Interest-rate risk and household portfolios

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This paper

- **Life-cycle portfolio choice model with stochastic interest rates**
  - High interest rates means higher lifetime consumption
  - Households hedge against lower rates by buying long-term assets
  - Older investors care less, so hedging demand decreases with age
  - Human capital and Social Security create substitution effects

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- **Partial equilibrium model: we want to explain the cross-section of long-term asset holdings**

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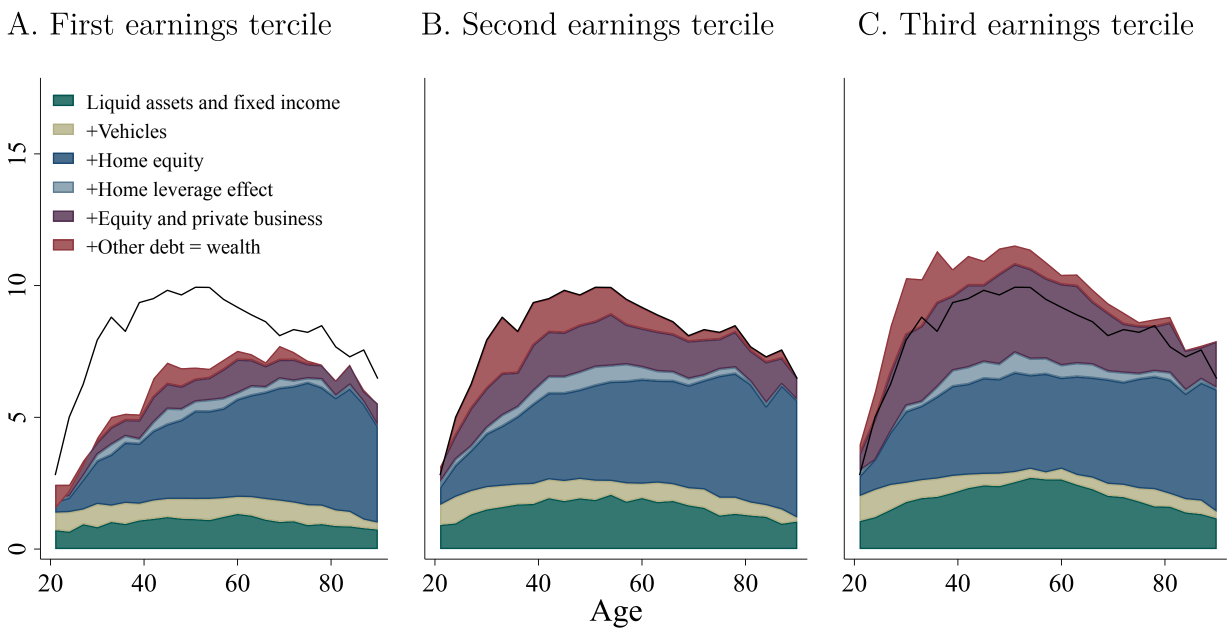
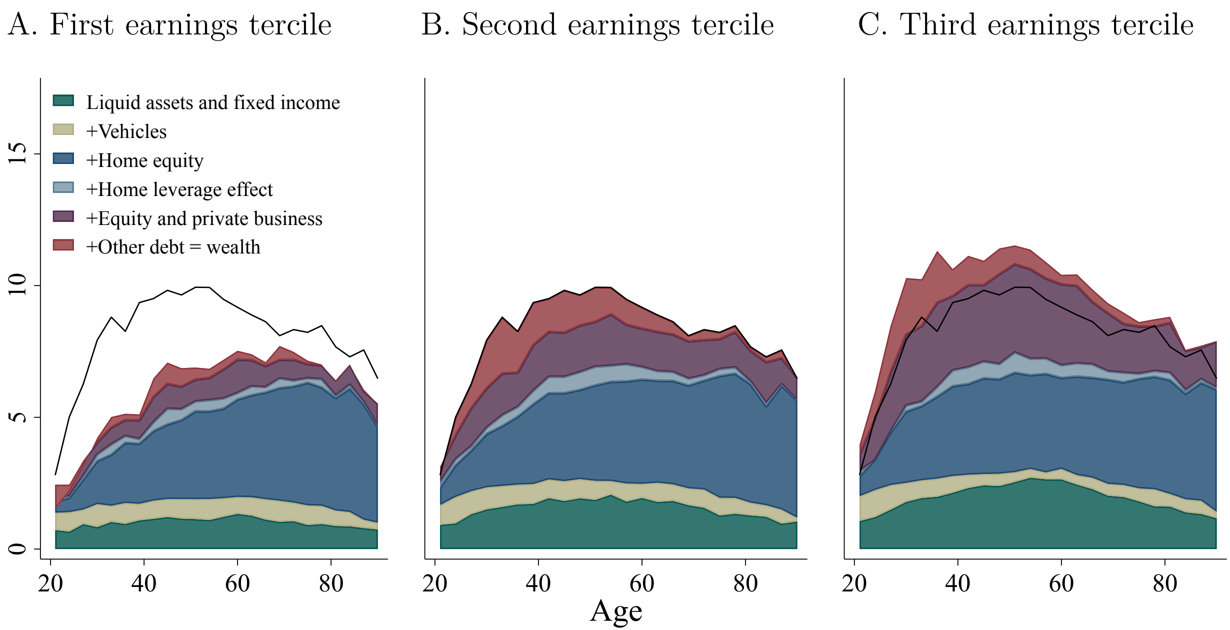
  - Human capital and Social Security create substitution effects
- **Partial equilibrium model: we want to explain the cross-section of long-term asset holdings**
- **Facts explained by the model:**
  - Cross-section of portfolios
  - Wealth inequality dynamics

