Uncovering Disaggregated Oil Market Dynamics: A Full-Information Approach to Granular Instrumental Variables

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Differences between local and aggregate outcomes can be an important source of identification.

Examples:

- Bartik instruments
- Granular instrumental variables (Gabaix and Koijen, JPE forthcoming)

- Our paper shows how to exploit the power of this idea using full-information maximum likelihood estimation.
- We illustrate with an analysis of the world oil market.

A model of the world oil market

Data from 1973:M1 to 2023:M2 (drop COVID)

q_{it} = growth rate of country i oil production

s_{qi} = share of country i in world total

$\sum_{i=1}^{n} s_{qi} q_{it}$ = approximate growth in global oil production

Our empirical analysis will use the three biggest producers (U.S., Saudi Arabia, Russia) plus the rest of the world ($n = 4$)

c_{jt} = growth rate of country j oil consumption

s_{cj} = share of country j in world total

$\sum_{j=1}^{m} s_{cj} c_{jt}$ = approximate growth in global oil consumption

Our empirical analysis will use the three biggest historical consumers (U.S., Japan, Europe) plus the rest of the world ($m = 4$)

Supply curve of country i

$$q_{it} = \phi_{qi} p_t + \mathbf{b}'_{qi} \mathbf{x}_{t-1} + u_{qit}$$

ϕ_{qi} = country i short-run supply elasticity

$\mathbf{x}_{t-1}$ contains intercept,12 lags production and consumption of every country in world, and 12 lags of world price

u_{qit} = supply shock for country i

Demand curve of country j

$$c_{jt} = \phi_{cj} p_t + \mathbf{b}_{cj}' \mathbf{x}_{t-1} + u_{cjt}$$

ϕ_{cj} = country j short-run demand elasticity

u_{cjt} = demand shock for country j

Inventory demand

$$v_t = \phi_v p_t + \mathbf{b}'_v \mathbf{x}_{t-1} + u_{vt}$$

This equals difference between global production and consumption

$$v_t = \sum_{i=1}^{n} s_{qi} q_{it} - \sum_{j=1}^{m} s_{cj} c_{jt}$$

v_t also includes measurement error

Structural model:

$$q_{it} = \phi_{qi} p_t + \mathbf{b}'_{qi} \mathbf{x}_{t-1} + u_{qit} \quad i = 1, \ldots, n$$

or $\underset{(n\times 1)}{\mathbf{q}_t} = \underset{(n\times 1)}{\boldsymbol{\phi}_q} \, p_t + \underset{(n\times k)}{\mathbf{B}_q} \, \mathbf{x}_{t-1} + \underset{(n\times 1)}{\mathbf{u}_{qt}}$

$$c_{jt} = \phi_{cj} p_t + \mathbf{b}'_{cj} \mathbf{x}_{t-1} + u_{cjt} \quad j = 1, \ldots, m$$

or $\underset{(m\times 1)}{\mathbf{c}_t} = \underset{(m\times 1)}{\boldsymbol{\phi}_c} \, p_t + \underset{(m\times k)}{\mathbf{B}_c} \, \mathbf{x}_{t-1} + \underset{(m\times 1)}{\mathbf{u}_{ct}}$

$$\mathbf{s}'_q \mathbf{q}_t - \mathbf{s}'_c \mathbf{c}_t = \phi_v p_t + \mathbf{b}'_v \mathbf{x}_{t-1} + u_{vt}$$

$$\underset{(1\times N)}{\mathbf{y}_t'} = \begin{bmatrix} \underset{(1\times n)}{\mathbf{q}_t'} & \underset{(1\times m)}{\mathbf{c}_t'} & \underset{(1\times 1)}{p_t} \end{bmatrix}$$

$$\underset{(1\times N)}{\mathbf{u}_t'} = \begin{bmatrix} \underset{(1\times n)}{\mathbf{u}_{qt}'} & \underset{(1\times m)}{\mathbf{u}_{ct}'} & \underset{(1\times 1)}{u_{vt}} \end{bmatrix}$$

