

# INEQUALITY AND OPTIMAL MONETARY POLICY IN THE OPEN ECONOMY

Sushant Acharya<sup>1</sup>    Edouard Challe<sup>2</sup>

<sup>1</sup>Bank of Canada and CEPR

<sup>2</sup>Paris School of Economics and CEPR

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The views expressed herein are those of the authors and not necessarily those of the Bank of Canada

## MOTIVATION

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- How do aggregate shocks affect domestic inequality in SOE, and how should CB respond?

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- This paper: **normative** perspective on monetary policy in Open-Eco HANK

# MAIN TRADEOFFS AND RESULTS

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Aggregate shocks  $\Rightarrow$  output, national income  $\Rightarrow$  consumption risk & inequality

## TRADE-OFFS

Stabilizing consumption inequality

**VS**

Closing output gap + stabilizing inflation + manipulating ToT

closed-eco RANK

open-eco RANK

## RESULTS

Conditions for “SOE-HANK divine coincidence”

Plausible calibration  $\Rightarrow$  More output and exch-rate stabilization than in RANK

**Model**

# HOUSEHOLDS

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- SOE à la Galí Monacelli (2005) + incomplete markets
- Perpetual youth demographics with turnover rate  $1 - \vartheta$
- 2 groups of HHs:
  - **Unconstrained** (share  $1 - \theta$ )  $\Rightarrow$  trade **non-state contingent** 1-period real actuarial bond
  - **Hand-to-Mouth** (share  $\theta$ )  $\Rightarrow$  cannot access asset markets
- All HHs subject to uninsured idiosyncratic shocks – in addition to aggregate shocks
- **CARA-Normal** structure as in Acharya et al. (2023)

- CARA-Normal structure  $\Rightarrow$  linear policy rules  $\Rightarrow$  **linear aggregation**
- Group- $u$  Euler equation:

details

$$\Delta c_{t+1}(u) = \underbrace{\frac{1}{\gamma} \ln \left( \frac{\beta R_t}{1 + \tau^*} \right)}_{\text{intertemporal substitution}} + \underbrace{\frac{\gamma}{2} \sigma_{c_u, t+1}^2}_{\text{prec. saving}}$$

where

$$c_t(u) = (1 - \vartheta) \sum_{s=-\infty}^t \vartheta^{t-s} \int c_t^s(i, u) di$$

and

$$\sigma_{c_u, t} \approx \mu \sigma_{y, t} + (1 - \mu) \sigma_{c_u, t+1}, \quad \sigma_{y, t} = \sigma_y e^{-\varphi \hat{y}_t}$$



# HAND-TO-MOUTH HOUSEHOLDS

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- Consume current income so that

$$\begin{aligned}c_t(h) &= (1 - \vartheta) \sum_{s=-\infty}^t \vartheta^{t-s} \int c_t^s(i, h) di \\ &= \frac{P_{H,t}}{P_t} y_t\end{aligned}$$

where

$$\frac{P_{H,t}}{P_t} = \left( \frac{1 - \alpha Q_t^{1-\eta}}{1 - \alpha} \right)^{\frac{1}{1-\eta}} \equiv p_H(Q_t)$$

- Consumption of HtM highly responsive to  $Q_t$  (“real income channel”)

# HOUSEHOLDS: DEMAND SYSTEM AND LABOUR SUPPLY

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- Demand system as in Galí-Monacelli with home bias  $1 - \alpha$  and elasticities

details

- $\eta$  btw. H vs. F goods
- $\nu$  across countries
- $\varepsilon$  across varieties

- Utilitarian unions set wages and demand uniform labor from HHs

details

- Flexible wages + sticky prices as in Galí-Monacelli (2005)

## REST OF MODEL

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- **NKPC:**

$$\pi_{H,t} = \kappa (\hat{w}_t - z_t - \hat{p}_{H,t}) + \beta \pi_{H,t+1}$$

- **Home goods:**

$$\frac{z_t n_t}{1 + \frac{\Psi}{2} (\ln \Pi_{Ht})^2} = y_t = c_{Ht}(\textcolor{red}{Q}_t, c_t) + c_{Ht}^*(\textcolor{red}{Q}_t, c^*)$$

