A Quantitative Theory of Hard and Soft Sovereign Defaults

Grey Gordon and Pablo Guerron-Quintana
Federal Reserve Bank of Richmond and Boston College*

February 5, 2024
First version: July, 2018
Preliminary

Abstract

Empirical research on sovereign default has shown "hard defaults," characterized by large haircuts and aggressive negotiations, are associated with worse outcomes for GDP growth than "soft defaults." We propose a model capable of capturing these and other empirical regularities. In it, the sovereign chooses whether to negotiate with creditors or not. When negotiating, the sovereign proposes a haircut, taking into account the likelihood of it being accepted and the default costs incurred if it is not accepted. Creditors weigh the offer, trading off accepting against the future expected path of haircut offers if they decline. The model generates hard and soft defaults endogenously as sovereigns in poor circumstances push for larger haircuts. Is also generates the data's (1) positive correlation between haircuts and default duration and (2) the mild effects associated with "preemptive restructurings," where debt is voluntarily restructured without a missed payment. We shed light on the underlying mechanism both theoretically and quantitatively. A crucial ingredient for generating large haircuts is large and persistent real exchange rate depreciations in response to negative growth shocks. We use the model to assess the actual path of the Argentinean economy from 1980 to 2020, and show the exchange rate targeting from 1993-2001 is essential for explaining Argentina's massive haircut in the 2001 default. While conventional wisdom views the difference between hard and soft defaults largely as a consequence of aggressive negotiation, we show more than half of the observed difference is due to selection. Last, we establish that some of the tests researchers have used for checking reverse causality are uninformative from the model's perspective.

JEL Codes: C51,C68,F34,F41,F43,F44
Keywords: Default, Sovereign, Debt, Autarky, Growth, Argentina, Nonlinear Filtering

*Grey Gordon, greygordon@gmail.com; Pablo Guerron, pguerron@gmail.com. We thank our discussant Miguel Faria-e-Castro, as well as George Alessandria, Tamon Asonuma, Yan Bai, Javier Bianchi, Andres Blanco, Pablo D'Erasmo, Jonathan Eaton, Leo Martinez, Gabriel Mihalache, Andy Neumeyer, Radek Paluszynski, Bruce Preston, Richard Reis, David Wiczer, Jeromin Zettelmeyer, participants at LACEA/LAMES Ecuador, Midwest Macro Tennessee, NBER DSGE Chicago, UVA-FRBR, the Cleveland Fed, EEA / ESEM Manchester, and seminar participants at the IMF, Melbourne University, Rochester, and Stony Brook for helpful comments.

The views expressed are the authors' and do not necessarily reflect those of the Federal Reserve Bank of Richmond or the Federal Reserve System.

1 Introduction

Recent research (Trebesch and Zabel, 2017) has revealed a striking pattern in the data that can be seen in the top left panel of Figure 1. In particular, the path for output following hard defaults—i.e., defaults characterized by large haircuts—and soft defaults—defaults characterized by small haircuts—are completely different. Whereas hard defaults are associated with a sharp and extremely persistent decline in output relative to a year before default, soft defaults are associated with a small decline on impact and growing output post default. We extend the results to show hard defaults are also characterized by larger real exchange rate (RER) depreciations (the top right panel of Figure 1) and longer default duration (the bottom left panel). The benchmark sovereign default models (Arellano, 2008; Hatchondo and Martinez, 2009; Chatterjee and Eyigungor, 2012) have nothing to say about this pattern as all defaults result in 100% haircuts, default duration is exogenous, and real exchange rates are constant. In this paper, we construct a default model with an intensive margin of default that rationalizes these patterns while simultaneously shedding light on how much of these patterns are causal—i.e., hard (soft) defaults literally reduce output—versus how much of these patterns are driven by selection—i.e., persistently low output growth leads to hard defaults.

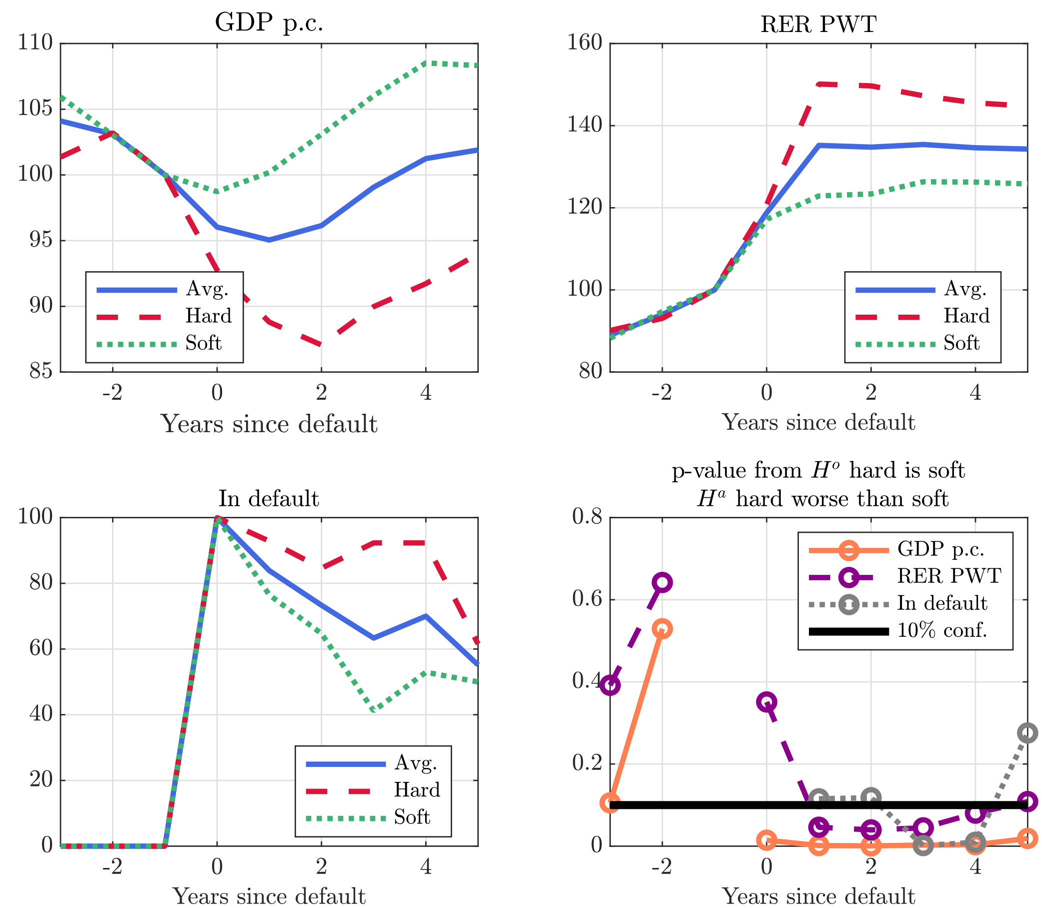


Figure 1: Hard and Soft Default Episodes

In the standard model, the sovereign's debt repudiation decision is a once-and-for-all choice to never repay any existing debt. We replace this assumption with a repeated bargaining problem between the sovereign and creditors. When choosing to negotiate, the sovereign proposes a haircut. Creditors evaluate whether they want to accept that or not, having rational expectations about the future haircut offers. Large haircuts occur when the sovereign proposes a large haircut both in the current period and in expectation. Quantitatively, the inclusion of growth shocks, transitory shocks, and endogenous real exchange rates (RER) lets creates a pattern where (1) bad growth shocks cause debt-GDP to explode higher and remain high for a long time—leading to large haircuts, while (2) bad transitory shocks cause a smaller and less persistent increase in debt-GDP, resulting in smaller haircuts. This selection into default, however, is only part of the story as default costs also play a direct role in reducing output. Disciplining the model lets us determine how much of the pattern is driven by selection versus causal effects. We find more than 60-85% of the difference between hard and soft defaults is due to selection.

In addition to capturing the differences between hard and soft defaults, the model also produces two key empirical regularities. First, it generates a strong positive correlation between default duration and haircuts (Benjamin and Wright, 2013). We show this is primarily attributable to a simple feature of the data, namely, the growth of missed payments and interest while in default. Second, the model generates the very mild effects associated with preemptive restructurings, where debt is restructured without any missed payments. This arises when the sovereign is in not in default and makes a positive haircut offer that is accepted. In this case, the sovereign is able to restructure without a (legal) default and its associated default costs.

The model also is a large quantitative success in matching the business cycle dynamics of Argentina including its spreads, RER, debt, GDP, default, and haircuts with few exceptions. Moreover, while flexibly parameterized default costs have been a crucial feature of quantitative models, our model generates all these features with only a single proportional default cost. The key, however, is that this default cost is on tradable goods, which generates a large RER depreciation in default. When using filtered shocks from a linearized version of the model, the model's implied dynamics are close to the data's but understates the severity of the 2001-2005 default. We show this can be rectified by explicitly modeling the pegged system Argentina put in place from 1993 to 2001. Exiting the peg causes a huge RER depreciation that drastically increases debt-GDP and ultimately creates a massive haircut, like in the data.

While quite rich, our proposed model is also considerably simpler along some dimensions than existing models of debt negotiation. E.g., many sovereign debt models have used an alternative offer approach to bargaining, where the sovereign gets to propose, then the creditors, and so on. In our novel bargaining approach, the sovereign proposes every time but creditors have shocks that make the sovereign uncertain about what offer creditors will accept. We show theoretically that the noisiness of these shocks controls the bargaining power creditors have.

Related Literature

Our work is closely related to Arellano, Mateo-Planas, and Ríos-Rull (2013). They show that default in the data is always partial in that defaulted debt relative to payments due is always less than one. Further, they propose a model that explicitly keeps track of debt in arrears, which rolls over at an exogenous rate. Relative to them, we have three main contributions. First, we show that the composition of growth and transitory shocks is the main driver for how "partial" default is (in the sense of how large the haircuts are). Second, our model captures many of the features of partial default without having to *explicitly* keep track of debt in arrears (it is still included, but in the overall stock of debt). Third, our model is computationally more facile, and the long-term debt specification can be computed without trouble (in their online draft, only the short-term debt version has results as of this writing). This tractability makes the model amenable to estimation and extensions along other dimensions. We also decompose the observed negative correlation between haircut size and output growth into causal and selection effects. Our work is also related to a substantial literature on debt renegotiation including Yue (2010), Asonuma and Trebesch (2016), and many other papers.

The rest of the paper proceeds as follows. We show the model and theoretical results in Section 2. The calibration exercise is in Section 3. In Section 4, we discuss the model's implications for hard and soft defaults. Section 5 presents estimates for the paths of several variables during the defaults in the 1980s and 2001 in Argentina. We conclude that section with a narrative account of economic events in Argentina and relate them to the filtered shocks and decomposition from our model.

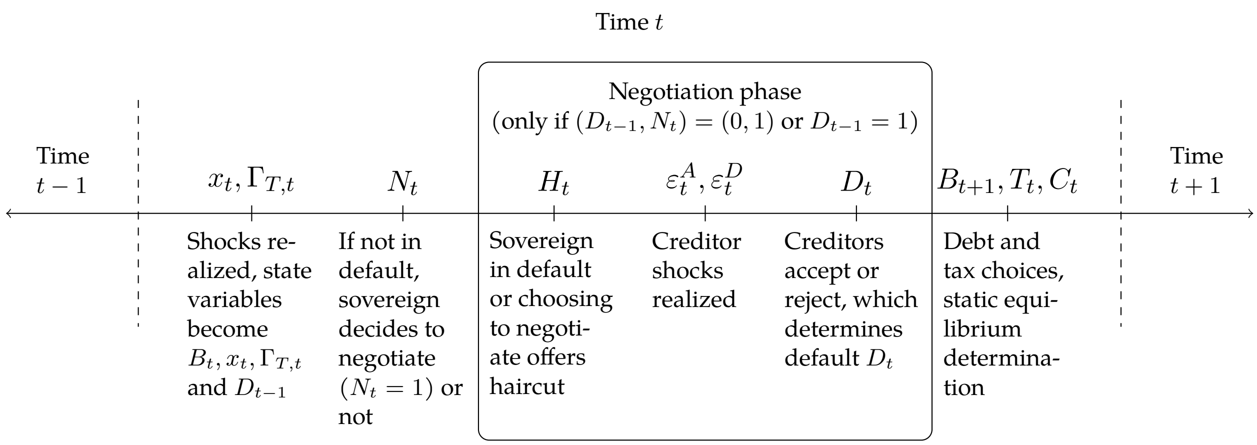
2 Quantitative model

Before going into details, we briefly describe the structure of the quantitative model. There are two types of goods, tradables and nontradables. The country is endowed with a stochastic amount of tradables and a deterministic amount of nontradables, both of which grow secularly over time. The tradable endowment is subject to both growth shocks and transitory shocks. All debt is tradable-denominated. There are three types of agents:

- **Consumers**. These take all prices as given and choose the optimal level of tradable and nontradable consumption.
- **The sovereign**. This agent takes as given the behavior of consumers and creditors and seeks to maximize consumer welfare using debt, taxes, and sovereign debt negotation.
- **Creditors**. These (foreign) agents own and competitively price all the sovereign debt and negotiate with the sovereign.

