Overcoming Discrimination: Harassment and Discrimination Dynamics *

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Abstract

A common feature of historical episodes in which integration was successful, as well as episodes where integration was unsuccessful, is the aggravated harassment of the early pathbreakers who put themselves at risk by violating the previous segregated norm. Examples abound including Jackie Robinson and Larry Doby in the case of Major League Baseball, Autherine Lucy who was the first Black student at the University of Alabama, and Jane Chastain and Melissa Ludtke who were early female sports reporters. In this paper, we explore from a theoretical perspective the role of harassment of what we refer to as integration pathbreakers in the success and speed with which integration occurs. In our model of labor market discrimination, harassment occurs because the harassers potentially receive direct and immediate utility from harassing, but also because harassment has the potential to slow down or even stop integration. Our main result is that such a setting can exhibit path dependence, where the success or failure of the early integration pathbreakers can be pivotal for the success and speed of the subsequent integration process. That is, early success is more likely to be followed by successful and faster integration than early failure, even when the early success is not due to aspects of the environment that make integration easier. In addition to our formal theoretical analysis of the role of harassment in the success and speed of integration, we apply our results to various historical episodes including the integration of Major League Baseball in the 1940s, 1950s, and 1960s.

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"Plenty of times I wanted to haul off when somebody insulted me for the color of my skin, but I had to hold to myself. I knew I was kind of an experiment. The whole thing was bigger than me."

–Jackie Robinson, Hall of Fame Baseball Player

"Jackie, we've got no army. There's virtually nobody on our side. No owner, no umpires, very few newspapermen. And I'm afraid that many fans may be hostile. We'll be in a tough position. We can win only if we can convince the world that I am doing this because you're a great ballplayer, and a fine gentleman."

–Branch Rickey, ex-General Manager, Brooklyn Dodgers

"Jackie Robinson made my success possible. Without him, I would never have been able to do what I did."

–Martin Luther King Jr., Nobel Peace Prize laureate

1 Introduction

Every American knows the story of Jackie Robinson — the only major league baseball player to have his number, 42, retired across all major league baseball teams. Robinson broke the "color barrier" in major league baseball in 1947 as a member of the Brooklyn Dodgers and went on to have a Hall of Fame career — he was inducted into the Hall in 1962 in his first year of eligibility. Also, starting with Larry Doby who was the first Black player in the American League becoming a member of the Cleveland baseball team later in 1947, Robinson's successful breaking of the color barrier resulted in one or more Black players on half the major league teams by 1953 and on all teams by 1959.

It was not simply, however, that Robinson showed that Black players could successfully compete against White players. Robinson was severely harassed by fans, players on rival teams, and to an extent even his own teammates, when he first started playing for the Dodgers. As captured by the quotes above, both Robinson and Branch Rickey, the baseball executive who made the decision to hire Robinson, correctly anticipated the harassment and felt that continuing to play in the face of the harassment and playing well were key for Robinson's breaking of the color barrier to be successful. And that is exactly what Robinson did. He was outstanding in terms of the standard ways that major league baseball players are measured, as evidenced by his winning the Rookie of the Year Award for his first season,[1] and the National League's Most Valuable

[1]The award was renamed the Jackie Robinson Rookie of the Year Award in 1987. Also, see Simon (2002) and Kahn (2014) for discussions of Jackie Robinson, Branch Rickey, and the integration of Major League

Player Award for his third season. But just as important, he persevered in the face of the harassment, and he eventually won over fans and opponents and the harassment became less severe over time.

And this story of aggravated harassment of what we will call integration pathbreakers is not unique to Robinson and Major League Baseball. There are numerous examples in which an integration pathbreaker faced aggravated harassment and the success of the pathbreaker in terms of both continued participation and high levels of performance eventually led to reduced levels of harassment and eventual integration, although in many cases significant harassment and discrimination remained for many years. In addition to Robinson, other examples include Autherine Lucy who was the first Black student at the University of Alabama, and Jane Chastain and Melissa Ludtke who were early female sports reporters.

Most of the existing economic theories of discrimination, discussed in detail later, do not give a prominent role to the aggravated harassment of integration pathbreakers as a determinant of the success or failure of integration. Based on Robinson's story and that of other integration pathbreakers, and consistent with the quote from Marin Luther King Jr. above, we believe that this type of harassment does indeed play an important role concerning whether integration succeeds or fails and the speed with which integration occurs when it succeeds. In this paper, we explore this issue from a theoretical perspective. In particular, we construct and analyze a model of integration and segregation in which early integration pathbreakers are harassed in an aggravated fashion, and the outcome of the harassment helps determine the success and speed of integration in subsequent periods. The key assumption in the model is that one of the goals of the early harassers is to stop integration from occurring. The result is that when the early pathbreakers succeed as in the case of Robinson and Doby, minority workers are more likely to be hired in subsequent periods and integration progresses. But if the partnership between the hiring firm and the pathbreaker fails, then integration is delayed possibly for an extended period.

We consider an infinite-period model characterized by one or more long-lived firms, where in each period a firm has multiple vacancies. Workers live a single period and there is a new cohort of workers in every period, where a cohort consists of multiple identical majority workers and a single minority worker.[2] The minority worker's ability is randomly drawn from a distribution and the realization, which is publicly observed, is frequently higher than that of the majority workers. In each period, a firm can hire majority workers only, or hiring can include the minority worker. But the minority worker is not necessarily employed even with higher ability because of the possibility of harassment by a third party, who can be thought of as a co-worker, customer of the firm, or someone else in the broader society.

Baseball.

[2]Throughout the paper we employ the term minority to refer to the discriminated against group, even if, as in the case of female workers, the group does not constitute a clear minority of workers in the overall population.

We call the employment relationship between a firm and a minority worker hired by the firm an integration partnership. The third-party individual incurs disutility if the minority worker is hired and the partnership survives till the end of the period. This third-party individual, however, can harass the partnership, where this harassment inflicts monetary and potentially other types of harm on the partnership. When a partnership is formed the third-party individual chooses between not harassing, an intermediate level of harassment, and an aggravated level of harassment, where the third-party individual incurs a cost when choosing the aggravated level of harassment. Also, the magnitude of the harm caused by the aggravated level of harassment is a random draw from a distribution governed by an unknown state, where the state can be either high or low. The high (low) state means a high (low) probability that the aggravated level of harassment results in an extreme rather than intermediate level of harm. Once the harms imposed on the contracting parties are observed by the firm and the minority worker, the two parties decide whether to dissolve the partnership and avoid a portion of the harm, or to keep the partnership intact till the end of the period and incur the full harm.

In our model, there is segregation in period t when the probability the minority worker is hired is zero, while we define integration as occurring when aggravated harassment stops and the minority worker is hired whenever her ability exceeds a given level. We say there is discrimination, but not segregation, when there is a positive probability the minority worker is hired, but such hiring only occurs when the minority worker's ability exceeds majority worker ability by a large enough amount. Also, we say discrimination increases (decreases) when the minimum minority worker ability required for the minority worker to be hired increases (decreases). Note that in our model integration can be associated with either no discrimination or a limited amount of discrimination.

If a minority worker is hired, the harassment level, the harm level, and whether the partnership survives or dissolves are all publicly observed. The result is a learning process concerning the state of the world that depends on realized harm levels. Also, we focus on the symmetric Markov Perfect Nash equilibrium of the game for two market structures — monopoly and oligopoly.

Here we find that, given either type of market structure, when the belief of the various economic agents is that there is a low probability that the state of the world is the high harm state, the third-party individual chooses either no harassment or the intermediate level of harassment if a partnership is formed, and the result is that a partnership does not dissolve once formed. Absent the risk of aggravated harassment, the firm(s) hire minority workers exhibiting either no discrimination or a limited level of discrimination, and as indicated a partnership once formed never dissolves. Learning is stuck but in a good sense. We call this belief range the integration region.

On the other hand, when the belief of the various economic agents is that there is a higher probability

that the state of the world is the high harm state, the third-party individual chooses the aggravated level of harassment when a partnership is formed. If this choice results in an extreme level of harm, then the partnership dissolves once formed. If the belief concerning the probability that the state of the world is the high harm state is sufficiently high, then the minority worker is never hired and learning concerning the state of the world does not occur because partnerships never form. We call this belief range the segregation region. For beliefs in between, aggravated harassment is chosen by the third-party individual when a partnership is formed, but the minority worker is sometimes hired even though this is the case and learning concerning the state of the world occurs when a partnership is formed and the harm level is realized. We call this belief range the learning region.

The dynamic features of equilibrium exhibit path dependence. That is, consider what determines whether a given period is consistent with integration, segregation, or the severity of discrimination when equilibrium is intermediate on this dimension. This depends to an extent on the random realizations of stochastic outcomes in previous periods rather than solely on the true state of the world. Specifically, consider how the economy evolves when beliefs are in the learning region. If a partnership is formed, which in equilibrium is always followed by aggravated harassment, there are two possibilities. If the harm level is intermediate in which case the partnership survives, then the learning process lowers the belief that the true state of the world is the high harm state. The result is that discrimination decreases, which means the probability a partnership is formed in the following period rises. In fact, it is possible that the belief probability enters the integration region in which case aggravated harassment stops and partnerships that are formed never dissolve.

In contrast, suppose beliefs are in the learning region, a partnership is formed, and the partnership is dissolved because the aggravated harassment that follows translates into the extreme level of harm. Then the result is that discrimination increases, which means the probability a partnership is formed in the following period falls. In fact, the belief probability may enter the segregation region in which case partnerships are never formed after the current period and the economy is characterized by permanent segregation.

The other main result is that integration is more likely in the monopoly case than in the oligopoly case. In the monopoly case, the firm benefits from reaching the integration region because it receives surplus from hiring the minority worker when she is high ability. This is not true in the oligopoly case due to competition between firms for the services of the minority worker. The result is that in the monopoly case integration partnerships form under a wider range of values for beliefs concerning the state of the world, and this translates into a higher probability that the economy eventually achieves permanent integration.

In addition to the main analysis described above, we also provide analyses of two extensions. In the first, we allow for the possibility that minority workers are altruistic towards future minority workers. We show that in the oligopoly case such altruism can increase both the likelihood beliefs reach the integration region

and also the speed of integration. The logic here is similar to why in the main analysis integration is more likely in the monopoly case. That is, altruism is similar to making minority workers long-lived players, and in the oligopoly case this leads to minority workers having a higher incentive to form partnerships, because future minority workers receive the surplus if beliefs reach the integration region. In the second extension, we consider a variant of the model in which harm levels are not publicly observable, and third-party individuals learn by observing whether previous integration partnerships survived or dissolved. We show that in the oligopoly case this alternative assumption concerning observability results in identical equilibrium behavior.

As discussed later in the paper in detail, the first extension can help explain the timing of Jackie Robinson's hiring by the Brooklyn Dodgers and other moves towards integration that occurred after World War II. As argued, for example, in Parker (2009a,b), upon returning to civilian life after military service in World War II, Black military veterans had changed preferences and attitudes which led to a higher willingness to challenge discrimination and segregation. Our first extension provides a formal theoretical framework that shows how changed preferences, even in the absence of any change concerning the bigotry of the majority community, can transform a labor market over time from one characterized by seemingly permanent segregation to one characterized by integration.

As a final introductory point, it is worth noting that our analysis not only captures the dynamics of how segregation ended in Major League Baseball and other settings where there was a transition from segregation to integration, but also to an extent captures why Jackie Robinson and Branch Rickey were so focused on doing everything possible to guarantee Robinson's success as a Major League Baseball player. That is, our analysis of the extension in which third-party individuals learn by observing whether previous partnerships survived or dissolved shows why they were so focused on his success. By succeeding he sped up the integration process in Major League Baseball and likely elsewhere in the economy. That is why he is widely celebrated as a national hero!

The outline for the paper is as follows. Section 2 discusses related literature. Section 3 presents the model and a preliminary analysis concerning equilibrium behavior when the state of the world is known with certainty at the beginning of the game. Section 4 conducts preliminary analysis for the continuation game where offers are made. Section 5 solves the model for both monopoly and oligopoly market structures. Section 6 presents analyses of two extensions and also possible public policy remedies. Section 7 discusses a number of real-world applications. Section 8 provides concluding remarks.

2 Related Literature

This paper contributes to a broad literature on the sources and severity of labor market discrimination. An early branch of this literature started by Becker (1957) is that of taste-based discrimination (see also, for example, Welch (1967), Arrow (1972a,b), Black (1995), and Lang, Manove, and Dickens (2005)). In models of this sort, a relevant individual in the economy receives disutility from the employment of workers in the minority group. There are various possibilities including that the biased individual is the employer, that there is a biased group consisting of potential co-workers, while a third possibility is that the biased group is the firm's customers. The specifics of equilibrium behavior depend to an extent on which individual or set of individuals exhibit bias, but the basic outcome is similar across models of this sort which is that minority workers are less likely to be employed and/or equilibrium wages for minority workers are reduced.

Another well-known approach in the labor market discrimination literature is that of statistical discrimination which dates back to early papers of Arrow (1972a,b) and Phelps (1972). The initial argument in this part of the literature is that, if there are differences in average ability or productivity across groups and individual productivity is hard to observe, then high productivity individuals in low productivity groups can be hurt. This occurs because beliefs about the productivity of these individuals are reduced due to the lower average productivity of the members of the individuals' group. This argument has also been extended in various ways. For example, Aigner and Cain (1977) argue that minority groups can be hurt when ability indicators are less informative for minority groups. Also, Coate and Loury (1993) show that self-fulfilling beliefs concerning investment returns can hurt minority groups. In both of these arguments the ability distributions are the same for the two groups.

