Regional Trade Policy Uncertainty[*]

Céline Poilly[†] Fabien Tripier[‡]

June 3, 2024

Abstract

Higher uncertainty about trade policy has recessionary effects in U.S. states. First, this paper builds a novel empirical measure of regional trade policy uncertainty, based on the volatility of national import tariffs at the sectoral level and the sectoral composition of imports in U.S. states. We show that a state which is more exposed to an unanticipated increase in the volatility of tariffs suffers from a larger drop in real output and employment, relative to the average U.S. state. We then build a regional open-economy model and we argue that the transmission channels of uncertainty shocks, in particular the precautionary-pricing channel, are magnified in regions that feature the highest import share and the strongest export intensity. Furthermore, we show that an expansionary monetary policy may amplify the regional divergence since – by raising expected inflation – it worsens the recession in the most-exposed region to trade policy uncertainty.

JEL classification: **E32, E52, F41.**

Keywords: Uncertainty Shocks, Regional Effects, Precautionary Pricing, Monetary Policy.

---

[*] Comments and remarks by Jonas Adolph, Giovanni Caggiano, Efrem Castelnuovo, Andreas Dibiasi, Francesco Furnaletto, Gernot Müller, Daniele Siena, Morten Olsen are gratefully acknowledged. We would like to thank the participants at the DEMUR workshop (Marseille), the Heidelberg-Tübingen-Hohenheim Workshop, the Workshop "Non-Linearities in Macro" (QMUL, London), the 5th Biennial Conference on “Uncertainty, Economic Activity, and Forecasting in a Changing Environment” (Padova, Italy), as well as all the seminar participants. Anastasiia Antonova provided outstanding research assistance. The project leading to this publication has received funding from the french government under the “France 2030” investment plan managed by the French National Research Agency (reference :ANR-17-EURE-0020) and from Excellence Initiative of Aix-Marseille University - A*MIDEX. It was also supported by French National Research Agency Grant ANR-20-CE26-DEMUR. Céline Poilly thanks the Institut Universitaire de France for its financial support.

[†] Aix-Marseille Univ., CNRS, AMSE, France. Email: celine.poilly@univ-amu.fr

[‡] Université Paris Dauphine, PSL Research University, LEDa, France, & CEPREMAP. Email: fabien.tripier@dauphine.fr

1 Introduction

Over the recent years, many countries have experienced economic and political disturbances initiated by the revision in some trade agreements and a general foster in trade protectionism. The most prominent events are the Brexit voted in 2016 or the trade war launched by the Trump administration in 2018. These events have led to an exceptional boost in uncertainty regarding the future market conditions, especially in the import-export market. The impact of higher trade policy uncertainty – which might source from unpredictable trade agreements negotiations or from a rising volatility in import tariffs – has been extensively analyzed in the literature (see Handley & Limão (2022) for a survey). Trade policy uncertainty has individual effects on firms' investment, employment or export decisions as well as aggregate effects on GDP at the country level. While tariffs on imports are decided at the national level, one might suspect that changes in tariffs and the uncertainty surrounding them have differentiated effects at the regional level, depending on the composition of imports of each region. In this paper, we tackle this question by analyzing the heterogeneous effects of uncertainty at the regional level, focusing on the U.S. states level.

One contribution of this paper is to propose a novel empirical measure of trade policy uncertainty, at the U.S. state level. We capture trade policy uncertainty through the volatility of import tariffs. In a first step, we construct national series of tariffs at the sectoral level based on the share of duties (collected by U.S. customs) over imports for each sector. We then estimate the time-varying volatility of those sectoral tariffs series using a stochastic volatility model as previously done by Born & Pfeifer (2014) and Fernández-Villaverde et al. (2015). In a second step, we build the exposure to uncertainty at the state level by weighting the sector-based volatility by the share of sectoral imports in total imports within the state. We find for instance that the sectors "Stone and Glass" or "Textile" are those who suffer the most from tariff volatility and some states, like the New-York state for instance, are especially exposed to trade policy uncertainty since they intensively import those products. With our U.S. state-level measure of exposure to trade policy uncertainty in hands, we investigate the effects of uncertainty shocks on U.S. states' economic activity. Our estimation shows a significant and sizable negative effect of a higher exposure to trade policy uncertainty on the economic activity whether this is measured by employment or GDP cumulative growth at the regional level.

We then develop an open two-region model to investigate the mechanisms by which trade policy uncertainty affects the activity of U.S. states. In our model, both regions make up the Home country – illustrating the United States – and each region trades with the rest of the world. In order to capture the heterogeneous exposure to trade policy uncertainty, we assume that the two regions differ in the import share (i.e. the ratio import-to-GDP) as well as the degree of export intensity that we define as the value of exports relative to total value of exports in the country. The model is calibrated using the empirical evidence emphasized in the empirical part, by splitting the sample between the exposed and less-exposed U.S. states to trade policy uncertainty. We feed the model with the empirical relevant series of trade policy uncertainty shocks so as to extract model-based series of output. This strategy allows us to confront the model to the data. We find that our stylized model, when it is submitted to the same data-generating process than the empirical data, is able to reproduce the empirical finding that higher exposure to trade policy uncertainty deteriorates the economic activity. The fact that our model is regional allows us to further analyze how trade integration heterogeneity leads different regions to be differently impacted by trade policy uncertainty. In our model, Region 1 is the most open region since it has the highest *(i)* import-to-GDP share and, *(ii)* degree of export intensity.

We obtain several results. First, we find that Region 1, when it is hit by volatility shocks on import tariffs to a same amount than Region 2, features a stronger recession. This result is explained by the feedback loop between the Foreign country and the most exposed region, combined with a precautionary-pricing behavior of firms. Indeed, firms in this region are more exposed to uncertainty and therefore are more uncertain about their future marginal cost. As a result, they adopt a stronger precautionary-pricing behavior, which leads to a magnified increase in markups – which can be seen as a self-insurance mechanism– and a substantial recession. Combined with the precautionary-saving behavior of households, aggregate demand in the Home country drops, reducing the demand for Foreign goods, especially for the region featuring the largest import share (Region 1). As the Foreign country also imports less, this region – which has also the highest degree of export intensity – needs to produce less. Due to the presence of nominal rigidities in the Home country, prices do not adjust as quickly as they should, leading to a stronger recession and a even larger increase in markups in the exposed region. Our second result concerns the role of monetary policy in the transmission channel of uncertainty shocks. Indeed, we show that the monetary policy may yield counterproductive outcomes in Region 1, the most exposed region. We assume that the central bank directly reacts to trade policy uncertainty shocks by reducing its nominal interest rate, on top of the reaction to inflation and output. This policy experiment is motivated by the pressures exerted by Donald Trump on the Fed Chairman Jérôme Powell in 2019, in the midst of trade tensions with China. When regions are symmetric, the uncertainty shock causes a strong recession that can indeed be mitigated by an expansionary monetary policy intervention. This is no longer the case, when regions are asymmetric. In this context, only the less-open Region, which is weakly exposed to the uncertainty shock, benefits from the monetary impulse and even experiences an expansion. Why is the recession in the most-open region worsened when the central bank intervenes to counter the recessionary effects of uncertainty shocks? By reducing the nominal interest rate, the central bank rises expected inflation in the exposed region, which magnifies the upward-pricing channel and makes the recession deeper. We are not the first to emphasize the importance of the precautionary-pricing behavior in the transmission channels of uncertainty shocks (see for instance Fernández-Villaverde et al. (2015) and Born & Pfeifer (2021)). We argue here that through this channel, an accommodative monetary policy might have damaging effects in the economy depending on the regional degree of exposure to uncertainty.

Contributions to the literature

Our first contribution is to enrich the literature on measures of uncertainty by providing an original measure of trade policy uncertainty at the regional level. On the one hand, existing measures of uncertainty about trade policy do not have a regional dimension, and on the other hand, existing measures of uncertainty at the regional level do not specifically address the issue of trade policy.

There is a large literature that measures trade policy uncertainty based on various data sources and concepts of uncertainty. Handley & Limão (2022) provides an extensive survey of the tools to measure trade policy uncertainty and its economic effects. For instance, uncertainty about trade policy can be measured through a change in trade agreements (Handley (2014); Handley & Limão (2015, 2017)), textual analysis of newspapers (Baker et al. (2016)), Economist Intelligence Unit country reports (Ahir et al. (2022)) or finally text analysis of earnings conference calls by firms (Caldara et al. (2020)). Our contribution to this literature is to provide a measure of uncertainty at the regional level based on the volatility of import tariffs. It is a common practice in macroeconomics to use tax volatility as a measure of uncertainty in the tradition of Born & Pfeifer (2014) and Fernández-Villaverde et al. (2015). These authors resort to a stochastic volatility model to estimate the time-varying volatility of taxes and exogenous shocks to this volatility. Caldara et al. (2020) apply this process to measure tariff uncertainty at the national level for the U.S. economy. We show in the paper how this methodology can be enriched to deliver measure of tariff uncertainty at the state level which we refer to as regional trade policy uncertainty.

In this way, we are also joining the important branch of the literature devoted to the disaggregation of uncertainty, below the national level. Baker et al. (2022) provide historical series economic policy uncertainty at the U.S. State level.[1] We differ from them in the methodology since they resort to textual analysis on local newspapers, which allows them to distinguish whether uncertainty comes from local events or national/international events. On top of this,

---

[1] Elkamhi et al. (2023) provide a similar index, using a different set of newspapers and removing the nationwide information. Shoag & Veuger (2016) adopt a similar approach focusing in the Great Recession.

we contribute to the literature by providing a U.S. state-level measure of uncertainty that is specific to trade policy.

Our second contribution is to extend the literature on regional macroeconomic models. To our best knowledge, the analysis of trade policy uncertainty in a regional perspective has not yet been studied in this literature. There is an increasing interest in macroeconomics for regional/state data to understand global phenomenon – see Chodorow-Reich (2020) for a discussion on the use of regional data in macroeconomics. The interest for regional data is to improve the identification of causal relationships compared to macroeconomic data which generally aggregate several simultaneous events. The literature on regional local fiscal multipliers is reviewed in Chodorow-Reich (2019), notably the leading contribution by Chodorow-Reich et al. (2012), Nakamura & Steinsson (2014) and Dupor & Guerrero (2017). Theoretical models with a special focus on regional economies are developed for instance in House et al. (2021); House et al. (2018); Beraja et al. (2018); Beraja et al. (2019). The local effects of trade policy have also been extensively studied in this literature to better understand the consequences of China's integration into international trade by Autor et al. (2013) and of the protectionist measures put in place by the USA in 2018 by Fajgelbaum et al. (2020). However, this literature on local effects of trade policy is almost silent when it comes the uncertainty about this policy, partly because of the lack of data. Since we develop a measure of this type of uncertainty at regional level, we can supplement this literature by considering the role of uncertainty shocks using regional data in a structural open economy model. We show how trade policy uncertainty at national level can be a source of inequalities between regions, which can be exacerbated by expansionary monetary policy due to heterogeneity between regions.

We contribute to the broad literature that studies the effects of uncertainty shocks in macroeconomics models, as surveyed by Fernández-Villaverde & Guerrón-Quintana (2020) and Castelnuovo (2023), for instance. In particular, we shed light on the role of the precautionary-pricing behavior of firms when they face uncertainty. There is an increasing attention towards this channel that has been revealed by Fernández-Villaverde et al. (2015) who show that under a standard CES production, profit is a concave and asymmetric function in prices. Therefore, when price-maker firms have to set a price in expectation, they tend to choose a price that is too high compared to the optimal price – or equivalently, to increase markups – in order to reduce their lost profits damages. Higher volatility reinforces this behavior since firms are risk averse. Oh (2020) shows that the precautionary-pricing behavior is strong in a model featuring Calvo-price setting, even at the steady state, because of the dispersion of relative prices. Born & Pfeifer (2021) make a empirical assessment of this channel, showing that their model-based measure of price markups vary little with uncertainty. Cho et al. (2021) investigate optimal monetary policy in presence of precautionary pricing. In line with our result, they show that monetary policy matters when expected inflation is volatile. Andreasen et al. (2024) argue that precautionary pricing bevhavior is stronger during recessions than expansions, partly because inflation volatility is usually stronger during recessions. Our paper contributes to this literature by showing that the precautionary-pricing behavior might account for understanding the regional effects of trade policy uncertainty shocks.

The paper is organized as follows. Section 2 builds an empirical measure of regional trade policy uncertainty and it investigates the effects on the U.S. State economies. Section 3 describes the theoretical model while Section 4 explains its parametrization. Section 5 analyzes the transmission channels of trade policy uncertainty shocks and Section 6 concludes.

2 Empirical Analysis

In this section, we lay out the methodology to build a novel measure of trade policy uncertainty (TPU, henceforth) based on sector-specific volatility of import tariffs and we highlight the effects of TPU shocks on the economic activity at the U.S. state level. We proceed in three steps. First, we construct a measure of tariff uncertainty by extracting the time-varying volatility of (national) import tariff series at the sectoral level. Second, we use these series to measure the exposure to tariffs uncertainty of U.S. states, taking into account the sectoral composition of imports by state. Third, we resort to a panel estimation to quantify the effects of TPU on U.S. states economic indicators.[2]

2.1 A Measure of Tariff Volatility

We construct sector-specific series of tariff rate by using data from the Census Bureau Foreign Trade Database (USA Trade Online). We extract the U.S. monthly data on imports by type of commodity.[3] Using the Harmonized System Codes of the World Customs Organization of 2017,

---

[2] All data sources are detailed in the online appendix.

