investment and productivity? Firm-level evidence from China," *American Economic Journal: Economic Policy*, 2019, *11* (3), 261–91.

Lohawala, Nafisa, "Roadblock or accelerator? The effect of electric vehicle subsidy elimination," Working Paper 2023.

Lyon, Thomas P, "Drivers and impacts of renewable portfolio standards," *Annual Review of Resource Economics*, 2016, *8*, 141–155.

Maffini, Giorgia, Jing Xing, and Michael P Devereux, "The impact of investment incentives: evidence from UK corporation tax returns," *American Economic Journal: Economic Policy*, 2019, *11* (3), 361–89.

Mcdonald, Garrett, "Oilseeds and Products Update: The People's Republic of China," Technical Report, United States Department of Agriculture 2022.

Miravete, Eugenio J, Katja Seim, and Jeff Thurk, "One markup to rule them all: Taxation by liquor pricing regulation," *American Economic Journal: Microeconomics*, 2020, *12* (1), 1–41.

Nyberg, Per, Trond Thorvaldsen, and Jan Greni, "The Power of Nature: Taxation of Wind Power - 2022 A Country Overview," Technical Report, KPMG Law Advokatfirma 2020.

O'Connell, Martin and Kate Smith, "Optimal sin taxation and market power," Technical Report, IFS Working Paper 2021.

Ohrn, Eric, "The effect of corporate taxation on investment and financial policy: Evidence from the DPAD," *American Economic Journal: Economic Policy*, 2018, *10* (2), 272–301.

Organization for Economic Co-Operation and Development (OECD), "Renewable Energy Feed-in-tariffs," Technical Report, Organization for Economic Co-Operation and Development 2022.

Parish, Ross M and Keith Robert McLaren, "Relative cost-effectiveness of input and output subsidies," *Australian Journal of Agricultural Economics*, 1982, *26* (1), 1–13.

Persson, Torsten and Lars EO Svensson, "Why a stubborn conservative would run a deficit: Policy with time-inconsistent preferences," *The Quarterly Journal of Economics*, 1989, *104* (2), 325–345.

Petersen, Claire, Mar Reguant, and Lola Segura, "Measuring the Impact of Wind Power and Intermittency," Working Paper 2022.

Pigou, Arthur C, *The economics of welfare*, MacMillan and Co - London, United Kingdom, 1920.

Raymond, Leigh, "Carbon pricing and economic populism: the case of Ontario," *Climate Policy*, 2020, *20* (9), 1127–1140.

Rennert, Kevin, Frank Errickson, Brian C. Prest, Lisa Rennels, Richard G. Newell, William Pizer, Cora Kingdon, Jordan Wingenroth, Roger Cooke, Bryan Parthum, David Smith, Kevin Cromar, Delavane Diaz, Frances C. Moore, Ulrich K. Müller, Richard J. Plevin, Adrian E. Raftery, Hana Ševčíková, Hannah Sheets, James H. Stock, Tammy Tan, Mark Watson, Tony E. Wong, and David Anthoff, "Comprehensive evidence implies a higher social cost of CO2," *Nature*, October 2022, *610* (7933), 687–692.

Rothschild, Casey and Florian Scheuer, "Optimal taxation with rent-seeking," *The Review of Economic Studies*, 2016, *83* (3), 1225–1262.

Sandmo, Agnar, "Optimal taxation in the presence of externalities," *The Swedish Journal of Economics*, 1975, pp. 86–98.

Scheuer, Florian and Alexander Wolitzky, "Capital taxation under political constraints," *American Economic Review*, 2016, *106* (8), 2304–28.

_ **and Iván Werning**, "Mirrlees meets diamond-mirrlees," Technical Report, National Bureau of Economic Research 2016.

Schmacker, Renke and Sinne Smed, "Do prices and purchases respond similarly to soft drink tax increases and cuts?," *Economics & Human Biology*, 2020, *37*, 100864.

_ **and** _ , "Sin taxes and self-control," *American Economic Journal: Economic Policy*, 2023, *15* (3), 1–34.

Schmalensee, Richard, "Evaluating policies to increase electricity generation from renewable energy," *Review of Environmental Economics and Policy*, 2012.

Seto, Karen C, Steven J Davis, Ronald B Mitchell, Eleanor C Stokes, Gregory Unruh, and Diana Ürge-Vorsatz, "Carbon lock-in: types, causes, and policy implications," *Annual Review of Environment and Resources*, 2016, *41*, 425–452.

Sexton, Steven, A Justin Kirkpatrick, Robert I Harris, and Nicholas Z Muller, "Heterogeneous solar capacity benefits, appropriability, and the costs of suboptimal siting," *Journal of the Association of Environmental and Resource Economists*, 2021, *8* (6), 1209–1244.

Sun, Liyang and Sarah Abraham, "Estimating dynamic treatment effects in event studies with heterogeneous treatment effects," *Journal of Econometrics*, 2021, *225* (2), 175–199.

The Economist, "Chicago's soda tax is repealed," Technical Report, The Economist July 2017.

The White House (White House), "Building a Clean Energy Economy: A Guidebook to the Inflation Reduction Act's Investments in Clean Energy Climate Action," Technical Report 1 December 2022.

United States Congressional Research Service (CRS), "Federal Excise Taxes on Alcoholic Beverages: A Summary of Present Law and a Brief History," Website 1999.

_ , "U.S. Dairy Programs After the 2014 Farm Bill (P.L. 113-79)," Technical Report, Congressional Research Service October 2014.

United States Department of the Treasury (TREAS), "2021 Instructions for Form 8835 Renewable Electricity, Refined Coal, and Indian Coal Production Credit," Website 2021.

United States Energy Information Administration (EIA), "Form EIA 860: 2001-2021," Data 2001-2021. https://www.eia.gov/electricity/data/eia860/ (Accessed 2022).

_ , "Form EIA 906/923," Data 2001-2021. https://www.eia.gov/electricity/data/eia923/ (Accessed 2022).

_ , "How much electricity does an American home use?," Website 2022. https://www.eia.gov/tools/faqs/faq.php?id=97&t=3 (Accessed 2023).

_ , "Renewable generation surpassed coal and nuclear in the U.S. electric power sector in 2022," Website 2023. https://www.eia.gov/todayinenergy/detail.php?id=61107.

United States Environmental Protection Agency (EPA), "Greenhouse gases equivalencies calculator - Calculations and references," Website 2022.

Urban Institute, "Soda Taxes," Technical Report, Urban Institute 2023.

Wang, Mengyu, Jeffrey Wurgler, and Hong Zhang, "Policy Uncertainty Reduces Green Investment," *Available at SSRN 4474726*, 2023.

Wessel, David, "What is reconciliation in Congress?," Website 2021.

Wiser, Ryan and Mark Bolinger, "Land-Based Wind Market Report: 2021 Edition," Technical Report, Lawrence Berkeley National Laboratory 2021.

Wooldridge, Jeffrey M, "Two-way fixed effects, the two-way mundlak regression, and difference-in-differences estimators," *Available at SSRN 3906345*, 2021.

Yi, Fujin, C-YC Lin Lawell, and K Thome, "A dynamic model of subsidies: Theory and application to ethanol industry," Working Paper 2018.

Zwick, Eric and James Mahon, "Tax policy and heterogeneous investment behavior," *American Economic Review*, 2017, *107* (1), 217–48.

A. Appendix Tables and Figures - For Online Publication

Table A.1: US Policies with Limited and Uncertain Durations

Panel A: Policies with Limited Durations

Policy	Industry	Duration
Renewable Energy Production Tax Credit*	Energy	10 years/firm
* supersceeded in Clean Electricity Production Credit		
Advanced Manufacturing Production Tax Credit	Energy	7 years
Residential Clean Energy Credit	Energy	13 years
Carbon Oxide Sequestration Credit	Energy	12 years
Clean Vehicle Credit	Transportation	10 years
Sustainable Aviation Fuel Credit	Transportation	3 years
Credit for New Energy-Efficient Homes	Construction	10 years

Panel B: Policies with Uncertain Durations

Policy	Industry	Duration Before Repeal
Excise Whiskey Tax of 1791*	Commercial	11 years
* later policies levied, raised, and lowered rates regularly		
West Virginia Soda Tax	Commercial	73 years
Chicago Soda Tax	Commercial	4 months
Marihuana [sic] Tax	Agriculture	32 years
Margin Protection Program - Dairy*	Agriculture	4 years
* rolled into Dairy Margin Coverage		

Note: This table gives some examples of current and former US tax and subsidy programs that have time limits or were repealed. Note that the capital life of most of these investments is 20-40 years. Consider wind turbines (20-30 years), manufacturing plants (5-15 years), furnaces or water heaters (15-30 years), new homes (50-70) years, etc. for examples.