$$\mathbf{A}\mathbf{y}_t = \mathbf{B}\mathbf{x}_{t-1} + \mathbf{u}_t$$

$$\underset{(N\times N)}{\mathbf{A}} = \begin{bmatrix} \mathbf{I}_n & \mathbf{0}_{nm} & -\boldsymbol{\phi}_q \\ \mathbf{0}_{mn} & \mathbf{I}_m & -\boldsymbol{\phi}_c \\ \mathbf{s}_q' & -\mathbf{s}_c' & -\phi_v \end{bmatrix} \quad \underset{(N\times k)}{\mathbf{B}} = \begin{bmatrix} \mathbf{B}_q \\ \mathbf{B}_c \\ \mathbf{b}_v' \end{bmatrix}$$

Given any value for $\mathbf{u}_t$, there exists a value of $p_t, \mathbf{q}_t, \mathbf{c}_t$ for which all N equations hold. Identification comes from assumptions about correlations between the structural shocks in $\mathbf{u}_t$

Example: suppose supply shocks are uncorrelated with demand shocks,

$E(\mathbf{u}_{qt}\mathbf{u}'_{ct}) = \mathbf{0}_{nm}$,

and elasticities are homogeneous across countries:

$$\underset{(n\times 1)}{\boldsymbol{\phi}_q} = \underset{(1\times 1)}{\phi_q} \underset{(n\times 1)}{\mathbf{1}_n} \qquad \underset{(m\times 1)}{\boldsymbol{\phi}_c} = \underset{(1\times 1)}{\phi_c} \underset{(m\times 1)}{\mathbf{1}_m}$$

Let $\mathbf{s}_q$ be the $(n \times 1)$ vector of global production shares.

Let $\mathbf{w}_q$ be any other $(n \times 1)$ vector for which $\mathbf{w}_q'\mathbf{1}_n = 1$.

$$\mathbf{q}_t = \phi_q \mathbf{1}_n p_t + \mathbf{B}_q \mathbf{x}_{t-1} + \mathbf{u}_{qt}$$

$$(\mathbf{s}_q - \mathbf{w}_q)'\mathbf{q}_t = (\mathbf{s}_q - \mathbf{w}_q)'\mathbf{B}_q \mathbf{x}_{t-1} + (\mathbf{s}_q - \mathbf{w}_q)'\mathbf{u}_{qt}$$

$$(\mathbf{s}_q - \mathbf{w}_q)'\mathbf{q}_t = (\mathbf{s}_q - \mathbf{w}_q)'\mathbf{B}_q\mathbf{x}_{t-1} + (\mathbf{s}_q - \mathbf{w}_q)'\mathbf{u}_{qt}$$

Conclusion:

$(\mathbf{s}_q - \mathbf{w}_q)'\mathbf{q}_t$ is uncorrelated with $\mathbf{u}_{ct}$.

Could estimate ϕ_c by IV

$$\mathbf{w}_c'\mathbf{c}_t = \phi_c p_t + \tilde{\mathbf{B}}_c\mathbf{x}_{t-1} + \tilde{u}_{ct}.$$

Instruments: $(\mathbf{s}_q - \mathbf{w}_q)'\mathbf{q}_t$ and $\mathbf{x}_{t-1}$

$\mathbf{w}_c$ is any $(m \times 1)$ vector with $\mathbf{w}_c'\mathbf{c}_t = 1$.

Example:

$\mathbf{w}_q = n^{-1}\mathbf{1}_n$

$(\mathbf{s}_q - \mathbf{w}_q)'\mathbf{q}_t$ is difference between share-weighted and arithmetic average production. This is the granular instrument insight of Gabaix and Koijen (JPE forthcoming).

Could also find supply elasticity ϕ_q from regression of $\mathbf{w}_q'\mathbf{q}_t$ on p_t and $\mathbf{x}_{t-1}$ using $(\mathbf{s}_c - \mathbf{w}_c)'\mathbf{c}_t$ and $\mathbf{x}_{t-1}$ as instruments.

