Why is Asset Demand Inelastic?

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April 13, 2024

Motivation: the demand elasticity gap

- **Demand elasticity (DE)**: investor demand sensitivity to price
  - Definition: DE = 10: price $\downarrow$ 1% $\rightarrow$ holdings $\uparrow$ 10%
  - Relevance: if DE is high, flows barely matter for prices
    - Selling 1% of shares $\Rightarrow \frac{1}{DE}$% lower price
- **Problem:** theory and empirics disagree about the magnitude
  - Theory predictions $\approx$ 7,000 (e.g., Gabaix and Koijen, 2021)
  - Empirical estimates $\approx$ 2 (e.g., Shleifer, 1986; Koijen and Yogo, 2019)
- **Simple Example:** (Why do we care?)
  - Say stock price is "efficient" at $100
  - Then 10% outflow for non-fundamental reasons (e.g., ESG, behavioral)
  - Workhorse CARA/CRRA/Epstein-Zin models, price $\approx$ $99.999
  - If elasticity $\approx$ 2, price is now $95
  - NOT a straw man
- **Question:** what explains this large gap?

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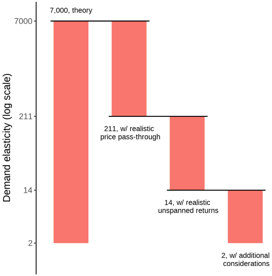
This paper

- **Goal:** reconcile the theoretical and empirical DE estimates
- **Decomposition:** for *optimizing investors*, DE has two determinants

$$DE_{i,t} \approx 1 + (\text{pass-through}) \times (\text{weight responsiveness})$$

- If we use realistic estimates, DE predictions are close to empirical findings



- **Main message**: empirically observed DE is not "too low", but largely consistent with *optimal* investor behavior
- **What we do**: consider a hypothetical mean-variance investor
- **What we DO NOT do**: we DO NOT estimate demand

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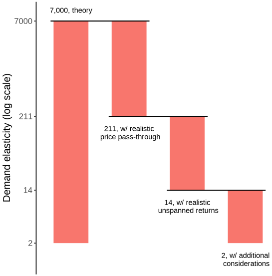

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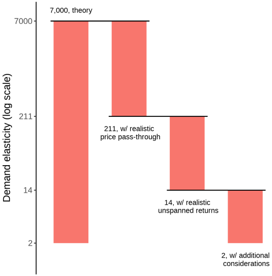

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Related literature

- Theoretical predictions of DE are *high* (e.g., 7,000)
  - Gabaix and Koijen (2021), Davis (2023)

- Empirical estimates of DE are *low* (around 2)
  - Price impact of demand:
    - Shleifer (1986), Lou (2012), Chang, Hong, and Liskovich (2015), Schmickler (2020), Pavlova and Sikorskaya (2023)
  - Direct estimates of DE:
    - Koijen and Yogo (2019), Haddad, Huebner, and Loualiche (2022)

This paper: reconcile the difference

Outline

1. Theory: what determines DE?

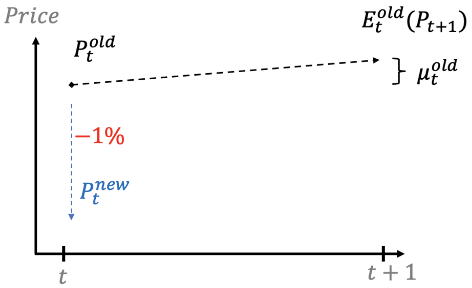
2. Empirical estimates

3. Additional implications

What determines DE? Intuition

Imagine you are an investor. Stock i price declined by 1% without cash flow-relevant news. How much more would you buy?

- **Q1:** How much does *expected return* $\mu_{i,t}$ change?



- **Q2:** how *substitutable* is stock i?
  - If stock i is *well spanned* by other stocks, this is almost an *arbitrage*: should aggressively buy stock i and short the replicating portfolio

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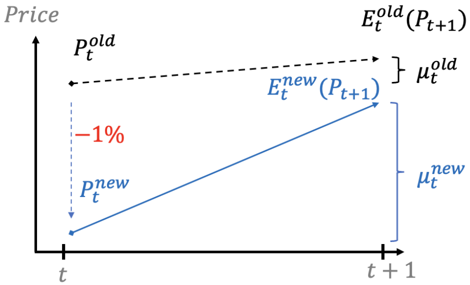

