Competing for the Quiet Life: An Organizational Theory of Market Structure^{*}

Patrick Legros[†] Andy Newman[‡] Zsolt Udvari[§]

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Abstract

We develop an incomplete-contracts model of endogenous market structure for a homogeneous-good industry. A large number of identical incentiveconstrained producers each decide whether to stand alone as price-taking competitors, or to horizontally integrate with others by selling their assets to profitmotivated professional managers, who then Cournot compete in the product market. Despite the absence of significant technological non-convexities, the equilibrium market structure is typically an oligopoly, demonstrating that contracting imperfections are a distinct source of market power. Unlike standard endogenous entry models, concentration may increase with the size of the market. We discuss implications for competition policy.

Keywords: Horizontal integration; incomplete contracts; theory of the firm; property rights; global market power; endogenous entry; hold-out; merger paradox; OIO; coalition formation

[‡]Boston University and C.E.P.R.

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[†]Université libre de Bruxelles (ECARES), and C.E.P.R

[§]Positive Competition

1 Introduction

The theory of industrial structure, salient as ever amid mounting evidence and growing unease about market concentration, is a work in progress. Even for its most basic concerns – the number, sizes, and market power of firms supplying a homogeneous good – there are open questions. While many drivers of those outcomes, notably technology, demand, and entry barriers, are well understood, others have so far eluded due scrutiny. In particular, it remains unclear whether anything of consequence turns on what is meant by a "firm."

For industrial organization (IO), firms are primitive and unitary entities. The predominant account of market structure is the endogenous entry model (EEM), in which these actors first decide to enter a market and then supply it in competition with one another. A zero-profit balance of market demand, efficient scale, and entry cost determines how many firms will occupy the market. The many variants of EEM all point to significant non-convexities in technology as the source of market power.

In organizational economics (OE), firms are neither primitive nor unitary: they comprise more fundamental, not to say willful, elements that inhabit a world rife with contracting frictions. Moreover, entry is not the only engine of market structure. So is the recombination of incumbents: mergers and acquisitions, spin-offs and divestitures – in short, the reconstitution of firm boundaries, among the fields's earliest and most enduring preoccupations. Thus it is fair to ask what OE might add to our understanding of industrial structure.

An alternative to the EEM that accommodates both the composite nature of the firm and the malleability of its boundaries has been straddling the IO/OE nexus for quite some time: the "coalitional approach." The elements are a population of producers already serving the market. The industrial structure question boils down to asking how they organize themselves into a collection of disjoint "firms," each a subset of the population, that then compete in the product market.

Under certain assumptions about contracting possibilities, it doesn't matter which way one tackles the market structure problem. Indeed, the coalitional approach is often interpreted as foundational for standard models. But as this paper shows, in the presence of contracting frictions and notions of the firm emanating from them, thinking along coalitional lines can make a substantial difference. First, those frictions may be sufficient for market power to emerge, even *without* significant economies of scale. Second, even a strongly consumer-oriented policy may encourage firms to operate with market power and well above the efficient scale. And third, predictions about the effects of fundamentals such as market size may differ markedly from those of standard models.

We consider an industry populated by a continuum of identical, capacity-constrained production units that we refer to as "teams," each consisting of a productive asset and several partners.¹ The team is the minimum efficient scale of operation, infinitesimal relative to market demand. Its output requires non-contractible choices such as managerial effort that entail private costs for the team's partners, and (constant) marginal monetary costs, up to the capacity constraint. Because of the non-contractibility, the team suffers internal incentive problems, to which there are a limited number of second-best solutions. We focus on the most classical: authority, which is wielded by a professional manager (HQ) who is motivated only by monetary profit and neglects her subordinates' private costs.²

While every team must be owned and operated by an HQ, we allow for the possibility of *horizontal integration* by supposing that an HQ can manage arbitrarily many production units, using the authority gained from ownership to enforce desired output levels produced by its team(s). As in the property-rights literature, we refer to the union of assets owned by one HQ as a *firm*; team members remain attached to the asset, but now operate as subordinates of the HQ (thus, a firm can range in size from a single team – a "stand-alone firm" – up to the entire population). To avoid stacking things in favor of industry concentration, we assume there is no (technological) gain or loss from this kind of consolidation: output aggregates linearly across teams regardless of whether they belong to the same firm.³ Moreover, HQ's are abundant, with zero opportunity costs.⁴

A fundamental classical argument (Stigler, 1950) predicts that without contracting frictions, this environment delivers firms operating at their small efficient scale, with vanishing market power. Since a stand-alone concern produces with the same

¹Examples might be a restaurant, with the building and kitchen equipment as the asset and the chef and maître d'hôtel as partners; a medical practice with the equipped office and its doctors; or a machine tool manufacturer, with a plant and separate managers handling sales, operations, and engineering. While team members have common goals in the form of profit, there are also conflicts of interest and consequent incentive problems that arise from free-riding motives or disagreements about the direction of the business.

 $^{^{2}}$ We omit others imperfect remedies, such as profit sharing, for simplicity. Allowing for them does not change the main conclusions.

³There may of course be other gains to horizontal integration, but as in the property-rights literature that inspires this paper (Grossman and Hart, 1986; Hart and Moore, 1990; Hart and Holmström, 2010), these are driven by the changes in incentives and behavior that result from changes in ownership structure.

⁴Indeed, at the cost of some algebraic complexity, instead of introducing the HQs as a separate class of players, we could let their managerial authority function be exercised by some of the team members, with little change to our results.

technology as would as part of a larger firm or syndicate, and can sell its output at the same market price, it would do better to remain outside any oligopoly, selling all the output it likes at that price, rather than subject itself to the oligopoly's output restrictions.⁵ This generates a standard anticipatory free-riding ("hold-out") problem: every team would prefer to remain outside the oligopoly (or insist on a buyout price that compensates for the profit that would accrue outside the oligopoly), taking advantage of the high price without contributing to its maintenance. Individual producers holding out in this manner would undermine any market structure other than an already competitive one.

Compelling as this reasoning may be, it does turn on the assumption that a production unit that stands alone can replicate whatever activity it would undertake as part of a larger firm.⁶ Contracting imperfections invalidate that premise. They limit the commitment power of team members and HQs, restricting them to incentive-compatible sets of output levels and payoffs. Those sets in turn depend on their firm's size and competitive environment. An action (e.g., expected quantity generated by managerial effort) that is available to a large firm's members may not be available to a stand-alone, and vice versa.

More specifically, since HQs are interested only in monetary profit, they undervalue their subordinates' private costs when making production decisions. In the case of a stand-alone firm, its "small" HQ will use her authority to induce them to over-provide effort and output. Meanwhile, a larger HQ that owns many assets has no more commitment ability than her small counterpart, for she still fails to internalize her subordinates' private costs. But her behavior differs, due to her market power. By ordering her subordinates to provide lower effort and output levels than those imposed by a small, price-taking HQ, she felicitously provides them a "quiet life" (Hicks, 1935) as a byproduct of her pursuit of oligopoly profit.

Thus, even though stand-alone producers have exactly the same technology as the oligopoly, and freely access the market price it sets, they may be unable to replicate the actual choices and payoffs they would achieve as members of the oligopoly because

⁵Because we assume a team is very small, its decision to remain outside the oligopoly does not affect the market price. The case in which single teams are not negligible, wherein the decision to remain outside the oligopoly may have some impact on the market price, although more subtle, leads to similar conclusions that firms operate at close to minimum efficient scale, unless that scale is already quite large, i.e., on the order of one-half the market demand at marginal cost.

⁶Some analysts have considered situations where a "coalitional entrepreneur" commits to an offer to one producer in which the payment depends on another producer's response to its offer, or where a pre-specified coalition-formation protocol essentially replicates this commitment. For instance, unless everyone agrees to join a large firm, the deal is off. As we explain below, we avoid such "multilateral" mechanisms in this paper and, indeed, do not need them for oligopoly to emerge.

of the different incentives their HQ's face. This removes the guarantee that they can improve upon those payoffs.

Thus, we now have a tradeoff: Stigler's holdout effect, which is so strong as to run unopposed in a world where a contract for any input can be costlessly enforced, is weakened enough when there are contracting frictions to be countervailed by Hicks's quiet-life effect. The first contribution of this paper is to provide a set of sufficient conditions on demand and private and monetary cost functions for an oligopoly to survive despite the lack of non-convexities. Contracting imperfections can be a distinct source of market power.

Typically there will be a large set of oligopolistic market structures (each represented by a partition of the set of teams into a finite number of positive measure subsets, which are the firms with market power, and a collection of singletons, the price-taking competitive fringe) that provide weakly more surplus to a team than standing alone (the "team rationality" condition), with concentration bounded both above and below. Excessive concentration will generally result in prices so high that producing large quantities as a stand-alone firm will be too enticing to join the oligopoly, despite the high private cost (Stigler's effect dominates). Insufficient concentration results in low prices and a not-so-quiet life inside the oligopoly, resulting in payoffs that are dominated by shutting down production altogether.

However, not all team rational structures are viable if there is sufficient competition among HQs at the firm formation stage. Indeed, modeling that competition via a farsighted stability concept (c.f. Ray-Vohra, 2015) pins down the predicted outcome to one that is essentially unique in outcome (market structure, price, and surpluses of teams). Farsighted stability captures a pseudo-dynamic process, where starting from a putative market structure that is not a stable structure, a set of teams may deviate and generate a new market structure while anticipating that other sets of teams will deviate from the modified market structure, ending up at the a market structure that is payoff dominant to every deviating coalition. It must be the case that, along the sequence, all teams in any deviating set are better off in the stable structure than in the market structure they deviated from. A second condition is that there is no stable structure that is far-sightedly dominated by another stable structure. The second condition will be trivially satisfied in our environment.

