Welfare Accounting

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Abstract

This paper develops a welfare accounting decomposition that identifies and quantifies the ultimate origins of welfare gains and losses in general economies with heterogeneous individuals and disaggregated production. The decomposition—exclusively based on preferences and technologies—first separates efficiency from redistribution considerations. Efficiency comprises exchange efficiency, which traces gains and losses to reallocating consumption and factor supply across individuals, and production efficiency, which captures allocative efficiency gains and losses due to adjusting intermediate inputs and factors, as well as technical efficiency gains and losses from primitive changes in technologies and factor endowments. Leveraging the decomposition, we characterize efficiency conditions in disaggregated production economies with heterogeneous individuals, extending classic efficiency results. In competitive economies, prices (and wedges) are directly informative about the welfare-relevant statistics that shape the welfare accounting decomposition, which allows us to characterize a generalized Hulten's theorem and its converse. We present several minimal examples and a rich application to monetary policy.

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1 Introduction

Identifying the sources of welfare gains and losses is critical to assess the impact of shocks and the desirability of policy interventions. This is a challenging task, however, especially in realistic economies where different individuals have different preferences, consumption baskets, and factor supply patterns, and where production technologies may rely on multiple factors and a complex network of intermediate inputs.

In light of these complexities, this paper introduces a decomposition of welfare assessments that applies to general economies with heterogeneous individuals and disaggregated production technologies. We refer to this approach as *welfare accounting* in contrast to traditional growth accounting, which seeks to trace the sources of output growth, not welfare. Welfare accounting is useful i) to identify and quantify the ultimate origins of welfare gains and losses induced by changes in allocations, technologies, or factor endowments and ii) to characterize efficiency conditions.

We consider a static environment in which heterogeneous individuals consume different goods and supply different factors, and goods can be produced using other goods and factors. Our results allow for elastic and fixed factors and make no assumptions about the homotheticity of utility and production functions. Critically and in contrast to existing work, the welfare accounting decomposition is exclusively based on preferences and technologies, and does not rely on assumptions about the (optimizing) behavior of agents, firm objectives, individual budget constraints, prices, or the notion of equilibrium.

The welfare accounting decomposition applies to welfare assessments under general social welfare functions (SWF). We initially leverage the results of Dávila and Schaab (2022) to separate welfare assessments into an aggregate efficiency and a redistribution component.[1] A central property of this decomposition is that aggregate efficiency does not depend on the choice of SWF—only redistribution does. For that reason, we study aggregate efficiency in Sections 3 through 5, and redistribution in Section 6.

Aggregate efficiency consists of *exchange efficiency* and *production efficiency*. Exchange efficiency captures efficiency welfare gains and losses due to the reallocation of consumption and factor supply among individuals. Theorem 1a decomposes exchange efficiency into two components. First, *cross-sectional consumption efficiency* measures welfare gains associated with reallocating consumption of a good from individuals who value it less to individuals who value it more, for a given level of aggregate consumption of the good. Second, *cross-sectional factor supply efficiency* measures welfare gains from reallocating the supply of a factor from individuals for whom supplying it is more costly to individuals for whom supplying it is less costly, for a given level of aggregate supply of the factor.

Production efficiency captures efficiency welfare gains and losses associated with the economy's

[1] While that paper takes the mapping between allocations and policies or shocks as given and focuses on how different planners trade off different normative considerations, this paper exploits resource constraints and production technologies to identify the ultimate origins of welfare gains and losses.

production side. It comprises allocative efficiency gains due to adjusting intermediate inputs and factors as well as technical efficiency gains from primitive changes in technologies and factor endowments. Theorem 1b decomposes production efficiency into six components. First, *cross-sectional intermediate input efficiency* measures the welfare gains from reallocating intermediate inputs from less to more socially desirable uses, for a given level of aggregate intermediate use. Second, *aggregate intermediate input efficiency* measures the welfare gains from adjusting the share of output that is consumed instead of used in production, for a given level of output. Third, *cross-sectional factor efficiency* measures the welfare gains from reallocating factors from less to more socially desirable uses, for a given level of aggregate factor use. Fourth, *aggregate factor efficiency* measures the welfare gains from adjusting the supply of factors. Finally, the *technology growth* and *factor endowment growth* components measure the direct welfare gains from primitive changes in technologies or factor endowments.

A central contribution of the welfare accounting decomposition is to identify the precise welfare-relevant statistics that translate physical changes in allocations, technologies, and endowments into welfare changes. These statistics are i) marginal rates of substitution (MRS), which measure the value of increases in individual consumption or factor supply; ii) aggregate marginal rates of substitution ($AMRS$), which measure the social value of changes in aggregate consumption or factor supply; iii) marginal welfare products (MWP), which measure the value of increasing the use of an input or factor in production; iv) aggregate marginal welfare products ($AMWP$), which measure the social value of changes in aggregate intermediate use or factor use; and v) marginal social values of output (MSV), which measure the social value of having an additional unit of a particular good. When combined with changes in allocations or primitives, these statistics are sufficient to compute the welfare impact of any perturbation. The MSV of output is a central object for welfare accounting because it is the sole determinant of the efficiency gains from pure technological change, and it governs marginal welfare products, which in turn determine each component of production efficiency.

In Section 4, we leverage the welfare accounting decomposition to characterize efficiency conditions, generalizing the classic efficiency conditions in Lange (1942), Samuelson (1947), and Mas-Colell et al. (1995). This is, to our knowledge, the first general characterization of efficiency conditions for disaggregated production economies with heterogeneous individuals.

Theorems 2a and 2b summarize the necessary conditions for exchange and production efficiency. Exchange efficiency requires the equalization of marginal rates of substitution for all individuals who consume a good or supply a factor, allowing individuals who do not consume a good (or supply a factor) to have lower (higher) marginal rates of substitution. Cross-sectional intermediate input and factor efficiency require the equalization of MWP across all uses of an input or a factor, allowing for potentially lower MWP when a good or factor is not used to produce another. Aggregate intermediate input efficiency requires the equalization of $AMWP$ with $AMRS$ for all mixed goods. For pure final goods, the $AMRS$ must be higher than the highest MWP of using the good in production. For pure intermediate goods, the $AMWP$ must be higher than the highest individual MRS when consuming it. Finally, aggregate factor efficiency requires the equalization of $AMWP$ with $AMRS$ for all factors in positive elastic supply.

A central message of this paper is that properly accounting for the non-negativity constraints that define feasible allocations is critical to characterize efficiency conditions and trace the origins of welfare gains and losses. This is particularly important when production is disaggregated—since disaggregated production networks are sparse—and when individuals are heterogeneous and consume different (disjoint) consumption bundles. In particular, we show that the classic efficiency conditions apply to interior links between mixed goods and/or elastic factors, but fail to hold otherwise, in particular when pure intermediate goods are involved. In general, we show that MWP and MRS are the appropriate objects to characterize efficiency conditions, rather than MRS and MRT (marginal rates of transformation), as in the classic approach.

After identifying the conditions that define allocative efficiency, Theorem 2c characterizes the MSV of output—which determines the technology growth component of the welfare accounting decomposition—at an efficient allocation. Since efficiency ensures that the value of a good must be the same whether it is consumed or used as an input, we show that MSV exactly corresponds to $AMRS$ for final goods, to $AMWP$ for intermediate goods, and to both for mixed goods.

Our results until Section 5 require no assumptions about the (optimizing) behavior of agents, individual budget constraints, prices, or notions of equilibrium. It is nonetheless valuable to specialize the welfare accounting decomposition to competitive economies, which we do in Section 5, since prices are directly informative about the welfare-relevant statistics. Starting from our baseline environment, we assume that individuals maximize utility and technologies are operated by firms that minimize costs. To allow for distortions, we saturate all choice margins with wedges.

Theorem 3a characterizes the MSV of output in competitive economies with wedges: It equals the competitive price augmented by an aggregate output wedge term that captures average distortions in consumption and intermediate input use. Intuitively, the presence of aggregate consumption and intermediate input wedges implies that there are goods that are over- or under-produced. Hence, the MSV of output for goods that ultimately increase the output of under-produced (over-produced) goods is higher (lower) than the price. Theorem 3a provides a converse result to Hulten's theorem that has been missing from the existing literature: The condition that ensures that prices equal the MSV of a good is that aggregate output wedges are zero.

We provide a new general Hulten's theorem, which applies to frictionless competitive economies with heterogeneous individuals, elastic and fixed factors, arbitrary preferences and technologies, and arbitrary social welfare functions. Its generality allows us to systematically discuss the many qualifications associated with this result. In particular, we show that Hulten's theorem applies to frictionless competitive economies, not to efficient economies, as typically formulated. Moreover, we show that Hulten's theorem is fundamentally a result about aggregate efficiency, not about final output or welfare.

Theorem 3b specializes the allocative efficiency components of the welfare accounting decomposition to competitive economies with wedges. A central takeaway from this analysis is that equalization of marginal revenue products is not sufficient for cross-sectional intermediate input or factor efficiency: Efficiency requires the equalization of marginal *welfare* products across uses of an intermediate input or a factor, while competition—when intermediate input or factor use wedges are zero—only enforces the equalization of marginal *revenue* products across uses.

Our analysis up to Section 5 focuses on aggregate efficiency. However, perturbations with identical efficiency implications may have different distributional implications. Theorem 4a in Section 6 decomposes redistribution gains or losses into four components: *Cross-sectional consumption* and *factor supply redistribution* capture redistribution gains due to the reallocation of consumption and factor supply shares, for given aggregate levels of consumption and factor supply. And *aggregate consumption* and *factor supply redistribution* capture redistribution gains due to changes in aggregate consumption and factor supply, for given shares. Critically, the choice of SWF will directly impact the welfare gains from redistribution and its components.

Before concluding, we explicitly discuss the relation between welfare accounting, as developed in this paper, and the well-established approach of growth accounting. Growth accounting measures the contribution of different inputs to final output, indirectly computing technological growth as a residual. Instead, welfare accounting attributes aggregate welfare gains to different sources. We also show how to use the welfare accounting decomposition to conduct growth accounting.

Finally, we conclude with a quantitative application that illustrates the welfare accounting decomposition in a disaggregated model of the macroeconomy along the lines of La'O and Tahbaz-Salehi (2022). We compute the optimal monetary policy response to a technology shock in a static, multi-sector heterogeneous-agent New Keynesian model with an input-output production network. We contrast the aggregate efficiency welfare gains from stabilization policy with its impact on redistribution and decompose the former into its allocative efficiency components.

Related literature. Our results are most closely related to the classic studies of efficiency—see Lange (1942), Samuelson (1947) or, for a modern treatment, Section 16.E of Mas-Colell et al. (1995). This body of work proves the welfare theorems by first characterizing conditions for efficiency in a planned economy and then showing that allocations in frictionless competitive economies satisfy these conditions.[2] While the classic approach to efficiency assumes that all goods are final or mixed, our general results show that allowing for pure intermediate goods substantially changes the nature of efficiency conditions.[3] Moreover, while Lange (1942) characterizes efficiency conditions, neither that paper nor subsequent literature presents welfare decompositions of the form introduced in Theorems 1a and 1b.

[2] While the classic proofs of the first welfare theorem provide useful insights into the relation between competition and efficiency, they are not the most general—see instead Arrow (1951) and Debreu (1951). Our approach is subject to the same advantages and disadvantages as the classic approach—see Geanakoplos (1989) for a discussion.

[3] By emphasizing the critical role played by pure intermediate goods, our results connect to the recent work on global value chains—see Antràs and Chor (2022) for a recent survey.

The welfare accounting decomposition introduced in this paper is also related to the vast literature on growth accounting that follows Solow (1957) and includes Hall (1990), Basu and Fernald (2002), and Baqaee and Farhi (2020), among many others. At times and to different degrees, this body of work draws connections between output growth and welfare gains—see for instance Basu and Fernald (2002), Basu et al. (2022), or Baqaee and Burstein (2022a). A common challenge for this literature is to aggregate among heterogeneous individuals. By using the approach introduced in Dávila and Schaab (2022), we are able to make aggregate welfare assessments and to separate efficiency from redistribution considerations without relying on prices. This in turn allows us to characterize the welfare accounting decomposition exclusively in terms of preferences and technologies, making no assumptions about the (optimizing) behavior or budget constraints of agents, prices, or notions of equilibrium. This contrasts our results from Baqaee and Farhi (2020), whose decomposition is based on markups and prices and assumes cost minimization, as well as Baqaee and Burstein (2022b), whose welfare results also rely on prices. More broadly our paper continues an agenda that seeks to understand the origins of welfare gains in general economies.

Our results build on the literature on multi-sector production networks.[4] A central result of this literature is Hulten's theorem (Hulten, 1978), which characterizes the aggregate impact of technological change. Instead of directly imposing a competitive structure, we provide a first-principles characterization of the MSV of output—which in turn determines the aggregate impact of technology—based on preferences and technologies. Liu (2019) presents a statistic that summarizes the social value of subsidizing inputs and factors. While related, our characterization of MSV differs because it i) makes no assumptions about optimizing behavior, budget constraints, or prices, and ii) considers a perturbation in the level of output rather than price subsidies. By specializing the MSV of output to competitive environments, we are then able to provide the most general Hulten-style result to date. We show that Hulten's theorem is fundamentally a result about aggregate efficiency—not about final output or welfare—that applies to frictionless competitive economies—not efficient economies. Bigio and La'O (2020) show that Hulten's theorem is valid for production efficiency, rather than output, in an environment with a single individual and elastic factor supply.

Finally, our results also relate to the work that defines measures of changes in living standards, potentially refining popular notions like GDP. See Nordhaus and Tobin (1973) for an earlier account of these ideas and Fleurbaey (2009), Jones and Klenow (2016), and Basu et al. (2022) for modern treatments. The welfare accounting decomposition can be used to show that GDP changes only correspond to welfare changes in very specific scenarios. In general, welfare assessments must also account for exchange efficiency, factor supply costs, and redistribution. We hope that the welfare accounting decomposition spurs future measurement efforts.

[4] This literature includes, among many others, Gabaix (2011), Jones (2011), Acemoglu et al. (2012), Bigio and La'O (2020), Liu (2019), Baqaee and Farhi (2018, 2020), Acemoglu and Azar (2020), and Kopytov et al. (2022). See Carvalho and Tahbaz-Salehi (2019) and Baqaee and Rubbo (2022) for recent surveys.

2 Environment and Social Welfare

We first introduce preferences, technologies, and resource constraints, and then define feasible allocations and perturbations. We conclude this section by describing our approach to welfare aggregation, which separates efficiency from redistribution considerations.

2.1 Preferences, Technologies, and Resource Constraints

We consider a static economy populated by a finite number $I \geq 1$ of individuals, indexed by $i \in \mathcal{I} = \{1, \ldots, I\}$.[5] There are $J \geq 1$ goods, indexed by $j \in \mathcal{J} = \{1, \ldots, J\}$ and $F \geq 0$ factors, indexed by $f \in \mathcal{F} = \{1, \ldots, F\}$. While goods are produced using goods and factors as inputs, factors are either directly supplied by individuals or appear as an endowment.

An individual i derives utility from consuming goods and (dis)utility from supplying factors, according to the utility function

$$(\textit{Preferences}) \qquad V_i = u_i\Big(\Big\{c^{ij}\Big\}_j, \Big\{n^{if,s}\Big\}_f\Big), \tag{1}$$

where c^{ij} denotes the final consumption of good j by individual i and $n^{if,s}$ denotes the amount of factor f supplied by individual i (the superscript s stands for factor supply).

Goods are produced using technologies that take goods and factors as inputs. The production function for good j, denoted by $G^j(\cdot) \geq 0$, is given by

$$(\textit{Technologies}) \qquad y^j = G^j\Big(\Big\{x^{jk}\Big\}_k, \Big\{n^{if,d}\Big\}_f; \theta\Big), \tag{2}$$

where y^j denotes the output of good j, x^{jk} denotes the amount of good k used in the production of good j, and $n^{jf,d}$ denotes the amount of factor f used in the production of good j (the superscript d stands for factor demand). We use the index $k \in \mathcal{J}$ to refer to goods used as intermediates. For clarity, we typically use K to denote the number of intermediate inputs, although $K = J$. We parametrize production functions by θ to consider perturbations to technology, as described below.

The resource constraint for good j is

$$(\textit{Resource Constraints: Goods}) \qquad y^j = \sum_i c^{ij} + \sum_k x^{kj}, \tag{3}$$

where $c^j = \sum_i c^{ij}$ represents the amount of good j that is consumed (aggregate consumption), while $x^j = \sum_k x^{kj}$ represents the amount of good j used in production (aggregate intermediate use).

[5] In this paper, we exclusively consider static economies with a finite number of individuals, goods and factors. In ongoing work, we extend the approach of this paper to dynamic stochastic economies with incomplete markets, which opens a new set of nontrivial considerations. All our results straightforwardly generalize to the case with a continuum of individuals, goods, and factors; see Section 8 for an illustration.

Equation (3) can also be written as $y^j = c^j + x^j$. The resource constraint for factor f is

$$(\textit{Resource Constraints: Factors}) \qquad \sum_i n^{if,s} + \sum_i \bar{n}^{if,s}(\theta) = \sum_j n^{jf,d}, \tag{4}$$

where $\bar{n}^{if,s}(\theta)$ represents i's endowment of factor f. We denote by $\bar{n}^f(\theta) = \sum_i \bar{n}^{if,s}(\theta)$ the aggregate endowment of factor f, and by $n^{f,s} = \sum_i n^{if,s}$ and $n^{f,d} = \sum_j n^{jf,d}$ its aggregate (elastic) supply and factor use. Equation (4) can also be written as $n^{f,s} + \bar{n}^f(\theta) = n^{f,d}$. We parametrize $\bar{n}^{if,s}(\theta)$ by θ to consider perturbations to factor endowments.

2.2 Feasible Allocations and Perturbations

Definition 1 describes a feasible allocation. Binding non-negativity constraints play a central role in our analysis.

Definition 1 (Feasible allocation)**.** *An allocation* $\{c^{ij}, n^{if,s}, x^{jk}, n^{jf,d}, y^j\}$ *is feasible if equations* (2) *through* (4) *hold and the non-negativity constraints* $c^{ij} \geq 0$, $n^{if,s} \geq 0$, $x^{jk} \geq 0$, $n^{jf,d} \geq 0$, *and* $y^j \geq 0$ *are satisfied.*

We assume that preferences and technologies are differentiable and that all variables are smooth functions of a perturbation parameter $\theta \in [0, 1]$, so derivatives such as $\frac{dc^{ij}}{d\theta}$, $\frac{dx^{jk}}{d\theta}$, or $\frac{dn^{if,d}}{d\theta}$ are well-defined.[6]

Feasible perturbations $d\theta$ have a dual interpretation. First, a perturbation may capture exogenous changes in technologies or endowments, but also changes in policies (e.g., taxes, subsidies, transfers, etc.) or any other primitive of a fully specified model (e.g., trade costs, markups, bargaining power, etc.). Under this interpretation, the mapping between allocations and θ emerges endogenously and accounts for equilibrium effects. Second, a perturbation may alternatively capture changes in feasible allocations directly chosen by a planner. This second interpretation is useful to characterize efficiency conditions, as we explain in Section 4.

In contrast to most prior work on disaggregated production economies, our environment features heterogeneous individuals, allows for elastic factor supplies, and imposes no assumption on the homotheticity of utility and production functions. Until Section 5, we also make no assumptions about the (optimizing) behavior of agents, firm objectives, individual budget constraints, prices, or the notion of equilibrium.

2.3 Social Welfare: Aggregate Efficiency and Redistribution

We use conventional social welfare functions (SWF) to make aggregate welfare assessments and leverage the welfare decomposition introduced in Dávila and Schaab (2022) to separate efficiency

[6] To simplify the exposition, we assume throughout that i) consumption is (weakly) desirable but supplying factors is not, i.e., $\frac{\partial u_i}{\partial c^{ij}} \geq 0$ and $\frac{\partial u_i}{\partial n^{if,s}} \leq 0$; ii) the marginal products of using intermediates and factors are (weakly) positive, i.e., $\frac{\partial G^j}{\partial x^{jk}} \geq 0$ and $\frac{\partial G^j}{\partial n^{jf,d}} \geq 0$; and iii) the no-free-lunch property holds, i.e., $G^j(\cdot) = 0$ if $x^{jk} = 0$, $\forall k$, and $n^{jf,d} = 0$, $\forall f$. Many of our results, including the welfare accounting decomposition, do not require such restrictions.

from redistribution considerations. Formally, social welfare for a welfarist planner with SWF $\mathcal{W}(\cdot)$ is given by

$$(\textit{Social Welfare Function}) \qquad W = \mathcal{W}(V_1, \ldots, V_i, \ldots, V_I), \tag{5}$$

where individual utilities V_i are defined in (1). A welfare assessment can thus be expressed as

$$\frac{dW}{d\theta} = \sum_i \frac{\partial \mathcal{W}}{\partial V_i} \frac{dV_i}{d\theta} = \sum_i \alpha^i \lambda^i \frac{\frac{dV_i}{d\theta}}{\lambda^i}, \tag{6}$$

where $\alpha_i = \partial \mathcal{W} / \partial V_i$ denotes the social marginal welfare gain of increasing individual i's utility, which we assume to be strictly positive, and λ^i is an individual normalizing factor that allows us to express individual welfare gains or losses in units of a common welfare numeraire.[7] In particular, since the units of λ^i are $\dim(\lambda^i) = \frac{\text{utils of individual } i}{\text{units of numeraire}}$, individual welfare gains or losses $\frac{dV_i}{d\theta}/\lambda^i$ are measured in units of the common welfare numeraire, with $\dim\left(\frac{dV_i}{d\theta}/\lambda^i\right) = \frac{\text{units of numeraire}}{\text{units of } \theta}$, $\forall i$. The only restriction when choosing the welfare numeraire is that λ^i must be strictly positive for all individuals, so $\lambda^i > 0$.[8]

Lemma 1 derives Dávila and Schaab (2022)'s aggregate additive decomposition of welfare assessments for a normalized welfarist planner in our environment.

Lemma 1 (Welfare Decomposition: Aggregate Efficiency vs. Redistribution)**.** *The aggregate welfare assessment of a perturbation for a normalized welfarist planner, $\frac{dW^\lambda}{d\theta}$, can be decomposed into an aggregate efficiency component, Ξ^{AE}, and a redistribution component, Ξ^{RD}, as*

$$\frac{dW^\lambda}{d\theta} = \frac{\frac{dW}{d\theta}}{\frac{\sum_i \alpha^i \lambda^i}{I}} = \sum_i \omega^i \frac{\frac{dV_i}{d\theta}}{\lambda^i} = \underbrace{\sum_i \frac{\frac{dV_i}{d\theta}}{\lambda^i}}_{=\Xi^{AE}} + \underbrace{\mathbb{C}ov_i^{\Sigma}\left[\omega^i, \frac{\frac{dV_i}{d\theta}}{\lambda^i}\right]}_{=\Xi^{RD}}, \tag{7}$$

where $\omega^i = \frac{\alpha^i \lambda^i}{\sum_i \alpha^i \lambda^i / I}$ and where $\mathbb{C}ov_i^{\Sigma}[\cdot, \cdot] = I \cdot \mathbb{C}ov_i[\cdot, \cdot]$ denotes a cross-sectional covariance-sum among all individuals.

The aggregate efficiency component Ξ^{AE} corresponds to the (unweighted) sum of individual gains or losses expressed in units of the common welfare numeraire. It can thus be interpreted as an aggregate willingness-to-pay for the perturbation, which corresponds to a Kaldor-Hicks interpretation

[7] As in Boadway and Bruce (1984), Kaplow (2011), or Saez and Stantcheva (2016), we refer to the use of SWFs—typically traced back to Bergson (1938) and Samuelson (1947)—as the *welfarist* approach. The framework in Dávila and Schaab (2022) encompasses welfare objectives more general than the welfarist approach, and it is straightforward to extend our results to those. As explained there, this may be helpful to make global welfare assessments based on equivalent or compensating variations in additive or multiplicative form.

[8] While we derive our results for a general normalizing factor λ^i, the marginal value of wealth expressed in nominal units (e.g., dollars) is the most natural normalization. In that case, λ^i is measured in $\frac{\text{utils of individual } i}{\text{dollars}}$ and $\frac{dV_i}{d\theta}/\lambda^i$ is measured in $\frac{\text{dollars}}{\text{units of } \theta}$. In particular applications, it may be useful to consider alternative welfare numeraires. For instance, one may choose the marginal utility of consuming a particular good or supplying a particular factor; e.g., if good 1 is chosen as welfare numeraire, then $\lambda^i = \frac{\partial u_i}{\partial c^{i1}}$, $\forall i$.

of efficiency. The redistribution component Ξ^{RD} corresponds to the cross-sectional covariance-sum of normalized individual weights ω^i with changes in individual utilities expressed in units of the common numeraire. The individual weights ω^i—which average to one, so $\sum_i \omega^i/I = 1$—capture the social marginal valuation of welfare changes for individual i in units of the common numeraire.

The decomposition of Lemma 1 satisfies several properties relevant for our subsequent analysis—see Dávila and Schaab (2022). Three are worth highlighting. First, the efficiency component Ξ^{AE} of the welfare assessment of any perturbation is *invariant* to the choice of SWF. Therefore, discrepancies in the welfare assessments of welfarist planners are exclusively due to redistribution considerations. This property motivates the structure of our analysis, first studying efficiency in Sections 3 through 5, and then revisiting redistribution in Section 6. Second, given a choice of welfare numeraire, the aggregate efficiency component Ξ^{AE} is also invariant to increasing transformations of individual utilities. Finally, the redistribution component Ξ^{RD} is zero when there is a single individual, so $I = 1$, or when the planner can costlessly implement lump-sum transfers across individuals, which is again consistent with a Kaldor-Hicks interpretation. Together, these properties support the view that Ξ^{AE} captures the aggregate welfare impact of a perturbation, while Ξ^{RD} captures how a particular SWF trades off the differential impact of the perturbation across individuals.

3 Accounting for the Origins of Efficiency Gains and Losses

This section develops the central welfare accounting result for efficiency: a decomposition that accounts for the origins of efficiency welfare gains and losses. In particular, the aggregate efficiency component of a welfare assessment, Ξ^{AE}, can be decomposed into an *exchange efficiency* component, $\Xi^{AE,X}$, and a *production efficiency* component, $\Xi^{AE,P}$, as

$$\Xi^{AE} = \Xi^{AE,X} + \Xi^{AE,P}, \tag{8}$$

where both components can be further decomposed, as illustrated in Figure 1 and explained in detail in the remainder of this paper. We study exchange efficiency in Section 3.1 and production efficiency in Section 3.2. We explore broader insights from the welfare accounting decomposition in Section 3.3 and illustrate each of its components with examples in Section 3.4.

3.1 Exchange Efficiency

3.1.1 Allocation Shares: Consumption and Factor Supply

To study exchange efficiency, we first introduce consumption and factor supply allocations shares. Working with shares, instead of directly with levels, allows us to distinguish welfare gains and losses due to reallocation from those due to changes in aggregates.

Formally, we define the (individual) *consumption share* of good j for individual i, χ_c^{ij}, and the

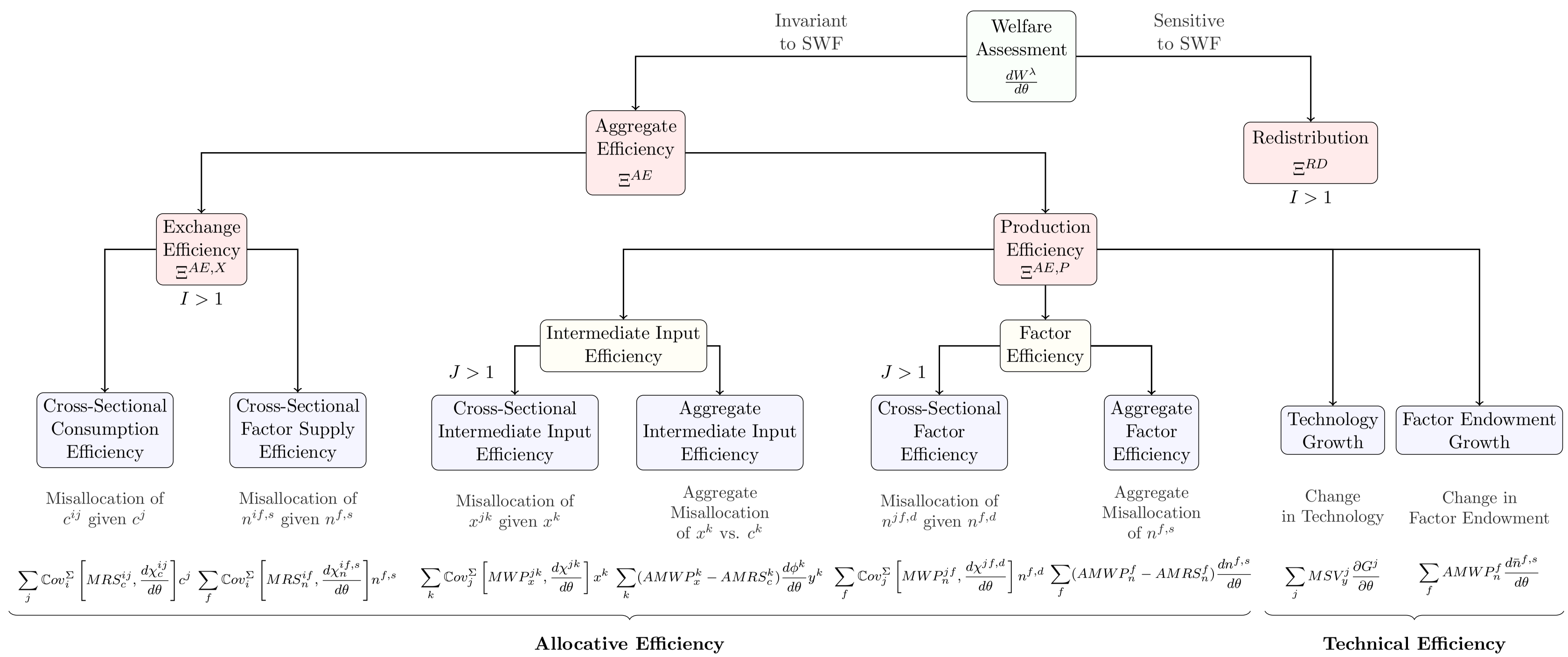


Figure 1. Welfare Accounting Decomposition: Aggregate Efficiency

Note. Figure 1 illustrates the welfare accounting decomposition introduced in this paper. Lemma 1 decomposes welfare assessments into aggregate efficiency and redistribution components. Aggregate efficiency welfare gains comprise exchange efficiency and production efficiency. Theorem 1a decomposes exchange efficiency into cross-sectional consumption and factor supply efficiency. Theorem 1b decomposes production efficiency into intermediate input efficiency, factor efficiency, technology growth, and factor endowment growth. Theorems 2a and 2b leverage this decomposition to characterize efficiency conditions. Theorem 2c characterizes the marginal social value of output at efficient allocations. Theorems 3a and 3b leverage this decomposition to characterize the technology growth component and the allocative efficiency components in competitive economies with wedges. Theorem 4a, which decomposes the redistribution component, is illustrated in Figure 5 in Appendix C.3, which is the complement of this figure.

factor supply share of factor f for individual i, $\chi_n^{if,s}$, as

$$\chi_c^{ij} = \begin{cases} \frac{c^{ij}}{c^j} & \text{if } c^j > 0 \\ \frac{\frac{dc^{ij}}{d\theta}}{\frac{dc^j}{d\theta}} & \text{if } c^j = 0 \quad \text{and} \quad \frac{dc^j}{d\theta} > 0 \\ 0 & \text{if } c^j = 0 \quad \text{and} \quad \frac{dc^j}{d\theta} = 0 \end{cases} \quad \text{and} \quad \chi_n^{if,s} = \begin{cases} \frac{n^{if,s}}{n^{f,s}} & \text{if } n^{f,s} > 0 \\ \frac{\frac{dn^{if,s}}{d\theta}}{\frac{dn^{f,s}}{d\theta}} & \text{if } n^{f,s} = 0 \quad \text{and} \quad \frac{dn^{f,s}}{d\theta} > 0 \\ 0 & \text{if } n^{f,s} = 0 \quad \text{and} \quad \frac{dn^{f,s}}{d\theta} = 0. \end{cases} \quad (9)$$

Consumption shares χ_c^{ij} represent either the share of aggregate consumption c^j consumed by individual i, when $c^j > 0$, or the share of the change in aggregate consumption $\frac{dc^j}{d\theta}$ consumed by individual i, when $c^j = 0$ and $dc^j/d\theta > 0$. Factor supply shares $\chi_n^{if,s}$ are defined analogously. The definitions of shares in equation (9) ensure that changes in individual consumption and factor supply can be expressed as

$$\frac{dc^{ij}}{d\theta} = \frac{d\chi_c^{ij}}{d\theta} c^j + \chi_c^{ij} \frac{dc^j}{d\theta} \qquad \text{and} \qquad \frac{dn^{if,s}}{d\theta} = \frac{d\chi_n^{if,s}}{d\theta} n^{f,s} + \chi_n^{if,s} \frac{dn^{f,s}}{d\theta}, \qquad (10)$$

even when $c^j = 0$ or $n^{f,s} = 0$.