Price pass-through:

$$-\frac{\partial \mu_{i,t}}{\partial \log(P_{i,t})}$$

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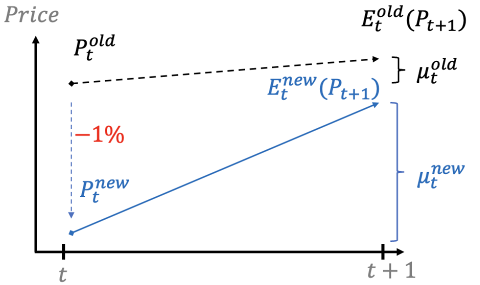

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Proposition

- **Decomposition:** DE for any asset i with positive weight is

$$\mathrm{DE}_{i,t} = -\frac{\partial \log(Q_{i,t})}{\partial \log(P_{i,t})} \approx 1 + \underbrace{\left(-\frac{\partial \mu_{i,t}}{\partial \log(P_{i,t})}\right)}_{\text{price pass-through}} \times \underbrace{\frac{\partial \log(w_{i,t})}{\partial \mu_{i,t}}}_{\text{weight responsiveness}}$$

- **Proposition:** For a MV investor, this becomes:

$$\mathrm{DE}_{i,t} \approx 1 + \underbrace{\left(-\frac{\partial \mu_{i,t}}{\partial \log(P_{i,t})}\right)}_{\text{price pass-through}} \times \underbrace{\left(\mu_{i,t} - \beta'_{-i,t} \cdot \mu_{-i,t}\right)^{-1}}_{1/\text{unspanned returns}}$$

- Intuition:
  1. Price pass-through: speed of price reversal
  2. Unspanned return: *distinctiveness* of the asset

- Both terms need to be small

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A model of μ_t and Σ_t

- **Perspective:** a price-taking, unconstrained active quantitative fund that forms MV portfolios, rebalancing monthly
- **Model:**
  - Expected return μ_t = function of characteristics
  - Covariance Σ_t: rolling 1 year estimates using daily data, with Ledoit and Wolf (2004) shrinkage:

$$\Sigma_t = (1-h) \cdot \Sigma_t^{\text{sample}} + h \cdot \Sigma_t^{\text{target}}, \quad h = 0.01$$

- **Data:** monthly U.S. stocks, 1970 - 2019
  - Require $> 20\%$ NYSE size, avg 1,633 stocks/month

summary stats

1) Estimate price pass-through ($-\frac{\partial \mu_{i,t}}{\partial \log P_{i,t}}$)

- Model: $\mu_{i,t} = \sum_k Z_{i,k,t} \cdot \pi_k$
  - Fama-MacBeth regression
- Implied monthly price pass-through:

$$-\frac{\partial \mu_{i,t}}{\partial \log P_{i,t}} = -\sum_k \frac{\partial Z_{i,k,t}}{\partial \log P_{i,t}} \pi_k$$
$$= \pi_{\text{lagged } r} - \pi_{\log(M)} + \pi_{\log(B/M)}$$
$$= 0.03 \quad \text{(S.E. 0.005)}$$

  - Main contributor is lagged ret

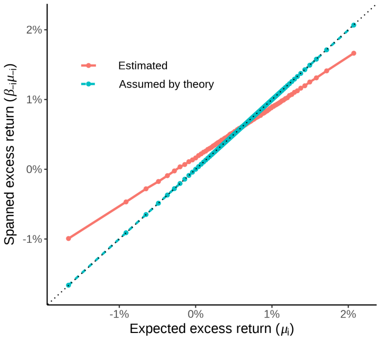
	Dependent variable: $r_{i,t}$ (%)	
	(1)	(2)
Intercept	−0.42	0.25
	(0.41)	(0.30)
$r_{i,t-1}$	-1.95^{***}	-2.73^{***}
	(0.49)	(0.50)
$\log(M_{i,t-1})$	0.09^{**}	0.02
	(0.04)	(0.03)
$\log(B/M)_{i,t-1}$	0.23^{**}	0.30^{***}
	(0.09)	(0.07)
Mom		1.23^{***}
		(0.19)
Beta		-0.29^{**}
		(0.13)
Investment		-1.26^{***}
		(0.18)
Profitability		1.27^{***}
		(0.21)
Obs	968,634	968,634
Average R^2	4.04%	8.51%

- Static theory models assume pass-through ≈ 1
- Many dynamic models have too much short-term discount rate variation (De la O, Han, and Myers, 2023)