The theory of screening discrimination is similar to that of statistical discrimination in that it relies on imperfect information concerning worker productivities (see, e.g., Cornell and Welch (1996)). Rather than differences across groups in terms of average ability or the informativeness of ability indicators, the screening discrimination argument posits that individuals making hiring decisions are better able to judge the productivity of individuals in their own group. So, if for example, managers making hiring and promotion decisions are mostly White workers, then Blacks can be disadvantaged in hiring and promotion decisions even though everyone realizes that the two groups are identical in terms of the distribution of abilities.[3]

Our paper belongs to the taste-based part of this literature. We assume the existence of a bigoted third party, which could represent various different economic players such as co-workers or the firm's customers, where the third-party individual is bigoted in that he receives disutility from the employment of a minority

[3]A third argument that depends on imperfect information concerning the ability levels of minority workers is the invisibility argument due initially to Milgrom and Oster (1987). See also, for example, Cassidy, DeVaro, and Kauhanen (2016) and Dato, Grunewald, and Kräkel (2021).

worker and also possibly receives direct utility from harassing minority partnerships. What distinguishes our approach from previous papers focused on taste-based discrimination is that the third party in our modeling is active rather than passive. That is, rather than the presence of a bigoted third-party individual affecting market prices which disadvantages potential minority employees, in our framework the bigoted individual can directly harass minority employees and their employers, and part of the goal of the harassment can be stopping or at least slowing down the integration process. As discussed in the introduction and further in Section 7, there are numerous real-world examples consistent with our approach concerning the aggravated harassment of integration pathbreakers.

It is also worth noting that our approach is not subject to a standard criticism of the taste-based approach. Standard models of taste-based discrimination predict that firms that discriminate will be at a disadvantage, and that competition will thus favor firms that do not discriminate which, in turn, should lead to reduced levels of discrimination over time. This prediction has been criticized as being inconsistent with the evidence (see, for example, Arrow (1998)). Our taste-based model is not subject to this criticism. In our model, firms are profit maximizing and more competition in our analysis, i.e., moving from the monopoly case to the oligopoly case, does not reduce discrimination and/or segregation. In other words, discrimination and segregation are the result of profit and utility maximizing decisions of the various parties, and more competition does not drive out firms that employ discriminatory practices.

Another point worth noting is that our paper does not rely on the basic ideas of the statistical discrimination or screening approaches in which firms are uncertain about an individual's ability or productivity. In our model, in each period both majority worker productivity and the minority worker's productivity are known with certainty. And in equilibrium the minority worker might not be employed even when the individual's productivity is higher and this is known with certainty. As indicated, the mechanism resulting in this outcome is related to the costs borne by the minority worker and her employer caused by third-party harassment.

The paper is related to prior papers that model discrimination dynamics, where most of the earlier papers on discrimination dynamics use the statistical discrimination idea as a building block. There are various ideas that have been explored in this part of the discrimination literature. Antonovics (2006) looks at how discrimination evolves across generations given human capital investments and statistical discrimination to analyze intergenerational income mobility. Fryer Jr (2007) employs a multi-stage game that incorporates promotion decisions that occur after workers are hired, and shows that discrimination against a minority group at the hiring stage can turn into that group being favored at the promotion stage. Blume (2006) and Levin (2009) introduce learning into dynamic models, where the unknown state to be learned is the distribution of abilities in each group. Also, Mailath, Samuelson, and Shaked (2000) and Eeckhout (2006)

are a few of the search-and-match models where coordination failure can lead to persistent discrimination. As indicated above, our model does not include a statistical discrimination component and learning concerns the severity of harassment related to a taste-based bias. We show how the learning process is governed by strategic interactions among three parties, which ultimately leads to either integration or segregation.

The theoretical model closest to our approach is the model found in Folke and Rickne (2022), which is a recent theoretical and empirical study focused on the role of sexual harassment in explaining sex segregation and pay differences in the workplace.[4] We feel the two papers are complementary. Their focus is harassment in a static theoretical model and showing that nationally representative survey data from Sweden, as well as data from a survey experiment on employed Swedes, are consistent with their theoretical predictions. Our focus is instead dynamics when harassment is part of the environment, and showing that path dependence can be an important factor for understanding how a labor market evolves over time in terms of integration, segregation, and discrimination.

Finally, the paper also contributes to the large literature on social learning and information cascades. Rothschild (1974) is a groundbreaking early paper on the topic which led to a large literature exploring the implications of social learning in a variety of settings. For example, Aghion et al. (1991) comprehensively studies a single decision maker's learning problem, while Bolton and Harris (1999) and Keller, Rady, and Cripps (2005) pioneered models of learning with strategic interaction among multiple players. In our modeling, learning involves multiple players, where our addition to the social learning literature is that the learning is influenced not only by the transacting parties (firms and minority workers), but also by other players who indirectly influence the speed of learning. In the equilibrium in our model there is a gradual information cascade, where successful hiring of a minority worker encourages more minority hiring in the future and weakens discrimination, while unsuccessful minority hiring results in the opposite outcome.

3 The Model

In this section we present our main model. Time is discrete and the horizon is infinite, where $t \geqslant 0$ denotes the time period. There are $N \geqslant 1$ long-lived firms, $n = 1, \ldots, N$, characterized by discount factor $\delta \in (0, 1)$. We consider both cases of monopoly ($N = 1$) and oligopoly ($N > 1$). As discussed in more detail below, in each period a cohort of workers enters the labor market and is in the market only for a single period. A cohort consists of Z identical majority workers and one single minority worker. Each period is also characterized by a short-lived third-party individual who receives disutility if any firm employs the

[4]Earlier related research includes Gutek and Cohen (1987), Gruber (1998), and Jackson and Newman (2004).

minority worker that period, and who may harass the minority worker and the hiring firm.

We assume that all majority workers have identical ability $\theta^\circ \geqslant 0$, which is constant across cohorts and commonly known. On the other hand, the ability of the minority worker in any given cohort is a random draw, θ_t, from a continuous distribution function F with support $[\underline{\theta}, \overline{\theta}]$, where $\underline{\theta} < \theta^\circ < \overline{\theta}$. At the beginning of each period, the realized θ_t becomes public knowledge.

A firm employs $k \geqslant 1$ workers per period, and we assume $Z > Nk$ to guarantee excess majority workers in each period sufficient to fill all firms' positions. Firm j's output in period t, denoted Y_{jt}, is given by:

$$Y_{jt} = a_1 \Theta_{jt}, \tag{1}$$

where $a_1 > 0$ and Θ_{jt} is the sum of the abilities of the k workers firm j employs in period t. If a worker of ability θ is not employed by a firm, then the worker's outside option is self-employment with output $a_0\theta$, where $a_0 \in (0, a_1)$. That is, a worker is more productive working at a firm, but due to excess workers, some wind up self-employed in each period.

The wage determination process is as follows. At the beginning of each period, all firms simultaneously make wage offers to every worker in that period's cohort. Each worker then accepts at most one offer. If in any period t the minority worker applies and is hired, we say that in period t the minority worker and the hiring firm form an *integration partnership*.

As indicated earlier, each period is characterized by a single additional market participant we refer to as the third-party or bigoted individual. This individual receives disutility $-b < 0$ if the minority worker is hired by any firm and the integration partnership survives till the end of the period. However, every time the minority worker is hired, this individual chooses the degree of harassment it imposes on the worker-firm partnership possibly in an attempt to dissolve it. It is not costly for the individual to choose an *intermediate* level of harassment, but it costs the individual $c > 0$ to choose an *aggravated* level.[5]

Harassment inflicts damages on the partnership. The intermediate level of harassment always leads to a total harm of $d_t = \underline{d}$, while the aggravated level causes a random total harm of either $d_t = \underline{d}$ or $d_t = \overline{d}$, with $\overline{d} > \underline{d} > 0$. There is an underlying state of the world denoted $\lambda \in [L, H]$, unknown by all parties at the beginning of the game. In state $\lambda = L, H$, the aggravated harassment yields total harm $d_t = \overline{d}$ with probability $q^\lambda \in (0, 1)$ and total harm $d_t = \underline{d}$ with probability $1 - q^\lambda$. Without loss of generality, we let $q^H > q^L$ with the interpretation that $\lambda = L$ is the low harm state, whereas $\lambda = H$ is the high harm state.

[5]One interpretation is that aggravated harassment entails illegal actions, and c is the expected cost of being caught and fined or prosecuted. Also, allowing the third-party individual to choose no harassment would not change equilibrium behavior, so for ease of exposition we do not introduce no harassment as a choice.

When harassment occurs, either the minority worker or the hiring firm can choose to dissolve the partnership. If neither party dissolves the partnership, production proceeds, and the total harm d_t is shared between the parties such that the firm incurs $(1-\beta)d_t$ and the minority worker incurs βd_t, where $\beta \in (0,1)$. If the partnership is dissolved, the firm replaces the minority worker with a majority worker and the minority worker moves to self-employment.[6] When dissolution occurs, the minority worker and the hiring firm avoid the full harm d_t, but instead suffer residual harm levels $d_W > 0$ and $d_F > 0$, respectively. Meanwhile, the third-party individual no longer incurs the disutility b. We assume the harassment level, the realized total harm level, and whether the partnership survives are all publicly observed by the end of the period.

Some assumptions on the parameters follow.

Assumption 1 (Parameters)

(i) Dissolution is costly, i.e., $d_F + d_W > (a_1 - a_0)(\overline{\theta} - \theta^\circ)\frac{1-q^H}{q^H}$.

(ii) The high damage dissolves all partnerships, i.e., $\overline{d} > \overline{\theta} + d_F + d_W$.

(iii) Harassment decision is not trivial, i.e., $\frac{c}{b} \in (q^L, q^H)$.

Assumption 1(i) states that $d_F + d_W$ is sufficiently large that when the state is known to be high harm, the firm does not hire even the most able minority worker. Assumption 1(ii) means $\overline{d}$ is prohibitively high that the partnership necessarily dissolves even with the highest-ability minority worker. Assumption 1(iii) confines our investigation to the interesting range where neither intermediate nor aggravated harassment is a dominant strategy. We also assume firms, workers, and the third-party individual are all risk neutral.

At the beginning of period 0, Nature chooses the state of the world $\lambda \in \{L, H\}$, where $p_0 \in (0,1)$ is the common prior of all parties that $\lambda = H$. Afterwards, the common belief p_t evolves according to Bayes rule. Note that this means $p_{t+1} = p_t$ if either an integration partnership is not formed in period t, or an integration partnership is formed but the third-party individual does not choose aggravated harassment.

A public history in the beginning of period t includes all minority workers' abilities, firms' wage offers, workers' offer choices, the third party's harassment levels, realized harms, and the dissolution decisions up to period $t-1$. However, our focus is stationary Markov Perfect Nash equilibrium where the public history matters only through the payoff-relevant state, the common belief p_t. We characterize all symmetric stationary Markov perfect equilibria. Before ending this section, we impose the following tie-break rules. None of these tie-break assumptions is important for the qualitative nature of our results.

[6] In terms of replacing a minority worker when an integration partnership is dissolved, we assume the firm simply hires one of the majority workers who applied to the firm at the beginning of the period but was not hired.

Assumption 2 (Tie-breaking)

(i) When a firm or a worker is indifferent about dissolving the partnership, they break the tie in favor of not dissolving.

(ii) When the third party is indifferent about the level of harassment, it breaks the tie in favor of aggravated harassment.

(iii) When the firm is indifferent about hiring a minority worker or not, it breaks the tie in favor of hiring.

4 Preliminary Analysis: After Wage Offers

As a first step in analyzing the model and solving the game, we conduct a preliminary analysis of continuation play after offers are made.

4.1 Worker and Firm: Dissolution Decisions

Consider a minority worker who has accepted employment at a firm. Given wage w_t and realized harm d_t, the worker does not dissolve the integration partnership if and only if:

$$w_t - \beta d_t \geqslant a_0\theta_t - d_W. \tag{2}$$

The firm is long-lived, but it learns d_t and updates its belief concerning p_t before the dissolution decision. Therefore, with Markov strategies, the dissolution decision relies solely on its intra-period tradeoffs. Given ability θ_t, wage w_t and harm d_t, it sustains the partnership if and only if:

$$a_1\theta_t - w_t - (1-\beta)d_t \geqslant (a_1 - a_0)\theta^\circ - d_F. \tag{3}$$

Adding these two inequalities, a necessary condition for the partnership to survive the harassment is that the ability of the minority worker is sufficiently high relative to the harm realization:

$$d_t \leqslant (a_1 - a_0)(\theta_t - \theta^\circ) + d_F + d_W.$$

By Assumption 1, no partnership satisfies this condition if the harm realization is $\overline{d}$, but some partnerships can survive the harassment, with proper wages, if the harm is $\underline{d}$.

4.2 Third Party: Harassment Decisions

The third-party individual weighs cost and benefit within period. From the above analysis of dissolution decisions, a high harm realization $d_t = \overline{d}$ always destroys the partnership, while a low harm $d_t = \underline{d}$ cannot if (2) and (3) both hold when evaluated at $d_t = \underline{d}$.