- **Home savings:**

$$\underbrace{(1 - \theta) \vartheta a_{t+1}}_{A_{t+1}} = R_t \left[ \underbrace{(1 - \theta) \vartheta a_t}_{A_t} + \textcolor{red}{p}_{Ht} y_t - c_t \right]$$

- **RIR parity:**

$$\ln R_t = \ln \textcolor{red}{R}_t^* + \ln \frac{Q_{t+1}}{Q_t} - \textcolor{red}{\wp} a_{t+1}$$

**Optimal policy**

# SOCIAL WELFARE FUNCTION

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Planner maximises

$$\mathbb{W}_0 = \sum_{t=0}^{\infty} \beta^t \left[ \underbrace{(1 - \vartheta) \sum_{s=-\infty}^t \vartheta^{t-s} \int u(c_t^s(i)) di}_{\text{flow utility to planner at time } t} - v(n_t) \right]$$

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**RANK:**  $\Sigma_t = 1$

**HANK:**  $\Sigma_t > 1$



## WELFARE COST OF INEQUALITY $\Sigma_t$

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- Overall index  $\Sigma_t$  combines **within** ( $\Sigma_{u,t}, \Sigma_{h,t}$ ) and **between** ( $\Upsilon_t$ ) group inequalities

$$\Sigma_t = (1 - \theta) e^{-\gamma\theta\Upsilon_t} \Sigma_{u,t} + \theta e^{\gamma(1-\theta)\Upsilon_t} \Sigma_{h,t}$$

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- Within **unconstrained**:

$$\Sigma_{u,t} = e^{\frac{\gamma^2 \sigma_{cu,t}^2}{2}} [1 - \vartheta + \vartheta \Sigma_{u,t-1}]$$

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- Within **HtM**:

$$\Sigma_{h,t} = \frac{1 - \vartheta}{e^{-\frac{\gamma^2 \sigma_{y,t}^2}{2}} - \vartheta}$$

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- Between**:

$$\Upsilon_t = c_t(u) - c_t(h)$$

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$$\Sigma_{h,t} = \frac{1 - \vartheta}{e^{-\frac{\gamma^2 \sigma_{y,t}^2}{2}} - \vartheta}$$

- Between**:

$$\Upsilon_t = c_t(u) - c_t(h)$$

- $\Upsilon_t > 0 \Rightarrow$  relatively less weight on inequality within group  $u$

# POLICY INSTRUMENTS

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- **Fiscal policy:**  $\{\tau, \tau^*, \tau^w, \tau_t^a\}$  optimally set ex ante and unresponsive to aggregate shocks
- **Monetary policy:**  $\{i_t\}$  adjusted optimally in response to aggregate shocks

# POLICY INSTRUMENTS

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- **Fiscal policy:**  $\{\tau, \tau^*, \tau^w, \tau_t^a\}$  optimally set ex ante and unresponsive to aggregate shocks
  - $\tau$  balances monopolistic distortions
  - $\tau^w$  balances labour-wedge distortions
  - $\tau^*$  kills steady-state capital outflow
  - $\tau_0^a$  kills unhedged interest-rate exposure
  - results in **constrained-efficient** steady state
- **Monetary policy:**  $\{i_t\}$  adjusted optimally in response to aggregate shocks

## **Domestic productivity shocks**



# DOMESTIC PRODUCTIVITY SHOCK

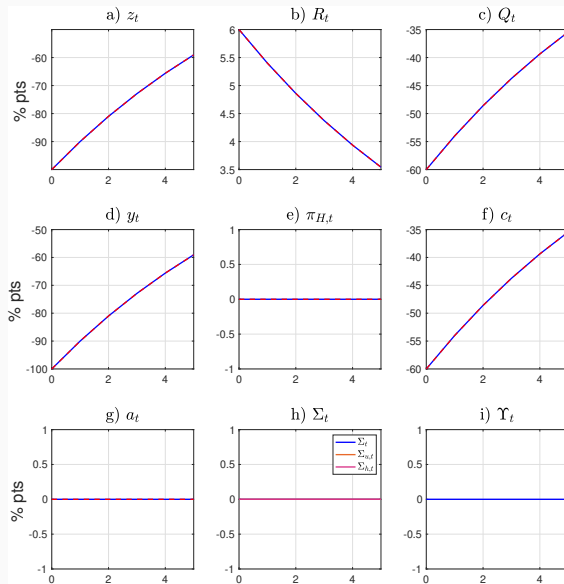
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- **RANK** benchmark: Galí & Monacelli (2005)
- With  $\gamma = \eta = \nu = 1$ , **domestic PPI stability** is optimal  $\Rightarrow$  “inward-looking” policy
- Optimal allocation features