2) Estimate unspanned returns $(\mu_{i,t} - \beta'_{-i,t} \cdot \mu_{-i,t})$

- For stocks with positive weight
  - Excess return $\mu_{i,t} \approx 0.7\%$
  - Spanned excess return: $\beta'_{-i,t} \cdot \mu_{-i,t} \approx 0.47\%$

details



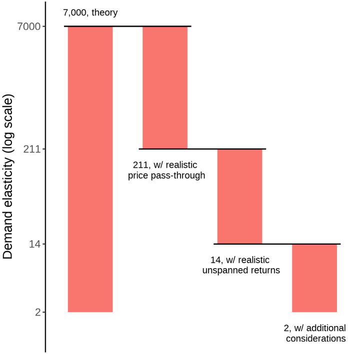
- Finding consistent with literature: Lopez-Lira and Roussanov (2023), Baba-Yara, Boyer, and Davis (2022)

- Existing models often assume almost-perfect-spanning
  - Petajisto (2009): monthly $\mu_{i,t} = 42\text{bp}$, $\beta'_{-i,t} \cdot \mu_{-i,t} = 41.9\text{bp}$

Implied optimal DE

$$1 + \underbrace{\left(-\frac{\partial \mu_{i,t}}{\partial \log(P_{i,t})}\right)}_{\text{price pass-through}} \times \left[\underbrace{\mu_{i,t} - \beta'_{-i,t} \cdot \mu_{-i,t}}_{\text{unspanned returns}}\right]^{-1}$$

$$\approx 1 + 0.03 \times \frac{1}{0.23\%} \approx 14$$



- **Takeaway**: using realistic estimates explains *most of the gap*

Important Robustness 1: price pass-through

- van Binsbergen, Boons, Opp, and Tamoni (2023) decomposition:

$$\log(P_{i,t}) = \log(\tilde{P}_{i,t}) + \underbrace{\log(P_{i,t}/\tilde{P}_{i,t})}_{\text{price wedge}}$$

  - $\tilde{P}_{i,t}$: cash flow-based valuation

- Estimate price pass-through, FM regressions:

$$r_{i,t+1\to t+H} = \alpha_H + \beta_H \cdot \log(\tilde{P}_{i,t}/P_{i,t}) + \epsilon_{i,t+1\to t+H}$$

Independent variable	Estimated coefficient β_H				Obs	Implied monthly price pass-through (β_H/H)			
	$H=1$	3	6	12		$H=1$	3	6	12
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
van Binsbergen et al. (2023) price wedge	0.014*** (0.003)	0.040*** (0.014)	0.079** (0.038)	0.157 (0.102)	1,270,646	0.014*** (0.003)	0.013*** (0.005)	0.013** (0.006)	0.013 (0.009)
Bartram and Grinblatt (2018) price wedge	0.001 (0.001)	0.002 (0.002)	0.003 (0.005)	0.006 (0.012)	782,431	0.001 (0.001)	0.001 (0.001)	0.000 (0.001)	0.000 (0.001)
Koijen and Yogo (2019)-instrumented $\log(P_{i,t})$	−0.002*** (0.000)	−0.004*** (0.002)	−0.009*** (0.003)	−0.021* (0.011)	1,519,519	−0.002*** (0.000)	−0.001*** (0.001)	−0.002*** (0.001)	−0.002* (0.001)
FIT-instrumented $\log(P_{i,t})$	0.006 (0.005)	0.020 (0.016)	0.040 (0.045)	0.094 (0.084)	1,443,296	0.006 (0.005)	0.007 (0.005)	0.007 (0.008)	0.008 (0.007)

Note: *p<0.1; **p<0.05; ***p<0.01

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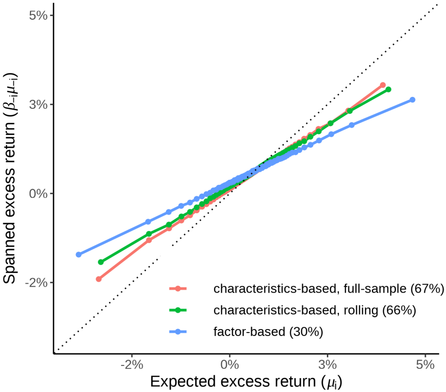
Robustness 2: unspanned returns

- Robust to using alternative models of μ_t
  - Characteristics-based, rolling 10y FM predictions
  - Factor-based: $\mu_{i,t} = \sum_k \beta_{i,k,t-1} \cdot \mu_k^{\text{factor}}$
    - 2y rolling loadings, full-sample FF5+mom+rev μ_k^{factor}