During negotiations, the sovereign proposes a haircut amount that creditors decide to accept or reject. In their accept/reject decision, creditors take into account expected future haircut offerings, shocks, and sovereign choices. This is formally modeled as a dynamic game, and we will seek a Markov perfect equilibrium of that game.

Figure 2: Timeline



2.1 Endowments

For tractability reasons, we assume that the nontradable endowment $Y_{N,t}$ grows deterministically at a rate μ with

$$\begin{aligned} Y_{N,t} &= \Gamma_{N,t} \\ \Gamma_{N,t} &= \mu \Gamma_{N,t-1}. \end{aligned} \tag{1}$$

There is a "potential" tradable endowment $Y_{T,t}$ that has both transitory and permanent shocks.[1] Specifically, we will assume it evolves according to the following process:

$$\begin{aligned} Y_{T,t} &= z_t \Gamma_{T,t} \\ \Gamma_{T,t} &= g_t \Gamma_{T,t-1} \\ \log z_t &= \rho_z \log z_{t-1} + \varepsilon_{z,t}, \quad \varepsilon_{z,t} \overset{i.i.d.}{\sim} N(0, \sigma_z^2). \end{aligned} \tag{2}$$

The transitory shock is z_t, while g_t is the growth shock. The growth shock evolves according to

$$\log g_t = (1 - \rho_g) \log \mu + \rho_g \log g_{t-1} + (\rho_e - 1) \log e_{t-1} + \varepsilon_{g,t}, \quad \varepsilon_{g,t} \overset{i.i.d.}{\sim} N(0, \sigma_g^2) \tag{3}$$

for $\rho_e \in [0, 1)$ where

$$e_t = \frac{\Gamma_{T,t}}{\Gamma_{N,t}}.$$

Incorporating a drift in g_t using $(\rho_e - 1) \log e_{t-1}$ ensures that log difference between the trend components shrinks at rate ρ_e in expectation, thus keeping $\frac{\Gamma_{T,t}}{\Gamma_{N,t}}$ (which is e_t) stationary. Strictly

[1] An important source of income in the countries studied in the related literature are commodity exports. The stochastic nature of the tradable endowment aims to capture fluctuations in commodity prices.

speaking, this means the growth shocks are not permanent. However, we will set ρ_e close to one numerically, approximating permanence. Jointly, the vector $x_t := [z_t, g_t, e_t]'$ in logs follows a VAR(1) with correlated innovations.

In the case the sovereign is in a state of default, there is a cost χ that reduces the potential tradable endowment to the realized tradable endowment. We let D_t denote whether the sovereign is in default, in which case the realized endowment is

$$(1 - D_t \chi) Y_{T,t}.$$

In our model, default is *not* a choice variable, but rather the result of creditors rejecting an offer tendered by the sovereign. Unlike in most of the literature, we will not need a flexibly parameterized default cost to reproduce the data's behavior. Second, because the default cost falls on tradables only, a default will itself generate a real exchange rate depreciations, consistent with the data.

2.2 The household problem

In describing the household problem, we will focus on the no-default case where the tradable endowment is $Y_{T,t}$. The default case is the same but with $Y_{T,t}$ everywhere replaced by $(1 - \chi_t) Y_{T,t}$. Preferences over consumption will be given by a time separable utility $\mathbb{E}_t \sum_\tau \beta^{\tau - t} u(C_t)$ for C_t a CES aggregator of tradable and nontradable consumption:

$$C_t = \left(\alpha_T C_{T,t}^{\frac{\rho-1}{\rho}} + \alpha_N C_{N,t}^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}}.$$

Here, $\rho > 0$ is the elasticity of substitution with the limit case of $\rho = 1$ given by Cobb-Douglas and smaller ρ making tradables and non-tradables more complementary. In order to detrend the model, we will need CRRA preferences over C_t with $u(C_t) = \frac{C_t^{1-\sigma}}{1-\sigma}$.

We choose nontradables as the numeraire so $p_{N,t} \equiv 1$, and let $p_{T,t}$ denote the relative price of tradables at time t. Households take prices and lump-sum taxes T_t (paid in tradables) as given, and can neither borrow nor save, resulting in the budget constraint

$$\sum_{i \in \{T,N\}} p_{i,t} C_{i,t} = \sum_{i \in \{T,N\}} p_{i,t} Y_{i,t} - p_{T,t} T_t.$$

The household maximization problem (of choosing $\{C_{i,t}\}$ subject to $\{Y_{i,t}\}$) is static, and is characterized by the budget constraint and the FOC

$$p_{T,t} = \frac{\alpha_T}{\alpha_N} \left(\frac{C_{N,t}}{C_{T,t}} \right)^{1/\rho}.$$

The government will not consume any goods, only transferring resources to households. Therefore, market clearing for non-tradables further requires that $C_{N,t} = Y_{N,t}$.

Define p_t as the price index

$$p_t = \left(\sum_{i,t} \alpha_i^\rho p_{i,t}^{1-\rho} \right)^{\frac{1}{1-\rho}}.$$

Define Y_t so that $p_t Y_t = \sum_{i \in \{T,N\}} p_{i,t} Y_{i,t}$. Then it is easy to verify that the consumption allocations given by

$$C_{i,t} = \alpha_i^\rho \left(\frac{p_{i,t}}{p_t} \right)^{-\rho} (Y_t - \frac{p_{T,t}}{p_t} T_t)$$

satisfy the FOC and the budget constraint. With these good-specific allocations, aggregate consumption is

$$C_t = \left(\sum_i \alpha_i C_{i,t}^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}} = Y_t - \frac{p_{T,t}}{p_t} T_t$$

Hence, we can write the budget constraint as

$$C_t = Y_t - \frac{p_{T,t}}{p_t} T_t.$$

2.3 Real exchange rate determination

To see the determination of the real exchange rate, consider a nominal exchange rate of $E_{ARG/USD}$ (where we've labeled with ARG for Argentina and USD for the US anticipating the application). We will assume the law of one price holds for tradable goods, so that 1 USD buys the same units of the tradable good as 1 USD converted to ARG. That is, $1/p^*_{T,t} = E_{ARG/USD}/p_{T,t}$, or

$$E_{ARG/USD} \frac{p^*_{T,t}}{p_{T,t}} = 1.$$

where the * denotes foreign (US) prices. The RER uses the same logic, expressing the relative price of the foreign good $1/p_t^*$ compared to the domestic good, which costs $E_{ARG/USD}/p_t$:

$$RER_t := \frac{E_{ARG/USD}/p_t}{1/p_t^*}.$$

Following the discussion in Uribe and Schmitt-Grohe,

$$RER_t := \frac{E_{ARG/USD}/p_t}{1/p_t^*} \frac{p_{T,t}}{p_{T,t}} = \frac{E_{ARG/USD}/p_t}{1/p_t^*} \frac{p_{T,t}}{E_{ARG/USD} p^*_{T,t}} = \frac{p_{T,t}/p_t}{p^*_{T,t}/p^*_t}.$$

We will take $p^*_{T,t}/p^*_t$ as exogenous and normalize it to 1. Consequently, the real exchange rate is simply

$$RER_t = \frac{p_{T,t}}{p_t}.$$

2.4 Static equilibrium in the domestic model block

Since the household problem is static, we can characterize the equilibrium conditions for their optimality and market clearing conditional on a tax T_t and the shock $Y_{T,t}$. As we will argue the $RER_t = p_{T,t}/p_t$ is key, we emphasize its role. The conditions are given by market clearing,

$$C_{N,t} = Y_{N,t},\ C_{T,t} = Y_{T,t} - T_t, \tag{4}$$

optimality,

$$RER_t = p_t^{-1} \frac{\alpha_T}{\alpha_N} \left(\frac{Y_{N,t}}{Y_{T,t} - T_t} \right)^{1/\rho}, \tag{5}$$

and aggregation,

$$\begin{aligned} 1 &= \alpha_T^{\rho} RER_t^{1-\rho} + \alpha_N^{\rho} p_t^{\rho-1} \\ Y_t &= RER_t Y_{T,t} + p_t^{-1} Y_{N,t} \\ C_t &= \left(\alpha_T C_{T,t}^{\frac{\rho-1}{\rho}} + \alpha_N C_{N,t}^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}}. \end{aligned} \tag{6}$$

Using some manipulation, one can find the two key expressions we will use in the sovereign's problem:[2]

$$\begin{aligned} RER_t &= \alpha_T \left(\alpha_T + \alpha_N \left(\frac{Y_{N,t}}{Y_{T,t} - T_t} \right)^{\frac{\rho-1}{\rho}} \right)^{\frac{1}{\rho-1}} \\ C_t &= (Y_{T,t} - T_t) \alpha_T^{-\rho} RER_t^{\rho} \end{aligned} \tag{7}$$

At risk of being redundant, the conditions in default are the same but everywhere replacing $Y_{T,t}$ with $(1-\chi)Y_{T,t}$.

2.5 Overview and timing of the negotiation game

We model the interaction between the sovereign and a large creditor (or multiple coordinating creditors) as an extensive form game, and we will look for a Markov perfect equilibrium. The timing of the model is as follows:

1. The sovereign comes into the period with debt B_t.
2. Innovations are realized, resulting in the exogenous state variables z_t, g_t, e_t.
3. The sovereign decides whether to negotiate with creditors ($N_t = 1$) or not.
4. If the sovereign chooses to negotiate, they enter the negotiation phase:
   (a) The sovereign *proposes* a haircut $H_t \in [0, 1]$.
   (b) Shocks affecting the accept / reject decision of creditors are realized.

[2] We establish this in proposition 2 in the appendix.

(c) Creditors decide to accept the offer or not.

(d) If the offer is accepted, the sovereign is not in default; if the offer is rejected, the sovereign is in default and suffers an output cost.

5. The sovereign chooses debt issuance (if applicable), levies taxes, and agents consume. If in default, debt accumulates with interest R^D.

In our model, a default is when a payment is missed, which occurs when the haircut offer H_t is not accepted. Our formulation captures two important features of the data. First, it endogenously generates a positive correlation between haircuts and default duration.[3] Second, it allows for the possibility that an offered haircut $H > 0$ is accepted but no debt payments are missed and therefore no default occurs. This is what Asonuma and Trebesch (2016) refer to as a *preemptive restructuring*. Consistent with their findings, preemptive restructurings in the model will have very mild effects on GDP.

2.6 The government problem

The sovereign can levy taxes T_t in tradable goods. The government internalizes the effect of its policies on prices and consumption allocations. Consequently, it knows that changing taxes T_t or a defaulting will change static equilibrium allocations as summarized in (7). To parsimoniously capture these equilibrium effects, we define

$$\psi(m) = \left(\alpha_T + \alpha_N \left(m^{-1}\right)^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}}$$

so that $\psi((Y_{T,t} - T_t)/Y_{N,t})$ gives $(RER_t/\alpha_T)^\rho$.

At any point in time the sovereign has a stock of tradable-good-denominated debt B_t. The debt is long-term maturing at a geometric rate λ with a coupon κ on unmatured debt. We define $\tilde{\lambda} = \lambda + (1-\lambda)\kappa$ as the debt-service due per unit of debt. When not in default, the government can issue debt. Any net debt issuance $B_{t+1} - (1-\lambda)B_t$ is valued at $Q(B_{t+1}, x_t, \Gamma_{T,t})$ per unit. Consequently, the government budget constraint is

$$T_t + Q(B_{t+1}, x_t, \Gamma_{T,t})(B_{t+1} - (1-\lambda)B_t) = \tilde{\lambda} B_t.$$

When in default, debt grows at a rate R^D, capturing the accumulation of missed coupons and interest on the defaulted debt.[4]

[3]If the sovereign and creditors Nash bargained over the haircut instead of our sequential bargaining approach, an agreement would be met immediately (provided a positive surplus existed). In deterministic models of sequential bargaining (Rubinstein, 1982), an agreement is met right away. However, here the size of the "pie" is time varying. As Merlo and Wilson (1995) analyze, the stochastic case is much more complicated and an agreement is not necessarily met right away. An alternative approach to generating delay is to have alternating as proposed in Benjamin and Wright and now adopted in many papers.

[4]Rather than separately model the stock of debt and debt in arrears, as is done in Arellano, Mateos-Planas, and Rios-Rull, we combine these into a single state variable and approximate the cost using R^D. This saves a continuous state variable.