As a result, if the wage w_t is set to violate either (2) or (3) when evaluated at $d_t = \underline{d}$, intermediate harassment suffices to dissolve the partnership. Instead, if the wage w_t satisfies both (2) and (3) when evaluated at $d_t = \underline{d}$, then the third party's harassment choice hinges on the common belief p_t. With an intermediate level of harassment, his utility remains at $-b$. With aggravated harassment, his utility is $-c$ if $d_t = \overline{d}$ because the partnership dissolves, or $-c - b$ if $d_t = \underline{d}$ because the partnership perseveres. With risk neutrality and Assumption 2(ii), the third-party individual harasses at the aggravated level if and only if the resulting expected utility is weakly higher than $-b$, i.e.,

$$p_t \geqslant p^I \equiv \xi^{-1}\left(\frac{c}{b}\right) \in (0,1), \tag{4}$$

where $\xi(p) \equiv pq^H + (1-p)q^L$ is the total probability of high harm $\overline{d}$ when $\Pr(\lambda = H) = p$.

4.3 Workers: Offer Selection

A majority worker does not suffer from harassment and therefore accepts one of the offers that pay at least $a_0\theta^\circ$, their outside option.

A minority worker's decision is more involved. Facing all offers $\{w_t^n\}_{n=1}^N$, the worker anticipates the terminal outcome associated with accepting each of them. Specifically, for a wage so low that (2) fails even when $d_t = \underline{d}$, the worker will have to eventually dissolve the partnership regardless of the realized harm and incur the residual harm d_W. Expecting this, she should not have accepted the offer in the first place. On the other hand, a wage so high that (3) fails (evaluated at $d_t = \underline{d}$) is "too good to be true," because now it is the firm that will eventually dissolve the partnership regardless of the realized harm. For the same reason, the minority worker should not have accepted the offer. In sum, only wages between these two extremes allows the partnership to persevere through the low harm realization $\underline{d}$.

Additional to the above results that follow from ex post individual rationality, the wage must also satisfy the ex ante version of the constraint, i.e., the minority worker's expected payoff is no less than $a_0\theta_t$. According to the previous subsection, when (2) and (3) are both satisfied, if $p_t < p^I$, the harassment is intermediate and the worker's payoff is $w_t - \beta\underline{d}$, while if $p_t \geqslant p^I$, the harassment is aggravated and the worker's expected payoff is $\xi(p_t)(a_0\theta_t - d_W) + (1 - \xi(p_t))(w_t - \beta\underline{d})$.

Therefore, the minority worker will only consider offers in the *acceptance set*, defined as:

$$\mathcal{W}(p_t, \theta_t) \equiv \begin{cases} \left[\beta \underline{d} + a_0\theta_t,\ a_1\theta_t - (1-\beta)\underline{d} - (a_1 - a_0)\theta^\circ + d_F\right] & \text{if } p_t < p^I, \\ \left[\beta \underline{d} + a_0\theta_t + \frac{d_W \xi(p_t)}{1-\xi(p_t)},\ a_1\theta_t - (1-\beta)\underline{d} - (a_1 - a_0)\theta^\circ + d_F\right] & \text{if } p_t \geqslant p^I. \end{cases}$$

Among all offers in the acceptance set, the minority worker will select the highest one, or one of the highest if multiple offers are tied at the highest level.

The above analysis of continuation play is summarized in the following lemma, based on which the wage offer stage is carried out as the first move.

Lemma 1 (Continuation Play)

The minority worker accepts an offer if and only if it is the highest among all offers in the acceptance set. Following acceptance, the third party harasses at the aggravated level if and only if $p \geqslant p^I$.

5 Main Analysis

Knowing the outcome of the continuation game in each period, we now use backward induction to return to the very first decision: hiring. This step drives the entire dynamic game because a long-lived firm cares about inter-temporal tradeoffs.

5.1 Complete Information

In this subsection, we consider the special case in which the state of the world λ is known at the beginning of the game. Because there is no learning, equilibrium behavior does not vary from period to period. We start with the monopoly case, $N = 1$.

Proposition 1 (Monopoly) *Suppose* $N = 1$ *and the state of the world is common knowledge.*
(i) If $\lambda = L$*, then the minority worker is hired (at wage* $\beta\underline{d} + a_0\theta_t$*) if and only if* $\theta_t \geqslant \theta_1^*(0)$*, where* $\theta_1^*(0) \equiv \theta^\circ + \frac{\underline{d}}{a_1 - a_0}$*. Majority workers always fill remaining positions at wage* $a_0\theta^\circ$*. The third-party individual chooses the intermediate harassment level, and the integration partnership survives till the end of the period.*
(ii) If $\lambda = H$*, then the firm hires* k *majority workers at wage* $a_0\theta^\circ$.

Proposition 1 tells us that in the monopoly case, if the state of the world is known at the beginning of the game, then there is no learning and behavior in each period is the same. If the state is high harm ($\lambda = H$), then the firm hires only majority workers each period, which means harassment is not part of equilibrium

behavior. Also, due to the excess majority workers, the equilibrium wage equals their reservation value. In contrast, suppose the state of the world is the low harm state. Then in period t the minority worker is hired if her ability is sufficiently above that of the majority workers, and the third-party individual chooses the intermediate level of harassment. Moreover, the minority worker's wage is above the outside option because the worker foresees the harm caused by the harassment and requires compensation.

Another result captured in Proposition 1 is that, when the state of the world is known, aggravated harassment is never chosen in equilibrium and integration partnerships, once formed, never dissolve. The former follows given our parameter restrictions concerning q^H and q^L. That is, q^L is low enough that the third-party individual does not find it worthwhile to choose aggravated harassment against an integration partnership, while q^H is high enough that aggravated harassment deters the firm from hiring the minority worker of any ability level.

We now consider the oligopoly case, $N > 1$.

Proposition 2 (Oligopoly) *Suppose $N > 1$ and the state of the world is common knowledge.*
(i) If $\lambda = L$, then the minority worker is hired by one firm (at wage $a_1\theta_t - (1-\beta)\underline{d} - (a_1 - a_0)\theta^\circ$) if and only if $\theta_t \geqslant \theta_N^*(0)$, where $\theta_N^*(0) = \theta_1^*(0)$. Majority workers always fill remaining positions of all firms at wage $a_0\theta^\circ$. The third-party individual chooses the intermediate harassment level, and the integration partnership survives till the end of the period.
(ii) If $\lambda = H$, then all firms hire k majority workers at wage $a_0\theta^\circ$.

Proposition 2 tells us that equilibrium behavior given oligopoly is qualitatively similar to the monopoly case. The main difference is the wage for the minority worker whenever she is hired. In the monopoly case, the firm has all the bargaining power, such that the minority worker receives no surplus in Proposition 1. In contrast, in the oligopoly case, while majority workers still receive no surplus, the unique minority worker now earns all the surplus provided she is hired, thanks to the Bertrand competition across firms.

In summary, in this subsection we considered how the model works in the absence of learning. In the low harm state, the minority worker is hired if their ability is high enough, although there is still discrimination reflected by $\theta_N^*(0) > \theta^\circ$ for all $N \geqslant 1$. In the high harm state, there is full segregation in the sense that the minority worker is never hired.

5.2 Incomplete Information

In this subsection, we explore equilibrium behavior when the state of the world is not known at the beginning of the game. We start with a result concerning how the public belief p_t evolves over time based on equilibrium behavior described in Section 4.

Lemma 2 (Bayesian Update) *Suppose the public belief at the beginning of period t is $p_t \in (0,1)$. Then:*
(i) $p_{t+1} = p_t$ when either an integration partnership is not formed in period t, or it is formed in period t and intermediate harassment is chosen.
(ii) $p_{t+1} = P_+(p_t) \equiv \frac{p_t q^H}{\xi(p_t)} > p_t$ when an integration partnership is formed in period t, aggravated harassment is chosen, and the realized harm is $\overline{d}$.
(iii) $p_{t+1} = P_-(p_t) \equiv \frac{p_t(1-q^H)}{1-\xi(p_t)} < p_t$ when an integration partnership is formed in period t, aggravated harassment is chosen, and the realized harm is $\underline{d}$.

In our model, learning concerns whether the probability of high harm is q^H or q^L when aggravated harassment is chosen. Thus, there is no learning if no integration partnership is formed, or a partnership is formed but the third-party individual does not choose aggravated harassment. This explains (i). On the other hand, learning does occur when an integrated partnership is formed and the third-party individual chooses aggravated harassment. When this is the case and the realized total harm level is $\overline{d}$, then Bayesian updating causes the belief about the high harm state to rise. This explains (ii). In contrast, when the realized total harm level is $\underline{d}$, then Bayesian updating causes the belief to fall. This explains (iii).

The next step of the analysis is to characterize equilibrium behavior in period t as a function of p_t.

Proposition 3 (Equilibrium) *In the stationary Markov equilibrium of the game, there exists a belief threshold $p_N^S \in [p^I, 1)$ and a cutoff ability $\theta_N^*(p)$ for all p such that:*
(i) If $p_t < p^I$, then belief is no longer updated, and behavior is the same as in Proposition 1(i) (if $N = 1$) or Proposition 2(i) (if $N > 1$) with $\theta_N^*(p_t) = \theta_N^*(0)$.
(ii) If $p_t \geqslant p_N^S$, then belief is no longer updated, and behavior is the same as in Proposition 1(ii) (if $N = 1$) or Proposition 2(ii) (if $N > 1$) with $\theta_N^*(p_t) \geqslant \overline{\theta}$.
(iii) If $p_t \in [p^I, p_N^S)$, then $\theta_N^*(p_t) < \overline{\theta}$. The minority worker is hired by one firm if and only if $\theta_t \geqslant \theta_N^*(p_t)$, while majority workers fill all remaining positions at wage $a_0\theta^\circ$. The minority worker's wage is $\beta\underline{d} + a_0\theta_t + \frac{d_W\xi(p_t)}{1-\xi(p_t)}$ if $N = 1$, and is $a_1\theta_t - (1-\beta)\underline{d} - (a_1 - a_0)\theta^\circ - \frac{d_F\xi(p)}{1-\xi(p)}$ if $N > 1$. Also, the third-party individual chooses aggravated harassment, and the partnership survives if and only if the realized harm is $\underline{d}$.

Proposition 3 tells us that, if the belief is sufficiently concentrated on the low harm state, i.e., $p_t < p^I$, then behavior is no different from the complete information case where the state is known to be $\lambda = L$. We refer to this range of beliefs as the *integration region*. On the other hand, if the belief is sufficiently concentrated on the high harm state, i.e., $p_t \geqslant p_N^S$, then behavior in period t is no different from the complete information case where the state is known to be $\lambda = H$. We refer to this range of beliefs as the *segregation region*. Thus, there are parameterizations of the model in which the economy is sometimes "stuck" from the

beginning of the game with permanent segregation even though the true state is $\lambda = L$, or with permanent integration even though the true state is $\lambda = H$.

Now suppose p_t is in the intermediate range, i.e., $p^I \leqslant p_t < p_N^S$. Then, equilibrium behavior is different from the complete information case. In particular, aggravated harassment arises as part of equilibrium behavior and integration partnerships sometimes dissolve. We call this range of beliefs the *learning region*. Intuitively, for any belief p_t in this region, the minority worker is hired if her ability is above some cutoff $\theta^*(p_t)$, which occurs with positive probability because $\theta^*(p_t) < \overline{\theta}$. When this is the case, the third-party individual chooses aggravated harassment, and the integration partnership survives or dissolves depending on whether the realized harm is $\underline{d}$ or $\overline{d}$. Therefore, in contrast to the complete information case, here equilibrium behavior is sometimes characterized by aggravated harassment, which is followed by the partnership surviving or dissolving.

In the following corollaries, we provide more details concerning equilibrium behavior. For ease of exposition, define $p_\infty \equiv \lim_{t\to\infty} p_t$ as the limiting belief on path.

Corollary 1 (Dynamics)

(i) If $p_0 \in [p^I, p_N^S)$*, then* p_∞ *almost surely exists,* $\Pr(p_\infty < p^I) > 0$*,* $\Pr(p_\infty \geqslant p_N^S) > 0$*, and* $\Pr(p_\infty < p^I) + \Pr(p_\infty \geqslant p_N^S) = 1$.

(ii) In any period t *such that* $p_t \in [p^I, p_N^S)$*, if a partnership survives harm* $d_t = \underline{d}$*, then* $\theta_N^*(p_{t+1}) < \theta_N^*(p_t)$ *or* $\theta_N^*(p_{t+1}) < p^I$*. Also,* $\Pr(p_\infty < p^I | p_{t+1}) > \Pr(p_\infty < p^I | p_t)$.

(iii) In any period t *such that* $p_t \in [p^I, p_N^S)$*, if a partnership dissolves from harm* $d_t = \overline{d}$*, then* $\theta_N^*(p_{t+1}) > \theta_N^*(p_t)$*. Also,* $\Pr(p_\infty < p^I | p_{t+1}) < \Pr(p_\infty < p^I | p_t)$.

Corollary 1 provides results concerning the dynamics of the model when the initial belief is in the learning region. Part (i) states that eventually learning pushes the economy into either permanent integration or permanent segregation, both with positive probability. This implies that the learning process is asymptotically inadequate, in the sense that learning can eventually result in permanent segregation in the L state, or permanent integration in the H state.