This competition among the HQ's as captured by our stability concept will lead to market structures that not only satisfy team rationality, but also lead to zeroprofits for HQ's, equal payoffs for all teams, and symmetry (equal size and identical behavior) among the large firms. Farsighted stability results in the highest per-team surplus structure among the market structures satisfying these conditions.

There are two value functions over the set of market structures representing the payoffs of being in stand-alone firms and joining the oligopoly. A stable market structure corresponds to the largest (most concentrated) intersection of these two functions when it exists. For higher concentrations, the stand-alone value exceeds the oligopoly value. (If there is no intersection, as will be the case for some demands, then there are two extreme possibilities: perfect competition or monopoly).

Just as in the EEM the comparative statics of equilibrium are understood in terms of the equation of a fixed entry cost with the oligopoly profit, the comparative static properties of our model can be understood with the aid of these value functions. But there are crucial differences. First, unlike the fixed cost in the EEM, the stand-alone value is endogenous since it depends on the market price, which depends on the degree of competition. Second, the stand-alone value cuts the oligopoly value from below since the stand-alone value dominates at the highest concentrations. This implies that parameter changes that increase the oligopoly payoff without altering the stand-alone payoff (or raise it less than they raise the oligopoly payoff) will increase concentration, opposite to how such parameters affect concentration in the EEM.

While some comparative-static effects may be similar to those generated by the EEM (for instance changes in a willingness-to-pay parameter in the product demand), others may strikingly different. Specifically we find that changes in market size (say, from an increase in the population of consumers) have essentially opposite effects from those predicted by the EEM. In the latter, larger size leads to lower concentration and lower markups. In our model, by contrast, increasing market size can *raise* concentration and markups. To see this, recall that market size does not affect Cournot equilibrium price whenever there are constant monetary costs and no fringe: firms expand production to accommodate the increased demand. Thus stand-alone value does not change with market size: its HQ is a price taker, so no change in price implies no change in quantity produced and, therefore no change in monetary profit or private cost. Meanwhile, there is an increase in the oligopoly payoff near the equilibrium concentration: monetary profits increase, and because of the strict convexity of the private cost function, there is only a slight increase in private cost to offset that. The new equilibrium is therefore found by traveling up the stand-alone value function, i.e., in the direction of higher concentration. Thus increasing market size is a force for growing concentration in our model.

Policy implications of our model also differ from those of the EEM. Often the

concern there is with excessive entry: when making their entry decisions, firms undervalue the business-stealing externality they inflict on competitors. Thus a regulator who cannot control prices may prefer to cap the number of firms at less than the laissez-faire outcome to economize on entry costs;⁷ one who can regulate price might prefer even less entry. By contrast, as we have already suggested, there is a range of team-rational concentrations for which the payoff dominance of oligopoly over standing alone applies. This would give a regulator with only firm size/number as policy instruments some space in which to operate. Rather than being content with the laissez-faire outcome, or something more concentrated, a consumer-oriented regulator would instead want to aim for the most competitive team-rational oligopoly. Even one with objectives that include producer as well as consumer welfare would typically opt for something more competitive than the laissez-faire outcome. The reason, of course, is that in both instances, equilibrium prices will be lower and quantities higher than in the laissez-faire outcome, enhancing consumer welfare. What is perhaps more surprising is that the outcome is more desirable than perfect competition, even though that is viable under our assumptions. Again, the organizational nature of firms drives this result: perfect competition entails that the industry operates with a mix of stand-alone firms and idle units, with the price being high enough to make these a matter of indifference to the teams. This implies that while the teams in stand-alone firms are over-producing, a significant fraction of the teams are operating at zero output, and consumers suffer as a consequence. Whereas the most competitive oligopoly has all teams operating at moderately high outputs, resulting in a lower equilibrium price. Finally, we show that this most competitive oligopolistic outcome is likely to become more concentrated with market size, just as the laissezfaire outcome will: with firms operating at high output, profit gains generated by increased size will tend to be swamped by the increases in private costs, lowering the payoff from oligopoly relative to standing alone as a self-managed team.

All of these outcomes occur, as we have said, without any fixed costs, where the set of existing teams could, but for the contracting problems, serve the entire market without any efficiency losses at all. The lesson for the empirical understanding of trends in market structure (e.g., increasing concentration) is that changes in technology need not be the only source. Increases in concentration could stem from increasing market size, a result of globalization, or economic or population growth. The model may also explain why concentration is being observed to rise in economies

⁷Imposing perfect competition in such a world is not even a possibility unless entry costs are vanishingly small, but then competition would be close to the laissez-faire outcome anyway.

with fairly aggressive competition policies and those where such policy is laxer.

So much for what IO can learn from OE. What about the reverse? Traditionally, the theory of the firm considers technologically complementary assets when considering the extent of firm boundaries/size of the firm, as in the study of vertical or lateral integration (e.g., auto body factories and car assembly plants; operating system development and device design). The limits to firm size stem from some form of internal diseconomies. Examples include "Coasian" costs such as bureaucratic rigidity or hierarchical control losses. Or in the Arrovian tradition, large organizations may also be too diverse to be efficiently or cohesively governed. Either way, the costs of firm formation are increasing in the firm's size, at least after some point. Less essentially, but just as commonly, internal diseconomies are typically independent of the environment in which the firm is sitting.

Our model differs from these two distinct ways. First, the assets are substitutes (or at least "independent"), yet there are still benefits to bringing them together within firm boundaries. So there is some broadening of the scope of the theory of ownership to include assets of this type.

Second, the limits to firm size here do not originate with internal diseconomies. Indeed, because of the quiet-life effect, larger firms have a (private) cost advantage over smaller ones, essentially opposite to the Arrow/Coase one. Instead, Stigler's holdout effect keeps firm size in check.⁸ If the Hicks effect were not present, and instead, private or other firm formation costs were globally increasing in firm size, then just as with complete contracting, the only outcome would be perfect competition, since any producer could do strictly better by staying outside a firm, enjoying the price set by a putative oligopoly while benefiting from lower costs.

Literature

The coalitional approach to market structure originates with the literature on coalition production economies (e.g., Hildenbrand, 1968; Boehm, 1973; Ichiishi, 1977), where the chief interest was in foundations for competitive analysis. Closer to the issues discussed here is a literature on syndicates (Aumann, 1973; Guesnerie, 1977; d'Aspremont et al., 1983; Legros, 1987) which analyzes when a coalition of teams benefits from forming a monopoly, as well as the literature on coalition formation

⁸In contrast to technologically driven costs of firm size, the behavioral benefits to private costs depend on the firm's environment. For instance, the number of firms in the market matters: for a firm of a given size (measure of teams), having more competitors raises the private cost because of the induced change in HQ behavior.

with or without externalities (see for instance Bloch, 1995; Ray and Vohra, 1997; Ray and Vohra, 2015*b* provide a survey of this literature). More recently, though not aimed at the market structure question, virtually all of the formal literature on firm boundaries (particularly those descending from Grossman and Hart, 1986 and especially Hart and Moore, 1990) adopt some form of this approach.

Our approach to integration is in the "property rights" tradition launched by those two papers. The model, however, is closer to property-rights models such as that in Hart and Holmström (2010) where the party with authority makes key production decisions, rather than bargaining over them after prior investments, as in the classical "hold-up" versions Grossman and Hart (1986); Hart and Moore (1990). Moreover, those decisions are assumed non-contractible rather than merely ex-ante non-contractible. See Legros and Newman (2014) for more discussion.

Integration changes the nature of the incentive problem. Non-integrated teams tend to under-value coordination and over-value private cost savings. If the team relinquishes control to a third party-HQ that behaves competitively, she will favor coordination and high effort from the team members (as in Hart and Holmström, 2010 and Legros and Newman, 2013). The modeling innovation in this paper is to allow an HQ to integrate many teams and achieve market power, the benefits of which include a high price for all teams and a quiet life (Hicks, 1935) for those that join. Since the HQ has authority over the use of the assets it owns, horizontal integration provides a mechanism by which the "cartel problem" – free riding ex-post on the high price induced by output restrictions – is resolved. But as already discussed, this merely transforms the cartel problem into the ex-ante holdout problem. Of course, the role of third parties in resolving incentive problems enjoys its own literature, including Alchian and Demsetz, 1972 and Holmstrom (1982).

As discussed, a key economic force constraining the formation of firms, limiting their size, and driving the comparative statics results, is the "hold-out problem." Beside Stigler's early contribution in IO, this idea has been expressed in several other literatures, notably corporate finance (Grossman and Hart, 1980) where it undermines takeover threats and with that, managerial discipline. As mentioned, it is a form of free-riding, a dominant theme in the abstract theory of coalition formation. As we have suggested, the "public good" (for teams) is the market price, and contributing to it entails restricting output. The market structure context, with large coalitions (firms) forming under the direction of HQ with imperfect commitment power, naturally leads to a subsidy, in the form of a quiet life, for joining a coalition and contributing to the public good. This is what prevents the total collapse of an oligopoly from free riding. Put another way, the oligopoly provides two benefits: a public one (nonrival, nonexcludable), namely high market price, and an excludable one, the quiet life, which cannot be enjoyed outside the oligopoly due to the incomplete-contract commitment problem. As is well known from the theory of public goods, excludability is often the key to overcoming underprovision.