Stylized facts

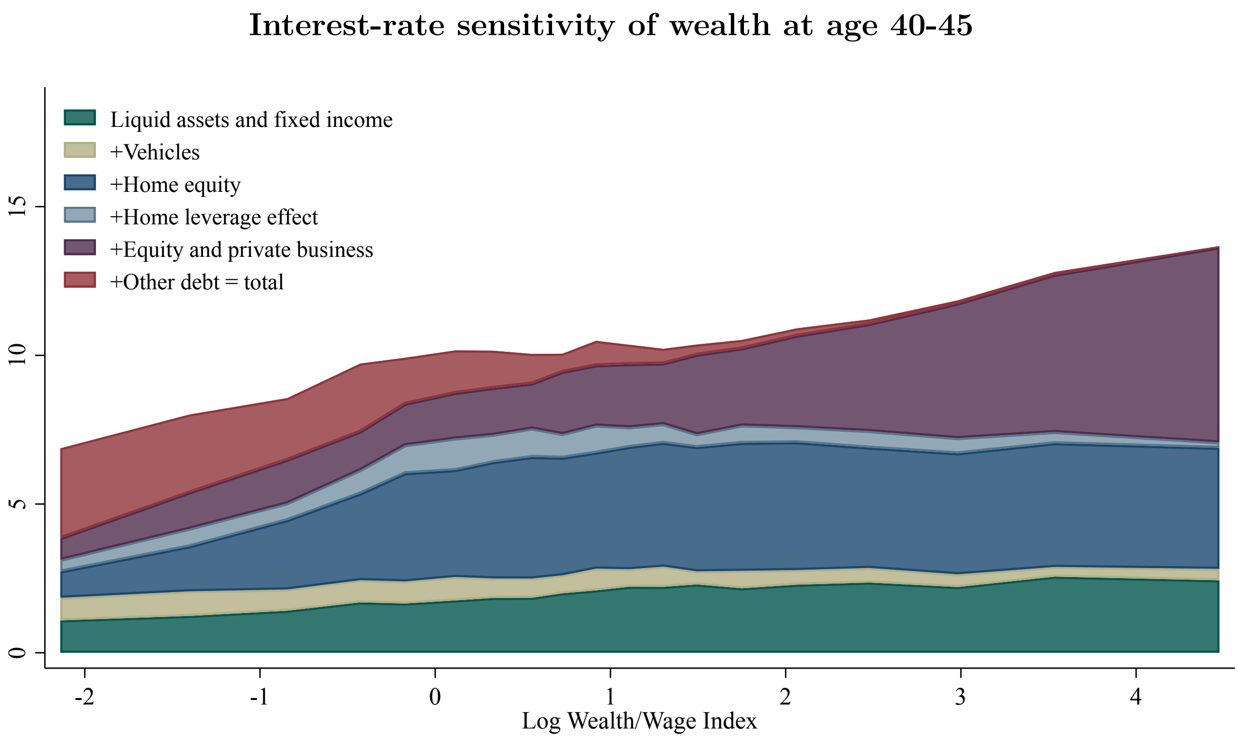
Fact 1: The life-cycle profile of the interest-rate sensitivity of wealth is hump shaped