Parts (ii) and (iii) focus on how outcomes in period t influence the behavior in the subsequent period and in the long run. In the learning region, one firm hires the minority worker given ability above $\theta^*(p_t)$ and aggravated harassment ensues. If the realized harm is $\underline{d}$, then the partnership survives and $p_{t+1} < p_t$ according to Lemma 2. Either p_{t+1} moves into the integration region permanently, or $\theta^*(p_{t+1}) < \theta^*(p_t)$ such that an integration partnership is more likely to form in the next period. Furthermore, the probability the economy eventually achieves full integration is higher due to the additional information revealed in period t. In contrast, if the realized harm is $\overline{d}$, then the partnership dissolves and $p_{t+1} > p_t$. An integration

partnership is less likely to form in the next period since $\theta^*(p_{t+1}) > \theta^*(p_t)$, and the probability the economy eventually achieves full integration is lower. In sum, equilibrium exhibits path dependence: although the true state and the game primitives do not change over time, beliefs change whenever integrated partnerships are formed and aggravated harassment is chosen.

The next corollary focuses on how behavior changes with N, the number of firms.

Corollary 2 (Market Structure)

(i) $\theta_1^*(p) < \theta_2^*(p)$ *for all* $p \in [p^I, p_1^S)$, *and* $p_1^S > p_2^S$.

(ii) p_N^S *and* $\theta_N^*(p)$ *are constant in* $N \geqslant 2$.

(iii) If $p_0 \in [p^I, p_1^S)$, *then* $\Pr(p_\infty < p^I | N = 1) \geqslant \Pr(p_\infty < p^I | N > 1)$.

Corollary 2 tells us that while equilibrium behavior in the oligopoly case is invariant to the number of firms, this is not true when moving from oligopoly to monopoly. First, in the monopoly case the learning region for p_t is larger. Second, when p_t is in the learning region for $N = 1$, integration partnerships are more likely to form in the monopoly case. Third, fixing p_0, the economy is more likely to eventually achieve permanent integration in the monopoly case.

The intuition for the above comparison stems from the fact that a monopoly firm receives the surplus from partnership with a minority worker. Because the monopolist is long-lived, the monopolist is willing to form an integration partnership that sacrifices some period-t profit in order to speed up the learning process. There is no similar incentive for any firm in the case of oligopoly: it is the minority worker who receives the surplus from the partnership in the case of oligopoly, and thus there is no incentive for an oligopoly firm to speed up the learning process.

This logic also sheds light on labor market discrimination. It can be shown that $\theta^*(p) > \theta^\circ$ for all $p \in [p^I, p_N^S)$ given $N > 1$, but that need not be the case given $N = 1$. When $N > 1$, as just discussed, firms have no incentive to speed up the learning process. As a result, when p is in the learning region the economy always exhibits discrimination, i.e., the minority worker is only hired when her ability is sufficiently above majority worker ability. This follows because the expected harm associated with aggravated harassment means it is only worthwhile forming an integration partnership when the minority worker's ability significantly exceeds majority worker ability. In contrast, when $N = 1$, it is theoretically possible that $\theta^*(p) \leqslant \theta^\circ$ for some values for p in the learning region. That is, because of the monopolist's incentive to speed up the learning process, an integration partnership may sometimes form when θ_t is equal to or slightly below θ° given p is in the learning region, but close to p^I.

In summary, in this section we derived a number of results concerning equilibrium behavior when the state of the world is not known at the beginning of the game. First, if the initial belief is sufficiently

concentrated on the high (low) harm state, then the economy is characterized by permanent segregation (permanent integration) starting in the first period. Second, if this belief is in an intermediate range, then in early periods integration partnerships sometimes form and when they form they are followed by aggravated harassment. Third, if the aggravated harassment results in the low (high) level of harm, then the partnership survives (dissolves) and the formation of an integration partnership become more (less) likely in the next period. In fact, it is even possible that the economy transitions into permanent integration (permanent segregation). Fourth, this learning process can proceed for a number of periods, but eventually the economy almost surely moves into either full integration or full segregation. Fifth, holding all other parameters fixed and assuming beliefs start in the learning region, the economy is more (less) likely to eventually achieve permanent integration (segregation) given the monopoly rather than the oligopoly market structure.

6 Extensions and Discussion

In this section we explore two extensions to the model analyzed in the previous section. In the first subsection, we consider how equilibrium behavior changes when minority workers are altruistic towards future minority workers. In the second, we consider a variant of the model in which realized harm levels are not publicly observed and learning is based on whether integration partnerships survive or dissolve rather than on realized harm levels. In the third subsection, we consider public policy.

6.1 Altruistic Minority Workers

In this subsection, we explore how equilibrium changes when minority workers are altruistic towards future minority workers. The specific change to the model is that the expected total welfare of the minority worker in any period t, denoted U_t, is recursively given by:

$$U_t = (1 - \gamma\delta)\ u_t + \gamma\delta\ U_{t+1}. \tag{5}$$

In this specification, u_t is the ex ante stage utility of the period-t minority worker before the type is drawn, while $\gamma \in [0, 1]$ is the altruism parameter. Note that this specification nests the main model when $\gamma = 0$. We still focus on Markov strategies where the public belief p_t summarizes previous harm realizations from harassment.

The first result concerning this extension is that in the monopoly case, introducing altruism does not change equilibrium behavior. The logic for this result is straightforward. In the monopoly case, as discussed

earlier, all the surplus associated with the hiring of minority workers is captured by the firm. So the behavior of a minority worker in any period t has no effect on the expected utility of future minority workers, which means altruism plays no role in overall equilibrium behavior.[7]

In contrast, as captured in Proposition 4 below, in the case of oligopoly, introducing minority worker altruism does impact equilibrium behavior.

Proposition 4 (Altruism: Equilibrium) *Suppose minority workers are altruistic as captured in* (5)*.*
(i) If $N = 1$, then equilibrium behavior is the same as in Proposition 3.
(ii) If $N > 1$, then equilibrium behavior is the same as in Proposition 3, except p_N^S is strictly higher and $\theta_N^*(p_t)$ strictly lower than in the main model ($\gamma = 0$), for any p_t in the learning region of the altruism case.

Proposition 4 captures the result discussed above that equilibrium behavior in the monopoly case is not affected by the introduction of altruistic minority workers, but also shows that in the oligopoly case behavior does change. In particular, introducing altruism among minority workers causes the learning region to expand, and given any belief in the learning region, increases the range of minority worker abilities for which minority partnerships form.

The logic for the results concerning oligopoly is related to the logic behind equilibrium behavior in the monopoly case in Section 5. That is, introducing minority worker altruism is similar to making minority workers long-lived, in the sense that altruism causes minority workers to care about future welfare. In the oligopoly case, since it is the minority workers who receive the surplus associated with permanent integration, the minority worker in any period t is willing to sacrifice some stage utility for increasing expected minority worker surplus in future periods. The overall outcome is that minority partnerships are more likely to form which is manifested by an increase in p_N^S and a decrease in $\theta^*(\cdot)$ in the learning region.

In the next corollary we consider how the introduction of altruism affects the probability the economy eventually achieves permanent integration.

Corollary 3 (Altruism: Effect) *When $N > 1$, introducing minority worker altruism ($\gamma > 0$) strictly increases $\Pr(p_\infty < p^I)$ when p_0 is in the learning region of the altruism case.*

This corollary states that minority worker altruism increases the probability that the economy eventually achieves permanent integration. It also implies that, if the economy is in a state of permanent segregation

[7]In the monopoly case, all surplus goes to the firm because of our assumption that the firm each period makes a take-it or leave-it offer to the workers. A different wage-setting assumption such as Nash bargaining would result in minority workers receiving a share of the surplus, and thus with such an alternative assumption the introduction of minority worker altruism would cause equilibrium behavior to change.

without altruism, then introducing altruism reduces the segregation region and potentially pushes the economy into the learning region. As a result, there is a positive probability that the economy eventually achieves permanent integration.

The analysis in this subsection indicates that in our framework an economy can move from segregation to integration solely due to changes in the preferences of minority workers, i.e., without any change in the distribution of minority worker ability nor in the preferences of the employers and the third-party individuals. The only factor that changes is the preferences of the minority workers. But it is possible that this single change causes the economy to move from a state of permanent segregation to a state of permanent integration. This is, in contrast, to most discussions such as in Becker (1957)'s original argument where changes in the extent of discrimination are the result of changes concerning employers, coworkers, or customers, as opposed to changes in the minority workers themselves.

As discussed briefly in the introduction, this result is consistent with various discussions such as found in Parker (2009a,b) concerning an important cause of the transition from segregation to integration in many U.S. labor markets following World War II. In particular, these scholars attribute much of the transition not to changes in the attitudes of the White community, but rather to changes in the attitudes of Black veterans after returning from military service at the end of World War II. These returning veterans, according to the argument, were much more willing than was true for Black workers prior to the war to confront and challenge the severe labor market discrimination which was common at the time. In turn, as in the case of Jackie Robinson and Major League Baseball, over time this led to the integration of many labor markets and an overall decrease in labor market discrimination.

6.2 Realized Harm Levels Not Publicly Observable

In this subsection, we study a variant of the model in which realized harm levels are not publicly observable and therefore beliefs of the third-party individuals depend on whether partnerships survive or dissolve. This case is more challenging in terms of tractability, and as a result we do not present a full analysis. Rather, our goal in this subsection is simply to show that qualitatively similar results can arise when realized harm levels are not publicly observable, and it is the survival or dissolution of minority partnerships that influence the frequency with which minority partnerships form and survive in future periods.

Our focus here is the case of oligopoly, i.e., $N > 1$, where we make a slight change to the main model that wages and realized harm are not publicly observed. To be specific, the history of wages and realized harm levels are observed only by firms and minority workers, but not the third-party individual.[8] The third-party

[8]The harm realization is deterministic if the harassment level is intermediate, and information plays no

individual only observes whether partnerships formed and survived in each of the earlier periods. We refer to this information structure as partially publicly observed harm.[9]

In Proposition 5 we show that in the oligopoly case equilibrium behavior is the same whether realized harm levels are publicly observed or partially publicly observed.

Proposition 5 *Suppose $N > 1$. Then equilibrium behavior is independent of whether the realized harm level in any period t in which a minority partnership forms and aggravated harassment is chosen is publicly observed or partially publicly observed.*

In the oligopoly case with public observability of realized harm, as discussed in the main model, competition across firms pushes all surplus from a partnership to the minority worker. As a result, partnerships form whenever the expected surplus is positive and survive whenever survival yields higher surplus than dissolution. With partial public observability of realized harm there is not asymmetric information between firms, and so all the expected surplus associated with the partnership still goes to the minority worker due to competition. The result is that firms have no incentives to manipulate the belief of the third-party individual by changing their behavior. In other words, as found in Proposition 5, equilibrium behavior is independent of whether realized harm levels are publicly observed or partially publicly observed.

That being said, there is an important difference between the cases. In the case of public observability, it is the realization of the harm level when aggravated harassment is chosen that influences the frequency with which minority partnerships form and succeed in future periods. In contrast, given partial public observability, the survival or dissolution of minority partnerships are themselves important for this frequency because they influence the beliefs of future minority workers and future third-party individuals. In other words, our analysis of the partial public observability case suggests why, for example, it was important for Jackie Robinson to behave in a way that ensured that the partnership survived.

Note that we are not arguing in this subsection that the case considered in Proposition 5 is the only interesting case to consider in which realized harm levels are not publicly observable. For example, in the partial publicly observable case when $N = 1$, the monopolist can have an incentive to employ the survival/dissolution decision in order to influence future third party individuals' beliefs concerning the state of the world. And in the case of pure private learning about realized harm levels and $N > 1$, there is a similar possibility that a firm might use the survival/dissolution decision to influence beliefs concerning the state of the world of both other firms and future third-party individuals. We feel these are interesting possibilities, but they are beyond the scope of the current paper.

role in that case.

[9]This assumption is more tractable than assuming that realized harm levels are privately observed by the parties that form the minority partnership because it avoids asymmetric information between the firms.

6.3 Policy Remedies

One question that our analysis raises is, what can the government do to reduce aggravated harassment, and in this way make it more likely that labor markets attain a state of permanent integration rather than segregation? According to our analysis, two standard policies used to combat discrimination should be effective in attaining these goals — affirmative action and penalties for those who practice aggravated harassment.

The standard rationale for affirmative action is that it combats inequality and exclusion.[10] In particular, in the case of hiring practices, the goal is to ensure equal employment opportunities for applicants and employees in groups that historically have faced discriminatory hiring practices. There are also a number of other complementary rationales that have been put forth. One that has been particularly emphasized in the education context is the returns to diversity. This rationale has been emphasized in the education context since a 1970s Supreme Court case concerning affirmative action in which Justice Lewis Powell's opinion upholding the use of affirmative action in college admissions relied on the returns to having a diverse student body.[11] But there is also evidence that a diverse workforce can be beneficial from the standpoint of worker productivity and firm productivity.

Our theory provides a variant of the standard rationale that affirmative action ensures equal employment opportunities. In our theory an important factor leading to segregated outcomes is the aggravated harassment of minority partnerships which have the potential to stop such partnerships from forming, where one of the goals of the bigoted third-party individual in choosing aggravated harassment is to stop or at least slow down the process of integration. Our argument is also characterized by path dependence. That is, in our framework the formation and survival of an integration partnership today leads to a higher expected number of successful partnerships in subsequent periods.