Table A.2: Estimates of Changes in Production after the Subsidy Period

Panel A: Main Effects	Capacity Factor	Net Generation (MWh)	Exit: $\mathbb{1}$(Net Generation = 0)
Overall Effect	-2.32	-1072	0.00
	(0.67)	(388)	(0.01)
Short-Term (Years 11-12)	-1.45	-733	0.00
	(0.54)	(352)	(0.00)
Long-Term (Years 13-15)	-3.16	-1405	0.00
	(0.87)	(492)	(0.01)
Average in Year 10	31.3	16,858	0.02

Panel B: Heterogeneity by Vintage	2002-2006	2007-2008	2009-2010
Short-Term (Years 11-12)	-1.63	-0.54	-1.20
	(1.12)	(0.61)	(0.63)
Average in Year 10	32.4	32.0	29.1

Panel C: Effect Heterogeneity	1603 Firms (Placebo)	Low Price	High Price
Overall Effect	-	-2.37	-2.32
		(1.03)	(0.51)
Short-Term (Years 11-12)	-0.33	-1.97	-1.09
	(0.44)	(0.87)	(0.40)
Long-Term (Years 13-15)	-	-2.73	-3.65
		(1.23)	(0.77)
Average in Year 10	28.2	32.0	30.7

Note: This table reports estimates from event study analyses of the change in production after the ten-year PTC subsidy duration. All estimates are weighted averages of event-coefficients relative to the end of the subsidy period. Panel A reports the main results across three different measures of production, the capacity factor, net generation, and an indicator for whether there was zero production in a given month (a measure of exit). Panel B reports the differences in short-term effects between older and newer facilities (began production in 2002-2006 versus 2007-2008 versus 2009-2011). Panel C reports placebo and heterogeneity tests, including wind firms that elected to receive the 1603 investment grant and were therefore not eligible for a PTC, and separately by firms who receive lower and higher average wholesale prices. For all regressions standard errors are two-way cluster corrected for arbitrary variance-covariance structure at the facility level and month-of-year level. All regressions control for facility and state-by-month-by-year fixed effects.

B. Proofs for the Optimal Tax Model - For Online Publication

B.1 Simplified Model

Assumption 1. Assume (1) $\lambda = 1$ (2) that $q(x, v)$ is increasing in both arguments with decreasing returns such that there exists an interior solution (x^f, v_1^f, v_2^f); and (3) that the firm choices (x^f, v_1^f, v_2^f) are implicit functions of the policy parameters (τ^i, τ^o, T) such that all first order conditions are continuously differentiable with respect to all arguments and produce a matrix $F = (f_x, f_{v_1}, f_{v_2}) = 0$ with a non-singular Jacobian with respect to x and v_t.

In order to prove the main results in Propositions 1-3, we first prove a helpful Lemma.

Lemma 1. Under assumption 1, the marginal increase in the firm's variable input (v_2^f) with respect to a marginal change in a policy parameter is equal to the marginal increase in the capital input (x^f) scaled by the ratio of the second derivatives of the production function.

$$\frac{\partial v_2^f}{\partial \tau^i} = -\frac{q_{xv}(x^f, v_2^f)}{q_{vv}(x^f, v_2^f)}\frac{\partial x}{\partial \tau^i}$$
$$\frac{\partial v_2^f}{\partial \tau^o} = -\frac{q_{xv}(x^f, v_2^f)}{q_{vv}(x^f, v_2^f)}\frac{\partial x}{\partial \tau^o}$$
$$\frac{\partial v_2^f}{\partial T} = -\frac{q_{xv}(x^f, v_2^f)}{q_{vv}(x^f, v_2^f)}\frac{\partial x}{\partial T}$$

Proof. The firm's first order condition for v_2 is given by

$$q_v(x^f, v_2^f) = m$$

Totally differentiating with respect to τ^i, τ^o and T respectively proves the lemma. □

Proof. Proof of Proposition 1, 2, and 3

The optimal investment and output subsidies for a given T are derived from the first order conditions from Equation 2. Taking the derivatives and setting $\lambda = 1$, τ^{o*} and τ^{i*} are defined by the following equations:

$$\tau^{o*} = \frac{\gamma \frac{\mathrm{d}Q}{\mathrm{d}\tau^o}}{T \frac{\mathrm{d}q(x^f, v_1^f)}{\mathrm{d}\tau^o}} - \tau^i \frac{\frac{\partial x^f}{\partial \tau^o}}{T \frac{\mathrm{d}q(x^f, v_1^f)}{\mathrm{d}\tau^o}} \tag{4}$$

$$\tau^{i*} = \frac{\gamma \frac{\mathrm{d}Q}{\mathrm{d}\tau^i}}{\frac{\partial x^f}{\partial \tau^i}} - \tau^o \frac{T \frac{\mathrm{d}q(x^f, v_1^f)}{\mathrm{d}\tau^i}}{\frac{\partial x^f}{\partial \tau^i}} \tag{5}$$

Setting $\tau^o = 0$ and $T = 0$ in Equation 5 and simplifying proves Proposition 1. Setting $\tau^i = 0$ in Equation 4 and simplifying proves Proposition 2.

To prove Proposition 3, substitute 4 into 5 and rearrange for τ^i:

$$\tau^{i*} = \gamma \left(\frac{\frac{\mathrm{d}Q}{\mathrm{d}\tau^i} \frac{\mathrm{d}q(x^f, v_1^f)}{\mathrm{d}\tau^o} - \frac{\mathrm{d}Q}{\mathrm{d}\tau^o} \frac{\mathrm{d}q(x^f, v_1^f)}{\mathrm{d}\tau^i}}{\frac{\partial x^f}{\partial \tau^i} \frac{\mathrm{d}q(x^f, v_1^f)}{\mathrm{d}\tau^o} - \frac{\partial x^f}{\partial \tau^o} \frac{\mathrm{d}q(x^f, v_1^f)}{\mathrm{d}\tau^i}} \right)$$

Using Lemma 1 to simplify further leads to the following final expression:

$$\tau^{i*} = \gamma \frac{(1-T) \frac{\mathrm{d}q(v_2^f)}{\mathrm{d}\tau^i}}{\frac{\partial x}{\partial \tau^i}} \tag{6}$$

To solve for τ^{o*}, substitute Equation 6 into Equation 4:

$$\tau^{o*} = \frac{\gamma}{T} \frac{\frac{\mathrm{d}Q}{\mathrm{d}\tau^o} \frac{\partial x^f}{\partial \tau^o} - (1-T) \frac{\partial x^f}{\partial \tau^o} \frac{\mathrm{d}q(v_2^f)}{\mathrm{d}\tau^i}}{\frac{\mathrm{d}q(v_1^f)}{\mathrm{d}\tau^o} \frac{\partial x^f}{\partial \tau^i}}$$

Again expanding $\frac{\mathrm{d}Q}{\mathrm{d}\tau^o}$, cancelling terms and simplifying with Lemma 1 gives

$$\tau^{o*} = \frac{\gamma}{T} \frac{T \frac{\mathrm{d}q(v_1^f)}{\mathrm{d}\tau^o} \frac{\partial x^f}{\partial \tau^i}}{\frac{\mathrm{d}q(v_1^f)}{\mathrm{d}\tau^o} \frac{\partial x^f}{\partial \tau^i}} = \gamma$$

□

Proof. Proof of Proposition 4 Interior Solution

The optimal subsidy duration, T, is found by differentiating Equation 2 with respect to

T. Setting $\lambda = 1$, the first order condition is

$$\begin{aligned}\frac{\partial W}{\partial T} =& q(x^f, v_1^f) - q(x^f, v_2^f) - m(v_1 - v_2) + \gamma\Big(q(x^f, v_1^f) - q(x^f, v_2^f)\Big) + \\ & \gamma\Big(T\frac{\mathrm{d}q(x^f, v_1^f)}{\mathrm{d}T} + (1-T)\frac{\mathrm{d}q(x^f, v_2^f)}{\mathrm{d}T}\Big) - \frac{\partial x^f}{\partial T}\tau^{i*} - T\tau^{o*}\frac{\mathrm{d}q(x^f, v_1^f)}{\mathrm{d}T}\end{aligned}$$

Using the expressions for τ^{i*} and τ^{o*} from Proposition 3 and the result from Lemma 1 to simplify $\frac{\partial x^f}{\partial T}\tau^{i*}$ the first order condition becomes

$$\frac{\partial W}{\partial T} = -(\Delta q - m\Delta v) - \gamma\Delta q - \phi'(T)$$

Here $\Delta q = q(x^f, v_2^f) - q(x^f, v_1^f)$ is used to denote the change in output at the end of the output subsidy resulting from a change in the variable input $\Delta v = v_2 - v_1$.