Fact 2: High earners have more rate-sensitive portfolios at all ages

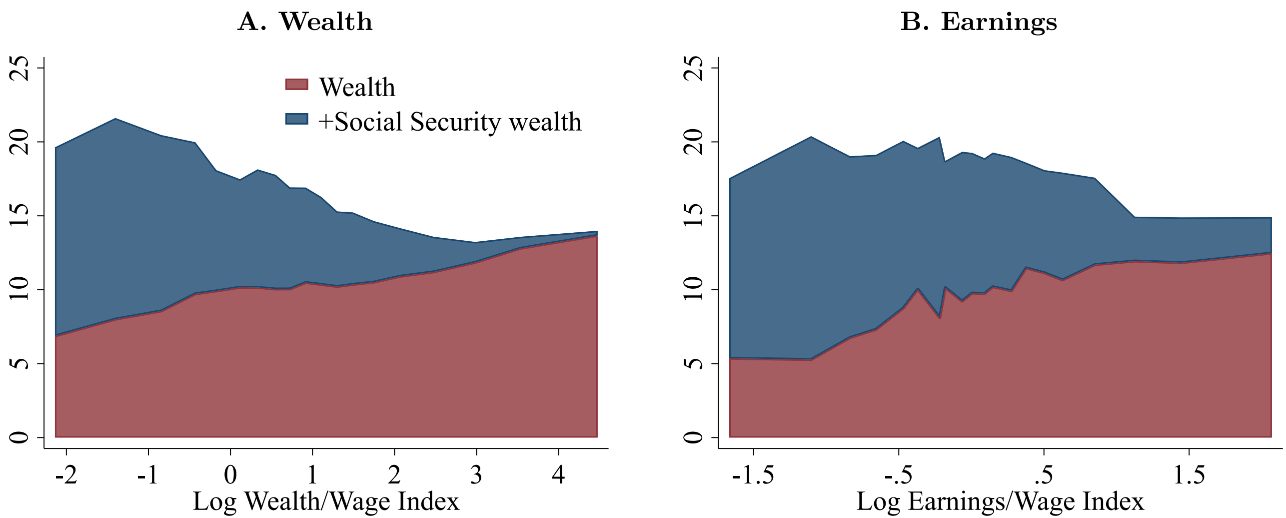


Fact 3: Wealthier households have more rate-sensitive portfolios

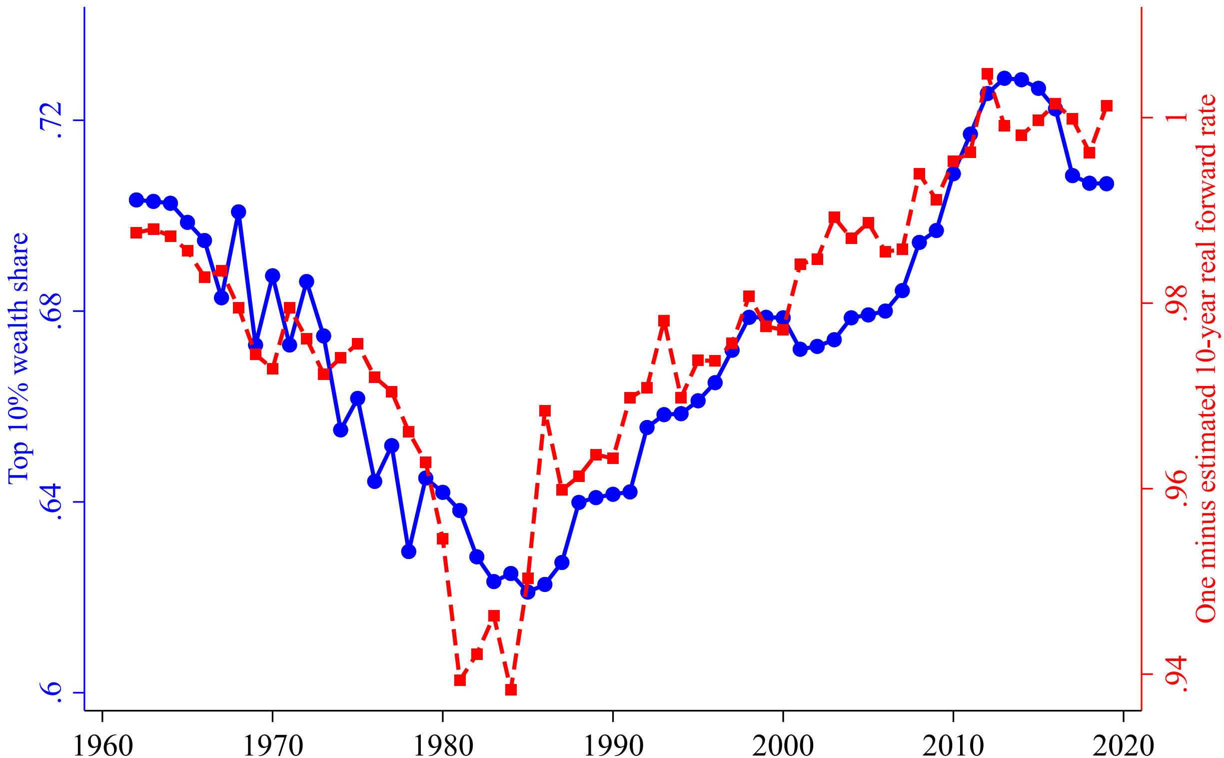


Fact 4: Social Security offsets cross-sectional differences in the rate-sensitivity of portfolios

Interest-rate sensitivity of wealth at ages 40–45: Role of Social Security



Fact 5: Wealth inequality follows interest rates



Life-cycle model

Households

- **Households choose consumption C_i and portfolio allocation π_i**

$$V_{it} = \max_{\{C_{is},\pi_{is}\}} \mathbb{E}_t \sum_{s=t}^{t_{\max}} \beta^{s-t} \left[(1-m_{is}) \frac{C_{is}^{1-\gamma}}{1-\gamma} + m_{is} b(W_{is}, r_{fs}) \right]$$

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- **Portfolio choice and return**
  - π and $1 - \pi$ are the portfolio shares of the n-year and 1-year real zero-coupon bonds
  - Return on wealth portfolio

$$R_{W,it+1} = R_{ft} + \pi_{it}(R_{n,t+1} - R_{ft})$$

  - Portfolio separation theorem: these two assets are sufficient to characterize the optimal interest-rate sensitivity of wealth

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- **Evolution of wealth:**

$$W_{i,t+1} = (W_{it} + L_{it} + B_{it} - T_{it} - C_{it}) R_{W,it+1}$$

Interest-rate risk and asset returns

- **Dynamics of 1-year log risk-free rate:**

$$r_{f,t+1} = (1-\varphi)\bar{r}_f + \varphi r_{ft} + \sigma_r \varepsilon_{r,t+1}.$$

- **Long-term asset is a zero coupon payable in n years, with log return:**

