(Most) global and country shocks are in fact sector shocks.*

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Abstract

We construct a multi-country multi-sector model with a rich international input-output structure to separate out the global, country, and sector-level components of economic fluctuations. The model is designed to distinguish country or global shocks from sector-level shocks that propagate via the supply chain. We compare the model-implied decompositions to standard reduced form results and show that the magnitude of true global shocks is significantly smaller than usual. We then show that a closed economy version of the structural model implies a higher contribution of country-level shocks to volatility than is actually the case, because the closed-economy model apportions to domestic aggregate shocks any foreign shock with consequences on more than a few domestic sectors. This is even the case in large economies like the US that are usually assumed to be closed, which we illustrate using the example of the 2011 Japanese earthquake. It is the emergence of vertical trade, in capital and in intermediate goods that explains the bulk of this misallocation.

Keywords: Shock Propagation, Global Supply Chains, Sector Shocks, Aggregate Shocks, Regional Shocks, Global Shocks

JEL Codes: D24, E23, F44, E32, L14

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1 Introduction

A large empirical literature suggests that there exists substantial co-movement of economic fluctuations on a global scale. This can mean that multi-country and global shocks are rampant in the data, for example wars, swings in commodity prices, or climate change. Or it can mean that the global trade network transmits idiosyncratic sector shocks across countries, which synchronizes business cycles on a global scale as formulated for example by Johnson (2014). This paper aims at separating "true" global, multi-country (which we call regional) and country shocks from idiosyncratic sector-level shocks in the presence of global production networks. Our purpose is to gauge the contribution to volatility of each type of shock, at the global, regional, country, and sector level.

For a student of economic fluctuations, paying careful attention to the propagation of idiosyncratic shocks is important. Firstly, not doing so is bound to give too much importance to global shocks, as sector shocks that propagate across countries replicate the statistical behavior of regional or global shocks.[1] A simple reduced form statistical approach is not able to distinguish between true multi-country vs. propagating idiosyncratic shocks. In fact this mis-labeling extends potentially to country-level shocks if propagation happens between the sectors of one country: In that case a reduced form statistical approach gives too much importance to country shocks.

Secondly, even a structural approach can result in substantial mis-labeling of shocks if it ignores the international dimension. A closed-economy structural model mechanically ascribes any global development to domestic aggregate shocks, unless the effect of the global shock is confined to a few domestic sectors. For example inasmuch as it can affect many US sectors, a closed-economy structural model of input-output linkages will

[1] In this paper, "sector" shocks refer to sector-specific developments that are not common across countries, what Huo et al. (2023) label"country-sector" shocks.

classify an earthquake in Japan as an aggregate US shock. Even with careful modeling of propagation between domestic sectors, a closed-economy model ascribes to country shocks a diverse set of events originating abroad, none of which is truly happening at country level: any supra-national shock, of course, but also any foreign sector shock that propagates into more than a few sectors in the domestic economy. A structural closed-economy model correctly apportions domestic sector shocks if they propagate across domestic sectors, but this identification does not extend to similar shocks that originate abroad. The limitation can matter even for so-called "closed" countries that do not trade much directly, because the propagation mechanism at play here pertains to high order trade linkages.

We explore the empirical relevance of these concerns. We construct a multi-country, multi-sector model of input-output linkages in final and intermediate goods with trade costs. The model distinguishes between trade in capital goods vs. trade in other intermediates. Fluctuations are driven by supply shocks, prices are flexible, and markets are complete. We allow for non-unitary elasticities of substitution following the closed-economy model in Atalay (2017). We show that the dynamics of output in equilibrium can be characterized by a international factor model, an extension of Foerster et al. (2011). The factor model expresses observed fluctuations as a function of the structural parameters and unobserved productivity shocks, akin to the "influence matrix" proposed in Baqaee and Farhi (2019a) or Huo et al. (2023). Inverting this expression yields a filtering equation that we use to estimate the true productivity shocks implied by the data. Crucially, these imputed shocks exclude propagation effects through the channels we specify in the model. We then compare a decomposition of economic fluctuations into their global, regional, country, and sector components as implied by the structural model, with a conventional reduced form decomposition. In addition, we compare the decomposition implied by the international structural model to that implied by a closed-economy version.

Our first result points to a large over-estimation of the importance of global, regional, and country shocks in reduced form variance decompositions. For example in reduced form on average 89 percent of the country-level variance comes from global, regional, and country shocks. But their true contribution as implied by the international structural model is only two third of that. Similarly, while the reduced form model estimates that virtually all of the global component comes from global shocks, the structural model concludes that a quarter of these are actually constituted by sector-level shocks that propagate globally.

Our second result compares the predictions of the open economy structural model with those of its closed economy counterpart. As is well-known, the closed economy model concludes that country-level shocks explain the bulk of country level volatility: We find that aggregate shocks explain 50 percent of US volatility, as in Foerster et al. (2011). But that share falls dramatically, to 18 percent, in the international version of the model. This is because all foreign shocks - at global, regional, and (propagating) sector levels - are classified as country shocks in the closed economy model. We illustrate the issue with the well-known example of the Japanese earthquake of 2011, which we demonstrate is labelled an aggregate US shock by a closed economy version of the model. Even though a closed economy model is customarily assumed to be appropriate to model the US economy, these results suggest that the international dimension is crucial even there. This happens because direct trade constitutes a small fraction of the US economy, but indirect trade does not and the international exposure of the US economy is in fact considerable.

We modify the trade costs in the model to simulate shutting down different components of international trade, with a view to identifying which linkages matter for our result. The simulations suggest that trade in final goods is essentially irrelevant in the sense that a model without final goods trade implies a set of variance decompositions that are virtually identical to those in the full model. It is trade in intermediate goods that matters, mostly across countries. We distinguish between trade in capital goods vs. other intermediates. Almost half of global sector comovement comes from trade in capital goods. This extends to an international setting the result in Foerster et al. (2011) that trade in capital goods constitutes a quantitatively important shock propagation mechanism.

We obtain these results in a unique international dataset with quarterly information at sector level, constructed from UNIDO data on quarterly industrial production (IP) data for 29 countries and 21 sectors over the period 2006Q1 to 2022Q3. We rely on the World Input Output Database (WIOD) to parameterize international sector linkages in intermediate and final trade. In addition we approximate international trade in capital goods at sector level combining data from the Bureau of Economic Analysis, Eurostat, Refinitiv Eikon, and WIOD. This gives rise to a quarterly inter-country, inter-sector dataset on economic fluctuations combined with corresponding vertical trade in capital and intermediate goods, whose coverage is to our knowledge unprecedented.

We contribute to three literatures. Firstly, the statistical methodology we implement is relatively standard, adapted from a rich literature, see for instance Kose et al. (2003, 2008); Del Negro and Otrok (2008); Crucini et al. (2011); Kose et al. (2012); Norrbin and Schlagenhauf (1996); Karadimitropoulou and León-Ledesma (2013), or Hirata et al. (2013). Foerster et al. (2011) implement a similar approach, but for the fact that ours includes the international dimension.

Secondly, our results are related to the empirical literature on shock propagation via trade. Many papers have documented the correlation between trade and co-fluctuations, starting with the seminal empirical analysis in Frankel and Rose (1998). The importance of vertical trade has been widely documented, see for instance Burstein et al. (2008) or Di Giovanni and Levchenko (2010) in reduced form. In response to the reduced form evidence, a literature has proposed models of multi-sector linkages building from the seminal contribution in Long and Plosser (1983). This includes for instance Ambler et al. (2002), Horvath (1998), Dupor (1999), or Carvalho (2010).

Finally we contribute to a third literature that has emerged in response to the need for sophisticated theories of input-output linkages, sometimes at the level of individual firms, sometimes with an international dimension, sometimes mixing ingredients from macroeconomics and trade theories. This includes for instance Johnson (2014), Acemoglu et al. (2016), Eaton et al. (2016a), Eaton et al. (2016b), Grassi (2017), Baqaee (2018), Baqaee and Farhi (2019c), Baqaee and Farhi (2019b), Bigio and La'O (2020), Di Giovanni et al. (2014), Di Giovanni et al. (2018) Giovanni et al. (Fortcoming), Boehm et al. (2019); Barrot and Sauvagnat (2016), and Carvalho et al. (2021). Most recently, Ho et al. (2023) introduce nominal rigidities to study demand shocks in this environment.

Huo et al. (2023) consider a multi-sector multi-country model with incomplete markets to study how much production networks drive co-movements across countries. Like us, they use the model to filter out propagation mechanisms from observed fluctuations, and infer the corresponding "true" supply shocks. Our purposes are different, however: Huo et al. (2023) focus on the determinants of cross-country business cycle correlations, an important question in international macroeconomics, while we focus on the origins of fluctuations and in particular the decomposition of variance into global, regional, country, and sector shocks. To do so, we construct an international, inter-sector quarterly dataset on industrial production, whereas Huo et al. (2023) have annual data (but of course they have data on all sectors in GDP).

Ho et al. (2023) introduce a rich multi-country model with nominal rigidities and multilateral trade linkages to inspect the driving forces behind the international correlations in inflation rates and in aggregate fluctuations between five regions. Like us, they find that propagation mechanisms are powerful even though observed direct trade flows are not large relative to GDP: Most of aggregate co-movements originate in country-level idiosyncratic shocks. They do not speak to the importance of idiosyncratic shocks at sector level. Interestingly, Ho et al. (2023) estimate minimal frictions in financial markets, i.e., an environment close to complete markets is adequate to replicate the data.[2] This helps justify the assumption og complete markets, which enables us to use factor analysis akin to what exists in the literature.

2 A first look at the data

2.1 Data

This section details our data. We rely on quarterly sector data on Industrial Production (IP) for the period Q1 2006 to Q3 2022. For 29 countries, we compile and homogenize IP data on the 21 sectors that make up national IP indices. The sectors correspond to industries at the 2-digit level, as defined by the ISIC rev. 4 standard. This list is the finest aggregation for which there is compatible data across IP indices from various national sources and the World Input Output Database. See Appendix A for a detailed account of the sector classification, aggregation, the mapping between classifications across data sources, and the treatment of missing data.

Industrial Production is the only measure of economic activity available at the quarterly sector-level that is comparable across countries. Industrial production also implies that we exclude agriculture and services from the analysis, which means the analysis pertains

[2]The consumption exchange rate puzzle introduced by **?** is resolved recognizing that financial frictions can have level effects on consumption.

on average to about one third of GDP, ranging from 20 percent in the UK to 52 percent in South Korea. Putative idiosyncratic shocks that emerge from sectors that are not included in the model but that do propagate into sectors that are included in the model will be mislabeled. For instance a global technology shock in Business Services that propagates into a single manufacturing sector would mistakenly be labeled as idiosyncratic, or as common if it propagates throughout manufacturing.

The main reason why we focus on IP is data availability. It is worth noting that input-output linkages are by far the strongest between manufacturing sectors: For example the World Input Output Database suggests that in 2014, linkages within manufacturing were on average three times larger than they were between services and manufacturing, and twice larger than between agriculture and manufacturing. This suggests the main activities where shock propagation between sectors and countries occurs are included in industrial production.

There are $N = 29$ countries, denoted by subscripts m or n and $J = 21$ economic sectors denoted by superscripts i or j. We categorize the countries into $R = 4$ regions, denoted by subscripts r: the Americas, Asia, Western Europe and Central and Eastern Europe. See Appendix B for the definitions of regions. We define aggregate world IP growth as $\sum_n^N \sum_j^J w_n^j \, g_{n,t}^j$ where $g_{n,t}^j$ is the IP growth for sector j in country n at time t and w_n^j is the value added weight of the sector in the aggregate IP index (averaged over time).[3] Aggregate regional and country IP growth rates are defined analogously.

Table 1 reports standard deviations for those different indices at the national, regional and global level. Standard deviations are also computed setting all covariances (within and between countries) to zero. The Table suggests that covariances are an important

[3]The use of constant aggregating weights comes directly from Foerster et al. (2011). We have verified that the fraction of aggregate IP volatility that comes from time-varying weights is negligible in our data, as it is in theirs.

magnifier of IP fluctuations: for example the standard deviation of global IP drops by a factor of five when the covariance terms are excluded from its calculation. Since covariances appear to be relevant for of IP fluctuations at country, regional and global levels, a challenge is to understand what drives this covariation – common or propagating shocks. The prominence of covariance terms in global or regional fluctuations makes it interesting to extend the analysis in Foerster et al. (2011) to an international setting.[4]

2.2 Reduced Form Factor Analysis

We start with a simple dynamic multi-factor model with the aim of estimating a set of unobserved common factors associated with sector IP growth. The model is standard, similar to Kose et al. (2003, 2008, 2012) or Norrbin and Schlagenhauf (1996). It is estimated directly on the original IP data and does therefore label as "common" those shocks that are in fact propagated via the global value chain.

Denote $\triangle \mathbf{y}_t$ a vector that stores $N \times J$ sectoral industrial production growth rates. Then

$$\triangle \mathbf{y}_t = \mathbf{\Lambda} \mathbf{F}_t + \mathbf{u}_t, \tag{1}$$

where $\mathbf{F_t}$ is a $k \times 1$ vector of k unobserved latent factors and $\mathbf{\Lambda}$ is a $(N \times J) \times k$ matrix that stores the factor loading coefficients. $\mathbf{u_t}$ is an $(N \times J)$ vector of sector-specific disturbances with a diagonal covariance matrix. The covariance matrix of the vector of IP growth rates $\mathbf{Y_t}$ is $\mathbf{\Sigma}_{YY} = \mathbf{\Lambda}\mathbf{\Sigma}_{FF}\mathbf{\Lambda}' + \mathbf{\Sigma}_{uu}$ where $\mathbf{\Sigma}_{FF}$ and $\mathbf{\Sigma}_{uu}$ are the covariance matrices of the unobserved factors and the disturbances respectively. Note that because in a factor model $\mathbf{\Sigma}_{uu}$ is assumed to be diagonal, all covariance in $\mathbf{\Sigma}_{YY}$ comes from the common factors $\mathbf{F}_t$.

[4] As us, Foerster et al. (2011) study the co-movement of Industrial Production at a quarterly frequency, while Atalay (2017) studies the co-movement of value added. His focus on value added allows his sample to include service sectors, yet comes with the significant disadvantage of limiting the data frequency to annual.

Table (1) Standard deviation of IP growth

	without sectorial covariance	with sectorial covariance
Country		
USA	2.5	5.5
CAN	5.8	9.3
MEX	3.7	6.5
DEU	5.0	9.9
ITA	4.0	10.5
GBR	6.4	7.7
FRA	3.1	6.6
CHN	1.4	3.2
JPN	3.9	8.8
KOR	6.1	10.6
Average	**3.6**	**7.0**
Region		
Western Europe	2.1	7.6
Central and Eastern Europe	2.3	9.0
Americas	2.2	5.6
Asia	1.5	4.6
Average	**1.9**	**5.8**
Global	**1.0**	**5.0**

Column *with sectorial covariance* reports the standard deviation of IP. The entries in column *without sectorial covariance* are calculated setting all covariances to zero. The table reports only results for the largest 10 countries in the sample. Averages are value added weighted averages.

The factor model is designed to identify whether common variation in IP growth is country specific, region specific, or global. The model includes one set of latent global factor common to all sectors, four sets of latent regional factors common to the sectors belonging to each of the four regions, and 29 sets of latent country factors common only to the sectors within a given country. We estimate the number of factors using the Ahn and Horenstein (2013) information criterion for static approximate factor models, and find evidence for the presence of one global factor and one regional factor per region. Estimates for the number of country factors vary between one and nine across countries with an average of three. We set to three the number of country factors per country, which implies a total of $k = 92$ common factors.[5]

At sector level equation (1) can be rewritten as

$$\triangle y_{n,t}^{j} = \Lambda_n^{j,\mathrm{G}} F_t^{\mathrm{G}} + \Sigma_r \Lambda_n^{j,\mathrm{r}} F_t^{\mathrm{r}} + \Sigma_m \mathbf{\Lambda}_n^{j,m} \mathbf{F}_t^m + u_{n,t}^{j}, \tag{2}$$

where F_t^{G} is the global latent factor, F_t^r is the regional specific factor, and $\mathbf{F}_t^m$ is the 3×1 vector of country factors, whose vector of loadings is denoted by $\mathbf{\Lambda}_n^{j,m}$.

Identification is obtained through a block structure imposing that the factor loadings are zero when a country n does not belong to a region r for the regional, and by imposing that the loadings are zero when $m \neq n$ for the country factors. We estimate the factor model with the Principal Components Analysis estimator from Jackson et al. (2016). We also estimated the multi-factor model using the Bayesian approach from Kose et al. (2003) and the maximum likelihood estimator from Delle Chiaie et al. (2021). The results are similar.

As standard in this literature, we gauge the importance of common factors using variance

[5] $92 = 1 \times 1 + 4 \times 1 + 29 \times 3$. Appendix G.1 reports the results for a model that allows for different numbers of country factors per country. The results are similar.

decompositions. We compute the fraction of IP variance explained by common shocks at three different levels of aggregation: global, regional and country. We decompose each volatility, measured at each level of aggregation, into the contributions of global, regional, and country-level factors. For example, we denote $R_r^2(F^{\text{Global}})$, $R_r^2(F^{\text{Region}})$, and $R_r^2(F^{\text{Country}})$ the fraction of IP growth variation in region r due to global, regional, and country factors. Analogous definitions hold at country and global level.

Denote Σ^G the variance covariance matrix of global shocks, Σ^R the variance covariance matrix of regional shocks, Σ^C the variance covariance of country shocks, and Σ^S the (diagonal) variance covariance of idiosyncratic sector shocks. We define as Σ^{All} the sum of all four, i.e., the variance covariance of industrial production as predicted by the reduced form factor model.

The results in Table 2 document well-known facts. Common factors (i.e., global, regional, and country-level) dominate economic fluctuations at all levels of aggregation: 89 percent in the average country, 95 percent in the average region, and 98 percent globally. The global factor dominates among the three possible common factors we consider: country and regional factors are small, even for country-level variance, sometimes negligible even as in some European countries. Through the lens of the statistical factor model, almost all of economic fluctuations are explained by common shocks. This is in line with well-known findings, for example by Kose et al. (2003) or Kose et al. (2012).

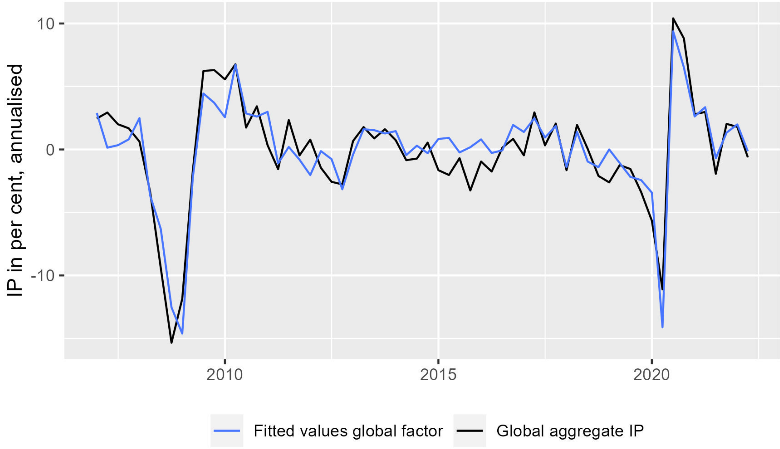
These results beg the question of the nature and origin of such prominent global shocks. Figure 1 plots the observed growth rate in world IP as against the fitted value of the common global factor on global IP growth. The Global Financial Crisis of 2008 and the COVID crisis in 2020 are very recognizable, but other sources of fluctuations remain somewhat mysterious historically.

Table (2) Results from reduced form factor models

	Global (1)	Region (2)	Country (3)	Total (4)	Sector (5)
Country					
USA	59%	10%	23%	92%	8%
CAN	51%	1%	35%	87%	13%
MEX	55%	4%	27%	87%	13%
DEU	80%	5%	4%	89%	11%
ITA	67%	3%	25%	95%	5%
GBR	23%	3%	23%	49%	51%
FRA	74%	5%	9%	88%	12%
CHN	6%	48%	39%	94%	6%
JPN	40%	28%	25%	93%	7%
KOR	40%	25%	22%	87%	13%
Average	**43%**	**19%**	**26%**	**89%**	**11%**
Region					
Western Europe	91%	2%	3%	96%	4%
Central and Eastern Europe	85%	3%	9%	97%	3%
Americas	66%	8%	20%	94%	6%
Asia	39%	52%	5%	96%	4%
Average	**61%**	**23%**	**10%**	**95%**	**5%**
Global	**84%**	**10%**	**4%**	**98%**	**2%**

Note: This table presents variance decompositions of country-level volatility in the ten largest countries in the sample, then of regional volatility for the four regions, and then of global volatility. The country average is a value added weighted average over all 29 countries in the sample. Columns (1) - (3) indicate variance decompositions with respect to global, regional and country common shocks. Column (4) sums over the three factors. Column (5) gives the residual importance of sector shocks.

Figure (1) The global business cycle and the global common factor



Note: The graph shows world IP growth in the data and its counterpart as predicted by global shocks. IP growth is annualised. World IP growth is calculated as $\sum_{n}^{N} \sum_{j}^{J} w_{n}^{j} g_{n,t}^{j}$.

3 The Model

The reduced form factor model in Section 2.2 crucially relies on the assumption that the sector innovations in $\mathbf{u}_t$ are weakly cross-sectionally correlated.[6] This implies that a statistical factor model attributes all the covariation in sector IP growth rates to common shocks. But in reality idiosyncratic shocks propagate via industrial linkages, which can create co-movement across sectors in a closed or an open economy.[7] Statistical factor models that do not account for propagation via trade linkages mechanically tend to overestimate the role of common (country, regional, or global) shocks.

To address this problem we construct a multi-sector multi-country dynamic general equilibrium model that generalizes Foerster et al. (2011) and Atalay (2017). The aim is to account explicitly for the propagation of shocks across sectors and countries and to produce estimates of sector shocks that are filtered from such spillovers. We allow for

[6] See Chamberlain and Rothschild (1983); Connor and Korajczyk (1986); Stock and Watson (2002).

[7] See Long and Plosser (1983); Horvath (1998); Carvalho (2010); Atalay (2017) and Foerster et al. (2011) in a closed economy context and Huo et al. (2023) in an international production network

propagation via three channels: trade in final goods, trade in intermediate inputs, and trade in capital goods. We generalize Foerster et al. (2011) by demonstrating that the international model with trade costs can be mapped into a factor model akin to the one described in 2.2. This allows us to estimate a standard factor model on the series produced by the model, which filters out propagation mechanisms while still allowing for common shocks in "true" productivity shocks. The results can then be usefully compared with those in Section 2.2.

3.1 An international model with sector linkages

Consider a global economy with N countries denoted either by m or n and J sectors denoted by either i or j. Throughout the rest of the paper, subscripts denote countries and superscripts denote sectors. Both indexes are ordered so that the first identifies the origin of production, and the second identifies the destination of use. Each sector j located in country n produces an amount $Y^j_{n,t}$ in period t. For that, the sector uses capital $K^j_{n,t}$ and labour $L^j_{nn,t}$, which is supplied by the representative agent in country n. Sectors also use intermediate inputs that are produced by other sectors located either domestically or abroad; let $M^j_{n,t}$ denote the bundle of intermediate goods that sector j in country n uses for production. Define

$$Y^j_{n,t} = A^j_{n,t}\left[(1-\mu^j_n)^{\frac{1}{\epsilon_Q}}\left(\left(\frac{K^j_{n,t}}{\alpha^j_n}\right)^{\alpha^j_n}\left(\frac{L^j_{n,t}}{1-\alpha^j_n}\right)^{1-\alpha^j_n}\right)^{\frac{\epsilon_Q-1}{\epsilon_Q}} + (\mu^j_n)^{\frac{1}{\epsilon_Q}}(M^j_{n,t})^{\frac{\epsilon_Q-1}{\epsilon_Q}}\right]^{\frac{\epsilon_Q}{\epsilon_Q-1}}, \tag{3}$$

where α^j_n is the capital share in value added for sector j in country n and μ^j_n (a typical element of the vector $\boldsymbol{\mu}$) denotes a technology shifter that reflects country n industry

j's usage of intermediate goods for production. ϵ_Q denotes the elasticity of substitution between the factors of production.

$A^j_{n,t}$ is the productivity index for country-sector nj and vector $\mathbf{A}_t$ stores all sector productivity indices. We assume that $\mathbf{A}_t$ follow a Random-Walk process

$$\ln(\mathbf{A}_t) = \ln(\mathbf{A}_{t-1}) + \boldsymbol{\zeta}_t, \tag{4}$$

where $\boldsymbol{\zeta}_t = (\zeta^1_{1,t}, ..., \zeta^J_{N,t})^\top$ are sector productivity shocks with covariance matrix $\mathbf{\Sigma}_{\zeta\zeta}$. We explicitly allow the $\zeta^j_{n,t}$ to be correlated in any arbitrary fashion and $\mathbf{\Sigma}_{\zeta\zeta}$ to be a non-diagonal matrix. In other words, productivity shocks are allowed to have sector, country, regional, or global components.

The intermediate good bundle $M^j_{n,t}$ aggregates intermediate good bundles of different varieties i available in country n, $M^{ij}_{n,t}$, according to:

$$M^j_{n,t} = \left(\sum_i (\mu^{ij}_n)^{\frac{1}{\epsilon_M}} (M^{ij}_{n,t})^{\frac{\epsilon_M - 1}{\epsilon_M}} \right)^{\frac{\epsilon_M}{\epsilon_M - 1}}, \quad \sum_i^J \mu^{ij}_n = 1, \tag{5}$$

where ϵ_M denotes the elasticity of substitution between different varieties of intermediate goods available in country n, μ^{ij}_n specifies country n industry j's shifter for intermediate inputs produced by industry i, stored in the matrix $\mathbf{\Gamma^{M1}}$.

In turn, M^{ij}_n aggregates varieties over countries, as in

$$M^{ij}_{n,t} = \left(\sum_m (\mu^{ij}_{mn})^{\frac{1}{\epsilon_T}} (M^{ij}_{mn,t})^{\frac{\epsilon_T - 1}{\epsilon_T}} \right)^{\frac{\epsilon_T}{\epsilon_T - 1}}, \quad \sum_m^N \mu^{ij}_{mn} = 1, \tag{6}$$

where M^{ij}_{mn} denotes intermediate goods produced in country-industry (m, i) and used for production in country-industry (n, j). ϵ_T denotes the elasticity of substitution between inputs produced in different countries m. μ^{ij}_{mn} specifies a shifter in country-industry (n, j) for intermediate inputs produced in country-industry (m, i), stored in the matrix $\mathbf{\Gamma^{M2}}$.

The capital stock in country-sector (m, j) follows the law of motion

$$K^j_{n,t} = X^j_{n,t} + (1 - \delta)K^j_{n,t}, \tag{7}$$

where $X^j_{n,t}$ denotes investment in sector (n, j) and δ is the depreciation rate of capital.

Investment in country-sector (n, j) is produced using capital goods bought from other -domestic or foreign- origins as described in

$$X^j_{n,t} = \left(\sum_m \sum_i (\kappa^{ij}_{mn})^{\frac{1}{\epsilon_X}} (X^{ij}_{mn,t})^{\frac{\epsilon_X - 1}{\epsilon_X}} \right)^{\frac{\epsilon_X}{\epsilon_X - 1}}, \quad \sum_m^N \sum_i^J \kappa^{ij}_{mn} = 1, \tag{8}$$

where $X^{ij}_{mn,t}$ denotes the amount of investment goods produced by country-sector (m, i) and used for investment by country-sector (n, j) and ϵ_X denotes the elasticity of substitution across investment goods. κ^{ij}_{mn} denotes country-industry (n, j)'s shifter for investment goods produced by country-industry (m, i), stored in the matrix $\mathbf{\Gamma^X}$. The model allows for explicit input-output linkages in investment goods.

There is one representative agent in each country who receives utility from the consumption of final goods from all sectors everywhere. We denote by $C^j_{nm,t}$ the consumption by the household in country m of a good j produced in country n. Utility at time zero is given by

$$U_{0,m} = E_0 \Bigg[\sum_{t=0}^{\infty} \beta^t \Bigg(\sum_n^N \sum_j^J \xi^j_{nm} \ln(C^j_{nm,t}) - \frac{\epsilon_{LS}}{\epsilon_{LS} + 1} \big(\sum_j^J L^j_{mm,t} \big)^{\frac{\epsilon_{LS}+1}{\epsilon_{LS}}} \Bigg) \Bigg], \tag{9}$$

where $\xi^j_{nm,t}$ denotes a country specific taste shifter, and we have assumed that labor is mobile across sectors but not across countries. ϵ_{LS} is the Frisch elasticity of labour supply.

The resource constraint is given by

$$\sum_{m=1}^{N}[\tau^C]^j_{nm}C^j_{nm,t} + \sum_{m=1}^{N}\sum_{i=1}^{J}[\tau^M]^{ji}_{nm}M^{ji}_{nm,t} + \sum_{m=1}^{N}\sum_{i=1}^{J}[\tau^X]^{ji}_{nm}X^{ji}_{nm,t} = Y^j_{n,t}. \tag{10}$$

The resource constraint accounts for the existence of iceberg trade costs, where $[\tau^M]^{ji}_{nm}$ denotes iceberg trade costs that arise for country-sector mi's use of intermediate goods produced by country-sector nj. In the same spirit, τ^X and τ^C denote iceberg trade costs for capital and final goods respectively.

We assume complete markets and perfect competition. Under these assumptions, we compute the competitive equilibrium by solving a social planning problem as in Eaton et al. (2016a). The social planner maximises the sum of the utilities of the representative agents in each country:

$$W = \sum_{m}^{N} U_m \tag{11}$$

In Appendix C we review the key steps to computing the competitive equilibrium.

The deterministic steady state of the model is analytically tractable and a linear approximation of the model's first-order conditions and resource constraints around the steady state yields a vector ARMA(1, 1) model for sector output growth:

$$\triangle\, \mathbf{y}_{t+1} = \mathbf{\Pi}_1 \triangle\, \mathbf{y}_t + \mathbf{\Pi}_2\zeta_t + \mathbf{\Pi}_3\zeta_{t+1} \tag{12}$$

where $\triangle \mathbf{y}_t$ is a vector of IP growth rates $(\triangle \ln(Y_{n,t}^j), ..., \triangle \ln(Y_{N,t}^J))$ and $\mathbf{\Pi}_1$, $\mathbf{\Pi}_2$ and $\mathbf{\Pi}_3$ are $NJ \times NJ$ matrices that depend on the model parameters, in particular those that define sector linkages in the model. ζ denotes the "true" sector productivity innovations as defined in equation 4: They are allowed to co-vary across sectors, countries, and regions.

Solving equation (12) for $\boldsymbol{\zeta}_{t+1}$ yields the filter:

$$\boldsymbol{\zeta}_{t+1} = \mathbf{\Pi}_3^{-1} \triangle \boldsymbol{y}_{t+1} - \mathbf{\Pi}_3^{-1}\mathbf{\Pi}_1 \triangle \boldsymbol{y}_t - \mathbf{\Pi}_3^{-1}\mathbf{\Pi}_2\boldsymbol{\zeta}_t. \tag{13}$$

After solving for the $\mathbf{\Pi}$ matrices, we can calculate sector level productivity shocks at each point in time from observed IP growth (see Appendix C for details). This is akin to Huo et al. (2023) who also use a model to extract TFP shocks from observable variables at international level with a focus on international correlations between aggregate fluctuations.