3.1.2 Exchange Efficiency Decomposition

Exchange efficiency captures efficiency welfare gains and losses associated with the reallocation of consumption and factor supply among individuals.

Theorem 1a (Exchange Efficiency)**.** *The exchange efficiency component of aggregate efficiency, $\Xi^{AE,X}$, can be decomposed into i) cross-sectional consumption efficiency and ii) cross-sectional factor supply efficiency, as*

$$\Xi^{AE,X} = \underbrace{\sum_j \mathbb{C}ov_i^{\Sigma} \left[MRS_c^{ij}, \frac{d\chi_c^{ij}}{d\theta} \right] c^j}_{\substack{\text{Cross-Sectional} \\ \text{Consumption Efficiency}}} \underbrace{- \sum_f \mathbb{C}ov_i^{\Sigma} \left[MRS_n^{if}, \frac{d\chi_n^{if,s}}{d\theta} \right] n^{f,s}}_{\substack{\text{Cross-Sectional} \\ \text{Factor Supply Efficiency}}},$$

where (individual) marginal rates of substitution for individual i between good j and the numeraire, MRS_c^{ij}, and between factor f and the numeraire, MRS_n^{if}, are given by

$$MRS_c^{ij} = \frac{\frac{\partial u_i}{\partial c^{ij}}}{\lambda^i} \qquad \text{and} \qquad MRS_n^{if} = -\frac{\frac{\partial u_i}{\partial n^{if,s}}}{\lambda^i}, \qquad (11)$$

and where $\mathbb{C}ov_i^{\Sigma}[\cdot,\cdot] = I \cdot \mathbb{C}ov_i[\cdot,\cdot]$ denotes a cross-sectional covariance-sum among all individuals.

Cross-sectional consumption efficiency measures the welfare gains associated with reallocating consumption of good j from individuals who value it less (with a lower MRS_c^{ij}) to individuals who value it more (with a higher MRS_c^{ij}), for a given level of aggregate consumption c^j.[9] Analogously,

[9] The marginal rate of substitution MRS_c^{ij} measures the value of a marginal increase in consumption of good j for individual i in units of the numeraire. Analogously, MRS_n^{if} measures the cost of a marginal increase in the supply of factor f for individual i in units of the numeraire.

cross-sectional factor supply efficiency measures the welfare gains associated with reallocating the supply of factor f from individuals for whom increasing factor supply is more costly (with a higher MRS_n^{if}) to individuals for whom increasing factor supply is less costly (with a lower MRS_n^{if}), for a given aggregate (elastic) supply of factor f, $n^{f,s}$.

Exchange efficiency satisfies several desirable properties of practical relevance.

Corollary 2 (Properties of Exchange Efficiency Decomposition)**.**

(a) (Single Individual) *In economies with a single individual ($I = 1$), exchange efficiency is zero.*

(b) (Zero Factor Supply) *In economies in which factors are not elastically supplied, so $n^{f,s} = 0$ for all factors, cross-sectional factor supply efficiency is zero.*

(c) (Equalized MRS_c^{ij} or MRS_n^{if}) *If marginal rates of substitution for good j (factor f) are identical across individuals for all goods (factors) with $c^j > 0$ ($n^{f,s} > 0$), then cross-sectional consumption (factor supply) efficiency is zero.*

Since exchange efficiency welfare gains arise by reallocating consumption and factor supply across individuals, exchange efficiency must be zero in single individual economies.[10] Relatedly, in economies in which individuals do not derive (dis)utility from factor supply, cross-sectional factor supply efficiency is zero. Lastly, only when individuals value consuming the same good or supplying the same factor differently is there scope to find welfare gains from reallocating either.

3.2 Production Efficiency

3.2.1 Allocation Shares: Intermediate Input and Factor Use

To study production efficiency, we first introduce allocation shares for intermediate input and factor use. Once again, working with shares allows us to distinguish welfare gains and losses due to reallocation from those due to changes in aggregates.

Formally, we define the *intermediate share* of good k, ϕ_x^k, and the *intermediate use share* of good k used to produce good j, χ_x^{jk}, as

$$\phi_x^k = \begin{cases} \frac{x^k}{y^k} & \text{if } y^k > 0 \\ \frac{\frac{dx^k}{d\theta}}{\frac{dy^k}{d\theta}} & \text{if } y^k = 0 \quad \text{and} \quad \frac{dy^k}{d\theta} > 0 \\ 0 & \text{if } y^k = 0 \quad \text{and} \quad \frac{dy^k}{d\theta} = 0 \end{cases} \quad \text{and} \quad \chi_x^{jk} = \begin{cases} \frac{x^{jk}}{x^k} & \text{if } x^k > 0 \\ \frac{dx^{jk}}{dx^k} & \text{if } x^k = 0 \quad \text{and} \quad \frac{dx^k}{d\theta} > 0 \\ 0 & \text{if } x^k = 0 \quad \text{and} \quad \frac{dx^k}{d\theta} = 0. \end{cases} \tag{12}$$

The intermediate share of good k, ϕ_x^k, represents either the share of output y^k devoted to production, when $y^k > 0$, or the share of the change in output $\frac{dy^k}{d\theta}$ devoted to production, when $y^k = 0$ and $\frac{dy^k}{d\theta} > 0$. When $\phi_x^k > 0$, its complement defines the *aggregate consumption share* $\phi_c^k = 1 - \phi_x^k$. The

[10] Exchange efficiency and redistribution are completely different notions, even though both require individual heterogeneity. In particular, the choice of SWF does not affect exchange efficiency but it directly impacts redistribution.

intermediate use share of good k, χ_x^{jk}, represents either the share of good k's aggregate intermediate use devoted to the production of good j, when $x^k > 0$, or its counterpart in changes when $x^k = 0$ and $\frac{dx^k}{d\theta} > 0$.[11]

Finally, we also define the *intermediate-output share* of good k by $\xi^{jk} = \chi_x^{jk}\phi_x^k$, which corresponds to $\frac{x^{jk}}{y^k}$ when $y^k > 0$ or to its counterpart in changes when $y^k = 0$ and $\frac{dy^k}{d\theta} > 0$. These definitions of shares ensure that changes in intermediate use can be expressed as

$$\frac{dx^{jk}}{d\theta} = \frac{d\xi^{jk}}{d\theta} y^k + \xi^{jk}\frac{dy^k}{d\theta}, \qquad \text{where} \qquad \frac{d\xi^{jk}}{d\theta} = \frac{d\chi_x^{jk}}{d\theta}\phi_x^k + \chi_x^{jk}\frac{d\phi_x^k}{d\theta}, \tag{13}$$

even when $y^k = 0$ and $x^k = 0$. Expression (13) initially decomposes level changes in the use x^{jk} of good k in the production of good j into two terms. First, changes in the intermediate-output share $\frac{d\xi^{jk}}{d\theta}$ change x^{jk} in proportion to the level of output y^k. Second, changes in the level of output $\frac{dy^k}{d\theta}$ change x^{jk} in proportion to the intermediate-output share ξ^{jk}. In turn, changes in the intermediate-output share $\frac{d\xi^{jk}}{d\theta}$ can occur either due to reallocation of good k across different intermediate uses—a change in the intermediate use share χ_x^{jk}—or due to reallocation from consumption to production—a change in the intermediate share ϕ_x^k.

At last, we define the *factor use share* of factor f used to produce good j, $\chi_n^{jf,d}$, as

$$\chi_n^{jf,d} = \begin{cases} \frac{n^{jf,d}}{n^{f,d}} & \text{if } n^{f,d} > 0 \\ \frac{\frac{dn^{jf,d}}{d\theta}}{\frac{dn^{f,d}}{d\theta}} & \text{if } n^{f,d} = 0 \quad \text{and} \quad \frac{dn^{f,d}}{d\theta} > 0 \\ 0 & \text{if } n^{f,d} = 0 \quad \text{and} \quad \frac{dn^{f,d}}{d\theta} = 0. \end{cases} \tag{14}$$

The factor use share $\chi_n^{jf,d}$ represents the share of factor f's aggregate use $n^{f,d}$ devoted to the production of good j, or its counterpart in changes when $n^{f,d} = 0$ and $\frac{dn^{f,d}}{d\theta} > 0$.[12] In this case, equation (14) ensures that changes in factor use can be expressed as

$$\frac{dn^{jf,d}}{d\theta} = \frac{d\chi_n^{jf}}{d\theta} n^{f,d} + \chi_n^{jf,d}\frac{dn^{f,d}}{d\theta}. \tag{15}$$

even when $n^{jf,d} = 0$. Equation (15) decomposes level changes in the use $n^{jf,d}$ of factor f in the production of good j into a change in the factor use share, $\frac{d\chi_n^{jf}}{d\theta}$, and a change in the aggregate factor use, $\frac{dn^{f,d}}{d\theta}$.

[11] Depending on ϕ_x^k, good k can be i) *pure final*, when $\phi_x^k = 0$; ii) *pure intermediate*, when $\phi_x^k = 1$; or iii) *mixed*, when $\phi_x^k \in (0,1)$. Equivalently, good k can be i) *final* when $\phi_x^k \in [0,1)$ or ii) *intermediate*, when $\phi_x^k \in (0,1]$, with mixed goods being simultaneously final and intermediate. These categorizations are only meaningful when $y^k > 0$ or $\frac{dy^k}{d\theta} > 0$. Depending on χ_x^{jk}, an intermediate input k is i) *specialized*, when $\chi_x^{jk} = 1$ for some j; or *diversified*, when $\chi_x^{jk} \in (0,1)$ for some j.

[12] A factor f is i) *specialized*, when $\chi_n^{jf,d} = 1$ for some j; or *diversified*, when $\chi_n^{jf,d} \in (0,1)$ for some j.

3.2.2 Network Propagation: Output Inverse Matrix

To study production efficiency it is necessary to understand how perturbations propagate through the production network. Lemma 3 introduces the *output inverse matrix* $\boldsymbol{\Psi}_y$, which characterizes the ultimate change in output induced by a unit impulse in output levels.[13]

Lemma 3 (Output Inverse Matrix)**.** *Changes in output can be expressed in terms of changes in intermediate-output shares* $\frac{d\xi^{jk}}{d\theta}$*, changes in factor use* $\frac{dn^{jf,d}}{d\theta}$*, and changes in technology* $\frac{\partial G^j}{\partial\theta}$*, as*

$$\frac{dy^j}{d\theta} = \underbrace{\sum_k \frac{\partial G^j}{\partial x^{jk}} \xi^{jk} \frac{dy^k}{d\theta}}_{\text{Propagation}} + \underbrace{\sum_k \frac{\partial G^j}{\partial x^{jk}} \frac{d\xi^{jk}}{d\theta} y^k + \sum_f \frac{\partial G^j}{\partial n^{jf,d}} \frac{dn^{jf,d}}{d\theta} + \frac{\partial G^j}{\partial \theta}}_{\text{Impulse}}. \tag{16}$$

Equivalently, in matrix form,

$$\frac{d\boldsymbol{y}}{d\theta} = \underbrace{\boldsymbol{\Psi}_y}_{\text{Propagation}} \underbrace{\left(\boldsymbol{G}_x \frac{d\boldsymbol{\xi}}{d\theta}\boldsymbol{y} + \boldsymbol{G}_n \frac{d\boldsymbol{n}^d}{d\theta} + \boldsymbol{G}_\theta\right)}_{\text{Impulse}} \quad \textit{where} \quad \underbrace{\boldsymbol{\Psi}_y = (\boldsymbol{I}_J - \boldsymbol{G}_x\boldsymbol{\xi})^{-1}}_{\text{Output Inverse}}, \tag{17}$$

where $\frac{d\boldsymbol{y}}{d\theta}$ *denotes the* $J \times 1$ *vector of* $\frac{dy^j}{d\theta}$*, and* $\boldsymbol{\Psi}_y = (\boldsymbol{I}_J - \boldsymbol{G}_x\boldsymbol{\xi})^{-1}$ *defines the* $J \times J$ *output inverse matrix. The remaining matrices are defined in Appendix A.*

Lemma 3 characterizes how much output is ultimately produced in response to changes in intermediate-output shares, factor use, and technology, accounting for network propagation. Consider the three "impulse" terms of equation (16), which represent the first-round impact of the perturbation on the level of output. First, a perturbation that changes intermediate-output shares by $\frac{d\xi^{jk}}{d\theta}$ raises at impact the amount of good k used as input for good j in proportion to y^k, which in turn increases output at impact by $\frac{\partial G^j}{\partial x^{jk}}$. Similarly, a perturbation that changes the use of factor f in the production of good k by $\frac{dn^{kf,d}}{d\theta}$ increases output at impact by $\frac{\partial G^j}{\partial n^{jf,d}}$. A change in technology simply increases output at impact by $\frac{\partial G^j}{\partial \theta}$.

Such first-round changes in the level of output in turn induce further changes in the level of intermediate inputs, which in turn induce further changes in output. These knock-on effects through the output network are captured by the output inverse matrix $\boldsymbol{\Psi}_y$. Under regularity conditions, which we assume hold at all times, $\boldsymbol{\Psi}_y$ admits the series representation

$$\boldsymbol{\Psi}_y = (\boldsymbol{I}_J - \boldsymbol{G}_x\boldsymbol{\xi})^{-1} = \boldsymbol{I}_J + \boldsymbol{G}_x\boldsymbol{\xi} + (\boldsymbol{G}_x\boldsymbol{\xi})^2 + (\boldsymbol{G}_x\boldsymbol{\xi})^3 + \dots\ . \tag{18}$$

The first term in the expansion, $\boldsymbol{I}_J$, represents the first round of output changes we just described. As

[13] In Appendix C.2, we introduce two related propagation matrices: the *intermediate inverse matrix* $\boldsymbol{\Psi}_x$, which characterizes network propagation for changes in the level of intermediates; and the *proportional output inverse matrix* $\tilde{\boldsymbol{\Psi}}_y = \hat{\boldsymbol{y}}^{-1}\boldsymbol{\Psi}_y\hat{\boldsymbol{y}}$, where $\hat{\boldsymbol{y}} = \operatorname{diag}(\boldsymbol{y})$, which characterizes network propagation for proportional changes in output. To simplify the exposition, we exclusively use the output inverse matrix in the body of the paper, but all three matrices are useful to understand network propagation, as explained in the Appendix.

output adjusts, the level of intermediate inputs x^{jk} changes in proportion to the intermediate-output share ξ^{jk}, or $\boldsymbol{\xi}$ in matrix form. In turn, changes in the level of intermediate inputs translate into a second round of changes in output in proportion to the marginal products of each input $\frac{\partial G^j}{\partial x^{jk}}$, or $\boldsymbol{G}_x$ in matrix form. This explains the second term $\boldsymbol{G}_x\boldsymbol{\xi}$ in (18), which generates knock-on effects in proportion to $(\boldsymbol{G}_x\boldsymbol{\xi})^2$ and so on. We refer to the conclusion of this fixed point of network propagation as the ultimate output change induced by the original perturbation.

3.2.3 Welfare-Relevant Statistics

In this section, we introduce the three sets of welfare-relevant statistics that determine production efficiency. These objects represent the welfare impact of specific perturbations in the levels of consumption, factor supply, output, intermediate use, or factor use. First, we introduce *aggregate marginal rates of substitution* ($AMRS$).

Definition 2 (Aggregate Marginal Rate of Substitution)**.** *We define the aggregate marginal rate of substitution (AMRS) between good j and the numeraire and between factor f and the numeraire as*

$$AMRS_c^j = \sum_i \chi_c^{ij} MRS_c^{ij} \quad \text{and} \quad AMRS_n^f = \sum_i \chi_n^{if,s} MRS_n^{if,s}, \tag{19}$$

where consumption and factor supply shares χ_c^{ij} and $\chi_n^{if,s}$ are defined in (9) and individual marginal rates of substitution MRS_c^{ij} and $MRS_n^{if,s}$ are defined in (11). We denote the $1 \times J$ and $1 \times F$ vectors of $AMRS_c^j$ and $AMRS_n^f$ by $\boldsymbol{AMRS}_c$ and $\boldsymbol{AMRS}_n$.

Aggregate marginal rates of substitution for goods and factors are cross-sectional weighted averages of individual marginal rates of substitution. For goods with $c^j > 0$ or $\frac{dc^j}{d\theta} > 0$, $AMRS_c^j$ corresponds to the welfare gain associated with increasing aggregate consumption of good j by a unit, making individuals consume in proportion to their consumption shares. For factors with $n^{f,s} > 0$ or $\frac{dn^{f,s}}{d\theta} > 0$, $AMRS_n^f$ corresponds to the welfare cost associated with increasing the aggregate supply of factor f by a unit, making individuals supply the factor in proportion to their factor supply shares.[14]

Second, we introduce the *marginal social value of output* (MSV).

Definition 3 (Marginal Social Value of Output)**.** *We define the marginal social value of output of good j, MSV_y^j, as the jth element of the $1 \times J$ vector $\boldsymbol{MSV}_y$, given by*

$$\boldsymbol{MSV}_y = \boldsymbol{AMRS}_c \boldsymbol{\phi}_c \boldsymbol{\Psi}_y, \tag{20}$$

where $\boldsymbol{AMRS}_c$ is defined in (19), $\boldsymbol{\phi}_c$ is the $J \times J$ diagonal matrix of aggregate consumption shares defined in Appendix A, and $\boldsymbol{\Psi}_y$ is the $J \times J$ output inverse matrix defined in (17).

[14] When $c^j = \frac{dc^j}{d\theta} = 0$ or $n^{f,s} = \frac{dn^{f,s}}{d\theta} = 0$, the definition of shares in (9) implies that $AMRS_c^j = 0$ and $AMRS_n^f = 0$, so these $AMRS$ cannot correspond to the welfare gain or loss associated with changing aggregate consumption or factor supply. This is purely a notational convention to simplify the exposition: we will show that the production efficiency decomposition does not depend on the values of $AMRS_c^j$ and $AMRS_n^f$ in those cases.

The marginal social value of output corresponds to the welfare gain associated with having an additional unit of a good in the economy. As just described in Section 3.2.2, a unit impulse in output levels generates an ultimate increase in output given by the output inverse matrix $\boldsymbol{\Psi}_y$. However, a fraction of output is used in the production of other goods, so only the aggregate consumption share $\boldsymbol{\phi}_c$ is consumed by individuals. And the $\boldsymbol{AMRS}_c$ captures the marginal welfare gain associated with increasing aggregate consumption, so the marginal social value of output is the product of these three objects. The definition of MSV highlights that the social value of a good emanates from the consumption—potentially of other goods—it ultimately generates.

Third, we introduce *marginal welfare products* (MWP).

Definition 4 (Marginal Welfare Product)**.** *We define the marginal welfare products (MWP) of input k and of factor f for technology j as*

$$MWP_x^{jk} = MSV_y^j \frac{\partial G^j}{\partial x^{jk}} \quad \text{and} \quad MWP_n^{jf} = MSV_y^j \frac{\partial G^j}{\partial n^{jf,d}}, \tag{21}$$

where the marginal social value of output of good j, MSV_y^j, is defined in (20).

Marginal welfare products correspond to the welfare gain associated with increasing the use of an input or factor in the production of a good. Marginal increases in x^{jk} or $n^{jf,d}$ increase output at impact by their physical marginal products, $\frac{\partial G^j}{\partial x^{jk}}$ and $\frac{\partial G^j}{\partial n^{jf}}$. As just described, the social value of a unit impulse in output is summarized by the marginal social value of output, MSV_y^j. Hence, marginal welfare products of inputs and factors are given by the product of physical marginal products and the marginal social value of output of the good produced.

Finally, we introduce *aggregate marginal welfare products* (AMWP).

Definition 5 (Aggregate Marginal Welfare Product)**.** *We define the aggregate marginal welfare product (AMWP) of good j and factor f as*

$$AMWP_x^k = \sum_j \chi_x^{jk} MWP_x^{jk} \quad \text{and} \quad AMWP_n^f = \sum_j \chi_n^{jf,d} MWP_n^{jf}, \tag{22}$$

where intermediate input use and factor use shares χ_x^{jk} and $\chi_n^{jf,d}$ are defined in (13) and (14) and marginal welfare products in (21).

The aggregate marginal welfare product of an input or factor is a cross-sectional weighted average of marginal welfare products. For inputs with $x^k > 0$ or $\frac{dx^k}{d\theta} > 0$, it corresponds to the welfare gain associated with increasing the aggregate intermediate use of good k in proportion to the intermediate use shares. For factors with $n^{f,d} > 0$ or $\frac{dn^{f,d}}{d\theta} > 0$, it corresponds to the welfare gain associated with increasing the factor use of factor f in proportion to the factor use shares.[15]

[15] When $x^k = \frac{dx^{jk}}{d\theta} = 0$ or $n^{f,d} = \frac{dn^{f,d}}{d\theta} = 0$, AMWP's as defined here do not correspond to the welfare gain or loss associated with changing aggregate intermediate or factor use. This is inconsequential, since the welfare accounting decomposition does not depend on the values of $AMWP_x^k$ and $AMWP_n^f$ in those cases.

At last, note that the marginal social value of output can be expressed in terms of aggregate marginal rates of substitution and aggregate marginal welfare products as

$$MSV_y^j = \phi_c^j AMRS_c^j + \phi_x^j AMWP_x^j. \tag{23}$$

This equation, which provides an alternative definition for MSV_y^j, shows that the value of a unit of output corresponds to the value of consuming its aggregate consumption share ϕ_c^j and using its aggregate intermediate use share ϕ_x^j in production. This definition is recursive since $AMWP_x^j$ is a function of the marginal social value of output for all goods.

3.2.4 Production Efficiency Decomposition

Production efficiency captures efficiency welfare gains associated with the economy's production side. It comprises i) allocative efficiency gains due to adjusting inputs and factors and ii) technical efficiency gains from primitive changes in technologies and factor endowments.[16]

Theorem 1b (Production Efficiency)**.** *Production efficiency $\Xi^{AE,P}$ can be decomposed into i) cross-sectional intermediate input efficiency, ii) aggregate intermediate input efficiency, iii) cross-sectional factor efficiency, iv) aggregate factor efficiency, v) technology growth, and vi) factor endowment growth, as*

$$\begin{aligned}
\Xi^{AE,P} = & \overbrace{\underbrace{\sum_k \mathbb{C}ov_j^{\Sigma}\left[MWP_x^{jk}, \frac{d\chi_x^{jk}}{d\theta}\right]x^k}_{\substack{\text{Cross-Sectional}\\ \text{Intermediate Input Efficiency}}} + \underbrace{\sum_k \left(AMWP_x^k - AMRS_c^k\right)\frac{d\phi_x^k}{d\theta}y^k}_{\substack{\text{Aggregate}\\ \text{Intermediate Input Efficiency}}}}^{\text{Intermediate Input Efficiency}} \\
& + \overbrace{\underbrace{\sum_f \mathbb{C}ov_j^{\Sigma}\left[MWP_n^{jf}, \frac{d\chi_n^{jf,d}}{d\theta}\right]n^{f,d}}_{\substack{\text{Cross-Sectional}\\ \text{Factor Efficiency}}} + \underbrace{\sum_f \left(AMWP_n^f - AMRS_n^f\right)\frac{dn^{f,s}}{d\theta}}_{\substack{\text{Aggregate}\\ \text{Factor Efficiency}}}}^{\text{Factor Efficiency}} \\
& + \underbrace{\sum_j MSV_y^j \frac{\partial G^j}{\partial \theta}}_{\substack{\text{Technology}\\ \text{Growth}}} + \underbrace{\sum_f AMWP_n^f \frac{d\bar{n}^{f,s}}{d\theta}}_{\substack{\text{Factor Endowment}\\ \text{Growth}}},
\end{aligned}$$

[16] Production efficiency gains ultimately correspond to higher aggregate consumption and lower aggregate factor supply. In fact, $\Xi^{AE,P}$ is given by

$$\Xi^{AE,P} = \sum_j AMRS_c^j \frac{dc^j}{d\theta} - \sum_f AMRS_n^f \frac{dn^{f,s}}{d\theta}.$$

This formulation shows that production efficiency can be interpreted as higher aggregate consumption/value added after appropriately netting the cost of supplying factors—see Nordhaus and Tobin (1973) for the importance of subtracting the cost of supplying factors to connect aggregate consumption/value added/GDP and welfare. In that sense, part of the contribution of Theorem 1b is to express changes in aggregate consumption net of factor supply costs in terms of changes in the allocation of intermediates, factors, technologies, and factor endowments.

where marginal welfare products, MWP_x^{jk} and MWP_n^{jf}, aggregate marginal rates of substitution, $AMRS_c^k$ and $AMRS_n^f$, aggregate marginal welfare products, $AMWP_x^k$ and $AMWP_n^f$, and the marginal social value of output, MSV_y^j, are defined in Section 3.2.3.

First, cross-sectional intermediate input efficiency measures welfare gains from reallocating intermediate inputs from low to high marginal welfare product uses, for a given level of aggregate intermediate use. Hence, for good k it corresponds to the covariance across uses between MWP_x^{jk} and the change in the intermediate use shares, $\frac{d\chi_x^{jk}}{d\theta}$, in proportion to the good's aggregate intermediate use, x^k.

Second, aggregate intermediate input efficiency measures the welfare gains from adjusting the share of output devoted to final consumption relative to production, for a given level of output. Hence, for good k it corresponds to the difference between $AMWP_x^k$ and $AMRS_c^k$, which captures the net welfare impact of reducing consumption of good k and using it in production, multiplied by the change in the intermediate use share, $\frac{d\phi_x^k}{d\theta}$, in proportion to the output of the good, y^k.

Third, cross-sectional factor efficiency measures the welfare gains from reallocating factors from low to high marginal welfare product uses, for a given level of aggregate factor use. Hence, for factor f it corresponds to the covariance across uses between MWP_n^{jf} and the change in the factor use shares, $\frac{d\chi_n^{jf,d}}{d\theta}$, in proportion to the aggregate use of the factor, $n^{f,d}$.

Fourth, aggregate factor efficiency measures the welfare gains from adjusting the (elastic) supply of factors. Hence, for factor f it corresponds to the difference between $AMWP_n^f$ and $AMRS_n^f$, which captures the net welfare impact of supplying an additional unit of factor f and putting it to use, multiplied by the change in factor supply, $\frac{dn^{f,s}}{d\theta}$.[17]

The final two components of the production efficiency decomposition measure welfare gains due to primitive changes in technology and factor endowments. The technology growth component measures the welfare gains from having more output (at no cost) for given allocation shares and factor supplies. Hence, for good j it corresponds to the output change induced by the change in technology, $\frac{\partial G^j}{\partial\theta}$, valued at its marginal social value MSV_y^j. Finally, the factor endowment growth component measures the welfare gains from having more factors (at no cost) for given allocation shares, elastic factor supplies, and technologies. Hence, for factor f it corresponds to the change in the supply of factor f, $\frac{d\bar{n}^{f,s}}{d\theta}$, valued at the welfare gain associated with increasing factor use, $AMWP_n^f$.

Corollary 4 shows that production efficiency satisfies several desirable properties. These properties are helpful to quickly analyze particular economies, as we do in Section 3.4.

Corollary 4 (Properties of Production Efficiency Decomposition)**.**

(a) (Single Good Economies) *In economies with a single good ($J = 1$), cross-sectional intermediate input efficiency and cross-sectional factor efficiency are zero.*

[17] In general, aggregate factor efficiency must be expressed in terms aggregate factor supply and not factor use. If the endowment of an elastically supplied factor is zero or does not change in a given perturbation, then $\frac{dn^{f,s}}{d\theta} = \frac{dn^{f,d}}{d\theta}$.

