The Surge in Emerging Markets' International Reserves and the Rise in Global Volatility *

(preliminary & incomplete)

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Abstract

Holdings of international reserves surged in emerging economies since the 1990's Sudden Stops. Research has shown that the unilateral accumulation of reserves can improve financial and macroeconomic stability in individual countries. In contrast, we show that the simultaneous accumulation of reserves by several countries has the opposite effect through general equilibrium effects. We propose a model with emerging and advanced economies in which the private sector issues defaultable debt which has a productive use for its holders. We show via quantitative counterfactuals that, by reducing the world interest rate, the observed increase in reserves induced higher private-sector leverage which in turn undermined financial and macroeconomic stability in *both* emerging and advanced economies.

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1 Introduction

The holdings of foreign reserves by emerging market economies (EMEs) increased significantly during the last three decades, as shown in the first panel of Figure 1. The sharp increase is especially notable after the 1990s Sudden Stops: FX reserves increased from 10 percent of GDP in 1997 to 30 percent in 2009. Foreign reserves also increased in advanced economies but at a much slower pace.

Foreign reserves are mainly held in the form of short-term public debt instruments issued by advanced economies. The second panel of Figure 1 shows that the public debt of advanced economies (AEs) rose sharply following the 2008 global financial crisis, after being relatively stable since the mid 1990s. It increased from about 60 percent of GDP in 2007 to about 95 percent in 2012.

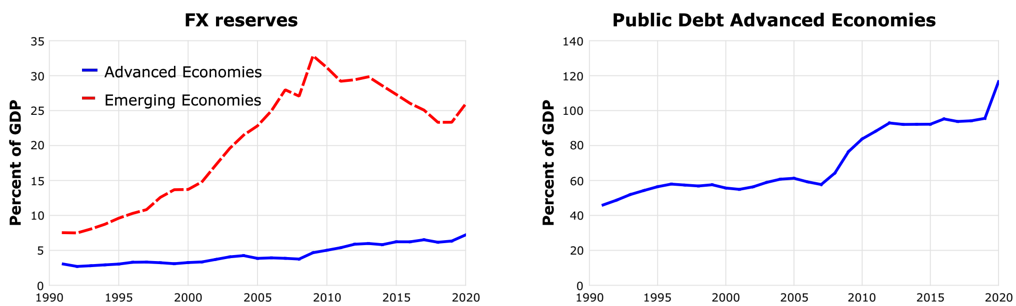

Figure 1: Foreign Exchange Reserves of Advanced and Emerging economies and Public Debt of Advanced economies.

Note: Data for FX reserves is from External Wealth of Nations database (Lane and Milesi-Ferretti (2018)). Advanced economies: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, United States. Emerging economies: Algeria, Argentina, Brazil, Bulgaria, Chile, China, Czech Republic, Colombia, Estonia, Hong Kong, Hungary, India, Indonesia, Israel, South Korea, Latvia, Lithuania, Malaysia, Mexico, Pakistan, Peru, Philippines, Poland, Romania, Russia, Saudi Arabia, Singapore, South Africa, Thailand, Turkey, Ukraine, Venezuela. Data on public debt is from IMF Global Debt Database. We use the series Central Government Debt which is available for thirteen countries: Canada, Finland, France, Germany, Italy, Japan, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, United States. The Global Debt Database provides two series: 'Central Government Debt' and 'General Government Debt'. We use the former. Data for all years 1991-2020 are available only for thirteen of the advanced economies (listed above). Hence, our measure of debt-to-GDP ratio for advanced economies results from the aggregation of these thirteen countries.

From a global perspective, these changes are important because the increased accumulation of FX reserves by EMEs raised the demand for risk-free financial assets (contributing to a lower world interest rate), while more issuance of public debt by AEs increased their supply (contributing to a higher interest rate). The goal of this paper is to understand how these changes affected the financial structure of both emerging and advanced economies and impacted their financial and macroeconomic volatility.

To this end, we develop a quantitative model that features two regions, representing AEs and EMEs, respectively. In each region, there are borrowers (issuers of financial liabilities by both private and public sectors) and lenders (buyers of liabilities from both private and public sectors). Two characteristics of these liabilities are central to our analysis. First, lenders in the private sector hold financial assets (the liabilities issued by borrowers) because they provide a convenience yield by facilitating production. Second, the liabilities issued by the private sector are defaultable.

Since private liabilities are defaultable, a financial crisis occurs when the debt is not fully repaid. Default arises in states in which the debt exceeds the liquidation value of the real assets owned by borrowers, and generates haircuts that redistribute wealth from creditors to debtors. This redistribution of wealth is critical for our findings, because it causes adverse real macroeconomic effects by wiping out some of the financial assets held by producers. The magnitude of these effects depends on the financial structure of the economy: When leverage is high, a financial crisis generates a larger redistribution of wealth and, thus, stronger macroeconomic effects.

How does the accumulation of foreign reserves affect the magnitude of financial crises and global macroeconomic volatility? A sizable increase in reserves relatively to the size of the world economy causes a reduction in the world interest rate, which in turn leads to higher private sector leverage in both emerging and advanced economies. Because of the higher leverage, financial crises cause larger wealth redistribution and stronger effects on the real economy (higher macroeconomic volatility). On the other hand, an increase in the supply of public debt increases the supply of assets, raising the interest rate and reducing the severity of financial crises.

The increase in EME's foreign reserves and AE's public debt were not the only significant global macroeconomic developments of the last three decades. For example, emerging economies grew faster than advanced economies and financial innovation also contributed to the changes in the structure of financial markets. These developments interacted with the changes in FX reserves and public debt issuance to produce the observed macro outcomes. Therefore, in order to quantify the impact of reserves accumulation and public debt issuance on financial and real sectors globally, we need to take into account the quantitative impact of these other changes. In particular, we consider exogenous changes in (i) productivity, (ii) innovations that affected the private demand of financial assets, (iii) innovations that affected the private supply of financial assets. The last two changes are captured by exogenous variations in two structural parameters. Counterfactual simulations over the period 1991-2020 show that the observed surge in reserves caused a sharp increase in macroeconomic and financial volatility while the increase in public debt reduced it.

We also consider the possibility that FX reserves may be used to provide liquidity and stabilize the economy in the event of a financial crisis. In particular, since the adverse real effects of a financial crisis in our model are due to the destruction in entrepreneurial wealth (i.e., the defaulted debt), we assume that FX reserves are used to bail out a fraction of the financial losses incurred by entrepreneurs. This arrangement has negligible effects on the volatility of advanced economies, because they do not hold large stocks of FX reserves relatively to the size of their economy, and hence bailouts are relatively small. For emerging economies, however, aggregate volatility drops markedly.

Related literature. Our work is related to three important strands of literature: (i) financial and macroeconomic implications of FX reserves; (ii) financial crises or Sudden Stops; (iii) scarcity of financial assets.

There is an extensive literature on the financial and macroeconomic implications of FX reserves. One branch focuses on foreign exchange interventions and their effects on exchange rates and financial stability (see the detailed survey by Popper (2022)). A second branch focuses on the implications of reserves for sovereign borrowing, vulnerability to financial crises, and design of macroprudential policy (e.g., Alfaro and Kanczuk (2009), Durdu, Mendoza, and Terrones (2009), Devereux and Wu (2022), Bianchi, Hatchondo, and Martinez (2018), Bianchi and Lorenzoni (2022), Bianchi and Sosa Padilla (2024)).

The above studies analyze the role of reserves in the context of small open economies, and thus treat the world risk-free interest rate as exogenous. In contrast, our analysis deviates from the small open economy assumption by proposing a mechanism that operates through general equilibrium changes in the world interest rate. We study how the observed surge in reserves, which is exogenous in the model, contributed to the fall in the world real interest rate, the expansion of private credit, and the increase in global macroeconomic volatility since the 1990s.

We show that a *collective* increase in reserves by EMEs makes the world economy more vulnerable to financial crises with larger output volatility. By contrast, the existing literature finds that unilateral increases in reserves by *individual* countries improve financial stability by reducing the likelihood of self-fulfilling sovereign debt crises or allowing countries to provide liquidity to the private sector in the eventuality of a crisis. An exception is Das, Gopinath, Kim, and Stein (2023) who study a model with currency mismatch where central banks act as lenders of last resort. Central banks may over-accumulate reserves because they do not internalize the impact on the dollar interest rate and this encourages higher currency mismatch. Our paper also emphasizes the general equilibrium impact of reserve accumulation on the world interest rate but the goal of our paper is to quantify the impact of this mechanism on global volatility rather the optimality of reserves accumulation. The mechanism through which the world interest rate affects the economy is also different as it operates through financial leverages rather than currency mismatch.

Various studies in the Sudden Stops literature examine the role of financial globalization, credit booms and high leverage as causing factors of financial crises. Examples include Calvo and Mendoza (1996), Caballero and Krishnamurthy (2001), Gertler, Gilchrist, and Natalucci (2007), Edwards (2004), Mendoza and Quadrini (2010), Mendoza and Smith (2014), Fornaro (2018).[1] Some of these studies emphasize mechanisms that cause financial crises because of equilibrium multiplicity due to self-fulfilling expectations as in Aghion, Bacchetta, and Banerjee (2001) and Perri and Quadrini (2018). Crises in our model also follow from periods of fast credit and leverage growth, and they are the result of self-fulfilling expectations.

Several studies in the corporate finance literature document and provide explanations for the raising demand of financial assets. An example is the literature on the growing cash holdings of nonfinancial businesses (e.g., Riddick and Whited (2009), Busso, Fernández, and Tamayo (2016) and Bebczuk and Cavallo (2016)). Our model has a similar feature in that some businesses, but not all, hold positive positions in financial assets. Our focus, however, is on the macroeconomic implications. The increase in net

[1] See Bianchi and Mendoza (2020) for a survey of the literature.

demand for financial assets due to the increased FX reserves, depresses the interest rate—a general equilibrium effect—which in turn strengthens incentives to leverage. While the higher leverage allows for sustained levels of financial intermediation and economic activity, it also makes both emerging and advanced economies more vulnerable to crises (global instability).

The remaining of the paper is organized as follows: Section 2 describes the model and characterizes the equilibrium. Section 3 uses the model in conjunction with the data to construct empirical series for productivity and exogenous structural changes that affect demand and supply of financial assets. We then conduct counterfactual simulations to quantify the role played by FX reserves and public debt in generating the observed trends. Section 4 analyzes the implications of changes in FX reserves and public debt for macro and financial volatility. Section 5 studies an extension of the model in which reserves are used to cover part of the entrepreneurs' losses in the eventuality of a crisis. Section 6 concludes.

2 Model

Consider a world economy that consists of two regions indexed by $j \in \{1, 2\}$. Region 1 represents advanced economies and Region 2 emerging economies. In each region there are three sectors: (i) a business sector with two types of firms; (ii) a household sector that supplies labor; (iii) a public sector that holds financial assets in the form of FX reserves and, in Region 1, issues liabilities (public debt).

We model two types of firms as a means to generate private borrowing and lending within a region (in addition to cross region borrowing and lending). We distinguish the private demand for financial assets by firms with positive financial positions from the private supply by firms with negative positions. The public sector allows us to study how the issuance of public debt and accumulation of FX reserves affect the economies of the two regions.

Regions are heterogeneous in three key dimensions: (i) economic *size*, captured by differences in aggregate productivity, $z_{j,t}$; (ii) a financial parameter that affects directly the *demand* for financial assets, $\phi_{j,t}$; and (iii) a financial parameter that affects directly the *supply* of financial assets, $\kappa_{j,t}$. Regions also differ in their stocks of foreign reserves, $FX_{j,t}$, and government debt issued by advanced economies, $D_{p,t}$.

Differences in economic size could be generated by other factors besides productivity (e.g., population, real exchange rates, etc.). For the questions addressed in this paper, however, other factors are isomorphic to productivity differences. This will become clear in the quantitative exercise. Productivity $z_{j,t}$, financial parameters $\phi_{j,t}$ and $\kappa_{j,t}$, foreign reserves $FX_{j,t}$, and public debt $D_{p,t}$ are time varying but not stochastic. Thus, their evolution over time is fully anticipated. The only source of uncertainty in the model comes from "sunspot" shocks that will be described later in this Section.

2.1 Household sector

In each region, there is a unit mass of households that maximize the following expected lifetime utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(c_{j,t} - z_{j,t-1}^{\frac{1}{\gamma}} \frac{h_{j,t}^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} \right),$$

where $c_{j,t}$ is consumption, $h_{j,t}$ is the supply of labor, and $z_{j,t-1}$ is lagged productivity in the production of final goods. The assumption that the utility is linear in consumption simplifies the characterization of the equilibrium. It allows us to derive analytic results without altering significantly the properties of the model that are central for the questions addressed in this paper. The dependence of labor dis-utility on productivity supports balanced growth. The utility function depends on two parameters: ν is the elasticity of labor supply and, as we will see, γ is the labor share in production of intermediate goods.

The households' budget constraint is

$$c_{j,t} = w_{j,t} h_{j,t} + \text{div}_{j,t} + T_{j,t}.$$

Consumption is paid for with wage income, $w_{j,t}h_{j,t}$, dividends distributed by firms owned by households, $\text{div}_{j,t}$, and government transfers, $T_{j,t}$.

The only relevant decision made by households is the supply of labor, which is determined by this first-order condition:

$$z_{j,t-1}^{\frac{1}{\gamma}} h_{j,t}^{\frac{1}{\nu}} = w_{j,t}. \tag{1}$$

Because the utility function is linear in consumption and additively-separable in labor disutility, the marginal rate of substitution between $c_{j,t}$ and $h_{j,t}$ does not depend on consumption. Hence, there is no income effect on the supply of labor. As we show later, the long run growth of wages is $z_{j,t-1}^{1/\gamma}$ and, therefore, the labor supply is constant in the long run.

2.2 Business sector

The business sector has two types of firms: producers of intermediate goods and producers of final goods. The former are owned by households, and the latter are operated by entrepreneurs. An important difference between them is that capital—which is pledgeable as collateral—is used in production only by intermediate-goods firms. Final-goods producers lack collateral assets. At equilibrium, then, the first type of firms are net borrowers and the second are net lenders (i.e., they have a positive position in financial assets). In this way, we generate borrowing and lending within the business sector.[2] We begin with the description of intermediate-goods producers.

2.2.1 Intermediate goods producers

Intermediate-goods firms produce inputs $x_{j,t}$ using labor, $l_{j,t}$, and capital, $k_{j,t}$, with the following Cobb-Douglas technology:

$$x_{j,t} = l_{j,t}^{\gamma} k_{j,t}^{1-\gamma}.$$

Although firms solve a dynamic problem that maximizes the discounted value of dividends paid to households, the choice of labor solves a static problem. Given the stock of capital $k_{j,t}$, firms choose the input of labor to

[2]Differences in financial structure could reflect the tangibility of capital. Firms that are intensive in intangible capital do not have enough collateral assets to borrow and, as a result, accumulate financial assets or cash. Falato, Kadyrzhanova, Sim, and Steri (2022) show the importance of this mechanism for explaining the rising cash holdings of US corporations during the last four decades. These types of firms are captured in the model by the final-goods producers. However, the fact that in the model intermediate-goods producers are net borrowers and final-goods producers are net lenders should not be interpreted literally when mapping the model to the data. What really matters is that there is production complementarity between the two groups of firms. This implies that, when firms in one group cut production due to financial conditions, the other firms also cut their production due to lower demand.

maximize profits $p_{j,t}x_{j,t} - w_{j,t}l_{j,t}$, where $w_{j,t}$ is the wage rate and $p_{j,t}$ is the price at which they sell the intermediate goods to final producers in competitive markets. The optimal demand for labor is then determined by the first order condition that equates the marginal revenue product of labor to the wage rate,

$$\gamma p_{j,t} l_{j,t}^{\gamma-1} k_{j,t}^{1-\gamma} = w_{j,t}.$$

Capital is reproducible without adjustment costs. Thus, in normal conditions, the price of capital is 1. To keep the model tractable, however, we assume that investment evolves exogenously.[3]

Borrowing and default. Intermediate-goods firms can also borrow. At the end of period $t-1$, firms borrow $d_{j,t}/R_{j,t-1}$, where $R_{j,t-1}$ is the gross interest rate and $d_{j,t}$ is the debt (promised repayment) due at time t. At the beginning of period t, when the debt is due, they could default. Under default, creditors have the right to liquidate the capital $k_{j,t}$. However, the liquidation value of capital could be insufficient to fully repay the debt $d_{j,t}$.

Denote by $\ell_{j,t}$ the liquidation price of capital at the beginning of period t. If the debt is bigger than the liquidation value, that is, $d_{j,t} > \ell_t k_{j,t}$, the debt is renegotiated. Under the assumption that borrowers have the whole bargaining power, the renegotiated debt is

$$\tilde{d}(d_{j,t}, \ell_{j,t}k_{j,t}) = \min\Big\{ d_{j,t} \ , \ \ell_{j,t}k_{j,t} \Big\}. \tag{2}$$

After renegotiation, the market for capital returns to normal at the end of the period (i.e., there is no market exclusion).

A key assumption is that there are states of nature in which the market for liquidated capital freezes and the liquidation price at the beginning of the period drops below its normal price of 1. More specifically, with probability $1-\lambda$ the liquidation price remains at its normal price $\ell_{j,t} = 1$. With probability λ, however, it drops to $\ell_{j,t} = \kappa_{j,t}$. The variable $\kappa_{j,t}$ is time-varying but exogenous. Importantly, $\kappa_{j,t}$ is always smaller than the normal price, that is, $\kappa_{j,t} < 1$. As we explain below, $\kappa_{j,t}$ is a key variable driving the private supply of financial assets.

[3] In the quantitative analysis, we impose that investment matches the series observed in the data. An alternative approach would be to make investment endogenous while adding time-varying investment wedges set to replicate the empirical series of investment.

Appendix D describes the mechanism that generates a freeze in the market for liquidated capital as a result of self-fulfilling expectations about the liquidation price of capital, which depends on the borrowers' leverage. In particular, when $d_{j,t} > \kappa_{j,t} k_{j,t}$, there are two equilibria. In one equilibrium, the market does not freeze and the liquidation price is 1. In the other, the market freezes and the liquidation price drops to $\kappa_{j,t} < 1$. The selection between the two equilibria is determined by the draw of a sunspot shock $\varepsilon_j \in \{0, 1\}$, and λ is the exogenous probability that the draw of the sunspot shock is the one associated with the market freeze.

