

Why is Asset Demand Inelastic?

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Motivation: the demand elasticity gap

Demand elasticity (DE): investor demand sensitivity to price

- Definition: $DE = 10 \times \frac{\text{price} \Delta}{\text{holdings} \Delta}$
- Relevance: if DE is high, flows barely matter for prices
 - Selling 1% of shares $\rightarrow \frac{1}{DE}\%$ lower price

Problem: theory and empirics disagree about the magnitude

- Theory predictions 7,000 (e.g., Gabaix and Koijen, 2021)
- Empirical estimates 2 (e.g., Shleifer, 1986; Koijen and Yogo, 2019)

Simple Example: (Why do we care?)

- Say stock price is "efficient" at \$100
- Then 10% outflow for non-fundamental reasons (e.g., ESG, behavioral)
- Workhorse CARA/CRRA/Epstein-Zin models, price \$99.999
- If elasticity 2, price is now \$95
- NOT a straw man

Question: what explains this large gap?

Motivation: the demand elasticity gap

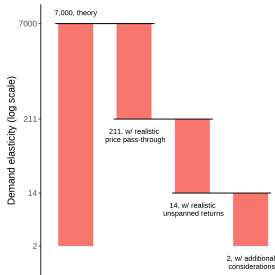
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This paper

- | **Goal:** reconcile the theoretical and empirical DE estimates
- | **Decomposition:** for *optimizing investors*, DE has two determinants

$$DE_{i,t} = 1 + (\text{pass-through}) \cdot (\text{weight responsiveness})$$

- | If we use realistic estimates, DE predictions are close to empirical findings



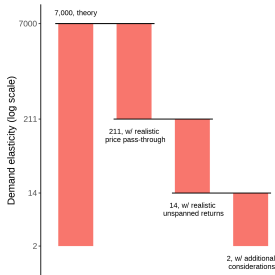
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- | What we do: consider a hypothetical mean-variance investor
- | What we DO NOT do: we DO NOT estimate demand

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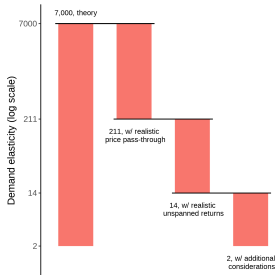
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Related literature

- | Theoretical predictions of DE are *high* (e.g., 7,000)
 - | Gabaix and Koijen (2021), Davis (2023)

- | Empirical estimates of DE are *low* (around 2)
 - | Price impact of demand:
 - | Shleifer (1986), Lou (2012), Chang, Hong, and Liskovich (2015), Schmickler (2020), Pavlova and Sikorskaya (2023)
 - | Direct estimates of DE:
 - | Koijen and Yogo (2019), Haddad, Huebner, and Loualiche (2022)

This paper: reconcile the difference

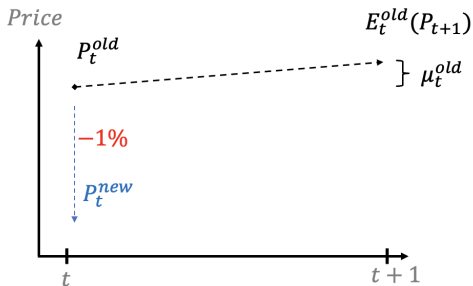
Outline

1. Theory: what determines DE?
2. Empirical estimates
3. Additional implications

What determines DE? Intuition

Imagine you are an investor. Stock i price declined by 1% without cash flow-relevant news. How much more would you buy?

| Q1: How much does *expected return* $\mu_{i,t}$ change?



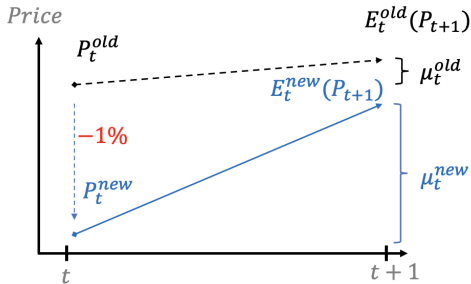
| Q2: how *substitutable* is stock i ?

| If stock i is *well spanned* by other stocks, this is almost an *arbitrage*: should aggressively buy stock i and short the replicating portfolio

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Price pass-through:

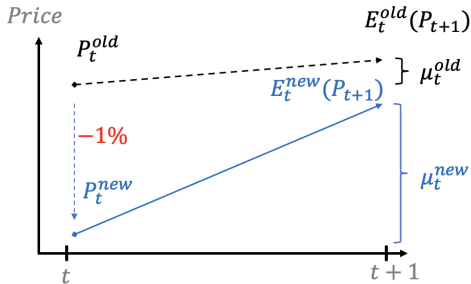
$$\frac{\partial \mu_{i,t}}{\partial \log(P_{i,t})}$$

- Q2: how *substitutable* is stock i ?
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Proposition

- Decomposition: DE for any asset i with positive weight is

$$DE_{i,t} = \frac{\partial \log(Q_{i,t})}{\partial \log(P_{i,t})} \underbrace{1 + \left(\frac{\partial \mu_{i,t}}{\partial \log(P_{i,t})} \right)}_{\text{price pass-through}} \underbrace{\frac{\partial \log(W_{i,t})}{\partial \mu_{i,t}}}_{\text{weight responsiveness}}$$

- Proposition: For a MV investor, this becomes:

$$DE_{i,t} = \underbrace{1 + \left(\frac{\partial \mu_{i,t}}{\partial \log(P_{i,t})} \right)}_{\text{price pass-through}} \underbrace{\left(\mu_{i,t} \quad \beta^0_{i,t} \quad \mu_{i,t} \right)^{-1}}_{\text{1/unspanned returns}}$$

- Intuition:

- Price pass-through: speed of price reversal
- Unspanned return: *distinctiveness* of the asset

- Both terms need to be small

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A model of μ_t and Σ_t

- | **Perspective:** a price-taking, unconstrained active quantitative fund that forms MV portfolios, rebalancing monthly
- | **Model:**
 - | Expected return μ_t = function of characteristics
 - | Covariance Σ_t : rolling 1 year estimates using daily data, with Ledoit and Wolf (2004) shrinkage:

$$\Sigma_t = (1 - h) \Sigma_t^{\text{sample}} + h \Sigma_t^{\text{target}}, \quad h = 0.01$$

- | **Data:** monthly U.S. stocks, 1970 - 2019
 - | Require > 20% NYSE size, avg 1,633 stocks/month

▶ summary stats

1) Estimate price pass-through ($\frac{\partial \mu_{i,t}}{\partial \log P_{i,t}}$)

- Model: $\mu_{i,t} = \sum_k Z_{i,k,t} \pi_k$
 - Fama-MacBeth regression

- Implied monthly price pass-through:

$$\begin{aligned} \frac{\partial \mu_{i,t}}{\partial \log P_{i,t}} &= \sum_k \frac{\partial Z_{i,k,t}}{\partial \log P_{i,t}} \pi_k \\ &= \pi_{\text{lagged } r} + \pi_{\log(M)} + \pi_{\log(B/M)} \\ &= 0.03 \quad (\text{S.E. } 0.005) \end{aligned}$$

- Main contributor is lagged ret

	Dependent variable: $r_{i,t}$ (%)	
	(1)	(2)
Intercept	-0.42 (0.41)	0.25 (0.30)
$r_{i,t-1}$	-1.95*** (0.49)	-2.73*** (0.50)
$\log(M_{i,t-1})$	0.09** (0.04)	0.02 (0.03)
$\log(B/M)_{i,t-1}$	0.23** (0.09)	0.30*** (0.07)
Mom		1.23*** (0.19)
Beta		-0.29** (0.13)
Investment		-1.26*** (0.18)
Profitability		1.27*** (0.21)
Obs	968,634	968,634
Average R^2	4.04%	8.51%

- Static theory models assume pass-through 1
- Many dynamic models have too much short-term discount rate variation (De la O, Han, and Myers, 2023)

2) Estimate unspanned returns ($\mu_{i,t}$, $\beta^0_{i,t}$, $\mu_{i,t}$)

- For stocks with positive weight

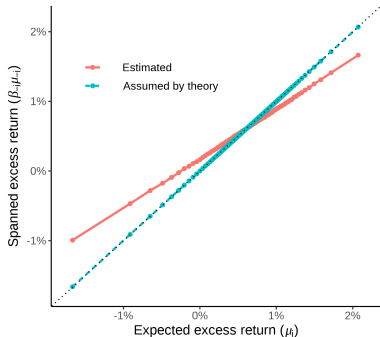
- Excess return $\mu_{i,t}$ 0.7%

- Spanned excess return:

- $\beta^0_{i,t} \mu_{i,t}$ 0.47%

[▶ details](#)

- Finding consistent with literature: Lopez-Lira and Roussanov (2023), Baba-Yara, Boyer, and Davis (2022)

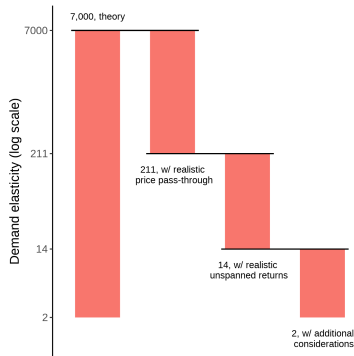


- Existing models often assume almost-perfect-spanning
 - Petajisto (2009): monthly $\mu_{i,t} = 42\text{bp}$, $\beta^0_{i,t} \mu_{i,t} = 41.9\text{bp}$

Implied optimal DE

$$1 + \underbrace{\left(\frac{\partial \mu_{i,t}}{\partial \log(P_{i,t})} \right)}_{\text{price pass-through}} \left[\underbrace{\mu_{i,t} \quad \beta^0_{i,t} \quad \mu_{i,t}}_{\text{unspanned returns}} \right]^1$$

$$1 + 0.03 \quad \frac{1}{0.23\%} \quad 14$$



| **Takeaway:** using realistic estimates explains *most of the gap*

Important Robustness 1: price pass-through

- van Binsbergen, Boons, Opp, and Tamoni (2023) decomposition:

$$\log(P_{i,t}) = \log(\tilde{P}_{i,t}) + \underbrace{\log(P_{i,t}/\tilde{P}_{i,t})}_{\text{price wedge}}$$

- $\tilde{P}_{i,t}$: cash flow-based valuation

- Estimate price pass-through, FM regressions:

$$r_{i,t+1|t+H} = \alpha_H + \beta_H \log(\tilde{P}_{i,t}/P_{i,t}) + \epsilon_{i,t+1|t+H}$$

Independent variable	Estimated coefficient β_H				Obs	Implied monthly price pass-through (β_H/H)				
	$H=1$	3	6	12		$H=1$	3	6	12	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	
van Binsbergen et al. (2023) price wedge	0.014 (0.003)	0.040 (0.014)	0.079 (0.038)	0.157 (0.102)	1,270,646	0.014 (0.003)	0.013 (0.005)	0.013 (0.006)	0.013 (0.009)	
Bartram and Grinblatt (2018) price wedge	0.001 (0.001)	0.002 (0.002)	0.003 (0.005)	0.006 (0.012)	782,431	0.001 (0.001)	0.001 (0.001)	0.000 (0.001)	0.000 (0.001)	
Kojien and Yogo (2019)-instrumented $\log(P_{i,t})$	0.002 (0.000)	0.004 (0.002)	0.009 (0.003)	0.021 (0.011)	1,519,519	0.002 (0.000)	0.001 (0.001)	0.002 (0.001)	0.002 (0.001)	
FIT-instrumented $\log(P_{i,t})$	0.006 (0.005)	0.020 (0.016)	0.040 (0.045)	0.094 (0.084)	1,443,296	0.006 (0.005)	0.007 (0.005)	0.007 (0.008)	0.008 (0.007)	
Note:							p<0.1;	p<0.05;	p<0.01	

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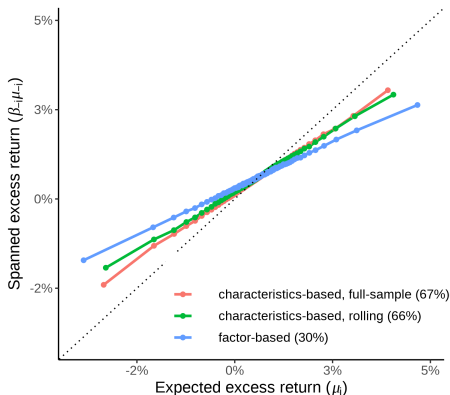
Robustness 2: unspanned returns

Robust to using alternative models of μ_t

Characteristics-based, rolling 10y FM predictions

Factor-based: $\mu_{i,t} = \sum_k \beta_{i,k,t} \mu_k^{\text{factor}}$

2y rolling loadings, full-sample FF5+mom+rev μ_k^{factor}



Alternative Σ_t :