(*b*) (No Intermediate Input Economies) *In economies with no intermediate goods ($x^{jk} = \xi^{jk} = 0$), cross-sectional and aggregate intermediate input efficiency are zero.*

(*c*) (Fixed Factor Supply Economies) *In economies in which all factors are in fixed supply ($\frac{dn^{f,s}}{d\theta} = 0$), aggregate factor efficiency is zero.*

(*d*) (Specialized Intermediate/Factor Economies) *In economies in which all intermediate inputs (factors) are specialized with $\chi_x^{jk} = 1$ ($\chi_n^{jf} = 1$) for some j, cross-sectional intermediate input (factor use) efficiency is zero.*

(*e*) (Equalized MWP_x^{jk} or MWP_n^{jf}) *If marginal welfare products for good k (factor f) are identical across uses for all goods (factors) with $x^k > 0$ ($n^{f,d} > 0$), then cross-sectional intermediate (factor) efficiency is zero.*

Since both cross-sectional intermediate input and factor efficiency rely on reallocating intermediate inputs and factors towards different uses in production, it is necessary to have at least two goods that can be produced. Relatedly, economies with no intermediate inputs cannot feature cross-sectional or aggregate intermediate input efficiency gains, since $\frac{d\chi_x^{jk}}{d\theta} = 0$ and $\frac{d\phi_x^k}{d\theta} = 0$, while economies with factors in fixed supply, cannot feature aggregate factor efficiency gains, since $\frac{dn^{f,s}}{d\theta} = 0$. Finally, cross-sectional intermediate input (factor) use efficiency must be zero when i) intermediate inputs (factors) are specialized, since there is no scope for reallocating intermediate input (factor) shares towards alternative uses, or ii) the social value of using a good (factor) is identical across uses, since there is no scope to find welfare gains from reallocating goods (factors).

3.3 Insights from Welfare Accounting Decomposition

We present several of the insights that emerge from the welfare accounting decomposition in a series of remarks.

Remark 1 (*Technological and preference origins of welfare gains and losses*)**.** Theorem 1 traces the origins of efficiency gains and losses under any perturbation to the reallocation of resources and to primitive changes in technology and endowments. Its main contribution is to characterize the *welfare-relevant social valuations* for each of these changes. In fact, Theorem 1 identifies a small set of summary statistics—MRS, MWP, $AMRS$, $AMWP$, and MSV—that are sufficient to translate physical changes in allocations, technologies, and endowments into welfare gains and losses. This decomposition is written purely in terms of preferences and technologies, and makes no reference to prices, individual budget constraints, or notions of equilibrium.

Remark 2 (*Social Value of Technology*)**.** Theorem 1b identifies the efficiency gains from pure technological change with the marginal social value of output, MSV_y^j, without making assumptions about the (optimizing) behavior or budget constraints of individuals, prices, or equilibrium notions. In fact, since MSV_y^j can be computed at the original allocation, Theorem 1b characterizes the efficiency gains from technology growth without the need to specify, compute, or measure a perturbation.[18] The technology growth component of the welfare accounting decomposition is always positive since $MSV_y^j > 0$. However, a technological improvement may decrease aggregate efficiency overall if its impact on allocative efficiency is sufficiently negative, which can only happen in inefficient allocations (see Section 4).

Remark 3 (*Allocative vs. Technical Efficiency; Efficiency vs. Misallocation*)**.** We refer to the welfare gains due to exchange efficiency and the first four components of production efficiency as *allocative efficiency* gains, because these involve changes in allocations (allocation shares and factor supplies). We could have alternatively used the term misallocation. That is, a perturbation that increases, say, cross-sectional or aggregate factor efficiency can be described as reducing cross-sectional or aggregate factor *misallocation*. In fact, the factor efficiency components are the marginal counterpart of the notions of misallocation in Hsieh and Klenow (2009). Since the welfare gains associated with technology and factor endowment growth do not involve changes in allocations but instead capture the pure effect of changes in primitives, we refer to these as *technical efficiency* gains.

Remark 4 (*Allocation Shares, Efficiency Conditions and Planning Problem*)**.** By design, the allocative efficiency components of the welfare accounting decomposition—with the exception of aggregate factor efficiency—are written in terms of changes in allocation shares. Working with shares allow us to separate changes due to reallocation (holding consumption, factor supply, output, intermediate input use, or factor use fixed) from changes in aggregates (aggregate factor supply, technology, or endowment growth).[19] Moreover, each allocative efficiency component maps directly into a particular optimality condition for the planning problem for this economy—as shown in Appendix C.1. This occurs because at an efficient allocation, reallocating resources cannot generate efficiency gains. We characterize these efficiency conditions next in Section 4.

Remark 5 (*Informational Requirements*)**.** What are the informational requirements to implement the welfare accounting decomposition, either by computing it in a structural model or by empirically measuring its components? To compute exchange efficiency, it is sufficient to know i) aggregate consumption and factor supply, ii) changes in individual consumption and factor supply shares, and iii) individual marginal rates of substitution. Conditional on these objects, the economy's production structure does not independently determine exchange efficiency. To compute production efficiency, it is sufficient to know i) total output, intermediate use, and factor use; ii) changes in intermediate use, and factor use shares, and changes in aggregate factor supply, technology, and endowment growth; iii) marginal welfare products; iv) aggregate marginal welfare products and marginal rates of

[18] By contrast, computing the welfare gains from individual, intermediate input, and factor efficiency requires knowledge of changes in allocations, which must be measured empirically or computed within a model.

[19] This separation is not possible working with levels, since perturbations that change the level of output necessarily change consumption and/or intermediate input use levels, via (3), while perturbations that change the level of factor supply necessarily change factor use levels, via (4).

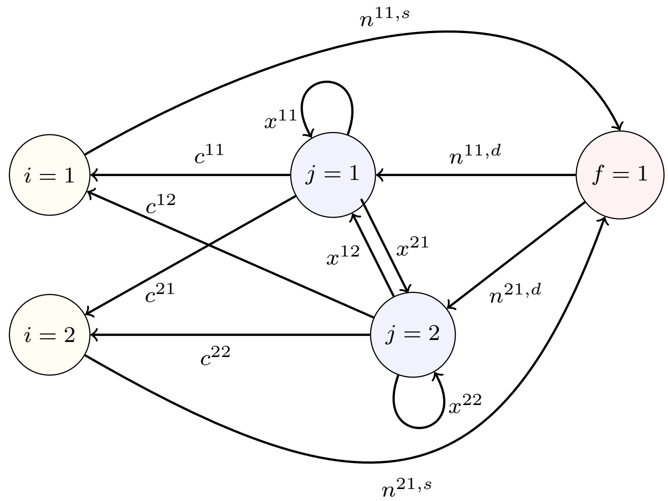


Figure 2. Minimal Welfare Accounting Economy

Note. Figure 2 illustrates the minimal economy in which all components of the welfare accounting decomposition can take non-zero values. We summarize special cases of this economy in Table 1 and study them in Appendix D.

substution; and v) the marginal social value of output. Conditional on these objects, the distribution of consumption and factor supply does not independently determine production efficiency. In Section 5, we show how prices can be used to infer these objects.

3.4 Examples: Minimal Welfare Accounting Economy

We conclude this section by applying Theorems 1a and 1b to simple economies. This is helpful to illustrate the economic forces that underlie each of the components of the decomposition. Figure 2 summarizes *the minimal welfare economy*, which is the simplest economy in which each component of the welfare accounting decomposition can take non-zero values. In Appendix D, we present seven special cases of this economy in which particular components of the welfare accounting decomposition are non-zero. Table 1 summarizes these special cases.

4 Efficiency

In this section, we leverage the welfare accounting decomposition to characterize and study efficient allocations. This is, to our knowledge, the first general characterization of efficiency conditions for disaggregated production economies with heterogeneous individuals.

4.1 Efficiency Conditions

We adopt the conventional definition of (Pareto) efficiency: an allocation is efficient when there is no perturbation that makes every individual (weakly) better off. Equivalently, given Theorems 1a and 1b, an allocation is efficient if there is no feasible perturbation for which any of the allocative

	Exchange Efficiency		Production Efficiency			
	Cross-Sectional Consumption Efficiency	Cross-Sectional Factor Supply Efficiency	Cross-Sectional Intermediate Input Efficiency	Aggregate Intermediate Input Efficiency	Cross-Sectional Factor Efficiency	Aggregate Factor Efficiency
Vertical	×	×	×	×	×	×
Robinson Crusoe	×	×	×	×	×	✓
Horizontal	×	×	×	×	✓	×
Roundabout	×	×	×	✓	×	×
Diversified Intermediate	×	×	✓	✓	×	×
Multiple Factor Suppliers	×	✓	×	×	×	✓
Edgeworth Box	✓	×	×	×	×	×

Table 1. Summary of Minimal Welfare Accounting Special Cases

Note. Table 1 illustrates the components of the welfare accounting decomposition that can be non-zero in special cases of the minimal welfare accounting economy introduced in Figure 2. All economies are formally defined in Appendix D.

efficiency components are positive. Theorems 2a and 2b respectively provide the necessary conditions for exchange and production efficiency.[20]

Theorem 2a (Efficiency Conditions: Exchange Efficiency)**.** *An efficient allocation must satisfy the following exchange efficiency conditions*:

(*a*) (Cross-sectional consumption efficiency) *For goods with* $c^j > 0$*, it must be that*

$$MRS_c^{ij} = \begin{cases} = AMRS_c^j & \forall i \quad s.t. \quad \chi_c^{ij} > 0 \\ \leq AMRS_c^j & \forall i \quad s.t. \quad \chi_c^{ij} = 0. \end{cases} \tag{24}$$

(*b*) (Cross-sectional factor supply efficiency) *For factors with* $n^{f,s} > 0$*, it must be that*

$$MRS_n^{if} = \begin{cases} = AMRS_n^f & \forall i \quad s.t. \quad \chi_n^{if,s} > 0 \\ \geq AMRS_n^f & \forall i \quad s.t. \quad \chi_n^{if,s} = 0. \end{cases} \tag{25}$$

Efficiency requires the equalization of MRS_c^{ij} across all consumers of good j, with MRS_c^{ij} potentially lower for individuals for whom $c^{ij} = 0$. Otherwise, it is feasible and welfare-improving to reallocate consumption from low to high MRS_c^{ij} individuals, for given aggregate consumption c^j. At the corner where individual i does not consume good j, it is not feasible to reallocate consumption away from individual i, even though marginal rates of substitution are not equalized. Similarly, efficiency requires the equalization of MRS_n^{if} across all suppliers of factor f, with MRS_n^{if} potentially lower for individuals for whom $n^{if,s} = 0$. Otherwise, it is feasible and welfare-improving to reallocate

[20] To simplify the exposition, we assume in the body of the paper that $y^j > 0$ and $n^{f,d} > 0$. We allow efficient allocations to feature $y^j = 0$ and $n^{f,d} = 0$ in the Appendix.

factor supply from high to low MRS_n^{if} individuals, for given aggregate factor supply $n^{f,s}$. At the corner where individual i does not supply factor f, it is not feasible to reallocate factor supply away from individual i, even though marginal rates of substitution are not equalized.

While the exchange efficiency conditions in Theorem 2a are arguably standard (see e.g. Mas-Colell et al., 1995), the production efficiency conditions that we characterize in Theorem 2b are novel.

Theorem 2b (Efficiency Conditions: Production Efficiency)**.** *An efficient allocation must satisfy the following production efficiency conditions*:

(a) (Cross-sectional intermediate input efficiency) *For goods with* $x^k > 0$*, it must be that*

$$MWP_x^{jk} = \begin{cases} = AMWP_x^k & \forall j \quad s.t. \quad \chi_x^{jk} > 0 \\ \leq AMWP_x^k & \forall j \quad s.t. \quad \chi_x^{jk} = 0. \end{cases} \tag{26}$$

(b) (Aggregate intermediate input efficiency) *For goods with* $y^k > 0$*, it must be that*

$$\begin{aligned} \max_j \{MWP_x^{jk}\} \leq AMRS_c^k & \qquad \forall k \quad s.t. \quad \phi_x^k = 0. \\ AMWP_x^k = AMRS_c^k & \qquad \forall k \quad s.t. \quad \phi_x^k \in (0,1) \\ AMWP_x^k \geq \max_i \{MRS_c^{ik}\} & \qquad \forall k \quad s.t. \quad \phi_x^k = 1. \end{aligned} \tag{27}$$

(c) (Cross-sectional factor efficiency) *For factors with* $n^{f,d} > 0$*, it must be that*

$$MWP_n^{jf} = \begin{cases} = AMWP_n^f & \forall j \quad s.t. \quad \chi_n^{jf} > 0 \\ \leq AMWP_n^f & \forall j \quad s.t. \quad \chi_n^{jf} = 0. \end{cases} \tag{28}$$

(d) (Aggregate factor efficiency) *For factors with* $n^{f,d} > 0$*, it must be that*

$$\begin{aligned} AMWP_n^f = AMRS_n^f & \qquad \forall f \quad s.t. \quad n^{f,s} > 0 \\ AMWP_n^f \leq \min_i \{MRS_n^{if}\} & \qquad \forall f \quad s.t. \quad n^{f,s} = 0. \end{aligned} \tag{29}$$

While the formal statement of the conditions for production efficiency is somewhat involved, the underlying economics are simple. First, cross-sectional intermediate input efficiency requires the equalization of MWP_x^{jk} across all uses of good k in production. Otherwise, it is feasible and welfare-improving to reallocate intermediate inputs from low to high MWP_x^{jk} uses, for given aggregate intermediate input use x^k. When good k is not used to produce good j, MWP_x^{jk} must be weakly lower.

Second, aggregate intermediate input efficiency for mixed goods with $\phi_x^k \in (0,1)$ requires the equalization of the marginal rate of substitution from consuming good k with its marginal welfare product as an input. For pure final goods with $\phi_x^k = 0$, the marginal rate of substitution from consuming good k must be higher than its highest marginal welfare product if used as an input. For pure intermediate goods with $\phi_x^k = 1$, the marginal welfare product of good k must be higher than its highest marginal rate of substitution if consumed. If these conditions are not satisfied, it is feasible and welfare-improving to reallocate good k from final consumption to intermediate input use, or vice versa, for a given level of output y^k.

A similar logic applies to factors. Third, cross-sectional factor efficiency requires the equalization of MWP_n^{jf} across all uses of factor f, with MWP_n^{jf} potentially lower when factor f is not used to produce good j. Otherwise, it is feasible and welfare-improving to reallocate factors from low to high MWP_n^{jf} uses, for a given level of fixed aggregate factor use $n^{f,d}$.

Finally, aggregate factor efficiency requires the equalization of the marginal welfare product of elastic factor f with its marginal rate of substitution, which captures the utility cost of supplying the factor. When factor f is not elastically supplied, its marginal welfare product must be weakly lower than the lowest marginal rate of substitution, which captures the cheapest cost of supplying the factor.

Theorems 2a and 2b highlight that carefully incorporating non-negativity constraints is critical to characterize the conditions for allocative efficiency in disaggregated economies. These issues become more relevant at finer levels of disaggregation, since heterogeneous individuals typically do not consume most goods and production networks with heterogenous producers become increasingly sparse. We elaborate on these issues in subsections 4.3 and 4.4.

4.2 Technology Growth under Efficiency

The marginal social value of output is a central object for welfare accounting. It is a key determinant of marginal welfare products and thus governs each component of production efficiency. It is furthermore the single determinant of the technology growth component of the welfare accounting decomposition. Theorem 2c characterizes the marginal social value of output at efficient allocations.[21]

Theorem 2c (MSV under Efficiency). *At an allocation that satisfies aggregate intermediate input efficiency, the marginal social value of output for good j is given by*

$$MSV_y^j = \begin{cases} AMRS_c^j & \text{if} \quad \phi_c^j > 0 \\ AMWP_x^j & \text{if} \quad \phi_x^j > 0. \end{cases} \tag{30}$$

At an allocation that additionally satisfies cross-sectional consumption and cross-sectional interme-

[21] Characterizing the endowment growth component under efficiency is straightforward. When $n^{f,d} > 0$, efficiency requires that $AMWP_n^f = MWP_n^{jf}$, $\forall j$ with $\chi_n^{jf,d} > 0$.

diate input efficiency, the marginal social value of output for good j is given by

$$MSV_y^j = \begin{cases} MRS_c^{ij} & \forall i \ \ s.t. \ \ \chi_c^{ij} > 0 \quad if \quad \phi_c^j > 0 \\ MWP_x^{kj} & \forall k \ \ s.t. \ \ \chi_x^{kj} > 0 \quad if \quad \phi_x^j > 0. \end{cases} \tag{31}$$

The marginal social value of output for a good derives from its consumption value when the good is final and from its production value when the good is used as an input. Aggregate intermediate input efficiency guarantees that these are equalized for mixed goods, i.e., $AMRS_c^j = AMWP_x^j$ for j mixed. When j is a final good with $\phi_c^j > 0$, therefore, its marginal social value equals its consumption value $AMRS_c^j$. When j is an intermediate good with $\phi_x^j > 0$, its marginal social value equals its production value $AMWP_x^j$. And when good j is mixed with $\phi_c^j > 0$ and $\phi_x^j > 0$, consumption and production value must be equalized, so $MSV_y^j = AMRS_c^j = AMWP_x^j$.

Conversely, the marginal social value of a pure final (pure intermediate) good is *not* equal to its production (consumption) value. As long as aggregate intermediate input efficiency is satisfied, $MSV_y^j > AMRS_c^j$ when j is a pure intermediate with $\phi_x^j = 1$ and $MSV_y^j > AMWP_x^j$ when j is a pure final good with $\phi_c^j = 1$.

Cross-sectional consumption efficiency furthermore guarantees that $MRS_c^{ij} = AMRS_c^j$ are equalized across all individuals i that consume good j ($\chi_c^{ij} > 0$). The MSV of a final good must therefore coincide with the valuation of each individual. Similarly, cross-sectional intermediate input efficiency guarantees that $MWP_x^{kj} = AMWP_x^j$ are equalized for good j across all its intermediate uses k ($\chi_x^{kj} > 0$). The MSV of goods used as intermediate inputs must then coincide with the marginal welfare product of each use. More broadly, efficiency requires that the value of using a good must be equalized across all uses and coincide with the MSV of the good.

4.3 Interior Economies: Revisiting Lange (1942) and Mas-Collel et al. (1995)

The classic approach to characterizing efficiency conditions is typically traced back to Lange (1942)—see also Samuelson (1947)—and is summarized in a modern treatment in Section 16.E of Mas-Colell et al. (1995). One contribution of our paper is to generalize these classic conditions to general environments with disaggregated production.

Definition 6 (Classic Efficiency Conditions)**.** *The classic (production) efficiency conditions for an intermediate link jk and a factor link jf hold if*

$$MRS_c^{ij} \frac{\partial G^j}{\partial x^{jk}} = MRS_c^{ik} \quad and \quad MRS_c^{ij} \frac{\partial G^j}{\partial n^{jf,d}} = MRS_n^{if}. \tag{32}$$

Critically, the classic approach exclusively studies interior production economies, in which every good is mixed and used in the production of every other good, i.e., $\chi_x^{jk} \in (0, 1)$ and $\phi_x^k \in (0, 1)$.[22]

[22] The classic approach typically allows for consumption or factor supply to be zero for some (but not all) individuals. For that reason, our contribution is to study non-interior economies in the production sense. Mas-Colell et al. (1995)

In that case, the classic efficiency conditions in equation (32) imply i) equalized marginal rates of substitution across individuals, ii) equalized marginal rates of transformation (MRT) across goods, and iii) the equalization of MRS with MRT.[23] In Corollary 5, we show that the classic efficiency conditions emerge as a special case of Theorems 2a and 2b in interior economies. We then show in subsection 4.4 that the classic efficiency conditions are typically invalid in disaggregated production economies that are not interior.

Corollary 5 (Revisiting Lange 1942 and Mas-Colell et al. 1995)**.** *In interior economies, the efficiency conditions of Theorems 2a and 2b collapse to those in Section 16.E of Mas-Colell et al. (1995).*

By construction, all (production) non-negativity constraints are slack in interior economies. Since $\chi_x^{jk} \in (0,1)$ and $\phi_x^k \in (0,1)$, it follows directly from Theorems 2a and 2b (conditions (26) and (27)) that $MWP_x^{jk} = MRS_c^{ik}$, $\forall i,j$ for every good k. Similarly for factors, conditions (28) and (29) imply that $MRS_n^{if} = MWP_x^{jf}$, $\forall i,j$ for every factor f. Both sets of conditions imply that the classic efficiency conditions in equation (32) are satisfied for all links.

4.4 Non-Interior Economies

What then distinguishes the conditions for production efficiency in economies that are not interior, and why do the classic conditions not apply to these environments?

Consider increasing x^{jk}, the use of good k in the production of good j. Assuming this is a feasible perturbation, efficiency requires that its social cost—the marginal social value of good k—is equalized with its social benefit—the marginal social value of good j multiplied by the marginal product $\frac{\partial G^j}{\partial x^{jk}}$. The classic efficiency conditions (32) use marginal rates of substitution to measure the social benefit (32 LHS) and cost (32 RHS). This is appropriate for interior efficient economies where all goods are mixed, since $MSV = MRS$ for final goods as we showed above. When j or k is a pure intermediate, however, marginal rates of substitution no longer represent the good's marginal social value, even at an efficient allocation (Theorem 2c). Since pure intermediates are not

justify their restriction to interior production economies as follows:

> *"(...) every commodity is both an input and an output of the production process. Because this is unrealistic, we emphasize that no more than expositional ease is involved here. Recall that for expositional ease we are not imposing any boundary constraints on the vectors of inputs/outputs."*

Our results show that exclusively considering interior economies is insufficient to properly understand efficiency conditions in disaggregated economies.

[23] Recall that we define marginal rates of substitution in units of the numeraire in this paper, i.e., $MRS_c^{ij} = \frac{\partial u_i}{\partial c^{ij}} / \lambda^i$. If condition (32) holds, then $MRS_c^{ik}/MRS_c^{ij} = \frac{\partial G^j}{\partial x^{jk}}$ must be equal across individuals since marginal products do not depend on i. This implies that two individuals' valuation of good k, expressed in units of good j, is equalized. Since (32) applies for all j and k, it also implies the equalization of MRS in units of the welfare numeraire. To derive the equalization of MRT, notice that (32) can be rewritten as

$$MRS_c^{ij} \frac{\partial G^j}{\partial x^{jk}} = MRS_c^{ij'} \frac{\partial G^{j'}}{\partial x^{j'k}} \quad \Longrightarrow \quad MRS_c^{ij}/MRS_c^{ij'} = \frac{\partial G^{j'}}{\partial x^{j'k}} / \frac{\partial G^j}{\partial x^{jk}} \equiv MRT^{jj',k}$$

where the RHS defines the marginal rate of transformation (MRT). Condition (32) therefore implies both $MRS = MRT$ (after a change of units) and the equalization of MRT across uses since the LHS does not depend on k. A similar argument applies to factor use.

consumed, efficiency requires their MRS to be lower than their MSV. The marginal social value of a pure intermediate instead derives from the consumption value it eventually generates *downstream* as it is used in the production of other goods throughout the network.

There is a second, more mechanical reason why the classic efficiency conditions do not extend to non-interior economies. If good k is not used in the production of good j, the associated efficiency condition is determined by the inequality in (26): efficiency at the jk link then requires that MWP_x^{jk} be lower than the marginal social value of good k.

We summarize the implications of Theorems 2a and 2b for non-interior economies in two corollaries. Corollary 6 concludes that the classic efficiency conditions still hold at the level of an intermediate input link, as long as that link itself is interior.

Corollary 6 (Classic Efficiency Conditions Hold for Interior Links)**.** *The classic efficiency conditions hold for the jk and jf links when*

(*a*) *a mixed good k is used to produce a mixed (or a pure final) good j*

(*b*) *an elastically supplied factor f is used to produce a mixed (or a pure final) good j.*

Intuitively, the classic efficiency conditions (32) extend to all interior links jk and jf because the MSV of mixed goods coincides with their MRS, even when there are non-interior links elsewhere in the network. Corollary 7 characterizes the scenarios in which the classic conditions fail to hold.

Corollary 7 (Scenarios in which Classic Efficiency Conditions Do Not Hold)**.** *The classic efficiency conditions generically*[24] *fail to hold for links jk and jf that feature pure intermediate goods, i.e.,*

(*a*) *a mixed good k is used to produce a pure intermediate good j*

(*b*) *a pure intermediate good k is used to produce any good j*

(*c*) *a factor f is used to produce a pure intermediate good j.*

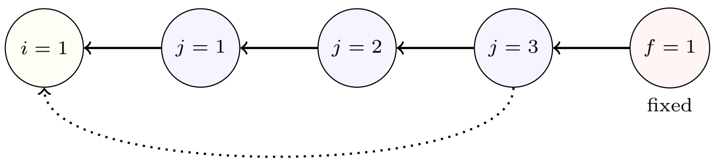
Trivially, the classic conditions also fail to hold for links jk and jf when good k and factor f are not used in the production of good j.

The first and third items of Corollary 7 highlight that the classic efficiency conditions may fail at links in which the efficiency conditions take the form of an equality, as long as an intermediate good is produced. This observation implies that properly characterizing production efficiency is more subtle than simply considering a set of inequalities, as in the case of exchange efficiency.

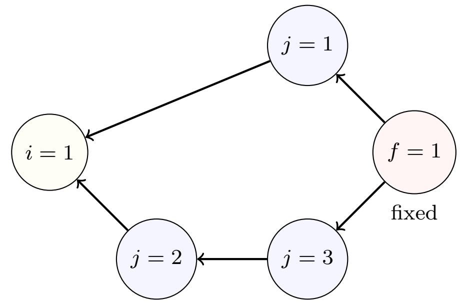
We illustrate Corollary 7 in two simple examples—see also Figure 3.

Example 1 (*Pure Intermediates*)**.** Example 1 features a single individual ($I = 1$), three goods ($J = 3$), and a single factor in fixed supply ($F = 1$). The individual's preferences are $V^1 = u_1(c^{11}, c^{13})$, which implies that $MRS^{12} = 0$. Technologies for each of the goods are $y^1 = G^1(x^{12})$, $y^2 = G^2(x^{23})$, and $y^3 = G^3(n^{31,d})$, which already imposes that many marginal products are zero, e.g., $\frac{\partial G^1}{\partial x^{13}} = 0$.

[24] The qualifier "generically" captures that it is always possible to find production structures for which these results hold.



(a) Example 1



(b) Example 2

Figure 3. Scenarios in which Classic Efficiency Conditions Do Not Hold

Note. Figure 3 illustrates Corollary 7 in two simple scenarios. The left panel shows a mixed good (good 3) used to produce a pure intermediate (good 2), as well as a a pure intermediate (good 2) used to produce a final good (good 1). The right panel shows a factor used to produce both a pure intermediate (good 3) and a final good (good 1).

The welfare accounting decomposition for this economy only features aggregate intermediate input efficiency: exchange efficiency is zero since $I = 1$, cross-sectional intermediate input and factor efficiency are zero since all inputs and factors are specialized, and aggregate factor efficiency is zero since the single factor is in fixed supply.[25] Plugging into Theorem 1b,

$$\Xi^{AE} = \Xi^{AE,P} = \sum_k \left(AMWP_x^k - AMRS_c^k\right)\frac{d\phi_x^k}{d\theta}y^k = \Bigg(\underbrace{MRS_c^{11}\frac{\partial G^1}{\partial x^{12}}\frac{\partial G^2}{\partial x^{23}}}_{AMWP_x^3} - \underbrace{MRS_c^{13}}_{AMRS_c^3}\Bigg)\frac{d\phi_x^3}{d\theta}y^3.$$

For the mixed good 3 with $\phi_x^3 \in (0,1)$, aggregate intermediate input efficiency requires that $AMWP_x^3 = AMRS_c^3$, or equivalently $MRS_c^{11}\frac{\partial G^1}{\partial x^{12}}\frac{\partial G^2}{\partial x^{23}} = MRS_c^{13}$. The classic efficiency condition would instead require $MRS_c^{12}\frac{\partial G^2}{\partial x^{23}} = MRS_c^{13}$, which is invalid since good 2 is a pure intermediate and $MRS_c^{11}\frac{\partial G^1}{\partial x^{12}} > MRS_c^{12} = 0$. At the efficient allocation, the classic condition would lead one to conclude good 3's intermediate use is inefficiently high. This illustrates Corollary 7a.

This example also illustrates Corollary 7b since it features a pure intermediate (good 2) that is used in the production of another good. Since $\phi_x^2 = 1$, aggregate intermediate input efficiency requires that $MRS_c^{11}\frac{\partial G^1}{\partial x^{12}} > MRS^{12} = 0$, i.e., the consumption value of good 2 must be lower than its production value. The classic efficiency condition $MRS_c^{11}\frac{\partial G^1}{\partial x^{12}} = MRS_c^{12}$ would lead one to conclude that, at the efficient allocation, $MSV_y^2 = AMWP_x^2 = AMRS_c^2$, which is incorrect.

Example 2 (*Factor Used to Produce Pure Intermediate*)**.** Example 2 features one individual ($I = 1$), three goods ($J = 3$), and one factor in fixed supply ($F = 1$). Preferences are $V^1 = u_1(c^{11}, c^{12})$ and technologies for each of the goods are $y^1 = G^1(n^{11,d})$, $y^2 = G^2(x^{23})$, and $y^3 = G^3(n^{31,d})$.