Readers interested in the micro-foundation of the market freeze may wish to treat Appendix D as an integral part of the current section. Otherwise, the Appendix can be skipped. What is essential for the analysis that follows is that the liquidation price of capital $\ell_{j,t}$ takes the value of 1 with probability $1 - \lambda$ and $\kappa_{j,t}$ with probability λ. The sunspot variables ε_1 and ε_2 are the only exogenous stochastic variables (shocks) in the model.[4]

The individual issuance of new debt $d_{j,t+1}$ carries a convex cost that takes the form

$$\varphi\left(d_{j,t+1}, \kappa_{j,t+1} k_{j,t+1}\right) = \eta \left[\frac{\max\{\, 0 \;,\; d_{j,t+1} - \kappa_{j,t+1} k_{j,t+1} \,\}}{d_{j,t+1}} \right]^2 d_{j,t+1}. \tag{3}$$

Figure 2 provides a graphical illustration of this cost. As long as the debt repayment promised in the next period, $d_{j,t+1}$, exceeds the minimum liquidation value, $\kappa_{j,t+1} k_{j,t+1}$, the cost is zero. Beyond that point, the cost rises at an increasing rate. This cost plays a similar role as a borrowing limit: it ensures that borrowing is bounded at equilibrium. The parameter η determines, for a given stock of capital, the speed at which the cost rises with debt. Thus, it captures the flexibility with which borrowing responds to changing market conditions (e.g., the interest rate). For very high values of η we have, effectively, a standard borrowing limit, that is, $d_{j,t+1} \leq \kappa_{j,t+1} k_{j,t+1}$.[5]

[4]Benhabib, Dong, Wang, and Xu (2024) develop an interesting model that generates self-fulfilling default cycles. The mechanism that in their model generates multiple equilibria relies on the survival of active firms, a number that changes with crises. Our mechanism, instead, relies on the liquidation value of collateral.

[5]The soft borrowing limit allows the model to generate an endogenous response of debt to changes in the interest rate. With a hard borrowing limit, instead, the interest rate would not impact the equilibrium debt (unless the borrowing limit also changes).

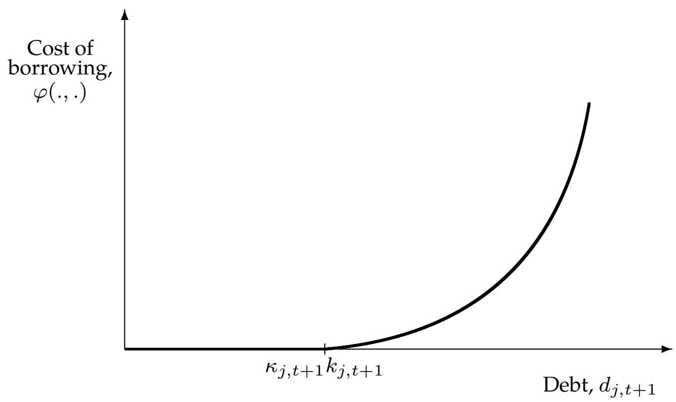

Figure 2: Convex cost of borrowing as a function of debt.

The budget constraint for intermediate-goods firms, after the renegotiation of the debt, is

$$\begin{aligned} \text{div}_{j,t} &= p_{j,t} l_{j,t}^{\gamma} k_{j,t}^{1-\gamma} - w_{j,t} l_{j,t} - i_{j,t} - \tilde{d}(d_{j,t}, \ell_{j,t} k_{j,t}) + \\ &\qquad \frac{d_{j,t+1}}{R_{j,t}} - \varphi(d_{j,t+1}, \kappa_{j,t+1} k_{j,t+1}). \end{aligned} \tag{4}$$

where $i_{j,t} = k_{j,t+1} - (1-\tau)k_{j,t}$ is investment and τ the depreciation rate.

Note that the deterministic path of $\kappa_{j,t}$ for the current and future periods is known. Also note that both $\kappa_{j,t}$ and $\kappa_{j,t+1}$ are relevant for date-t decisions: $\kappa_{j,t}$ matters for the repayment of the existing debt, while $\kappa_{j,t+1}$ matters for the cost of issuing new debt.

The gross interest rate $R_{j,t}$ depends on individual borrowing decisions. If the firm borrows more, relatively to the ownership of capital, the expected repayment will be lower in the next period. This will be reflected in a higher interest rate on the newly issued debt.

Denote by $\overline{R}_{j,t}$ the *expected* gross return from buying a diversified portfolio of debt issued by all intermediate-goods firms in Region j at time t. Since firms are atomistic and financial markets are competitive, the expected return on the debt issued by an 'individual' firm must be equal to the expected return from the diversified portfolio, that is,

$$\frac{d_{j,t+1}}{R_{j,t}} = \frac{1}{\overline{R}_{j,t}} \mathbb{E}_t \tilde{d}(d_{j,t+1}, \ell_{j,t+1} k_{j,t+1}). \tag{5}$$

The left-hand-side is the amount borrowed in period t while the right-hand-side is the expected repayment in period $t+1$, discounted by the market return $\overline{R}_{j,t}$. Since an intermediate-goods firm renegotiates the debt when $d_{j,t+1} > \ell_{j,t+1}k_{j,t+1}$, the actual repayment $\tilde{d}(d_{j,t+1}, \ell_{j,t+1}k_{j,t+1})$ could be lower than $d_{j,t+1}$. Competition in financial markets requires that the left-hand-side equals the right-hand-side.

Equation (5) determines the interest rate $R_{j,t}$ for an individual borrower. It can also be viewed as determining the borrowing spread paid by the borrower, $R_{j,t}/\overline{R}_{j,t} = d_{j,t+1}/\mathbb{E}_t\tilde{d}(d_{j,t+1}, \ell_{j,t+1}k_{j,t+1})$. For a firm expected to fully repay with certainty, the spread is zero ($R_{j,t}/\overline{R}_{j,t} = 1$). For a firm that is expected to repay in full only with some probability, $R_{j,t}$ exceeds $\overline{R}_{j,t}$. The higher rate depends on how much the contracted repayment, $d_{j,t+1}$, falls below the expected repayment after renegotiation, that is, $\mathbb{E}_t\tilde{d}(d_{j,t+1}, \ell_{j,t+1}k_{j,t+1})$. At equilibrium, all firms make the same decisions and they all borrow at the same rate. In order to characterize the optimal borrowing, however, we need to allow for individual deviations.

Firms' decisions. Intermediate-goods firms make decisions sequentially. At the beginning of the period they decide whether to default and renegotiate the debt. After that, they choose the input of labor $l_{j,t}$ and produce $x_{j,t}$. Finally, they choose the new debt $d_{j,t+1}$. Since the default and production decisions have already been characterized, we focus here on the optimality condition for the choice of the new debt.

Appendix C presents the optimization problem solved by an individual firm. The first-order condition for the optimal choice of $d_{j,t+1}$ is

$$\frac{1}{\overline{R}_{j,t}} = \beta + \Phi\left(\frac{d_{j,t+1}}{\kappa_{j,t+1}k_{j,t+1}}\right). \tag{6}$$

The function $\Phi(.)$ embeds expectations of future variables. The explicit functional form is provided in Appendix C. The only source of uncertainty in the model is the realization of sunspot shocks. Since future repayments conditional on default and the probability of default are known in advance, we can calculate analytically the expected repayment, which is incorporated in $\Phi(.)$.

The function $\Phi(.)$ increases when the ratio $d_{j,t+1}/\kappa_{j,t+1}k_{j,t+1}$ rises, mirroring the increasing cost of borrowing showed in Figure 2. The ratio is a measure of *effective* leverage: debt over the minimum liquidation value of capital. Because $\Phi(.)$ is an increasing function, condition (6) posits a *negative* relationship between the expected cost of the debt (the interest rate) and the effective leverage. This relationship is central to our finding that lower interest rates, resulting from the surge in FX reserves, increase leverage and worsen financial instability.

2.2.2 Final goods producers (entrepreneurs)

In each region, there is a unit mass of atomistic entrepreneurs that produce final goods with the aim to maximize logarithmic expected lifetime utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \ln(c^e_{j,t}),$$

where $c^e_{j,t}$ denotes the entrepreneur's consumption in Region j at time t.

Entrepreneurs are business owners producing homogeneous goods that can be traded internationally. Although they resemble privately-owned firms, we should think of them more broadly and including also some publicly-traded companies. Entrepreneurial consumption, then, can be interpreted as dividend payments and the concavity of the utility function could derive from the risk aversion of managers and/or major shareholders. Although not explicitly modeled, the concavity could also reflect, in reduced form, the cost associated with financial distress: even if shareholders and managers are risk-neutral, a convex cost of financial distress would make the objective of the business concave. Since there are no idiosyncratic shocks in the model, we can focus on the representative entrepreneur.

The production function of final-goods producers takes the form

$$y_{j,t} = z_{j,t} x_{j,t}, \tag{7}$$

where $y_{j,t}$ is production, $z_{j,t}$ is the region-specific aggregate productivity, and $x_{j,t}$ is the input of intermediate goods purchased from intermediate-goods firms. In the long-run, $z_{j,t}$ grows in both regions at the common rate g so as to support world balanced growth. In the short-run, however, the growth rate of productivity can deviate from its long-run value.

Working capital and accumulation of financial resources. Production of final goods also requires financial resources that increase with the purchase of intermediate goods. For this purpose, entrepreneurs accumulate financial wealth $m_{j,t}$ in order to satisfy the constraint

$$m_{j,t} \geq \phi_{j,t} p_{j,t} x_{j,t}. \tag{8}$$

A narrow interpretation of this constraint is that it represents advanced payment of a fraction $\phi_{j,t}$ of the cost of production (working capital). However, we give it a broader interpretation. In addition to funding advanced factor payments, financial wealth facilitates production through other channels that are not explicitly modelled here. For example, it provides insurance against earning risks, increasing the willingness to operate larger firms (Angeletos (2007)). Also, firms with more favorable financial positions may find easier to attract new workers (Monacelli, Quadrini, and Trigari (2023)) or to retain existing workers (Baghai, Silva, Thell, and Vig (2021)). We will get back to this broader interpretation of $m_{j,t}$ in the quantitative section of the paper.

The time-varying parameter $\phi_{j,t}$ plays an important role in determining the demand for financial assets. The higher the value of $\phi_{j,t}$, the higher the need for those assets and, hence, the larger the holdings of $m_{j,t}$.

The financial wealth held by entrepreneurs is in the form of liabilities issued by intermediate-goods firms (either domestic or foreign) and liabilities issued by the government of advanced economies. Even though we are assuming perfect capital mobility, the price of private liabilities differs from the price of public liabilities because they have different repayment risks. While private bonds are defaultable, public bonds issued by advanced economies are always repaid in full. We denote by $q_{j,t}$ the price of bonds issued by intermediate firms in Region j, and by $q_{p,t}$ the price of public bonds issued by Region 1 (advanced economies).

Entrepreneurial decisions. The representative entrepreneur in Region j enters period t with bonds issued by firms in Region 1, $b_{1,j,t}$, bonds issued by firms in Region 2, $b_{2,j,t}$, and government bonds issued by advanced economies, $b_{p,j,t}$. The first subscript denotes the issuer (Region 1 or Region 2 for private bonds, and p for public bonds), while the second subscript denotes the residence of the holder. In the event of default, entrepreneurs incur financial losses proportional to their ownership of private bonds (but not public bonds since they are risk-free).

Denote by $\delta_{1,t}$ and $\delta_{2,t}$ the fractions of private bonds repaid, respectively, by Region 1 and Region 2. The post-default values of the two bonds are then $\delta_{1,t}b_{1,j,t}$ and $\delta_{2,t}b_{2,j,t}$. The repayment fractions $\delta_{1,t}$ and $\delta_{2,t}$ are endogenous stochastic variables that are determined in the general equilibrium. After their realization at the beginning of the period, the entrepreneur's wealth becomes

$$m_{j,t} = \delta_{1,t} b_{1,j,t} + \delta_{2,t} b_{2,j,t} + b_{p,j,t}.$$

This is the entrepreneurial wealth that enters the financial constraint (8).

After production, the end-of-period wealth is

$$a_{j,t} = m_{j,t} + z_{j,t} x_{j,t} - p_{j,t} x_{j,t}.$$

This is in part allocated to consumption and in part to new bonds, in accordance to the budget constraint

$$c^e_{j,t} + q_{1,t} b_{1,j,t+1} + q_{2,t} b_{2,j,t+1} + q_{p,t} b_{p,j,t+1} = a_{j,t}. \tag{9}$$

While the production scale depends on $m_{j,t}$ (through constraint (8)), portfolio decisions, $b_{1,j,t+1}$, $b_{2,j,t+1}$ and $b_{p,j,t+1}$, depend on $a_{j,t}$. The following lemma characterizes the production decision.

Lemma 2.1 *If constraint (8) binds, then $p_{j,t} < z_{j,t}$ and the demand for intermediate goods chosen by final-goods producers is*

$$x_{j,t} = \left(\frac{1}{\phi_{j,t} p_{j,t}} \right) m_{j,t}.$$

If (8) does not bind, $p_{j,t} = z_{j,t}$ and the demand for intermediate-goods is determined by the supply from intermediate-goods firms.

Proof 2.1 *Appendix A.*

When the marginal productivity of the intermediate input exceeds its cost, that is, $z_{j,t} > p_{j,t}$, the firm makes a profit on each unit of final output (see Appendix A). It is then optimal for the entrepreneur to expand the scale of production until the financial constraint binds, that is, $m_{j,t} = \phi_{j,t} p_{j,t} x_{j,t}$. Solving the binding constraint for $x_{j,t}$ returns the expression reported in Lemma 2.1.

For the financial constraint not to be binding, profits must be zero, that is, $z_{j,t} = p_{j,t}$. In this case, the financial wealth of the entrepreneur $m_{j,t}$, and the financial parameter $\phi_{j,t}$ are irrelevant for the final production chosen by an individual firm. Only the aggregate production is determined in equilibrium (by the supply of intermediate-goods firms).

Under what conditions is constraint (8) binding? In general, the constraint is binding when entrepreneurs have low financial wealth ($m_{j,t}$ is small), the production input requires more funds ($\phi_{j,t}$ is high), and entrepreneurial firms are more productive ($z_{j,t}$ is high). As shown in Appendix 2.1, when this constraint binds, entrepreneurs earn positive profits that are proportional to $m_{j,t}$. This implies that bond holdings have a convenience yield—the profit—over and above the contracted market yield.

The next step is to characterize the optimal saving and portfolio choices made at the end of the period.

Lemma 2.2 *The entrepreneur allocates the end-of-period wealth $a_{j,t}$ as follows:*

$$\begin{aligned} c^e_{j,t} &= (1-\beta)a_{j,t}, \\ q_{1,t}b_{1,j,t+1} &= \beta\theta_{1,t}a_{j,t}, \\ q_{2,t}b_{2,j,t+1} &= \beta\theta_{2,t}a_{j,t}, \\ q_{p,t}b_{p,j,t+1} &= \beta(1-\theta_{1,t}-\theta_{2,t})a_{j,t}, \end{aligned}$$

where $\theta_{1,t}$ and $\theta_{2,t}$ solve the first-order conditions

$$\begin{aligned} \mathbb{E}_t\left\{\frac{\frac{\delta_{1,t+1}}{q_{1,t}}}{\theta_{1,t}\frac{\delta_{1,t+1}}{q_{1,t}}+\theta_{2,t}\frac{\delta_{2,t+1}}{q_{2,t}}+(1-\theta_{1,t}-\theta_{2,t})\frac{1}{q_{p,t}}}\right\} &= 1, \\ \mathbb{E}_t\left\{\frac{\frac{\delta_{2,t+1}}{q_{2,t}}}{\theta_{1,t}\frac{\delta_{1,t+1}}{q_{1,t}}+\theta_{2,t}\frac{\delta_{2,t+1}}{q_{2,t}}+(1-\theta_{1,t}-\theta_{2,t})\frac{1}{q_{p,t}}}\right\} &= 1. \end{aligned}$$

Proof 2.2 *Appendix B.*

Lemma 2.2 establishes that entrepreneurs split the end-of-period wealth between consumption and saving according to the fixed factor β. This derives from the logarithmic specification of the utility function. A fraction $\theta_{1,t}$ of saved wealth ($\beta a_{j,t}$) is then allocated to private bonds issued by Region 1, a fraction $\theta_{2,t}$ to private bonds issued by Region 2, and the remaining fraction $1-\theta_{1,t}-\theta_{2,t}$ to public bonds issued by Region 1 (advanced economies). We would like to point out that the three bonds are not perfect substitutes because they face different probabilities of default. Thus, there is a gain from diversification that explains why the portfolio shares are well defined.

We would also like to point out that the portfolio shares $\theta_{1,t}$ and $\theta_{2,t}$ change over time as recovery rates and bond prices vary. However, they are the same for entrepreneurs in Region 1 and in Region 2. This is indicated by the fact that $\theta_{1,t}$ and $\theta_{2,t}$ do not have the region subscript j. Thus, entrepreneurs in both regions choose the same portfolio composition.[6]

2.3 Public sector

The government of Region 1 issues risk-free bonds (public debt), and the governments of both regions hold some of these bonds as foreign reserves (henceforth FX reserves). Governments also pay lump-sum transfers to (or raise taxes from) households in order to balance their budgets.

The reason we focus on public debt issued by advanced economies is in part due to data limitations for emerging economies. More importantly, however, our choice is motivated by considerations related to two key differences between the public debt issued by the two regions. First, sovereign default in advanced economies is rare and public bonds issued by countries like Germany, Japan, the United Kingdom, and the United States are considered to be risk-free. This makes the public debt of these countries very different from their private debt, which is not risk-free. Because of their negligible repayment risk, these government bonds are important for liquidity and accumulation of FX reserves. U.S. public debt, in particular, represents roughly 60% of the assets held as FX reserves worldwide (see Ito and McCauley (2020)). Also, because the public debt of advanced economies is large relatively to the size of the world economy, it could have important general equilibrium implications.

Second, while governments in emerging economies do issue public debt, the debt is not risk-free and sovereign default arises often in conjunction with private default. Hence, from the perspective of an investor, there may be less significant differences between private and public debt issued by emerging economies. Their public debt is also much smaller than the public debt of advanced economies.

The budget constraint of the government of Region 1 (AEs) is

$$FX_{1,t} + q_{p,t} D_{p,t+1} = q_{p,t} FX_{1,t+1} + D_{p,t} + T_{1,t}. \tag{10}$$

[6]It is important to emphasize that, because $\theta_{1,t}$ and $\theta_{2,t}$ do not have the j subscript, the last three conditions in Lemma 2.2 are not just accounting identities.

The left-hand-side includes the sources of funds, and contains two terms. The first is the stock of foreign reserves accumulated in the previous period, $FX_{1,t}$. The second is the funds raised with the issuance of new debt $D_{p,t+1}$ sold at price $q_{p,t}$. The right-hand-side contains the uses of funds. The first term is the purchase of new reserves. The second is the repayment of the public debt issued in the previous period. The third is the transfer $T_{1,t}$ to domestic households (or taxes if negative). Notice that reserves are only in the form of public bonds issued by Region 1. Therefore, what matters for Region 1 is the net debt,[7] that is, $D_{p,t} - FX_{1,t}$.

The budget constraint for the government of Region 2 (EMEs) is

$$FX_{2,t} = q_{p,t} FX_{2,t+1} + T_{2,t}. \tag{11}$$

The variables $D_{p,t}$, $FX_{1,t}$ and $FX_{2,t}$ are time varying but exogenous. In the quantitative exercise, these variables replicate the observed dynamics of public debt in advanced economies and FX reserves in both advanced and emerging economies.