We assume complete markets following Johnson (2014), Eaton et al. (2016a) or Eaton et al. (2016b). In contrast Huo et al. (2023) assume financial autarky. Johnson (2014) proposes two versions of a multi-country multi-sector model, with complete and incomplete markets. In simulations, he shows that propagation is less prevalent with incomplete markets. We replicated the exercise and fail to reject that simulated output co-movements in complete markets are significantly different from co-movements simulated under incomplete markets. Using simulated method of moments techniques, Ho et al. (2023) conclude that a complete market version of their model generates moments that are closer to the data.

3.2 Structural Factor model

We now put some structure on the covariance matrix $\mathbf{\Sigma}_{\zeta\zeta}$. The filtered sector productivity shocks can reflect both common (country, regional, or global) and idiosyncratic sector shocks. But propagation mechanisms are now filtered out. We can express $\boldsymbol{\zeta}_t$ as

$$\boldsymbol{\zeta}_t = \mathbf{\Lambda}_S \boldsymbol{S}_t + \boldsymbol{\epsilon}_t \tag{14}$$

where $\boldsymbol{\epsilon}_t$ is a $(N \times J)$ vector that contains only idiosyncratic sector shocks, $\boldsymbol{S}_t$ is a $k \times 1$ vector that accounts for k common shocks and $\mathbf{\Lambda}_S$ is a $(N \times J) \times k$ matrix with the related factor loadings that capture the various common shocks contained in the productivity series. We impose a multi-factor structure on $\boldsymbol{S}_t$ and $\mathbf{\Lambda}_S$ where $\boldsymbol{S}_t$ includes one global, four regional and 29 country-specific common shocks. As in the statistical model of Section 2.2, the identification of the different common factors requires to restrict certain elements of $\mathbf{\Lambda}_S$ to 0, for example those that represent the factor loadings of $\zeta^j_{n,t}$ to a regional factor different from the region country n belongs to, or to a country factor different from country n.

Combining equation (12) and (14) yields a structural dynamic factor model that can be mapped into a formulation of IP growth akin to the factor model introduced in section 2.2. To see this first rewrite equation (12) as

$$\triangle\, \boldsymbol{y}_t = (\boldsymbol{I} - \mathbf{\Phi}_1 L)^{-1}(\mathbf{\Phi}_2 + \mathbf{\Phi}_3 L)\boldsymbol{\zeta}_t \tag{15}$$

Then, equation (14) and (15) imply the structural factor model:

$$\triangle\, \boldsymbol{y}_t = (\boldsymbol{I} - \mathbf{\Phi}_1 L)^{-1}(\mathbf{\Phi}_2 + \mathbf{\Phi}_3 L)(\mathbf{\Lambda}_S \boldsymbol{S}_t + \boldsymbol{\epsilon}_t) \tag{16}$$

Rearranging yields a formulation of IP growth that maps with a conventional statistical factor model:

$$\triangle \boldsymbol{y}_t = \boldsymbol{\Lambda} \boldsymbol{S}_t + \boldsymbol{v}_t, \tag{17}$$

where

$$\boldsymbol{\Lambda} = (\boldsymbol{I} - \boldsymbol{\Phi}_1 L)^{-1} (\boldsymbol{\Phi}_2 + \boldsymbol{\Phi}_3 L) \boldsymbol{\Lambda}_S$$

and

$$\boldsymbol{v}_t = (\boldsymbol{I} - \boldsymbol{\Phi}_1 L)^{-1} (\boldsymbol{\Phi}_2 + \boldsymbol{\Phi}_3 L) \boldsymbol{\epsilon}_t.$$

The structural model has a conventional factor structure, but its residual v_t does not isolate true idiosyncratic sector shocks. Instead v_t embeds both the true idiosyncratic shocks ϵ_t *and* their propagation through trade linkages. A proper statistical factor model should make allowances for this propagation: the reduced form factor model in Section 2.2 does not. Instead it attributes all of the common components in v_t to common factors even though some of them are in fact an outcome of the propagation of idiosyncratic sector shocks, country shocks, or regional shocks. Thus the reduced form factor model over-estimates the magnitude of common shocks.

We quantify the magnitude of the bias by comparing the properties of the structural form residuals $\boldsymbol{\epsilon}$ and the reduced form residuals from Section 2.2. We first solve for $\boldsymbol{\Pi_1}$, $\boldsymbol{\Pi_2}$ and $\boldsymbol{\Pi_3}$ implied by the multi-country multi-sector RBC model as described in Appendix C. Using this set of matrices, we filter $\boldsymbol{\zeta}_t$ from observed IP growth rates, using the filtering equation (13).[8] Finally, we apply a factor model to the estimated productivity shocks

[8] When they identify $\boldsymbol{\zeta}_t$ Foerster et al. (2011) initialise $\boldsymbol{\zeta}_0$ with 0. In our application we experienced that a few eigenvalues of $\boldsymbol{\Pi_3}^{-1}\boldsymbol{\Pi_2}$ lie outside the unit circle. Atalay (2017) notes that in those cases, data on IP growth rates cannot fully identify alone the productivity shocks since the "poor man's

as described in equation (14) in order to separate $\boldsymbol{\zeta}_t$ into common and true idiosyncratic shocks. This yields an empirical estimate of ϵ_t, which we then compare with the results from the naive factor analysis presented in Section 2.2.

3.3 Calibration

The estimation of the filtered series of productivity shocks $\boldsymbol{\zeta}_t$ in equation (13) requires the calibration of the matrices $\mathbf{\Pi}_1$, $\mathbf{\Pi}_2$ and $\mathbf{\Pi}_3$. Following Atalay (2017) we set $\beta = 0.96$, and $\delta = 0.1$. We now describe our other calibration choices.

Factor shares

Estimates of α_n^j are obtained from the Socio-Economic Accounts published as part of WIOD. They are computed as (one minus) the shares of labor compensation as a fraction of sector value added. Negative measures are set to zero.

Elasticities of Substitution

In our baseline specification, we assume $\epsilon_Q = 1$ and $\epsilon_{LS} = 2$ which correspond to the values in Atalay (2017). For the elasticities of substitution between the various intermediate goods, we follow Bonadio et al. (2023) and Baqaee and Farhi (2019a), setting $\epsilon_M = 0.2$. This follows from the estimates of ϵ_M obtained in Atalay (2017) or Cravino and Sotelo (2019). We set $\epsilon_T = 0.5$ based on the estimates in Boehm et al. (2023): They estimate $\epsilon_T = 0.26$ on impact and $\epsilon_T = 0.76$ after one year. We have quarterly data and so take an average between these two values. We set ϵ_X to 0.8, which corresponds to the

invertibility condition" in Fernández-Villaverde et al. (2007) is violated. He shows that treating the initial productivity shock as unknown state and applying the Kalman filter in order to use IP growth data in each period to iteratively calculate the productivity innovation in each date is a robust approach. We follow Atalay (2017) and and apply the Kalman filter to produce estimates for the initialisation of the productivity innovations.

estimate for a non-differentiated intermediate input substitution elasticity in Huo et al. (2023). Appendix G.4 presents extensive robustness analysis around alternative values for the elasticities.

Intermediate and Final Expenditure Shares

We calibrate the taste shifters $\boldsymbol{\mu}$, $\boldsymbol{\Gamma^{M1}}$ and $\boldsymbol{\Gamma^{M2}}$ jointly with the corresponding trade costs. Appendix E details how the model's equilibrium conditions at the steady state can be manipulated to yield a mapping between the shifters-cum-trade-costs and observed expenditure shares. In particular, we show that

$$\mu_{mn}^{ij}([\tau^M]_{mn}^{ij})^{1-\epsilon_T} = \left(\frac{[P^M]_n^{ij}}{P_m^i}\right)^{1-\epsilon_T} \times \frac{[\tau^M]_{mn}^{ij} P_m^i M_{mn}^{ij}}{[P^M]_n^{ij} M_n^{ij}}, \tag{18}$$

so that the shifter-cum-trade-cost $\mu_{mn}^{ij}([\tau^M]_{mn}^{ij})^{1-\epsilon_T}$ maps into the fraction of intermediate inputs of variety i that are imported from country m into country-sector nj. Similarly,

$$\mu_n^{ij} = \left(\frac{[P^M]_n^j}{[P^M]_n^{ij}}\right)^{1-\epsilon_M} \times \frac{[P^M]_n^{ij} M_n^{ij}}{[P^M]_n^j M_n^j}, \tag{19}$$

so that μ_n^{ij} maps into the fraction of intermediate inputs used in country-sector nj that are purchased from sector i. And

$$\mu_n^j = \left(\frac{P_n^j}{[P^M]_n^j}\right)^{1-\epsilon_Q} \times \frac{[P^M]_n^j M_n^j}{P_n^j Y_n^j}, \tag{20}$$

so that μ_n^j maps into the cost of intermediate inputs in country-sector nj.

The final expenditure share are given by the (steady state) equilibrium condition

$$\xi_{mn}^j = \frac{[\tau^C]_{mn}^j P_m^j C_{mn}^j}{[P^C]_n C_n^j}, \tag{21}$$

whose derivation is left for Appendix E. Because utility is logarithmic, ξ^{j}_{mn} can be measured directly as the share of consumption expenditure by households, government and non-profit organisations in country n on goods produced by country-sector mi.

All expenditure shares are calibrated from long-run averages of the World Input Output Database (Timmer et al., 2015). The World Input Output Database (WIOD) is available for the period 2000 - 2014 at an annual frequency. Three out of the four conditions include expressions for relative prices, whose values at the steady state are also inferred from WIOD. WIOD contains information on sectors and countries for which we do not have industrial production data. In these instances, we set the relevant values in WIOD data to zero. We then normalize the non-zero values of $\tilde{\mu}^{ij}_{mn}$ by its column-wise sum, so that $\sum_m \tilde{\mu}^{ij}_{mn}$ continues to equal one. This is necessary to maintain the assumption of constant returns to scale in production.

Capital Expenditures Shares

By analogy with the previous section, the calibration of the taste shifter for capital goods κ^{ij}_{mn} has to be done jointly with capital trade costs τ^X. Appendix E derives the following mapping between the shifter-cum-trade-cost and observable expenditure shares:

$$\kappa^{ij}_{mn}([\tau^X]^{ij}_{mn})^{1-\epsilon_X} = \left(\frac{[P^X]^j_n}{P^i_m}\right)^{1-\epsilon_X} \times \frac{[\tau^X]^{ij}_{mn} P^i_m X^{ij}_{mn}}{[P^X]^j_n X^j_n}. \tag{22}$$

The difference with the calibration of the other expenditure shares is that there is no international data for capital expenditures that could help measure the importance of output in country-sector mi as a source of investment for country-sector nj. WIOD does include information on Gross Fixed Capital Formation (GFCF), collecting the value of production at the source country-sector mi, and the value of its sales to a destination country n. However, there is no breakdown into destination sectors of sale, which is instrumental to calibrating $\mathbf{\Gamma}^{\boldsymbol{X}}$ and $\boldsymbol{\tau}^{\boldsymbol{X}}$.

We use three proxies to attribute capital good sales to sector level destinations: (i) BEA capital flow data, (ii) Eurostat outward Foreign Direct Investment data and (iii) Refinitiv Eikon Mergers and Acquisitions data.

BEA capital flow data specifies the value of purchases by US sector j of investment goods produced in sector i located in any country, including the US. We exploit the data to add the sector dimension missing from the destination of GFCF as reported in WIOD. We do this imposing that the sector-level breakdown of investment destinations observed by the BEA applies to WIOD data into the US. In particular, we apply the fraction

$$\frac{\sum_m \tilde{X}^{ij}_{m,US}}{\sum_{j,m} \tilde{X}^{ij}_{m,US}},$$

computed from BEA data, to GFCF from country-sector mi into the US as reported by WIOD. The assumption is that all exporting countries m supply capital goods from sector i to US sector j following the pattern reported by the BEA. This gives rise to a breakdown of GFCF from country-sector mi into US sectors j.[9]

We use the same combination of WIOD and BEA data to impute domestic investment flows across sectors for all countries in the sample. We impose that the sector-level breakdown of investment destinations observed by the BEA applies to WIOD data for any country m. In particular, we now apply the fraction

$$\frac{\sum_m \tilde{X}^{ij}_{m,US}}{\sum_{j,m} \tilde{X}^{ij}_{m,US}},$$

computed from BEA data, to GFCF from country-sector mi into country m as reported by WIOD. This gives rise to a breakdown of GFCF from country-sector mi to the sectors

[9] BEA capital flow tables are also used for the paramaterization of sector to sector investment linkages in the closed economy models of Foerster et al. (2011) and Atalay (2017).

j of country m. The underlying assumption is that the domestic investment structure across sectors is the same in the US and in other countries. Similarly Atalay (2017) uses BEA capital flow tables when applying his closed economy model to non-US countries.

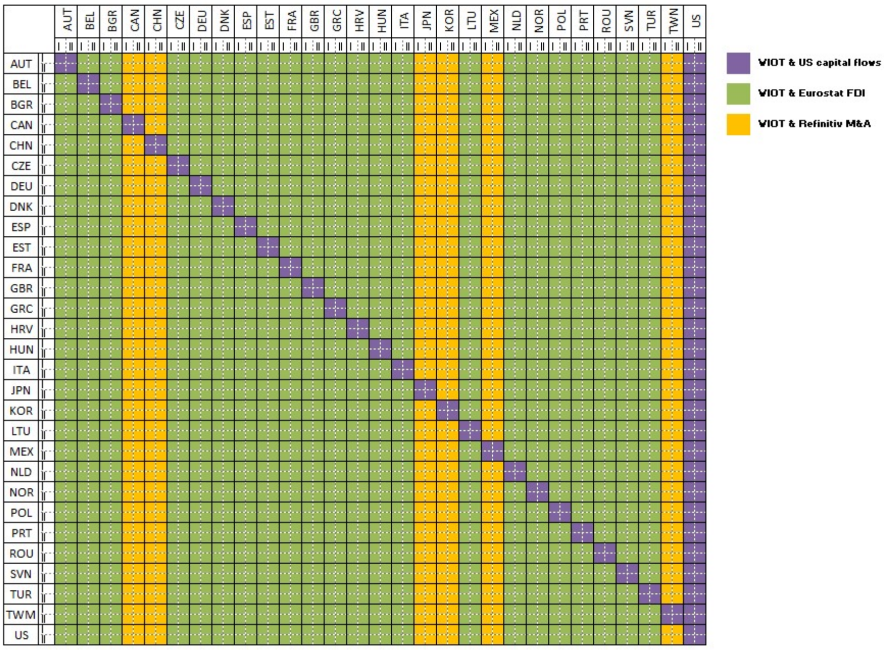
This approximates international investment linkages from the world into the US and domestic investment linkages within all countries in the sample. For other bilateral flows, we resort to Foreign Direct Investment data as reported by Eurostat. Eurostat reports how much country-sector nj purchases (foreign direct) investment goods from country m, across all of its sectors i. The coverage for country n is typically limited to European Union members, even though m includes other countries, e.g., the US. We compute the ratio

$$\frac{\sum_i \tilde{X}^{i,j}_{m,n}}{\sum_{j,i} \tilde{X}^{i,j}_{m,n}},$$

using Eurostat data, which characterizes the allocation of (foreign direct) investment purchased by country-sector nj across supplying countries m. Now GFCF from WIOD measures how much country n buys investment goods from country-sector mi, i.e., it is missing the very dimension that is provided by the ratio implied by Eurostat data. Applying this ratio to GFCF flows approximates the bilateral dimension we need to infer $\mathbf{\Gamma}^{\boldsymbol{X}}$ and $\boldsymbol{\tau}^{\boldsymbol{X}}$. The approximation is valid under the assumption that the observed international allocation of FDI is proportional to the unobserved breakdown of international capital goods buyers at sector level. Eurostat FDI data is only available for 2008 - 2012 so we compute the ratio $\frac{\sum_i \tilde{X}^{i,j}_{m,n}}{\sum_{j,i} \tilde{X}^{i,j}_{m,n}}$ averaging Eurostat data over this period.

Given that Eurostat data are restricted to investment outflows from European countries, we resort to Mergers and Acquisitions (a subset of FDI) data as reported by Refinitiv Eikon for invesment outflows from Canada, China, Japan, Mexico, Korea and Taiwan. The Mergers and Acquisitions data are available at the same granularity as Eurostat data and for 2006 - 2022. We compute again the ratio $\frac{\sum_i \tilde{X}^{i,j}_{m,n}}{\sum_{j,i} \tilde{X}^{i,j}_{m,n}}$ averaging Refinitiv

Figure (2) Data availability and sources for investment matrix



Note: Schematic overview of data availability for the construction of $\tilde{\Gamma^X}$.

Eikon data over this period. Figure 2 presents the availability and sources of data used for the construction of the capital flow table.

McGrattan and Schmitz (1999) show that maintenance expenditures are largely absent from capital flow tables. They document that much maintenance and repair happens intra-sector. Following Foerster et al. (2011), we add 25 percent of sector investment expenditures to the diagonal of the estimated capital flow table. We complete the computation of the matrix of capital expenditure shares by normalizing its elements by their column-wise sum.

Figure 3 displays log-values of the expenditure shares in intermediate, capital, and final goods for the G7 countries. Each chart shows in the form of a heatmap the importance of national and foreign production sources for intermediate, investment, and final goods. All heatmaps confirm the prevalence of domestic linkages. Both trade in intermediates and final consumption goods showcase the importance of the US and Germany, as well as trade within Europe. Canada and the US are also exceptionally linked together in intermediate goods trade. The heatmap for final consumption goods does not have variation across columns, since final goods destinations are recorded at country level only. Trade in capital goods concentrates in a few sectors, e.g., machinery and equipment, computer electronics, electronic and optical products, and fabricated metal products.[10]

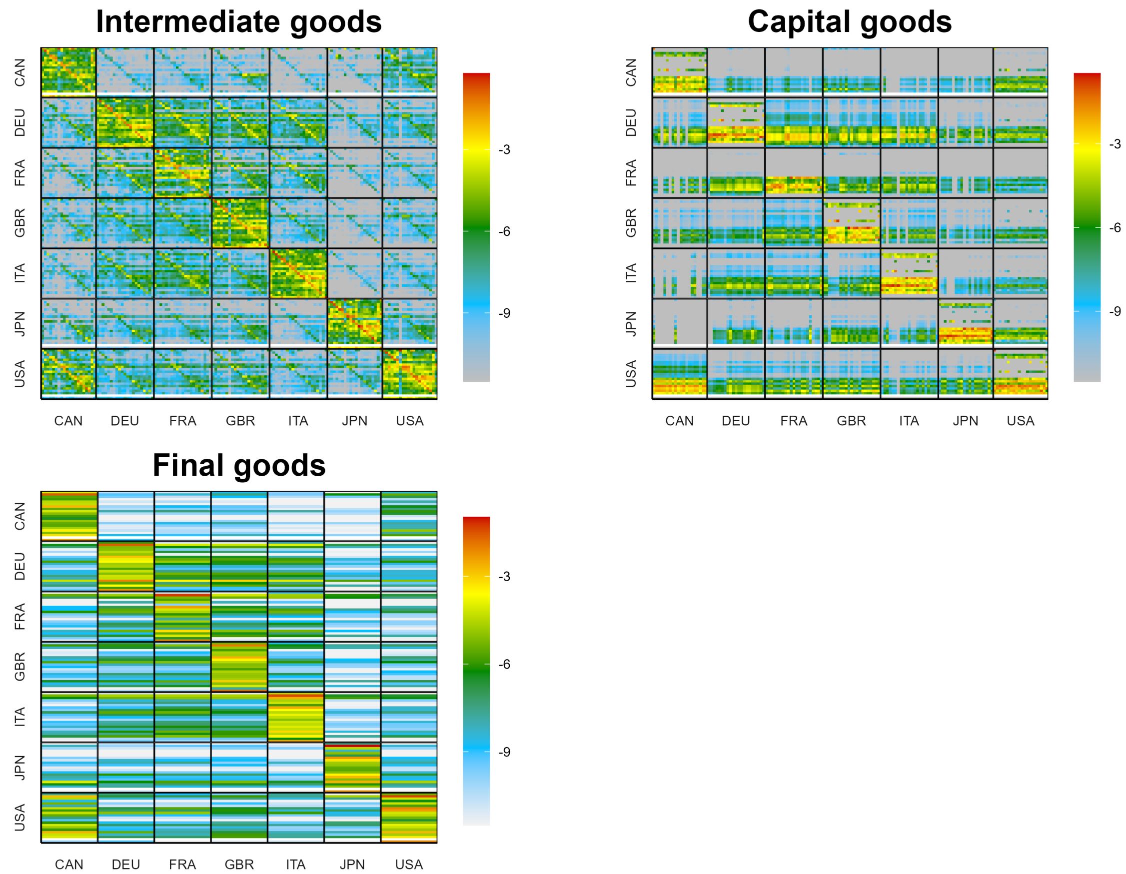
4 Results

We first verify how the factor model fits the data. We then exploit the decompositions implied by the factor models to document the proportions of empirical moments that are explained by global, regional, country, and sector shocks. We compare the decompositions implied by the structural and reduced form models. The structural model apportions propagating shocks where they belong, i.e., to the level at which the original shock occurs. But the reduced form model apportions propagating shock to common, i.e., country, regional, or global shocks.

We then examine the decompositions implied by a closed economy version of the structural model in which there is no specific external sector. By construction the shocks that exist in such an environment can occur at either sector or "aggregate" level, where the latter is often taken to mean country level. In the closed economy, any shock that propagates from the external sector and affects more than a few domestic sectors is an "aggregate" shock. This is mislabeling at two levels: First, it is not recognizing the shock's true geographic origin, second it is not recognizing the shock's true aggregation

[10] The investment matrix suggests that certain sectors do not supply any investment goods domestically, while the same sectors appear to supply investment goods internationally. The reason is that the BEA tables exhibit a number of rows with zero entries, e.g., manufacturing of food, beverage and tobacco products, basic pharmaceutical products and pharmaceutical preparations or coke and refined petroleum products. On the other hand, Eurostat data has non-zero entries for these sectors, but they are in fact very close to zero, below 0.01 percent.

Figure (3) The Calibration of sector linkages



The shading refers to the logarithm of the value of intermediate goods, capital goods and final goods shares. For a better readability of the heatmaps, we bounded small values at 10e-6.

level - viz. foreign sector, foreign country, foreign region, or global shock.

4.1 The Fit of the Model

We verify if the full model is able to replicate empirical moments. For example, the assumption that the residuals $\boldsymbol{\epsilon}_t$ in the structural factor model are idiosyncratic does not necessarily hold in the data: In practice it is possible that some correlation persists in the residual $\boldsymbol{\epsilon}$, which would affect the fit of the model.

Table 3 presents empirical and theoretical estimates of standard deviations, computed at country, regional and global levels of aggregation, and pairwise Pearson correlation coefficients, computed between sectors and averaged at the same three levels of aggregation. Columns (1) and (2) document the empirical values of both moments. The average standard deviation at country level is 6.9 percent, down to 5.9 percent at regional level, and 5.0 percent globally. Pairwise correlation coefficients are on average 0.29 at country level, 0.20 within regions, and 0.16 globally. Both moments fall with aggregation, a direct implication of the law of large numbers.

Columns (3) and (4) report the same moments as implied by the full model. The numbers suggest that the propagation mechanisms considered in the model, along with the factor structure in equation (14) do a good job of reproducing the data at all three levels of aggregation. If anything the model tends to over-predict the data. We checked that this comes from some negative covariance in the residuals $\boldsymbol{\epsilon}$, which exists in the data but not in the model and tends to push up simulated moments relative to empirical ones. We computed the empirical standard deviations of $\boldsymbol{\epsilon}$ at country level, with or without covariance terms. In most countries the two measures are closely related, which vindicates the factor model. But the difference is large in the UK and China, where

Table (3) Fit of the Model

	Data		Full Model	
	$\bar{\rho}$	σ	$\bar{\rho}$	σ
	(1)	(2)	(3)	(4)
Country				
USA	0.33	5.5	0.34	6.4
CAN	0.26	9.3	0.26	9.8
MEX	0.24	6.5	0.30	8.0
DEU	0.28	9.8	0.36	11.2
ITA	0.34	10.4	0.38	11.6
GBR	0.09	7.7	0.15	10.3
FRA	0.26	6.5	0.33	8.3
CHN	0.34	3.2	0.43	5.6
JPN	0.32	8.7	0.38	10.0
KOR	0.21	10.5	0.31	13.5
Average	**0.29**	**6.9**	**0.35**	**8.5**
Region				
Western Europe	0.17	7.5	0.19	8.8
Central and Eastern Europe	0.17	9.0	0.17	9.7
Americas	0.25	5.6	0.26	6.4
Asia	0.19	4.6	0.21	6.2
Average	**0.20**	**5.9**	**0.22**	**7.1**
Global	**0.16**	**5.0**	**0.15**	**5.6**

$\bar{\rho}$ indicates the average pairwise correlation of sector IP growth within countries, regions or globally. σ indicates the standard deviation of aggregated IP growth at the country, region or global level. Rows "Average" report a value added weighted average across all *within* country (region) pairwise correlations and fluctuations of aggregate country (region) IP growth.

the standard deviations without covariance is larger. This explains the slight over-fit documented in Table 3.

4.2 Variance Decompositions

We exploit the model to generate variance decompositions at various aggregation levels. Our purpose is to compare model generated moments that account for endogenous propagation with reduced form moments computed directly from the data.

The elements of the structural factor model in equation (14) are orthogonal by construction. Appendix D describes in detail how we exploit this property to perform a decomposition of the overall variance covariance matrix of industrial production into the variances of global, regional, country, and sector shocks. The decomposition is performed on “filtered” series, i.e., using the model to eliminate propagation mechanisms from productivity shocks.

Analogously to section 2.2, we decompose volatility into the contributions of true global, true regional and true country factors. We perform variance decomposition of volatility measured at each level of aggregation, that is at the country, region and global level. See Appendix D for details.

Table 4 presents variance decompositions at various aggregation levels as predicted by the reduced form model in Section 2.2, compared with the structural model in Section 3.2. The table presents evidence for the largest 10 countries in the sample, see Appendix G.2 for results for the full sample. Columns (1) to (4) presents the variance decompositions according to the reduced form factor model, columns (5) to (8) report the decompositions according to the structural model, and columns (9) to (12) present the absolute difference between the two.

The first result is that propagation makes a large difference for the magnitude of overall "common" shocks. At country level, the prevalence of common shocks goes on average from 89 to 58 percent of variance when propagation is accounted for. In virtually all cases, common shocks explain less than two thirds of country-level volatility after correction for shock propagation, down from very high levels. For example, in the US common shocks explain 92 percent of aggregate fluctuations, but only 64 percent after the correction. In Germany the proportion falls from 89 to 69 percent; from 94 to 39 percent in China. These are large corrections, which imply that a large share of country-level volatility is in fact due to sector shocks. And these corrections concern all countries in the sample, large or small: Global propagation mechanisms affect the decomposition of variance for so-called closed and open economies alike. That is because closed economies are often construed to be so on the basis of direct measures of trade whereas what is at play in Table 4 is the prevalence of indirect trade.

The correction at country level is caused by the fact that naive factor models wrongly apportion propagating sector shocks to global, regional and country factors: Of the three common shocks we consider the difference between naive and corrected variance decomposition is on average largest for global shocks (see columns (9) and (10)). This is particularly clear for large European economies - France, Germany, or Italy - where almost all of the change in the common component comes from a change in the global component. This result probably comes from the emergence of global supply chains, which make it easy to confuse sector with global shocks absent a structural model. Some differences exist across countries, however. For example in Korea, Japan, Canada, or Mexico it is country shocks whose importance decreases dramatically in the structural model. These countries are exceptionally open to intermediate trade, across many sectors, so that propagating sector shocks are mistaken for specific developments at country level. The correction in China, the largest in our sample, comes from the most part from a

Table (4) Variance Decomposition in Structural and Reduced Form Models

	$\boldsymbol{F}$ (1)	$\boldsymbol{F}^{\mathrm{G}}$ (2)	$\boldsymbol{F}^{\mathrm{R}}$ (3)	$\boldsymbol{F}^{\mathrm{C}}$ (4)	$\boldsymbol{S}$ (5)	$\boldsymbol{S}^{\mathrm{G}}$ (6)	$\boldsymbol{S}^{\mathrm{R}}$ (7)	$\boldsymbol{S}^{\mathrm{C}}$ (8)	Δ (9)	Δ^{G} (10)	Δ^{R} (11)	Δ^{C} (12)
Country												
USA	92%	59%	10%	23%	64%	36%	9%	18%	28%	22%	1%	5%
CAN	87%	51%	1%	35%	67%	49%	6%	12%	20%	2%	-6%	24%
MEX	87%	55%	4%	27%	75%	44%	21%	10%	12%	11%	-16%	17%
DEU	89%	80%	5%	4%	69%	43%	14%	13%	20%	37%	-9%	-9%
ITA	95%	67%	3%	25%	66%	40%	9%	17%	29%	27%	-6%	8%
GBR	49%	23%	3%	23%	46%	21%	0%	24%	3%	2%	3%	-2%
FRA	88%	74%	5%	9%	65%	35%	11%	19%	23%	38%	-5%	-10%
CHN	94%	6%	48%	39%	39%	2%	10%	28%	54%	5%	38%	12%
JPN	93%	40%	28%	25%	65%	18%	37%	9%	28%	22%	-10%	16%
KOR	87%	40%	25%	22%	59%	21%	29%	9%	28%	19%	-4%	13%
Country	**89%**	**43%**	**19%**	**26%**	**58%**	**26%**	**13%**	**19%**	**31%**	**17%**	**6%**	**7%**
Region												
W. Europe	96%	91%	2%	3%	77%	53%	10%	13%	19%	37%	-8%	-10%
C.&E. Europe	97%	85%	3%	9%	79%	70%	3%	6%	18%	15%	0%	3%
Americas	94%	66%	8%	20%	70%	44%	11%	15%	24%	22%	-3%	5%
Asia	96%	39%	52%	5%	62%	13%	34%	15%	34%	26%	18%	-10%
Average	**95%**	**61%**	**23%**	**10%**	**69%**	**34%**	**19%**	**15%**	**26%**	**27%**	**4%**	**-5%**
Global	**98%**	**84%**	**10%**	**4%**	**78%**	**54%**	**12%**	**11%**	**20%**	**30%**	**-2%**	**-8%**

Columns (1) to (4) report variance decompositions for the reduced form model as implied by global, regional and country common factors. Column (1) summarizes the contribution of a "common" factor, summing columns (2), (3), and (4). Columns (5) - (8) report the corresponding results for the structural form factor model. Columns (9) - (12) report the absolute differences. "Average" reports value added weighted averages of the country and region-level variance decompositions.

misallocation of regional shocks: There, too, it is intuitive to ascribe this to the regional integration between China and its satellite trade partners in Southeast or East Asia.

The importance of common shocks is also vastly overestimated at regional and global levels. A naive statistical model ascribes more than 95 percent of the volatility at supranational level (regional or global) to common shocks. For example, in Western Europe, according to a naive factor model 96 percent of volatility is explained by common shocks, but only 77 percent in the structural model. In the Americas (Canada, Mexico, the US) the percentage falls from 94 to 70 percent. Strikingly, common shocks explain "only" 78 percent of global fluctuations, against 98 percent in a naive model: A quarter of the volatility in the global cycle should in fact be ascribed to the propagation of sector-level shocks.