$$r_{n,t+1} = r_{ft} + \mu_n - \sigma_n \varepsilon_{r,t+1}$$

where

$$\sigma_n = \frac{1-\varphi^{n-1}}{1-\varphi}\sigma_r$$

and $\mu_n = -\sigma_n^2/2 \Rightarrow \mathbb{E}[R_n] = \mathbb{E}[R_f]$

  - Interest-rate sensitivity increases with maturity n and shock persistence φ
  - Household can target any interest-rate sensitivity by mixing the two assets

Rest of the model

- **Stochastic income process**
- **Progressive Social Security system**: higher replacement rate to workers with low lifetime earnings
- **Income tax on earnings and Social Security benefits**

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- **Progressive Social Security system**: higher replacement rate to workers with low lifetime earnings
- **Income tax on earnings and Social Security benefits**
- **Bequest motive is a utility flow of $\bar{b}$ years**

$$b(W_{it}, r_{ft}) = \bar{b}\frac{\bar{C}_{it}^{1-\gamma}}{1-\gamma},$$

where $\bar{C}_i$ is the coupon implicit in a $\bar{b}$-year annuity of value W_{it}:

$$W_{it} = \bar{C}_{it} \sum_{k=0}^{\bar{b}} P_{kt}.$$

Economic intuition

We solve a linearized version of the model with no income risk and no bequest

No labor earnings or Social Security

- **Long-term asset share:**

$$\pi^*_{it} = \underbrace{\frac{1}{\gamma}\frac{\mu_n + \frac{1}{2}\sigma_n^2}{\sigma_n^2}}_{\text{myopic demand}} + \underbrace{\left(1 - \frac{1}{\gamma}\right)\varrho_{rt}\left(\frac{\sigma_n}{\sigma_r}\right)^{-1}}_{\text{duration/hedging demand}}$$