Different shrinkage parameters

shrinkage parameter

Characteristics-based covariance (e.g., Barra)

Average fraction spanned in parentheses

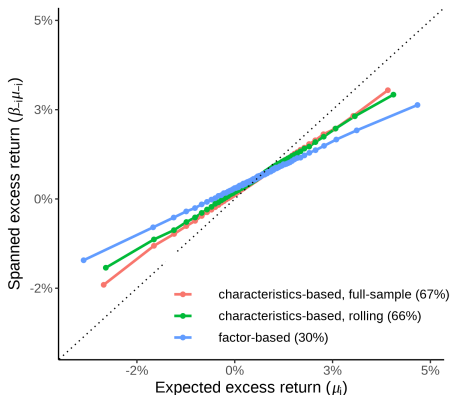
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Unspanned Returns and Maximum Sharpe

- | **Relationship:** unspanned returns relates to max Sharpe

$$S^2 = \mu_{\text{unspanned}}^0 (D\Sigma D) \mu_{\text{unspanned}}$$

where $D = \begin{pmatrix} 1/\sigma_{1j}^2 & 1 & \dots & \dots \\ & 1/\sigma_{2j}^2 & 2 & \dots \\ & \dots & & \\ 0 \dots & \dots & & 1/\sigma_{Nj}^2 & N \end{pmatrix}$

- | Note that $D\Sigma D$ only depends on the covariance matrix
- | Sharpe of MV strategy 1.3
- | If unspanned variance is high, it offsets high unspanned returns
- | Even with sensitive MV weights, we have inelastic demand

What about the remaining gap?

| Formula:

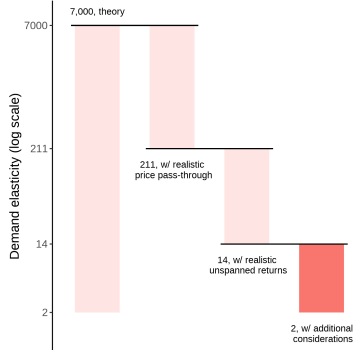
$$DE_{i,t} = 1 + (\text{pass-through}) \quad (\text{weight responsiveness}) \\ + \text{other terms}$$

| **Other terms** matter little

- | Variance/Covariance effects
- | Epstein-Zin hedging
- | Wealth effects

| **Weight responsiveness**

- | Frictionless MV demand:
 $\mu_{i,t} \text{ " } 0.01\% \Rightarrow \text{dollar " } 4\%$
- | For $DE_{i,t} \text{ } 2 \Rightarrow \text{ " } 0.3\%$
- | Frictions/constraints/biases:
 - | Short-sale/leverage constraints
 - | Heuristic demand
 - | Trading costs
 - | Behavioral effects (e.g., Giglio, Maggiori, Stroebe, and Utkus, 2021)



Other Terms

| Formula:

$$DE_{i,t} = 1 + (\text{pass-through}) \quad (\text{weight responsiveness}) + \text{other terms}$$

| With covariances:

$$\frac{\partial \log(Q_{i,t})}{\partial \log(P_{i,t})} = 1 + \underbrace{\frac{\partial \log(w_{i,t})}{\partial \mu_{i,t}} \left(\frac{\partial \mu_{i,t}}{\partial \log(P_{i,t})} \right)}_{\text{Main Decomposition}}$$

$$\underbrace{\sum_j \frac{\partial \log(w_{i,t})}{\partial \Sigma_{i,j,t}^1} \frac{\partial \Sigma_{i,j,t}^1}{\partial \log(P_{i,t})}}_{\text{Covariance Effects}}$$

| This matters, but 7000 - 3 does not get us there

| Idiosyncratic volatility effects reduce demand elasticity a bit

| β_t effects (i.e. $\Sigma_{i,t} = \beta_t \Omega_t \beta_t^0 + \text{Diag}(\sigma_t)$) are small

| Epstein-Zin consumption hedging is small

Applications of our Decomposition

$$DE_{i,t} = 1 + \underbrace{\left(\frac{\partial \mu_{i,t}}{\partial \log(P_{i,t})} \right)}_{\text{price pass-through}} \underbrace{\left(\mu_{i,t} \quad \beta^0_{i,t} \quad \mu_{i,t} \right)}_{1/\text{unspanned returns}}^1$$