For small changes in v, we can Taylor expand $q(x, v_1^f)$ around $q(x, v_2^f)$. This leads to $\Delta q = \Delta v q_v(x^f, v_2^f) = m\Delta v$. The first order condition therefore simplifies further and the optimal T is defined by

$$\phi'(T) = -\gamma\Delta q$$

$\square$

Proof. Sufficient Conditions for Uniqueness and Corner Solutions for Proposition 4

Proposition 4 provides a unique solution if $\frac{\partial^2 W}{\partial T^2} < 0 \ \forall T \in [0,1]$.

$$\frac{\partial^2 W}{\partial T^2} = -\gamma\frac{\partial \Delta q}{\partial T} - \phi''(T)$$

By assumption ϕ is convex so $\frac{\partial \Delta q}{\partial T} \geq 0$ is a sufficient condition for a unique solution.

$$\begin{aligned}\frac{\partial \Delta q}{\partial T} =& \frac{\partial q(x, v_2)}{\partial T} - \frac{\partial q(x, v_1)}{\partial T} = q_x(x, v_2)\frac{\partial x}{\partial T} + q_v(x, v_2)\frac{\partial v_2}{\partial T} - \Big(q_x(x, v_1)\frac{\partial x}{\partial T} + q_v(x, v_1)\frac{\partial v_1}{\partial T}\Big) \\ =& \Big(q_x(x, v_2) - q_x(x, v_1)\Big)\frac{\partial x}{\partial T} + q_v(x, v_2)\frac{\partial v_2}{\partial T} - q_v(x, v_1)\frac{\partial v_1}{\partial T}\end{aligned}$$

For small changes in v we can use the following Taylor expansions of $q_v(x^f, v_1^f)$ and

$q_x(x^f, v_1^f)$ around (x^f, v_2^f):

$$\begin{aligned} q_v(x^f, v_1^f) &\approx q_v(x^f, v_2^f) - \Delta v\, q_{vv}(x^f, v_2^f) \\ q_x(x^f, v_1^f) &\approx q_x(x^f, v_2^f) - \Delta v\, q_{xv}(x^f, v_2^f) \end{aligned}$$

Using the firm's first order conditions and the Taylor expansion of q_v, we find that

$$\Delta v = \frac{m\tau^o}{q_{vv}(x^f, v_2^f)(1+\tau^o)}$$

Therefore, if q_v is locally linear then Δv does not depend on T and $\frac{\partial v_1}{\partial T} = \frac{\partial v_2}{\partial T}$. Using the implicit definitions of v_1 and v_2, as well as the Taylor expansion of $q_x(x, v_1)$, the expression simplifies to

$$\frac{\partial \Delta q}{\partial T} = q_{xv}(x^f, v_2^f)\Delta v \frac{\partial x}{\partial T} + \frac{m\tau^o}{1+\tau^o}\frac{\partial v_2}{\partial T}$$

Substituting in the expression for Δv and using Lemma 1, the above expression cancels out and we are left with $\frac{\partial \Delta q}{\partial T} = 0$. We are then left with $\frac{\partial^2 W}{\partial T^2} = -\phi''(T)$ which is negative for all T and therefore the solution in Proposition 4 is unique. □

B.2 Generalized Model

Proof of Propositions 5 and 6

Before beginning proofs, it is helpful to extend the exposition in Section 3 with additional definitions.

Definition 1. Let $\mathcal{J} = \lim_{t\to\infty} \mathcal{J}_t$ be the set of all firms and the distribution of $\mathcal{J}$ be $F(j)$.

Additionally, define (present-value weighted) expectation and covariance operators across three domains (all firms, all subsidized production space, and all unsubsidized production space) as follows:

Definition 2.

$$\begin{aligned} \mathbb{E}_0[g(\cdot)] &= \int_{\mathcal{J}} \frac{e^{-\beta s_j}}{N} g(\cdot)\,\mathrm{d}\,F(j) \\ \mathbb{E}_1[g(\cdot)] &= \int_{\mathcal{J}} \int_{s_j}^{T+\kappa} \frac{e^{-\beta t} g(\cdot)}{N_1}\,\mathrm{d}\,t\,\mathrm{d}\,F(j) \\ \mathbb{E}_2[g(\cdot)] &= \int_{\mathcal{J}} \int_{T+\kappa}^{\infty} \frac{e^{-\beta t} g(\cdot)}{N_2}\,\mathrm{d}\,t\,\mathrm{d}\,F(j) \end{aligned}$$

$$Cov_0(X,Y) = \mathbb{E}_0[(X - \mathbb{E}_0[X])(Y - \mathbb{E}_0[Y])]$$
$$Cov_1(X,Y) = \mathbb{E}_1[(X - \mathbb{E}_1[X])(Y - \mathbb{E}_1[Y])]$$
$$Cov_2(X,Y) = \mathbb{E}_2[(X - \mathbb{E}_2[X])(Y - \mathbb{E}_2[Y])]$$

$$N = \int_{\mathcal{J}} e^{-\beta s_j} dF(j)$$
$$N_1 = N\mathbb{E}_0[\frac{1 - e^{-\beta(T+\kappa-s_j)}}{\beta}]$$
$$N_2 = N\mathbb{E}_0[\frac{e^{-\beta(T+\kappa-s_j)}}{\beta}]$$

Lemma 2. Firm's Problem The firm's initial capital X_j and variable input at time t ($v_{j,t}$) are defined by

$$\begin{aligned} c_{s_j} - \tau^i = &\int_s^{T+\kappa} e^{-\beta(t-s)} \left[(p_t + \tau^o)\frac{\partial q_{j,t}(x_{j,t}, v_{j,t})}{\partial x}\delta(t-s)\,\mathrm{d}\,t + \right. \\ &\int_{T+\kappa}^{\infty} e^{-\beta(t-s)} \left[p_t \frac{\partial q_{j,t}(x_{j,t}, v_{j,t})}{\partial x}\delta(t-s)\right]\mathrm{d}\,t \end{aligned}$$

$$\begin{cases} (p_{t'} + \tau^o)\frac{\partial q_j(x_{t'}, v_{t'})}{\partial v} = m_{t'} & t' \in [s, T+\kappa] \\ p_{t'}\frac{\partial q_j(x_{t'}, v_{t'})}{\partial v} = m_{t'} & t' \in (T+\kappa, \infty). \end{cases}$$

Lemma 3. Consumer Demand In equilibrium, the representative consumer's marginal utility at time t is simply equal to the output price p_t and the Langrange multiplier on their budget constraint L_t is equal to one.

Proof. The quasi-linear representative conumser's probelm in period t is

$$\max_{q_t, w_t} u_t(q_t) + w_t + L_t(y_t - p_t q_t - w_t),$$

where y_t is their period t income and w_t is numeraire good consumption. The lemma follows from the standard solution. □

Proof. Proof of Propositions 5 and 6

Note that the welfare function can be rewritten as

$$
\begin{aligned}
&\mathcal{W}(\tau^o,\tau^i,T) = \\
&\quad \int_{\mathcal{J}}\int_{s_j}^{\infty} e^{-\beta t}\pi_{j,t}\,\mathrm{d}\,t\,\mathrm{d}\,F + \int_0^{\infty} e^{-\beta t}U_t + L_t(y_t - p_t q_t - w_t)\,\mathrm{d}\,t \\
&\quad + \int_{\mathcal{J}}\int_{s_j}^{\infty} e^{-\beta t} q_{j,t}\gamma_{j,t}\,\mathrm{d}\,t\,\mathrm{d}\,F - \int_{\mathcal{J}}\left(e^{-\beta s_j}X_j\tau^i + \tau^o\int_{s_j}^{T+\kappa} e^{-\beta t}q_{j,t}\,\mathrm{d}\,t\right)\mathrm{d}\,F - \phi(T)
\end{aligned}
$$