- **Without myopic demand (no term premium):**

$$\underbrace{\pi^*_{it}\frac{\sigma_n}{\sigma_r}}_{\text{rate sens. of wealth}} = \left(1 - \frac{1}{\gamma}\right)\underbrace{\varrho_{rt}}_{\text{rate sens. of constant cons. plan}}$$

- **Trade-off:**
  - Option value of reinvesting when rates of return are high makes the long-term asset undesirable
  - Hedging value of capital gains when rates fall make the long-term asset desirable
  - Hedging effect dominates for $\gamma > 1$

Adding labor income and Social Security

- **Same portfolio rule applies to <u>total</u> wealth**

$$\frac{\pi_{it}W_{it} + \pi_{it}^{H}H_{it} + \pi_{it}^{S}S_{it}}{W_{it} + H_{it} + S_{it}} = \pi_{it}^{*}$$

  - π_{it}^{*} is the optimal long-term asset share without background assets
  - H and S are the certainty equivalents of human capital and Social Security
  - $\pi_{it}^{H}H_{it}$ and $\pi_{it}^{S}S_{it}$ are implicit background holdings of the long-term asset

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- **Human capital and Social Security substitution effects**

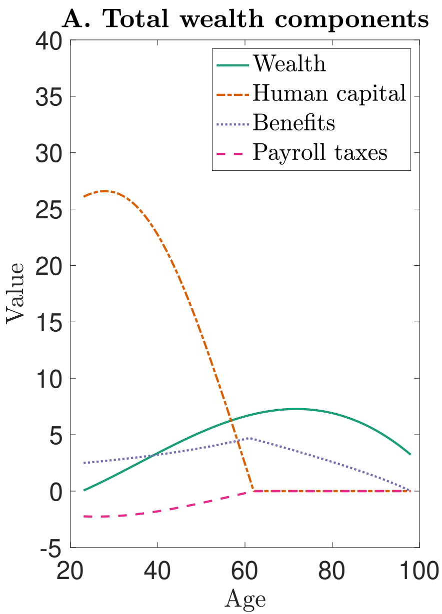
$$\pi_{it} = \pi_{it}^{*} + (\pi_{it}^{*} - \pi_{it}^{H}) \frac{H_{it}}{W_{it}} + (\pi_{it}^{*} - \pi_{it}^{S}) \frac{S_{it}}{W_{it}}.$$

  - Human capital reduces the hedging demand for long-term asset if it's implicit long-term share exceeds the optimal target ($\pi_{it}^{*} < \pi_{it}^{H}$)
  - Same intuition for Social Security

- **Equivalently, in terms of rate-sensitivity:**

$$\varepsilon_r(W_{it}) = \varepsilon_{r,it}^{*} - (\varepsilon_r(H_{it}) - \varepsilon_{r,it}^{*}) \frac{H_{it}}{W_{it}} - (\varepsilon_r(S_{it}) - \varepsilon_{r,it}^{*}) \frac{S_{it}}{W_{it}}, \qquad (1)$$

Substitution effects shape the life cycle of the long-term asset share



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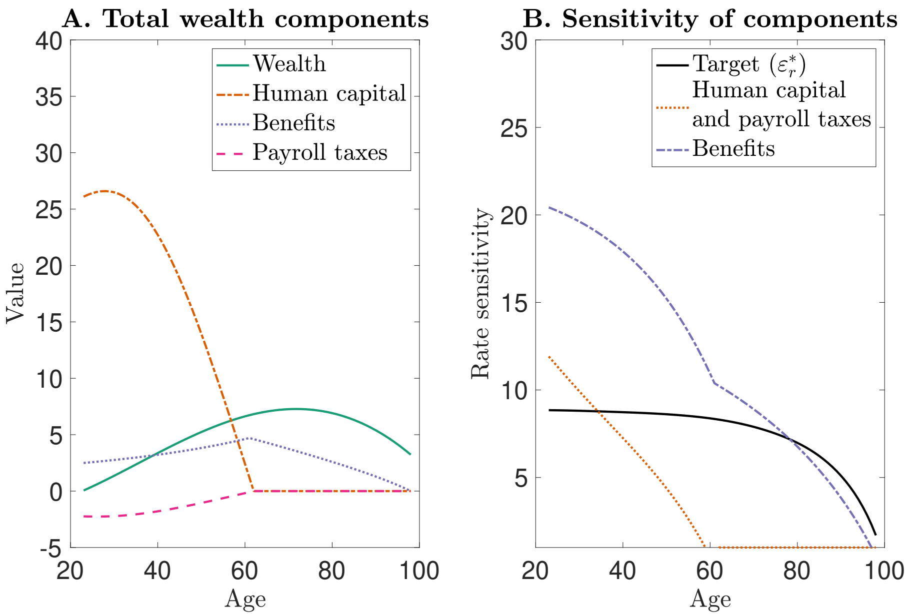