The welfare accounting decomposition for this economy only features cross-sectional factor

[25] Formally, we assume here that the efficient production structure is as in Figure 3a. The full set of efficiency conditions also features inequalities to ensure that, for example, it is not efficient to consume good 2 or use it in the production of good 3.

efficiency: exchange efficiency is zero since $I = 1$, cross-sectional intermediate input efficiency is zero since all inputs are specialized, aggregate factor efficiency is zero since the single factor is in fixed supply, and aggregate intermediate input efficiency is zero since $\phi_c^1 = \phi_x^2 = \phi_x^3 = 1$ by construction. Therefore,

$$\Xi^{AE} = \Xi^{AE,P} = \mathbb{C}ov_j^{\Sigma}\left[MWP_n^{j1}, \frac{d\chi_n^{j1,d}}{d\theta}\right]n^{1,d} = \left(MSV_y^1 \frac{\partial G^1}{\partial n^{11,d}} \frac{d\chi_n^{11,d}}{d\theta} + MSV_y^3 \frac{\partial G^3}{\partial n^{31,d}} \frac{d\chi_n^{31,d}}{d\theta}\right)n^{1,d}$$

where $MSV_y^1 = MRS_c^{11}$ and $MSV_y^3 = MRS_c^{12}\frac{\partial G^2}{\partial x^{23}}$. Since labor is in fixed supply but used in the production of two goods, a feasible perturbation is $\frac{d\chi_n^{11,d}}{d\theta} = -\frac{d\chi_n^{31,d}}{d\theta}$. Cross-sectional factor efficiency therefore requires that $MRS_c^{11}\frac{\partial G^1}{\partial n^{11,d}} = MRS_c^{12}\frac{\partial G^2}{\partial x^{23}}\frac{\partial G^3}{\partial n^{31,d}}$. The classic efficiency condition would instead associate the marginal social value of pure intermediate good 3 with its MRS and require $MRS_c^{11}\frac{\partial G^1}{\partial n^{11,d}} = MRS_c^{13}\frac{\partial G^3}{\partial n^{31,d}}$. Since $MRS_c^{12}\frac{\partial G^2}{\partial x^{23}} > MRS_c^{13} = 0$ at the efficient allocation, the classic condition would lead one to conclude the use of labor in the production of good 3 is inefficiently high, illustrating Corollary 7c.

We conclude the study of non-interior economies with a remark that highlights the importance of characterizing efficiency conditions in terms of MWP and MRS instead of MRS and MRT.

Remark 6 (*$MWP \gtreqless MRS$ generalizes $MRS \gtreqless MRT$*)**.** One central takeaway from this section is that MWP and MRS are the appropriate objects to characterize efficiency conditions, rather than MRS and MRT, as in the classic approach. For instance, when good k is mixed or factor f is in elastic supply, efficiency requires that

$$MWP_x^{jk} = MRS_c^{ik} \quad \text{and} \quad MWP_n^{jf} = MRS_n^{if}, \tag{33}$$

for all i such that $\chi_c^{ij} > 0$ and for all j such that $\chi_x^{jk} > 0$, but the classic efficiency conditions in (32) would not be valid if j is a pure intermediate. More generally, the correct inequalities that characterize production efficiency (see Theorem 1b) can be written in terms of MWP and MRS, but not MRS and MRT. This insight is useful to understand the distinction between marginal revenue products and marginal welfare products in Section 5.3.

4.5 Planning Problem

We have emphasized that the welfare accounting decomposition can be leveraged to derive efficiency conditions directly. An equivalent alternative approach is to set up the planning problem.

Definition 7 (Planning Problem)**.** *The planning problem—formally stated in Appendix C.1–maximizes the SWF in (5), with preferences V_i defined in (1), subject to technologies and resource constraints, defined in (2), (3) and (4), as well as non-negativity constraints. We denote the Lagrange multipliers on the goods and factor resource constraints by ζ_y^j and ζ_n^f, respectively.*

There are three reasons why studying the planning problem is useful. First, it provides an equivalent characterization of the efficiency conditions in Theorems 2b and 2a. As we show in the Appendix, the restriction to feasible perturbations that underlie our characterization of efficiency conditions is implied by the Kuhn-Tucker multipliers on the constraints of the planning problem. Second, and more importantly for this paper, the planning problem provides a justification for the welfare accounting decomposition. As we show in the Appendix, each of the components of the decomposition can be interpreted as a particular perturbation of the planning problem. Finally, the planning problem provides an interpretation of the technology growth and factor endowment growth components of the welfare accounting decomposition in terms of the Lagrange multipliers ζ_y^j and ζ_n^f, since $\zeta_y^j = MSV_y^j$ when $y^j \neq 0$ and $\zeta_n^f = AMWP_n^f$ when $n^{f,d} \neq 0$. In fact, one could interpret the contribution of this section as fully characterizing the Lagrange multipliers of the planning problem.

Remark 7 (Socialist Calculation Debate)**.** The results in this section directly speak to the socialist calculation debate, which discusses the feasibility of central planning—see e.g. Lange (1936), Lerner (1944), or Hayek (1945). Our results illustrate how computing efficiency conditions in production economies is significantly harder than efficiently allocating goods across individuals, especially in economies that feature pure intermediates. In particular, Theorem 3a below shows that computing MSV_y^j for pure intermediates requires knowledge of the entire production network—to compute the output inverse—while computing MSV_y^j for mixed or pure final goods only requires knowledge of aggregate individual valuations via marginal rates of substitution.

5 Welfare Accounting in Competitive Economies

Our results so far have made no assumptions about the (optimizing) behavior of agents, individual budget constraints, prices, or notions of equilibrium. In this section, we specialize the welfare accounting decomposition to competitive economies with and without wedges. This provides new insights by shedding light on the relation between efficiency and competition and by relating prices to the welfare-relevant statistics we have identified in this paper.

5.1 Competitive Equilibrium with Wedges

Starting from the physical environment described in Section 2, we now assume that individuals maximize utility and technologies are operated with the objective of minimizing costs and maximizing profits. To allow for distortions, we saturate all choices with wedges, which we take as primitives.

Individual i faces a budget constraint of the form

$$\sum_j p^j \left(1 + \tau_c^{ij}\right) c^{ij} = \sum_f w^f \left(1 + \tau_n^{if,s}\right)\left(n^{if,s} + \bar{n}^{if,s}\right) + \sum_j \nu^{ij} \pi^j + T^{ij}, \tag{34}$$

where p^j denotes the price of good j, w^f denotes factor f's compensation per unit supplied, $\nu^{ij}\pi^j$ denotes the profit associated with the operation of technology j received by individual i, and T^{ij} is a lump-sum transfer that rebates wedges back to individuals. Individual i faces individual-specific consumption and factor supply wedges τ_c^{ij} and $\tau_n^{if,s}$.

Firms operate technologies to minimize costs, which defines the cost functions

$$\mathcal{C}^j\Big(y^j;\big\{w^f\big\}_f,\big\{p^k\big\}_k\Big)=\min_{n^{jf,d},\,x^{jk}}\sum_f w^f\Big(1+\tau_n^{jf,d}\Big)n^{jf,d}+\sum_k p^k\Big(1+\tau_x^{jk}\Big)x^{jk}, \tag{35}$$

subject to equation (2), facing technology-specific factor wedges $\tau_n^{if,d}$ and technology-specific intermediate input wedges τ_x^{jk}. We assume that the supply of good j can be expressed as the solution to a profit maximization problem given by

$$\pi^j=\max_{y^j}p^j\Big(1+\tau_y^j\Big)y^j-\mathcal{C}^j\Big(y^j;\big\{w^f\big\}_f,\big\{p^k\big\}_k\Big), \tag{36}$$

where τ_y^j denotes a markup wedge for technology j.

Definition 8 (Competitive Equilibrium with Wedges)**.** *A competitive equilibrium with wedges comprises a feasible allocation $\{c^{ij},n^{if,s},x^{jk},n^{jf,d},y^j\}$ and prices $\{p^j,w^f\}$ that satisfy resource constraints (3) and (4), such that individuals optimize,*

$$MRS_c^{ij}\le p^j\left(1+\tau_c^{ij}\right),\quad\forall i,\forall j\quad\text{and}\quad MRS_n^{if}\ge w^f\left(1+\tau_n^{if,s}\right),\quad\forall i,\forall f,$$

where the equations hold with equality when $c^{ij}>0$ and $n^{if,s}>0$, respectively, and firms minimize costs and maximize profits,

$$p^j\frac{\partial G^j}{\partial x^{jk}}\le p^k\frac{1+\tau_x^{jk}}{1+\tau_y^j},\quad\forall j,\forall k\quad\text{and}\quad p^j\frac{\partial G^j}{\partial n^{jf,d}}\le w^f\frac{1+\tau_n^{jf,d}}{1+\tau_y^j},\quad\forall j,\forall f,$$

where the equations hold with equality when $x^{jk}>0$ and $n^{jf,d}>0$, respectively.[26]

In a competitive equilibrium, individuals equalize marginal rates of substitution with prices or wages cum wedges, while firms equalize marginal revenue products with marginal costs cum wedges.[27] We can compactly represent the optimality conditions in matrix form as

$$\begin{array}{ccc}\boldsymbol{MRS}_c\le\boldsymbol{p}\left(\mathbf{1}_c+\boldsymbol{\tau}_c\right) & & \boldsymbol{pG}_x\le\boldsymbol{p}\left(\mathbf{1}_x+\boldsymbol{\tau}_x\right)\\ \boldsymbol{MRS}_n\ge\boldsymbol{w}\left(\mathbf{1}_{n^s}+\boldsymbol{\tau}_{n^s}\right) & \text{and} & \boldsymbol{pG}_n\le\boldsymbol{w}\left(\mathbf{1}_{n^d}+\boldsymbol{\tau}_{n^d}\right),\end{array} \tag{37}$$

where all matrices are defined in Appendix A. The matrices $\boldsymbol{\tau}_x$ and $\boldsymbol{\tau}_x$ include markup wedges τ_y^j in addition to intermediate input use wedges τ_x^{jk} and factor use wedges $\tau_n^{jf,d}$. We refer to economies with no wedges ($\tau_c^{ij}=\tau_n^{if,s}=\tau_x^{jk}=\tau_n^{jf,d}=\tau_y^j=0$) as *frictionless competitive economies*. In these

[26] In this section, we implicitly choose the nominal numeraire (i.e. the unit in which prices, wages, and profits are defined) to be the welfare numeraire. This is without loss of generality since we can always renormalize MRS.

[27] In parallel to the definition of marginal welfare products, we define marginal revenue products as $MRP_x^{jk}=p^j\frac{\partial G^j}{\partial x^{jk}}$ and $MRP_n^{jf}=p^j\frac{\partial G^j}{\partial n^{jf,d}}$. In matrix form, $\boldsymbol{MRP}_x=\boldsymbol{pG}_x$ and $\boldsymbol{MRP}_n=\boldsymbol{pG}_n$.

economies, the First Welfare Theorem holds, so any competitive equilibrium allocation is efficient.[28]

Prices and wages (cum wedges) are helpful to recover the welfare-relevant statistics in competitive economies. Conditions (37) link prices to marginal rates of substitution and marginal products, an insight that we exploit repeatedly in this section.

5.2 Marginal Social Value of Output in Competitive Economies

5.2.1 Competitive Economies with Wedges

Characterizing the marginal social value of output in competitive economies with wedges is critical because it directly determines the efficiency gains from technology growth as well as marginal welfare products, which in turn govern all production efficiency components.

Theorem 3a (Marginal Social Value of Output)**.** *In competitive economies with wedges, the marginal social value of output, defined via a $1 \times J$ matrix $\boldsymbol{MSV}_y$, is given by*

$$\boldsymbol{MSV}_y = \boldsymbol{p} + \boldsymbol{p}\bar{\boldsymbol{\tau}}_y \boldsymbol{\Psi}_y \quad \textit{where} \quad \bar{\boldsymbol{\tau}}_y = \boldsymbol{\phi}_x \bar{\boldsymbol{\tau}}_x + \boldsymbol{\phi}_c \bar{\boldsymbol{\tau}}_c, \tag{38}$$

where $\boldsymbol{p}$ denotes the $1 \times J$ vector of prices, $\bar{\boldsymbol{\tau}}_x$ and $\bar{\boldsymbol{\tau}}_c$ denote $J \times J$ diagonal matrices of aggregate intermediate input and consumption wedges, with elements given by $\bar{\tau}_x^j = \sum_k \chi_x^{kj} \tau_x^{kj}$ and $\bar{\tau}_c^j = \sum_i \chi_c^{ij} \tau_c^{ij}$, $\boldsymbol{\phi}_x$ and $\boldsymbol{\phi}_c$ are $J \times J$ diagonal matrices of aggregate intermediate use and consumption shares, $\bar{\boldsymbol{\tau}}_y$ defines the aggregate output wedge, and $\boldsymbol{\Psi}_y$ is the output inverse matrix defined in (17)*.*[29]

Equation (38) shows that the marginal social value of output equals the vector of prices augmented by a term that captures the average of the aggregate wedges in consumption and intermediate input use. Aggregate consumption and intermediate input use wedges are weighted averages of individual consumption wedges, $\bar{\tau}_c^j = \sum_i \chi_c^{ij} \tau_c^{ij}$, and intermediate input use wedges, $\bar{\tau}_x^j = \sum_k \chi_x^{kj} \tau_x^{kj}$. The aggregate output wedge is in turn a weighted average of the two.

In order to understand why $\boldsymbol{MSV}_y$ takes this form in competitive economies, it is useful to start from its definition, $\boldsymbol{MSV}_y = \boldsymbol{AMRS}_c \boldsymbol{\phi}_c \boldsymbol{\Psi}_y$, and proceed gradually. First, using the optimality conditions for individual consumption, $\boldsymbol{MSV}_y$ can be written as

$$\boldsymbol{MSV}_y = \boldsymbol{p}\boldsymbol{\phi}_c \boldsymbol{\Psi}_y + \underbrace{(\boldsymbol{AMRS}_c - \boldsymbol{p})}_{\boldsymbol{p}\bar{\boldsymbol{\tau}}_c} \boldsymbol{\phi}_c \boldsymbol{\Psi}_y. \tag{39}$$

Intuitively, a unit impulse in output ultimately increases aggregate consumption by $\boldsymbol{\phi}_c \boldsymbol{\Psi}_y$, for given allocation shares and factor supplies. The social value of this change in aggregate consumption can be split into its market value and the deviation between the the true social value, given by $\boldsymbol{AMRS}_c$, and the market value. This difference is precisely determined the aggregate consumption wedge, $\bar{\boldsymbol{\tau}}_c$.

[28] While the general proofs of the First Welfare Theorem by Arrow (1951) and Debreu (1951) apply to the economy considered here, our resusts provide an alternative constructive proof. Under standard convexity assumptions, a Second Welfare Theorem also holds.

[29] In sum form, we can express an element of $\boldsymbol{MSV}_y$ as $MSV_y^k = p^k + \sum_j p^j \bar{\tau}_y^j \psi_y^{jk}$, where $\bar{\tau}_y^j = \phi_c^j \bar{\tau}_c^j + \phi_x^j \bar{\tau}_x^j$.

Next, the market value of the change in aggregate consumption, can be expressed as

$$\boldsymbol{p}\boldsymbol{\phi}_c\boldsymbol{\Psi}_y = \boldsymbol{p} + \underbrace{(\boldsymbol{p}\boldsymbol{G}_x\boldsymbol{\chi}_x - \boldsymbol{p})}_{\boldsymbol{p}\bar{\boldsymbol{\tau}}_x}\boldsymbol{\phi}_x\boldsymbol{\Psi}_y. \tag{40}$$

Intuitively, the ultimate change in aggregate consumption induced by a unit impulse in output, $\boldsymbol{\phi}_c\boldsymbol{\Psi}_y$, can be expressed as the ultimate change in output net of aggregate intermediate use.[30] Hence, the ultimate market value of a unit impulse in output corresponds to the sum of the market value of the impulse, given by $\boldsymbol{p}$, and the market value of the knock-on effects net of aggregate intermediate use, given by $\boldsymbol{p}\boldsymbol{G}_x\boldsymbol{\chi}_x - \boldsymbol{p}$. This difference is precisely determined by the aggregate intermediate input wedge, $\bar{\boldsymbol{\tau}}_x$.

Combining (39) and (40), we can reformulate (38) as

$$\boldsymbol{MSV}_y = \boldsymbol{p} + \underbrace{(\boldsymbol{p}\boldsymbol{G}_x\boldsymbol{\chi}_x - \boldsymbol{p})}_{=\boldsymbol{p}\bar{\boldsymbol{\tau}}_x}\boldsymbol{\phi}_x\boldsymbol{\Psi}_y + \underbrace{(\boldsymbol{AMRS}_c - \boldsymbol{p})}_{=\boldsymbol{p}\bar{\boldsymbol{\tau}}_c}\boldsymbol{\phi}_c\boldsymbol{\Psi}_y.$$

This expression illustrates that aggregate consumption (intermediate input use) of good j is too low when $\bar{\tau}_c^j > 0$ ($\bar{\tau}_x^j > 0$), and output of good j is too low when $\bar{\tau}_y^j = \phi_c^j\bar{\tau}_c^j + \phi_x^j\bar{\tau}_x^j > 0$. Hence, the marginal social value of output for goods that ultimately increase the output of goods with positive aggregate output wedges is higher than the price.

Given Theorem 3a, the technology growth component of the welfare accounting decomposition is simply given by

$$\boldsymbol{MSV}_y\boldsymbol{G}_\theta = \sum_j MSV_y^j\frac{\partial G^j}{\partial\theta} = \sum_j\left(p^j + \sum_k p^k\bar{\tau}_y^k\psi_y^{kj}\right)\frac{\partial G^j}{\partial\theta}.$$

The following remarks discuss insights that emerge from Theorem 3a for competitive economies with wedges. We then revisit its implications for frictionless competitive economies in Section 5.2.2.

Remark 8 (*Condition for $\boldsymbol{MSV}_y = \boldsymbol{p}$*)**.** It is well understood that prices capture the social value of technology growth in frictionless competitive economies—see Corollary 8 below. Theorem 3a implies a converse result that has been missing from the existing literature: The condition that ensures $\boldsymbol{MSV}_y = \boldsymbol{p}$ is that aggregate output wedges are zero, that is,

$$\bar{\boldsymbol{\tau}}_y = \boldsymbol{\phi}_c\bar{\boldsymbol{\tau}}_c + \boldsymbol{\phi}_x\bar{\boldsymbol{\tau}}_x = 0. \tag{41}$$

While frictionless competition guarantees that (41) is satisfied, this condition may also hold otherwise, possibly at inefficient allocations. For instance, prices will capture the marginal social value of output as long as aggregate output wedges are zero, even when intermediate input and consumption wedges are non-zero ($\boldsymbol{\tau}_x \neq 0$ and $\boldsymbol{\tau}_c \neq 0$) and the competitive equilibrium is inefficient.[31]

[30] Formally, (40) uses the following physical identity, which follows from (18):

$$\boldsymbol{\phi}_c\boldsymbol{\Psi}_y = \boldsymbol{\Psi}_y - \boldsymbol{\phi}_x\boldsymbol{\Psi}_y = \boldsymbol{I}_J + \boldsymbol{G}_x\boldsymbol{\xi}\boldsymbol{\Psi}_y - \boldsymbol{\phi}_x\boldsymbol{\Psi}_y = \boldsymbol{I}_J + (\boldsymbol{G}_x\boldsymbol{\chi}_x - \boldsymbol{I}_J)\,\boldsymbol{\phi}_x\boldsymbol{\Psi}_y,$$

where the ultimate change in output, $\boldsymbol{\Psi}_y$, is decomposed into the unit impulse, $\boldsymbol{I}_J$, and knock-on effects, $\boldsymbol{G}_x\boldsymbol{\xi}\boldsymbol{\Psi}_y$.

[31] Aggregate output wedges can be zero when aggregate consumption and intermediate use wedges cancel out,

Remark 9 (*Invariance of $\boldsymbol{MSV}_y$ to Factor Wedges*)**.** Theorem 3a also implies that the marginal social value of output does not depend directly on factor supply or factor use wedges. This result underscores the asymmetry between consumption and intermediate input distortions on the one hand and factor supply and use distortions on the other. Because $\boldsymbol{MSV}_y$ enters in the definition of marginal welfare products, all production efficiency components are non-zero when $\bar{\boldsymbol{\tau}}_y \neq 0$, but only factor efficiency components directly depend on factor wedges, as we show in Theorem 3b below.

Remark 10 ($\boldsymbol{MSV}_y$ *and Network Propagation*)**.** Theorem 3a has two important implications for network propagation. First, when $\bar{\boldsymbol{\tau}}_y = 0$, the marginal social value of output can be read exclusively off prices and does not require knowledge of the entire production network. This observation is made at times in frictionless competitive economies—see Corollary 8—which Theorem 3a shows applies more generally. Second, when $\bar{\boldsymbol{\tau}}_y \neq 0$, the output inverse matrix $\boldsymbol{\Psi}_y$ contains the necessary information on network propagation to determine $\boldsymbol{MSV}_y$. While it is possible to characterize $\boldsymbol{\Psi}_y$ in terms of prices, allocations, and intermediate input wedges—as we do in Appendix C.2—this is only relevant insofar as it captures ultimate changes in output.[32]

Remark 11 (*Relation to Cost-Based Domar Weights*)**.** A central result of Baqaee and Farhi (2020) is that cost-based Domar weights summarize the impact of pure technological change on final output in an environment with a single individual, factors in fixed supply, and markup wedges. Their result is a special case of Theorem 3a. Formally, under the assumptions in that paper,

$$\underbrace{\frac{1}{\sum_j p^j c^j}}_{\text{Normalization}} \quad \underbrace{\boldsymbol{MSV}_y \boldsymbol{G}_\theta}_{\substack{\text{Technology Growth}\\ \text{Component}}} = \underbrace{\frac{1}{\sum_j p^j c^j}\boldsymbol{p}\,\hat{\boldsymbol{c}}}_{\substack{\text{Final Expenditure}\\ \text{Shares}}} \quad \underbrace{\tilde{\boldsymbol{\Psi}}_y}_{\substack{\text{Cost-Based}\\ \text{Leontief Inverse}}}, \tag{42}$$

where $\hat{\boldsymbol{c}} = \text{diag}(\boldsymbol{c})$ and $\tilde{\boldsymbol{\Psi}}_y$ is the proportional output inverse, which in turn maps to the intermediate input block of the cost-based Leontief inverse defined in Baqaee and Farhi (2020)—see Appendix C.2. Relative to equation (42), Theorem 3a illustrates how competitive forces guarantee that $MSV_y^j = p^j$ when $\bar{\boldsymbol{\tau}}_y = 0$. Crucially, away from the assumptions in Baqaee and Farhi (2020), Theorem 3a highlights that cost-based Domar weights cease to capture the efficiency gains from pure technological change, for instance in the presence of aggregate consumption wedges.

5.2.2 Frictionless Competitive Economies: Hulten's Theorem Revisited

Theorem 3a allows us to revisit the impact of technology changes in the frictionless competitive case. This is the widely studied Hulten's theorem (Hulten, 1978), a result that has played a prominent

or when both are zero. In turn, aggregate consumption and intermediate use wedges can be zero when its elements cancel out, or when all its constituents are zero. For cancelations to occur, it must be that some wedges are positive and other negative.

[32] Interestingly, only intermediate input wedges directly enter $\boldsymbol{\Psi}_y$, which echoes existing insights highlighting the outsized role that intermediate input distortions play in production—see e.g. Ciccone (2002) or Jones (2011).

role in the literatures on the macroeconomic impact of microeconomic shocks and growth accounting (Gabaix, 2011; Acemoglu et al., 2012; Baqaee and Farhi, 2020; Bigio and La'O, 2020).[33]

Corollary 8 (Hulten's Theorem Revisited)**.** *In frictionless competitive economies, the aggregate efficiency impact of a proportional Hicks-neutral technology change j is*

$$\frac{1}{\sum_j p^j c^j}\Xi^{AE} = \underbrace{\frac{p^j y^j}{\sum_j p^j c^j}}_{\text{Sales Share}}, \tag{43}$$

where $\frac{p^j y^j}{\sum_j p^j c^j}$ is the Domar weight or sales share of good j in $\sum_j p^j c^j$.

Corollary 8 provides, to our knowledge, the most general Hulten-style result to date, which applies to frictionless competitive economies with heterogeneous individuals, elastic factor supplies, arbitrary preferences and technologies, and arbitrary social welfare functions. Its generality allows us to systematically discuss the many qualifications associated with this result in the following remarks.

Remark 12 (*Welfare vs. Aggregate Efficiency vs. Production Efficiency vs. Output*)**.** Hulten's theorem is typically formulated in terms of final output (often via TFP). This is in contrast to Corollary 8, which highlights that Hulten's theorem is at its core a result about aggregate efficiency (via production efficiency) and neither about final output nor welfare. Why is this the case? In economies with a single individual ($I = 1$) and in which supplying factors causes no disutility ($\frac{\partial u_i}{\partial n^{if,s}} = 0$), changes in final output, production efficiency, aggregate efficiency, and welfare coincide, which has justified the use of Hulten's theorem as a result about final output. In economies with a single individual, redistribution and exchange efficiency are zero, so aggregate efficiency and welfare coincide and are exclusively determined by production efficiency. And when supplying factors causes no disutility, there is no need to subtract the social cost of supplying factors to transform final output changes into welfare changes, so production efficiency exclusively captures changes in final output (i.e. aggregate consumption).[34] Corollary 8 highlights that, in frictionless competitive economies, sales shares capture the impact of technology on aggregate efficiency, not final output or overall welfare.[35]

[33] Hulten's theorem is typically stated—see Baqaee and Farhi (2019)—as:

> *"For efficient economies and under minimal assumptions, the impact on aggregate TFP of a microeconomic TFP shock is equal to the shocked producer's sales as a share of GDP (Domar weight)"*

[34] It is common to state that Hulten's theorem does not apply to economies with elastic factor supplies. For instance, Baqaee and Farhi (2018) state that

> *"Hulten's theorem fails when factors supplies are elastic".*

While this is true when Hulten's theorem is formulated in terms of final output, Corollary 8 highlights that Hulten's theorem does apply to economies with elastic factors when formulated in terms of aggregate efficiency. Bigio and La'O (2020) already show that Hulten's theorem is valid for aggregate efficiency in an environment with a single individual and elastic labor supply; see also Basu and Fernald (2002).

[35] Away from frictionless competition, Hulten's Theorem applies to production efficiency (i.e. sales shares capture the production efficiency impact of a proportional Hicks-neutral technology change) if i) all production wedges and aggregate consumption wedges are zero and ii) aggregate output wedges are zero at an allocation that satisfies

Remark 13 (*Efficient vs. Frictionless Competitive vs. Efficient Interior Economies*)**.** Hulten's theorem is typically stated as applying to efficient economies, which is incorrect. Corollary 8 shows instead that Hulten's theorem applies to *frictionless competitive economies*, which is a subset of efficient economies.[36] Why is this the case? When an allocation is efficient, all allocative efficiency components are necessarily zero, which guarantees that aggregate efficiency is exclusively due to technology growth and factor endowment growth. However, efficiency is not enough to guarantee that $\boldsymbol{MSV}_y = \boldsymbol{p}$: this only occurs when $\bar{\boldsymbol{\tau}}_y = 0$, which is the case in frictionless competitive economies. There are efficient allocations in which $\bar{\boldsymbol{\tau}}_y \neq 0$ and Hulten's theorem does not hold. Intuitively, it is possible to have efficient non-interior allocations in which marginal welfare products and input prices are misaligned. Hence, while Hulten's theorem does apply to efficient interior allocations, it can fail in efficient non-interior allocations. This result further underscores the importance of carefully dealing with non-interior allocations when studying disaggregated economies.

Example 3 (*Failure of Hulten's Theorem in an Efficient Equilibrium*)**.** We consider the same environment as in Example 1, and focus on a technology change for good 2, so $\frac{\partial G^2}{\partial \theta} \neq 0$. For simplicity, we set all wedges to zero, with the exception of $\tau_x^{12} \neq 0$. The competitive equilibrium of this economy is efficient, with the relevant efficiency condition here being $MRS_c^{11} \frac{\partial G^1}{\partial x^{12}} > 0$. In this case, competition ensures that $p^1 \frac{\partial G^1}{\partial x^{12}} = p^2(1 + \tau_x^{12})$. But note that

$$MSV_y^2 = MRS_c^{11} \frac{\partial G^1}{\partial x^{12}} = p^1 \frac{\partial G^1}{\partial x^{12}} = p^2(1 + \tau_x^{12}) \neq p^2,$$

so prices do not capture the marginal social value of output and Hulten's theorem fails in this efficient economy. This example illustrates that $\bar{\tau}_y^2 = \bar{\tau}_x^2 = \tau_x^{12} = 0$ is the condition that ensures $MSV_y^2 = p^2$, not efficiency.[37]

Remark 14 (*Normalizations behind Domar Weights*)**.** Comparing Theorem 3a and Corollary 8 highlights why Hulten's theorem is typically stated in terms of Domar weights. First, considering proportional Hicks-neutral technology shocks implies that $\frac{\partial G^j}{\partial \theta} = y^j$, which ensures that the numerator of the Domar weight in (43) is $p^j y^j$. Second, Hulten's theorem is typically stated using nominal GDP as numeraire, which ensures that the denominator of the Domar weight in (43) is $\sum_j p^j c^j$. These are arbitrary normalization; in fact, normalizing by the aggregate value of output $\sum_j p^j y^j$ would define alternative weights that add up to one.

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production efficiency. Away from frictionless competition, Hulten's theorem applies to production efficiency (i.e. sales shares capture the production efficiency impact of a proportional Hicks-neutral technology change) if i) all production wedges and aggregate consumption wedges are zero and ii) aggregate output wedges are zero at an allocation that satisfies production efficiency.

[36] This logic applies regardless of whether Hulten's theorem is expressed in terms of aggregate efficiency or final output. The fact that frictionless competition is a more stringent condition than efficiency is well understood (Edgeworth, 1881; Debreu and Scarf, 1963). One reason that explains why the existing literature has been imprecise about the scope of Hulten's theorem is that prior to the results in Section 4 there had been no characterization of efficiency conditions for general disaggregated production economies with heterogeneous individuals.

[37] Baqaee and Farhi (2020) already provide an example of an efficient economy in which Hulten's theorem fails.

5.3 Allocative Efficiency in Competitive Economies

In this subsection, we specialize the allocative efficiency components of the welfare accounting decomposition to competitive economies with wedges.