In such a setting, affirmative action can have both important direct effects and important indirect effects. The direct effect is obvious. It outlaws discrimination which should increase the frequency with which minority partnerships form. In addition, however, it would potentially decrease the incentive for bigoted third parties to practice aggravated harassment since these third-party individuals would realize that achieving permanent segregation is not achievable given the affirmative action policy. And if aggravated harassment becomes less frequent, then our theory predicts that permanent integration should become more common.

The other policy is to more severely penalize aggravated harassment. In our model, if aggravated

[10] See Anderson (2004) for a history of affirmative action.

[11] The 2023 Supreme Court decision in the college admissions cases involving Harvard and the University of North Carolina reject the diversity argument.

harassment is more costly to the individual doing the harassing, then the range of beliefs for which the bigoted individual chooses aggravated harassment when a minority partnership forms becomes smaller. The result is that, holding fixed the initial belief concerning the state of the world (and assuming it is in the learning region), the higher is the probability the economy eventually achieves permanent integration. More generally, if the penalty for aggravated harassment is higher, our theoretical approach suggests both direct and indirect reasons why an integration outcome is more likely.

7 Applications

In this section we discuss three applications of our analysis. The first is the case of the integration of Major League Baseball discussed briefly in the introduction. The second is the case of female sports reporters and broadcasters. The third is women in firefighting.

7.1 The Integration of Major League Baseball

The birth of Major League Baseball can be traced back to 1876 with the founding of the National League that consisted of eight teams.[12] There was rapid turnover of teams in the early years and the birth and death of a number of rival leagues. Professional baseball resembling something similar to how it is organized today started in the early 1900s with the establishment of the American League, and the first "World Series" which was played in 1903 with Boston's American League champions defeating the National League champions from Pittsburgh.

At the time of the founding of the National League there were no clear prohibitions on Black players in professional baseball. And there are a few examples of Black professional players around 1880 including John "Bud" Fowler who played for a minor league team in 1878 and Moses Fleetwood Walker who played for a major league team, the Toledo Blue Stockings, in 1884. But Cap Anson, who was a star on the Chicago team of that time period, refused to play against Walker's team if Walker was on the field. And the resulting attention to the issue eventually led to Blacks being excluded from playing for both major league teams and affiliated minor league teams by the end of the 1880s. There was no official written policy, but rather the prohibition on Black players seemed to be enforced through a tacit understanding among the teams. There was also a vote in 1887 among high minor league teams against allowing new contracts with Black players that eventually led to the complete disappearance of Blacks from both high minor and low minor league

[12]See Constantino (2021) for a history of Black baseball players in professional baseball, and Kleinknecht (1977) for a discussion specifically focused on the history of Black players in professional baseball in the nineteenth century.

teams soon afterwards.

With Blacks being denied the opportunity to play in both major league and minor league baseball, in the early part of the twentieth century numerous Black teams were formed and these teams established their own negro leagues starting around 1920 with stars such as Josh Gibson, Cool Papa Bell, "Smokey" Joe Williams, and Satchel Paige. This situation with successful major and minor league teams that excluded Blacks and Negro league teams consisting of only Black players lasted till the end of World War II.

In 1946 Jackie Robinson signed a contract with the Brooklyn Dodgers and that season he played with the Dodgers affiliated minor league team, the Montreal Royals. He had a successful season for the Royals and the following season he played for the Dodgers where he faced aggravated harassment from rival players, fans, and to an extent his own teammates. Branch Rickey who was the baseball executive who signed him had gotten him to agree not to retaliate when verbally and physically harassed. Robinson stuck to the agreement and persevered in the face of aggravated harassment, and also had an excellent first season winning the Rookie of the Year Award for his on field performance.

Robinson's debut was quickly followed in the same year by the signing of two more Black players to major league contracts – Larry Doby by Cleveland's team and Hank Thompson by the St. Louis Browns. By 1954 twelve of the then sixteen major league teams had signed a Black player, and by 1959 all major league teams had signed at least one Black player. The overall percentage of Black players in the major leagues was also growing rapidly during this time period. It was 0.9% in 1947, 1.7% in 1950, 5.6% in 1954, and 8.8% in 1959. And it continued to grow for many years peaking at 18.7% in 1981.[13] Note that the 8.8% figure for 1959 was only a little below the overall Black percentage in the U.S. population at the time, while the 1981 figure of 18.7% exceeded the overall percentage of Blacks in the U.S. population at that later date according to census statistics.

From the standpoint of our theory, the other interesting aspect of this history concerns the nature of the harassment faced by the early Black baseball players, and how the extent of harassment evolved over time. As already discussed, Jackie Robinson faced aggravated harassment when he first started playing for the Dodgers in 1947, and similar levels of harassment were also faced by the other early Black major league players such as Larry Doby, the first Black player in the American League, who started his career with Cleveland's team just eleven weeks after Robinson's debut with the Dodgers. Like Robinson, Doby persevered in the face of aggravated harassment and also had an outstanding career, including seven All Star games and being elected to the Baseball Hall of Fame in 1998.

The harassment of Robinson, Doby, and others did not stop immediately or after a short period of time, but rather there were incidents throughout their careers. But many of the most severe cases of harassment

[13]These statistics were taken from Armour and Levitt (2016).

occurred very early in their major league careers, such as the vicious racist verbal taunting of Robinson by the Philadelphia manager, Ben Chapman which occurred at Ebbets Field in Brooklyn during the first month of the 1947 season – the episode is depicted in the 2013 Hollywood movie, 42, which provides an account of the Robinson and Rickey story. There was also a petition started by Dodger players while Robinson was still on the Montreal team urging the Dodgers not to promote him to the majors. According to reports, the petition was quashed by Branch Rickey. Doby similarly faced various forms of aggravated harassment early in his career with the Cleveland team.

Of course, even today Black sports figures are frequently harassed because of their race including, for example, the verbal racial harassment in 2019 of the NBA star Russell Westbrook by a White Utah Jazz fan whose behavior was sufficiently egregious that the fan was subsequently banned from attending Jazz home games. But the harassment faced early on by Robinson, Doby, and others was of a different severity and included frequent harassment by fans, opposing players, and teammates. Also, Robinson, Doby and others frequently had to stay in different hotels and eat at different restaurants than other team members when traveling with their respective teams. In fact, some speculate that in the case of Robinson the stress caused by the aggravated harassment helped contribute to his early death at the age of 53.

There are many reasons that harassment was so severe early on for Robinson, Doby, and others, and became less severe over time. Our argument/theory is that part of this dynamic is related to the goals of the harassment. That is, early on one of the goals was to stop integration – a clear example is the Dodgers' player petition referred to above. And when it became evident that this goal would not be achieved, as must have been clear within a year or two of Robinson's debut with the Dodgers, the harassment dissipated to an extent which served to speed up the integration process.

7.2 Women Sports Reporters and Broadcasters

Sports reporting and broadcasting were initially occupations dominated by men. In the 1940s, 1950s and earlier there few women in these occupations.[14] Starting in the 1960s and 1970s the situation started to change with pioneering women such as Jane Chastain, Melissa Ludtke, Jeanne Morris, and Lesley Visser. The situation in the 1960s, 1970s, and 1980s is quite different than the situation today. Although female sports reporters and broadcasters are still in the minority, the percentages are much higher and numerous women working in these fields today are very popular and highly respected. This includes Erin Andrews who is a sideline reporter for the NFL on FOX, Hannah Storm who among other roles has worked as an anchor on ESPN's SportsCenter, Lisa Salters who is currently a sideline reporter for ABC's coverage of the

[14]See Lannin (2022) for a recent discussion of the history of women in sports journalism.

NBA, and Doris Burke who does NBA and college basketball announcing for ESPN and ABC.

There are various factors for why women have become more prominent in these professions over time. For example, the enactment of *Title IX* in 1972 which prohibited sex-based discrimination in schools and educational programs that receive federal funding led to a large increase in female sports programs at the college level, which, in turn, led to increases in the number of women interested in sports journalism and sports reporting.

There were also various court cases that served to even the playing field for women in these professions. Probably the best known is a 1978 court decision in a case brought by the *Sports Illustrated* reporter Melissa Ludtke against Major League Baseball concerning her not being given access to the Yankee Stadium clubhouses during the 1977 World Series. After the ruling in favor of Ludtke, Major League Baseball and other sports leagues gave female reporters much better access to male athlete clubhouses and locker rooms. This eliminated a competitive disadvantage faced by women in the profession.

Another factor consistent with our argument is that there has been a substantial decrease over time in the severity and frequency of harassment of women in these professions. Many of the early female sports reporters and sportscasters faced aggravated harassment. There are numerous reports of sexual harassment and other forms of harassment by players, fans, and coaches. As reported in *Sports Illustrated*, a particularly egregious case that led the reporter to leave the profession concerned a female reporter for the *Boston Herald* who at the time of the harassment covered the New England Patriots football team and the Boston Bruins hockey team. The following quote taken from the *Sports Illustrated* article describes some of the aggravated harassment she was subject to.[15]

> "People spit on her head from the luxury boxes, the only seats in the Garden located above the hockey press box. She started to wear a hat for protection. Someone spray-painted... on the front of her apartment house. Her tires were slashed. The letters continued. She changed her phone number again and again. In Hartford, a group of male fans seated in the front of the press box chanted all night at her..."
>
> –Montville (1991)

Another egregious example of aggravated harassment concerns an incident between Dave Kingman, a star baseball player for the Oakland Athletics in the 1980s, and Susan Fornoff who covered the A's as a sports reporter for the *Sacramento Bee* (see Fornoff (1993) for a description of her experiences as a sports reporter during this time period). In addition to sometimes refusing to give interviews when Fornoff was in the group of reporters, he at one point sent Fornoff a live rat while she was working in the press box at one

[15]We have left out of the quote terms that we regard as offensive.

of the A's games. And according to Gammons (1986), Fornoff was not the only female sports reporter that Kingman harassed. Of interest, Kingman was fined $3,500 for the rat incident, which probably had little deterrent effect given his high compensation as a star baseball player at the time.

Of course, there is still harassment of female sports reporters and broadcasters today as discussed, for example, in Mazel (2021). But there is evidence suggesting that the extent of harassment is much less than was faced by the early women in these professions. For example, Swanson (2009) interviewed a number of prominent female sports reporter in 2009 and concluded that significant progress had been made. Also, of interest from the standpoint of our theory, is that many of the women interviewed by Swanson pointed to the early women in the field and their willingness to persevere in the face of aggravated harassment as an important driver of this progress. For example, one of the interviewees, Laura Behnke who worked at NBC Sports at the time, told Swanson the following.

> "What the first women who went into locker rooms had to deal with was horrible, but those women stuck it out. Even when it was awful, even when players were flashing them, even when coaches were trying to kick them out of locker rooms, they didn't go anywhere. That laid the groundwork for us. I mean, I am so thankful to all of those women that were doing this 15 to 20, even 30 years ago in some cases."
>
> –Swanson (2009), p.53

We are not arguing that the decrease in the severity of harassment in these occupations is the only factor behind the substantial increase in female participation over time. But our reading of the evidence indicates that this is an important part of the story.

7.3 Women in Firefighting

In contrast to our first two applications in which over time harassment reduced and minority or female participation substantially increased, our third application does not have as positive a history. Specifically, there is a history of substantial harassment of various sorts of women in firefighting and limited female participation. And harassment seems to have reduced little over time and the percentage of female firefighters remains very low.[16]

The first female career firefighters were hired in the 1970s. Sandra Forcier was hired in North Carolina in 1973 as a Public Safety Officer, which was a combination of police officer and firefighter, while Judith Livers was hired as a career firefighter in 1974 in Arlington, Virginia. The numbers have grown from that early start, but the growth has been slow and harassment seems to be an important factor.

[16]See Moseley (2017) for a discussion of the history of women in firefighting.

According to Hulett, Bendick Jr, Thomas, and Moccio (2008a,b), in the 2000 census, of the 350,000 career firefighters only 3.7% were female. This figure placed firefighting towards the bottom of all occupations in terms of the percentage of employees that were women. The authors of that study also developed a benchmark using the 2000 census for expected female representation in firefighting based on representation in other occupations that resemble firefighting in terms of requirements concerning strength, stamina, etc. According to this benchmark, the expected female representation among firefighters was 17% which far exceeds the actual percentage which as indicated was 3.7. They also found that the actual percentage of female firefighters varied substantially across locations. In a number of metropolitan areas such as Minneapolis, Madison, and San Francisco, representation was close to the 17% expected level of representation. But in many other locations the percentage was zero or close to zero —- in fact, they found that over half of the 291 metropolitan areas in the 2000 census had no paid women firefighters.

Hulett, Bendick Jr, Thomas, and Moccio (2008a,b) along with other sources such as Banks (2006) make it clear that harassment of various sorts was an important factor behind the low female representation in firefighting found in various locations in the 2000 census. Hulett, Bendick Jr, Thomas, and Moccio (2008b) report results from surveys and interviews which show that discrimination and harassment was common among women firefighters. In their survey, 84.7% of women firefighters reported being treated differently due to their gender, 50.8% reported being shunned/isolated, 42.9% reported being subject to verbal harassment, etc. Banks (2006) reports details concerning various incidents of harassment of female firefighters in the Los Angeles Fire Department which had a very low percentage of female firefighters at the time — 2.6%.

More recent articles and studies indicate that improvement over the last 15 to 20 years has been limited. Fahy, Evarts, and Stein (2022) report that in 2020 approximately 5% of career firefighters were women, while numerous sources such as Hom, Stanley, Spencer-Thomas, and Joiner (2017), Perket and McQueeney (2022), Tracy and Greene (2023) report that harassment of women in various forms is still an important concern in this industry.