- Alternative Σ_t:
  - Different shrinkage parameters
    - shrinkage parameter
  - Characteristics-based covariance (e.g., Barra)

Average fraction spanned in parentheses

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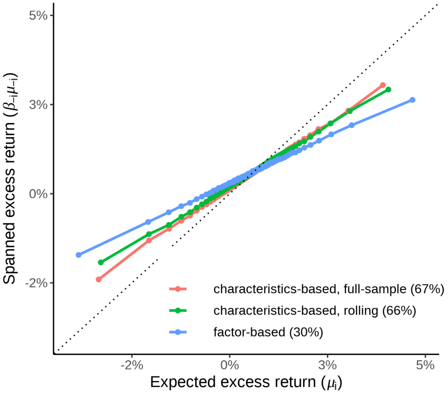

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Unspanned Returns and Maximum Sharpe

- **Relationship:** unspanned returns relates to max Sharpe

$$S^2 = \mu'_{\text{unspanned}} \cdot (D\Sigma D) \cdot \mu_{\text{unspanned}}$$

$$\text{where } D = \begin{pmatrix} 1/\sigma^2_{1|-1} & \dots & \dots \\ & 1/\sigma^2_{2|-2} & \dots \\ \dots & & \\ 0\dots & \dots & 1/\sigma^2_{N|-N} \end{pmatrix}$$

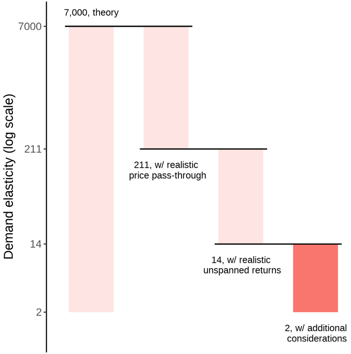
  - Note that $D\Sigma D$ only depends on the covariance matrix
- Sharpe of MV strategy ≈ 1.3
- If unspanned variance is high, it offsets high unspanned returns
- Even with sensitive MV weights, we have inelastic demand

What about the remaining gap?

- Formula:

  $$DE_{i,t} = 1 + (\text{pass-through}) \times (\text{weight responsiveness}) + \text{other terms}$$

- Other terms matter little
  - Variance/Covariance effects
  - Epstein-Zin hedging
  - Wealth effects
- Weight responsiveness
  - Frictionless MV demand: $\mu_{i,t} \uparrow 0.01\% \implies$ dollar $\uparrow 4\%$
  - For $DE_{i,t} \approx 2 \implies \approx \uparrow 0.3\%$
  - Frictions/constraints/biases:
    - Short-sale/leverage constraints
    - Heuristic demand
    - Trading costs
    - Behavioral effects (e.g., Giglio, Maggiori, Stroebel, and Utkus, 2021)



Other Terms

- Formula:

$$DE_{i,t} = 1 + (\text{pass-through}) \times (\text{weight responsiveness}) + \text{other terms}$$

- With covariances:

$$-\frac{\partial \log(Q_{i,t})}{\partial \log(P_{i,t})} = \underbrace{1 + \frac{\partial \log(w_{i,t})}{\partial \mu_{i,t}} \left(-\frac{\partial \mu_{i,t}}{\partial \log(P_{i,t})} \right)}_{\text{Main Decomposition}}$$

$$\underbrace{-\sum_j \frac{\partial \log(w_{i,t})}{\partial \Sigma^{-1}_{i,j,t}} \frac{\partial \Sigma^{-1}_{i,j,t}}{\partial \log(P_{i,t})}}_{\text{Covariance Effects}}$$

  - This matters, but 7000 - 3 does not get us there
    - Idiosyncratic volatility effects reduce demand elasticity a bit
    - β_t effects (i.e. $\Sigma_{i,t} = \beta_t \Omega_t \beta_t' + \text{Diag}(\sigma_t)$) are small
- Epstein-Zin consumption hedging is small

Applications of our Decomposition

$$\mathrm{DE}_{i,t} \approx 1 + \underbrace{\left(-\frac{\partial \mu_{i,t}}{\partial \log(P_{i,t})}\right)}_{\text{price pass-through}} \times \underbrace{\left(\mu_{i,t} - \beta'_{-i,t} \cdot \mu_{-i,t}\right)}_{1/\text{unspanned returns}}^{-1}$$

- Davis (2023) statistical arbitrage demand elasticity is inelastic
  - All stat arb models exhibit inelastic demand
  - In many models (e.g., IPCA), cannot separately decompose mean/covariance
  - Alpha/beta and idiosyncratic/systematic DE are low
- Demand elasticity can be very high (Li, Fu, and Chaudhary, 2023)
- DE is not a primitive structural parameter! (Lucas critique)
  - In IO, consumers have *direct* preferences on goods
  - In asset pricing, investors have *indirect* preference over securities

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2) Implications for *equilibrium* models

- Most cross-sectional equilibrium models generate very high DE
  1. Many (not all) are static models, and thus assume price pass-through = 1.
  2. Essentially all of them generate *almost-perfect spanning*.

