A Macroeconomic Perspective on Taxing Multinational Enterprises^{*}

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Abstract

We study the macroeconomic consequences of tax policies designed to reduce international profit shifting by multinational enterprises (MNEs) using a model that emphasizes transfer pricing of intangible capital. We prove analytically that such policies would reduce MNEs' intangible investment, reducing output both at home and abroad. We then quantify the effects of the OECD's proposed reforms: reallocating the rights to tax MNEs' profits to the countries where they sell their products; and a minimum global corporate income tax. These policies would reduce profit shifting by more than two-thirds, but would also reduce output in all regions of the global economy.

Keywords: Multinational enterprise; transfer pricing; profit shifting; base erosion; intangible capital; corporate tax.

JEL Codes: F23, H25, H26

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1 Introduction

Multinational enterprises (MNEs) shift large portions of their profits to foreign tax havens, costing governments in their home countries hundreds of billions of dollars per year in tax revenue. In October 2021, 136 countries signed onto a policy designed by the OECD and G20 governments to reduce profit shifting, making it the largest international tax reform in history. We analyze the macroeconomic consequences of OECD/G20 reform using a new model of profit shifting that emphasizes transfer pricing of intangible capital. We find that this reform would substantially reduce profit shifting and increase tax revenues in high-tax countries, but it would also cause global output to fall substantially.

Base erosion and profit shifting (BEPS) refers to MNEs' use of tax planning strategies to exploit gaps and mismatches in tax rules to artificially shift profits to low- or no-tax countries where they conduct little or no economic activity, or to erode tax bases through deductible payments such as interest or royalties. The scale of profit shifting is striking. For example, Tørsløv, Wier and Zucman (2022) estimate that 36 percent of worldwide multinational profits are shifted to tax havens, while Guvenen, Mataloni, Rassier and Ruhl (2022) find that 38 percent of foreign income reported by U.S. MNEs is actually generated at home in the United States. The implications for public finances are equally striking: Clausing (2020a) estimates that about a third of U.S. corporate income taxes are lost to profit shifting, which is equivalent to more than \$100 billion per year. According to the OECD, profit shifting reduces global corporate income tax revenues by as much as 10 percent per year, or \$240 billion (Johansson et al., 2017).

Addressing this issue is a top priority for policymakers in high-tax countries where many of the biggest MNEs are based. The OECD/G20 Inclusive Framework on BEPS outlines two major policy changes, or "pillars."¹ The first pillar is revenue-based profit allocation, which allocates the rights to tax some of an MNE's profits to the countries in which it operates in proportion to these countries' shares of the MNE's global sales. The second is a global minimum corporate income tax, which would require that all corporate income, regardless of where it is booked, be effectively taxed at no lower than 15 percent. At the time of this writing, none of the 136 signatory countries have made either pillar into law. However, in December 2022, the Council of the European Union approved a directive that requires E.U. countries to implement the second pillar by the end of 2023, and policymakers anticipate that this directive could lead to a wave of implementation around the world.²

MNEs can use a variety of strategies to shift profits, but the most important one centers

¹The press statement and a description of these pillars can be found here. We provide an executive summary of the current international tax regime and the OECD/GS20 Framework in Appendix A.

²See the press statement of the Council of the EU here.

around intangible capital.³ U.S. Senator Carl Levin puts it eloquently in a 2013 statement this issue: "More and more, intellectual property is the dominant source of value in the global economy. It is also highly mobile—unlike more tangible, physical assets, its value can be transferred around the globe, often with just a few keystrokes...The key to offshore tax avoidance is transferring the profit-generating potential of that valuable intellectual property offshore so that the profits are directed not to the United States, but to an offshore tax haven." One of the most prominent examples is Apple. Levin states that "95 percent of Apple's R&D... is conducted in the United States... [During] 2009 to 2012, [Apple Ireland] paid... \$5 billion to [Apple USA] as its share of the R&D costs. Over that same time period, ASI received profits of \$74 billion. The difference between ASI's costs and the profits, almost \$70 billion, is how much taxable income [should] have flowed to the United States."⁴ In addition to this anecdote, there is a wide variety of empirical evidence that intangible capital plays a central role in profit shifting. Guvenen et al. (2022) show that profit shifting is concentrated in the most intangible-intensive industries such as electronics manufacturing, pharmaceuticals, and information technology. Gumpert et al. (2016) and Delis et al. (2021) find similar relationships between intangible intensity and profit shifting at the firm level. More direct evidence comes from Accoto et al. (2021), who document that profit shifting is associated with imports of intellectual property services from recognized tax havens countries, and Dischinger and Riedel (2011), who find that MNEs transfer ownership of intangible capital to subsidiaries in low-tax countries.

Motivated by this evidence, we develop a theory that (i) explicitly describes how MNEs shift profits by transferring the rights to intangible capital, and (ii) connects profit shifting to MNEs' production decisions. As in McGrattan and Prescott (2010), intangible capital is nonrival: MNEs produce it by doing research and development at home, but use it to produce simultaneously in all of their foreign subsidiaries around the world.⁵ According to transfer pricing rules, these subsidiaries pay licensing fees to use this capital. Normally, these fees are paid to the domestic parent corporation, but the rights to this capital can be transferred—at a cost—to subsidiaries in a tax haven. The end result is that the income generated by MNEs' intangible capital is taxed at a lower rate, which increases an MNE's optimal level

 $^{^{3}}$ We provide a discussion of profit shifting channels and their importance in the Appendix A. The existing empirical evidence suggest that up to 80 percent of profit shifting is related to intangible capital and manipulating transfer prices and the rest is associated with debt payment manipulations within MNE.

⁴See Levin's full statement here.

⁵Some MNEs do R&D in their foreign affiliates as well, but this is not quantitatively important. See Arkolakis et al. (2018), who write: "most of the R&D is still done in the multinationals' home country. For example, according to BEA data for 2009, the parents of U.S. multinationals accounted for 85 percent of its total RD expenditure but only 70 percent of its value-added. See also Bilir and Morales (2020), which concludes that the parent RD is a substantially more important determinant of firm performance than affiliate R&D."

of intangible investment. Because intangible capital is nonrival, this leads to higher output in all of an MNE's subsidiaries, both foreign and domestic. This illustrates the tradeoff that profit shifting presents to global policymakers: although it artificially redistributes MNEs' income to foreign tax havens, it also increases the amount of worldwide income that they actually generate. Moreover, we show that the size of this effect is increasing in the difference between the corporate tax rates in the MNE's home country and the tax haven. This has direct implications for the second pillar of the OECD/G20 plan: the higher the minimum tax rate, the larger the reduction in intangible investment and global economic activity. Similarly, our theory shows that sales-based profit reallocation, the first pillar of this plan, will also have adverse macroeconomic effects.

To quantify the macroeconomic effects of the OECD/G20 proposal, we embed our theory into a multi-country, general-equilibrium framework. Each country in our quantitative model is populated by a representative household, a government, and a measure of firms. Households choose how much to work and how much to consume. Governments levy taxes on corporate profits to finance lump-sum transfers. Firms are heterogeneous in productivity and make four choices: where to export; where to establish foreign affiliates; intangible investment; and profit shifting. Exporting and FDI are subject to fixed costs, so only the most productive firms engage in multinational production in equilibrium as in Helpman et al. (2004). All firms invest in intangible capital, but its nonrival nature makes the return greater for MNEs, and so they account for the lion's share of intangible investment, consistent with the empirical evidence.⁶ However, firms gain access to the profit-shifting technology only if they have an affiliate in a tax-haven country. Our quantitative model makes several methodological contributions in its own right: incorporating nonrival intangible capital into a heterogeneous-firm environment; allowing individual firms to make joint decisions about multinational production and innovation; and, of course, incorporating our theory of profit shifting.

In our calibration, we discipline the model's parameters so that it reproduces micro- and macroeconomic data on production, trade, multinational activity, and, most importantly, profit shifting. We split the world into five regions. The countries identified as tax havens make up two of these regions: the first is a productive low-tax region that includes Ireland, Switzerland, and other countries where most of the economy is not devoted to profit shifting; while the second is a "true" tax haven that includes the Caribbean, the Channel Islands, and other small countries whose economies rely heavily on profit shifting. The other three regions are North America, Europe (minus countries in the low-tax region), and the rest of

 $^{^{6}}$ According to the BEA data U.S. MNEs' parent companies account for about 75% of all R&D in the United States post 2000. See Foley et al., eds (2021) for an extensive discussion on the importance of the MNEs.

the world. Firms in these three regions can shift profits to the low-tax region and/or the tax haven—provided they have paid the cost of establishing foreign affiliates there. We choose the costs of profit shifting to match Tørsløv et al. (2022)'s estimates lost profits at the country level. Our model also reproduces three other sets of facts about profit shifting that we do not target in our calibration: the share of low-tax countries' corporate income taxes that are paid by foreign MNEs; the aggregate compensation of employees hired by MNEs to engage in profit shifting; and the MNE-level relationship between profits reported by the domestic parent division and the tax differential between the home country and the tax haven.

We use our calibrated model to simulate the effects of the two pillars of the OECD/G20proposal, both together and in isolation. We find that this proposal would go a long way toward eliminating profit shifting: lost profits would fall by 77 percent in North America, 82 percent in Europe, and 90 percent in the rest of the world. However, it would also materially reduce intangible investment and overall macroeconomic performance around the world: GDP would fall by 0.17 percent in North America, 0.16 percent in Europe, 0.13 percent in the low-tax productive region, and 0.14 percent in the rest of the world. Our model predicts a decline in global GDP more than twice as large the OECD estimates (OECD, 2020), highlighting the quantitative importance of our methodological innovations. Additionally, we show that although both pillars of the OECD/G20 plan would reduce profit shifting, the first (sales-based profit reallocation) would have significantly larger macroeconomic consequences than the second (a global minimum corporate income tax). This is because the former affects firms that do not shift profits and even some firms that do not engage in multinational production at all. We find that the same reduction in profit shifting could be accomplished at a much smaller macroeconomic cost by scrapping the first pillar entirely and slightly increasing the minimum tax rate, and we recommend that policymakers seriously consider this alternative.

2 Related Literature

This paper contributes to two strands of literature on profit shifting. First, we draw on two groups of empirical studies on profit shifting. The first group documents the striking scope of this phenomenon at the aggregate level. Guvenen et al. (2022) estimate that 38 percent of U.S. MNEs' foreign income is the result of profit shifting and should be re-attributed to domestic GDP, while Tørsløv et al. (2022) report a similar figure at the global level. Clausing (2020a) concludes that profit shifting reduces U.S. corporate tax revenues by as much as a

third.⁷ We draw on these estimates to discipline our quantitative model. The second group provides microeconomic evidence about which firms engage in profit shifting and how they do so. Gumpert et al. (2016) and Delis et al. (2021) show that firms with high shares of intangible capital in total assets are more likely to shift profits, and Accoto et al. (2021) document that profit-shifting firms import intellectual property services from recognized tax-havens. Using cross-country data on bilateral royalty payments from 1995 to 2012, Santacreu (2023) finds that differences in taxation impact international technology licensing. These studies provide support for our theory of profit shifting that centers around transfer pricing of intangible capital.

Second, we contribute to the literature on the real economic consequences of profit shifting. Most studies in this literature use models that are based on Hines and Rice (1994), in which MNEs can pay a cost to reduce their effective tax rate by shifting profits to a low-tax country. For example, Suárez Serrato (2018) illustrates the effects of a policy that limited U.S. MNEs' ability to shift profits to Puerto Rico and finds that it reduces the affected firms' investment activity within the US. Bilicka et al. (2022) add a fixed profit-shifting cost to study the extensive margin of this phenomenon.⁸ In these reduced-form models, profit shifting has real effects simply because investment is not tax deductable (which implies that it is decreasing in the effective tax rate), but the mechanism by which firms shift profits is left unspecified. By contrast, our micro-founded model explicitly spells out the transfer pricing transactions MNEs use to engage in profit shifting. Further, these studies restrict attention to partial-equilibrium, firm-level analysis, whereas we embed our theory into a general equilibrium framework and quantify the macroeconomic effects of profit shifting.

More broadly, this paper contributes to the literature on globalization and multinational activity. One strand of this literature emphasizes the role of firm heterogeneity and selection. The seminal papers of Melitz (2003) and Chaney (2008) study selection into exporting. Helpman et al. (2004) develop a model of the "proximity-concentration tradeoff," in which firms can serve foreign markets by exporting, which requires a small fixed cost but larger variable costs, or by establishing foreign affiliates, which requires a large fixed cost but smaller variable costs. More recent work incorporates additional margins like export platforms (Tintelnot, 2017), endogenous product creation (Arkolakis et al., 2018), selection dynamics (Garetto et al., 2019), and offshoring (Spencer, 2021). Another strand emphasizes the role of nonrival intangible capital in shaping the aggregate effects of foreign direct investment (FDI). Mc-

⁷Blouin and Robinson (2020) and Clausing (2020a) discuss the methodological challenges associated with estimating the magnitude of profit shifting. Bolwijn et al. (2018) and Crivelli et al. (2015) study the impact of profit shifting on tax revenues for developing countries. See Dowd et al. (2017), Clausing (2016), and OECD (2015) for extensive reviews of the profit-shifting literature and the estimates found therein.

⁸Other studies in this line of research documenting real effects of transfer pricing and profit shifting include: Buettner et al. (2018), de Mooij and Liu (2020) or Schwab and Todtenhaupt (2021).

Grattan and Prescott (2009) build a neoclassical growth model in which the representative multinational invests in intangible capital that can be used simultaneously to produce output at home and abroad, and show that this channel substantially increases the gains to openness to FDI.⁹ McGrattan and Waddle (2020) use a multi-country version of this model to study the macroeconomic consequences of FDI restrictions caused by Brexit. We synthesize these two approaches by developing a model in which heterogeneous firms choose where to export, where to establish foreign affiliates, and how much to invest in nonrival intangible capital. On top of this new framework, we incorporate our theory of profit shifting, allowing firms to additionally choose whether to establish affiliates in a tax haven and how much intangible capital to shift.

In terms of quantitative methodology, the most similar papers to ours in this literature are Arkolakis et al. (2018) and Wang (2020). Arkolakis et al. (2018) studies the relationship between innovation and multinational production. In their model, innovation is used to create new firms, which then go on to make decisions about where to establish foreign affiliates. Importantly, innovation occurs at the aggregate level (it is pinned down by a sort of free-entry condition) and is completely separate from the firm's individual optimization problem.¹⁰ In our model, each individual firm chooses its own level of intangible investment as well as where to establish foreign affiliates. Moreover, the nonrival nature of intangible capital in our model creates an important interaction between these two decisions: the more foreign affiliates a firm establishes, the greater the return to intangible investment. Wang (2020)extends Arkolakis et al. (2018) to incorporate a reduced-form kind of profit shifting similar to Hines and Rice (1994). Due to the structure of the Arkolakis et al. (2018) framework, there is no interaction in this paper between profit-shifting decisions and innovation; the only decision that profit shifting affects in this paper is the choice of where to establish foreign affiliates. Importantly, profit shifting has no effect on an MNE's output in its home country (or in any other foreign country in which it would operate even in the absence of profit shifting). Our approach to modeling profit shifting has two significant advantages: (i) it explicitly micro-founds the transfer pricing transactions used to reallocate ownership of intangible capital and the income that it generates; and (ii) it directly affects firms' incentives to invest in intangible capital.

⁹McGrattan and Prescott (2010) show that nonrival intangible capital also has important measurement implications. Specifically, they show that it accounts for the high profitability of foreign subsidiaries of U.S. MNEs relative to U.S. subsidiaries of foreign MNEs. This helps explain why U.S. net foreign payments are positive despite the United States' negative current account and net foreign asset position—nonrival intangible capital is Hausmann and Sturzenegger (2007)'s "dark matter."

¹⁰This is true even in the extension of their model described in their online appendix in which they allow for innovation that improves productivity. In this version of their model, entrants can augment the expected productivity of the firms they create, but cannot do any further productivity-enhancing innovation once these firms' productivities are drawn.

Our paper also relates to the lines of research on corporate income taxation and tax competition. The literature on the corporate taxes is vast and dates back to seminal contributions by Harberger (1962) and Auerbach (1983). More recent contributions include Barro and Furman (2018), who assess the macroeconomic consequences of the Tax Cuts and Jobs Act of 2017; Kaymak and Schott (2018), who argue that falling corporate income taxes across are the main driver behind the decline of the labor share; Fajgelbaum, Morales, Serrato and Zidar (2019) who quantify the degree of spatial misallocation resulting from state taxes in the U.S. using a general equilibrium framework; and Bhandari and McGrattan (2020), who quantify the impact of reducing corporate income taxes in a model where firms choose their legal form of organization. So far, however, little attention has been paid in this literature to the macroeconomic effects of international profit shifting and its impact on international tax competition; see Keen and Konrad (2013) for an extensive review. In a companion paper (Dyrda et al., 2022), we integrate our theory of profit shifting into the game-theoretic tax competition framework.

3 Theory of Profit Shifting and Intangible Investment

In order to study the real effects of profit shifting, we develop a theory of transfer pricing based on McGrattan and Prescott (2009)'s model of multinational production with nonrival intangible capital. An MNE based in a high-tax country invests in intangible capital at home and uses it to produce simultaneously in its subsidiaries abroad, which pay licensing fees to use this capital according to transfer pricing rules. The MNE can shift profits by selling intangible capital property rights to a subsidiary in a tax haven, so that licensing fees flow to this subsidiary instead of the MNE's domestic parent company. The end result is that the after-tax return on intangible investment increases, which incentivizes the MNE to do more of this investment and ultimately produce more output.

3.1 Environment

Consider an MNE that operates subsidiaries in I regions. Each region k = 1, ..., I is characterized by population N_k , total factor productivity A_k , and corporate tax rate $\tau_k \in [0, 1]$. The MNE's home region is denoted by i. Without loss of generality, we normalize the entire population across regions to unity, i.e. $\sum_{k=1}^{I} N_k = 1$. We refer to the region with the lowest tax rate, which we denote by i^* , as the tax haven, i.e., $\tau_{i^*} = \min{\{\tau_1, ..., \tau_I\}}$.

In each region, the MNE has access to a production technology F_k in that transforms

labor ℓ_k and intangible capital z into a final good:

$$F_k(z,\ell_k) = A_k \left(N_k z\right)^{\phi} \ell_k^{\gamma}.$$
(1)

As in McGrattan and Prescott (2009), intangible capital is nonrival: it is purchased in the headquarters region i at the local price p_i , but it can be used in all I locations simultaneously. Its productivity is determined by the local population N_k , which proxies for the number of production locations in a given region where the intangible capital can be deployed. Labor is rented in a competitive market at wage rate w_k . We assume decreasing returns to scale, i.e., $\phi + \gamma < 1$.¹¹

As a starting point, we begin by defining the MNE's profits in the standard setup (e.g., as in McGrattan and Prescott, 2009) in which foreign subsidiaries use intangible capital free of charge:

$$\pi_i = p_i \left(A_i \left(N_i z \right)^{\phi} \ell_i^{\gamma} \right) - w_i \ell_i - p_i z \tag{2}$$

$$\pi_k = p_k \left(A_k \left(N_k z \right)^{\phi} \ell_k^{\gamma} \right) - w_k \ell_k, \quad \forall k \neq i.$$
(3)

We refer to this as the *free transfer* (FT) scenario and denote the allocation of intangible capital in this case by z^{FT} . Our methodological innovation is to add two new ingredients to this setup: transfer pricing and profit shifting, which we do one at a time.

In the *transfer pricing* (TP) scenario, the parent division retains legal ownership of the MNE's stock of intangible capital and licenses the right to use this capital to its foreign affiliates. The accounting profits in each of the MNE's divisions in this scenario are

$$\pi_i^{TP} = \pi_i + \sum_{k \neq i} \vartheta_k(z) \, z, \tag{4}$$

$$\pi_k^{TP} = \pi_k - \vartheta_k(z) z \quad \forall k \neq i.$$
(5)

According to the arm's length principle, the licensing fees, ϑ_k , are set to the affiliates' marginal revenue products of intangible capital,

$$\vartheta_k(z) \equiv \phi p_k N_k \left(A_k \left(N_k z \right)^{\phi - 1} \ell_k^{\gamma} \right).$$
(6)

We denote the allocation of intangible capital in this case by z^{TP} . In this section, we assume that the MNE takes $\vartheta_k(z)$ as given according to the spirit of the arm's length principle;

¹¹In our quantitative model we assume constant returns to scale and monopolistic competition. In this partial equilibrium setting, the two approaches are isomorphic. We choose a decreasing returns approach here for its analytical simplicity.

it does not internalize the effect of its choice of z on $\vartheta_k(z)$. Mathematically speaking, the MNE does not take the derivative of $\vartheta_k(z)$ when taking the first-order condition of its profit function with respect to z. This keeps our key equations relatively simple, which allows us to highlight the important economic forces at work behind our results. In Appendix E.3, we show that all of our analytical results hold when the MNE does internalize the effect of its choice of z on $\vartheta_k(z)$, and we allow for this effect in our quantitative analysis as well.

In the profit shifting (PS) scenario, the MNE's headquarter sells a fraction λ of its intangible capital to its affiliate in the tax haven, which then licenses the rights to use this capital to the parent division and the other non-haven foreign affiliates. We assume that the tax-haven affiliate buys intangible capital from the headquarters at a markdown $\varphi \leq 1$ below the competitive price, which is equal to the sum total of the licensing fees that this capital can generate, i.e., the sum of the marginal revenue products across all of the regions in which the MNE operates.¹² Manipulating transfer prices in this way is assumed to be costly, as the multinational needs to modify its books, and possibly its real trade and investment patterns, to be able to justify the distorted transfer prices to the tax authorities. We impose the following assumption on the cost function $C(\lambda)$.

Assumption 1 Let $C(\lambda) \equiv \lambda + (1 - \lambda) \log (1 - \lambda)$, implying $C'(\lambda) = -\log (1 - \lambda)$, C(0) = 0, C(1) = 1, and $\lambda \in [0, 1)$.

It is important to note that $C(\lambda)$ captures direct costs of profit shifting (e.g. increased spending on lawyers, accountants, and transfer pricing consultants), but also, in a reducedform way, the increased risk of penalization by the government (see, e.g., Allingham and Sandmo, 1972; Rotberg and Steinberg, 2022).

Pre-tax profits in the *profit shifting* scenario are thus:

$$\pi_{i}^{PS} = \pi_{i} + z \left[\varphi \lambda \sum_{k} \vartheta_{k} \left(z \right) - \lambda \vartheta_{i} \left(z \right) + \left(1 - \lambda \right) \sum_{k \neq i} \vartheta_{k} \left(z \right) - \sum_{k} \vartheta_{k} \left(z \right) \mathcal{C} \left(\lambda \right) \right], \quad (7)$$

$$\pi_{i^{*}}^{PS} = \pi_{i^{*}} + z \left[\lambda \sum_{k \neq i^{*}} \vartheta_{k} \left(z \right) - \left(1 - \lambda \right) \vartheta_{i^{*}} \left(z \right) - \varphi \lambda \sum_{k} \vartheta_{k} \left(z \right) \right], \tag{8}$$

$$\pi_k^{PS} = \pi_k - \vartheta_k \left(z \right) z \quad \forall k \neq i, i^*.$$
(9)

The first term in the square brackets in (7), $\varphi \lambda \sum_k \vartheta_k(z)$, is the revenue from selling intangible capital to the tax haven. The second term, $-\lambda \vartheta_i(z)$, denotes the licensing fee that the

¹²Instead of marking down the price at which the tax haven buys intangible capital, we could mark up the licensing fees that the parent (and other subsidiaries) pay to the tax haven. One can show that for any value of the markdown φ , there exists a markup on licensing fees paid to the tax haven that yields exactly the same solutions for λ and z. Thus, these two formulations are equivalent.

headquarter pays to the tax haven for the right to use the fraction λ of intangible capital that has changed ownership. The third term, $(1 - \lambda) \sum_{k \neq i} \vartheta_k(z)$, represents the licensing fees that the headquarter collects from the other affiliates for the remaining intangible capital that the headquarter retains. The term $\mathcal{C}(\lambda) \sum_k \vartheta_k(z)$ captures the costs of shifting intangible capital to the tax haven. The terms in (8) have analogous interpretations. We denote the allocation of intangible capital in this scenario by z^{PS} .

Consider the problem of maximizing after-tax profits in each scenario:

$$\max_{z^{s}, \left\{\ell_{k}^{s}\right\}_{k=1}^{I}, \lambda} \sum_{k=1}^{I} \left(1 - \tau_{k}\right) \pi_{k}^{s} \tag{10}$$

where $s \in \{FT, TP, PS\}$. Note that λ is only chosen in the profit shifting scenario. We first characterize the MNE's optimal choice of λ in this scenario, and then characterize how this choice alters the MNE's intangible investment decision. The formal proofs of these results are relegated to Appendix E.1.

3.2 Optimal profit shifting

In the *profit shifting* scenario, the MNE's optimal choice of λ is given by

$$\lambda = 1 - \exp\left(-\frac{(1-\varphi)\left(\tau_i - \tau_{i^*}\right)}{1-\tau_i}\right). \tag{11}$$

The following lemma provides a formal characterization of how this solution depends on the profit shifting technology, which is governed by the markdown φ , and the potential gain from shifting profits, which is governed by the tax haven's tax rate τ_{i^*} .