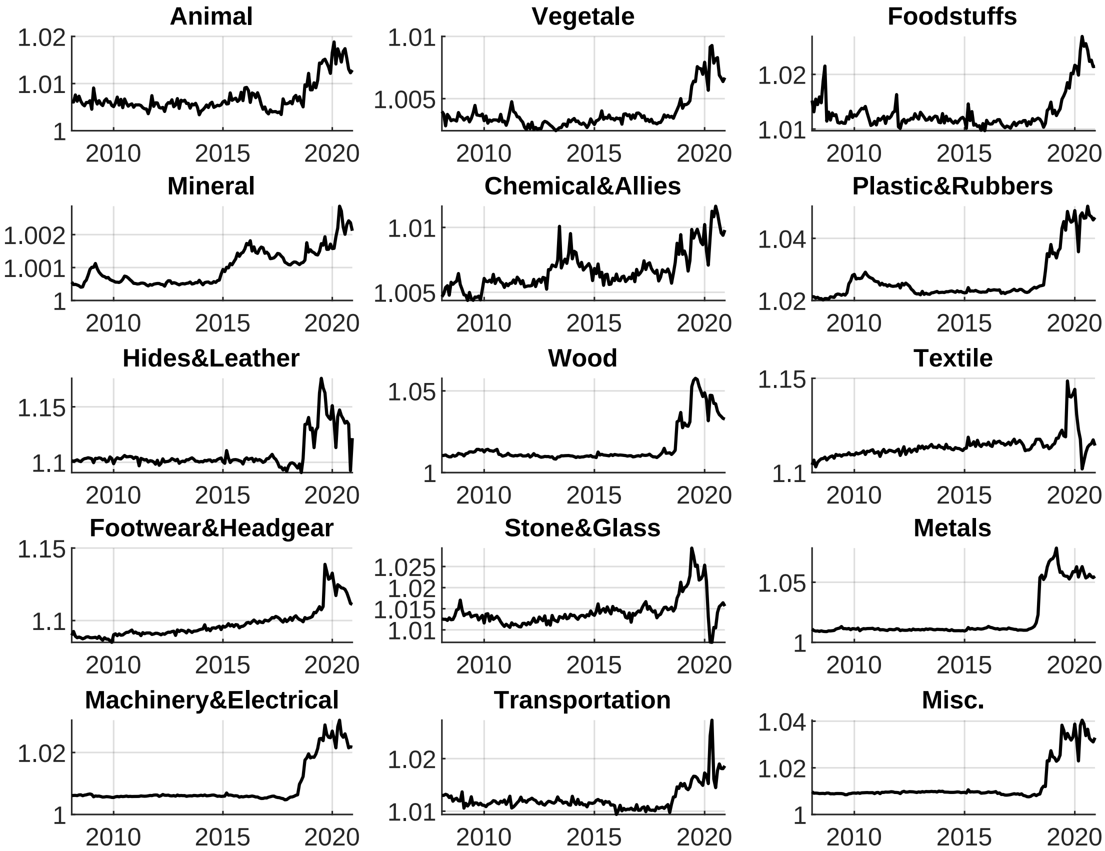
[3] Notice that the database covers trade in the manufacturing and the agricultural sectors, excluding therefore trade in services. We use imports for consumption, i.e. "Measures the total of merchandise that has physically cleared through Customs either entering consumption channels immediately or entering after withdrawal for consumption from bonded warehouses or Foreign Trade Zones under U.S. Customs and Border Protection (CBP) custody" (source: Census).

we convert commodity types into 15 sectors.[4] The data sample goes from 2008m2 to 2020m12. Let $p = 1, ..., P$, denote the sector index with $P = 15$ and $t = 1, ..., T$, the time index. Let $M_{p,t}$ denote the total real value of imports in sector p at time t, and $CD_{p,t}$, the estimates of calculated duty.[5] We define the gross rate of the national effective tariff at the sectoral level, $\tau_{p,t}$, as

$$\tau_{p,t} = 1 + \frac{CD_{p,t}}{M_{p,t}}. \tag{1}$$

Figure 1 displays the gross import tariff rate, $\tau_{p,t}$, for the 15 sectors over our sample.

Figure 1: Sector-specific gross tariff rate



Note: The sector-specific gross tariff rate, $\tau_{p,t}$, is defined in Equation (1).

---

[4] The 15 sectors are classified using the 2-digit HS code list: "01-05 Animal & Animal Products", "06-15 Vegetable Products", "16-24 Foodstuffs", "25-27 Mineral Products", "28-38 Chemicals & Allied Industries", "39-40 Plastics / Rubbers", "41-43 Raw Hides, Skins, Leather, & Furs", "44-49 Wood & Wood Products", "50-63 Textiles", "64-67 Footwear / Headgear", "68-71 Stone / Glass", "72-83 Metals", "84-85 Machinery / Electrical", "86-89 Transportation", "90-97 Miscellaneous". The Harmonized System Codes of the World Customs Organization of 2017 is a natural candidate since our raw data are classified using the Harmonized System. Santacreu et al. (2023) also construct a sectoral classification, which in their case aggregates their raw data classified with the NAICS (2012) classification. In total, we share more than 70% of their 16 sectors.

[5] The value of imports and custom duties at the sectoral level are computed by summing all types of commodity holding to a sector. All series are seasonally adjusted.

Unsurprisingly, most of the sectors experienced a surge in their import tariffs during the U.S. Trade War initiated in 2018 by President D. Trump mostly against China, but also against other countries. The first wave of tariffs increase started in Spring 2018 as the Federal government started to impose tariffs on steel and aluminum imports against several countries – including China but also Canada –, which led to an average tariff rate in the metal sector of 1.2% in February 2018 and 7.8% in March 2019. In September 2019, supplementary tariffs has been imposed on imports, targeting textile products from China, leading to an average tariff rate in the textile sector of 11.9% in August 2019 and 14.9% in September 2019. Despite this exceptional period of high volatility, Figure 1 also shows some heterogeneity in tariffs across sectors over the full sample. For example, in September 2009, the Obama administration signed China-specific safeguard actions against imports of tires for three years by imposing tariffs of 35% in the first year, 30% in the second year and 25% in the third year.[6] The Figure reports this protective trade policy through an average tariff rate in the sector "Plastic and Rubbers" changed from 2.1% in August 2009 to 2.8% in December 2009.

The stochastic volatility model is an attractive setup to disentangle level shocks from volatility shocks and therefore to extract time-varying uncertainty (see Fernández-Villaverde & Guerrón-Quintana (2020) for a discussion). On top of this, this statistical process is in line with typical way of modeling time-varying volatility shocks in the DSGE-type of models (see Section 3). Fernández-Villaverde et al. (2011) applied this methodology to extract time-varying volatility on the real interest rates, Fernández-Villaverde et al. (2015) on fiscal instruments, and closer to us, Caldara et al. (2020) estimated a stochastic-volatility process on import tariffs. In this spirit, we assume that $\Delta\tau_{p,t}$ – the monthly growth rate of $\tau_{p,t}$ – follows an AR(1) process with time-varying volatility[7]

$$\Delta\tau_{p,t} = \rho_p^{\tau}\Delta\tau_{p,t-1} + \alpha_p\Delta\tau_{-p,t} + \exp(\sigma_{p,t}^{\tau})\varepsilon_{p,t}^{\tau}, \tag{2}$$

with

$$\sigma_{p,t}^{\tau} = (1-\rho_p^{\sigma})\bar{\sigma}_p^{\tau} + \rho_p^{\sigma}\sigma_{p,t-1}^{\tau} + \eta_p\varepsilon_{p,t}^{\sigma}, \tag{3}$$

where $\varepsilon_{p,t}^{\tau} \sim \mathcal{N}(0,1)$ and $\varepsilon_{p,t}^{\sigma} \sim \mathcal{N}(0,1)$ for $p = 1,...P$. Let $\sigma_{p,t}^{\tau}$ denote the measure of trade policy uncertainty in sector p. Following Fernández-Villaverde et al. (2011), $\sigma_{p,t}^{\tau}$ is expressed in exponential in Equation (2) to ensure that trade policy uncertainty remains positive. Level (first-order) shocks, $\varepsilon_{p,t}^{\tau}$, correspond to unexpected variations in the sector-p tariff growth rate and volatility (second-order) shocks, $\varepsilon_{p,t}^{\sigma}$, correspond to unexpected variations in its standard

---

[6] These tariffs applied to products 4011.10.10, 4011.10.50, 4011.20.10, and 4011.20.50 of the Harmonized Tariff Schedule of the United States. See Bown & Kolb (2023) for a discussion about the reform.

[7] Notice that $\Delta\tau_{p,t}$ is demeaned in the estimation of Equation (2). We estimate the stochastic-volatility process on the growth rate of the tariff rate so as to mitigate the non stationarity of the series resulting from the 2018' trade war.

deviation. Parameter ρ_p^τ drives the persistence of level shocks and ρ_p^σ drives the persistence of volatility shocks, $\varepsilon_{p,t}^\sigma$. $\bar{\sigma}_p^\tau$ gives the average standard deviation of innovations $\varepsilon_{p,t}^\tau$ and η_p governs the magnitude of tariff volatility shocks. The process (2)-(3) is estimated sector by sector. Because we are interested in sector-specific volatility, we control for the common variations in tariffs coming from others sectors than p. Precisely, we sum the gross effective tax rates over all sectors but p and we compute the associated growth rate, denoted $\Delta\tau_{-p,t}$. Thus, parameter α_p captures variations in $\Delta\tau_{p,t}$ which are explained by the aggregate dynamics of all other sectors.

Regressions (2)-(3) capture the dynamics of sectoral growth rate tariffs as well as their time-varying standard deviation. Caldara et al. (2020) estimate a similar stochastic volatility process on U.S. aggregated series of tariffs while we consider herein sector-p specific tariff uncertainty. Using the algorithm provided by Born & Pfeifer (2014), Equations (2)-(3) are estimated jointly with Bayesian methods using sequential Monte Carlo Methods. We obtain priors directly from the data using simple least-squares regressions. The historical series of the unobserved volatility shock are extracted using a non-linear particle filter from the posterior distribution computed using the MCMC algorithm. The posterior distributions of parameters are computed from 20 000 draws where the first 5 000 are discarded. The Metropolis-Hastings parameter is adjusted to get an acceptance rate of roughly 25%.

Figure 2 plots the historical series of tariffs uncertainty as estimated in Equation (3).[8] The solid line is the median historical smoothed estimate of $\sigma_{p,t}^\tau$, expressed in exponential and multiplied by 100. The grey area displays the first and last deciles, respectively. The figure confirms that if tariff volatility is sector specific, all sectors have experienced a peak in volatility starting in 2018. As mentioned above, “Footwear and Headgear”, “Foodstuffs”and “Textile”display a high volatility in average. However, some sectors concentrate high volatility in tariffs during the 2018 trade war peak, as for instance “Stone and Glass” or “Metals”. We now exploit this heterogeneity across sectors to capture U.S. states exposure to TPU.

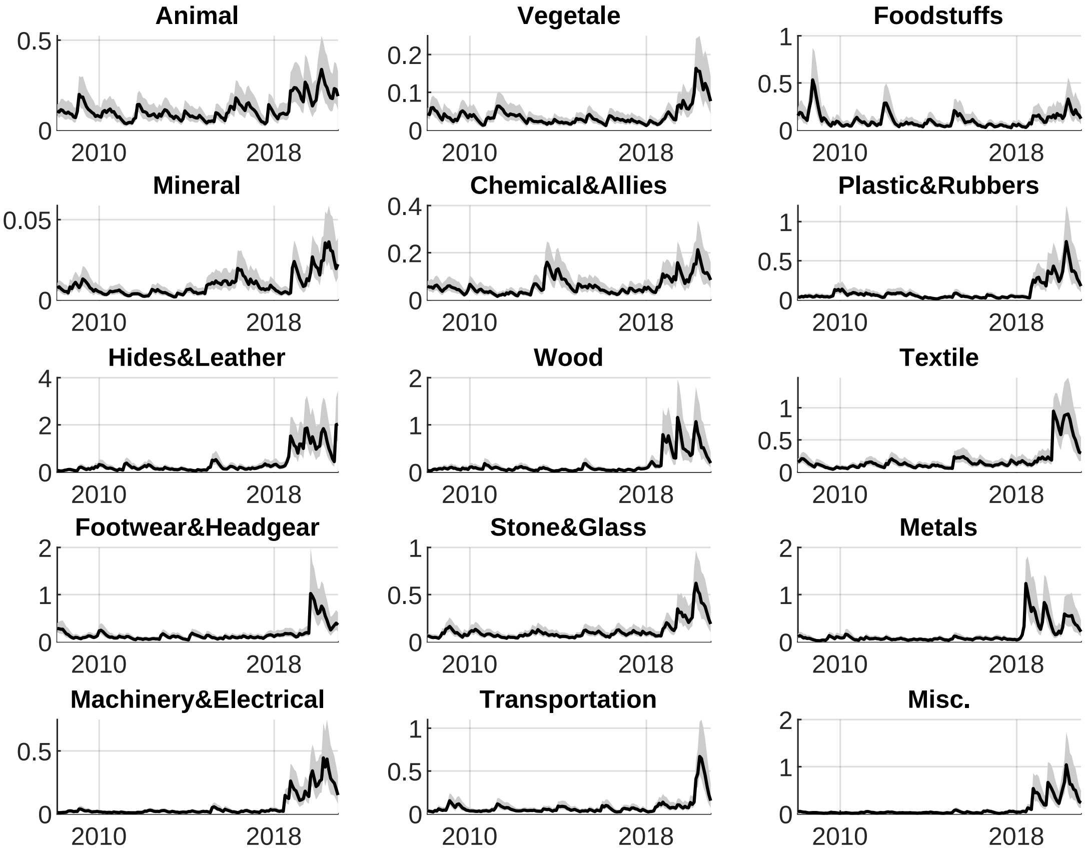
2.2 State-Level Exposure to Uncertainty

In the spirit of the shift-share design, we assess the effects of trade policy uncertainty on the U.S. states by using the weighted average of the sectoral shocks as a regressor, the weight being the state-exposure share (see Adão et al. (2019) and Goldsmith-Pinkham et al. (2020) for recent discussion of shift-share settings). This strategy allows us to proxy state-level TPU shocks by assigning volatility shocks to the sectoral composition of imports. In pratice, we build a series of exposure to TPU shocks in state-s ($TPU_{s,t}$), weighting volatility shocks ($\varepsilon_{p,t}^\sigma$) by the state-specific

---

[8] The posterior estimates are reported in the online appendix.

Figure 2: Sector-specific tariff volatility series



Note: The tariff volatility $\sigma^{\tau}_{p,t}$ is defined in Equations (2)-(3). The displayed series are $(100\times\exp(\sigma^{\tau}_{p,t}))$ based on the posterior estimation results reported in the online appendix. The solid line is the median historical smoothed series and the grey area displays the $[10^{th},90^{th}]$ percentile probability interval.

sectoral composition of imports ($\mu_{p,s,t}$) defined as

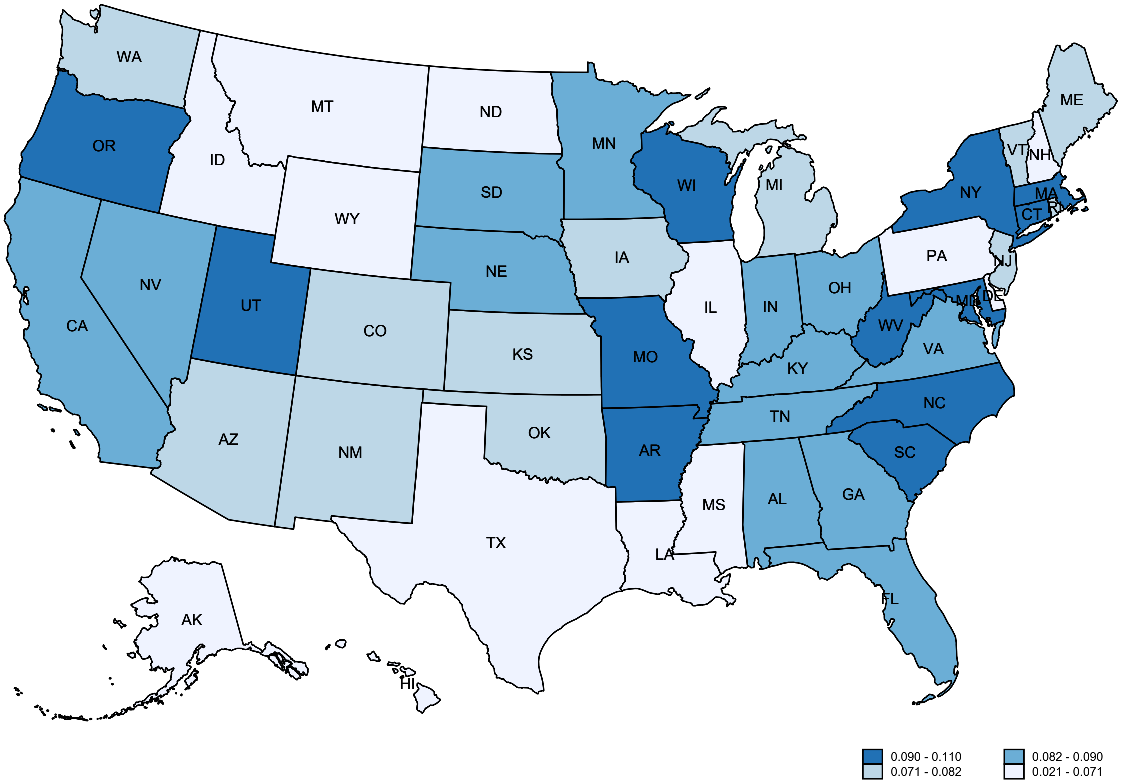
$$\mu_{p,s,t} \equiv \frac{M_{p,s,t}}{M_{s,t}}. \tag{4}$$

$M_{p,s,t}$ corresponds to the sector-p custom values collected from imports in state s at time t and $\mu_{p,s,t}$ is therefore the share of sector-p imports in total imports for state s at time t. Series of imports at the state level, $M_{p,s,t}$, are here again extracted from the Census Bureau Foreign Trade Database and are also available at a monthly frequency. In order to mitigate endogeneity issues, we use the constant share $\bar{\mu}_{p,s}$ equal to the average value of the import share over the first three years of the sample (2009m1-2011m12) for each state. The series of exposure to TPU shocks in state-s ($TPU_{s,t}$) is then

$$TPU_{s,t} = \sum_{p=1}^{P} \bar{\mu}_{p,s} \times \varepsilon^{\sigma}_{p,t}, \tag{5}$$

where $s = 1, ..., 50$, $\bar{\mu}_{p,s} = (1/36) \times \sum_{t=1}^{36} \mu_{p,s,t}$ using the definition of $\mu_{p,s,t}$ given by Equation (4), and $\varepsilon^{\sigma}_{p,t}$ estimated from Equation (3).