- 1) is easy to fix. 2) less so.
  - For tractability, equilibrium models use CARA-normal setups which predict CAPM (or slight variations)

- To deliver realistic *equilibrium* price impact, two possible ways:
  1. Market segmentation
     - Gârleanu, Panageas, and Yu (2015), Iachan, Silva, and Zi (2022)
  2. Low spanning, possibly due to "complexity":
     - Martin and Nagel (2022), Baba-Yara et al. (2022), Da, Nagel, and Xiu (2022)

Summary

- **Research question:** reconcile the theory predictions and empirical estimates of DE (7,000 vs 2)

- **Main result:** low DE is consistent with optimal response to realistic price processes:
  1. Low price pass-through
  2. High unspanned returns

- Existing equilibrium models are useful for *qualitative*, but not *quantitative* understanding of price impact
  - They need to be adjusted to generate realistic price processes

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Appendix

Unspanned Returns and Factor Models

- Consider some factor model ex. returns f with weights W ($N \times F$)
- Normalize weights so $\mathrm{Var}^{-1}(f)\mathrm{E}[f] = \mathbf{1}$
- Then we have:

$$\mu_{i,\text{unspanned}} = \underbrace{\alpha_i - \beta'_{-i}\alpha_{-i}}_{\alpha_{i,\text{unspanned}}} + \underbrace{\Sigma'_i(W - W^*_{-i})\mathbf{1}}_{\text{weight dependence on } i}$$

where

$$W^*_{-i} = \underbrace{I'_{-i}\Sigma^{-1}_{-i,-i}I_{-i}\Sigma}_{\text{projection-like matrix}} \; W$$

- **Take-away**: high unspanned returns means either
  1. High unspanned alpha or
  2. High factor model weight dependence
- If f is MVE (i.e. $W = w^* = \Sigma^{-1}\mu$):

$$\mu_{i,\text{unspanned}} = \underbrace{\Sigma'_i(w^* - w^*_{-i})}_{\text{weight dependence on } i}$$

where $w^*_{-i} = I'_{-i}\Sigma^{-1}_{-i,-i}\mu_{-i}$

Summary statistics

Variable	N	Mean	SD	5%	25%	50%	75%	95%
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Market cap ($ bn)	1,633	4.58	13.96	0.30	0.51	1.08	3.02	17.65
Monthly excess return	1,633	0.72%	9.77%	-13.83%	-4.68%	0.38%	5.67%	16.24%
Lagged monthly return	1,633	1.49%	9.88%	-12.82%	-4.16%	0.86%	6.31%	17.69%
Log(B/M)	1,633	-0.59	0.82	-2.02	-1.06	-0.52	-0.06	0.63
Asset growth	1,633	0.13	0.19	-0.09	0.03	0.09	0.19	0.52
Dividend/book	1,633	2.91%	2.84%	0.00%	0.74%	2.11%	4.30%	9.36%
Profitability	1,633	0.24	0.22	-0.03	0.15	0.24	0.34	0.59

back

Not sensitive to shrinkage parameter

- Ledoit and Wolf (2004) shrinkage parameter h:

$$\Sigma_t = (1 - h) \cdot \Sigma_t^{\text{sample}} + h \cdot \Sigma_t^{\text{target}}$$