In IO, the free-riding force has been noted before, not only in the cartel literature but more saliently in the literature on the "merger paradox" (Salant et al., 1983; Perry and Porter, 1985; Deneckere and Davidson, 1985). This literature has focused on conditions under which a pair of unitary firms could capture enough post-merger rents to make a single merger worthwhile and has not really been developed into a theory of market structure.

There has been active research on the role of competition on the provision of incentives in principal-agent models (starting with Hart, 1983, see also Martin, 1993; Raith, 2003; Vives, 2008; Legros and Newman, 2014 provide a survey of this literature.) There, the boundaries of firms, as well as their market behavior (e.g., Cournot or competition), are unchanged when the market structure (in particular, the number of firms or demand) conditions change. By contrast, firm boundaries and market behavior are jointly determined with the market structure in the present paper.

In oligopoly theory, firms are assumed to have market power, and market structures are obtained by assuming a fixed cost of entry. The higher the fixed cost, the higher the concentration in the industry. An exception is the theory of market structure with endogenous sunk costs (Sutton, 1991), in which firms invest and create an industry-wide externality: when the number of firms increases, incentives to invest decrease, and this reduces the benefit of entry. For some demand functions, there exist equilibria in which a finite number of firms enter, even if the fixed cost is close to zero. Although achieved by a very different mechanism, this resembles the upper bound on the number of powerful firms in our model. However, unlike our model, it provides no lower bound to the degree of competition. Other exceptions are competition among firms taking the form of an initial investment in R&D, or cost reduction, followed by Bertrand or Cournot competition (see the survey in (Vives, 2008)). For some demand systems, the number of varieties may decrease with market size because the investment incentives decrease and as a result firms face a more competitive environment. In models with heterogeneous firms, changes in market size ambiguously affect the number and scale of firms that select for export (Melitz and Ottaviano, 2008).

In general, whether there is a fixed cost of entry or an industry-wide externality

that limits the benefits of entry, higher demand – specifically in the form of greater market size – will lead to more entry and lower concentration in the industry. The contrast with our result is striking. In our model, when size increases, the hold-out constraint is relaxed: the payoff from being in the fringe is largely independent of size, while oligopoly payoffs rise quickly with size, giving it a comparative advantage and allowing for increased concentration.

A growing empirical literature is documenting a decades-long trend of increased market concentration across a wide variety of industries, countries, and regulatory regimes (e.g., De Loecker et al., 2020; De Loecker and Eeckhout, 2018; Autor et al., 2020). So far, when accounting for causes, researchers have emphasized supply-side factors: technological changes that increase entry costs or efficient scales in all of these contexts. Our model suggests there could also be a role for the demand side: increases in market size that comes with growth or globalization.

2 Model

Our model proceeds in two stages. First, partitioning the teams into one or more firms is accomplished in an asset market, where all HQ's compete to purchase teams by making offers to buy their assets, and each team decides which offer to accept. Second, those HQ's that own teams after the asset market closes compete à la Cournot with the other HQ's in the product market. In equilibrium, all asset market participants correctly anticipate this second stage's outcome when making their offer and acceptance decisions.

Demand. The inverse demand is a decreasing, differentiable, log-concave function of total output. We will index the market size by the demand shifter S. Hence if industry output is Q, the inverse demand is $P(Q; S) = P\left(\frac{Q}{S}; 1\right)$.

Production units are identical teams, indexed by t on the interval [0, 1] equipped with the Lebesgue measure. Each team has a unit production capacity and can produce as a stand-alone firm or (horizontally) integrate with other teams. There is no other cost of production.⁹

For a team to be productive, it needs to give control to an HQ; there is a large measure of potential, and identical, HQs. HQs have authority on teams and can

⁹We can also allow for positive monetary input costs of the form cx(t) per team where $x(t) \le 1$ is team t's output. We normalize c = 0 for now.

Teams care not only about their monetary revenue but also their private cost. By contrast, HQs care only about their monetary revenue, internalizing the private cost of their teams only lexicographically: if there are several distinct $x_i(t)$ that generate the same monetary profit, the HQ will choose one that minimizes the aggregate private costs of the teams she owns; (with identical teams and strictly convex $\phi(\cdot)$ this entails setting $x_i(t)$ equal to a constant).

Basic team building block. A simple example of team production that borrows freely from Legros and Newman (2013) will be useful for the reader to keep in mind. Production is overseen by a team of two symmetric "managers" who jointly own an indivisible asset (possible interpretations already offered in the introduction – the generalization to more than two, or even to one if output is non-contractible, is straightforward). With probability min $\{a_1 + a_2, 1\}$, production succeeds, generating a unit of output with value P; otherwise it fails, yielding 0. Here $(a_1, a_2) \in [0, 1]^2$ represent non-contractible "efforts" that incur strictly convex and increasing private costs $\psi(a_i)$, with $\psi(0) = 0$. Manager j's payoff is $y_j - \psi(a_j)$, where $y_j \ge 0$ is income satisfying a limited liability constraint.¹⁰

Given the symmetry of the technology and costs, efficient production (maximizing the joint payoff) requires setting $a_1 = a_2$, with $a_j = \min\{\psi'^{-1}(P), 1\}$. Since this cannot be generally be achieved within our contracting environment, the team makes use of a profit-oriented HQ with preference y_H : they sell their asset to her, and she subsequently exercises the authority thereby aquired to choose (a_1, a_2) for them. As there is a continuum of (a_1, a_2) pairs that generate a given profit, we assume the HQ opts for the one that minimizes her subordinates' aggregate private cost. This too entails $a_1 = a_2$ because of the convexity of $\psi(\cdot)$.¹¹

Since we will only be interested in effort allocations that are symmetric between the managers, define $e = a_1 + a_2$. Then expected output $x = \min\{e, 1\}$ and its private cost is $\phi(e) = 2\psi(\frac{e}{2})$. For the remainder of the paper we take $e, \phi(\dot{j})$ and

¹⁰Putting $b_1 \equiv 1 - a_1$ transforms the teams's joint payoff into $P \min\{1 - |b_1 - a_2|, 1\} - \psi(1 - b_1) - \psi(a_2)$, facilitating an interpretation of the inputs as choices of standards, working conditions, or projects over which the managers disagree, but which only need "coordination" (small distance between a_2 and b_1) for successful production. As our earlier examples may suggest, this is often our preferred interpretation.

¹¹Profit sharing between the two managers who do without an HQ, which as a simplification we have already ruled out, would also require equal sharing and consequently $a_1 = a_2$.

output/revenue to be the primitives of team production.

As we have said, HQ cannot commit to a choice of e (therefore x) via contract. To ensure compliance by HQ's subordinates, the ownership contract may specify all or part of the asset payment be withheld until the end of the production period.¹² Finally, an HQ that owns many teams will make the same effort choice for each manager, both within and across her teams, a consequence of her lexicographic preference first for profit, then for private cost minimization.

A market structure is a partition of [0, 1]. If all partition elements have zero measure, the market structure is competitive. If there are n elements with positive measures, the structure is oligopolistic with n oligopolistic firms. We will be concerned by the emergence of oligopolistic market structures that are not outcome equivalent to competition, in which firms with positive measures produce less per active team than under competition. We call such firms "powerful." Hence, relevant market structure have n powerful firms $E_i, i = 1, \dots, n$, each with measure $m_i := \mu(E_i)$, and a fringe E_0 . The maximum measure of E_0 is $1 - \sum_{i=1}^n m_i$. A market structure is denoted $\omega = (E_0, E_1, \dots, E_n)$, with the understanding that E_0 is the competitive fringe.

The timing of events is the following:

- 0. (Teams enter the market)
- 1. Firm formation in asset market: HQs make offers to buy assets; teams respond. HQs obtain authority to choose quantities to be produced.
- 2a. HQs compete in the product market. HQs of stand-alone teams produce as price-takers, while HQs of oligopolistic firms behave in a Cournot fashion.
- 2b. HQs impose production decisions to implement desired quantities.
- 2c. The private costs are borne, output is realized, profits accrue to HQ's and asset prices are paid off.

¹²We assume the private costs are not incurred until after HQ makes her choices; any attempt by the managers to limit HQ's choices by exiting the relationship would only result in forfeiture of the payment without avoiding the (sunk) costs. Alternatively, the capacity constraint could be viewed as the maximum value of e that HQ could impose before inducing the subordinates to exit.

Offers and production decisions. Firm formation in the first period is orchestrated only by the same players who will eventually be operating the firms in the second period, i.e., HQs and teams. We restrict HQ's offers for the assets she wishes to purchase to be *bilateral*, in the sense that the offer to one team cannot depend on the responses by other teams to their own offers. This seems to us appropriate for modeling a competitive market for assets; offers that are conditioned on other teams' responses require more commitment than seems plausible in such an environment, where renegotiation among teams that have accepted offers (possibly facilitated by competition from other HQs) when a deal falls through would be difficult to forestall.