Lemma 1 The share λ of intangible capital sold to the tax haven is decreasing in φ with elasticity

$$\varepsilon_{\varphi}^{\lambda} = -\left(\frac{1-\lambda}{\lambda}\right) \left(\frac{\tau_i - \tau_{i^*}}{1 - \tau_i}\right) \varphi < 0, \tag{12}$$

decreasing in τ_{i^*} with elasticity

$$\varepsilon_{\tau_{i^*}}^{\lambda} = -\left(\frac{1-\lambda}{\lambda}\right) \left(\frac{1-\varphi}{1-\tau_i}\right) \tau_{i^*} < 0.$$
(13)

The first part of this lemma says that the smaller the markdown below the competitive price (i.e. the larger φ is), the smaller the fraction of intangible capital that is shifted to the tax haven. In particular, if the MNE has to sell the rights to intangible capital at the competitive price with no markdown (i.e., $\varphi = 1$), then no profit shifting takes place at all. The second part says that λ is decreasing in the tax haven's tax rate, τ_{i^*} . The elasticity of λ with respect to τ_{i^*} depends on four terms. First, the closer λ is to 1, the larger the reduction. Second, λ is more responsive to τ_{i^*} if the markdown φ is smaller. Third, the elasticity is increasing in the level of the tax rate in the headquarters, τ_i . Finally, it is proportional to τ_{i^*} itself.

3.3 The Effect of Profit Shifting on Intangible Investment

Having characterized the MNE's decision about how much intangible capital to transfer to the tax haven, we can now characterize the effect of this decision on the MNE's intangible investment choice. The optimal intangible capital allocations in the three scenarios are

$$z^{FT} = \left(\frac{\sum_{k} \left(1 - \tau_{k}\right) \Lambda_{k}}{\left(1 - \tau_{i}\right) p_{i}}\right)^{\frac{1 - \gamma}{1 - \phi - \gamma}},\tag{14}$$

$$z^{TP} = \left(\frac{\sum_k \Lambda_k}{p_i}\right)^{\frac{1-\gamma}{1-\phi-\gamma}},\tag{15}$$

$$z^{PS} = z^{TP} \left(1 - \mathcal{C}\left(\lambda\right) + \frac{\lambda \left(1 - \varphi\right) \left(\tau_i - \tau_{i^*}\right)}{1 - \tau_i} \right)^{\frac{1 - \gamma}{1 - \phi - \gamma}},\tag{16}$$

where $\Lambda_k \equiv \phi \gamma^{\frac{\gamma}{1-\gamma}} p_k^{\frac{1}{1-\gamma}} A_k^{\frac{1}{1-\gamma}} \left(\frac{1}{w_k}\right)^{\frac{\gamma}{1-\gamma}} N_k^{\frac{\phi}{1-\gamma}}$. The following proposition summarizes the relationships between these allocations.

Proposition 1 The following hold:

- 1. if $\tau_i = \max{\{\tau_k\}_{k=1}^K}$ then $z^{TP} < z^{FT}$;
- 2. if $\varphi < 1$ then $z^{PS} > z^{TP}$;
- 3. z^{PS} is decreasing in φ ;
- 4. z^{PS} is decreasing in τ_{i^*} with elasticity

$$\varepsilon_{\tau_{i^*}}^{z^{PS}} = -\left(\frac{1-\gamma}{1-\phi-\gamma}\right) \frac{1}{\left(1+\frac{1-\mathcal{C}(\lambda)}{\lambda \mathcal{C}'(\lambda)}\right)} \left(\frac{\tau_{i^*}}{\tau_i-\tau_{i^*}}\right) < 0.$$
(17)

The first part of the proposition states that if the MNE's home country has the highest tax rate across all of the jurisdictions in which the MNE operates, transfer pricing reduces intangible investment, i.e., $z^{TP} < z^{FT}$. Intuitively, requiring foreign affiliates to pay licensing fees to use intangible capital reallocates intangible income to the headquarters, and if the headquarters' income is taxed at a higher rate, the MNE's global profits decline. This

demonstrates that asymmetries in tax rates across jurisdictions are more distortionary when MNEs are required to account for intangible income according to the arm's length principle.

The second part of the proposition states that, relative to the transfer pricing scenario, profit shifting increases intangible investment, i.e., $z^{PS} > z^{TP}$, if and only if intangible capital can be sold to the tax haven below the competitive price, i.e., $\varphi < 1$. In this case, as can be seen in (11), $\lambda \in (0, 1)$, and we show in the Appendix that this implies the term in parentheses in (16) is strictly greater than one. Intuitively, profit shifting allows the MNE to partially undo the impact of transfer pricing. Transfer pricing forces the MNE to book foreign affiliates' intangible income at the home tax rate, while profit shifting allows the MNE to book some of this income at the tax haven's tax rate instead. In fact, if the MNE's home country has the highest tax rate, then one can show that $z^{TP} < z^{PS} < z^{FT}$.

The third and fourth parts of the proposition characterize the size of the effect described in the second part. As shown in Lemma 1, the smaller the markdown (the larger φ is), the smaller the fraction λ of intangible capital that is sold to the tax haven. This implies that the MNE's profit is decreasing in φ ; the closer the transfer price is to the competitive price, the lower the incentive to purchase intangible capital. In turn, this implies that z^{PS} is decreasing in φ . Similarly, z^{PS} is decreasing in the tax haven's tax rate τ_{i^*} . As this rate increases, λ falls, and with it falls the extra gain from intangible investment relative to the transfer pricing scenario. The elasticity of this margin is negative and given by (17). It is a product of three terms: (i) technological parameters; (ii) the profit shifting cost function; and (iii) the difference between the tax rates in the tax haven and the MNE's home country.

These results are crucial for understanding the central economic trade-off we uncover in this paper: profit shifting erodes high-tax countries' tax bases, but also boosts economic activity by increasing MNEs' intangible investment. This trade-off has important implications for the OECD/G20 BEPS framework. Specifically, a global minimum corporate income tax—which in this simple environment acts like an increase in the tax haven's tax rate—will reduce profit shifting, but this reduction will come at the cost of lower economic performance.

3.4 Effects of Sales-Based Profit Allocation

We can also use our theory of profit shifting to illustrate the impact of the first pillar of the OECD/G20 framework, which allocates the rights to tax a portion of an MNE's global profits to the regions in which it operates in proportion to these regions' shares of the MNE's overall sales. Under this rule, the tax base of a subsidiary in region k is the sum of local routine profit π_k^r , a share $(1 - \theta)$ of local residual profit π_k^R , and a fraction of total global residual profit Π^R that is based on this region's share of the MNE's total global sales:

$$T_k = \pi_k^r + (1 - \theta) \cdot \pi_k^R + \theta \cdot \frac{p_k y_k}{\sum_k p_k y_k} \cdot \Pi^R.$$
 (18)

Routine profit is defined as the fraction μ of the revenues in jurisdiction k: $\pi_k^r = \mu p_k y_k$. Residual profit is defined as the complementary fraction: $\pi_k^R = \pi_k^{PS} - \pi_k^r$. Global residual profit is the sum of residual profits across regions: $\Pi^R = \sum_i \pi_i^R$. The two key parameters are: (i) the fraction of residual profits that are allocated across regions based on sales, θ ; and (ii) the routine profitability margin, μ . Under the OECD/G20 proposal, these are set to $\theta = 0.25$ and $\mu = 0.1$, but in what follows we will analyze comparative statics with respect to their values.

Consider now the MNE's modified profit-maximization problem in the *profit shifting* scenario under the profit allocation rule:

$$\max_{z^{PS}, \left\{\ell_k^{PS}\right\}_{k=1}^I, \lambda} \sum_{k=1}^I \left(\pi_k^{PS} - \tau_k T_k\right).$$
(19)

The share of intangible capital that is sold to the tax haven is now given by

$$\hat{\lambda} = 1 - \exp\left(-\frac{(1-\varphi)\left(1-\theta\right)\left(\tau_{i}-\tau_{i^{*}}\right)}{1-\left((1-\theta)\tau_{i}+\theta\hat{\tau}\right)}\right).$$
(20)

where $\hat{\tau}$ is the sales-weighted average tax rate across regions:

$$\widehat{\tau} \equiv \sum_{i} \tau_i \cdot \frac{p_i y_i}{\sum_k p_k y_k}.$$
(21)

The MNE's optimal choice of intangible capital is given by

$$\hat{z}^{PS} = \hat{z}^{TP} \left(1 - \mathcal{C}\left(\lambda\right) + \frac{\left(1 - \theta\right)\lambda\left(1 - \varphi\right)\left(\tau_{i} - \tau_{i^{*}}\right)}{\left(1 - \left(\left(1 - \theta\right)\tau_{i} + \theta\widehat{\tau}\right)\right)} \right)^{\frac{1 - \gamma}{1 - \phi - \gamma}}.$$
(22)

We are now ready to characterize how the profit allocation rule affects the MNE's intangible investment decision.

Proposition 2 The following hold:

- 1. the allocation of intangible capital under the profit allocation rule, for any $0 < \theta \leq 1$, is smaller than under the current regime, i.e. $\hat{z}^{PS} < z^{PS}$;
- 2. \hat{z}^{PS} is decreasing in θ with elasticity

$$\varepsilon_{\theta}^{\hat{z}^{PS}} = \varepsilon_{\theta}^{\hat{\lambda}} \left(\frac{1 - \gamma}{1 - \phi - \gamma} \right) \left(\frac{\hat{\lambda}}{\mathcal{C}(\hat{\lambda})(1 - \hat{\lambda})} \right) \left(\frac{1}{1 + \frac{1 - \mathcal{C}(\hat{\lambda})}{\hat{\lambda}\mathcal{C}'(\hat{\lambda})}} \right) < 0;$$
(23)

3. \hat{z}^{PS} is decreasing in τ_{i^*} , and if the MNE's sales in the tax haven are sufficiently small then

$$\left|\varepsilon_{\tau_{i^*}}^{\hat{z}^{PS}}\right| < \left|\varepsilon_{\tau_{i^*}}^{z^{PS}}\right|. \tag{24}$$

The first part of the proposition states that the profit allocation rule will reduce intangible investment relative to the current regime, i.e., $\hat{z}^{PS} < z^{PS}$. This can be seen by comparing the solution for z^{PS} in (15) with the solution for \hat{z}^{PS} in (22). The second part states that intangible investment is decreasing in the fraction of residual profits allocated based on sales, θ . The elasticity of this margin is given by (23). It is proportional to the elasticity of $\hat{\lambda}$ with respect to θ given by (E.13), which itself is negative as shown in the Appendix. Finally, the third part of the proposition states that intangible investment under the profit allocation rule is decreasing in the tax haven's tax rate, which is also true under the current regime. However, as with the share of intangible capital sold to the tax haven, the size of this effect is smaller under the profit allocation rule, provided that the tax haven is sufficiently small.¹³

These findings reveal an important interaction between the two OECD/G20 pillars and provide a deeper understanding of the trade-offs that policymakers face. On the one hand, the profit allocation rule decreases profit shifting. On the other hand, although it decreases intangible investment, it also alleviates the negative impact of the global minimum tax. As we will see, these margins play important roles in our quantitative analysis, which we take up in the next two sections of the paper.

4 Quantitative Model

In order to assess the macroeconomic implications of our theory of profit shifting, we integrate it into a general equilibrium model with heterogeneous firms in the tradition of the international economics literature. Our quantitative framework synthesizes Helpman et al. (2004)

¹³We show in the Appendix that the profit allocation rule reduces the fraction of intangible capital sold to the tax haven, i.e. λ ; and λ decreases in the allocation share, θ , and the tax rate of the tax haven, τ_{i^*} .

and McGrattan and Prescott (2009). There are I "productive" regions, each populated by a representative household, a measure of heterogeneous firms, and a government. Regions, indexed by i and j, differ in population, total factor productivity, trade costs, FDI costs, and corporate income taxes. Firms in each region decide the following: where to export and where to establish foreign subsidiaries; how much labor to hire in the parent division and each foreign subsidiary; and how much intangible capital to produce in the parent division. Intangible capital is nonrival and is used simultaneously in all of a firm's divisions.

As in section 3, multinational firms (firms with foreign affiliates) use transfer pricing to allocate the costs of producing intangible capital across their foreign affiliates in proportion to the scale at which these affiliates use this capital. Affiliates license the right to use intangible capital from the division that owns this capital, and MNEs can shift profits by selling their intangible capital to affiliates in lower-tax regions. We denote the "productive" region with the lowest corporate income tax rate by LT. Additionally, there is an "unproductive" tax haven that is populated by a representative household and a government, labelled as TH, where no economic activity takes place. MNEs based in high-tax regions can transfer their intangible capital rights to either (or both) of these regions, provided that they have established subsidiaries there.

As in the stylized model in section 3, we restrict attention to a static economy, although one can interpret our quantitative model's equilibrium as the long-run steady state of a dynamic model. Studying the transition dynamics that would follow corporate tax reforms, and how profit shifting would shape these dynamics, would be worthwhile pursuit, but we leave this for future research as our model is already quite computationally complex.

4.1 Households

Each region *i* has a representative household with preferences over consumption, C_i , and labor supply, L_i , given by

$$u\left(\frac{C_i}{N_i}, \frac{L_i}{N_i}\right) = \log\left(\frac{C_i}{N_i}\right) + \psi_i \log\left(1 - \frac{L_i}{N_i}\right).$$
(25)

Households choose consumption and labor supply to maximize utility subject to a budget constraint

$$P_i C_i = W_i L_i + D_i + T_i, (26)$$

where W_i is the wage, D_i is the aggregate dividend payment from firms based in region *i*, and T_i is a transfer from the government.

Consumption is a constant-elasticity-of-substitution aggregate of products from different

source countries,

$$C_{i} = \left[\sum_{j=1}^{J} \int_{\Omega_{ji}} q_{ji}(\omega)^{\frac{\rho-1}{\rho}} d\omega\right]^{\frac{\rho}{\rho-1}},$$
(27)

where $q_{ji}(\omega)$ is the quantity of variety ω from region j, Ω_{ji} is the set of goods from j available in i, and ρ is the elasticity of substitution between varieties. The demand curve for each variety can be written as

$$p_{ji}(\omega) = P_i C_i^{\frac{1}{\rho}} q_{ji}(\omega)^{-\frac{1}{\rho}}.$$
 (28)

The aggregate price index is

$$P_i = \left[\sum_{j=1}^J \int_{\Omega_{ji}} p_{ji}(\omega)^{1-\rho} d\omega\right]^{\frac{1}{1-\rho}}.$$
(29)

4.2 Firms

Each productive region *i* has a unit measure Ω_i of firms that compete monopolistically as in Melitz (2003) and Chaney (2008). Each firm is associated with a product variety ω . Firms are heterogeneous in productivity, *a*, which is drawn from a distribution $F_i(a)$. Firms produce their products using labor and intangible capital. Intangible capital, which we denote by *z*, is nonrival: it is produced in the home country but can be used to produce abroad as well, provided that a firm pays the cost of setting up a foreign affiliate in another productive region. Foreign affiliates pay licensing fees to use intangible capital according to the rules of transfer pricing. Firms can shift the profits associated with these fees to the low-tax region and/or the tax haven by transferring the rights to intangible capital to affiliates in these regions. Profit shifting is costly, however, and the more capital that is transferred, the larger the cost. Throughout this subsection, we index firms by their productivities instead of their varieties to economize on notation; all firms from a given region with the same productivity make the same decisions.

Production. A firm from region i with productivity a and intangible capital z can produce its good in any productive region j using the technology

$$y_{ij} = \sigma_{ij} A_j a \left(N_j z \right)^{\phi} \ell_j^{\gamma}. \tag{30}$$

This technology is the same as in the theory developed in section 3 with two modifications: it depends on the firm's idiosyncratic productivity as well as region j's aggregate productivity; and the firm's ability to deploy its productivity and intangible capital abroad may be limited by FDI barriers, σ_{ij} , as in McGrattan and Waddle (2020). We assume that $\sigma_{ij} \in [0, 1]$ and

that $\sigma_{ii} = 1$.

Research & development. Firms hire workers in their domestic parent corporations to produce intangible capital. We assume that labor productivity in R&D is the same as TFP in production. In other words, it takes $1/A_i$ workers in region *i* to produce one unit of intangible capital, i.e., the cost to produce *z* units of intangible capital is $W_i z/A_i$. Following McGrattan and Waddle (2020), we assume that R&D expenditures are tax-deductible, which is how they are treated under most countries' tax codes.

Trade and foreign direct investment. Firms can sell in the domestic market freely, but serving foreign markets is costly. There are two options for serving foreign markets: (i) pay a fixed cost κ_i^F to export domestically produced goods; and (ii) pay a fixed cost κ_i^F to open a foreign affiliate and produce locally. Fixed costs are denominated in units of the home country's labor. Each unit of goods shipped abroad incurs an iceberg transportation cost ξ_{ij} . Firms can simultaneously export to, and produce locally for, the same foreign country; exports and locally produced products are considered distinct varieties as in Garetto et al. (2019) and McGrattan and Waddle (2020).¹⁴ Let $J_X \subseteq I \setminus \{i\}$ denote the set of foreign regions to which a firm exports, and let $J_F \subseteq I \setminus \{i\}$ denote the set of regions in which it operates a foreign affiliate. The firm's resource constraints can then be written as follows:

$$y_{ii} = q_{ii} + \sum_{j \in J_X} \xi_{ij} q_{ij}^X,$$
(31)

$$y_{ij} = q_{ij}, \ j \in J_F,\tag{32}$$

where we distinguish exported goods, denoted as q_{ij}^X , from goods that are produced and consumed in the same location, q_{ij} .

Transfer pricing. As in section 3, foreign subsidiaries pay licensing fees to use intangible capital. The licensing fee of a subsidiary in region j is given by $\vartheta_{ij}z$, where $\vartheta_{ij} \equiv \gamma p_{ij}y_{ij}/z$ is the marginal revenue product of intangible capital, and the total amount of licensing fees across the conglomerate is $\nu_i z \equiv \sum_{j \in J_F \cup \{i\}} \vartheta_{ij} z$. Note again that this includes the licensing fee for the parent corporation's use of its own intangible capital.

¹⁴We have also studied a version of the model in which firms must choose whether to export or produce locally for each foreign market as in Helpman et al. (2004). The results of our policy experiments in this model are similar to our baseline results, but non-MNEs' share of gross value added is too large relative to the data. Additionally, we have experimented with allowing for export platforms as in Arkolakis et al. (2018), but this greatly increases the computational complexity of the firm's problem. We conjecture that this would increase the macroeconomic bite of profit shifting, as MNEs with export platforms would have even higher returns on intangible capital due to nonrivalrly.

Profit shifting. Also as in section 3, a firm based in a high-tax region can shift its profits by transferring ownership of its intangible capital to its affiliates in low-tax jurisdictions (provided that the firm has paid the fixed costs to establish these affiliates). Here, we allow firms to shift profits to the low-tax productive region and/or the tax haven region. Suppose the firm sells a fraction λ^{LT} of its intangible capital to the former and a fraction λ^{TH} to the latter. Its affiliate in the former collects licensing fees of $\lambda_{LT} \sum_{j \in J_F \cup \{i\} \setminus \{LT\}} \vartheta_{ij} z$, while its affiliate in the latter collects $\lambda_{TH} \sum_{j \in J_F \cup \{i\}} \vartheta_{ij} z$. The domestic parent collects the remaining fees, $(1 - \lambda_{LT} - \lambda_{TH}) \sum_{j \in J_F} \vartheta_{ij} z$.

In our quantitative model, the cost (in units of home-country labor) to sell a fraction λ of intangible capital to country $j \in \{LT, TH\}$ is specified as $C_{ij}(\lambda) = [\lambda + (1 - \lambda) \log(1 - \lambda)] \psi_{ij}$; the total cost to sell a fraction λ_{LT} to the low-tax region and a fraction λ_{TH} to the tax haven is $C_{i,LT}(\lambda_{LT}) + C_{i,TH}(\lambda_{TH})$. Recall that in section 3, the profit shifting technology is governed by the discount φ at which the MNE sells its intangible capital the tax haven. Here, the parameter ψ_{ij} captures this discount as well as the resource cost of shifting profits.¹⁵¹⁶ Setting up an affiliate in the tax haven also requires a fixed cost κ_i^{TH} .

4.3 The firm's problem

The firm's objective is to maximize its dividend payout. We describe the firm's problem in three steps: first, in a standard environment without transfer pricing or profit shifting; second, with transfer pricing but without profit shifting; and third, with profit shifting.

4.3.1 Free transfer scenario

Here, the firm chooses where to export (J_X) ; where to open a foreign affiliate (J_F) ; how much intangible capital to produce (z); how much labor to hire in each of its divisions (ℓ_{ij}) ; and how much to sell to each of its markets (q_{ij}, q_{ij}^X) . We can break this problem into two stages, working backward. In the second stage, the firm maximizes each division's gross operating

 $^{{}^{15}\}psi_{ij}$ enters the first-order conditions for λ and z in exactly the same way as φ , so we could not separately identify the two parameters if they were both included. For example, the solution for λ_j is now given by $\lambda_j = 1 - \exp\left(-\frac{1-\varphi}{\psi_{ij}}\frac{\tau_i - \tau_i^*}{1-\tau_i}\right)$. We could pick any value for φ and recalibrate ψ_{ij} to match our target moments, and the equilibrium would always be identical.

¹⁶Strictly speaking, we assume that affiliates in the low-tax region and tax haven acquire intangible capital property rights from the parent for free (a discount of 100%, i.e., $\varphi = 0$). What matters from the MNE's perspective at the micro level is not what the affiliate pays to acquire this capital per se, but the overall effect on the MNE's global after-tax profit, which is subsumed by the parameter ψ_{ij} in our quantitative model as described in footnote 7. Regardless, this assumption is a reasonable approximation of reality. For example, according to the testimony of U.S. Senator Carl Levin referenced above, Apple's Irish subsidiary paid approximately \$5 billion during 2009–2012 to acquire the rights to license Apple's IP and received \$74 billion in licensing fees. These figures imply a discount of more than 93%.

profits taking J_X , J_F , and z as given. The domestic parent corporation's profits are

$$\pi_{i}^{D}(a, z; J_{X}) = \max_{q_{ii}, \{q_{ij}^{X}\}_{j \in J_{X}}, \ell_{i}} \left\{ p_{ii}(q_{ii})q_{ii} + \sum_{j \in J_{X}} p_{ij}(q_{ij}^{X})q_{ij}^{X} - W_{i}\ell_{i} \right\}$$

s.t $q_{ii} + \sum_{j \in J_{X}} \xi_{ij}q_{ij} = y_{i} = A_{i}a(N_{i}z)^{\gamma}\ell_{i}^{\phi}.$ (33)

Foreign subsidiaries' profits are

$$\pi_{ij}^{F}(a,z) = \max_{q_{ij},\ell_j} p_{ij}(q_{ij})q_{ij} - W_j\ell_j, \ j \in J_F.$$
(34)

Note that these objects will not change when we incorporate transfer pricing and profit shifting.

In the first stage, the firm chooses J_X , J_F , and z to maximize its global net profits, taking into account the cost of producing intangible capital, as well as the fixed costs of exporting and opening foreign affiliates:

$$d_{i}^{FT}(a) = \max_{z,J_{X},J_{F}} \left\{ (1-\tau_{i}) \left[\pi_{i}^{D}(a,z;J_{X}) - W_{i} \left(z/A_{i} + \sum_{j \in J_{X}} \kappa_{ijX} + \sum_{j \in J_{F}} \kappa_{ijF} \right) \right] + \sum_{j \in J_{F}} (1-\tau_{j}) \pi_{ij}^{F}(a,z) \right\}.$$
(35)

The fact that z is nonrival creates interdependence across regions in the firm's export participation and multinational production decisions as in Tintelnot (2017), making this a computationally demanding combinatorial problem. It is highly parallelizable across firms, however, and brute force is feasible provided one has enough computer cores. We use π_{ii}^{FT} and π_{ij}^{FT} to denote the firm's taxable profits in its domestic parent division and foreign subsidiaries, respectively, in this scenario.

4.3.2 Transfer pricing scenario

Here, the firm makes the same choices as in the *free transfer* scenario, but it takes into account the licensing fees that its foreign affiliates pay to the parent corporation. The first stage of the firm's problem in this scenario is

$$d_{i}^{TP}(a) = \max_{z,J_{X},J_{F}} \left\{ (1-\tau_{i}) \left[\pi_{i}^{D}(a,z;J_{X}) - W_{i} \left(z/A_{i} + \sum_{j \in J_{X}} \kappa_{ijX} + \sum_{j \in J_{F}} \kappa_{ijF} \right) + \sum_{j \in J_{F}} \vartheta_{ij}(z) \right] + \sum_{j \in J_{F}} (1-\tau_{j}) \left[\pi_{ij}^{F}(a,z) - \vartheta_{ij}(z)z \right] \right\}.$$
(36)

We make explicit the dependence of the licensing fees on the firm's choice of intangible capital by writing $\vartheta_{ij}(z)$ as a function of z. In contrast to our simple static framework, firms in our quantitative model internalize the effects of their choices of z on transfer prices. In this scenario, π_{ii}^{TP} and π_{ij}^{TP} denote the firm's taxable profits in its domestic and foreign divisions, respectively. The difference between these objects and their counterparts in the free transfer scenario is intangible capital licensing fees, which increase taxable profits in the parent and reduce them in foreign subsidiaries. As we will see, these fees will be crucial in defining the amount of lost profits in our model with profit shifting.