The recursive formulation of the sovereign's problem (keeping the time subscripts for aid in interpretation) conditional on repayment as

$$V^R(B_t, x_t, \Gamma_{T,t}) = \max_{B_{t+1}} u(C_t) + \beta \mathbb{E}_{[x_{t+1}|x_t]} \left[\max_{N_{t+1} \in \{0,1\}} \left\{ \begin{array}{r} N_{t+1} \, V(B_{t+1}, x_{t+1}, g_{t+1}\Gamma_{T,t}) \\ +(1 - N_{t+1}) \, V^R(B_{t+1}, x_{t+1}, g_{t+1}\Gamma_{T,t}) \end{array} \right] \right\}$$

$$\text{s.t. } C_t = (Y_{T,t} - T_t)\psi((Y_{T,t} - T_t)/Y_{N,t})$$

$$T_t = -Q(B_{t+1}, x_t, \Gamma_{T,t})(B_{t+1} - (1-\lambda)B_t) + \tilde{\lambda} B_t$$

$$Q(B_{t+1}, x_t, \Gamma_{T,t}) \geq \underline{q}$$

Next period's negotiation decision N_{t+1} appears in the continuation utility and allows the sovereign to either continue in repayment next period or enter into negotiation. Hence, while in repayment, the sovereign can avoid the risk of failed negotiations and default by staying current on payments. The final constraint $Q \geq \underline{q}$ prevents the sovereign from issuing debt at spreads above a certain level. With negotiation, it's important to limit the ability of the sovereign to dilute existing debt holders as otherwise infinite dilution may be optimal.[5] Let the optimal savings policy and negotiation policies be denoted $\mathcal{B}(B, x, \Gamma)$ and $v(B, x, \Gamma_T)$, respectively.

The value conditional on default is

$$V^D(B_t, x_t, \Gamma_{T,t}) = u(C_t) + \beta \mathbb{E}_{[x_{t+1}|x_t]} V(B_{t+1}, x_{t+1}, g_{t+1}\Gamma_{T,t})$$

$$\text{s.t. } C_t = (1-\chi)Y_{T,t}\psi(((1-\chi)Y_{T,t})/Y_{N,t})$$

$$B_{t+1} = R^D B_t$$

This embeds the endowment loss $\chi Y_{T,t}$ and the associated RER depreciation associated with default in $\psi(((1-\chi)Y_{T,t})/Y_{N,t})$. When in default, debt continues to grow at rate R^D, which reflects the growth of liabilities from continued missed principal and coupon payments as well as interest on debt in arrears.

The value of negotiation is

$$V(B_t, x_t, \Gamma_{T,t}) = \max_{\hat{H}_t \in [0,1]} \left[\begin{array}{r} A(\hat{H}_t; B_t, x_t, \Gamma_{T,t}) V^R((1-\hat{H}_t)B_t, x_t, \Gamma_{T,t}) \\ +(1 - A(\hat{H}_t; B_t, x_t, \Gamma_{T,t}))V^D(B_t, x_t, \Gamma_{T,t}) \end{array} \right],$$

where the sovereign internalizes the haircut offer's role in the probability of acceptance, A. Let the optimal policy be denoted $H(B, x, \Gamma_T)$.

2.7 Creditors' problem

Creditors face two types of shocks when evaluating a haircut offer. With probability $1-\bar{\alpha}$, creditors reject *any* offer. This shock ensures two important features. First, it ensures negotiations are always costly for the sovereign by ensuring the negotiations might fail and result in default. Second, it

[5] Hatchondo, Martinez, and Sosa-Padilla (JME, 2014) show it can be optimal to infinitely dilute debt.

controls how much commitment creditors have. For instance, as this probability goes to one, default becomes permanent creating a sort of grim trigger punishment for the sovereign. Conversely, as the probability goes to zero, debt could be constantly renegotiated leading to no realized default and therefore no punishment.

The second type of shock creditors face captures purely idiosyncratic motives for accepting or rejecting. Specifically, when considering a haircut offer H_t that would carry fundamental value (per unit) of Q_t^A and imply a value of Q_t^D if rejected, idiosyncratic valuation shocks make accepting worth $Q_t^A + \sigma_\alpha \epsilon_t^A$ and rejecting worth $Q^D + \sigma_\alpha \epsilon_t^D$. The shocks ϵ_t^A and ϵ_t^D are i.i.d. Type 1 extreme value, and so are purely idiosyncratic. In reality, considering a restructuring has many idiosyncratic components such as the patience of creditors, their willingness to try to holdout for better terms, or different risk exposures such as through credit default swaps. These shocks capture such idiosyncratic differences. Together, the shocks imply the ex-ante probability of accepting the offer is

$$A(\hat{H}_t; B_t, x_t, \Gamma_{T,t}) = \bar{\alpha} \frac{1}{1 + e^{-(Q^A(\hat{H}_t, B_t, x_t, \Gamma_{T,t}) - Q^D(B_t, x_t, \Gamma_{T,t}))/\sigma_\alpha}}.$$

An accepted offer's value Q^A is given by

$$Q^A(\hat{H}_t, B_t, x_t, \Gamma_{T,t}) = (1 - \hat{H}_t)\left(\tilde{\lambda} + (1-\lambda) Q(\underbrace{\mathcal{B}((1-\hat{H}_t)B_t, x_t, \Gamma_{T,t})}_{B_{t+1} \text{ if } \hat{H}_t \text{ accepted}}, x_t)\right),$$

which takes into account the haircut size and that the debt must be serviced at least once. One interesting feature of the model is creditors take into account the effects of debt *concentration*: Q^A moves less than proportionately with $1 - \hat{H}_t$ because larger haircuts reduce B_{t+1} (since $\mathcal{B}((1 - \hat{H}_t)B_t, x_t)$ is monotone increasing in its first argument) and hence increase the market value of long-term debt $Q(B_{t+1}, x_t, \Gamma_{T,t})$. This debt concentration effect is the reverse of debt dilution.

The one period ahead debt pricing when in repayment is

$$Q(B_{t+1}, x_t, \Gamma_{T,t}) = \frac{1}{1+r^*} \mathbb{E}_{x_{t+1}|x_t} \begin{bmatrix} (1 - N_{t+1}) Q^A(0, B_{t+1}, x_{t+1}, \Gamma_{T,t+1}) \\ + N_{t+1} A_{t+1} Q^A(H_{t+1}, B_{t+1}, x_{t+1}, \Gamma_{T,t+1}) \\ + N_{t+1}(1 - A_{t+1}) Q^D(B_{t+1}, x_{t+1}, \Gamma_{T,t+1}) \end{bmatrix} \tag{8}$$

where

$$\begin{aligned} H_{t+1} &= H(B_{t+1}, x_{t+1}, \Gamma_{T,t+1}), \\ N_{t+1} &= N(B_{t+1}, x_{t+1}, \Gamma_{T,t+1}), \text{ and} \\ A_{t+1} &= A(H(B_{t+1}, x_{t+1}, \Gamma_{T,t+1}); B_{t+1}, x_{t+1}, \Gamma_{T,t+1}). \end{aligned} \tag{9}$$

Note that in pricing the rejected offer, the Markov policies H and A are used, consistent with the equilibrium concept.[6]

[6] A subtle point is that the ϵ^A and ϵ^D shocks enter only via the acceptance probabilities. Hence, (15) is generally not the expected discounted value of (**??**). We do this so the pricing always reflects the fundamental values, i.e., the net present value of the stream of payments. It is trivial to reconcile the two equations by adding a correction term to (**??**)

A rejected offer's value Q^D is given by

$$Q^D(B_t, x_t, \Gamma_{T,t}) = R^D \tilde{Q}(\underbrace{R^D B_t}_{B_{t+1}}, x_t, \Gamma_{T,t}). \tag{10}$$

where $\tilde{Q}$ is the continuation value of a unit of defaulted debt. The reason R^D appears is that Q^D is the price per unit of debt and the missed payments are being added to the stock of debt. $\tilde{Q}$ is analogous to Q but always features negotiation:

$$\tilde{Q}(B_{t+1}, x_t, \Gamma_{T,t}) = \frac{1}{1+r^*}\mathbb{E}_{x_{t+1}|x_t}\begin{bmatrix} A_{t+1}Q^A(H_{t+1}, B_{t+1}, x_{t+1}, \Gamma_{T,t+1}) \\ (1-A_{t+1})Q^D(B_{t+1}, x_{t+1}, \Gamma_{T,t+1}) \end{bmatrix} \tag{11}$$

where A_{t+1} and H_{t+1} are as in (9).

One convenient feature of our setup is that the haircut H exactly corresponds to the Sturzenegger and Zettlemeyer (2008) (SZ) haircut measure.[7] The SZ haircut H_{sz} is one minus the ratio of the net present value (NPV) of the new debt relative to the NPV of the old debt inclusive of interest on arrears, with both discounted using the IRR of the new debt. At the time of the debt exchange, the old debt with interest on arrears is summarized by B_t, while the new debt is $(1-\hat{H}_t)B_t$. Since these debt amounts prescribe exactly the same profile of payments ($\tilde{\lambda}$ next period, $(1-\lambda)\tilde{\lambda}$ in the second, and so on), the SZ haircut is

$$H_{sz} = 1 - \frac{\sum_{j=1}^{\infty}(1+r)^{-j}\tilde{\lambda}(1-\lambda)^{j-1}(1-\hat{H}_t)B_t}{\sum_{j=1}^{\infty}(1+r)^{-j}\tilde{\lambda}(1-\lambda)^{j-1}B_t} = \hat{H}_t$$

(where the r, though evidently irrelevant here, corresponds to the IRR from the new debt).

2.8 Equilibrium

An equilibrium is policies and values for the sovereign $\mathcal{B}, N, H, V, V^D, V^R$ and acceptance probabilities and values for creditors $A, Q^A, Q^D, Q, \tilde{Q}$ solving their respective problems that the other agents' policies and values as given. The detrended version of the model is shown in appendix B.

3 Data, estimation, and calibration

This section describes how we determine the parameters of the model. We set some of the parameters apriori, others we estimate using a subset of the model's equations, and the rest we calibrate by matching moments.

that depends only on Q^A, Q^D, and σ_α and that leaves all the other model equations unaltered.

[7] There are two other primary measures of haircuts in the data. The first is a nominal haircut measure which compares old and new debts at their face values. The second is a market haircut measure that compares the face value of the old debt with the market value of the new debt. Tomz and Wright (2013) say in their data all the haircuts deliver surprisingly similar results.

3.1 Exogenously determined parameters

We set the constant relative risk aversion (CRRA) parameter to 2 in line with most of the literature. We set the coupon payment $\kappa = .03$, the maturity rate $\lambda = .05$, and the real risk-free rate $r^* = .01$ following Chatterjee and Eyigungor (2012). We set the arrears rate $R^D = 1.021$ to match the rate implied by κ, λ, and r^* when default lasts for 14 quarters.[8] We choose $\underline{q}$ so that spreads in repayment are less than 100%. Lastly, we fix the persistence of the transitory shock $\rho_z = .95$. Doing this allows z to capture business cycle frequency movement, makes the results more comparable to the literature (e.g., Arellano, 2008; Chatterjee and Eyigungor, 2012), and aids in identification. Similarly, we set $\rho_e = .99$ so that the half-life of a $\epsilon_{g,t}$ shock is almost 20 years.

3.2 Estimation

We estimate the shock parameters $(\mu, \rho_g, \sigma_z, \sigma_g)$ and the CES parameters $(\theta_T, \alpha_N, \rho)$—where $\theta_T = \alpha_T/(\alpha_N + \alpha_T)$ is the share of tradables absent shocks and default—using time series for the real exchange rate, GDP in constant national prices, and default indicators.[9] These estimates are all conditional on a value of χ (and since we calibrate χ, we must reestimate the model many times). We incorporate results from the literature using priors to improve identification and efficiency. First, we center the estimate for the elasticity of substitution 0.5 but with substantial support in $[.3, .7]$ in keeping with the survey in Akinci (2011). We center the share of tradables θ_T around the 40% used in Bianchi and Sosa-Padilla (2020). Last, we use a diffuse prior for σ_z that has a mode at 0.05, twice the size of in Arellano (2008) to account for the shock only hitting tradables.

The estimation equations are detrended and log-linearized versions of the endowment process (1-3), the static equilibrium equations (4-7) evaluated with $T_t = 0$, and a final equation that specifies a default policy, $\delta_t = .95\delta_{t-1} + .5\epsilon_{\delta,t}$ where $\delta_t = e^{D_t}$. We log-linearize with respect to RER_t, $y_{T,t} = Y_{T,t}/\Gamma_{T,t}$, $z_t, e_t, g_t, \Gamma_{T,t}$, and δ_t. So default has real effects, we log-linearize about $D_t = 1$ ($\delta_t = e^1$). We assume the measurement error of the log RER and GDP are 5% and 1%, respectively. We also allow for a small probability that default is mismeasured by using a measurement error of 25% for our default indicator (hence, for $D_t = 1$ to result in a no-default measurement requires a highly unlikely 4 standard deviation deviation).[10] The complete estimation system and additional

[8] To see where this number comes from, let the debt in arrears be denoted A_t. Then debt in arrears evolves according to $A_t = A_{t-1}(1+r^*) + (\lambda + \kappa(1-\lambda))B_{t-1}$. The remaining stock of debt that hasn't defaulted yet dwindles at rate $1-\lambda$: $B_t = (1-\lambda)B_{t-1}$. Consequently, the total debt stock (arrears plus non-defaulted debt) grows according to

$$(A_t + B_t) = (A_{t-1} + B_{t-1}) \left(1 + r^* \frac{A_{t-1}}{A_{t-1} + B_{t-1}} + \kappa(1-\lambda) \frac{B_{t-1}}{A_{t-1} + B_{t-1}} \right).$$

(Note that interest begins at the larger value $\kappa(1-\lambda)$ initially but converges to r^* eventually.) We choose $R^D = 1.021$ to match the average interest rate after 14 quarters.