For regional and global volatilities, the corrections arising from the structural model have mostly to do with the magnitude of global and regional shocks. For global volatility for example, roughly the entire reduction in the importance of common shocks comes from a reduction in global, not country or regional, shocks. In Europe, most of the decrease in the importance of common shocks comes from a decrease in the importance of global shocks, as seen from columns (9) and (10). But in Asia it is regional shocks that reflect propagation mechanisms, another indication of the strong regional dimension of supply chains in these regions of the world.

In unreported results, we examined the decompositions of correlations computed at various aggregation levels. We performed the same comparisons between the reduced form vs. the structural models as in Table 4 and unsurprisingly reached similar conclusions in terms of the importance of different shocks. One specific moment of great importance is the international co-movements in macroeconomic fluctuations, the main object of analysis in Huo et al. (2023). Like them, we find that roughly four-fifths of these correlations are driven by correlated shocks. It is interesting that we should reach such similar conclusions, since our model assumes complete markets whereas Huo et al. (2023) impose financial autarky.

4.3 Closed economy comparison

So far the comparison has been between two classes of open economy models: The naive statistical model that decomposes fluctuations into global, regional, country, and sector factors, vs. the structural factor model that accounts for international propagation mechanisms. An equally interesting comparison is between the open economy model with rich international propagation mechanisms and a closed economy model with only domestic propagation. There are reasons why the latter model can give exaggerated importance to aggregate shocks (i.e., to country shocks in a closed economy environment).

In a closed economy model, any foreign shock that affects more than a few domestic sectors will be mis-categorized: Because of its foreign origin the shock is disqualified from the domestic propagation mechanisms that the closed economy model allows for. Inasmuch as it affects several domestic sectors simultaneously, this foreign shock is simply categorized as an aggregate, or country shock. This can be true of global, regional, or simply foreign idiosyncratic shocks. The issue is typically assumed away on grounds that for large economies like the US the external sector is often thought to be small and largely irrelevant. But because of existing intricate international linkages, foreign shocks, even if they occur at sector level, do tend to affect several domestic sectors simultaneously. And therefore end up being classified as country shocks in the closed economy model that only accounts for domestic input-output linkages. This has potentially far ranging implications for the prevalence of aggregate, country-level shocks, even in economies that are customarily assumed to be closed.

To evaluate this potential concern, we construct a version of the model that assumes away the external sector. In practice, the model replicates the closed economy version setup proposed by Atalay (2017) for all the individual countries in our sample, assuming away all their international linkages. We do this following exactly the treatment of the external sector in Foerster et al. (2011): For each country-sector, we aggregate all supplying sectors whether they are located in the domestic country or abroad. For example, we calibrate intermediate cost shares in the model (inclusive of trade costs) by computing $\mu^{ij}_{nn} = \sum_m \mu^{ij}_{mn}$.[11] The elasticities ϵ_Q, ϵ_{LS} and ϵ_X take the same values as in the open economy model. On the other hand, ϵ_T is not well defined in the closed economy. We consider $\epsilon_M = 1$, 0.5, and 0.2, which covers most of the estimates in the literature. As in the open economy model, we decompose the model-implied productivity shocks into a common and and an idiosyncratic component. We calculate model implied moments and variance decompositions using the thus defined closed economy model individually for each country in our sample.[12]

Table 5 describes the ability of the closed economy model to replicate empirical moments. Columns (1) and (2) replicate the empirical moments already reported in Table 3. Columns (3) and (4) report the model-implied moments. The fit is good, which is not surprising since the ARMA(1,1) structure used to filter out structural shocks continues to hold in a closed economy, and the factor model is identical to its open economy version but for the fact that it subsumes regional and global shocks into "aggregate" shocks.[13] Care is in order when interpreting this relative goodness-of-fit: it suggests the model adequately identifies aggregate shocks, which appear to matter to a first order in the data.

[11] We also explore setting all intermediate trade to zero whenever it crosses a border, which requires adequate normalization as discussed in Section 3.3. The paper's conclusions are largely unchanged.

[12] In closed economy models, we apply the calibration approach described in Foerster et al. (2011), which requires an adequate normalisation of sector output. The normalisation approach does not affect the differences in results between the closed economy and international model; If anything the gap between the two models increases somewhat. See Appendix F for details.

[13] The model is still over-fitting somewhat the data, for the same reason as in the open economy version.

Table (5) Fit of the Closed Economy Model

	Data		Full Model	
	$\bar{\rho}$	σ	$\bar{\rho}$	σ
	(1)	(2)	(3)	(4)
USA	0.33	5.5	0.35	6.0
CAN	0.26	9.3	0.32	10.8
MEX	0.24	6.5	0.42	10.3
DEU	0.28	9.8	0.37	11.8
ITA	0.34	10.4	0.41	12.4
GBR	0.09	7.7	0.22	12.4
FRA	0.26	6.5	0.37	9.1
CHN	0.34	3.2	0.46	5.2
JPN	0.32	8.7	0.42	10.9
KOR	0.21	10.5	0.36	14.6
Country	**0.29**	**6.9**	**0.38**	**8.9**

$\bar{\rho}$ indicates the average pairwise correlation of sector IP growth within countries. σ indicates the standard deviation of aggregated IP growth at the country level. Row "Average" reports a value added weighted average across all *within* country pairwise correlations and fluctuations.

Table (6) Variance decompositions in the closed economy model

	International model				Closed economy model		
	S	S^{G}	S^{R}	S^{C}	$\epsilon_M = 1$	$\epsilon_M = 0.5$	$\epsilon_M = 0.2$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
USA	64%	36%	9%	18%	69%	59%	35%
CAN	67%	49%	6%	12%	66%	56%	6%
MEX	75%	44%	21%	10%	70%	61%	40%
DEU	69%	43%	14%	13%	78%	65%	3%
ITA	66%	40%	9%	17%	76%	50%	3%
GBR	46%	21%	0%	24%	6%	7%	1%
FRA	65%	35%	11%	19%	61%	42%	2%
CHN	39%	2%	10%	28%	56%	5%	4%
JPN	65%	18%	37%	9%	79%	61%	4%
KOR	59%	21%	29%	9%	68%	46%	12%
Country	**58%**	**26%**	**13%**	**19%**	**63%**	**40%**	**20%**

Columns (1) to (4) repeat the variance decompositions of country volatility as implied by the full international model. Column (5) reports the variance decompositions with respect to aggregate shocks in the closed economy model. Row "Average" reports a value added weighted average across variance decompositions.

But here aggregate shocks conflate shocks at the global, regional, and country levels, as well as foreign sector-level shocks that propagate across sectors of the domestic economy. So the decomposition is problematic even though it delivers a good fit.

Table 6 compares the variance decompositions implied by the full international model to a version with domestic linkages only. Columns (1)-(4) replicate the decompositions in the complete model, repeated from Table 4. Column (5)-(7) present the contribution of common shocks to country volatility as implied by the single factor on "aggregate" shocks, which subsumes country, regional, and global factors. Column (5) assumes a substitution elasticity of $\epsilon_M = 1$, columns (6) and (7) set $\epsilon_M = 0.5$ and $\epsilon_M = 0.2$.

There are two issues with the allocation of shocks implied by the closed economy model. Firstly, the closed economy model allocates global and regional shocks into "aggregate" shocks, which are typically interpreted as country-level shocks. So mechanically in the closed model country shocks include all global and regional shocks, on top of "true" country shocks (what we call "common" shocks elsewhere in the paper). Secondly, any foreign development that affects the domestic economy in more than a few sectors will also be classified as aggregate, i.e., a country shock. This acts to increase the importance of aggregate shocks in the closed economy, potentially above "common" (i.e., global, regional, and country) shocks in the full model.

The calibration of substitution elasticities affect both misallocations. High substitutability means that the propagation of foreign common and sector shocks is theoretically limited, so the closed economy model tends to interpret these shocks as common. As a result, the importance of common shocks in the closed economy model increases with the calibrated elasticity ϵ_M.

These mechanisms are apparent from Table 6: With $\epsilon_M = 1$ country shocks in the closed economy represent a significantly larger share of country volatility than "common" shocks do in the full model, 63 vs. 58 percent on average. This happens because with high elasticity, the closed model concludes there are many global and regional shocks, the only way to rationalize co-movements between sectors when propagation mechanisms are weak. When $\epsilon_M = 0.5$, the same calibration than in the full model, country shocks in the closed model actually represent less than common shocks do in the full model, 40 vs. 58 percent. This becomes even more salient with high complementarities.

Most interestingly, the importance of country shocks in the closed economy model is always larger than in the full model, irrespective of the calibration of ϵ_M. With high substitutability, the misallocation of shocks can be very large: The closed model ascribes 69 percent of US volatility to country shocks, while it is only 18 percent. In Japan, the closed model says 79 percent of volatility come from country shocks, as against 9 percent. The misallocations continue to be large for $\epsilon_M = 0.5$, for example in the US (59 vs. 18 percent) or in Japan (61 vs. 9 percent); They are even larger for "open" European economies or China. The discrepancy persists even under strong complementarities; For example the contribution of country shocks to US volatility is 35 percent in the closed economy model with $\epsilon_M = 0.2$, vs. 18 percent in the full model. [14]

4.4 Alternative specifications

In this section, we investigate the importance of two sets of assumptions. First, the calibration of the model relies on long-run averages of cost shares computed from the World Input Output Database. A concern is that averaging the cost shares over time may mask a decrease in trade costs, which in turn could affect the cost shares of goods produced abroad, with potential end effects on the results. To alleviate this concern, we perform two alternative calibrations of the model: once using WIOD tables for 2006 (the starting point of our sample) and once using WIOD tables for 2014 (the last year for which WIOD tables are available). We report results in Appendix G.3. We find that the variance decompositions corresponding to the alternative calibrations closely resemble that of the baseline model, so that falling trade costs within our sample period do not appear to have large consequences.

Next, we investigate how the variance decompositions implied by the full model depend on the calibrations of the elasticities of substitution $\epsilon_L S$, ϵ_Q, ϵ_M, ϵ_T, and ϵ_X. The results are reported in Appendix G.4. Table G.6 suggests $\epsilon_L S$ and ϵ_Q have minimal impact on the variance decompositions implied by the full model. On the other hand, the elasticities in input substitution ϵ_M, ϵ_T, and ϵ_X, do have an effect: The extreme cases $\epsilon_M = \epsilon_T = \epsilon_X = 1$ and $\epsilon_M = \epsilon_T = \epsilon_X = 0.2$ have substantially different implications

[14] When $\epsilon_M = 0.5$ the decomposition of US volatility implied by the closed economy model in Table 6 is very close to the result in Foerster et al. (2011). And it is very close to Atalay (2017) when $\epsilon_M = 0.2$, which is the calibration he uses.

on the role of global and regional shocks. This is not surprising since these parameters govern the extent of shock propagation.

For that reason, Figure G.3 plots the contributions of common shocks to global and to average country volatilities for values in the three input elasticities between 0 and 2. A salient result is that it is the value of ϵ_T that matters to a first order, whereas even relatively extreme values for ϵ_M and ϵ_X have little consequences. This is to be expected, since ϵ_T governs international shock propagation via intermediate trade. We know little about the value for this elasticity, especially independently from ϵ_M in the context of a two-tiered nested CES setup. However it is important to note that, even for high values of ϵ_T, the reduced form statistical model still gives too much prominence to global shocks.

5 A Historical Decomposition

We perform a historical decomposition implied by the structural factor model around well-known shocks and compare the results with those suggested by a reduced form international factor model, and by a structural model of a closed economy. The aim of this exercise is to illustrate how the different models categorize well-known historic events and assess the corresponding foreign spillovers.

We focus on the triple disaster in Japan: on March 11^{th}, 2011, the northeast of Japan was hit by the Tohoku earthquake and tsunami which ultimately led to a meltdown of a nuclear reactor in Fukushima. The economic effects were devastating. Japanese industrial production declined (in annualised terms) by 15% in Q1 2011 and by 16% in Q2 2011. Sectoral effects were even higher: Industrial production growth declined by up to 54% in the automotive sector and 33% in the electronics sector.

The tragedy of the event notwithstanding, the disaster represents a good example of a large exogenous shock with substantial effects on a single economy, which propagated through production networks to other countries. Unsurprisingly the event is much studied in the literature. For example, Boehm et al. (2019) find that U.S. affiliates of Japanese multinationals suffered large losses in their US output due to disruptions in the production of intermediate inputs in Japan.

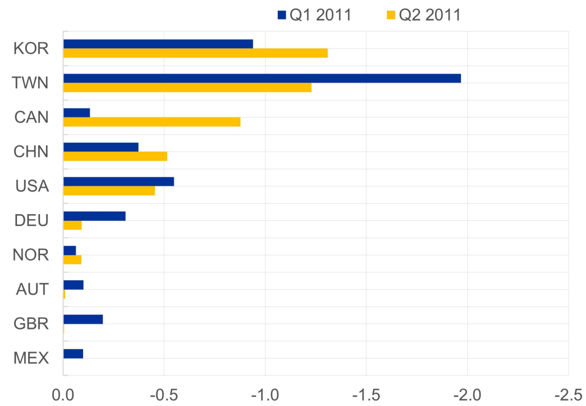
To discipline the selection of identified shocks that correspond to this disaster, we consider any (country or sector) shock identified in Japan that has a negative contribution to IP growth in 2011Q2. This procedure singles out a Japan-wide country shock and shocks in: manufacturing of refined petroleum, motor vehicles, electronics, basic metals, fabricated metals, printing, other transport equipment, and minerals and paper. Using the full model, we then single out the contributions of these shocks to the historical decompositions of 2011Q2 industrial production growth in all the countries and sectors in the sample. For completeness, we also consider historical decompositions of IP growth in 2011Q1 using these same shocks.[15]

We find significant and plausible international propagation effects from the natural disaster across countries. Figure 4 presents the contributions of the Japanese shocks associated with the Tohoku earthquake to aggregate country IP growth in other economies. For example in Q2 2011 the Tohoku shock decreased Taiwan's and Korea's industrial production growth by almost 1.5 percentage points. US growth was less but still significantly affected, with a contribution of -0.5 percentage points. We also find that country spillovers are higher for most countries in Q2 2011 than Q1 2011.

Next, we study the spillovers of the disaster at sector level. Figure 5 presents the contributions to US sector-level activity of Japanese shocks associated with the Tohoku

[15] Since the earthquake hit in March 2011, it is not clear whether its main consequences were recorded in 2011Q1 or Q2. In the main text, we have considered 2011Q2 because this is the quarter with largest changes in IP growth in Japan. The conclusions of the exercise are virtually identical if we use 2011Q1 instead as the criterion to select the earthquake shocks.

Figure (4) Contribution to country IP growth from Tohoku shocks



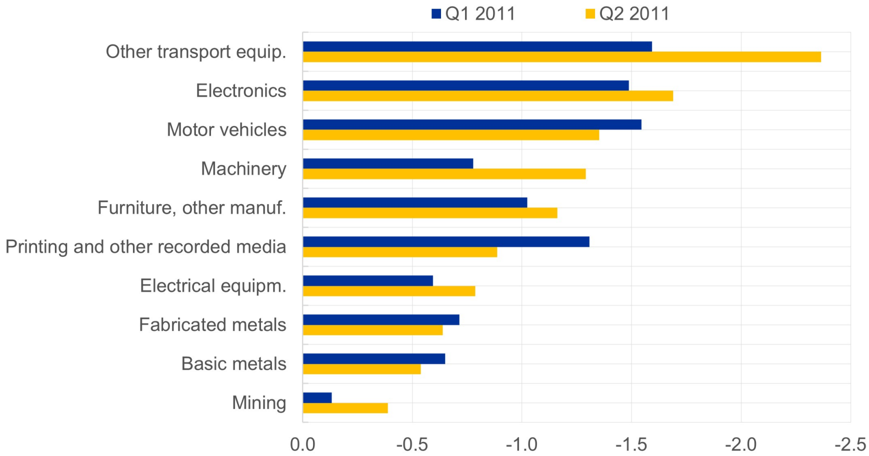
Note: IP growth is annualised. Aggregate IP growth for country n is calculated as $\sum_j^J w_n^j g_{n,t}^j$. The bars represent the effect from shocks associated with the Japanese triple disaster to the 10 most affected countries.

earthquake. We first note that sector effects are larger than the country effects in Figure 4, up to roughly 2.5 percentage points in the US. Second, the US industries most affected by the Tohoku shocks are once again plausible: Trade intensive manufacturing sectors, such as the machinery, electronics or other transport equipment sector. Third, the majority of the production losses in these sectors is due to either energy shocks in Japan caused by the earthquake that brought about significant power outages, or to disruptions in similar sectors in Japan and in the US. For example disruption in the motor vehicle and other transport equipment sectors in Japan caused roughly 25 percent of the total effect on the US other transport equipment sector.

Given our finding that a reduced form international factor model or a structural closed economy factor model engender substantial misallocation of idiosyncratic or aggregate shocks, it is natural to ask how these models perform in allocating the Tohoku earthquake shock. Figure 6 decomposes US aggregate country growth in Q2 2011 into its constituent shocks, through the lenses of the international structural, the international reduced form, and the closed economy structural models.[16]

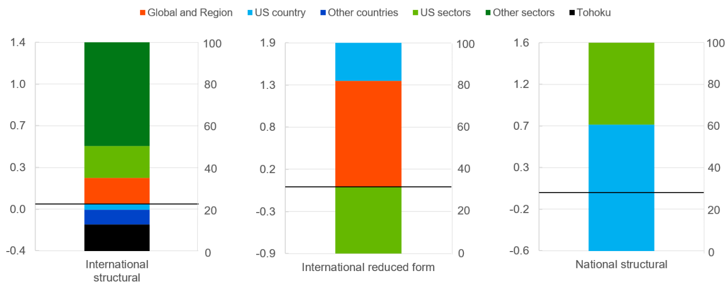
[16]The calibration of the closed economy model is the same as in the previous section. Since the three

Figure (5) Contribution to US sector IP growth from Tohoku shocks



Note: IP growth is annualised. The bars represent the effect from shocks associated with the Japanese triple disaster to the 10 individual most affected US sectors.

Figure (6) Decomposition of aggregate US IP growth in Q2 2011, by model



Note: The left y-axis indicates the model-implied IP growth (normalised to 1) and the right y-axis indicates the relative share of shock contributions. Tohoku refer to Japanese shocks associated with the earthquake. The implied US growth across models differs as both structural models slightly overestimate US growth.

The international structural model concludes that more than half of the negative shocks affecting the US in 2011Q2 originated in the Tohoku earthquake; The other half cam from (mostly foreign) country shocks. The bulk of the shocks affecting the US economy that quarter comes happened at sector level, mostly foreign but also domestic. Finally, there were some positive global developments, but quite small. Including the Tohoku shock,

models have different fits, they imply different IP growth rates in 2011Q2 in the US: They are normalized to 1 in the figure.

about three-quarters of the shocks affecting the US economy in Q2 2011 happened at sector level.

The reduced form factor model ascribes two-thirds of US IP growth in 2011Q2 to global and domestic country shocks. This corresponds to the inability of the statistical model to recognize the channels of propagation of (domestic or foreign) sector shocks, and labeling them global or country shocks according to the co-movements they generate. The result is a variance decomposition that gives undue importance to global shocks. The reduced form model also attributes all of the negative IP growth in 2011Q2 to domestic sector shocks, presumably because the Tohoku shock affected a limited number of sectors in the US, which the reduced form model classifies accordingly.

The closed economy model ignores foreign shocks altogether and interpret all contributions to US IP growth as either sector or country shocks. All of the US industrial production growth in 2011Q2 is forced to originate in US shocks only. For example, foreign shocks (that actually constitute half of the shocks affecting the US) are ascribed either to US country or sector shocks, depending on the cross-sector co-movements they actually generate. Similarly, global, foreign country, and Tohoku shocks are in fact classified as US country shocks, presumably because those shocks do actually affect many US sectors simultaneously. The closed economy model implies two severe misallocations: Firstly, global or foreign country shocks are part of US aggregate shocks for lack of an international dimension. Secondly, foreign sectors shocks (such as Tohoku) are wrongly allocated to US aggregate shocks because of their far ranging effects on US sectors. The result is a decomposition that gives an enormously biased importance to US country shocks.

Table (7) Second Moments under Alternative Scenarios

		$\bar{\rho}$	$\bar{\rho}/\bar{\rho}_{Benchmark}$	σ	σ(diag)	σ(scaled)	σ(scaled) / $\sigma_{Benchmark}$
		(1)	(2)	(3)	(4)	(5)	(6)
1.	Data	0.16		4.95			
2.	Benchmark	0.15		5.56	1.23		
No sector linkages (domestic and international):							
3.	τ^M,τ^X,τ^C	0.04	28%	2.69	1.24	2.66	48%
4.	τ_M	0.05	34%	2.54	1.05	2.97	53%
5.	τ_X	0.11	73%	5.21	1.36	4.72	85%
No international trade:							
6.	τ^M,τ^X,τ^C	0.08	53%	4.14	1.50	3.41	61%
7.	τ^M	0.09	59%	4.27	1.40	3.76	68%
8.	τ^X	0.14	93%	5.52	1.28	5.32	96%
9.	τ^C	0.15	100%	5.57	1.23	5.57	100%

Notes: The list of τs in each row indicate which transport costs have been modified in alternative scenarios. $\bar{\rho}$ and σ refer to average pairwise correlations and standard deviations of global implied IP growth respectively.

6 Understanding the mechanism

The results in the previous section hinge on three types of international trade linkages: in intermediates, in capital, and in final goods. A natural question is what propagation channels are determinant for the key results in this paper. We perform a number of alternative calibrations to evaluate the roles of different types of trade.

A natural way to do so is by modifying the trade costs in the model-implied cost shares for intermediate, capital, or final goods.[17] Given the modified trade costs we solve the international RBC model for a set of alternative $\mathbf{\Pi}_1^*$, $\mathbf{\Pi}_2^*$, and $\mathbf{\Pi}_3^*$ matrices. We then apply the recovered productivity shocks $\boldsymbol{\zeta}$ from the baseline scenario to the alternative $\mathbf{\Pi}^*$ matrices and compute the moments of implied (global) industrial production growth.

Table 7 reports average bilateral correlations computed across sector-level IP growth,

[17]Since in the representative agents' utility functions we have unit elasticities of substitution between final goods, trade costs for final goods would naturally cancel out in the final cost shares. Similarly to Huo et al. (2023), we construct the alternative scenario for zero trade in final goods as a limiting case with $\tau^{\mathbf{C}} \to \infty$ and the substitution elasticity between final goods approaching 1.

and the aggregate variance of IP growth for each of the alternative model specifications. Consider first the consequence on correlations, in columns (1) and (2). In row 3, we modify the costs of both domestic and international trade in all types of goods by setting to extreme values $[\tau^M]^{ij}_{mn}$ and $[\tau^M]^{ij}_{mn}$ for all $mi \neq nj$, and $[\tau^C]^{j}_{mn}$ for all $m \neq n$. This effectively shuts down any form trade in the model.[18] Row 3 in Table 7 indicates that the effects on pairwise correlations of sector growth rates would decrease to 28% of their initial value in the baseline model.

To differentiate what drives this, row 4 of Table 7 reports the pairwise correlation coefficient for an economy in which there is no domestic or international trade in intermediate good, while row 5 shuts down trade in capital goods. The strongest transmission mechanism works via intermediate trade: shutting down intermediate goods trade results in an average pairwise correlation equal to 34 percent of its value in the baseline case with trade, while it is still 73 percent of its baseline level without trade in capital goods.

Row 6 of Table 7 reports summary statistics for a scenario in which we restrict only *international* trade in intermediate, capital and final consumption goods, i.e., setting extreme values for $[\tau^M]^{ij}_{mn}$, $[\tau^M]^{ij}_{mn}$, and $[\tau^C]^{j}_{mn}$ for all $m \neq n$. Not surprisingly, we find that international trade drives a substantial amount of co-movement in the data: pairwise correlations drop by almost half compared to the benchmark scenario. Lines 7, 8 and 9 then investigate to what degree international trade in intermediates, capital, or final goods is associated with this loss in propagation. The Table shows unambiguously that it is international intermediate goods trade that is most important, with average pairwise correlations decreasing by 41 percent. Restricting international trade in capital goods lowers correlations by roughly 10 percent, and international trade in final consumption

[18] The extreme values for trade costs depend on the elasticities of substitution between intermediate, capital and final goods. With complementarities, the elasticities are below 1 and shutting down trade means setting trade costs close to zero. This is apparent from the expenditure shares: For example the expenditure share for capital goods $\pi^{ij}_{mn} = \frac{\kappa^{ij}_{mn}([\tau^X]^{ij}_{mn}P^i_m)^{1-\epsilon_X}}{\sum_{i,m}\kappa^{ij}_{mn}([\tau^X]^{ij}_{mn}P^i_m)^{1-\epsilon_X}}$ increases with τ^X when $\epsilon_X < 1$.

goods barely lowers them at all.

The results are very similar as regards the response of simulated volatility to the same changes in trade costs, as documented in columns (3) to (6) in Table 7. There is one methodological difference here, though: With different trade costs in the model the scaling of the filtered shocks change, since both the (diagonal) elements in the matrices $\mathbf{\Pi}$ and the filtered shocks depend on the values of the trade costs. Therefore in order to meaningfully compare variances across calibrations of trade costs, it is important to normalize the shocks so that the variance of simulated sector-level IP growth (but not their covariances) takes the same value as the one it takes in the benchmark in row 2. For example, in row 4, the shocks are scaled by a factor (1.17) so that the model-implied variance of IP growth without intermediate trade is equal to its benchmark value (1.17).[19] Then, any difference across models in the variance of simulated IP growth comes from differences in the covariance terms between sector growth rates, i.e., from differences in propagation.[20] Column (6) in Table 7 confirms that trade, and especially international trade has dramatic propagation effects: It is these propagation mechanisms that can explain the importance of covariances between sectors that Table 1 in Section 2.1 emphasized. Rows 6-9 confirm that it is mostly trade in intermediate goods that channels this propagation.

Table 7 documents the effect the different forms of trade have on average sector-level co-movements and volatility, across all dimensions. Table 8 focuses the question onto correlations within countries and within regions. The structure of the table is similar to Table 7, but focused on correlation coefficients for these two dimensions of the data. Column (1) replicates the correlation results in Table 7). It is immediately apparent

[19] $1.17 = \frac{1.23}{1.05}$

[20] This follows directly from Foerster et al. (2011). Of course the problem also exists when simulating correlations, but then the scaling of the shocks enters both the numerator and denominator, and therefore cancels out.

Table (8) Correlations under Alternative Scenarios: Regions and Countries

	Sector (1)	Regional Average (2)	Country Average (3)
No sector linkages (domestic and international):			
3. τ^M,τ^X,τ^C	28%	25%	22%
4. τ_M	34%	33%	31%
5. τ_X	73%	72%	74%
No international trade:			
6. τ^M,τ^X,τ^C	53%	55%	80%
7. τ^M	59%	61%	85%
8. τ^X	93%	94%	97%
9. τ^C	100%	100%	100%

Notes: The table states $\bar{\rho}/\bar{\rho}_{Benchmark}$ for alternative model specifications and for implied IP growth at the global, region and country level. Column (1) corresponds to column (2) in Table 7. Column (2) and (3) give the equivalent ratio for (value-added) region and country averages.

from the table that restrictions on domestic and international trade have broadly the same effect on correlations between sectors and between regions or countries. This is not necessarily surprising since here propagation channels are restricted both between and within countries. Interestingly, the correlation coefficients within country continue to fall dramatically, by about 20 percent, even when it is only international trade that is restricted. In other words, shocks that propagate through international sector linkages do also have a significant effect on sector co-movements within countries. This appears to be due mostly to international trade in intermediate goods.

7 Conclusion

Global shocks are not as prevalent as implied by simple statistical factor analyses. A quarter of the volatility in the global cycle originates in fact from idiosyncratic shocks, in specific sector and specific locations, that propagate through the global value chain. The role of global, regional, or country shocks in country-level volatility is similarly over-estimated, by about 30 percent. We demonstrate these facts in a multi-sector multi-country model with trade costs, which we use to filter out the propagation of shocks via supply chains from observed fluctuations in output. We also show that the model implies a factor structure akin to a simple statistical factor model, which facilitates the comparison between our results and those implied by reduced form estimations.

The fact that global shocks are not as important as we thought does not imply that closed economy structural models are sufficient to identify shocks precisely. The issue raised in reduced form factor models (mis-labeling sectors shocks as global shocks) carries through in closed-economy structural models, which mis-label sector shocks as country shocks. In an increasingly networked world, foreign shocks are increasingly likely to statistically resemble a domestic aggregate shock, even if domestic input-output linkages are modeled accurately, since a closed economy model is blind to the very origin of the shock. We show that the importance of country shocks is vastly over-estimated by a conventional structural closed-economy model, even for such large and typically "closed" economies as the US. Both over-estimations (of country and of global shocks) depend on the intensity of intermediate trade. The actual importance of country and global shocks is intimately related to the substitutability between intermediate goods, whose estimation across sectors and across countries should be high on the agenda for any student of economic fluctuations.

References

Acemoglu, D., U. Akcigit, and W. Kerr. 2016. Networks and the macroeconomy: An empirical exploration. *NBER Macroeconomics Annual* 30:273–335.

Ahn, S. C., and A. R. Horenstein. 2013. Eigenvalue ratio test for the number of factors. *Econometrica* 81:1203–1227.

Ambler, S., E. Cardia, and C. Zimmermann. 2002. International transmission of the business cycle in a multi-sector model. *European Economic Review* 46:273–300.

Atalay, E. 2017. How important are sectoral shocks? *American Economic Journal: Macroeconomics* 9:254–280.

Baqaee, D., and E. Farhi. 2019a. Networks, Barriers, and Trade. Tech. rep., National Bureau of Economic Research, Cambridge, MA.

Baqaee, D. R. 2018. Cascading Failures in Production Networks. *Econometrica* 86:1819–1838.

Baqaee, D. R., and E. Farhi. 2019b. Macroeconomics with Heterogeneous Agents and Input-Output Networks. *NBER Working Paper n.24684* pp. 1–60.

Baqaee, D. R., and E. Farhi. 2019c. The Macroeconomic Impact of Microeconomic Shocks: Beyond Hulten's Theorem. *Econometrica* 87:1155–1203.

Barrot, J. N., and J. Sauvagnat. 2016. Input specificity and the propagation of idiosyncratic shocks in production networks. *Quarterly Journal of Economics* 131:1543–1592.

Bigio, S., and J. La'O. 2020. Distortions in Production Networks*. *The Quarterly Journal of Economics* 135:2187–2253.

Blanchard, O. J., and C. M. Kahn. 1980. The Solution of Linear Difference Models under Rational Expectations. *Econometrica* 48:1305.

Boehm, C. E., A. Flaaen, and N. Pandalai-Nayar. 2019. Input Linkages and the Transmission of Shocks: Firm-Level Evidence from the 2011 Tohoku Earthquake. *Review of Economics and Statistics* 101:60–75.

Boehm, C. E., A. A. Levchenko, and N. Pandalai-Nayar. 2023. The Long and Short (Run) of Trade Elasticities. *American Economic Review* 113:861–905.

Bonadio, B., Z. Huo, A. A. Levchenko, and N. Pandalai-Nayar. 2023. Globalization, Structural Change and International Comovement. Tech. rep., Mimeo, Michigan, Yale, and UT Austin.

Burstein, A., C. Kurz, and L. Tesar. 2008. Trade, production sharing, and the international transmission of business cycles. *Journal of Monetary Economics* 55:775–795.

Carvalho, V. M. 2010. Aggregate Fluctuations and the Network Structure of Intersectoral Trade. *October* pp. 1–54.