Maximum likelihood estimation:

$$\mathbf{u}_t \sim N(\mathbf{0}, \mathbf{D})$$

$$\underset{(N\times N)}{\mathbf{D}} = \begin{bmatrix} \underset{(n\times n)}{\mathbf{D}_q} & \mathbf{0}_{nm} & \mathbf{0}_{n1} \\ \mathbf{0}_{mn} & \underset{(m\times m)}{\mathbf{D}_c} & \mathbf{0}_{m1} \\ \mathbf{0}_{1n} & \mathbf{0}_{1m} & \underset{(1\times 1)}{\sigma_v^2} \end{bmatrix}$$

MLE is function of

$$\hat{\boldsymbol{\Pi}} = \left[\sum_{t=1}^{T} \mathbf{y}_t \mathbf{x}'_{t-1}\right]\left[\sum_{t=1}^{T} \mathbf{x}_{t-1} \mathbf{x}'_{t-1}\right]^{-1}$$

$$\underset{(N\times 1)}{\hat{\boldsymbol{\epsilon}}_t} = \mathbf{y}_t - \hat{\boldsymbol{\Pi}}\mathbf{x}_{t-1} = \begin{bmatrix} \underset{(n\times 1)}{\hat{\boldsymbol{\epsilon}}_{qt}} \\ \underset{(m\times 1)}{\hat{\boldsymbol{\epsilon}}_{ct}} \\ \underset{(1\times 1)}{\hat{\varepsilon}_{pt}} \end{bmatrix}$$

Proposition 2: FOC for MLE are

$$\hat{\phi}_c = \frac{\sum_{t=1}^{T} \tilde{z}_{ct}\tilde{c}_t}{\sum_{t=1}^{T} \tilde{z}_{ct}\hat{\varepsilon}_{pt}}$$

$$\tilde{c}_t = \hat{\mathbf{w}}_c'\hat{\boldsymbol{\epsilon}}_{ct} \qquad \hat{\mathbf{w}}_c' = \mathbf{1}_m'\hat{\mathbf{D}}_c^{-1}/(\mathbf{1}_m'\hat{\mathbf{D}}_c^{-1}\mathbf{1}_m)$$

$$\hat{\mathbf{D}}_c = T^{-1}\sum_{t=1}^{T}\left(\hat{\boldsymbol{\epsilon}}_{ct} - \hat{\phi}_c\mathbf{1}_m\hat{\varepsilon}_{pt}\right)\left(\hat{\boldsymbol{\epsilon}}_{ct} - \hat{\phi}_c\mathbf{1}_m\hat{\varepsilon}_{pt}\right)'$$

$$\tilde{z}_{ct} = -(\mathbf{s}_q - \hat{\mathbf{w}}_q)'\hat{\boldsymbol{\epsilon}}_{qt} - (\tilde{q}_t - \hat{\phi}_q\hat{\varepsilon}_{pt}) + (\hat{\varepsilon}_{vt} - \hat{\phi}_v\hat{\varepsilon}_{pt})$$

$$\tilde{q}_t = \hat{\mathbf{w}}_q'\hat{\boldsymbol{\epsilon}}_{qt} \qquad \hat{\mathbf{w}}_q' = \mathbf{1}_n'\hat{\mathbf{D}}_q^{-1}/(\mathbf{1}_n'\hat{\mathbf{D}}_q^{-1}\mathbf{1}_n)$$

$$\hat{\mathbf{D}}_q = T^{-1}\sum_{t=1}^{T}\left(\hat{\boldsymbol{\epsilon}}_{qt} - \hat{\phi}_q\mathbf{1}_n\hat{\varepsilon}_{pt}\right)\left(\hat{\boldsymbol{\epsilon}}_{qt} - \hat{\phi}_q\mathbf{1}_n\hat{\varepsilon}_{pt}\right)'$$