Figure 3: Tariff-volatility exposure across the U.S. states



Note: Colors illustrate states' exposure to tariff volatility shocks, $TPU_{s,t}$, which are computed from Equation (5), averaged over the sample 2008m2-2020m12. Darker colors represent states which are the most exposed to tariff volatility shocks.

Figure 3 plots the average degree of exposure to trade policy uncertainty, $TPU_{s,t}$, of each state. Darker colors correspond to the states which are the most exposed to sectors displaying high volatility shocks in import tariffs. The most exposed state is the state of New York (NY) which is also in the top 5 of the largest importers in the U.S. Note that 28% of imports reaching NY comes from the sector "Stone and Glass " and 11% from the sector "Textile". As mentioned above, these two sectors feature a quite high volatility (see Figure 2). They also explain the high degree of exposure of Utah (UT) – 39 % of its import are "Stone and Glass "– and North Carolina (NC) since 16 % of its imports are "Textile". The sector "Metal "also explains the high exposure to tariff volatility. For example, Connecticut (CT) and Maryland (MD) are the two largest importers in this sector (16 % and 15 % of their imports, respectively). As shown in Figure 2, "Metal" has been extensively affected by the 2018 trade war as its volatility of tariffs has jumped.

2.3 Regional Effects of Trade Policy Uncertainty

Armed with a state-level measure of exposure to tariff volatility, we now turn to investigate the economic effects of TPU shocks.

2.3.1 Baseline Estimation

We estimate a cross-sectional regression in the spirit of the "local multipliers" literature (see Chodorow-Reich (2019) and Canova (2024) for instance), which allows us to use the heterogeneity across states to assess the dynamic effects of a policy change. We estimate the following baseline regression

$$y_{s,t+h} - y_{s,t-1} = \delta_{s,h} + \delta_{t,h} + \beta_h TPU_{s,t} + \gamma_h X_{s,t} + v_{s,t+h}, \tag{6}$$

where β_h corresponds to the relative impact of TPU exposure shocks on the outcome variable across states for a given period of time and $y_{s,t+h}$ is the outcome in state s at time $t+h$ with $h \geq 0$. Notice that β_h cannot be interpreted as the aggregate response at the U.S. level. Indeed, it measures the relative change in the outcome variable between two states in response to an increase in TPU exposure in a treated (or shocked) region relative to an untreated one. By resorting to state-level estimation, we therefore depart from Caldara et al. (2020) who estimate a bi-variable SVAR model on U.S. data and show that that positive shocks on tariff volatility are recessionary at the national level. We consider two measures of economic activity, namely the real GDP and employment.[9] The former consists in state-level GDP in private industries and it is available from the Bureau of Economic Analysis. Employment is the total number of employees in the private sector, reported by the Bureau of Labor Statistics. Both measures are expressed in log and Equation (6) is estimated weighting observations by the population share of states in 2008 as in Dupor & Guerrero (2017). Since the real state-level GDP is available at a quarterly frequency, we convert our series of volatility exposure, $TPU_{s,t}$, from monthly to quarterly by an averaging transformation. We include time fixed effects, $\delta_{t,h}$, to absorb all the common shocks in the U.S., as well as state fixed effects, $\delta_{s,h}$, to control for unobservable state-specific factors which might affect the outcome variable. In order to mitigate a potential omitted variable bias, we also include $X_{s,t}$, a vector of state-level control variables. We include as a first control variable the exposure to level shocks, which corresponds to the state exposure to the sectoral tariff shocks as built in Equation (5) where we replace $\varepsilon^{\sigma}_{p,t}$ by $\varepsilon^{\tau}_{p,t}$. By doing so, we seek to isolate the pure effects of a rise in volatility exposure by removing all variations that would come from a level shock, even through the stochastic process (2)-(3) disentangles volatility and level shocks, which are supposed to be uncorrelated. Second, we control for the business cycle by using the growth rate of the Coincident Economic Activity Index provided by the Federal Reserve Bank of Philadelphia (ΔCI). Last, since

---

[9] The online appendix reports robustness results with alternative outcome and control variables.

we focus on trade policy uncertainty, we want to make sure that we do not capture other sources of uncertainty at the national level and so we use EPU as a control variable, i.e. the state-level National Economic Policy Uncertainty index from Baker et al. (2022). The sample goes from 2008q1 to 2020q4. Equation (6) is estimated for horizons $h = 0, 4, 8$ by pooling the 50 states over the 64 quarters.

Table 1 reports the estimation results. The first three columns provide the relative effect of an exogenous rise in the exposure to TPU on state-level real GPD at impact ($h = 0$), after a year ($h = 4$) and after two years ($h = 8$). We find that a higher exposure to trade policy uncertainty have recessionary effects, although it takes one year to materialize. The maximum impact is reached after two years, with the estimate $\beta_h = -0.052$, significant at 1%. If we consider a high shock of 0.068 (corresponding to the 75 percentile of the shock distribution), this means a 0.35 percent cumulated drop in output over two years. When it comes to employment, we also find that a higher exposure to uncertainty significantly reduces employment after a year. The value of the estimates indicates that a shock to exposure of 0.068 reduces employment by 0.19 percent after 8 quarters and it is highly significant.

Uncertainty about trade policy thus has persistent and high magnitude effects on the economic activity of U.S. states. The magnitude of these effects is assessed by controlling for the state's cyclical situation at the time of the shock, measured by the change in the coincident index ΔCI, which is naturally highly correlated with the future evolution of output and employment in states. Interestingly, the economic policy uncertainty at the state level, as measured by the EPU variable, is not significantly related to economic activity in our estimation. This indicates that trade policy uncertainty, as measured by states' exposure to tariff volatility, contains distinct information from the EPU uncertainty variable, which is based on textual analysis and, above all, concerns all aspects of economic policy, not just tariff policy. The last interesting result from Table 1 is that a higher exposure to tariffs (in level) has a positive and significant impact on output, but only at a quarter-4 horizon. This leads us to conclude that a rise in protectionism might have some beneficial effects on the regional economies, but they are very short in time.

2.3.2 Robustness Checks

We now explore the robustness of our findings. Table 2 reports the estimation results for GDP for several alternative specifications. For the sake of space, we focus here on the effects of TPU shocks on real GDP after two years ($h = 8$). In the online appendix, we report the estimation results for all horizons and for both GDP and employment.

Table 1: Effects of TPU shocks on the economic activity

	Real GDP			Employment		
	(I)	(II)	(III)	(IV)	(V)	(VI)
	$h = 0$	$h = 4$	$h = 8$	$h = 0$	$h = 4$	$h = 8$
Volatility exposure (TPU)	-0.005	-0.033^{**}	-0.052^{***}	-0.003	-0.026^{***}	-0.029^{***}
	(0.005)	(0.016)	(0.018)	(0.005)	(0.007)	(0.009)
Level exposure	0.003	0.009^{**}	0.007	0.002	0.002	0.004
	(0.002)	(0.004)	(0.008)	(0.001)	(0.004)	(0.006)
EPU	0.000	0.000	0.000	0.000	-0.000	0.000
	0.000	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
ΔCI	0.151^{***}	0.897^{***}	0.792^{**}	0.333^{***}	0.663^{***}	0.737^{***}
	(0.031)	(0.215)	(0.333)	(0.042)	(0.169)	(0.251)
Observations	$2,550$	$2,350$	$2,150$	$2,550$	$2,350$	$2,150$
State FE	Yes	Yes	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes	Yes	Yes

Note: Table reports the effects of TPU shocks on real GDP in Columns (I)-(III) and employment in Columns (IV)-(VI). Each column reports the results from regression (6) for the horizon $h = 0, 4, 8$ quarters. The coefficient of interest β_h associated to $TPU_{s,t}$ is reported in the first line. All regressions include state- and time-FE. Robust standard errors are clustered at the state level. *** p<0.01, ** p<0.05, * p<0.1.

(I). Spillover Effects The cross-sectional dimension of our analysis helps us to assess the extent to which a stronger exposure of a state to TPU shocks alters its own economic activity, relative to the average state's exposure. Therefore, it fails to capture the spillover effects across units – U.S. states, here – that might emerge because these units are interconnected (see Auerbach et al. (2020), for instance). We check the robustness of our results when we take into account the geographical and the trade spillover effects across states. One state can be affected by its own idiosyncratic TPU shocks but also indirectly by TPU shocks hitting its neighboring states. In this case, our estimation would not fully estimate the economic impact of TPU shocks since these indirect effects are not taken into account. First, we enrich the baseline regression (6) by adding a new variable that corresponds to the sum of all TPU exposure indexes ($TPU_{s,t}$) of the neighboring states. Column (Ia) of Table 2 shows that TPU shocks in the neighboring states do not significantly impact the economic activity of a specific state. This finding suggests that geographical spillovers are small. Interestingly, when we assume that TPU shocks do not come from the neighboring states but from the main economic partner of one state, as defined by Dupor & Guerrero (2017), one find a strong positive effect (see Column (Ib) of Table 2). However, our main estimates is not altered and still significant. This positive spillover effect suggests that the total effect of trade policy uncertainty could be somehow smaller than in our benchmark.

(II). Retaliation Effects Any change in the level and the volatility of import tariffs might lead trade partners to retaliate. In order to control for the reaction of foreign countries, we re-run the benchmark regression by including the increase in retaliation tariff as provided by Fajgelbaum et al. (2020), which is sector specific. More precisely, we aggregate their measure of retaliation tax increase over the 15 sectors presented in Section 2.1 and we build a state-specific retaliation exposure index using as a weight the state's export share to total U.S. exports. Due to data availability, we re-run the estimation over a shorter sample, i.e. 2013q1-2019q2. Column (II) in Table 2 shows that state's GDP reacts negatively but not significantly to retaliation tax at quarter 8. More importantly, a stronger exposure to TPU still significantly reduces GDP while the less pronounced effect is mostly explained by the shorter sample.

(III). Swing-states Effects The last alternative control variable that we consider is related to political considerations. In order to check the exogeneity of our measure of tariff volatility exposure, we investigate the role of swing states in our baseline regression. We use data from the MIT Election Data and Science Lab (MEDSL) which provides the state-level returns for elections to the U.S. presidency from 1976 to 2020. We create a dummy equals to one when a state is considered as a swing state, i.e. the election returns have shifted from Democrat to Republican (or vice-versa) at least once over our period 2008-2020. The swing states are Arizona (AZ), Florida (FL), Georgia (GA), Indiana (IN), Iowa (IA), Michigan (MI), North Carolina (NC), Ohio (OH), Pennsylvania (PA), Wisconsin (WI). We then create an interaction term between the tariff volatility exposure series and the swing-states dummy. This interaction term is negative and significantly different from the zero at the impact (reported in the online appendix), but not at longer horizons, as in shown in Column (III) of Table 2. The estimation of economic effects of TPU shocks are close to those reported in our benchmark regression.

(IV). Sample Without Trade War We estimate the baseline regression over the sample 2008q1-2017q4 to abstract from the 2018 trade war. As shown in Column (IV) of Table 2, estimated coefficients are slightly lower and significantly different from zero only at 10%-level for real GDP, against 1% for the benchmark, but remains significant at the 1% level for employment (reported in the online appendix). It is remarkable that even if we discard from the sample the trade war episode, we identify significant economic effects of shocks to TPU that occurred before this period suggesting that the recessionary effects of trade policy uncertainty is not only driven by this exceptional jump in tariff volatility.

(V). Placebo Shocks The degree of exposure to trade policy uncertainty is built based on the state-specific sectoral composition of imports, as explained in Equation (4). In order to check that our results capture exogenous variations in tariff volatility, instead of being driven by the heterogeneous composition of imports, we run a placebo test by randomly resampling the vector of sectoral tariffs volatility and then building a counterfeit measure of exposure to TPU. Column (V) in Table 2 shows the results. We find that when the baseline regression is estimated using this placebo series, there is no longer significant effects of volatility exposure shocks on the state economic activity.

Table 2: Robustness exercises

	(Ia)	(Ib)	(II)	(III)	(IV)	(V)
Volatility exposure (TPU)	-0.052^{***}	-0.051^{***}	-0.028^{**}	-0.050^{***}	-0.038^{*}	-0.004
	(0.018)	(0.018)	(0.013)	(0.020)	(0.021)	(0.008)
Level exposure	0.008	0.006	0.004	0.007	-0.001	0.003
	(0.008)	(0.008)	(0.009)	(0.008)	(0.008)	(0.008)
EPU	0.000	0.000	0.000	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
ΔCI	0.786^{**}	0.771^{**}	0.798^{***}	0.791^{***}	0.834^{***}	0.804^{***}
	(0.326)	(0.324)	(0.256)	(0.334)	(0.337)	(0.333)
Geographical spillovers	-0.007					
	(0.006)					
Trade spillovers		0.029^{***}				
		(0.011)				
Retaliation effect			-2.780			
			(2.769)			
Swing-state interaction				-0.008		
				(0.024)		
Observations	2,150	2,150	1,200	2,150	1,950	2,150
State FE	Yes	Yes	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes	Yes	Yes

Note: Table reports the effects of TPU shocks on real GDP. Each column reports the results from regression (6) for the horizon $h = 8$ quarters. The coefficient of interest β_h associated to $TPU_{s,t}$ is reported in the first line. Columns (Ia)-(Ib)-(II)-(III) extend the benchmark regression reported in Column (III) of Table 1 (namely, $h = 8$ for Real GDP) by sequentially adding the variables associated to geographic spillovers, trade spillovers, retaliation effect, and swing-state interaction (see text for definitions and sources). Column (IV) corresponds to the pre-2017 sample estimation of the benchmark regression reported in Column (III) of Table 1. Column (V) is the placebo regression of the benchmark regression reported in Column (III) of Table 1 . All regressions include state- and time-FE. Robust standard errors are clustered at the state level. *** p<0.01, ** p<0.05, * p<0.1.