Carvalho, V. M., M. Nirei, Y. U. Saito, and A. Tahbaz-Salehi. 2021. Supply Chain Disruptions: Evidence from the Great East Japan Earthquake. *Quarterly Journal of Economics* .

Chamberlain, G., and M. Rothschild. 1983. Arbitrage, Factor Structure, and Mean-Variance Analysis on Large Asset Markets. *Econometrica* 51:1281.

Connor, G., and R. A. Korajczyk. 1986. Performance measurement with the arbitrage pricing theory. A new framework for analysis. *Journal of Financial Economics* 15:373–394.

Crucini, M. J., M. A. Kose, and C. Otrok. 2011. What are the driving forces of international business cycles? *Review of Economic Dynamics* 14:156–175.

Del Negro, M., and C. M. Otrok. 2008. Dynamic Factor Models with Time-Varying

Parameters: Measuring Changes in International Business Cycles. *FRB of New York Staff Report* .

Delle Chiaie, S., L. Ferrara, and D. Giannone. 2021. Common factors of commodity prices. *Journal of Applied Econometrics* .

Di Giovanni, J., and A. A. Levchenko. 2010. Putting the parts together: trade, vertical linkages, and business cycle comovement. *American Economic Journal: Macroeconomics* 2:95–124.

Di Giovanni, J., A. A. Levchenko, and I. Mejean. 2014. Firms, destinations, and aggregate fluctuations. *Econometrica* 82:1303–1340.

Di Giovanni, J., A. A. Levchenko, and I. Mejean. 2018. The micro origins of international business-cycle comovement. *American Economic Review* 108:82–108.

Dupor, B. 1999. Aggregation and irrelevance in multi-sector models. *Journal of Monetary Economics* 43:391–409.

Eaton, J., S. Kortum, and B. Neiman. 2016a. Obstfeld and Rogoff[U+05F3]s international macro puzzles: a quantitative assessment. *Journal of Economic Dynamics and Control* 72:5–23.

Eaton, J., S. Kortum, B. Neiman, and J. Romalis. 2016b. Trade and the global recession. *American Economic Review* 106:3401–3438.

Fernández-Villaverde, J., J. F. Rubio-RAMÍREZ, T. J. Sargent, and M. W. Watson. 2007. ABCs (and Ds) of understanding VARs.

Foerster, A. T., P. D. G. Sarte, and M. W. Watson. 2011. Sectoral versus aggregate shocks: A structural factor analysis of industrial production. *Journal of Political Economy* 119:1–38.

Frankel, J. A., and A. K. Rose. 1998. The endogenity of the optimum currency area criteria. *The economic journal* 108:1009–1025.

Giovanni, J. D., A. Levchenko, and I. Mejean. Fortcoming. Foreign shocks as granular fluctuations. *Journal of Political Economy* .

Grassi, B. 2017. IO in I-O: Size, Industrial Organization, and the Input-Output Network Make a Firm Structurally Important. *Working Paper* .

Hirata, H., M. A. Kose, C. Otrok, and M. E. Terrones. 2013. Global House Price Fluctuations: Synchronization and Determinants. *SSRN Electronic Journal* .

Ho, P., P.-D. G. Sarte, F. Schwartzman, et al. 2023. How Does Trade Impact the Way GDP Growth and Inflation Comove Across Countries? *Richmond Fed Economic Brief* 23.

Horvath, M. 1998. Cyclicality and sectoral linkages: Aggregate fluctuations from independent sectoral shocks. *Review of Economic Dynamics* 1:781–808.

Huo, Z., A. Levchenko, and N. Pandalai-Nayar. 2023. International Comovement in the Global Production Network. *National Bureau of Economic Research* 13796.

Jackson, L. E., M. A. Kose, C. Otrok, and M. T. Owyang. 2016. Specification and estimation of Bayesian dynamic factor models: A Monte Carlo analysis with an application to global house price comovement. In *Dynamic Factor Models*, vol. 35, pp. 361–400. Emerald Group Publishing Limited.

Johnson, R. C. 2014. Trade in intermediate inputs and business cycle comovement. *American Economic Journal: Macroeconomics* 6:39–83.

Karadimitropoulou, A., and M. León-Ledesma. 2013. World, country, and sector factors in international business cycles. *Journal of Economic Dynamics and Control* 37:2913–2927.

King, R. G., and M. W. Watson. 2002. System Reduction and Solution Algorithms for Singular Linear Difference Systems under Rational Expectations. *Computational Economics* 20:57–86.

Kose, M. A., C. Otrok, and E. Prasad. 2012. Global business cycles: Convergence or decoupling. *International Economic Review* 53:511–538.

Kose, M. A., C. Otrok, and C. H. Whiteman. 2003. International business cycles: World, region, and country-specific factors. *American Economic Review* 93:1216–1239.

Kose, M. A., C. Otrok, and C. H. Whiteman. 2008. Understanding the evolution of world business cycles. *Journal of International Economics* 75:110–130.

Long, J. B., and C. I. Plosser. 1983. Real Business Cycles. *Journal of Political Economy* 91:39–69.

McGrattan, E. R., and J. A. Schmitz. 1999. Maintenance and Repair: Too Big to Ignore. *Quarterly Review* 23:2–13.

Norrbin, S. C., and D. E. Schlagenhauf. 1996. The role of international factors in the business cycle: A multi-country study. *Journal of International Economics* 40:85–104.

Stock, J. H., and M. W. Watson. 2002. Forecasting using principal components from a large number of predictors. *Journal of the American Statistical Association* 97:1167–1179.

Timmer, M. P., E. Dietzenbacher, B. Los, R. Stehrer, and G. J. de Vries. 2015. An Illustrated User Guide to the World Input-Output Database: The Case of Global Automotive Production. *Review of International Economics* 23:575–605.

FOR ONLINE PUBLICATION

A Data sources and treatment

Data sources

Where possible we downloaded quarterly seasonal adjusted time series for industrial production (IP) indices at the 2-digit ISIC Rev.4 level, starting in 2006 Q1 and until 2022 Q2. We retrieved data from the UNIDO quarterly IIP dataset. We computed growth rates in annual terms, i.e. $y^j_{n,t} = 400 * ln(IP^j_{n,t}/IP^j_{s,t-1})$ for sector j and country n, where $IP^j_{n,t}$ is the quarterly IP reading.

Sectors and classification

The analysis is based on Revision 4 of the International Standard for Industrial Classification of All Economic Activities (ISIC Rev. 4) at division level (2-digit). In particular, we focus on sections belonging to the divisions Mining and quarrying (B), Manufacturing (C) and Electricity, gas, steam and air conditioning supply (D). See Table A.1 for the list of included sections (referred to as sectors in the main text). This list is the largest intersection of sectors for which there is both production data largely available at quarterly frequency and coverage in the World Input Output Database (Timmer et al., 2015).

Seasonal adjustment

For countries or individual sectors where UNIDO does not publish seasonally adjusted data, we perform seasonal and working day adjustments using the X-13ARIMA-SEATS software developed by the United States Census Bureau.

Missing data and data treatment

For the vast majority of sectors data is available. However, there were three cases of missing data:

1. For some countries, certain sectors are missing completely. For example, sectors C33 (Repair and installation of machinery and equipment), and C19 (Manufacture of coke and refinement) are particularly prone not to be reported by National Statistical Offices.

Table (A.1) Sectors included in the dataset

Division	**Section code**	**Section name**
B	B	Mining and quarrying
C	C10-C12	Manufacture of food products, beverages and tobacco products
	C13-C15	Manufacture of textiles, wearing apparel and leather products
	C16	Manufacture of wood and of products of wood and cork, except furniture; manufacture of articles of straw and plaiting materials
	C17	Manufacture of paper and paper products
	C18	Printing and reproduction of recorded media
	C19	Manufacture of coke and refined petroleum products
	C20	Manufacture of chemicals and chemical products
	C21	Manufacture of basic pharmaceutical products and pharmaceutical preparati
	C22	Manufacture of rubber and plastic products
	C23	Manufacture of other non-metallic mineral products
	C24	Manufacture of basic metals
	C25	Manufacture of fabricated metal products, except machinery and equipment
	C26	Manufacture of computer, electronic and optical products
	C27	Manufacture of electrical equipment
	C28	Manufacture of machinery and equipment n.e.c.
	C29	Manufacture of motor vehicles, trailers and semi-trailers
	C30	Manufacture of other transport equipment
	C31-C32	Manufacture of furniture; other manufacturing
	C33	Repair and installation of machinery and equipment
D	D35	Electricity, gas, steam and air conditioning supply

2. Certain sectors show large publication gaps, usually at the beginning or the end of the time series

3. Few sectors are not reported for a given quarter. Occurrences of 0s are treated as missing data.

In all three cases, we attempt to fill data gaps with information from other sources: we revert to Eurostat and National Statistical Offices and/or non-seasonal adjusted datasets.

If no other data source publishes the data in question, we treat entirely missing sectors with a sector weight of 0 for the aggregation to country industrial production indices (see following section). In case of partially missing data, we drop the sector in the same way. But in the case of individual data gaps, we apply a linear interpolation to replace the gap, which only happens four times in our dataset. The impact of any resulting measurement error for the outcome of the analysis is likely to be minimal. All in all, we lack full time series for 17 out of 609 sectors leaving us with a sample of 592 sectors. Figure A.1 shows the final availability of sector data per country in our sample.

Factor models are sensitive to extreme values. For that reason, we detect and remove outliers by replacing growth rates above (below) the median plus (minus) 5 times the interquartile range (calculated per sector over time) by the sector-specific median growth rate computed over a 7 quarter rolling window.

Sector aggregation, compatibility with WIOD data and IP weights

For the structural factor model the sectoral industrial production data needs to be compatible with the sector classification of the World Input Output Database. The 2016 WIOD release uses the ISIC Rev. 4 classification. However, WIOD aggregates a number of sectors, namely *C10-C12, C13-C15* and *C31-C32*.

For concordance between the datasets we therefore aggregate IP time series to the same sector composition. To this end, we either downloaded data in aggregated form, and if not available, aggregate the time series using value added weights from UNIDO as shown in figure A.2.

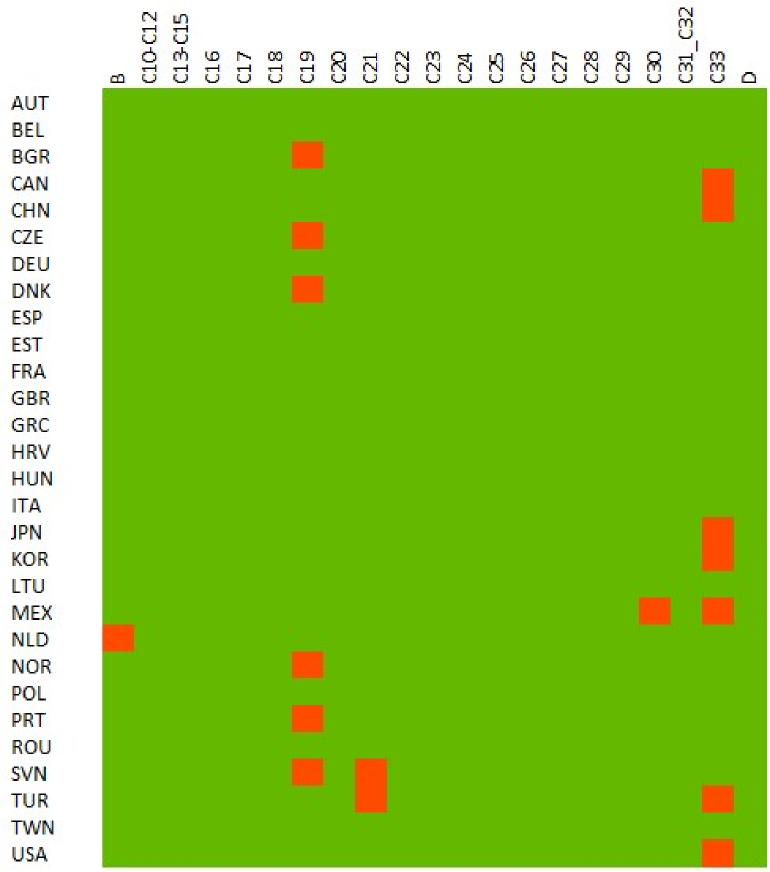
For the aggregation of sector data we follow the approach of Foerster et al. (2011): Assume aggregate sector C is composed of sectors A and B, then the growth rate of C is approximated as:

$$ln(\frac{IP_{C,t}}{IP_{C,t-1}}) = \frac{w_{A,t-1}ln(IP_{A,t}/IP_{A,t-1}) + w_{B,t-1}ln(IP_{B,t}/IP_{B,t-1})}{w_{A,t-1} + w_{B,t-1}} \quad \text{(A.23)}$$

For 2-digit sectors that are published in two separate sub-series, we apply the same procedure to achieve concordance between data sets.

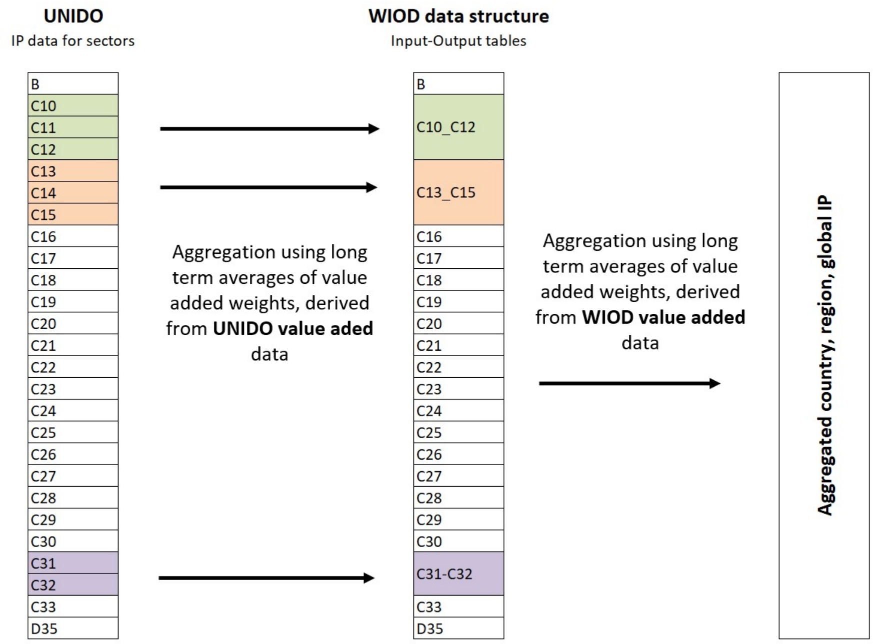
With a view to aggregating our sector-level results to country, region and world level, we

Figure (A.1) Availability of data per country and 2-digit sector



(a) Green boxes indicate availability of sector data for country, red boxed indicate missing sector data

Figure (A.2) Use of value added weights



define three sets of weights: vector $\mathbf{w_m}$ with elements $[w_m]_n^j$ defines weights to aggregate to country level m. Vector $\mathbf{w_r}$ with elements $[w_r]_n^j$ defines weights to aggregate to region level r. And vector $\mathbf{w_G}$ with elements $[w_G]_n^j$ defines weights to aggregate to world level. All weights rely on WIOD value added data and are averaged over time (2006 to 2014).

B The Definitions of Regions

We allocate countries into four different regions (1) the Americas (Canada, Mexico, USA), (2) Asia (China, Japan, South Korea, Taiwan), (3) Western Europe (Austria, Belgium, Germany, Spain, France, UK, Greece, Italy, the Netherlands and Portugal), and (4) Central and Eastern Europe (Bulgaria, Czech Republic, Croatia, Denmark, Estonia, Hungary, Lithuania, Poland, Romania, Slovenia). Not included in any region are Turkey (as its geographic/economic allocation is less clear) and Norway (as its business cycle is rather uncorrelated with other Western European countries given the high weight of oil production in Norway's IP index).

C Equilibrium in the multi-country multi-sector RBC model

Consider the social planner problem that maximises the sum of country-level utilities:

$$\max_{\{C_t, L_t, K_{t+1}, M_t, X_t\}_{t=0}^{\infty}} \quad E_t \sum_{m=1}^{N} \left(\sum_{t=0}^{\infty} \beta^t \big[C_{m,t} - L_{m,t} \big] \right) \tag{C.24}$$

with

$$C_{m,t} = \sum_{n=1}^{N} \sum_{j=1}^{J} \xi_{nm}^{j} \ln C_{nm,t}^{j} \quad \text{and} \quad \sum_{n=1}^{N} \sum_{j=1}^{J} \xi_{nm}^{j} = 1,$$

$$L_{m,t} = \frac{\epsilon_{LS}}{\epsilon_{LS}+1} \big(\sum_{j}^{J} L_{mm,t}^{j} \big)^{\frac{\epsilon_{LS}+1}{\epsilon_{LS}}}$$

subject to:

$$\sum_{m=1}^{N} [\tau^C]_{nm}^{j} C_{nm,t}^{j} + \sum_{m=1}^{N} \sum_{i=1}^{J} [\tau^M]_{nm}^{ji} M_{nm,t}^{ji} + \sum_{m=1}^{N} \sum_{i=1}^{J} [\tau^X]_{nm}^{ji} X_{nm,t}^{ji} = Y_{n,t}^{j}, \tag{C.25}$$

$$K_{n,t+1}^{j} = X_{n,t}^{j} + (1-\delta) K_{n,t}^{j}, \tag{C.26}$$

$$X_{n,t}^{j} = \left(\sum_{m} \sum_{i} (\kappa_{mn}^{ij})^{\frac{1}{\epsilon_X}} (X_{mn,t}^{ij})^{\frac{\epsilon_X - 1}{\epsilon_X}} \right)^{\frac{\epsilon_X}{\epsilon_X - 1}}, \quad \sum_{m}^{N} \sum_{i}^{J} \kappa_{mn}^{ij} = 1, \tag{C.27}$$

$$\begin{aligned} Y_{n,t}^{j} = A_{n,t}^{j} \Bigg[& (1-\mu_n^j)^{\frac{1}{\epsilon_Q}} \left(\left(\frac{K_{n,t}^{j}}{\alpha_n^j} \right)^{\alpha_n^j} \left(\frac{L_{n,t}^{j}}{1-\alpha_n^j} \right)^{1-\alpha_n^j} \right)^{\frac{\epsilon_Q - 1}{\epsilon_Q}} + \\ & (\mu_n^j)^{\frac{1}{\epsilon_Q}} (M_{n,t}^{j})^{\frac{\epsilon_Q - 1}{\epsilon_Q}} \Bigg]^{\frac{\epsilon_Q}{\epsilon_Q - 1}}, \end{aligned} \tag{C.28}$$

$$M_{n,t}^{j} = \left(\sum_{i} (\mu_n^{ij})^{\frac{1}{\epsilon_M}} (M_{n,t}^{ij})^{\frac{\epsilon_M - 1}{\epsilon_M}} \right)^{\frac{\epsilon_M}{\epsilon_M - 1}}, \quad \sum_{i}^{J} \mu_n^{ij} = 1, \tag{C.29}$$

$$M_{n,t}^{ij} = \left(\sum_{m} (\mu_{mn}^{ij})^{\frac{1}{\epsilon_T}} (M_{mn}^{ij})^{\frac{\epsilon_T - 1}{\epsilon_T}} \right)^{\frac{\epsilon_T}{\epsilon_T - 1}} \quad \sum_{m}^{N} \mu_{mn}^{ij} = 1. \tag{C.30}$$

The proof for equation (12) consists of the following steps: First write out the Lagrangian for the social planner. Then solve for the steady state allocation and log-linearize around the steady state. This yields a set of $5 \times (N \times J) + 2 \times (N \times J)^2 + 1 \times (N^2 \times J)$ equations.

We then reduce this system of equations to a set of $2 \times (N \times J)$ equations with a view to solving the dynamics of the system using standard linear rational expectations tool kits as described in Blanchard and Kahn (1980) and King and Watson (2002). Using the resulting policy functions, we can then solve for the model filter.

C.1 Optimization problem

$$
\begin{aligned}
\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \Bigg\{ \sum_{m=1}^{N} \Bigg[\sum_{n}^{N} \sum_{j}^{J} \xi^j_{nm,t} \ln(C^j_{nm,t}) - \frac{\epsilon_{LS}}{\epsilon_{LS}+1} \Big(\sum_{j}^{J} L^j_{mm,t} \Big)^{\frac{\epsilon_{LS}+1}{\epsilon_{LS}}} \Bigg] - \\
\sum_{n}^{N} \sum_{j}^{J} [P^X]^j_{n,t} \Bigg[K^j_{n,t+1} - (X^j_{n,t} + (1-\delta_K) K^j_{n,t}) \Bigg] - \\
\sum_{n}^{N} \sum_{j}^{J} P^j_{n,t} \Bigg[\sum_{m}^{M} [\tau^C]^j_{nm} C^j_{nm,t} + \sum_{m}^{M} \sum_{i}^{J} [\tau^M]^{ji}_{nm} M^{ji}_{nm,t} + \sum_{m}^{M} \sum_{i}^{J} [\tau^X]^{ji}_{nm} X^{ji}_{nm,t} - Y^j_{n,t} \Bigg] \Bigg] \Bigg\},
\end{aligned}
\tag{C.31}
$$

where $P^j_{n,t}$ and $[P^X]^j_{n,t}$ are shadow prices. As common in this literature (for example in Eaton et al., 2016a), the shadow prices $P^j_{n,t}$ and $[P^X]^j_{n,t}$ in the central planner's problem are interpreted as the competitive price for output from country-sector nj and as the rental rate of capital in country-sector nj.

C.1.1 Necessary first order conditions

$$
[C^j_{nm,t}]: \quad \xi^j_{nm} \, (C^j_{nm,t})^{-1} = [\tau^C]^j_{nm} P^j_{n,t}. \tag{C.32}
$$

$$
[M^{ij}_{mn}]: \quad \frac{[\tau^M]^{ij}_{mn} P^i_{m,t}}{P^j_{n,t}} = (A^j_{n,t})^{\frac{\epsilon_Q - 1}{\epsilon_Q}} \left(\frac{Y^j_{n,t} \mu^j_n}{M^j_{n,t}} \right)^{\frac{1}{\epsilon_Q}} \left(\frac{M^j_{n,t} \mu^{ij}_n}{M^{ij}_{n,t}} \right)^{\frac{1}{\epsilon_M}} \left(\frac{M^{ij}_{n,t} \mu^{ij}_{mn}}{M^{ij}_{mn,t}} \right)^{\frac{1}{\epsilon_T}} \tag{C.33}
$$

$$
[X^{ij}_{mn}]: \quad [\tau^X]^{ij}_{mn} P^i_{m,t} = [P^X]^j_{n,t} \left(\frac{X^j_{n,t} \kappa^{ij}_{mn}}{X^{ij}_{mn,t}} \right)^{\frac{1}{\epsilon_X}} \tag{C.34}
$$

$$[L^j_{nn,t}] : \quad \left(\sum_j^J L^j_{nn,t}\right)^{\frac{1}{\epsilon_{LS}}} = P^j_{n,t}(A^j_{n,t})^{\frac{\epsilon_Q-1}{\epsilon_Q}}(Y^j_{n,t}(1-\mu^j_n))^{\frac{1}{\epsilon_Q}} \left(\frac{K^j_{n,t}}{\alpha^j_n}\right)^{\alpha^j_n\frac{\epsilon_Q-1}{\epsilon_Q}}\left(\frac{L^j_{nn,t}}{1-\alpha^j_n}\right)^{\frac{\alpha^j_n-1-\alpha^j_n\epsilon_Q}{\epsilon_Q}} \tag{C.35}$$

$$[K^j_{n,t+1}] : \quad [P^X]^j_{n,t} = \beta E_t\left[P^j_{n,t+1}(Y^j_{n,t+1}(1-\mu^j_n))^{\frac{1}{\epsilon_Q}}(A^j_{n,t+1})^{\frac{\epsilon_Q-1}{\epsilon_Q}} \left(\frac{K^j_{n,t+1}}{\alpha^j_n}\right)^{-1+\alpha^j_n\frac{\epsilon_Q-1}{\epsilon_Q}}\left(\frac{L^j_{nn,t+1}}{1-\alpha^j_n}\right)^{\frac{(1-\alpha^j_n)(\epsilon_Q-1)}{\epsilon_Q}}\right]+ \beta(1-\delta_K)E_t([P^X]^j_{n,t+1}) \tag{C.36}$$

C.1.2 Firm cost minimization

With a view to deriving steady state shares in the next step, here we solve the firm cost minimization problem (excluding the firm cost minimization problems for $K^j_{n,t+1}$ and $M^{ij}_{mn,t}$ as the resulting FOCs coincide with those in the central planner problem above). Equalising marginal costs and marginal revenue products yields:

$$[M^j_{n,t}] \quad : \quad [P^M]^j_{n,t} = P^j_{n,t}\frac{\partial Y^j_{n,t}}{\partial M^j_{n,t}}$$

$$\frac{[P^M]^j_{n,t}}{P^j_{n,t}} = (A^j_{n,t})^{\frac{\epsilon_Q-1}{\epsilon_Q}}\left(\frac{Y^j_{n,t}\mu^j_n}{M^j_{n,t}}\right)^{\frac{1}{\epsilon_Q}} \tag{C.37}$$

$$[M^{ij}_{m,t}] \quad : \quad [P^M]^{ij}_{m,t} = P^j_{n,t}\frac{\partial Y^j_{n,t}}{\partial M^{ij}_{m,t}}$$

$$\frac{[P^M]^{ij}_{m,t}}{P^j_{n,t}} = (A^j_{n,t})^{\frac{\epsilon_Q-1}{\epsilon_Q}}\left(\frac{Y^j_{n,t}\mu^j_n}{M^j_{n,t}}\right)^{\frac{1}{\epsilon_Q}}\left(\frac{M^j_{n,t}\mu^{ij}_m}{M^{ij}_{m,t}}\right)^{\frac{1}{\epsilon_M}} \tag{C.38}$$

$$[L^j_{nn,t}] : w^j_{n,t} = P^j_{n,t} \frac{\partial Y^j_{n,t}}{\partial L^j_{nn,t}}$$

$$w^j_{n,t} = P^j_{n,t}(A^j_{n,t})^{\frac{\epsilon_Q - 1}{\epsilon_Q}} (Y^j_{n,t}(1-\mu^j_n))^{\frac{1}{\epsilon_Q}} \left(\frac{K^j_{n,t}}{\alpha^j_n}\right)^{\alpha^j_n \frac{\epsilon_Q-1}{\epsilon_Q}} \left(\frac{L^j_{nn,t}}{1-\alpha^j_n}\right)^{\frac{\alpha^j_n - 1 - \alpha^j_n \epsilon_Q}{\epsilon_Q}} \tag{C.39}$$

Firm cost minimisation also yields a standard set of aggregate price indices. Prices indices for country-sector nj's aggregate intermediate good bundle ($[P^M]^j_n$), country-sector nj's aggregate intermediate good bundle of variety i ($[P^M]^{ij}_n$) and the capital good bundle ($[P^X]^j_n$) are defined as:

$$[P^M]^j_{n,t} = \left(\sum_i \mu^{ij}_m ([P^M]^{ij}_{n,t})^{1-\epsilon_M}\right)^{\frac{1}{1-\epsilon_M}} \tag{C.40}$$

$$[P^M]^{ij}_{n,t} = \left(\sum_m \mu^{ij}_{mn} ([\tau^M]^{ij}_{mn} P^i_{m,t})^{1-\epsilon_T}\right)^{\frac{1}{1-\epsilon_T}} \tag{C.41}$$

$$[P^X]^j_{n,t} = \left(\sum_m \sum_i \kappa^{ij}_{mn} ([\tau^X]^{ij}_{mn} P^i_{m,t})^{1-\epsilon_X}\right)^{\frac{1}{1-\epsilon_X}} \tag{C.42}$$

where $\kappa^{ij}_{mn}([\tau^X]^{ij}_{mn})^{1-\epsilon_X}$ and $\mu^{ij}_{mn}([\tau^M]^{ij}_{mn})^{1-\epsilon_T}$ are trade-cost-cum-shifter parameters.

C.2 Steady State

We determine steady state allocations as follows: First, we rewrite the model's basic equations and first order conditions in the steady state. We then substitute the steady state equations for firms' cost-minimizing choice of capital, labour and intermediate goods into sector production, in order to solve for the equilibrium prices. With those at hand, we solve for the steady state quantities using the remaining first order conditions in the steady state.