Theorem 3b (Production Efficiency in Competitive Economies)**.** *In competitive economies with wedges, in the absence of technology and factor endowment growth, production efficiency is given by*

$$\Xi^{AE,P} = \underbrace{\sum_k \mathbb{C}ov_j^{\Sigma}\left[\tau_x^{jk}, \frac{d\chi_x^{jk}}{d\theta}\right] p^k x^k + \sum_k \mathbb{C}ov_j^{\Sigma}\left[\left(MSV_y^j - p^j\right)\frac{\partial G^j}{\partial x^{jk}}, \frac{d\chi_x^{jk}}{d\theta}\right] x^k}_{\text{Cross-Sectional Intermediate Input Efficiency}}$$

$$+ \underbrace{\sum_k \left(p^k\left(\bar{\tau}_x^k - \bar{\tau}_c^k\right) + \sum_j \left(MSV_y^j - p^j\right)\frac{\partial G^j}{\partial x^{jk}}\chi_x^{jk}\right)\frac{d\phi_x^k}{d\theta} y^k}_{\text{Aggregate Intermediate Input Efficiency}}$$

$$+ \underbrace{\sum_f \mathbb{C}ov_j^{\Sigma}\left[\tau_{n^d}^{jf}, \frac{d\chi_n^{jf,d}}{d\theta}\right] w^f n^{f,d} + \sum_f \mathbb{C}ov_j^{\Sigma}\left[\left(MSV_y^j - p^j\right)\frac{\partial G^j}{\partial n^{jf,d}}, \frac{d\chi_n^{jf,d}}{d\theta}\right] n^{f,d}}_{\text{Cross-Sectional Factor Efficiency}}$$

$$+ \underbrace{\sum_f \left(w^f\left(\bar{\tau}_{n^s}^f - \bar{\tau}_{n^d}^f\right) + \sum_j \left(MSV_y^j - p^j\right)\frac{\partial G^j}{\partial n^{jf,d}}\chi_n^{jf,d}\right)\frac{dn^{f,s}}{d\theta}}_{\text{Aggregate Factor Efficiency}}.$$

Theorem 3b follows from imposing the equilibrium conditions in (37) into the production efficiency decomposition in Theorem 2b. In line with Remark 13, Theorem 3b further underscores the asymmetry between aggregate output wedges, which directly impact all production efficiency components (via the terms that contain $MSV^j - p^j$, since $\boldsymbol{MSV}_y - \boldsymbol{p} = \boldsymbol{p}\bar{\boldsymbol{\tau}}_y\boldsymbol{\Psi}_y$) and other wedges. Hence, any changes in inputs or factors that increase the output of goods with high aggregate output wedges have a separate impact on the aggregate efficiency components. Since these effects are identical across all components, we focus on describing the remaining terms.

First, cross-sectional intermediate input efficiency directly depends on the dispersion in intermediate input use wedges. Intuitively, reallocating intermediate inputs towards uses with higher wedges is valuable since the competitive equilibrium features too litle of those input uses. Second, aggregate intermediate input efficiency directly depends on the difference between aggregate intermediate input and consumption wedges. Intuitively, if $\bar{\tau}_x^k > (<)\, \bar{\tau}_c^k$, the aggregate intermediate use of good k is inefficiently high relative to its consumption use. Third, cross-sectional factor efficiency directly depends on the dispersion in factor use wedges. Intuitively, reallocating factors towards uses with higher wedges is valuable since the competitive equilibrium features too litle of those factor uses. Finally, aggregate factor efficiency directly depends on the difference between aggregate factor supply and factor use wedges. Intuitively, if $\bar{\tau}_{n^s}^f > (<)\, \bar{\tau}_{n^d}^f$, the aggregate supply of factor f is inefficiently low (high) relative to its use. In the Appendix, we characterize the factor endowment growth component.

Remark 15 (*Equalization of Marginal Revenue Products Does Not Ensure Cross-Sectional Factor Efficiency*)**.** In frictionless competitive economies, marginal revenue products are equalized across all uses and the cross-sectional factor efficiency component is zero. However, equalization of marginal revenue products is not sufficient to ensure that the cross-sectional factor efficiency component is zero in competitive economies with wedges, even when factor use wedges are zero. A similar logic applies to cross-sectional input efficiency. Why is this the case? As explained in Section 4, efficiency requires the equalization of marginal *welfare* products across uses of a factor, while competition when factor use wedges are zero ensures the equalization of marginal *revenue* products across uses. If $MSV_y^j \neq p^j$ for some goods that use a particular factor, the marginal welfare products of that factor won't be equalized across uses, allowing for cross-sectional factor efficiency to be non-zero. We illustrate this possibility in Example 4.

Example 4 (*Marginal Welfare Product vs. Marginal Revenue Product*)**.** We consider the same environment as in Example 2. All wedges are zero except $\tau_x^{23} \neq 0$. In this case, competition ensures that $MRS_c^{11} = p^1$ and $MRS_c^{12} = p^2$, as well as $p^1 \frac{\partial G^1}{\partial n^{11,d}} = w^1$ and $p^3 \frac{\partial G^3}{\partial n^{31,d}} = w^1$. The only equilibrium condition with a wedge is $p^2 \frac{\partial G^2}{\partial x^{23}} = \left(1 + \tau_x^{23}\right) p^3$. Consequently, competition implies that marginal revenue products are equalized across uses, so $MRP_n^{11} = MRP_n^{31}$. Therefore,

$$p^1 \frac{\partial G^1}{\partial n^{11,d}} = p^3 \frac{\partial G^3}{\partial n^{31,d}} \implies p^1 \frac{\partial G^1}{\partial n^{11,d}} = \frac{1}{1 + \tau_x^{23}} p^2 \frac{\partial G^2}{\partial x^{23}} \frac{\partial G^3}{\partial n^{31,d}}.$$

However, this condition is inconsistent with cross-sectional factor efficiency,

$$p^1 \frac{\partial G^1}{\partial n^{11,d}} = p^2 \frac{\partial G^2}{\partial x^{23}} \frac{\partial G^3}{\partial n^{31,d}},$$

which requires the equalization of marginal welfare products. This discrepancy is due to the fact that marginal social value of good 3 does not equal its price, since $\bar{\tau}_y^3 = \tau_x^{23} > 0$.

Theorem 3c (Exchange Efficiency in Competitive Economies)**.** *In competitive economies with wedges, exchange efficiency is given by*

$$\Xi^{AE,\,X} = \underbrace{\sum_j \mathbb{C}ov_i \left[\tau_c^{ij}, \frac{d\chi_c^{ij}}{d\theta}\right] p^j c^j}_{\substack{\text{Cross-Sectional} \\ \text{Consumption Efficiency}}} - \underbrace{\sum_f \mathbb{C}ov_i \left[\tau_n^{if,s}, \frac{d\chi_n^{if,s}}{d\theta}\right] w^f n^{f,s}}_{\substack{\text{Cross-Sectional} \\ \text{Factor Supply Efficiency}}}. \tag{44}$$

Equation (44) highlights that cross-sectional dispersion in consumption and factor supply wedges is necessary for exchange efficiency to be non-zero. Intuitively, reallocating consumption towards individuals with higher consumption wedges is valuable since these individuals consume too little in equilibrium. Similarly, reallocating factor supply towards individuals with lower factor supply wedges is valuable since these individuals' factor supply is too high in equilibrium. Finally, note that intermediate input wedges, factor use wedges, or the aggregate levels of consumption and factor supply wedges do not determine exchange efficiency directly.

6 Redistribution

Our analysis has so far focused on aggregate efficiency, which is invariant to the choice of a SWF, as explained in Section 2.3. However, two different perturbations with identical efficiency implications may have completely different distributional implications, as we explain next. Theorem 4a presents a decomposition of the redistribution component of the welfare accounting decomposition using the definitions of allocation shares for consumption and factor supply. Figure 5 illustrates this decomposition and is the counterpart to Figure 1.

Theorem 4a (General Redistribution Decomposition)**.** *The redistribution component of the welfare accounting decomposition, Ξ^{RD}, can be decomposed into*

$$\begin{aligned}\Xi^{RD} = & \overbrace{\sum_j \mathbb{C}ov_i^{\Sigma}\left[\omega^i, MRS_c^{ij}\frac{d\chi_c^{ij}}{d\theta}\right]c^j}^{\substack{\text{Cross-Sectional} \\ \text{Consumption Redistribution}}} + \overbrace{\sum_j \mathbb{C}ov_i^{\Sigma}\left[\omega^i, MRS_c^{ij}\chi_c^{ij}\right]\frac{dc^j}{d\theta}}^{\substack{\text{Aggregate} \\ \text{Consumption Redistribution}}} \\ & \underbrace{-\sum_f \mathbb{C}ov_i^{\Sigma}\left[\omega^i, MRS_n^{if}\frac{d\chi_n^{if,s}}{d\theta}\right]n^{f,s}}_{\substack{\text{Cross-Sectional} \\ \text{Factor Supply Redistribution}}} \underbrace{-\sum_f \mathbb{C}ov_i^{\Sigma}\left[\omega^i, MRS_n^{if}\chi_n^{if,s}\right]\frac{dn^{f,s}}{d\theta}}_{\substack{\text{Aggregate} \\ \text{Factor Supply Redistribution}}}.\end{aligned}$$

The cross-sectional terms capture redistribution gains or losses due to the reallocation of consumption and factor supply, for given c^j and $n^{f,s}$. In particular, cross-sectional consumption redistribution is positive for good j when individuals with high normalized individual weight ω^i—those relatively favored by the planner—see their consumption shares increase; MRS_c^{ij} captures potentially different marginal consumption values.[38] The aggregate terms capture redistribution gains due to changes in aggregates, for given allocation shares. In particular, aggregate consumption redistribution is positive for good j when aggregate consumption increases and individuals with high ω^i consume a relatively larger share of the good. The logic is parallel for factor supply redistribution.

The cross-sectional terms parallel exchange efficiency since they are driven by changes in consumption or factor supply shares given aggregates, while the aggregate terms parallel production efficiency since they are driven by changes in aggregates consumption and factor supply. While it is possible to further decompose the aggregate terms, this is not particularly useful. Instead, in

[38] In economies that satisfy exchange efficiency, Theorem 4a simplifies to

$$\sum_j AMRS_c^j(\mathbb{C}ov_i^{\Sigma}[\tilde{\omega}^i, \frac{d\chi_c^{ij}}{d\theta}]c^j + \mathbb{C}ov_i^{\Sigma}[\tilde{\omega}^i, \chi_c^{ij}]\frac{dc^j}{d\theta}) + \sum_f AMRS_n^f(\mathbb{C}ov_i^{\Sigma}[\tilde{\omega}^i, \frac{d\chi_n^{if,s}}{d\theta}]n^{f,s} + \mathbb{C}ov_i^{\Sigma}[\tilde{\omega}^i, \chi_n^{if,s}]\frac{dn^{f,s}}{d\theta})$$

Theorem 4b in Appendix C.3 we provide an alternative decomposition in competitive economies with wedges based on distributive pecuniary effects and individual distortions.

7 Welfare Accounting *vs.* Growth Accounting

Before concluding, we would like to discuss the relation between welfare accounting, as developed in this paper, and the well-established approach of growth accounting. Growth accounting measures the contribution of different inputs to final output (i.e. aggregate consumption), indirectly computing technological growth as a residual. Instead, welfare accounting attributes aggregate welfare gains to different sources, which brings it closer to the "beyond GDP" literature (Fleurbaey, 2009; Jones and Klenow, 2016).

Heuristically, the welfare accounting decomposition can be expressed as

$$\text{Welfare} = \text{Exchange Efficiency} + \underbrace{\text{Final Output} - \text{Factor Supply Cost}}_{\text{Production Efficiency}} + \text{Redistribution},$$

where the goal is to compute welfare changes by computing or measuring all right-hand side elements. Instead, growth accounting abstracts from exchange efficiency, factor supply costs, and redistribution, and exploits a relation of the form

$$\text{Final Output} = \text{Intermediate Inputs} + \text{Factors} + \text{Technology}, \tag{45}$$

where the goal is to measure both final output (left-hand side) and the intermediate input and factor components (part of the right-hand side) to back out the technology component. These are distinct exercises which are nonetheless related. For instance, when $I = 1$, exchange efficiency and redistribution are zero, and when factors are not supplied by individuals, the welfare cost of factor supply is also zero. In that case, welfare and final output are identical.

Moreover, when directly measuring the components of the welfare accounting decomposition, growth accounting can be used to measure technology growth. Through the lens of the welfare accounting decomposition, the adequate counterpart of the growth accounting relation in (45), solving for the technology growth component, is

$$\underbrace{\boldsymbol{MSV}_y \boldsymbol{G}_\theta}_{\text{Technology}} = \underbrace{\boldsymbol{AMRS}_c \frac{d\boldsymbol{c}}{d\theta}}_{\text{Final Output}} - \underbrace{\left(\boldsymbol{AMWP}_x - \boldsymbol{AMRS}_c\right) \frac{d\boldsymbol{\phi}_x}{d\theta} \boldsymbol{y}}_{\text{Intermediate Input Use}} - \underbrace{\boldsymbol{MWP}_n \frac{d\boldsymbol{n}^d}{d\theta}}_{\text{Factor Use}}, \tag{46}$$

where $\boldsymbol{AMRS}_c \frac{dc}{d\theta}$ becomes the welfare-relevant change in final output, which is a welfare-analog of GDP. Equation (46) is stated exclusively in terms of preferences and technologies. Additional assumptions about market structure would make it possible to conduct a growth accounting exercise by measuring all right-hand side components of (46), a task we leave for future work.

8 Application: Monetary Policy

In this section, we apply the welfare accounting decomposition to trace the welfare gains from optimal monetary stabilization policy. To that end, we develop a static, multi-sector heterogeneous-agent New Keynesian model with an input-output production network—a static "HANK-IO" model (Schaab and Tan, 2023). Our model builds on La'O and Tahbaz-Salehi (2022) and Rubbo (2023) but allows for household heterogeneity in addition to sectoral heterogeneity. The purpose of this application is to illustrate how to apply the welfare accounting decomposition to a disaggregated model of the macroeconomy.

Model. There are I (types of) households indexed by i. Each has mass μ^i, with $\sum_i \mu^i = 1$. There are N production sectors indexed by j. Each comprises a continuum of firms indexed by $\ell \in [0,1]$. Each firm produces a distinct good, indexed by $j\ell$.

The preferences of household i are given by

$$V_i = \frac{1}{1-\gamma}(c^i)^{1-\gamma} - \frac{1}{1+\varphi}(n^i)^{1+\varphi} \,, \qquad \text{where} \tag{47}$$

$$c^i = \left(\sum_j (\Gamma_c^{ij})^{\frac{1}{\eta_c}} (c^{ij})^{\frac{\eta_c - 1}{\eta_c}} \right)^{\frac{\eta_c}{\eta_c - 1}} \qquad \text{and} \qquad c^{ij} = \left(\int_0^1 (c^{ij\ell})^{\frac{\epsilon^j - 1}{\epsilon^j}} d\ell \right)^{\frac{\epsilon^j}{\epsilon^j - 1}},$$

where c^i denotes a final consumption aggregator, c^{ij} denotes a sectoral consumption aggregator, and $c^{ij\ell}$ is household i's consumption of good $j\ell$. Each household is endowed with a unique labor factor and n^i denotes hours of work. The household budget constraint is given by $\sum_j \int_0^1 p^{j\ell} c^{ij\ell} d\ell = W^i n^i + T^i$, where $p^{j\ell}$ is the price of good $j\ell$, W^i is the wage paid to factor i, and T^i is a lump-sum transfer that accounts for profits. Household optimization implies $(n^i)^\varphi (c^i)^\gamma = W^i / P^i$.

Firm ℓ in sector j produces according to the nested CES production function

$$y^{j\ell} = A^j \left((1-\vartheta^j)^{\frac{1}{\eta}} (n^{j\ell})^{\frac{\eta-1}{\eta}} + (\vartheta^j)^{\frac{1}{\eta}} (x^{j\ell})^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}, \quad \text{where} \quad n^{j\ell} = \left(\sum_i (\Gamma_w^{ji})^{\frac{1}{\eta_w}} (n^{j\ell i})^{\frac{\eta_w - 1}{\eta_w}} \right)^{\frac{\eta_w}{\eta_w - 1}}, \tag{48}$$

$$x^{j\ell} = \left(\sum_k (\Gamma_x^{jk})^{\frac{1}{\eta_x}} (x^{j\ell k})^{\frac{\eta_x - 1}{\eta_x}} \right)^{\frac{\eta_x}{\eta_x - 1}} \qquad \text{and} \qquad x^{j\ell k} = \left(\int_0^1 (x^{j\ell k\ell'})^{\frac{\epsilon^k - 1}{\epsilon^k}} d\ell' \right)^{\frac{\epsilon^k}{\epsilon^k - 1}}.$$

We denote by A^j a sector-specific, Hicks-neutral technology shifter, ϑ^j governs sector j's intermediate input share, and η is the elasticity of substitution between labor and inputs. Firm ℓ in sector j uses a bundle of labor $n^{j\ell}$ that is itself a CES aggregate of its use of labor factors i, $n^{j\ell i}$. It also uses a bundle of intermediate inputs $x^{j\ell}$, which is a CES aggregate of sectoral bundles $x^{j\ell k}$, where $x^{j\ell k\ell'}$ denotes firm $j\ell$'s use of good $k\ell'$ in production.

Firms are monopolistically competitive. They choose labor and inputs to minimize costs, and prices to maximize profits. Each firm ℓ is small and takes as given aggregate and sectoral variables. Profits are $\Pi^{j\ell} = (1-\tau^j) p^{j\ell} y^{j\ell} - \sum_k \int_0^1 p^{k\ell'} x^{j\ell k\ell'} d\ell' - \sum_i W^i n^{j\ell i} = (1-\tau^j) p^{j\ell} y^{j\ell} - mc^j y^{j\ell}$, where

τ^j is a revenue tax. Marginal cost mc^j is uniform across firms in each sector as we show in Appendix E.1. If prices are flexible, firms set prices as a markup over marginal cost, $p^{j\ell} = p^j = \frac{\epsilon^j}{\epsilon^j - 1}\frac{1}{1-\tau^j} mc^j$. To introduce nominal rigidities, we assume that only a fraction $\delta^j \in [0,1]$ of firms in sector j can reset their prices in response to a shock. Otherwise, prices remain fixed at some initial level $\bar{p}^j$, which we specify in the Appendix. The sectoral price distribution is thus given by

$$p^{j\ell} = \begin{cases} \frac{\epsilon^j}{\epsilon^j - 1}\frac{1}{1-\tau^j} mc^j & \text{for } \ell \in [0, \delta^j] \\ \bar{p}^j & \text{for } \ell \in (\delta^j, 1]. \end{cases} \tag{49}$$

We model monetary policy by assuming that aggregate nominal expenditures are constrained by a cash-in-advance constraint of the form $\sum_j \int_0^1 p^{j\ell} y^{j\ell} d\ell \leq M$, where M is the monetary policy instrument. Finally, the markets for goods and labor factors have to clear, requiring

$$y^{j\ell} = \sum_i \mu_i c^{ij\ell} + \sum_k \int_0^1 x^{k\ell' j\ell} d\ell' \quad \text{and} \quad \mu^i n^i = \sum_j \int_0^1 n^{j\ell i} d\ell. \tag{50}$$

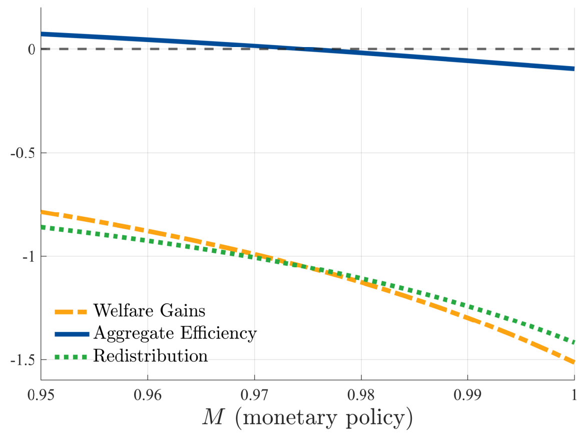
We formally define competitive equilibrium in Appendix E.2.

Calibration. We calibrate a model with $N = 66$ sectors and $I = 10$ household types, corresponding to deciles of the income distribution, as in Schaab and Tan (2023). We use data from the Consumer Expenditure Survey to calibrate Γ_c^{ij} so the model matches consumption expenditure shares. Similarly, we use data from the American Community Survey and the BEA's I-O and GDP tables to calibrate ϑ^j, Γ_x^{jk} and Γ_w^{ji} so the model matches sectoral input-output data and payroll shares. We calibrate ϵ^j to match sectoral markup data from Baqaee and Farhi (2020) and δ^j to match Pasten et al. (2017)'s data on sectoral price rigidities. We allow revenue taxes τ^j to offset initial markups and study the case with $\tau^j = 0$ in Appendix E.4. Finally, we assume an equal-weighted utilitarian SWF. Appendix E.3 presents a detailed discussion of our calibration.

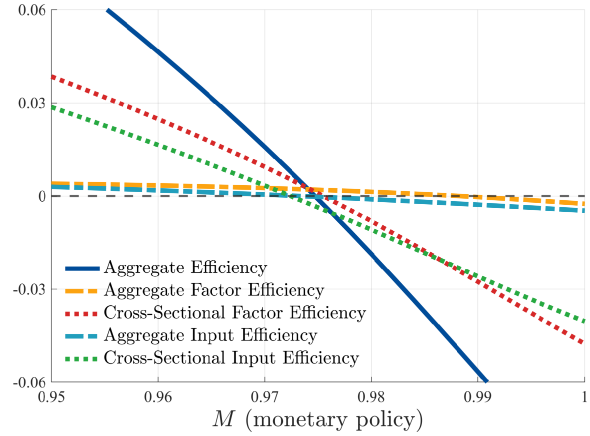
Results. We study monetary policy in response to a 2% technology shock that is uniform across sectors. When households and sectors are symmetric, Divine Coincidence holds and there exists an optimal monetary policy M^* that closes output and inflation gaps. Through the lens of the welfare accounting decomposition, Divine Coincidence implies that each allocative efficiency term of Theorems 1a and 1b is zero. We discuss this case in Appendix E.4.

When households and sectors are heterogeneous, Divine Coincidence fails. Figure 4 plots the welfare accounting decomposition, treating M as the perturbation parameter (θ).

Panel (4a) decomposes welfare gains (yellow) into gains from aggregate efficiency (blue) and redistribution (green). The blue line intersects 0 at around $M^{AE} = 0.974$, which is the policy that maximizes aggregate efficiency. Redistribution is negative at this point, indicating that the redistribution motive of the utilitarian SWF calls for a more contractionary policy (lower M).



(a) Welfare Gains



(b) Aggregate Efficiency Gain

Figure 4. Optimal Monetary Policy

Panel (4b) decomposes aggregate efficiency into its four allocative efficiency components: cross-sectional and aggregate factor and intermediate input efficiency. Several additional insights emerge. First, factor and input efficiency are both quantitatively important determinants of the production efficiency gains from monetary policy. Second, at $M^{AE} = 0.974$, aggregate (light blue) and cross-sectional (green) input efficiency are negative. These two motives call for more contractionary policy. Third, aggregate (yellow) and cross-sectional (red) factor efficiency are positive at $M^{AE} = 0.974$, calling for more expansionary policy. The policy that maximizes efficiency trades off and balances these considerations. Lastly, Appendix E.4 illustrates the role of revenue taxes. When they are not available to offset initial markup distortions, aggregate input and factor efficiency become quantitatively more important and call for expansionary policy.

9 Conclusion

This paper introduces a welfare accounting framework that applies to general economies with heterogeneous agents and disaggregated production technologies. The welfare accounting decomposition is useful to identify and quantify the ultimate origins of welfare gains and losses induced by changes in allocations or primitive changes in technologies or factor endowments. It is also useful to characterize efficiency conditions, which allows us to provide the first general characterization of efficiency conditions for disaggregated production economies. Our results underscore the importance of properly accounting for non-negativity constraints in feasible allocations, especially in the context of sparse production networks and diverse consumption patterns among individuals. By specializing our results to competitive economies, we show that prices and wedges can be used to recover the welfare-relevant statistics required to implement the welfare accounting decomposition. We illustrate the use of the welfare accounting decomposition across a range of minimal examples and in a rich application to monetary stabilization policy.

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Online Appendix

A Matrix Definitions

This section defines all matrices used in the body of the paper and in this Appendix. To simplify the exposition, we represent all matrices for the $I = 2$, $J = 3$, $F = 2$ case, although we define matrix dimensions for the general case.

Allocations. We collect output and aggregate consumption of all goods, y^j and c^j, in $J \times 1$ vectors $\boldsymbol{y}$ and $\boldsymbol{c}$ and $J \times J$ diagonal matrices $\hat{\boldsymbol{y}}$ and $\hat{\boldsymbol{c}}$, as follows:

$$\boldsymbol{y} = \begin{pmatrix} y^1 \\ y^2 \\ y^3 \end{pmatrix}_{J\times 1}, \quad \boldsymbol{c} = \begin{pmatrix} c^1 \\ c^2 \\ c^3 \end{pmatrix}_{J\times 1}, \quad \hat{\boldsymbol{y}} = \begin{pmatrix} y^1 & 0 & 0 \\ 0 & y^2 & 0 \\ 0 & 0 & y^3 \end{pmatrix}_{J\times J}, \quad \hat{\boldsymbol{c}} = \begin{pmatrix} c^1 & 0 & 0 \\ 0 & c^2 & 0 \\ 0 & 0 & c^3 \end{pmatrix}_{J\times J}.$$

We also define $J \times J$ diagonal matrices of aggregate consumption and intermediate shares, $\boldsymbol{\phi}_c$ and $\boldsymbol{\phi}_x$, as follows:

$$\boldsymbol{\phi}_c = \begin{pmatrix} \phi_c^1 & 0 & 0 \\ 0 & \phi_c^2 & 0 \\ 0 & 0 & \phi_c^3 \end{pmatrix}_{J\times J}, \quad \boldsymbol{\phi}_x = \begin{pmatrix} \phi_x^1 & 0 & 0 \\ 0 & \phi_x^2 & 0 \\ 0 & 0 & \phi_x^3 \end{pmatrix}_{J\times J}.$$

We collect intermediate-output shares $\xi^{jk} = \frac{x^{jk}}{y^k}$ in the $JK \times J$ matrix $\boldsymbol{\xi}$, and intermediate uses in a $JK \times 1$ vector $\boldsymbol{x}$ and in a $JK \times J$ matrix $\check{\boldsymbol{x}}$, as follows:

$$\boldsymbol{\xi} = \begin{pmatrix} \xi^{11} & 0 & 0 \\ \xi^{21} & 0 & 0 \\ \xi^{31} & 0 & 0 \\ 0 & \xi^{12} & 0 \\ 0 & \xi^{22} & 0 \\ 0 & \xi^{32} & 0 \\ 0 & 0 & \xi^{13} \\ 0 & 0 & \xi^{23} \\ 0 & 0 & \xi^{33} \end{pmatrix}_{JK\times J}, \quad \boldsymbol{x} = \begin{pmatrix} x^{11} \\ x^{21} \\ x^{31} \\ x^{12} \\ x^{22} \\ x^{32} \\ x^{13} \\ x^{23} \\ x^{33} \end{pmatrix}_{JK\times 1}, \quad \check{\boldsymbol{x}} = \begin{pmatrix} x^{11} & 0 & 0 \\ x^{21} & 0 & 0 \\ x^{31} & 0 & 0 \\ 0 & x^{12} & 0 \\ 0 & x^{22} & 0 \\ 0 & x^{32} & 0 \\ 0 & 0 & x^{13} \\ 0 & 0 & x^{23} \\ 0 & 0 & x^{33} \end{pmatrix}_{JK\times J}.$$

We define the $J \times 1$ vector of aggregate intermediate use as $\bar{\boldsymbol{x}} = \mathbf{1}_x \boldsymbol{x}$, and in the form of a $J \times J$ diagonal matrix as $\hat{\bar{\boldsymbol{x}}} = \mathbf{1}_x \check{\boldsymbol{x}}$. Note that $\boldsymbol{\phi}_c \boldsymbol{y} = \boldsymbol{c}$ and $\boldsymbol{\phi}_x \boldsymbol{y} = \bar{\boldsymbol{x}}$. We collect factor demands in a $JK \times 1$ vector $\boldsymbol{n}^d$, and aggregate factor demand, aggregate factor supply, and factor endowments in

$F \times 1$ vectors, $\boldsymbol{n}^{f,d}$, $\boldsymbol{n}^{f,s}$, and $\bar{\boldsymbol{n}}^{f,s}$, as follows:

$$\boldsymbol{n}^d = \begin{pmatrix} n^{11,d} \\ n^{21,d} \\ n^{31,d} \\ n^{12,d} \\ n^{22,d} \\ n^{32,d} \end{pmatrix}_{JF\times 1}, \quad \boldsymbol{n}^{f,d} = \begin{pmatrix} n^{1,d} \\ n^{2,d} \end{pmatrix}_{F\times 1}, \quad \boldsymbol{n}^{f,s} = \begin{pmatrix} n^{1,s} \\ n^{2,s} \end{pmatrix}_{F\times 1}, \quad \bar{\boldsymbol{n}}^{f,s} = \begin{pmatrix} \bar{n}^{1,s} \\ \bar{n}^{2,s} \end{pmatrix}_{F\times 1}.$$

Marginal products/technology change. We collect marginal products of intermediates in a $J \times JK$ matrix $\boldsymbol{G}_x$, marginal products of factors in a $J \times JF$ matrix $\boldsymbol{G}_n$, and technology changes in a $J \times 1$ vector $\boldsymbol{G}_\theta$, as follows:

$$\boldsymbol{G}_x = \begin{pmatrix} \frac{\partial G^1}{\partial x^{11}} & 0 & 0 & \frac{\partial G^1}{\partial x^{12}} & 0 & 0 & \frac{\partial G^1}{\partial x^{13}} & 0 & 0 \\ 0 & \frac{\partial G^2}{\partial x^{21}} & 0 & 0 & \frac{\partial G^2}{\partial x^{22}} & 0 & 0 & \frac{\partial G^2}{\partial x^{23}} & 0 \\ 0 & 0 & \frac{\partial G^3}{\partial x^{31}} & 0 & 0 & \frac{\partial G^3}{\partial x^{32}} & 0 & 0 & \frac{\partial G^3}{\partial x^{33}} \end{pmatrix}_{J\times JK}$$

$$\boldsymbol{G}_n = \begin{pmatrix} \frac{\partial G^1}{\partial n^{11,d}} & 0 & 0 & \frac{\partial G^1}{\partial n^{12,d}} & 0 & 0 \\ 0 & \frac{\partial G^2}{\partial n^{21,d}} & 0 & 0 & \frac{\partial G^2}{\partial n^{22,d}} & 0 \\ 0 & 0 & \frac{\partial G^3}{\partial n^{31,d}} & 0 & 0 & \frac{\partial G^3}{\partial n^{32,d}} \end{pmatrix}_{J\times JF}, \quad \boldsymbol{G}_\theta = \begin{pmatrix} \frac{\partial G^1}{\partial \theta} \\ \frac{\partial G^2}{\partial \theta} \\ \frac{\partial G^3}{\partial \theta} \end{pmatrix}_{J\times 1}.$$