Differentiating, employing the envelope theorem, substituting in $L_t = 1$ to cancel $\frac{\partial p_t}{\partial \tau}$ terms, and rewriting with expectation operators gives the following first order conditions for τ^o,

$$
\begin{aligned}
&\underbrace{N_1\mathbb{E}_1[\tau^o\lambda\frac{\partial q_{j,t}}{\partial \tau^o}]}_{\text{Direct fiscal externality}} + \underbrace{N\mathbb{E}_0[\lambda\tau^i\frac{\partial X_j}{\partial \tau^o}]}_{\text{Cross fiscal externality}} \\
&\qquad = \underbrace{N_1\mathbb{E}_1[\gamma_{j,t}\frac{\partial q_{j,t}}{\partial \tau^o}] + N_2\mathbb{E}_2[\gamma_{j,t}\frac{\partial q_{j,t}}{\partial \tau^o}]}_{\text{Environmental externality benefit}} + \underbrace{N_1\mathbb{E}_1[(1-\lambda)q_{j,t}]}_{\text{Direct transfer effect}},
\end{aligned}
$$

and τ^i,

$$
\begin{aligned}
&\underbrace{N\mathbb{E}_0[\lambda\tau^i\frac{\partial X_j}{\partial \tau^i}]}_{\text{Direct fiscal externality}} + \underbrace{N_1\mathbb{E}_1[\tau^o\lambda\frac{\partial q_{j,t}}{\partial \tau^i}]}_{\text{Cross fiscal externality}} + \\
&\qquad = \underbrace{N_1\mathbb{E}_1[\gamma_{j,t}\frac{\partial q_{j,t}}{\partial \tau^i}] + N_2\mathbb{E}_2[\gamma_{j,t}\frac{\partial q_{j,t}}{\partial \tau^i}]}_{\text{Environmental externality benefit}} + \underbrace{N\mathbb{E}_0[(1-\lambda)X_j]}_{\text{Direct transfer effect}}.
\end{aligned}
$$

Combining equations and employing a firm specific version of Lemma 1, we find the optimal expressions are

$$
\begin{aligned}
\tau^{i*} =& \frac{1-\tilde{T}_\kappa}{\beta}\frac{\overline{\gamma}_2\ \mathbb{E}_2\left[\frac{\mathrm{d}q_{j,t}}{\mathrm{d}X_j}\right]}{\lambda} \\
&+ \frac{\tilde{T}_\kappa}{\beta}\Omega_{\frac{\gamma}{\lambda},\frac{\partial q1}{\partial \tau^i}} + \frac{1-\tilde{T}_\kappa}{\beta}\Omega_{\frac{\gamma}{\lambda}\frac{\mathrm{d}q2}{\mathrm{d}X},\frac{\partial X}{\partial \tau^i}} + \frac{1-\lambda}{\lambda}\Psi_{\tau^i} \\
\tau^{o*} =& \frac{\overline{\gamma}_1}{\lambda} + \Omega_{\frac{\gamma}{\lambda},\frac{\partial q1}{\partial \tau^o}} + \frac{1-\tilde{T}_\kappa}{\tilde{T}_\kappa}\Omega_{\frac{\gamma}{\lambda}\frac{\mathrm{d}q2}{\mathrm{d}X},\frac{\partial X}{\partial \tau^o}} + \frac{1-\lambda}{\lambda}\Psi_{\tau^o}
\end{aligned}
$$

where

$$
\begin{aligned}
\Omega_{\frac{\gamma}{\lambda},\frac{\partial q1}{\partial \tau^i}} =& \frac{Cov_1(\frac{\gamma_{j,t}}{\lambda},\frac{\partial q_{j,t}}{\partial \tau^i}) - \eta_{q1} Cov_1(\frac{\gamma_{j,t}}{\lambda},\frac{\partial q_{j,t}}{\partial \tau^o})}{\mathbb{E}_0[\frac{\partial X_j}{\partial \tau^i} - \eta_{q1}\frac{\partial X_j}{\partial \tau^o}]} \\
\Omega_{\frac{\gamma}{\lambda},\frac{\partial q1}{\partial \tau^o}} =& \frac{Cov_1(\frac{\gamma_{j,t}}{\lambda},\frac{\partial q_{j,t}}{\partial \tau^o}) - \eta_X Cov_1(\gamma_{j,t},\frac{\partial q_{j,t}}{\partial \tau^i})}{\mathbb{E}_1[\frac{\partial q_{j,t}}{\partial \tau^o} - \eta_X\frac{\partial q_{j,t}}{\partial \tau^i}]} \\
\Omega_{\frac{\gamma}{\lambda}\frac{\mathrm{d}q2}{\mathrm{d}X},\frac{\partial X}{\partial \tau^i}} =& Cov_2(\frac{\gamma_{j,t}}{\lambda},\frac{\mathrm{d}q_{j,t}}{\mathrm{d}X_j}) + \frac{Cov_2(\frac{\gamma_{j,t}}{\lambda}\frac{\mathrm{d}q2}{\mathrm{d}X_j},\frac{\partial X_j}{\partial \tau^i}) - \eta_{q1} Cov_2(\frac{\gamma_{j,t}}{\lambda}\frac{\mathrm{d}q2}{\mathrm{d}X_j},\frac{\partial X_j}{\partial \tau^o})}{\mathbb{E}_0[\frac{\partial X_j}{\partial \tau^i} - \eta_{q1}\frac{\partial X_j}{\partial \tau^o}]} \\
\Omega_{\frac{\gamma}{\lambda}\frac{\mathrm{d}q2}{\mathrm{d}X},\frac{\partial X}{\partial \tau^o}} =& \frac{Cov_2(\frac{\gamma_{j,t}}{\lambda}\frac{\mathrm{d}q2}{\mathrm{d}X_j},\frac{\partial X_j}{\partial \tau^o}) - \eta_X Cov_2(\frac{\gamma_{j,t}}{\lambda}\frac{\mathrm{d}q2}{\mathrm{d}X_j},\frac{\partial X_j}{\partial \tau^i})}{\mathbb{E}_1[\frac{\partial q_{j,t}}{\partial \tau^o} - \eta_X\frac{\partial q_{j,t}}{\partial \tau^i}]} \\
\Psi_{\tau^i} =& \frac{N\mathbb{E}_0[X_j] - \eta_{q1} N_1 \mathbb{E}[q_{j,t}]}{N\lambda\mathbb{E}_0[\frac{\partial X_j}{\partial \tau^i} - \eta_{q1}\frac{\partial X_j}{\partial \tau^o}]} \\
\Psi_{\tau^o} =& \frac{N_1\mathbb{E}_1[q_{j,t}] - \eta_X N\mathbb{E}_0[X_j]}{\lambda N_1\mathbb{E}_1[\frac{\partial q_{j,t}}{\partial \tau^o} - \eta_X\frac{\partial q_{j,t}}{\partial \tau^i}]} \\
\frac{\mathrm{d}q_{j,t}}{\mathrm{d}X_j} =& \delta(t-s)\Big(q_x(x_{j,t},v_{j,t}) - q_v(x_{j,t},v_{j,t})\frac{q_{xv}(x_{j,s,t},v_{j,s,t})}{q_{vv}(x_{j,s,t},v_{j,s,t})}\Big) \\
\eta_{q1} =& \frac{\mathbb{E}_1[\frac{\partial q_{j,t}}{\partial \tau^i}]}{\mathbb{E}_1[\frac{\partial q_{j,t}}{\partial \tau^o}]} \\
\eta_X =& \frac{\mathbb{E}_0[\frac{\partial X_j}{\partial \tau^o}]}{\mathbb{E}_0[\frac{\partial X_j}{\partial \tau^i}]} \\
\tilde{T}_\kappa =& \mathbb{E}_0[e^{-\beta(T+\kappa-s)}]
\end{aligned}
$$

When firms are homogeneous and all enter at $s_j = 0$, the externality is constant, and $\lambda = 1$, then the Ω and Ψ terms can all be ignored, proving Proposition 5. □

Proof of Proposition 7

Before proving Proposition 7, it is useful to define the present-value expectation across firms, at the moment that the firm's subsidy period ends:

Definition 3.

$$\mathbb{E}_{T^*+\kappa}[f_j(\cdot)] = \mathbb{E}_0[e^{-\beta(T+\kappa-s_j)} f_{j,T+\kappa}(\cdot)]$$

It is also necessary to define the instantaneous change in output and the variable input for firm j at the end of their subsidy period:

Definition 4.

$$\begin{aligned}\Delta q_j =& q_j(x_{j,T+\kappa}, v_{j,T+\kappa+\varepsilon}) - q_j(x_{j,T+\kappa}, v_{j,T+\kappa-\varepsilon})\\ \Delta v_j =& v_{j,T+\kappa+\varepsilon} - v_{j,T+\kappa-\varepsilon}.\end{aligned}$$