From the model's basic equations C.26 and C.28 and first order conditions (equations C.32-C.36), we have the following steady-state equations:

$$Y^j_n = \left[(1-\mu^j_n)^{\frac{1}{\epsilon_Q}} \left(\left(\frac{K^j_n}{\alpha^j_n}\right)^{\alpha^j_n} \left(\frac{L^j_n}{1-\alpha^j_n}\right)^{1-\alpha^j_n}\right)^{\frac{\epsilon_Q-1}{\epsilon_Q}} + (\mu^j_n)^{\frac{1}{\epsilon_Q}} (M^j_n)^{\frac{\epsilon_Q-1}{\epsilon_Q}}\right]^{\frac{\epsilon_Q}{\epsilon_Q-1}} \tag{C.43}$$

$$\delta_K K_n^j = X_n^j \tag{C.44}$$

$$[\tau^C]_{nm}^j P_n^j = \xi_{nm}^j (C_{nm}^j)^{-1} \tag{C.45}$$

$$\frac{[\tau^M]_{mn}^{ij} P_m^i}{P_n^j} = \left(\frac{Y_n^j \mu_n^j}{M_n^j}\right)^{\frac{1}{\epsilon_Q}} \left(\frac{M_n^j \mu_n^{ij}}{M_n^{ij}}\right)^{\frac{1}{\epsilon_M}} \left(\frac{M_n^{ij} \mu_{mn}^{ij}}{M_{mn}^{ij}}\right)^{\frac{1}{\epsilon_T}} \tag{C.46}$$

$$[\tau^X]_{mn}^{ij} P_m^i = [P^X]_n^j \left(\frac{X_n^j \kappa_{mn}^{ij}}{X_{mn}^{ij}}\right)^{\frac{1}{\epsilon_X}} \tag{C.47}$$

$$\left(\sum_j^J L_{nn}^j\right)^{\frac{1}{\epsilon_{LS}}} = P_n^j (Y_n^j (1-\mu_n^j))^{\frac{1}{\epsilon_Q}} \left(\frac{K_n^j}{\alpha_n^j}\right)^{\alpha_n^j \frac{\epsilon_Q - 1}{\epsilon_Q}} \left(\frac{L_{nn}^j}{1-\alpha_n^j}\right)^{(1-\alpha_n^j)\frac{(\epsilon_Q-1)}{\epsilon_Q} - 1} \tag{C.48}$$

$$\begin{aligned} \frac{1-\beta(1-\delta_K)}{\beta} [P^X]_{n,t}^j = & P_n^j (Y_n^j (1-\mu_n^j))^{\frac{1}{\epsilon_Q}} \times \\ & \left(\frac{K_n^j}{\alpha_n^j}\right)^{-1+\alpha_n^j \frac{\epsilon_Q - 1}{\epsilon_Q}} \left(\frac{L_{nn}^j}{1-\alpha_n^j}\right)^{\frac{(1-\alpha_n^j)(\epsilon_Q-1)}{\epsilon_Q}} \end{aligned} \tag{C.49}$$

From the first order conditions of the firm cost minimisation problem (equations C.37 - C.39), we have the following steady state equations:

$$(\mu_n^j)^{\frac{1}{\epsilon_Q}} (M_n^j)^{\frac{\epsilon_Q - 1}{\epsilon_Q}} = \mu_n^j (Y_n^j)^{\frac{\epsilon_Q - 1}{\epsilon_Q}} \left(\frac{P_n^j}{[P^M]_n^j}\right)^{\epsilon_Q - 1} \tag{C.50}$$

$$\frac{[P^M]_m^{ij}}{P_n^j} = \left(\frac{Y_n^j \mu_n^j}{M_n^j}\right)^{\frac{1}{\epsilon_Q}} \left(\frac{M_n^j \mu_m^{ij}}{M_m^{ij}}\right)^{\frac{1}{\epsilon_M}} \tag{C.51}$$

$$w_n^j = P_n^j (Y_n^j (1-\mu_n^j))^{\frac{1}{\epsilon_Q}} \left(\frac{K_n^j}{\alpha_n^j}\right)^{\alpha_n^j \frac{\epsilon_Q - 1}{\epsilon_Q}} \left(\frac{L_{nn}^j}{1-\alpha_n^j}\right)^{\frac{\alpha_n^j - 1 - \alpha_n^j \epsilon_Q}{\epsilon_Q}} \tag{C.52}$$

Combining equation C.52 and C.49 yields:

$$(1-\mu_n^j)^{\frac{1}{\epsilon_Q}}\left(\left(\frac{K_n^j}{\alpha_n^j}\right)^{\alpha_n^j}\left(\frac{L_{nn}^j}{1-\alpha_n^j}\right)^{1-\alpha_n^j}\right)^{\frac{\epsilon_Q-1}{\epsilon_Q}} = (1-\mu_n^j)(Y_n^j)^{\frac{\epsilon_Q-1}{\epsilon_Q}}\times$$

$$\left(\frac{(w_n^j)^{1-\alpha_n^j}\left(\frac{1-\beta(1-\delta_K)}{\beta}\right)^{\alpha_n^j}\left([P^X]_{n,t}^j\right)^{\alpha_n^j}}{P_n^j}\right)^{1-\epsilon_Q} \tag{C.53}$$

Next, we solve for steady state prices. For this, substitute equations C.53 and C.50, together with the steady state equations for the aggregate price indices from equations C.40 - C.42, into the steady state production functions (equation C.43).

$$\begin{aligned}(P_n^j)^{1-\epsilon_Q} =& (1-\mu_n^j)(w_n^j)^{(1-\alpha_n^j)(1-\epsilon_Q)}\left(\beta^{-1}(1-\beta(1-\delta_K))\right)^{\alpha_n^j(1-\epsilon_Q)}\times\\ &\left(\sum_m\sum_i \kappa_{mn}^{ij}([\tau^X]_{mn}^{ij}P_m^i)^{1-\epsilon_X}\right)^{\alpha_n^j\frac{1-\epsilon_Q}{1-\epsilon_X}} + \\ &\mu_n^j\left(\sum_i \mu_n^{ij}\left(\sum_m \mu_{mn}^{ij}([\tau^M]_{mn}^{ij}P_m^i)^{1-\epsilon_T}\right)^{\frac{1-\epsilon_M}{1-\epsilon_T}}\right)^{\frac{1-\epsilon_Q}{1-\epsilon_M}}\end{aligned} \tag{C.54}$$

Note that

- We assume exogenous country-level labour supply in the steady state. We define the steady state labour supply in country n $\left(\sum_j L_{nn}^j\right)^{\frac{1}{\epsilon_{LS}}}$ to be the numeraire good. We calibrate steady state labour supply in all other countries relative to the numeraire country based on labour force data from the World Bank. As equations C.52 and C.48 suggest that $w_n^j = \left(\sum_j L_{nn}^j\right)^{\frac{1}{\epsilon_{LS}}}$, the calibration defines steady state wages.
- We calibrate the technology parameters μ_n^j, the shifters μ_n^{ij} and the trade-cost-cum-shifter parameters $\kappa_{mn}^{ij}([\tau^X]_{mn}^{ij})^{1-\epsilon_X}$ and $\mu_{mn}^{ij}([\tau^M]_{mn}^{ij})^{1-\epsilon_T}$ on observed cost shares and model-implied relative prices. Section E derives the equations used for calibration.

Equation C.54 describes a system of $(N\times J)\times(N\times J)$ equations for the $(N\times J)$ steady state price levels (P_n^j), exogenous variables and parameters (w_n^j, α_n^j, ϵ_Q, ϵ_M,ϵ_T and ϵ_X) and endogenous technology, shifter and trade-cost-cum-shifter parameters (μ_n^j, μ_n^{ij}, $[\tau^M]_{mn}^{ij})^{1-\epsilon_T}$ and $[\tau^X]_{mn}^{ij})^{1-\epsilon_X}$). We numerically solve for the $(N\times J)$ price levels, the technology, shifter and the trade cost-cum-shifter parameters jointly, using a fixed point algorithm.[21] This yields a first characterisation of the steady state in terms of

[21] Starting with an initial guess for steady state prices P_m^i and the technology, shifter and trade-

prices.

Next, we solve for steady state output. For that, we start by writing out the right hand side of the steady state market clearing condition from equation C.25 for good j produced in country n:

$$Y_n^j = \sum_m [\tau^C]_{nm}^j C_{nm}^j + \sum_m \sum_i \left[[\tau^M]_{nm}^{ji} M_{nm}^{ji} + [\tau^X]_{nm}^{ji} X_{nm}^{ji} \right] \tag{C.55}$$

We then substitute in turns for the equilibrium quantities on the right hand side of equation C.55 in terms of output quantities, known parameters and prices. Starting with C_{nm}^j we slightly rewrite equation C.45:

$$C_{nm}^j = \xi_{nm}^j ([\tau^C]_{nm}^j P_n^j)^{-1} \tag{C.56}$$

Second, with the goal of solving for M_{nm}^{ji}, start by rewriting C.51:

$$M_m^{ji} = (Y_m^i \mu_m^i)^{\frac{\epsilon_M}{\epsilon_Q}} (M_m^i)^{\frac{\epsilon_Q - \epsilon_M}{\epsilon_Q}} \mu_m^{ji} \left(\frac{P_m^i}{[P^M]_m^{ji}} \right)^{\epsilon_M} \tag{C.57}$$

And substitute C.50 into C.57:

$$M_m^{ji} = (Y_m^i \mu_m^i) \mu_m^{ji} ([P^M]_m^{ji})^{-\epsilon_M} ([P^M]_m^i)^{\epsilon_M - \epsilon_Q} (P_m^i)^{\epsilon_Q} \tag{C.58}$$

Dividing equation C.46 by C.51 gives:

$$M_{nm}^{ji} = M_m^{ji} \mu_m^{ji} \left(\frac{[\tau^M]_{nm}^{ji} P_n^j}{[P^M]_m^{ji}} \right)^{-\epsilon_T} \tag{C.59}$$

Substituting equation C.58 in equation C.59 then yields:

$$\begin{aligned} M_{nm}^{ji} = & Y_m^i \mu_m^i \mu_m^{ji} \mu_{nm}^{ji} ([\tau^M]_{nm}^{ji} P_n^j)^{-\epsilon_T} \\ & ([P^M]_m^i)^{\epsilon_M - \epsilon_Q} ([P^M]_m^{ji})^{\epsilon_T - \epsilon_M} (P_m^i)^{\epsilon_Q} \end{aligned}$$

cost-cum-shifter parameters μ_n^j, μ_n^{ij}, $\kappa_{mn}^{ij}([\tau^X]_{mn}^{ij})^{1-\epsilon_X}$ and $\mu_{mn}^{ij}([\tau^M]_{mn}^{ij})^{1-\epsilon_T}$ we compute the implied $(N \times J)$ price levels from C.54. Given the updated set of prices, we update the calibration of μ_n^j, μ_n^{ij}, $\kappa_{mn}^{ij}([\tau^X]_{mn}^{ij})^{1-\epsilon_X}$ and $\mu_{mn}^{ij}([\tau^M]_{mn}^{ij})^{1-\epsilon_T}$ using equations E.147-E.150. We plug the updated set of parameters and prices back into equation C.54 and repeat the process until the technology, shifter and taste-cost-cum-shifter parameters converge.

And replace the price indices using equations C.40 and C.41 in steady state:

$$M_{nm}^{ji} = Y_m^i \mu_m^i \mu_m^{ji} \mu_{nm}^{ji} ([\tau^M]_{nm}^{ji} P_n^j)^{-\epsilon_T}$$
$$\left(\sum_j \mu_m^{ji} \left(\sum_n \mu_{nm}^{ji} ([\tau^M]_{nm}^{ji} P_n^j)^{1-\epsilon_T} \right)^{\frac{1-\epsilon_M}{1-\epsilon_T}} \right)^{\frac{\epsilon_M - \epsilon_Q}{1-\epsilon_M}}$$
$$\left(\sum_n \mu_{nm}^{ji} ([\tau^M]_{nm}^{ji} P_{n,t}^j)^{1-\epsilon_T} \right)^{\frac{\epsilon_T - \epsilon_M}{1-\epsilon_T}} (P_m^i)^{\epsilon_Q} \tag{C.60}$$

Third, rewrite capital good purchases X_{nm}^{ji} in terms of steady state prices, output and exogenous parameters. For this, rewrite the equilibrium quantity of capital stocks in country n sector j by combining equations C.49 and C.53:

$$\left(\frac{K_n^j}{\alpha_n^j} \right) = (1 - \mu_n^j) Y_n^j (P_n^j)^{\epsilon_Q} (w_n^j)^{(1-\alpha_n^j)(1-\epsilon_Q)} \times$$
$$\left(\frac{1 - \beta(1 - \delta_K)}{\beta} [P^X]_n^j \right)^{-1+\alpha_n^j(1-\epsilon_Q)} \tag{C.61}$$

And substitute C.44 into C.61:

$$X_n^j = (1 - \mu_n^j) Y_n^j \alpha_n^j \delta_K (P_n^j)^{\epsilon_Q} (w_n^j)^{(1-\alpha_n^j)(1-\epsilon_Q)} \times$$
$$\left(\frac{1 - \beta(1 - \delta_K)}{\beta} [P^X]_n^j \right)^{-1+\alpha_n^j(1-\epsilon_Q)} \tag{C.62}$$

Then, rearrange C.47:

$$X_{nm}^{ji} = X_m^i \kappa_{nm}^{ji} ([P^X]_m^i)^{\epsilon_X} ([\tau^X]_{nm}^{ji} P_n^j)^{-\epsilon_X} \tag{C.63}$$

Now substitute equation C.62 into equation C.63 and replace the capital price indices using C.42 in steady state:

$$X_{nm}^{ji} = (1 - \mu_m^i) Y_m^i \alpha_m^i \delta_K \left(\frac{1 - \beta(1 - \delta_K)}{\beta} \right)^{-1+\alpha_m^i(1-\epsilon_Q)} \times$$
$$\kappa_{nm}^{ji} \left(\sum_n \sum_j \kappa_{nm}^{ji} ([\tau^X]_{nm}^{ji} P_n^j)^{1-\epsilon_X} \right)^{\frac{\epsilon_X - 1 + \alpha_m^i(1-\epsilon_Q)}{1-\epsilon_X}} ([\tau^X]_{nm}^{ji} P_n^j)^{-\epsilon_X} \times$$
$$(P_m^i)^{\epsilon_Q} (w_m^i)^{(1-\alpha_m^i)(1-\epsilon_Q)} \tag{C.64}$$

We have now rewritten the equilibrium quantities for C_{nm}^j, M_{nm}^{ji} and X_{nm}^{ji} in terms of output and known parameters and prices. Finally, we substitute equations C.56, C.60 and C.64 into the resource constraint in equation C.55. This gives:

$$Y_n^j - \sum_m \sum_i \tilde{\Gamma}_{nm}^{ji} Y_m^i = \sum_m \xi_{nm}^j (P_n^j)^{-1} \tag{C.65}$$

where

$$\begin{aligned}
\tilde{\Gamma}_{nm}^{ji} =& (P_m^i)^{\epsilon_Q} \Bigg(\mu_m^i \mu_m^{ji} \mu_{nm}^{ji} ([\tau^M]_{nm}^{ji})^{1-\epsilon_T} (P_n^j)^{-\epsilon_T} \\
& \left(\sum_j \mu_m^{ji} \left(\sum_n \mu_{nm}^{ji} ([\tau^M]_{nm}^{ji} P_n^j)^{1-\epsilon_T} \right)^{\frac{1-\epsilon_M}{1-\epsilon_T}} \right)^{\frac{\epsilon_M-\epsilon_Q}{1-\epsilon_M}} \times \\
& \left(\sum_n \mu_{nm}^{ji} ([\tau^M]_{nm}^{ji} P_{n,t}^j)^{1-\epsilon_T} \right)^{\frac{\epsilon_T-\epsilon_M}{1-\epsilon_T}} + \\
& (1-\mu_m^i) \alpha_m^i \delta_K \left(\frac{1-\beta(1-\delta_K)}{\beta} \right)^{-1+\alpha_m^i(1-\epsilon_Q)} \times \\
& \kappa_{nm}^{ji} ([\tau^X]_{nm}^{ji})^{1-\epsilon_X} \left(\sum_n \sum_j \kappa_{nm}^{ji} ([\tau^X]_{nm}^{ji} P_n^j)^{1-\epsilon_X} \right)^{-1+\alpha_m^i \frac{(1-\epsilon_Q)}{1-\epsilon_X}} \times \\
& (P_n^j)^{-\epsilon_X} (w_m^i)^{(1-\alpha_m^i)(1-\epsilon_Q)} \Bigg)
\end{aligned}$$

We can now solve the equation for steady state output using linear algebra. With steady state output at hand, we can then solve for the steady state shares.

C.2.1 Defining the steady state shares

Substituting equation C.53 into C.48 yields:

$$\begin{aligned}
L_{nn}^j =& (P_n^j)^{\epsilon_Q} Y_n^j (1-\alpha_n^j)(1-\mu_n^j)(w_n^j)^{(1-\alpha_n^j)(1-\epsilon_Q)} \times \\
& (\frac{1-\beta(1-\delta_K)}{\beta})^{\alpha_n^j(1-\epsilon_Q)} ([P^X]_n^j)^{\alpha_n^j(1-\epsilon_Q)} (\sum_j (L_{nn}^j)^{\frac{1}{\epsilon_{LS}}})^{-1}
\end{aligned}$$

using $w_n^j = (\sum_j L_{nn}^j)^{\frac{1}{\epsilon_{LS}}}$ gives:

$$\begin{aligned} L_{nn}^j =& (P_n^j)^{\epsilon_Q} Y_n^j (1-\alpha_n^j)(1-\mu_n^j)(w_n^j)^{(1-\alpha_n^j)(1-\epsilon_Q)-1} \times \\ & (\frac{1-\beta(1-\delta_K)}{\beta})^{\alpha_n^j(1-\epsilon_Q)}([P^X]_n^j)^{\alpha_n^j(1-\epsilon_Q)} \end{aligned} \tag{C.66}$$

And define the steady state labour share:

$$\begin{aligned} \frac{L_{nn}^j}{\sum_j L_{nn}^j} =& (\sum_j L_{nn}^j)^{-1}(P_n^j)^{\epsilon_Q} Y_n^j (1-\alpha_n^j)(1-\mu_n^j)(w_n^j)^{(1-\alpha_n^j)(1-\epsilon_Q)-1} \times \\ & (\frac{1-\beta(1-\delta_K)}{\beta})^{\alpha_n^j(1-\epsilon_Q)}([P^X]_n^j)^{\alpha_n^j(1-\epsilon_Q)} \end{aligned} \tag{C.67}$$

Using equations C.60 and C.64 we also define (for the next section):

$$\begin{aligned} \frac{[\tau^M]_{nm}^{ji} M_{nm}^{ji}}{Y_n^j} =& Y_n^{j-1}(Y_m^i \mu_m^i)\mu_m^{ji}\mu_{nm}^{ji}([\tau^M]_{nm}^{ji})^{1-\epsilon_T}(P_n^j)^{-\epsilon_T} \\ & ([P^M]_m^i)^{\epsilon_M-\epsilon_Q}([P^M]_m^{ji})^{\epsilon_T-\epsilon_M}(P_m^i)^{\epsilon_Q} \end{aligned} \tag{C.68}$$

$$\begin{aligned} \frac{[\tau^X]_{nm}^{ji} X_{nm}^{ji}}{Y_n^j} =& Y_n^{j-1}(1-\mu_m^i)Y_m^i \alpha_m^i \delta_K \left(\frac{1-\beta(1-\delta_K)}{\beta}\right)^{-1+\alpha_m^i(1-\epsilon_Q)} \times \\ & \kappa_{nm}^{ji}\left(\sum_n \sum_j \kappa_{nm}^{ji}([\tau^X]_{nm}^{ji} P_n^j)^{1-\epsilon_X}\right)^{\frac{\epsilon_X-1+\alpha_m^i(1-\epsilon_Q)}{1-\epsilon_X}} \times \\ & ([\tau^X]_{nm}^{ji})^{1-\epsilon_X}(P_n^j)^{-\epsilon_X}(P_m^i)^{\epsilon_Q}(w_m^i)^{(1-\alpha_m^i)(1-\epsilon_Q)} \end{aligned} \tag{C.69}$$

For future reference, define the following matrices:

- $\mathbf{S^L}$: Vector with elements $[S^L]_n^j$ that stores in its nj entry the fraction of labour the reprentative agent in country n supplies to sector j: $[S^L]_n^j = \frac{L_{nn}^j}{\sum_j L_{nn}^j}$
- $\mathbf{S_M^Y}$: Matrix with elements $[S_M^Y]_{nm}^{ji}$ that stores in its nj, mi entry the fraction of good j produced in country n that is sold to industry i in country m as an intermediate input: $[S_M^Y]_{nm}^{ji} = \frac{[\tau^M]_{nm}^{ji} M_{nm}^{ji}}{Y_n^j}$
- $\mathbf{S_X^Y}$: Matrix with elements $[S_X^Y]_{nm}^{ji}$ that stores in its nj, mi entry the fraction of good j produced in country n that is sold to industry i in country m as an investment input: $[S_X^Y]_{nm}^{ji} = \frac{[\tau^X]_{nm}^{ji} X_{nm}^{ji}}{Y_n^j}$
- $\mathbf{S_C^Y}$: Matrix with elements $[S_C^Y]_{nm}^j$ that stores in its nj, m entry the fraction of good j produced in country n that is consumed by the representative agent in country

m: $S_C^Y]_{nm}^j = \frac{[\tau^C]_{nm}^j C_{nm}^j}{Y_n^j}$.

- $\mathbf{S_1^M}$: matrix with elements $[S_1^M]_n^{ij} = \mu_n^{ij} \left(\frac{[P^M]_n^j}{[P^M]_n^{ij}}\right)^{\epsilon_M - 1}$
- $\mathbf{S_2^M}$: matrix with elements $[S_2^M]_{mn}^{ij} = \mu_{mn}^{ij} ([\tau^M]_{mn}^{ij})^{1-\epsilon_T} \left(\frac{[P^M]_n^{ij}}{P_m^i}\right)^{\epsilon_T - 1}$
- $\mathbf{S_1^X}$: matrix with elements $[S_1^X]_{mn}^{ij} = \kappa_{mn}^{ij} [\tau^X]_{mn}^{ij\,1-\epsilon_X} \left(\frac{[P^X]_n^j}{P_m^i}\right)^{\epsilon_X - 1}$

C.3 Log-linearized Equations

Next, we log-linearize the first order conditions in equations C.32 - C.36 and the basic equations C.25, C.26 and C.28. Linearizing these equations yields the following conditions in percentage deviations from the steady state, denoted with hatted variables:

$$\hat{x}_{n,t}^j = \delta_K^{-1} \hat{k}_{n,t+1}^j + (1 - \delta_K^{-1}) \hat{k}_{n,t}^j \tag{C.70}$$

$$\hat{y}_{n,t}^j = \sum_m \left([S_C^Y]_{nm}^j \hat{c}_{nm,t}^j\right) + \left(\sum_m \sum_i [S_M^Y]_{nm}^{ji} \hat{m}_{nm,t}^{ji} + [S_X^Y]_{nm}^{ji} \hat{x}_{nm,t}^{ji}\right) \tag{C.71}$$

$$\hat{p}_{n,t}^j = -\hat{c}_{nm,t}^j \tag{C.72}$$

$$\hat{m}_{mn,t}^{ij} = \frac{\epsilon_T}{\epsilon_Q}(\epsilon_Q - 1)\hat{a}_{n,t}^j + \frac{\epsilon_T}{\epsilon_Q}\hat{y}_{n,t}^j + \left(1 - \frac{\epsilon_T}{\epsilon_Q}\right)\hat{m}_{n,t}^j + \tag{C.73}$$

$$\left(\epsilon_M - \epsilon_T\right)[\hat{p}^M]_n^j - \left(\epsilon_M - \epsilon_T\right)[\hat{p}^M]_n^{ij} + \epsilon_T(\hat{p}_{n,t}^j - \hat{p}_{m,t}^i) \tag{C.74}$$

$$\hat{p}_{m,t}^i = [\hat{p}^X]_{n,t}^j + \frac{1}{\epsilon_X}(\hat{x}_{n,t}^j - \hat{x}_{mn,t}^{ij}) \tag{C.75}$$

$$\frac{1}{\epsilon_{LS}} \sum_j^J [S^L]_n^j \hat{l}_{nn,t}^j = \hat{p}_{n,t}^j + \frac{\epsilon_Q - 1}{\epsilon_Q}\hat{a}_{n,t}^j + \frac{1}{\epsilon_Q}\hat{y}_{n,t}^j +$$

$$\frac{(\epsilon_Q - 1)(1 - \alpha_n^j)}{\epsilon_Q}\hat{l}_{nn,t}^j + \left[-1 + \alpha_n^j \frac{\epsilon_Q - 1}{\epsilon_Q}\right]\hat{k}_{n,t}^j \tag{C.76}$$

$$
\begin{aligned}
[\hat{p}^X]^j_{n,t} =& \beta(1-\delta_K)[\hat{p}^X]^j_{n,t+1} + (1-\beta(1-\delta_K))\Bigg[\hat{p}^j_{n,t+1} + \frac{1}{\epsilon_Q}\hat{y}^j_{n,t+1} + \\
& \frac{\epsilon_Q - 1}{\epsilon_Q}\hat{a}^j_{n,t+1} + \left(-1 + \alpha^j_n \frac{\epsilon_Q - 1}{\epsilon_Q}\right)\hat{k}^j_{n,t+1} + \\
& (1-\alpha^j_n)\frac{\epsilon_Q - 1}{\epsilon_Q}\hat{l}^j_{n,t+1}\Bigg]
\end{aligned} \tag{C.77}
$$

$$
\begin{aligned}
\hat{y}^j_{n,t} =& \hat{a}^j_{n,t} + \alpha^j_n(1 - S_{M^j_n})\hat{k}^j_{n,t} + \\
& (1-\alpha^j_n)(1 - S_{M^j_n})\hat{l}^j_{nn,t} + S_{M^j_n}\hat{m}^j_{n,t}
\end{aligned} \tag{C.78}
$$

Note that equation C.73 follows from

(i) log-linearizing the result from dividing equation C.51 by C.50:

$$
[\hat{p}^M]^{ij}_n - [\hat{p}^M]^j_n = \frac{1}{\epsilon_M}(\hat{m}^j_{n,t} - \hat{m}^{ij}_{n,t}), \tag{C.79}
$$

(ii) log-linearizing equation C.33

$$
\begin{aligned}
\hat{p}^i_{m,t} - \hat{p}^j_{n,t} =& \frac{\epsilon_Q - 1}{\epsilon_Q}\hat{a}^j_{n,t} + \frac{1}{\epsilon_Q}\hat{y}^j_{n,t} + \left(\frac{1}{\epsilon_M} - \frac{1}{\epsilon_Q}\right)\hat{m}^j_{n,t} + \left(\frac{1}{\epsilon_T} - \frac{1}{\epsilon_M}\right)\hat{m}^{ij}_{n,t} - \\
& \frac{1}{\epsilon_T}\hat{m}^{ij}_{mn,t},
\end{aligned} \tag{C.80}
$$

and (iii) rearranging C.79 for $\hat{m}^{ij}_{n,t}$, substituting the expression into C.80 and rearranging for $\hat{m}^{ij}_{mn,t}$.

Rewriting the log-linearized equations C.70 - C.78 in matrix notation:

$$
\hat{\mathbf{k}}_{t+1} = \delta_K^{-1}\hat{\mathbf{X}}_t + (1-\delta_K^{-1})\hat{\mathbf{k}}_t \tag{C.81}
$$

$$
\hat{\mathbf{y}}_t = \tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{C}}\hat{\mathbf{c}}_t + \tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{M}}\hat{\mathbf{m}}_t + \tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{X}}\hat{\mathbf{x}}_t \tag{C.82}
$$

$$
\hat{\mathbf{c}}_t = \boldsymbol{\Omega}_{\mathbf{cp}}\hat{\mathbf{p}}_t \tag{C.83}
$$

$$\hat{\mathbf{m}}_t = \frac{(\epsilon_Q - 1)}{\epsilon_Q}\epsilon_T \mathbf{T_1}\hat{\mathbf{a}}_t + \frac{\epsilon_T}{\epsilon_Q}\mathbf{T_1}\hat{\mathbf{y}}_t + \left(1 - \frac{\epsilon_T}{\epsilon_Q}\right)\mathbf{T_1}\hat{\mathbf{M}}_t + (\epsilon_M - \epsilon_T)\mathbf{T_1}\hat{\mathbf{p}}_t^{\mathbf{M1}} - (\epsilon_M - \epsilon_T)\mathbf{T_3}\hat{\mathbf{p}}_t^{\mathbf{M2}} + \epsilon_T \mathbf{T_1}\hat{\mathbf{p}}_t - \epsilon_T \mathbf{T_2}\hat{\mathbf{p}}_t \tag{C.84}$$

$$\hat{\mathbf{x}}_t = \mathbf{T_1}\hat{\mathbf{X}}_t + \epsilon_X \mathbf{T_1}\hat{\mathbf{p}}_t^X - \epsilon_X \mathbf{T_2}\hat{\mathbf{p}}_t \tag{C.85}$$

$$\frac{1}{\epsilon_{LS}}\tilde{\mathbf{S}}^{\mathbf{L}}\hat{\mathbf{l}}_t = \hat{\mathbf{p}}_t + \frac{\epsilon_Q - 1}{\epsilon_Q}\hat{\mathbf{a}}_t + \frac{1}{\epsilon_Q}\hat{\mathbf{y}}_t + \frac{\epsilon_Q - 1}{\epsilon_Q}\boldsymbol{\alpha}\hat{\mathbf{k}}_t + \frac{\boldsymbol{\alpha} - \mathbf{I} - \boldsymbol{\alpha}\epsilon_Q}{\epsilon_Q}\hat{\mathbf{l}}_t \tag{C.86}$$

$$\hat{\mathbf{p}}_t^X = \beta(1-\delta_K)\hat{\mathbf{p}}_{t+1}^{\mathbf{X}} + (1 - \beta(1-\delta_K))\Bigg[\hat{\mathbf{p}}_{t+1} + \frac{1}{\epsilon_Q}\hat{\mathbf{y}}_{t+1} + \frac{\epsilon_Q - 1}{\epsilon_Q}\hat{\mathbf{a}}_{t+1} + \left(-\mathbf{I} + \boldsymbol{\alpha}\frac{\epsilon_Q - 1}{\epsilon_Q}\right)\hat{\mathbf{k}}_{t+1} + (\mathbf{I} - \boldsymbol{\alpha})\frac{\epsilon_Q - 1}{\epsilon_Q}\hat{\mathbf{l}}_{t+1}\Bigg] \tag{C.87}$$

$$\hat{\mathbf{y}}_t = \hat{\mathbf{a}}_t + \boldsymbol{\alpha}(\mathbf{I} - \mathbf{S_M})\hat{\mathbf{k}}_t + (\mathbf{I} - \boldsymbol{\alpha})(\mathbf{I} - \mathbf{S_M})\hat{\mathbf{l}}_t + \mathbf{S_M}\hat{\mathbf{M}}_t \tag{C.88}$$

$\hat{\mathbf{X}}_t$ and $\hat{\mathbf{M}}_t$ refer to $(N \times J) \times 1$ vectors of the investment and intermediate good bundles, that are employed by each sector, respectively. $\hat{\mathbf{x}}_t$ and $\hat{\mathbf{m}}_t$ refer to the $(N \times J)^2 \times 1$ vectors that capture the flows of intermediate and investment inputs across pairs of industries, with ordering $\hat{\mathbf{m}}_t = (\hat{m}_{11,t}^{11}, \hat{m}_{11,t}^{12}, ..., \hat{m}_{1N,t}^{1J}, \hat{m}_{11,t}^{21}, \hat{m}_{11,t}^{22}, ..., \hat{m}_{1N,t}^{2J}, ..., \hat{m}_{NN,t}^{JJ})^T$. $\hat{\mathbf{c}}_t$ refers to the $N^2J \times 1$ consumption vector by the representative agents, with ordering $\hat{\mathbf{c}}_t = (\hat{c}_{11,t}^1, \hat{c}_{11,t}^2, ..., \hat{c}_{N1,t}^J, \hat{c}_{12,t}^1, \hat{c}_{12,t}^2, ..., \hat{c}_{N2,t}^J, ..., \hat{c}_{NN,t}^J)^T$. Moreover, equations C.81 - C.88 use the following matrix definitions:

1. Define $\tilde{\mathbf{S}}^{\mathbf{L}}$ as the $(N \times J) \times (N \times J)$ matrix with steady state labour shares:

$$\tilde{\mathbf{S}}^{\mathbf{L}} = \begin{bmatrix} \frac{L_{11}^1}{\sum_j L_{11}^j} & ... & \frac{L_{11}^J}{\sum_j L_{11}^j} & 0 & 0 & 0 & ... & ... \\ & ... & & & & ... & & \\ \frac{L_{11}^1}{\sum_j L_{11}^j} & ... & \frac{L_{11}^J}{\sum_j L_{11}^j} & 0 & 0 & 0 & ... & ... \\ 0 & ... & 0 & \frac{L_{22}^1}{\sum_j L_{22}^j} & ... & \frac{L_{22}^J}{\sum_j L_2^j} & ... & ... \\ & ... & & & & & & \\ 0 & ... & 0 & 0 & ... & 0 & ... & \frac{L_{NN}^J}{\sum_j L_{NN}^j} \end{bmatrix} \tag{C.89}$$

2. Define $\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{M}}$ as the $(N \times J) \times (N \times J)^2$ matrix with steady state intermediate shares in output:

$$\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{M}} = \begin{bmatrix} \frac{[\tau^M]^{11}_{11} M^{11}_{11}}{Y^1_1} & \frac{[\tau^M]^{12}_{11} M^{12}_{11}}{Y^1_1} & \dots & \frac{[\tau^M]^{1J}_{1N} M^{1J}_{1N}}{Y^1_1} & 0 & \dots & 0 & \dots & \dots \\ 0 & 0 & \dots & 0 & \frac{[\tau^M]^{21}_{11} M^{21}_{11}}{Y^2_1} & \dots & \frac{[\tau^M]^{2J}_{1N} M^{2J}_{1N}}{Y^2_1} & \dots & \dots \\ & & \dots & & & & & \dots & \dots \\ 0 & 0 & \dots & 0 & 0 & 0 & 0 & \dots & \frac{[\tau^M]^{JJ}_{NN} M^{JJ}_{NN}}{Y^j_n} \end{bmatrix} \tag{C.90}$$

3. Define $\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{X}}$, the $(N \times J) \times (N \times J)^2$ matrix with steady state capital shares in output, analogous to $\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{M}}$

4. Define $\tilde{\mathbf{S}}^{\mathbf{C}}_{\mathbf{Y}}$ as the $(N \times J) \times (N^2 \times J)$ matrix with steady state consumption shares in output.

$$\tilde{\mathbf{S}}^{\mathbf{C}}_{\mathbf{Y}} = \begin{bmatrix} \frac{[\tau^C]^1_{11} C^1_{11}}{Y^1_1} & 0 & 0 & \dots & \frac{[\tau^C]^1_{12} C^1_{12}}{Y^1_1} & & \\ 0 & \frac{[\tau^C]^2_{11} C^2_{11}}{Y^2_1} & 0 & \dots & 0 & \frac{[\tau^C]^2_{12} C^2_{12}}{Y^2_1} & \\ & & \dots & \frac{[\tau^C]^J_{N1} C^J_{N1}}{Y^J_N} & & \dots & \frac{[\tau^C]^J_{NN} C^J_{NN}}{Y^J_N} \end{bmatrix} \tag{C.91}$$

5. $\mathbf{T_1}$ is a $(N \times J)^2 \times (N \times J)$ matrix equal to the Kronecker product $\mathbf{1}_{(N \times J)} \otimes \mathbf{I}_{(N \times J)}$, where $\mathbf{1}_{(N \times J)}$ is a column vector of ones. Similarly $\mathbf{T_2}$ is $\mathbf{I}_{(N \times J)} \otimes \mathbf{1}_{(N \times J)}$. Finally, $\mathbf{T_3}$ is a $(N \times J)^2 \times (N^2 \times J)$ matrix that maps $\hat{\mathbf{m}}_t$ to $\hat{\mathbf{p}}^{\mathbf{M2}}_t$.