Analogous FOC for $\hat{\phi}_q$ and $\hat{\phi}_v$

$$\hat{\phi}_q = \frac{\sum_{t=1}^{T} \tilde{z}_{qt}\tilde{q}_t}{\sum_{t=1}^{T} \tilde{z}_{qt}\hat{\varepsilon}_{pt}}$$

$$\tilde{z}_{qt} = (\mathbf{s}_c - \hat{\mathbf{w}}_c)'\hat{\boldsymbol{\epsilon}}_{ct} + (\tilde{c}_t - \hat{\phi}_c\hat{\varepsilon}_{pt}) + (\hat{\varepsilon}_{vt} - \hat{\phi}_v\hat{\varepsilon}_{pt})$$

$$\hat{\phi}_v = \frac{\sum_{t=1}^{T} \tilde{z}_{vt}\hat{\varepsilon}_{vt}}{\sum_{t=1}^{T} \tilde{z}_{vt}\hat{\varepsilon}_{pt}}$$

$$\hat{\varepsilon}_{vt} = \mathbf{s}_q'\hat{\boldsymbol{\epsilon}}_{qt} - \mathbf{s}_c'\hat{\boldsymbol{\epsilon}}_{ct}$$

$$\tilde{z}_{vt} = (\mathbf{s}_c - \hat{\mathbf{w}}_c)'\hat{\boldsymbol{\epsilon}}_{ct} - (\mathbf{s}_q - \hat{\mathbf{w}}_q)'\hat{\boldsymbol{\epsilon}}_{qt} - (\tilde{q}_t - \hat{\phi}_q\hat{\varepsilon}_{pt}) + (\tilde{c}_t - \hat{\phi}_c\hat{\varepsilon}_{pt})$$

Iterated 3SLS

$$\hat{\phi}_c^{(1)} = \frac{\sum_{t=1}^{T} \tilde{z}_{ct}^{(1)} \tilde{c}_t^{(1)}}{\sum_{t=1}^{T} \tilde{z}_{ct}^{(1)} \hat{\varepsilon}_{pt}}$$

$$\tilde{c}_t^{(1)} = \mathbf{s}_c' \hat{\boldsymbol{\epsilon}}_{ct} \qquad \tilde{z}_{ct}^{(1)} = (n^{-1}\mathbf{1}_n - \mathbf{s}_q)' \hat{\boldsymbol{\epsilon}}_{qt}$$

$$\hat{\mathbf{D}}_c^{(1)} = T^{-1} \sum_{t=1}^{T} \left(\hat{\boldsymbol{\epsilon}}_{ct} - \hat{\phi}_c^{(1)} \mathbf{1}_m \hat{\varepsilon}_{pt} \right) \left(\hat{\boldsymbol{\epsilon}}_{ct} - \hat{\phi}_c^{(1)} \mathbf{1}_m \hat{\varepsilon}_{pt} \right)'$$

$$\tilde{c}_t^{(2)} = \hat{\mathbf{w}}_c^{(2)\prime} \hat{\boldsymbol{\epsilon}}_{ct}$$

$$\hat{\mathbf{w}}_c^{(2)\prime} = \mathbf{1}_m' \left(\hat{\mathbf{D}}_c^{(1)} \right)^{-1} \div \left[\mathbf{1}_m' \left(\hat{\mathbf{D}}_c^{(1)} \right)^{-1} \mathbf{1}_m \right]$$

Comparison of plain-vanilla granular IV (step 1 of 3SLS) and MLE (iterate on 3SLS to convergence)

Parameter	IV	MLE
Demand elasticity ϕ_c	-0.106 (0.252)	-0.130 (0.026)
Supply elasticity ϕ_q	-3.699 (7.717)	0.054 (0.009)
Inventory demand elasticity ϕ_v		-0.373 (0.052)

(standard errors in parentheses)

Likelihood ratio test rejects the model's 21 overidentifying assumptions.

A more general model with heterogeneous elasticities is also rejected.

Reason: there do not exist (4×1) vectors $\boldsymbol{\phi}_q$ and $\boldsymbol{\phi}_c$ for which

$$T^{-1} \sum_{t=1}^{T} (\hat{\boldsymbol{\epsilon}}_{qt} - \boldsymbol{\phi}_q \hat{\varepsilon}_{pt})(\hat{\boldsymbol{\epsilon}}_{ct} - \boldsymbol{\phi}_c \hat{\varepsilon}_{pt})' \simeq \mathbf{0}_{nm}.$$

Supply shocks $\mathbf{u}_{qt}$ and demand shocks $\mathbf{u}_{ct}$ appear to be correlated.