Marginal rates of substitution. We collect marginal rates of substitution in $1 \times IJ$ and $1 \times IF$ vectors $\boldsymbol{MRS}_c$ and $\boldsymbol{MRS}_n$, as follows:

$$\boldsymbol{MRS}_c = \begin{pmatrix} MRS_c^{11} & MRS_c^{21} & MRS_c^{12} & MRS_c^{22} & MRS_c^{13} & MRS_c^{23} \end{pmatrix}_{1\times IJ}$$

$$\boldsymbol{MRS}_n = \begin{pmatrix} MRS_n^{11} & MRS_n^{21} & MRS_n^{12} & MRS_n^{22} \end{pmatrix}_{1\times IF}.$$

Matrices of zero and ones. We define several matrices with zeros and ones. First, we denote identity matrices of dimension J, JK, and F by $\boldsymbol{I}_J$, $\boldsymbol{I}_{JK}$, and $\boldsymbol{I}_F$, respectively. We denote a $1 \times J$ vector of ones by ι_J. We define a $J \times JK$ matrix $\mathbf{1}_x$, a $F \times JF$ matrix $\mathbf{1}_{n^d}$, a $J \times IJ$ matrix $\mathbf{1}_c$, and $F \times IF$ matrix $\mathbf{1}_{n^s}$ as

$$\mathbf{1}_x = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}_{J\times JK}, \quad \mathbf{1}_{n^d} = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}_{F\times JF}$$

$$\mathbf{1}_c = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}_{J\times IJ}, \quad \mathbf{1}_{n^s} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}_{F\times IF}$$

We also define a $J \times JK$ matrix $\mathbb{I}_x$ as

$$\mathbb{I}_x = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}_{J \times JK}.$$

Finally, we use $\boldsymbol{I}_c$ and $\boldsymbol{I}_{n^s}$ to respectively denote $J \times J$ and $F \times F$ indicator matrices, as follows:

$$\boldsymbol{I}_c = \begin{pmatrix} 1\left[c^1 > 0\right] & 0 & 0 \\ 0 & 1\left[c^2 > 0\right] & 0 \\ 0 & 0 & 1\left[c^3 > 0\right] \end{pmatrix}_{J \times J}, \quad \boldsymbol{I}_{n^s} = \begin{pmatrix} 1\left[n^{1,s} > 0\right] & 0 \\ 0 & 1\left[n^{2,s} > 0\right] \end{pmatrix}.$$

Allocation shares. We collect allocations shares for intermediate uses in a $JK \times J$ matrix $\boldsymbol{\chi}_x$, for factor demands in a $JF \times F$ matrix $\boldsymbol{\chi}_{n^d}$, for aggregate consumption in a $IJ \times J$ matrix $\boldsymbol{\chi}_c$, and for factor supplies in a $IF \times F$ matrix $\boldsymbol{\chi}_{n^s}$, as follows:

$$\boldsymbol{\chi}_x = \begin{pmatrix} \chi_x^{11} & 0 & 0 \\ \chi_x^{21} & 0 & 0 \\ \chi_x^{31} & 0 & 0 \\ 0 & \chi_x^{12} & 0 \\ 0 & \chi_x^{22} & 0 \\ 0 & \chi_x^{32} & 0 \\ 0 & 0 & \chi_x^{13} \\ 0 & 0 & \chi_x^{23} \\ 0 & 0 & \chi_x^{33} \end{pmatrix}_{JK \times J}, \quad \boldsymbol{\chi}_{n^d} = \begin{pmatrix} \chi_n^{11,d} & 0 \\ \chi_n^{21,d} & 0 \\ \chi_n^{31,d} & 0 \\ 0 & \chi_n^{12,d} \\ 0 & \chi_n^{22,d} \\ 0 & \chi_n^{32,d} \end{pmatrix}_{JF \times F}, \quad \boldsymbol{\chi}_c = \begin{pmatrix} \chi_c^{11} & 0 & 0 \\ \chi_c^{21} & 0 & 0 \\ 0 & \chi_c^{12} & 0 \\ 0 & \chi_c^{22} & 0 \\ 0 & 0 & \chi_c^{13} \\ 0 & 0 & \chi_c^{23} \end{pmatrix}_{IJ \times J}$$

$$\boldsymbol{\chi}_{n^s} = \begin{pmatrix} \chi_n^{11,s} & 0 \\ \chi_n^{21,s} & 0 \\ 0 & \chi_n^{12,s} \\ 0 & \chi_n^{22,s} \end{pmatrix}_{IF \times F}.$$

Given allocation shares, aggregate marginal rates of substitution can be written as $\boldsymbol{AMRS}_c = \boldsymbol{MRS}_c \boldsymbol{\chi}_c$ and $\boldsymbol{AMRS}_n = \boldsymbol{MRS}_n \boldsymbol{\chi}_{n^s}$, and aggregate marginal welfare products as $\boldsymbol{AMWP}_x = \boldsymbol{MWP}_x \boldsymbol{\chi}_x$ and $\boldsymbol{AMWP}_n = \boldsymbol{MWP}_n \boldsymbol{\chi}_{n^d}$.

Marginal social value of output. The marginal social value of output is given by a $1 \times J$ vector $\boldsymbol{MSV}_y$, as follows:

$$\boldsymbol{MSV}_y = \begin{pmatrix} MSV_y^1 & MSV_y^2 & MSV_y^3 \end{pmatrix}_{1 \times J}$$

Output inverse matrix. We define the elements of the output inverse $\boldsymbol{\Psi}_y$ as follows:

$$\boldsymbol{\Psi}_y = \begin{pmatrix} \psi_y^{11} & \psi_y^{12} & \psi_y^{13} \\ \psi_y^{21} & \psi_y^{22} & \psi_y^{23} \\ \psi_y^{31} & \psi_y^{32} & \psi_y^{33} \end{pmatrix}_{J\times J}.$$

Competitive economies. In competitive economies, we define a $1 \times J$ vector of prices $\boldsymbol{p}$, a $J \times J$ matrix of prices $\hat{\boldsymbol{p}}$, and $1 \times F$ vector of wages $\boldsymbol{w}$ as

$$\boldsymbol{p} = \begin{pmatrix} p^1 & p^2 & p^3 \end{pmatrix}_{1\times J}, \qquad \hat{\boldsymbol{p}} = \begin{pmatrix} p^1 & 0 & 0 \\ 0 & p^2 & 0 \\ 0 & 0 & p^3 \end{pmatrix}_{J\times J}, \qquad \boldsymbol{w} = \begin{pmatrix} w^1 & w^2 \end{pmatrix}_{1\times F}$$

We also define a $JK \times JK$ matrix of prices as $\check{\boldsymbol{p}} = \hat{\boldsymbol{p}} \otimes \boldsymbol{I}_J$. Finally, we also define a $J \times JK$ vector of intermediate use wedges, a $F \times JF$ vector of factor demand wedges, a $J \times IJ$ vector of consumption wedges, an a $F \times IF$ vector of factor supply wedges, as follows:

$$\boldsymbol{\tau}_x = \begin{pmatrix} \frac{\tau_x^{11}-\tau_y^1}{1+\tau_y^1} & \frac{\tau_x^{21}-\tau_y^2}{1+\tau_y^2} & \frac{\tau_x^{31}-\tau_y^3}{1+\tau_y^3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\tau_x^{12}-\tau_y^1}{1+\tau_y^1} & \frac{\tau_x^{22}-\tau_y^2}{1+\tau_y^2} & \frac{\tau_x^{32}-\tau_y^3}{1+\tau_y^3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\tau_x^{13}-\tau_y^1}{1+\tau_y^1} & \frac{\tau_x^{23}-\tau_y^2}{1+\tau_y^2} & \frac{\tau_x^{33}-\tau_y^3}{1+\tau_y^3} \end{pmatrix}_{J\times JK},$$

$$\boldsymbol{\tau}_{n^d} = \begin{pmatrix} \frac{\tau_n^{11,d}-\tau_y^1}{1+\tau_y^1} & \frac{\tau_n^{12,d}-\tau_y^1}{1+\tau_y^1} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\tau_n^{21,d}-\tau_y^2}{1+\tau_y^2} & \frac{\tau_n^{22,d}-\tau_y^2}{1+\tau_y^2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\tau_n^{31,d}-\tau_y^3}{1+\tau_y^3} & \frac{\tau_n^{32,d}-\tau_y^3}{1+\tau_y^3} \end{pmatrix}_{F\times JF}$$

$$\boldsymbol{\tau}_c = \begin{pmatrix} \tau_c^{11} & \tau_c^{21} & 0 & 0 & 0 & 0 \\ 0 & 0 & \tau_c^{12} & \tau_c^{22} & 0 & 0 \\ 0 & 0 & 0 & 0 & \tau_c^{13} & \tau_c^{23} \end{pmatrix}_{J\times IJ}, \qquad \boldsymbol{\tau}_{n^s} = \begin{pmatrix} \tau_n^{11,s} & \tau_n^{21,s} & 0 & 0 \\ 0 & 0 & \tau_n^{12,s} & \tau_n^{22,s} \end{pmatrix}_{F\times IF}$$

B Proofs and Derivations

Proof of Lemma 1. (Welfare Decomposition: Aggregate Efficiency vs. Redistribution)

Proof. For any welfarist planner with Social Welfare Function $\mathcal{W}(V_1, \ldots, V_I)$, we can express $\frac{dW}{d\theta}$ as

$$\frac{dW}{d\theta} = \sum_i \frac{\partial \mathcal{W}}{\partial V_i} \frac{dV_i}{d\theta} = \sum_i \alpha^i \lambda^i \frac{\frac{dV_i}{d\theta}}{\lambda_i},$$

where $\alpha^i = \frac{\partial \mathcal{W}}{\partial V_i}$ and where λ^i is an individual normalizing factor with units $\dim(\lambda^i) = \frac{\text{utils of individual } i}{\text{units of numeraire}}$ that allows us to express individual welfare assessments into a common unit/numeraire. We can therefore write

$$\frac{dW^{\lambda}}{d\theta}=\frac{\frac{dW}{d\theta}}{\frac{\sum_i \alpha^i\lambda^i}{I}}=\sum_i \omega^i\frac{\frac{dV_i}{d\theta}}{\lambda_i}=\underbrace{\frac{\sum_i \omega^i}{I}}_{=1}\sum_i\frac{\frac{dV_i}{d\theta}}{\lambda_i}+I\mathbb{C}ov_i\left[\omega^i,\frac{\frac{dV_i}{d\theta}}{\lambda_i}\right]=\underbrace{\sum_i\frac{\frac{dV_i}{d\theta}}{\lambda_i}}_{=\Xi^{AE}}+\underbrace{\mathbb{C}ov_i^{\Sigma}\left[\omega^i,\frac{\frac{dV_i}{d\theta}}{\lambda_i}\right]}_{=\Xi^{RD}},$$

where $\omega^i=\frac{\alpha^i\lambda^i}{\frac{\sum_i\alpha^i\lambda^i}{I}}$, which implies that $\frac{\sum_i\omega^i}{I}=1$. ∎

Proof of Theorem 1a. (Exchange Efficiency)

Proof. Given the definition of V_i in equation (1), we can express $\frac{\frac{dV_i}{d\theta}}{\lambda_i}$ as

$$\frac{\frac{dV_i}{d\theta}}{\lambda_i}=\sum_j\frac{\frac{\partial u_i}{\partial c^{ij}}}{\lambda_i}\frac{dc^{ij}}{d\theta}+\sum_f\frac{\frac{\partial u_i}{\partial n^{if,s}}}{\lambda_i}\frac{dn^{if,s}}{d\theta}=\sum_j MRS_c^{ij}\frac{dc^{ij}}{d\theta}-\sum_f MRS_n^{if}\frac{dn^{if,s}}{d\theta}.$$

Hence, from Lemma 1, it follows that

$$\Xi^{AE}=\sum_i\frac{\frac{dV_i}{d\theta}}{\lambda_i}=\sum_j\sum_i MRS_c^{ij}\frac{dc^{ij}}{d\theta}-\sum_f\sum_i MRS_n^{if}\frac{dn^{if,s}}{d\theta}.$$

Given our definitions of χ_c^{ij} and $\chi_n^{if,s}$, we can write

$$\sum_i MRS_c^{ij}\frac{dc^{ij}}{d\theta}=\mathbb{C}ov_i^{\Sigma}\left[MRS_c^{ij},\frac{d\chi_c^{ij}}{d\theta}\right]c^j+AMRS_c^j\frac{dc^j}{d\theta},$$

where $AMRS_c^j$ is defined in (19). Similarly, we can write

$$\sum_i MRS_n^{if,s}\frac{dn^{if,s}}{d\theta}=\mathbb{C}ov_i^{\Sigma}\left[MRS_n^{if,s},\frac{d\chi_n^{if,s}}{d\theta}\right]n^{f,s}+AMRS_n^f\frac{dn^{f,s}}{d\theta},$$

where $AMRS_n^f$ is also defined in (19). Hence, exchange efficiency, $\Xi^{AE,X}$, can be expressed as

$$\Xi^{AE,I}=\underbrace{\mathbb{C}ov_i^{\Sigma}\left[MRS_c^{ij},\frac{d\chi_c^{ij}}{d\theta}\right]c^j}_{\substack{\text{Cross-Sectional}\\\text{Consumption Efficiency}}}\underbrace{-\mathbb{C}ov_i^{\Sigma}\left[MRS_n^{if,s},\frac{d\chi_n^{if,s}}{d\theta}\right]n^{f,s}}_{\substack{\text{Cross-Sectional}\\\text{Factor Supply Efficiency}}},$$

while production efficiency corresponds to

$$\Xi^{AE,P}=\sum_j AMRS_c^j\frac{dc^j}{d\theta}-\sum_f AMRS_n^f\frac{dn^{f,s}}{d\theta}.$$

∎

Proof of Corollary 2. (Properties of Exchange Efficiency Decomposition)

Proof. a) When $I = 1$, $\mathbb{C}ov_i^{\Sigma}\left[MRS_c^{ij}, \frac{dc^{ij}}{d\theta}\right] = \mathbb{C}ov_i^{\Sigma}\left[MRS_n^{if,s}, \frac{dn^{if,s}}{d\theta}\right] = 0$ for all j and f. b) When $n^{f,s} = 0$, $\mathbb{C}ov_i^{\Sigma}\left[MRS_n^{if,s}, \frac{d\chi_n^{if,s}}{d\theta}\right] n^{f,s} = 0$ for all f. c) When MRS_c^{ij} is identical for all i, $\mathbb{C}ov_i^{\Sigma}\left[MRS_c^{ij}, \frac{dc^{ij}}{d\theta}\right] = 0$. When MRS_n^{if} is identical for all f, $\mathbb{C}ov_i^{\Sigma}\left[MRS_n^{if}, \frac{dn^{if,s}}{d\theta}\right] = 0$. ■

Proof of Lemma 3. (Output Inverse Matrix)

Proof. Given (13) and (16) we can write $\frac{dy^j}{d\theta}$ and $\frac{dx^{jk}}{d\theta}$ in matrix form, as

$$\frac{d\boldsymbol{y}}{d\theta} = \boldsymbol{G}_x \frac{d\boldsymbol{x}}{d\theta} + \boldsymbol{G}_n \frac{d\boldsymbol{n}^d}{d\theta} + \boldsymbol{G}_\theta \quad \text{and} \quad \frac{d\boldsymbol{x}}{d\theta} = \frac{d\boldsymbol{\xi}}{d\theta}\boldsymbol{y} + \boldsymbol{\xi}\frac{d\boldsymbol{y}}{d\theta}. \tag{51}$$

Combining both expressions, we find that

$$\frac{d\boldsymbol{y}}{d\theta} = \boldsymbol{G}_x\left(\frac{d\boldsymbol{\xi}}{d\theta}\boldsymbol{y} + \boldsymbol{\xi}\frac{d\boldsymbol{y}}{d\theta}\right) + \boldsymbol{G}_n\frac{d\boldsymbol{n}^d}{d\theta} + \boldsymbol{G}_\theta = (\boldsymbol{I}_J - \boldsymbol{G}_x\boldsymbol{\xi})^{-1}\left(\boldsymbol{G}_x\frac{d\boldsymbol{\xi}}{d\theta}\boldsymbol{y} + \boldsymbol{G}_n\frac{d\boldsymbol{n}^d}{d\theta} + \boldsymbol{G}_\theta\right),$$

where $\boldsymbol{\Psi}_y = (\boldsymbol{I}_J - \boldsymbol{G}_x\boldsymbol{\xi})^{-1}$, which exactly corresponds to equation (17) in the text. ■

Proof of Theorem 1b. (Production Efficiency)

Proof. Note that we can express $\Xi^{AE,\,P}$ in matrix form as

$$\Xi^{AE,\,P} = \boldsymbol{AMRS}_c\frac{d\boldsymbol{c}}{d\theta} - \boldsymbol{AMRS}_n\frac{d\boldsymbol{n}^{f,s}}{d\theta}.$$

Using the resource constraints for goods and the production function we can express changes in aggregate consumption as

$$\begin{aligned}\frac{d\boldsymbol{c}}{d\theta} &= \frac{d\boldsymbol{y}}{d\theta} - \boldsymbol{1}_x\frac{d\boldsymbol{x}}{d\theta} = \frac{d\boldsymbol{y}}{d\theta} - \boldsymbol{1}_x\left(\boldsymbol{\xi}\frac{d\boldsymbol{y}}{d\theta} + \frac{d\boldsymbol{\xi}}{d\theta}\boldsymbol{y}\right) = \boldsymbol{\phi}_c\frac{d\boldsymbol{y}}{d\theta} - \boldsymbol{1}_x\frac{d\boldsymbol{\xi}}{d\theta}\boldsymbol{y}\\ &= \boldsymbol{\phi}_c\boldsymbol{\Psi}_y\left(\boldsymbol{G}_x\frac{d\boldsymbol{\xi}}{d\theta}\boldsymbol{y} + \boldsymbol{G}_n\frac{d\boldsymbol{n}^d}{d\theta} + \boldsymbol{G}_\theta\right) - \boldsymbol{1}_x\frac{d\boldsymbol{\xi}}{d\theta}\boldsymbol{y} = (\boldsymbol{\phi}_c\boldsymbol{\Psi}_y\boldsymbol{G}_x - \boldsymbol{1}_x)\frac{d\boldsymbol{\xi}}{d\theta}\boldsymbol{y} + \boldsymbol{\phi}_c\boldsymbol{\Psi}_y\boldsymbol{G}_n\frac{d\boldsymbol{n}^d}{d\theta} + \boldsymbol{\phi}_c\boldsymbol{\Psi}_y\boldsymbol{G}_\theta,\end{aligned}$$

where we use the fact that $\boldsymbol{\phi}_x = \boldsymbol{1}_x\boldsymbol{\xi}$ and $\boldsymbol{\phi}_c = \boldsymbol{I}_J - \boldsymbol{\phi}_x$ Hence, combining the above expression with the expression for $\Xi^{AE,\,P}$, and using the resource constraint for factors, which implies that $\frac{d\boldsymbol{n}^{f,s}}{d\theta} = \boldsymbol{1}_n\frac{d\boldsymbol{n}^d}{d\theta} - \frac{d\bar{\boldsymbol{n}}^{f,s}}{d\theta}$, we find that

$$\begin{aligned}\Xi^{AE,\,P} = &(\boldsymbol{MWP}_x - \boldsymbol{AMRS}_c\boldsymbol{1}_x)\frac{d\boldsymbol{\xi}}{d\theta}\boldsymbol{y} + (\boldsymbol{MWP}_n - \boldsymbol{AMRS}_n\boldsymbol{1}_n)\frac{d\boldsymbol{n}^d}{d\theta} +\\ &\boldsymbol{MSV}_y\boldsymbol{G}_\theta + \boldsymbol{AMRS}_n\frac{d\bar{\boldsymbol{n}}^{f,s}}{d\theta},\end{aligned}$$

where we define $\boldsymbol{MWP}_x = \boldsymbol{MSV}_y\boldsymbol{G}_x$, $\boldsymbol{MWP}_n = \boldsymbol{MSV}_y\boldsymbol{G}_n$, and $\boldsymbol{MSV}_y = \boldsymbol{AMRS}_c\boldsymbol{\phi}_c\boldsymbol{\Psi}_y$.

For intermediate input efficiency,

$$(\boldsymbol{MWP}_x - \boldsymbol{AMRS}_c \mathbf{1}_x)\frac{d\boldsymbol{\xi}}{d\theta}\boldsymbol{y} = (\boldsymbol{MWP}_x - \boldsymbol{AMRS}_c \mathbf{1}_x)\left(\frac{d\boldsymbol{\chi}_x}{d\theta}\boldsymbol{\phi}_x + \boldsymbol{\chi}_x \frac{d\boldsymbol{\phi}_x}{d\theta}\right)\boldsymbol{y}$$

$$= \underbrace{\boldsymbol{MWP}_x \frac{d\boldsymbol{\chi}_x}{d\theta}\check{\boldsymbol{x}}}_{\substack{\text{Cross-Sectional} \\ \text{Intermediate Input Efficiency}}} + \underbrace{(\boldsymbol{AMWP}_x - \boldsymbol{AMRS}_c)\frac{d\boldsymbol{\phi}_x}{d\theta}\boldsymbol{y}}_{\substack{\text{Aggregate} \\ \text{Intermediate Input Efficiency}}}$$

where we use the fact that $\mathbf{1}_x \frac{d\boldsymbol{\chi}_x}{d\theta} = \mathbf{0}_{J\times J}$ and $\mathbf{1}_x \boldsymbol{\chi}_x = \boldsymbol{I}_J$.

For factor input efficiency,

$$(\boldsymbol{MWP}_n - \boldsymbol{AMRS}_n \mathbf{1}_n)\frac{d\boldsymbol{n}^d}{d\theta}$$

$$+\boldsymbol{AMRS}_n \frac{d\bar{\boldsymbol{n}}^{f,s}}{d\theta} = (\boldsymbol{MWP}_n - \boldsymbol{AMRS}_n \mathbf{1}_n)\left(\frac{d\boldsymbol{\chi}_{n^d}}{d\theta}\boldsymbol{n}^{f,d} + \boldsymbol{\chi}_{n^d}\frac{d\boldsymbol{n}^{f,d}}{d\theta}\right) + \boldsymbol{AMRS}_n \frac{d\bar{\boldsymbol{n}}^{f,s}}{d\theta}$$

$$= \underbrace{\boldsymbol{MWP}_n \frac{d\boldsymbol{\chi}_{n^d}}{d\theta}\boldsymbol{n}^{f,d}}_{\substack{\text{Cross-Sectional} \\ \text{Factor Efficiency}}} + \underbrace{(\boldsymbol{AMWP}_n - \boldsymbol{AMRS}_n)\frac{d\boldsymbol{n}^{f,s}}{d\theta}}_{\substack{\text{Aggregate} \\ \text{Factor Efficiency}}} + \underbrace{\boldsymbol{AMWP}_n \frac{d\bar{\boldsymbol{n}}^{f,s}}{d\theta}}_{\substack{\text{Factor Endowment} \\ \text{Growth}}}.$$

where we use the fact that $\mathbf{1}_n \frac{d\boldsymbol{\chi}_{n^d}}{d\theta} = \mathbf{0}_{F\times F}$ and $\mathbf{1}_n \boldsymbol{\chi}_{n^d} = \boldsymbol{I}_F$. So in matrix form

$$\Xi^{AE,P} = \underbrace{\boldsymbol{MWP}_x \frac{d\boldsymbol{\chi}_x}{d\theta}\check{\boldsymbol{x}}}_{\substack{\text{Cross-Sectional} \\ \text{Intermediate Input Efficiency}}} + \underbrace{(\boldsymbol{AMWP}_x - \boldsymbol{AMRS}_c)\frac{d\boldsymbol{\phi}_x}{d\theta}\boldsymbol{y}}_{\substack{\text{Aggregate} \\ \text{Intermediate Input Efficiency}}}$$

$$+ \underbrace{\boldsymbol{MWP}_n \frac{d\boldsymbol{\chi}_{n^d}}{d\theta}\boldsymbol{n}^{f,d}}_{\substack{\text{Cross-Sectional} \\ \text{Factor Efficiency}}} + \underbrace{(\boldsymbol{AMWP}_n - \boldsymbol{AMRS}_n)\frac{d\boldsymbol{n}^{f,s}}{d\theta}}_{\substack{\text{Aggregate} \\ \text{Factor Efficiency}}} + \underbrace{\boldsymbol{MSV}_y \boldsymbol{G}_\theta}_{\substack{\text{Technology} \\ \text{Growth}}} + \underbrace{\boldsymbol{AMWP}_n \frac{d\bar{\boldsymbol{n}}^{f,s}}{d\theta}}_{\substack{\text{Factor Endowment} \\ \text{Growth}}},$$

which exactly corresponds to the same equation as in the text when expressed in sum form. ■

Proof of Corollary 4. (Properties of Production Efficiency Decomposition)

Proof. a) When $J = 1$, $\mathbb{C}ov_j^{\Sigma}[\cdot,\cdot] = 0$.b) In this case, $x^k = \frac{d\phi_x^k}{d\theta} = 0, \forall k$. c) In this case, $\frac{dn^{f,s}}{d\theta} = 0, \forall s$. d) In this case, $\frac{d\chi_x^{jk}}{d\theta} = 0, \forall j,k$, or $\frac{d\chi_n^{jf,d}}{d\theta} = 0, \forall j,f$. e) When marginal welfare products are equalized for intermediates: $\mathbb{C}ov_j^{\Sigma}\left[MWP_x^{jk}, \frac{d\chi_x^{jk}}{d\theta}\right] = 0$; for factors: $\mathbb{C}ov_j^{\Sigma}\left[MWP_n^{jf}, \frac{d\chi_n^{jf,d}}{d\theta}\right] = 0$. ■

Proof of Theorem 2a. (Efficiency Conditions: Exchange Efficiency)

Proof. If MRS_c^{ij} is different across any two individuals with $\chi_c^{ij} > 0$ for good j with $c^j > 0$, then there exists a perturbation of consumption shares in which cross-sectional consumption efficiency is positive. If MRS_c^{ij} is less than $AMRS_c^j$ when $\chi_c^{ij} = 0$, then there is no feasible perturbation that reduces the share of consumption for individual i. The exact same logic applies to cross-sectional factor supply efficiency. ■

Proof of Theorem 2b. (Efficiency Conditions: Production Efficiency)

Proof. If MWP_x^{jk} is different across any two intermediate uses of good k two individuals with $\chi_x^{jk} > 0$, then there exists a perturbation of intermediate use shares in which cross-sectional intermediate input efficiency is positive. The same logic applies to cross-sectional factor use efficiency.

When $\phi_x^k \in (0,1)$, then there exists a perturbation of ϕ_x^k such that aggregate intermediate input efficiency is positive unless $AMWP_x^k = AMRS_c^k$. If $\phi_x^k = 0$, it must be that $AMWP_x^k \leq AMRS_c^k$ for the best possible combination of intermediate use shares, which is the one that allocates good k to its highest marginal welfare product intermediate use. If $\phi_x^k = 1$, it must be that $AMWP_x^k \geq AMRS_c^k$ for the possible combinations of consumption shares, which is the one that allocates the consumption of good j to the individual with the highest MRS_c^{ik}.

When $n^{f,s} > 0$ (and $n^{f,d} > 0$), then there exists a perturbation of $n^{f,s}$ such that aggregate factor supply efficiency is positive unless $AMWP_n^f = AMRS_n^f$. If $n^{f,s} = 0$, it must be that $AMWP_n^f \leq AMRS_n^f$ for the best possible combination of factor supply shares, which is the one that allocates the consumption of good j to the individual with the lowest MRS_n^{if}. If $n^{f,s} = n^{f,d} = 0$, then it must be that the most costly way of supplying a factor is higher than the highest marginal welfare product of doing so, formally: $\max_j \left\{MWP_n^{jf}\right\} \leq \min_i \left\{MRS_n^{if}\right\}$.[39] ■

Proof of Theorem 2c. (Technology Growth under Efficiency)

Proof. In matrix form, it follows from Equation (23) that

$$\boldsymbol{MSV}_y = \boldsymbol{AMRS}_c\boldsymbol{\phi}_c + \boldsymbol{MSV}_y\boldsymbol{G}_x\boldsymbol{\xi} = \boldsymbol{AMRS}_c\boldsymbol{\phi}_c + \boldsymbol{AMWP}_x\boldsymbol{\phi}_x,$$

where $\boldsymbol{\xi} = \boldsymbol{\chi}_x\boldsymbol{\phi}_x$ and $\boldsymbol{AMWP}_x = \boldsymbol{MSV}_y\boldsymbol{G}_x\boldsymbol{\chi}_x$. Therefore, equation (30) follows immediately when aggregate intermediate input efficiency holds. Equation (31) follows directly from the cross-sectional efficiency conditions. ■

Proof of Corollary 5. (Revisiting Lange 1942 and Mas-Colell, Whinston and Green 1995)

Proof. Follows from derivations in footnote 23. ■

Proof of Corollary 6. (Classic Efficiency Conditions Hold for Interior Links)

Proof. At an interior link, Theorems 2b and 2c immediately imply that both equations in (32) hold. The result follows then from the same logic as in Corollary 5. ■

[39] When $n^{f,d} = 0$, the value of a marginal unit of endowment of factor f is simply $\max_j\{MWP_n^{jf}\}$

Proof of Corollary 7. (Scenarios in which Classic Efficiency Conditions Do Not Hold)

Proof. i) If good j is a pure intermediate, then $MSV_y^j \neq AMRS_c^j$, which implies that the classic efficiency conditions cannot hold, since efficiency requires that $MSV_y^j \frac{\partial G^j}{\partial x^{jk}} = MRS_c^{ij}$. ii) If good k is a pure intermediate, then last condition of equation 27 already implies that the classic efficiency conditions cannot hold. iii) As in i), $MSV_y^j \neq AMRS_c^j$, which implies that the classic efficiency conditions cannot hold, since efficiency requires that $MSV_y^j \frac{\partial G^j}{\partial n^{jf,d}} = MRS_n^{if}$. ■

Proof of Theorem 3a. (Marginal Social Value of Output)

Proof. In a competitive equilibrium with wedges, we can express $\boldsymbol{AMRS}_c = \boldsymbol{MRS}_c \boldsymbol{\chi}_c$ as $\boldsymbol{AMRS}_c = \boldsymbol{p}\left(\boldsymbol{I}_c + \bar{\boldsymbol{\tau}}_c\right)$, where $\boldsymbol{I}_c$ is $J \times J$ diagonal matrix in which the j'th element is 1 when $c^j > 0$ and 0 if $c^j = 0$, and where we define a $J \times J$ matrix of aggregate consumption wedges as $\bar{\boldsymbol{\tau}}_c = \boldsymbol{\tau}_c \boldsymbol{\chi}_c$. It is also the case that $\boldsymbol{p}\boldsymbol{G}_x \boldsymbol{\chi}_x = \boldsymbol{p}\left(\boldsymbol{I}_x + \boldsymbol{\tau}_x \boldsymbol{\chi}_x\right) = \boldsymbol{p}\left(\boldsymbol{I}_x + \bar{\boldsymbol{\tau}}_x\right)$, where $\boldsymbol{I}_x$ is $J \times J$ diagonal matrix in which the j'th element is 1 when $x^j > 0$ and 0 if $x^j = 0$, and where we define a $J \times J$ matrix of aggregate intermediate use wedges as $\bar{\boldsymbol{\tau}}_x = \boldsymbol{\tau}_x \boldsymbol{\chi}_x$.