[9] One of α_T or α_N is a normalization, which is identified from the average level of the RER in the data.

[10] We have annual default indicators throughout our quarterly sample and tried with some success to get the exact dates of default. However, the literature has measured default episodes in different ways, leading to very different results with respect to how long default episodes last and at what frequency it occurs. For example, as Tomz and Wright (2013) relay, the criteria of using the criterias of Borenstein and Panizza (2009), Cruces and Trebesch (2013), and Arteta and Hale (2008) record one, four, five, and 23 defaults during the 1980s, respectively.

details are given in the appendix.

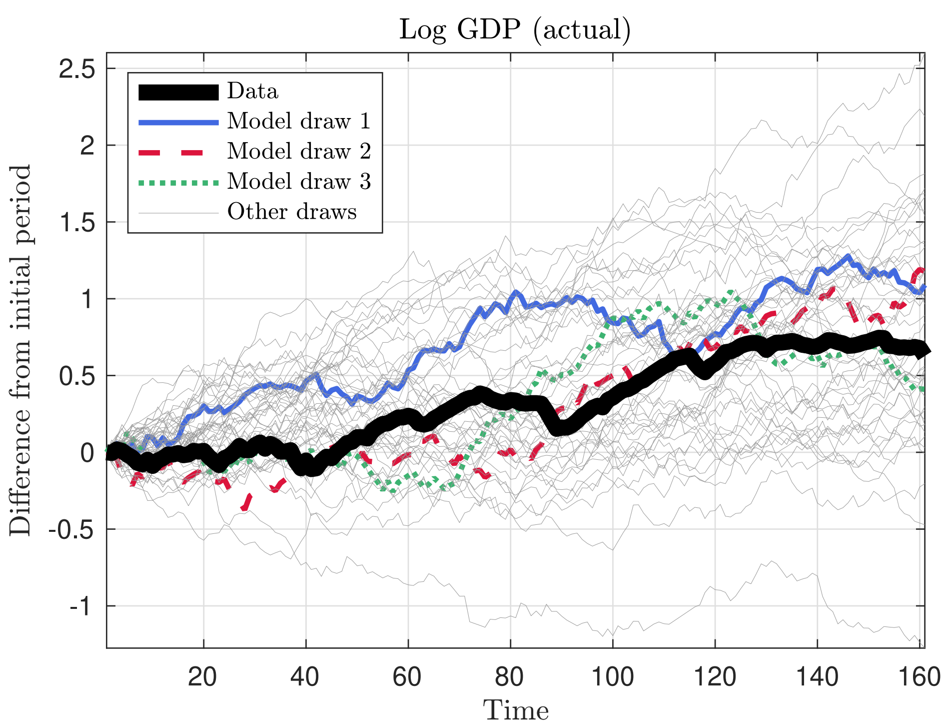
Table 1 reports estimates of the posterior modes and the associated standard errors alongside the prior distributions with some of its summary statistics.

Table 1: Estimates from the linearized model

Parameter	Estimate	SE	Prior	Mode	Mean	Stdev.
ρ	0.779	0.041	Beta(20,20)	0.500	0.500	0.078
θ_T	0.228	0.006	Beta(30,44.5)	0.400	0.403	0.056
σ_z	0.017	0.013	Beta(2,20)	0.050	0.091	0.060
σ_g	0.045	0.008				
ρ_g	0.666	0.096				
μ	1.005	0.000				
α_T	0.140	0.016				

Figure 3 plots the data's path for log GDP alongside some simulated paths from the model. The data's path exhibits 10 year periods of stagnation punctuated by fast growth, along with periods of sharp declines. The estimated GDP process reproduces these features, having multi-year periods of sharp growth followed by decade long stagnations. The very precise estimate of the positive trend does mean these paths all grow secularly over time, but over 160 quarters GDP can on net fall with a non-trivial probability.

Figure 3: Simulated GDP paths



The linearized model has a few key estimates. One the elasticity of the RER to endowment shocks, and this is -1.04. Another is the elasticity of the RER with respect to default, which is 0.287. The last is the elasticity of GDP to default, which is -0.073.

3.3 Calibration

The remaining four parameters are a default cost parameter χ, the time discount factor β, the probability creditors reject any offer $\bar{\alpha}$, and the idiosyncratic valuation shocks σ_α. We identify these by matching five moments: mean spreads and debt levels conditional on non-default periods, default duration conditional, average haircut sizes, and time in default. (The last moment aids in identifcation as the other four moments are conditional on either no default or default; we could alternatively match average spreads unconditionally, but prefer our approach to be comparable to the literature.)

The targeted and untargeted moments are displayed in Table 2. The model reproduces the targeted moments well, and also reproduces a host of untargeted ones. Some of the key untargeted moments the model reproduces are the excess volatility of consumption, countercyclical net exports, countercyclical real exchange rates, and a large dispersion of haircuts.

In sum, the model matches important targeted moments and many of the untargeted ones. We now look more in depth at the model's predictions.

4 Equilibrium hard and soft defaults

In this section, we discuss additional quantitative implications of our model.

4.1 Hard and soft defaults and preemptive restructurings

Figure 4 plots the paths of several macroeconomic variables following hard (red dashed lines) and soft defaults (green dots), as distinguished by being above or below median haircut defaults. For completeness, we plot alongside the paths for average defaults (blue lines) and preemptive restructurings (orange circles). Like in Trebesch and Zabel, GDP has not recovered in hard defaults even after 5 years. In contrast, soft defaults recover in around 2 years. Moreover, preemptive restructuring results in better outcomes than an average default, which in turn is less damaging that hard defaults.

4.2 Duration and default intensity

One of the key patterns generated by the model is a strong link between haircut intensity and default duration. The model's scatter plot is compared with the data's (using Trebesch and Zabel's data) in Figure 5. The pattern closely mirrors that in the data.

4.3 Inspecting the mechanism

We now look at the mechanism in a few different ways.

Targeted moments	Model	Target	Parameter	Value
Debt to GDP | no default	0.97	0.96	χ	0.28
Spreads | no default	0.08	0.08	β	0.94
Log default duration	2.84	2.87	$\bar{\alpha}$	0.06
Log default duration s.d.	1.12	0.91		
Fraction of time in default	0.38	0.40	σ_α	0.15

Untargeted moments	Model	Data
Debt to GDP s.d. | no default	0.32	0.34
Debt to GDP	2.22	1.23
Debt to GDP s.d.	2.58	0.75
Debt service to GDP | no default	0.08	0.06
Debt service to GDP s.d. | no default	0.03	-
Spreads s.d. | no default	0.08	0.04
Haircut size	0.66	0.38
Haircut size s.d.	0.18	0.21
Corr. of haircut and duration	0.64	0.31
Fraction of time with pre-emptive restructuring	0.00	0.02
RER	1.61	1.75
RER s.d.	0.71	0.68
RER | no default	1.31	-
RER s.d. | no default	0.52	-
Log GDP s.d.*	0.14	0.09
S.d. log consumption / s.d. log GDP*	1.03	1.09
Corr. of spreads and log GDP*	-0.01	-0.34
Corr. of NX/GDP and log GDP*	-0.14	-0.46
Corr. of RER and log GDP*	-0.23	-0.36
Corr. of spreads and log GDP* | no default	-0.30	-0.47
Corr. of NX/GDP and log GDP* | no default	-0.25	0.10
Corr. of RER and log GDP* | no default	-0.12	0.15
Corr. of spreads and log GDP* | spreads<.2	-0.24	-0.51
Corr. of NX/GDP and log GDP* | spreads<.2	-0.25	-0.27
Corr. of RER and log GDP* | spreads<.2	-0.09	-0.25

Note: $*$ means the variable has been detrended using Hamilton's Not-HP filter.

Table 2: Targeted and untargeted moments

Figure 4: Hard and soft defaults and preemptive restructurings

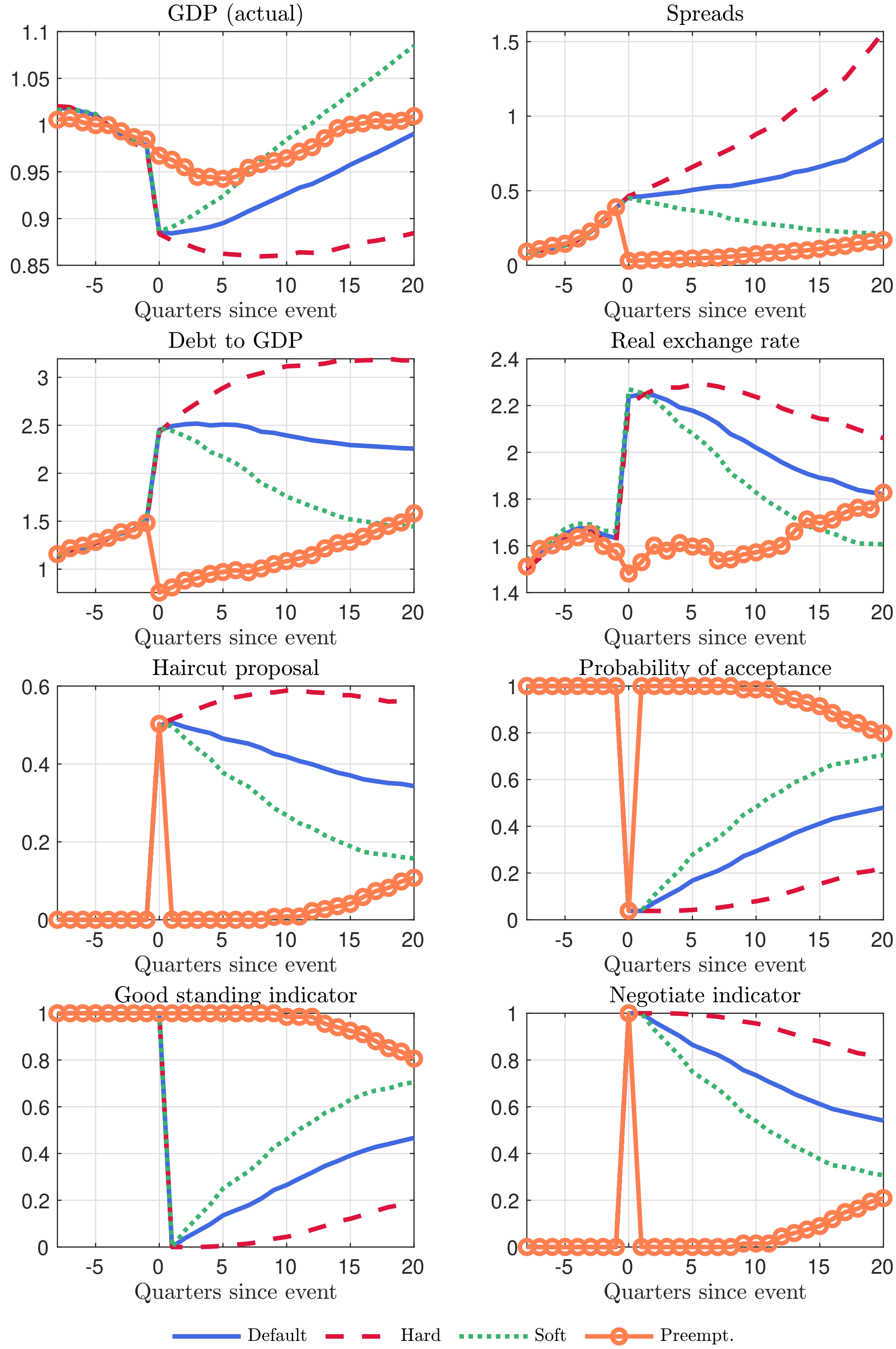
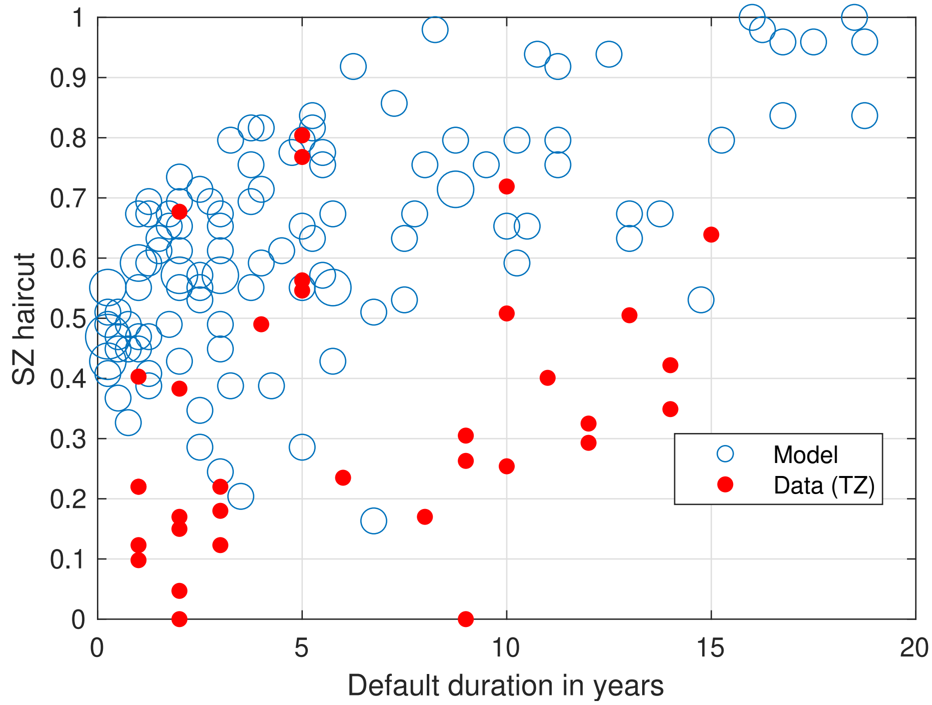