6. $\mathbf{S_M}$ is a diagonal matrix with the intermediate cost shares $\frac{M^j_n}{Y^j_n}$ along the diagonal; and $\boldsymbol{\alpha}$ is a diagonal matrix with α^j_n along the diagonal.

7. $\mathbf{\Omega_{cp}}$ is a $(N^2 \times J) \times (N \times J)$ matrix that stacks

$$\mathbf{\Omega_{cp}} = \begin{bmatrix} -\mathbf{I}_{(N \times J)} \\ -\mathbf{I}_{(N \times J)} \\ \dots \\ -\mathbf{I}_{(N \times J)} \end{bmatrix} \tag{C.92}$$

Also note that log-linearizations of the equations C.40 - C.42 for prices of the different good bundles yield:

$$[\hat{p}^X]^j_{n,t} = \sum_{m,i} [S^X_1]^{ij}_{mn} \hat{p}^i_{m,t} \tag{C.93}$$

$$[\hat{p}^M]^j_{n,t} = \sum_{i} [S^M_1]^{ij}_n [\hat{p}^M]^{ij}_{n,t} \tag{C.94}$$

$$[\hat{p}^M]^{ij}_{n,t} = \sum_{m} [S^M_2]^{ij}_{mn} \hat{p}^i_{m,t} \tag{C.95}$$

Expressed in matrix notation:

$$\hat{\mathbf{p}}^{\mathbf{x}}_t = \mathbf{S^X_1} \hat{\mathbf{p}}_{\mathbf{t}} \tag{C.96}$$

$$\hat{\mathbf{p}}^{\mathbf{M1}}_t = \tilde{\mathbf{S}}^{\mathbf{M}}_{\mathbf{1}} \hat{\mathbf{p}}^{\mathbf{M2}}_t \tag{C.97}$$

$$\hat{\mathbf{p}}^{\mathbf{M2}}_t = \tilde{\mathbf{S}}^{\mathbf{M}}_{\mathbf{2}} \hat{\mathbf{p}}_t \tag{C.98}$$

so that:

$$\hat{\mathbf{p}}^{\mathbf{M1}}_t = \tilde{\mathbf{S}}^{\mathbf{M}}_{\mathbf{1}} \tilde{\mathbf{S}}^{\mathbf{M}}_{\mathbf{2}} \hat{\mathbf{p}}_t \tag{C.99}$$

Using the following matrix definitions

- Define $\tilde{\mathbf{S}}^{\mathbf{M}}_{\mathbf{1}}$ as the $(N \times J) \times (N \times J^2)$ matrix

$$\tilde{\mathbf{S}}^{\mathbf{M}}_{\mathbf{1}} = \begin{bmatrix} [S^M_1]^{11}_1 & ... & [S^M_1]^{J1}_1 & 0 & ... & 0 & & & \\ 0 & ... & 0 & [S^M_1]^{12}_1 & ... & [S^M_1]^{J2}_1 & & & \\ & ... & & & & & & & \\ 0 & ... & 0 & & ... & & [S^M_1]^{1J}_N & ... & [S^M_1]^{JJ}_N \end{bmatrix} \tag{C.100}$$

- Define $\tilde{\mathbf{S}}_{\mathbf{2}}^{\mathbf{M}}$ as the $(N \times J^2) \times (N \times J)$ matrix

$$
\tilde{\mathbf{S}}_{\mathbf{2}}^{\mathbf{M}} = \begin{bmatrix}
[S_2^M]_{11}^{11} & 0 & 0 & & \cdots & [S_2^M]_{N1}^{11} & 0 & & \\
0 & [S_2^M]_{11}^{21} & 0 & & \cdots & & [S_2^M]_{N1}^{21} & & \\
 & & \cdots & & & & & \cdots & \\
 & & & [S_2^M]_{11}^{J1} & \cdots & & & & [S_2^M]_{N1}^{J1} \\
[S_2^M]_{11}^{12} & 0 & 0 & & \cdots & [S_2^M]_{N1}^{12} & 0 & & \\
0 & [S_2^M]_{11}^{22} & 0 & & \cdots & & [S_2^M]_{N1}^{22} & & \\
 & & \cdots & & & & & \cdots & \\
 & & & [S_2^M]_{11}^{J2} & \cdots & & & & [S_2^M]_{N1}^{J2} \\
\cdots & & & & & & & & \\
[S_2^M]_{12}^{11} & 0 & 0 & & \cdots & [S_2^M]_{N2}^{11} & 0 & & \\
0 & [S_2^M]_{12}^{21} & 0 & & \cdots & & [S_2^M]_{N2}^{21} & & \\
 & & \cdots & & & & & \cdots & \\
 & & & [S_2^M]_{12}^{J1} & \cdots & & & & [S_2^M]_{N2}^{J1} \\
\cdots & & & & & & & & \\
 & & & [S_2^M]_{1N}^{JJ} & \cdots & & & & [S_2^M]_{NN}^{JJ}
\end{bmatrix} \tag{C.101}
$$

- Define $\hat{\mathbf{p}}_t^{\mathbf{M2}}$ as the column vector
$\hat{\mathbf{p}}_t^{\mathbf{M2}} = ([\hat{p}^M]_1^{11}, [\hat{p}^M]_1^{21}, ..., [\hat{p}^M]_1^{J1}, [\hat{p}^M]_1^{12}, ..., [\hat{p}^M]_1^{J2}, ..., [\hat{p}^M]_1^{JJ}, [\hat{p}^M]_2^{11}, ..., [\hat{p}^M]_N^{JJ})^T$

C.4 System Reduction

The model solution is described by a set of $5 \times (N \times J) + 2 \times (N \times J)^2 + 1 \times (N^2 \times J)$ equations, summarized in equations C.81-C.88. There are $3 \times (N \times J)$ equations from the model's basic setup: equations C.81, C.82 and C.88. C.86 and C.87 represent two sets of $(N \times J)$ equations, C.83 one set of $(N^2 \times J)$ equations and C.84 and C.85 two sets of $(N \times J)^2$ equations from the model's first order conditions. With a view to solving the dynamics of the model in the next section, we now reduce the system of $5 \times (N \times J)$ $+ 2 \times (N \times J)^2 + 1 \times (N^2 \times J)$ equations to a system with a set of $2 \times (N \times J)$ equations featuring capital, prices and productivity.

Step 1: Substitute out $\hat{\mathbf{c}}_t$, $\hat{\mathbf{m}}_t$ and $\hat{\mathbf{x}}_t$ in equation C.82 using equations C.83, C.84 and C.85. This reduces the system of equations in C.81-C.88 to the following set:

$$\begin{aligned}
\hat{\mathbf{k}}_{t+1} =& \delta_K^{-1}\hat{\mathbf{X}}_t + (1-\delta_K^{-1})\hat{\mathbf{k}}_t && \text{(C.102)}\\
\left(\mathbf{I} - \frac{\epsilon_T}{\epsilon_Q}\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{M}}\mathbf{T_1}\right)\hat{\mathbf{y}}_t =& \tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{C}}{}_t\boldsymbol{\Omega}_{\mathbf{cp}}\hat{\mathbf{p}}_t + \tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{X}}\mathbf{T_1}\hat{\mathbf{X}}_t + \frac{(\epsilon_Q - 1)}{\epsilon_Q}\epsilon_T\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{M}}\mathbf{T_1}\hat{\mathbf{a}}_t\\
&+ \left(1 - \frac{\epsilon_T}{\epsilon_Q}\right)\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{M}}\mathbf{T_1}\hat{\mathbf{M}}_t + (\epsilon_M - \epsilon_T)\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{M}}\mathbf{T_1}\hat{\mathbf{p}}^{\mathbf{M1}}_t\\
&- (\epsilon_M - \epsilon_T)\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{M}}\mathbf{T_3}\hat{\mathbf{p}}^{\mathbf{M2}}_t + \epsilon_T\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{M}}(\mathbf{T_1} - \mathbf{T_2})\hat{\mathbf{p}}_t\\
&+ \epsilon_X\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{X}}\mathbf{T_1}\hat{\mathbf{p}}^{\mathbf{X}}_t - \epsilon_x\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{X}}\mathbf{T_2}\hat{\mathbf{p}}_t && \text{(C.103)}\\
\frac{1}{\epsilon_{LS}}\tilde{\mathbf{S}}^{\mathbf{L}}\hat{\mathbf{l}}_t =& \hat{\mathbf{p}}_t + \frac{\epsilon_Q - 1}{\epsilon_Q}\hat{\mathbf{a}}_t + \frac{1}{\epsilon_Q}\hat{\mathbf{y}}_t + \frac{\epsilon_Q - 1}{\epsilon_Q}\boldsymbol{\alpha}\hat{\mathbf{k}}_t + \frac{\boldsymbol{\alpha} - \mathbf{I} - \boldsymbol{\alpha}\epsilon_Q}{\epsilon_Q}\hat{\mathbf{l}}_t && \text{(C.104)}\\
\hat{\mathbf{p}}^{\mathbf{X}}_t =& \beta(1-\delta_K)\hat{\mathbf{p}}^{\mathbf{X}}_{t+1} + (1 - \beta(1-\delta_K))\Bigg[\hat{\mathbf{p}}_{t+1} + \frac{1}{\epsilon_Q}\hat{\mathbf{y}}_{t+1}\\
&+ \frac{\epsilon_Q - 1}{\epsilon_Q}\hat{\mathbf{a}}_{t+1} + \left(-\mathbf{I} + \boldsymbol{\alpha}\frac{\epsilon_Q - 1}{\epsilon_Q}\right)\hat{\mathbf{k}}_{t+1}\\
&+ (\mathbf{I} - \boldsymbol{\alpha})\frac{\epsilon_Q - 1}{\epsilon_Q}\hat{\mathbf{l}}_{t+1}\Bigg] && \text{(C.105)}\\
\hat{\mathbf{y}}_t =& \hat{\mathbf{a}}_t + \boldsymbol{\alpha}(\mathbf{I} - \mathbf{S_M})\hat{\mathbf{k}}_t + (\mathbf{I} - \boldsymbol{\alpha})(\mathbf{I} - \mathbf{S_M})\hat{\mathbf{l}}_t + \mathbf{S_M}\hat{\mathbf{M}}_t && \text{(C.106)}
\end{aligned}$$

Step 2: Using equations C.96-C.98, substitute $\hat{\mathbf{p}}^{\mathbf{X}}_t = \mathbf{S}^{\mathbf{X}}_{\mathbf{1}}\hat{\mathbf{p}}_t$ in equation C.103 and C.105, and substitute $\hat{\mathbf{p}}^{\mathbf{M1}}_t = \mathbf{S}^{\mathbf{M}}_{\mathbf{1}}\hat{\mathbf{p}}^{\mathbf{M2}}_t$ and $\hat{\mathbf{p}}^{\mathbf{M2}}_t = \mathbf{S}^{\mathbf{M}}_{\mathbf{2}}\hat{\mathbf{p}}_t$ in equation C.103. Define $\tilde{\beta} \equiv 1-\beta(1-\delta_K)$ and substitute in C.105. Rearrange equation C.102 to $\hat{\mathbf{X}}_t = \delta_K^{-1}\hat{\mathbf{k}}_{t+1} + (1-\delta_K^{-1})\hat{\mathbf{k}}_t$, and substitute also in C.105. This reduces equations C.102-C.106 to the following set of equations:

$$
\begin{aligned}
\left(\mathbf{I}-\frac{\epsilon_T}{\epsilon_Q}\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{M}}\mathbf{T_1}\right)\hat{\mathbf{y}}_t =& \tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{C}}{}_t\boldsymbol{\Omega}_{\mathbf{cp}}\hat{\mathbf{p}}_t+\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{X}}\mathbf{T_1}\delta_K^{-1}\hat{\mathbf{k}}_{t+1}+\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{X}}\mathbf{T_1}(1-\delta_K^{-1})\hat{\mathbf{k}}_t \\
&+\frac{(\epsilon_Q-1)}{\epsilon_Q}\epsilon_T\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{M}}\mathbf{T_1}\hat{\mathbf{a}}_t+\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{M}}\left(I-\frac{\epsilon_T}{\epsilon_Q}\right)\mathbf{T_1}\hat{\mathbf{M}}_t \\
&+\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{M}}(\epsilon_M-\epsilon_T)\mathbf{T_1}\mathbf{S}^{\mathbf{M}}_{\mathbf{1}}\mathbf{S}^{\mathbf{M}}_{\mathbf{2}}\hat{\mathbf{p}}_t-(\epsilon_M-\epsilon_T)\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{M}}\mathbf{T_3}\mathbf{S}^{\mathbf{M}}_{\mathbf{2}}\hat{\mathbf{p}}_t \\
&+\epsilon_T\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{M}}(\mathbf{T_1}-\mathbf{T_2})\hat{\mathbf{p}}_t+\epsilon_X\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{X}}\mathbf{T_1}\mathbf{S}^{\mathbf{X}}_{\mathbf{1}}\hat{\mathbf{p}}_t-\epsilon_x\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{X}}\mathbf{T_2}\hat{\mathbf{p}}_t \qquad \text{(C.107)}\\
\frac{1}{\epsilon_{LS}}\tilde{\mathbf{S}}^{\mathbf{L}}\hat{\mathbf{l}}_t =& \hat{\mathbf{p}}_t+\frac{\epsilon_Q-1}{\epsilon_Q}\hat{\mathbf{a}}_t+\frac{1}{\epsilon_Q}\hat{\mathbf{y}}_t+\frac{\epsilon_Q-1}{\epsilon_Q}\boldsymbol{\alpha}\hat{\mathbf{k}}_t+\frac{\boldsymbol{\alpha}-\mathbf{I}-\boldsymbol{\alpha}\epsilon_Q}{\epsilon_Q}\hat{\mathbf{l}}_t \qquad \text{(C.108)}\\
\mathbf{S}^{\mathbf{X}}_{\mathbf{1}}\hat{\mathbf{p}}_t =& \left[\tilde{\beta}\mathbf{I}+\beta(1-\delta_K)\mathbf{S}^{\mathbf{X}}_{\mathbf{1}}\right]\hat{\mathbf{p}}_{t+1}+\tilde{\beta}\frac{1}{\epsilon_Q}\hat{\mathbf{y}}_{t+1} \\
&+\tilde{\beta}\frac{\epsilon_Q-1}{\epsilon_Q}\hat{\mathbf{a}}_{t+1}+\tilde{\beta}\left(-\mathbf{I}+\boldsymbol{\alpha}\frac{\epsilon_Q-1}{\epsilon_Q}\right)\hat{\mathbf{k}}_{t+1} \\
&+\tilde{\beta}(\mathbf{I}-\boldsymbol{\alpha})\frac{\epsilon_Q-1}{\epsilon_Q}\hat{\mathbf{l}}_{t+1} \qquad \text{(C.109)}\\
\hat{\mathbf{y}}_t =& \hat{\mathbf{a}}_t+\boldsymbol{\alpha}(\mathbf{I}-\mathbf{S_M})\hat{\mathbf{k}}_t+(\mathbf{I}-\boldsymbol{\alpha})(\mathbf{I}-\mathbf{S_M})\hat{\mathbf{l}}_t+\mathbf{S_M}\hat{\mathbf{M}}_t \qquad \text{(C.110)}
\end{aligned}
$$

Step 3 : Log-linearizing equation C.37 yields $\hat{\mathbf{M}}_t = (\epsilon_Q-1)\hat{\mathbf{a}}_t+\hat{\mathbf{y}}_t+\epsilon_Q(\hat{\mathbf{p}}_t-\hat{\mathbf{p}}^{\mathbf{M1}}_t)$. Using C.97, substitute $\hat{\mathbf{p}}^{\mathbf{M1}}_t=\tilde{\mathbf{S}}^{\mathbf{M}}_{\mathbf{1}}\tilde{\mathbf{S}}^{\mathbf{M}}_{\mathbf{2}}\hat{\mathbf{p}}_t$ which gives $\hat{\mathbf{M}}_t=(\epsilon_Q-1)\hat{\mathbf{a}}_t+\hat{\mathbf{y}}_t+\epsilon_Q(\mathbf{I}-\tilde{\mathbf{S}}^{\mathbf{M}}_{\mathbf{1}}\tilde{\mathbf{S}}^{\mathbf{M}}_{\mathbf{2}})\hat{\mathbf{p}}_t$. We then substitute this equation into equations C.107 and C.110. Now, the set of equations becomes:

$$\Big(\mathbf{I}-\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{M}}\mathbf{T_1}\Big)\hat{\mathbf{y}}_t = \tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{C}t}\boldsymbol{\Omega}_{\mathbf{cp}}\hat{\mathbf{p}}_t + \tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{X}}\mathbf{T_1}\delta_K^{-1}\hat{\mathbf{k}}_{t+1} + \tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{X}}\mathbf{T_1}(1-\delta_K^{-1})\hat{\mathbf{k}}_t$$
$$+ (\epsilon_Q - 1)\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{M}}\mathbf{T_1}\hat{\mathbf{a}}_t + \Big[\epsilon_Q\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{M}}\mathbf{T_1}(\mathbf{I}-\mathbf{S}^{\mathbf{M}}_{\mathbf{1}}\mathbf{S}^{\mathbf{M}}_{\mathbf{2}})$$
$$+ (\epsilon_M - \epsilon_T)\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{M}}\mathbf{T_1}\mathbf{S}^{\mathbf{M}}_{\mathbf{1}}\mathbf{S}^{\mathbf{M}}_{\mathbf{2}} - (\epsilon_M - \epsilon_T)\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{M}}\mathbf{T_3}\mathbf{S}^{\mathbf{M}}_{\mathbf{2}}$$
$$+ \epsilon_T\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{M}}(\mathbf{T_1}\mathbf{S}^{\mathbf{M}}_{\mathbf{1}}\mathbf{S}^{\mathbf{M}}_{\mathbf{2}} - \mathbf{T_2}) + \epsilon_x\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{X}}(\mathbf{T_1}\mathbf{S}^{\mathbf{X}}_{\mathbf{1}} - \mathbf{T_2})\Big]\hat{\mathbf{p}}_t \tag{C.111}$$

$$\frac{1}{\epsilon_{LS}}\tilde{\mathbf{S}}^{\mathbf{L}}\hat{\mathbf{l}}_t = \hat{\mathbf{p}}_t + \frac{\epsilon_Q - 1}{\epsilon_Q}\hat{\mathbf{a}}_t + \frac{1}{\epsilon_Q}\hat{\mathbf{y}}_t + \frac{\epsilon_Q - 1}{\epsilon_Q}\boldsymbol{\alpha}\hat{\mathbf{k}}_t + \frac{\boldsymbol{\alpha} - \mathbf{I} - \boldsymbol{\alpha}\epsilon_Q}{\epsilon_Q}\hat{\mathbf{l}}_t \tag{C.112}$$

$$\mathbf{S}^{\mathbf{X}}_{\mathbf{1}}\hat{\mathbf{p}}_t = \big[\tilde{\beta}\mathbf{I} + \beta(1-\delta_K)\mathbf{S}^{\mathbf{X}}_{\mathbf{1}}\big]\hat{\mathbf{p}}_{t+1} + \tilde{\beta}\frac{1}{\epsilon_Q}\hat{\mathbf{y}}_{t+1}$$
$$+ \tilde{\beta}\frac{\epsilon_Q - 1}{\epsilon_Q}\hat{\mathbf{a}}_{t+1} + \tilde{\beta}\Big(-\mathbf{I} + \boldsymbol{\alpha}\frac{\epsilon_Q - 1}{\epsilon_Q}\Big)\hat{\mathbf{k}}_{t+1}$$
$$+ \tilde{\beta}(\mathbf{I}-\boldsymbol{\alpha})\frac{\epsilon_Q - 1}{\epsilon_Q}\Big(\hat{\mathbf{l}}_{t+1}\Big) \tag{C.113}$$

$$\hat{\mathbf{y}}_t = (\mathbf{I}-\mathbf{S_M})^{-1}(\mathbf{I}+\mathbf{S_M}(\epsilon_Q - 1))\hat{\mathbf{a}}_t + \boldsymbol{\alpha}\hat{\mathbf{k}}_t + (\mathbf{I}-\boldsymbol{\alpha})\hat{\mathbf{l}}_t$$
$$+ (\mathbf{I}-\mathbf{S_M})^{-1}\mathbf{S_M}\epsilon_Q(\mathbf{I}-\tilde{\mathbf{S}}^{\mathbf{M}}_{\mathbf{1}}\tilde{\mathbf{S}}^{\mathbf{M}}_{\mathbf{2}})\hat{\mathbf{p}}_t \tag{C.114}$$

Step 4: Next, we substitute out $\hat{\mathbf{y}}_t$ using equation C.114. First, rewrite equation C.114 twice:

$$\frac{1}{\epsilon_Q}\hat{\mathbf{y}}_t = \frac{1}{\epsilon_Q}(\mathbf{I}-\mathbf{S_M})^{-1}(\mathbf{I}+\mathbf{S_M}(\epsilon_Q - 1))\hat{\mathbf{a}}_t + \frac{\boldsymbol{\alpha}}{\epsilon_Q}\hat{\mathbf{k}}_t$$
$$+ \frac{1}{\epsilon_Q}(\mathbf{I}-\boldsymbol{\alpha})\hat{\mathbf{l}}_t + (\mathbf{I}-\mathbf{S_M})^{-1}\mathbf{S_M}(\mathbf{I}-\tilde{\mathbf{S}}^{\mathbf{M}}_{\mathbf{1}}\tilde{\mathbf{S}}^{\mathbf{M}}_{\mathbf{2}})\hat{\mathbf{p}}_t \tag{C.115}$$

$$\Big(\mathbf{I}-\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{M}}\mathbf{T_1}\Big)\hat{\mathbf{y}}_t = \Big(\mathbf{I}-\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{M}}\mathbf{T_1}\Big)(\mathbf{I}-\mathbf{S_M})^{-1}(\mathbf{I}+\mathbf{S_M}(\epsilon_Q - 1))\hat{\mathbf{a}}_t$$
$$+ \Big(\mathbf{I}-\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{M}}\mathbf{T_1}\Big)\boldsymbol{\alpha}\hat{\mathbf{k}}_t + \Big(\mathbf{I}-\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{M}}\mathbf{T_1}\Big)(\mathbf{I}-\boldsymbol{\alpha})\hat{\mathbf{l}}_t$$
$$+ \Big(\mathbf{I}-\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{M}}\mathbf{T_1}\Big)(\mathbf{I}-\mathbf{S_M})^{-1}\mathbf{S_M}(\mathbf{I}-\tilde{\mathbf{S}}^{\mathbf{M}}_{\mathbf{1}}\tilde{\mathbf{S}}^{\mathbf{M}}_{\mathbf{2}})\epsilon_Q\hat{\mathbf{p}}_t \tag{C.116}$$

Then, substitute equation C.115 into equation C.112 and C.113, and substitute equation

C.116 into equation C.111.

$$
\begin{aligned}
0 &= \tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{C}t}\boldsymbol{\Omega}_{\mathbf{cp}}\hat{\mathbf{p}}_t + \tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{X}}\mathbf{T_1}\delta_K^{-1}\hat{\mathbf{k}}_{t+1} + [\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{X}}\mathbf{T_1}(1-\delta_K^{-1}) - (\mathbf{I} - \tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{M}}\mathbf{T_1})\boldsymbol{\alpha}]\hat{\mathbf{k}}_t \\
&+ \left[(\epsilon_Q - 1)\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{M}}\mathbf{T_1} - (\mathbf{I} - \tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{M}}\mathbf{T_1})(\mathbf{I} - \mathbf{S_M})^{-1}(\mathbf{I} + \mathbf{S_M}(\epsilon_Q - 1))\right]\hat{\mathbf{a}}_t \\
&- \left(\mathbf{I} - \tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{M}}\mathbf{T_1}\right)(\mathbf{I} - \boldsymbol{\alpha})\hat{\mathbf{l}}_t \\
&+ \Big[\epsilon_Q\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{M}}\mathbf{T_1}(\mathbf{I} - \mathbf{S}^{\mathbf{M}}_{\mathbf{1}}\mathbf{S}^{\mathbf{M}}_{\mathbf{2}}) + (\epsilon_M - \epsilon_T)\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{M}}\mathbf{T_1}\mathbf{S}^{\mathbf{M}}_{\mathbf{1}}\mathbf{S}^{\mathbf{M}}_{\mathbf{2}} - (\epsilon_M - \epsilon_T)\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{M}}\mathbf{T_3}\mathbf{S}^{\mathbf{M}}_{\mathbf{2}} \\
&+ \epsilon_T\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{M}}(\mathbf{T_1}\mathbf{S}^{\mathbf{M}}_{\mathbf{1}}\mathbf{S}^{\mathbf{M}}_{\mathbf{2}} - \mathbf{T_2}) + \epsilon_x\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{X}}(\mathbf{T_1}\mathbf{S}^{\mathbf{X}}_{\mathbf{1}} - \mathbf{T_2}) \\
&- \left(\mathbf{I} - \tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{M}}\mathbf{T_1}\right)\epsilon_Q(\mathbf{I} - \mathbf{S_M})^{-1}\mathbf{S_M}(\mathbf{I} - \tilde{\mathbf{S}}^{\mathbf{M}}_{\mathbf{1}}\tilde{\mathbf{S}}^{\mathbf{M}}_{\mathbf{2}})\Big]\hat{\mathbf{p}}_t \qquad (C.117) \\
\mathbf{S}^{\mathbf{X}}_{\mathbf{1}}\hat{\mathbf{p}}_t &= [\tilde{\beta}\mathbf{I} + \beta(1-\delta_K)\mathbf{S}^{\mathbf{X}}_{\mathbf{1}} + \tilde{\beta}(\mathbf{I} - \mathbf{S_M})^{-1}\mathbf{S_M}(\mathbf{I} - \tilde{\mathbf{S}}^{\mathbf{M}}_{\mathbf{1}}\tilde{\mathbf{S}}^{\mathbf{M}}_{\mathbf{2}})]\hat{\mathbf{p}}_{t+1} \\
&+ \tilde{\beta}\left[\frac{1}{\epsilon_Q}(\mathbf{I} - \mathbf{S_M})^{-1}(\mathbf{I} + \mathbf{S_M}(\epsilon_Q - 1)) + \frac{\epsilon_Q - 1}{\epsilon_Q}\mathbf{I}\right]\hat{\mathbf{a}}_{t+1} \\
&+ \tilde{\beta}(-\mathbf{I} + \boldsymbol{\alpha})\hat{\mathbf{k}}_{t+1} + \tilde{\beta}(\mathbf{I} - \boldsymbol{\alpha})\hat{\mathbf{l}}_{t+1} \qquad (C.118) \\
(I - \boldsymbol{\alpha})\hat{l}_t &= \boldsymbol{v}\boldsymbol{\alpha}\hat{\mathbf{k}}_t + \boldsymbol{v}\left[\frac{\epsilon_Q - 1}{\epsilon_Q}\mathbf{I} + \frac{1}{\epsilon_Q}(\mathbf{I} - \mathbf{S_M})^{-1}(\mathbf{I} + \mathbf{S_M}(\epsilon_Q - 1))\right]\hat{\mathbf{a}}_t \\
&+ \boldsymbol{v}[(\mathbf{I} - \mathbf{S_M})^{-1}\mathbf{S_M}(\mathbf{I} - \tilde{\mathbf{S}}^{\mathbf{M}}_{\mathbf{1}}\tilde{\mathbf{S}}^{\mathbf{M}}_{\mathbf{2}}) + \mathbf{I}]\hat{\mathbf{p}}_t \qquad (C.119)
\end{aligned}
$$

where $\boldsymbol{v} = (\mathbf{I} - \boldsymbol{\alpha})[\frac{1}{\epsilon_{LS}}\tilde{\mathbf{S}}^{\mathbf{L}} + \boldsymbol{\alpha}]^{-1}$

Step 5 : Substituting equation C.119 into equations C.117 and C.118 reduces the system of equations further :

$$
\begin{aligned}
0 =& \tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{C}t}\boldsymbol{\Omega}_{\mathbf{cp}}\hat{\mathbf{p}}_t + \tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{X}}\mathbf{T_1}\delta_K^{-1}\hat{\mathbf{k}}_{t+1} + [\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{X}}\mathbf{T_1}(1-\delta_K^{-1}) - (\mathbf{I}-\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{M}}\mathbf{T_1})(1+\boldsymbol{\upsilon})\boldsymbol{\alpha}]\hat{\mathbf{k}}_t \\
&+ \Big[(\epsilon_Q - 1)\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{M}}\mathbf{T_1} - (\mathbf{I}-\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{M}}\mathbf{T_1})(\mathbf{I}+\boldsymbol{\upsilon}\epsilon_Q^{-1})(\mathbf{I}-\mathbf{S_M})^{-1}(\mathbf{I}+\mathbf{S_M}(\epsilon_Q-1))\Big]\hat{\mathbf{a}}_t \\
&- \Big(\mathbf{I}-\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{M}}\mathbf{T_1}\Big)\boldsymbol{\upsilon}\frac{\epsilon_Q-1}{\epsilon_Q}\hat{\mathbf{a}}_t \\
&+ \Big[\epsilon_Q\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{M}}\mathbf{T_1}(\mathbf{I}-\mathbf{S}^{\mathbf{M}}_{\mathbf{1}}\mathbf{S}^{\mathbf{M}}_{\mathbf{2}}) + (\epsilon_M-\epsilon_T)\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{M}}\mathbf{T_1}\mathbf{S}^{\mathbf{M}}_{\mathbf{1}}\mathbf{S}^{\mathbf{M}}_{\mathbf{2}} - (\epsilon_M-\epsilon_T)\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{M}}\mathbf{T_3}\mathbf{S}^{\mathbf{M}}_{\mathbf{2}} \\
&+ \epsilon_T\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{M}}(\mathbf{T_1}\mathbf{S}^{\mathbf{M}}_{\mathbf{1}}\mathbf{S}^{\mathbf{M}}_{\mathbf{2}} - \mathbf{T_2}) + \epsilon_x\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{X}}(\mathbf{T_1}\mathbf{S}^{\mathbf{X}}_{\mathbf{1}} - \mathbf{T_2}) \\
&- \Big(\mathbf{I}-\tilde{\mathbf{S}}^{\mathbf{Y}}_{\mathbf{M}}\mathbf{T_1}\Big)\epsilon_Q(\mathbf{I}-\mathbf{S_M})^{-1}\mathbf{S_M}(\mathbf{I}-\tilde{\mathbf{S}}^{\mathbf{M}}_{\mathbf{1}}\tilde{\mathbf{S}}^{\mathbf{M}}_{\mathbf{2}}) \\
&+ \boldsymbol{\upsilon}[(\mathbf{I}-\mathbf{S_M})^{-1}\mathbf{S_M}(\mathbf{I}-\tilde{\mathbf{S}}^{\mathbf{M}}_{\mathbf{1}}\tilde{\mathbf{S}}^{\mathbf{M}}_{\mathbf{2}}) + \mathbf{I}]\Big]\hat{\mathbf{p}}_t \qquad \text{(C.120)} \\
\mathbf{S}^{\mathbf{X}}_{\mathbf{1}}\hat{\mathbf{p}}_t =& \big[\beta(1-\delta_K)\mathbf{S}^{\mathbf{X}}_{\mathbf{1}} + \tilde{\beta}(\mathbf{I}+\boldsymbol{\upsilon})(\mathbf{I}+(\mathbf{I}-\mathbf{S_M})^{-1}\mathbf{S_M}(\mathbf{I}-\tilde{\mathbf{S}}^{\mathbf{M}}_{\mathbf{1}}\tilde{\mathbf{S}}^{\mathbf{M}}_{\mathbf{2}}))\big]\hat{\mathbf{p}}_{t+1} \\
&+ \tilde{\beta}(\mathbf{I}+\boldsymbol{\upsilon})\Big[\frac{1}{\epsilon_Q}(\mathbf{I}-\mathbf{S_M})^{-1}(\mathbf{I}+\mathbf{S_M}(\epsilon_Q-1)) + \frac{\epsilon_Q-1}{\epsilon_Q}\mathbf{I}\Big]\hat{\mathbf{a}}_{t+1} \\
&+ \tilde{\beta}(-\mathbf{I}+\boldsymbol{\alpha}+\boldsymbol{\upsilon}\boldsymbol{\alpha})\hat{\mathbf{k}}_{t+1} \qquad \text{(C.121)}
\end{aligned}
$$

We have now reduced the system of equations into one that involves only prices, capital, and the exogenous shocks.