- | Davis (2023) statistical arbitrage demand elasticity is inelastic
 - | All stat arb models exhibit inelastic demand
 - | In many models (e.g., IPCA), cannot separately decompose mean/covariance
 - | Alpha/beta and idiosyncratic/systematic DE are low
- | Demand elasticity can be very high (Li, Fu, and Chaudhary, 2023)
- | DE is not a primitive structural parameter! (Lucas critique)
 - | In IO, consumers have *direct* preferences on goods
 - | In asset pricing, investors have *indirect* preference over securities

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2) Implications for *equilibrium* models

- | Most cross-sectional equilibrium models generate very high DE
 1. Many (not all) are static models, and thus assume price pass-through = 1.
 2. Essentially all of them generate *almost-perfect spanning*.

- | 1) is easy to fix. 2) less so.
 - | For tractability, equilibrium models use CARA-normal setups which predict CAPM (or slight variations)

- | To deliver realistic *equilibrium* price impact, two possible ways:
 1. Market segmentation
 - | Gârleanu, Panageas, and Yu (2015), Iachan, Silva, and Zi (2022)
 2. Low spanning, possibly due to “complexity”:
 - | Martin and Nagel (2022), Baba-Yara et al. (2022), Da, Nagel, and Xiu (2022)

Summary

- | **Research question:** reconcile the theory predictions and empirical estimates of DE (7,000 vs 2)
- | **Main result:** low DE is consistent with optimal response to realistic price processes:
 1. Low price pass-through
 2. High unspanned returns
- | Existing equilibrium models are useful for *qualitative*, but not *quantitative* understanding of price impact
 - | They need to be adjusted to generate realistic price processes

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Appendix

Unspanned Returns and Factor Models

- | Consider some factor model ex. returns f with weights $W (N \times F)$
- | Normalize weights so $\text{Var}^{-1}(f)E[f] = \mathbf{1}$
- | Then we have:

$$\mu_{i,\text{unspanned}} = \underbrace{\alpha_i + \beta_i \alpha_i}_{\alpha_{i,\text{unspanned}}} + \underbrace{\sum_i^0 (W_i^T W_i)}_{\text{weight dependence on } i} \mathbf{1}$$

where

$$W_i = \underbrace{I^0 \Sigma^{-1} \mathbf{1}_i \mathbf{1}_i^T \Sigma^{-1}}_{\text{projection-like matrix}} W$$

- | **Take-away:** high unspanned returns means either
 1. High unspanned alpha or
 2. High factor model weight dependence
- | If f is MVE (i.e. $W = W = \Sigma^{-1} \mu$):

$$\mu_{i,\text{unspanned}} = \underbrace{\sum_i^0 (W_i^T W_i)}_{\text{weight dependence on } i}$$

where $W_i = I^0 \Sigma^{-1} \mathbf{1}_i \mu_i$

Summary statistics

Variable	N	Mean	SD	5%	25%	50%	75%	95%
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Market cap (\$ bn)	1,633	4.58	13.96	0.30	0.51	1.08	3.02	17.65
Monthly excess return	1,633	0.72%	9.77%	-13.83%	-4.68%	0.38%	5.67%	16.24%
Lagged monthly return	1,633	1.49%	9.88%	-12.82%	-4.16%	0.86%	6.31%	17.69%
Log(B/M)	1,633	-0.59	0.82	-2.02	-1.06	-0.52	-0.06	0.63
Asset growth	1,633	0.13	0.19	-0.09	0.03	0.09	0.19	0.52
Dividend/book	1,633	2.91%	2.84%	0.00%	0.74%	2.11%	4.30%	9.36%
Profitability	1,633	0.24	0.22	-0.03	0.15	0.24	0.34	0.59

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Not sensitive to shrinkage parameter

| Ledoit and Wolf (2004) shrinkage parameter h :

$$\Sigma_t = (1 - h) \Sigma_t^{\text{sample}} + h \Sigma_t^{\text{target}}$$

Covariance shrinkage (h)	Stock-level				Portfolio-level	
	Expected return	Spanned return	Unspanned return	Unspanned fraction	Weight responsiveness	Demand elasticity
	(μ_i)	$(\beta'_{-i}\mu_{-i})$	$(\mu_i - \beta'_{-i}\mu_{-i})$		$\theta = \frac{\partial \log(w)}{\partial \mu}$	$(1 + \theta\psi)$
	(1)	(2)	(3)	(4)	(5)	(6)
0.01	0.70%	0.47%	0.23%	32.9%	471.2	15.2
0.025	0.70%	0.47%	0.23%	32.8%	471.7	15.2
0.05	0.70%	0.47%	0.23%	32.6%	472.7	15.2
0.25	0.72%	0.50%	0.23%	31.3%	478.6	15.4
0.50	0.75%	0.53%	0.23%	29.8%	479.3	15.4