Figure 5: Haircut size and default duration



4.3.1 Theoretical insights

The equilibrium determination of haircuts involves several complicated equilibrium objects. To shed some light on the mechanism, it is useful to focus on a stylized case that captures some of its ingredients. So consider a continuous time model where the sovereign is risk neutral, begins in default, suffers an output cost χy while remaining in default, but can escape default at some rate $\hat{\alpha}(\hat{h})$, which is a function of the value of acceptance to creditors $\hat{q}^A$ and the defaulted debt value q^D, which takes into account expected future haircut offers and acceptance rates. Assume the debt is short-term, and let the debt stock be b, and so the haircut offer $\hat{h}$ is worth $\hat{q}^A = 1 - \hat{h}$ to creditors. Let y evolve according to Brownian motion with variance σ_y^2. Let the time discount rate of the sovereign and creditors be ρ. Use hats to denote off-equilibrium offer and acceptance rates $(\hat{q}^A, \hat{h}, \hat{\alpha}(\hat{h}))$, and non-hatted variables (q^A, h, α) to denote their on-equilibrium steady state values. Then the steady state equilibrium conditions are

$$\begin{aligned}
&\rho V = \max_{\hat{h}} -\chi y + \frac{V_{yy}\sigma_y^2}{2} + \hat{\alpha}(\hat{h})(-(1-\hat{h})b - V) \\
&\hat{q}^A = 1 - \hat{h} \\
&q^A = 1 - h \\
&\rho q^D = \alpha \cdot (q^A - q^D) \\
&\alpha = \hat{\alpha}(h)
\end{aligned} \tag{12}$$

together with a final equation specifying the off-equilibrium acceptance probabilities $\hat{\alpha}$, which we defer for a moment. The first equation is the HJB, the second is the off-equilibrium valuation of haircuts (which since debt is short-term is just $1 - \hat{h}$), the third is the belief about the value of accepting future haircut policies, the fourth is the value of declining the current offer, and the fifth is the expectation of future acceptance probabilities.

Note several properties here. First, note that

$$q^D = \frac{\alpha}{\alpha + \rho} q^A.$$

So it's always better accept an offer that has $\hat{h} = h$ rather than defer because one will do the same in the future and creditors are impatient. This means that if $\alpha(h)$ is noiseless, with $\alpha = \bar{\alpha}\mathbf{1}[q^A \geq q^D]$, the sovereign always offers $q^A = q^D$ and so $q^A = q^D = 0$ since this is the only way to satisfy the above equality: Haircuts are total.

Now, suppose the acceptance rate has the form

$$\hat{\alpha}(\hat{h}) = \bar{\alpha}\sigma_\alpha(\hat{q}^A - q^D)^{1/\sigma_\alpha}. \tag{13}$$

Then proposition 1 characterizes the equilibrium level of h and α:

Proposition 1. *In the simple model given by (12) and (13), equilibrium haircuts are*

$$h = 1 - \frac{\chi y}{b(\sigma_\alpha + 1)\rho}$$

and acceptance rates are given implicitly by

$$\alpha^{\sigma_\alpha} \frac{\alpha + \rho}{\rho} = (\bar{\alpha}\sigma_\alpha)^{\sigma_\alpha} (1 - h)$$

or

$$\alpha^{\sigma_\alpha}(\alpha + \rho) = (\bar{\alpha}\sigma_\alpha)^{\sigma_\alpha} \frac{\chi y}{b(\sigma_\alpha + 1)}$$

The proof is in the appendix.

This characterization admits a few immediate insights. First, as output increases, haircuts decrease. Second, as debt increases, haircuts increase. This suggests that when interest in arrears grows overtime, thereby growing b, haircuts increase. Third, as the accept reject becomes more inelastic from σ_α increasing, haircuts increase. Intuitively, this is because with a less elastic acceptance probability, the sovereign has less incentive to propose an attractive offer. More impatience from higher ρ makes creditors compete against their future selves in accepting offers, resulting in the acceptance of larger haircuts. Last, note that $\bar{\alpha}$ does not enter into the haircut decision at all. However, it does affect $\alpha(h)$ proportionately. So in the full model with interest on arrears growing, smaller $\bar{\alpha}$ results in more periods of interest accumulation and therefore larger haircuts (through the growth of b).

In the equation characterizing α, the lefthand side is an increasing function of α. So anything that decreases the righthand side (apart from changes in ρ) will necessary result in lower acceptance probabilities and longer delays. So, low default costs, low output, or large debts increase the probability of delay.

One final insight can be derived. Note that $(1-h)b$ is $q^A b$, i.e., the market value of the debt if an agreement is reach. Conversely, if no agreement is ever reached, the debt is worth 0. Similarly, $\frac{\chi y}{\rho}$ is the net present value of default costs. The surplus to the sovereign from reaching an agreement versus never reaching an agreement is $\frac{\chi y}{\rho} - (1-h)b$, the creditor's surplus from reaching an agreement is $(1-h)b$, while the total surplus is $\frac{\chi y}{\rho}$. So, in equilibrium, the creditor's total share of the surplus is

$$\frac{(1-h)b}{\chi y/\rho} = \frac{1}{\sigma_\alpha + 1}.$$

Hence, as the accept/reject decision becomes more or less noisy from σ_α, the share of the creditor's surplus varies from 0 (when the acceptance rate is infinitely elastic) to 1 (when the acceptance rate is linear). Note that because $\bar{\alpha}$ can be adjusted up or down to achieve a given level of haircut, this statement really is about the elasticity rather than the level of the acceptance rate.

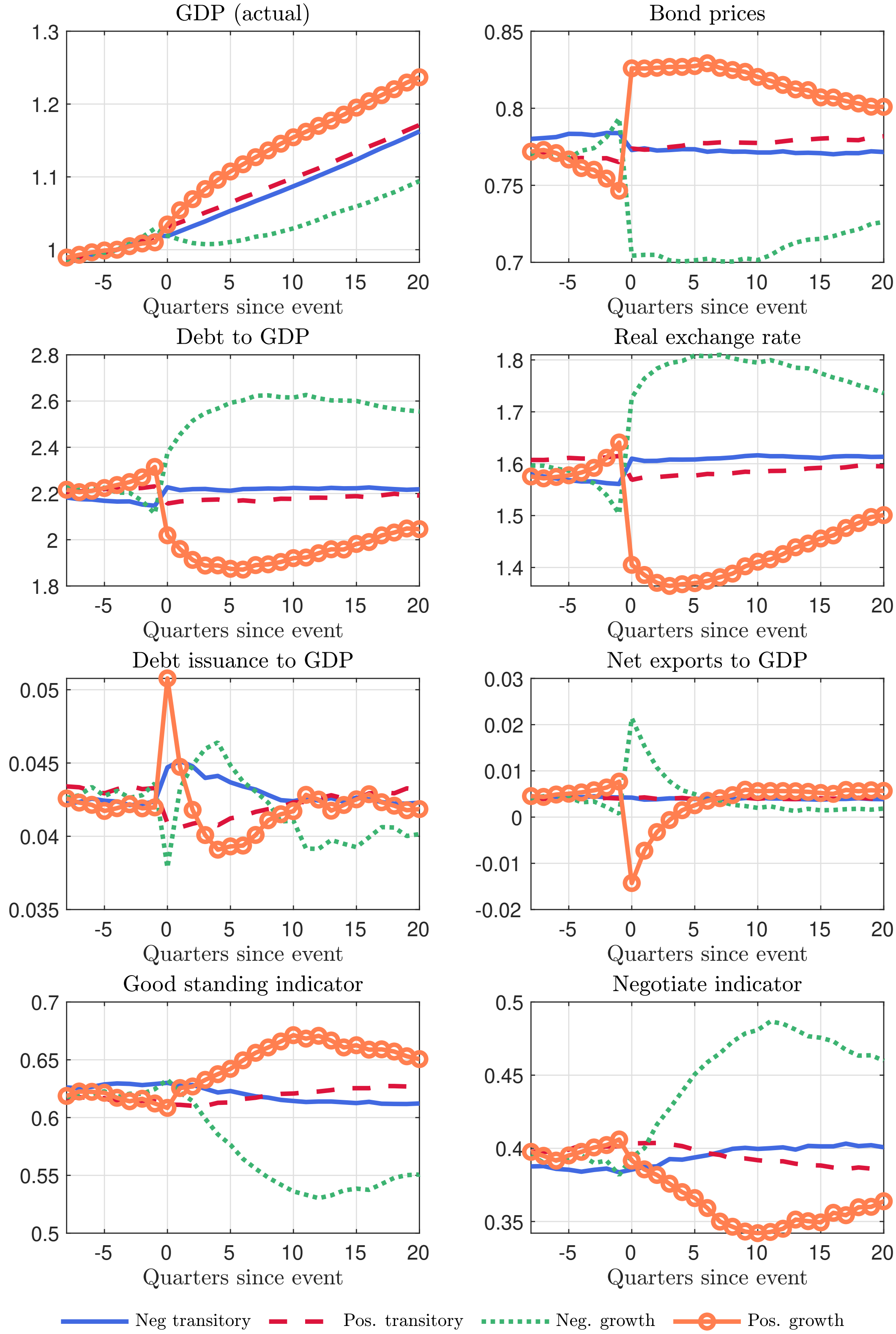
4.3.2 Impulse Response Functions – IRFs

Figure 6 plots the IRFs to transitory, z, and permanent, g, shocks. To highlight the asymmetric nature of default episodes, we allow for positive and negative disturbances. A positive growth shock (orange circles) leads to a permanent expansion. More important, there is a prolonged appreciation of the domestic currency accompanied by a drop in the debt-to-GDP ratio in spite of the higher issuance of debt. The boom results in higher bond prices and a worsening of the trade balance. Although a negative growth shock (green dots) has qualitatively similar but with flipped signs, they have stronger long-term effects on the economy than a positive growth innovation. For example, while during an expansion the local currency appreciates by 6%, it depreciates by 9% 20 quarters after an adverse growth shock.

4.3.3 Shutting down channels

Table 3 plots some key summary statistics for different model parameters that shut down or reduce different mechanisms in the model. Without debt in arrears growing ($R^D = 1$), the model generate a negative correlation between haircuts and duration and much smaller haircuts. However, this does not matter to creditors at all, as spreads and debt when not in default are unchanged. This is because how much the sovereign can afford / wants to pay creditors is an amount that scales in output. It's independent of the how big the debt is, per se. E.g., if the sovereign can at most afford to service a debt/GDP ratio of 1, then moving the current debt/GDP in arrears from 3 to 300 only increases the necessary haircut. It does not change the value to creditors, and therefore does not affect debt pricing ex ante. Hence much of the link between delay in the data and haircuts can be driven simply by how debt evolves post default, which is mechanical. The negative correlation

Figure 6: Response to z and g innovations conditional on good standing



between haircuts and delay comes from creditors knowing that when a shock is permanent, there's no point in delay (output will not recover)—hence bigger haircuts are offered when output has fallen permanently.

Another insight from this is that when creditors never automatically reject an offer ($\bar{\alpha} = 1$), default duration shrinks tremendously and the sovereign spends far less time in default. Moreover, spreads are ex-ante much lower but much less debt can be supported. Time spent in preemptive restructuring also increases by an order of magnitude. This increase in negotiation allows for a more efficient outcome to be met. The preemptive restructuring allows the sovereign to offer better terms in a negotiation because they can pass on some of the savings from default costs they won't incur. However, this benefit ex-post makes restructuring attractive ex ante, leading to debt being less sustainable.