One interesting aspect of our theory is that it is consistent with the wide variation in female representation in this occupation across locations. In the baseball case, learning is mostly at the national level. In particular, Jackie Robinson's breaking of the color barrier was reported in all the major newspapers across the country, so when that partnership survived it had a positive impact on the willingness of all the other teams to hire Black baseball players. In contrast, learning in firefighting is likely more local. And our theory predicts that in an occupation where learning is at the local level there can be wide variation across local labor markets concerning whether the market exhibits integration, segregation, or something in between.

8 Conclusion

What determines whether a labor market is integrated or segregated given the presence of a minority group that some individuals in the economy are bigoted against? This question has received substantial prior attention in the economics literature, but previous theories have ignored the role played by the harassment of early integration pathbreakers such as Jackie Robinson and Larry Doby in the case of Major League Baseball. We construct and analyze a model in which early integration pathbreakers are potentially harassed by bigoted individuals in the economy, and then derive the role that this type of harassment plays in labor market dynamics concerning whether the market exhibits integration, segregation, or something in between.

In our model, there is a third-party individual who is bigoted and can harass the hiring firm and minority worker when a minority worker is hired, and the hiring firm and minority worker choose whether or not to dissolve the partnership after the harassment level is chosen and the level of harm is realized. Our main result is that labor market dynamics exhibit path dependence, where the path dependence is driven by a learning process. When an integration partnership is formed and aggravated harassment is chosen, if the extreme level of harm is realized, then the partnership dissolves and market participants come to believe that an aggravated level of harassment has a higher likelihood of resulting in extreme harm. The result is that a partnership is less likely to form in subsequent periods, and it is even possible that the labor market permanently moves into a state of full segregation. In contrast, if the partnership survives, then beliefs change consistent with a lower probability that aggravated harassment results in extreme harm. In this case, partnerships are more likely to form in subsequent periods and it is possible that the labor market moves into a state of permanent integration characterized by either no discrimination or a limited amount of discrimination.

In addition to our main result showing that incorporating harassment into a model of labor market discrimination yields a role for path dependence, we derive a number of related results. For example, we show that altruism on the part of early integration pathbreakers towards future minority workers can be important for the labor market to successfully transition from a situation characterized by segregation to one characterized by integration. This result provides a potential explanation for the movement towards integration in many labor markets after World War II when returning Black veterans had changed attitudes concerning their role in society. We also consider a variant of the model in which harms are not publicly observed, and learning to an extent is based on whether partnerships that are formed survive or dissolve. Our analysis of this case indicates that dynamics can be similar even when realized harm levels are not publicly observable.

There are numerous directions in which the analysis in this paper could be extended. One direction we

feel is of particular interest is focusing on markets other than labor markets. The example of the integration of Major League Baseball starting with Jackie Robinson is consistent with the focus in our formal theoretical analysis on the dynamics of integration and segregation in a labor market setting. But there are important examples of aggravated harassment in other types of settings such as the case of Autherine Lucy who was the first Black student at the University of Alabama, and the recent behavior by protesters towards Anheuser-Busch and Target related to marketing campaigns conducted by these firms aimed at selling to the LGBTQ community. We believe our theoretical approach concerning learning, harassment, and path dependence can potentially provide insights concerning the operation of these other types of markets, and we plan to investigate this in future research.

Appendix: Proofs

Proof of Lemma 1

Proof. For $p_t \leqslant p^I$, if w_t satisfies both (2) and (3) evaluated at $d_t = \underline{d}$, then the third-party individual harasses at intermediate level because $-c + (-b)(1 - \xi(p_t)) \leqslant -b$. If w_t violates either (2) or (3) evaluated at $d_t = \underline{d}$, then the partnership will dissolve under any harm realization. Knowing this, the minority worker only considers offers $w_t \in \left[\beta\underline{d} + a_0\theta_t - d_W,\ a_1\theta_t - (1-\beta)\underline{d} - (a_1 - a_0)\theta^\circ + d_F\right]$. Meanwhile, the minority worker's expected utility from an offer in this interval is $w_t - \beta\underline{d} \geqslant a_0\theta_t$, so that $w_t \geqslant \beta\underline{d} + a_0\theta_t$. Since $\beta\underline{d} + a_0\theta_t > \beta\underline{d} + a_0\theta_t - d_W$, the conditions for offer acceptance are combined into $w_t \in \mathcal{W}(p_t, \theta_t)$.

For $p_t > p^I$, if w_t satisfies both (2) and (3) evaluated at $d_t = \underline{d}$, then the third-party individual harasses at aggravated level because $-c + (-b)(1 - \xi(p_t)) > -b$. If w_t violates either (2) or (3) evaluated at $d_t = \underline{d}$, then the partnership will dissolve under any harm realization. Knowing this, the minority worker only considers offers $w_t \in \left[\beta\underline{d} + a_0\theta_t - d_W,\ a_1\theta_t - (1-\beta)\underline{d} - (a_1 - a_0)\theta^\circ + d_F\right]$. Meanwhile, the minority worker's expected utility from an offer in this interval is $(w_t - \beta\underline{d})(1 - \xi(p_t)) + (a_0\theta_t - d_W)\xi(p_t) \geqslant a_0\theta_t$, so that $w_t \geqslant \beta\underline{d} + a_0\theta_t + \frac{d_W\xi(p_t)}{1-\xi(p_t)}$. Since $\beta\underline{d} + a_0\theta_t + \frac{d_W\xi(p_t)}{1-\xi(p_t)} > \beta\underline{d} + a_0\theta_t - d_W$, the conditions for offer acceptance are combined into $w_t \in \mathcal{W}(p_t, \theta_t)$. ■

Proof of Proposition 1

Proof. (i) Suppose $\lambda = L$ is commonly known. If the firm hires k majority workers, it optimally sets offers at $a_0\theta^\circ$ and earns stage profit $k(a_1 - a_0)\theta^\circ$. Alternatively, if it hires the minority worker of ability θ_t and $k-1$ majority workers, then it pays each majority worker $a_0\theta^\circ$ and the minority worker $\min \mathcal{W}(0, \theta_t) = \beta\underline{d} + a_0\theta_t$. According to Lemma 1, the third party harasses at intermediate level, which does not dissolve the partnership. The firm's stage profit is $(k-1)(a_1 - a_0)\theta^\circ + a_1\theta - (\beta\underline{d} + a_0\theta_t) - (1-\beta)\underline{d}$. In order for the firm to hire the minority worker, a necessary condition is $\theta_t \geqslant \theta_1^*(0)$. For sufficiency, we also verify that for all $\theta_t \geqslant \theta_1^*(0)$, $\mathcal{W}(0, \theta_t)$ is non-empty.

(ii) Suppose $\lambda = H$ is commonly known. If the firm hires k majority workers, it optimally sets offers at $a_0\theta^\circ$ and earns stage profit $k(a_1 - a_0)\theta^\circ$. Alternatively, if it hires the minority worker of ability θ_t and $k-1$ majority workers, then it pays each majority worker $a_0\theta^\circ$ and the minority worker $\min \mathcal{W}(0, \theta_t) = \beta\underline{d} + a_0\theta_t + \frac{d_W\xi(p_t)}{1-\xi(p_t)}$. According to Lemma 1, the third party harasses at aggravated level, which dissolves the partnership with probability $\xi(1) = q^H$. The firm's stage profit is $(k-1)(a_1 - a_0)\theta^\circ + (a_1\theta - (\beta\underline{d} + a_0\theta_t + \frac{d_W q^H}{1-q^H}) - (1-\beta)\underline{d})(1-q^H) + ((a_1 - a_0)\theta^\circ - d_F)q^H$. Because $\overline{\theta} < \theta^\circ + \frac{d_F + d_W}{a_1 - a_0}\frac{q^H}{1-q^H}$, hiring the minority worker is not optimal for the firm. ■

Proof of Proposition 2

Proof. There is at least one majority worker who does not receive any offer at least $a_0\theta^\circ$, because otherwise at least one firm ends up hiring more than k workers and can improve by deleting an offer. This also means the wage of any employed majority worker must be exactly $a_0\theta^\circ$.

(i) Suppose $\lambda = L$ is commonly known. If $\theta_t < \theta_N^*(0)$, then in equilibrium no firm hires the minority worker because at wage $w_t = \min \mathcal{W}(0, \theta_t)$ the firm earns $(k-1)a_1\theta^\circ + a_1\theta_t - w_t - (1-\beta)\underline{d} < (a_1 - a_0)\theta^\circ$. If $\theta_t \geqslant \theta_N^*(0)$, then the minority worker must be hired by one firm because otherwise a firm can deviate and offer $w_t = \min \mathcal{W}(0, \theta_t)$ to the minority worker and replace one majority worker, and make a profit (when indifferent, Assumption 2 ensures an offer to the minority worker). According to Lemma 1, the third party harasses at intermediate level, which does not dissolve the partnership. The minority worker's wage must be $a_1\theta_t - (1-\beta)\underline{d} - (a_1 - a_0)\theta^\circ$ because the hiring firm is indifferent at this wage. If the wage is any lower, another firm can outbid this firm and replace one majority worker.

(ii) Suppose $\lambda = H$ is commonly known. If the minority worker is hired by one firm, then the third-party individual harasses at aggravated level, which dissolves the partnership with probability $\xi(1) = q^H$. Even at the lowest acceptable wage $w_t = \min \mathcal{W}(0, \theta_t)$, that firm's stage profit is $(k-1)(a_1 - a_0)\theta^\circ + (a_1\theta - w_t - (1-\beta)\underline{d})(1-q^H) + ((a_1 - a_0)\theta^\circ - d_F)q^H$. Because $\overline{\theta} < \theta^\circ + \frac{d_F + d_W}{a_1 - a_0}\frac{q^H}{1-q^H}$, hiring the minority worker is not optimal for this firm. ■

Proof of Lemma 2

Proof. (i) If there is no harassment or it is at intermediate level, then $d_t = \overline{d}$ never arises. Therefore, there is no update in belief.

(ii) If the realized harm is $\overline{d}$ following aggravated harassment, then Bayes' rule gives

$$p_{t+1} = \Pr(\lambda = H | p_t, \overline{d}) = \frac{\Pr(\overline{d} | \lambda = H, p_t)\Pr(\lambda = H | p_t)}{\Pr(\overline{d} | p_t)} = \frac{q^H p_t}{q^H p_t + q^L(1-p_t)} = P_+(p_t).$$

(iii) If the realized harm is $\underline{d}$ following aggravated harassment, then Bayes' rule gives

$$p_{t+1} = \Pr(\lambda = H | p_t, \underline{d}) = \frac{\Pr(\underline{d} | \lambda = H, p_t)\Pr(\lambda = H | p_t)}{\Pr(\underline{d} | p_t)} = \frac{(1-q^H)p_t}{(1-q^H)p_t + (1-q^L)(1-p_t)} = P_-(p_t).$$

■

Proof of Proposition 3

Proof. Define $v^\circ \equiv k(a_1 - a_0)\theta^\circ$ as the default profit of a firm when it hires k majority workers. Define $V_N(p)$ as the value function of one of the N firms in a Markov equilibrium in excess of the default profit v° when the belief is p, before the worker's ability is drawn.

(i). If $p < p^I$, the third party individual harasses the partnership at intermediate level provided $w_t \in \mathcal{W}(p_t, a_t)$, and there will be no learning. With Markov strategies, the firm(s) maximize stage profit. With the same reasoning as in the proofs of Proposition 1 and Proposition 2, the minority worker is hired if and only if $\theta_t \geqslant \theta_N^*(0)$. The wage is $w_t = \min \mathcal{W}(p_t, \theta_t) = \beta\underline{d} + a_0\theta_t$ if $N = 1$, and is $w_t = a_1\theta_t - (1-\beta)\underline{d} - (a_1 - a_0)\theta^\circ \in \mathcal{W}(p_t, \theta_t)$ if $N > 1$. Therefore, for $N = 1$ the value function at $p < p^I$ is:

$$V_1(p) = \overline{V} \equiv (a_1 - a_0)\int_{\theta_1^*(0)}^{\overline{\theta}} (\theta - \theta_1^*(0))\mathrm{d}F(\theta).$$

For $N > 1$, on the other hand, the wage competes away all excess profit from the hiring firm, so that $V_N(p) = v^\circ$.

(ii) and (iii). For $p \geqslant p^I$, the third party individual harasses the partnership at aggravated level provided $w_t \in \mathcal{W}(p_t, a_t)$.