An offer by an HQ *i* is a pair $\alpha_i = (a_i, x_i)$. The measurable map $a_i : E_i \to \mathcal{R}_+$ is contractible, and $a_i(t)$ is the uncontingent payment to team *t* for belonging to firm E_i .¹³ $x_i(t)$ is the intensity with which team *t* will be asked to work and the measurable map $x_i : E_i \to [0, 1]$ may be non-contractible. If x_i is not contractible, teams will believe the offer only if it is incentive compatible for the HQ (i.e., consistent with the HQ's Cournot behavior in the second period, given the market structure implied by the complete profile of offers being accepted). A configuration **c** is a pair (ω, α) . Offers in a configuration are credible if three basic conditions hold: (i) the HQ makes a non-negative profit, (ii) if the offers are optimal for the HQ, and (iii) if the surpluses teams earn are weakly greater than the maximum surplus that a team can achieve in a stand-alone firm. This last condition is dubbed "Team Rationality." In fact, since a team could opt for being inactive, thereby earning zero, we also impose a fourth condition on credible offers namely they must generate (iv) non-negative surplus. Configurations in which all offers are credible are admissible. The set of admissible configurations is denoted by \mathcal{A} .

2.1 The Concept of Stability

As in economies with widespread externalities (e.g., markets with adverse selection or platform competition), deviators must form beliefs about the resulting configuration that their deviation may generate, especially when the reorganization of the economy requires coordination among individuals. Indeed, many market structures can be realized when a deviation exists in our setting. For instance, if initially the market structure is $(E_0 = [0, 1/3); E_1 = [1/3, 2/3); E_2 = [2/3, 1])$ a deviation by a set D = [1/2, 1] could lead to a monopoly structure $(E'_0 = [0, 1/2); D)$ or a duopoly structure $(E'_0 = \emptyset, E'_1 = [0, 1/2), D)$. However, each of these new configurations may

¹³We can think of the payment as being made at the end of the production period so that the team is not tempted to flee the scene because it doesn't like the private costs its HQ imposes.

be subject to further deviations.

We adopt a concept of farsighted stability in the spirit of that developed in Ray and Vohra (2015*a*) for an environment without externalities across coalitions.¹⁴ Starting from an admissible configuration c, if a set D deviates, the market structure may change, either because D contains teams from different initial powerful firms whose sizes decrease or because teams in D decide to join the fringe, in which case the size of the oligopoly decreases. This change in market structure will typically affect the credibility of the initial offers; further deviations can then be triggered. Suppose this process of deviations converges to a configuration c' that is admissible. In that case, we say that c is (farsightedly) dominated by c' if given the configuration at which there is a deviation, all teams in the deviating sets are better off in c' than in the configuration they deviated from.

Formally, consider a configuration $\mathbf{c} \in \mathcal{A}$. Another admissible configuration $\mathbf{c}' \in \mathcal{A}$ dominates \mathbf{c} , denoted $\mathbf{c}' > \mathbf{c}$, if there exists a sequence of configurations in \mathbf{C} (not necessarily admissible) $\{\mathbf{c}_0, \mathbf{c}_1, \ldots, \mathbf{c}_{\ell}, \ldots, \mathbf{c}_L\}$ with $\mathbf{c}_0 = \mathbf{c}$, $\mathbf{c}_L = \mathbf{c}'$, such that \mathbf{c}_{ℓ} is obtained from $\mathbf{c}_{\ell-1}$ by the deviation of a set D_{ℓ} , and for each $\ell = 1, \ldots, L$, and each $t \in D_{\ell}, u(t, \mathbf{c}') > u(t, \mathbf{c}_{\ell-1})$. For a set, $\Sigma \subseteq \mathcal{A}$, the set of admissible configurations that are dominated by some element of Σ is denoted $dom(\Sigma) = {\mathbf{c} \in \mathbf{C} : \exists \mathbf{c}' \in \Sigma, \mathbf{c}' > \mathbf{c}}$. Then, a stable set is such that $\Sigma = \mathbf{C} - dom(\Sigma)$.¹⁵

Complete Contracting — The TR Constraint yields Competition. Suppose that within each firm, the effort map $x_i(t)$ is contractible. If the profile of accepted offers were to lead to a powerful oligopoly sustaining price p, a team \hat{t} that had been offered $(a_i(\hat{t}), x_i(\hat{t}))$ to participate in powerful firm i could instead contract with a stand-alone HQ to produce $x_i(\hat{t})$. Together they would obtain the oligopoly payoff $\pi_O = px_i(\hat{t}) - \phi(x_i(\hat{t}))$. Any other contracted choice of x made by the stand-alone would leave p unaffected. So \hat{t} could contract on the choice $x^*(p)$ that maximizes $px - \phi(x)$, generating a surplus that almost surely strictly exceeds π_O .¹⁶ This violates admissibility, unless the "oligopoly" already replicates the competitive outcome $x^*(p)$.

We now turn to the case of incomplete contracting on efforts. There, the set \mathcal{A}

¹⁴See their discussion on the drawbacks of myopic stability concepts like Von Neumann and Morgenstern (2007), Harsanyi (1974).

¹⁵Note the final requirement both rules out the possibility of two elements of the stable set dominating each other (which could lead to cycles), often called "internal stabiluty" in the literature, and that elements of the stable set are not dominated by elements outside of it (external stability).

¹⁶It is enough that $P(\cdot)$ is strictly decreasing and $\phi(\cdot)$ strictly convex; then $x^*(p) \neq x_i(\hat{t})$, and therefore $px^*(p) - \phi(x^*(p)) > \pi_O$.

is large, and contains many oligopolistic structures, but as we will show, the stable set has a simple characterization and is essentially unique.

3 Incomplete contracting

When HQs cannot commit to their decisions $x_i(t)$, teams anticipate that their HQs will maximize the monetary profit, ignoring the private costs of the teams in the firm. Let the Cournot quantity per powerful firm be $q_i(\mathbf{c})$, and the industry price be $p(\mathbf{c})$. The HQs of teams in the fringe take this price as given and will therefore impose a maximum effort on their teams: $\arg \max_x p(\mathbf{c})x = 1$. The HQ of a powerful firm will choose an output level $q_i(\mathbf{c})$ that maximizes the residual profit of the firm and will choose efforts to produce $\int_{t \in E_i} x_i(t) dt = q_i(\mathbf{c})$. While this HQ is indifferent among all intensities that satisfy the output constraint, teams are not. As noted above, we assume the HQs will choose intensities to minimize the aggregate private costs of the teams it owns (a "minimum blowback" assumption). By (strict) convexity of $\phi(x)$, all teams will exert the same effort in a powerful firm. In addition, the HQ of each powerful firm must make non-negative profits. It follows that offers (ω, α) in the set \mathcal{A} satisfy:

- (i) Non-negative payoff to HQ: $a_0(t) \le p(c)$ and $\forall i \ge 1$, $\int_{t \in E_i} a_i(t) \le p(c)q_i(c)$
- (ii) Best response effort: $\forall i, \forall t \in E_0, x_i(t) = x_i$, where $x_0 = 1$, and $\forall i \ge 1$, $m_I x_i = q(c)$.
- (iii) Team-Rationality: $\forall i, \forall t \in E_i, a_i(t) \phi(x_i) \ge p \phi(1)$.
- (iv) Non-negative surplus: $\forall t, u(t, c) \ge 0$

We denote the set of admissible configurations by \mathcal{A} .

3.1 Characterization of Stable Configurations

In our environment, we can show that the stable set exists and is essentially unique. To focus on the economic and regulatory implications of the model, we postpone the complete proof of this result to the Appendix.

It is convenient to introduce the set Σ_0 of configurations in \mathcal{A} that satisfies the following conditions.

- Zero profit for HQs: for each E_i , $\int_{t \in E_i} a_i(t) = q(c)p(c)$.

- Non-Negative Surpluses: for each *i* and each $t \in E_i$, $a_i(t) \phi(x_i(t)) \ge 0$.
- Equal Treatment: for all t, u(t, c) = u(t', c)
- Symmetry, i.e., equal Size of powerful firms: for each $i = 1, ..., n, m_i = \frac{1-m_0}{n}$.

The credible offers in such configurations are fully determined by the number n of powerful firms, the size m of the oligopoly, and the measure f of productive teams in the fringe.¹⁷ Indeed, by symmetry, $q_i(\mathbf{c}) = q(\mathbf{c})$ for all $i \ge 1$, and the payment is equal to $a = p(\mathbf{c}) \frac{nq(\mathbf{c})}{m}$ in powerful firms and to $a_0 = p(\mathbf{c})$ in the fringe. The Cournot quantity for each powerful firm is $q(\mathbf{c})$, and the equilibrium price is $p(\mathbf{c}) = P(nq(\mathbf{c}) + f)$ are functions of (n, m, f). Note that all teams in the oligopoly have the same surplus.

$$U(c) := p(c)\frac{nq(c)}{m} - \phi\left(\frac{nq(c)}{m}\right).$$

Proposition 1. The stable set exists and is $\Sigma^* := \arg \max_{c \in \Sigma_0} U(c)$. At stable configurations with powerful firms, all teams, including any in the fringe, produce, and the team rationality condition binds whenever the measure of teams in the fringe is positive.

The proof of this result is postponed to the Appendix. Starting from the competitive structure, it is easy to improve the surplus of each team by having powerful firms form sequentially until we obtain a market structure and corresponding offers in Σ^* . All teams that deviate are better off in the final configuration than in the configuration they deviate from. Starting from another configuration **c** that is not in Σ^* , it is necessary to exhibit a process in which a sequence of deviations attains Σ^* . A first step in this direction is to show that every configuration that does not satisfy one of the conditions for Σ_0 is dominated by a configuration satisfying that condition. For the zero profit condition, this is immediate: if it is violated in a firm, another HQ can offer a greater surplus to the teams. The arguments for symmetry and equal treatment use the convexity of the private costs.