Table 3: Alternative parameter values

Statistic	Bench.	$R^D = 1$	$\bar{\alpha} = 1$	$\rho \downarrow$	$\theta_T \uparrow$	$\sigma_\alpha \downarrow$	$\chi \uparrow$	$\beta \uparrow$
Debt to GDP | no default	0.965	0.990	0.121	0.951	1.065	0.956	1.024	0.978
Spreads | no default	0.076	0.102	0.132	0.074	0.077	0.081	0.082	0.054
Haircut size	0.663	0.403	0.551	0.672	0.657	0.659	0.654	0.676
Haircut size s.d.	0.184	0.197	0.091	0.174	0.182	0.174	0.179	0.182
RER	1.607	1.638	1.398	0.922	2.498	1.598	1.608	1.576
RER s.d.	0.714	0.740	0.575	0.454	1.107	0.718	0.778	0.729
Corr. of haircut and duration	0.645	-0.324	0.083	0.668	0.663	0.665	0.642	0.659
Log default duration	2.836	2.887	0.510	2.837	2.787	2.811	2.788	2.900
Fraction of time in default	0.377	0.460	0.037	0.371	0.383	0.386	0.378	0.312
Fraction of time with pre-emptive restructuring	0.001	0.001	0.021	0.001	0.001	0.001	0.000	0.000

4.4 Causation vs. selection

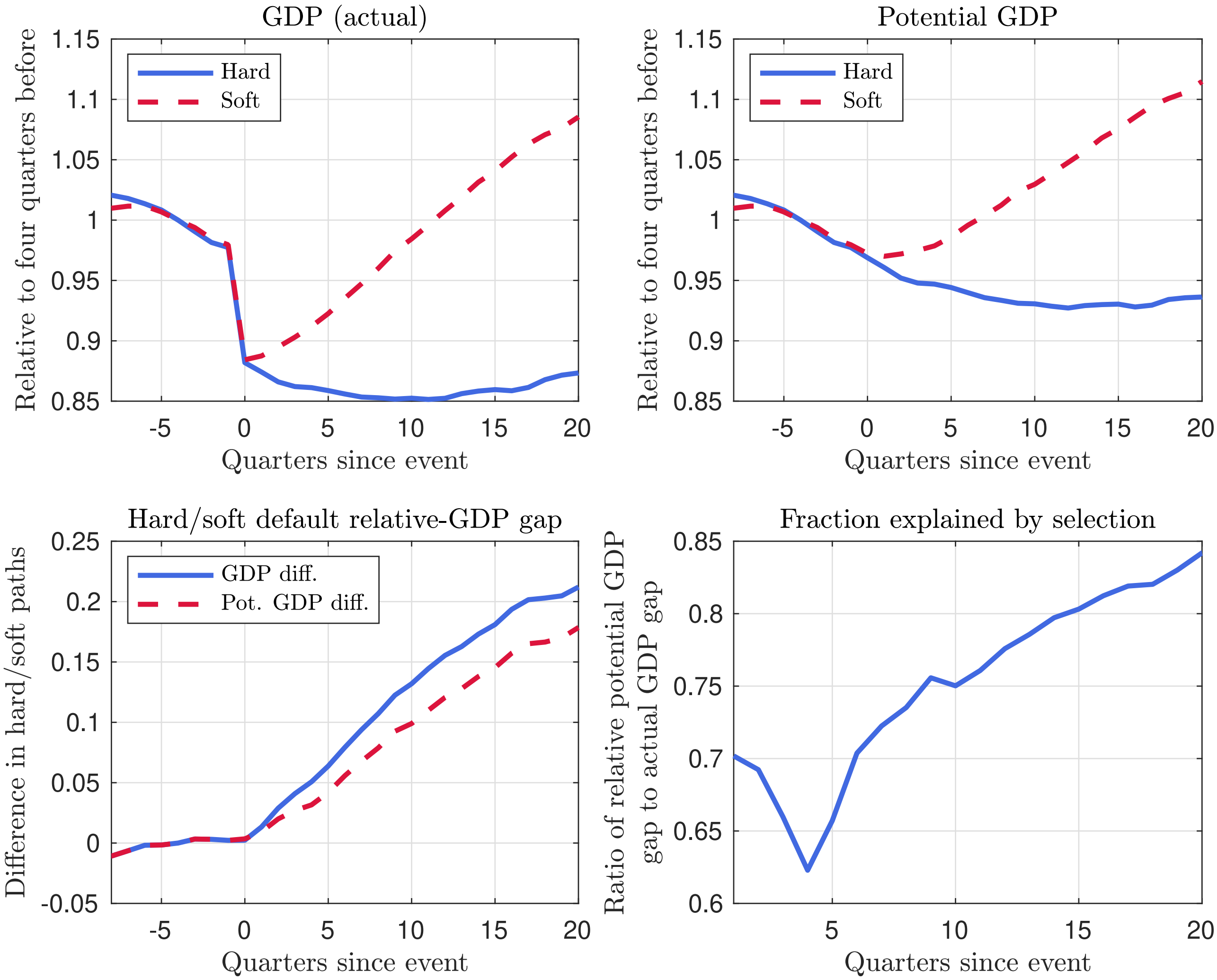
Trebesch and Zabel, while acknowledging the possibility of reverse causality, view the gap in outcomes between hard and soft defaults as predominantly causal. Numerous policy papers on sovereign debt have similarly argued that sovereigns should negotiate default to avoid the negative consequences of hard default [PAPERS TO BE LISTED]. In the model, the gap between hard and soft default paths is a mix of actual causal differences—driven by being in default and default costs—and selection, i.e., different shocks that result in hard vs. soft defaults. And, using the model, we can decompose how much of the observed difference between hard and soft defaults is causal vs selection by looking at the difference between realized GDP Y_t and potential GDP, $\tilde{Y}_t$, that is the GDP prevailing absent default costs. This, as well as the potential GDP in hard and soft defaults, is plotted in the top panels figure 7.

To be concrete, relative to a default at time t, the difference $Y_{t+h}^{soft}/Y_{t-4}^{soft} - Y_{t+h}^{hard}/Y_{t-4}^{hard}$ is the difference of the two series in the top left panel of figure 4. Taking the ratio of the potential GDP and actual gap

$$\theta_h = \frac{\mathbb{E}[\tilde{Y}_{t+h}^{soft}/\tilde{Y}_{t-4}^{soft} - \tilde{Y}_{t+h}^{hard}/\tilde{Y}_{t-4}^{hard}]}{\mathbb{E}[Y_{t+h}^{soft}/Y_{t-4}^{soft} - Y_{t+h}^{hard}/Y_{t-4}^{hard}]} \tag{14}$$

gives the fraction of the observed gap explained by selection. For instance, if $\tilde{Y}_t = Y_t$, then the observed difference is entirely driven by shocks, not default costs, and $\theta = 1$. On the other hand, if $\tilde{Y}_t^{hard} = \tilde{Y}_t^{soft}$, then all the difference is driven by default costs and $\theta = 0$. The numerator and denominator of (14) are displayed in the bottom left panel of figure 7, and θ itself is plotted in the bottom right panel.

Figure 7: Causation vs. selection



The estimates suggest 60-85% of the observed gap is driven by selection, with selection increasingly responsible for the difference at longer horizons. This indicates a substantial role for both causation and selection, but a much greater role for selection than may be expected based on previous work. Trebesch and Zabel were aware of the possibility of reverse causation and try to test for its presence. We can run these tests in the model, albeit approximately.

To run these tests, we must first discuss Trebesch and Zabel's coerciveness index is from Enderlein, Trebesch, and von Daniels (2012). For each of the following questions, one point is added:

1. Were payments missed?
2. Were payments unilaterally suspended?
3. Was there a full suspension (including interest)?
4. Was there a freeze on assets?
5. Was there a breakdown or refusal of negotations?
6. Was there an explicit moratorium declaration?
7. Were there explicit threates to repudiate?
8. Where there data disclosure problems?
9. Was there forced and non-negotiated restructuring?
10. Were all the above satisfied?

The maximum score is a 10, corresponding to a yes for all of these questions. The minimum score, in a crisis, is 1, which is also assigned when no criteria are fulfilled but there is a crisis. While not all the questions have model counterparts, we can construct a similar coerciveness index in the model. Questions 1 and 2 correspond to $D_t = 1$. Qualitatively, question 5, 6, 7, and 9 correspond to haircut offers that are so high the acceptance probability is virtually zero. In their coerciveness index, questions 1-4 correspond to payment behavior during the crisis and are worth 4 points; question 5-9 are about negotiations during the crisis and account for 5 points. So, we define our coerciveness index as

$$coerce_t = \max\{\mathbf{1}[H_t > 0], \quad 4 \cdot \mathbf{1}[D_t = 1] + 5 \cdot \mathbf{1}[A_t < \underline{A}] + \mathbf{1}[D_t = 1, A_t < \underline{A}].\}$$

For $\underline{A}$ we use $\bar{\alpha}/2$, which makes negotiations aggressive if and only if $Q^A < Q^D$: i.e., if and only if the sovereign is offering creditor's a worse deal than no deal.

One of the authors' tests for reverse causality is regressing the coerciveness index on lagged annual growth rates. The idea here is that if there is no correlation, then current coerciveness is not a function of the current economic state; instead, the argument goes, current coerciveness affects the economic state. In TZ's regressions, all the coefficients are negative but not statistically significant. We run the regressions of $coerce_t$ on lagged values of annualized GDP growth in percent (as TZ do), and the results are displayed in models (1)-(3) of table 4. Despite a large number of differences (they include time and country FEs as well as other macro controls), the magnitude of the coefficients is similar to the TZ estimates. However, from the model lens, the interpretation is very different.

In the model, the small magnitude of lagged growth rates on current coerciveness is driven by a lack of explanatory power from growth rates to coerciveness. To see this, first consider that annualized GDP growth rates are highly autocorrelated (0.68). Hence, lagged growth rates are highly

Table 4: Model regressions corresponding to Trebesch and Zabel (2017)

	coerce				
varname2	(1)	(2)	(3)	varname	annGdpGrowth
L.annGdpGrowth	-0.009	-0.009	-0.008	negotiate	7.468
L2.annGdpGrowth		-0.009	-0.009	coerce	-2.090
L3.annGdpGrowth			-0.009		
R-squared	0.007	0.014	0.021	R-squared	0.010

predictive of current rates. However, the link from growth rates to coerciveness does not have large explanatory power. This can be seen in a regression of GDP growth on negotiation and coercion, as is done in the right panel of table 4. The coefficients have the same sign as in TZ's table 1 (model 2) and are very statistically significant (not displayed, but we have arbitrarily large samples in the simulation), but the R^2 is very low (0.01). This does not seem to be counterfactually low, either: TZ do not report a regression without time fixed effects, but even with those the R^2 is 0.135. While the model does suggest aggressive negotiations cause a gap between hard and soft defaults (through prolonged default duration and its accompanying default csots), these results call for caution in interpreting the difference between hard and soft defaults as a causal relationship: Perhaps more than half of it is driven by selection.

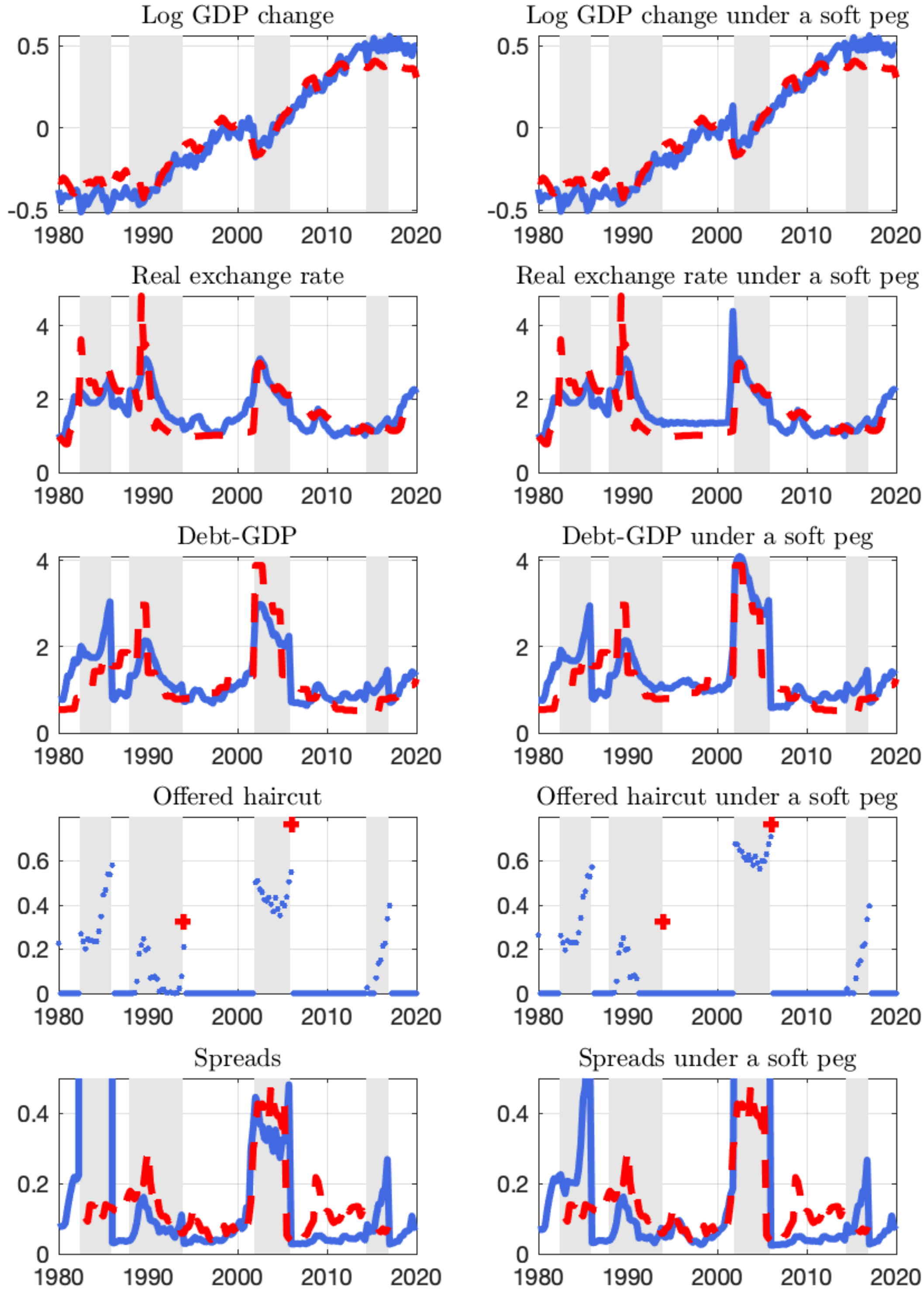
5 Argentina's historical experience

The left panels of Figure 8 plot the simulated path of the economy given the estimated states from the Kalman filter while forcing default or repayment when the data had default or repayment, respectively. The model's behavior of log GDP and the RER (which are observables in the estimation) closely mirrors the data's counterparts. The model almost reproduces exactly the haircut that Trebesch and Zabel report for the end of the 1993 default.[11] Additionally, debt/GDP and spreads have some strong similarities to the data, with notable exceptions. First, the 2001Q4 default. This event was exceptional in ways we will discuss shortly. Second, the 2014Q3 selective default. In the 2014 default, the issues were quite complicated with ongoing legal disputes from the 2001 default. Notably, there were several restructurings in the 2006-2014 period that are not captured as defaults, making the debt/GDP series run above the data's and resulting in greater spreads.