(VI). State-by-State Estimation Canova (2024) highlights potential bias in cross-sectional estimation of average dynamic effect when data are pooled. We then run a state-by-state estimation of Equation (6) where the coefficient of TPU becomes state-specific, denoted by $\beta_{i,h}$. As we cannot consider time-fixed effects in this case, we add the Fed Funds rate, the unemployment rate, and the growth rate of imports for the U.S. economy as national controls. Figure 8 in the appendix reports the state-specific coefficient estimates, $\beta_{s,h}$ where s is the index of the U.S. state and for $h = 8$, using real GDP as the outcome variable. It shows that coefficients are negative for 49 states as there is a single state where the estimated coefficient is positive, still not significantly different from zero. The relatively low dispersion between estimated coefficients suggests that our baseline estimates are not affected by a bias associated with the heterogeneity between states' responses to trade policy uncertainty.

2.3.3 Effects of TPU Shocks on Inflation

Our previous findings have shown that higher exposure to trade policy uncertainty significantly depresses state-level economic activity. However, a full picture of the effects of state-based TPU requires to assess how prices react to higher uncertainty, all the more so because higher volatility on import taxes is likely to alter expected firm's marginal cost and therefore price dynamics. To conduct this analysis, we use the state-specific quarterly inflation rates for the Consumer Price Index (CPI) provided by Hazell et al. (2022). Due to limited data availability, the sample covers the period 2008q1-2017q4. However, we showed in Section 2.3.2 that our results are not entirely driven by the 2018 trade war episode. In addition, CPI inflation series are provided for 32 states, which represent 86% of total U.S. GDP.

Table 3 presents the results when we estimate Regression (6) by setting the outcome variable, $y_{s,t}$ as the price index built from the CPI inflation measure provided by Hazell et al. (2022). Interestingly, the effect of the level tariff shock on trade policy at the impact is positive and highly significant, which is in line with the transmission of tariff shocks to prices e.g. Fajgelbaum et al. (2020). When it comes to the effects of uncertainty shocks about trade policy, we find that cumulative inflation becomes significantly negative after two years. This positive co-movement between output and prices suggests that (import tariffs) uncertainty shocks look like demand shocks, as argued by Leduc & Liu (2016) and Basu & Bundick (2017) for instance. In the next section, we theoretically emphasize that – besides this aggregate demand effect – uncertainty shocks about trade policy affect price-setting decisions through a higher volatility of expected future marginal costs.

Table 3: Effects of TPU shocks on inflation

	(I) $h = 0$	(II) $h = 4$	(III) $h = 8$
Volatility exposure (TPU)	-0.001 (0.010)	-0.050^{*} (0.030)	-0.062^{***} (0.031)
Level exposure	0.003^{***} (0.001)	0.007 (0.006)	0.002 (0.010)
EPU	0.000 0.000	0.000 (0.000)	-0.000 (0.000)
ΔCI	0.105^{***} (0.007)	-0.258 (0.322)	-1.162^{***} (0.408)
Observations	1,287	1,155	1,023
State FE	Yes	Yes	Yes
Time FE	Yes	Yes	Yes

Note: Table reports the effects of TPU shocks on inflation in Columns (I)-(III). Each column reports the results from regression (6) for the horizon $h = 0, 4, 8$ quarters. The coefficient of interest β_h associated to $TPU_{s,t}$ is reported in the first line. All regressions include state- and time-FE. Robust standard errors are clustered at the state level. *** p<0.01, ** p<0.05, * p<0.1.

3 Theoretical Model

We now describe the theoretical model which allows us to go further in the understanding of the transmission channels of TPU shocks at the regional level. The economy is made up of two countries, Home and Foreign. The Home country is split into two regions of size η_j, where $j = \{1, 2\}$ is the region-specific index. The Foreign country is considered as a whole, his size being η^*, such that we normalize $\eta_1 + \eta_2 + \eta^* = 1$. The two Home regions belong to a monetary union and they trade intermediary goods with the Foreign country only, imposing national import taxes on imported intermediate goods. We abstract from any trade across regions as well as labor and capital mobility.[10]

3.1 Production Sector

In each region of the Home country, at time t, a representative retailer produces a non-tradable final good (Y_t) by aggregating differentiated intermediate goods. Intermediate goods are produced by monopolistic competitors who combine domestic and imported individual goods through a CES production function. They face price adjustment costs à la Calvo (1983) and we consider a producer currency pricing setting implying that the law of one price holds. Finally, the individual

---

[10] The online appendix provides details of the computations.

goods are produced in each region by a representative firm who uses labor and capital. In the Foreign country, the production structure is the same, the representative retailer using a combination of domestic and imported individual goods, imported from both regions. Foreign variables are denoted with a star *. For notation convenience of trade-related variables, the lower index of these variables corresponds to the country/region of origin (for the production of goods) and the upper index corresponds to the country/region of destination. We start describing in details the production in the Home country and then we present briefly the Foreign country when it differs.

3.1.1 Retailers

In Region j of the Home country, a representative retailer produces a final good Y_t^j by combining type-i diffentiated intermediate goods, $Y_t^j(i)$ using a Dixit-Stiglitz production function

$$Y_t^j = \left[\int_0^1 Y_t^j(i)^{\frac{\varepsilon_p - 1}{\varepsilon_p}} \mathrm{d}i\right]^{\frac{\varepsilon_p}{\varepsilon_p - 1}}, \tag{7}$$

where $\varepsilon_p > 1$ is the elasticity of substitution between intermediate goods. Let P_t^j denote the aggregate price index of the Home Region-j final good Y_t^j and $P_t^j(i)$ denote the price of type-i region-j intermediate good, $Y_t^j(i)$. The profit maximization yields the typical demand function for intermediate goods $Y_t^j(i)$

$$Y_t^j(i) = \rho_t^j(i)^{-\varepsilon_p} Y_t^j, \tag{8}$$

where $\rho_t^j(i) \equiv P_t^j(i)/P_t^j$ is the relative price of differentiated goods.

3.1.2 Intermediate Good Production

In Region j of the Home country, we assume a continuum of monopolistic competitive firms, indexed by $i \in [0,1]$. Type-i firm produces a non-tradable intermediate goods $Y_t^j(i)$ by combining domestic and imported individual goods (Y_{Ht}^j and Y_{Ft}^j, respectively) through a CES production function

$$Y_t^j(i) = \left[(1-\omega_j)^{\frac{1}{\theta}}(Y_{Ht}^j)^{\frac{\theta-1}{\theta}} + (\omega_j)^{\frac{1}{\theta}}(Y_{Ft}^j)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}, \tag{9}$$

where $\theta > 1$ is the trade elasticity of substitution between domestic and foreign goods and parameter $\omega_j \in [0,1]$ is the degree of openness, i.e. it drives the magnitude of the home bias in Region j. Let P_{Ht}^j and P_{Ft} denote the price index of the two individual goods, Y_{Ht}^j and Y_{Ft}^j, respectively.[11]

---

[11] The imported quantity of good Y_{Ft}^j may differ according to the region j, but the price P_{Ft} is identical for the two regions.

In a first step, the type-i monopolistic producer chooses the amount of domestic, $Y^j_{Ht}(i)$, and imported, $Y^j_{Ft}(i)$, individual goods so as to minimize his real cost of production, denoted by $S^j_t(i)$, taking into account the production function (9), with

$$S^j_t \equiv \rho^j_{Ht} Y^j_{Ht} + (1+\tau_t)\, \rho^j_{Ft} Y^j_{Ft}, \tag{10}$$

where $\rho^j_{Ht} \equiv P^j_{Ht}/P^j_t$ and $\rho^j_{Ft} \equiv P_{Ft}/P^j_t$ are the relative prices of domestic and imported individual goods, respectively. We assume that importing individual goods, Y^j_{Ft}, is costly through the introduction of a national import tax denoted by τ_t. Since the tariff policy at Home is national, this rate is identical in both regions. The minimization program yields the demand for domestic and imported individual goods, expressed at the symmetric equilibrium,

$$Y^j_{Ht} = (1-\omega_j)\left(\frac{\rho^j_{H,t}}{\Lambda^j_t}\right)^{-\theta} Y^j_t(i), \quad \text{and} \quad Y^j_{Ft} = \omega_j \left(\frac{(1+\tau_t)\,\rho^j_{Ft}}{\Lambda^j_t}\right)^{-\theta} Y^j_t(i), \tag{11}$$

where Λ^j_t denotes the real marginal cost of the intermediate good producer in Region j.

In a second stage, the type-i monopolistic competitor in Region j chooses the price of its good, $P^j_t(i)$, by maximizing its profits $d^j_t(i)$. We assume that price setting is modeled through a Calvo price setting. Precisely, a type-i intermediate good producer keeps its previous price with probability α_p and resets its price with probability $1-\alpha_p$. This gives the standard non-linear New-Keynesian Phillips Curve (NKPC)

$$\pi^{j\star}_t = \pi^j_t \left(\frac{\varepsilon_p}{\varepsilon_p - 1}\right) \frac{X^j_{1t}}{X^j_{2t}}, \tag{12}$$

where $\pi^{j\star}_t \equiv P^{j\star}_t / P^j_{t-1}$ and $P^{j\star}_t$ denotes the optimal reset price, all expressed at the symmetric equilibrium. In addition, X^j_{1t} and X^j_{2t} are defined recursively as

$$X^j_{1t} = \Lambda^j_t Y^j_t + \alpha_p \mathrm{E}_t \left\{ \beta_{t,t+1} (\pi^j_{t+1})^{\varepsilon_p} X^j_{1t+1} \right\}, \tag{13}$$

$$X^j_{2t} = Y^j_t + \alpha_p \mathrm{E}_t \left\{ \beta_{t,t+1} (\pi^j_{t+1})^{\varepsilon_p - 1} X^j_{2t+1} \right\}, \tag{14}$$

where $\beta_{t,t+1}$ is the stochastic discount factor of the household and E_t is the expectation operator conditional to the information available in period t. Expressing the aggregate price index in terms of inflation rate yields

$$(\pi^j_t)^{1-\varepsilon_p} = (1-\alpha_p)\,(\pi^{j\star}_t)^{1-\alpha_p} + \alpha_p. \tag{15}$$

Notice that the real marginal cost Λ_t^j in Region j is increasing with import tariff, τ_t, with

$$\Lambda_t^j = \rho_{H,t}^j \frac{Y_{Ht}^j}{Y_t^j} + (1+\tau_t)\, \rho_{Ft}^j \frac{Y_{Ft}^j}{Y_t^j}. \tag{16}$$

As standard in the literature, the NKPC given by Equation (12) tells us that monopolistic competitive firms set their price taking into account the future expected real marginal cost. In our regional open-economy model, a region which is more open (ω_j large) will have its real marginal cost more sensitive to the import tax τ_t (see Equations (11) or (16)). We show in the next section that higher uncertainty about tariffs generates more dispersed marginal costs which in turn pulls the trigger of the precautionary pricing channel.

3.1.3 Wholesaler

In Region j of the Home country, we assume perfectly competitive wholesale producers who produce the good $X_{j,t}$ by using capital services, $k_{j,t}$, and labor $\ell_{j,t}$. The wholesale good is then split into intermediate goods used domestically, denoted Y_{Ht}^j, and abroad, denoted $Y_{Hj,t}^*$, such that

$$X_{j,t} = Y_{Ht}^j + Y_{Hj,t}^*. \tag{17}$$

The wholesale good $X_{j,t}$ is produced with a Cobb-Douglas production function

$$X_{j,t} = a_t \eta_j k_{j,t}^{\alpha} \left(\ell_{j,t}\right)^{1-\alpha}, \tag{18}$$

where a_t corresponds to a total factor productivity (TFP) shock, defined below. The representative wholesaler chooses the amount of capital and labor services, as well as the supply of intermediate goods, so as to maximize its real profits denoted by $\tilde{d}_{j,t}$, subject to the production function (18) and definition (17). The real profits write

$$\tilde{d}_{j,t} = \rho_{Ht}^j Y_{Ht}^j + Q_{j,t} \rho_{Hj,t}^* Y_{Hj,t}^* - w_{j,t} \eta_j \ell_{j,t} - z_{j,t} \eta_j k_{j,t}, \tag{19}$$

where $\rho_{Hj,t}^* \equiv P_{Hj,t}^*/P_t^*$, $w_{j,t}$ and $z_{j,t}$ denote the Region-j real wage and rental rate of capital, respectively. The real exchange rate in Region j is defined as the price of Foreign consumption goods basket relative to the price of Region-j consumption goods basket, such that $Q_{j,t} \equiv e_t P_t^*/P_t^j$, where e_t is the nominal exchange rate. The law of one price holds, such that

$$\rho_{Ht}^j = Q_{j,t} \rho_{Hj,t}^*. \tag{20}$$

Therefore, intermediate good producers charge the same price for both their domestic and their import markets. Profit maximization yields the labor-demand and capital-demand equations

$$w_{j,t} = (1-\alpha)\,\rho^{j}_{Ht}\frac{X_{j,t}}{\eta_j \ell_{j,t}}, \quad \text{and} \quad z_{j,t} = \alpha\rho^{j}_{Ht}\frac{X_{j,t}}{\eta_j k_{j,t}}. \tag{21}$$

3.1.4 Foreign Country

We now turn to describe the production sector in the Foreign country. For the sake of simplicity, we keep the Foreign country as stylized as possible, abstracting for instance from nominal rigidities. The Foreign market creates a trade connection between the two regions of the Home country since the Foreign final good, Y^*_t, is produced by using both domestic, $Y^*_{F,t}$, and imported individual goods, $\tilde{Y}^*_{H,t}$, using the following production function

$$Y^*_t = \left[(1-\omega^*)^{\frac{1}{\theta}}(Y^*_{F,t})^{\frac{\theta-1}{\theta}} + (\omega^*)^{\frac{1}{\theta}}(\tilde{Y}^*_{H,t})^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}, \tag{22}$$

where $\omega^* \in [0,1]$ is the Foreign degree of trade openness. We assume that the imported individual good, $\tilde{Y}^*_{H,t}$, is a bundle of imported goods from each Home region

$$\tilde{Y}^*_{H,t} = \left[(\tilde{\omega})^{\frac{1}{\theta}}(Y^*_{H1,t})^{\frac{\theta-1}{\theta}} + (1-\tilde{\omega})^{\frac{1}{\theta}}(Y^*_{H2,t})^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}. \tag{23}$$

Parameter $\tilde{\omega} \in [0,1]$ drives the degree of preference of Foreign producers for Region 1's individual goods. Let P^*_t denotes the price of the final good Y^*_t, P^*_{Ft} the price of domestic individual goods $Y^*_{F,t}$, and $P^*_{Hj,t}$ the price of individual goods imported from Region j. The profit maximization of the final good producer gives the demand equation for domestic goods as

$$Y^*_{F,t} = (1-\omega^*)\left(\rho^*_{F,t}\right)^{-\theta} Y^*_t, \tag{24}$$

with $\rho^*_{F,t} \equiv P^*_{Ft}/P^*_t$, the relative price of domestically produced individual goods, and the demand equation for imported goods as

$$Y^*_{H1,t} = \omega^*\tilde{\omega}\left(\rho^*_{H1,t}\right)^{-\theta} Y^*_t, \quad \text{and} \quad Y^*_{H2,t} = \omega^*(1-\tilde{\omega})\left(\rho^*_{H2,t}\right)^{-\theta} Y^*_t, \tag{25}$$

with $\rho^*_{Hj,t} \equiv P^*_{Hj,t}/P^*_t$, the relative price of the imported goods from region j. A stronger preference for Region 1 ($\tilde{\omega}$) increases the imports of goods coming from this region.