2.4 General equilibrium

Using capital letters to denote aggregate variables, the aggregate states include the bonds held by entrepreneurs, $B_{1,1,t}$, $B_{2,1,t}$, $B_{p,1,t}$, $B_{1,2,t}$, $B_{2,2,t}$, $B_{p,2,t}$, and the sunspot shocks $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$. The aggregate debt issued by the intermediate-goods firms of each country in the previous period are $D_{1,t} = B_{11,t} + B_{12,t}$ and $D_{2,t} = B_{21,t} + B_{22,t}$, respectively. The sequences of productivity—$z_{1,t}$ and $z_{2,t}$—financial variables—$\phi_{1,t}$, $\phi_{2,t}$, $\kappa_{1,t}$ and $\kappa_{2,t}$—public debt and reserves—$D_{p,t}$, $FX_{1,t}$ and $FX_{2,t}$—capital stocks—$K_{1,t}$ and $K_{2,t}$—are also relevant for the equilibrium. Since these variables are exogenous and perfectly anticipated, their full sequence going into the future is part of the state space. We denote the sequence of a variable starting at time t and going to infinity with subscript t and superscript ∞. For example, $z_{j,t}^{\infty}$

[7] Technically, the reserves of Region 1 are foreign assets, not the repurchase of its own public debt. However, since Region 1 is the aggregation of all advanced economies, it is not possible to clearly distinguish $D_{p,t}$ from $FX_{1,t}$. In reality, the reserves held by some advanced economies (for example European countries) could be in bonds issued by other advanced economies (for example, the US). Once we aggregate all advanced economies, without netting out the reserves from the public debt, it looks like advanced economies issue public bonds and then repurchase the same bonds as FX reserves.

represents the sequence of productivity for Region j from time t to ∞. To use a compact notation, we denote the state vector as

$$\mathbf{s}_t \equiv (z_{1,t}^{\infty}, z_{2,t}^{\infty}, \phi_{1,t}^{\infty}, \phi_{2,t}^{\infty}, \kappa_{1,t}^{\infty}, \kappa_{2,t}^{\infty}, D_{p,t}^{\infty}, FX_{1,t}^{\infty}, FX_{2,t}^{\infty}, K_{1,t}^{\infty}, K_{2,t}^{\infty}, \\ B_{1,1,t}, B_{2,1,t}, B_{p,1,t}, B_{1,2,t}, B_{2,2,t}, B_{p,2,t}, \varepsilon_{1,t}, \varepsilon_{2,t}).$$

Figure 3 sketches the steps to define an equilibrium by dividing the period in three sub-periods.

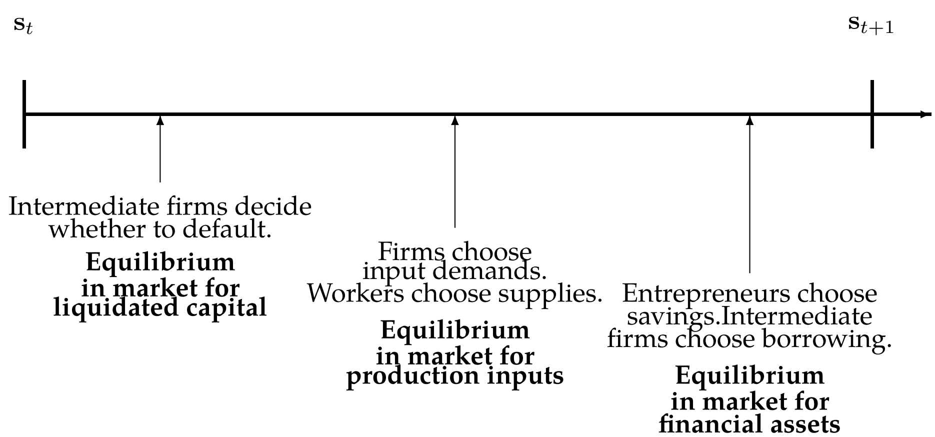

Figure 3: Timing within period t.

1. **Subperiod 1**: Given the realization of the sunspot shocks $\varepsilon_{j,t}$, intermediate-goods firms choose the fraction of debt to repay given by

$$\delta_{j,t} = \begin{cases} \frac{\kappa_{j,t} K_{j,t}}{D_{j,t}}, & \text{if} \quad D_{j,t} \geq \kappa_{j,t} K_{j,t} \text{ and } \varepsilon_{j,t} = 0 \\ 1, & \text{otherwise} \end{cases}.$$

   This expression illustrates that a financial crisis has a fundamental cause—the level of debt or leverage—and a self-fulfilling (or multiple equilibria) cause driven by sunspot shocks. Figure 4 plots the probability of a crisis as a function of the debt, $D_{j,t}$. Given the aggregate stock of capital $K_{j,t}$, the probability of a crisis is zero when the debt $D_{j,t}$ is below the threshold $\kappa_{j,t}K_{j,t}$. Above this threshold the crisis probability becomes λ, which corresponds to the probability of drawing the sunspot shock $\varepsilon_{j,t} = 0$. For values of $D_{j,t}$ greater than $K_{j,t}$ the crisis probability becomes 1 because the liquidation value of capital is always smaller than the debt. This shows that a financial crisis is not just the result of a negative sunspot shock but also the consequence of high leverage (the fundamental cause).

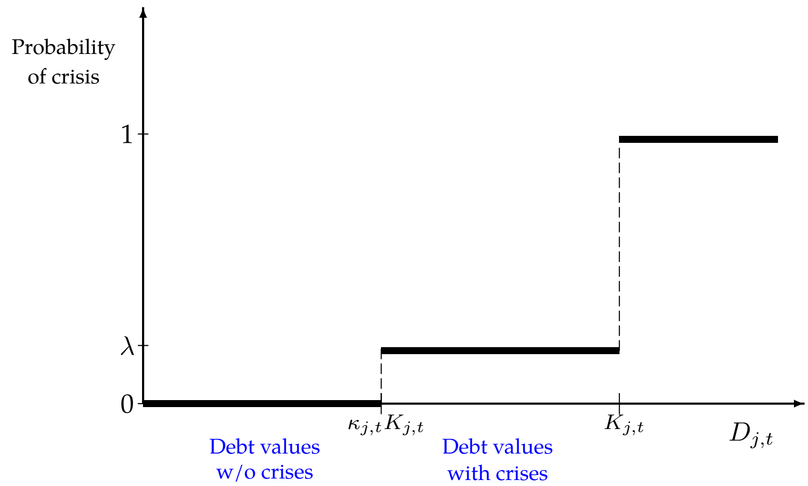

Figure 4: Probability of crisis: debt values with and without crises.

The post-default wealth of entrepreneurs is

$$M_{j,t} = \delta_{1,t}B_{1j,t} + \delta_{2,t}B_{2j,t} + B_{p,j,t}.$$

2. **Subperiod 2**: Intermediate-goods firms choose labor demand, entrepreneurs choose their demand for intermediate goods, and households choose the supply of labor. The demand for labor is

$$L_{j,t} = \left(\frac{\gamma p_{j,t}}{w_{j,t}}\right)^{\frac{1}{1-\gamma}} K_{j,t}.$$

The demand for intermediate goods depends on whether constraint (8) is binding (see Lemma 2.1). When binding, the aggregate demand for intermediate goods is

$$X_{j,t} = \left(\frac{1}{\phi_{j,t} p_{j,t}}\right) M_{j,t}.$$

If constraint (8) is not binding, the aggregate demand for intermediate inputs is determined by the supply, that is,

$$X_{j,t} = L_{j,t}^{\gamma} K_{j,t}^{1-\gamma}.$$

The aggregate supply of labor is derived from the household's firs-order condition (1), which we can re-arrange as

$$H_{j,t} = \left(\frac{w_{j,t}}{z_{j,t-1}^{1/\gamma}}\right)^{\nu}.$$

The stock of capital evolves exogenously. Market-clearing in the labor market and in the intermediate goods market determine the wage rate $w_{j,t}$ and the price for intermediate goods $p_{j,t}$, respectively.

3. **Subperiod 3**: The end-of-period wealth of entrepreneurs is

$$A_{j,t} = M_{j,t} + z_{j,t} X_{j,t} - p_{j,t} X_{j,t}.$$

According to Lemma 2.2, a fraction $1-\beta$ is consumed while the remaining fraction β is saved in new bonds: A fraction $\theta_{1,t}$ in private bonds issued by Region 1, a fraction $\theta_{2,t}$ in private bonds issued by Region 2, and the remaining fraction $1-\theta_{1,t}-\theta_{2,t}$ in public bonds issued by the government of Region 1. Intermediate firms choose the new debt $D_{j,t+1}$.

Market-clearing in the three financial markets requires

$$B_{1,1,t+1} + B_{1,2,t+1} = D_{1,t+1}, \quad (12)$$

$$B_{2,1,t+1} + B_{2,2,t+1} = D_{2,t+1}, \quad (13)$$

$$B_{p,1,t+1} + B_{p,2,t+1} + FX_{1,t+1} + FX_{2,t+1} = D_{p,t+1}. \quad (14)$$

Because of capital mobility and cross-region heterogeneity, the net foreign asset positions of the two regions could be different from zero. Formally, in Region 1 we could have $B_{1,1,t+1} + B_{2,1,t+1} + B_{p,1,t+1} +$

$FX_{1,t+1} \neq D_{1,t+1} + D_{p,t+1}$, and in Region 2 we could have $B_{1,2,t+1} + B_{2,2,t+1} + B_{p,2,t+1} + FX_{2,t+1} \neq D_{2,t+1}$. Appendix F derives the region-specific and world resource constraints implied by the market-clearing conditions of labor and financial markets, and the budget constraints of households, firms, entrepreneurs and governments. From these results, we can derive the region-specific trade balance, current account and NFA positions. For instance, Region 1's beginning-of-period NFA (after the borrower's default decision is made) is given by $\delta_{2,t}b_{2,1,t} - \delta_{1,t}b_{1,2,t} - (b_{p,2,t} + FX_{2,t})$.

Competition also implies that the price paid by entrepreneurs to purchase private debt is consistent with the interest rate, that is,

$$q_{j,t} = \frac{\mathbb{E}_{t+1}\delta_{j,t+1}}{\overline{R}_{j,t}}. \tag{15}$$

The above condition relates the price of private bonds $q_{j,t}$ to their expected return. A similar condition applies to public bonds, that is, $q_{p,t} = \frac{1}{\overline{R}_{j,t}}$.

The supply of private bonds is derived from the borrowing decisions of intermediate-goods firms (equation (6)),

$$\frac{1}{\overline{R}_{j,t}} = \beta + \Phi\left(\frac{D_{j,t+1}}{\kappa_{j,t+1}K_{j,t+1}}\right).$$

Using equation (15), we can rewrite the condition as

$$q_{j,t} = \left[\beta + \Phi\left(\frac{D_{j,t+1}}{\kappa_{j,t+1}K_{j,t+1}}\right)\right]\mathbb{E}\delta_{j,t+1}. \tag{16}$$

The assumption that $z_{j,t}, \phi_{j,t}, \kappa_{j,t}, FX_{j,t}, D_{p,t}, K_{j,t}$ are time-varying and the possibility that intermediate-goods firms could default imply that the economy does not converge to a steady state. Instead, it displays stochastic dynamics driven by the sunspot shocks.

These dynamics evolves as follows. The sunspot shocks can take two values: $\varepsilon_{j,t} = 0$ (with *possible* market freeze) and $\varepsilon_{j,t} = 1$ (no market freeze). The realization $\varepsilon_{j,t} = 0$ could generate a drop in the liquidation value of capital (if the leverage of the region is sufficiently high), which in turn leads to a financial crisis where bonds are only partially repaid.

This redistributes wealth from lenders (final-goods firms) to borrowers (intermediate-goods firms). The decline in entrepreneurs' wealth $M_{j,t}$, then, reduces the demand for intermediate goods which in turn lowers its price $p_{j,t}$. Intermediate-goods firms respond to the price drop by reducing their demand for labor and, at equilibrium, there is lower employment and production. This is the mechanism through which financial crises have real macroeconomic consequences.

2.5 Sequential property of the equilibrium

The particular structure of the model allows us to solve for the equilibrium at time t independently of future equilibria as if the model were static. More precisely, given the states s_t, we can find the values of all equilibrium variables at time t by solving the system of nonlinear equations listed in Appendix E. This allows us to solve the model sequentially. For example, to solve for the sequence of equilibria from $t = 1991$ to $t = 2020$, we first solve for the equilibrium at $t = 1991$. We then solve for the equilibrium at $t = 1992$, and continue until $t = 2020$. Note that this property would not hold if investments were endogenous and households were risk-averse.

The sequential property of the equilibrium allows us to reduce the sufficient set of state variables. In general, the equilibrium depends on the whole time-varying sequences $z_{j,t}$, $\phi_{j,t}$, $\kappa_{j,t}$, $FX_{j,t}$, $D_{p,t}$, $K_{p,t}$ from t to infinity. However, thanks to the sequential property of the equilibrium, variables at time t are only affected by $z_{j,t}$, $\phi_{j,t}$, $\kappa_{j,t}$, $\kappa_{j,t+1}$, $FX_{j,t+1}$, $D_{p,t+1}$, $K_{j,t}$ and $K_{j,t+1}$. Therefore, from now on, to characterize the equilibrium we redefine the sufficient set of state variables as

$$\begin{aligned}\mathbf{s}_t \equiv \big(&z_{1,t}, z_{2,t}, \phi_{1,t}, \phi_{2,t}, \kappa_{1,t}, \kappa_{2,t}, \kappa_{1,t+1}, \kappa_{2,t+1}, FX_{1,t+1}, FX_{2,t+1}, D_{p,t+1},\\ &K_{1,t}, K_{2,t}, K_{1,t+1}, K_{2,t+1}, B_{1,1,t}, B_{2,1,t}, B_{p,1,t}, B_{1,2,t}, B_{2,2,t}, B_{p,2,t}, \varepsilon_{1,t}, \varepsilon_{2,t}\big).\end{aligned}$$

This property will be useful for the quantitative application of the model. In particular, we will apply it to construct the sequences of exogenous productivity $z_{j,t}$ and financial variables $\phi_{j,t}$ and $\kappa_{j,t}$ for which the model replicates the targeted empirical data, given the calibrated parameters.

2.6 Other properties and remarks

Another property of the equilibrium worth emphasizing is that the risk-free interest rate is on average lower than the rate of time preference (or, equivalently, the price of a risk-free bond is higher than the inter-temporal discount factor β). In models with precautionary savings, this property holds because of the incentive to build a buffer for self-insurance. In our model, instead, entrepreneurs are willing to hold private and public debt even if the interest rate is lower than the rate of time preference because of its inside money-convenience yield: it is a financial asset that facilitates production. Provided that constraint (8) is binding, entrepreneurs receive a benefit from holding bonds that is additional to the payment of interests.

The equilibrium property by which final-goods firms are net savers and intermediate-goods firms are borrowers is important for the macroeconomic consequences of a financial crisis. Because final-goods producers have a positive financial position, a crisis redistributes wealth away from them and toward intermediate-goods producers. The drop in entrepreneurial net worth causes a decline in the demand for intermediate goods which, in turn, reduces the demand for labor and generates a macroeconomic contraction. In an environment in which final-goods producers are net borrowers, the lower repayments of debt associated with a financial crisis would increase the net worth of these firms and would have the opposite macroeconomic consequence.[8]

Having some producers with a positive financial position is consistent with the recent changes in the financial structure of US corporations characterized by higher holdings of financial assets. This suggests that the proportion of financially dependent firms has declined over time, which is consistent with the empirical findings of Shourideh and Zetlin-Jones (2012) and Eisfeldt and Muir (2016).

The large accumulation of financial assets by producers—often referred to 'cash'—is related to the significance of business savings. Although the rising savings of US corporations has attracted considerable attention in the literature (see, for example, Riddick and Whited (2009) and Begenau and Palazzo (2021)), this is not just a US phenomenon. Busso et al. (2016) document the share of savings done by firms both in advanced and emerging economies and present evidence that in Latin America this share is

[8]It is possible to rewrite the model so that intermediate-goods firms are net lenders and final-goods firms are net borrowers. What matters, however, is that (i) a crisis redistributes wealth from units that have a higher marginal value of wealth to those with a lower marginal value of wealth, and (ii) the productions of the two units are complementary. If the productions of the two units were substitutable, the contraction of adversely affected firms could be offset by the expansion of firms that were positively affected.

even larger than in advanced economies. The importance of business savings is also documented in Bebczuk and Cavallo (2016). Using data for 47 countries over 1995–2013, they show that the contribution of businesses to national savings is more than 50%.

The increase in corporate cash suggests that more and more firms borrow less than what could be available to them, and our entrepreneurial sector captures the growing importance of these firms. It also captures the significant heterogeneity among corporate firms as many of them are net borrowers and have become more leveraged over time. Most likely, those are firms that own substantial tangible assets. In our model, they are represented by intermediate-goods producers while corporations that own large amounts of cash are represented by final-goods producers.[9]

3 Quantitative analysis

We assess quantitatively through the lens of the model how the *observed* accumulation of FX reserves and issuance of public debt impacted financial and macroeconomic volatility over the past three decades. The quantitative exercise applies to the period 1991-2020 and follows these steps:

1. Calibration of structural parameters.
2. Construction of sequences for $z_{1,t}$, $z_{2,t}$, $\phi_{1,t}$, $\phi_{2,t}$, $\kappa_{1,t}$, $\kappa_{2,t}$.
3. Counterfactual simulations to assess the impact of changes in $FX_{1,t}$, $FX_{2,t}$ and $D_{p,t}$.

To perform the first two steps, we use various international data sources, primarily the World Bank's *World Development Indicators* and the *External Wealth of Nations* database from Lane and Milesi-Ferretti (2018). Aggregate variables for Advanced and Emerging Economies are constructed by aggregating individual country variables. The countries included in AEs and EMEs are listed at the bottom of Figure 5.

[9]See Kalemli-Ozcana, Sorensen, and Yesiltas (2012) for stylized facts about bank and firm leverage using international micro data.

3.1 Calibration of structural parameters

The model is calibrated at an annual frequency and the discount factor is set to $\beta = 0.93$, implying an annual intertemporal discount rate of about 7%. We set the elasticity of labor supply to $\nu = 1$, the labor share parameter in production to $\gamma = 0.6$, and the depreciation rate to $\tau = 0.08$. These values are commonly used in the literature to calibrate macroeconomic models.

The probability that the liquidation price of capital drops to $\kappa_{j,t} < 1$ (i.e., the probability of a realization of the sunspot shock $\varepsilon = 0$) is $\lambda = 0.04$. This is within the range of crisis probabilities used in the literature (see, for example, Bianchi and Mendoza (2018)). It implies that crises are low probability events, every twenty-five years on average. Since sunspot shocks are region-specific and independent across regions, a *global* financial crisis is an even rarer event, with a probability of $0.04 \times 0.04 = 0.0016$.

The last parameter we calibrate is η. This parameter determines the sensitivity of the borrowing cost to the borrowed amount. Unfortunately, we have limited information to pin down this parameter. We set it to $\eta = 0.1$ but we will conduct a sensitivity analysis to gauge its relevance for our results (see Appendix G).

Table 1: Parameter values.