We allow a single global factor on which both $\mathbf{u}_{qt}$ and $\mathbf{u}_{ct}$ can load without restriction.

Seems to be response of Saudi and OPEC production to global demand.

Proposed model:

ϕ_q and ϕ_c unrestricted (4×1) vectors

$$\mathbf{D} = E(\mathbf{u}_t \mathbf{u}_t') =$$

$$\begin{bmatrix} \mathbf{h}_q \mathbf{h}_q' + \mathbf{\Sigma}_q & \mathbf{h}_q \mathbf{h}_c' & \mathbf{0}_{n1} \\ \mathbf{h}_c \mathbf{h}_q' & \mathbf{h}_c \mathbf{h}_c' + \boldsymbol{\gamma}_c \boldsymbol{\gamma}_c' + \mathbf{\Sigma}_c & \mathbf{0}_{m1} \\ \mathbf{0}_{1n} & \mathbf{0}_{1m} & \sigma_v^2 \end{bmatrix}$$

$\mathbf{h}_q, \mathbf{h}_c, \boldsymbol{\gamma}_c$ are (4×1) vectors

$\mathbf{\Sigma}_q$ and $\mathbf{\Sigma}_c$ are diagonal (4×4) matrices

Model has 16 overidentifying restrictions that are not rejected.

Maximum likelihood estimates of elasticities and their standard errors

U.S. supply	0.021	(0.016)
Saudi supply	0.248	(0.058)
Russia supply	0.034	(0.010)
ROW supply	0.066	(0.020)
U.S. demand	-0.077	(0.025)
Japan demand	-0.001	(0.031)
Europe demand	-0.202	(0.037)
ROW demand	-0.139	(0.038)
Inventory demand	-0.355	(0.061)

Global supply elasticity: 0.077 (0.017)

Global demand elasticity: -0.119 (0.030)