Hence, marginal social value of output satisfies

$$\boldsymbol{MSV}_y = \boldsymbol{AMRS}_c \boldsymbol{\phi}_c \boldsymbol{\Psi}_y = \boldsymbol{p}\left(\boldsymbol{I}_c + \bar{\boldsymbol{\tau}}_c\right) \boldsymbol{\phi}_c \boldsymbol{\Psi}_y = \boldsymbol{p}\boldsymbol{\phi}_c \boldsymbol{\Psi}_y + \boldsymbol{p}\bar{\boldsymbol{\tau}}_c \boldsymbol{\phi}_c \boldsymbol{\Psi}_y = \boldsymbol{p} + \boldsymbol{p}\left(\bar{\boldsymbol{\tau}}_x \boldsymbol{\phi}_x + \bar{\boldsymbol{\tau}}_c \boldsymbol{\phi}_c\right) \boldsymbol{\Psi}_y,$$

where we use the fact that $\boldsymbol{I}_c \boldsymbol{\phi}_c = \boldsymbol{I}_J \boldsymbol{\phi}_c$ and that

$$\begin{aligned}
\boldsymbol{p}\boldsymbol{\phi}_c \boldsymbol{\Psi}_y &= \boldsymbol{p}\left(\left(\boldsymbol{G}_x - \boldsymbol{1}_x\right) \boldsymbol{\xi} \boldsymbol{\Psi}_y + \boldsymbol{I}_J\right) = \left(\boldsymbol{p}\boldsymbol{G}_x - \boldsymbol{p}\boldsymbol{1}_x\right) \boldsymbol{\chi}_x \boldsymbol{\phi}_x \boldsymbol{\Psi}_y + \boldsymbol{p} \\
&= \left(\boldsymbol{p}\boldsymbol{G}_x \boldsymbol{\chi}_x - \boldsymbol{p}\right) \boldsymbol{\phi}_x \boldsymbol{\Psi}_y + \boldsymbol{p} = \left(\boldsymbol{p}\left(\boldsymbol{I}_x + \bar{\boldsymbol{\tau}}_x\right) - \boldsymbol{p}\right) \boldsymbol{\phi}_x \boldsymbol{\Psi}_y + \boldsymbol{p} = \boldsymbol{p}\bar{\boldsymbol{\tau}}_x \boldsymbol{\phi}_x \boldsymbol{\Psi}_y + \boldsymbol{p}.
\end{aligned}$$

■

Proof of Corollary 4. (Welfare Hulten's Theorem)

Proof. Since frictionless competitive economies are efficient, Ξ^{AE} simply equals technology growth. When $\bar{\boldsymbol{\tau}}_c = \bar{\boldsymbol{\tau}}_x = 0$, it follows from Theorem 3a that $MSV_y^j = p^j$. Hence, $\Xi^{AE} = p^j \frac{\partial G^j}{\partial \theta} = p^j y^j$, where we use the fact that $\frac{\partial G^j}{\partial \theta} = y^j$ for proportional Hicks-neutral technology changes. Simply dividing by $\sum_j p^j c^j$ yields equation (38) in the text. ■

Proof of Theorem 3b. (Production Efficiency in Competitive Economies)

Proof. It follows from the optimality conditions for production and the definition of $\Xi^{AE,P}$. ■

Proof of Theorem 3c. (Exchange Efficiency in Competitive Economies)

Proof. It follows from the individual optimality conditions for consumption and factor supply and the definition of $\Xi^{AE,X}$. ■

Proof of Theorem 4a. (General Redistribution Decomposition)

Proof. It follows from the definition of allocation shares and the definition of Ξ^{RD}. ■

C Additional Results

C.1 Planning Problem

The Lagrangian of the planning problem can be expressed as

$$\begin{aligned}
\mathcal{L} &= \mathcal{W}(V_1, \ldots, V_i, \ldots, V_I) \\
&- \sum_j \zeta_y^j \left(\sum_i c^{ij} + \sum_k x^{kj} - G^j \left(\left\{ x^{jk} \right\}_k, \left\{ n^{jf,d} \right\}_f \right) \right) - \sum_f \zeta_n^f \left(\sum_j n^{jf,d} - \sum_i n^{if,s} - \sum_i \bar{n}^{if,s} \right) \\
&+ \sum_i \sum_j \kappa_c^{ij} c^{ij} + \sum_i \sum_f \kappa_n^{if,s} n^{if,s} + \sum_j \sum_k \kappa_x^{jk} x^{jk} + \sum_j \sum_f \kappa_n^{jf,d} n^{jf,d},
\end{aligned}$$

where V_i is defined in (1). Hence, the first-order conditions can be derived from a perturbation of the form

$$\begin{aligned}
d\mathcal{L} = &\sum_j \sum_i \left(\alpha^i \frac{\partial u_i}{\partial c^{ij}} - \zeta_y^j + \kappa_c^{ij} \right) dc^{ij} + \sum_i \sum_f \left(\alpha^i \frac{\partial u_i}{\partial n^{if,s}} + \zeta_n^f + \kappa_n^{if,s} \right) dn^{if,s} \\
&+ \sum_j \sum_k \left(\zeta_y^j \frac{\partial G^j}{\partial x^{jk}} - \zeta_y^k + \kappa_x^{jk} \right) dx^{jk} + \sum_j \sum_f \left(\zeta_y^j \frac{\partial G^j}{\partial n^{jf,d}} - \zeta_n^f + \kappa_n^{jf,d} \right) dn^{jf,d},
\end{aligned}$$

where we take good j' as numeraire, which allows us to substitute α^i for $\alpha^i \frac{\partial u_i}{\partial c^{ij'}} = \zeta_y^{j'} \Rightarrow \alpha^i = 1 / \frac{\frac{\partial u_i}{\partial c^{ij'}}}{\zeta_y^{j'}}$, and where we define $MWP_x^{jk} = \zeta_y^j \frac{\partial G^j}{\partial x^{jk}}$ and $MWP_n^{jf} = \zeta_y^j \frac{\partial G^j}{\partial n^{jf,d}}$. Formally, the Kuhn-Tucker conditions are

i) $\kappa_c^{ij} c^{ij} = 0 \Rightarrow \left(\zeta_y^j - MRS_c^{ij} \right) c^{ij} = 0$, with generically one of the the two terms > 0

ii) $\kappa_n^{if,s} n^{if,s} = 0 \Rightarrow \left(\zeta_n^f + MRS_n^{ij} \right) n^{if,s} = 0$, with generically one of the the two terms > 0

iii) $\kappa_x^{jk} x^{jk} = 0 \Rightarrow \left(\zeta_y^k - MWP_x^{jk} \right) x^{jk} = 0$, with generically one of the the two terms > 0

iv) $\kappa_n^{if,d} n^{if,d} = 0 \Rightarrow \left(\zeta_n^f - MWP_n^{jf} \right) n^{if,d} = 0$, with generically one of the the two terms > 0

By adding up the consumption optimality conditions for all individuals for good j:

$$\sum_i \left(\zeta_y^j - MRS_c^{ij} \right) c^{ij} = 0 \Rightarrow \sum_i MRS_c^{ij} c^{ij} - \zeta_y^j \sum_i c^{ij} \Rightarrow \sum_i MRS_c^{ij} c^{ij} = \zeta_y^j c^j.$$

If $c^j > 0$ (as long as one agent is consuming the good, so good j is final):

$$\zeta_y^j = \sum_i MRS_c^{ij} \frac{c^{ij}}{\sum_i c^{ij}} = \sum_i \chi_c^{ij} MRS_c^{ij} = AMRS_c^k.$$

If $c^j = 0$, we must have $\zeta_y^j > MRS_c^{ij}$, for all i, which means that $\zeta_y^j > \max_i \left\{MRS_c^{ij}\right\}$. By adding up the intermediate good optimality conditions for all uses j of good k:

$$\sum_j \left(MWP_x^{jk} - \zeta_y^k\right) x^{jk} = 0 \Rightarrow \sum_j MWP_x^{jk} x^{jk} - \zeta_y^k \sum_j x^{jk} \Rightarrow \sum_j MWP_x^{jk} x^{jk} = \zeta_y^k x^k.$$

If $x^k > 0$ (as long as one good j uses good k as input, so good k is intermediate):

$$\zeta_y^k = \sum_j MWP_x^{jk} \frac{x^{jk}}{\sum_j x^{jk}} = \sum_j \chi_x^{jk} MWP_x^{jk} = AMWP_x^k.$$

If $x^k = 0$, we must have $\zeta_y^k > MWP_x^{jk}$, for all j, which means that $\zeta_y^k > \max_j \left\{MWP_x^{jk}\right\}$. Combining consumption and intermediate good optimality:

$$\sum_i MRS_c^{ij} c^{ik} + \sum_j MWP_x^{jk} x^{jk} = \zeta_y^k y^k,$$

so if $y^k > 0$, it must be that $\zeta_y^k = AMRS_c^k \phi_c^k + \sum_j \zeta_y^j \frac{\partial G^j}{\partial x^{jk}} \xi^{jk}$, which can be written in matrix form as $\boldsymbol{\zeta}_y = \boldsymbol{AMRS}_c \boldsymbol{\phi}_c \boldsymbol{\Psi}_y$, where $\boldsymbol{\Psi}_y = (\boldsymbol{I}_J - \boldsymbol{G}_x \boldsymbol{\xi})^{-1}$.

Similarly, for factors, if $n^{f,s} > 0$ (as long as one agent is supplying factor f):

$$\zeta_n^f = \sum_i \frac{n^{if,s}}{n^{f,s}} MRS_n^{ij} = \sum_i \chi_n^{if,s} MRS_n^{ij} = AMRS_n^f$$

If $n^{f,s} = 0$, we must have $\zeta_n^f < MRS_n^{ij}$, for all i, which means that $\zeta_n^f < \max_i \left\{MRS_n^{ij}\right\}$. If $n^{jf,d} > 0$ (as long as factor f is used to produce a good j):

$$\zeta_n^f = \sum_j MWP_n^{jf} \frac{n^{jf,d}}{n^{f,d}} = \sum_j MWP_n^{jf} \chi_n^{jf,d} = AMWP_n^{jf}$$

If $n^{jf,d} = 0$, we must have $\zeta_n^f > \sum_j MWP_n^{jf} \chi_n^{jf,d}$, for all j, which means that $\zeta_n^f > \max_j \left\{MWP_n^{jf}\right\}$. If $n^{f,s} > 0$ and $n^{f,d} > 0$: $AMWP_n^f = AMRS_n^f$. If $n^{f,s} = 0$, it must be that $\zeta_n^f < MRS_n^{if}$, or $\zeta_n^f < \min_i \left\{MRS_n^{if}\right\}$. If $n^{f,d} = 0$, it must be that $MWP_n^{jf} < \zeta_n^f$, or $\max_j \left\{MWP_n^{jf}\right\} < \zeta_n^f$. Hence, for $n^{f,s} = 0 = n^{f,d}$, we must have that $\max_j \left\{MWP_n^{jf}\right\} < \min_i \left\{MRS_n^{if}\right\}$. Finally, for $y^j = 0$ to be optimal, it must be that $c^j = x^{kj} = 0$ on the use side and $x^{jk} = n^{jf,d} = 0$ on the input side. This condition can be written as

$$\max\left\{\max_i\left\{\frac{\partial u_i}{\partial c^{ij}}\right\},\max_k\left\{\zeta_y^k\frac{\partial G^k}{\partial x^{kj}}\right\}\right\}<\zeta_y^j<\min\left\{\min_f\left\{\left(\frac{\partial G^j}{\partial n^{jf,d}}\right)^{-1}\zeta_n^f\right\},\min_k\left\{\left(\frac{\partial G^j}{\partial x^{jk}}\right)^{-1}\zeta_y^k\right\}\right\}.$$

C.2 Alternative Propagation Matrices

Intermediate inverse matrix Following similar steps as in the Proof of Lemma 3, we can express changes in intermediate input use as follows. Using both equations in (51), we can instead solve for $\frac{d\boldsymbol{x}}{d\theta}$ as follows

$$\frac{d\boldsymbol{x}}{d\theta}=\frac{d\boldsymbol{\xi}}{d\theta}\boldsymbol{y}+\boldsymbol{\xi}\left(\boldsymbol{G}_x\frac{d\boldsymbol{x}}{d\theta}+\boldsymbol{G}_n\frac{d\boldsymbol{n}^d}{d\theta}+\boldsymbol{G}_\theta\right),$$

so we can define a $JK\times JK$ propagation matrix in the space of intermediate links $\boldsymbol{\Psi}_x$:

$$\frac{d\boldsymbol{x}}{d\theta}=\underbrace{\boldsymbol{\Psi}_x}_{\text{Propagation}}\underbrace{\left(\frac{d\boldsymbol{\xi}}{d\theta}\boldsymbol{y}+\boldsymbol{\xi}\left(\boldsymbol{G}_n\frac{d\boldsymbol{n}^d}{d\theta}+\boldsymbol{G}_\theta\right)\right)}_{\text{Impulse}},\quad\text{where}\quad\boldsymbol{\Psi}_x=\left(\boldsymbol{I}_{JK}-\boldsymbol{\xi}\boldsymbol{G}_x\right)^{-1}.\tag{52}$$

Propagation in the space of output and the space of intermediate links is connected. In particular, Woodbury's identity implies that $\boldsymbol{\Psi}_x=\boldsymbol{I}_{JK}+\boldsymbol{\xi}\boldsymbol{\Psi}_y\boldsymbol{G}_x$, and it is also the case that $\boldsymbol{\Psi}_x\boldsymbol{\xi}=\boldsymbol{\xi}\boldsymbol{\Psi}_y$, connecting propagation in the space of goods and the space of intermediate links. Leveraging (52), it is possible to solve for changes in consumption as

$$\begin{aligned}\frac{d\boldsymbol{c}}{d\theta}&=\frac{d\boldsymbol{y}}{d\theta}-\boldsymbol{1}_x\frac{d\boldsymbol{x}}{d\theta}=\boldsymbol{G}_x\frac{d\boldsymbol{x}}{d\theta}+\boldsymbol{G}_n\frac{d\boldsymbol{n}^d}{d\theta}+\boldsymbol{G}_\theta-\boldsymbol{1}_x\frac{d\boldsymbol{x}}{d\theta}\\&=\left(\boldsymbol{G}_x-\boldsymbol{1}_x\right)\boldsymbol{\Psi}_x\frac{d\boldsymbol{\xi}}{d\theta}\boldsymbol{y}+\left(\left(\boldsymbol{G}_x-\boldsymbol{1}_x\right)\boldsymbol{\Psi}_x\boldsymbol{\xi}+\boldsymbol{I}_J\right)\boldsymbol{G}_n\frac{d\boldsymbol{n}^d}{d\theta}+\left(\left(\boldsymbol{G}_x-\boldsymbol{1}_x\right)\boldsymbol{\Psi}_x\boldsymbol{\xi}+\boldsymbol{I}_J\right)\boldsymbol{G}_\theta.\end{aligned}$$

Proportional output inverse matrix While the intermediate output inverse is expressed in levels, at times, it may be useful to work with proportional propagation matrix. Starting from the definition of $\frac{d\boldsymbol{y}}{d\theta}$, it follows that

$$\hat{\boldsymbol{y}}^{-1}\frac{d\boldsymbol{y}}{d\theta}=\tilde{\boldsymbol{\Psi}}_y\left(\hat{\boldsymbol{y}}^{-1}\boldsymbol{G}_x\frac{d\boldsymbol{\xi}}{d\theta}\boldsymbol{y}+\hat{\boldsymbol{y}}^{-1}\boldsymbol{G}_n\frac{d\boldsymbol{n}^d}{d\theta}+\hat{\boldsymbol{y}}^{-1}\boldsymbol{G}_\theta\right),$$

where $\tilde{\boldsymbol{\Psi}}_y=\hat{\boldsymbol{y}}^{-1}\boldsymbol{\Psi}_y\hat{\boldsymbol{y}}$. In the competitive case, it is possible to express $\boldsymbol{MSV}_y$ as $\boldsymbol{MSV}_y=\boldsymbol{p}+\boldsymbol{p}\left(\hat{\boldsymbol{c}}\bar{\boldsymbol{\tau}}_x+\hat{\bar{\boldsymbol{x}}}\bar{\boldsymbol{\tau}}_c\right)\tilde{\boldsymbol{\Psi}}_y\hat{\boldsymbol{y}}^{-1}$ where an element of $\tilde{\boldsymbol{\Psi}}_y$ is given by $\tilde{\psi}^{jk}=\frac{p^k\left(1+\tau_x^{jk}\right)x^{jk}}{p^j\left(1+\tau_y^j\right)y^j}$, where $\bar{\boldsymbol{\tau}}_c=\boldsymbol{\tau}_c\boldsymbol{\chi}_c$ and $\bar{\boldsymbol{\tau}}_x=\boldsymbol{\tau}_x\boldsymbol{\chi}_x$. Formally, $\boldsymbol{\Psi}_y=\hat{\boldsymbol{y}}\left(\hat{\boldsymbol{p}}\hat{\boldsymbol{y}}-\left(\mathbb{I}_x+\tilde{\boldsymbol{\tau}}_x\right)\check{\boldsymbol{p}}\check{\boldsymbol{x}}\right)^{-1}\hat{\boldsymbol{p}}$ and $\tilde{\boldsymbol{\Psi}}_y=\left(\hat{\boldsymbol{p}}\hat{\boldsymbol{y}}-\left(\mathbb{I}_x+\tilde{\boldsymbol{\tau}}_x\right)\check{\boldsymbol{p}}\check{\boldsymbol{x}}\right)^{-1}\hat{\boldsymbol{p}}\hat{\boldsymbol{y}}$, where $\tilde{\boldsymbol{\tau}}_x$ is a $J\times JK$ matrix analogouse to $\bar{\boldsymbol{\tau}}_x$, but with the same ordering as $\mathbb{I}_x$.

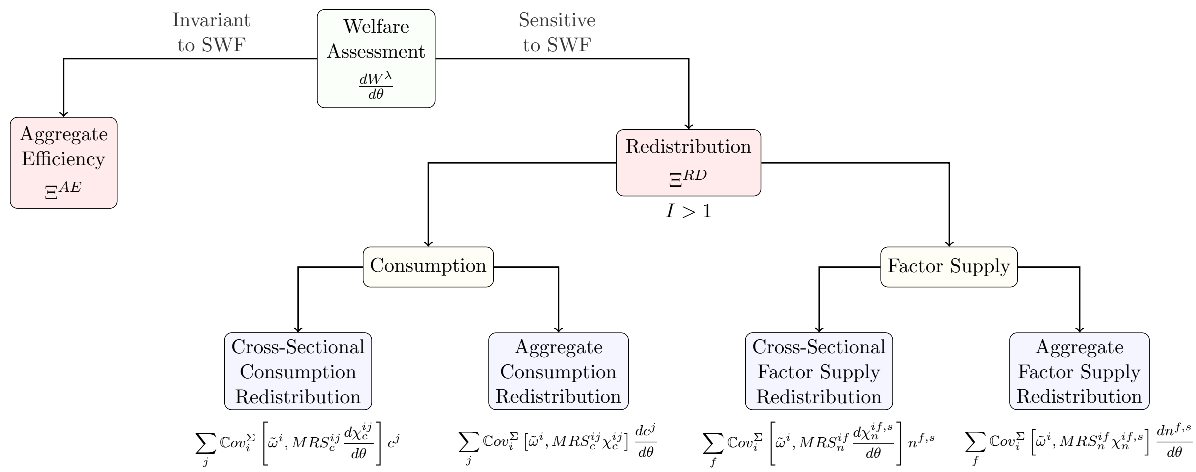


Figure 5. Welfare Accounting Decomposition: Redistribution

C.3 Alternative Redistribution Decomposition

Theorem 4b (Redistribution Decomposition in Competitive Economies)**.** *In competitive economies with wedges, the redistribution component of the welfare accounting decomposition,* Ξ^{RD}*, can be decomposed into distributive pecuniary and distortionary redistribution components, given by*

$$\Xi^{RD} = \overbrace{\mathbb{C}ov_i\left[\omega^i, -\sum_j \frac{dp^j}{d\theta} c^{ij} + \sum_f \frac{dw^f}{d\theta}\left(n^{if,s} + \bar{n}^{if,s}\right) + \sum_j \nu^{ij} \frac{d\pi^j}{d\theta}\right]}^{\text{Distributive Pecuniary Redistribution}}$$
$$+ \underbrace{\mathbb{C}ov_i\left[\omega^i, \sum_j p^j \tau_c^{ij} \frac{dc^{ij}}{d\theta} - \sum_f w^f \tau_n^{if,s}\left(\frac{dn^{if,s}}{d\theta} + \frac{d\bar{n}^{if,s}}{d\theta}\right)\right]}_{\text{Distortionary Redistribution}}.$$

The distributive pecuniary redistribution component captures the differential impact of changes in prices and profits on individual welfare—these are the distributive pecuniary effects present in any competitive economy. Intuitively, if a perturbation reduces the prices of goods consumed or increases the income earned by individuals with high ω^i, the distributive pecuniary redistribution component will be positive. Importantly, in the absence of technology or endowment growth, the sum across individuals of distributive pecuniary effects is zero (see e.g. Dávila and Korinek, 2018).

The distortionary redistribution component captures the differential impact on individual welfare of changes in the allocation of goods and factors that are distorted by individual wedges. This component is positive if a perturbation reallocates consumption (factor supply) towards individuals with higher consumption (lower factor supply) wedges. When $\tau_c^{ij} > 0$, for instance, individual i consumes too little of good j. An increase in c^{ij} when individual i is relatively favored by the planner contributes positively to the distortionary redistribution component. In contrast to the distributive effects, the sum across individuals of distortionary redistribution effects will typically not be zero.

D Minimal Welfare Accounting Economy: Special Cases

D.1 Minimal Welfare Accounting Economy

The minimal welfare accounting economy features two individuals, two goods, and single factor in elastic supply: $I = 2$, $J = 2$, $F = 1$. Individual preferences take the form $V_1 = u_1\left(c^{11}, c^{12}, n^{11,s}\right)$ and $V_2 = u_2\left(c^{21}, c^{22}, n^{21,s}\right)$ and technologies are given by$y^1 = G^1\left(x^{11}, x^{12}, n^{11,d}; \theta\right)$ and $y^2 = G^2\left(x^{21}, x^{22}, n^{21,d}; \theta\right)$. Finally, resource constraints are simply given by $y^1 = c^{11} + c^{21} + x^{11} + x^{21}$ and $y^2 = c^{12} + c^{22} + x^{12} + x^{32}$ and $n^{11,s} + n^{21,s} + \bar{n}^{11,s} + \bar{n}^{21,s} = n^{11,d} + n^{21,d}$. In this economy, all of the components of aggregate efficiency can be non-zero, as we illustrate in a series of special cases.[40]

D.2 Vertical Economy

This minimal vertical economy is a special case of the minimal welfare accounting economy. In this economy, there is a single individual who consumes a final good produced using an intermediate good, which is in turn produced by a single factor in fixed supply, so $I = 1$, $J = 2$, and $F = 1$. This is the simplest economy that illustrates the role played by pure intermediate goods. In this economy, individual preferences are given by $V_1 = u_1\left(c^{11}\right)$, technologies by $y^1 = G^1\left(x^{12}; \theta\right)$ and $y^2 = G^2\left(n^{21,d}; \theta\right)$, and resource constraints by $y^1 = c^{11}$, $y^2 = x^{12}$, and $\bar{n}^{1,s} = n^{21,d}$. By construction, all allocative efficiency components of the welfare accounting decomposition are zero, so this economy exclusively features technology growth and factor endowment growth.

Aggregate and production efficiency are given by

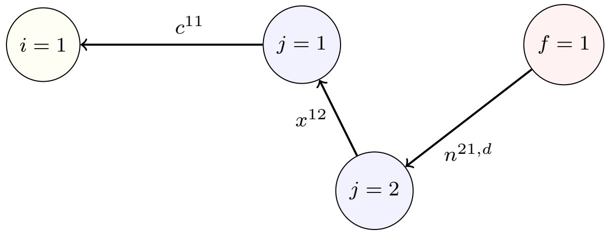
$$\Xi^{AE} = \Xi^{AE,P} = MSV_y^1 \frac{G^1}{\partial \theta} + MSV_y^2 \frac{G^2}{\partial \theta} + MSV_y^1 \frac{\partial G^1}{\partial n^{11,d}} \frac{d\bar{n}^{1,s}}{d\theta},$$

where $MSV_y^1 = MRS_c^{11}$ and $MSV_y^2 = MRS_c^{11} \frac{\partial G^1}{\partial x^{12}}$. In this economy, an efficient allocation must satisfy $MRS_c^{11} > 0$ and $MRS_c^{11} \frac{\partial G^1}{\partial x^{12}} > 0$.

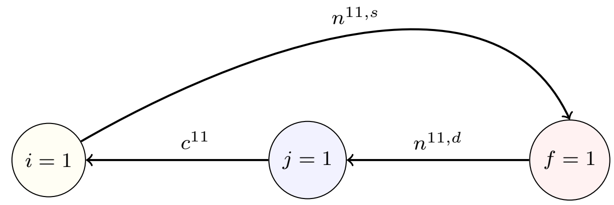
D.3 Robinson Crusoe Economy

One-producer one-consumer economies (i.e., Robinson Crusoe economies) are the simplest to study production—see Section 15.C of Mas-Colell et al. (1995). In these economies, a single individual consumes a single good and elastically supplies a single factor of production. A single production technology uses the supplied factor to produce the good, so $I = 1$, $J = 1$, and $F = 1$. Formally,

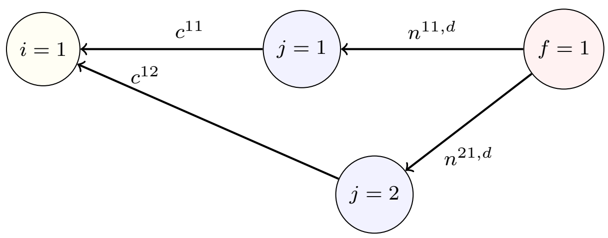
[40] At times, it is necessary to have $J = 3$ goods to represent some phenomena in production networks. For instance, three goods are necessary to have a pure intermediate good being used to produce another pure intermediate good. This is a relevant case in which classic efficiency conditions do not apply, as illustrated in examples 1 and 2.



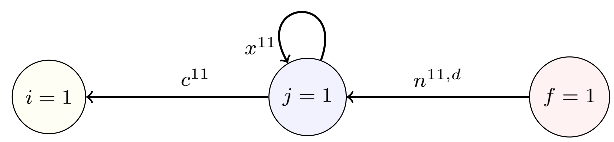
(a) Minimal Vertical



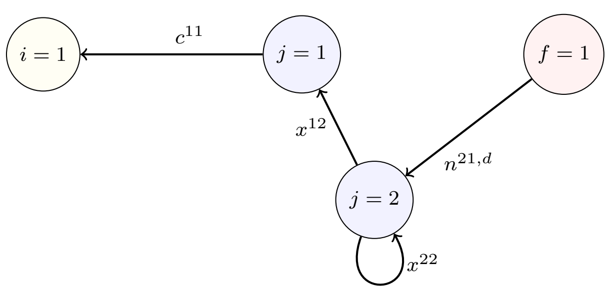
(b) Robinson Crusoe



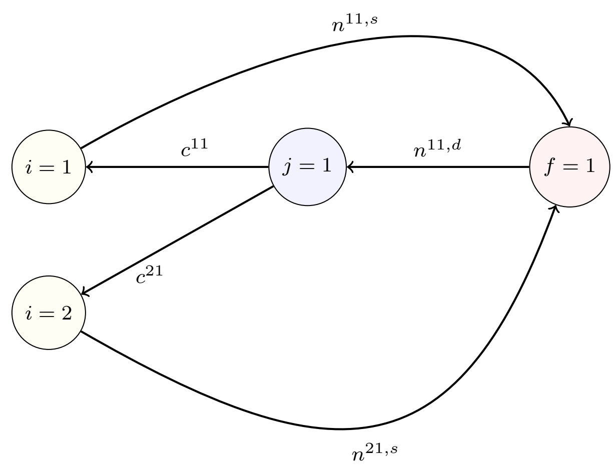
(c) Minimal Horizontal



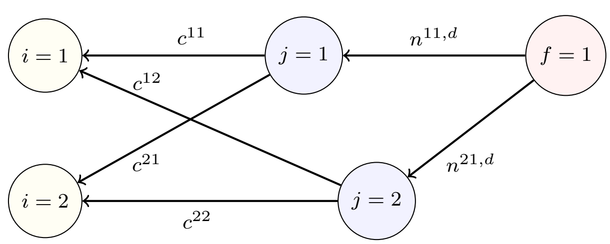
(d) Minimal Roundabout



(e) Minimal Diversified Intermediate



(f) Minimal Multiple Factor Suppliers



(g) Edgeworth Box

Figure 6. Minimal Welfare Accounting Economy: Special Cases

preferences, technology, and resource constraints are respectively given by $V_1 = u_1\left(c^{11}, n^{11,s}\right)$, $y^1 = G^1\left(n^{11,d};\theta\right)$, $y^1 = c^{11}$, and $n^{11,s} = n^{11,d}$. This economy exclusively features aggregate factor efficiency and technology growth.