Description	*Parameter*	*Value*
Discount factor	β	0.930
Share of labor in production	γ	0.600
Depreciation rate	τ	0.080
Elasticity of labor supply	ν	1.000
Probability of crises (low sunspot shock)	λ	0.040
Cost of borrowing	η	0.100
Long-run growth rate of productivity	g	0.010

In the long-run, productivity $z_{j,t}$ grows at rate g in both regions. Given the growth rate of productivity, the long-run growth rate of both capital and output is $(1 + g)^{1/\gamma} - 1$. We set g to the average growth rate of productivity in advanced economies, $z_{1,t}$, over the sample period 1991-2020. The construction of the productivity series will be described in the next subsection. Table 1 provides the full list of calibrated parameters.

3.2 Construction of sequences for $z_{j,t}$, $\phi_{j,t}$, $\kappa_{j,t}$

Regional differences in size and financial structure are generated by the deterministic sequences $z_{j,1991}^{2020}$, $\phi_{j,1991}^{2020}$, $\kappa_{j,1991}^{2021}$.

Productivity $z_{j,t}$. The productivity series $z_{j,1991}^{2020}$ are constructed as Solow residuals from the production function. To construct them, we need measures of production inputs and outputs. For output, we use GDP at nominal exchange rates, not PPP. Since movements in nominal exchange rates affect the purchasing power of a country in the acquisition of foreign assets, our productivity measures should also reflect movements in exchange rates. Another factor that contributes to generate differences in aggregate GDP is population growth. Since population is not explicitly modeled, the constructed sequences of productivity also capture changes in population.

Denote by $P_{j,t}$ the nominal price index for country j expressed in US dollars. The price is calculated by multiplying the price in local currency with the dollar exchange rate. We can then define the nominal (dollar) aggregate output of country j as

$$P_{j,t}Y_{j,t} = P_{j,t}\hat{z}_{j,t}L_{j,t}^{\gamma}K_{j,t}^{1-\gamma}N_{j,t},$$

where $\hat{z}_{j,t}$ is actual productivity, $L_{j,t}$ is per-capita employment, $K_{j,t}$ is per-capita capital, and $N_{j,t}$ is population.

The above expression for final output aggregates the whole business sector. Since intermediate-goods production is $X_{j,t} = L_{j,t}^{\gamma}K_{j,t}^{1-\gamma}$, replacing X_t in the final-goods production, $z_{j,t}X_{j,t}$, we obtain per-capita output $z_{j,t}L_{j,t}^{\gamma}K_{j,t}^{1-\gamma}$. Aggregate final production, $Y_{j,t}$, is the product of per-capita production and population.

Deflating the nominal GDP in both regions by the price index in country 1, we obtain

$$Y_{1,t} = \hat{z}_{1,t}L_{1,t}^{\gamma}K_{1,t}^{1-\gamma}N_{1,t},$$

$$\frac{P_{2,t}Y_{2,t}}{P_{1,t}} = \left(\frac{P_{2,t}\hat{z}_{2,t}}{P_{1,t}}\right)L_{2,t}^{\gamma}K_{2,t}^{1-\gamma}N_{2,t},$$

Thus, aggregate productivities in the model correspond to

$$z_{1,t} = \hat{z}_{1,t} N_{1,t},$$
$$z_{2,t} = \hat{z}_{2,t} \left(\frac{P_{2,t} N_{2,t}}{P_{1,t}} \right).$$

Since $P_{2,t}$ is the dollar price of output in emerging economies, the ratio $P_{2,t}/P_{1,t}$ corresponds to the real exchange rate. Thus, the above expressions show that $z_{1,t}$ and $z_{2,t}$ also reflect cross-region differences in real exchange rates and population, in addition to actual TFP.

The productivity sequences that we use in the model are calculated from the data as

$$z_{1,t} = \frac{Y_{1,t}}{L_{1,t}^{\gamma} K_{1,t}^{1-\gamma}}, \quad (17)$$

$$z_{2,t} = \frac{P_{2,t} Y_{2,t}/P_{1,t}}{L_{2,t}^{\gamma} K_{2,t}^{1-\gamma}}. \quad (18)$$

The numerator is total real GDP, deflated by the nominal price in advanced economies. If the real exchange rate of emerging economies appreciates, it will be reflected in higher relative productivity. Although this does not increase actual productivity, it raises the ability of these countries to purchase assets in advanced economies, which is important for the general equilibrium. Also notice that changes in relative prices could simply reflect movements in nominal exchange rates. Still, when the currencies of emerging economies appreciate, assets created in advanced economies become cheaper for emerging economies.

In order to use equations (17) and (18) to construct the productivity sequences, we need empirical counterparts for $Y_{1,t}$, $P_{2,t}Y_{2,t}/P_{1,t}$, $L_{1,t}$, $L_{2,t}$, $K_{1,t}$, and $K_{2,t}$, which we obtain from the *World Development Indicators*. These variables are plotted in the first three panels of Figure 5.

The output variables $Y_{1,t}$ and $P_{2,t}Y_{2,t}/P_{1,t}$ are obtained by aggregating the GDP of countries in advanced and emerging economies, respectively, both expressed at constant US dollars. For the labor input $L_{j,t}$ we use employment-to-population ratio (population over 15 years of age).

Capital $K_{j,t}$ is constructed from the investment and depreciation data by applying the perpetual inventories method. We have data on investment and depreciation, $I_{j,t}$ and $DEP_{j,t}$, from the *World Development Indicators*.

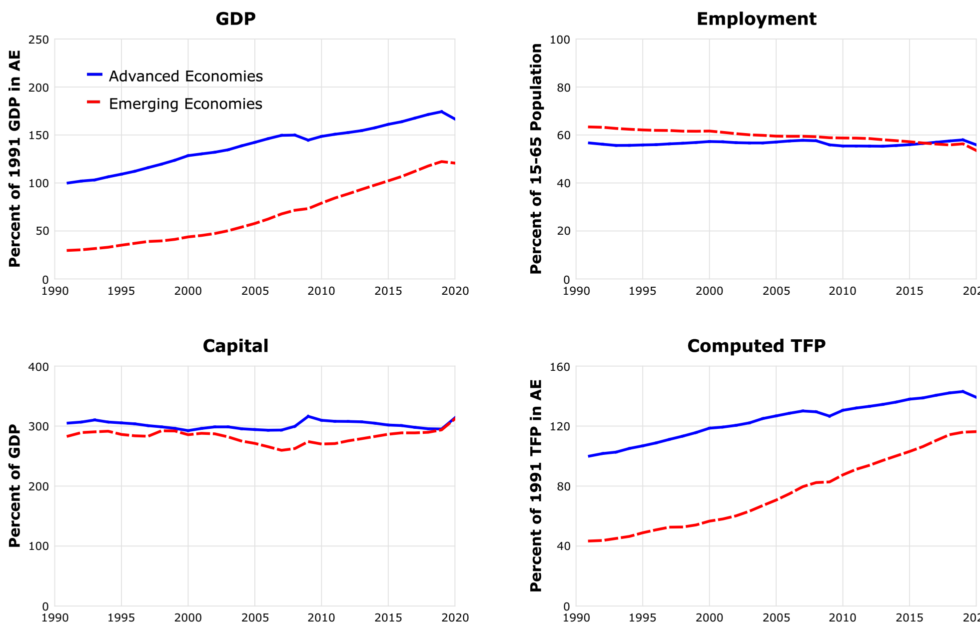

Figure 5: GDP, labor, capital and TFP in Advanced and Emerging Economics, 1991-2020.

Note: **Advanced economies**: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, United States. **Emerging economies**: Algeria, Argentina, Brazil, Bulgaria, Chile, China, Czech Republic, Colombia, Estonia, Hong Kong, Hungary, India, Indonesia, Israel, South Korea, Latvia, Lithuania, Malaysia, Mexico, Pakistan, Peru, Philippines, Poland, Romania, Russia, Saudi Arabia, Singapore, South Africa, Thailand, Turkey, Ukraine, Venezuela. **Sources**: World Development Indicators (World Bank).

After guessing the initial value of capital, $K_{j,0}$, we compute $K_{j,1} = K_{j,0} - DEP_{j,0} + I_{j,0}$. Given the calculated value of $K_{j,1}$, we then compute $K_{j,2} = K_{j,1} - DEP_{j,1} + I_{j,1}$, and continue until the end of the sample period. At this point we repeat the whole procedure after changing the guess for $K_{j,0}$ until the capital-GDP ratio displays no trend over the sample period. The last panel of Figure 5 plots the constructed productivity series.

Financial structure $\phi_{j,t}$ and $\kappa_{j,t}$. The time-varying parameter $\phi_{j,t}$ is important for the *demand* of financial assets, in the spirit of Mendoza, Quadrini, and Ríos-Rull (2009): Higher values of $\phi_{j,t}$ increase the demand because more financial assets are needed for production (working capital, etc.).

The time-varying parameter $\kappa_{j,t}$ is important for the *supply* of financial assets, in the spirit of Caballero, Farhi, and Gourinchas (2008): Higher values of $\kappa_{j,t}$ increase the incentive for intermediate firms to borrow.[10]

The sequences of $\phi_{1,t}$, $\phi_{2,t}$, $\kappa_{1,t}$, $\kappa_{2,t}$ are constructed so that the model replicates four empirical series over the period 1991-2020: (i) private domestic credit-to-GDP ratio in advanced economies, (ii) private domestic credit-to-GDP ratio in emerging economies, (iii) Net Foreign Asset position of advanced economies, (iv) US risk-free real interest rate. These empirical series are plotted in the top three panels of Figure 6.

Two important caveats are worth noting. First, the empirical series include private credit received by all private domestic sectors, not just businesses, while in the model we only have business debt. However, we should think of business debt in the model as the consolidation of businesses and households with negative financial positions (net borrowers). We did the same with physical capital: we assumed that physical capital is held by firms but in reality some of the capital—for example, residential capital—is directly held by households. At the cost of increasing notational complexity, we could split domestic credit and capital into the components directly held by businesses and households. Provided that households face the same increasing cost of borrowing and have similar ability to renegotiate the debt, the model would have the same properties.

A second caveat is that the financial liabilities issued in the model (private credit) are held by the business sector (entrepreneurial firms). In reality, they are also held directly or indirectly (through financial intermediaries) by households. Therefore, the financial assets in the model should be interpreted as resulting from the consolidation of business firms and households with positive financial positions (financial assets greater than financial liabilities).

The precise equations that map the four empirical targets to the corresponding variables in the model are as follows:

[10] Note that in contrast with these and other studies in the global imbalances literature, our goal is not to explain global imbalances but to understand how the surge in emerging markets demand for FX reserves affects financial and macroeconomic volatility. Theories of global imbalances proposed in the literature do not typically deal with their implications for financial and macroeconomic volatility.

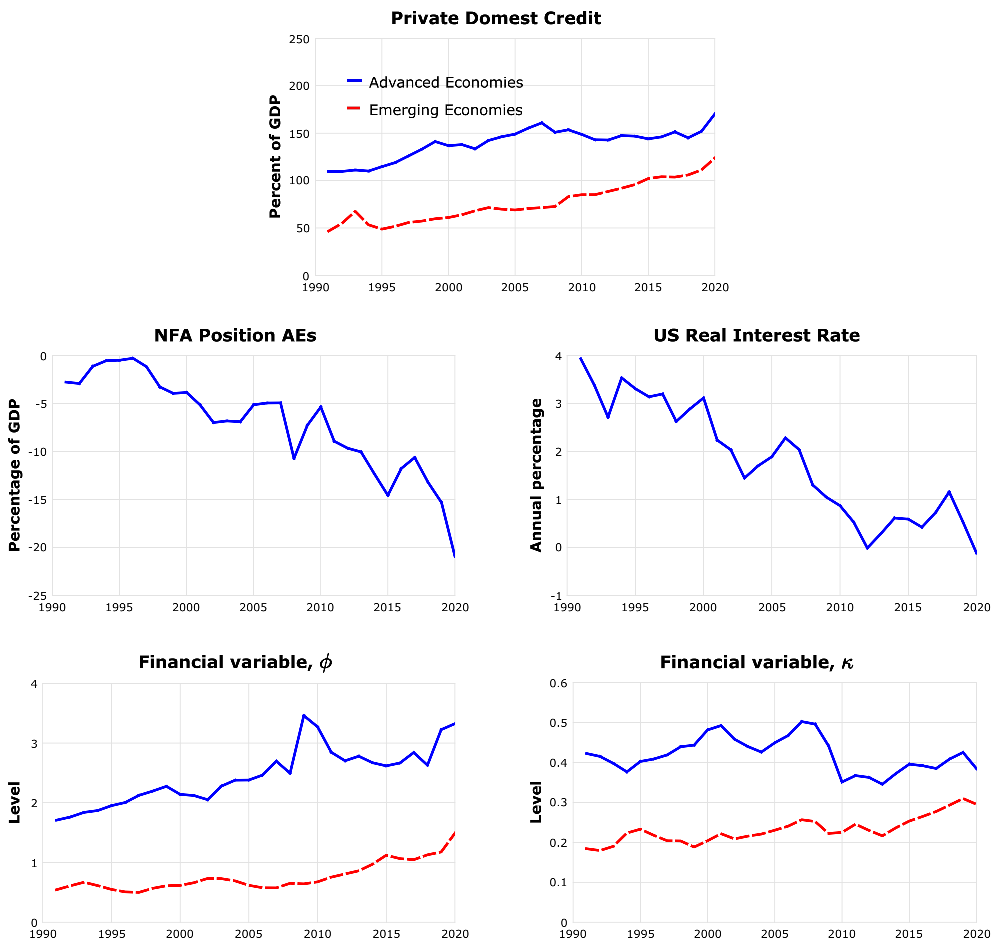

Figure 6: Private Domestic Credit, Net Foreign Asset Position, Interest Rate and Financial Parameters, 1991-2020.

Note: **Advanced economies**: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, United States. **Emerging economies**: Algeria, Argentina, Brazil, Bulgaria, Chile, China, Czech Republic, Colombia, Estonia, Hong Kong, Hungary, India, Indonesia, Israel, South Korea, Latvia, Lithuania, Malaysia, Mexico, Pakistan, Peru, Philippines, Poland, Romania, Russia, Saudi Arabia, Singapore, South Africa, Thailand, Turkey, Ukraine, Venezuela. **Sources**: World Development Indicators (World Bank) and External Wealth of Nations database (Lane and Milesi-Ferretti (2018)).

$$\text{Private Credit-to-GDP AEs} = \frac{q_{1,t}D_{1,t+1}}{Y_{1,t}}, \tag{19}$$

$$\text{Private Credit-to-GDP EMEs} = \frac{q_{2,t}D_{2,t+1}}{Y_{2,t}}, \tag{20}$$

$$\text{NFA-to-GDP in AEs} = \frac{q_{1,t}B_{1,1,t+1} + q_{2,t}B_{2,1,t+1} + q_{p,t}B_{p,1,t+1} + q_{p,t}FX_{1,t+1} - q_{1,t}D_{1,t+1} - q_{p,t}D_{p,t+1}}{Y_{1,t}}, \tag{21}$$

$$\text{US real interest rate} = \frac{1}{q_{p,t}} - 1. \tag{22}$$

The terms in the right-hand-side are equilibrium objects that we can compute from the model for given values of $\phi_{1,t}$, $\phi_{2,t}$, $\kappa_{1,t}$, $\kappa_{2,t}$, $\kappa_{1,t+1}$ and $\kappa_{2,t+1}$.[11] Given the sequential property of the equilibrium (see Section 2.5), we find the values of these variables in period t by solving the system of nonlinear equations listed in Appendix E. After initializing $\kappa_{1,1991}$ and $\kappa_{2,1991}$, we solve for $\phi_{1,1991}$, $\phi_{2,1991}$, $\kappa_{1,1992}$ and $\kappa_{2,1992}$ by applying two nested nonlinear solvers.[12] The inner solver finds the model's equilibrium given the values of $\phi_{1,1991}$, $\phi_{2,1991}$, $\kappa_{1,1992}$ and $\kappa_{2,1992}$. The outer solver uses the inner solution to check whether the equilibrium associated with the particular values of $\phi_{1,1991}$, $\phi_{2,1991}$, $\kappa_{1,1992}$ and $\kappa_{2,1992}$ satisfies conditions (19)-(22). It then updates the values of $\phi_{1,1991}$, $\phi_{2,1991}$, $\kappa_{1,1992}$ and $\kappa_{2,1992}$ until conditions (19)-(22) are satisfied. At this point we move to the next period and find the values of $\phi_{1,1992}$, $\phi_{2,1992}$, $\kappa_{1,1993}$ and $\kappa_{2,1993}$, and continue until we have solved for all sample years 1991-2020.

Figure 7 provides a stylized illustration of the financial market equilibrium that yields a clear intuition for how the values of the four time-varying financial parameters are identified using four data points at any given date.

The interest rate equalizes the global demand for assets (sum of the demands from both regions) to the global supply (sum of the supplies from both regions). Here demands and supplies contain both private and public components. In advanced economies, the demand for financial assets is given by $q_{1,t}B_{1,1,t+1} + q_{2,t}B_{2,1,t+1} + q_{p,t}B_{p,1,t+1} + q_{p,t}FX_{1,t+1}$, while the supply is $q_{1,t}D_{1,t+1} + q_{p,t}D_{p,t+1}$. They are plotted in the first panel of Figure 7. In emerging economies, instead, the demand for financial assets is given by $q_{1,t}B_{1,2,t+1}+q_{2,t}B_{2,2,t+1}+q_{p,t}B_{p,2,t+1}+q_{p,t}FX_{2,t+1}$ while the supply is $q_{2,t}D_{2,t+1}$. They are plotted in the second panel of Figure 7.

The parameters $\phi_{j,t}$ and $\kappa_{j,t+1}$ determine the positions of the demand and supply curves in region j. Given the public demand for financial assets, $FX_{j,t+1}$, and the public supply, $D_{p,t+1}$, an increase in $\phi_{j,t}$ shifts the *demand* of region j to the right while an increase in $\kappa_{j,t+1}$ shifts the *supply* of region j to the right. The four variables used to identify $\phi_{1,t}$, $\phi_{2,t}$, $\kappa_{1,t}$ and $\kappa_{2,t}$ are indicated in the graph with a circle: (i) the debt issued by region 1; (ii) the debt issued by region 2; (iii) the net foreign asset position of region 1; (iv) the world interest rate.

[11] It also requires the constructed productivity and capital series plotted in Figure 5, and the empirical series for FX reserves and public debt plotted in Figure 1.

[12] As long as the realizations of the sunspot shock in 1991 are not those causing a crisis (which is our assumption), the values of $\kappa_{1,1991}$ and $\kappa_{2,1991}$ are irrelevant as initial states.