Similarly to the Home country, we assume that a representative wholesaler produces an individual good which is sent domestically ($Y^*_{F,t}$) or exported in each region of the Home country (Y^1_{Ft} and Y^2_{Ft}), such that

$$X^*_t = Y^*_{F,t} + Y^1_{Ft} + Y^2_{Ft}. \tag{26}$$

The good X_t^* is produced using capital, k_t^*, and labor, ℓ_t^*, through the following Cobb-Douglas production function

$$X_t^* = \eta^* \left(k_t^*\right)^{\alpha} \left(\ell_t^*\right)^{1-\alpha}. \tag{27}$$

The demand for labor and capital are therefore given by

$$w_t^* = (1-\alpha)\, \rho_{Ft}^* \frac{X_t^*}{\eta^* \ell_t^*}, \quad \text{and} \quad z_t^* = \alpha \rho_{Ft}^* \frac{X_t^*}{\eta^* k_t^*}. \tag{28}$$

while the law of one price implies that

$$\rho_{Ft}^* = \frac{\rho_{Ft}^1}{Q_{1,t}}, \quad \text{and} \quad \rho_{Ft}^1 = \frac{Q_{1,t}}{Q_{2,t}} \rho_{Ft}^2. \tag{29}$$

The zero profit condition gives

$$\rho_{Ft}^* Y_{Ft}^* + \frac{\rho_{Ft}^1}{Q_{1,t}} Y_{Ft}^1 + \frac{\rho_{Ft}^2}{Q_{2,t}} Y_{Ft}^2 = w_t^* \ell_t^* + z_t^* k_t^*. \tag{30}$$

3.2 Rest of the model

The rest of the model is standard. We assume that households make their saving decisions under financial autarky, implying that they do not have access to international lending or borrowing and thus there is no opportunity to share risks across borders. For the sake of brevity, we focus here only on the Home country.

3.2.1 Household

The Home country is populated by a continuum of identical households. In each region, they choose a sequence of consumption ($c_{j,t}$), regional bonds ($b_{H,t}^j$) and they supply labor ($\ell_{j,t}$), capital services ($k_{j,t}$) to maximize the discounted lifetime utility. The objective of the representative household is thus to maximize

$$\mathrm{E}_t \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{1-\sigma} \left[\left(c_{j,t} - \psi \left(\ell_{j,t}\right)^{\omega_w} \mathcal{X}_{j,t} \right)^{1-\sigma} - 1 \right] \right\}, \tag{31}$$

where the term $\mathcal{X}_{j,t}$ is defined by

$$\mathcal{X}_{j,t} = c_{j,t}^{\sigma_X} \mathcal{X}_{j,t-1}^{1-\sigma_X}, \tag{32}$$

with $\beta \in (0,1)$ is the subjective discount factor, σ_X drives the strength of the wealth effect on labor supply, ω_w^{-1} is the Frisch elasticity of labor supply, ψ is a normalizing constant (which governs the relative disutility of labor effort) and σ is the inverse of the risk aversion coefficient. We assume financial autarky such that the representative household has access in $t-1$ to a one-period regional bond ($B^j_{H,t-1}$), which pays off a gross riskless interest rate R^j_t. She can also save by investing in physical capital by the amount $i_{j,t}$ and she receives labor income, with the real wage denoted $w_{j,t}$, and capital services revenue, with the real return denoted $z_{j,t}$. Last, the household perceives dividend income from differentiated goods producers and lump-sum transfers from the government. Accordingly, the budget constraint writes down as

$$c_{j,t} + b^j_{H,t} + i_{j,t} = \frac{R_{t-1}}{\pi_{j,t}} b^j_{H,t-1} + w_{j,t}\ell_{j,t} + z_{j,t}k_{j,t} + T_{j,t} + T^f_{j,t}, \tag{33}$$

where $b^j_{H,t} = B^j_{H,t}/P^j_t$ denotes the amount of bonds held in real terms, $T_{j,t}$ and $T^f_{j,t}$ are lump-sum transfers of profits from differentiated firms and of the international bond adjustment, respectively. We assume that the law of motion of capital is given by

$$k_{j,t+1} = (1-\delta)\,k_{j,t} + \left[1 - \frac{\varkappa}{2}\left(\frac{i_{j,t}}{i_{j,t-1}} - 1\right)^2\right] i_{j,t}, \tag{34}$$

where $\varkappa$ drives the strength of investment adjustment costs. Let λ_t, υ_t and $\breve{\xi}_t$ denote the Lagrangian multipliers associated to the budget constraint (33), the definition (32), and the law of motion of capital (34), respectively. For the sake of space, first-order conditions, which are standard, are reported in Appendix B..

3.2.2 Fiscal and Monetary Policy

State-specific government in each country runs a balanced budget

$$\eta_j T^f_{j,t} = \eta_j b^j_{H,t} - \frac{R^j_{t-1}}{\pi_{j,t}} \eta_j b^j_{H,t-1} + \tau \rho^j_{Ft} Y^j_{Ft} - g_{j,t}. \tag{35}$$

where $g_{j,t}$ corresponds to region-specific public spending, which follows an exogenous process as defined below. In addition, all regions in the Home country are part of a monetary union. The Home central bank sets the nominal interest rate rule according to the following Taylor Rule

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}}\right)^{\rho_r} \left(\frac{\pi_t}{\bar{\pi}}\right)^{(1-\rho_r)\rho_\pi} \tag{36}$$

where ρ_r drives the degree of interest rate inertia and ρ_π is the weight on CPI national inflation, defined as $\pi_t = \frac{\eta_1}{\eta_1+\eta_2}\pi_{1,t} + \frac{\eta_2}{\eta_1+\eta_2}\pi_{2,t}$. The real exchange rate at the national level is given by

$$Q_t = (Q_{1,t})^{\frac{\eta_1}{\eta_1+\eta_2}} (Q_{2,t})^{\frac{\eta_2}{\eta_1+\eta_2}}. \tag{37}$$

In the Foreign country, due to the absence of nominal rigidities, we assume that the monetary policy simply follows $\pi_t^* = 0$. Regional interest rates R_t^j are equal to the national interest R_t augmented by an inflation premium

$$\frac{R_t^j}{\bar{R}^j} = \frac{R_t}{\bar{R}} \left(\frac{\pi_t^j}{\bar{\pi}^j} \right)^{\varpi} \tag{38}$$

with $\varpi > 0$ and $\bar{\pi}^j$ is the steady-state gross inflation rate. The elasticity of the interest rate to the inflation rate, $\varpi > 0$, corresponds to the existence of an inflation premium that has been documented in financial markets; e.g. Bekaert & Wang (2010) and Hördahl & Tristani (2012). When inflation exceeds its steady-state value in Region j, the increase in the nominal interest rate R_t^j above the interest rate set by the national central bank, R_t, responds to a demand from investors for protection against the risk of inflation. Without explicitly modeling the behavior of these investors and the financial markets, Equation (38) is a shortcut for taking into account the inflation premium and it ensures model's stability.

3.2.3 Equilibrium and Shocks

The market clearing equation in Region j on the final goods market

$$Y_t^j = \eta_j c_{j,t} + \eta_j i_{j,t} + \eta_j g_{j,t}, \tag{39}$$

and under financial autarky, the trade balance of each region is zero implying that the value of exports from Region j equals the value of imports

$$Q_{j,t} \rho_{Hj,t}^* Y_{Hj,t}^* = \rho_{Ft}^j Y_{Ft}^j. \tag{40}$$

We assume that import tariffs follow an AR(1) process with stochastic volatility such that

$$\begin{aligned} (1+\tau_t) &= (1-\rho_\tau)(1+\bar{\tau}) + \rho_\tau (1+\tau_{t-1}) + \exp(\sigma_t^\tau) \varepsilon_t^\tau, & (41) \\ \sigma_t^\tau &= (1-\rho_\sigma)\bar{\sigma}^\tau + \rho_\sigma \sigma_{t-1} + \gamma \varepsilon_t^\sigma, & (42) \end{aligned}$$

where ρ_τ and ρ_σ are persistence parameters of level and volatility tariff shocks, respectively, γ drives the magnitude of tariff uncertainty shocks and $\bar{\sigma}^\tau$ is the steady-state standard deviation. The two other level shocks, namely the supply-driven TFP and the demand-driven public spending shocks follow an AR(1) process[12]

$$a_t = (1-\rho_a)\,\bar{a} + \rho_a a_{t-1} + \exp(\sigma^a)\varepsilon_t^a, \tag{43}$$

$$g_t = (1-\rho_g)\,\bar{g} + \rho_g g_{t-1} + \exp(\sigma^g)\varepsilon_t^g, \tag{44}$$

where ρ_x and σ^x are the persistence and standard-deviation parameters with $x = \{a, g\}$. All innovations $\varepsilon_t^z \sim \mathcal{N}(0,1)$ for $z = \{\tau, \sigma, a, g\}$.

4 Parametrization

We now calibrate the set of parameters following standard values in the literature and some empirical targets. A period in our regional model is a quarter. We assume that the two regions of the Home country as well as the Foreign country are perfectly symmetric, except regarding their size and some trade parameters, as explained below. Table 4 reports our parametrization. Only parameters which are specific to a given region or a country are subscripted by an index.

Common Parameters We start with parameters which are common to all location. As standard in the literature, the discount factor, β is set to 0.99 which fits an annual interest rate of 4%. The capital share, α, is equal to 0.33 so as to fit a labor income share of 0.67 and the capital depreciation rate $\delta = 0.025$ implying an annual depreciation rate of 10%. We set the elasticity of substitution between intermediate goods, ε_p, to 11, implying a markup of 10%. The trade elasticity equals $\theta = 1.5$, which is in line with the literature (see Caldara et al. (2020) for instance). The Calvo parameter is set to $\alpha_p = 0.75$ which corresponds to a price frequency change of one year. Household's preference parameters are standard in the literature. We set the degree of risk aversion $\sigma = 2$ and we calibrate the strength of the wealth effect ($\sigma_X = 0.001$) following Born & Pfeifer (2014), as for the Frisch elasticity ($\omega_w = 1$). The scale parameter ψ is set to ensure that steady-state hours worked correspond to $\bar{\ell} = 0.33$ of total hours. The investment adjustment cost is set to $\kappa = 6$. When it comes to the Taylor rule, we choose the standard value by assuming that inflation reaction parameter to the nominal interest rate is $\rho_\pi = 1.5$ and the interest rate inertia parameter is $\rho_r = 0.8$. Parameter $\varpi = 1.2$ needs to be set to a sufficiently high value to ensure stability of the model. We set the share of public spending-to-GDP on our sample mean and we get $\bar{g}/\bar{y} = 0.18$. Finally, we set the steady-state value of import tax to $\bar{\tau} = 5\%$ in both Home regions, which corresponds to our sample mean. We abstract from any retaliatory taxation.

---

[12] Although we are primarily interested in the effect of the uncertainty shock, the quantitative fit of our model requires to incorporate all standard sources of fluctuations, i.e. demand and supply shocks.

Table 4. Parametrization

Common Parameters			
β	Discount factor	0.99	Annual interest rate (4%)
α	Capital share	0.33	Labor income share (0.67)
δ	Capital depreciation rate	0.025	Annual depreciation rate (10%)
ε_p	Elasticity of substitution btw intermediate goods	11	Markup (10%)
θ	Trade elasticity	1.5	Caldara et al. (2020)
α_p	Calvo parameter	0.75	Price adj. of one year
σ	Degree of risk aversion	2	Born & Pfeifer (2014)
ω_w^{-1}	Frisch elasticity of labor supply	1	Born & Pfeifer (2014)
σ_X	Wealth effect	0.001	Born & Pfeifer (2014)
$\bar{\ell}$	Steady state labor	0.33	Standard calibration
κ	Investment adjustment cost	6	Standard calibration
ρ_π	Taylor rule parameter wrt inflation	1.5	Standard calibration
ρ_r	Interest rate inertia	0.8	Standard calibration
ϖ	Inflation premium	1.2	Assure model's stability
g/y	Share of government expenditure in output	0.18	Bureau of Economic Analysis
Region-specific Parameters			
$\tilde{\omega}$	Region-1 export intensity	0.84	Highest export intensity (0.68)
ω_1	Region-1 degree of trade openess	0.14	Import share (0.13)
ω_2	Region-2 degree of trade openess	0.08	Import share (0.08)
ω^*	Foreign degree of trade openess	0.25	Foreign import share (0.10)
η^*	Size of Foreign	0.5	U.S. population share (0.5)
η_1	Size of Region-1	0.315	Population share (0.63)
η_2	Size of Region-2	0.185	Population share (0.37)
$\bar{\tau}$	U.S. import tax	0.05	Foreign Trade Database
Shocks Parameters			
ρ_τ	Tariff level shock: persistence	0.99	Own estimation
$\bar{\sigma}_\tau$	Tariff level shock: s.d	-5.83	Own estimation
ρ_σ	Tariff volatility shock: persistence	0.99	Own estimation
η	Tariff volatility shock: magnitude	0.28	Own estimation
ρ_a	TFP shock: persistence	0.76	Own estimation
σ_a	TFP level shock: s.d	log(0.008)	Own estimation
ρ_g	Public spending shock: persistence	0.90	Own estimation
σ_g	Public spending shock: s.d	log(0.014)	Own estimation

Region-specific Parameters We now turn to the calibration of parameters which are region-specific. We first need to choose the values of a set of parameters which determine trade openness of the two Home regions as well as the Foreign country (ω_1, ω_2, ω^*, $\tilde{\omega}$). We pick these values to fit four targets from the empirical analysis presented in Section 2. More precisely, we need to differentiate the two regions in the Home country and so, we assume that they are exposed to international trade to a different extent. Notice that we define in this paper the degree of export intensity as the relative real export value to total U.S. real exports. We split our sample of 50 U.S. states into two groups of a same size. The first (second, resp.) group consists in states which have a degree of export intensity in 2008 below (above, resp.) the median.[13] The degree of export intensity gives us a target to calibrate $\tilde{\omega}$. Targeting the sum of export intensity (0.68 in Region 1 and thus 0.32 in Region 2) leads us to $\tilde{\omega} = 0.25$. We then use the same two groups of states to parametrize ω_j and η_j, with $j = 1, 2$. First, we pick values for ω_1 and ω_2 so as to target the 2008 average import-to-GDP share of high and low export intensity states, i.e. 0.13 and 0.08, respectively. These empirical targets lead to $\omega_1 = 0.14$ and $\omega_2 = 0.08$. We assume that the import-to-GDP share in the Foreign country is 10%, which implies $\omega^* = 0.25$. We also target the population share in the two group of U.S. states, based on the empirical finding that high-export intensity group consists in 63% of the total U.S. population share in 2008 (and thus 0.37% for the low-export intensity group). Assuming that the Foreign country consists in 50% of the world economy ($\eta^* = 0.5$), we obtain $\eta_2 = 0.315$ and $\eta_1 = 0.185$.