Proof. Proof of Proposition 7

Differentiate $\mathcal{W}$ with respect to T:

$$\begin{aligned}\frac{\partial \mathcal{W}}{\partial T} =& N\mathbb{E}_0[e^{-\beta(T+\kappa-s_j)}(-p_{T+\kappa}(\Delta q_j - m_{T+\kappa}\Delta v_j))] + (1-\lambda)\tau^o N\mathbb{E}_0[e^{-\beta(T+\kappa-s_j)}q_{j,T+\kappa}]\\ &+ N_1\mathbb{E}_1[\gamma_{j,t}\frac{\partial q_{j,t}}{\partial T}] - \lambda\tau^o N_1\mathbb{E}_1[\frac{\partial q_{j,t}}{\partial T}] + N_2\mathbb{E}_2[\gamma_{j,t}\frac{\partial q_{j,t}}{\partial T}] - \lambda\tau^i N\mathbb{E}_0[\frac{\partial X_j}{\partial T}]\\ &+ N\mathbb{E}_0[-e^{-\beta(T+\kappa-s_j)}\Delta q_j\gamma_{j,T+\kappa}] - \phi'(T).\end{aligned}$$

Using a first-order Taylor Approximation for the change in output for firm j after the subsidy ends, substituting in τ^{o*}, τ^{i*}, and noting that $\frac{\partial q_{j,t}}{\partial v} = m$ for $t \geq T+\kappa$ the first order condition becomes

$$\frac{\partial \mathcal{W}}{\partial T} = -\mathbb{E}_{T^*+\kappa}[\Delta q_j\gamma_j] + \Omega_{\gamma,\frac{\partial q1}{\partial T}} + \Omega_{\gamma\frac{\mathrm{d}q2}{\mathrm{d}X},\frac{\partial X}{\partial T}} + (1-\lambda)\Psi_T$$

where

$$\begin{aligned}\Omega_{\gamma,\frac{\partial q1}{\partial T}} =& N_1 Cov_1(\gamma_{j,t}, \frac{\partial q_{j,t}}{\partial T}) - \lambda N_1\mathbb{E}_1[\frac{\partial q_{j,t}}{\partial T}]\Omega_{\frac{\gamma}{\lambda},\frac{\partial q1}{\partial\tau^o}} - \lambda\frac{N\widetilde{T}_\kappa}{\beta}\mathbb{E}_0[\frac{\partial X_j}{\partial T}]\Omega_{\frac{\gamma}{\lambda},\frac{\partial q1}{\partial\tau^i}}\\ \Omega_{\gamma\frac{\mathrm{d}q2}{\mathrm{d}X},\frac{\partial X}{\partial T}} =& N_2\overline{\gamma}_2 Cov_2(\frac{\mathrm{d}q_{j,t}}{\mathrm{d}X}, \frac{\partial X_j}{\partial T}) + N_2 Cov_2(\gamma_{j,t}, \frac{\partial q_{j,t}}{\partial T})\\ &- \lambda\frac{N(1-\widetilde{T}_\kappa)}{\beta}\mathbb{E}_0[\frac{\partial X_j}{\partial T}]\Omega_{\frac{\gamma}{\lambda}\frac{\mathrm{d}q2}{\mathrm{d}X},\frac{\partial X}{\partial\tau^i}} - \lambda\frac{N_1(1-\widetilde{T}_\kappa)}{\widetilde{T}_\kappa}\mathbb{E}_1[\frac{\partial q_{j,t}}{\partial T}]\Omega_{\frac{\gamma}{\lambda}\frac{\mathrm{d}q2}{\mathrm{d}X},\frac{\partial X}{\partial\tau^o}}\\ \Psi_T =& \tau^o N\mathbb{E}_{T+\kappa^*}[q_j] - N\mathbb{E}_0[\frac{\partial X_j}{\partial T}]\Psi_{\tau^i} - \lambda N_1\mathbb{E}_1[\frac{\partial q_{j,t}}{\partial T}]\Psi_{\tau^o}.\end{aligned}$$

□

B.3 Proofs of Corollaries

Uncertainty

Lemma 4. Firm's Choice For an output subsidy with constant hazard rate and probability p of being overturned before $t = 1$, the optimal investment decision x^f, and variable inputs when the subsidy is (v_1) and is not (v_2) in place are defined by

$$
\begin{aligned}
&q_v(x^f, v_1^f) - \frac{m}{1+\tau^o} = 0 \\
&q_v(x^f, v_2^f) - m = 0 \\
&(1-p)q_x(x^f, v_1^f)(1+\tau^o) + pq_x(x^f, v_2^f) - (c - \tau^i) = 0.
\end{aligned}
$$

Proof. Treating τ^o as a random variable equals τ^o at time t with probability $1 - pt$ and zero otherwise, the firm's expected profits are

$$
\begin{aligned}
&\mathbb{E}\left[\int_0^1 [q(x, v_t)(1+\tau^o) - mv_t]dt - x(c - \tau^i)\right] \\
=&\int_0^1 [q(x, v_t) - mv_t]pt\, dt \\
&\quad + \int_0^1 [q(x, v_t)(1+\tau^o) - mv_t](1-pt)\, dt - x(c - \tau^i).
\end{aligned}
$$

The first order conditions for v_1 and v_2 give the first two equations of the lemma. Differentiating expected profits with respect to x and noting that v_1 and v_2 are constant proves the lemma. □

Proof. Proof of Corollary 1

The policymaker maximizes welfare treating v_t as a random variable equal to v_1 at time t with probability $1 - pt$ and equal to v_2 otherwise. The policymaker therefore maximizes

$$
\begin{aligned}
&\mathbb{E}\left[\int_0^1 [q(x, v_t)(1+\gamma) - mv_t]\, \mathrm{d}\, t - xc - \phi(T)\right] \\
=&\int_0^1 [(1+\gamma)q(x, v_2) - mv_2]pt\ \mathrm{d}\, t \\
&\quad + \int_0^1 [(1+\gamma)q(x, v_1) - mv_1](1-pt)\ \mathrm{d}\, t - xc - \phi(T).
\end{aligned}
$$

Noting from Lemma 4 that v_1 and v_2 are constant, the policymaker's objective function can be written as

$$
\begin{aligned}
(1-p)&\Big[q(x^f, v_1^f)(1+\tau^o) - mv_1^f\Big] + p\Big[q(x^f, v_2^f) - mv_2^f\Big] - x^f(c - \tau^i) \\
&+ \gamma\Big((1-p)q(x^f, v_1^f) + pq(x^f, v_2^f)\Big) - \lambda\Big(\tau^i x^f + (1-p)\tau^o q(x^f, v_1^f)\Big) - \phi(T).
\end{aligned}
$$

This model is therefore isomorphic to the model defined in Section 2, if $p = (1 - T)$ — thus proving Corollary 1. $\square$

Network Effects

Definition 5. Let the total prior investment in period t be $\mathcal{X}_t = \int_0^t \int_{\mathcal{J}_t} X_j \,\mathrm{d}\, F(j)$.

Proof. Proof of Corollary 2. When investment costs, production, demand, and the externalities are allowed to be functions of $\mathcal{X}_t$, the social planner's problem is

$$\max_{\tau^o,\tau^i,T} \mathcal{W}(\tau^o,\tau^i,T) = \max_{\tau^o,\tau^i,T} \int_0^\infty e^{-\beta t}\left[U_t(\mathcal{X}_t) + \int_{\mathcal{J}_t} \pi_{j,t} + \gamma^o_{j,t}(\mathcal{X}_t) q_{j,t}(\mathcal{X}_t) + \lambda T C_{j,t} \,\mathrm{d}\, F(j)\right] \mathrm{d}\, t - \phi(T).$$

The first order conditions are

$$\begin{aligned}\frac{\partial \mathcal{W}}{\partial \tau^o} = \int_0^\infty e^{-\beta t} \Bigg\{ &\int_{\mathcal{J}_t} (\gamma^o_{j,t} - \tau^o)\frac{\partial q_{j,t}}{\partial \tau^o} \,\mathrm{d}\, F(j) \\ &+ \frac{\partial \mathcal{X}_t}{\partial \tau^o}\left[p_t \frac{\partial U}{\partial \mathcal{X}} + \int_{\mathcal{J}_t} \frac{\partial \gamma^o}{\partial \mathcal{X}} q_{j,t} + \gamma^o_{j,t}\frac{\mathrm{d} q_{j,t}}{\mathrm{d}\mathcal{X}} - X_j \frac{\partial c}{\partial \mathcal{X}} \,\mathrm{d}\, F(j)\right]\Bigg\} \mathrm{d}\, t \\ &- \tau^i \int_{\mathcal{J}} \frac{\partial X_j}{\partial \tau^o} \,\mathrm{d}\, F(j) = 0\end{aligned}$$

$$\begin{aligned}\frac{\partial \mathcal{W}}{\partial \tau^i} = \int_0^\infty e^{-\beta t} \Bigg\{ &\int_{\mathcal{J}_t} (\gamma^o_{j,t} - \tau^o)\frac{\partial q_{j,t}}{\partial \tau^i} \,\mathrm{d}\, F(j) \\ &+ \frac{\partial \mathcal{X}_t}{\partial \tau^i}\left[p_t \frac{\partial U}{\partial \mathcal{X}} + \int_{\mathcal{J}_t} \frac{\partial \gamma^o}{\partial \mathcal{X}} q_{j,t} + \gamma^o_{j,t}\frac{\mathrm{d} q_{j,t}}{\mathrm{d}\mathcal{X}} - X_j \frac{\partial c}{\partial \mathcal{X}} \,\mathrm{d}\, F(j)\right]\Bigg\} \mathrm{d}\, t \\ &- \tau^i \int_{\mathcal{J}} \frac{\partial X_j}{\partial \tau^i} \,\mathrm{d}\, F(j) = 0\end{aligned}$$