	Stock-level				Portfolio-level	
Covariance shrinkage (h)	Expected return (μ_i)	Spanned return ($\beta'_{-i}\mu_{-i}$)	Unspanned return ($\mu_i - \beta'_{-i}\mu_{-i}$)	Unspanned fraction	Weight responsiveness $\theta = \frac{\partial \log(w)}{\partial \mu}$	Demand elasticity ($1 + \theta\psi$)
	(1)	(2)	(3)	(4)	(5)	(6)
0.01	0.70%	0.47%	0.23%	32.9%	471.2	15.2
0.025	0.70%	0.47%	0.23%	32.8%	471.7	15.2
0.05	0.70%	0.47%	0.23%	32.6%	472.7	15.2
0.25	0.72%	0.50%	0.23%	31.3%	478.6	15.4
0.50	0.75%	0.53%	0.23%	29.8%	479.3	15.4

back

Predicting return using decomposed $\mu_{i,t}$

	Dependent variable: $r_{i,t+1}$		$\mu_{\text{spanned},i,t}$ - $\mu_{\text{unspanned},i,t}$	
FM Specification:	Full-sample	Rolling	Full-sample	Rolling
	(1)	(2)	(1b)	(2b)
Intercept	0.001	0.003	0.204	0.139
	(0.002)	(0.003)	(0.176)	(0.189)
$\mu_{\text{spanned},i,t}$	0.992***	0.629***		
	(0.138)	(0.158)		
$\mu_{\text{unspanned},i,t}$	0.788***	0.490***		
	(0.110)	(0.104)		
Obs	957,550	806,652		
Average R^2	1.86%	1.82%		
Note:			*p<0.1; **p<0.05; ***p<0.01	

back

Implied Sharpe ratio

- One may worry that large unspanned returns at the stock-level implies infinite Sharpe ratios, per APT logic.

	Sharpe ratio	
Return predictor:	Full-sample FM	Rolling FM
Covariance shrinkage (h)	(1)	(2)
0.01	1.26	1.15
0.025	1.27	1.16
0.05	1.29	1.18
0.25	1.46	1.30
0.50	1.73	1.49

back

Leverage effect

MV-efficient portfolio weight for each stock i is:

$$w_i = \frac{1}{\gamma} \cdot \frac{\mu_i - \beta_{-i}\mu_{-i}}{\sigma_i^2 - \Sigma'_{-i}\Sigma^{-1}_{-i,-i}\Sigma_{-i}} = \frac{1}{\gamma} \cdot \frac{\mu_{i,\text{unspanned}}}{\sigma^2_{i,\text{unspanned}}}$$

$$\Rightarrow -\frac{1}{w_i} \cdot \frac{\partial w_i}{\partial \log P_i} = \frac{1}{\mu_{i,\text{unspanned}}} \cdot \left(-\frac{\mu_i}{\partial \log P_i}\right) + \frac{1}{\sigma^2_{i,\text{unspanned}}} \cdot \frac{\partial \sigma_i^2}{\partial \log P_i}$$

- $\sigma^2_{\text{idio},i,t}$: monthly variance of daily FF5 residuals

$$\sigma^2_{\text{idio},i,t} = b_1 r_{i,t-1} + b_2 \sigma^2_{\text{idio},i,t-1} + \epsilon_{i,t}$$

	Full sample		
	(1)	(2)	(3)
$r_{i,t-1}$	-0.077^{***}	-0.063^{***}	-0.065^{***}
	(0.003)	(0.003)	(0.002)
$\sigma^2_{\text{idio},i,t-1}$	0.633^{***}	0.472^{***}	0.456^{***}
	(0.010)	(0.011)	(0.011)
Stock FE	N	Y	Y
Time FE	N	N	Y
Obs	2,136,330	2,136,330	2,136,330
R^2	37.51%	43.55%	44.59%

back

Other terms in an Epstein-Zin framework

- Campbell, Chan, and Viceira (2003): Epstein-Zin investor, dynamic optimization

$$w_t = \frac{1}{\kappa}\Sigma_t^{-1}\left[\mu_t + \frac{1}{2}\sigma_t^2 - \frac{\vartheta}{\varsigma}\sigma_{c-w,t}\right],$$

  - $\sigma_{c-w,t} = Cov(r_t, \log(C_t/W_t))$

- Empirical implementation:
  - Σ_t depends on prices through characteristics
  - $\sigma_{c-w,t}$ estimated using cay (Lettau and Ludvigson, 2001)

- Both effects turn out to be very small

back

Restricting investment universe

- **Exercise:** restricting investment to 500/100/20 largest stocks
  - Recompute the fraction of returns unspanned

Fraction unspanned				
Investment	Stock size ranking			
universe	1 to 20	21 to 100	101 to 500	500+
full sample	0.21	0.26	0.28	0.34
largest 500	0.37	0.40	0.41	
largest 100	0.42	0.47		
largest 20	0.45			

- **Conclusion:** unspanned returns increase by $\approx$ 60%/90%/110%
  - Then, DE increases by $\approx$ 35%/47%/52%

back