Shocks parameters We parametrize the exogenous driving process outside the model by using available aggregate U.S. series and estimating an AR(1) over the sample 1980q1-2020q4. First, we build the quarterly series of U.S. tariffs rate as in Caldara et al. (2020), with $\tau_t = CD_t/(M_t - CD_t)$ where CD_t is the taxes on production and imports received by the Federal government and M_t is the real value of total imports.[14] We then estimate a stochastic volatility process as in Equations (41)-(42), here again using the algorithm of Born & Pfeifer (2014) and select the mean values of the posterior distribution of the parameters. This parametrization strategy gives us $\rho_\tau = 0.99$ and $\rho_\sigma = 0.99$ as well as $\gamma = 0.28$ and $\bar{\sigma}^\tau = -5.83$, which is in line with the results by Caldara et al. (2020). When it comes to the level shocks, TFP and public spending, we estimate an AR(1) as in Equations (43) and (44), respectively. The TFP series is borrowed to Fernald

---

[13] The degree of export intensity in a state $s \in [0, 50]$, denoted $\mathcal{D}_s$, is computed as $\mathcal{D}_s = \frac{X_{s,2008}}{\sum_{s=1}^{S} X_{s,2008}}$, where $X_{s,2008}$ is the value of real exports per capita in state s in 2008, which corresponds to the beginning of the sample. We compute the median value in 2008 for all states and we partition the states which have a value above the median (high intensity) or below the median (low intensity). The degree of export intensity would be $1/50$ for each state if they would be uniformly distributed. States which feature a degree of export intensity larger than the median are "AK"; "CA"; "CT"; "DE"; "IL"; "IN"; "IA"; "KS"; "KY"; "LA"; "MA"; "MI"; "MN"; "NJ"; "NY"; "ND"; "OH"; "OR"; "SC"; "TN"; "TX"; "UT"; "VT"; "WA"; "WI".

[14] Since our theoretical model is not a multi-sector model, it does not aim at capturing the sectoral heterogeneity in tariffs highlighted in Section 2. Therefore, we calibrate the tariff shocks – level and volatility shocks – by using U.S. aggregate data.

(2012) and corresponds to the utilization-adjusted measure. Public spending series is the real government consumption expenditures and gross investment provided from the U.S. Bureau of Economic Analysis. Both series are expressed in log and HP-filtered. We obtain $\rho_a = 0.76$ and $\sigma_a = \log(0.008)$ for TFP shocks as well as $\rho_g = 0.90$ and $\sigma_g = \log(0.014)$ for public spending shocks. Finally, we assume that the Foreign country is not directly hit by any shock.

5 Transmission Channels of Regional TPU

In this section, we investigate the regional effects of trade policy uncertainty shocks resorting to the theoretical model developed in Section 3. First, we solve and simulate the theoretical model to check its quantitative validity. Then, we analyze the Impulse Response Functions (IRFs) to trade policy uncertainty shocks in order to highlight the role of regional heterogeneity in the transmission channels of shocks. Finally, we investigate whether the monetary policy can mitigate the recessional effects of TPU shocks.

5.1 Solution Methods and Simulation

We solve the model using third-order pruned perturbation methods in order to track the effects of tariff volatility shocks on the economy. We first check the ability of our theoretical model to replicate the empirical evidence emphasized in Section 2.3. As explained in the calibration strategy, we split our empirical dataset into two groups so as to calibrate our theoretical model taking into account the heterogeneity between the two regions. Region 1 in the model corresponds to the group of U.S. states that features a degree of export intensity higher than the 2008-median i.e. the most open states. Region 2 in the model is the group of U.S. states with a degree of export intensity below the median. We proceed in three steps. First, for each group of U.S. states, we extract the average exposure to volatility tariff shocks $TPU_{s,t}$ defined by Equation (5), denoted by $TPU_{j,t}$ with $j = \{1, 2\}$.[15] This gives us two series of volatility shocks over the sample 2008q1-2020q4 ($T = 52$). We also extract the average series of exposure to *level* tariff shocks. Second, we simulate the theoretical model feeding it with these series, after a long enough burn-in period that allows us to start from the stochastic steady state. We extract the simulated series of output, $Y_{j,t}$ for $j = \{1, 2\}$, as defined in Equation (7) that we express in log. Third, we estimate the following regression

$$\log(Y_{j,t+h}) - \log(Y_{j,t-1}) = \delta_{j,h} + \delta_{t,h} + \beta_h^{model} TPU_{j,t} + v_{j,t+1}, \text{ for } h >= 0 \tag{45}$$

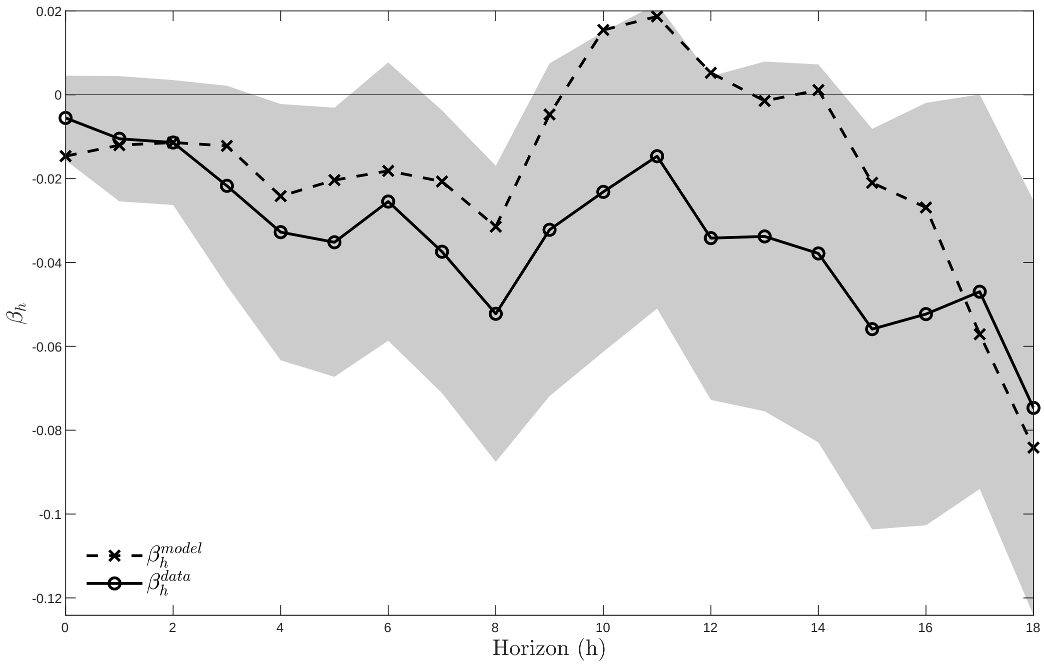
---

[15] More precisely, $TPU_{j,t} = \sum_{s \in S_j} TPU_{s,t} \times (1/25)$ where S_1 (respectively, S_2) denotes the set of states with a degree of export intensity above (respectively, below) the median.

for each $j = \{1, 2\}$, $\delta_{j,h}$ and $\delta_{t,h}$ are region- and time-fixed effects and finally, β_h^{model} is the model counterpart of β_h in Equation (6). This estimation strategy has the advantage that we apply the same data-generating process than in the empirical part.

Figure 4 reports the results. Each marker corresponds to the point estimates from the data (β_h) and the model (β_h^{model}).[16] The Figure shows that, despite its stylized dimension, the theoretical regional model does a good job in reproducing the magnitude of effects of tariff volatility shocks on output at several horizons. In particular, the model generates an impact elasticity of $\beta_0^{model} = -0.01$ while it is $\beta_0^{data} = -0.005$ in the data. More importantly, the model is able to reproduce the cumulated effect after 8 quarters ($\beta_0^{model} = -0.03$ in the model and $\beta_0^{data} = -0.05$ in the data), which is a statistically significant effect, as we emphasized previously. In the following subsection, we investigate the sources of heterogeneity which affect the transmission channels of tariff volatility shocks.

Figure 4: Effects of regional TPU shocks on output: Data versus model



Note: Each marker point corresponds to point estimates β_h estimated from Equation (6) for the data (solid line) and from Equation (45) for the model (dashed line). The grey area displays the $[10^{\text{th}}, 90^{\text{th}}]$ confidence interval of the empirical estimates.

---

[16] Therefore, the data-based markers for horizon $h = 0, 4, 8$ correspond to the values reported in Table 1.

5.2 IRFs Analysis

We investigate the transmission channels of trade policy uncertainty in a regional model by computing the IRFs to a national volatility tariff shock and by carrying out a set of counterfactual exercises. We compute the response of variables to the volatility shock in deviation from the EMAS, all shocks being shut down.[17]

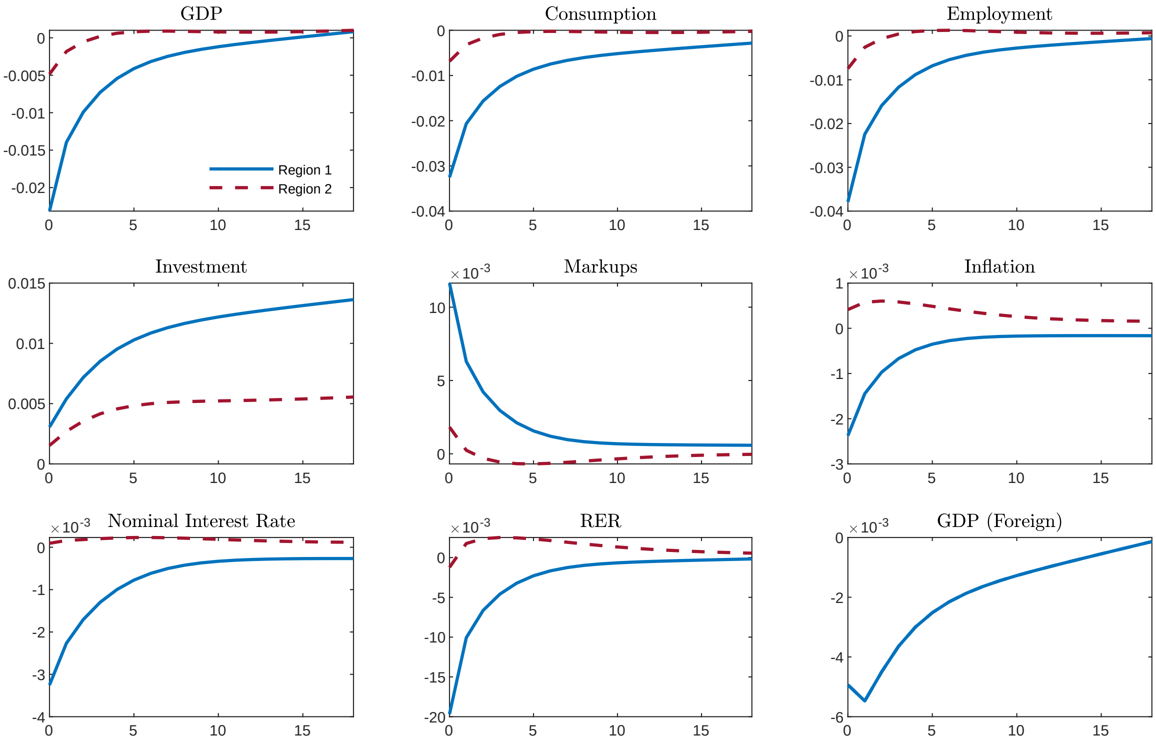
Baseline Calibration Figure 5 shows the IRFs of the main variables to a national TPU shock that hits both regions of the Home economy. Let us remind that the model is calibrated so that the two regions differ in our benchmark calibration according their size (η_j), their share of national imports (ω_j) and their share into national exports ($\tilde{\omega}$). Precisely, Region 1 features the largest degree of trade openness, through a large import-to-GDP share ($\omega_1 > \omega_2$) and a large degree of export intensity ($\tilde{\omega} > 0.5$).

We first look at the dynamics common to the two regions. Figure 5 shows that both regions experience a recession as well as a drop in consumption after a positive uncertainty shock on import tariffs. Concerning investment, it reacts positively to higher uncertainty. In Keynesian models where production is determined by demand, a fall in consumption leads to a fall in production. The reaction of investment depends on the relative magnitude of these two falls. Here, the fall in consumption is so large relative to realized output that unconsumed output available for investment increases. Note that this property results from the high persistence of the trade policy shock. For a lower value of this persistence, we find the usual comovement between consumption, investment and production in response to uncertainty shocks, without affecting our results. It is well documented in the literature that the recessionary effects of uncertainty shocks are regularly explained by the precautionary-saving motive of households and the precautionary-pricing behavior of firms (see Bianchi et al. (2023) for details). First, the convex marginal utility of consumption implies that households who face uncertainty about the future economic prospective postpone their current consumption and increase their savings to smooth consumption over time. As a consequence of this precautionary-saving channel, a higher volatility of import tariffs generates second-order effects which lead to a reduction in private consumption. Notice also that the potential precautionary labor-supply behavior does not generate a rise in labor since it is counteracted by the presence of nominal rigidities, as explained by Basu & Bundick (2017). Second, trade policy uncertainty makes also firms more uncertain about their expected real marginal cost since import tariffs have a direct impact on the

---

[17] Technically, we simulate the model starting from the deterministic steady state with all shocks imposed to be zero and burn in the 10 000 first observations to get the ergodic mean without shock (EMAS). We simulate once again the model, impulsing the volatility innovation ε_t after the burn-in period. The IRFs are then expressed in deviation from the EMAS. One alternative would be to compute the Generalized IRFs (GIRFs, see Fernández-Villaverde & Guerrón-Quintana (2020), for instance). Our results are robust to the methodology used to express IRFs.

Figure 5: IRFs to a TPU shock



Note: IRFs are computed in deviation from the ergodic mean and multiplied by 100.

real marginal cost, as shown in Equation (16). Therefore, the variance of future desired price is higher, which in turn makes firms more "prudent" in their pricing decisions, by setting a price higher than that they normally would do in the absence of uncertainty. This precautionary-pricing behavior of firms leads them to increase markups which reduces output since firms are demand-constrained.