$\square$

Then solving for τ and combining yields

$$
\begin{aligned}
\tau^{o*} &= \frac{\int_0^\infty e^{-\beta t}\left(\int_{\mathcal{J}_t}\gamma_{j,t}(\frac{\partial q_{j,t}}{\partial \tau^o}-\eta_x\frac{\partial q_{j,t}}{\partial \tau^i})\,\mathrm{d}\,F(j)\right)\mathrm{d}\,t}{\int_0^\infty e^{-\beta t}\int_{\mathcal{J}_t}\frac{\partial q_{j,t}}{\partial \tau^o}-\eta_x\frac{\partial q_{j,t}}{\partial \tau^i}\,\mathrm{d}\,F(j)\,\mathrm{d}\,t}\\
&+\frac{\int_0^\infty e^{-\beta t}\left(\left[p_t\frac{\partial U}{\partial \mathcal{X}}+\int_{\mathcal{J}_t}\frac{\partial \gamma^o}{\partial \mathcal{X}}q_{j,t}+\gamma^o_{j,t}\frac{\mathrm{d}q_{j,t}}{\mathrm{d}\mathcal{X}}-X_j\frac{\partial c}{\partial \mathcal{X}}\,\mathrm{d}\,F(j)\right](\frac{\partial \mathcal{X}_t}{\partial \tau^o}-\eta_x\frac{\partial \mathcal{X}_t}{\partial \tau^i})\right)\mathrm{d}\,t}{\int_0^\infty e^{-\beta t}\int_{\mathcal{J}_t}\frac{\partial q_{j,t}}{\partial \tau^o}-\eta_x\frac{\partial q_{j,t}}{\partial \tau^i}\,\mathrm{d}\,F(j)\,\mathrm{d}\,t}\\
&\equiv \tau^o_6+\omega_{\mathcal{X}}\overline{\gamma_{\mathcal{X}}}+\Omega_{\gamma_{\mathcal{X}},\frac{\partial \mathcal{X}}{\partial \tau^o}}\\
\tau^{i*} &= \frac{\int_0^\infty e^{-\beta t}\left(\int_{\mathcal{J}_t}\gamma_{j,t}(\frac{\partial q_{j,t}}{\partial \tau^i}-\eta_{q_1}\frac{\partial q_{j,t}}{\partial \tau^o})\,\mathrm{d}\,F(j)\right)}{\int_0^\infty e^{-\beta t}\int_{\mathcal{J}_t}\frac{\partial X_j}{\partial \tau^i}-\eta_{q_1}\frac{\partial X_j}{\partial \tau^o}\,\mathrm{d}\,F(j)\,\mathrm{d}\,t}\\
&+\frac{\int_0^\infty e^{-\beta t}\left(\int_{\mathcal{J}_t}\left[p_t\frac{\partial U}{\partial \mathcal{X}}+\int_{\mathcal{J}_t}\frac{\partial \gamma^o}{\partial \mathcal{X}}q_{j,t}+\gamma^o_{j,t}\frac{\mathrm{d}q_{j,t}}{\mathrm{d}\mathcal{X}}-X_j\frac{\partial c}{\partial \mathcal{X}}\,\mathrm{d}\,F(j)\right](\frac{\partial \mathcal{X}_t}{\partial \tau^i}-\eta_{q_1}\frac{\partial \mathcal{X}_t}{\partial \tau^o})\right)\mathrm{d}\,t}{\int_0^\infty e^{-\beta t}\int_{\mathcal{J}_t}\frac{\partial X_j}{\partial \tau^i}-\eta_{q_1}\frac{\partial X_j}{\partial \tau^o}\,\mathrm{d}\,F(j)\,\mathrm{d}\,t}\\
&\equiv \tau^i_6+\omega^i_{\mathcal{X}}\overline{\gamma_{\mathcal{X}}}+\Omega_{\gamma_{\mathcal{X}},\frac{\partial \mathcal{X}}{\partial \tau^i}}
\end{aligned}
$$

where $\gamma_{\mathcal{X}} = p_t\frac{\partial U}{\partial \mathcal{X}}+\int_{\mathcal{J}_t}\frac{\partial \gamma^o}{\partial \mathcal{X}}q_{j,t}+\gamma^o_{j,t}\frac{\mathrm{d}q_{j,t}}{\mathrm{d}\mathcal{X}}-X_j\frac{\partial c}{\partial \mathcal{X}}\,\mathrm{d}\,F(j)$ and $\omega^o_{\mathcal{X}} = \frac{\mathbb{E}[\frac{\partial \mathcal{X}_t}{\partial \tau^o}-\eta_x\frac{\partial \mathcal{X}_t}{\partial \tau^i}]}{\mathbb{E}[\frac{\partial q_{j,t}}{\partial \tau^o}-\eta_x\frac{\partial q_{j,t}}{\partial \tau^i}]}$ and $\omega^i_{\mathcal{X}} = \frac{\mathbb{E}[\frac{\partial \mathcal{X}_t}{\partial \tau^i}-\eta_{q_1}\frac{\partial \mathcal{X}_t}{\partial \tau^o}]}{\mathbb{E}[\frac{\partial X_j}{\partial \tau^i}-\eta_{q_1}\frac{\partial X_j}{\partial \tau^o}]}$ each reflect the relative effectiveness of increasing early investment with an output or investment subsidy.

Variable Input Subsidy

Consider an expanded policy space where during the subsidy period, the policy maker is now able to subsidize the variable input, v_t with a variable input subsidy denoted by τ^n. The firm's profit maximization problem is now given by

$$
\begin{aligned}
\max_{X,\{v_t\}}\int_s^{T+\kappa} & e^{-\beta t}\left[(p_t+\tau^o)q(x_t,v_t)-(m_t-\tau^n)v_t\right]\mathrm{d}\,t\\
&+\int_{T+\kappa}^\infty e^{-\beta t}\left[p_t q(x_t,v_t)-m_t v_t\right]\mathrm{d}\,t-X(c_s-\tau^i).
\end{aligned}
$$

The total fiscal costs of subsidizing firm j are now $TC_j = \tau^i X_j+\int_s^{T+\kappa}e^{-\beta t}\tau^o q_{j,t}+\tau^n v_{j,t}\,\mathrm{d}\,t$ but the welfare function is otherwise unchanged.

Corollary 3. For a cohort of homogeneous firms (described in Proposition 5) with a constant externality value, the optimal variable input subsidy will be zero if the output and investment subsidies are set optimally.

Proof. Denote welfare evaluated at the optimal output and investment subsidy as $\mathcal{W}(\tau^{o*}, \tau^{i*})$. The optimal variable input subsidy is characterized by the following first order condition:

$$\begin{aligned}\frac{\partial \mathcal{W}(\tau^{o*}, \tau^{i*})}{\partial \tau^n} &= \int_0^T e^{-\beta t} v_t \,\mathrm{d}\, t + \int_0^\infty e^{-\beta t} \gamma \frac{\partial q_t}{\partial \tau^n} \,\mathrm{d}\, t \\ &\quad - \Big[\frac{\partial X}{\partial \tau^n}\tau^{i*} + \int_0^T e^{-\beta t}\left(\tau^{o*}\frac{\partial q_t}{\partial \tau^n} + v_t + \tau^n \frac{\partial v_t}{\partial \tau^n}\right)\mathrm{d}\, t\Big] \\ &= \int_0^T e^{-\beta t}(\gamma - \tau^{o*})\frac{\partial q_t}{\partial \tau^n}\,\mathrm{d}\, t + \int_T^\infty e^{-\beta t}\gamma \frac{\mathrm{d} q_t}{\mathrm{d}\tau^n}\,\mathrm{d}\, t - \tau^{i*}\frac{\partial X}{\partial \tau^n} + \tau^n \int_0^T e^{-\beta t}\frac{\partial v_t}{\partial \tau^n}\,\mathrm{d}\, t = 0.\end{aligned}$$

Noting that $\tau^{o*} = \gamma$ and $\tau^{i*} = \int_T^\infty e^{-\beta t}\gamma \frac{\mathrm{d}q}{\mathrm{d}x}\,\mathrm{d}\, t$ and applying a input-related version of Lemma 1 reveals that the first order condition becomes

$$\tau^n \int_0^T e^{-\beta t}\frac{\partial v_t}{\partial \tau^n}\,\mathrm{d}\, t = 0.$$

Noting that $\frac{\partial v_t}{\partial \tau^n} > 0$, the first order condition is satisfied if and only if $\tau^n = 0$.