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Predicting return using decomposed $\mu_{i,t}$

FM Specification:	Dependent variable: $r_{i,t+1}$		$\mu_{spanned,i,t} - \mu_{unspanned,i,t}$	
	Full-sample	Rolling	Full-sample	Rolling
	(1)	(2)	(1b)	(2b)
Intercept	0.001 (0.002)	0.003 (0.003)	0.204 (0.176)	0.139 (0.189)
$\mu_{spanned,i,t}$	0.992*** (0.138)	0.629*** (0.158)		
$\mu_{unspanned,i,t}$	0.788*** (0.110)	0.490*** (0.104)		
Obs	957,550	806,652		
Average R^2	1.86%	1.82%		
<i>Note:</i>	*p<0.1; **p<0.05; ***p<0.01			

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Implied Sharpe ratio

- | One may worry that large unspanned returns at the stock-level implies infinite Sharpe ratios, per APT logic.

Return predictor:	Sharpe ratio	
	Full-sample FM	Rolling FM
Covariance shrinkage (h)	(1)	(2)
0.01	1.26	1.15
0.025	1.27	1.16
0.05	1.29	1.18
0.25	1.46	1.30
0.50	1.73	1.49

Leverage effect

MV-efficient portfolio weight for each stock i is:

$$w_i = \frac{1}{\gamma} \frac{\mu_i - \beta_i \mu_{i,unspanned}}{\sigma_i^2 - \beta_i^2 \sigma_{i,unspanned}^2} = \frac{1}{\gamma} \frac{\mu_{i,unspanned}}{\sigma_{i,unspanned}^2}$$

$$\frac{1}{w_i} \frac{\partial w_i}{\partial \log P_i} = \frac{1}{\mu_{i,unspanned}} \left(\frac{\mu_i}{\partial \log P_i} \right) + \frac{1}{\sigma_{i,unspanned}^2} \frac{\partial \sigma_i^2}{\partial \log P_i}$$

| $\sigma_{idio,i,t}^2$: monthly variance of daily
FF5 residuals

$$\sigma_{idio,i,t}^2 = b_1 r_{i,t-1} + b_2 \sigma_{idio,i,t-1}^2 + \epsilon_{i,t}$$

	Full sample		
	(1)	(2)	(3)
$r_{i,t-1}$	-0.077*** (0.003)	-0.063*** (0.003)	-0.065*** (0.002)
$\sigma_{idio,i,t-1}^2$	0.633*** (0.010)	0.472*** (0.011)	0.456*** (0.011)
Stock FE	N	Y	Y
Time FE	N	N	Y
Obs	2,136,330	2,136,330	2,136,330
R^2	37.51%	43.55%	44.59%

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Other terms in an Epstein-Zin framework

- | Campbell, Chan, and Viceira (2003): Epstein-Zin investor, dynamic optimization

$$w_t = \frac{1}{\kappa} \Sigma_t^{-1} \left[\mu_t + \frac{1}{2} \sigma_t^2 - \frac{\vartheta}{\zeta} \sigma_{c, w, t} \right],$$

- | $\sigma_{c, w, t} = \text{Cov}(r_t, \log(C_t/W_t))$
- | Empirical implementation:
 - | Σ_t depends on prices through characteristics
 - | $\sigma_{c, w, t}$ estimated using cay (Lettau and Ludvigson, 2001)
- | Both effects turn out to be very small

Restricting investment universe

- | **Exercise:** restricting investment to 500/100/20 largest stocks
 - | Recompute the fraction of returns unspanned

Investment universe	Fraction unspanned			
	Stock size ranking			
	1 to 20	21 to 100	101 to 500	500+
full sample	0.21	0.26	0.28	0.34
largest 500	0.37	0.40	0.41	
largest 100	0.42	0.47		
largest 20	0.45			

- | **Conclusion:** unspanned returns increase by 60%/90%/110%
 - | Then, DE increases by 35%/47%/52%

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