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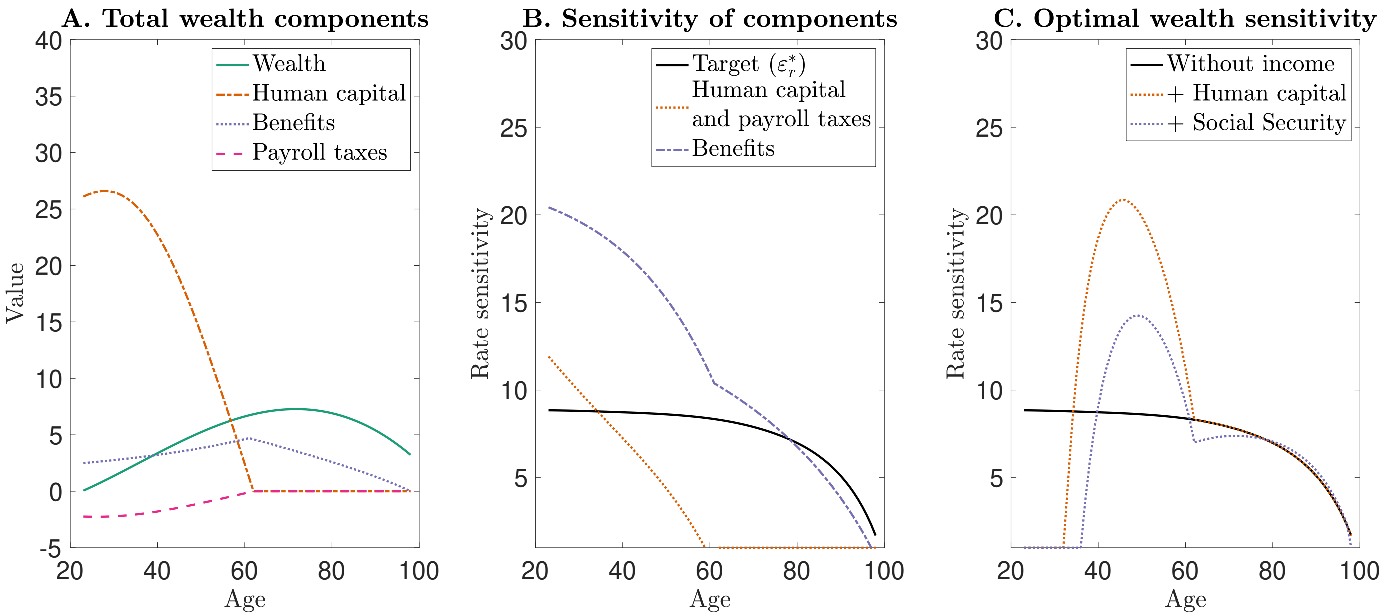


Duration-matching interpretation

- **Intertemporal budget constraint:**

$$W_{it} = \sum_{k=1}^{t_{\max}} P_{kt}(\bar{C}_{i,t+k} - Y_{i,t+k})$$

- **No interest-rate risk strategy**: for each period in k years, buy $\bar{C}_{i,t+k} - Y_{i,t+k}$ units of the corresponding zero-coupon at spot price P_{kt}
- **Intuition**: Hold that portfolio, ignore changes in r, just use the coupons to pay (or save) for excess consumption plan
- **We prove that when $\gamma \to +\infty$, the optimal portfolio converges to the duration-matching strategy**