First consider $N = 1$. The Bellman equation reads:

$$V_1(p) = \begin{cases} \overline{V} & \text{if } p < p^I, \\ \mathbb{E}\Big[\max\Big\{\delta V_1(p),\ \max\limits_{w\in\mathcal{W}(p,\theta)} \xi(p)(-d_F(1-\delta)+\delta V_1(P_+(p))) \\ \quad + (1-\xi(p))\Big((a_1\theta - w - (1-\beta)\underline{d} - (a_1-a_0)\theta^\circ)(1-\delta) + \delta V_1(P_-(p))\Big)\Big\}\Big] & \text{if } p \geqslant p^I. \end{cases} \tag{6}$$

Note that $V_1(p)$ is bounded for all $p \in [0,1]$. On one hand, $V_1(p) \leqslant \overline{V}$ because $\overline{V}$ is the highest possible stage profit. On the other hand, $V_1(p) \geqslant 0$ as the firm can guarantee default profit by only hiring majority workers. Define operator T as:

$$Tf(p) \equiv \begin{cases} \overline{V} & \text{if } p < p^I, \\ \mathbb{E}\Big[\max\Big\{\delta f(p),\ \max\limits_{w\in\mathcal{W}(p,\theta)} \xi(p)(-d_F(1-\delta)+\delta f(P_+(p))) \\ \quad + (1-\xi(p))\Big((a_1\theta - w - (1-\beta)\underline{d} - (a_1-a_0)\theta^\circ)(1-\delta) + \delta f(P_-(p))\Big)\Big\}\Big] & \text{if } p \geqslant p^I, \end{cases} \tag{7}$$

so that $TV_1(p)$ is also bounded. We can verify Blackwell's sufficient conditions for T to be a contraction: for any $\hat{f} \geqslant f$, $T\hat{f} \geqslant Tf$; and for any $\alpha > 0$, $T(f+\alpha)(p) \leqslant Tf(p) + \delta\alpha$. Therefore, V_1 as the unique fixed point of T exists by Contraction mapping theorem. If $f \leqslant \overline{V}$, then for any $p \geqslant p^I$, we have $\delta f \leqslant \delta\overline{V}$ and $\xi(p)(-d_F(1-\delta)+\delta f(P_+(p)))+(1-\xi(p))((a_1\theta-w-(1-\beta)\underline{d}-(a_1-a_0)\theta^\circ)(1-\delta)+\delta f(P_-(p))) < \overline{V}(1-\delta)+\delta\overline{V}$, so that $Tf(p) < \overline{V}$. If f is decreasing, then for any $\hat{p} \geqslant p^I > p$, we have $Tf(\hat{p}) \leqslant Tf(p) = \overline{V}$. If otherwise:

$$\begin{aligned} &\max_{w\in\mathcal{W}(\hat{p},a)} \xi(\hat{p})(-d_F(1-\delta)+\delta f(P_+(\hat{p}))) + (1-\xi(\hat{p}))((\theta - w - (1-\beta)\underline{d} - (a_1-a_0)\theta^\circ)(1-\delta) + \delta f(P_-(\hat{p}))) \\ &\leqslant \max_{w\in\mathcal{W}(p,\theta)} \xi(\hat{p})(-d_F(1-\delta)+\delta f(P_+(p))) + (1-\xi(\hat{p}))((\theta - w - (1-\beta)\underline{d} - (a_1-a_0)\theta^\circ)(1-\delta) + \delta f(P_-(p))) \\ &\leqslant \max_{w\in\mathcal{W}(p,\theta)} \xi(p)(-d_F(1-\delta)+\delta f(P_+(p))) + (1-\xi(p))((\theta - w - (1-\beta)\underline{d} - (a_1-a_0)\theta^\circ)(1-\delta) + \delta f(P_-(p))), \end{aligned}$$

which again implies $Tf(\hat{p}) \leqslant Tf(p)$. Therefore $V_1 = T^\infty 0$ is decreasing.

For $p \geqslant p^I$, a minority worker of ability θ is hired if and only if $\mathcal{W}(p,\theta) \neq \emptyset$ and $\xi(p)(-d_F(1-\delta) + \delta V_1(P_+(p))) + (1-\xi(p))(((a_1-a_0)(\theta-\theta^\circ) - \underline{d} - \frac{d_W\xi(p)}{1-\xi(p)})(1-\delta) + \delta V_1(P_-(p))) \geqslant \delta V_1(p)$. This reduces to:

$$\begin{aligned} \theta \geqslant \theta_1^*(p) &\equiv \max\{\theta_1^\dagger(p), \theta_1^\ddagger(p)\}, \quad \text{where} \\ \theta_1^\dagger(p) &\equiv \theta^\circ + \frac{\underline{d} - d_F}{a_1 - a_0} + \frac{d_W}{a_1-a_0}\frac{\xi(p)}{1-\xi(p)}, \\ \theta_1^\ddagger(p) &\equiv \theta^\circ + \frac{\underline{d}}{a_1-a_0} + \frac{d_W + d_F}{a_1-a_0}\frac{\xi(p)}{1-\xi(p)} - \frac{\delta I_1(p)}{(1-\delta)(1-\xi(p))(a_1-a_0)}, \\ I_1(p) &\equiv \xi(p)V_1(P_+(p)) + (1-\xi(p))V_1(P_-(p)) - V_1(p). \end{aligned} \tag{8}$$

Since V_1 is monotone, $\lim_{p\to 1} V_1(p) = \lim_{p\to 1} V_1(P_+(p)) = \lim_{p\to 1} V_1(P_-(p))$ exists, and from (6), V_1 is continuous at $p = 1$. Therefore, we know both I_1 and θ_1^* are continuous at $p = 1$, $\lim_{p\to 1} I_1(p) = I_1(1) = 0$ and $\lim_{p\to 1}\theta_1^*(p) = \theta_1^*(1) = \theta_1^\ddagger(1) = \theta^\circ + \frac{\underline{d}}{a_1-a_0} + \frac{d_W+d_F}{a_1-a_0}\frac{q^H}{1-q^H}$. Since $\overline{\theta} < \theta^\circ + \frac{d_W+d_F}{a_1-a_0}\frac{q^H}{1-q^H} < \theta_1^*(1)$, we define $p_1^S \equiv \inf\{\hat{p} \in [p^I, 1) : \theta_1^*(p) \geqslant \overline{\theta}, \forall\ p > \hat{p}\} < 1$.

When $p \geqslant p_1^S$, we have $\theta_1^*(p) \geqslant \overline{\theta}$ and the minority worker is never hired. There is no harassment and the belief is stuck at p. The firm hires k majority workers at wage $a_0\theta^\circ$ and earns default profit v° from now on.

When $p \in [p^I, p_1^S)$, we must have $\theta_1^*(p) < \overline{\theta}$. This is because if $\theta_1^*(p) \geqslant \overline{\theta}$ for some $p \in [p^I, p_1^S)$, learning is stuck and $V_1(p) = 0$. By definition of p_1^S, we must have some $\hat{p} \in (p, p_1^S)$ such that $\theta_1^*(\hat{p}) < \overline{\theta}$. This means $V_1(\hat{p}) > \delta V_1(\hat{p})$, i.e., $V_1(\hat{p}) > 0 = V_1(p)$, violating monotonicity. The minority worker's wage is $w = \min\mathcal{W}(p,\theta) = \beta\underline{d} + a_0\theta + \frac{d_W\xi(p)}{1-\xi(p)}$. The remainder of (iii) is implied by Lemma 1.

Next consider $N > 1$. Suppose towards contradiction that $V_N(p) > 0$ for some $p \geqslant p^I$. Then starting

from $p_0 = p$, there must exist a history with belief $\hat{p}$ and realized ability θ of the minority worker such that the firm can earn stage profit strictly higher than v°. That is, the wage w the minority worker accepts from this firm must satisfy $w < a_1\theta - (1-\beta)\underline{d} - (a_1 - a_0)\theta^\circ - \frac{d_F\xi(p)}{1-\xi(p)}$, which implies $w < \max \mathcal{W}(p,\theta)$. Then another firm which is earning default profit v° can deviate by offering $w + \varepsilon \in \mathcal{W}(p,\theta)$ to win over the minority worker and earn profit strictly higher than v°. Such a deviation does not affect learning because the third-party individual will choose aggravated harassment in either case. In a Markov equilibrium, the deviating firm's continuation value does not change, while its stage profit is improved.

Combining the above with (i), $V_N(p) = 0$ for all $p \in [0,1]$ when $N > 1$. For $p < p^I$, with logic similar to the proof of Proposition 2, we know the minority worker is hired if and only if $\theta \geqslant \theta_1^*(0)$. Moreover, at least two firms will compete the wage up to $a_1\theta - (1-\beta)\underline{d} - (a_1 - a_0)\theta^\circ$ so that they earn no excess profit.

For $p \geqslant p^I$, zero excess profit of the firms requires $w = a_1\theta - (1-\beta)\underline{d} - (a_1 - a_0)\theta^\circ - \frac{d_F\xi(p)}{1-\xi(p)}$, then $w \in \mathcal{W}(p,\theta)$ in turn requires $\theta \geqslant \theta_N^*(p) \equiv \theta^\circ + \frac{\underline{d}}{a_1-a_0} + \frac{d_F+d_W}{a_1-a_0}\frac{\xi(p)}{1-\xi(p)}$. Since $\theta_N^*(1) > \overline{\theta}$, let $p_N^S \in [p^I, 1)$ be the unique solution to $\theta_N^*(p_N^S) = \overline{\theta}$ if it exists, and $p_2^S = p^I$ otherwise. Then for all $p \geqslant p_N^S$, $\theta_N^*(p) \geqslant \overline{a}$, leading to hiring freeze for minority workers and no learning. For all $p \in [p^I, p_N^S)$, $\theta_N^*(p) < \overline{\theta}$, so that minority workers are still hired with positive probability and learning continues. ■

Proof of Corollary 1

Proof. (i). Since

$$\mathbb{E}p_{t+1} = (1 - \Pr(\text{Aggravated}))p_t + \Pr(\text{Aggravated})(\xi(p_t)P_+(p_t) + (1-\xi(p_t))P_-(p_t)) = p_t,$$

p_t is a bounded martingale. By Martingale convergence theorem, p_∞ exists almost surely. According to the equilibrium behavior, $p_\infty \in [p^I, p_N^S)$ with probability zero. Since $\mathbb{E}p_\infty = p_0 \in [p^I, p_N^S)$, it must be the case $\Pr(p_\infty < p^I) > 0$ and $\Pr(p_\infty \geqslant p_N^S) > 0$.

(ii) and (iii). First consider $N = 1$. A bounded function f on $[0,1]$ has *order property* if for any $p \geqslant p^I$, $\frac{f(P_+(p))}{1-\xi(P_+(p))} \leqslant \frac{f(p)}{1-\xi(p)} \leqslant \frac{f(P_-(p))}{1-\xi(P_-(p))}$. Using T defined in (7):

$$\frac{Tf(p)}{1-\xi(p)} = \begin{cases} \frac{\overline{V}}{1-\xi(p)} & \text{if } p < p^I, \\ \mathbb{E}\Big[\max\Big\{\frac{\delta f(p)}{1-\xi(p)}, \max\limits_{w\in\mathcal{W}(p,\theta)}(1-\delta)\Big(a_1\theta - w - (1-\beta)\underline{d} - (a_1-a_0)\theta^\circ - \frac{d_F\xi(p)}{1-\xi(p)}\Big) \\ \quad + \delta\xi(P_-(p))\frac{f(P_+(p))}{1-\xi(p)} + \delta(1-\xi(P_-(p)))\frac{f(P_-(p))}{1-\xi(p)}\Big\}\Big] & \text{if } p \geqslant p^I. \end{cases}$$

If f has order property, then for any $p \geqslant p^I$, the above equation implies $\frac{Tf(P_+(p))}{1-\xi(P_+(p))} \leqslant \frac{Tf(p)}{1-\xi(p)}$, and also $\frac{Tf(P_-(p))}{1-\xi(P_-(p))} \geqslant \frac{Tf(p)}{1-\xi(p)}$ if $P_-(p) \geqslant p^I$. If $P_-(p) < p^I$, then $\frac{Tf(P_-(p))}{1-\xi(P_-(p))} = \frac{\overline{V}}{1-\xi(P_-(p))} > \frac{\overline{V}}{1-\xi(P_-(p))}(1-(1-\delta)\xi(P_-(p))) = \frac{Tf(p)}{1-\xi(p)}$. In sum, the order property is preserved under T, and hence $V_1 = T^\infty 0$ also has the order property. Moreover, for p in the learning region where the worker is hired with positive probability, $\frac{V_1(P_+(p))}{1-\xi(P_+(p))} = \frac{TV_1(P_+(p))}{1-\xi(P_+(p))} < \frac{TV_1(p)}{1-\xi(p)} = \frac{V_1(p)}{1-\xi(p)}$ and $\frac{V_1(P_-(p))}{1-\xi(P_-(p))} = \frac{TV_1(P_-(p))}{1-\xi(P_-(p))} > \frac{TV_1(p)}{1-\xi(p)} = \frac{V_1(p)}{1-\xi(p)}$.

From (8), $\theta_1^*(p) \equiv \max\{\theta_1^\dagger(p), \theta_1^\ddagger(p)\}$. Note that $\theta_1^\dagger$ is strictly increasing. From (6) we know for all $p \geqslant p^I$, $\int_{\theta_1^*(p)}^{\overline{\theta}}(\theta - \theta_1^*(p))\mathrm{d}F(\theta) \leqslant \frac{V_1(p)}{(1-\xi(p))(a_1-a_0)} \leqslant \int_{\theta_1^\ddagger(p)}^{\overline{\theta}}(\theta - \theta_1^\ddagger(p))\mathrm{d}F(\theta)$. Therefore, for any p in the learning region, $\int_{\theta_1^*(P_+(p))}^{\overline{\theta}}(\theta - \theta_1^*(P_+(p)))\mathrm{d}F(\theta) \leqslant \frac{V_1(P_+(p))}{(1-\xi(P_+(p)))(a_1-a_0)} < \frac{V_1(p)}{1-\xi(p)} \leqslant \int_{\theta_1^\ddagger(p)}^{\overline{\theta}}(\theta - \theta_1^\ddagger(p))\mathrm{d}F(\theta)$, so that $\theta_1^\ddagger(p) < \theta_1^*(P_+(p))$. Because $\theta_1^\dagger(p) < \theta_1^\dagger(P_+(p)) \leqslant \theta_1^*(P_+(p))$, we know $\theta_1^*(p) < \theta_1^*(P_+(p))$. If $P_-(p) \geqslant p^I$, then the above logic also generates $\theta_1^*(P_-(p)) < \theta_1^*(p)$. If $P_-(p) < p^I$, then permanent integration is achieved.