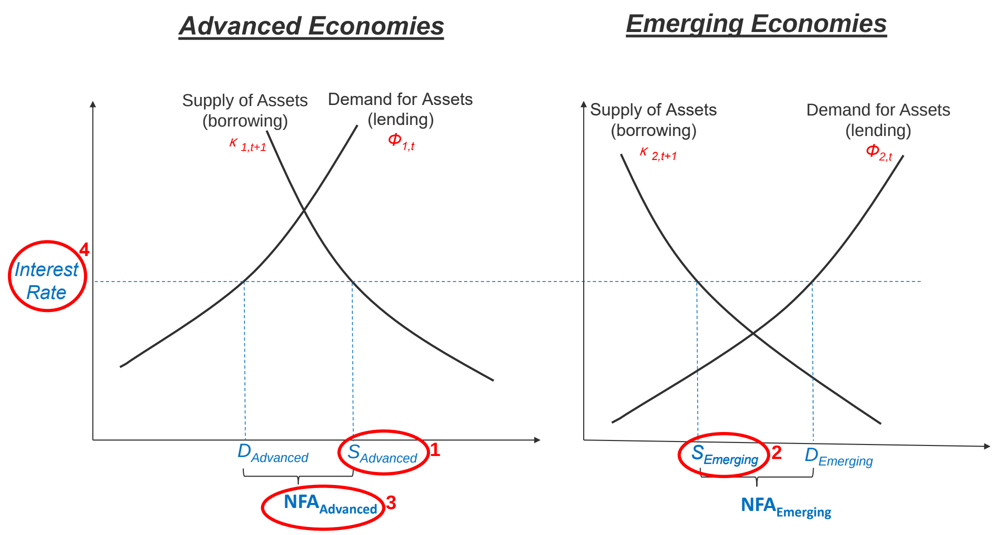

Figure 7: Identification of financial structure parameters.

As indicated in equations (19)-(22), the empirical counterparts of these four variables are: (i) Private domestic credit in AEs; (ii) Private domestic credit in EMEs; (iii) Net foreign asset position of AEs; (iv) US interest rate. The goal is to find the values of the four financial parameters so that the positions of the supply and demand curves in the two regions give rise to an equilibrium that matches the four empirical targets.

Public debt and FX reserves are important because they are part of the demands and supplies of assets. For example, an increase in FX reserves, either from advanced economies or emerging economies, moves the demand for assets to the right, leading to a reduction in the world interest rate. On the other hand, an increase in public debt issued by advanced economies shifts their supply of assets to the right. This leads to an increase in the world interest rate.

A complication in the construction of the sequences $\phi_{j,t}$ and $\kappa_{j,t}$ is that the constructed values of these variables depend on the stochastic realizations of the sunspot shocks $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$. Therefore, we have to choose a particular sequence of $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ over the 1991-2020 period. The chosen sequence contains $\varepsilon_{j,t} = 1$ (no crisis) in all simulated years with only few exceptions. For emerging economies it takes the value of zero in 1997 and 2009 ($\varepsilon_{2,1997} = 0$ and $\varepsilon_{2,2009} = 0$). These two years correspond, respectively, to the 1997 Sudden Stops in South-East Asia and to the Global Financial Crisis that started in 2008 and fully materialized in 2009. Both crises had an impact on emerging economies. For advanced economies, instead, it takes the value of zero only in 2009 ($\varepsilon_{1,2009} = 0$) reflecting, again, the Global Financial Crisis. It is important to point out that, even though we calibrate the model assuming a specific sequence of shocks, agents do not anticipate them and make decisions based on their stochastic properties.

The computed series for $\phi_{j,t}$ and $\kappa_{j,t}$ are plotted in the last two panels of Figure 6. The results show that, in order to match the observed paths of private domestic credit, NFA and world interest rate, the model requires a sustained increase in ϕ in both regions. Although $\phi_{1,t}$ is uniformly higher than $\phi_{2,t}$ and attained a value around 3.3 by 2020, growing by a factor of 1.8 since 1990, the increase was proportionally larger in emerging economies, where it grew by a factor of 3 from 0.5 to 1.5 over 1990-2020. Thus, the need for the convenience yield or productive use of financial assets grew faster in emerging economies.

The model predicts that $\kappa_{1,t}$ fluctuated in the 0.4-0.5 range with a drop around the time of the GFC and it reverted only partially ($\kappa_{1,t}$ fell roughly 20% from its 0.5 maximum in 2006-2007 to about 0.4 in 2010-2020). $\kappa_{2,t}$, instead, rose relatively steadily from 0.2 in 1990 to 0.3 in 2020, a 50% increase. This change in $\kappa_{2,t}$ sustains the faster growth of private sector credit in emerging economies during 2010-2020 shown in the top panel.

3.3 Counterfactual simulations

In this section, we explore how the evolution of FX reserves and public debt issued by AEs during the 1991-2020 period (Figure 1) affected the observed macroeconomic dynamics. We do so by conducting counterfactual simulations in which we impose that either FX reserves or public debt grow at their lower long-run rate while productivity and financial parameters are still allowed to display the dynamics we solved for earlier.

FX reserves. To explore the role played by the accumulation of FX reserves, we assume that in the first simulation year, 1991, $FX_{1,t}$ and $FX_{2,t}$ takes their actual values in the data. Afterwards, we impose that $FX_{1,t}$ and $FX_{2,t}$ grow at the constant rate $(1+g)^{1/\gamma}-1$. This is the long-run growth of output (recall that g is the long-run growth rate of productivity indicated in Table 1). This keeps FX reserves constant as a percentage of the trend level of output. All other time-varying exogenous variables—$z_{j,t}$, $\phi_{j,t}$, $\kappa_{j,t}$, $K_{j,t}$ and $D_{p,t}$—change as in the baseline model.

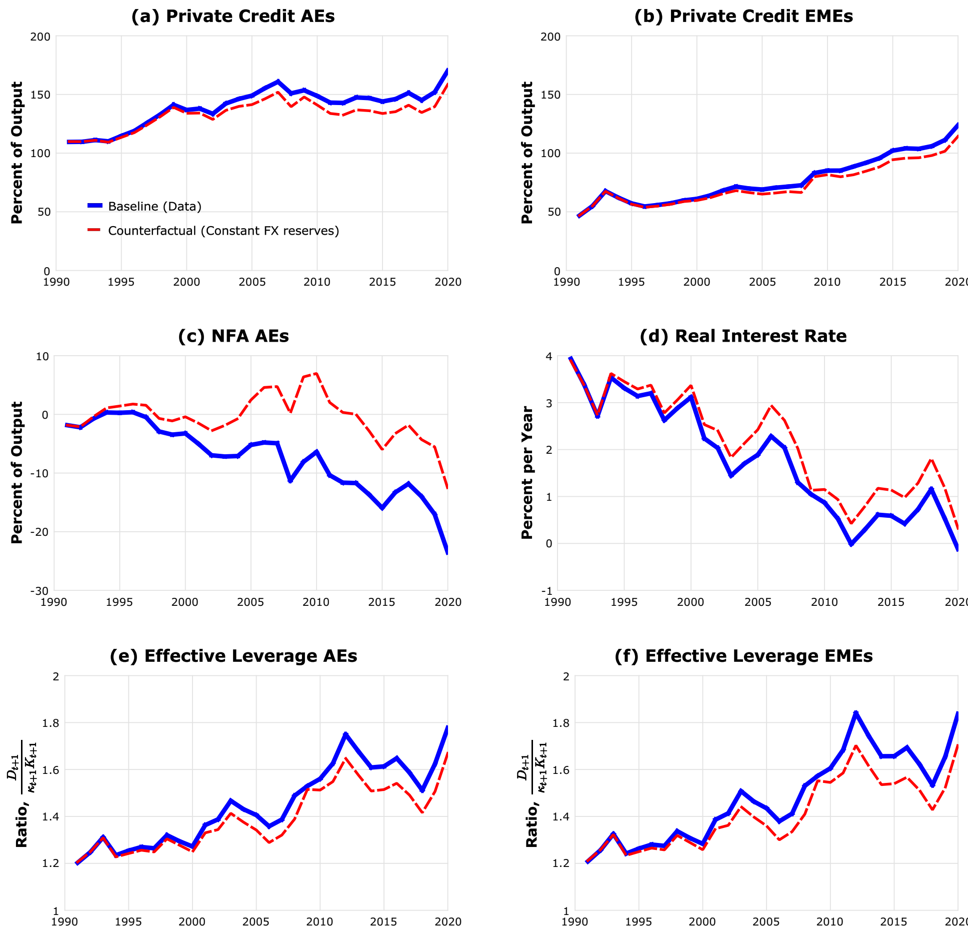

Figure 8: Counterfactual simulation with FX reserves growing at the constant long-run rate from its 1991 value.

The simulated variables are plotted in Figure 8. Each panel in this Figure plots two lines. The continuous line is for the baseline case which FX reserves take the empirical values shown in Figure 1. Hence, the continuous line in panels (a)-(d) coincides with the empirical data since the exogenous variables in the baseline case were chosen to replicate these series. The dashed line, instead, shows the dynamics generated by the model when FX reserves grow, counter-factually, at the constant long-run growth rate. The difference between the dashed and continuous lines shows the effect of removing the observed growth in reserves in excess of the long-run growth in GDP.

Panels (a) and (b) show that private credit, as a percentage of output, would have grown less in absence of the surge in FX reserves growth. Panel (d) shows why: without growth in FX reserves, the cost of borrowing—the interest rate—would have fallen by a smaller amount and the private sector would have borrowed less. With slower growth in borrowing, Panels (e) and (f) show that effective leverage—the ratio of private debt over the liquidation value of capital during crises—would have risen less. This is important for understanding the implication for aggregate volatility, as we will discuss shortly.

Looking now at panel (c), we observe that a sizable portion of the decline in the net foreign asset position of advanced economies can be attributed to the accumulation of FX reserves by emerging economies. Recall that most of the increase in FX reserves came from emerging economies.

It is worth noting that the differences between continuous and dashed lines all widen in the later years. This occurs because the surge in reserves becomes more relevant as productivity and the production need for financial assets rose (see last panel of Figure 5 and the left-bottom panel of Figure 6). It implies that the growing private demand for financial assets had to compete with the large demand for FX reserves, and this despite the fact that the reservses of EMEs actually declined between 2009 and 2020 (see Fig. 1). As we will see, however, this competition was partially alleviated by the growth in the supply of public debt from advanced economies.

Public debt. To explore the role of public debt, we conduct a similar counterfactual exercise but focusing on $D_{p,t}$—the public debt issued by AEs. We start by assuming that in the starting year 1991, $D_{p,t}$ takes the same value as in the data. Afterwards, it grows at the long-run rate $(1+g)^{1/\gamma}-1$. As for FX reserves, this assumption guarantees that public debt remains constant in the long run as a percentage of output. All other time-varying exogenous variables—$z_{j,t}$, $\phi_{j,t}$, $\kappa_{j,t}$, $K_{j,t}$ and $FX_{j,t}$—change as in the baseline model. The simulated variables are plotted in Figure 9.

As in the previous figure, the continuous line is for the baseline model while the dashed line is for the counterfactual simulation in which public debt grows at the long-run rate. The difference between the dashed line

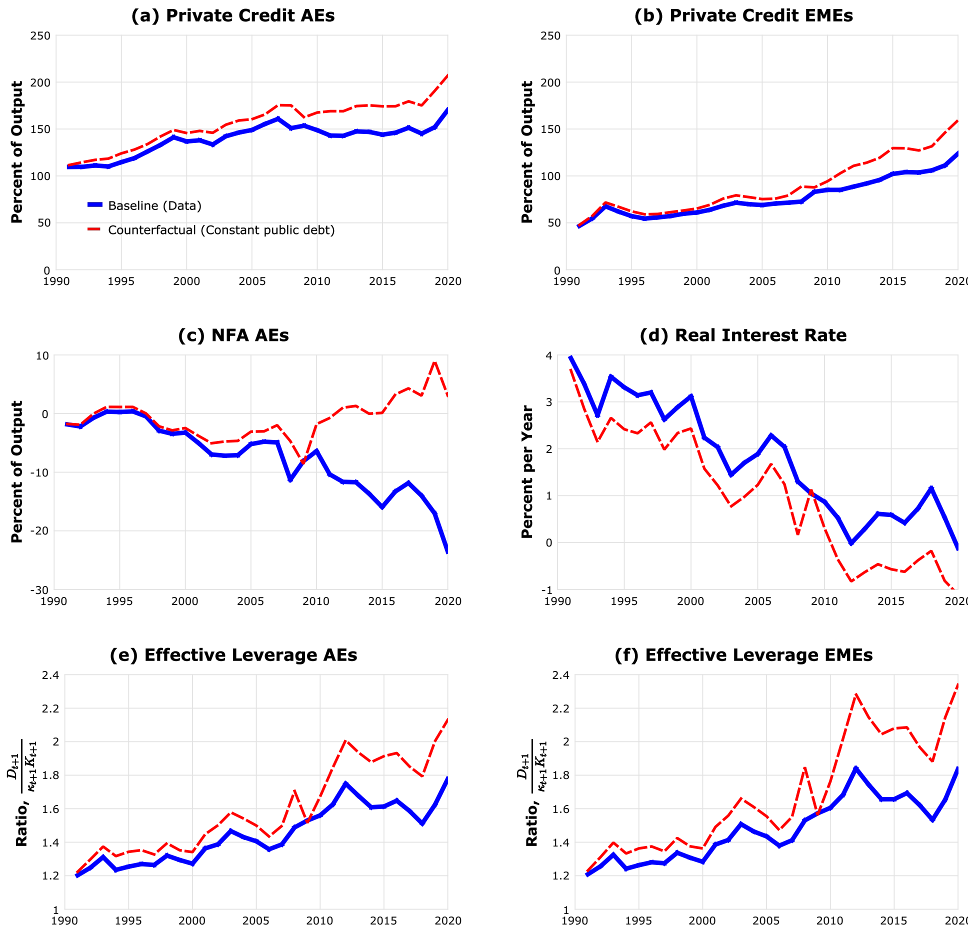

Figure 9: Counterfactual simulation with public debt of advanced economies growing at the constant long-run rate from its 1991 value.

and the continuous line captures the impact of the actual rise in public debt shown in the second panel of Figure 1.

Panels (a) and (b) show that private credit, as a percentage of output, would have grown more in absence of public debt, both in advanced and emerging economies. This is because without public debt growth, the interest rate would have fallen even more than it actually did (see Panel (d)). In addition, panels (e) and (f) show that effective leverage would have been significantly higher without public debt growth. This is important for understanding the implication for aggregate volatility (next section).

Regarding the NFA position of advanced economies, panel (c) shows that the increase in public debt contributed to the sharp decline in NFA over 2010-2020 (when public debt grew the most). Starting in 2010, the NFA of advanced economies would have *improved* without the increase in public borrowing. Therefore, both public borrowing in AEs and FX reserves accumulation in EMEs contributed to global imbalances.

As with the case of FX reserves, the role of public debt becomes more relevant quantitatively in the later years. This is partly because of the growth in private demand for financial assets driven by the rising $z_{j,t}$ and $\phi_{j,t}$, but also because 2010-2020 is when public debt in advanced economies grew the most.

4 Financial and macroeconomic volatility

We now explore how the accumulation of FX reserves and the issuance of public debt impacted financial and macroeconomic volatility. To compute measures of volatility, we simulate the model for 130 years in response to random draws of the sunspot shocks in each region: $\varepsilon_{j,t} = 0$ with probability $\lambda = 0.04$, and $\varepsilon_{j,t} = 1$ with probability $1 - \lambda = 0.96$. As explained earlier, when $\varepsilon_{j,t} = 0$ *and* leverage is sufficiently high, the liquidation price of capital drops to $\kappa_{j,t} < 1$ and the outstanding debt of private borrowers is renegotiated.

The initial 100 years of simulation are use to derive the invariant distribution of the states. During these 100 years, productivity, FX reserves and public debt all grow at the constant long-run rate, and the financial structure parameters, $\phi_{j,t}$ and $\kappa_{j,t}$, are kept constant at their 1991 values. The focus of these experiments is on the subsequent 30 years, which correspond to the 1991-2020 period. During this period $z_{j,t}$, $\phi_{j,t}$, $\kappa_{j,t}$, $K_{j,t}$, $FX_{j,t}$ and $D_{p,t}$ take the values plotted in Figures 1, 5 and 6. The whole simulation is then repeated 10,000 times, each time with a new sequence of random draws of the sunspot shocks $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ over 130 years.

The repeated simulations generate 10,000 "cross-sectional" data points for each of the 130 years. The mean of region-j output in every year t is the cross-sectional arithmetic average computed as $\overline{Y}_{j,t} = \frac{1}{10{,}000}\sum_{i=1}^{10{,}000} Y_{j,t}^i$.

We also compute the 5th and 95th percentiles of the 10,000 data points in each year. The difference between these two percentiles provides a measure of output volatility. The 5th percentile for region j, denoted by $P_{j,t}(5)$, is the threshold value for which 5 percent of the 10,000 realizations of the variable are smaller than $P_{j,t}(5)$. Formally, $\frac{1}{10{,}000}\sum_{i}^{10{,}000}\Big(1\Big|Y_{j,t}^{i} < P_{j,t}(5)\Big) = 0.05$. Similarly for the 95th percentile. We then construct a time-varying index of output volatility as the difference between the 5th and 95th percentiles, normalized by the mean of output,

$$VOL_{j,t} = \left(\frac{P_{j,t}(95) - P_{j,t}(5)}{\overline{Y}_{j,t}}\right) \times 100. \tag{23}$$

The volatility index for the period 1991-2020 is plotted in Figure 10. The continuous lines are for the baseline model while the dashed lines are for counterfactual simulations.

We conduct two counterfactuals. In the first, *detrended* FX reserves remain constant at their 1991 value, and the resulting volatility series are plotted in panels (a) and (b) for advanced and emreging economies, respectively. In the second, we keep *detrended* public debt fixed at the 1991 value, and the resulting volatility series are plotted in panels (c) and (d).

Consider first the volatility measure generated by baseline model. This is shown by the continuous lines. The model predicts that output volatility has increased significantly over the sample period. It started in 1991 at around 2% (0.8%) in advanced (emerging) economies, and ended around 5% (3.9%) in 2020. Hence, proportionally, the increase was larger in emerging than in advanced economies (a factor of 4.9 instead of 2.5).

The higher volatility generated by the baseline model is the equilibrium outcome of various factors that caused the decline in the interest rate and incentivized higher leverage in both regions (see bottom panels in Figures 8 and 9). When the economy is more leveraged, a financial crisis generates a larger redistribution of wealth from creditors to debtors, with in turn has a larger macroeconomic impact. More specifically, entrepreneurs lose a larger share of their financial wealth, forcing them to reduce the demand for intermediate goods. This, in turn, causes a larger decline in employment and production.