Real-world implementation of duration-matching

- **Low earners:Social Security implements duration-matching**
  - Working years ($Y_{i,t+k} > \bar{C}_{i,t+k}$): Pay Social Security contributions
  - Retirement ($Y_{i,t+k} < \bar{C}_{i,t+k}$): Receive Social Security benefits
- **Middle-class earners: Add homeownership with fixed-rate mortgage**
  - Working years: Mortgage payments
  - Retirement: Receive in-kind coupon payment (rent-free residence)
- **High earners: Complement the above with a retirement account** (...)
  - Working years: Invest in long-term assets (stocks)
  - Retirement: Consume out of retirement account
- **Note: Duration of market wealth increases with earnings... but not the duration of total endowment (W+H+S)**

Model Fit

Calibration: Preferences & Income process

- Preferences:
  - Discount factor $\beta = 0.95$
  - Bequest motive $\bar{b} = 10$ years
  - Relative risk aversion $\gamma = 6$
    consistent with many recent portfolio choice studies (Benzoni et al., 2007; Lynch and Tan, 2011; Catherine, 2022; Calvet et al., 2021; Meeuwis, 2022).
    $\Rightarrow$ we could match households' exposure to other risk factors (systematic risk, real-estate) with this calibration
- Income process from Guvenen et al. (2022)
- Differences in life expectancy across earnings distribution from Chetty et al. (2016)

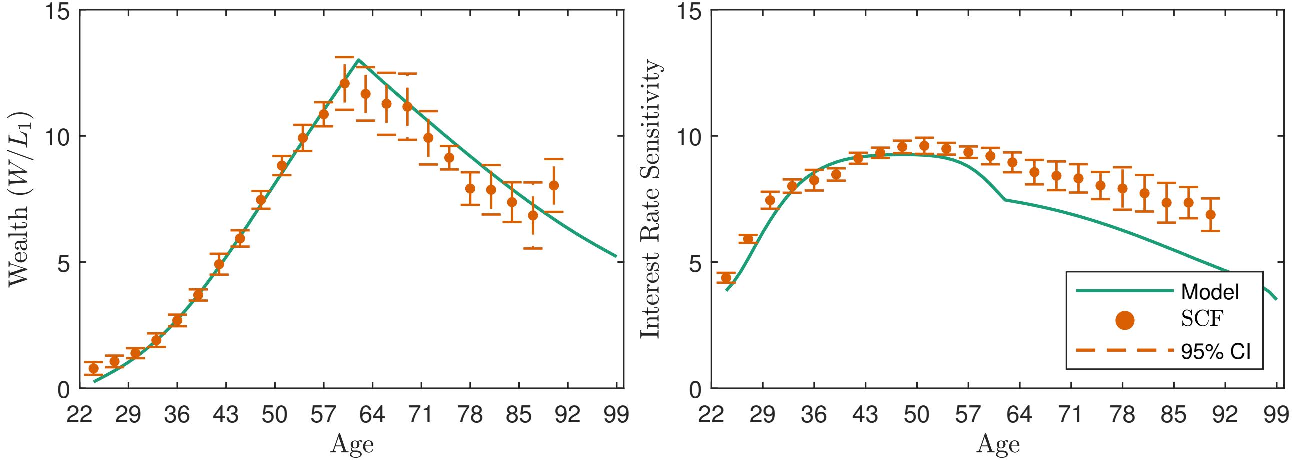
Calibration: Interest rates

- **We match moments from real yield curve data from 1989-2019 so our calibration reflect the beliefs of market participants**