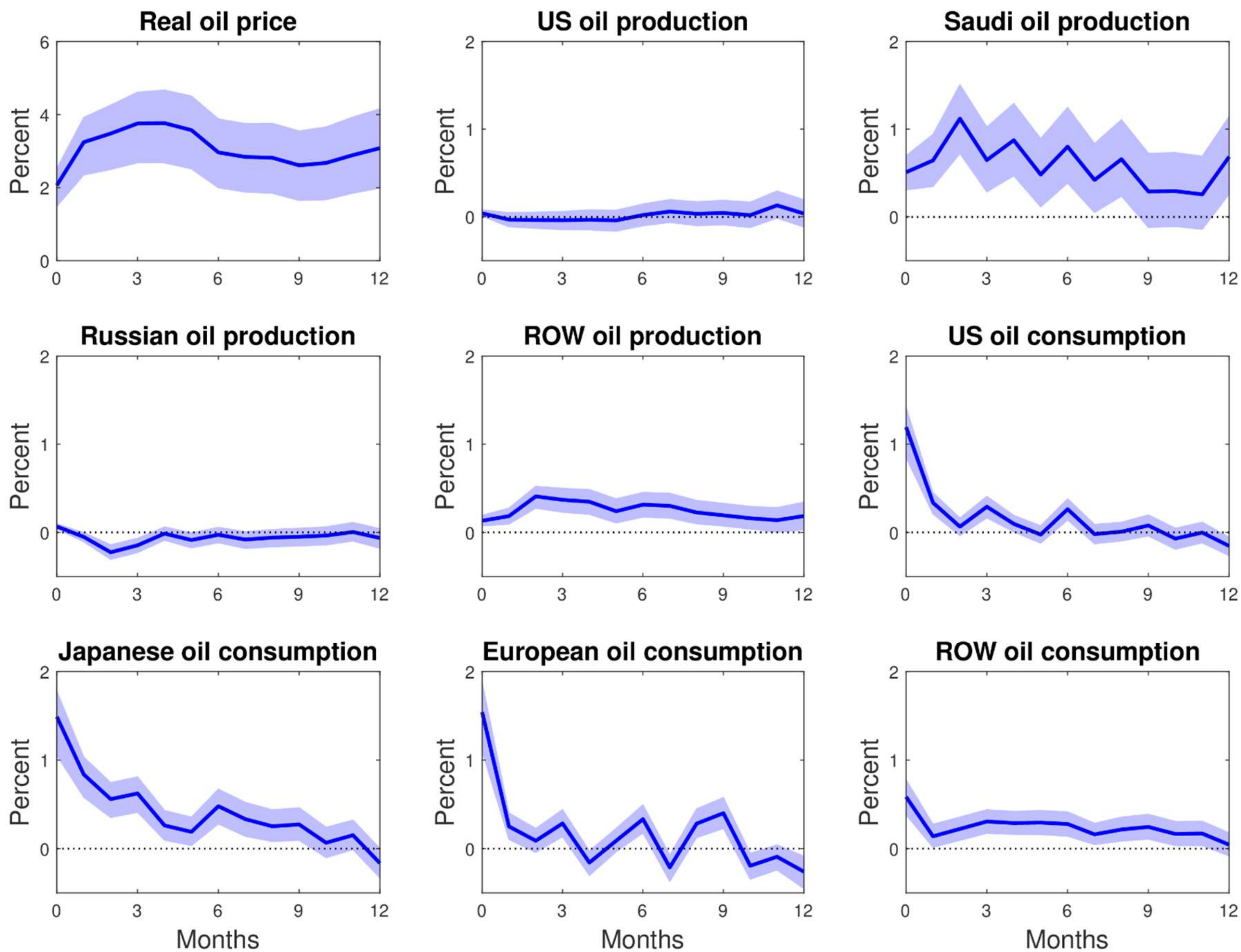
Loadings on global demand factor

U.S.	1.367	(0.425)
Japan	1.495	(0.499)
Europe	1.981	(0.537)
Rest of world	0.881	(0.321)

Impact effect of one-standard-deviation increase in global demand factor

Variable	as % of country			% of world
	direct effect (1)	response to price (2)	net effect (3)	net effect (4)
p	2.055			
q_{US}	0	0.044	0.044	0.005
q_{Saudi}	0	0.509	0.509	0.061
q_{Russia}	0	0.070	0.070	0.010
q_{ROW}	0	0.135	0.135	0.082
q				0.159
c_{US}	1.367	-0.159	1.208	0.302
c_{Japan}	1.495	-0.002	1.493	0.105
c_{Europe}	1.981	-0.416	1.565	0.125
c_{ROW}	0.881	-0.286	0.595	0.357
c				0.889
v				0.730