4.3.3 Profit shifting scenario

Profit shifting adds an additional decision: how much intangible capital to shift to affiliates in the low-tax region and/or tax haven. This problem can be written as

$$d_{i}^{PS}(a) = \max_{z,J_{X},J_{F},\lambda_{TL},\lambda_{TH}} \left\{ (1-\tau_{i}) \left[\pi_{i}^{D}(a,z;J_{X}) - W_{i} \left(z/A_{i} + \sum_{j \in J_{X}} \kappa_{ijX} + \sum_{j \in J_{F}} \kappa_{ijF} + \kappa_{iTH} \mathbb{1}_{\{\lambda_{TH} > 0\}} \right) \right. \\ \left. + \underbrace{\sum_{j \in J_{F}} (1-\lambda_{LT} - \lambda_{TH}) \vartheta_{ij}(z) z}_{U_{LT} - \lambda_{TH}} \right] \left\{ \sum_{j \in J_{F}} (1-\lambda_{LT} - \lambda_{TH}) \vartheta_{ij}(z) z - \underbrace{\sum_{j \in J_{F}} (1-\lambda_{LT} - \lambda_{TH}) \vartheta_{ii}(z) z}_{U_{LT} - W_{i}(C_{i,TH}(\lambda_{TH}) + C_{i,LT}(\lambda_{LT})) \nu_{i}(z) z} \right] \right] \\ \left. + (1-\tau_{LT}) \left[\pi_{i,LT}^{F}(a,z) + \underbrace{\sum_{j \in J_{F} \cup \{i\} \setminus \{LT\}} (1-\tau_{TH}) \vartheta_{ij}(z) z}_{U_{LT} - \psi_{ij}(z) z} \right] \mathbb{1}_{\{\lambda_{TH} > 0\}} \\ \left. + \underbrace{\sum_{j \in J_{F} \setminus \{LT\}} (1-\tau_{j}) \left[\pi_{ij}^{F}(a,z) - \underbrace{\vartheta_{ij}(z) z}_{U_{LT} - \psi_{ij}(z) z} \right] \right] \right\}$$

$$(37)$$

subject to $\lambda_{LT} + \lambda_{TH} \leq 1$ and $\lambda_{LT} \leq \mathbb{1}_{\{LT \in J_F\}}$. The last inequality simply says that you cannot shift profits to the low-tax region if you do not have an affiliate there. The first square-bracketed term represents the profits of the parent division, π_{ii}^{PS} in this scenario, the second term represents the profits of the low-tax affiliate, $\pi_{i,LT}^{PS}$, the third represents the profits of the tax-haven affiliate, $\pi_{i,TH}^{PS}$, and the fourth represents the profits of affiliates in other high-tax regions, π_{ij}^{PS} .¹⁷ Since firms in the low-tax region cannot shift profits, their

¹⁷We abstract in our model from the Global Intangible Low Tax Income (GILTI), adopted by the U.S. government in 2017, for two reasons. First, once we take the model to the data (see next section) we treat North America as a single region. Second, according to the scarce literature on GILTI, see Clausing (2020b) and Garcia-Bernardo et al. (2022), it had limited impact on profit shifting of the U.S. multinationals.

problem in this scenario is the same as in the previous one.

4.4 Aggregation and accounting measures

Several national and international accounting measures are required to close the model and compare it to the data. Here, we revert to expressing firms' choices as functions of their varieties (ω) for notational brevity.

Gross domestic product. Nominal GDP is the total value of goods produced in a given region:

$$GDP_i = \sum_{j=1}^{I} \int_{\omega \in \Omega_j, i \in J_F(\omega)} p_{ji}(\omega) y_{ji}(\omega) \, d\omega.$$
(38)

We compute real GDP by deflating by the consumer price index P_i defined in (29).

Goods trade. Aggregate goods trade flows are given by

$$EX_i^G = \sum_{j \neq i} \int_{\Omega_i} p_{ij}^X(\omega) \left(1 + \xi_{ij}\right) q_{ij}^X(\omega) \ d\omega, \tag{39}$$

$$IM_i^G = \sum_{j \neq i} \int_{\Omega_j} p_{ji}^X(\omega) \left(1 + \xi_{ji}\right) q_{ji}^X(\omega) \ d\omega.$$

$$\tag{40}$$

Services trade. Consistent with Guvenen et al. (2022) and Accoto et al. (2021), intangible capital licensing fees enter the national accounts as exports or imports of intellectual property services. High-tax regions' services trade flows are given by

$$EX_i^S = \sum_{j \neq i} \int_{\Omega_i} \left[1 - \lambda_{LT}(\omega) - \lambda_{TH}(\omega) \right] \vartheta_{ij}(\omega) z(\omega) \ d\omega, \tag{41}$$

$$IM_{i}^{S} = \sum_{j \neq i} \int_{\Omega_{i}} \left[\lambda_{LT}(\omega) + \lambda_{TH}(\omega) \right] \vartheta_{ij}(\omega) z(\omega) \ d\omega + \sum_{j \neq i} \int_{\Omega_{j}} \vartheta_{ji}(\omega) z(\omega) \ d\omega.$$
(42)

The low-tax region's services trade flows are

$$EX_{LT}^{S} = \sum_{j \neq i} \int_{\Omega_{i}} \left[1 - \lambda_{TH}(\omega) \right] \vartheta_{ij}(\omega) z(\omega) \ d\omega + \sum_{j \neq i} \int_{\Omega_{j}} \lambda_{LT} \vartheta_{ji}(\omega) z(\omega) \ d\omega, \tag{43}$$

$$IM_{LT}^{S} = \sum_{j \neq i} \int_{\Omega_{i}} \lambda_{TH}(\omega) \vartheta_{ij}(\omega) z(\omega) \ d\omega + \sum_{j \neq i} \int_{\Omega_{j}} [1 - \lambda_{LT}(\omega)] \vartheta_{ji}(\omega) z(\omega) \ d\omega.$$
(44)

Note that in the *TP* scenario, $\lambda_{LT}(\omega) = \lambda_{TH}(\omega) = 0$. The tax haven's services exports (it

has no imports because foreign affiliates located there do not produce anything) are

$$EX_{TH}^{S} = \sum_{j=1}^{I} \int_{\Omega_{j}} \lambda_{TH} \vartheta_{ji}(\omega) z(\omega) \ d\omega.$$
(45)

Net factor receipts and payments. Net factor receipts from (payments to) foreigners are the sum total of the dividends paid by foreign subsidiaries of domestic multinationals (domestic subsidiaries of foreign multinationals):

$$NFR_i = \sum_{j \neq i} \int_{\Omega_i} (1 - \tau_j) \pi_{ij}^{PS}(\omega) \ d\omega,$$
(46)

$$NFP_i = \sum_{j \neq i} \int_{\Omega_j} (1 - \tau_i) \pi_{ji}^{PS}(\omega) \ d\omega.$$
(47)

In the *FT* and *TP* scenarios, we use π_{ij}^{FT} and π_{ij}^{TP} , respectively, to calculate these objects.

Shifted profits. We define the profits shifted out of region j by a firm ω that is based in region i by comparing the profits the firm books in j in the PS scenario to the profits it would book in the TP scenario: $\tilde{\pi}_{ij}(\omega) = \pi_{ij}^{TP}(\omega) - \pi_{ij}^{PS}(\omega)$. When $\tilde{\pi}_{ij}(\omega) > 0$, this indicates that the firm would book more profits in region j in the absence of profit shifting, i.e., the firm has shifted profits away from region j. Aggregating shifted profits by firms at the region level yields the total profits shifted out of region j:

$$\tilde{\Pi}_j = \sum_{i=1}^{I} \int_{\Omega_i} \tilde{\pi}_{ij}(\omega) \ d\omega.$$
(48)

Note that $\pi_{ij}^{TP}(\omega)$ is a counterfactual object that can be computed in partial equilibrium or general equilibrium. In partial equilibrium, we calculate it while holding fixed firms' decision rules from the *PS* scenario. In general equilibrium, on the other hand, we re-solve the firm's problem for the *TP* scenario, which changes allocations at the micro level and ultimately at the macro level as well. We use the partial-equilibrium version of this measure in our calibration procedure, but we use the general-equilibrium version when analyzing the implications of the two pillars of the OECD proposal.

4.5 Market clearing and equilibrium

In a general equilibrium of our model, the labor market must clear, the government's budget constraint must be satisfied, and the balance of payments must hold in each productive region. Labor market. Labor demand comes from four sources: production of intermediate goods; production of intangible capital; fixed costs of exporting and setting up foreign affiliates; and the costs of transferring intangible capital. The labor market clearing condition can be written as

$$L_{i} = \sum_{j=1}^{I} \int_{\Omega_{j}} \ell_{ji}(\omega) \ d\omega + \int_{\Omega_{i}} z(\omega)/A_{i} \ d\omega + \int_{\Omega_{i}} \left(\sum_{j \in J_{X}(\omega)} \kappa_{i}^{X} + \sum_{j \in J_{F}(\omega)} \kappa_{i}^{F} + \mathbb{1}_{\{\lambda_{TH}(\omega) > 0\}} \kappa_{i}^{TH} \right) \ d\omega + \underbrace{\int_{\Omega_{i}} \left(\mathcal{C}_{i,TH}(\lambda_{TH}) + \mathcal{C}_{i,LT}(\lambda_{LT}) \right) \nu(\omega) z(\omega) \ d\omega}_{\text{costs of shifting } z}$$
(49)

Note that at the macro level, profit shifting diverts labor from goods production and R&D to wasteful administrative costs, potentially offsetting the positive macroeconomic effects of increased R&D at the micro level, and policies that reduce profit shifting such as the OECD/G20 proposal free up some of these wasted resources. We discuss the quantitative importance of this channel in section 4.7.2 below.

Government budget constraint. We assume that revenue from corporate income taxation is rebated lump-sum to households.¹⁸ In the benchmark PS model, lump-sum transfers are given by

$$T_i = \tau_i \sum_{j=1}^{I} \int_{\Omega_j} \pi_{ji}^{PS}(\omega) \ d\omega.$$
(50)

In the *FT* and *TP* scenarios, π_{ji}^{FT} and π_{ji}^{TP} are used instead.

Balance of payments. The balance of payments requires that each region's current account must be zero:

$$EX_{i}^{G} + EX_{i}^{S} - IM_{i}^{G} - IM_{i}^{S} + NFR_{i} - NFP_{i} = 0.$$
(51)

Note that several things happen to the balance of payments when a firm shifts profits away from its home region. First, that region's services trade balance worsens: the firm receives fewer licensing fees from its foreign subsidiaries and makes more licensing payments. Second, net factor receipts rise: the firm's profits in the tax haven (or low-tax region) rise, and these increased profits are ultimately rebated back to the home country. These two effects offset one another, but not completely: some of the shifted profits are taxed and therefore remain

 $^{^{18}\}mbox{We}$ have also analyzed a version of the model in which labor income taxes adjust to clear the government's budget constraint, and the results are similar.

in the tax haven and/or low-tax region. Thus, the net effect is that the current account worsens. The reduction in the services trade balance and the increase in net factor income are consistent with the accounting of Guvenen et al. (2022). The net negative effect on the balance of payments is consistent with the findings of Hebous et al. (2021). To regain equilibrium, that trade balance and/or net factor income balance must improve, which shows up in our model as a real exchange rate depreciation.

Competitive equilibrium. Given a set of parameters and a scenario (*FT*, *TP*, or *PS*), an equilibrium in our model is a set of aggregate prices and quantities $\{W_i, P_i, C_i, L_i\}$ and a set of firm decision rules $\{J_X(\omega), J_F(\omega), z(\omega), \ell(\omega), q(\omega), \lambda_{LT}(\omega), \lambda_{TH}(\omega)\}$ for each productive region $i \in J$ that satisfy the household's problem, the firm's problem, and the market-clearing conditions.

4.6 Calibration

We calibrate our model's parameters so that its equilibrium, given the current international tax regime, reproduces salient facts about production, international trade, foreign direct investment, and, most importantly, profit shifting. Some of the parameters, like elasticities of substitution, are assigned externally to standard values, while others, like population, can be set directly to exact data analogues. The remaining parameters are jointly calibrated by matching a set of target moments. These parameters influence all of the target moments to some degree, but there is one target that provides most of the identification for each parameter. Thus, in what follows, we describe each calibrated parameter alongside its main target. Table 1 lists target moments and calibrated parameter values for each region. Appendix B provides details on the data sources we use to discipline the model.

Regions. We partition the world into five regions. The countries identified as tax havens by Tørsløv et al. (2022) are split into two regions: a low-tax productive region, LT, including Belgium, Ireland, Hong Kong, the Netherlands, Singapore, and Switzerland; and an unproductive tax-haven region, TH, including Luxembourg, small European countries and territories like Cyprus, Malta, and the Isle of Man, and a number of Carribean countries. The other three regions are North America, Europe (except for the countries in the low-tax and tax-haven regions), and the rest of the world. Data for each region are obtained by aggregating or averaging country-level data. See Appendix B for more details.

Assigned parameters. The elasticity of substitution between varieties, ρ , is set to the standard value of 5. Each region's population, N_i , is set by aggregating country-level data from the World Bank's World Development Indicators database. Corporate income tax rates,

 τ_i , are set by averaging country-level estimates of effective corporate income tax rates from Tørsløv et al. (2022).

Technology capital share (ϕ). We set the technology capital share in the production function (30) to match the share of foreign-owned firms' income that accrues to intangible capital, which is estimated by Cadestin et al. (2021) to be 28%. Note that domestic-owned firms have lower intangible income shares, at around 22%. Although we do not target this moment in our calibration, our model is consistent with this fact. This is because technology capital is nonrival, which means that multinational firms have a greater incentive to invest in it than non-MNEs. Thus, our model captures the extent to which nonrivalry creates increasing returns at the MNE level.

Total factor productivity (A_i) . Each region's TFP is set to match its aggregated real GDP based on PPP-adjusted data from the World Development Indicators database.

Productivity distribution ($F_i(a)$). We assume that firms' productivities are drawn from Pareto distributions with region-specific tail parameters η_i . We calibrate these tail parameters to match the share of aggregate employment that is accounted for by firms with fewer than 100 times the average number of employees, which is equal to 58.9% in data published by the U.S. Census Bureau. Although this is the only moment of the firm-size distribution that we target, our model's Lorenz curve is very close to its empirical counterpart.

Utility weight on leisure (ψ_i) . We choose the weight on leisure in the utility function (25) so that the representative household in each country works for one-third of its time endowment, i.e., $L_i = N_i/3$.

Variable trade cost (ξ_{ij}) . We set the iceberg trade barriers to match aggregate bilateral imports of goods (agriculture, resource extraction, and manufacturing) relative to nominal GDP. Import data are from the World Input Output Database. Nominal GDP data are from the World Development Indicators. For both, we sum across the countries within each region.

Fixed export cost (κ_i^X) . Each region's fixed cost of exporting is chosen so that 22.7% of firms export, as reported by Alessandria et al. (2021).

Variable FDI cost (σ_{ij}) . We calibrate the parameters that govern the efficiency with which intangible capital can be deployed abroad to match the share of each region's gross value added that is accounted for by foreign multinationals. These data come from the OECD AMNE database. This share is equal to 11.12% in North America, 19.82% in Europe, 28.74% in the low-tax region, and 9.55% in the rest of the world. Fixed FDI cost to productive regions (κ_i^F). The fixed costs of establishing foreign affiliates in other productive regions are set to match the average employment of multinational firms (i.e., firms with foreign affiliates) relative to the overall average employment of all firms. This ratio is equal to 444. The former is calculated using Compustat, while the latter is calculated using data from the U.S. Census.¹⁹

Variable profit-shifting costs (ψ_{iLT}, ψ_{iTH}). The parameters that govern the cost of transferring technology capital are calibrated by matching Tørsløv et al. (2022)'s estimates of (i) total lost profits, and (ii) the share of lost profits that are shifted to countries in our tax-haven region. As with production and trade data, we obtain region-level measures by summing the country-level estimates reported in this paper. Total lost profits are \$143bn for North America, \$216bn for Europe, and \$257bn for the rest of the world. The shares of these totals that are shifted to the tax-haven region are 66.39%, 44.50%, and 71.69%, respectively.

Fixed FDI cost to tax haven (κ_i^{TH}) . The fixed costs of establishing affiliates in the tax haven region are set to match the average employment of firms that have affiliates in at least one country in our tax haven region. This ratio is equal to 981. It is also calculated using Compustat.

4.7 External validation

We have calibrated the key parameters of our model—the profit-shifting costs, ψ_{ij} —to match macroeconomic estimates of aggregate lost profits. However, our calibrated model also matches very closely several other facts about profit shifting that we did not target in our calibration. This indicates that it is well suited to measuring the macroeconomic effects of profit shifting and the OECD/G20 reform.

4.7.1 Foreign MNEs' share of corporate income tax revenues

One way to validate our calibration is to measure the share of corporate income tax revenues in each region that are paid by local affiliates of foreign MNEs. The idea is that foreign MNEs should pay a large share of taxes in the low-tax region because the profits they report in this region have been artificially increased. Panel (a) of Table 2 reports the share of corporate income tax revenues paid by foreign MNEs in each region in our model according to the OECD's Corporate Tax Statistics Database (OECD, 2022a). The rest of the world has the lowest share (16.3%), followed closely by North America (16.6%). Europe has a higher share (41.6%), but, as one expects, the low-tax region has the highest share by a large margin

¹⁹Compustat contains data on public firms only. We do not have information on employment of private multinational firms. Our approach assumes that private multinationals are similar in size to public multinationals.

(72.4%). Our model matches the data for Europe, the rest of the world, and the low-tax region very closely, although it overshoots the data for North America. The low-tax region's share is the most informative moment about the suitability of our calibration, however, and we reproduce this figure almost exactly.

4.7.2 Global spending by MNEs to engage in profit shifting

Another way to validate our calibration is to measure MNEs' spending on profit shifting costs. Although MNEs do not report this spending in reality for obvious reasons, Tørsløv et al. (2022) argue that it can be inferred by estimating the salaries earned by transfer pricing specialists. Using data from LinkedIn and Glassdoor, they estimate that transfer pricing specialists employed in the private sector earned \$25 billion in base salary income worldwide in 2020. We compute the model counterpart of this figure by summing profit shifting costs, $W_i(\mathcal{C}_{i,TH}(\lambda_{TH}) + \mathcal{C}_{i,LT}(\lambda_{LT}))$ across all firms in all regions, which yields a value of \$76 billion. Both figures are reported in panel (b) of Table 2. Although our model's figure is about three times larger than Tørsløv et al. (2022)'s estimate, the latter is likely to be biased downward for several reasons. First, it does not include benefits, bonuses, and other components of transfer pricing specialists' compensation. Second, it excludes salaries earned by lawyers, accountants, compliance officers, executives in tax-haven affiliates, and other workers that facilitate transfer pricing but are not classified as transfer pricing specialists. Finally, it also excludes non-salary profit shifting costs such as transfer pricing software.²⁰ The fact that our model produces a figure of the same magnitude as Tørsløv et al. (2022)'s estimate indicates that our calibrated profit shifting costs are empirically reasonable.

4.7.3 Firm-level evidence on profit shifting

Yet another way to validate our calibration is to analyze the determinants of profit shifting at the firm level. One of the key objects of interest in the empirical literature on profit shifting is the semi-elasticity of reported pre-tax profits in a MNE's domestic parent division to the tax differential between the home country and a foreign tax haven. Panel (c) of Table 2 reports three estimates of this elasticity. They range from 0.8 to 1.1, which means that one-percentage-point increase in the tax differential is associated with a 0.8% to 1.1% increase in profits reported at home.²¹ We estimate this elasticity in our model by solving for counterfactual equilibria with different tax rates, constructing simulated datasets from these

²⁰See, for example, the specialized transfer pricing solutions offered by Thomson Reuters, Workiva, and Insight Software.

 $^{^{21}\}mathrm{Appendix}\ \mathrm{C}$ provides a detailed discussion of this literature and the methodology employed in the three studies cited in the table.

equilibria, and running the following regression:

$$\log \pi_i^{k,PS}(\omega) = \beta_0 + \beta_\ell \log \ell_i^k(\omega) + \beta_z \log z^k(\omega) - \beta_\tau \hat{\tau}_i^k + \epsilon_i^k(\omega),$$
(52)

where k denotes the index of the counterfactual economy and $\hat{\tau}_i^k$ denotes the tax differential between an MNE's home region and the profit-shifting destination region (either the lowtax region or the tax haven).²² The parameter of interest is β_{τ} . We obtain an estimate of $\beta_{\tau} = 0.87$, which lies comfortably within the narrow bounds of the empirical estimates.

5 Quantitative Results

Having described the model and its calibration, we turn now to the results of our quantitative analysis. First, we illustrate the effects of transfer pricing and profit shifting by comparing our baseline model to counterfactuals without these ingredients. Second, we analyze the effects of the two pillars of the OECD/G20 policy framework.

5.1 Inspecting the mechanism

Before using the calibrated model to analyze the consequences of changing the global corporate income tax landscape, it is helpful to illustrate the effects of our model's novel ingredients—transfer pricing and profit shifting—under the current tax system. We do this by comparing our baseline model, in which MNEs license technology capital to foreign affiliates according to transfer pricing rules and shift profits by selling technology capital to their affiliates in the tax haven, to two counterfactual models. In the first, the domestic parent corporation retains ownership of technology capital but still licenses this capital to foreign affiliates according to the arm's length principle. We refer to this version as the *no-shifting* counterfactual. In the second, the cost of foreign affiliates' usage of technology capital is not accounted for at all (licensing fees are set to zero). We refer to this version as the *no-transfer-pricing* counterfactual.

5.1.1 Effects of transfer pricing

To illustrate the effects of transfer pricing, panel (a) of Table 3 shows how the no-shifting counterfactual compares to the no-transfer-pricing counterfactual. In the highest-tax region in our model, North America, MNEs reduce R&D and produce less output, consistent with part 1 of Proposition 1. In other regions, however, MNEs' R&D actually increases. North America, as a large, high-productivity region, is an important FDI destination for these

 $^{^{22}}$ Appendix C.3 contains more details on how we produce the model-generated data and specify the empirical regression.

other regions' MNEs. Transfer pricing allows these MNEs to book the returns to intangible capital in their North American subsidaries, which face the highest tax rates, as profits in their domestic parent divisions, which face lower tax rates. This effect is most pronounced in the low-tax region; this is effectively the reverse of part 1 of Proposition 1. In this case, there is also a notable general equilibrium effect for non-MNEs that operates in the opposite direction: greater labor demand by MNEs increases prices, crowding out non-MNEs.

Although the effects of transfer pricing on R&D differ across regions, output falls in equilibrium everywhere, albeit for different reasons. In North America, the decline in output is driven by the response of domestic MNEs. Note that output of foreign MNEs' North American subsidiaries actually rises, but because foreign MNEs account for a relatively small share of overall North American output, this increase is not enough to offset the decline in domestic firms' output. In other regions, the output decline is driven primarily by foreign MNEs, specifically those from North America whose R&D falls.

The effects on corporate tax revenues are heterogeneous across regions. Revenues rise in high-tax North America because licensing fees reallocate income from domestic MNEs' foreign subsidiaries to their parent divisions. In Europe and the rest of the world, revenues fall for the opposite reason: profits of foreign MNEs' subsidiaries in these regions fall when they must pay to use intangible capital. In the low-tax region, revenues rise because domestic MNEs do more R&D and earn more profits globally, which return home in the form of licensing fees.

5.1.2 Effects of profit shifting

Panel (b) of Table 3 demonstrates the effects of profit shifting by comparing the baseline model to the no-shifting counterfactual. These effects are easier to explain, as they are the same in the three high-tax regions, North America, Europe, and the rest of the world. In these regions, MNEs increase R&D and produce more output, consistent with part 2 of Proposition 1, and this ultimately leads to higher aggregate output. At the same time, profit shifting reduces corporate tax revenues, with the largest effect in Europe.

In the low-tax region, profits shifted in from the high-tax regions amount to almost 4 percent of GDP and tax revenues rise by a full 23.5 percent. In equilibrium, this increase in income raises prices, reducing R&D among both MNEs and non-MNEs. However, the effect of this reduction on aggregate output is offset to a large degree by higher production by foreign MNEs' subsidiaries in this region.

5.2 Policy experiments

We use our calibrated model to conduct four experiments to analyze the macroeconomic consequences of the policies proposed in the OECD/G20 Inclusive Framework on BEPS described in detail in Appendix A.3. In the first experiment we focus on the first pillar of

this framework, which allocates a portion of an MNE's overall global profit to its subsidiaries based on these subsidiaries' revenues. The second experiment focuses on the second pillar, which imposes a global minimum corporate income tax rate of 15 percent. In the third experiment, we analyze the combined effects of these two pillars together. In the fourth experiment (which is really a set of sub-experiments) we study the combined effects of both pillars under different values for the profit reallocation share and global minimum tax rate. In all four experiments, we restrict attention to long-run analysis, comparing the steady state under the current regime to the steady state after the policy is implemented. Table 4 and Figure 1 show the results of these experiments, which we will explain in detail below. Appendix D shows that our main results are robust to a wide range of alternative setups and calibrations, such as different intangible capital shares and profit-shifting costs.