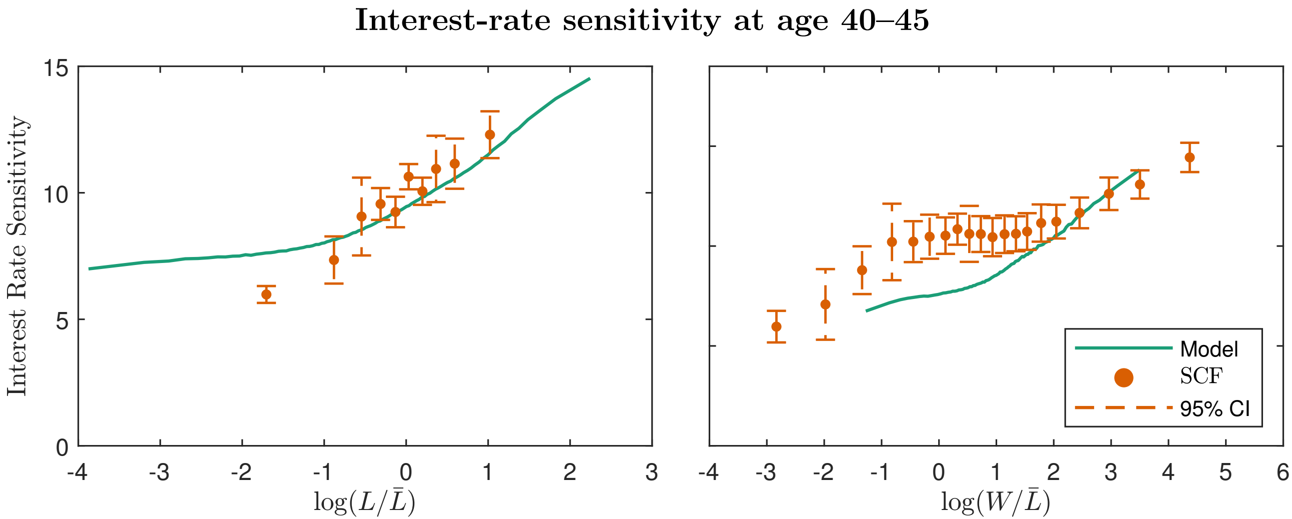
  1. Regression coefficient of 30-year forward rate on 1-year rate identifies the persistence of shocks φ
  2. Mean 30-years forward rate identifies the long-run average historical rate $\bar{r}_f$
  3. Variance 1-year rate identifies rate volatility σ_r

Moment condition			Estimate	
Data moment	Model equiv.	Data value	Parameter	Value
$\text{cov}(f_{30,t}, r_{ft})/\text{var}(r_{ft})$	φ^{30}	0.2569	φ	0.9557
$f_{30,t}$	$\bar{r}_f$	0.0193	$\bar{r}_f$	0.0193
$\text{var}(r_{ft})$	$\sigma_r^2/(1-\varphi^2)$	0.0167	σ_r	0.0049

Wealth & interest-rate sensitivity over the life cycle

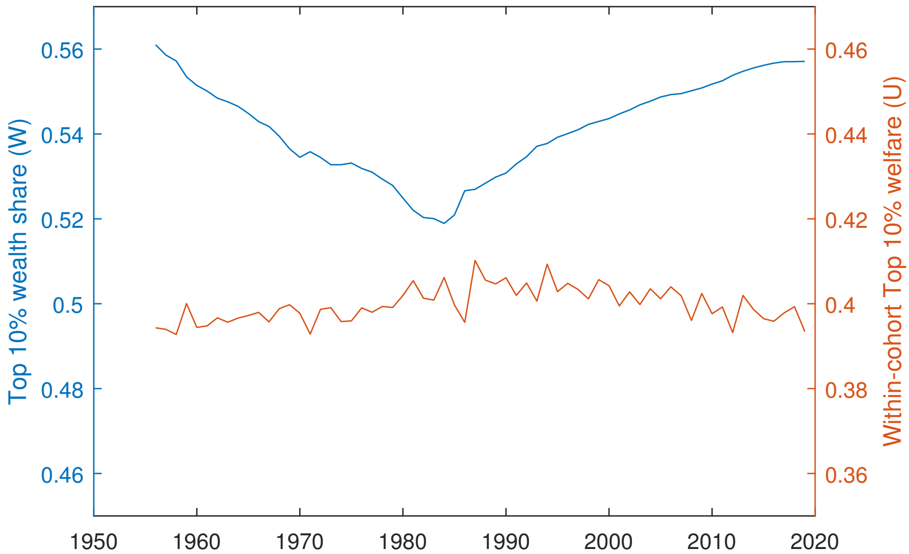


Interest-rate sensitivity of wealth within a cohort



Evolution of wealth inequality in OLG framework

- We simulate data starting with the cohort of 1880, using the historical path of real interest rates
- We generate 40% of the long-run variations in the top 10% share
- 66% of the top 10% share within the bottom 99%



Conclusion

- **We provide micro-foundations for the cross-section of the interest-rate sensitivity of wealth**
  - Hedging demand against rate risk can explain who invests in long-term assets
  - Substitution effects from human capital and Social Security play key roles
  - Differences in life expectancy are less important
- **Our model helps understand trends in wealth inequality**
  - Rates explain 40% of long-run variations in the top 10% wealth share since 1960
  - We shed light on welfare implications