The production efficiency decomposition takes the form

$$\Xi^{AE,P} = \left(\underbrace{MSV_y^1 \frac{\partial G^1}{\partial n^{11,d}}}_{AMWP_n^1} - \underbrace{MRS_n^{11}}_{AMRS_n^1}\right)\frac{dn^{1,s}}{d\theta} + MSV_y^1 \frac{\partial G^1}{\partial \theta},$$

where the marginal social value of output of good 1 is given by $MSV_y^1 = MRS_c^{11}$. In this economy, an efficient allocation must satisfy $MSV_y^1 \frac{\partial G^1}{\partial n^{11,d}} = MRS_n^{11}$.

D.4 Horizontal Economy

This minimal horizontal economy is the simplest to illustrate the role played by the possibility of reallocating factors across different uses. This economy generalizes to many well-known frameworks, including Heckscher-Ohlin, Armington (1969), and Hsieh and Klenow (2009). In this economy, a single individual consumes two different goods that can be produced using the same factor, which we assume to be in fixed supply, so $I = 1$, $J = 2$, and $F = 1$. Formally, preferences, technology, and resource constraints are given by $V_1 = u_1\left(c^{11}, c^{12}\right)$, $y^1 = G^1\left(n^{11,d};\theta\right)$, $y^2 = G^2\left(n^{21,d};\theta\right)$, $y^1 = c^{11}$, $y^2 = c^{12}$, and $\bar{n}^{1,s} = n^{11,d} + n^{21,d}$. This economy exclusively features cross-sectional factor efficiency, technology growth, and factor endowment growth

The production efficiency decomposition takes the form

$$\Xi^{AE,P} = \mathbb{C}ov_j^{\Sigma}\left[\underbrace{MSV_y^j \frac{\partial G^j}{\partial n^{j1,d}}}_{MWP_n^{j1}}, \frac{d\chi_n^{j1,d}}{d\theta}\right] n^{1,d} + MSV_y^1 \frac{\partial G^1}{\partial \theta} + MSV_y^2 \frac{\partial G^2}{\partial \theta} + AMWP_n^1 \frac{d\bar{n}^{1,s}}{d\theta}$$

where $AMWP_n^1 = \chi_n^{11,d} MSV_y^1 \frac{\partial G^1}{\partial n^{11,d}} + \chi_n^{21,d} MSV_y^2 \frac{\partial G^2}{\partial n^{21,d}}$, and where $MSV_y^1 = MRS_c^{11}$ and $MSV_y^2 = MRS_c^{12}$. In this economy, an efficient allocation must satisfy $MSV_y^1 \frac{\partial G^1}{\partial n^{11,d}} = MSV_y^2 \frac{\partial G^2}{\partial n^{21,d}}$.

D.5 Minimal Roundabout Economy

Roundabout economies have been used to illustrate the impact of intermediate goods on production—see e.g., Jones (2011). The minimal roundabout economy is the simplest economy in which aggregate intermediate input efficiency can exist. In this economy a single individual consumes a single mixed good, which is at the same time final and intermediate to itself, so $I = 1$, $J = 1$, and $F = 1$. Formally, preferences, technology, and resource constraints are given by $V_1 = u_1\left(c^{11}\right)$, $y^1 = G^1\left(x^{11}, n^{11,d};\theta\right)$, $y^1 = c^{11} + x^{11}$, and $\bar{n}^{1,s} = n^{11,d}$. This economy only features aggregate intermediate input efficiency, technology growth, and factor endowment growth.

The production efficiency decomposition takes the following form

$$\Xi^{AE,P} = \left(\underbrace{MSV_y^1 \frac{\partial G^1}{\partial x^{11}}}_{AMWP_x^1} - \underbrace{MRS_c^{11}}_{AMRS_c^1} \right) \frac{d\phi_x^1}{d\theta} y^1 + MSV_y^1 \frac{\partial G^1}{\partial \theta} + AMWP_n^1 \frac{d\bar{n}^{1,s}}{d\theta},$$

where $AMWP_n^1 = MSV_y^1 \frac{\partial G^1}{\partial n^{11,d}}$, and $MSV_y^1 = \frac{MRS_c^{11}}{1-\xi^{11}\frac{\partial G^1}{\partial x^{11}}}$. In this economy, an efficient allocation must satisfy $MSV_y^1 = MSV_y^1 \frac{\partial G^1}{\partial x^{11}} = MRS_c^{11}$.

D.6 Diversified Intermediate

This minimal diversified intermediate economy is the simplest economy in which cross-sectional intermediate input efficiency can exist. In this economy, a single individual consumes a final good, which is exclusively produced by a pure intermediate that can be also used for roundabout production. This pure intermediate is produced using a single factor in fixed supply, so $I = 1$, $J = 2$, and $F = 1$. Formally, preferences, technology, and resource constraints are given by $V_1 = u_1\left(c^{11}\right)$, $y^1 = G^1\left(x^{12}; \theta\right)$, $y^2 = G^2\left(x^{22}, n^{21,d}; \theta\right)$, $y^1 = c^{11}$, $y^2 = c^{12} + x^{12} + x^{22}$, and $\bar{n}^{1,s} = n^{21,d}$. This economy features cross-sectional intermediate input efficiency, aggregate intermediate input efficiency, technology growth, and factor endowment growth.

The production efficiency decomposition takes the form

$$\begin{aligned}\Xi^{AE,P} =& \mathbb{C}ov_j^{\Sigma}\left[\underbrace{MSV^j \frac{\partial G^j}{\partial x^{j2}}}_{MWP_x^{j2}}, \frac{d\chi_n^{j2}}{d\theta}\right] x^2 + \underbrace{\left(\chi_x^{12} MSV_y^1 \frac{\partial G^1}{\partial x^{12}} + \chi_x^{22} MSV_y^2 \frac{\partial G^2}{\partial x^{22}}\right)}_{AMWP_x^2} \frac{d\phi_x^2}{d\theta} y^2 \\ &+ MSV_y^1 \frac{\partial G^1}{\partial \theta} + MSV_y^2 \frac{\partial G^2}{\partial \theta} + \underbrace{\left(MSV_y^2 \frac{\partial G^2}{\partial n^{21,d}}\right)}_{AMWP_n^1} \frac{d\bar{n}^{1,s}}{d\theta},\end{aligned}$$

where $MSV_y^1 = MRS_c^{11}$ and $MSV_y^2 = MRS_c^{11} \frac{\frac{\partial G^1}{\partial x^{12}}\xi^{12}}{1-\xi^{22}\frac{\partial G^2}{\partial x^{22}}}$

D.7 Two Factor Supplier Economy

This minimal two factor supplier economy (we could also call it Robinson Crusoe and Friday economy) is the simplest economy in which cross-sectional factor supply efficiency can exist. In this economy, we assume that two individuals have identical linear preferences for consumption of a single produced good, which we use as numeraire. This eliminates potential gains from cross-sectional consumption efficiency, since $MRS_c^{11} = MRS_c^{21} = 1$. We also assume that there is a single production technology that uses a single factor that can be supplied either of the two individuals, with in principle different disutility, so $I=2$, $J=1$, and $F=1$. Formally, preferences, technology, and resource constraints are given by $V_1 = c^{11} + u_1\left(n^{11,s}\right)$, $V_2 = c^{21} + u_2\left(n^{21,s}\right)$ $y^1 = G^1\left(n^{11,d};\theta\right)$ $y^1 = c^{11} + c^{21}$, and $n^{11,s} + n^{21,s} = n^{31,d}$. This economy features cross-sectional factor supply efficiency, aggregate factor efficiency, and technology growth.

The exchange efficiency decomposition takes the form

$$\Xi^{AE,\,X} = -\mathbb{C}ov_i^{\Sigma}\left[MRS_n^{i1}, \frac{d\chi_n^{i1,s}}{d\theta}\right] n^{1,s}.$$

The production efficiency decomposition takes the form

$$\Xi^{AE,\,P} = \left(\underbrace{MSV_y^1\frac{\partial G^1}{\partial n^{11,d}}}_{AMWP_n^1} - \underbrace{\left(\chi_n^{11,s}MRS_n^{11} + \chi_n^{21,s}MRS_n^{21}\right)}_{AMRS_n^1}\right)\frac{dn^{1,s}}{d\theta} + MSV_y^1\frac{\partial G^1}{\partial\theta} + AMWP_n^1\frac{d\bar{n}^{1,s}}{d\theta},$$

where $AMWP_n^1 = MSV_y^1\frac{\partial G^1}{\partial n^{11,d}}$ where the marginal social value of output for good 1 is

$$MSV_y^1 = \chi_c^{11}MRS_c^{11} + \chi_c^{21}MRS_c^{21} = 1.$$

D.8 Edgeworth Box Economy

Pure exchange economies (i.e., Edgeworth Box economies) are the simplest to study most phenomena in general equilibrium and welfare economics. In this economy, two individuals consume two different goods, which appear as endowments. To formalize endowments, we assume that there is a single factor in fixed supply and that factor uses are predetermined, so $I=2$, $J=2$, and $F=1$. Formally, preferences, technologies, and resource constraints are respectively given by $V_1 = u_1\left(c^{11}, c^{12}\right)$, $V_2 = u_2\left(c^{21}, c^{22}\right)$, $y^1 = G^1\left(n^{11,d};\theta\right)$, $y^2 = G^2\left(n^{21,d};\theta\right)$, $y^1 = c^{11} + c^{21}$, $y^2 = c^{12} + c^{22}$, and $\bar{n}^{1,s} = n^{11,d} + n^{12,d}$. This economy features cross-sectional consumption efficiency, technology growth, and factor endowment growth, where the last two should be interpreted as changes in endowments.

The exchange efficiency component takes the form

$$\Xi^{AE,\,X} = \mathbb{C}ov_i^{\Sigma}\left[MRS_c^{i1}, \frac{d\chi_c^{i1}}{d\theta}\right]c^1 + \mathbb{C}ov_i^{\Sigma}\left[MRS_c^{i2}, \frac{d\chi_c^{i2}}{d\theta}\right]c^2.$$

The production efficiency component takes the form

$$\Xi^{AE,\,P} = MSV_y^1\frac{\partial G^1}{\partial\theta} + MSV_y^2\frac{\partial G^2}{\partial\theta} + AMWP_n^1\frac{d\bar{n}^{1,s}}{d\theta} + AMWP_n^2\frac{d\bar{n}^{2,s}}{d\theta}.$$

where the marginal social value of output is

$$\boldsymbol{MSV}_y = \left(\chi_c^{11} MRS_c^{11} + \chi_c^{21} MRS_c^{21} \quad \chi_c^{12} MRS_c^{12} + \chi_c^{22} MRS_c^{22}\right).$$

E Appendix for Monetary Policy Application

This Appendix presents additional model details in E.1, competitive equilibrium in E.2, a self-contained quantitative calibration in E.3, and additional numerical results in E.4.

E.1 Additional Model Details

Households. Household preferences (47) give rise to the usual CES demand functions

$$c^{ij} = \Gamma_c^{ij} \left(\frac{p^j}{P^i}\right)^{-\eta_c} c^i \qquad \text{and} \qquad c^{ij\ell} = \left(\frac{p^{j\ell}}{p^j}\right)^{-\epsilon^j} c^{ij}.$$

Under homothetic CES consumption preferences, each household i faces an ideal price index

$$P^i = \left[\sum_j \Gamma_c^{ij} (p^j)^{1-\eta_c}\right]^{\frac{1}{1-\eta_c}}.$$

Production. The production function (48) features three nests of CES aggregates. Taking as given prices and wages, firms choose inputs to minimize cost

$$\mathcal{C}^{j\ell} = \min_{\{x^{j\ell k\ell'}\}_{k\ell'}, \{n^{j\ell i}\}_i} \sum_k \int_0^1 p^{k\ell'} x^{j\ell k\ell'} d\ell' + \sum_i W^i n^{j\ell i},$$

subject to the CES production structure in (48). This problem gives rise to labor demand

$$n^{j\ell} = (A^j)^{\eta-1} (1-\vartheta^j) \left(\frac{W^{j\ell}}{mc^j}\right)^{-\eta} y^{j\ell} \qquad \text{and} \qquad n^{j\ell i} = \Gamma_w^{ji} \left(\frac{W^i}{W^{j\ell}}\right)^{-\eta^w} n^{j\ell}$$

and intermediate input demand

$$x^{j\ell} = (A^j)^{\eta-1} \vartheta^j \left(\frac{p_x^{j\ell}}{mc^j}\right)^{-\eta} y^{j\ell}, \qquad x^{j\ell k} = \Gamma_x^{jk} \left(\frac{p^k}{p_x^{j\ell}}\right)^{-\eta_x} x^{j\ell} \qquad \text{and} \qquad x^{j\ell k\ell'} = \left(\frac{p^{k\ell'}}{p^k}\right)^{-\epsilon_k} x^{j\ell k}.$$

Nominal marginal cost is given by

$$mc^j = \frac{1}{A^j} \left[(1-\vartheta^j)(W^j)^{1-\eta} + \vartheta^j (p_x^j)^{1-\eta}\right]^{\frac{1}{1-\eta}},$$

which is symmetric across firms ℓ in sector j. Marginal cost is not affected by the revenue tax, which is the only wedge in this application. Finally, the cost indices are given by

$$W^j = \left[\sum_k \Gamma_w^{ji}(W^i)^{1-\eta_w}\right]^{\frac{1}{1-\eta_w}} \qquad \text{and} \qquad p_x^j = \left[\sum_k \Gamma_x^{jk}(p^k)^{1-\eta_x}\right]^{\frac{1}{1-\eta_x}}.$$

Since production functions are homogeneous of degree one, total cost is given by $\boldsymbol{C}^{j\ell} = mc^j y^{j\ell}$.

Sectoral aggregation. Firms set prices according to (49). Aggregating to the sectoral level, the price of sector j's good is

$$\begin{aligned} p^j &= \left(\int_0^1 (p^{j\ell})^{1-\epsilon^j} d\ell\right)^{\frac{1}{1-\epsilon^j}} = \left[\int_0^{\delta^j} \left(\frac{\epsilon^j}{\epsilon^j - 1}\frac{1}{1-\tau^j} mc^j\right)^{1-\epsilon^j} d\ell + \int_{\delta^j}^1 (\bar{p}^j)^{1-\epsilon_j} d\ell\right]^{\frac{1}{1-\epsilon^j}} \\ &= \left[\delta^j\left(\frac{\epsilon^j}{\epsilon^j - 1}\frac{1}{1-\tau^j} mc^j\right)^{1-\epsilon^j} + (1-\delta^j)(\bar{p}^j)^{1-\epsilon^j}\right]^{\frac{1}{1-\epsilon^j}} \\ &= \frac{\epsilon^j}{\epsilon^j - 1}\frac{1}{1-\tau^j}\left[\delta^j (mc^j)^{1-\epsilon^j} + (1-\delta^j)(\bar{mc}^j)^{1-\epsilon^j}\right]^{\frac{1}{1-\epsilon^j}}, \end{aligned}$$

where the very first equality follows since

$$p^j c^{ij} = \int_0^1 p^{j\ell} c^{ij\ell} d\ell = \int_0^1 p^{j\ell}\left(\frac{p^{j\ell}}{p^j}\right)^{-\epsilon^j} c^{ij} d\ell \implies p^j = \left(\int_0^1 (p^{j\ell})^{1-\epsilon^j} d\ell\right)^{\frac{1}{1-\epsilon^j}}.$$

Aggregating the goods market clearing condition, we have

$$p^j y^j \equiv \int_0^1 p^{j\ell} y^{j\ell} d\ell = \sum_i \mu^i \int_0^1 p^{j\ell} c^{ij\ell} d\ell + \sum_k \int_0^1 \int_0^1 p^{j\ell} x^{k\ell' j\ell} d\ell' d\ell,$$

where $\int_0^1 p^{j\ell} y^{j\ell} d\ell$ denotes total nominal expenditures on sectoral good j. This also implies a resource constraint at the sectoral level, given by $y^j = \sum_i \mu^i c^{ij} + \sum_k \int_0^1 x^{k\ell j} d\ell$. All this relies on our assumption that all agents buying in sector j share the same homothetic demand aggregators over varieties ℓ. In particular, it implies that we also have

$$y^{j\ell} = \left(\frac{p^{j\ell}}{p^j}\right)^{-\epsilon^j} y^j \qquad \text{and} \qquad y^j = \left(\int_0^1 (y^{j\ell})^{\frac{\epsilon^j - 1}{\epsilon^j}} d\ell\right)^{\frac{\epsilon^j}{\epsilon^j - 1}}.$$

Fiscal rebates. In the absence of fiscal policy, the rebate T^i that household i receives simply corresponds to total corporate profits plus the proceeds from the revenue tax. That is,

$$\sum_i \mu^i T^i = \sum_j \int_0^1 \Pi^{j\ell} d\ell + \sum_j \int_0^1 \tau^j p^{j\ell} y^{j\ell} d\ell = \sum_j \int_0^1 (p^{j\ell} - mc^j) y^{j\ell} d\ell$$

Assuming a uniform rebate, we simply have $T^i = \sum_j \int_0^1 (p^j - mc^j) y^{j\ell} d\ell$.

E.2 Equilibrium

Definition 9 (Competitive Equilibrium)**.** *Taking as given an initial price distribution $\{\bar{p}^{j\ell}\}_{j\ell}$, a realization of technology shocks $\{A^j\}_j$, revenue taxes $\{\tau^j\}_j$, and monetary policy M, a competitive equilibrium comprises an allocation $\{c^{ij\ell}, n^i, x^{i\ell k\ell'}, y^{j\ell}\}_{i,j\ell,k\ell'}$ and prices $\{p^{j\ell}, W^i\}_{i,j\ell}$ such that (i) households optimize consumption and labor supply, (ii) firms $\ell \in [0, \delta^j)$ in sector j reset their prices optimally, and (iii) markets for goods and factors clear*

$$y^{j\ell} = \sum_i \mu^i c^{ij\ell} + \sum_k \int_0^1 x^{k\ell' j\ell} d\ell' \qquad \textit{and} \qquad \mu^i n^i = \sum_j \int_0^1 n^{j\ell i} d\ell.$$

Notice that each sector features two representative firms ex post since all firms are symmetric ex ante and those firms that reset prices all choose the same reset price. At the sector level, there is consequently a representative price-adjusting firm and a representative fixed-price firm.

Computing competitive equilibrium requires an initial price distribution $\{\bar{p}^{j\ell}\}_{j\ell}$. We assume that initial prices are given by

$$\bar{p}^{j\ell} = \bar{p}^j = \frac{\epsilon^j}{\epsilon^j - 1} \frac{1}{1 - \tau^j} \bar{mc}^j = \frac{\epsilon^j}{\epsilon^j - 1} \frac{1}{1 - \tau^j} mc^j \Big(1, \Big\{\bar{p}^{k\ell'}\Big\}_{k\ell'}, \Big\{\bar{W}^i\Big\}_i\Big).$$

That is, $\bar{p}^j$ corresponds to the price firms in sector j would set if all technologies remain at their default level $A^j = \bar{A}^j$. This initialization is heuristically consistent with the zero-inflation steady state of a dynamic New Keynesian model. In the absence of technology shocks, therefore, no firm faces an incentive to adjust prices. If $A^j \neq \bar{A}^j$, a fraction δ^j of firms in each sector reset their price.

Numeraire. We take as our numeraire total nominal expenditures in the absence of shocks, i.e., $\bar{M} = \sum_j p^j y^j = 1$. Therefore, $\bar{M} = 1$ provides a benchmark stance for monetary policy. In the absence of technology shocks, setting $M = \bar{M} = 1$ implies production efficiency and therefore aggregate efficiency since all firms are symmetric.

Macro block. To compute this model, it is particularly convenient to characterize a macro block by aggregating to the sectoral level. To that end, we aggregate several key equilibrium conditions.

The aggregate labor market clearing condition (aggregated to the level of household type) is

$$\mu^i n^i = \sum_j \int_0^1 n^{j\ell i} d\ell = \sum_j \int_0^1 \Gamma_w^{ji} \left(\frac{W^i}{W^j} \right)^{-\eta_w} n^{j\ell} d\ell$$

$$= \sum_j \Gamma_w^{ji} \left(\frac{W^i}{W^j} \right)^{-\eta_w} (A^j)^{\eta-1} (1-\vartheta^j) \left(\frac{W^j}{mc^j} \right)^{-\eta} \int_0^1 y^{j\ell} d\ell$$

$$= \sum_j \Gamma_w^{ji} \left(\frac{W^i}{W^j} \right)^{-\eta_w} (A^j)^{\eta-1} (1-\vartheta^j) \left(\frac{W^j}{mc^j} \right)^{-\eta} D^j y^j$$

where

$$D^j = \int_0^1 \left(\frac{p^{j\ell}}{p^j} \right)^{-\epsilon^j} d\ell$$

is a measure of sectoral price dispersion.

Aggregating the goods market clearing condition yields

$$y^{j\ell} = \sum_i \mu^i c^{ij\ell} + \left(\frac{p^{j\ell}}{p^j} \right)^{-\epsilon^j} \sum_k \Gamma_x^{kj} \left(\frac{p^j}{p_x^k} \right)^{-\eta_x} \int_0^1 (A^k)^{\eta-1} \vartheta^k \left(\frac{p_x^k}{mc^k} \right)^{-\eta} y^{k\ell'} d\ell'.$$

And plugging in for CES demand functions implies

$$y^j = \sum_i \mu^i c^{ij} + \sum_k \Gamma_x^{kj} \left(\frac{p^j}{p_x^k} \right)^{-\eta_x} (A^k)^{\eta-1} \vartheta^k \left(\frac{p_x^k}{mc^k} \right)^{-\eta} y^k D^k,$$

yielding sectoral goods market clearing conditions written as a fixed point in y^j.

Finally, the budget constraint can be written as

$$P^i c^i = W^i n^i + \sum_j (p^j - D^j mc^j) y^j.$$

Computationally, it is now easiest to solve the macro block as a separate system of equations. Firm-level allocations can then be obtained from CES demand functions.

E.3 Calibration

Our calibration broadly follows Schaab and Tan (2023) and is summarized in Table 2. It is based on 66 production sectors and 10 household types, which we associate with deciles of the household income distribution.

For household preferences, we set the coefficient of relative risk aversion to $\gamma = 2$ and the inverse Frisch elasticity to $\varphi = 2$. We use an elasticity of substitution of $\eta_c = 1$, so the consumption aggregator is Cobb-Douglas, and we calibrate the consumption weights Γ_c^{ij} to match consumption expenditure shares across household types in the CEX.

	Parameters	Value / Target	Source
	Household preferences		
γ	Relative risk aversion	2	Standard
φ	Inverse Frisch elasticity	2	Standard
η_c	Elasticity of substitution across goods	1	Cobb-Douglas
Γ_c^{ij}	CES consumption weights	Consumption expenditure shares	CEX
	Production and nominal rigidities		
η	Elasticity of substitution across inputs and labor	1	Cobb-Douglas
ϑ^j	CES input bundle weight	Sectoral input share	BEA
η_x	Elasticity of substitution across inputs	1	Cobb-Douglas
η_w	Elasticity of substitution across factors	1	Cobb-Douglas
Γ_x^{ij}	CES input use weights	Input-output network	BEA I-O
Γ_w^{ij}	CES factor use weights	Payroll shares	ACS
ϵ^j	Elasticities of substitution across varieties	Sectoral markups	Baqaee and Farhi (2020)
δ^j	Sectoral price adjustment probabilities	Price adjustment frequencies	Pasten et al. (2017)

Table 2. List of Calibrated Parameters

On the production side, we set the elasticity of substitution between the labor and intermediate input bundles to $\eta = 1$. Therefore, ϑ^j and $1 - \vartheta^j$ correspond respectively to the input and labor shares in production, which we obtain from the BEA GDP-by-Industry data. We compute the input share ϑ^j as input expenditures relative to gross output, averaged between 1997 and 2015, and treat the labor share as its complement. We set the elasticities of substitution across intermediate inputs and factors to $\eta_x = \eta_w = 1$. We calibrate Γ_x^{ij} and Γ_w^{ij} to match data on input-output linkages and payroll shares. For the former, we use data from the BEA Input Output "Use" Table to compute input shares as a sector j's expenditures on goods from sector k as a share of j's total expenditures on inputs, averaged between 1997 and 2015. We obtain payroll shares from a linked ACS-IO dataset as type i's earnings from sector j as a share of total earnings, averaged between 1997 and 2015.

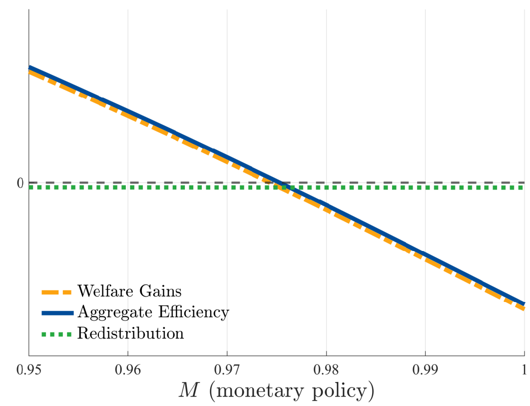
We use data from Baqaee and Farhi (2020) on sectoral markups to calibrate the elasticity of substitution across sectoral varieties ϵ^j. Sectoral markups are computed as $\mu^j = \frac{\epsilon^j}{\epsilon^j - 1}$.

Finally, we use data from Pasten et al. (2017) on price adjustment frequencies to calibrate δ^j. They estimate monthly price adjustment frequencies using the data underlying the Bureau of Labor Statistics' Producer Price Index for 754 industries from 2005 to 2011. First, we link these estimates to the 66 sectors in our data. Second, we obtain quarterly adjustment probabilities as $1 - (1 - \frac{\text{monthly adjustment frequency}}{100})^3$. Finally, we bin these estimates into quintiles. This allows us to solve our model assuming that each of the 66 sectors consists of 5 firms.

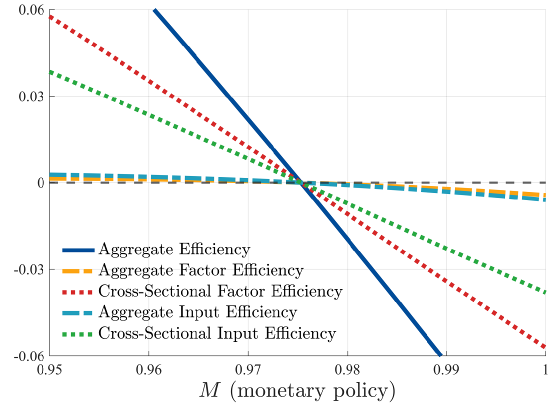
E.4 Additional Results

In this subsection, we present additional numerical results that are referenced in the main text.

Divine coincidence. Consider an alternative calibration where households and sectors are symmetric, so there exist a representative household and a representative sector. Our model then collapses to the standard, one-sector New Keynesian model, albeit with roundabout production.



(a) Welfare Gains



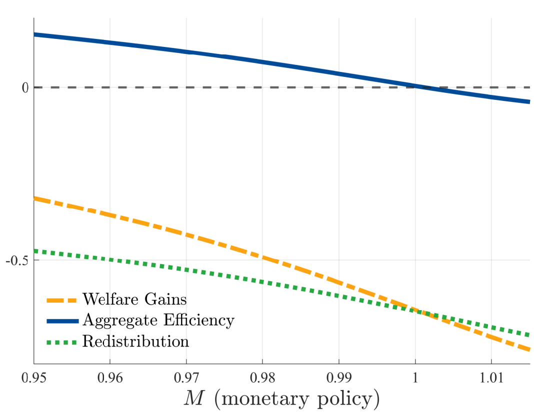
(b) Aggregate Efficiency Gain

Figure 7. Optimal Monetary Policy under Divine Coincidence

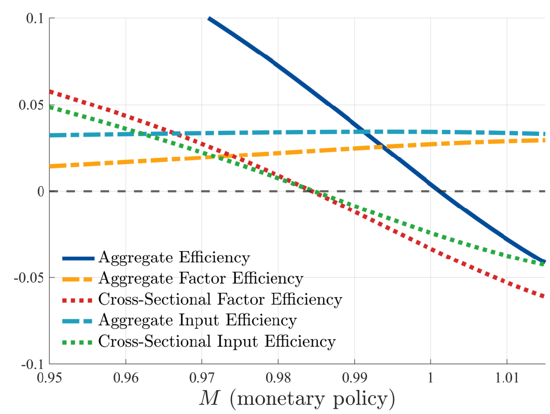
Divine Coincidence holds in this model. That is, the optimal monetary policy response to an aggregate technology shock closes both output and inflation gaps.

Figure 7 illustrates this benchmark from the perspective of our welfare accounting decomposition. In that context, Divine Coincidence implies that each allocative efficiency component is 0, indicating that optimal policy can attain an efficient allocation. Moreover, since households are symmetric, there is no scope for redistribution gains, so welfare and aggregate efficiency coincide.

Importance of markup distortions. Figure 4 in the main text corresponds to a calibration of the model that assumes revenue taxes are available to eliminate initial markups. We reproduce our main experiment in Figure 8 below, assuming that revenue taxes are not available.



(a) Welfare Gains



(b) Aggregate Efficiency Gain

Figure 8. Optimal Monetary Policy with Markup Distortions

It is well known from the New Keynesian literature that monopolistic competition implies inefficiently low steady state employment. In that context, optimal monetary policy under discretion, which is heuristically comparable to the static optimization problem we consider, seeks to raise employment via expansionary monetary policy. We revisit this result from the perspective of our welfare accounting decomposition. Figure 8 demonstrates that, in the presence of initial markup distortions, aggregate factor and input use efficiency considerations push optimal monetary policy towards a more expansionary stance. In the one-sector New Keynesian model (without roundabout production), aggregate factor efficiency corresponds to the standard labor wedge. In this multi-sector variant, aggregate factor and input use efficiency formally capture that aggregate employment and aggregate activity are inefficiently low.

Cross-sectional factor and input use efficiency, on the other hand, push monetary policy towards a relatively more contractionary stance. Optimal policy therefore trades off the gains from stimulating aggregate activity in the presence of markup distortions against the cost of creating misallocation in the form of price dispersion, captured by cross-sectional factor and input use efficiency.