□

Changing Output Subsidies

Corollary 4. An output subsidy that can be differentiated across firms and over time (before the end of the subsidy period) should equal the marginal externality value plus a constant value to account for the relatively effectiveness of the output subsidy to target the period 2 production.

Proof. A time and firm varying output subsidy and investment subsidy together must satisfy the following first order conditions (for $\lambda = 1$):

$$\begin{aligned}N_1\mathbb{E}_1[(\tau^o_{j,t} - \gamma_{j,t})\frac{\partial q_{j,t}}{\partial \tau^o}] + N\mathbb{E}_0[\lambda\tau^i\frac{\partial X_j}{\partial \tau^o}] &= N_2\mathbb{E}_2[\gamma_{j,t}\frac{\partial q_{j,t}}{\partial \tau^o}] \\ N\mathbb{E}_0[\lambda\tau^i\frac{\partial X_j}{\partial \tau^i}] + N_1\mathbb{E}_1[(\tau^o_{j,t} - \gamma_{j,t})\frac{\partial q_{j,t}}{\partial \tau^i}] &= N_2\mathbb{E}_2[\gamma_{j,t}\frac{\partial q_{j,t}}{\partial \tau^i}].\end{aligned}$$

If $\tau^o_{j,t} = \gamma_{j,t} + \bar{\tau^o}$ then the first order conditions become

$$\bar{\tau^o} N_1\mathbb{E}_1[\frac{\partial q_{j,t}}{\partial \tau^o}] + N\mathbb{E}_0[\lambda\tau^i\frac{\partial X_j}{\partial \tau^o}] = N_2\mathbb{E}_2[\gamma_{j,t}\frac{\partial q_{j,t}}{\partial \tau^o}]$$

and

$$N\mathbb{E}_0[\lambda\tau^i\frac{\partial X_j}{\partial\tau^i}] + N_1\bar{\tau^o}\mathbb{E}_1[\frac{\partial q_{j,t}}{\partial\tau^i}] = N_2\mathbb{E}_2[\gamma_{j,t}\frac{\partial q_{j,t}}{\partial\tau^i}].$$

The optimal combined subsidy is therefore

$$\tau^o_{j,t} = \gamma_{j,t} + \frac{1-\tilde{T}_\kappa}{\tilde{T}_\kappa}\Omega_{\frac{\gamma}{\lambda}\frac{\mathrm{d}q2}{\mathrm{d}X},\frac{\partial X}{\partial\tau^o}}$$

$$\tau^i = \frac{1-\tilde{T}_\kappa}{\beta}\frac{\overline{\gamma 2}\ \mathbb{E}_2\left[\frac{\mathrm{d}q_{j,t}}{\mathrm{d}X_j}\right]}{\lambda} + \frac{1-\tilde{T}_\kappa}{\beta}\Omega_{\frac{\gamma}{\lambda}\frac{\mathrm{d}q2}{\mathrm{d}X},\frac{\partial X}{\partial\tau^i}}$$

□

C. The Social Cost of Longer Subsidy Periods - For Online Publication

There are many possible frictions that could produce time limits in practice. This appendix details a number of these frictions and connects them to the $\phi(T)$ duration cost term. We also present conditions necessary for the convexity of $\phi(T)$.

C.1 Sources of Duration Costs

Administrative and Compliance Costs. The most straightforward cost of added duration are firm-level administrative and compliance costs. These costs are very intuitive as real-world subsidy receipt requires firms to learn new statutes, keep records, fill out and submit paperwork, and work with the possibility of audits and responses. Similarly, the government needs to process paperwork, monitor firms, and administer payments. Dharmapala et al. (2011) propose a model where the government faces fixed administrative costs of tax collection for each firm in each time period. They also suggest that this type of cost may be prevalent enough to explain common phenomena such as the empirical distribution of firm sizes in countries with size-based tax exemptions. Fixed-costs per firm imply increasing $\phi(T)$ that is convex under the assumptions of Lemma 5.[50]

***Ex Post* Uncertainty Induced Duration Costs**. An uncertain subsidy duration could induce *ex post* welfare costs if the firm expects the subsidy duration to be different than the actual realized duration. Intuitively, if the firm expects the subsidy duration to differ from the realized duration, their initial investment decision will not be optimal *ex post*. Lemma 6 shows that the welfare costs of *ex post* uncertainty are quadratic in the difference between the actual investment level made under uncertain beliefs, x^u, and the optimal investment decision if the subsidy duration was known with certainty, x^*. These welfare costs could be increasing and convex in (actual) subsidy duration if longer subsidy durations cause firms to make bigger "mistakes" in their investment decisions, a condition that could arise if longer subsidy durations are less predictable.

Statutory Difficulties and Policy Processes. There are statutory aspects of the policy making process in many countries making time-limited policies a political expediency (often at a "round" number like 10 or 20 years after the policy is passed). These policies suggest a discontinuous $\phi(T)$ where the global costs (i.e., across all firms) are constant or slightly increasing before some threshold $\tilde{t}$, after which there is a discrete jump. A globally piecewise $\phi(T)$ would be increasing, but because it is discontinuous, the optima would be a

[50] Note that if the marginal compliance costs are decreasing in T, $\phi(T)$ may still be convex, but would require the mass of firms to increase at a faster rate.

corner solution $T^* \in \{0, \tilde{t}, \infty\}$.[51]

Policymaker Career Concerns. In the political economy of policy making, policymakers seeking reelection may not value future social benefits at the same rate as the social planner. If any policy has to pass a budget vote, policymakers may discount future gains more quickly than they do present-value costs—because not all gains improve election prospects. In this case $\phi(T)$ would be increasing but not globally convex, suggesting a corner solution of subsidizing only investment if the career concern is relatively strong or only output if the social value of the policy to constituents is relatively strong.

C.2 Proofs of Duration Cost Microfoundations

Lemma 5. If Assumption 1 holds for all firms, if there is a constant, positive administrative or compliance cost ϕ_0 for each firm in each time period during the subsidy period, and $\kappa = 0$, then $\phi(T)$ is increasing. Furthermore, if the number of firms is exponentially growing at a rate faster than β, $\phi(T)$ will be convex.

Proof. Let the number of firms present at time period t be $J(t) = \int_{\mathcal{J}_t} dF(j)$. Note that $J'(t) > 0$. Furthermore, let $\phi(T)$ be the sum of all constant administrative costs paid in all periods by firms form each cohort receiving the subsidy.

$$\phi(T) = \int_0^T e^{-\beta t} \phi_0 J(t) \, \mathrm{d}\, t.$$

Differentiating with respect to T:

$$\phi'(T) = e^{-\beta T} \phi_0 J(t).$$

Given that $\phi_0 > 0$, then $\phi'(T) > 0$ and $\phi(T)$ is increasing in T. Differentiating again yields

$$\phi''(T) = \beta e^{-\beta T} \phi_0 (J'(t) - \beta J(T)).$$

Therefore $\phi(T)$ is convex if and only if the number of firms, $J(t)$, is exponentially growing at rate faster than β $\left(\frac{J'(T)}{J(T)} > \beta\right)$. □

Lemma 6. Uncertainty, creates *ex post* welfare loss when firms expect the duration of an output subsidy to differ from the true subsidy duration. The welfare loss is quadratic in the

[51] Note that an unknown $\tilde{t}$ can generate a convex $\phi(T)$ if we allow $\phi(T)$ to instead represent the expected costs given the uncertain political constraints. In this case $\phi(T)$ would be convex wherever the PDF of the prior over $\tilde{t}$ is increasing.

difference between firm investment in the fixed input under the uncertain (wrong) duration beliefs, x^u, and the counterfactual investment if they knew the duration with certainty x^*. How these loses vary with the true subsidy duration, T, depends on how the uncertainty induced investment wedge, $x^u - x^*$, varies as T increases.