C.5 Solution and Policy Functions

Blanchard Kahn

Equations C.120 and C.121 express the reduced system as

$$
\begin{bmatrix} E_t(\hat{\mathbf{p}}_{t+1}) \\ \hat{\mathbf{k}}_{t+1} \end{bmatrix} = \boldsymbol{\Psi}\begin{bmatrix} \hat{\mathbf{p}}_t \\ \hat{\mathbf{k}}_t \end{bmatrix} + \boldsymbol{\Phi}\begin{bmatrix} \hat{\mathbf{a}}_t \\ \hat{\mathbf{b}}_t \end{bmatrix}, \qquad \text{(C.122)}
$$

where $\hat{b}_t$ denotes a $(N \times J) \times 1$ vector with 0s. We use a linear rational expectation toolkit to solve the dynamics of the system (using the methods proposed by King and Watson (2002)) As this expression is identical to the closed economy model in Atalay (2017), we apply his derivation of the model filter. For the convenience of the reader, the remaining section C.5 restates the section from the original paper.

In the last expression, $\boldsymbol{\Psi}$ and $\boldsymbol{\Phi}$ have $N \times J$ stable and unstable eigenvalues. Using a Jordan decomposition, we write $\boldsymbol{\Psi} = \mathbf{VDV}^{-1}$ where $\mathbf{D}$ is diagonal and is ordered such that the $N \times J$ explosive eigenvalues are ordered first and the $N \times J$ stable eigenvalues are ordered last. Re-write:

$$\begin{aligned}\Upsilon_{t+1} \equiv \mathbf{V}^{-1}\begin{bmatrix}E_t(\hat{\mathbf{p}}_{t+1})\\ \hat{\mathbf{k}}_{t+1}\end{bmatrix} &= \mathbf{D}\mathbf{V}^{-1}\begin{bmatrix}\hat{\mathbf{p}}_t\\ \hat{\mathbf{k}}_t\end{bmatrix} + \mathbf{V}^{-1}\mathbf{\Phi}\begin{bmatrix}\hat{\mathbf{a}}_t\\ \hat{\mathbf{b}}_t\end{bmatrix}\\ &\equiv \mathbf{D}\boldsymbol{\Upsilon}_t + \tilde{\mathbf{\Phi}}\begin{bmatrix}\hat{\mathbf{a}}_t\\ \hat{\mathbf{b}}_t\end{bmatrix}\end{aligned}$$

Partition $\boldsymbol{\Upsilon}_t$ into the first $(N \times J) \times 1$ block, $\boldsymbol{\Upsilon}_{1t}$, and the lower $(N \times J) \times 1$, $\boldsymbol{\Upsilon}_{2t}$. Similarly, partition $\tilde{\mathbf{\Phi}}$ and $\mathbf{D}$.

$$\boldsymbol{\Upsilon}_{1t} = \mathbf{D}_1^{-1}E_t[\boldsymbol{\Upsilon}_{1,t+1}] - \mathbf{D}_1^{-1}\tilde{\mathbf{\Phi}}\begin{bmatrix}\hat{\mathbf{a}}_t\\ \hat{\mathbf{b}}_t\end{bmatrix}$$

Substitute recursively

$$\boldsymbol{\Upsilon}_{1t} = -\mathbf{D}_1^{-1}\sum_{s=0}\mathbf{D}_1^{-s}\tilde{\mathbf{\Phi}}\begin{bmatrix}\hat{\mathbf{a}}_t\\ \hat{\mathbf{b}}_t\end{bmatrix} = -\mathbf{D}_1^{-1}(I - \mathbf{D}_1^{-1})\tilde{\mathbf{\Phi}}\begin{bmatrix}\hat{\mathbf{a}}_t\\ \hat{\mathbf{b}}_t\end{bmatrix} \tag{C.123}$$

For $\boldsymbol{\Upsilon}_{2t}$:

$$\boldsymbol{\Upsilon}_{2t} = \mathbf{D}_2^{-1}\boldsymbol{\Upsilon}_{2,t-1}\tilde{\mathbf{\Phi}}_2\begin{bmatrix}\hat{\mathbf{a}}_t\\ \hat{\mathbf{b}}_t\end{bmatrix}$$

Remember that

$$\begin{bmatrix}\boldsymbol{\Upsilon}_{1t}\\ \boldsymbol{\Upsilon}_{2t}\end{bmatrix} = \mathbf{V}^{-1}\begin{bmatrix}\hat{\mathbf{p}}_t\\ \hat{\mathbf{k}}_t\end{bmatrix}$$

This implies:

$$\hat{\mathbf{p}}_t = -(\mathbf{V}_{11}^{-1})^{-1}\mathbf{V}_{12}^{-1}\hat{\mathbf{k}}_t + (\mathbf{V}_{11}^{-1})^{-1}\boldsymbol{\Upsilon}_{1t}$$

Substituting equation C.123:

$$\hat{\mathbf{p}}_t = -(\mathbf{V}_{11}^{-1})^{-1}\mathbf{V}_{12}^{-1}\hat{\mathbf{k}}_t - (\mathbf{V}_{11}^{-1})^{-1}\mathbf{D}_1^{-1}(I - \mathbf{D}_1^{-1})^{-1}\tilde{\mathbf{\Phi}}_1\begin{bmatrix}\hat{\mathbf{a}}_t\\ \hat{\mathbf{b}}_t\end{bmatrix} \tag{C.124}$$

Equation C.122 implies that the endogenous state evolves as follows:

$$\hat{\mathbf{k}}_{t+1} = \mathbf{\Psi}_{22}\hat{\mathbf{k}}_t + \mathbf{\Psi}_{21}\hat{\mathbf{p}}_t + \mathbf{\Phi}_2\begin{bmatrix}\hat{\mathbf{a}}_t\\ \hat{\mathbf{b}}_t\end{bmatrix} \tag{C.125}$$

Substitute C.124 into C.125:

$$\hat{\mathbf{k}}_{t+1} = \underbrace{(\mathbf{\Psi}_{22} - \mathbf{\Psi}_{21}(\mathbf{V}_{11}^{-1})^{-1}\mathbf{V}_{12}^{-1})}_{\equiv \mathbf{M_{kk}}}\hat{\mathbf{k}}_t + \underbrace{\Big(- \mathbf{\Psi}_{21}(\mathbf{V}_{11}^{-1})^{-1}\mathbf{D}_1^{-1}(I - \mathbf{D}_1^{-1})^{-1}\tilde{\mathbf{\Phi}}_1 + \mathbf{\Phi}_2\Big)}_{\equiv [\mathbf{M_{ka}}, \mathbf{M_{kb}}]}\begin{bmatrix}\hat{\mathbf{a}}_t \\ \hat{\mathbf{b}}_t\end{bmatrix} \tag{C.126}$$

For future reference, equation C.125 implies:

$$\hat{\mathbf{p}}_t = \mathbf{\Psi}_{21}^{-1}\hat{\mathbf{k}}_{t+1} - \mathbf{\Psi}_{21}^{-1}\mathbf{\Psi}_{22}\hat{\mathbf{k}}_t - \mathbf{\Psi}_{21}^{-1}\mathbf{\Phi}_2\begin{bmatrix}\hat{\mathbf{a}}_t \\ \hat{\mathbf{b}}_t\end{bmatrix} \tag{C.127}$$

C.6 Obtaining the model filter

Substitute equation C.119 in C.114 to write $\hat{\mathbf{y}}_t$ as a function of exogenous variables, $\hat{\mathbf{k}}$ and $\hat{\mathbf{p}}$

$$\begin{aligned}\hat{\mathbf{y}}_t = &(\mathbf{I} + \boldsymbol{v})\boldsymbol{\alpha}\hat{\mathbf{k}}_t \\ &+ \left[\frac{\epsilon_Q - 1}{\epsilon_Q}\boldsymbol{v} + \left(\frac{\boldsymbol{v}}{\epsilon_Q} + \mathbf{I}\right)(\mathbf{I} - \mathbf{S_M})^{-1}(\mathbf{I} + \mathbf{S_M}(\epsilon_Q - 1))\right]\hat{\mathbf{a}}_t \\ &+ [(\boldsymbol{v} + \epsilon_Q\mathbf{I})(\mathbf{I} - \mathbf{S_M})^{-1}\mathbf{S_M}(\mathbf{I} - \tilde{\mathbf{S}}_{\mathbf{1}}^{\mathbf{M}}\tilde{\mathbf{S}}_{\mathbf{2}}^{\mathbf{M}}) + \boldsymbol{v}]\hat{\mathbf{p}}_t\end{aligned} \tag{C.128}$$

With a view of expressing $\hat{\mathbf{y}}_t$ only in terms of exogenous shocks, we first substitute out $\hat{\mathbf{p}}_t$. Start by substituting C.126 in C.127:

$$\begin{aligned}\hat{\mathbf{p}}_t =&\mathbf{\Psi}_{21}^{-1}(\mathbf{\Psi}_{22} - \mathbf{\Psi}_{21}(\mathbf{V}_{11}^{-1})^{-1}\mathbf{V}_{12}^{-1})\hat{\mathbf{k}}_t - \mathbf{\Psi}_{21}^{-1}\mathbf{\Psi}_{22}\hat{\mathbf{k}}_t - \mathbf{\Psi}_{21}^{-1}\mathbf{\Phi}_2\begin{bmatrix}\hat{\mathbf{a}}_t \\ \hat{\mathbf{b}}_t\end{bmatrix} \\ &+ \mathbf{\Psi}_{21}^{-1}\Big(- \mathbf{\Psi}_{21}(\mathbf{V}_{11}^{-1})^{-1}\mathbf{D}_1^{-1}(I - \mathbf{D}_1^{-1})^{-1}\tilde{\mathbf{\Phi}}_1 + \mathbf{\Phi}_2\Big)\begin{bmatrix}\hat{\mathbf{a}}_t \\ \hat{\mathbf{b}}_t\end{bmatrix} \\ =&(\mathbf{V}_{11}^{-1})^{-1}\mathbf{V}_{12}^{-1}\hat{\mathbf{k}}_t - (\mathbf{V}_{11}^{-1})^{-1}\mathbf{D}_1^{-1}(I - \mathbf{D}_1^{-1})^{-1}\tilde{\mathbf{\Phi}}_1\begin{bmatrix}\hat{\mathbf{a}}_t \\ \hat{\mathbf{b}}_t\end{bmatrix}\end{aligned} \tag{C.129}$$

Next, we substitute C.129 back into C.128:

$$\begin{aligned}\hat{\mathbf{y}}_t =& \Big\{(\mathbf{I}+\boldsymbol{\upsilon})\boldsymbol{\alpha} - [(\boldsymbol{\upsilon}+\epsilon_Q\mathbf{I})(\mathbf{I}-\mathbf{S_M})^{-1}\mathbf{S_M}(\mathbf{I}-\tilde{\mathbf{S}}_\mathbf{1}^\mathbf{M}\tilde{\mathbf{S}}_\mathbf{2}^\mathbf{M})+\boldsymbol{\upsilon}](\mathbf{V}_{11}^{-1})^{-1}\mathbf{V}_{12}^{-1}\Big\}\hat{\mathbf{k}}_t \\ &+ \Big\{\frac{\epsilon_Q-1}{\epsilon_Q}\boldsymbol{\upsilon} + \left(\frac{\boldsymbol{\upsilon}}{\epsilon_Q}+\mathbf{I}\right)(\mathbf{I}-\mathbf{S_M})^{-1}(\mathbf{I}+\mathbf{S_M}(\epsilon_Q-1)) \\ &- [(\boldsymbol{\upsilon}+\epsilon_Q\mathbf{I})(\mathbf{I}-\mathbf{S_M})^{-1}\mathbf{S_M}(\mathbf{I}-\tilde{\mathbf{S}}_\mathbf{1}^\mathbf{M}\tilde{\mathbf{S}}_\mathbf{2}^\mathbf{M})+\boldsymbol{\upsilon}](\mathbf{V}_{11}^{-1})^{-1}\mathbf{D}_1^{-1}(I-\mathbf{D}_1^{-1})^{-1}\tilde{\boldsymbol{\Phi}}_{11}\Big\}\hat{\mathbf{a}}_t \\ &- \Big\{[(\boldsymbol{\upsilon}+\epsilon_Q\mathbf{I})(\mathbf{I}-\mathbf{S_M})^{-1}\mathbf{S_M}(\mathbf{I}-\tilde{\mathbf{S}}_\mathbf{1}^\mathbf{M}\tilde{\mathbf{S}}_\mathbf{2}^\mathbf{M})+\boldsymbol{\upsilon}](\mathbf{V}_{11}^{-1})^{-1}\mathbf{D}_1^{-1}(I-\mathbf{D}_1^{-1})^{-1}\tilde{\boldsymbol{\Phi}}_{12}\Big\}\hat{\mathbf{b}}_t,\end{aligned}$$

which we can rewrite as:

$$\hat{\mathbf{y}}_t = \boldsymbol{\Phi_{kq}}\hat{\mathbf{k}}_t + \boldsymbol{\Phi_{aq}}\hat{\mathbf{a}}_t + \boldsymbol{\Phi_{bq}}\hat{\mathbf{b}}_t \tag{C.130}$$

Next, we substitute out $\hat{\mathbf{k}}_t$. First note, that as long as $\boldsymbol{\Phi_{kq}}$ is invertible, equation C.130 is equivalent to:

$$\hat{\mathbf{k}}_t = \boldsymbol{\Phi_{kq}}^{-1}\hat{\mathbf{y}}_t - \boldsymbol{\Phi_{kq}}^{-1}\boldsymbol{\Phi_{bq}}\hat{\mathbf{b}}_t - \boldsymbol{\Phi_{kq}}^{-1}\boldsymbol{\Phi_{aq}}\hat{\mathbf{a}}_t \tag{C.131}$$

Take equation C.130 one period ahead:

$$\hat{\mathbf{y}}_{t+1} = \boldsymbol{\Phi_{kq}}\hat{\mathbf{k}}_{t+1} + \boldsymbol{\Phi_{bq}}\hat{\mathbf{b}}_{t+1} + \boldsymbol{\Phi_{aq}}\hat{\mathbf{a}}_{t+1} \tag{C.132}$$

Now, we first substitute equation C.126 in equation C.132:

$$\hat{\mathbf{y}}_{t+1} = \boldsymbol{\Phi_{bq}}\hat{\mathbf{b}}_{t+1} + \boldsymbol{\Phi_{aq}}\hat{\mathbf{a}}_{t+1} + \boldsymbol{\Phi_{kq}}\big(\mathrm{M}_\mathbf{kk}\hat{\mathbf{k}}_t + \mathrm{M}_\mathbf{ka}\hat{\mathbf{a}}_t + \mathrm{M}_\mathbf{kb}\hat{\mathbf{b}}_t\big),$$

followed by substituting out $\hat{\mathbf{k}}_t$ using equation C.131:

$$\begin{aligned}\hat{\mathbf{y}}_{t+1} =& \boldsymbol{\Phi_{bq}}\hat{\mathbf{b}}_{t+1} + \boldsymbol{\Phi_{aq}}\hat{\mathbf{a}}_{t+1} + \boldsymbol{\Phi_{kq}}\mathrm{M}_\mathbf{ka}\hat{\mathbf{a}}_t + \boldsymbol{\Phi_{kq}}\mathrm{M}_\mathbf{kb}\hat{\mathbf{b}}_t + \boldsymbol{\Phi_{kq}}\mathrm{M}_\mathbf{kk}\boldsymbol{\Phi_{kq}}^{-1}\hat{\mathbf{y}}_t \\ &- \boldsymbol{\Phi_{kq}}\mathrm{M}_\mathbf{kk}\boldsymbol{\Phi_{kq}}^{-1}\boldsymbol{\Phi_{bq}}\hat{\mathbf{b}}_t - \boldsymbol{\Phi_{kq}}\mathrm{M}_\mathbf{kk}\boldsymbol{\Phi_{kq}}^{-1}\boldsymbol{\Phi_{aq}}\hat{\mathbf{a}}_t,\end{aligned}$$

and simplify:

$$\begin{aligned}\hat{\mathbf{y}}_{t+1} =& \boldsymbol{\Phi_{bq}}\hat{\mathbf{b}}_{t+1} + \boldsymbol{\Phi_{aq}}\hat{\mathbf{a}}_{t+1} + \boldsymbol{\Phi_{kq}}\mathrm{M}_\mathbf{kk}\boldsymbol{\Phi_{kq}}^{-1}\hat{\mathbf{y}}_t + [\boldsymbol{\Phi_{kq}}\mathrm{M}_\mathbf{ka} - \boldsymbol{\Phi_{kq}}\mathrm{M}_\mathbf{kk}\boldsymbol{\Phi_{kq}}^{-1}\boldsymbol{\Phi_{aq}}]\hat{\mathbf{a}}_t \\ &+ [\boldsymbol{\Phi_{kq}}\mathrm{M}_\mathbf{kb} - \boldsymbol{\Phi_{kq}}\mathrm{M}_\mathbf{kk}\boldsymbol{\Phi_{kq}}^{-1}\boldsymbol{\Phi_{bq}}]\hat{\mathbf{b}}_t\end{aligned}$$

Finally, take two consecutive periods, and use the definitions of $\boldsymbol{\zeta^A_{t+1}} \equiv \hat{\mathbf{a}}_{t+1} - \hat{\mathbf{a}}_t$ and set $\boldsymbol{\zeta^B_{t+1}} \equiv \hat{\mathbf{b}}_{t+1} - \hat{\mathbf{b}}_t = 0$. Then, it follows that:

$$\Delta \hat{\mathbf{y}}_{t+1} = \underbrace{\boldsymbol{\Phi}_{\boldsymbol{kq}} \mathbf{M}_{\mathbf{kk}} \boldsymbol{\Phi}_{\boldsymbol{kq}}{}^{-1}}_{\Pi_1} \Delta \hat{\mathbf{y}}_t + \underbrace{[\boldsymbol{\Phi}_{\boldsymbol{kq}} \mathbf{M}_{\mathbf{ka}} - \boldsymbol{\Phi}_{\boldsymbol{kq}} \mathbf{M}_{\mathbf{kk}} \boldsymbol{\Phi}_{\boldsymbol{kq}}{}^{-1} \boldsymbol{\Phi}_{\boldsymbol{aq}}]}_{\Pi_2} \boldsymbol{\zeta}_{\boldsymbol{t}}^{\boldsymbol{A}} + \underbrace{\boldsymbol{\Phi}_{\boldsymbol{aq}}}_{\Pi_3} \boldsymbol{\zeta}_{\boldsymbol{t+1}}^{\boldsymbol{A}} \tag{C.133}$$

D Derivations of Structural Variance Covariance Matrices

Solving equation (C.133) by recursion:

$$\triangle \boldsymbol{y}_{t+1} = \sum_{v=0} [\boldsymbol{\Pi}_1^{v+1}\boldsymbol{\Pi}_3\boldsymbol{\zeta}_{t-v} + \boldsymbol{\Pi}_1^{v}\boldsymbol{\Pi}_2\boldsymbol{\zeta}_{t-v}] + \boldsymbol{\Pi}_3\boldsymbol{\zeta}_{t+1} \tag{D.134}$$

Taking variances

$$Var(\triangle \boldsymbol{y}_{t+1}) = Var\Big(\sum_{v=0} [\boldsymbol{\Pi}_1^{v+1}\boldsymbol{\Pi}_3\boldsymbol{\zeta}_{t-v} + \boldsymbol{\Pi}_1^{v}\boldsymbol{\Pi}_2\boldsymbol{\zeta}_{t-v}] + \boldsymbol{\Pi}_3\boldsymbol{\zeta}_{t+1}\Big) \tag{D.135}$$

where $Var(\triangle \boldsymbol{y}_{t+1})$ is the variance covariance matrix of sector IP growth. Using equation (14), decompose $\boldsymbol{\zeta}_t$ into its common and idiosyncratic components so that:

$$\begin{aligned} Var(\triangle \boldsymbol{y}_{t+1}) &= Var\Big(\sum_{v=0} [\boldsymbol{\Pi}_1^{v+1}\boldsymbol{\Pi}_3(\boldsymbol{\Lambda}_S\boldsymbol{S}_{t-v} + \boldsymbol{\epsilon}_{t-v}) + \boldsymbol{\Pi}_1^{v}\boldsymbol{\Pi}_2(\boldsymbol{\Lambda}_S\boldsymbol{S}_{t-v} + \boldsymbol{\epsilon}_{t-v})] + \boldsymbol{\Pi}_3(\boldsymbol{\Lambda}_S\boldsymbol{S}_{t+1} + \boldsymbol{\epsilon}_{t+1})\Big) \\ &= Var\Big(\sum_{v=0} [\boldsymbol{\Pi}_1^{v+1}\boldsymbol{\Pi}_3\boldsymbol{\Lambda}_S\boldsymbol{S}_{t-v} + \boldsymbol{\Pi}_1^{v}\boldsymbol{\Pi}_2\boldsymbol{\Lambda}_S\boldsymbol{S}_{t-v}] + \boldsymbol{\Pi}_3\boldsymbol{\Lambda}_S\boldsymbol{S}_{t+1} \\ &+ \sum_{v=0} [\boldsymbol{\Pi}_1^{v+1}\boldsymbol{\Pi}_3\boldsymbol{\epsilon}_{t-v} + \boldsymbol{\Pi}_1^{v}\boldsymbol{\Pi}_2\boldsymbol{\epsilon}_{t-v}] + \boldsymbol{\Pi}_3\boldsymbol{\epsilon}_{t+1}\Big) \end{aligned} \tag{D.136}$$

As defined in section 3.2, $\boldsymbol{\epsilon}_t$ is a vector capturing country-sector level idiosyncratic shocks and has elements $\epsilon^1_{1,t}, \epsilon^2_{1,t}, ..., \epsilon^J_{N,t}$ while $\boldsymbol{S}_t$ is a vector capturing global, regional and country common shocks with elements $S_{1,t}, ..., S_{k,t}$. By assumption factors and idiosyncratic shocks are orthogonal to each other, and both the factors and idiosyncratic shocks are uncorrelated, $E(\epsilon^i_{m,t}\epsilon^j_{n,t}{}') = 0$ for all $i, m \neq j, n$, $E(S_{l,t}S'_{k,t}) = 0$ for all $l \neq k$, $E(\epsilon^j_{n,t}S'_{k,t}) = 0$ for all j, n, k. Therefore, D.136 becomes:

$$\begin{aligned} Var(\triangle y_{t+1}) &= \sum_{k=1} Var\left(\sum_{v=0}\left[\boldsymbol{\Pi}_1^{v+1}\boldsymbol{\Pi}_3[\boldsymbol{\Lambda}_S]_k S_{k,t-v} + \boldsymbol{\Pi}_1^{v}\boldsymbol{\Pi}_2[\boldsymbol{\Lambda}_S]_k S_{k,t-v}\right] + \boldsymbol{\Pi}_3[\boldsymbol{\Lambda}_S]_k S_{k,t+1}\right) \\ &+ \sum_{n=1}\sum_{j=1} Var\left(\sum_{v=0}\left[\boldsymbol{\Pi}_1^{v+1}[\boldsymbol{\Pi}_3]_{nj}\epsilon^j_{n,t-v} + \boldsymbol{\Pi}_1^{v}[\boldsymbol{\Pi}_2]_{nj}\epsilon^j_{n,t-v}\right] + [\boldsymbol{\Pi}_3]_{nj}\epsilon^j_{n,t+1}\right) \end{aligned} \tag{D.137}$$

Where $[\boldsymbol{\Pi_3}]_{nj}$ is a vector with the elements $[\boldsymbol{\Pi}_3]^{1j}_{1n}, [\boldsymbol{\Pi}_3]^{2j}_{1n}, ..., [\boldsymbol{\Pi}_3]^{JJ}_{NN}$, $[\boldsymbol{\Pi_2}]_{nj}$ is a vector with the elements $[\boldsymbol{\Pi}_2]^{1j}_{1n}, [\boldsymbol{\Pi}_2]^{2j}_{1n}, ..., [\boldsymbol{\Pi}_2]^{JJ}_{NN}$ and $[\boldsymbol{\Lambda}_S]_k$ is a vector with $\boldsymbol{\Lambda}_S$'s k column.

Given that both sector and factor innovations are not serially correlated, rearranging and rewriting yields

$$Var(\triangle \mathbf{y}_{t+1}) =$$
$$\sum_{k=1} \Bigg(\underbrace{\sum_{v=0} \Big[(\mathbf{\Pi}_1^{v+1}\mathbf{\Pi}_3[\mathbf{\Lambda}_S]_k)(\mathbf{\Pi}_1^{v+1}\mathbf{\Pi}_3[\mathbf{\Lambda}_S]_k)' + (\mathbf{\Pi}_1^{v}\mathbf{\Pi}_2[\mathbf{\Lambda}_S]_k)(\mathbf{\Pi}_1^{v}\mathbf{\Pi}_2[\mathbf{\Lambda}_S]_k)' \Big](\sigma_k^S)^2 + (\mathbf{\Pi}_3[\mathbf{\Lambda}_S]_k)(\mathbf{\Pi}_3[\mathbf{\Lambda}_S]_k)'(\sigma_k^S)^2}_{\text{Variance covariance matrix of implied IP growth associated with common shocks from factor } k} \Bigg)$$
$$+ \sum_{n=1} \sum_{j=1} \Bigg(\underbrace{\sum_{v=0} \Big[(\mathbf{\Pi}_1^{v+1}[\mathbf{\Pi}_3]_{nj})(\mathbf{\Pi}_1^{v+1}[\mathbf{\Pi}_3]_{nj})' + (\mathbf{\Pi}_1^{v}[\mathbf{\Pi}_2]_{nj})(\mathbf{\Pi}_1^{v}[\mathbf{\Pi}_2]_{nj})' \Big](\sigma_{nj}^\epsilon)^2 + ([\mathbf{\Pi}_3]_{nj})([\mathbf{\Pi}_3]_{nj})'(\sigma_{nj}^\epsilon)^2}_{\text{Variance covariance matrix of implied IP growth associated with idiosyncratic shocks from country-sector } nj} \Bigg), \tag{D.138}$$

where σ_k^S and σ_{nj}^ϵ are the standard deviations of the common shock associated with factor k and of the idiosyncratic country-sector shock nj respectively.