Turning specifically to the 2001Q4 default, the benchmark model misses quite badly the 76.8% haircut in the data, while also understating the extent to which debt to GDP explodes in the default. Statistically, this haircut was unusually large (Edwards, 2015). However, it was also preceeded by an unusual period in Argentina's history. From 1991Q2 to 2001Q4, Argentina ran a currency board (Frank, 2004). During this period, the currency board attempted to peg nominal exchange rate on a 1:1 basis with the USD, and in that period the nominal exchange rate varied from 0.99 to 1. This

[11] Trebesch and Zabel count a single default from 1982-1993.

Figure 8: Path along estimated shocks



also kept the RER between 1.31 and 0.98 and between 1.02 to .98 after 1993Q1 (as can be seen in the graph). However, eventually the currency board could not sustain the peg and abandoned it in 2002Q1, with the exchange rate exploding to 2.95 and a commensurate increase in inflation. This made debt/GDP explode.

The model can capture this by adding the objective of RER stability to the sovereign's problem. The sovereign can then use fiscal policy, specifically, taxes, transfers, and international borrowing, to try to reach the RER target. We do this in the model by creating exogenous regime changes where, when in a pegged regime, the sovereign gets a large reward from keeping the RER close enough to 1. Specifically, when in a pegged regime, flow utility becomes $u(C_t)+\psi\mathbf{1}[|RER_t-1|<\delta]$. Specifying the flexible regime as an absorbing state and using a persistence of 0.95 for the pegged regime, we solve the model with ψ large and choose δ to deliver a devaluation in the 2001 default like in the data.[12] For this, we found $\delta = .4$ was required.

The right panels of Figure 8 plot the resulting series when we solve the model with and without this "soft peg" and impose an unanticipated transition to the regime in 1991Q2 (with an exit in 2001Q4). Imposing these regime changes corrects the RER behavior and lets the model capture the magnitude of the debt-GDP increase from 2001 to 2002. The largest failure of the soft peg model is it implies a zero haircut early on as the sovereign is desperate to leave default and start using taxes to hit the RER target. Crucially, however, the implied haircut in 2005 is very similar to the model's, thus providing a rationale for why the 2001 haircut was so much larger than normal.

6 Conclusion

We proposed a novel theory of hard and soft sovereign defaults that, while quite simple, captures many features of the data. First and foremost of these is the low output growth following hard defaults and comparatively high output growth following soft ones. The model rationalizes this feature by having negative growth shocks lead to protracted periods of non-payment and hence hard defaults with less negative growth shocks resulting in shorter period periods of non-payment. Using a historical shock decomposition to recover the shocks in, before, and after Argentina's defaults, the model successfully reproduced the behavior of the observable spreads and output as well as non-observables such as the the paths of debt, the magnitude of haircuts, and the timing of defaults episodes.

[12]The 2001 default occurred in late December, which is presumably why the RER in the data does not explode until 2002.

References

O. Akinci. A note on the estimation of the atemporal elasticity of substitution between tradable and nontradable goods. http://www.columbia.edu/~oa2140/ElasticityofSubstitution_TRvsNTR_revised.pdf, retrieved 2/5/2022, February 2011. Mimeo.

C. Arellano. Default risk and income fluctuations in emerging economies. *American Economic Review*, 98(3):690–712, 2008.

C. Arellano, X. Mateo-Planas, and J.-V. Ríos-Rull. Partial default. Mimeo, 2013.

T. Asonuma and C. Trebesch. Sovereign debt restructurings: Preemptive or post-default. *Journal of the European Economic Association*, 14(1):175–214, 2016.

J. Bianchi and C. Sosa-Padilla. Reserve accumulation, macroeconomic stabilization, and sovereign risk. https://sosapadilla.github.io/files/papers/Working_papers/reserves_macro_stabilization_web.pdf, retrieved 2/5/2022, September 2020.

S. Chatterjee and B. Eyigungor. Maturity, indebtedness and default risk. *American Economic Review*, 102(6):2674–2699, 2012.

S. Edwards. Sovereign default, debt, and recovery rates: Was the argentinian "haircut" excessive? Technical Report Working Paper 20964, National Bureau of Economic Research, 2015.

D. Frank. How currency boards collapse — the case of argentina. Technical report, University of Basel, 2004.

J. C. Hatchondo and L. Martinez. Long-duration bonds and sovereign defaults. *Journal of International Economics*, 79:117–125, 2009.

M. Tomz and M. L. Wright. Empirical research on sovereign debt and default. *Annual Review of Economics*, 5:247–272, 2013.

C. Trebesch and M. Zabel. The output costs of hard and soft sovereign default. *European Economic Review*, 92:416–432, 2017.

V. Z. Yue. Sovereign default and debt renegotiation. *Journal of International Economics*, 80(2):176–187, 2010.

A Omitted theory results and proofs

Proof of proposition 1. From the HJB,

$$(\rho+\alpha)V = -\chi y + \frac{V_{yy}\sigma_y^2}{2} - \alpha(1-h)b$$

Because of risk-neutrality, $(\rho+\alpha)V_y = -\chi$ and $V_{yy} = 0$. So,

$$\frac{V}{b} = \frac{-\chi y/b - \alpha(1-h)}{\rho+\alpha}.$$

From the first order condition characterizing optimality of h requires

$$\begin{aligned}
0 &= [\hat{\alpha}(\hat{h})(-(1-\hat{h})b - V)]' \\
&= \alpha'(-(1-h)b - V) + \alpha b \\
1 &= \frac{\alpha'}{\alpha}((1-h) + \frac{V}{b})
\end{aligned}$$

Plugging the HJB expression for V/b into the FOC gives

$$\begin{aligned}
1 &= \frac{\alpha'}{\alpha}((1-h) + \frac{-\chi y/b - \alpha(1-h)}{\rho+\alpha}) \\
&= \frac{\alpha'}{\alpha}(\frac{\rho}{\rho+\alpha}(1-h) + \frac{-\chi y/b}{\rho+\alpha}) \qquad (*) \\
&= \frac{\alpha'}{\alpha}\frac{1}{\rho+\alpha}(\rho(1-h) - \chi y/b)
\end{aligned}$$

Debt pricing of $\rho q_D = \alpha(q^A - q^D)$ implies $q^D = \frac{\alpha}{\alpha+\rho}q^A$ giving

$$q^A - q^D = q^A(1 - \frac{\alpha}{\alpha+\rho}) = (1-h)\frac{\rho}{\alpha+\rho}.$$

The functional form for $\hat{\alpha}$ gives

$$\hat{\alpha}(\hat{h}) = \bar{\alpha}\sigma_\alpha(\hat{q}^A - q^D)^{1/\sigma_\alpha} \Rightarrow \hat{\alpha}'(\hat{h}) = -\bar{\alpha}(\hat{q}^A - q^D)^{1/\sigma_\alpha - 1}$$

So,

$$\begin{aligned}
\frac{\alpha'}{\alpha} &= -\frac{1}{\sigma_\alpha(q^A - q^D)} = -\frac{\alpha+\rho}{(1-h)\sigma_\alpha\rho} \\
\frac{\alpha'}{\alpha}\frac{1}{\alpha+\rho} &= -\frac{1}{\rho\sigma_\alpha}\frac{1}{1-h}
\end{aligned}$$

Substituting this into (*),

$$
\begin{aligned}
1 &= -\frac{1}{\rho\sigma_\alpha}\frac{1}{1-h}(\rho(1-h) - \chi y/b) \\
-\sigma_\alpha &= \frac{1}{\rho}\frac{1}{1-h}(\rho(1-h) - \chi y/b) \\
-\sigma_\alpha &= 1 - \frac{\chi y}{b}\frac{1}{\rho}\frac{1}{1-h} \\
1 + \sigma_\alpha &= \frac{\chi y}{b}\frac{1}{\rho}\frac{1}{1-h} \\
\rho(1+\sigma_\alpha) &= \frac{\chi y}{b}\frac{1}{1-h} \\
1 - h &= \frac{\chi y}{b}\frac{1}{\rho(1+\sigma_\alpha)} \\
h &= 1 - \frac{\chi y}{b}\frac{1}{\rho(1+\sigma_\alpha)}.
\end{aligned}
$$

From the definition of $\hat{\alpha}(\cdot)$ and $q^A - q^D = (1-h)\frac{\rho}{\alpha+\rho}$,

$$\alpha = \bar{\alpha}\sigma_\alpha((1-h)\frac{\rho}{\alpha+\rho})^{1/\sigma_\alpha}$$

$$\alpha^{\sigma_\alpha}\frac{\alpha+\rho}{\rho} = (\bar{\alpha}\sigma_\alpha)^{\sigma_\alpha}(1-h)$$

$$\alpha^{\sigma_\alpha}(\alpha+\rho) = (\bar{\alpha}\sigma_\alpha)^{\sigma_\alpha}\frac{\chi y/b}{\sigma_\alpha+1}$$

□

Proposition 2.

$$RER_t = \alpha_T\left(\alpha_T + \alpha_N\left(\frac{Y_{N,t}}{Y_{T,t}-T_t}\right)^{\frac{\rho-1}{\rho}}\right)^{\frac{1}{\rho-1}}$$

$$c_t = (Y_{T,t} - T_t)\alpha_T^{-\rho} RER_t^{\rho}$$

Proof. From the FOC of the household's N and T choice and the normalization that N is the numeraire, we have

$$p_{T,t} = \frac{\alpha_T}{\alpha_N}\left(\frac{Y_{N,t}}{Y_{T,t}-T_t}\right)^{1/\rho}$$

Dividing both sides by $p_{T,t}$, multiplying by p_t, and using $RER_t = p_{T,t}/p_t$,

$$p_t = RER_t^{-1}\frac{\alpha_T}{\alpha_N}\left(\frac{Y_{N,t}}{Y_{T,t}-T_t}\right)^{1/\rho}.$$

Plugging that expression for p_t into

$$1 = \alpha_T^{\rho} RER_t^{1-\rho} + \alpha_N^{\rho} p_t^{\rho-1},$$

one arrives at

$$
\begin{aligned}
1 &= \alpha_T^\rho RER_t^{1-\rho} + \alpha_N^\rho \left(RER_t^{-1} \frac{\alpha_T}{\alpha_N} \left(\frac{Y_{N,t}}{Y_{T,t} - T_t} \right)^{1/\rho} \right)^{\rho-1} \\
&= RER_t^{1-\rho} \left(\alpha_T^\rho + \alpha_N^\rho \left(\frac{\alpha_T}{\alpha_N} \left(\frac{Y_{N,t}}{Y_{T,t} - T_t} \right)^{1/\rho} \right)^{\rho-1} \right) \\
&= RER_t^{1-\rho} \left(\alpha_T^\rho + \alpha_N^\rho \frac{\alpha_T^{\rho-1}}{\alpha_N^{\rho-1}} \left(\frac{Y_{N,t}}{Y_{T,t} - T_t} \right)^{\frac{\rho-1}{\rho}} \right) \\
&= \left(\frac{RER_t}{\alpha_T} \right)^{1-\rho} \left(\frac{\alpha_T^\rho}{\alpha_T^{\rho-1}} + \alpha_N \left(\frac{Y_{N,t}}{Y_{T,t} - T_t} \right)^{\frac{\rho-1}{\rho}} \right) \\
\Rightarrow \left(\frac{RER_t}{\alpha_T} \right)^{\rho-1} &= \left(\alpha_T + \alpha_N \left(\frac{Y_{N,t}}{Y_{T,t} - T_t} \right)^{\frac{\rho-1}{\rho}} \right) \\
RER_t &= \alpha_T \left(\alpha_T + \alpha_N \left(\frac{Y_{N,t}}{Y_{T,t} - T_t} \right)^{\frac{\rho-1}{\rho}} \right)^{\frac{1}{\rho-1}}.
\end{aligned}
$$