5.2.1 OECD Pillar 1: revenue-based profit allocation

The first pillar of the OECD BEPS project allocates, for the purposes of taxation, a fraction of a firm's global profits to the countries in which the firm sells its products. Following the OECD proposal, this allocation is based on these countries' shares of the firm's overall global sales. Importantly, it is independent of whether the firm has a physical presence in these countries, which implies that non-MNE exporters are also subject to this rule.²³ The firm's problem under this rule can be written as

$$d_{i}^{TP}(a) = \max_{z,J_{X},J_{F}} \left\{ \pi_{i}^{D}(a,z;J_{X}) - W_{i} \left(z/A_{i} + \sum_{J \in J_{X}} \kappa_{ijX} + \sum_{j \in J_{F}} \kappa_{ijF} \right) + \sum_{j \in J_{F}} \vartheta_{ij}(z)z + \sum_{j \in J_{F}} \left[\pi_{ij}^{F}(a,z) - \vartheta_{ij}(z)z \right] - \sum_{j \in J_{F} \cup J_{X} \cup \{i\}} \tau_{j}T_{ij}(a,z) \right\},$$
(53)

where $T_{ij}(a, z)$ represents the tax base for region j under the profit allocation rule. In Appendix F.4, we show that $T_{ij}(a, z)$ is given by

$$T_{ii}(a,z) = (1-\theta) \cdot \pi_i^D(a,z;J_X) + \theta \cdot \hat{S}_{ii}(a,z) \cdot \left[\pi_i^D(a,z;J_X) + \sum_{j \in J_F} \pi_j^F(a,z)\right],$$
(54)

$$T_{ij}(a,z) = (1-\theta) \cdot \pi_{ij}^F(a,z) + \theta \cdot \hat{S}_{ij}(a,z) \cdot \left[\pi_i^D(a,z;J_X) + \sum_{j \in J_F} \pi_j^F(a,z) \right], \ j \in J_F, \quad (55)$$

$$T_{ij}(a,z) = \theta \cdot \hat{S}_{ij}(a,z) \cdot \left[\pi_i^D(a,z;J_X) + \sum_{j \in J_F} \pi_j^F(a,z) \right], \ j \in J_X \setminus \{J_F\}$$
(56)

²³According to the current implementation timeline of Pillar 1, its scope will be initially limited to the largest MNEs. However, OECD intends to extend the coverage in future years. Our approach reflects these long-term policy goals.

where θ is the fraction of residual profits that are reallocated and $\hat{S}_{ij}(a, z)$ is region j's share of the firm's total global sales. The OECD's proposal sets θ to 25%.

Panel (a) of Table 4 shows the effects of this pillar. It would indeed make a large dent in international profit shifting and materially raise high-tax countries' corporate income tax revenues. Lost profits would fall by 34–40% in North America, Europe, and the rest of the world, and tax revenues would increase by 1.6–2.6%. In the low-tax region, profits shifted inward would fall by 31% and tax revenues would fall by 11.4%. At the same time, however, this pillar would decrease output globally. MNEs based in all three high-tax regions would reduce R&D and produce less output, and although non-MNEs would expand slightly in equilibrium, overall output in these regions would decline. The effects would be largest in North America, where MNEs' R&D would fall by 0.8% and aggregate output would fall by 0.13%. In the low-tax region, domestic MNEs would increase R&D, but the decline in foreign MNEs' output in this region would ultimately drag overall output downward as well.

5.2.2 OECD Pillar 2: Global minimum corporate income tax

The second pillar is a global minimum corporate income tax. Following the OECD guidance, we implement this policy through top-up taxes levied by the governments of MNEs' home countries. Specifically, if a firm based in jurisdiction *i* reports profits in a jurisdiction *j* where the tax rate is below the global minimum tax rate $\underline{\tau}$, such profits are taxed in jurisdiction *i* at a rate equal to the tax differential, $\underline{\tau} - \tau_j$. Thus, region *i*'s tax revenue is is then

$$T_i = \tau_i \sum_{j=1}^{I} \int_{\Omega_j} \pi_{ji}^{PS}(\omega) \ d\omega + \sum_{j=1}^{I} \int_{\Omega_i} \max\left(\left(\underline{\tau} - \tau_j\right), 0\right) \pi_{ij}^{PS}(\omega) \ d\omega.$$
(57)

The rest of the equilibrium conditions stay unchanged. Panel (b) of Table 4 shows the effects of the second pillar. This policy has even larger effects on high-tax countries' lost profits and tax revenues than the first pillar. Lost profits in North America, Europe, and the rest of the world would fall by 63–85% and tax revenues would rise by 2.6–4.9%. On the other hand, the macroeconomic effects would be smaller. Although European MNEs and MNEs from the rest of the world would reduce R&D by more, North American MNEs' R&D would fall less, and low-tax MNEs' R&D would rise more. The net effect would be negligible effects on GDP in all four regions. Finally, note that in the low-tax region, profits shifted inward from the high-tax regions would fall by 51% and corporate income tax revenues would fall by 9.7%, but there would be little effect on aggregate output. This is due to the fact that while domestic firms would actually increase R&D slightly, output produced by foreign MNEs in this region would fall.

5.2.3 Both pillars combined

Panel (c) of Table 4 shows the effects of implementing both pillars simultaneously. Consistent with Proposition 2, the effects are not additive but larger than in either of the first two experiments. Profit shifting would be mostly eliminated: lost profits would fall by 77% in North America, 82% in Europe, and 90% in the rest of the world. Corporate income tax revenues would rise more than under either pillar alone, especially in North America. In the low-tax region, profits shifted inward would fall by 67% and tax revenues by 16.5%. The macroeconomic effects would be slightly larger than under Pillar 1 in all regions.

Figure 1 shows how the effects of the two pillars change when their parameters are varied. The x-axis in each plot is Pillar 1's profit reallocation share and the y-axis is Pillar 2's global minimum tax rate. The first column of plots in the figure shows how the effects on lost profits change and the second column shows how the effects on output change. In both columns, darker shades of red indicate "worse" outcomes: smaller reductions in lost profits in the first column and larger output losses in the second column. The results of this analysis clearly show that a global minimum tax rate is better policy than profit reallocation. Both pillars are effective at reducing profit shifting, but profit reallocation causes much larger output losses. A 17 percent minimum tax rate would essentially eliminate profit shifting entirely but would not reduce output much more than the benchmark 15 percent rate. It would take a profit reallocation share of 90 percent or greater to achieve the same reduction in lost profits, but the output losses from this policy would be an order of magnitude greater.

5.2.4 Why does Pillar 1 have larger macroeconomic consequences?

Our results show that either pillar of the OECD/G20 Framework would reduce profit shifting substantially, but Pillar 1 would have larger effects on output. The explanation for this is that Pillar 1 affects firms that do not shift profits—and even some firms that do not engage in multinational production at all—while Pillar 2 only affects MNEs that actually shift profits. Specifically, Pillar 1 allocates taxation rights based on where firms make their sales, including export markets. This aspect of the rule increases effective tax rates for firms in Europe, the low-tax region, and the rest of the world because North America, the largest, richest export market, has the highest tax rate. In Appendix D, we show that a version of Pillar 1 that allocates taxation rights based on production instead of sales would achieve the same reduction in profit shifting at a lower macroeconomic cost in these regions. In the case of North America, Pillar 1 has larger macroeconomic effects because the home share of revenue is larger than the home share of profits for firms based in this region, as intangible investment expenses (as well as the fixed costs of selling abroad) are incurred at home.

5.2.5 Comparison with the OECD's estimates

The OECD predicts that implementing the two pillars will increase global corporate income tax revenue by 3.0%–5.1% and reduce global GDP by about 0.07% (OECD, 2020).²⁴ They arrive at these figures by combining a formulaic application of the pillars to pre-reform corporate profits with a partial-equilibrium, reduced-form estimate of how profit shifting and tangible investment will change based on relevant elasticities from the empirical literature. In comparison, our results indicate a 3.24% increase in global corporate income tax revenue and a 0.15% decrease in global GDP. Thus, while our estimate of the revenue effect is similar to the OECD's, our estimate of the macroeconomic effect is more than twice as large. The reason for this difference is twofold. First, the decrease in non-rival intangible capital in our model affects production both at home and abroad, whereas tangible investment affects domestic production only; OECD (2020) considers the latter but not the former. Second, our general-equilibrium model takes into account price changes and reallocation from more productive MNEs towards less productive domestic firms, both of which further reduce global output. This highlights the importance of using a general-equilibrium framework with a microfounded model of profit shifting to evaluate this kind of policy change.

6 Conclusion

We have developed a theory of international profit shifting by multinational enterprises (MNEs) to study the macroeconomic implications of this phenomenon. In our model, MNEs invest in nonrival intangible capital which they can use simultaneously in all of their divisions around the world. MNEs charge their foreign affiliates licensing fees to use intangible capital according to transfer pricing rules, and they can shift profits by transferring the rights to this capital to affiliates in low-tax jurisdictions.

In addition to the methodological contribution that our theory represents, we make two substantive contributions. First, we prove that profit shifting presents a trade-off between economic performance and tax revenues. On the one hand, profit shifting erodes the corporate income tax base in the jurisdiction in which an MNE is based. On the other hand, it incentivizes MNEs to invest in more intangible capital, which boosts output at home as well as abroad. Second, we calibrate our model to match empirical facts about profit shifting under the current international tax regime and use it to quantify the impact of the OECD's plan to eliminate profit shifting. This plan features two pillars: taxing MNEs in the countries in which they sell their products rather than the countries in which they book their profits;

 $^{^{24}}$ Note that the OECD's headline figure for the global tax revenue effect of the policy, reported in Table 1.1 of OECD (2020), is 2.3%-4.0%. However, this number is calculated using a 12.5% minimum tax rate in Pillar. Panel B of Table 3.15 reports results for a minimum tax rate of 15%.

and a global minimum corporate income tax rate. We find that this reform would indeed largely eliminate profit shifting and boost tax revenues in high-tax jurisdictions. However, it would also materially reduce intangible capital investment and overall macroeconomic performance.

To put our quantitative results in context, it is helpful to compare them to the effects of other major international policy changes that have been analyzed elsewhere in the literature. Caliendo and Parro (2014) estimate that the North American Free Trade Agreement increased welfare by 0.08% in the United States and reduced it by 0.06% in Canada, while di Giovanni et al. (2014) find that the average country gained 0.13% from liberalizing trade with China. Caliendo et al. (2021) find that the 2004 EU enlargement, which liberalized international labor markets as well as trade, increased welfare in the original EU member states by 0.04%. Despite the small number of firms involved in profit-shifting—far fewer firms engage in multinational production than trade, and only a small fraction of the former shift profits—we find that the macroeconomic effects of the OECD/G20 BEPS framework would be even larger than these examples.

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Statistic or parameter value	North America	Europe	Low-tax	RoW	Tax haven
(a) Assigned parameters and target moments					
Population $(NA = 100)$	100	92	11	1,323	_
Real GDP (NA $= 100$)	100	80.78	14.57	297.10	_
Corporate tax rate $(\%)$	22.5	17.3	11.4	17.4	3.3
Labor tax rate $(\%)$	22.4	22.4	22.4	22.4	_
Foreign MNEs' VA share $(\%)$	11.12	19.82	28.73	9.55	—
Total lost profits (\$B)	143	216	_	257	—
Lost profits to TH $(\%)$	66.4	44.5	_	71.1	_
Imports from (% GDP)					
North America	—	1.28	1.77	1.74	_
Europe	1.70	_	12.39	3.78	—
Low tax	0.35	2.98	_	0.59	_
Row	6.15	7.96	6.78	_	_
(b) Calibrated parameter values					
TFP (A_i)	1.00	0.89	1.58	0.20	_
Prod. dispersion (η_i)	4.28	4.31	4.83	4.12	_
Utility weight on leisure (ψ_i)	1.06	1.08	1.09	1.06	_
Fixed export cost (κ_i^X)	1.7e-3	3.5e-3	1.0e-3	1.4e-2	_
Variable FDI cost (σ_i)	0.47	0.56	0.52	0.53	—
Fixed FDI cost (κ_i^F)	1.80	1.59	0.46	8.75	—
Cost of shifting profits to LT (ψ_{iLT})	3.40	0.38	_	2.35	—
Cost of shifting profits to TH (ψ_{iTH})	2.25	1.25	_	1.76	_
Fixed FDI cost to TH (κ_i^{TH})	0.09	0.06	_	0.59	—
Variable export cost (ξ_{ij}) from					
North America	_	3.21	3.41	2.07	_
Europe	1.89	_	1.69	1.33	—
Low tax	2.04	1.59	_	1.56	_
RoW	2.26	2.59	3.01	_	_

Table 1: Calibration

Notes: Population and real GDP from World Bank WDI. Corporate tax rate from Tørsløv et al. (2022). Foreign MNEs' VA share from OECD AMNE database. Fractions of firms with foreign affiliates from Compustat. Lost profits from Tørsløv et al. (2022). Imports/GDP from WIOD. Dashes (–) represent "not applicable."

(a) Share of corporate taxes paid by foreign MNEs $(\%)$								
Source	North America	Europe	Low tax	RoW				
OECD (2022a)	16.65	41.58	72.40	16.32				
Model	24.40	40.56	73.30	18.54				

Table 2: Validation

(b) Global profit-shifting costs (\$bn)

Source	Estimate
Tørsløv et al. (2022)	25
Model	76

(c) Firm-level semi-elasticity of profit shifting

Source	Estimate
Johansson et al. (2017)	1.11
Heckemeyer and Overesch (2017)	0.79
Beer et al. (2020)	0.98
Model	0.87

Notes: Panel (a): Data source is OECD Corporate Tax Statistics Database (OECD, 2022a). Shares are first calculated at the country level, and then aggregated to the region level by averaging, weighting by total corporate tax revenues. Panel (b): Model value calculated by summing $C(\lambda)$ across all firms, dividing by world GDP in the model, and multiplying by 2020 world GDP in the data from the World Bank (\$84.91 tn). Panel (c): See Appendix C.2 for empirical estimates and Appendix C.3 for model estimate.

			Value added (% chg.)				Tech. capital (% chg.)		
Region	Lost profits (% GDP)	Corp. tax rev. (% chg.)	Total	Non MNEs	Domestic MNEs	Foreign MNEs	Total	Non MNEs	Domestic MNEs
(a) Effects of tra	ansfer pricing ((no transfer pric	ing vs. n	no shifting	ı)				
North America	0.00	4.33	-0.16	0.36	-0.85	0.34	-0.54	0.58	-1.34
Europe	0.00	-2.43	-0.17	-0.15	-0.11	-0.31	0.12	0.06	0.17
Low tax	0.00	-2.10	-0.25	-0.72	1.10	-0.56	0.74	-0.75	2.28
Rest of world	0.00	-0.43	-0.18	-0.18	-0.15	-0.31	0.05	0.00	0.08
(b) Effects of profit shifting (no shifting vs. baseline)									
North America	0.68	-3.79	0.08	-0.00	0.15	0.15	0.21	-0.11	0.45
Europe	1.05	-5.51	0.06	-0.02	0.17	0.04	0.26	-0.07	0.54
Low tax	-4.37	23.94	-0.04	-0.33	-0.29	0.64	-0.55	-0.60	-0.49
Rest of world	0.50	-2.66	0.04	-0.01	0.08	0.07	0.12	-0.06	0.27

Table 3: Inspecting the mechanism

Table 4: Effects of OECD BEPS pillars

				Value added (% chg.)				Tech. capital (% chg.)		
Region	Lost profits $(\text{benchmark} = 1)$	Corp. tax rev. (% chg.)	Total	Non MNEs	Domestic MNEs	Foreign MNEs	Total	Non MNEs	Domestic MNEs	
(a) Pillar 1: Pr	ofit reallocation									
North America	0.60	2.50	-0.13	-0.00	-0.30	-0.05	-0.40	0.15	-0.80	
Europe	0.66	2.67	-0.14	-0.10	-0.18	-0.17	-0.10	0.04	-0.21	
Low tax	0.69	-11.63	-0.13	-0.10	0.36	-0.57	0.79	0.23	1.35	
Rest of world	0.63	1.65	-0.13	-0.11	-0.14	-0.19	-0.05	0.02	-0.10	
(b) Pillar 2: Global minimum tax rate										
North America	0.37	3.20	-0.06	0.01	-0.10	-0.13	-0.15	0.08	-0.31	
Europe	0.26	4.95	-0.02	0.04	-0.11	-0.01	-0.22	0.06	-0.45	
Low tax	0.49	-9.84	0.02	0.23	0.19	-0.46	0.32	0.36	0.28	
Rest of world	0.15	2.71	-0.01	0.04	-0.05	-0.03	-0.11	0.06	-0.24	
(c) Pillars 1 & 2 together										
North America	0.23	4.28	-0.17	-0.02	-0.36	-0.11	-0.48	0.17	-0.94	
Europe	0.18	5.51	-0.16	-0.09	-0.24	-0.18	-0.21	0.06	-0.43	
Low tax	0.33	-16.79	-0.13	0.07	0.50	-0.99	1.00	0.48	1.51	
Rest of world	0.10	3.25	-0.14	-0.09	-0.17	-0.21	-0.10	0.05	-0.22	

Notes: Lost profits are measured relative to the benchmark. Note that for the low-tax region, lost profits are negative in both the benchmark equilibrium and in the policy counterfactuals, i.e., profits are shifted inward to the low-tax region. However, the magnitude of these lost profits are smaller in the counterfactuals. For example, in panel (b), the amount of profits shifted into the low-tax region under pillar 2 is about half of the amount in the benchmark.



Figure 1: Varying the sizes of the pillars

Notes: Each column reports effects on one variable for each region. First column: Lost profits (reported relative to the benchmark equilibrium). Second column: real GDP (reported as a percent change from the benchmark equilibrium). X-axis in each plot represents the reallocation share for Pillar 1. Y-axis in each plot represents the global minimum corporate income tax rate for Pillar 2.

Appendix (For Online Publication)

A Institutional Background

In this section we provide a brief overview of the current international tax regime and describe the main features of the two-pillar reform proposed by the OECD. We aim here to deliver an executive summary, rather than an exhaustive discussion, of these immensely complex issues.²⁵ Understanding the main components of the international tax architecture is crucial since they largely dictate the setup of our theory and impose restrictions on what any reform proposal can achieve.

A.1 The Current International Tax Regime

Existing international law entitles a country to tax persons, either natural or legal, with which it has sufficient ties. In practice, taxing rights are a product of multiple national laws and international treaties that often contradict one another. The following are the most important characteristics of the current regime.

Legal separation of entities. The current regime treats subsidiaries within one MNE as separate legal entities. Thus, any transaction between parts of an MNE in different tax jurisdictions, such as for example an asset purchase, has real tax consequences. This characteristic coupled with heterogeneity of the tax systems across jurisdictions and manipulation of transfer prices gives rise to profit-shifting opportunities.

Allocation of taxing rights. There are at least four possible locations where a multinational company might in principle be taxed: the location of its shareholders, parent companies, affiliates, or customers. According to the current regime MNEs are taxed primarily in the third location (affiliates' location), but sometimes also in the second. This is achieved by a combination of legal rules allowing the countries concerned to tax on to a source or residence basis.²⁶

Transfer prices. Within-MNE transactions occur at transfer prices, which are disciplined by the so-called *arm's length principle (ALP)*. The basic idea behind the ALP is that within-MNE prices should reflect the market prices that would have been charged by two independent parties of transactions. There are five core methods to achieve the ALP standard: the comparable uncontrolled price (CUP), resale price minus, cost plus, profit split, and transactional net margin methods. The practical implementation of this principle is challenging and requires complex guidelines published regularly by the OECD which member countries should obey—see OECD (2022b) for the latest guide.

Treatment of intangibles. The method preferred by the OECD to implement ALP is CUP, which simply employs the price charged on comparable transactions between independent parties. CUP however is hard to implement in case of trading intangibles, since most of the time a comparable transaction is non-existent. In such cases the preferred method is the profit split method, which essentially inspects the relative financial or other contributions made by the two companies entering into a transaction. A profit split is then determined

 $^{^{25}\}mathrm{Our}$ summary is largely based on Devereux et al. (2021), OECD (2015), OECD (2017), and OECD (2022b).

²⁶From a legal perspective, a country taxes on a residence basis when it taxes companies that are resident in that country for tax purposes on income arising in that or in another country. A country taxes on a source basis when it taxes companies that are not resident in that country for tax purposes on income deemed to arise in that country. For a thorough discussion of these concepts see Devereux et al. (2021).

based on these contributions. OECD (2014) provides extensive guidelines on pricing transactions involving intangibles.

A.2 Tax Avoidance and Profit Shifting Channels

Tax avoidance and profit shifting is conducted by MNEs using variety of channels. In what follows we briefly discuss the most important ones.²⁷

- 1. Transfer Pricing Manipulations. The global allocation of tax base between source and residence countries is impacted by the valuation of intracompany transactions within an MNE. The arm's length principle, which requires that internal prices between related parties be similar to prices that would exist between independent parties, is used by most countries. However, this principle can be subject to significant subjective interpretation and may not have a "correct" arm's length price if there are no comparable third-party transactions. MNEs can manipulate transfer pricing by charging lower prices for exports from high-tax to low-tax countries or higher prices for inputs from low-tax countries, thus reducing their global tax liability.
- 2. Strategic Location of IP. The global tax of an MNE can also be reduced through the strategic location of IPs. A company may carry out R&D in one country and then transfer ownership of the resulting patent to a subsidiary in a low-tax jurisdiction, where the income generated from it will be taxed at a lower rate. Determining the arm's length price for intangible transactions within a company can be difficult, as there are often no comparable transactions between unrelated parties. This creates room for transfer pricing manipulation for tax purposes.
- 3. *International Debt Shifting*. Intracompany loans can also serve as a tool for reducing the tax bill of an MNE. The variation in corporate tax rates across countries creates the possibility of lending from low-tax countries to high-tax affiliates or borrowing from external sources in high-tax countries. The deductibility of interest payments on debt from taxable income results in a reduced tax bill for the group, without affecting its overall debt exposure and hence, bankruptcy risk.
- 4. Tax Treaty Shopping. Considerable variation in the withholding taxes (WHT) in more than 3000 bilateral double tax treaties creates opportunities of treaty shopping. This enables MNCs to link different treaties and divert cross-border payments through the country with the lowest WHT rate.
- 5. *Tax Treaty Shopping.* The presence of the withholding taxes (WHT) disparities in 3000-plus bilateral double tax agreements produces prospects for treaty shopping. This gives MNEs the capability to tie together varied pacts and divert cross-border payments through the nation with the least possible WHT rate.

A.2.1 The importance of intangible capital and strategic IP location.

In an influential paper Grubert (2003) examines the links between intangible income, income shifting, intercompany transactions, and location choices by utilizing data from U.S. parent corporations and their manufacturing subsidiaries. He finds that income from R&D-based intangible assets makes up roughly half of the income that is shifted from high-tax to low-tax jurisdictions. Heckemeyer and Overesch (2017) distinguish between the tax response through financial planning, such as inter-company debt financing, and the response through transfer pricing and licensing. They specifically compare the tax sensitivity of pre-tax

 $^{^{27}}$ This discussion is largely based on Johansson et al. (2017) and Beer et al. (2020).

profits, which encompasses shifting through various means, with the tax sensitivity of earnings before interest and taxes (EBIT), which only captures non-financial shifting mechanisms. The results of their stylized calculations indicate that transfer pricing and licensing, not inter-company debt financing, is the principal channel for profit shifting. The overall semi-elasticity of pre-tax profits is estimated to be 0.786, and the non-financial component of this response accounts for 82% of the total response. In their headline analysis OECD (Johansson et al., 2017) concludes that a comprehensive analysis of the allocation of third-party debt, among MNEs presents evidence of debt manipulation, accounting for 20% of profit shifting. Thus the rest, the vast majority, is accounted for by mispricing and strategic location of intangible capital. Moreover OECD finds that tax sensitivity of profit is almost twice as high among patenting MNEs than other MNEs (see Table A5.4 in their study). Beer, de Mooij and Liu (2020) extend the meta analysis conducted by Heckemeyer and Overesch (2017) by almost doubling the sample size of primary estimates. They conclude that debt-shifting channel plays, on average, a minor role.

A.3 OECD Base Erosion and Profit Shifting Project

In what follows we briefly summarize the key provisions of reform proposed by OECD/G20 Inclusive Framework on BEPS, as they were at the time of the writing of this paper.²⁸

A.3.1 Pillar 1: Profit allocation and nexus

The general principle behind Pillar 1 is to allocate taxing rights more closely where the customers and users of the in-scope MNEs are located. The key elements of Pillar 1 are as follows.

Scope. The new profit allocation rule will apply to groups with greater than $\in 20$ billion in worldwide revenues and a profitability before tax margin of at least 10 percent. There are some exclusions for extractive industries and regulated financial services.