Proof. Using the model from Section 2, if a firm mistakenly believes the output subsidy will last for duration T^u instead of true duration T at the time of investment, then the *ex post* welfare loss is

$$\begin{aligned}
&\mathcal{W}^u - \mathcal{W}^* = \\
&\Big[T(q(x^u, v_1^u)(1+\gamma) - mv_1^u) + (1-T)(q(x^u, v_2^u)(1+\gamma) - mv_2^u) - c\, x^u\Big] \\
&- \Big[T(q(x^*, v_1^*)(1+\gamma) - mv_1^*) + (1-T)(q(^*, v_2^*)(1+\gamma) - mv_2^*) - c\, x^*\Big].
\end{aligned}$$

Making second-order Taylor Approximation around $q(x^*, v_1^*)$ and $q(x^*, v_2^*)$, and substituting in firm's first order conditions and the expressions for optimal policy from Proposition 3, the welfare wedge simplifies to

$$\begin{aligned}
&\mathcal{W}^u - \mathcal{W}^* = \\
&\gamma(1-T)\Big(q_v(x^*, v_2^*)\frac{q_{xv}(x^*, v_2^*)}{q_{vv}(x^*, v_2^*)}(x^u - x^*) + q_v(x^*, v_2^*)(v_2^u - v_2^*)\Big) \\
&+ T(1+\gamma)\Big(q_{xx}(x^*, v_1^*)(x^u - x^*)^2 + q_{vv}(x^*, v_1^*)(v_1^u - v_1^*)^2 + q_{xv}(x^*, v_1^*)(x^* - x^u)(v_1^u - v_1^*)\Big) \\
&+ (1-T)(1+\gamma)\Big(q_{xx}(x^*, v_2^*)(x^u - x^*)^2 + q_{vv}(x^*, v_2^*)(v_2^u - v_2^*)^2 + q_{xv}(x^*, v_2^*)(x^* - x^u)(v_2^u - v_2^*)\Big).
\end{aligned}$$

Noting that $q_v(x^u, v_t^u) = q_v(x^*, v_t^*)$ and a first order Taylor Approximation for $q_v(x^u, v_t^u)$ imply that $q_{xv}(x^*, v_t^*)(x^u - x^*) = q_{vv}(x^*, v_t^*)(v_t^u - v_t^*)$. Substituting in this expression and simplifying, the welfare wedge becomes

$$\mathcal{W}^u - \mathcal{W}^* = (1+\gamma)\Big(Tq_{xx}(x^*, v_1^*)(x^u - x^*)^2 + (1-T)q_{xx}(x^*, v_2^*)(x^u - x^*)^2\Big).$$

As $q_{xx} < 0$, this expression shows the wedge is negative if $T^u \neq T^*$ and the loss is increasing quadratically in $(x^u - x^*)$. If the returns to scale in x are constant in v ($q_{xx}(x*, v_1^*) = q_{xx}(x*, v_2^*)$ or equivalently $q_{xxv} = 0$ locally between v_1 and v_2) then the *ex post* welfare costs of firm's duration uncertainty is increasing in T if $(x^u - x^*)$ is increasing in T and it will be convex in T if $(x^u - x^*)$ is increasing at a rate faster than $\sqrt{T}$.

□

D. The Optimality of the PTC - For Online Publication

D.1 Determining the External Value of Wind

Although there are external benefits to offsetting other pollutants, reducing CO_2 is the main benefit in most locations (see calculations in Cullen, 2013). If 1 MWh of wind energy reduces average CO_2 emissions by 0.71 metric tons (as estimated by EPA, 2022), then the external value of wind is between \$35 (based on the EPA's social cost of carbon estimate of \$51 per ton) and \$131 (based on recent academic work like \$87 average in Cai and Lontzek (2019) or \$185 in Rennert et al. (2022)). Note that computing the true external value of wind from these average figures is complicated by two considerations. First, the average CO_2 and pollution offsets reported by the EPA may not reflect the marginal offset in the short run (Cullen, 2013, although in the long run the "marginal" effect of clean energy will be the average difference in pollution as dirty firms exit). Second, there is heterogeneity across time and space in the value of one MWh of wind energy (e.g., Hollingsworth and Rudik, 2019; Fell et al., 2021; Sexton et al., 2021).

D.2 Inverse Optima

Having measured how energy production changes after the PTC subsidy period, we can return to the theory to consider whether existing subsidies for wind energy are designed optimally. To assess the optimality, we will assume that the current policy is calibrated appropriately and will consider what model primitives would justify each policy. For simplicity we assume that production technology and externalities do not vary across firms enough to make the Ω terms quantitatively meaningful and that corporate taxation accounts for the net Ψ terms. Further assume that $\lambda = 1.5$, a relatively large marginal cost of public funds, and that a wind turbine has a capital life of 25 years.

First, we consider the \$25 rate of the PTC which can only be justified by a small external value or large marginal cost of public funds. With the assumptions above, Proposition 6 suggests that if all policy parameters are optimally chosen, the output subsidy should be $\tau^o = \frac{\gamma_1}{\lambda}$; thus, optimality requires that $\frac{\gamma}{\lambda}$ =\$25/MWh. Although there are additional external benefits to offsetting other pollutants, reducing CO_2 is the main benefit in most locations (see calculations in Cullen, 2013). If 1 MWh of wind energy reduces average CO_2 emissions by 0.709 metric tons (as estimated by EPA, 2022), then the optimal subsidy should be $\tau^o = \frac{0.709\text{SCC}}{1.5}$. As such \$25/MWh subsidy benefit implies a social cost of carbon of \$53 per ton. Although this estimate is very similar to the EPA's estimate (\$51 per ton), it is much lower than recent academic work (\$87 average in Cai and Lontzek (2019) or \$185 in

Rennert et al. (2022)).[52]

Second, we consider the PTC's ten-year subsidy period and show that it could be optimal under large institutional frictions. Recall, that at the optimum $\phi'(T) = -\mathbb{E}[\gamma \Delta q_j]$. We estimate that the average change in production is about 1000 MWh/month/firm. For context, this means that (again assuming $\frac{\gamma}{\lambda} = \$25$) extending the PTC by one year would cost society over \$366 million.[53]

Finally, we consider the policy of accelerated and bonus depreciation, and show it is only an optimal investment subsidy if the average capital share of output is very small. This subsidy was probably worth about 7-12% of investment costs in our sample period. Recall that the optimal investment subsidy should be $\tau^i = (1-T)\frac{gamma_2}{\lambda}\mathbb{E}\frac{\mathrm{d}q}{\mathrm{d}X}$. Given an output subsidy with a ten years time limit, and assuming the life of a wind turbine is twenty five years, $T \approx 0.4$. If the average investment cost for producing an additional MWh over the capital life is between \$10-20,[54] and if $\frac{\gamma_2}{\lambda}$ is also \$25, then the bonus depreciation is optimally subsidizing investment only if $\frac{\mathrm{d}q}{\mathrm{d}X} < 0.16$.[55] Whereas such a small fixed-inputs share on the margin might be true in some industries, it seems implausible in the wind industry. If γ_2 is smaller the investment share may be larger.

[52] Considering REC prices as an added output subsidy raises this to \$84 in markets with renewable portfolio standards and REC markets.

[53] That is, \$37.5 in value for about 12,800 MWh forgone in the eleventh year for each firm. This linear approximation of $\phi(T)$ around $T = 0.4$ underestimates the administrative cost from a convex $\phi(T)$ (and convexity is necessary for an interior solution like $T = 0.4$ to be optimal).

[54] 1 MW of capacity operates for 8760 hours each year for 25 years with an average capacity factor of 31.3, it will produce just under 70,000 MWh, so to produce 1 Mwh over the capital life it requires $\frac{1}{70{,}000}$ MW of capacity. Recalling that 1 MW of capacity costs about \$0.8-1.5 M, this means the cost will be \$11-20.

[55] If $\frac{\tau^i}{c} \in [0.07, 0.12]$ is optimal, then Plugging in $T \approx 0.4$, $\frac{\gamma_2}{\lambda} = \$25$, and $c \in \ (10, 20)$ for bounds we can simplify $\frac{(1-T)\gamma}{\lambda c}\mathbb{E}[\frac{\mathrm{d}q}{\mathrm{d}X}]$ to $\mathbb{E}[\frac{\mathrm{d}q}{\mathrm{d}X} \in (0.09, 0.16)]$.