$$c_t = p_H(Q_t)y_t \quad a_t = 0 \quad \Pi_{H,t} = 1 \quad \forall t \geq 0$$

- Implementable by monetary policy **with or without** international risk sharing  
(in latter case, HHs **choose** not to borrow/lend from abroad)

# $z_t$ -SHOCK (RANK)



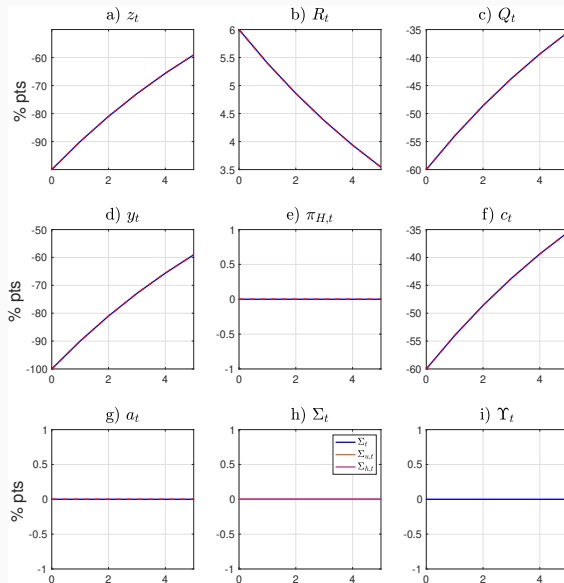
**Proposition:** Under “Cole-Obstfeld” elasticities ( $\gamma = \eta = \nu = 1$ ) and acyclical income risk ( $\varphi = 0$ ), optimal monetary policy implements strict producer price stability in SOE-HANK, regardless of the fraction of HtM households ( $\theta$ ) or the size of income risk ( $\sigma_{y,t}$ ).

**Intuition:**

- Acyclical risk  $\Rightarrow$  within-group inequality cannot be manipulated by the central bank
- Cole-Obstfeld  $\Rightarrow$  unconstrained as a whole do not save  $\Rightarrow$  no between-group inequality

**Sketch of proof:** Show that dynamics under planner’s FOCs replicates flex-price allocation

# $z_t$ -SHOCK (HANK, COLE-OBSTFELD, ACYCLICAL RISK)



# BREAKDOWN OF DIVINE COINCIDENCE

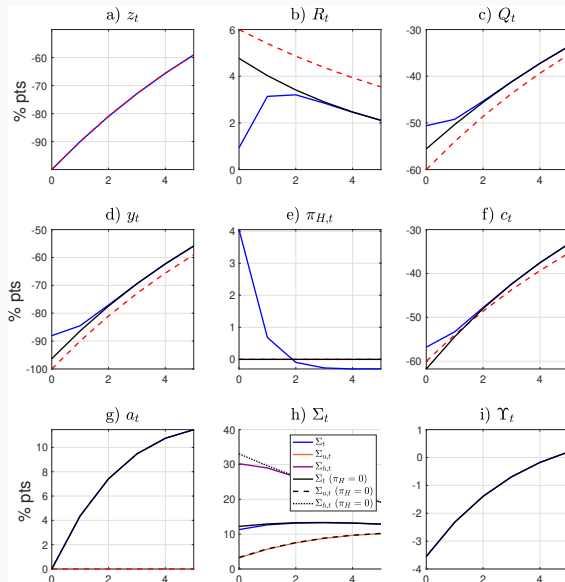
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Former calibration is an (unrealistic) **benchmark**