Nexus. The allocation key is based on the revenue that is sourced to each jurisdiction. It will be sourced to the end-market jurisdictions, where goods or services are used or consumed, permitting allocation to a market jurisdiction from which the in-scope MNE derives at least $\in 1$ million in revenues.

Quantum. For in-scope MNEs, 25% of residual profit (i.e. profit in excess of 10% of revenue) will be allocated to market jurisdictions with nexus using a revenue-based allocation key.

Elimination of double taxation. Profit allocated to a market jurisdiction will be dispensed from double taxation through direct exemption of credit method.

Unilateral Measures. The agreement requires all parties to remove all digital services taxes and other relevant, similar measures with respect to all companies and to commit not to introduce such measures in the future.

A.3.2 Pillar 2: Global minimum taxation

The second pillar consists of two sets of rules granting jurisdictions additional taxing rights: (i) interlocking domestic rules termed Global anti-Base Erosion (GloBE) rules, and (ii) a treaty-based Subject to Tax Rule (STTR). Their key features are as follows.

²⁸The details of both pillars as well as the exact implementation plan are very much a work in progress at the time of writing this paper. Since November 2021 the OECD has been organizing a series of public consultation meetings in order to work out technical details and parameters of the reform.

Scope. GloBe rules apply to multinational enterprise groups with a total consolidated group revenue above \notin 750 million in at least two of the four preceding years.

Minimum tax rate. GloBE rules apply a system of top-up taxes that brings the total amount of taxes paid on an MNE's profit in a jurisdiction up to the minimum rate of 15%.

Exclusions. GloBe rules will also provide for an exclusion for those jurisdictions where the MNE has revenues of less than EUR 10 million and profits of less than EUR 1 million.

Subject to Tax Rule (STTR). This complements the GloBE rules by targeting intra-MNE payments exploiting certain provisions of the treaty to shift profits from source countries to payee jurisdictions where those payments are subject to no or low rates of nominal taxation. In such cases, it reallocates taxing rights to source jurisdictions. It applies to such payments as covered payments—interest, royalties, brokerage, marketing, procurement, agency or other intermediary services, and so on. The minimum rate for the STTR will be 9 percent.

B Data sources

Region definitions. We take the list of tax havens from Tørsløv et al. (2022). The complete list of countries in the tax-haven region is: Andorra, Anguilla, Antigua, Aruba, the Bahamas, Bahrain, Barbados, Belize, Bermuda, British Virgin Islands, Cayman Islands, Curacao, Cyprus, Gibraltar, Grenada, Guernsey, the Isle of Man, Jersey, Lebanon, Liechtenstein, Luxembourg, Malta, Marshall Islands, Mauritius, Monaco, the Netherlands Antilles, Panama, Puerto Rico, Samoa, Seychelles, Sint Maartin, St. Kitts & Nevis, St. Vincent & the Grenadines, St. Lucia, the Turks & Caicos, and Vanuatu.

World Development Indicators. Data on population and output come from the World Bank's World Development Indicators database. The specific series that we use are total population (SP.POP.TOTL), GDP in current US dollars (NY.GDP.MKTP.CD), and GDP at purchasing power parity in constant 2011 international dollars (NY.GDP.MKTP.PP.KD). For each of these variables, when constructing regional aggregates, we sum across countries within a region following McGrattan and Waddle (2020), and then average over the period 2014–2017.

World Input-Output Database. International goods trade data are taken from the World Input-Output Database (Timmer et al., 2015). For each bilateral import relationship, we sum all intermediate inputs and final uses of goods (industries 1–23, which represent agriculture, resource extraction, and manufacturing) from countries in the source region by countries in the destination region. We use data from 2014, the most recent year available.

OECD AMNE Database. This is a new dataset provided by the OECD which distinguishes between three types of firms: foreign affiliates (firms with at least 50% foreign ownership), domestic MNEs (domestic firms with foreign affiliates), and domestic firms not involved in international investment. It includes a full matrix of the output of foreign affiliates in 59 countries plus the rest of the world (in the host country, industry, parent country dimension), as well as matrices for value-added and for exports and imports of intermediate inputs (host country and industry). A second set of matrices in the database provides information on output, value-added, and exports and imports of intermediate inputs of domestic MNEs and non-MNE domestic firms (from 2008 onwards). In addition, split Inter-Country Input-Output tables are provided distinguishing for all countries the transactions of domestic-owned and foreign-owned firms. These tables can be used to analyze

multinational production in value-added terms. We exploit them to discipline our model and make sure it replicates the share of each region's gross value added that is accounted for by foreign multinationals. We first map the set of 59 countries from the AMNE dataset to our five regions and then compute the average value-added shares for three types of firms (foreign affiliates, domestic MNES, and domestic non-MNEs) in each region over the time period 2008–2016. The data can be accessed at OECD AMNE Database.

Compustat. Data on sales, employment, and country of origin of parent companies come from the Compustat North America Fundamentals Annual database. This database contains data of North American companies parsed from SEC filings. Data on subsidiaries come from the Wharton Research and Data Services (WRDS) Subsidiary Data. These data also come from SEC filing, particularly Exhibit 21, in which firms filing with the SEC must list the names of all existing Significant Subsidiaries. For a detailed, legal definition of Significant Subsidiaries, see here. Roughly, if the parent company controls at least 10% of the subsidiary, it is considered Significant. The WRDS data are available from 1995 to 2019, and contain identifying information for the parent company, as well as the name and country of residence of all Significant Subsidiaries. These two datasets were linked using a common identifier of the parent company, the gvkey. Mean and median sales and employment statistics were computed for the years 2010-2019. The unit of observation was parent company-year.

U.S. Census Data. To discipline the firm size distribution we exploit data from the Statistics of U.S. Businesses (SUSB). SUSB is an annual series that provides national and subnational data on the distribution of economic data by establishment industry and enterprise size. SUSB covers most of the country's economic activity. The series excludes data on nonemployer businesses, private households, railroads, agricultural production, and most government entities. We construct a Lorenz employment curve for the U.S. at the firm level using two Excel spreadsheets available at the Census website. We combine the table with detailed employment sizes with the table with larger employment sizes (20,000+ employees), both from 2019 SUSB. This allows us to account for a long right tail of the firm size distribution in our model, which is crucial given that average MNE is three orders of magnitude larger than the average firm in the U.S. economy. Both Excel files can be downloaded from the SUSB website.

OECD Corporate Tax Statistics Database. To compute the share of corporate income tax revenue in each region that is paid by local affiliates of foreign MNEs, we use data from the OECD Corporate Tax Statistics Database (OECD, 2022a). First, for each country in the database, we compute two numbers from Table 1: (i) corporate income taxes paid by domestic MNEs' affiliates; and (ii) corporate income taxes paid by foreign MNEs' affiliates. Second, we compute (i) as a share of (ii). Third, for each region in our model, we compute the average of these shares across the countries in that region, weighting by (i) + (ii). The data can be accessed here.

Tørsløv et al. (2022). Two kinds of data are taken from this paper: lost profits and effective corporate income tax rates. Total lost profits are from sheet Table3 of the Main Data Excel file. We first sum across all countries within the North America and Europe regions, and then set the rest of the world's lost profits by subtracting the North America and Europe totals from the overall world total. The share of lost profits that are shifted to the tax haven region is constructed in the same way using sheet TableC2 in the Replication Guide Tables Excel file. The effective corporate income tax rates come from sheet DataF2 in the Main Tables Excel file. Here, we take the average across countries within each region. Both Excel files can be downloaded from https://missingprofits.world/.

C Firm-level profit shifting estimates

In this section we briefly discuss the empirical literature on profit shifting, which aims to estimate the elasticity of reported profits with respect to the tax rate differentials across jurisdictions. We begin with an overview of the empirical strategy adopted in this line of research, then move to the discussion of the headline, consensus estimates emerging from the literature. Finally we link our structural modelling approach to the empirical strategy.

C.1 Empirical strategy

Most of the empirical literature on elasticity of the profit shifting margin follows the concept presented by Grubert and Mutti (1991) and Hines and Rice (1994) that the reported pre-tax profit of a multinational entity, Π_i^R , is a sum of the "true" profit, Π_i^T , and the profit shifted for tax reasons, Π_i^S

$$\Pi_i^R = \Pi_i^T + \Pi_i^S. \tag{C.1}$$

This shifted profit would be positive in low-tax countries and negative in high-tax countries. The idea here is that the actual profitability of multinational enterprises with similar characteristics (e.g. size, industry, country etc.) is similar. However, the opportunities to shift profits differ since they depend on such characteristics as locations of the other subsidiaries and statutory tax rates in these locations. Thus, the entities linked to low-tax jurisdictions are more likely to shift profits and the entities linked to high-tax jurisdictions are more likely to receive profits. The fundamental challenge for estimating the elasticity of profit shifting margin is that neither "true" profits nor shifted ones are directly observable in the firm-level data. To tackle this problem the literature usually assumes that "true" profits are equal to output minus the wage bill, with the wage being equal to marginal product of labor (see for example Huizinga and Laeven (2008)). As for the shifted profits, the literature typically specifies some stylized framework that allows linking shifted profits to tax differentials between jurisdiction j and other operating jurisdictions. This strategy leads to the following generic equation to identify shifting profits:

$$\pi_{i,j,t}^R = \beta X_{i,j,t} - \gamma C_{i,j,t} + \delta_t + \varepsilon \tag{C.2}$$

where $\pi_{i,j,t}^R = \ln \prod_{i,j,t}^R$ are log reported profits of a multinational *i* located in jurisdiction *j* at time *t*, $X_{i,j,t}$ is a vector of determinants of true profitability, which includes capital and labor inputs among others. It may also include a number of macroeconomic variables, such as GDP growth, exchange rate, or inflation. $C_{i,j,t}$ is a composite variable that summarizes the tax differentials between jurisdiction *j* and other jurisdictions in which the MNE located in jurisdiction *i* has subsidiaries. The specific formula for $C_{i,j,t}$ differs across papers but in all of them it reflects the tax incentives to shift profits away from or into jurisdiction *j*. Finally δ_t denotes time fixed-effect and ε denotes the residual term. The coefficient of interest is then γ which reflects the extent to which the multinational shifts profits into or out of affiliate *i*. It is important to note that this estimate represents a marginal effect – i.e. the change in reported profits associated with a small change in tax rates, holding all else constant. We can interpret γ in equation (C.2) as the semi-elasticity of observed profits π_i^O with respect to the composite tax variable $C_{i,j,t}$. The semi-elasticity indicates the percentage change of reported profit in response to a one percentage point change in the tax differential vis-a-vis other international locations, reflecting the incentive to shift profits abroad.

C.2 Empirical estimates

A number of papers estimate different versions of equation (C.2) for a variety of datasets and time periods. A thorough and detailed review of this literature is beyond the scope of this paper.²⁹ Instead, we focus here on the two most recent survey papers, which conduct meta analyses of existing estimates, and on the main OECD estimate, all of which report the headline semi-elasticity number.

Johansson et al. (2017) provide the main estimate of the magnitude of the profit shifting used by the OECD. They conduct a comprehensive study using firm-level data from the ORBIS database to assess international tax planning by multinational enterprises (MNEs). Their results are based on an impressively large sample of firms (1.2 million observations of MNE accounts) in 46 OECD and G20 countries and a sophisticated procedure to identify MNE groups. Their headline estimate of the semi-elasticity of the profit shifting margin with respect to the tax differential is **1.11** (see Table 1, column 1 and footnote 31 in their paper). Hence, reported profits decrease by about 1.1% if the international tax rate differential increases by one percentage point. The estimated elasticities combined with a number of assumptions are then used to estimate the effect of international tax planning on corporate tax revenues: the estimated net tax revenue loss ranges from 4% to 10% of global corporate tax revenues.

Heckemeyer and Overesch (2017) construct a meta-database containing 203 primary estimates sampled from 27 empirical studies identified by means of article search engines. All of the included studies estimate the empirical relationship between reported parent and subsidiary profitability and the tax incentive to shift profits abroad. Therefore, this meta-analysis reviews the literature, providing indirect evidence for profit shifting without specifying directly the shifting methods. They find a tax semi-elasticity of pre-tax profit of about 0.79, in absolute terms. They conclude that across all specifications the predicted semi-elasticities turn out to be statistically significant and rather robust in magnitude. They also provide a 95% confidence interval in addition to the point estimate and conclude that conditional on a hypothetical state-of-the-art study design, the set of semi-elasticities that should not be rejected at the 5% significance level ranges from 0.546 to 1.026.

Beer, de Mooij and Liu (2020) extend the analysis conducted by Heckemeyer and Overesch (2017) and include 11 additional studies and 199 additional primary estimates. They also reduce specification bias, and adopt an enhanced estimation method that corrects for within-study correlation of primary estimates. Their results indicate that a semielasticity of reported pretax profits with respect to international tax differentials equal to **0.98** is a good reflection of the literature. This means that a one-percentage-point larger tax rate differential reduces reported pretax profits of an affiliate by 1%.

C.3 Model counterpart of semi-elasticity

We now describe how we estimate the model counterpart of the semi-elasticity summarized above. We view this as a validation exercise of the cost function $C(\lambda)$ upon which the extent of profit shifting in the presence of tax differentials between jurisdictions heavily depends. Since our parsimonious model of only four productive regions does not provide sufficient variation in cross-jurisdiction differences in corporate tax rates (regressor $C_{i,j,t}$ in equation (C.2)), we conduct a simulation exercise as follows.

We simulate 100 counterfactual economies, raising the corporate tax rate of the LT region incrementally for the first 50 economies and the rate of the TH region for the latter 50. We set the highest counterfactual corporate tax rate to 15%, equal to the global minimum tax rate of OECD Pillar 2. In each of these

 $^{^{29}}$ See Dharmapala (2014), Heckemeyer and Overesch (2017), Johansson et al. (2017) and Beer et al. (2020) for extensive reviews of this line of research.

counterfactual economies, we hold fixed the set of firms' FDI and exporting destinations, J_F and J_X , as well as the final good price and wage rate of each region, P_i and W_i . We allow firms to solve for their optimal choices of labor ℓ , intangible capital z and shifting shares λ_{LT} and λ_{TH} . In other words, the firms' problem is re-solved in a partial equilibrium setting, which allows us to isolate the relationship of reported profits in home divisions to tax rate differentials relative to the profit-shifting destination.

Denote k as the index of a counterfactual economy. We follow the empirical specification of equation (C.2) and run the regression using the model-simulated dataset:

$$\log \pi_i^{k,PS}(\omega) = \beta_0 + \beta_\ell \log \ell_i^k(\omega) + \beta_z \log z^k(\omega) - \beta_\tau \hat{\tau}_i^k + \epsilon_i^k(\omega)$$
(C.3)

where we denote by τ_i^k the counterfactual tax differential defined as $\hat{\tau}_i^k = \tau_i - \tau_{LT}^k$ for $k \leq 50$ and $\hat{\tau}_i^k = \tau_i - \tau_{TH}^k$ otherwise. For each experiment k, we include in the regression only home divisions of firms doing FDI in the region for which we change the corporate tax rate. We only include home divisions of profit-shifting MNEs because we do not model profit shifting originating from a foreign subsidiary. Nonetheless, such regression informs us of how reported profit responds to changes in profit-shifting relevant tax differentials, which is captured by the coefficient of interest β_{τ} . We report the coefficient estimate of β_{τ} in Table 2.

D Sensitivity Analysis

Our quantitative results are robust to a variety of alternative assumptions and calibrations. Here, we describe the results of three sensitivity analyses that illustrate the impact of some of the most important elements of our model and policy experiments. Table D.1 shows the results of these sensitivity analyses.

Alternative profit reallocation rules. The first pillar of the OECD's BEPS project reallocates the rights to tax a portion of a firm's global profits to the regions in which it operates in accordance with these regions' shares of the firm's global sales. Importantly, some of these rights are allocated to a firm's export markets, even if the firm does not operate foreign affiliates in these markets. This aspect of the rule increases effective tax rates for firms based in Europe, the low-tax region, and the rest of the world because North America, which is a large, rich export market, has the highest corporate income tax rate. This reduces these firms' incentives to invest in intangible capital, even if they do not shift profits at all. This partly explains why Pillar 1 has larger macroeoconomic consequences than Pillar 2, despite having smaller effects on profit shifting. To explore the importance of this aspect of Pillar 1, we have analyzed the effects of alternative versions in which profit taxation rights are allocated for MNEs only, or are based on output shares instead of sales shares. Panel (a) of Table D.1 shows the effects of a profit allocation rule that applies only to MNEs, as opposed to firms that export but do not operate foreign affiliates. The effects on profit shifting are the same as the OECD's version but the macroeconomic consequences are smaller, especially outside of North America. Panel (b) of Table D.1 shows what happens when profit-taxation rights are allocated based on output rather than sales. Under this version of the pillar, export destinations do not receive any taxation rights at all. The results are almost identical to panel (a). These results indicate that allocating taxation rights based on export sales should be avoided.

Intangible share. We have set the share of intangible capital in production, ϕ , to match the share of income that accrues to intangible capital in MNEs' foreign affiliates. This approach ensures that our model captures the extent to which nonrivalry governs MNEs' incentives to invest in intangible capital. This share is the key determinant of the potential scope for profit shifting; a greater intangible share means more licensing

fee income that can be transferred to the low-tax region and/or the tax haven. Of course, it is also the key determinant of the macroeconomic impact of policies that affect incentives to invest in intangible capital, including the policies designed to reduce profit shifting that we study. Panels (c) and (d) of Table D.1 show the results of our experiments under alternative calibrations with different intangible shares. In each, we recalibrate all model parameters except for those that govern profit shifting. This allows us to explore how the intangible share affects profit shifting under the current international tax system as well as the effects of changes to this system. The results of these analyses show that a lower intangible share reduces macroeconomic effects of transfer pricing and profit shifting, reduces the amount of profit shifting under the current tax code, and reduces the macroeconomic consequences of the OECD BEPS pillars; the reverse is true for a higher intangible share. However, the extent to which the BEPS pillars reduce profit shifting is about the same as in the baseline model. For example, with a lower intangible share, lost profits in North America fall by 1 - 0.27/0.45 = 40% under Pillar 1, exactly the same as in the baseline model.

Labor supply. In our baseline model we assume that preferences are log-separable in consumption and leisure as in McGrattan and Waddle (2020). This means that labor supply is driven partly by income effects. This is particularly important for the low-tax region, where lump-sum transfers of taxes on profits shifted away from high-tax countries are fairly sizeable. Here, we explore alternative versions of the model without income effects on labor supply. Panel (e) of Table D.1 shows the results of when preferences are assumed to be GHH, and panel (f) shows the results from an even starker assumption of perfectly inelastic labor supply. Both versions yield similar results. For the three high-tax regions, the results are broadly similar to the baseline, with all variables moving in the same direction in all experiments. In the low-tax region, though, profit shifting now raised GDP instead of lowering it, and the OECD/G20 BEPS pillars reduce GDP much more than in the baseline. This highlights a somewhat important point for the low-tax country: profit shifting causes something akin to Dutch disease (which is kind of funny, since the Netherlands is in our low-tax region).

Profit-shifting costs. We have set the costs of profit shifting, ψ_{iLT} and ψ_{iTH} , to match estimates in the literature about the amount of profit shifting and the extent to which profits are shifted to low-tax "productive" regions versus "unproductive" tax havens. These estimates are inferred from information about the profitability and labor shares of MNEs' foreign affiliates in these regions—it is impossible to measure lost profits directly without access to detailed information about intra-MNE transactions—so there is some uncertainty about how much profit shifting truly occurs. To determine the sensitivity of our results to these key parameters, we have conducted our experiments in alternative calibrations within which these parameters are set to higher or lower values. Panel (g) of Table D.1 shows the results when ψ_{iLT} and ψ_{iTH} are halved, while panel (h) shows the results when they are doubled. With lower profit-shifting costs, there is more profit shifting under the current tax system and the OECD BEPS pillars have larger macroeconomic effects; the reverse holds with lower costs. As in the previous exercise, the BEPS pillars reduce profit shifting by about the same amount as in the baseline. For example, with lower shifting costs, lost profits in North America fall by 1 - 1.26/2.03 = 38% under Pillar 1.

	Lost	Lost profits (benchmark $= 1$)			GDP (% chg.)				
Experiment	North America	Europe	Low tax	Rest of World	North America	Europe	Low tax	Rest of World	
(a) Profit reallocation rule	applies to	MNEs only	/						
Pillar 1	0.60	0.66	0.69	0.63	-0.12	-0.10	-0.06	-0.09	
Pillars 1 & 2 together	0.23	0.18	0.33	0.10	-0.16	-0.12	-0.06	-0.10	
(b) Production-based profit	reallocatio	n rule							
Pillar 1	0.60	0.66	0.69	0.63	-0.12	-0.09	-0.06	-0.09	
Pillars 1 & 2 together	0.23	0.18	0.33	0.10	-0.16	-0.11	-0.06	-0.09	
(c) Low intanaible share									
Effects of transfer pricing	_	_	_	_	-0.06	-0.08	-0.12	-0.08	
Effects of profit shifting	0.45	0.45	0.48	0 44	0.02	0.02	-0.04	0.01	
Pillar 1	0.10	0.30	0.33	0.28	-0.08	-0.11	-0.12	-0.10	
Pillar 2	0.17	0.12	0.23	0.07	-0.02	-0.01	0.02	0.00	
Pillars 1 & 2 together	0.10	0.08	0.16	0.04	-0.10	-0.12	-0.13	-0.10	
(a) High intangiole share					0.07	0.90	0.90	0.00	
Effects of transfer pricing	-	-	-	-	-0.27	-0.26	-0.38	-0.28	
Effects of profit shifting	1.60	1.60	1.60	1.63	0.16	0.12	-0.04	0.09	
Pillar I	0.96	1.06	1.10	1.03	-0.19	-0.18	-0.14	-0.17	
Pillar 2	0.59	0.42	0.78	0.25	-0.11	-0.05	0.02	-0.03	
Pillars 1 & 2 together	0.36	0.28	0.53	0.16	-0.25	-0.22	-0.15	-0.18	
(e) GHH preferences									
Effects of transfer pricing	-	-	-	-	-0.11	-0.21	-0.20	-0.18	
Effects of profit shifting	1.00	1.00	1.00	1.00	0.01	-0.03	0.31	-0.01	
Pillar 1	0.60	0.66	0.69	0.63	-0.09	-0.10	-0.35	-0.10	
Pillar 2	0.37	0.26	0.49	0.15	0.00	0.06	-0.16	0.03	
Pillars 1 & 2 together	0.23	0.18	0.33	0.10	-0.09	-0.07	-0.46	-0.09	
(f) Fixed labor supply									
Effects of transfer pricing	_	_	_	_	-0.09	-0.21	-0.21	-0.18	
Effects of profit shifting	1.00	1.00	1.00	1.00	-0.02	-0.05	0.24	-0.02	
Pillar 1	0.60	0.66	0.69	0.63	-0.06	-0.09	-0.32	-0.10	
Pillar 2	0.37	0.26	0.49	0.15	0.02	0.07	-0.11	0.04	
Pillars 1 & 2 together	0.23	0.18	0.33	0.10	-0.06	-0.06	-0.38	-0.08	
(a) Low profit-shifting cost	s								
Effects of profit shifting	2.03	1.96	1.90	2.05	0.15	0.10	-0.09	0.06	
Pillar 1	1.00	1.00	1 33	1 32	-0.16	-0.16	-0.10	-0.14	
Pillar 2	0.79	0.53	0.96	0.33	-0.10	-0.03	0.10	-0.01	
Pillars 1 & 2 together	0.48	0.35	0.66	0.33 0.21	-0.23	-0.19	-0.09	-0.15	
(h) High profit shifting and	to								
(<i>n</i>) myn projn-snijung cos	0.40	0 50	0 51	0.47	0.04	0.04	0.00	0.02	
Dillog 1	0.40	0.00	0.91	0.47	0.04	0.04	-0.02	0.05	
Filler 9	0.28	0.32	0.30	0.29	-0.12	-0.13	-0.14	-0.13	
Pillars 1 & 2 together	0.17	0.13	0.20 0.17	0.07	-0.03	-0.01	-0.15	-0.00	

Table D.1: Sensitivity analysis

Notes: Panel (a): profit-reallocation rule for pillar 1 applies only to MNEs (not firms that export but do not operate foreign affiliates). Panel (b): rule is based on value added rather than sales; profits are not reallocated to export destinations. Panels (c) and (d): intangible share is changed and all parameters except for profit-shifting costs are recalibrated. Panel (e): households have GHH preferences (no income effects on labor supply). Panel (f): labor supply is fixed. Panel (g): parameters that govern marginal cost of profit shifting, ψ_{ij} , are halved. Panel (h): ψ_{ij} are doubled. Lost profits measured relative to benchmark equilibrium in baseline calibration.

Proofs of Analytical Results \mathbf{E}

This Appendix contains the proofs of the lemmas and propositions from the main body of the paper.

Optimal Profit-Shifting and Effect on Intangible Investment E.1

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Proof of Lemma 1.