This characterization allows us to perform comparative statics on the stable set configurations. The stylized example below assumes that the number of firms is a continuous variable, which allows a simple way to highlight the positive covariation of market size and the degree of industry concentration.

¹⁷It is a priori possible that some units in the fringe do not produce.

Example 1. We assume that n is a continuous variable and that there is no fringe. It is an admittedly strong assumption but useful to develop our intuition. There is a unit measure of the teams that can be part of an oligopoly with n symmetric firms. The inverse demand is linear: P(Q; S) = A - Q/S, where is S is market size. The private cost quadratic: $\phi(x) = \phi x^2$. The HQ of a powerful enterprise E_i chooses a total production q_i to maximize $P(\sum_{j=1}^n q_j; S)q_i$, resulting in a Cournot equilibrium $q(n, S) = \frac{AS}{n+1}$ and a market price $P(nq(n, S)); S) = \frac{A}{n+1}$.

For the oligopoly to be effective, output per team $\frac{ASn}{n+1}$ must be less than the capacity 1, else it just replicates the competitive outcome. Under zero payoffs for HQ, the expected payoffs of a team in a firm are for each i = 1, ..., n,

$$u_i(n,S) := \underbrace{\frac{A^2 n S}{(n+1)^2}}_{monetary \ profit} - \underbrace{\phi \cdot \left(\frac{AnS}{n+1}\right)^2}_{private \ cost},$$

which is non-negative when

$$n \le \overline{n}(S) := \frac{1}{S\phi}.$$
(1)

Thus, if

$$(A - \phi)S < 1, \tag{2}$$

the oligopoly is effective whenever it delivers positive surplus. A stand-alone team would obtain an expected payoff of

$$u_0(n) = \max\left(0, \frac{A}{n+1} - \phi\right).$$

Note that oligopoly surplus is a decreasing function of n, and the team rationality condition can be written $\frac{A}{1+n} \leq \phi(1 + \frac{ASn}{n+1})$. Hence, if the team rationality condition does not bind, n can be decreased, raising the oligopoly payoff. The surplus maximizing configuration binds the team rationality constraint, and $\forall i \geq 1$, $u_i(n, S) = u_0(n, S)$. Therefore the size of the stable oligopoly is

$$\underline{n}(S) = \frac{A - \phi}{\phi(1 + AS)}.$$
(3)

It is immediate that $\underline{n}(S) < \overline{n}(S)$. Moreover, it is enough that

$$(A - 2\phi)S < 1 \tag{4}$$

for the oligopoly to be effective at $\underline{n}(S)$, which of course is weaker than (2).

For values of S larger than that satisfying (4), equilibrium market structures replicate the competitive outcome. We shall have more to say about this below.

The comparative statics of this surplus maximizing market structure and the equilibrium price with respect to market size are illustrated in Figure 1b. Observe



Figure 1: Stable market structures and equilibrium price

that $\underline{n}(S)$ is a decreasing function of S in the effective oligopoly region: as market size increases, concentration increases. This result is opposite to the usual effect of the market size in competition or oligopoly settings and reflects the Quiet Life induced by firms exerting market power.¹⁸

In this example, we treat n as a continuous variable with no fringe firms. For this reason, when there is an oligopoly of a given size, the equilibrium price, and therefore the surplus of a stand-alone team, is not sensitive to market size. By contrast, the surplus of a team in a powerful firm is responsive to a change in market size because the effort intensity varies with S. One can show that with local increases in S, the surplus function $u_o(n, S)$ increases for $n < n_0(S)$ and decreases for $n > n_0(S)$, where $\underline{n}(S) < n_0(S) \equiv \frac{1}{2\phi S} < \overline{n}(S)$, whenever (4) holds, implying that $\underline{n}(S)$ must decrease, as is evident from (3).¹⁹ Notice that this "rotation" of the oligopoly value about n_0 implies that $\underline{n}(S)$ is also decreasing. Thus, the set of team-rational, non-negative surplus oligopoly structures represented by the interval $[n(S), \overline{n}(S)]$ decreases.

For the continuous n case, the negative relationship between market size and n can be generalized to log-concave demand functions and strictly convex private costs.

¹⁸On the other hand, some comparative statics of this model – notably, changes in the demand parameter A – closely resemble their EEM counterparts. We do not emphasize them here.

¹⁹Monetary profits for oligopoly teams increase with S; because $\phi(\cdot)$ is convex, this comes at little additional private cost when production is low, i.e., where n is small, so team surplus increases. But where n is large, so is output per team, and expanding output further raises the private cost more than it increases profit, reducing the surplus.

Indeed, a configuration in Σ^* has a number $n^*(S)$ of oligopolistic firms (corresponding to $\underline{n}(S)$ in the example above), a Cournot outcome in which each team produces $x^*(n^*(S), S)$, equal to the total industry output:

$$P'(x^*(n^*(S),S);S)x^*(n^*(S),S) + n^*(S)P(x^*(n^*(S),S);S) = 0$$
(5)

The stable market structure $n^*(S)$ solves the program

$$\max_{n} P(x(n,S);S)x(n,S) - \phi(x(n,S))$$

$$P(x(n,S);S) \le G_{\phi}(x(n,S)) := \frac{\phi(1) - \phi(x(n,S))}{1 - x(n,S)}.$$
(6)

where (6) is the team rationality condition, taking account of zero profits for HQ's. For log-concave demand functions, x(n, S) is increasing in n and S; the objective is decreasing in n, as in the example $((P'(x(n, S))x(n, S)+P(x(n, s)-\phi'(x(n, S))x_n(n, s)) < (P'(x(n, s))x(n, s) + nP(x(n, s)))x_n(n, S) = 0$ for the (unique) solution x(n, S) to the Cournot equilbrium condition.) By convexity of $\phi(\cdot)$, the function $G_{\phi}(x)$ is an increasing function of x. Hence, the optimum is obtained at the value $n^*(S)$ that binds the constraint (6). We also need the effective oligopoly condition

to be satisfied (indeed, $G_{\phi}(1)$ is not well-defined), which is dependent only on properties of the demand (and if relevant, monetary cost) functions, and not on the private cost function $\phi(\cdot)$.

Now, as S increases slightly to S', x(n, S') > x(n, S), and therefore $P(x(n(S), S'); S') < G_{\phi}(x(n(S), S')))$, and to bind the TR constraint it is necessary to decrease n.

Remark 1. Strict convexity of $\phi(\cdot)$ is needed for the existence of an effective oligopoly. Indeed, TR and non-negative surplus together require $\frac{\phi(x)}{x} \leq P \leq G(x)$. Then if $\phi(\cdot)$ is linear $(=\phi x)$, then we must have

$$\frac{\phi(x)}{x} = G_{\phi}(x) = \phi,$$

and therefore $P = \phi$, the competitive outcome.

Remark 2. Suppose then that $\phi(\cdot)$ is strictly convex. Then if there is an effective oligopoly for $\phi(\cdot)$, that is, there is \hat{n} such that $\frac{\phi(x(\hat{n},S))}{x(\hat{n},S)} \leq P(\hat{n},S) \leq G(x(\hat{n},S))$, with $x(\hat{n},S) < 1$, then \hat{n} is also an effective oligopoly for any strict convexification

 $\psi(\cdot)$ of $\phi(\cdot)$.²⁰ Moreover if the private cost function increases from ϕ to $\lambda\phi$ for some $\lambda > 1$, if there is an effective oligopoly \hat{n} for ϕ , there is an effective oligopoly $\hat{n}' \leq \hat{n}$ for $\lambda\phi$.²¹ In short, higher and more convex private costs lead to more concentration.

4 Extensions

We use the continuous-n model to analyze various extensions.

4.1 Regulation

Absent regulation, the stable configuration is $n^*(S)$ when the TR constraint binds. But because the market price is a decreasing function of n, a regulator who puts weight only on consumer surplus may want to prevent the formation of oligopolies with less than \overline{n} firms. Would this regulator prefer to prevent *any* oligopoly? The answer is no whenever demand at the competitive price is less than the aggregate industry capacity i.e., $P(1/S) < \phi(1)$

Consider the (unconstrained) oligopoly output Q(n, S). This output is an increasing function of n, and there exists $n_0(S)$ such that the capacity constraint $Q(n, S) \leq 1$ is binding if, and only if, n exceeds $n_0(S)$.

Suppose that a positive measure of teams does not produce under competition. Hence, the competitive price is equal to $\phi(1)$, and there is a proportion α of teams producing under competition, where α solves,

$$P(\alpha; S) = \phi(1).$$

The team surplus

$$u(n, S) := P(Q(n, S); S)Q(n, S) - \phi(Q(n, S)),$$

is decreasing in n, and there exists a unique $\overline{n}(S)$ such that u(n, S) = 0.

We claim that the surplus is zero at an oligopoly size smaller than the size at which the capacity constraint binds, that is $\overline{n}(S) < n_0(S)$. The capacity constraint

 $^{^{20}\}psi:[0,1] \to [0,c] \subset \mathbb{R}$ is a strict convexification of ϕ if $\psi(0) = \phi(0) = 0$, $\psi(1) = \phi(1) = c$, and there exists a strictly convex function $v:[0,c] \to [0,c]$ such that $\psi \equiv v \circ \phi$, where all three functions are increasing. The result follows from the fact that v(y) < y on $(0,\alpha)$ and therefore $\psi(x)/x < \phi(x)/x$ and $G_{\psi}(x) > G_{\phi}(x)$ on (0,1).