The two counterfactual exercises, illustrated by the dashed lines, shed light on the contributions of FX reserves and public debt to the model's predicted increase in volatility. The dashed lines in panels (a) and (b) show that, if FX reserves had grown at the lower pace assumed in the counterfactual exercise, volatility would have increased less in both regions. Thus, the surge in FX reserves across EMEs as a region has contributed to *in-*

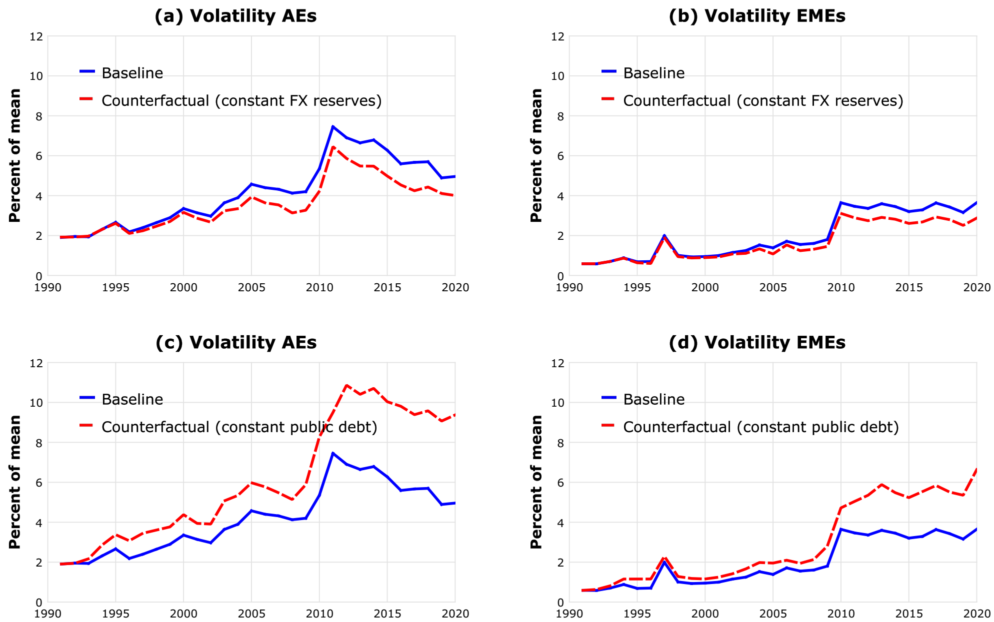

Figure 10: Output volatility and mean of effective leverage over the period 1991-2020. The volatility measure is the difference between the 5th and 95th percentiles as a percentage of the output mean. Effective leverage is the ratio of debt over the liquidation value of capital in a crisis.

creasing output volatility, instead of reducing it, and it has done so in both emerging and advanced economies.

The explanation for this finding is illustrated in Figure 8: With lower accumulation of reserves, the interest rate would have fallen less and the increase in leverage would have been lower. Financial crises, then, would have less severe macroeconomic consequences.

Another important result is that, according to the model, the surge in demand for FX reserves by emerging economies not only increased volatility in both regions, but it also caused a larger increase in volatility in AEs than in EMEs. In particular, between 2010 and 2020, panels (a) and (b) of Figure 10 show that re-introducing the surge in reserves increases volatility in advanced economies by about 1 percentage point, compared with roughly half of a percentage point in emerging economies. Hence, the surge in reserves by emerging economies as a group worsened output volatility not only for this group but, especially, for advanced economies.

This result is largely due to differences in the predicted evolution of ϕ and κ in each of the two regions during 2010-2020 (see the bottom panels of Figure 6). As noted earlier, ϕ rose in both regions, strengthening demand for financial assets for their productive use globally. In contrast, $\kappa_{1,t}$ fell about 20% while $\kappa_{2,t}$ rose about 50%, making advanced (emerging) economies more (less) vulnerable to financial crises as the liquidation price of capital dropped (rose). The larger price drop in a crisis implies a larger wealth redistribution from creditors to debtors and thus a larger drop in factor demands and output for the former.

Consider next the effects of public debt. The dashed lines in panels (c) and (d) are for the counterfactual simulation in which we keep the detrended public debt of advanced economies at the 1991 value. As we can see, with slower public debt growth, output volatility would have been significantly larger. Thus, the growth in public debt had a stabilizing effect.

The intuition is simple and it is illustrated in Figure 9: When the governments of AEs issue more debt, the supply of debt increases and its price decreases. This results in uniformly higher interest rates in the baseline case relative to the counterfactual with detrended public debt constant at its 1991 value. Intuitively, AEs' governments have to pay a higher interest rate to attract investors. But with a higher interest rate it becomes more costly for the private sector to borrow and hence leverage declines. With lower leverage, financial crises have a smaller macroeconomic impact.

Again we see a larger effect later in the sample and larger on AEs than EMEs. The reasons are the same as in the case of FX reserves: The effects are larger later in the sample because volatility increases endogenously with the growth in productivity and with the production need for financial assets. The effect is larger for AEs because $\kappa_{1,t}$ falls while $\kappa_{2,t}$ rises.

5 Government bailout

Thus far, we have examined a setup in which the accumulation of reserves does not play any direct role in the outcomes of the region that accumulates them. But, of course, FX reserves are a form of publicly-owned liquidity that could facilitate government interventions when needed. Financial crises are examples of situations in which this use of FX reserves could be especially desirable.

In this section, we extend the model by assuming that governments use FX reserves to provide liquidity and thereby contribute to stabilize the economy. In particular, since the main channel through which a financial crisis affects the macro-economy is by depleting entrepreneurial wealth, we assume that the government uses FX reserves to bail out a fraction of the financial losses incurred by entrepreneurs. For simplicity, the bailout mechanism is specified as an exogenous rule that depends on the stock of FX reserves.

5.1 Bailout mechanism

With the bailout mechanism present, the government budget constraint in Region 1 (AEs) becomes

$$FX_{1,t} + q_{p,t}D_{p,t+1} = q_{p,t}FX_{1,t+1} + D_{p,t} + T_{1,t} + Bail_{1,t}. \tag{24}$$

This is the same budget constraint as the one specified in equation (10) but with the additional variable $Bail_{1,t}$ on the right-hand-side as a new use of funds. This variable denotes transfers that the government makes to domestic entrepreneurs in the eventuality of a bailout. A similar modification arises in the budget constraint of the government of Region 2 (EMEs),

$$FX_{2,t} = q_{p,t}FX_{2,t+1} + T_{2,t} + Bail_{2,t}. \tag{25}$$

We now specify how the bailout transfers are determined. Consider first the aggregate losses incurred by entrepreneurs in region j,

$$Loss_{j,t} = (1 - \delta_{1,t})B_{1,j,t} + (1 - \delta_{2,t})B_{2,j,t}. \tag{26}$$

The government of region j uses part of its FX reserves to cover the losses according to the following exogenous policy rule:

$$Bail_{j,t} = Loss_{j,t} \cdot \left[1 - e^{-\alpha\left(\frac{FX_{j,t}}{Loss_{j,t}}\right)}\right]. \tag{27}$$

The term in square brackets is the fraction of losses covered by the bailout. This fraction is always smaller than 1 and converges to 1 when reserves $FX_{j,t}$ converge to infinity. The overall bailout spending converges to zero when either the losses are zero or the reserves are zero. The parameter α captures the easiness with which the region can use the accumulated reserves for bailout policies. If $\alpha \leq 1$, the size of the bailout, $Bail_{j,t}$, is never greater than the reserves, $FX_{j,t}$. When $\alpha = 0$ we get back to the model studied earlier where FX reserves were not used for bailout.

The bailout transfers are paid to entrepreneurs in proportion to their residual, after-default wealth. Denoting by $\chi_{j,t}$ the transfer rate, an individual entrepreneur in region j receives $\chi_{j,t}[\delta_{1,t}b_{1,j,t} + \delta_{2,t}b_{2,j,t} + b_{p,j,t}]$.

The transfer rate $\chi_{j,t}$ is determined so that the total funds allocated to a bailout, $Bail_{j,t}$, are equal to the total transfers paid to entrepreneurs, $\chi_{j,t}[\delta_{1,t}B_{1,j,t} + \delta_{2,t}B_{2,j,t} + B_{p,j,t}]$. Equalizing these two quantities we determine the transfer or subsidy rate as

$$\chi_{j,t} = \frac{Bail_{j,t}}{\delta_{1,t}B_{1,j,t} + \delta_{2,t}B_{2,j,t} + B_{p,j,t}}. \tag{28}$$

Notice that the bailout funds $Bail_{j,t}$ are zero if there is no default, that is, $\delta_{1,t} = \delta_{2,t} = 1$ and, therefore, the subsidy rate is also zero.

The assumption that bailout transfers are proportional to individual entrepreneurial wealth but the subsidy rate $\chi_{j,t}$ does not depend on individual bond composition is made for analytical convenience. An alternative assumption would be that the entrepreneurs' losses are covered with lump-sum transfers. Under this assumption, however, entrepreneurs in the two regions would choose different portfolio compositions, which would complicate the analysis significantly. Another possible assumption is that the transfers are proportional to the bond holdings that generated the losses. Again, this would lead to non-symmetric portfolio choices.

The variables $D_{p,t}$, $FX_{1,t}$ and $FX_{2,t}$ are time varying but exogenous. Instead, the bailout funds $Bail_{1,t}$ and $Bail_{2,t}$ are endogenously determined by condition (27), and the households' transfers $T_{j,t}$ are determined by the two budget constraints, equations (24) and (25). The intuition is that in subperiod 1, when default occurs and entrepreneurs are bailed out, the government uses $FX_{j,t}$ to provide the required resources (recall that the assumed policy rule implies $Bail_{j,t} \leq FX_{j,t}$), and then in subperiod 3 the government adjusts $T_{j,t}$ as needed so that the exogenous $FX_{j,t+1}$ is still attained at the end of the period (i.e., reserves are only used within-the-period to finance the bailout). See Figure 3 for the definition of the three subperiods.

As an alternative, we could assume that households' transfers $T_{j,t}$ are unchanged and $FX_{j,t+1}$ responds endogenously after the bailout. We did not adopt this assumption for simplicity. However, our assumption raises the question of why FX reserves are needed and whether the government could not just reduce transfers (or raise taxes) directly to fund the bailout. The assumed policy is motivated by the idea that changing $T_{j,t}$ requires time. By the time the government has raised funds, the bailout may no longer be needed. By holding liquid reserves, instead, the government has the flexibility to intervene in a timely fashion. More generally, we could envisage a situation more akin to reality in which the change in $T_{j,t}$ (i.e., the tax hike needed to fund bailouts) occurs over time, so that the stock of FX reserves drops in the short run after the government intervention. The specification proposed here is a limiting case of this scenario in which taxes cannot adjust in subperiod 1 but they can adjust in subperiod 3.

5.2 Portfolio choice of entrepreneurs

Recall that the representative entrepreneur in Region j enters period t with bonds issued by firms in Regions 1 and 2, $b_{1,j,t}$ and $b_{2,j,t}$, respectively, and government bonds issued by advanced economies, $b_{p,j,t}$. In the original setup, default by intermediate-goods producers caused entrepreneurs to incur financial losses proportional to their ownership of private bonds, with the post-default values given by $\delta_{1,t}b_{1,j,t}$ and $\delta_{2,t}b_{2,j,t}$. In this extension of the model, however, the government bails out entrepreneurs by covering some of their losses with the transfer $\chi_{j,t}[\delta_{1,t}b_{1,j,t} + \delta_{2,t}b_{2,j,t} + b_{p,j,t}]$. Thus, the entrepreneur's wealth after the repayment of the bonds and after government transfers is

$$m_{j,t} = \Big[\delta_{1,t}b_{1,j,t} + \delta_{2,t}b_{2,j,t} + b_{p,j,t}\Big](1 + \chi_{j,t}).$$

This is the entrepreneurial wealth that enters the financial constraint (8). Besides this, all the conditions that define the entrepreneur's problem remain unchanged, including the equation that defines wealth at the end of the period,

$$a_{j,t} = m_{j,t} + z_{j,t}x_{j,t} - p_{j,t}x_{j,t}.$$

We can show that Lemmas 2.1 and 2.2 also remain unchanged. This is also true for the equilibrium conditions derived earlier with the aggregate wealth of entrepreneurs defined as

$$M_{j,t} = \Big[\delta_{1,t}B_{1,j,t} + \delta_{2,t}B_{2,j,t} + B_{p,j,t}\Big](1 + \chi_{j,t}).$$

5.3 Quantitative results

We simulate this variant of the model using the same sequences of $z_{j,t}$, $\phi_{j,t}$, $\kappa_{j,t}$, $z_{j,t}$, $K_{j,t}$, $FX_{j,t}$ and $D_{p,t}$ as in the baseline case. We only need to choose the bailout parameter α. Since we do not have direct empirical evidence about this parameter, we show results for alternative values of α.

Simulation results. Figure 11 plots the output volatility measures for advanced and emerging economies. The continuous line is for the baseline case where FX reserves are not used for bailout interventions. This is the same as the continuous line shown in the previous Figure 10. The dashed lines, instead, are for the model with bailouts, for two values of α, 0.1 and 0.3. As explained earlier, α captures the extent to which the government uses FX reserves to bail out entrepreneurs during financial crises. Given the accumulated FX reserves, the higher is the value of α, the bigger is the size of the bailout. When $\alpha = 0$ there is no bailout and we revert to the baseline model.

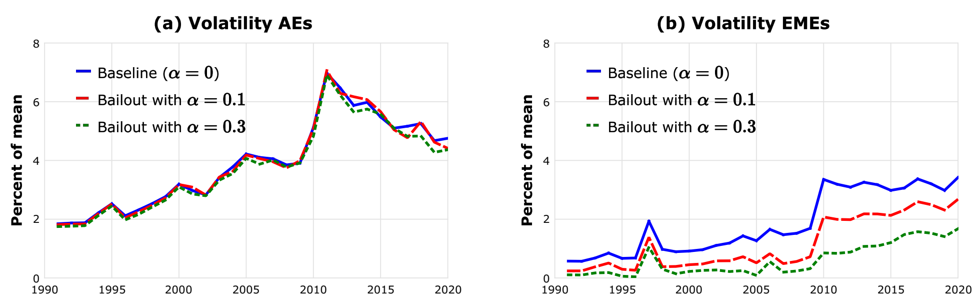

Figure 11: Counterfactual simulation when FX reserves are used for bailouts, 1991-2020.

Panel (s) shows that, for advanced economies, the volatility measure is only marginally affected by the parameter α. This is a straightforward result because AEs do not hold large stocks of reserves relatively to the size of their economy. Therefore, bailouts are relatively small.[13]

For emerging economies, however, the picture is quite different. Even with $\alpha = 0.1$ (dashed line with longer segments), aggregate volatility

[13]One could consider alternative means by which AEs could provide liquidity and bailouts to entrepreneurs, by, for example, swapping defaulted private obligations for newly-issued (risk-free) public debt paid for by future taxes (akin to the 2008 TARP program of the U.S. treasury).

drops visibly. With $\alpha = 0.3$ (dashed line with shorter segments), volatility drops to less than half of what it was in the baseline. This is intuitive since with a higher α EMEs use a larger fraction of FX reserves for bailouts.

The larger FX reserves held by EMEs give them a bigger liquidity buffer for stabilization policies than AEs. But even if the overall EMEs volatility declines with bailouts, we still see an increasing trend.

5.4 Bailout policies and moral hazard

Although bailout policies could alleviate the consequences of crises, their anticipation could create undesired distortions. The standard argument is that the anticipation of a bailout, that is the anticipation that some of the investment losses will be covered by government, may induce investors to demand a lower expected return from borrowers. This reduces the cost of borrowing and creates the conditions for higher leverage which, in turn, makes financial crises more damaging. Although the paper does not address welfare questions, we can explore the possible effects of this mechanism in the context of our model. The main question is whether the anticipation of bailouts affects equilibrium borrowing.

It turns out that in our model the anticipation of bailouts has a small effect on the interest rate, and therefore, on the equilibrium debt. In part this derives from the fact that bailout subsidies are conditional on the materialization of a financial crisis, which is a very low probability event. But there is also another reason.

When a crisis materializes, entrepreneurs receive extra funds, part of which are saved to the next period. This should reduce the interest rate, at least after a crisis. However, because entrepreneurs have more funds after the bailout, they can purchase more intermediate inputs (this was the intent of the bailout), which raises the price of the intermediate inputs. The higher price reduces entrepreneurs' profits and, therefore, the end-of-period net worth that can be saved. It turns out that the two effects (transfers from the government and lower profits per unit of wealth) almost cancel each other out. As a result, the impact on the equilibrium interest rate is negligible.

6 Discussion and conclusion

An implication of the increased size of emerging economies is that, collectively, they play a more influential role in driving global capital markets and macroeconomic dynamics. The view that emerging markets are a collection of small open economies with negligible impact on advanced economies is no longer a valid approximation. One way in which emerging economies affect the world economy is through financial markets. In this paper we focused on one channel: the accumulation of foreign reserves.

Since the 1990s, emerging economies have sharply increased their reserves as a percentage of both their own GDP and global GDP. This represents a large increase in world demand for financial assets (typically government bonds issued by advanced economies). Through a counterfactual analysis, we showed that this surge in reserves contributed to the observed fall in the world interest rate. As the cost of borrowing fell, the private sector became more leveraged and this increased financial and macro volatility *globally*.

While the accumulation of reserves by EMEs contributed to lower interest rates and greater global volatility, it also provided these economies with liquidity usable for stabilization purposes. The end result in the model is that the significant accumulation of FX reserves by EMEs reduced their financial and macroeconomic volatility but increased the volatility of advanced economies that did not accumulate reserves as EMEs did.

During the same period, we also observed that governments in advanced economies increased public borrowing, raising the supply of financial assets. This had the opposite effect from the surge in EMEs' reserves: it propped up the world interest rate, which in turn discouraged private borrowing (crowding out). Lower private leverage, then, contributed to reduce global economic instability.

In our counterfactual exercises we used changes in FX reserves and public debt as exogenous inputs. In reality, these variables are chosen by governments. Since they can have non-negligible welfare effects, it would be interesting to explore how governments choose these policies. In an integrated world economy, these policies depend on the size of the country. For example, if a country is small compared to the world economy and chooses to increase its FX reserves, the economy of that country may become more stable. However, if many countries implement a similar policy, the world interest rate could fall inducing more leverage and higher macroeconomic instability (as shown in the paper). This suggests that, without cross-country policy coordination, we may have too much accumulation of reserves because individual countries, being relatively small compared to the world economy, do not internalize the impact of their policies on the world interest rate.

The idea that emerging countries over-accumulate reserves is consistent with the theoretical analysis of Das et al. (2023). However, there is also another side of the story. Low interest rates encourage the governments of advanced economies to issue more public debt. The higher supply of public debt could make the portfolio composition of savers (entrepreneurs in our model) less vulnerable to crises. This is because in equilibrium a larger share of private portfolios will be allocated to non-defaultable public bonds.

There is also another consideration. The fact that emerging economies have experienced faster growth than advanced economies means that today they are relatively bigger. This has increased the world demand for public debt issued by advanced economies, which could encourage AEs to issue more public debt. Again, to the extent that the public debt of advanced economies remains safe, the portfolios of private savers become less risky and, as a result, economies in both advanced and emerging world could become less vulnerable to financial crises. These and other related issues could be the subject of future research.

Appendix

A Proof of Lemma 2.1

The optimization problem of an entrepreneur in region j is

$$\max_{\{x_{j,t},c^e_{j,t},b_{1,j,t+1},b_{2,j,t+1},b_{p,j,t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \ln(c^e_{j,t}) \tag{29}$$

subject to

$$\begin{aligned} m_{j,t} &= \delta_{1,t}b_{1,j,t} + \delta_{2,t}b_{2,j,t} + b_{p,j,t}, \\ m_{j,t} &\geq \phi_{j,t}p_{j,t}x_{j,t}, \\ a_{j,t} &= m_{j,t} + z_{j,t}x_{j,t} - p_{j,t}x_{j,t}, \\ c^e_{j,t} &= a_{j,t} - q_{1,t}b_{1,j,t+1} - q_{2,t}b_{2,j,t+1} - q_{p,t}b_{p,j,t+1}. \end{aligned}$$

The first-order condition for $x_{j,t}$ is

$$z_{j,t} = (1 + \hat{\xi}_{j,t}\phi_{j,t})p_{j,t}, \tag{30}$$

where $\hat{\xi}_{j,t} \equiv \xi_{j,t}/u'(c^e_{j,t}) = \xi_{j,t}c^e_{j,t}$ and $\xi_{j,t}$ is the Lagrange multiplier associated with the working capital constraint in the above optimization problem.