Rewrite the problem (10) using definitions of profits as

$$\max_{z,\lambda,\{\ell_i\}_{i=1}^{I}} (1-\tau_i) \left(p_i \left(A_i \left(N_i z \right)^{\phi} \ell_i^{\gamma} \right) - w_i \ell_i - p_i z + z \left[\varphi \lambda \sum_k \vartheta_k \left(z \right) - \lambda \vartheta_i \left(z \right) + (1-\lambda) \sum_{k \neq i} \vartheta_k \left(z \right) - \sum_k \vartheta_k \left(z \right) \mathcal{C} \left(\lambda \right) \right] \right) + (1-\tau_{i^*}) \left(p_{i^*} \left(A_{i^*} \left(N_{i^*} z \right)^{\phi} \ell_{i^*}^{\gamma} \right) - w_{i^*} \ell_{i^*} + z \left[\lambda \sum_{k \neq i^*} \vartheta_k \left(z \right) - (1-\lambda) \vartheta_{i^*} \left(z \right) - \varphi \lambda \sum_k \vartheta_k \left(z \right) \right] \right) + \sum_{k \neq i, i^*} (1-\tau_k) \left(p_k \left(A_k \left(N_k z \right)^{\phi} \ell_k^{\gamma} \right) - w_k \ell_k - \vartheta_k \left(z \right) z \right).$$
(E.1)

The FOCs are then:

$$\ell_{i}: 0 = \gamma p_{i} A_{i} (N_{i} z)^{\phi} \ell_{i}^{\gamma-1} - w_{i}, \ i = 1, 2, ..., I,$$

$$(E.2)$$

$$z: 0 = \sum_{k} (1 - \tau_{k}) \phi N_{k} p_{k} A_{k} (N_{k} z)^{\phi-1} \ell_{k}^{\gamma} + (1 - \tau_{i}) \left[-p_{i} + \varphi \lambda \sum_{k} \vartheta_{k} (z) - \lambda \vartheta_{i} (z) + (1 - \lambda) \sum_{k \neq i} \vartheta_{k} (z) - \sum_{k} \vartheta_{k} (z) \mathcal{C} (\lambda) \right]$$

$$+ (1 - \tau_{i^{*}}) \left[\lambda \sum_{k \neq i^{*}} \vartheta_{k} (z) - (1 - \lambda) \vartheta_{i^{*}} (z) - \varphi \lambda \sum_{k} \vartheta_{k} (z) \right] - \sum_{k \neq i, i^{*}} (1 - \tau_{k}) \vartheta_{k} (z),$$

$$(E.3)$$

$$\lambda: 0 = (1 - \tau_{i}) z \left[\varphi \sum_{k} \vartheta_{k} (z) - \vartheta_{i} (z) - \sum_{k \neq i} \vartheta_{k} (z) - \sum_{k} \vartheta_{k} (z) \mathcal{C}' (\lambda) \right]$$

$$+ (1 - \tau_{i^{*}}) z \left[\sum_{k \neq i^{*}} \vartheta_{k} (z) + \vartheta_{i^{*}} (z) - \varphi \sum_{k} \vartheta_{k} (z) \right].$$

Inspect the FOC wrt to λ :

$$0 = (1 - \tau_i) z \left[\varphi \sum_k \vartheta_k \left(z \right) - \vartheta_i \left(z \right) - \sum_{k \neq i} \vartheta_k \left(z \right) - \sum_k \vartheta_k \left(z \right) \mathcal{C}' \left(\lambda \right) \right] + (1 - \tau_{i^*}) z \left[\sum_{k \neq i^*} \vartheta_k \left(z \right) + \vartheta_{i^*} \left(z \right) - \varphi \sum_k \vartheta_k \left(z \right) \right] \right]$$

which yields

$$\lambda = \left(\mathcal{C}'\right)^{-1} \left[\left(1 - \varphi\right) \frac{\left(\tau_i - \tau_{i^*}\right)}{1 - \tau_i} \right].$$
(E.5)

Under Assumption 1 this can be written as

$$\lambda = 1 - \exp\left(-\frac{(1-\varphi)\left(\tau_i - \tau_{i^*}\right)}{1-\tau_i}\right).$$
(E.6)

Now towards proving the lemma, we have

$$\frac{\partial \lambda}{\partial \varphi} = -\exp\left(-\frac{(1-\varphi)\left(\tau_{i}-\tau_{i^{*}}\right)}{1-\tau_{i}}\right)\left(\frac{\tau_{i}-\tau_{i^{*}}}{1-\tau_{i}}\right) < 0,$$
$$\frac{\partial \lambda}{\partial \tau_{i^{*}}} = -\exp\left(-\frac{(1-\varphi)\left(\tau_{i}-\tau_{i^{*}}\right)}{1-\tau_{i}}\right)\left(\frac{1-\varphi}{1-\tau_{i}}\right) < 0,$$

and therefore the elasticities are

$$\begin{split} \varepsilon_{\varphi}^{\lambda} &= \frac{\partial \lambda}{\partial \varphi} \frac{\varphi}{1 - \exp\left(-\frac{(1-\varphi)(\tau_i - \tau_{i^*})}{1 - \tau_i}\right)} = -\exp\left(-\frac{(1-\varphi)(\tau_i - \tau_{i^*})}{1 - \tau_i}\right) \left(\frac{\tau_i - \tau_{i^*}}{1 - \tau_i}\right) \frac{\varphi}{1 - \exp\left(-\frac{(1-\varphi)(\tau_i - \tau_{i^*})}{1 - \tau_i}\right)} \\ &= -\left(\frac{1-\lambda}{\lambda}\right) \left(\frac{\tau_i - \tau_{i^*}}{1 - \tau_i}\right) \varphi, \end{split}$$

$$\begin{split} \varepsilon_{\tau_i^*}^{\lambda} &= \frac{\partial \lambda}{\partial \tau_{i^*}} \frac{\tau_{i^*}}{1 - \exp\left(-\frac{(1-\varphi)(\tau_i - \tau_{i^*})}{1 - \tau_i}\right)} = -\exp\left(-\frac{(1-\varphi)(\tau_i - \tau_{i^*})}{1 - \tau_i}\right) \left(\frac{1-\varphi}{1 - \tau_i}\right) \frac{\tau_{i^*}}{1 - \exp\left(-\frac{(1-\varphi)(\tau_i - \tau_{i^*})}{1 - \tau_i}\right)} \\ &= -\left(\frac{1-\lambda}{\lambda}\right) \left(\frac{1-\varphi}{1 - \tau_i}\right) \tau_i^*, \end{split}$$

which proves 1. and 2. \blacksquare

The following lemma will be useful in our further derivations.

Lemma 2 The allocations of intangible capital are as follows:

$$z^{FT} = \left(\frac{\sum_{k} \left(1 - \tau_{k}\right) \Lambda_{k}}{\left(1 - \tau_{i}\right) p_{i}}\right)^{\frac{1 - \gamma}{1 - \phi - \gamma}},\tag{E.7}$$

$$z^{TP} = \left(\frac{\sum_k \Lambda_k}{p_i}\right)^{\frac{1-\gamma}{1-\phi-\gamma}},\tag{E.8}$$

$$z^{PS} = z^{TP} \left(\left(1 - \mathcal{C}\left(\lambda\right) \right) + \frac{\lambda \left(1 - \varphi \right) \left(\tau_i - \tau_{i^*} \right)}{1 - \tau_i} \right)^{\frac{1 - \gamma}{1 - \phi - \gamma}}.$$
 (E.9)

Proof. Free transfer of z requires $\vartheta_k(z) = 0$ thus the (E.3) becomes

$$z^{FT} = \left(\frac{\sum_{k} \left(1 - \tau_{k}\right) \Lambda_{k}}{\left(1 - \tau_{i}\right) p_{i}}\right)^{\frac{1 - \gamma}{1 - \phi - \gamma}},$$

and hence we obtain (E.7). For the transfer pricing case we have $\lambda = 0$ and the (E.3) becomes

$$0 = z^{\frac{\phi \pm \gamma - 1}{1 - \gamma}} \sum_{k} (1 - \tau_k) \Lambda_k - (1 - \tau_i) p_i - \sum_{k} \vartheta_k (z) (\tau_i - \tau_k),$$

where

$$\vartheta_k\left(z\right) = \phi p_k N_k \left(A_k \left(N_k z\right)^{\phi-1} \ell_k^{\gamma}\right) = \phi \gamma^{\frac{\gamma}{1-\gamma}} p_k^{\frac{1}{1-\gamma}} A_k^{\frac{1}{1-\gamma}} \left(\frac{1}{w_k}\right)^{\frac{\gamma}{1-\gamma}} N_k^{\frac{\phi}{1-\gamma}} \left(z\right)^{\frac{\phi+\gamma-1}{1-\gamma}} = \Lambda_k \left(z\right)^{\frac{\phi+\gamma-1}{1-\gamma}}$$

Thus, we have

$$z^{TP} = \left(\frac{\sum_k \Lambda_k}{p_i}\right)^{\frac{1-\gamma}{1-\phi-\gamma}}$$

•

Hence we obtain (E.8). Now, for the profit shifting case, we can rewrite (E.3) as

$$0 = z^{\frac{\phi + \gamma - 1}{1 - \gamma}} \sum_{k} (1 - \tau_k) \Lambda_k - (1 - \tau_i) p_i - z^{\frac{\phi + \gamma - 1}{1 - \gamma}} \sum_{k} \Lambda_k \left[(\tau_i - \tau_k) - \lambda (1 - \varphi) (\tau_i - \tau_{i^*}) + (1 - \tau_i) \mathcal{C}(\lambda) \right],$$

and thus

$$z^{PS} = \left(\frac{\sum_{k} \Lambda_{k} \left(1 - \tau_{i}\right) \left[\left(1 - \mathcal{C}\left(\lambda\right)\right) + \frac{\lambda\left(1 - \varphi\right)\left(\tau_{i} - \tau_{i^{*}}\right)}{1 - \tau_{i}}\right]}{\left(1 - \tau_{i}\right) p_{i}}\right)^{\frac{1 - \gamma}{1 - \phi - \gamma}} = z^{TP} \left(\left(1 - \mathcal{C}\left(\lambda\right)\right) + \frac{\lambda\left(1 - \varphi\right)\left(\tau_{i} - \tau_{i^{*}}\right)}{1 - \tau_{i}}\right)^{\frac{1 - \gamma}{1 - \phi - \gamma}}$$

thus we have (E.9). \blacksquare

Proof of Proposition 1. Note we have derived the formulas for z^{FT} , z^{TP} and z^{PS} and we have the following formulas for λ and $C(\lambda)$:

$$\lambda = 1 - \exp\left(-\frac{(1-\varphi)\left(\tau_i - \tau_{i^*}\right)}{1-\tau_i}\right),\tag{E.10}$$

$$\mathcal{C}(\lambda) = \lambda - (\lambda - 1)\log(1 - \lambda), \tag{E.11}$$

where

$$\tau_{i^*} \equiv \min\left\{\tau_1, \dots, \tau_K\right\}.$$

Start with showing 1. Let

$$\tau_i \equiv \max\left\{\tau_1, ..., \tau_K\right\},\,$$

then

$$\frac{1-\tau_i}{1-\tau_i} < \frac{1-\tau_k}{1-\tau_i} \quad \forall k$$

Thus

$$z^{FT} = \left(\frac{\sum_{k} \left(1 - \tau_{k}\right) \Lambda_{k}}{\left(1 - \tau_{i}\right) p_{i}}\right)^{\frac{1 - \gamma}{1 - \phi - \gamma}} > \left(\frac{1}{p_{i}}\right)^{\frac{1 - \gamma}{1 - \phi - \gamma}} \left(\sum_{k} \frac{\left(1 - \tau_{i}\right)}{\left(1 - \tau_{i}\right)} \Lambda_{k}\right)^{\frac{1 - \gamma}{1 - \phi - \gamma}} = z^{TP}.$$

Now, towards showing 2. Start with (\Leftarrow) direction, and let $0 < \varphi < 1$. Then, by (E.10) we have $0 < \lambda < 1$. Take any $\lambda \in (0, 1)$ and notice that $z^{PS} > z^{TP}$ iff

$$\mathcal{C}(\lambda) < \frac{\lambda \left(1 - \varphi\right) \left(\tau_i - \tau_{i^*}\right)}{1 - \tau_i}.$$
(E.12)

Note that $\forall x > 0$ we have

$$\begin{aligned} x &< -\ln\left(1-x\right) \\ \left(1 + \frac{\exp\left(-x\right)}{1 - \exp\left(-x\right)}\right) &> \frac{1}{x}. \end{aligned}$$

Now, set

$$x \equiv \frac{(1-\varphi)\left(\tau_i - \tau_{i^*}\right)}{(1-\tau_i)},$$

which implies

$$\frac{\left(1-\varphi\right)\left(\tau_{i}-\tau_{i^{*}}\right)}{\left(1-\tau_{i}\right)}\left[1+\frac{\left(\exp\left(-\frac{a(1-\varphi)(\tau_{i}-\tau_{i^{*}})}{1-\tau_{i}}\right)\right)}{1-\exp\left(-\frac{a(1-\varphi)(\tau_{i}-\tau_{i^{*}})}{1-\tau_{i}}\right)}\right]>1$$

$$\frac{\left(1-\exp\left(-\frac{(1-\varphi)(\tau_{i}-\tau_{i^{*}})}{1-\tau_{i}}\right)-1\right)}{1-\exp\left(-\frac{(1-\varphi)(\tau_{i}-\tau_{i^{*}})}{1-\tau_{i}}\right)}\log\left(1-1+\exp\left(-\frac{(1-\varphi)(\tau_{i}-\tau_{i^{*}})}{1-\tau_{i}}\right)\right)>1-\frac{(1-\varphi)(\tau_{i}-\tau_{i^{*}})}{(1-\tau_{i})},$$

which using (E.10) can be written as

$$\frac{(\lambda-1)}{\lambda}\log\left(1-\lambda\right) > 1 - \frac{(1-\varphi)\left(\tau_i - \tau_{i^*}\right)}{(1-\tau_i)} > 0,$$

which through the series of iff inequalities can be transformed as follows

$$\mathcal{C}(\lambda) < \frac{\lambda \left(1 - \varphi\right) \left(\tau_i - \tau_{i^*}\right)}{1 - \tau_i}.$$

This proves (E.12) and hence establishes $z^{PS} > z^{TP}$. Given that all the inequalities are iffs the reverse argument holds immediately. To show 3. and 4. notice from (E.10) first, that $\frac{\partial \lambda}{\partial \varphi} < 0$. Now, we want to show

$$\frac{\partial z^{PS}}{\partial \varphi} = \frac{1-\gamma}{1-\phi-\gamma} \cdot z^{PS} \left((1-\mathcal{C}(\lambda)) + \frac{\lambda (1-\varphi) (\tau_i - \tau_{i^*})}{1-\tau_i} \right)^{-1} \left(\frac{\partial \lambda}{\partial \varphi} \left[\frac{(1-\varphi) (\tau_i - \tau_{i^*})}{1-\tau_i} - \mathcal{C}'(\lambda) \right] - \lambda \frac{(\tau_i - \tau_{i^*})}{1-\tau_i} \right) < 0.$$

This is negative if

$$\frac{\left(1-\varphi\right)\left(\tau_{i}-\tau_{i^{*}}\right)}{1-\tau_{i}}-\mathcal{C}'\left(\lambda\right)\leq0,$$

and it holds with equality, since it is the condition equalizing marginal cost with marginal benefit of profit shifting λ . Thus we prove 3. Notice, that proof for 4. follows analogously. Now towards deriving the elasticity

$$\varepsilon_{\tau_{i^*}}^{z} = \frac{1-\gamma}{1-\phi-\gamma} \left(\frac{-\tau_{i^*}}{\tau_i-\tau_{i^*}}\right) \frac{1}{\left[1+\frac{1-\mathcal{C}(\lambda)}{\lambda \mathcal{C}'(\lambda)}\right]} < 0.$$

E.2 Effects of Sales-Based Profit Allocation (OECD/G20 Pillar 1)

We first establish the following lemma characterizing how $\hat{\lambda}$ depends on the parameters of the profit allocation rule and how it differs from the share λ that is transferred under the existing tax regime. With slight abuse of notation we denote in the appendix:

$$\hat{\tau}_i(\theta) = (1-\theta)\tau_i + \theta \sum_i \tau_i \cdot \frac{p_i y_i}{\sum_k p_k y_k}$$

Lemma 3 Under Assumption 1, the following hold:

- 1. the fraction of intangible capital sold to the tax haven under the profit allocation rule is smaller than under the current regime, i.e., $\hat{\lambda} < \lambda$;
- 2. $\hat{\lambda}$ is decreasing in θ with elasticity given by

$$\varepsilon_{\theta}^{\hat{\lambda}} = -\mathcal{C}'\left(\hat{\lambda}\right)\left(\frac{1-\hat{\lambda}}{\hat{\lambda}}\right)\left(\frac{\theta}{1-\theta}\right)\frac{(1-\hat{\tau})}{1-((1-\theta)\,\tau_i+\theta\hat{\tau})} < 0; \tag{E.13}$$

3. $\hat{\lambda}$ is decreasing in τ_{i^*} , and if the MNE's sales in the tax haven are sufficiently small, then

$$\left|\varepsilon_{\tau_{i^*}}^{\hat{\lambda}}\right| > \left|\varepsilon_{\tau_{i^*}}^{\lambda}\right|. \tag{E.14}$$

Proof of Lemma 3. Start with derivation of the optimal λ in the case of profit reallocation. The profit maximization problem of the MNE in this case is

$$\begin{split} \max_{z,\lambda,\{\ell_i\}_{i=1}^{I}} \left(1 - \tau_i \left(1 - \theta\right)\right) \left(p_i \left(A_i \left(N_i z\right)^{\phi} \ell_i^{\gamma}\right) - w_i \ell_i - p_i z \right. \\ \left. + z \left[\varphi \lambda \sum_k \vartheta_k \left(z\right) - \lambda \vartheta_i \left(z\right) + \left(1 - \lambda\right) \sum_{k \neq i} \vartheta_k \left(z\right) - C(\lambda) \sum_k \vartheta_k \left(z\right) \right] \right) \right. \\ \left. + \left(1 - \tau_{i^*} \left(1 - \theta\right)\right) \left(p_{i^*} \left(A_{i^*} \left(N_{i^*} z\right)^{\phi} \ell_{i^*}^{\gamma}\right) - w_k \ell_{i^*} \right. \\ \left. + z \left[\lambda \sum_{k \neq i^*} \vartheta_k \left(z\right) - \left(1 - \lambda\right) \vartheta_{i^*} \left(z\right) - \varphi \lambda \sum_k \vartheta_k \left(z\right) \right] \right) \right. \\ \left. + \sum_{k \neq i, i^*} \left(1 - \tau_k \left(1 - \theta\right)\right) \left(p_k \left(A_k \left(N_k z\right)^{\phi} \ell_k^{\gamma}\right) - w_k \ell_k - \vartheta_k \left(z\right) z\right) \right. \\ \left. - \left(1 - \left(1 - \theta\right)\right) \sum_i \tau_i \cdot \frac{p_i y_i}{\sum_k p_k y_k} \cdot \left[\sum_k \left(p_k \left(A_k \left(N_k z\right)^{\phi} \ell_k^{\gamma}\right) - w_k \ell_k\right) - p_i z - C(\lambda) \sum_k \vartheta_k \left(z\right) \right]. \end{split}$$

We can derive the following from the FOC with respect to λ :

$$\hat{\lambda} = 1 - \exp\left(-\frac{(1-\varphi)\left(1-\theta\right)\left(\tau_i - \tau_{i^*}\right)}{1-\hat{\tau}_i(\theta)}\right).$$
(E.15)

Towards proving 1., pick any $0 < \theta \leq 1$. Using (E.6) we have that the following sequence of inequalities

holds:

$$\begin{split} 1 - \exp\left(-\frac{\left(1-\varphi\right)\left(\tau_{i}-\tau_{i^{*}}\right)}{1-\tau_{i}}\right) > 1 - \exp\left(-\frac{\left(1-\varphi\right)\left(1-\theta\right)\left(\tau_{i}-\tau_{i^{*}}\right)}{1-\hat{\tau}_{i}(\theta)}\right) \\ & \frac{1}{1-\tau_{i}} > \frac{1-\theta}{1-\hat{\tau}_{i}(\theta)} \\ & \sum_{i} \tau_{i} \cdot \frac{p_{i}y_{i}}{\sum_{k} p_{k}y_{k}} < 1. \end{split}$$

The last inequality holds, since $\tau_k < 1 \ \forall k$ and all sales shares are by construction less than one. This proves that $\hat{\lambda} < \lambda$. Now, towards showing 2, inspect how θ affects $\hat{\lambda}$, i.e.

$$\frac{\partial \hat{\lambda}}{\partial \theta} = -\left(1-\varphi\right)\left(\tau_i - \tau_{i^*}\right) \exp\left(-\frac{\left(1-\varphi\right)\left(1-\theta\right)\left(\tau_i - \tau_{i^*}\right)}{1-\hat{\tau}_i(\theta)}\right) \frac{1-\sum_i \tau_i \cdot \sum_k^{p_i y_i}}{\left[1-\hat{\tau}_i(\theta)\right]^2} < 0,$$

and the elasticity is given

$$\varepsilon_{\theta}^{\hat{\lambda}} = -\left(\frac{1-\hat{\lambda}}{\hat{\lambda}}\right) \mathcal{C}'\left(\hat{\lambda}\right) \frac{\theta}{1-\theta} \frac{1-\sum_{i} \tau_{i} \cdot \frac{p_{i}y_{i}}{\sum_{k} p_{k}y_{k}}}{1-\hat{\tau}_{i}(\theta)} < 0,$$

where in the last equality we used the first order condition of the profit function with respect to $\hat{\lambda}$. Hence, we have established 2. Now, inspect how τ_{i^*} affects $\hat{\lambda}$ to prove 3. First, compute the relevant partial derivative

$$\begin{aligned} \frac{\partial \hat{\lambda}}{\partial \tau_{i^*}} &= -\exp\left(-\frac{\left(1-\varphi\right)\left(1-\theta\right)\left(\tau_i-\tau_{i^*}\right)}{1-\hat{\tau}_i(\theta)}\right) \cdot \\ &\left(-1\right)\left(\frac{-\left(1-\varphi\right)\left(1-\theta\right)\left[1-\hat{\tau}_i(\theta)\right] + \left(1-\varphi\right)\left(1-\theta\right)\left(\tau_i-\tau_{i^*}\right)\left[-\frac{p_{i^*}y_{i^*}}{\sum_k p_k y_k}\right]}{\left[1-\hat{\tau}_i(\theta)\right]^2}\right) < 0, \end{aligned}$$

and hence the elasticity

$$\varepsilon_{\tau_{i^*}}^{\hat{\lambda}} = -\left(\frac{1-\hat{\lambda}}{\hat{\lambda}}\right) \frac{(1-\varphi)}{(1-\tau_i)} \tau_{i^*} \left[(1-\tau_i) \left(1-\theta\right) \left(\frac{\left[1-\hat{\tau}_i(\theta)\right] + \left(\tau_i - \tau_{i^*}\right) \left(\frac{p_i * y_{i^*}}{\sum_k p_k y_k}\right)}{\left[1-\hat{\tau}_i(\theta)\right]^2}\right) \right].$$

Suppose that the size of tax-haven is negligible i.e. $p_{i^*}y_{i^*} \approx 0$, then we have

$$\varepsilon_{\tau_{i^*}}^{\hat{\lambda}} = -\left(\frac{1-\hat{\lambda}}{\hat{\lambda}}\right) \frac{(1-\varphi)}{(1-\tau_i)} \tau_{i^*} \left(1-\tau_i\right) \left(1-\theta\right) \left(\frac{1-\hat{\tau}_i(\theta)}{\left[1-\hat{\tau}_i(\theta)\right]^2}\right) = \varepsilon_{\tau_{i^*}}^{\lambda} \cdot \left(\frac{1-\hat{\lambda}}{\hat{\lambda}}\right) \left(\frac{1-\lambda}{\lambda}\right)^{-1} \left(\frac{1-\tau_i\left(1-\theta\right)-\theta}{1-\hat{\tau}_i(\theta)}\right).$$

We want to show that $\left|\varepsilon_{\tau_{i^*}}^{\hat{\lambda}}\right| > \left|\varepsilon_{\tau_{i^*}}^{\lambda}\right|$; it suffices to show that:

$$\left(\frac{1-\hat{\lambda}}{\hat{\lambda}}\right)\left(\frac{1-\lambda}{\lambda}\right)^{-1}\left(\frac{1-\tau_i\left(1-\theta\right)-\theta}{\left[1-\hat{\tau}_i(\theta)\right]}\right) > 1.$$

Plugging equations (E.6) and (E.15) into the inequality we want to show, we have:

$$\left(\frac{\hat{\lambda}}{1-\hat{\lambda}}\right) \cdot \left(\frac{[1-\hat{\tau}_{i}(\theta)]}{1-\theta}\right) < \frac{\lambda}{1-\lambda} \cdot (1-\tau_{i})$$

$$\left[\exp\left(\frac{(1-\varphi)\left(\tau_{i}-\tau_{i^{*}}\right)}{(1-\hat{\tau}_{i}(\theta))/(1-\theta)}\right) - 1\right] \cdot \left(\frac{[1-\hat{\tau}_{i}(\theta)]}{1-\theta}\right) < \left[\exp\left(\frac{(1-\varphi)\left(\tau_{i}-\tau_{i^{*}}\right)}{1-\tau_{i}}\right) - 1\right] \cdot (1-\tau_{i}).$$

We have shown that

$$\frac{\left[1-\hat{\tau}_i(\theta)\right]}{1-\theta} > \left(1-\tau_i\right).$$

Therefore, the inequality holds if $f(x) = \left(\exp\left(\frac{(1-\varphi)(\tau_i-\tau_{i^*})}{x}\right) - 1\right) \cdot x$ is an decreasing function when x > 0, i.e. f'(x) < 0, x > 0. Taking the derivative of f(x), we have

$$f'(x) = \exp\left(\frac{(1-\varphi)(\tau_i - \tau_{i^*})}{x}\right) - \exp\left(\frac{(1-\varphi)(\tau_i - \tau_{i^*})}{x}\right) \cdot \frac{(1-\varphi)(\tau_i - \tau_{i^*})}{x} - 1$$

Now, let $y = \frac{(1-\varphi)(\tau_i - \tau_{i^*})}{x} > 0$, it's straight forward to show that $g(y) = exp(y) - exp(y) \cdot y - 1 < 0$, y > 0, as g(0) = 0 and $g'(y) = -exp(y) \cdot y < 0$, y > 0.