Equation (D.137) can be expressed as

$$\mathbf{\Sigma}^{\text{all}} = \mathbf{\Sigma}^G + \mathbf{\Sigma}^R + \mathbf{\Sigma}^C + \mathbf{\Sigma}^S \tag{D.139}$$

where $\mathbf{\Sigma}^{\text{all}} = Var(\triangle \mathbf{y}_{t+1})$. $\mathbf{\Sigma}^G$ captures the $(N \times J) \times (N \times J)$ variance covariance matrix of sector IP growth induced by the global common shock:

$$\begin{aligned} \mathbf{\Sigma}^G = & \sum_{v=0} \Big[(\mathbf{\Pi}_1^{v+1}\mathbf{\Pi}_3[\mathbf{\Lambda}_S]_{\text{Global}})(\mathbf{\Pi}_1^{v+1}\mathbf{\Pi}_3[\mathbf{\Lambda}_S]_{\text{Global}})' \\ & + (\mathbf{\Pi}_1^{v}\mathbf{\Pi}_2[\mathbf{\Lambda}_S]_{\text{Global}})(\mathbf{\Pi}_1^{v}\mathbf{\Pi}_2[\mathbf{\Lambda}_S]_{\text{Global}})' \Big] (\sigma_{\text{Global}}^S)^2 \\ & + (\mathbf{\Pi}_3[\mathbf{\Lambda}_S]_{\text{Global}})(\mathbf{\Pi}_3[\mathbf{\Lambda}_S]_{\text{Global}})'(\sigma_{\text{Global}}^S)^2 \end{aligned} \tag{D.140}$$

$\mathbf{\Sigma}^R$ captures the $(N \times J) \times (N \times J)$ variance covariance matrix of sector IP growth induced by the 4 common regional shocks:

$$\begin{aligned} \mathbf{\Sigma}^R = & \sum_{r=1} \Bigg(\sum_{v=0} \Big[(\mathbf{\Pi}_1^{v+1}\mathbf{\Pi}_3[\mathbf{\Lambda}_S]_{\text{Region } r})(\mathbf{\Pi}_1^{v+1}\mathbf{\Pi}_3[\mathbf{\Lambda}_S]_{\text{Region } r})' \\ & + (\mathbf{\Pi}_1^{v}\mathbf{\Pi}_2[\mathbf{\Lambda}_S]_{\text{Region } r})(\mathbf{\Pi}_1^{v}\mathbf{\Pi}_2[\mathbf{\Lambda}_S]_{\text{Region } r})' \Big] (\sigma_{\text{Region } r}^S)^2 \\ & + (\mathbf{\Pi}_3[\mathbf{\Lambda}_S]_{\text{Region } r})(\mathbf{\Pi}_3[\mathbf{\Lambda}_S]_{\text{Region } r})'(\sigma_{\text{Region } r}^S)^2 \Bigg) \end{aligned} \tag{D.141}$$

$\mathbf{\Sigma}^C$ captures the $(N \times J) \times (N \times J)$ variance covariance matrix of sector IP growth induced by the 29 common country shocks:

$$\begin{aligned}\boldsymbol{\Sigma}^C = \sum_{m=1} \Bigg(\sum_{v=0} \Big[&(\boldsymbol{\Pi}_1^{v+1}\boldsymbol{\Pi}_3[\boldsymbol{\Lambda}_S]_{\text{Country } m})(\boldsymbol{\Pi}_1^{v+1}\boldsymbol{\Pi}_3[\boldsymbol{\Lambda}_S]_{\text{Country } m})' \\ &+ (\boldsymbol{\Pi}_1^{v}\boldsymbol{\Pi}_2[\boldsymbol{\Lambda}_S]_{\text{Country } m})(\boldsymbol{\Pi}_1^{v}\boldsymbol{\Pi}_2[\boldsymbol{\Lambda}_S]_{\text{Country } m})' \Big] (\sigma^S_{\text{Country } m})^2 \\ &+ (\boldsymbol{\Pi}_3[\boldsymbol{\Lambda}_S]_{\text{Country } m})(\boldsymbol{\Pi}_3[\boldsymbol{\Lambda}_S]_{\text{Country } m})'(\sigma^S_{\text{Country } m})^2 \Bigg)\end{aligned} \tag{D.142}$$

$\boldsymbol{\Sigma}^S$ captures the the $(N \times J) \times (N \times J)$ variance covariance matrix of sector IP growth induced by idiosyncratic sector shocks:

$$\begin{aligned}\boldsymbol{\Sigma}^S = \sum_{n=1}\sum_{j=1} \Bigg(&\sum_{v=0} \Big[(\boldsymbol{\Pi}_1^{v+1}[\boldsymbol{\Pi}_3]_{nj})(\boldsymbol{\Pi}_1^{v+1}[\boldsymbol{\Pi}_3]_{nj})' + (\boldsymbol{\Pi}_1^{v}[\boldsymbol{\Pi}_2]_{nj})(\boldsymbol{\Pi}_1^{v}[\boldsymbol{\Pi}_2]_{nj})' \Big] (\sigma^{\epsilon}_{nj})^2 \\ &+ ([\boldsymbol{\Pi}_3]_{nj})([\boldsymbol{\Pi}_3]_{nj})'(\sigma^{\epsilon}_{nj})^2 \Bigg)\end{aligned} \tag{D.143}$$

Additionally we define variance decompositions of global IP growth with respect to global, regional and country common shocks by:

$$\begin{aligned} R^2_{\text{G}}(S^{\text{Global}}) &= \frac{\mathbf{w}_G' \boldsymbol{\Sigma}^G \mathbf{w}_G}{\mathbf{w}_G' \boldsymbol{\Sigma}^{\text{all}} \mathbf{w}_G} \\ R^2_{\text{G}}(S^{\text{Region}}) &= \frac{\mathbf{w}_G' \boldsymbol{\Sigma}^R \mathbf{w}_G}{\mathbf{w}_G' \boldsymbol{\Sigma}^{\text{all}} \mathbf{w}_G} \\ R^2_{\text{G}}(S^{\text{Country}}) &= \frac{\mathbf{w}_G' \boldsymbol{\Sigma}^C \mathbf{w}_G}{\mathbf{w}_G' \boldsymbol{\Sigma}^{\text{all}} \mathbf{w}_G}. \end{aligned} \tag{D.144}$$

Equivalently, we define variance decompositions of IP growth in region r with respect to global, regional and country common shocks by:

$$
\begin{aligned}
R^2_{\mathrm{r}}(S^{\mathrm{Global}}) &= \frac{\mathbf{w}'_r \mathbf{\Sigma}^G_r \mathbf{w}_r}{\mathbf{w}'_r \mathbf{\Sigma}^{\mathrm{all}}_r \mathbf{w}_r} \\
R^2_{\mathrm{r}}(S^{\mathrm{Region}}) &= \frac{\mathbf{w}'_r \mathbf{\Sigma}^R_r \mathbf{w}_r}{\mathbf{w}'_r \mathbf{\Sigma}^{\mathrm{all}}_r \mathbf{w}_r} \\
R^2_{\mathrm{r}}(S^{\mathrm{Country}}) &= \frac{\mathbf{w}'_r \mathbf{\Sigma}^C_r \mathbf{w}_r}{\mathbf{w}'_r \mathbf{\Sigma}^{\mathrm{all}}_r \mathbf{w}_r}
\end{aligned} \tag{D.145}
$$

where $\mathbf{\Sigma}^G_r$, $\mathbf{\Sigma}^R_r$, $\mathbf{\Sigma}^C_r$, and $\mathbf{\Sigma}^{\mathrm{all}}_r$ are those subsets of $\mathbf{\Sigma}^G$, $\mathbf{\Sigma}^R$, $\mathbf{\Sigma}^C$, and $\mathbf{\Sigma}^{\mathrm{all}}$ that span over the sectors located in region r. Finally, the fraction of IP growth in country m explained by (the same set of) common shocks S_t is given by:

$$
\begin{aligned}
R^2_m(S^{\mathrm{Global}}) &= \frac{\mathbf{w}'_m \mathbf{\Sigma}^G_m \mathbf{w}_m}{\mathbf{w}'_m \mathbf{\Sigma}^{\mathrm{all}}_m \mathbf{w}_m} \\
R^2_m(S^{\mathrm{Region}}) &= \frac{\mathbf{w}'_m \mathbf{\Sigma}^R_m \mathbf{w}_r}{\mathbf{w}'_m \mathbf{\Sigma}^{\mathrm{all}}_m \mathbf{w}_m} \\
R^2_{\mathrm{m}}(S^{\mathrm{Country}}) &= \frac{\mathbf{w}'_m \mathbf{\Sigma}^C_m \mathbf{w}_m}{\mathbf{w}'_m \mathbf{\Sigma}^{\mathrm{all}}_m \mathbf{w}_m}
\end{aligned} \tag{D.146}
$$

where $\mathbf{\Sigma}^G_m$, $\mathbf{\Sigma}^R_m$, $\mathbf{\Sigma}^C_m$, and $\mathbf{\Sigma}^{\mathrm{all}}_m$ are those subsets of $\mathbf{\Sigma}^G$, $\mathbf{\Sigma}^R$, $\mathbf{\Sigma}^C$, and $\mathbf{\Sigma}^{\mathrm{all}}$ that span over the sectors located in country m. $\mathbf{w}_m$,$\mathbf{w}_r$ and $\mathbf{w}_G$ and define value-added sector weights at aggregate country, region, and world level, as discussed in Appendix A.

E Parameterization of the open economy model

We choose steady state values for $\mathbf{\Gamma}^{M1}$, $\mathbf{\Gamma}^{M2}$, $\mathbf{\Gamma}^X$, ξ, μ, $\tau^{\mathbf{M}}$ and $\tau^{\mathbf{X}}$ by matching model-implied cost shares in the steady state to long-run averages of cost shares in the data. Below, we rearrange the resulting equations for the technology and shifter parameters.

In practise, these relationships do not allow to identify the level of the shifter parameters for intermediate and capital goods $\mathbf{\Gamma}^{\mathbf{M2}}$ and $\mathbf{\Gamma}^{\mathbf{X}}$ separately from the intermediate and capital trade cost $\tau^{\mathbf{M}}$ and $\tau^{\mathbf{X}}$. Instead we identify the trade cost - cum - preference parameters $\mu^{ij}_{mn}([\tau^M]^{ij}_{mn})^{1-\epsilon_T}$ and $\kappa^{ij}_{mn}([\tau^X]^{ij}_{mn})^{1-\epsilon_X}$, which is sufficient for the calibration of the model. For that reason, the five equations below determine five unknown technology, shifter and shifter-cum-trade cost parameters.

Rearranging equation C.47 and multiplying by $\frac{[\tau^X]^{ij}_{mn} P^i_m}{[P^X]^j_n} \frac{[P^X]^j_n}{[\tau^X]^{ij}_{mn} P^i_m}$ yields:

$$\kappa^{ij}_{mn}([\tau^X]^{ij}_{mn})^{1-\epsilon_X} = \left(\frac{[P^X]^j_n}{P^i_m}\right)^{1-\epsilon_X} \times \underbrace{\frac{[\tau^X]^{ij}_{mn} P^i_m X^{ij}_{mn}}{[P^X]^j_n X^j_n}}_{\text{Capital cost share in the data}} \tag{E.147}$$

Dividing equation C.46 by equation C.51, rearranging and multiplying by $\frac{[\tau^M]^{ij}_{mn} P^i_m}{[P^M]^{ij}_n} \frac{[P^M]^{ij}_n}{[\tau^M]^{ij}_{mn} P^i_m}$ yields:

$$\mu^{ij}_{mn}([\tau^M]^{ij}_{mn})^{1-\epsilon_T} = \left(\frac{[P^M]^{ij}_n}{P^i_m}\right)^{1-\epsilon_T} \times \underbrace{\frac{[\tau^M]^{ij}_{mn} P^i_m M^{ij}_{mn}}{[P^M]^{ij}_n M^{ij}_n}}_{i \text{ specific cost share wrt to country } m \text{ in the data}} \tag{E.148}$$

Dividing equation C.51 by equation C.50, rearranging and multiplying by $\frac{[P^M]^{ij}_n}{[P^M]^j_n} \frac{[P^M]^j_n}{[P^M]^{ij}_n}$ yields:

$$\mu^{ij}_n = \left(\frac{[P^M]^j_n}{[P^M]^{ij}_n}\right)^{1-\epsilon_M} \times \underbrace{\frac{[P^M]^{ij}_n M^{ij}_n}{[P^M]^j_n M^j_n}}_{\text{Cost share wrt to good } i \text{ in the data}} \tag{E.149}$$

Rearranging equation C.50 and multiplying by $\frac{[P^M]^j_n}{P^j_n} \frac{P^j_n}{[P^M]^j_n}$ yields:

$$\mu^j_n = \left(\frac{P^j_n}{[P^M]^j_n}\right)^{1-\epsilon_Q} \times \underbrace{\frac{[P^M]^j_n M^j_n}{P^j_n Y^j_n}}_{\text{Interm. cost share in the data}} \tag{E.150}$$

Maximising utility of the representative agents with respect to any given budget constraint yields:

$$\xi^j_{mn} = \underbrace{\frac{[\tau^C]^j_{mn} P^j_m C^j_{mn}}{[P^C]_n C^j_n}}_{\text{Final cost share in the data}} \tag{E.151}$$

All of the above preference parameters and trade cost - cum - preference parameters are defined by observed cost shares in the data and model-implied steady state prices. Section C.2 discusses how we derive model-implied prices and preference and trade cost - cum - preference parameters jointly.

F Parameterization of the closed economy model

For the parameterization in the closed economy model we apply the calibration approach described in Foerster et al. (2011). In a first step, we normalise sector output by the sum of sector value added and intermediate consumption of goods in domestic mining and manufacturing sectors, excluding other sectors in the economy. In a second step, we calibrate capital, labour, and intermediate shares based on the normalised sector output. This approach contrasts to the parameterization in the international model that relies on observed capital, labour and intermediate shares in sector output on the basis of all sectors in the economy (as in Atalay (2017) or Huo et al. (2023)).

G Additional tables

G.1 Allowing for country-specific numbers of country factors

Table (G.2) Variance Decompositions in Structural and Reduced Form Models

	$\boldsymbol{F}$ (1)	$\boldsymbol{F}^{\mathrm{G}}$ (2)	$\boldsymbol{F}^{\mathrm{R}}$ (3)	$\boldsymbol{F}^{\mathrm{C}}$ (4)	$\boldsymbol{S}$ (5)	$\boldsymbol{S}^{\mathrm{G}}$ (6)	$\boldsymbol{S}^{\mathrm{R}}$ (7)	$\boldsymbol{S}^{\mathrm{C}}$ (8)	Δ (9)	Δ^{G} (10)	Δ^{R} (11)	Δ^{C} (12)
Country												
USA	87%	60%	10%	18%	47%	36%	9%	3%	40%	24%	1%	15%
CAN	64%	59%	1%	3%	52%	45%	6%	2%	11%	15%	-5%	2%
MEX	91%	57%	5%	29%	83%	49%	23%	11%	8%	8%	-18%	18%
DEU	89%	80%	5%	4%	69%	43%	14%	12%	20%	37%	-9%	-9%
ITA	98%	68%	3%	27%	81%	42%	10%	29%	17%	25%	-6%	-1%
GBR	42%	24%	3%	14%	42%	21%	0%	21%	0%	3%	3%	-6%
FRA	85%	73%	5%	7%	60%	33%	10%	16%	25%	39%	-5%	-9%
CHN	94%	6%	48%	39%	39%	2%	10%	28%	54%	5%	38%	12%
JPN	92%	40%	27%	25%	58%	18%	37%	2%	34%	22%	-10%	22%
KOR	93%	41%	26%	26%	79%	28%	37%	14%	14%	14%	-11%	12%
Average	**86%**	**44%**	**19%**	**23%**	**53%**	**26%**	**13%**	**14%**	**33%**	**18%**	**6%**	**9%**
Region												
Western Europe	96%	91%	2%	3%	76%	53%	10%	13%	20%	38%	-8%	-10%
Central and Eastern Europe	97%	86%	3%	8%	85%	73%	4%	8%	12%	13%	0%	0%
Americas	91%	70%	8%	13%	57%	44%	11%	2%	34%	26%	-2%	10%
Asia	96%	39%	52%	5%	62%	13%	35%	14%	34%	26%	17%	-9%
Average	**93%**	**62%**	**24%**	**7%**	**64%**	**35%**	**20%**	**10%**	**29%**	**28%**	**4%**	**-3%**
Global	**96%**	**85%**	**10%**	**3%**	**76%**	**55%**	**13%**	**9%**	**20%**	**30%**	**-2%**	**-6%**

Columns (1) to (4) report variance decompositions for the reduced form model as implied by global, regional and country common factors. Column (1) summarizes the contribution of a "common" factor, summing columns (2), (3), and (4). Columns (5) - (8) report the corresponding results for the structural form factor model. Columns (9) - (12) report the absolute differences. "Average" reports value added weighted averages of the country and region-level variance decompositions.

G.2 Variance decomposition for full set of countries

Table (G.3) Variance Decomposition in Structural and Reduced Form Models

	$\boldsymbol{F}$ (1)	$\boldsymbol{F}^{\mathrm{G}}$ (2)	$\boldsymbol{F}^{\mathrm{R}}$ (3)	$\boldsymbol{F}^{\mathrm{C}}$ (4)	$\boldsymbol{S}$ (5)	$\boldsymbol{S}^{\mathrm{G}}$ (6)	$\boldsymbol{S}^{\mathrm{R}}$ (7)	$\boldsymbol{S}^{\mathrm{C}}$ (8)	Δ (9)	Δ^{G} (10)	Δ^{R} (11)	Δ^{C} (12)
Country												
AUT	90%	74%	1%	15%	69%	59%	3%	7%	21%	14%	-2%	8%
BEL	78%	57%	1%	20%	60%	42%	9%	9%	18%	15%	-9%	11%
BGR	84%	55%	1%	28%	53%	42%	7%	4%	32%	13%	-6%	24%
CAN	87%	51%	1%	35%	67%	49%	6%	12%	20%	2%	-6%	24%
CHN	94%	6%	48%	39%	39%	2%	10%	28%	54%	5%	38%	12%
CZE	88%	79%	0%	9%	66%	54%	5%	6%	22%	24%	-5%	2%
DEU	89%	80%	5%	4%	69%	43%	14%	13%	20%	37%	-9%	-9%
DNK	71%	14%	0%	57%	48%	15%	2%	31%	23%	-1%	-2%	27%
ESP	91%	65%	3%	23%	52%	36%	8%	7%	39%	29%	-6%	15%
EST	82%	45%	2%	35%	41%	33%	3%	5%	41%	12%	-1%	30%
FRA	88%	74%	5%	9%	65%	35%	11%	19%	23%	38%	-5%	-10%
GBR	49%	23%	3%	23%	46%	21%	0%	24%	3%	2%	3%	-2%
GRC	85%	35%	18%	32%	60%	42%	2%	16%	26%	-7%	16%	16%
HRV	69%	37%	10%	21%	50%	38%	1%	11%	19%	-1%	10%	10%
HUN	88%	79%	0%	9%	73%	61%	3%	9%	15%	18%	-3%	0%
ITA	95%	67%	3%	25%	66%	40%	9%	17%	29%	27%	-6%	8%
JPN	93%	40%	28%	25%	65%	18%	37%	9%	28%	22%	-10%	16%
KOR	87%	40%	25%	22%	59%	21%	29%	9%	28%	19%	-4%	13%
LTU	65%	56%	1%	8%	34%	26%	1%	7%	31%	30%	0%	1%
MEX	87%	55%	4%	27%	75%	44%	21%	10%	12%	11%	-16%	17%
NLD	75%	60%	1%	14%	55%	39%	2%	13%	20%	21%	-1%	0%
NOR	34%	0%	0%	33%	54%	1%	0%	52%	-20%	-1%	0%	-19%
POL	90%	54%	7%	29%	53%	39%	5%	9%	36%	15%	2%	20%
PRT	78%	46%	4%	28%	62%	42%	0%	19%	16%	4%	4%	9%
ROU	84%	62%	5%	17%	52%	47%	1%	4%	32%	14%	4%	13%
SVN	91%	72%	0%	19%	69%	61%	2%	5%	22%	11%	-2%	14%
TUR	93%	57%	0%	37%	80%	51%	1%	28%	13%	6%	-1%	9%
TWN	76%	34%	29%	13%	61%	13%	29%	19%	15%	22%	-1%	-6%
USA	92%	59%	10%	23%	64%	36%	9%	18%	28%	22%	1%	5%
Average	**89%**	**43%**	**19%**	**26%**	**58%**	**26%**	**13%**	**19%**	**31%**	**17%**	**6%**	**7%**
Region												
W. Europe	96%	91%	2%	3%	77%	53%	10%	13%	19%	37%	-8%	-10%
C.&E. Europe	97%	85%	3%	9%	79%	70%	3%	6%	18%	15%	0%	3%
Americas	94%	66%	8%	20%	70%	44%	11%	15%	24%	22%	-3%	5%
Asia	96%	39%	52%	5%	62%	13%	34%	15%	34%	26%	18%	-10%
Average	**95%**	**61%**	**23%**	**10%**	**69%**	**34%**	**19%**	**15%**	**26%**	**27%**	**4%**	**-5%**
Global	**98%**	**84%**	**10%**	**4%**	**78%**	**54%**	**12%**	**11%**	**20%**	**30%**	**-2%**	**-8%**

Columns (1) to (4) report variance decompositions for the reduced form model as implied by global, regional and country common factors. Column (1) summarizes the contribution of a "common" factor, summing columns (2), (3), and (4). Columns (5) - (8) report the corresponding results for the structural form factor model. Columns (9) - (12) report the absolute differences. "Average" reports value added weighted averages of the country and region-level variance decompositions.

G.3 Alternative WIOD tables

2006

Table (G.4) Variance Decompositions in Structural and Reduced Form Models, 2008 WIOD

	$\boldsymbol{F}$	$\boldsymbol{F}^{\mathrm{G}}$	$\boldsymbol{F}^{\mathrm{R}}$	$\boldsymbol{F}^{\mathrm{C}}$	$\boldsymbol{S}$	$\boldsymbol{S}^{\mathrm{G}}$	$\boldsymbol{S}^{\mathrm{R}}$	$\boldsymbol{S}^{\mathrm{C}}$	Δ	Δ^{G}	Δ^{R}	Δ^{C}
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Country												
USA	92%	59%	10%	23%	63%	37%	9%	17%	29%	21%	1%	6%
CAN	87%	51%	1%	35%	68%	51%	6%	12%	18%	0%	-5%	23%
MEX	87%	55%	4%	27%	77%	46%	22%	9%	10%	9%	-18%	18%
DEU	89%	80%	5%	4%	68%	43%	14%	11%	21%	37%	-8%	-7%
ITA	95%	67%	3%	25%	66%	41%	9%	15%	30%	26%	-6%	10%
GBR	49%	23%	3%	23%	43%	21%	0%	22%	5%	2%	3%	1%
FRA	88%	74%	5%	9%	65%	35%	10%	20%	23%	39%	-5%	-11%
CHN	94%	6%	48%	39%	47%	3%	13%	30%	47%	3%	35%	9%
JPN	93%	40%	28%	25%	70%	18%	38%	14%	23%	22%	-10%	11%
KOR	87%	40%	25%	22%	63%	23%	30%	10%	24%	17%	-5%	12%
Average	**89%**	**43%**	**19%**	**26%**	**60%**	**27%**	**14%**	**19%**	**29%**	**16%**	**6%**	**7%**
Region												
W. Europe	96%	91%	2%	3%	76%	54%	10%	12%	20%	37%	-8%	-8%
C.&E. Europe	97%	85%	3%	9%	78%	71%	2%	5%	18%	13%	1%	4%
Americas	94%	66%	8%	20%	69%	45%	10%	14%	25%	21%	-2%	6%
Asia	96%	39%	52%	5%	68%	14%	37%	17%	28%	25%	15%	-11%
Average	**95%**	**61%**	**23%**	**10%**	**71%**	**36%**	**20%**	**15%**	**24%**	**25%**	**3%**	**-4%**
Global	**98%**	**84%**	**10%**	**4%**	**79%**	**56%**	**13%**	**11%**	**19%**	**29%**	**-3%**	**-7%**

Columns (1) to (4) report variance decompositions for the reduced form model as implied by global, regional and country common factors. Column (1) summarizes the contribution of a "common" factor, summing columns (2), (3), and (4). Columns (5) - (8) report the corresponding results for the structural form factor model. Columns (9) - (12) report the absolute differences. "Average" reports value added weighted averages of the country and region-level variance decompositions.

2014

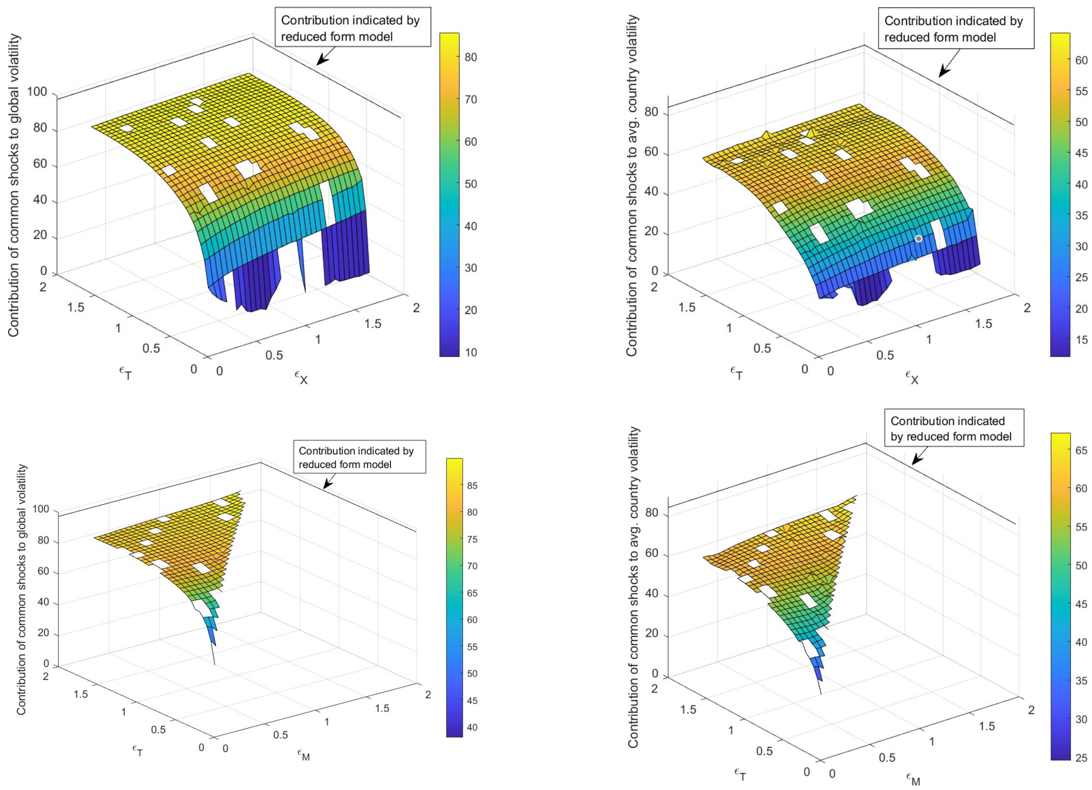
Table (G.5) Variance Decompositions in Structural and Reduced Form Models; 2014 WIOD

	$\boldsymbol{F}$	$\boldsymbol{F}^{\mathrm{G}}$	$\boldsymbol{F}^{\mathrm{R}}$	$\boldsymbol{F}^{\mathrm{C}}$	$\boldsymbol{S}$	$\boldsymbol{S}^{\mathrm{G}}$	$\boldsymbol{S}^{\mathrm{R}}$	$\boldsymbol{S}^{\mathrm{C}}$	Δ	Δ^{G}	Δ^{R}	Δ^{C}
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Country												
USA	92%	59%	10%	23%	63%	35%	8%	21%	28%	24%	1%	3%
CAN	87%	51%	1%	35%	66%	47%	6%	12%	21%	3%	-6%	23%
MEX	87%	55%	4%	27%	75%	43%	21%	11%	12%	12%	-16%	16%
DEU	89%	80%	5%	4%	71%	43%	14%	14%	19%	37%	-8%	-10%
ITA	95%	67%	3%	25%	70%	38%	8%	24%	25%	29%	-5%	1%
GBR	49%	23%	3%	23%	43%	23%	0%	20%	5%	0%	3%	2%
FRA	88%	74%	5%	9%	66%	37%	11%	18%	22%	36%	-6%	-9%
CHN	94%	6%	48%	39%	35%	1%	8%	26%	59%	5%	40%	13%
JPN	93%	40%	28%	25%	62%	20%	39%	3%	31%	20%	-11%	22%
KOR	87%	40%	25%	22%	57%	20%	28%	9%	30%	21%	-3%	13%
Average	**89%**	**43%**	**19%**	**26%**	**57%**	**26%**	**12%**	**19%**	**32%**	**18%**	**7%**	**7%**
Region												
W. Europe	96%	91%	2%	3%	78%	54%	10%	13%	18%	36%	-8%	-10%
C.&E. Europe	97%	85%	3%	9%	80%	69%	4%	7%	16%	15%	-1%	2%
Americas	94%	66%	8%	20%	69%	42%	10%	17%	25%	24%	-2%	3%
Asia	96%	39%	52%	5%	59%	13%	33%	14%	37%	26%	19%	-9%
Average	**95%**	**61%**	**23%**	**10%**	**68%**	**34%**	**18%**	**15%**	**27%**	**27%**	**5%**	**-5%**
Global	**98%**	**84%**	**10%**	**4%**	**78%**	**54%**	**12%**	**12%**	**21%**	**30%**	**-2%**	**-8%**

Columns (1) to (4) report variance decompositions for the reduced form model as implied by global, regional and country common factors. Column (1) summarizes the contribution of a "common" factor, summing columns (2), (3), and (4). Columns (5) - (8) report the corresponding results for the structural form factor model. Columns (9) - (12) report the absolute differences. "Average" reports value added weighted averages of the country and region-level variance decompositions.

G.4 Alternative substitution elasticities

Figure (G.3) Contribution of global, region and country shocks to volatility under alternative substitution elasticities for ϵ_T, ϵ_X and ϵ_M



The surface plots report the contribution of global, regional and country shocks to global and average country volatility subject to different parameterizations of the key substitution elasticities. Gaps in the surface plots reflect that for a handful of parameter combinations the model does not solve.

Table (G.6) Variance Decompositions in Structural Models

ϵ_{LS}	ϵ_Q	ϵ_M	ϵ_T	ϵ_X	S (1)	S^G (2)	S^R (3)	S^C (4)	Δ (5)
Country average									
2	1.01	0.2	0.5	0.8	58%	26%	13%	19%	31%
2	1.01	1.01	1.01	1.01	69%	33%	17%	19%	20%
2	1.01	0.2	0.2	0.2	44%	11%	10%	23%	44%
2	0.8	0.2	0.5	0.8	60%	28%	14%	18%	29%
2	1.25	0.2	0.5	0.8	55%	25%	13%	17%	34%
2	1.5	0.2	0.5	0.8	52%	23%	12%	17%	36%
0.5	1.01	0.2	0.5	0.8	64%	30%	15%	19%	24%
0.75	1.01	0.2	0.5	0.8	62%	29%	15%	18%	26%
1.01	1.01	0.2	0.5	0.8	61%	28%	14%	18%	28%
1.25	1.01	0.2	0.5	0.8	60%	28%	14%	18%	29%
1.5	1.01	0.2	0.5	0.8	59%	27%	14%	18%	30%
1.75	1.01	0.2	0.5	0.8	58%	27%	13%	18%	30%
Region average									
2	1.01	0.2	0.5	0.8	69%	34%	19%	15%	31%
2	1.01	1.01	1.01	1.01	81%	44%	26%	11%	14%
2	1.01	0.2	0.2	0.2	46%	13%	10%	23%	49%
2	0.8	0.2	0.5	0.8	71%	36%	20%	14%	24%
2	1.25	0.2	0.5	0.8	66%	33%	19%	15%	29%
2	1.5	0.2	0.5	0.8	63%	31%	18%	15%	31%
0.5	1.01	0.2	0.5	0.8	77%	41%	23%	13%	18%
0.75	1.01	0.2	0.5	0.8	75%	39%	23%	13%	20%
1.01	1.01	0.2	0.5	0.8	73%	38%	22%	14%	22%
1.25	1.01	0.2	0.5	0.8	72%	37%	21%	14%	23%
1.5	1.01	0.2	0.5	0.8	70%	36%	21%	14%	24%
1.75	1.01	0.2	0.5	0.8	69%	35%	20%	14%	25%
Global									
2	1.01	0.2	0.5	0.8	78%	54%	12%	11%	20%
2	1.01	1.01	1.01	1.01	90%	68%	15%	6%	9%
2	1.01	0.2	0.2	0.2	53%	18%	11%	24%	46%
2	0.8	0.2	0.5	0.8	80%	57%	13%	10%	18%
2	1.25	0.2	0.5	0.8	76%	52%	12%	12%	22%
2	1.5	0.2	0.5	0.8	74%	49%	12%	12%	25%
0.5	1.01	0.2	0.5	0.8	86%	66%	13%	8%	12%
0.75	1.01	0.2	0.5	0.8	84%	63%	13%	9%	14%
1.01	1.01	0.2	0.5	0.8	83%	61%	13%	9%	16%
1.25	1.01	0.2	0.5	0.8	81%	59%	13%	10%	17%
1.5	1.01	0.2	0.5	0.8	80%	57%	13%	10%	18%
1.75	1.01	0.2	0.5	0.8	79%	56%	13%	11%	19%

Columns (1) to (4) report variance decompositions for the structural model as implied by global, regional and country common factors. Column (1) summarizes the contribution of a "common" factor, summing columns (2), (3), and (4). Columns (5) reports the absolute differences between the total role of common factors in the specification of the structural model and the reduced form factor model. Country and Region "averages" report value added weighted averages of the country and region-level variance decompositions.