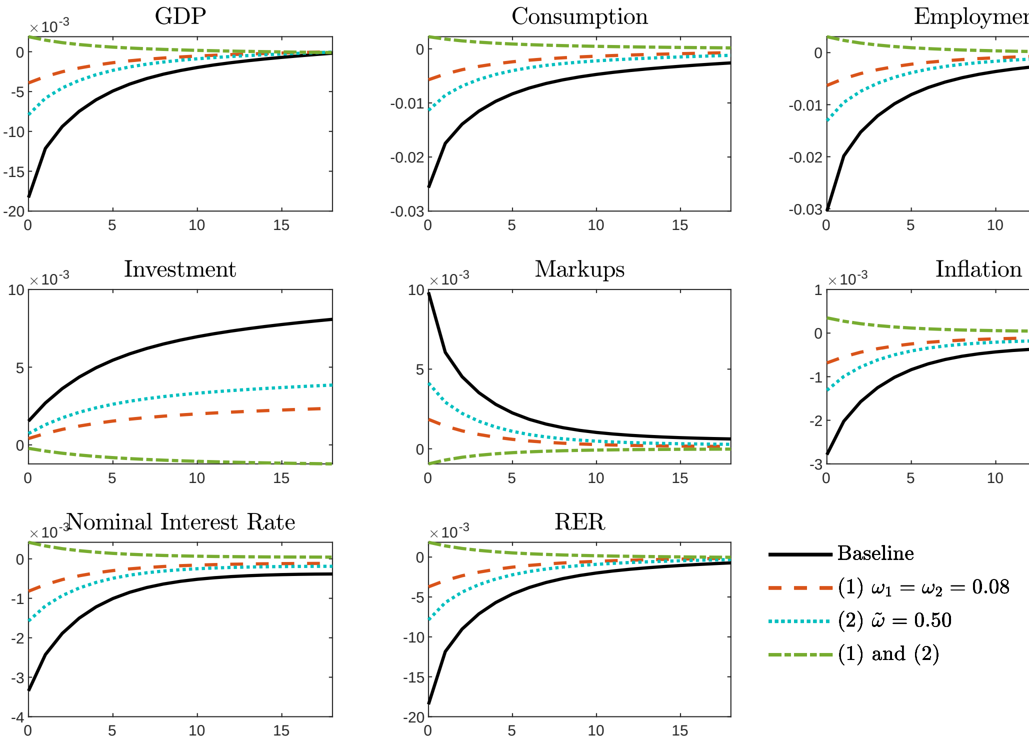
We now seek to understand why Region 1 experiences a larger drop in output than Region 2. To do so, one need to show how these traditional transmission channels of uncertainty shocks operate in the presence of heterogeneous regions. As shown in Equation (11), a larger degree of trade openness, ω_1, makes the demand function of imported good by Region 1 more sensitive to a high volatility in tariffs. Therefore, the precautionary-pricing behavior of firms in this region is likely to be stronger as they are more exposed to tariffs uncertainty, which in turn generates a larger appreciation of the real exchange rate and a larger recession. There is an additional effect which explains why the drop in Region 1's output is stronger and it comes from the reaction of the Foreign country. Higher uncertainty about tariffs in the Home economy leads to a collapse of its imports, which means for the Foreign economy a contraction in demand for its own production. As a result, the Foreign country also enters a recession. Since Region 1 is also the region which features the highest degree of export intensity – i.e. most of the Foreign imports come from this region – it suffers the most from the reduction in Foreign demand, which strongly reduces its exports. Setting $\tilde{\omega}$ to 0.84 means that Region 1 accounts for 84% in the Foreign production process of $\tilde{Y}_t$ defined by Equation (23) against 16% for Region 2. Region 1 will therefore be more strongly impacted than Region 2 by the contraction of activity in the Foreign economy. In total, the reduction in aggregate demand generates a drop in inflation in Region 1. Region 2 is hit by the same uncertainty shock than Region 1 but it experiences a lower increase in markups and a more limited recession since it is less exposed to the Foreign recession. Unlike Caldara et al. (2020), our model does not incorporate the firms' export participation decision. Even without this mechanism, our model captures the main expected effects of an uncertainty shock on tariffs, as the authors show that the extensive margin of exports mostly affect the magnitude of the responses rather than the transmission channels.

Assessing the Role of Regional Heterogeneity We now turn to assess more in details how trade openness can lead to differentiated regional responses to the national TPU shock. In order to highlight the heterogeneity across regions, we compute the inter-regional gap in IRFs to the uncertainty shock, defined as the difference between the two region-specific IRFs. We alternately modify the key parameters which affect the regional exposure to trade – namely ω_j and $\tilde{\omega}$ – and we compare the wedge with the one observed in the baseline calibration. Figure 6 displays the results.

The solid black lines show the inter-region IRFs gap for the benchmark calibration (solid lines). The dashed red lines show the wedge for $\omega_1 = \omega_2 = 0.08$, which corresponds to a low import share in both regions. The dotted blue lines represent the IRFs gap when $\tilde{\omega} = 0.5$, i.e. both regions feature the same degree of export intensity. Finally, the dashed-dot green lines are when $\omega_1 = \omega_2 = 0.08$ and $\tilde{\omega} = 0.5$, i.e. regions are slightly open to trade and the only source of heterogeneity is given by the size of the region.

The negative wedge in the output responses displayed in Figure 6 illustrates that Region 1 experiences a greater recession than Region 2 in the benchmark calibration (see also Figure 5). The rational behind this result is that Region 1 features a larger degree of trade openness ($\omega_1 > \omega_2$) and a larger degree of export intensity ($\tilde{\omega} > 0.5$). These two sources of heterogeneity, linked to the regions' imports and exports respectively, are complementary and mutually reinforcing.
The gap in output between the two regions is driven by the degree of trade openness, ω_j, since the wedge is strongly attenuated when we assume that both regions feature an identical and large home bias (dashed red lines). One reason is that the larger rise in Region 1's markups observed

Figure 6: Inter-regional gap of IRFs to a TPU shock: international trade channels



Note: Each line depicts the difference multiplied by 100 between the IRFs of Region 1 and the IRFs of Region 2 to a TPU shock. The solid black lines correspond to the benchmark calibration. The dashed red lines correspond to a low import share calibration ($\omega_1 = \omega_2 = 0.08$). The dotted blue lines correspond to an equalized degree of export intensity ($\tilde{\omega} = 0.50$). The dashed-dotted green lines correspond to $\omega_1 = \omega_2 = 0.08$ and $\tilde{\omega} = 0.50$.

in the benchmark calibration is less pronounced when $\omega_1 = \omega_2 = 0.08$. Indeed, a low value of ω_j means that the domestic production technology is not strongly dependent on imported goods (see Equation (9)). As uncertainty here relates to the marginal cost of production through import tariffs, monopolistic producers tend to less protect themselves from this uncertainty if they rely less heavily on these imports. Said differently, the rise in markups in Region 1 is lower when this region has a larger home bias which illustrates a lower exposure to the upward-pricing behavior. Another reason for explaining the attenuated heterogeneity comes from the household perspective. Indeed, increasing the home bias in Region 1 results in a lower drop in consumption as the precautionary motive of savers is dampened.[18] In total, the source of heterogeneity is lessened when both regions feature an identical and small import share. However, the slightly stronger recession in Region 1 is explained by the fact that this region still features a larger degree of export exposure ($\tilde{\omega} > 0.5$) and it is therefore more exposed to Foreign spillovers.

Figure 6 shows that the degree of export intensity, $\tilde{\omega}$, is also an important driver of regional

---

[18] Even though imported goods are not directly consumed by households, they appear in the production function of intermediate goods which are then used for the production of final consumption goods.

heterogeneity in the transmission channels of uncertainty shocks. The dotted blue lines correspond to the difference of Region-1 and Region-2 IRFs when we assume that both regions contribute equally to the total amount of exports of the Home country ($\tilde{\omega} = 0.5$). Shutting down the export intensity assumption reduces by half the amount of heterogeneity in regional output responses. The reason is that Region 1 is less exposed to the Foreign recession than in the benchmark calibration since its good is not preferred as much by the Foreign country. On the opposite, Region 2 suffers from a larger recession since it is more intensive in exports relative to the baseline calibration. Mechanically, the gaps between the two regions is reduced.

Finally, the dotted-dashed green lines in Figure 6 show the results when we abstract from the two international trade channels, i.e. we assume a low import share in both regions and an equal degree of export intensity. The only source of heterogeneity comes from the size of the regions, which slightly matters for the transmission of uncertainty shocks since the wedge between the two IRFs is quite small for most of the variables. Interestingly, the sign of the gap for all responses is reversed compared to the benchmark calibration. Focusing on the response of output, this means that Region 1, i.e. the largest region, experiences a smaller recession that Region 2 in response a higher TPU uncertainty. This can be easily explained by the fact that the largest region needs less imported goods to produce the wholesale good and therefore, it is less exposed to import tariffs uncertainty, which helps to dampen the recession.

5.3 Monetary Policy Reaction to Trade Policy Uncertainty

As explained above, trade policy uncertainty affects the expected marginal cost, which forces firms to adopt a precautionary pricing behavior. Even though tariffs are set nationally, a region which is more exposed to trade suffers from a stronger recession since the upward-pricing bias is amplified. One natural question the arise: to which extend monetary policy can reduce the heterogeneous effects of TPU among regions? This question is reminiscent of the famous tweet by U.S. President Donald Trump on August 23, 2019, in the midst of trade tensions with China: "’Who is our bigger enemy,’ Fed Chairman Powell or Chinese President Xi?". As interpreted in the press, this tweet reflected the President’s dissatisfaction with the monetary policy conducted by Jerome Powell, the chairman of the Federal Reserve. Donald Trump argued that the Fed chairman raised interest rates too swiftly and has not moved quickly enough to cut them.[19] The policy experiment we consider is a deviation from the Taylor rule in response to the uncertainty shock on trade policy. The Taylor rule is is modified as follows

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}}\right)^{\rho_r} \left[\left(\frac{\pi_t}{\bar{\pi}}\right)^{\rho_\pi} \exp(-\rho_{r,\tau} \times \varepsilon_t^{\sigma})\right]^{(1-\rho_r)} . \tag{46}$$

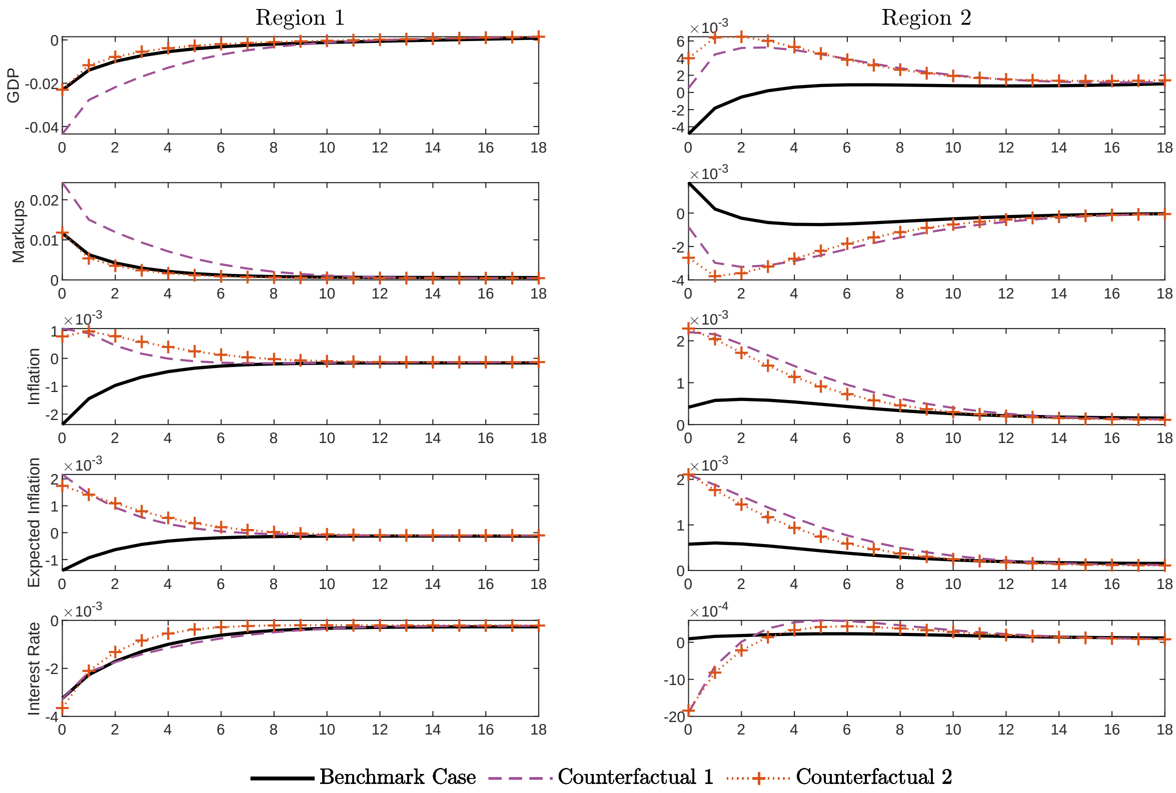
---

[19] https://www.cnbc.com/2019/08/23/trump-tweets-who-is-our-bigger-enemy-fed-chairman-powell-or-chinese-president-xi.html

The uncertainty shock on trade policy is used here as a monetary policy shock, with $\rho_{r,\tau} > 0$, as the scaling parameter. A positive shock ϵ_t^τ that increases tariffs uncertainty triggers a decline in the interest rate, the intensity of which is determined by $\rho_{R,\tau} = 0.00005$. This specification is close to the Taylor rule estimates including an uncertainty variable realized by Evans et al. (2015) and Caggiano et al. (2018). The difference is that we introduce here directly the uncertainty shock on trade policy and not the level of this uncertainty. This means that we consider that the uncertainty shock on trade policy also acts as a monetary policy shock leading to a discretionary deviation from the Taylor rule, which is precisely what Donald J. Trump asked the Federal Reserve to do in the previously mentioned tweet.

The solid black lines in Figure 7 remind the effects of a TPU shock in the benchmark model ($\rho_{r,\tau} = 0$). The dashed purple lines (counterfactual 1) show the responses in the baseline model when we augment the monetary policy rule with a reaction to TPU shocks ($\rho_{r,\tau} > 0$). The dotted-market orange lines (counterfactual 2) report the responses when we shut off the precautionary-pricing channel by linearizing the New Keynesian Phillips Curve in the augmented model ($\rho_{r,\tau} > 0$ and linearized Equation (12)).

Figure 7: IRFs to TPU shock and monetary policy reaction



Note: IRFs are computed in deviation from the ergodic mean and multiplied by 100.

Counterfactual 1 (dashed lines) shows that a reduction in the nominal interest rate in response to higher uncertainty has asymmetric effects on the two regions. Indeed, expansionary monetary policy stimulates output in Region 2 while it makes the recession even stronger in Region 1. Let remind that Region 2 is less exposed to trade. Therefore, the reduction in the national nominal interest rate stimulates demand in this region, which in turn overcomes the negative effects of trade policy uncertainty shocks. In total, both output and inflation increase in this region. More surprisingly, making the Taylor rule responsive to TPU shocks leads to a stronger recession in Region 1. The intuition behind this result is that expansionary monetary policy, which can be seen as a positive demand shock, strongly increases expected inflation. Since firms are more exposed to uncertainty about tariffs imports, they react to a higher expected inflation, both in level and volatility terms, by strongly increasing their markups. This precautionary-pricing behavior of firms is large enough to depress even further the economy and the central bank amplifies the heterogeneity among regions. Said differently, when the upward-pricing is strong in a region – because for instance it is more exposed to uncertainty – the reduction in the nominal interest rate is ineffective to stimulate the economy. Indeed, it generates a large rise in expected inflation which magnifies the precautionary-pricing behavior of firms that they adopt to hedge against the risk of choosing a price too low.