We now move on to prove Proposition 2. We first derive the formulas for allocation of the intangible capital under the profit allocation rule. The following lemma summarizes them.

Lemma 4 The allocations of intangible capital and share of shifted intangible capital under the profit allocation rule are as follows:

$$\begin{aligned} \hat{z}^{FT} &= \left(\frac{\sum_{k} \left(1 - \tau_{k}\right) \Lambda_{k}}{\left[1 - \hat{\tau}_{i}(\theta)\right] p_{i}}\right)^{\frac{1 - \gamma}{1 - \phi - \gamma}}, \\ \hat{z}^{TP} &= \left(\frac{\sum_{k} \Lambda_{k}}{p_{i}}\right)^{\frac{1 - \gamma}{1 - \phi - \gamma}}, \\ \hat{z}^{PS} &= \hat{z}^{TP} \left(1 - \mathcal{C}\left(\hat{\lambda}\right) + \frac{\left(1 - \theta\right) \lambda \left(1 - \varphi\right) \left(\tau_{i} - \tau_{i^{*}}\right)}{1 - \hat{\tau}_{i}(\theta)}\right)^{\frac{1 - \gamma}{1 - \phi - \gamma}} \end{aligned}$$

Proof of Lemma 4. The proof follows the same procedure as Lemma 2. \blacksquare

Proof of Proposition 2. We begin with proving 1. The proof relies on the following sequence of iff inequalities:

$$\begin{aligned} \hat{z}^{PS} &< z^{PS} \\ 1 - \mathcal{C}\left(\hat{\lambda}\right) + \hat{\lambda} \left[\frac{\left(1 - \varphi\right)\left(\tau_{i} - \tau_{i^{*}}\right)}{1 - \tau_{i}}\right] \frac{\left(1 - \tau_{i}\right)\left(1 - \theta\right)}{1 - \hat{\tau}_{i}(\theta)} < 1 - \mathcal{C}\left(\lambda\right) + \frac{\lambda\left(1 - \varphi\right)\left(\tau_{i} - \tau_{i^{*}}\right)}{1 - \tau_{i}} \\ \lambda \frac{\left(1 - \varphi\right)\left(\tau_{i} - \tau_{i^{*}}\right)}{1 - \tau_{i}} - \hat{\lambda} \frac{\left(1 - \varphi\right)\left(\tau_{i} - \tau_{i^{*}}\right)\left(1 - \theta\right)}{1 - \hat{\tau}_{i}(\theta)} > \mathcal{C}\left(\lambda\right) - \mathcal{C}\left(\hat{\lambda}\right), \end{aligned}$$

To simplify notation, let's denote:

$$\begin{split} \hat{A} &= \frac{\left(1-\varphi\right)\left(1-\theta\right)\left(\tau_{i}-\tau_{i^{*}}\right)}{1-\hat{\tau}_{i}(\theta)}, \\ A &= \frac{\left(1-\varphi\right)\left(\tau_{i}-\tau_{i^{*}}\right)}{1-\tau_{i}}. \end{split}$$

Plugging equations (E.6) and (E.15) and assumption 1 to the inequality we want to show, we have

$$\begin{split} 1 - \exp\left(-A\right) + \exp\left(-A\right)\left(-A\right) - 1 + \exp\left(-\hat{A}\right) - \exp\left(-\hat{A}\right)\left(-\hat{A}\right) < \left(1 - \exp\left(-A\right)\right)A - \left(1 - \exp\left(-\hat{A}\right)\right)\hat{A} \\ \hat{A} + \exp\left(-\hat{A}\right) < A + \exp\left(-A\right). \end{split}$$

We have shown that $0 < \hat{A} < A$, $\forall \theta > 0$, thus proving the inequality above amounts to proving that function $f(x) = x + \exp(-x)$ is monotonically increasing when x > 0. Taking its derivative we get:

$$f'(x) = 1 - \exp(-x) > 0, \ x > 0.$$

To prove 2., we start with the partial derivative with respect to θ :

$$\begin{split} \frac{\partial \hat{z}^{PS}}{\partial \theta} &= \left(\frac{1-\gamma}{1-\phi-\gamma}\right) \hat{z}^{PS} \left(1-\mathcal{C}\left(\hat{\lambda}\right) + \frac{\left(1-\theta\right)\hat{\lambda}\left(1-\varphi\right)\left(\tau_{i}-\tau_{i^{*}}\right)}{\left[1-\hat{\tau}_{i}(\theta)\right]}\right)^{-1} \cdot \left\{\frac{\partial \hat{\lambda}}{\partial \theta} \left[\frac{\left(1-\theta\right)\left(1-\varphi\right)\left(\tau_{i}-\tau_{i^{*}}\right)}{\left(1-\hat{\tau}_{i}(\theta)\right)} - \mathcal{C}'\left(\hat{\lambda}\right)\right] + \frac{-\hat{\lambda}\left(1-\varphi\right)\left(\tau_{i}-\tau_{i^{*}}\right)\left(1-\hat{\tau}_{i}(\theta)\right) - \left(\left(\tau_{i}-\frac{\sum_{k}\Lambda_{k}\tau_{k}}{\sum_{k}\Lambda_{k}}\right)\right)\left(1-\theta\right)\hat{\lambda}\left(1-\varphi\right)\left(\tau_{i}-\tau_{i^{*}}\right)}{\left[\left(1-\hat{\tau}_{i}(\theta)\right)\right]^{2}}\right\}, \end{split}$$

and notice that the FOC w.r.t. $\hat{\lambda}$ is given by

$$\mathcal{C}'\left(\hat{\lambda}\right) = (1-\varphi) \, \frac{(1-\theta) \, (\tau_i - \tau_{i^*})}{1 - \hat{\tau}_i(\theta)}.$$

Thus to evaluate the sign, we need to sign the following

$$\frac{-\hat{\lambda}\left(1-\varphi\right)\left(\tau_{i}-\tau_{i^{*}}\right)\left(1-\hat{\tau}_{i}(\theta)\right)-\left(\left(\tau_{i}-\frac{\sum_{k}\Lambda_{k}\tau_{k}}{\sum_{k}\Lambda_{k}}\right)\right)\left(1-\theta\right)\hat{\lambda}\left(1-\varphi\right)\left(\tau_{i}-\tau_{i^{*}}\right)}{\left[\left(1-\hat{\tau}_{i}(\theta)\right)\right]^{2}} = \frac{\hat{\lambda}\left(1-\varphi\right)\left(\tau_{i}-\tau_{i^{*}}\right)\left[\frac{\sum_{k}\Lambda_{k}\tau_{k}}{\sum_{k}\Lambda_{k}}-1\right]}{\left[\left(1-\hat{\tau}_{i}(\theta)\right)\right]^{2}} < 0,$$

thus we have established that

$$\frac{\partial \widehat{z}^{PS}}{\partial \theta} < 0.$$

And the elasticity is

$$\varepsilon_{\theta}^{\hat{z}^{PS}} = \varepsilon_{\theta}^{\hat{\lambda}} \left(\frac{1 - \gamma}{1 - \phi - \gamma} \right) \left(\frac{\hat{\lambda}}{\mathcal{C}(\hat{\lambda})(1 - \hat{\lambda})} \right) \left(\frac{1}{1 + \frac{1 - \mathcal{C}(\hat{\lambda})}{\hat{\lambda}\mathcal{C}'(\hat{\lambda})}} \right) < 0.$$

Now, to show 3. consider the partial derivative of \hat{z}^{PS} with respect to τ_{i^*} ,

$$\begin{aligned} \frac{\partial \hat{z}^{PS}}{\partial \tau_{i^*}} &= \left(\frac{1-\gamma}{1-\phi-\gamma}\right) \hat{z}^{PS} \left(1-\mathcal{C}\left(\hat{\lambda}\right) + \frac{\left(1-\theta\right)\hat{\lambda}\left(1-\varphi\right)\left(\tau_i-\tau_{i^*}\right)}{\left[1-\hat{\tau}_i(\theta)\right]}\right)^{-1} \cdot \\ &\left[\frac{\partial \hat{\lambda}}{\partial \tau_{i^*}} \left(\frac{\left(1-\varphi\right)\left(\tau_i-\tau_{i^*}\right)\left(1-\theta\right)}{1-\left(\left(1-\theta\right)\tau_i+\theta\sum_k\tau_k\frac{\Lambda_k}{\sum_k\Lambda_k}\right)} - \mathcal{C}'\left(\hat{\lambda}\right)\right) + \\ &\left(\frac{\theta\frac{\Lambda_{i^*}}{\sum_k\Lambda_k}\left(1-\theta\right)\hat{\lambda}\left(1-\varphi\right)\left(\tau_i-\tau_{i^*}\right) - \left(1-\theta\right)\hat{\lambda}\left(1-\varphi\right)\left[1-\hat{\tau}_i(\theta)\right]}{\left[1-\hat{\tau}_i(\theta)\right]^2}\right)\right] \end{aligned}$$

Notice that the FOC wrt to $\hat{\lambda}$ implies

$$\mathcal{C}'\left(\hat{\lambda}\right) = (1-\varphi) \, \frac{(1-\theta) \, (\tau_i - \tau_{i^*})}{1 - \hat{\tau}_i(\theta)},$$

thus the elasticity becomes

$$\varepsilon_{\tau_{i^{*}}}^{\hat{z}^{PS}} = \left(\frac{-\tau_{i^{*}}}{\tau_{i} - \tau_{i^{*}}}\right) \left(\frac{1 - \gamma}{1 - \phi - \gamma}\right) \left(\frac{1}{\left[1 + \frac{1 - \mathcal{C}(\hat{\lambda})}{\hat{\lambda}\mathcal{C}'(\hat{\lambda})}\right]}\right) \left(\frac{1 - \tau_{i} - \theta \left[\frac{\Lambda_{i^{*}}}{\sum_{k}\Lambda_{k}} \left(\tau_{i} - \tau_{i^{*}}\right) + \sum_{k}\tau_{k}\frac{\Lambda_{k}}{\sum_{k}\Lambda_{k}} - \tau_{i}\right]}{(1 - \hat{\tau}_{i}(\theta))}\right)$$

Compare it to the elasticity of z^{PS}

$$\varepsilon_{\tau_{i^*}}^{z^{PS}} = \left(\frac{1-\gamma}{1-\phi-\gamma}\right) \left(\frac{-\tau_{i^*}}{\tau_i-\tau_{i^*}}\right) \frac{1}{\left[1+\frac{1-\mathcal{C}(\lambda)}{\lambda \mathcal{C}'(\lambda)}\right]},$$

and note that

$$\lim_{\theta \to 0} \varepsilon_{\tau_{i^*}}^{\hat{z}^{PS}} = \varepsilon_{\tau_{i^*}}^{z^{PS}}.$$

To show this, we have

$$\left(\frac{1-\tau_i-\theta\left[\frac{\Lambda_{i^*}}{\sum_k\Lambda_k}\left(\tau_i-\tau_{i^*}\right)+\sum_k\tau_k\frac{\Lambda_k}{\sum_k\Lambda_k}-\tau_i\right]}{(1-\hat{\tau}_i(\theta))}\right)=\left(\frac{\left(1-\hat{\tau}_i(\theta)\right)+\theta\left[\frac{\Lambda_{i^*}}{\sum_k\Lambda_k}\left(\tau_i-\tau_{i^*}\right)\right]}{(1-\hat{\tau}_i(\theta))}\right),$$

and under the assumption that sales to tax haven are negligible we have

$$\lim_{\Lambda_{i^*} \to 0} \left(\frac{(1 - \hat{\tau}_i(\theta)) + \theta \left[\frac{\Lambda_{i^*}}{\sum_k \Lambda_k} \left(\tau_i - \tau_{i^*} \right) \right]}{(1 - \hat{\tau}_i(\theta))} \right) = 1.$$

Finally, we want to prove $\left|\varepsilon_{\tau_{i^*}}^{\hat{z}^{PS}}\right| < \left|\varepsilon_{\tau_{i^*}}^{z^{PS}}\right|$. It suffices to show that

$$\frac{1}{\left(1+\frac{1-\mathcal{C}(\hat{\lambda})}{\hat{\lambda}\mathcal{C}'(\hat{\lambda})}\right)} < \frac{1}{\left(1+\frac{1-\mathcal{C}(\lambda)}{\lambda\mathcal{C}'(\lambda)}\right)}.$$

This is easy to prove given that $C'(\lambda) > 0$ and $C''(\lambda) < 0$.

E.3 Analytical Results when MNEs Take $\vartheta'_k(z)$ into Account

Here, we assume that MNEs internalize the effect of changing z on the licensing fee $\vartheta_k(z)$ and solve for optimal z under different scenarios (*FT*, *TP*, and *PS*). We then prove Proposition 2 under this assumption. As before, we start from the optimal z.

Lemma 5 The allocations of intangible capital are as follows:

$$\hat{z}^{FT} = \left(\frac{\sum_{k} \left(1 - \tau_k\right) \Lambda_k}{\left(1 - \tau_i\right) p_i}\right)^{\frac{1 - \gamma}{1 - \phi - \gamma}},\tag{E.16}$$

$$\hat{z}^{TP} = \left(\frac{\sum_{k} \left(1 - \hat{\tau}_{k}\left(\theta\right)\right) \Lambda_{k} - \frac{\phi}{1 - \gamma} \sum_{k} \left(1 - \theta\right) \left(\tau_{i} - \tau_{k}\right) \Lambda_{k}}{\left(1 - \hat{\tau}_{i}\left(\theta\right)\right) p_{i}}\right)^{\frac{1 - \gamma}{1 - \phi - \gamma}},\tag{E.17}$$

$$\hat{z}^{PS} = \left(\frac{-\frac{\phi}{1-\gamma}\mathcal{C}\left(\hat{\lambda}\right)\sum_{k}\Lambda_{k}}{p_{i}} + \frac{\sum_{k}\left(1-\hat{\tau}_{k}\left(\theta\right)\right)\Lambda_{k}-\frac{\phi}{1-\gamma}\sum_{k}\left(1-\theta\right)\left(\tau_{i}-\tau_{k}\right)\Lambda_{k}+\hat{\lambda}\frac{\phi}{1-\gamma}\left(1-\theta\right)\left(\tau_{i}-\tau_{i^{*}}\right)\left(1-\varphi\right)\sum_{k}\Lambda_{k}}{\left(1-\hat{\tau}_{i}\left(\theta\right)\right)p_{i}}\right)^{\frac{1-\gamma}{1-\phi-\gamma}}.$$
(E.18)

Proof of Lemma 5. The proof follows the same procedure as the one of Lemma 2

Proof of Proposition 2 under alternative assumption. We start from proving 1 from deriving a set of iff inequalities:

$$\begin{split} & \frac{\sum_{k}\Lambda_{k}}{p_{i}} - \frac{\phi + \gamma - 1}{1 - \gamma} \frac{\sum_{k}\left(1 - \theta\right)\left(\tau_{i} - \tau_{k}\right)\Lambda_{k}}{\left(1 - \hat{\tau}_{i}\left(\theta\right)\right)p_{i}} - \left(1 + \frac{\phi + \gamma - 1}{1 - \gamma}\right) \frac{\sum_{k}\Lambda_{k}}{p_{i}} \left[\mathcal{C}\left(\hat{\lambda}\right) - \frac{\hat{\lambda}\left(1 - \theta\right)\left(\tau_{i} - \tau_{i^{*}}\right)\left(1 - \varphi\right)}{\left(1 - \hat{\tau}_{i}\left(\theta\right)\right)}\right] \\ & < \frac{\sum_{k}\Lambda_{k}}{p_{i}} - \frac{\phi + \gamma - 1}{1 - \gamma} \frac{\sum_{k}\left(\tau_{i} - \tau_{k}\right)\Lambda_{k}}{\left(1 - \tau_{i}\right)p_{i}} - \left(1 + \frac{\phi + \gamma - 1}{1 - \gamma}\right) \frac{\sum_{k}\Lambda_{k}}{p_{i}} \left[\mathcal{C}\left(\lambda\right) - \frac{\lambda\left(\tau_{i} - \tau_{i^{*}}\right)\left(1 - \varphi\right)}{\left(1 - \tau_{i}\right)p_{i}}\right] \\ & - \frac{1 - \phi - \gamma}{1 - \gamma} \frac{\sum_{k}\left(\tau_{i} - \tau_{k}\right)\Lambda_{k}}{\left(1 - \tau_{i}\right)p_{i}} + \frac{\phi}{1 - \gamma} \frac{\sum_{k}\Lambda_{k}}{p_{i}} \left[\mathcal{C}\left(\lambda\right) - \frac{\lambda\left(\tau_{i} - \tau_{i^{*}}\right)\left(1 - \varphi\right)}{\left(1 - \tau_{i}\right)}\right] \\ & < - \frac{1 - \phi - \gamma}{1 - \gamma} \frac{\sum_{k}\left(1 - \theta\right)\left(\tau_{i} - \tau_{k}\right)\Lambda_{k}}{\left(1 - \hat{\tau}_{i}\left(\theta\right)\right)p_{i}} + \frac{\phi}{1 - \gamma} \frac{\sum_{k}\Lambda_{k}}{p_{i}} \left[\mathcal{C}\left(\hat{\lambda}\right) - \frac{\hat{\lambda}\left(1 - \theta\right)\left(\tau_{i} - \tau_{i^{*}}\right)\left(1 - \varphi\right)}{\left(1 - \hat{\tau}_{i}\left(\theta\right)\right)}\right]. \end{split}$$

We have proven before that $\mathcal{C}(\lambda) - \frac{\lambda(\tau_i - \tau_i^*)(1 - \varphi)}{(1 - \tau_i)} < \mathcal{C}(\hat{\lambda}) - \frac{\hat{\lambda}(1 - \theta)(\tau_i - \tau_i^*)(1 - \varphi)}{(1 - \hat{\tau}_i(\theta))}$. It suffices to prove that

$$-\frac{1-\phi-\gamma}{1-\gamma}\frac{\sum_{k}\left(\tau_{i}-\tau_{k}\right)\Lambda_{k}}{\left(1-\tau_{i}\right)p_{i}} < -\frac{1-\phi-\gamma}{1-\gamma}\frac{\sum_{k}\left(1-\theta\right)\left(\tau_{i}-\tau_{k}\right)\Lambda_{k}}{\left(1-\hat{\tau}_{i}\left(\theta\right)\right)p_{i}},$$

which simplifies to $1 > \sum_k \tau_k \frac{\Lambda_k}{\sum_i \Lambda_i}$. Thus, we have proven 1. We now prove 2:

$$\frac{d\hat{z}^{PS}}{d\theta} = \frac{1-\gamma}{1-\phi-\gamma} (\hat{z}^{PS})^{\frac{\phi}{1-\gamma}} \frac{1}{p_i} \left(\frac{\sum_k \tau_k \frac{\Lambda_k}{\sum_i \Lambda_i} - 1}{\left(1 - \hat{\tau}_i\left(\theta\right)\right)^2} \right) \left(\frac{\phi}{1-\gamma} \lambda \left(\tau_i - \tau_{i^*}\right) \left(1 - \varphi\right) + \frac{1-\phi-\gamma}{1-\gamma} \sum_k \left(\tau_i - \tau_k\right) \Lambda_k \right).$$

We have shown that

$$\sum_k \tau_k \frac{\Lambda_k}{\sum_i \Lambda_i} - 1 < 0$$

The other terms are all positive, thus we have proven 2. Now to prove 3 we can show that

$$\begin{split} \frac{\partial \hat{z}^{PS}}{\partial \tau_{i^*}} &= \left(\frac{1-\gamma}{1-\phi-\gamma}\right) \left(\hat{z}^{PS}\right)^{\frac{\phi}{1-\gamma}} \frac{1}{p_i} \Biggl\{ -\frac{\phi}{1-\gamma} \mathcal{C}'\left(\hat{\lambda}\right) \frac{\partial \hat{\lambda}}{\partial \tau_{i^*}} \sum_k \Lambda_k + \frac{\phi}{1-\gamma} \frac{\sum_k \Lambda_k}{\left(1-\hat{\tau}_i\left(\theta\right)\right)^2} \cdot \\ & \left[\left(\frac{\partial \hat{\lambda}}{\partial \tau_{i^*}} \left(1-\varphi\right) \left(\tau_i - \tau_{i^*}\right) \left(1-\theta\right) \sum_k \Lambda_k - \left(1-\theta\right) \hat{\lambda} \left(1-\varphi\right) \right) \left(1-\hat{\tau}_i\left(\theta\right)\right) + \left(\theta \frac{\Lambda_{i^*}}{\sum_k \Lambda_k}\right) \left(1-\theta\right) \hat{\lambda} \left(1-\varphi\right) \left(\tau_i - \tau_{i^*}\right) \Biggr] - \frac{1-\phi-\gamma}{1-\gamma} \left(1-\theta\right) \Lambda_{i^*} \frac{1-\tau_i}{\left(1-\hat{\tau}_i\left(\theta\right)\right)^2} \Biggr\}. \end{split}$$

We have shown in previous proof that the sum of first two terms in the big bracket is negative. It's clear that the last term is also negative. Hence we have proven 3, that is $\frac{\partial \hat{z}^{PS}}{\partial \tau_{i^*}} < 0$.