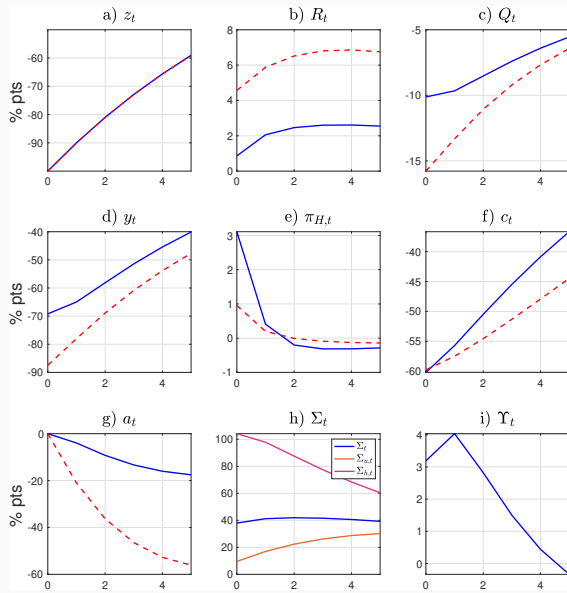
Switch to:

- countercyclical risk  $\Rightarrow \varphi = 5$  as in Acharya et al. (2023)
- Higher trade elasticities, smaller EIS  $\Rightarrow \eta = 1.5, \nu = 4, \gamma = 2$  as in Egorov-Mukhin (2023)

# $z_t$ -SHOCK (HANK, CO, COUNTERCYCLICAL RISK)



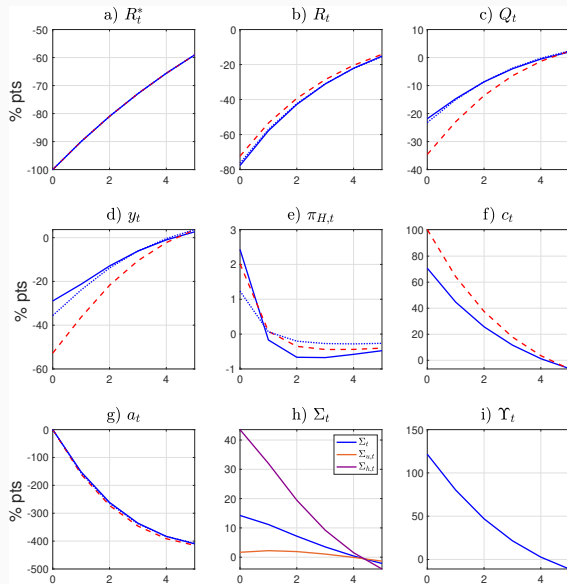
# $z_t$ -SHOCK (HANK, NON-CO, COUNTERCYCLICAL RISK)



## **Capital flow shocks**



# $R^*$ -SHOCK (HANK, NON-CO, COUNTERCYCLICAL RISK)



# CONCLUSION

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- Acyclical risk + Cole-Obstfeld  $\Rightarrow$  **SOE-HANK divine coincidence**  
(i.e., Cole-Obstfeld matters for ToT manipulation **and** for inequality)
- Breaks down under more plausible risk (counter)cyclicalities and (higher) trade elasticities
- Optimal policy implements **less volatile** exchange rate and output in HANK
  - **[unequal exposures]**  $\Rightarrow$  reduces differences in real incomes btw  $u$  and  $h$  HHs
  - **[countercyclical risk]**  $\Rightarrow$  reduces fluctuations of within-group inequality

Newborn  $i$  at date  $s$  max

$$\mathbb{E}_s \sum_{t=s}^{\infty} (\beta \vartheta)^{t-s} \left( u(c_t^s(i, u)) - v(n_t) \right)$$

s.t.

$$c_t^s(i, u) + (1 + \tau^*) \frac{\vartheta}{R_t} a_{t+1}^s(i) = \mathbf{y}_t^s(i, u) + (1 - \tau_t^a) a_t^s(i) \quad a_t^t(i) = a_t$$

Newborn  $i$  at date  $s$  max

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$$\mathbf{y}_t^s(i, u) = (1 - \tau^w) w_t n_t e_t^s(i, u) + \mathcal{D}_t + \mathcal{T}_t + \mathbb{T}_t$$