Dynamic effect of one-standard-deviation increase in global demand factor

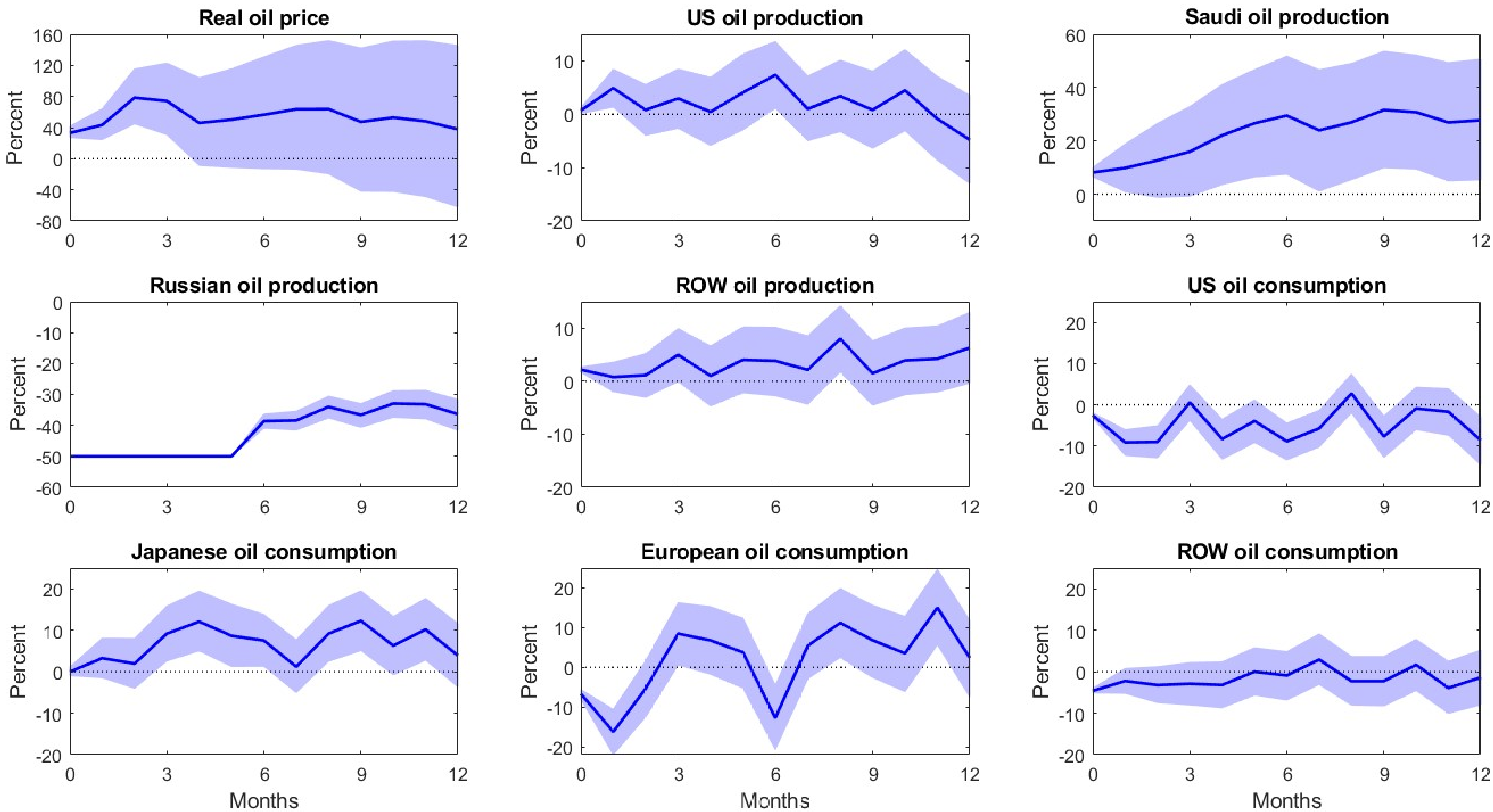


Shaded regions denote 68% confidence bands

Impact effect of 50% cut in Russian production (inventory change = 0)

Variable	as % of country			in mb/d
	direct effect (1)	response to price (2)	net effect (3)	net effect (4)
p	33.020			
q_{US}	0	0.699	0.699	0.086
q_{Saudi}	0	8.186	8.186	0.808
q_{Russia}	-50	0.000	-50.000	-5.350
q_{ROW}	0	2.165	2.165	1.069
q				-3.386
c_{US}	0.000	-2.554	-2.554	-0.420
c_{Japan}	0.000	-0.026	-0.026	-0.001
c_{Europe}	0.000	-6.679	-6.679	-0.275
c_{ROW}	0.000	-4.603	-4.603	-2.690
c				-3.386
v				0.000