When the financial constraint is binding we have that $\xi_{j,t} > 0$. Then condition (30) implies that $z_{j,t} > p_{j,t}$ and the entrepreneurs' profits, $\pi_{j,t} = (z_{j,t} - p_{j,t})x_{j,t}$, are positive. When the constraint is not binding, instead, $\xi_{j,t} = 0$ and the first-order condition becomes $z_{j,t} = p_{j,t}$. Profits are then zero, that is, $\pi_{j,t} = 0$.

Using the financial constraint $m_{j,t} = \phi_{j,t}p_{j,t}x_{j,t}$ and condition (30), we can write the profits as

$$\pi_{j,t} = \hat{\xi}_{j,t}m_{j,t}. \tag{31}$$

The lender's wealth is $a_{j,t} = m_{j,t} + \pi_{j,t}$. Using (31) it can be rewritten as

$$a_{j,t} = (1 + \hat{\xi}_{j,t})m_{j,t}$$

This shows how the multiplier $\hat{\xi}_{j,t}$ captures the notion of a convenience yield. When the working capital constraint binds, bonds yield a return over and above the yield implicit in their prices at rate $\hat{\xi}_{j,t}$ per unit of financial wealth $m_{j,t}$.

The entrepreneur's optimality conditions can also be used to express the above results in terms of factor prices instead of the shadow value $\hat{\xi}_{j,t}$ as follows:

$$p_{j,t}x_{j,t} = \frac{m_{j,t}}{\phi_{j,t}}$$

$$y_{j,t} = z_{j,t}\frac{m_{j,t}}{p_{j,t}\phi_{j,t}}$$
$$\pi_{j,t} = \frac{m_{j,t}}{\phi_{j,t}}\left(\frac{z_{j,t}}{p_{j,t}} - 1\right)$$
$$a_{j,t} = m_{j,t}\left[1 + \frac{1}{\phi_{j,t}}\left(\frac{z_{j,t}}{p_{j,t}} - 1\right)\right] \geq m_{j,t}$$

The results for profits then imply that the shadow value of the financial constraint satisfies this condition:

$$\hat{\xi}_{j,t} = \frac{1}{\phi_{j,t}}\left(\frac{z_{j,t}}{p_{j,t}} - 1\right)$$

This demonstrates that end-of-period wealth is linear in initial financial wealth, with a slope of 1 if the working capital constraint does not bind and with a slope of $1 + \hat{\xi}_{j,t}$ when it binds. In the latter case, the slope coefficient is a nonlinear function of productivity, factor prices and $\phi_{j,t}$. This linearity of wealth will be used in Appendix B to solve for the entrepreneur's portfolio allocation problem.

B Proof of Lemma 2.2

Given that at the optimum of the entrepreneur's problem $a_{j,t} = (1 + \hat{\xi}_{j,t})m_{j,t}$ and since $m_{j,t} = \delta_{1,t}b_{1,j,t} + \delta_{2,t}b_{2,j,t} + b_{p,j,t}$, we can write the end-of-period wealth at time t and at $t + 1$ as

$$\begin{aligned} a_{j,t} &= (1 + \hat{\xi}_{j,t})(\delta_{1,t}b_{1,j,t} + \delta_{2,t}b_{2,j,t} + b_{p,j,t}), \\ a_{j,t+1} &= (1 + \hat{\xi}_{j,t+1})(\delta_{1,t+1}b_{1,j,t+1} + \delta_{2,t+1}b_{2,j,t+1} + b_{p,j,t+1}). \end{aligned}$$

We derive next the first-order conditions for Problem (29) with respect to $b_{1,j,t+1}$, $b_{2,j,t+1}$ and $b_{p,j,t+1}$,

$$\frac{q_{1,t}}{c^e_{j,t}} = \beta\mathbb{E}_t\left(\frac{(1 + \hat{\xi}_{j,t+1})\delta_{1,t+1}}{c^e_{j,t+1}}\right), \tag{32}$$
$$\frac{q_{2,t}}{c^e_{j,t}} = \beta\mathbb{E}_t\left(\frac{(1 + \hat{\xi}_{j,t+1})\delta_{2,t+1}}{c^e_{j,t+1}}\right). \tag{33}$$
$$\frac{q_{p,t}}{c^e_{j,t}} = \beta\mathbb{E}_t\left(\frac{(1 + \hat{\xi}_{j,t+1})}{c^e_{j,t+1}}\right). \tag{34}$$

The right-hand-sides of these three Euler equations reflect again the convenience yield of financial wealth. The marginal benefit of buying bonds at t to carry over to $t+1$ increases by $(1+\hat{\xi}_{j,t+1})$ if the working capital constraint binds. This is because holding additional bonds relaxes the constraint, which is in addition to the contractual yield of each bond (the reciprocal of their prices). As shown earlier, this convenience yield is equal to profits per unit of financial wealth, but now in terms of expected profits at $t+1$.

We now guess that optimal consumption is a fraction $1-\beta$ of wealth,

$$c^e_{j,t} = (1-\beta)a_{j,t}.$$

The saved wealth is allocated to private bonds issued by region 1 and by region 2 and public debt issued by region 1. Denoting by $\theta_{1,j,t}$ and $\theta_{2,j,t}$ the portfolio shares allocated to private bonds issued by region 1 and region 2, respectively, we have

$$q_{1,t}b_{1,j,t+1} = \theta_{1,j,t}\beta a_{j,t}, \quad (35)$$

$$q_{2,t}b_{2,j,t+1} = \theta_{2,j,t}\beta a_{j,t}, \quad (36)$$

$$q_{p,t}b_{p,j,t+1} = (1-\theta_{1,j,t}-\theta_{2,j,t})\beta a_{j,t}. \quad (37)$$

We now multiply equation (32) by $b_{1,j,t+1}$, equation (33) by $b_{2,j,t+1}$, and equation (34) by $b_{p,j,t+1}$. Adding the resulting expressions and using the equations that define consumption and next period wealth, we obtain

$$q_{1,t}b_{1,j,t+1} + q_{2,t}b_{2,j,t+1} + q_{p,t}b_{p,j,t+1} = \beta a_{j,t}.$$

This is clearly satisfied given (35)-(37). Since we have derived this condition from the Euler equations (32)-(34), we have proved that, if consumption is a fraction $1-\beta$ of wealth, the three Euler equations are satisfied. This verifies our guess.

We now replace the guess for $c^e_{j,t}$ into equations (32) and (33), to obtain

$$\mathbb{E}_t\left\{\frac{\frac{\delta_{1,t+1}}{q_{1,t}}}{\theta_{1,j,t}\frac{\delta_{1,t+1}}{q_{1,t}}+\theta_{2,j,t}\frac{\delta_{2,t+1}}{q_{2,t}}+(1-\theta_{1,j,t}-\theta_{2,j,t})\frac{1}{q_{p,t}}}\right\} = 1. \quad (38)$$

$$\mathbb{E}_t\left\{\frac{\frac{\delta_{2,t+1}}{q_{2,t}}}{\theta_{1,j,t}\frac{\delta_{1,t+1}}{q_{1,t}}+\theta_{2,j,t}\frac{\delta_{2,t+1}}{q_{2,t}}+(1-\theta_{1,j,t}-\theta_{2,j,t})\frac{1}{q_{p,t}}}\right\} = 1. \quad (39)$$

These two conditions determine the shares of savings invested in the private bonds of the two regions. Since the conditions are the same for entrepreneurs in both regions, it must be that $\theta_{1,1,t}=\theta_{1,2,t}=\theta_{1,t}$ and $\theta_{2,1,t}=\theta_{2,2,t}=\theta_{2,t}$. ■

The above results show that the convenience yield plays two roles: First, in subperiod 2 of the lender's problem, it takes the form of profits as we showed in Appendix A (if the financial constraint binds, the ex-post payoff of a bond increases above its actual payout, inclusive of any haircut, because of the profits that are allowed by the bonds used as working capital). Second, in subperiod 3, if the financial constraint is expected to bind at date-t+1, the expected marginal return of the bonds purchased at date-t rises because of the expected convenience yield at t+1 (the expected profits the new portfolio of bonds will yield). To put it differently, the financial constraint induces both an atemporal wedge between market factor prices and their corresponding marginal products, and an intertemporal wedge between marginal costs and benefits of saving into bonds. The following proposition, however, establishes that the logarithmic utility neutralizes the intertemporal wedge.

Proposition B.1 *The intertemporal wedge of the working capital constraint does not enter the entrepreneur's Euler equations. In particular, the marginal benefit of saving into each of the three bonds in the right-hand-side of* (32)*-*(34) *is independent of* $\hat{\xi}_{j,t+1}$*.*

Proof B.1 *Consider the marginal benefit of buying an extra unit of* $b_{1,j,t+1}$ *with logarithmic utility, as expressed in the right-hand-side of* (32)*:*

$$\beta \mathbb{E}_t \left(\frac{(1+\hat{\xi}_{j,t+1})\delta_{1,t+1}}{c^e_{j,t+1}} \right)$$

Since $c^e_{j,t+1} = (1-\beta)a_{j,t+1}$ *and* $a_{j,t+1} = (1+\hat{\xi}_{j,t+1})m_{j,t+1}$*, the above expression can be re-written as:*

$$\beta \mathbb{E}_t \left(\frac{(1+\hat{\xi}_{j,t+1})\delta_{1,t+1}}{(1-\beta)(1+\hat{\xi}_{j,t+1})m_{j,t+1}} \right).$$

Using $m_{j,t+1} = \delta_{1,t+1}b_{1,j,t+1} + \delta_{2,t+1}b_{2,j,t+1} + b_{p,j,t+1}$ *and conditions* (35)*-*(37)*, we obtain:*

$$\mathbb{E}_t \left\{ \frac{\delta_{1,t+1}}{(1-\beta)a_t \left(\theta_{1,j,t}\frac{\delta_{1,t+1}}{q_{1,t}} + \theta_{2,j,t}\frac{\delta_{2,t+1}}{q_{2,t}} + (1-\theta_{1,j,t}-\theta_{2,j,t})\frac{1}{q_{p,t}} \right)} \right\}.$$

This is independent of $\hat{\xi}_{j,t+1}$ *because* $\delta_{1,t+1}$ *and* $\delta_{2,t+1}$ *are taken as given by the entrepreneur and the portfolio shares that solve* (38) *and* (39) *are independent of* $\hat{\xi}_{j,t+1}$*. A similar argument applies to the marginal benefit of saving into* $b_{2,j,t+1}$ *and* $b_{p,j,t+1}$*.* ■

C First-order conditions for intermediate goods producers

Producers of intermediate goods maximize the present value of the dividends they pay to households. Their optimization problem can be written recursively as

$$V(d,k) = \max_{l,d'} \left\{ \text{div} + \beta \mathbb{E} V(d',k') \right\},$$

subject to

$$\tilde{d}(d,\ell k) + \text{div} + \varphi(d',\kappa' k') \;=\; p l^{\gamma} k^{1-\gamma} - wl - k' + (1-\tau)k + \frac{1}{R}\mathbb{E}\tilde{d}(d',\ell' k'),$$

where k and the law of motion for k' are exogenous, $\tilde{d}(d,\ell k)$ is defined in (2) and $\varphi(d',\kappa' k')$ in (3). The firm discounts dividends at rate β, which corresponds to the households' discount factor. Because households have linear utility in c, the marginal utility of consumption is always 1.

The first-order conditions with respect to l and d' are:

$$\gamma k^{1-\gamma} l^{\gamma-1} = w,$$

$$\tfrac{1}{R}\mathbb{E}\left\{\tfrac{\partial \tilde{d}(d',\ell' k')}{\partial d'}\right\} - \tfrac{\partial \varphi(d',\kappa' k')}{\partial d'} + \beta \mathbb{E}\left\{\tfrac{\partial V(d',k')}{\partial d'}\right\} = 0$$

The envelope condition for debt is

$$\frac{\partial V(d,k)}{\partial d} \;=\; -\frac{\partial \tilde{d}(d,\ell k)}{\partial d}.$$

Updating this condition by one period and substituting in the first-order condition for debt, we obtain

$$\tfrac{1}{R} = \beta + \frac{\frac{\partial \varphi(d',\kappa' k')}{\partial d'}}{\mathbb{E}\left\{\frac{\partial \tilde{d}(d',\ell' k')}{\partial d'}\right\}} \tag{40}$$

We now derive the analytical expressions for the derivatives included in the right-hand-side of the above expression. To do so we use the functional forms for $\tilde{d}(d,\ell k)$ and $\varphi(d',\kappa' k')$ defined, respectively, in (2) and (3):

$$\frac{\partial \tilde{d}(d,\ell k)}{\partial d} \;=\; \begin{cases} 0, & \text{if} \quad d \geq \ell k \\ 1, & \text{otherwise} \end{cases}$$

$$\frac{\partial \varphi(d',\kappa' k')}{\partial d'} \;=\; \begin{cases} 2\eta\left(1-\frac{\kappa' k'}{d'}\right)\frac{\kappa' k'}{d'} + \eta\left(1-\frac{\kappa' k'}{d'}\right)^2, & \text{if} \quad d' \geq \kappa' k' \\ 0, & \text{otherwise} \end{cases}$$

If $d' > \kappa' k'$, the liquidation price ℓ is equal to κ with probability λ (probability of default). If $d' \leq \kappa' k'$, the liquidation price ℓ is always equal to 1. Using this, we can rewrite the expected value of the derivative of $\tilde{d}$ as

$$\mathbb{E}\left\{\frac{\partial \tilde{d}(d', \ell' k')}{\partial d'}\right\} = \begin{cases} 1-\lambda, & \text{if } d' \geq \kappa k' \\ 1, & \text{otherwise} \end{cases}$$

Using the above expressions in the first-order condition (40) we obtain

$$\frac{1}{R} = \beta + \Phi\left(\frac{d'}{\kappa' k'}\right), \tag{41}$$

where

$$\Phi\left(\frac{d'}{\kappa' k'}\right) = \begin{cases} \left(\frac{1}{1-\lambda}\right)\eta\left[1-\left(\frac{\kappa' k'}{d'}\right)^2\right], & \text{if } \frac{d'}{\kappa' k'} \geq 1 \\ 0, & \text{if } \frac{d'}{\kappa' k'} < 1 \end{cases}$$

The function $\Phi(.)$ is strictly increasing for $\frac{d'}{\kappa' k'} \geq 1$. In addition, for $\frac{d'}{\kappa' k'} \geq 1$, taking derivatives we can verify that it is increasing in d' and decreasing in both k' and κ'. Note also that with $\eta = 0$ (costless debt issuance), the debt Euler equation collapses to $\frac{1}{R} = \beta$ and hence debt and leverage would be indeterminate. ■

D Market for liquidated capital and equilibrium multiplicity

In the main body of the paper, we assumed that the liquidation price $\ell_{j,t}$ can be either $\kappa_{j,t}$ or 1 with constant probabilities λ and $1-\lambda$. In this section, we describe the market structure that provides the micro-foundation for the determination of ℓ_t. The specification admits two self-fulfilling equilibria and λ represents the probability of a sunspot shock that selects one of two equilibria.

The market for liquidated capital meets at the beginning of the period. We make two important assumptions about the operation of this market.

Assumption 1 *Capital can be sold to domestic intermediate-goods firms or final-goods firms (entrepreneurs). However, if sold to entrepreneurs, capital loses its functionality as a productive asset and it is converted to consumption goods at rate* $\kappa_{j,t} < 1$.

This assumption formalizes the idea that capital may lose value when reallocated to another sector or region. The assumption that capital loses its functionality also when reallocated abroad implies that a crisis could be local. However, even if a crisis takes place only in one region, it will have real economic consequences for the other region due to the cross-country diversification of bond portfolios.

Assumption 2 *Intermediate-goods firms can purchase liquidated capital only if the liquidation value of their own capital exceeds the debt obligations, $d_{j,t} < \ell_{j,t}k_{j,t}$.*

If an intermediate-goods firm starts with liabilities bigger than the liquidation value of the owned assets, that is, $d_{j,t} > \ell_{j,t}k_{j,t}$, it will be unable to raise additional funds to purchase the capital liquidated by other firms. Potential investors know that the new liabilities (as well as the outstanding liabilities) are not collateralized, and the debt will be renegotiated immediately after taking the new debt. We refer to an intermediate-goods firm with $d_{j,t} < \ell_{j,t}k_{j,t}$ as 'liquid' since it can raise extra funds at the beginning of the period. Instead, a firm with $d_{j,t} > \ell_{j,t}k_{j,t}$ is 'illiquid'.

To better understand Assumptions 1 and 2, consider the condition for not renegotiating, $d_{j,t} \leq \ell_{j,t}k_{j,t}$. If this condition is satisfied, intermediate-goods firms have the ability to raise funds to purchase additional capital. This ensures that the liquidation price is $\ell_{j,t} = 1$. If $d_{j,t} > \kappa_{j,t}k_{j,t}$ for all intermediate-goods firms, however, there will be no firms capable of buying the liquidated capital. Then, the liquidated capital can only be purchased by entrepreneurs at price $\ell_{j,t} = \kappa_{j,t}$.

This shows that the market price for liquidated capital depends on the financial decision of firms, $d_{j,t}$, which in turn depends on the liquidation price. This interdependence is critical for generating self-fulfilling equilibria.

Proposition D.1 *There exists multiple equilibria only if $d_{j,t} > \kappa_{j,t}k_{j,t}$.*

Proof D.1 *At the beginning of the period, firms choose whether to renegotiate the debt. Given the initial states d_t and k_t, renegotiation boils down to a take-it or leave-it offer made to creditors for the repayment of the debt.*

Denote by $\tilde{d}_t = \psi(d_t, k_t, \ell_t)$ the offered repayment. This depends on the individual liabilities, d_t, individual capital, k_t, and the price for liquidated capital, ℓ_t. The liquidation price is the price at which the lender could sell capital after rejecting the offer from the borrower. The best offer made by the intermediate-goods firm is

$$\psi(d_t, k_t, \ell_t) = \begin{cases} d_t, & \textit{if} \quad d_t \leq \ell_t k_t \\ \\ \ell_t k_t, & \textit{if} \quad d_t > \ell_t k_t \end{cases}, \tag{42}$$

which is accepted by creditors if they cannot sell at a price higher than ℓ_t.

We assume, for the moment, that the equilibrium is symmetric, that is, all intermediate-goods firms start with the same ratio d_t/k_t. At this stage this is only an assumption. However, we will show below that firms do not have an incentive to deviate from the choice of other firms.

Given the assumption that the equilibrium is symmetric, multiple equilibria arise if $d_t/k_t \in [\kappa_t, 1)$. If the market expects that the liquidation price is $\ell_t = \kappa_t$, all firms are illiquid and they choose to renege on their liabilities (given the renegotiation policy (42)). As a result, there will be no firms that can purchase the liquidated capital of other firms. The only possible liquidation price that is consistent with the expected price is $\ell_t = \kappa_t$. On the other hand, if the market expects $\ell_t = 1$, intermediate-goods firms are liquid and, if one firm reneges, creditors can sell the liquidated capital to other intermediate-goods firms at the liquidation price $\ell_t = 1$. Therefore, it is optimal for firms not to renegotiate.