²¹Both $\frac{\lambda\phi(x)}{x}$ and $G_{\lambda\phi}(x)$ are increasing in λ , but the latter increases faster, so the effective oligopoly price that binds TR and generates positive surplus for ϕ can be increased (and *n* lowered) to satisfy TR and non-negative surplus for $\lambda\phi$.

is satisfied at $\overline{n}(S)$. (Otherwise, the oligopoly output at $\overline{n}(S)$ equals 1, but the surplus is negative.)

Now, because the capacity constraint is not binding at $\overline{n}(S)$, we have

$$P(Q(\overline{n}(S), S); S) = \frac{\phi(Q(\overline{n}(S), S))}{Q(\overline{n}(S), S)}$$

< $\phi(1)$
= $P(\alpha; S)$.

and therefore, the oligopoly price at $\overline{n}(S)$ is strictly less than the price under competition.

If, under competition, all teams produce and have a positive surplus, then any oligopoly in which the capacity constraint is satisfied will lead to an industry price not smaller than the price P(1; S) under competition. Hence, the surplus of teams is positive. In this case, the maximum size of an oligopoly in Σ_0 is achieved at $n_0(S)$ when the capacity constraint binds. In this case, the regulator is indifferent between oligopoly with sizes greater than $n_0(S)$ and competition.

Because the competitive price when all firms produce is an increasing function of S, there exists a unique S_0 such that all teams produce under competition if, and only if, the market size is smaller than S_0 . In this case, a regulator will strictly prefer an oligopoly to competition.

Finally, a regulator whose objective is to maximize consumer+producer welfare can also find an optimal oligopolistic market structure. To see this, note that the market structure \overline{n} that achieves zero surplus for teams has $P(\overline{n}) = \frac{\phi(x(\overline{n}))}{x(\overline{n})} < \phi'(x(\overline{n}))$, while the team surplus maximizer n^* has $P(n^*) = G(x(n^*)) > \phi'(x(n^*))$. Thus by continuity, there is an (unique, by monotonicity of $P(\cdot)$) oligopoly $n_{FB} \in [n^*, \overline{n}]$ satisfying the first-best condition $P = \phi'(x(n_{FB}))$.²²

4.2 Entry

Unregulated oligopolies generate prices exceeding the competitive one. Moreover, indefinite increases in market size, given a fixed set of teams, eventually result in competitive-equivalent outcomes, though those will also prices that exceed the (pri-

²²In fact, n_{FB} is the "rotation" point n_0 for team oligopoly surplus $V = P(\frac{x}{S})x - \phi(x)$ referred to above: $V_S = P'\frac{x}{S}[x_S - \frac{x}{S}] + (P - \phi')x_S = 0 \iff P = \phi'$. The equivalence follows from the well-known unit size elasticity property $(x_S = \frac{x}{S})$ of Cournot output under constant marginal costs, obtained by differentiating the Cournot equilibrium condition (5) with respect to S and exploiting log-concavity of $P(\cdot)$.

22

vate) cost of production. In either case, teams earn rents, inviting entry. Here we consider the effects of entry by new teams into the industry.

Consider an extension of the continuous-n, linear demand Example 1. Denote by T the measure of teams in the industry, which will enter at stage 0, before the asset market opens. Each team incurs entry cost $E(T) \ge 0$, where the notation allows for the possibility that the cost may depend on how many teams enter the market (e.g., the cost of medical education is likely to be increasing in the number of aspiring doctors).

At the product market Stage 2, potential supply is now T rather than 1, but the analysis proceeds straightforwardly as before. Though the quantity produced each of n firms in an oligopoly is independent of T, not so the team level intensity x which is now

$$x_o = \frac{AnS/T}{n+1}$$

(this is obtained by noting it is now T/n teams producing the oligopoly quantity q = AnS/(n+1)).

In the asset market Stage 1, the (binding) TR constraint P = G(x) now implies

$$\underline{n}(S/T) = \frac{A - \phi}{\phi(1 + AS/T)}$$

with corresponding price

$$P = \frac{\phi(1+AR)}{1+\phi R}$$

At this value of n, the effective oligopoly condition $x_o < 1$ is satisfied as long as

$$(A - 2\phi)S/T < 1. \tag{7}$$

Since TR binds, the equilibrium equilibrium oligopoly surplus will equal the fringe payoff $P - \phi = \frac{\phi(A-\phi)S/T}{1+\phi S/T}$, and in equilibrium of the overall game with entry we must have

$$P - \phi = E(T).$$

It is straightforward to check that this equilibrium fringe payoff is increasing in S/T, so when S increases

- If $E(T) \equiv E$, T will increase proportionally, leaving market structure $\underline{n}(S/T)$ and price unchanged;
- If E(T) is increasing, concentration $\underline{n}(S/T)$, P, and entry cost all increase.

The first case is already a departure from EEM, which would predict falling concentration and prices with rising size. The second implies that population growth, or other sources of increasing market size such as trade agreements, could be a driver not only of increasing concentration and prices or markups, but also of entry costs. Note that the interpretation of entry costs here may be somewhat different from the ones usually invoked in applications of the EEM: here it is "small," incurred at the team level, like an education, medical office, or small plant, while in traditional oligopoly theory it is typically "large" compared to the size of the market, like building a giant assembly plant, or developing new technologies or markets.

Finally, if $E(T) < \phi$ for all finite T, then the free entry condition $P_O - \phi = \frac{\phi(A-\phi)S/T}{1+\phi S/T} = E(T)$ implies $(A-2\phi)S/T < 1$. In other words, if the entry cost is not too large over its entire domain, the equilibrium outcome is always an effective oligopoly, never competition, and the increasing concentration effect of growing markets, rather than being confined to a bounded range of market sizes as illustrated in Figure 1, pertains to arbitrary S.²³

4.3 Contractibility and Concentration

TBA

5 Discreteness and the Endogenous Competitive Fringe

While the model with continuous n and no fringe provides the basic intuitions for our results, there are some subtle differences from the "real" model in which n is discrete. In particular, the coexistence of the oligopoly and a competitive fringe is a generic outcome, and the comparative statics of market size are marked alternately by stasis, jumps and slow adjustments.

Let (N, m) be a market structure, with $N \in \mathbb{N}_+$ representing the number of oligopoly firms and $m \in [0, 1]$ the measure of teams they comprise. There is no loss in restricting attention to configurations in which all teams in the fringe produce (see the Appendix.) It is convenient to generalize our basic environment to this case.

²³If teams anticipated competitive behavior post-entry, then equilibrium would entail that the competitive price $P_c = A - T/S = \phi + E(T) < 2\phi$, so that $(A - 2\phi)S/T < 1$. But then by (7), after entry, the teams would form an effective oligopoly with $P_o > \phi + E(T)$, a contradiction.

5.1 The Example with a Fringe

Straightforward computations imply that (ignoring the capacity condition for the moment) the per-firm output is

$$q(N,m) = \frac{AS - (1-m)}{N+1},$$
(8)

and the per-team effort in an oligopoly firm is

$$x(N,m) = \frac{N}{N+1} \frac{AS - (1-m)}{m}$$

Total industry output and industry price are

$$Q(N,m) = \frac{NAS + (1-m)}{N+1}; \ p(N,m) = \frac{AS - (1-m)}{S(N+1)}.$$

The capacity condition x < 1 requires that

$$\frac{m}{N} > AS - 1. \tag{9}$$

The surplus of teams in the oligopoly equals $px - \phi x^2$ which must not be smaller than the surplus of fringe teams $p - \phi$. As before, stability requires the TR constraint to be satisfied:

$$p(N,m) \le \phi(1+x(N,m)),\tag{10}$$

with equality whenever m is less than one. There are two basic qualitative results of our model that this example will illustrate.

- (a) A market structure with powerful firms exists if the market size S is not too large relative to the measure of teams.
- (b) If (N, m) is a stable market structure with powerful firms, then as the market size increases slightly, the ratio N/m decreases at the new stable structure.

Stable Structures For each value of S consistent with an oligopoly with powerful enterprises, there exists a unique value of N maximizing U(N, S). Figure 2 summarizes the evolution of the stable market structures, the associated price, surplus, and efforts as the market size S varies. The stability condition generates a discontinuity in the price when there is an oligopoly, mirroring the discontinuity in the size of the fringe, but the price is monotonically non-decreasing in market



Figure 2: Index of market structure and price in stable structures as a function of market size

size. However, it is more illuminating to compare directly with the competitive price $p_c(S) := \max\left(\phi(1), A - \frac{1}{S}\right)$, since that too is non-decreasing in size. In the spirit of the Lerner index that contrasts the oligopoly price and the competitive price, a proxy for market power is the ratio $\frac{p_O(S)-p_c(S)}{p_c(S)}$. It is clear from Figure 2b that this index is increasing with S as long as the surplus of teams under competition is equal to zero (that is when $p_{comp}(S) = \phi(1)$) and is then decreasing as the industry hits capacity.