Dynamic effect of 50% cut in Russian production

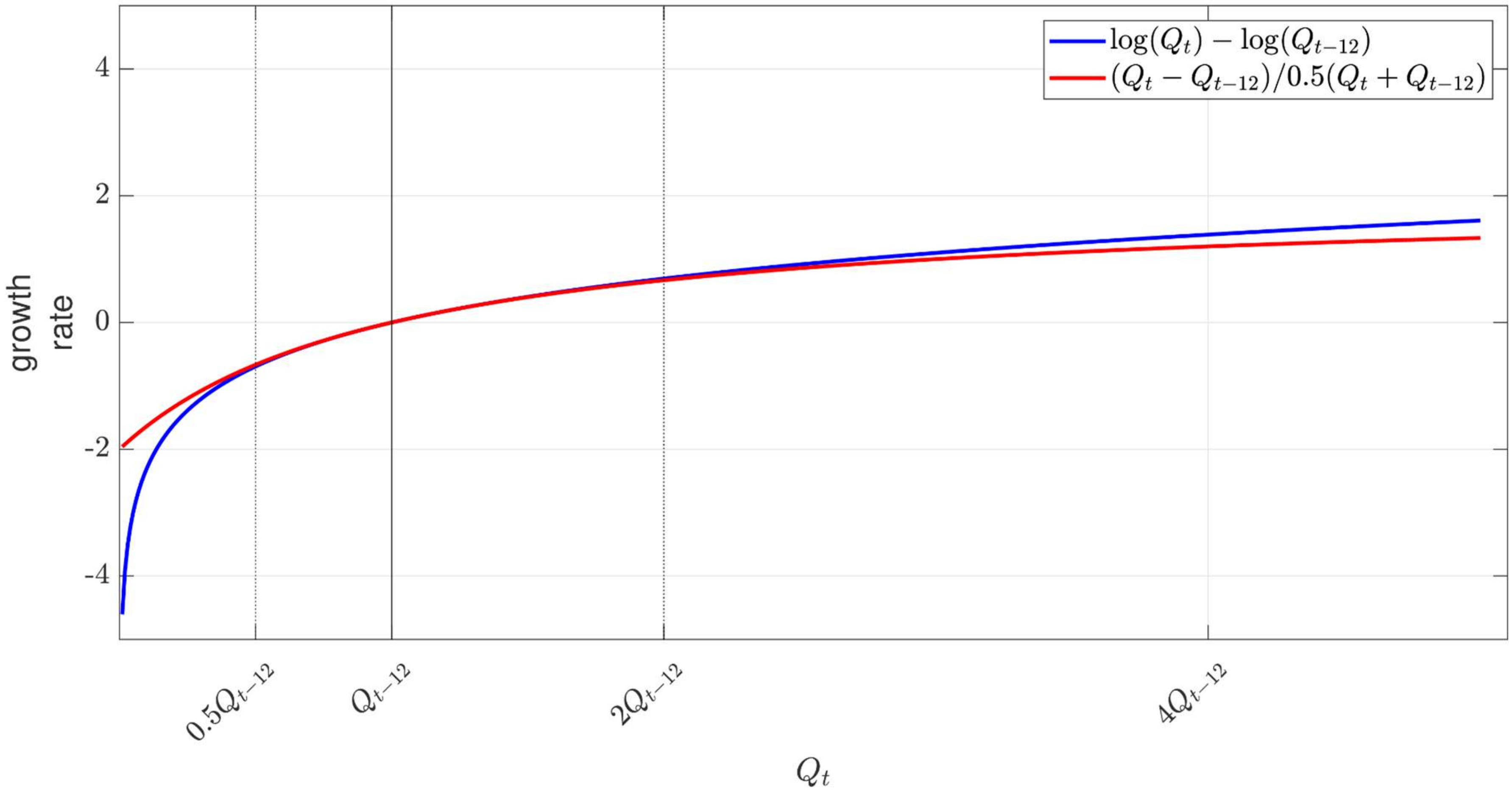


Assumes zero inventory change for first 6 months

Conclusion

- If correlations between supply and demand shocks can be described with low-order factor structure, can use correlations between price and country-specific production and consumption to estimate key elasticities.
- Next step: use regularization to apply to larger numbers of producers and consumers.

Additional slides


$\log(Q_t) - \log(Q_{t-12})$
$(Q_t - Q_{t-12})/0.5(Q_t + Q_{t-12})$
4
2
0
-2
-4
growth rate
$0.5Q_{t-12}$
Q_{t-12}
$2Q_{t-12}$
$4Q_{t-12}$
Q_t