To shed further light on the role played by the interaction between monetary policy and the upward-pricing channel, we now shut off this channel by solving the non-linear model while the New Keynesian Phillips Curve is linearized in the model with "enriched" monetary policy ($\rho_{r,\tau} > 0$). Linearizing Equation (12) allows us to get rid of the second- and third-order effect in the pricing decisions, which are at the origin of the precautionary-pricing behavior (see Born & Pfeifer (2021)). Counterfactual 2, illustrated by the dotted-marker orange lines in Figure 7, shows that the recession generated by the expansionary monetary policy in Region 1 is eliminated as soon as we shut off the precautionary-pricing behavior. Said differently, even though the reduction in the nominal interest rate boosts expected inflation, this does not materialize into a stronger recession because firms are not concerned about setting a higher price to smooth volatility in expected future real marginal costs. Still, the expansionary effects of the monetary policy are limited in Region 1 which still experiences a drop in output, as the recessionary effects of uncertainty shocks dominate in this region strongly exposed to trade uncertainty. This finding echoes the strength of the literature who argues that monetary policy and precautionary pricing are connected, like for instance Fasani & Rossi (2018), Born et al. (2020), Andreasen et al. (2024) or Castelnuovo et al. (2023). We show here that this tight connection, that comes from the volatility of expected inflation, might lead the central bank to worsen heterogeneity between regions (or countries).

6 Conclusion

This paper studies the the effects of trade policy uncertainty shocks – both empirically and theoretically – using a regional approach. We build a measure of U.S. states' exposure to trade policy uncertainty and we estimate the effect of a related shock on GDP and employment. We show that a positive shock on regional TPU exposure is recessionary. We rationalize this result using a two-region open-economy model and we show that regions which are the most exposed to trade policy uncertainty shocks suffers from the feedback loop of the Foreign recession combined with the fact that their importers are likely to adopt a precautionary-pricing behavior. Altogether, the most-exposed region suffers from a stronger recession. We also show that an expansionary monetary policy might have detrimental effects on the economic activity of the regions the most exposed to uncertainty, by raising expected inflation and thus generating volatility. Our findings complement the results already obtained in the literature at different levels of aggregation, such as at Caldara et al. (2020), which are at the level of the United States as a whole or of American firms. This analysis could be pursued in the analysis of local policies which could be useful to mitigate the heterogeneous effects of tariffs that are decided at national level.

References

Adão, R., Kolesár, M., & Morales, E. (2019). Shift-Share Designs: Theory and Inference. *The Quarterly Journal of Economics*, 134(4), 1949–2010.

Ahir, H., Bloom, N., & Furceri, D. (2022). The World Uncertainty Index. NBER Working Paper 29763.

Andreasen, M., Caggiano, G., Castelnuovo, E., & Pellegrino, G. (2024). Why Does Risk Matter More in Recessions than in Expansions? Implications for Monetary Policy. *Journal of Monetary Economics*, 143, 103533.

Auerbach, A.-J., Gorodnichenko, Y., & Murphy, D. (2020). Local Fiscal Multipliers and Fiscal Spillovers in the USA. *IMF Economic Review*, 68, 195–229.

Autor, D., Dorn, D., & Hanson, G. H. (2013). The China Syndrome: Local Labor Market Effects of Import Competition in the United States. *American Economic Review*, 103(6), 2121–68.

Baker, S. R., Bloom, N., & Davis, S. J. (2016). Measuring Economic Policy Uncertainty. *The Quarterly Journal of Economics*, 131(4), 1593–1636.

Baker, S. R., Davis, S. J., & Levy, J. A. (2022). State-Level Economic Policy Uncertainty. *Journal of Monetary Economics*, 132, 81–99.

Basu, S. & Bundick, B. (2017). Uncertainty Shocks in a Model of Effective Demand. *Econometrica*, 85(3), 937–958.

Bekaert, G. & Wang, X. (2010). Inflation Risk and the Inflation Risk Premium. *Economic Policy*, 25(64), 755–806.

Beraja, M., Fuster, A., Hurst, E., & Vavra, J. (2018). Regional Heterogeneity and the Refinancing Channel of Monetary Policy. *The Quarterly Journal of Economics*, 134(1), 109–183.

Beraja, M., Hurst, E., & Ospina, J. (2019). The Aggregate Implications of Regional Business Cycles. *Econometrica*, 87(6).

Bianchi, F., Kung, H., & Tirskikh, M. (2023). The Origins and Effects of Macroeconomic Uncertainty. *Quantitative Economics*, 14(3), 855–896.

Born, B., Muller, G., & Pfeifer, J. (2020). Uncertainty shocks in currency unions. CEPR Discussion Papers 15579.

Born, B. & Pfeifer, J. (2014). Policy Risk and the Business Cycle. *Journal of Monetary Economics*, 68, 68–85.

Born, B. & Pfeifer, J. (2021). Uncertainty-Driven Business Cycles: Assessing the Markup Channel. *Quantitative Economics*, 12(2), 587–623.

Bown, C. P. & Kolb, M. (2023). Trump's Trade War Timeline: An Up-to-Date Guide. PIIE Trade and Investment Policy Watch.

Caggiano, G., Castelnuovo, E., & Nodari, G. (2018). Risk Management-Driven Policy Rate Gap. *Economics Letters*, 171, 235–238.

Caldara, D., Iacoviello, M., Molligo, P., Prestipino, A., & Raffo, A. (2020). The Economic Effects of Trade Policy Uncertainty. *Journal of Monetary Economics*, 109, 38–59.

Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics*, 12(3), 383–398.

Canova, F. (2024). Should We Trust Cross-Sectional Multiplier Estimates? *Journal of Applied Econometrics*. In press.

Castelnuovo, E. (2023). Uncertainty Before and During COVID-19: A Survey. *Journal of Economic Survey*, 37, 821–864.

Castelnuovo, E., Pellegrino, G., & Særkjær, L. L. (2023). Uncertainty and the Business Cycle When Inflation is High. Mimeo.

Cho, D., Han, Y., Joonseok, O., & Rogantini Picco, A. (2021). Uncertainty Shocks, Precautionary Pricing, and Optimal Monetary Policy. *Journal of Macroeconomics*, 69, 103343.

Chodorow-Reich, G. (2019). Geographic Cross-Sectional Fiscal Spending Multipliers: What Have We Learned? *American Economic Journal: Economic Policy*, 11(2), 1–34.

Chodorow-Reich, G. (2020). Regional Data in Macroeconomics: Some Advice for Practitioners. *Journal of Economic Dynamics and Control*, 115, 103875.

Chodorow-Reich, G., Feiveson, L., Liscow, Z., & Woolston, W. G. (2012). Does State Fiscal Relief during Recessions Increase Employment? Evidence from the American Recovery and Reinvestment Acts. *American Economic Journal: Economic Policy*, 4(3), 118–45.

Dupor, B. & Guerrero, R. (2017). Local and Aggregate Fiscal Policy Multipliers. *Journal of Monetary Economics*, 92, 16 – 30.

Elkamhi, R., Jo, C., & Salerno, M. (2023). Measuring "State-Level" Economic Policy Uncertainty. *Journal of Financial and Quantitative Analysis*, in press.

Evans, C., Fisher, J., Gourio, F., & Krane, S. (2015). Risk Management for Monetary Policy near the Zero-Lower Bound. *Brookings Papers on Economic Activity*, (pp. 141–219).

Fajgelbaum, P. D., Goldberg, P. K., Kennedy, P. J., & Khandelwal, A. K. (2020). The Return to Protectionism. *The Quarterly Journal of Economics*, 135(1), 1–55.

Fasani, S. & Rossi, L. (2018). Are Uncertainty Shocks Aggregate Demand Shocks? *Economics Letters*, 167, 142–146.

Fernald, J. (2012). A Quarterly, Utilization-Adjusted Series on Total Factor Productivity. Federal Reserve Bank of San Francisco. Working Paper Series 2012-19.

Fernández-Villaverde, J., Guerrón-Quintana, P., Kuester, K., & Rubio-Ramírez, J. (2015). Fiscal Volatility Shocks and Economic Activity. *American Economic Review*, 105(11), 3352–84.

Fernández-Villaverde, J., Guerrón-Quintana, P., Rubio-Ramirez, J. F., & Uribe, M. (2011). Risk Matters: The Real Effects of Volatility Shocks. *American Economic Review*, 101(6), 2530–61.

Fernández-Villaverde, J. & Guerrón-Quintana, P. A. (2020). Uncertainty Shocks and Business Cycle Research. *Review of Economic Dynamics*, 37, 118–S146.

Goldsmith-Pinkham, P., Sorkin, I., & Swift, H. (2020). Bartik Instruments: What, When, Why, and How. *American Economic Review*, 110(8), 2586–2624.

Handley, K. (2014). Exporting Under Trade Policy Uncertainty: Theory and Evidence. *Journal of International Economics*, 94(1), 50–66.

Handley, K. & Limão, N. (2015). Trade and Investment under Policy Uncertainty: Theory and Firm Evidence. *American Economic Journal: Economic Policy*, 7(4), 189–222.

Handley, K. & Limão, N. (2017). Policy Uncertainty, Trade, and Welfare: Theory and Evidence for China and the United States. *American Economic Review*, 107(9), 2731–83.

Handley, K. & Limão, N. (2022). Trade Policy Uncertainty. *Annual Review of Economics*, 14, 363–395.

Hazell, J., Herreno, J., Nakamura, E., & Steinsson, J. (2022). The Slope of the Phillips Curve: Evidence from US States. *The Quarterly Journal of Economics*, 137(3), 1299–1344.

Hördahl, P. & Tristani, O. (2012). Inflation Risk Premia in the Term Structure of Interest Rates. *Journal of the European Economic Association*, 10(3), 634–657.

House, C. L., Proebsting, C., & Tesar, L. L. (2018). Quantifying the Benefits of Labor Mobility in a Currency Union. NBER Working Paper 25347.

House, C. L., Proebsting, C., & Tesar, L. L. (2021). Regional Effects of Exchange Rate Fluctuations. *Journal of Money, Credit and Banking*, 52, 429–463.

Leduc, S. & Liu, Z. (2016). Uncertainty Shocks are Aggregate Demand Shocks. *Journal of Monetary Economics*, 82, 20–35.

Nakamura, E. & Steinsson, J. (2014). Fiscal Stimulus in a Monetary Union: Evidence from US Regions. *American Economic Review*, 104(3), 753–92.

Oh, J. (2020). The Propagation Of Uncertainty Shocks: Rotemberg Versus Calvo. *International Economic Review*, 61(3), 1097–1113.

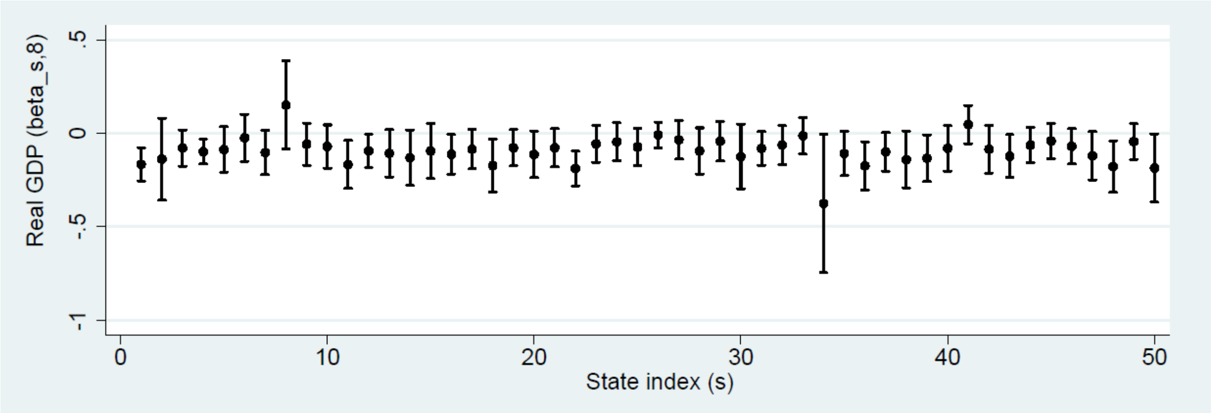
Santacreu, A. M., Sposi, M., & Zhang, J. (2023). What Determines State Heterogeneity in Response to US Tariff Changes? Mimeo.

Shoag, D. & Veuger, S. (2016). Uncertainty and the Geography of the Great Recession. *Journal of Monetary Economics*, 84, 84–93.

Appendix

A. State-to-State Estimation

Figure 8: State-to-state estimation results



Note: Each point corresponds to the estimates $\hat{\beta}$ of Regression (6) for the state s at horizon $h = 8$. The interval corresponds to the 90 percent confidence interval.

B. Households First-Order Conditions

Maximizing the discounted lifetime utility measure (31) subject to the definition 32), the budget constraint (33), the law of capital accumulation (34) yields the following first-order conditions

$$1 = \beta \mathrm{E}_t \left\{ \frac{\lambda_{j,t+1}}{\lambda_{j,t}} \frac{R_t^j}{\pi_{j,t+1}} \right\}, \tag{47}$$

$$\left(c_{j,t} - \psi \mathcal{X}_{j,t} \left(\ell_{j,t} \right)^{\omega_w} \right)^{-\sigma} + \sigma_X v_{j,t} c_{j,t}^{\sigma_X - 1} \mathcal{X}_{j,t-1}^{1-\sigma_X} = \lambda_{j,t}, \tag{48}$$

$$v_{j,t} + \psi \left(\ell_{j,t} \right)^{\omega_w} \left(c_{j,t} - \psi \left(\ell_{j,t} \right)^{\omega_w} \mathcal{X}_{j,t} \right)^{-\sigma} = \beta \left(1 - \sigma_X \right) \mathrm{E}_t \left\{ v_{j,t+1} c_{j,t+1}^{\sigma_X} \mathcal{X}_{j,t}^{-\sigma_X} \right\}, \tag{49}$$

$$\psi \omega_w \left(\ell_{j,t} \right)^{\omega_w - 1} \mathcal{X}_{j,t} \left(c_{j,t} - \psi \mathcal{X}_{j,t} \left(\ell_{j,t} \right)^{\omega_w} \right)^{-\sigma} = \lambda_{j,t} w_{j,t}, \tag{50}$$

$$\varsigma_{j,t} = \left[1 - \Phi \left(\frac{i_{j,t}}{i_{j,t-1}} \right) - \Phi' \left(\frac{i_{j,t}}{i_{j,t-1}} \right) \frac{i_{j,t}}{i_{j,t-1}} + \beta \mathrm{E}_t \left\{ \frac{\lambda_{j,t+1} \varsigma_{j,t+1}}{\lambda_{j,t} \varsigma_{j,t}} \left(\frac{i_{j,t+1}}{i_{j,t}} \right)^2 \Phi' \left(\frac{i_{j,t+1}}{i_{j,t}} \right) \right\} \right]^{-1}, \tag{51}$$

$$1 = \beta \mathrm{E}_t \left\{ \frac{\lambda_{j,t+1}}{\lambda_{j,t}} \left[\frac{z_{j,t+1} + (1 - \delta) \varsigma_{j,t+1}}{\varsigma_{j,t}} \right] \right\}, \tag{52}$$

$$\tag{53}$$

with $\varsigma_{j,t} \equiv \breve{\xi}_{j,t} / \lambda_{j,t}$, the relative price of capital and with $\Phi \left(\frac{i_{j,t}}{i_{j,t-1}} \right) = \frac{\varkappa}{2} \left(\frac{i_{j,t}}{i_{j,t-1}} - 1 \right)^2$ and $\Phi' \left(\frac{i_{j,t}}{i_{j,t-1}} \right) = \varkappa \frac{i_{j,t}}{i_{j,t-1}} \left(\frac{i_{j,t}}{i_{j,t-1}} - 1 \right)$.