Then, beginning with the definition of the consumption aggregator,

$$
\begin{aligned}
c_t^{\frac{\rho-1}{\rho}} &= \alpha_T (Y_{T,t} - T_t)^{\frac{\rho-1}{\rho}} + \alpha_N Y_{N,t}^{\frac{\rho-1}{\rho}} \\
&= (Y_{T,t} - T_t)^{\frac{\rho-1}{\rho}} \left(\alpha_T + \alpha_N \left(\frac{Y_{N,t}}{Y_{T,t} - T_t} \right)^{\frac{\rho-1}{\rho}} \right) \\
&= (Y_{T,t} - T_t)^{\frac{\rho-1}{\rho}} (RER_t/\alpha_T)^{\rho-1} \\
\Rightarrow c_t &= (Y_{T,t} - T_t) (RER_t/\alpha_T)^\rho .
\end{aligned}
$$

□

B Detrending the model

Define the detrended variables as

$$
\begin{aligned}
y_{T,t} &= Y_{T,t}/\Gamma_{T,t} && \Rightarrow y_{T,t} = z_t, \\
y_{N,t} &= Y_{N,t}/\Gamma_{N,t} && \Rightarrow y_{N,t} = 1, \\
c_t &= C_{T,t}/\Gamma_{T,t}, \\
\tau_t &= T_t/\Gamma_{T,t}, \\
b_{t+1} &= B_{t+1}/\Gamma_{T,t} && \Rightarrow \frac{b_t}{g_t} = \frac{B_t}{\Gamma_{T,t}}.
\end{aligned}
$$

Define

$$
e_t = \Gamma_{T,t}/\Gamma_{N,t} \Rightarrow e_{t+1} = e_t \frac{g_{t+1}}{\mu}.
$$

Now, we'll conjecture the form of the detrended problem and then show a solution to the detrended problem is a solution to the problem with trend.

$$\begin{gathered}
v^R(b_t, x_t, \hat{h}) = \max_{b_{t+1}} u(c_t) + \beta \mathbb{E}_{[x_{t+1}|x_t]} g_{t+1}^{1-\sigma} v(b_{t+1}, x_{t+1}) \\
\text{s.t. } c_t = (z_t - \tau_t)\psi((z_t - \tau_t)e_t) \\
\tau_t = -q_t(b_{t+1}, x_t)(b_{t+1} - (1-\lambda)\frac{b_t}{g_t}) + \tilde{\lambda}\frac{b_t}{g_t} + \zeta \mathbf{1}[\hat{h} > 0]
\end{gathered}$$

conditional on default as

$$\begin{gathered}
v^D(b_t, x_t) = u(c_t) + \beta \mathbb{E}_{[x_{t+1}|x_t]} g_{t+1}^{1-\sigma} v(b_{t+1}, x_{t+1}) \\
\text{s.t. } c_t = (1-\chi) z_t \psi((1-\chi) z_t e_t) \\
b_{t+1} = R^D \frac{b_t}{g_t}
\end{gathered}$$

with

$$v(b_t, x_t) = \max_{\hat{h}_t} \alpha(\hat{h}; b_t, x_t) v^R(b_t, x_t, \hat{h}) + (1 - \alpha(\hat{h}; b_t, x_t)) v^D(b_t, x_t).$$

Proposition 3. *A solution to the detrended problem is a solution to the problem with trend.*

Proof. (Sketch.) Consider the budget constraint in repayment. We have

$$\begin{gathered}
c_t \Gamma_{T,t} = (z_t - \tau_t)\Gamma_{T,t}\psi((z_t - \tau_t)\frac{\Gamma_{T,t}}{\Gamma_{N,t}}) \\
C_t = (Y_{T,t} - T_t)\psi((z_t - \tau_t)\frac{\Gamma_{T,t}}{\Gamma_{N,t}}) \\
C_t = (Y_{T,t} - T_t)\psi(\frac{Y_{T,t} - T_t}{Y_{N,t}}),
\end{gathered}$$

which is the same as in the problem with trend. Likewise,

$$\begin{gathered}
\tau_t = -q_t(b_{t+1}, x_t)(b_{t+1} - (1-\lambda)\frac{b_t}{g_t}) + \tilde{\lambda}\frac{b_t}{g_t} + \zeta \mathbf{1}[\hat{h} > 0] \\
T_t = -q_t(b_{t+1}, x_t)(B_{t+1} - (1-\lambda)\frac{b_t}{g_t}\Gamma_{T,t}) + \tilde{\lambda}\frac{b_t}{g_t}\Gamma_{T,t} + \zeta \mathbf{1}[\hat{h} > 0]\Gamma_{T,t} \\
T_t = -q_t(b_{t+1}, x_t)(B_{t+1} - (1-\lambda)\frac{B_t}{\Gamma_{T,t}}\Gamma_{T,t}) + \tilde{\lambda}\frac{B_t}{\Gamma_{T,t}}\Gamma_{T,t} + \zeta \mathbf{1}[\hat{h} > 0]\Gamma_{T,t} \\
T_t = -q_t(b_{t+1}, x_t)(B_{t+1} - (1-\lambda)B_t) + \tilde{\lambda}B_t + \zeta \mathbf{1}[\hat{h} > 0]\Gamma_{T,t}.
\end{gathered}$$

For the value function, use $\Gamma^{1-\sigma}v(b,x) = V(b/g\Gamma, x, \Gamma)$ to find

$$\Gamma_{T,t}^{1-\sigma} v^R(b_t, x_t, \hat{h}) = \max_{b_{t+1}} \Gamma_{T,t}^{1-\sigma} u(c_t) + \beta \mathbb{E}_{[x_{t+1}|x_t]} g_{t+1}^{1-\sigma} \Gamma_{T,t}^{1-\sigma} v(b_{t+1}, x_{t+1})$$

$$V^R(\frac{b_t}{g_t}\Gamma_{T,t}, x_t, \Gamma_{T,t}, \hat{h}) = \max_{b_{t+1}} u(C_t) + \beta \mathbb{E}_{[x_{t+1}|x_t]} (g_{t+1}\Gamma_{T,t})^{1-\sigma} v(b_{t+1}, x_{t+1})$$

$$V^R(B_t, x_t, \Gamma_{T,t}, \hat{h}) = \max_{b_{t+1}} u(C_t) + \beta \mathbb{E}_{[x_{t+1}|x_t]} \Gamma_{T,t+1}^{1-\sigma} v(b_{t+1}, x_{t+1})$$

$$V^R(B_t, x_t, \Gamma_{T,t}, \hat{h}) = \max_{b_{t+1}} u(C_t) + \beta \mathbb{E}_{[x_{t+1}|x_t]} V(\frac{b_{t+1}}{g_{t+1}}\Gamma_{T,t+1}, x_{t+1}, \Gamma_{T,t+1})$$

$$V^R(B_t, x_t, \Gamma_{T,t}, \hat{h}) = \max_{b_{t+1}} u(C_t) + \beta \mathbb{E}_{[x_{t+1}|x_t]} V(b_{t+1}\Gamma_{T,t}, x_{t+1}, \Gamma_{T,t+1})$$

$$V^R(B_t, x_t, \Gamma_{T,t}, \hat{h}) = \max_{B_{t+1}} u(C_t) + \beta \mathbb{E}_{[x_{t+1}|x_t]} V(B_{t+1}, x_{t+1}, \Gamma_{T,t+1})$$

□

Note that the restrictions we imposed on g_t make e_t, a component of x_t, stationary. Note that our definition of e_t gives $\log e_t = \log e_{t-1} + \log g_t - \log \mu$. And, since $\log g_t = (\rho_e - 1)\log e_{t-1} + \log \mu + \varepsilon_{g,t}$, we have

$$\log e_t = \rho_e \log e_{t-1} + \varepsilon_{g,t}.$$

Hence, $\log e \sim N(0, \frac{\sigma_g^2}{1-\rho_e^2})$.

A positive growth shock has two effects. It effectively reduces debt, reflected in b_t/g_t in the budget constraint. This reduces τ_t for a given level of b_{t+1} issuance. But it simultaneously

When creditors receive an offered haircut $\hat{h}$, they need to accept or reject the offer. Because of the Markov perfect equilibrium concept, they assume that future haircut offers will follow a policy $h(b,x)$. In equilibrium, this haircut offer policy must be the same as the solution to (**??**).

An accepted offer's value q^A is given by

$$q^A(\hat{h}, b, x) = (1-\hat{h})\left(\tilde{\lambda} + (1-\lambda) q(b'((1-\hat{h})b, x, \hat{h}), x)\right),$$

which takes into account the haircut size and that the debt must be serviced at least once. A subtle but important effect here is that when creditors evaluate the offer $\hat{h}$, they internalize the effects of *debt concentration*. Specifically, a bigger haircut lowers ex post debt resulting in less debt next period and greater continuation value. Mathematically, a larger $\hat{h}$ decreases $(1-\hat{h})b/g$, increasing $b'((1-\hat{h})b/g, x, \hat{h})$ and hence $q(b'((1-\hat{h})b/g, x, \hat{h}), x)$.

A rejected offer's value q^D is given by

$$q^D(b,x) = R^D q(R^D b/g, x). \tag{15}$$

The reason R^D appears is that we are expressing q^D as the price per unit of debt, and each debtholders' number of units grows at rate R^D.

Finally, the one period ahead debt pricing is given by

$$q(b', x) = \frac{1}{1+r^*}\mathbb{E}_{x'|x}[\alpha' q^A(h', b', x') + (1-\alpha')q^D(b', x')] \tag{16}$$

where

$$h' = h(b', x'), \text{ and } \alpha' = \alpha(h'; b', x').$$

Note that in pricing the rejected offer, the Markov policies h and α are used, consistent with the equilibrium concept.

B.1 Equilibrium

A Markov perfect equilibrium is $b', h, \alpha, q, q^A, q^D, V, V^R, V^D$ such that

1. V, V^R, V^D, b', h solve the sovereign's problem taking α, q, q^A, q^D as given, and
2. α, q, q^A, q^D solve the creditors' problem taking b', h as given.

B.2 Measurement

The table below gives the measurement of key variables.

Statistic	Not-detrended	Detrended
Real exchange rate	RER_t	$\alpha_T \psi((z_t - \tau_t)e_t)^{1/\rho}$
Price level	p_t	$\alpha_N^{\frac{\rho}{1-\rho}}(1 - \alpha_T^\rho RER_t^{1-\rho})^{\frac{1}{\rho-1}}$
Consumption	C_t	$c_t \Gamma_{T,t}$
GDP	Y_t	$(RER_t z_t + p_t^{-1}/e_t)\Gamma_{T,t}$
Debt level[a]	$RER_t B_t$	$RER_t(b_t/g_t)\Gamma_{T,t}$
Current account[b]	$RER_t T_t$	$RER_t \tau_t \Gamma_{T,t}$

(a) Since this variable is denominated in tradables, to map to aggregate consumption units one multiplies by $p_{T,t}/p_t$, which for us is the RER_t. (b) The current account is private savings less investment plus taxes less government expenditures, $(S-I)+(Taxes-G)$. Without private savings, capital or government expenditures, this is just taxes (measured in aggregate consumption unties), which is $RER_t T_t$.

B.2.1 Spreads

For spreads, we compare the internal rate of return (IRR), or the yield to maturity (YTM), with the riskfree rate. Specifically, the IRR is the interest rate that equates the cost of a project with the benefit. The IRR corresponding to of $q(b', x)$ in (16) is given by the interest rate r (not necessarily r^*)

that equates the amount borrowed $-q(b', x)b'$ with the no-default stream of payments discounted at r:

$$r = \frac{\tilde{\lambda}}{-q(b', x)} - \lambda \Leftrightarrow -q(b', x)b' = \sum_{j=1}^{\infty}(1+r)^{-j}\tilde{\lambda}(1-\lambda)^{j-1}b'.$$

Hence, our measure of spreads is $(1+r)^4 - (1+r^*)^4$. This is the same measure used in Chatterjee and Eyigungor (2012) and many other papers.

B.2.2 Default and default episodes

The literature has measured default episodes in different ways, leading to very different results with respect to how long default episodes last and at what frequency it occurs. For example, as Tomz and Wright (2013) relay, the criteria of using the criterias of Borenstein and Panizza (2009), Cruces and Trebesch (2013), and Arteta and Hale (2008) record one, four, five, and 23 defaults during the 1980s, respectively. Consequently, we need to determine the default episode in a way consistent with each data source we use.