To complete the proof we need to show that an individual firm does not have an incentive to deviate from the symmetric equilibrium and choose a different ratio d_t/k_t at $t-1$. Specifically, we want to show that, in the anticipation that the liquidation price could drop to $\ell_t = \kappa_t$, an intermediate-goods firm does not find optimal to borrow less at $t-1$ so that it will be able to purchase the liquidated capital at t.

The first point to consider is that, in equilibrium, capital is never liquidated. The low liquidation price κ_t represents the threat value for creditors. Since creditors accept the renegotiation offer, no capital is ever liquidated. What would happen if there is an intermediate-goods firm that is liquid and has the ability to purchase the capital at a price higher than κ_t? Debtors know that their creditors could liquidate the capital and sell it at a higher price than κ_t. Knowing this, debtors will offer a higher repayment and, as a result, capital will not be liquidated. The liquidation price, then, could be driven to 1. This shows that an intermediate-goods firm cannot gain from remaining liquid. Thus, there is no incentive to deviate from the symmetric equilibrium. ■

The proof of the proposition establishes that the equilibrium is symmetric and all intermediate-goods firms choose the same ratio d_t/k_t. Then, multiple equilibria determined by self-fulfilling expectations about the liquidation price exists if $d_t/k_t \in [\kappa_t, 1)$. On the one hand, if the market expects a liquidation price $\ell_t = \kappa_t$, all intermediate-goods firms are illiquid and choose to renege on their liabilities. As a result, there are no intermediate-goods firms that can purchase the liquidated capital and, therefore, the only liquidation price consistent with the expected price is $\ell_t = \kappa_t$. On the other hand, when the market expects $\ell_t = 1$, intermediate-goods firms are liquid and, if one firm reneges, creditors can sell the liquidated capital to other firms at price $\ell_t = 1$, which makes it optimal not to renege.

When multiple equilibria are possible, that is, when we have $d_{j,t} > \kappa_{j,t} k_{j,t}$, the equilibrium is selected by random draws of sunspot shocks. Let $\varepsilon_{j,t}$ be a variable that takes the value of 0 with probability λ and 1 with probability $1-\lambda$. If the condition for multiplicity is satisfied, agents coordinate their expectations on the low liquidation price $\kappa_{j,t}$ when $\varepsilon_{j,t} = 0$. This implies that the probability distribution of the low liquidation price is

$$f_{j,t}\Big(\ell_{j,t} = \kappa_{j,t}\Big) = \begin{cases} 0, & \text{if} \quad d_{j,t} \le \kappa_{j,t} k_{j,t} \\ \\ \lambda, & \text{if} \quad d_{j,t} > \kappa_{j,t} k_{j,t} \end{cases}$$

The ratio $d_{j,t}/\kappa_{j,t}k_{j,t}$ is the relevant measure of leverage. When it is sufficiently small, intermediate-goods firms remain liquid even if the (expected) liquidation price is $\kappa_{j,t}$. But then the liquidation price cannot be low and the realization of the sunspot shock is irrelevant for the equilibrium. Instead, when leverage is high, firms' liquidity depends on the liquidation price. The realization of the sunspot shock $\varepsilon_{j,t}$ then becomes important for selecting one of the two equilibria. When $\varepsilon_{j,t} = 0$—which happens with probability λ—the market expects that the liquidation price is $\kappa_{j,t}$, making the intermediate-goods sector illiquid. On the other hand, when $\varepsilon_{j,t} = 1$—which happens with probability $1 - \lambda$—the market expects that intermediate-goods firms are capable of participating in the liquidation market, validating the expectation of a higher liquidation price.

The above argument is based on the assumption that $\kappa_{j,t}$ is sufficiently low (implying a low liquidation price if the capital freezes). Also, the value of capital without a freeze, $k_{j,t}$, is always bigger than the debt $d_{j,t}$. Otherwise, firms would be illiquid with probability 1 and the equilibrium price would be always $\kappa_{j,t}$. Condition (5) guarantees that this does not happen at equilibrium: if the probability of default is 1, the anticipation of the renegotiation cost increases the interest rate, which deters intermediate-goods firms from borrowing too much.

E Equilibrium system of equations at time t

Given the state vector

$$\begin{aligned} \mathbf{s}_t \quad \equiv \quad & \big(z_{1,t}, z_{2,t}, \phi_{1,t}, \phi_{2,t}, \kappa_{1,t}, \kappa_{2,t}, \kappa_{1,t+1}, \kappa_{2,t+1}, FX_{1,t+1}, FX_{2,t+1}, D_{p,t+1}, \\ & K_{1,t}, K_{2,t}, K_{1,t+1}, K_{2,t+1}, B_{1,1,t}, B_{2,1,t}, B_{p,1,t}, B_{1,2,t}, B_{2,2,t}, B_{p,2,t}, \varepsilon_{1,t}, \varepsilon_{2,t}\big), \end{aligned}$$

we can find the values of $\delta_{j,t}$, $M_{j,t}$, $L_{j,t}$, $X_{j,t}$, $w_{j,t}$, $p_{j,t}$, $q_{j,t}$, $q_{p,t}$, $A_{j,t}$, $B_{j,1,t+1}$, $B_{j,2,t+1}$, $B_{p,j,t+1}$, $D_{j,t+1}$, $\theta_{1,t}$ and $\theta_{2,t}$, by solving the following system of equations:

$$\delta_{j,t} = \begin{cases} \min\left\{1\,,\ \frac{\kappa_{j,t}K_{j,t}}{D_{j,t}}\right\}, & \text{if} \quad \varepsilon_{j,t}=0 \\ 1, & \text{if} \quad \varepsilon_{j,t}=1 \end{cases} \tag{43}$$

$$M_{j,t} = \delta_{1,t}B_{1,t} + \delta_{2,t}B_{2,t} + B_{p,t} \tag{44}$$

$$L_{j,t} = \left(\frac{\gamma p_{j,t}}{w_{j,t}}\right)^{\frac{1}{1-\gamma}} K_{j,t}, \tag{45}$$

$$L_{j,t} = \left(\frac{w_{j,t}}{z_{j,t}^{1/\gamma}}\right)^{\nu}, \tag{46}$$

$$X_{j,t} = L_{j,t}^{\gamma}K_{j,t}^{1-\gamma}, \tag{47}$$

$$p_{j,t} = \begin{cases} \frac{M_{j,t}}{\phi_{j,t}X_{j,t}}, & \text{if} \quad M_{j,t} < \phi_{j,t}X_{j,t} \\ 1, & \text{if} \quad M_{j,t} = \phi_{j,t}X_{j,t} \end{cases} \tag{48}$$

$$A_{j,t} = M_{j,t} + z_{j,t}X_{j,t} - p_{j,t}X_{j,t}, \tag{49}$$

$$B_{1,j,t+1} = \frac{\theta_{1,t}\beta A_{j,t}}{q_{1,t}}, \tag{50}$$

$$B_{2,j,t+1} = \frac{\theta_{2,t}\beta A_{j,t}}{q_{2,t}}, \tag{51}$$

$$B_{p,j,t+1} = \frac{(1-\theta_{1,t}-\theta_{2,t})\beta A_{j,t}}{q_{p,t}}, \tag{52}$$

$$1 = \mathbb{E}_t\left\{\frac{\frac{\delta_{1,t+1}}{q_{1,t}}}{\theta_{1,t}\frac{\delta_{1,t+1}}{q_{1,t}} + \theta_{2,t}\frac{\delta_{2,t+1}}{q_{2,t}} + (1-\theta_{1,t}-\theta_{2,t})\frac{1}{q_{p,t}}}\right\}, \tag{53}$$

$$1 = \mathbb{E}_t\left\{\frac{\frac{\delta_{2,t+1}}{q_{2,t}}}{\theta_{1,t}\frac{\delta_{1,t+1}}{q_{1,t}} + \theta_{2,t}\frac{\delta_{2,t+1}}{q_{2,t}} + (1-\theta_{1,t}-\theta_{2,t})\frac{1}{q_{p,t}}}\right\}, \tag{54}$$

$$D_{j,t+1} = B_{j1,t+1} + B_{j2,t+1}, \tag{55}$$

$$q_{j,t} = \left[\beta + \Phi\left(\frac{D_{j,t+1}}{\kappa_{j,t+1}K_{j,t+1}}\right)\right]\mathbb{E}_t\delta_{j,t+1}. \tag{56}$$

$$D_{p,t+1} = FX_{1,t+1} + FX_{2,t+1} + B_{p,1,t+1} + B_{p,2,t+1}. \tag{57}$$

Equation (43) defines the optimal renegotiation strategy (the fraction of the debt repaid). Equation (44) defines entrepreneurial wealth after default. Equation (45) is the demand for labor from intermediate-goods firms. Equation (46) is the supply of labor from households. Equation (47) is the production of intermediate goods and (48) defines its price $p_{j,t}$, which depends on whether the working capital constraint is binding or not binding. Equation (49) defines the end-of-period wealth of entrepreneurs after production. This is allocated to private bonds issued by the two regions and public bonds issued by region 1 as indicated in equations (50)-(52). Equations (53) and (54) are the conditions that determine the investment shares $\theta_{1,t}$ and $\theta_{2,t}$. They are the Euler equations derived from the optimization problem of entrepreneurs. Equation (55) is equilibrium in the bond market. Equation (56) is the Euler equation for intermediate-goods firms for the issuance of debt. This determines the price of bonds. The final equation (57) is the market equilibrium for public bonds.

The list includes 15 equations. However, since 12 of them are for $j \in \{1, 2\}$, the total system has 27 equations. The number of unknown variables is also 27: $\delta_{j,t}$, $M_{j,t}$, $L_{j,t}$, $X_{j,t}$, $w_{j,t}$, $p_{j,t}$, $q_{j,t}$, $A_{j,t}$, $B_{j,1,t+1}$, $B_{j,2,t+1}$, $B_{p,j,t+1}$, $D_{j,t+1}$ for $j \in \{1, 2\}$, plus $q_{p,t}$, $\theta_{1,t}$ and $\theta_{2,t}$.

F Resource constraints & balance-of-payments accounting

Combining the budget constraints of households, producers, and governments, plus the market-clearing conditions for financial and labor markets, we obtain the following resource constraints for Country 1 and Country 2:[14]

$$\begin{aligned} C_{1,t} + C^e_{1,t} + I_{1,t} + \varphi(D_{1,t+1}, \kappa_{1,t+1}K_{1,t+1}) \\ = z_{1,t}L^{\gamma}_{1,t}K^{1-\gamma}_{1,t} - [q_{2,t}B_{2,1,t+1} - q_{1,t}B_{1,2,t+1} - q_{p,t}(B_{p,2,t+1} + FX_{2,t+1})] \\ + [\delta_{2,t}B_{2,1,t} - \delta_{1,t}B_{1,2,t} - (B_{p,2,t} + FX_{2,t})] \end{aligned} \tag{58}$$

$$\begin{aligned} C_{2,t} + C^e_{2,t} + I_{2,t} + \varphi(D_{2,t+1}, \kappa_{2,t+1}K_{2,t+1}) \\ = z_{2,t}l^{\gamma}_{2,t}K^{1-\gamma}_{2,t} - [q_{1,t}B_{1,2,t+1} - q_{2,t}B_{2,1,t+1} + q_{p,t}(B_{p,2,t+1} + FX_{2,t+1})] \\ + [\delta_{1,t}B_{1,2,t} - \delta_{2,t}B_{2,1,t} + (B_{p,2,t} + FX_{2,t})] \end{aligned} \tag{59}$$

The uses of resources in the left-hand-sides of these conditions represents domestic absorption, which includes final goods consumption of households and final goods producers (entrepreneurs), investment expenditures $I_{j,t} = K_{j,t+1} - (1 -$

[14] In deriving these results, we should note that the bond prices satisfy $q_{j,t} = 1/R_{j,t}$ and that $\delta_{j,t}D_{j,t} - \tilde{d}(D_{j,t}, \kappa_{j,t}K_{j,t}) = 0$ always (when $D_{j,t} < \kappa_{j,t}K_{j,t}$, we have $\delta_{j,t} = 1$ and $\tilde{d}(\cdot) = D_{j,t}$, and when $D_{j,t} \geq \kappa_{j,t}K_{j,t}$, we have $\delta_{j,t} = \kappa_{j,t}K_{j,t}/D_{j,t}$ and $\tilde{d}(\cdot) = \kappa_{j,t}K_{j,t}$).

$\tau)K_{j,t}$, and borrowing costs. The sources in the right-hand-sides include GDP and cross-border capital flows related to the three bonds traded by the two regions.

Adding the above constraints yields the world resource constraint, which simply states that global absorption must equal global output (because the cross border asset positions are an asset for one country and a liability for the other):

$$C_{1,t} + C^e_{1,t} + I_{1,t} + \varphi(D_{1,t+1}, \kappa_{1,t+1}K_{1,t+1}) + C_{2,t} + C^e_{2,t} + I_{2,t} + \varphi(D_{2,t+1}, \kappa_{2,t+1}K_{2,t+1}) = z_{1,t}L^{\gamma}_{1,t}K^{1-\gamma}_{1,t} + z_{2,t}L^{\gamma}_{2,t}K^{1-\gamma}_{2,t}$$

The country resource constraints can be re-written to show that the balance-of-payments accounting condition holds in each country:

$$\begin{aligned} NX_{1,t} &\equiv z_{1,t}L^{\gamma}_{1,t}K^{1-\gamma}_{1,t} - \Big[C_{1,t} + C^e_{1,t} + I_{1,t} + \varphi\left(D_{1,t+1}, \kappa_{1,t+1}K_{1,t+1}\right)\Big] \\ &= \Big[q_{2,t}B_{2,1,t+1} - q_{1,t}B_{1,2,t+1} - q_{p,t}(B_{p,2,t+1} + FX_{2,t+1})\Big] - \\ &\qquad \Big[\delta_{2,t}B_{2,1,t} - \delta_{1,t}B_{1,2,t} - (B_{p,2,t} + FX_{2,t})\Big] \end{aligned} \tag{60}$$

$$\begin{aligned} NX_{2,t} &\equiv z_{2,t}L^{\gamma}_{2,t}K^{1-\gamma}_{2,t} - \Big[C_{2,t} + C^e_{2,t} + I_{2,t} + \varphi(D_{2,t+1}, \kappa_{2,t+1}K_{2,t+1})\Big] \\ &= \Big[q_{1,t}B_{1,2,t+1} - q_{2,t}B_{2,1,t+1} + q_{p,t}(B_{p,2,t+1} + FX_{2,t+1})\Big] - \\ &\qquad \Big[\delta_{1,t}B_{1,2,t} - \delta_{2,t}B_{2,1,t} + (B_{p,2,t} + FX_{2,t})\Big] \end{aligned} \tag{61}$$

In these expressions, $NX_{j,t}$ denotes the trade balance (exports minus imports) which is equal to the gap between GDP and domestic absorption. The second equality in each expression shows that the trade balance equals the current account $CA_{j,t}$ minus net factor payments to the rest of the world. For instance, in Country 1, $\delta_{2,t}B_{2,1,t} - \delta_{1,t}B_{1,2,t} - (B_{p,2,t} + FX_{2,t})$ is the beginning of period net foreign asset position (NFA), after the borrower's default decision is made, and $[q_{2,t}B_{2,1,t+1} - q_{1,t}B_{1,2,t+1} - q_{p,t}(B_{p,2,t+1} + FX_{2,t+1})]$ is the end-of-period NFA position minus net factor payments ($NFP_{j,t}$), implicit in the fact that the bonds have a zero coupon so that the final holdings of each bond are discounted by the corresponding yield (i.e., the implied interest payment is netted out). Hence, $NX_{j,t} = CA_{j,t} - NFP_{j,t}$.

The above expressions are useful for quantifying the effects of the parameter changes we study on international trade and financial flows. In addition, they can be used to calculate gross and net foreign asset positions for each country.

G Sensitivity to the cost of borrowing, η

In this section we conduct a sensitivity analysis with respect to the parameter η. The parameter determines the elasticity with which the cost of borrowing increases with debt. In all simulations presented in the paper, we used the value of $\eta = 0.1$. We now show how the results change when we double the value of this parameter, that is, we set $\eta = 0.2$.

After changing η, we repeat all quantitative exercises, including the construction of the time-varying parameters $z_{j,t}$, $\phi_{j,t}$ and $\kappa_{j,t}$ to replicate the same empirical targets (domestic credit, NFA and interest rate).

Figure 12 plots the volatility measure when $\eta = 0.2$. The corresponding plots for the baseline model with $\eta = 0.1$ were shown in Figure 10. Both graphs use the same scale so they are easily comparable.

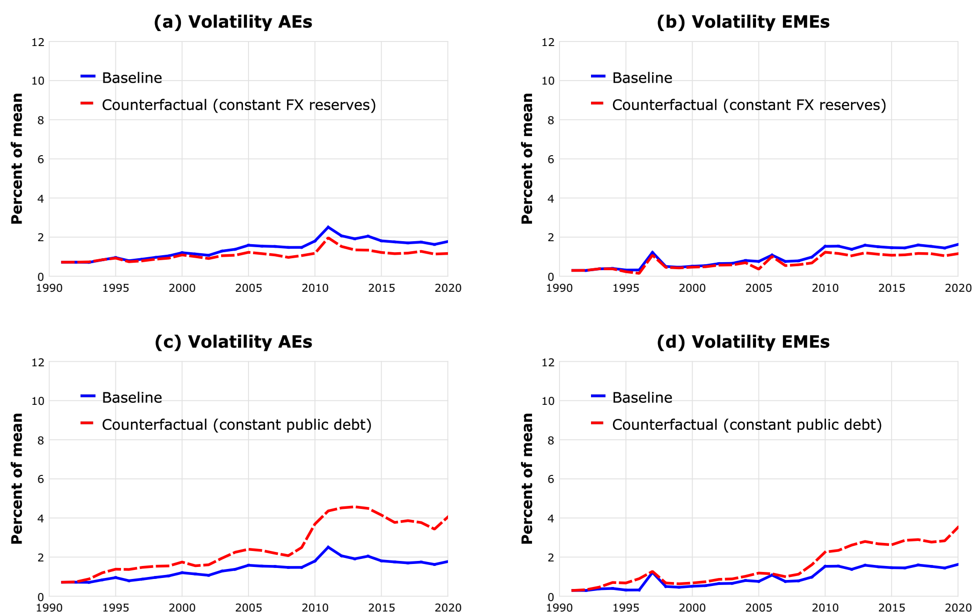

Figure 12: Sensitivity to cost of borrowing parameter η in Advanced Economies.

With a higher value of η, the cost of borrowing increases more rapidly with the stock of debt, and leverage responds less to the increase in reserves and public debt. As a result, the increase in output volatility is smaller. Qualitatively, however, the predictions of the model do not change. The impacts of the growth in FX reserves and public debt (difference between dashed and continuous lines) are smaller in absolute value but the proportional changes are similar.

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