Next consider $N > 1$. From the proof of Proposition 3, $\theta_N^*(p)$ is strictly increasing in p, so that the comparison of p_{t+1} and p_t carries over.

Also, note that for $p \in [p^I, p_N^S)$:

$$\Pr(p_\infty < p^I | p) = \xi(p)\Pr(p_\infty < p^I | P_+(p)) + (1-\xi(p))\Pr(p_\infty < p^I | P_-(p)).$$

Because of the monotonicity of P_+ and P_-, $\{p_\infty < p^I | P_+(p)\} \subset \{p_\infty < p^I | p\} \subset \{p_\infty < p^I | P_-(p)\}$. Meanwhile, $P_-(P_+(p)) > P_-(p)$, so that $\{p_\infty < p^I | P_+(p)\} \neq \{p_\infty < p^I | P_-(p)\}$, and $\Pr(p_\infty < p^I | P_+(p)) < \Pr(p_\infty < p^I | p) < \Pr(p_\infty < p^I | P_-(p))$. ■

Proof of Corollary 2

Proof. (ii). From the proof of Proposition 3, $\theta_N^*(p) = \theta^\circ + \frac{\underline{d}}{a_1 - a_0} + \frac{d_F + d_W}{a_1 - a_0} \frac{\xi(p)}{1 - \xi(p)}$, which is independent of $N \geqslant 2$.

(i). For $p \in [p^I, p_1^S)$, $\theta_2^*(p) - \theta_1^\dagger(p) = \frac{d_F}{(1-\xi(p))(a_1 - a_0)} > 0$. Also, $\theta_2^*(p) - \theta_1^\ddagger(p) = \frac{\delta I_1(p)}{(1-\delta)(1-\xi(p))(a_1 - a_0)}$. From (6) we have:

$$V_1(p) = \mathbb{E}\left[\max\left\{0, \max_{w \in \mathcal{W}(p,\theta)} \xi(p)(-d_F) + (1 - \xi(p))(a_1\theta - w - (1-\beta)\underline{d} - (a_1 - a_0)\theta^\circ) + \frac{\delta I(p)}{1-\delta}\right\}\right].$$

On the other hand, starting from any p in the learning region, consider the Markov strategy: always use the prior p to make myopic decisions, until the true belief falls into the integration region, where the firm switches to $\theta_1^*(0)$. Because the probability of reaching the integration region is positive and the stage profit thereafter is strictly higher than before, we have:

$$V_1(p) > \mathbb{E}\left[\max\left\{0, \max_{w \in \mathcal{W}(p,\theta)} \xi(p)(-d_F) + (1 - \xi(p))(a_1\theta - w - (1-\beta)\underline{d} - (a_1 - a_0)\theta^\circ)\right\}\right].$$

Therefore, $I_1(p) > 0$ on $p \in [p^I, p_1^S)$, and $\theta_2^*(p) - \theta_1^\ddagger(p) > 0$. In sum, $\theta_2^*(p) > \max\{\theta_1^\dagger(p), \theta_1^\ddagger(p)\} = \theta_1^*(p)$.

Moreover, note that $I_1(p_1^S) = (1 - \xi p_1^S) V_1(P_-(p_1^S)) > 0$, so that $\theta_2^*(p_1^S) > \theta_1^*(p_1^S) \geqslant \overline{\theta}$. Due to continuity of $\theta_2^*(\cdot)$, we have $p_2^S < p_1^S$.

(iii). Note that $p_\infty < p^I$ is equivalent to the existence of t such that $p_t < p^I$ and $p_{t'} < p_N^S$ for all $t' < t$. Then by definition, $p_N^S < p_1^S$ implies $\Pr(p_\infty < p^I | N > 1) \leqslant \Pr(p_\infty < p^I | N = 1)$ for all $N > 1$. ■

Proof of Proposition 4

Proof. In a Markov equilibrium with N firms, denote $\tilde{U}_N(p)$ and $\tilde{u}_N(p)$ the ex ante welfare and stage utility, respectively, of a minority worker in excess of her outside option $a_0 \mathbb{E}\theta$ when the public belief is p. Also, denote $\tilde{U}_N(p, \theta)$ and $\tilde{u}_N(p, \theta)$ the interim welfare and stage utility, respectively, of a minority worker with ability θ in excess of her outside option $a_0\theta$ when the public belief is p. Then $\tilde{U}_N(p) = \mathbb{E}\tilde{U}_N(p, \theta)$. Define $v^\circ \equiv k(a_1 - a_0)\theta^\circ$ as the default stage profit of a firm when it hires k majority workers.

(i). Suppose $N = 1$. First, if $p < p^I$, then learning stops and the minority worker's future excess welfare is fixed at $U_{t+1} = \tilde{U}_1(p)$. Therefore, the minority worker accepts a wage only in $\mathcal{W}(p, \theta)$. Knowing this, the firm makes the same offer as is described in Proposition 3, leaving $\tilde{u}_1(p, \theta) = 0$, which also implies $\tilde{U}_1(p) = 0$. Second, if $p \geqslant p^I$, then we must have $\tilde{U}_1(p) \geqslant 0$ because $\tilde{U}_1(p, \theta) \geqslant 0 + \gamma\delta\tilde{U}_1(p)$ implies $\tilde{U}_1(p) \geqslant \gamma\delta\tilde{U}_1(p)$. Suppose towards contradiction that $\sup_{p \geqslant p^I} \tilde{U}_1(p) > 0$. Since $\gamma\delta \in (0, 1)$ and $\tilde{u}_1(p)$ is uniformly bounded for $p \in [p^I, 1]$, so is $\tilde{U}_1(p)$. For any small $\varepsilon > 0$, select $p(\varepsilon) \in [p^I, 1]$ such that $\tilde{U}_1(p(\varepsilon)) > \sup_{p \geqslant p^I} \tilde{u}_1(p) - \varepsilon > 0$. With belief $p(\varepsilon)$, there must exist some θ s.t. $\tilde{U}_1(p(\varepsilon), \theta) > \gamma\delta\tilde{U}_1(p(\varepsilon))$; otherwise $\tilde{U}_1(p(\varepsilon)) \leqslant 0$, a contradiction. The minority worker with this ability θ must be hired at the wage $\beta\underline{d} + a_0\theta_t - d_W$ with ex post IR binding, because otherwise the firm should lower the wage further. Consider the minority worker's welfare improvement when deviating to not accepting the offer: $(1 - \gamma\delta)d_W + \gamma\delta\left(\tilde{U}_1(p(\varepsilon)) - \xi(p(\varepsilon))\tilde{U}_1(P_+(p(\varepsilon))) - (1 - \xi(p(\varepsilon)))\tilde{U}_1(P_-(p(\varepsilon)))\right)$. Taking $\varepsilon \to 0$, this improvement is positive, a contradiction. Therefore, $\tilde{U}_1(p)$ is identically zero, and the minority worker's equilibrium behavior and the firm's wage offering are the same as in Proposition 3.

(ii). When $N > 1$, we use the same argument as in the proof of Proposition 3 to conclude that the firms' interim stage profit always equals their default profit v° for all $p \in [0, 1]$ and all θ. Therefore, at least two firms will compete the wage up to $w = a_1\theta - (1 - \beta)\underline{d} - (a_1 - a_0)\theta^\circ$ if $p < p^I$, and $w =$

$a_1\theta-(1-\beta)\underline{d}-(a_1-a_0)\theta^\circ-\frac{d_F\xi(p)}{1-\xi(p)}$ if $p\geqslant p^I$. Given these wages, $\tilde{U}_N(p)$ satisfies:

$$\tilde{U}_N(p)=\begin{cases}(a_1-a_0)\int_{\theta_1^*(0)}^{\overline{\theta}}(\theta-\theta_1^*(0))\mathrm{d}F(\theta) & \text{if } p<p^I,\\ \mathbb{E}\Big[\max\Big\{\gamma\delta\tilde{U}_N(p),\ \xi(p)(-d_W(1-\gamma\delta)+\gamma\delta\tilde{U}_N(P_+(p)))\\ \quad+(1-\xi(p))\Big((w-\beta\underline{d}-a_0\theta)(1-\gamma\delta)+\gamma\delta\tilde{U}_N(P_-(p))\Big)\Big\}\Big] & \text{if } p\geqslant p^I,\end{cases}\tag{9}$$

where $w=a_1\theta-(1-\beta)\underline{d}-(a_1-a_0)\theta^\circ-\frac{d_F\xi(p)}{1-\xi(p)}$ if $a_1\theta-(1-\beta)\underline{d}-(a_1-a_0)\theta^\circ-\frac{d_F\xi(p)}{1-\xi(p)}\geqslant\beta\underline{d}+a_0\theta-d_W$, and w does not exist otherwise.

For $p\geqslant p^I$, a minority worker of ability θ is hired if and only if $a_1\theta-(1-\beta)\underline{d}-(a_1-a_0)\theta^\circ-\frac{d_F\xi(p)}{1-\xi(p)}\geqslant\beta\underline{d}+a_0\theta-d_W$ and $(1-\xi(p))\left(\left((a_1-a_0)(\theta-\theta^\circ)-\underline{d}-\frac{d_F\xi(p)}{1-\xi(p)}\right)(1-\gamma\delta)+\gamma\delta\tilde{U}_N(P_-(p))\right)+\xi(p)(-d_W(1-\gamma\delta)+\gamma\delta\tilde{U}_N(P_+(p)))\geqslant\gamma\delta\tilde{U}_N(p)$. This reduces to:

$$\begin{aligned}\theta\geqslant\theta_N^*(p) &\equiv \max\{\theta_N^\dagger(p),\theta_N^\ddagger(p)\},\ \text{ where}\\ \theta_N^\dagger(p) &\equiv \theta^\circ+\frac{\underline{d}-d_W}{a_1-a_0}+\frac{d_F}{a_1-a_0}\frac{\xi(p)}{1-\xi(p)},\\ \theta_N^\ddagger(p) &\equiv \theta^\circ+\frac{\underline{d}}{a_1-a_0}+\frac{d_W+d_F}{a_1-a_0}\frac{\xi(p)}{1-\xi(p)}-\frac{\gamma\delta I_1(p)}{(1-\gamma\delta)(1-\xi(p))(a_1-a_0)},\\ I_1(p) &\equiv \xi(p)\tilde{U}_N(P_+(p))+(1-\xi(p))\tilde{U}_N(P_-(p))-\tilde{U}_N(p).\end{aligned}$$

Now it is clear that the recursive problem the minority worker faces here is isomorphic to the one the monopoly firm faces in the proof of Proposition 3, where δ is replaced by $\gamma\delta$ and d_F and d_W are exchanged. Therefore, properties in Proposition 3 carry over. Moreover, Corollary 2 carries over too, so that p_N^S in the altruism case is strictly higher than that in the main model, while $\theta_N^*(p)$ is strictly lower for all p in the learning region. ■

Proof of Corollary 3

Proof. According to Proposition 4, p_N^S is larger in the altruism case. Therefore, the proof of Corollary 2(iii) carries over to imply $\Pr(p_\infty<P^I|\gamma=0)<\Pr(p_\infty<P^I|\gamma>0)$. ■

Proof of Proposition 5

Proof. In any PBE of the game in the unobservable harm case, we claim that all firms must earn zero excess stage profit, as is in the main model. Denote p the (correct) private belief of the firms and the minority worker, and $\hat{p}$ the (potentially incorrect) public belief of the third-party individual. If the third-party individual's belief is such that $\hat{p}<p^I$, then they will harass at intermediate level. If a firm earns positive excess stage profit, then it must be offering a wage $w<a_1\theta-(1-\beta)\underline{d}-(a_1-a_0)\theta^\circ<\max\mathcal{W}(p,\theta)$ and get accepted. However, another firm which earns default profit in equilibrium can deviate to $w+\varepsilon$ and win over the minority worker. This deviation does not change the information set nor the behavior of the third-party individual, and therefore is profitable, a contradiction. If the third-party individual's belief is such that $\hat{p}\geqslant p^I$, then they will harass at aggravated level. If a firm earns positive excess stage profit, then it must be offering a wage $w<a_1\theta-(1-\beta)\underline{d}-(a_1-a_0)\theta^\circ-\frac{d_F\xi(p)}{1-\xi(p)}<\max\mathcal{W}(p,\theta)$ and get accepted. However, another firm which earns default profit in equilibrium can deviate to $w+\varepsilon$ and win over the minority worker. Again, this deviation does not change the information set nor the behavior of the third-party individual, and therefore is profitable, a contradiction.

Therefore, in any PBE, the firms must earn zero excess stage profit in every period. At any history where a firm has to decide whether to keep or dissolve the partnership, the firm only considers within-period tradeoffs because its continuation payoff is fixed. Therefore, the firms' behavior is exactly the same as in the

main model. Given this, the short-lived minority worker has the same behavior as in the main model. Also, since the third-party individual's belief is correct on path, they behave the same as in the main model. ■

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