Comparison with the continuous-*n* **model.** Because of the binding TR condition, there is little reason to expect that a model with a continuous n approximates the discrete model perfectly. If there is no fringe in the discrete model and if $n^*(S)$ is an integer, it should be clear that $n^*(S) = N(S)$. However, as a fringe is possible, a better proxy for the degree of competition in the oligopoly is the ratio $\frac{N}{m}$. As the scale of the monopoly decreases, there is more "external" competition to the oligopoly; because fringe firms produce at capacity, the effect is different than in the continuous model. Indeed, as n varies in the oligopoly, there is no external competition, implying that even if $n = \frac{N}{m}$, there is a lower output in the continuous specification than in the discrete model. This implies that the price in the continuous model is above the price in the discrete model for any value of S. Intuitively, by ignoring the possibility of a fringe, the continuum model puts emphasis on the first order effect on surplus due to the change in the number of oligopolistic enterprises in response to a change in market size. By contrast, in the discrete model, a change in the size of the fringe, keeping the number of powerful firms the same, creates only second-order effects.



Figure 3: Continuous (dashed curve) versus Discrete models

5.2 Comparative Statics

In the continuum model, there is an unambiguous relationship between the market size and the power of the stable oligopoly (as S increases, n decreases). This is not the case in the model with a fringe, but the result continues to hold under additional conditions.

Proposition 2. Consider an infinitesimal change in market size from S to S' > S.

- (i) If there is an oligopoly (N, 1) in the stable set when the market size is S, then as the market size increases, either there is no change in market structure, or the number of firms and the scale of oligopoly decrease.
- (ii) If there is an oligopoly with a fringe (N,m) (m < 1) in the stable set when the market size is S', the scale of the oligopoly increases when the difference $u(N,m,S) - u_0(N,m,S)$ is locally increasing in S.
- (iii) The equilibrium price is non-decreasing in S. The margin of the oligopoly price above the competitive price increases in S whenever the surplus of teams under competition is equal to zero.

The argument for (ii) follows the fact that when there is a fringe, the surplus of teams increases in the scale of the oligopoly. Hence, if there is a fringe, and the teams are not at capacity, it must be the case that the TR constraint binds. But then, if for the same market structure, the TR constraint strictly holds for S' > S, it is possible to decrease the fringe, hence increasing the size of the oligopoly, without violating the TR constraint.

In the example, condition (ii) is satisfied. The next result provides a sufficient condition for this to happen. If x is the per-team effort in the oligopoly, the condition

can be restated as the difference

$$G(x) - P(mx + (1 - m)k; S)$$

being locally increasing in S (see the Appendix). For a quadratic private cost $\phi(x) = \phi \cdot x^2$, we have $G'(x) = \phi$; and for linear demand $P(Q) = A - \frac{Q}{S}$, equilibrium $P = \frac{A - \frac{1-m}{S}}{N+1}$ and $x = \frac{N(AS - 1+m)}{(N+1)m}$, so the condition can be restated as

$$\phi \frac{NA}{(N+1)m} - \frac{1-m}{(N+1)S^2} > 0,$$

which is satisfied whenever

$$\phi ANS^2 > m(1-m).$$

As the right side is bounded above by 1/4, we have

Corollary 1. In the quadratic private cost, linear demand example, the scale of the oligopoly increases with market size whenever $4\phi AS^2 > 1$.

Thus, if the private cost parameter ϕ is large enough relative to the standard IO fundamentals A and S, not only are oligopolies stable, but they respond to growth in market size by moving toward greater concentration.

A Appendix

Here we provide proofs of the main propositions. We denote the capacity by k; in the text we have k = 1.

A.1 Proof of Proposition 1

Consider the set $\Sigma_0 \subseteq \mathcal{A}$ of configurations satisfying Zero Profit (ZP), Equal Treatment (ET), and Symmetry (Sym). Such configurations can be summarized by the triple (N, m, f) where N is the number of powerful enterprises, m is the size of the oligopoly, and f is the measure of productive teams in the fringe. The Cournot equilibrium quantity per enterprise is q(N, m, f) and the price is p(N, m, f) =P(nq(n, m, f) + fk). For each $\mathbf{c} \in \Sigma_0$, all teams in the oligopoly have the same surplus equal to $U(\mathbf{c})$.

If in all configurations in Σ_0 , the resource constraint $\frac{nq(n,m,f)}{m} \leq k$ binds, the stable set contains all configurations in Σ_0 that are outcome equivalent to competition. Therefore, suppose that the set of configurations in Σ_0 for which enterprises produce at less than capacity (nq(n,m,f) < mk) is non-empty. The surplus of teams corresponding to such configurations is bounded above by the surplus under monopoly. Therefore, there exist configurations maximizing the per-team surplus in Σ_0 . We need to show that this subset $\Sigma^* := \arg \max_{c \in \Sigma_0} U(c)$ is the stable set, that is that (i) for each $c \notin \Sigma^*$, there exists $c' \in \Sigma^*$ such that c' > c, and (ii) that for each $c \in \Sigma^*$, there does not exist $c' \in \Sigma^*$ such that c' > c. It should be clear that (ii) is satisfied by the construction of the set Σ^* .

If the market structure is $\omega = (E_0, E_1, \dots, E_N)$, the measure of teams in enterprise i is denoted m_i while the total scale of the oligopoly is $m = \sum_{i=1}^{N} m_i$. Note that the equilibrium price and output depend only on m and not on its distribution among the N enterprises. However, the capacity constraint requires that m_i is not too small; otherwise, the per-team effort $\frac{q(c)}{m_i}$ will be greater than k.

Consider a configuration c that does not satisfy (ZP). There is a sequence of deviations where the market structure is the same and where the initial positive profit of HQs of enterprises is reallocated among the teams of its enterprise. Doing so will preserve Team Rationality and the capacity constraint. Hence, for any such configuration, another configuration c' satisfying (ZP) dominates it. If we show that c' is itself dominated by an element c'' of Σ^* , it follows that c'' dominates c.

There are three cases for the initial configuration c.

Case 1: m < 1 and f < 1 - m. Teams in the fringe that do not produce (there is a measure m - f of them) have a zero surplus. It must be the case that the surplus of teams that produce is also equal to zero. Otherwise, an HQ could make a credible offer to a non-producing team, improving that team's payoff without modifying the market structure. Hence, teams that produce in the fringe must make zero surpluses. But then, the equilibrium price $p(\mathbf{c})$ equals the average private cost $\phi(k)/k$. Such an outcome can be replicated under competition. Indeed, if there are powerful firms, $p(\mathbf{c})$ is greater than the competitive price; hence, if $p(\mathbf{c}) = \phi(k)/k$, we must have $P(k) < \phi(k)$. Under competition, the equilibrium will require that only a few teams produce up to the point where teams are indifferent between producing and not. Hence, the equilibrium price will equal $p(\mathbf{c})$. There is, therefore, no loss of generality in restricting attention to configurations in which, if there is a fringe of positive measure, all teams in the fringe produce.

Case 2: $\mathbf{c} \in \Sigma_0 - \Sigma^*$. By definition, for any $\mathbf{c}' \in \Sigma^*$, $U(\mathbf{c}) < U(\mathbf{c}')$. Let the market structures be (E_0, E_1, \ldots, E_N) and $(E'_0, E'_1, \ldots, E'_N)$ for \mathbf{c} and \mathbf{c}' respectively. Consider the sequence:

- $D_1 = \bigcup_{i \ge 1} E_i$, and all teams in D_1 join the fringe. HQs of teams in D_1 make offers equal to the competitive profit for each team in D_1 . The resulting configuration is c_1 .
- Teams in $D_2 = E'_1$ form an enterprise, and the resulting market structure is a fringe $[0, 1] E'_1$ together with an enterprise E'_1 . Suppose their HQ makes offers leading to a surplus of U(c'). The other offers stay the same. The resulting configuration is c_2 .
- $D_3 = E'_2$; the resulting market structure is a fringe $[0,1] (E'_1 \cup E'_2)$ and enterprises E'_1, E'_2 . The HQ of E'_2 offers its teams a surplus U(c').
- Replicating this construction, after N + 1 deviations, $D_{N+1} = E'_{N'}$, and the market structure is \mathbf{c}' . The offers made during the sequence are now credible for the market structure $(E'_0, E'_1, \ldots, E'_n)$, and $\mathbf{c}' > \mathbf{c}$. Each team in D_i is better off in \mathbf{c}' than in \mathbf{c}_i , hence $\mathbf{c}' > \mathbf{c}$.

Case 3: $c \notin \Sigma_0$. As argued above, there is no loss of generality in assuming that the offers in c satisfy ZP. Therefore, c must violate ET, Symm, or both conditions.