Newborn  $i$  at date  $s$  max

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$$e_t = 1 + \sigma_t \xi_t, \quad \xi_t = \xi_{t-1} + v_t$$

Newborn  $i$  at date  $s$  max

$$\mathbb{E}_s \sum_{t=s}^{\infty} (\beta \vartheta)^{t-s} \left( -\frac{1}{\gamma} e^{-\gamma c_t^s(i)} - v(n_t) \right)$$

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$$c_t^s(i, u) + (1 + \tau^*) \frac{\vartheta}{R_t} a_{t+1}^s(i) = \mathbf{y}_t^s(i, u) + (1 - \tau_t^a) a_t^s(i)$$

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$$e_t = 1 + \sigma_t \xi_t, \quad \xi_t = \xi_{t-1} + v_t, \quad v \sim \mathcal{N}(0, 1)$$

# UNCONSTRAINED HOUSEHOLDS

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Newborn  $i$  at date  $s$  max

$$\mathbb{E}_s \sum_{t=s}^{\infty} (\beta \vartheta)^{t-s} \left( -\frac{1}{\gamma} e^{-\gamma c_t^s(i)} - v(n_t) \right)$$

s.t.

$$c_t^s(i, u) + (1 + \tau^*) \frac{\vartheta}{R_t} a_{t+1}^s(i) = \mathbf{y}_t^s(i, u) + (1 - \tau_t^a) a_t^s(i)$$

$$\mathbf{y}_t^s(i, u) = \underbrace{\frac{P_{H,t}}{P_t} y_t}_{\text{national income}} + \mathbb{T}_t + \sigma_{y,t} \xi_t^s(i) \quad \sigma_{y,t} = \sigma_y e^{-\varphi \hat{y}_t}$$

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**Euler equation:**

$$e^{-\gamma c_t^s(i, u)} = \left( \frac{\beta R_t}{1 + \tau^*} \right) \mathbb{E}_t \left[ e^{-\gamma c_{t+1}^s(i, u)} \right]$$



## DEMAND SYSTEM

- Final cons. goods produced by competitive retailers aggregating varieties from all countries
- Their production functions are

$$c = \left[ \alpha^{\frac{1}{\eta}} c_F^{\frac{\eta-1}{\eta}} + (1-\alpha)^{\frac{1}{\eta}} c_H^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \quad c_H = \left[ \int_0^1 c_H(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad c_F = \left[ \int_0^1 c_k^{\frac{\nu-1}{\nu}} dk \right]^{\frac{\nu}{\nu-1}}$$

- Let  $p_{H,t}, p_{F,t}$  be the prices of the home and foreign baskets in terms of home consumption
- Profit minimisation + zero-profit condition gives the demands

$$c_{H,t} = (1-\alpha)p_{H,t}^{-\eta} c_t \quad c_{F,t} = (1-\alpha)p_{F,t}^{-\eta} c_t$$

where

$$(1-\alpha)p_{H,t}^{1-\eta} + \alpha p_{F,t}^{1-\eta} = 1 \quad \text{and} \quad p_{F,t} = Q_t$$

- Conversely, the demand for home goods by the RoW is

$$c_{Ht}^* = \alpha \left( \frac{p_{H,t}}{Q_t} \right)^{-\nu} c^*$$

# LABOUR SUPPLY

- Setup similar to Auclert et al. (2023): Each HH supplies a continuum of labour types to a continuum of unions, each of which demands the same number of hours from all members
- Each union is benevolent and utilitarian, and sets wages accordingly
- With flexible wages, the optimality condition boils down to

$$\underbrace{(1 - \tau^w) w_t}_{\text{post-tax wage}} = \underbrace{\mathcal{M}_w}_{\text{markup}} \times \underbrace{\frac{v'(n_t)}{u'(c_t) \Sigma_t}}_{\text{"avg. MRS"}}$$

where

$$\Sigma_t = (1 - \vartheta) \sum_{s=-\infty}^t \vartheta^{t-s} \int e^{-\gamma[c_t^s(i) - c_t]} di$$

captures the dispersion in marginal utility between the members of every union