Case 3a: Suppose that Symm is violated. Hence, there exist i, j such that $m_i < \frac{m}{n} < m_j$. By ZP

$$\sum_{i=1}^{N} \int_{t \in E_{i}} \left(o_{i}(t) - \phi\left(\frac{q(\mathsf{c})}{m_{i}}\right) \right) dt = p(\mathsf{c})nq(\mathsf{c}) - \sum_{i=1}^{N} m_{i}\phi\left(\frac{q(\mathsf{c})}{m_{i}}\right)$$

By convexity of $\phi(y)$, and the fact that $m_i \neq m_j$ for at least two enterprises,

$$\sum_{i=1}^{N} m_i \phi\left(\frac{q(c)}{m_i}\right) = m \sum_{i=1}^{N} \frac{m_i}{m} \phi\left(\frac{q(c)}{m_i}\right)$$
$$> m \phi\left(\frac{nq(c)}{m}\right)$$

Therefore,

$$\frac{1}{m}\sum_{i=1}^{N}\int_{t\in E_{i}}\left(o_{i}(t)-\phi\left(\frac{q(\mathsf{c})}{m_{i}}\right)\right)dt < p(\mathsf{c})\frac{nq(\mathsf{c})}{m}-\phi\left(\frac{nq(\mathsf{c})}{m}\right)$$

But the right-hand side is the per-team payoff in a configuration $\mathbf{c}' \in \Sigma_0$ with the same market structure as \mathbf{c} . Because the resource constraint is satisfied in \mathbf{c} , it is satisfied in \mathbf{c}' . Now, as we have seen in Case 2, and $\mathbf{c} \in \Sigma_0 - \Sigma^*$ is (weakly) dominated by a configuration \mathbf{c}'' in Σ^* , and $U(\mathbf{c}') \leq U(\mathbf{c}'')$ and the set $D_1 = \{t : u(t, \mathbf{c}) < U(\mathbf{c}'')\}$ is non-empty. Consider the sequence:

- $D_1 = \{t : u(t, c) < U(c'')\}$, and all teams in D_1 join the fringe. Letting ω_1 be the new market structure, HQs of D_1 make monetary offers equal to the surplus of a team under competition; the offers of the other HQs stay the same. The resulting market structure is $\omega_1 = (E_0 \cup D_1, E_1 - D_1, E_2 - D_1, \dots, E_N - D_1)$, and c_1 is the structure with the new offers for teams in D_1 and the old offers for the surviving enterprises.²⁴
- By the same argument as above, the average surplus among teams in the new oligopoly is inferior to U(c"). Hence the set D₂ = {t : u(t, c₁) < U(c")} has positive measure.
- All teams in D_2 join the fringe, and their HQs make offers leading to the surplus under competition; the new market structure has a fringe $E_0 \cup (D_1 \cup D_2)$ and

²⁴If the offer to teams in D_1 gives a surplus inferior to $U(\mathbf{c}'')$, teams in D_1 will be strictly better off at the end of the process. However, we need to ensure that the HQs of these teams are not worse off. Hence they should have a non-negative profit, and using the surplus of a team under competition ensures this.

enterprises $E_i - (D_1 \cup D_2)$.

- Continuing in this fashion, all teams join the fringe.
- Then, teams in E_1'' can form an enterprise with offers giving a surplus of U(c''), etc.
- This process leads to configuration c'' and, by construction, each set of teams that deviates is better off in c'' than in the configuration it deviates from.

Case 3b: ET is violated but not Sym. We can replicate the construction in case 3a. There exists a configuration $\mathbf{c}' \in \Sigma_0$ with the same (symmetric) market structure as \mathbf{c} . The average surpluses are the same in \mathbf{c} and \mathbf{c}' , but since ET is violated in \mathbf{c} , there exists a set of positive measure D_1 for which $u(t, \mathbf{c}) < U(\mathbf{c}')$, etc.

Binding TR constraint if m < 1. If a stable configuration c in Σ_0 is such that m < 1 and the TR constraint is not binding, there exists a new configuration c' in which a positive measure of teams in the fringe replace teams in the oligopoly and are strictly better of than in the initial configuration. Hence c' is not dominated by c, contradicting stability.

A.2 Variations with respect to market and oligopoly sizes

For convenience, we start by collecting in the following lemma some properties of the variables of interest in our Cournot model.

Lemma A.1. Consider a log-concave demand function and a convex private cost function. Suppose that x(n,m;S) < k and m < 1. Then, locally,

- (i) $\frac{q(N,m;S)}{S}$ is an increasing function of S.
- (ii) If m < 1, as m increases, the oligopoly output increases, and the total industry output decreases.

Proof. Log-concavity of the inverse demand function implies that the ratio

$$\Psi(y;S) := -\frac{P(y;S)}{P'(y;S)}$$

is a decreasing function of y and an increasing function of S. Let q(N, m; S) be the Cournot quantity of a powerful firm.

The Cournot quantity solves

$$\frac{q(N,m;S)}{S} = \Psi(Nq(N,m;S) + (1-m)k;S)$$

(i) At S' > S,

$$\frac{q(N,m;S)}{S} < \Psi\left(Nq(N,m;S) + (1-m)k;S'\right),$$

and therefore, to recover the first order condition, we must have $\frac{q(N,m;S')}{S'} > \frac{q(N,m;S)}{S}$. Note that while the oligopoly output increases, the industry output decreases, hence the price increases.

(ii) Differentiating the first order condition $\frac{q(N,m;S)}{S} = \Psi(Nq(N,m;S) + (1-m)k;S)$ with respect to m yields $q_m = (Nq_m - k)\Psi'$, where q_m denotes the partial derivative of q(N,m;S) with respect to m. Because $\Psi' < 0$, if $q_m < 0$ the right-hand side is negative only if $Nq_m > k$, a contradiction. Hence, q_m is positive and $Nq_m < k$, and the total output decreases with m to satisfy the first order condition. \Box

Note that while the output of oligopolistic firms increases with m, the per-team effort may increase or decrease with m.

A.3 Proof of Proposition 2

The surplus of teams in the oligopoly is

$$u(N,m;S) := P(mx(N,m;S) + (1-m)k;S)x(N,m;S) - \phi(x(N,m;S))$$

while the surplus of teams in the fringe is

$$u_0(N,m;S) := P(mx(N,m;S) + (1-m)k;S)k - \phi(k)$$

A stable configuration (N, m) solves

$$\begin{cases} \max_{N,m; m \in [0,1]} & P(mx(N,m;S) + (1-m)k;S) x(N,m;S) - \phi(x(N,m;S)) \\ & x(N,m;S) \le k \\ & u(N,m;S) = u_0(N,m;S) \text{ if } m < 1 \\ & u(N,1;S) \ge u_0(N,1;S) \end{cases}$$

and firms are powerful if the capacity constraint is not binding.

It is convenient to split this problem into two parts. First, keeping N constant, we can solve for the best (constrained) oligopoly size.

$$\begin{cases} \max_{m; m \in [0,1]} & P(mx(N,m;S) + (1-m)k;S) x(N,m;S) - \phi(x(N,m;S)) \\ & x(N,m;S) \le k \\ & u(N,m;S) = u_0(N,m;S) \text{ if } m < 1 \\ & u(N,1;S) \ge u_0(N,1;S) \end{cases}$$
(11)

This will give us the optimal size of the oligopoly M(N; S) and the indirect surplus of teams.

$$U(N;S) := u(N, M(N,S);S).$$

Suppose (N,1) is the stable configuration for a market size S. By the Cournot condition, $\frac{x(N,1;S)}{S} = N\Psi\left(\frac{x(N,1;S)}{S}\right)$ and therefore the ratio $\frac{x(N,1;S)}{S}$ is constant when S varies. It follows that if S' > S, $x(N,1;S') = \frac{S'}{S}x(N,1;S) > x(N,1;S)$.

The TR constraint (6) when m = 1 is

$$P(x(N,1;S);S) \le G(x(N,1;S)).$$
(12)

By convexity of ϕ , G(x) is an increasing function of $x \in [0, k]$. But then, at S',

$$P(x(N,1;S');S') = P(x(N,1;S);S) \le G(x(N,1;S)) < G(x(N,1;S')),$$

And the TR constraint holds with a strict inequality at S' if it is satisfied at S. Hence, as long as x(N, 1; S') satisfies the capacity constraint, (N, 1) is also a candidate for the stable configuration for a market size S' > S. Now, for each S', there exists a unique m(S') such that m(S')x(N - 1, m(S'); S') + (1 - m(S'))k = x(N, 1; S').²⁵ Because x(N - 1, m(S'); S') satisfies the capacity constraint, it must be inferior to x(N, 1; S').²⁶ For the market size S', the total output is the same – by construction – at (N - 1, m(S')) than at (N, 1). Because the effort per team is smaller, there is a larger surplus at (N - 1, m(S')) than at (N, 1). If (N - 1, m(S')) satisfies the TR condition, (N, 1) is no longer the stable configuration at S'. Note that if the TR constraint binds at (N, 1; S), it will not be satisfied at (N - 1, m(S'); S').

²⁵At m = 1, the left-hand side is equal to the maximum capacity. Hence greater than x(N, 1; S'); at m = 0 the left-hand side is the (full-) oligopoly intensity when there are N - 1 firms, which is smaller than the (full-) oligopoly output with N powerful firms.

²⁶Indeed, assuming that x(N, 1; S) < x(N-1, m(S'); S') would imply that x(N-1, m(S'); S') > k.

Therefore, if there exists \hat{S} such that the TR constraint binds at $(N-1, m(\hat{S}); \hat{S})$, then the market structure is (N, 1) for $S' \leq \hat{S}$ and is equal to $(N - 1, m(\hat{S}); \hat{S})$ at \hat{S} . If there is no such value \hat{S} , the capacity constraint $x(N, 1; S') \leq k$ will bind at a high enough value of S', and there will be competition.

Suppose (N, m), m < 1 is stable N for a market size S. Because the TR constraint binds at (N, m; S), the per-team surplus is equal to $u_0(N, m; S)$. Because the team payoff is an increasing function of m, if the TR constraint binds at (N, m, S) but is slack at (N, m, S'), for S' > S, it is possible to increase m, and a stable structure must have m' > m). Letting $x = \frac{Nq(N,m;S)}{m}$,

$$u(N,m;S) - u_0(N,m;S) = (k-x)(G(x) - P(mx + (1-m)k;S)))$$

Keeping the market structure, Lemma A.1 implies that as S increases, x increases. The variation in the right-hand side is $(G'x_S - P_S)(k - x) - (G - P)x_S$. But as TR binds, G - P = 0. Hence, the difference $u - u_0$ increases if and only if the difference G - P increases.

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