F Quantitative Model Derivations

F.1 Firm's problem with no transfer pricing or profit shifting

F.1.1 Scale choice: the parent division

We start from the parent division of a firm $\omega \in \Omega_i$'s scale choice here. A parent division that produces for the domestic market and exports to a set of J_X regions chooses its scale and how to allocate its output across its markets. Note that this problem nests the problem for firms only producing for the domestic markets when $J_X = \emptyset$. The parent division's problem can then be written as

$$\pi_i^D(a, z; J_X) = \max_{q_{ii}, (q_{ij})_{j \in J_X}, \ell} \left\{ p_{ii}(q_{ii})q_{ii} + \sum_{j \in J_X} p_{ij}(q_{ij}^X)q_{ij}^X - W_i \ell \right\},$$

s.t $q_{ii} + \sum_{j \in J_X} \xi_{ij}q_{ij}^X = y_i = A_i a(N_i z)^{\gamma} \ell^{\phi}.$

The solution is

$$q_{ij} = \left[\frac{\phi(\varrho-1)}{\varrho}\right]^{\varrho} \left[\frac{P_j Q_j^{\frac{1}{\varrho}} A_i a(N_i z)^{\gamma} \ell^{\phi-1}}{\xi_{ij} W_i}\right]^{\varrho} = \left[\frac{P_j Q_j^{\frac{1}{\varrho}}}{\xi_{ij}}\right]^{\varrho} \left[\frac{\phi(\varrho-1)}{\varrho}\right]^{\varrho} \left[\frac{A_i a(N_i z)^{\gamma} \ell^{\phi-1}}{W_i}\right]^{\varrho}.$$

Plugging this back into the resource constraint, can solve for labor as

$$\ell = \left\{ \left[P_i^{\varrho} Q_i + \sum_{j \in J_X} P_j^{\varrho} \tau_j^{1-\varrho} Q_j \right] \left[\frac{\phi(\varrho-1)}{\varrho} \right]^{\varrho} W_i^{-\varrho} (A_i a)^{\varrho-1} (N_i z)^{\gamma(\varrho-1)} \right\}^{\frac{1}{\phi+\varrho-\varrho\phi}}.$$
 (F.1)

F.1.2 Scale choice: foreign subsidiaries

Foreign subsidiaries are similar to domestic-only firms. They just choose scale to maximize profits from selling to the host market given the demand curve and production technology. The only difference is the presence of the FDI barrier σ_{ij} . The foreign subsidiary's problem is

$$\pi_{ij}^{F}(a,z) = \max_{q,\ell} p_{ij}(q)q - W_{i}\ell$$
$$= \max_{\ell} P_{j}Q_{j}^{\frac{1}{\varrho}} \left(\sigma_{ij}A_{j}a\right)^{\frac{\varrho-1}{\varrho}} \left(N_{j}z\right)^{\gamma\frac{\varrho-1}{\varrho}} \ell^{\phi\frac{\varrho-1}{\varrho}} - W_{j}\ell$$

The optimal ℓ , which yields all the other quantities immediately, is

$$\ell = \left\{ \left[\frac{\phi(\varrho - 1)}{\varrho} \right]^{\varrho} (P_j / W_j)^{\varrho} Q_j \left(\sigma_{ij} A_j a \right)^{\varrho - 1} (N_j z)^{\gamma(\varrho - 1)} \right\}^{\frac{1}{\varphi + \varrho - \phi_{\varrho}}}.$$
 (F.2)

F.1.3 Technology choice

Now that we have $\pi_i^D(a, z; J_X)$ of the parent division and $\pi_{ij}^F(a, z)$ of foreign affiliates, $j \in J_F$, we can determine how much R&D to do taking J_F and J_X as given. Note that we can ignore the fixed costs of exporting and FDI for now:

$$\hat{d}_i(a; J_X, J_F) = \max_z \left\{ (1 - \tau_i) \left[\pi_i^D(a, z; J_X) - W_i z / A_i \right] + \sum_{j \in J_F} (1 - \tau_j) \pi_{ij}^F(a, z) \right\}.$$

Using the first order conditions of this problem, we can express domestic parent revenues as

$$p_{ii}q_{ii} + \sum_{j \in J_X} p_{ij}q_{ij} = P_i Q_i^{\frac{1}{\varrho}} q_{ii}^{\frac{\varrho-1}{\varrho}} + \sum_{j \in J_X} P_j Q_j^{\frac{1}{\varrho}} q_{ij}^{\frac{\varrho-1}{\varrho}}$$

$$= \left[P_i Q_i^{\frac{1}{\varrho}} \bar{Q}_{ii}^{\frac{\varrho-1}{\varrho}} + \sum_{j \in J_X} P_j Q_j^{\frac{1}{\varrho}} \bar{Q}_{ij}^{\frac{\varrho-1}{\varrho}} \right]$$

$$\times \left\{ \left[P_i^{\varrho} Q_i + \sum_{j \in J_X} P_j^{\varrho} \tau_j^{1-\varrho} Q_j \right] \left[\frac{\phi(\varrho-1)}{\varrho} \right]^{\varrho} W^{-\varrho} \right\}^{\frac{\varrho-1}{\varrho} \frac{\phi}{\phi+\varrho-\varrho\phi}}$$

$$\times (A_i a)^{\frac{\varrho-1}{\phi+\varrho-\phi\varrho}} N_i^{\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}} z^{\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}}$$

$$= \bar{R}_{ii} z^{\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}}.$$
(F.3)

Similarly, domestic parent costs are

$$\begin{split} W_i\ell + W_i z/A_i &= W_i \left\{ \left[P_i^{\varrho} Q_i + \sum_{j \in J_X} P_j^{\varrho} \tau_j^{1-\varrho} Q_j \right] \left[\frac{\phi(\varrho-1)}{\varrho} \right]^{\varrho} W^{-\varrho} (A_i a)^{\varrho-1} (N_i z)^{\gamma(\varrho-1)} \right\}^{\frac{1}{\varphi+\varrho-\varrho\phi}} + W_i z/A_i \\ &= \bar{C}_{ii} z^{\frac{\gamma(\varrho-1)}{\bar{\varphi}+\varrho-\varrho\phi}} + W_i z/A_i. \end{split}$$

Foreign affiliate revenues are

$$p_{ij}q_{ij} = P_j Q_j^{\frac{1}{\varrho}} q_{ij}^{\frac{\varrho-1}{\varrho}}$$

$$= \left[P_j Q_j^{\frac{1}{\varrho}} \right] \left[(P_j/W_j) \frac{\phi(\varrho-1)}{\varrho} \right]^{\frac{\phi(\varrho-1)}{\phi+\varrho-\phi_\varrho}} Q_j^{\frac{\varrho-1}{\varphi+\varrho-\phi_\varrho}} (A_j \sigma_{ij} a)^{\frac{\varrho-1}{\phi+\varrho-\phi_\varrho}} N_j^{\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi_\varrho}} z^{\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi_\varrho}}$$

$$= \bar{R}_{ij} z^{\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi_\varrho}}.$$
(F.4)

Foreign affiliate costs are

$$W_{j}\ell = W_{j}\left\{\left[\frac{\phi(\varrho-1)}{\varrho}\right]^{\varrho} (P_{j}/W_{j})^{\varrho}Q_{j} (A_{j}\sigma_{ij}a)^{\varrho-1} (N_{j}z)^{\gamma(\varrho-1)}\right\}^{\frac{1}{\phi+\varrho-\phi\varrho}} = \bar{C}_{ij}z^{\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}}.$$

Tus total net revenues are

$$(1-\tau_i)p_{ii}q_{ii} + \sum_{j\in J_X} (1-\tau_j)p_{ij}q_{ij} + \sum_{j\in J_F} p_{ij}q_{ij} = (1-\tau_i)\bar{R}_{ii}z^{\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}} + \sum_{j\in J_F} (1-\tau_j)\bar{R}_{ij}z^{\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}}$$

And total costs are

$$(1-\tau_i)(W_i\ell_{ii}+W_iz/A_i) + \sum_{j\in J_F}(1-\tau_j)W_j\ell_{ij} = \left[(1-\tau_i)\bar{C}_{ii} + \sum_{j\in J_F}(1-\tau_j)C_{ij}\right]z^{\frac{\gamma(\varrho-1)}{\varphi+\varrho-\varrho\phi}} + (1-\tau_i)W_iz/A_i.$$

We can write the objective function as

$$\left[(1-\tau_i)(\bar{R}_{ii}-\bar{C}_{ii}) + \sum_{j\in J_F} (1-\tau_j) \left(\bar{R}_{ij}-\bar{C}_{ij}\right) \right] z^{\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}} - (1-\tau_i) W_i z / A_i.$$

Then the optimal choice of z is

$$z = \left\{ \left(\frac{\phi + \varrho - \phi\varrho}{\gamma(\varrho - 1)}\right) \left[\frac{(1 - \tau_i)W_i/A_i}{(1 - \tau_i)\left(\bar{R}_{ii} - \bar{C}_{ii}\right) + \sum_{j \in J_F}(1 - \tau_j)\left(\bar{R}_{ij} - \bar{C}_{ij}\right)}\right] \right\}^{\frac{\phi + \varrho - \phi\varrho}{\gamma\varrho + \phi\varrho - \gamma - \phi - \varrho}}$$

F.1.4 Market choice

Now that we have $\hat{d}_i(a; J_X, J_F), \forall J_X, J_F$, we can decide where to export and where to operate foreign subsidiaries:

$$d_i(a) = \max_{J_X, J_F} \left\{ \hat{d}_i(a; J_X, J_F) - W_i\left(\sum_{j \in J_X} \kappa_{jX} - \sum_{j \in J_F} \kappa_{jF}\right) \right\}.$$
 (F.5)

•

This is a combinatorial discrete choice problem as a firm's exporting and FDI choices are interdependent across regions. This problem is hard to solve since the number of potential decision sets grows exponentially in the number of regions. We limit the number of regions in the quantitative model to ease the computational burden.

F.2 Firm's problem with transfer pricing

Here, we solve the optimal nonrival technology allocation z in the environment with transfer pricing, taking J_X and J_F as given. We define total profit earned by a firm in this scenario as *transfer-pricing profit*, d_i^{TP} :

$$d_i^{TP}(a) = \max_{J_X, J_F} \left\{ \hat{d}_i^{TP}(a; J_X, J_F) - W_i \left(\sum_{j \in J_X} \kappa_{jX} - \sum_{j \in J_F} \kappa_{jF} \right) \right\},\tag{F.6}$$

where

$$\hat{d}_{i}^{TP}(a; J_{X}, J_{F}) = \max_{z} \left\{ (1 - \tau_{i}) \left[\pi_{i}^{D}(a, z; J_{X}) + (\sum_{j \in J_{F}} \vartheta_{j}(z) - W_{i}/A_{i})z \right] + \sum_{j \in J_{F}} (1 - \tau_{j})(\pi_{ij}^{F}(a, z) - \vartheta_{j}(z) \cdot z) \right\}.$$
(F.7)

Taking J_X and J_F as given, each firm chooses z to maximize $\hat{d}_i^{TP}(a; J_X, J_F)$. We can write the objective function as

$$\begin{split} \max_{z} & \left\{ (1-\tau_{i}) \left[\left(\bar{R}_{ii} - \bar{C}_{ii} \right) + \sum_{j \in J_{F}} \left(\frac{\gamma(\varrho - 1)}{\phi + \varrho - \phi \varrho} \right) \left(\bar{R}_{ij} - \bar{C}_{ij} \right) \right] \right. \\ & \left. + \sum_{j \in J_{F}} (1-\tau_{j}) \left(1 - \frac{\gamma(\varrho - 1)}{\phi + \varrho - \phi \varrho} \right) \left(\bar{R}_{ij} - \bar{C}_{ij} \right) \right\} z^{\frac{\gamma(\varrho - 1)}{\phi + \varrho - \phi \varrho}} - (1-\tau_{i}) W_{i} z / A_{i}. \end{split}$$

Then the optimal choice of z is

$$z = \left\{ \left(\frac{\phi + \varrho - \phi_{\varrho}}{\gamma(\varrho - 1)} \right) \left[\frac{(1 - \tau_i)W_i / A_i}{DENOM^{TP}} \right] \right\}^{\frac{\phi + \varrho - \phi_{\varrho}}{\gamma_{\varrho} + \phi_{\varrho} - \gamma - \phi - \varrho}}$$

where

$$DENOM^{TP} = (1 - \tau_i) \left[\left(\bar{R}_{ii} - \bar{C}_{ii} \right) + \sum_{j \in J_F} \left(\frac{\gamma(\varrho - 1)}{\phi + \varrho - \phi \varrho} \right) \left(\bar{R}_{ij} - \bar{C}_{ij} \right) \right] \\ + \sum_{j \in J_F} (1 - \tau_j) \left(1 - \frac{\gamma(\varrho - 1)}{\phi + \varrho - \phi \varrho} \right) \left(\bar{R}_{ij} - \bar{C}_{ij} \right).$$

F.3 Firm's problem with transfer pricing and profit shifting

Here, we solve the optimal nonrival technology allocation z and profit shifting shares λ_{TH} and λ_{LT} in the environment with transfer pricing and profit shifting, taking J_X and J_F as given. This problem nests the one with only transfer pricing and no profit shifting simply by setting λ and $C(\lambda)$ to zero for both LT and TH. Here we solve for the full problem where λ_{LT} and $\lambda_{TH} > 0$. It is easier to rewrite $\hat{d}^{PS}(a)$ as:

$$d_i^{PS}(a) = \max_{J_X, J_F} \left\{ \hat{d}_i^{PS}(a; J_X, J_F) - W_i \left(\sum_{j \in J_X} \kappa_{jX} - \sum_{j \in J_F} \kappa_{jF} - \kappa_{iTH} \mathbb{1}(\lambda_{TH} > 0) \right) \right\},$$

where

$$\begin{split} \hat{d}_{i}^{PS} &= \max_{z,\lambda_{LT},\lambda_{TH}} \left\{ (1-\tau_{i}) \Big[\pi_{i}^{D}(a,z;J_{X}) + \Big(-(\lambda_{LT}+\lambda_{TH})\vartheta_{i}(z) + (1-\lambda_{LT}-\lambda_{TH}) \sum_{j\in J_{F}} \vartheta_{j}(z) \\ &- W_{i}/A_{i} - W_{i} \big(\mathcal{C}_{i,LT}(\lambda_{LT}) + \mathcal{C}_{i,TH}(\lambda_{TH}) \big) \nu_{i}(z) \Big) z \Big] \\ &+ (1-\tau_{LT}) \Big[\pi_{i,LT}^{F}(a,z) + \Big(\lambda_{LT} \sum_{j\in J_{F}\cup\{i\}\setminus\{LT\}} \vartheta_{j}(z) - (1-\lambda_{LT})\vartheta_{iLT}(z) \Big) z \Big] \\ &+ (1-\tau_{TH}) \Big[\Big(\lambda_{TH} \sum_{j\in J_{F}\cup\{i\}} \vartheta_{j}(z) \Big) z \Big] \\ &+ \sum_{j\in J_{F}\setminus\{LT\}} (1-\tau_{j}) \left[\pi_{ij}^{F}(a,z) - \vartheta_{j}(z) z \right] \Big\}. \end{split}$$

Substituting in the optimal scale choices specified in equation (F.1) and (F.2) and letting $\lambda = \lambda_{TH} + \lambda_{LT}$, we can write \hat{d}_i^{PS} as

$$\begin{split} \max_{z,\lambda_{TH},\lambda_{LT}} & \left\{ \left(1-\tau_{i}\right) \left[\left(1-\lambda \left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}\right)\right) (\bar{R}_{ii}-\bar{C}_{ii}) + (1-\lambda) \sum_{j\in J_{F}} \left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}\right) (\bar{R}_{ij}-\bar{C}_{ij}) \right] \right. \\ & \left. - \left(1-\tau_{i}\right) \left[W_{i}\left(\mathcal{C}_{i,LT}(\lambda_{LT}) + \mathcal{C}_{i,TH}(\lambda_{TH})\right) \sum_{j\in J_{F}\cup\{i\}} \left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}\right) (\bar{R}_{ij}-\bar{C}_{ij}) \right] \right. \\ & \left. + \left(1-\tau_{LT}\right) \left[\left(1-\left(1-\lambda_{LT}\right) \left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}\right)\right) (\bar{R}_{i,LT}-\bar{C}_{i,LT}) + \right. \\ & \left. \lambda_{LT} \sum_{j\in J_{F}\cup\{i\}\setminus\{LT\}} \left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}\right) (\bar{R}_{ij}-\bar{C}_{ij}) \right] \right. \\ & \left. + \left(1-\tau_{TH}\right) \left[\lambda_{TH} \sum_{j\in J_{F}\cup\{i\}} \left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}\right) (\bar{R}_{ij}-\bar{C}_{ij}) \right] \right] \\ & \left. + \sum_{j\in J_{F}\setminus\{LT\}} \left(1-\tau_{j}\right) \left[\left(1-\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}\right) (\bar{R}_{ij}-\bar{C}_{ij}) \right] \right\} z^{\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}} - \left(1-\tau_{i}\right) W_{i}z/A_{i}. \end{split}$$

And further simplifying

$$\begin{split} \max_{z,\lambda_{TH},\lambda_{LT}} \bigg\{ \sum_{j \in J_F \cup \{i\}} (1-\tau_j) (\bar{R}_{ij} - \bar{C}_{ij}) - \sum_{j \in J_F \cup \{i\}} (\tau_i - \tau_j) \left(\frac{\gamma(\varrho - 1)}{\phi + \varrho - \phi \varrho} \right) (\bar{R}_{ij} - \bar{C}_{ij}) \\ &+ (\tau_i - \tau_{LT}) \lambda_{LT} \sum_{j \in J_F \cup \{i\}} \left(\frac{\gamma(\varrho - 1)}{\phi + \varrho - \phi \varrho} \right) (\bar{R}_{ij} - \bar{C}_{ij}) \\ &+ (\tau_i - \tau_{TH}) \lambda_{TH} \sum_{j \in J_F \cup \{i\}} \left(\frac{\gamma(\varrho - 1)}{\phi + \varrho - \phi \varrho} \right) (\bar{R}_{ij} - \bar{C}_{ij}) \\ &- (1 - \tau_i) W_i \left(\mathcal{C}_{i,LT}(\lambda_{LT}) + \mathcal{C}_{i,TH}(\lambda_{TH}) \right) \sum_{j \in J_F \cup \{i\}} \left(\frac{\gamma(\varrho - 1)}{\phi + \varrho - \phi \varrho} \right) (\bar{R}_{ij} - \bar{C}_{ij}) \bigg\} z^{\frac{\gamma(\varrho - 1)}{\phi + \varrho - \phi \varrho}} - (1 - \tau_i) W_i z/A_i. \end{split}$$

Recall that the λ values can be solved independent of z:

$$\lambda_{LT} = \left(\mathcal{C}_{i,LT}'\right)^{-1} \left[\frac{1}{W_i} \frac{\left(\tau_i - \tau_{LT}\right)}{1 - \tau_i}\right],$$
$$\lambda_{TH} = \left(\mathcal{C}_{i,TH}'\right)^{-1} \left[\frac{1}{W_i} \frac{\left(\tau_i - \tau_{LT}\right)}{1 - \tau_i}\right].$$

The FOC for z is

$$\begin{split} (1-\tau_i)W_i/A_i &= \left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}\right) z^{\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}-1} \bigg\{ \sum_{j\in J_F\cup\{i\}} (1-\tau_j)(\bar{R}_{ij}-\bar{C}_{ij}) \\ &- \sum_{j\in J_F\cup\{i\}} (\tau_i-\tau_j)\left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}\right)(\bar{R}_{ij}-\bar{C}_{ij}) \\ &+ (\tau_i-\tau_{LT})\lambda_{LT}\sum_{j\in J_F\cup\{i\}} \left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}\right)(\bar{R}_{ij}-\bar{C}_{ij}) \\ &+ (\tau_i-\tau_{TH})\lambda_{TH}\sum_{j\in J_F\cup\{i\}} \left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}\right)(\bar{R}_{ij}-\bar{C}_{ij}) \\ &- (1-\tau_i)W_i\left(\mathcal{C}_{i,LT}(\lambda_{LT})+\mathcal{C}_{i,TH}(\lambda_{TH})\right)\sum_{j\in J_F\cup\{i\}} \left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}\right)(\bar{R}_{ij}-\bar{C}_{ij})\bigg\}. \end{split}$$

We can solve the optimal z as:

$$z = \left\{ \left(\frac{\phi + \varrho - \phi \varrho}{\gamma(\varrho - 1)} \right) \left[\frac{(1 - \tau_i) W_i / A_i}{DENOM^{PS}} \right] \right\}^{\frac{\phi + \varrho - \phi \varrho}{\gamma \varrho + \phi \varrho - \gamma - \phi - \varrho}},$$

where $DENOM^{PS}$ is the stuff inside the big brackets above.

F.4 Firm's problem under sales-based profit allocation

As before, we solve for the full problem where $\lambda_{LT} > 0$ and $\lambda_{TH} > 0$. It's easier to state the firm's problem as:

$$d_i^{PR}(a) = \max_{z, J_X, J_F, \lambda_{TL}, \lambda_{TH}} \left\{ \hat{d}_i^{PR}(a; J_X, J_F) - W_i \left(\sum_{j \in J_X} \kappa_{jX} - \sum_{j \in J_F} \kappa_{jF} - \kappa_{iTH} \mathbb{1}(\lambda_{TH} > 0) \right) \right\}.$$

Each firm, taking J_X and J_F as given, chooses z and λ to maximize

$$\hat{d}_i^{PR}(a; J_X, J_F, \varrho) = \max_{z, \lambda_{TH}, \lambda_{LT}} \left\{ \sum_{j \in \{i\} \cup J_X \cup J_F} \left(\pi_j^{PR}(a, z) - \tau_j T_j \right) \right\},\tag{F.8}$$

where

$$\begin{split} \hat{d}_{i}^{PR} &= \max_{z,\lambda_{LT},\lambda_{TH}} \left\{ \left[\pi_{i}^{D}(a,z;J_{X}) + \left(-(\lambda_{LT} + \lambda_{TH})\vartheta_{i}(z) + (1 - \lambda_{LT} - \lambda_{TH})\sum_{j\in J_{F}}\vartheta_{j}(z) \right. \\ &\left. - W_{i}/A_{i} - W_{i} \big(\mathcal{C}_{i,LT}(\lambda_{LT}) + \mathcal{C}_{i,TH}(\lambda_{TH})\big)\nu_{i}(z) \Big) z - \tau_{i}T_{i} \right] \right. \\ &\left. + \left[\pi_{i,LT}^{F}(a,z) + \left(\lambda_{LT}\sum_{j\in J_{F}\cup\{i\}\setminus\{LT\}}\vartheta_{j}(z) - (1 - \lambda_{LT})\vartheta_{iLT}(z) \right) z - \tau_{LT}T_{LT} \right] \right. \\ &\left. + \left[\left(\lambda_{TH}\sum_{j\in J_{F}\cup\{i\}}\vartheta_{j}(z) \right) z - \tau_{TH}T_{TH} \right] \right. \\ &\left. + \sum_{j\in J_{F}\setminus\{LT\}} \left[\pi_{ij}^{F}(a,z) - \vartheta_{j}(z) z - \tau_{j}T_{j} \right] + \sum_{j\in J_{X}\setminus J_{F}} \left[-\tau_{j}T_{j} \right] \right\}. \end{split}$$

The term T_j is the tax base in region j, which consists of local firms' routine profit, a proportion of local firms' residual profits and reallocated residual profits to this region:

$$T_{j} = \Pi_{j}^{r} + (1 - \theta) \cdot \Pi_{j}^{R} + \theta \cdot \frac{R_{j}}{\sum_{k} R_{k}} \cdot \Pi^{R}$$

$$= \vartheta R_{j} + (1 - \theta) \cdot (\pi_{j}(a, z; J_{X}) - \mu R_{j}) + \theta \cdot \frac{R_{j}}{\sum_{k} R_{k}} \cdot \sum_{k \in \{i\} \cup J_{X} \cup J_{F}} (\pi_{k}(a, z; J_{X}) - \mu R_{k})$$

$$= (1 - \theta) \cdot \pi_{j}(a, z; J_{X}) + \theta \cdot \frac{R_{j}}{\sum_{k} R_{k}} \cdot \sum_{k \in \{i\} \cup J_{X} \cup J_{F}} \pi_{k}(a, z; J_{X}).$$

Profit π_j is the profit earned in region j and it is zero if the firm does not operate in the region. Revenue earned in region j, denoted as R_j , include sales of both goods produced locally (by parent division or FDI) and goods exported to the region. Formally:

$$\begin{aligned} R_{i} &= p_{ii}\left(q_{ii}\right)q_{ii}, \\ R_{j} &= p_{ij}^{F}\left(q_{ij}\right)q_{ij}, j \in J_{F}, j \notin J_{X}, \\ R_{j} &= p_{ij}^{X}\left(q_{ij}^{X}\right)q_{ij}^{X}, j \in J_{X}, j \notin J_{F}, \\ R_{j} &= p_{ij}^{F}\left(q_{ij}\right)q_{ij} + p_{ij}^{X}\left(q_{ij}^{X}\right)q_{ij}^{X}, j \in J_{X} \cap J_{F}, \\ R_{j} &= 0, \quad j \notin \{i\} \cup J_{F} \cup J_{X}. \end{aligned}$$

We can rewrite firm's problem as:

$$\hat{d}_i^{PR} = \max_{z,\lambda_{TH},\lambda_{LT}} \left\{ \sum_{j \in \{i\} \cup J_X \cup J_F} \left((1 - \tau_j(1 - \theta)) \,\pi_j(a, z; J_X) - \tau_j \theta \cdot \frac{R_j}{\sum_j R_j} \cdot \sum_k \pi_k(a, z; J_X) \right) \right\}.$$

Further, substituting in π_i and denoting $\lambda = \lambda_{TH} + \lambda_{LT}$, we get

$$\begin{split} \max_{z,\lambda_{TH},\lambda_{LT}} & \left\{ (1-(1-\theta)\tau_i) \left[\left(1 - \lambda \left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho} \right) \right) (\bar{R}_{ii} - \bar{C}_{ii}) + (1-\lambda) \sum_{j \in J_F} \left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho} \right) (\bar{R}_{ij} - \bar{C}_{ij}) \right] \right. \\ & \left. - (1-(1-\theta)\tau_i) \left[W_i \left(\mathcal{C}_{i,LT}(\lambda_{LT}) + \mathcal{C}_{i,TH}(\lambda_{TH}) \right) \sum_{j \in J_F \cup \{i\}} \left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho} \right) (\bar{R}_{ij} - \bar{C}_{ij}) \right] \right. \\ & \left. + (1-(1-\theta)\tau_{LT}) \left[\left(1 - (1-\lambda_{LT}) \left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho} \right) (\bar{R}_{ij} - \bar{C}_{ij}) \right] \right. \\ & \left. + (1-(1-\theta)\tau_{TH}) \left[\lambda_{TH} \sum_j \left(\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho} \right) (\bar{R}_{ij} - \bar{C}_{ij}) \right] \right. \\ & \left. + \sum_{j \in J_F \setminus \{LT\}} (1-\tau_j) \left[\left(1 - \frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho} \right) (\bar{R}_{ij} - \bar{C}_{ij}) \right] \right\} z^{\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}} - (1-(1-\theta)\tau_i) W_i z / A_i \\ & \left. - \sum_{j \in \{i\} \cup J_X \cup J_F} \tau_j \theta \cdot \frac{R_j}{\sum_j R_j} \cdot \sum_k \pi_k(a, z; J_X). \end{split}$$

Here we define \tilde{R}_{ij} as the revenue shifter in region j for firms from region i, depending on region j is served. These terms are defined analogously of \bar{R}_{ij} in equations (F.3) and (F.4):

$$\begin{split} \tilde{R}_{ii} &= P_i Q_i^{\frac{1}{\varrho}} \bar{Q}_{ii}^{\frac{\varrho-1}{\varrho}} \left\{ \left[P_i^{\varrho} Q_i + \sum_{j \in J_X} P_j^{\varrho} \tau_j^{1-\varrho} Q_j \right] \left[\frac{\phi(\varrho-1)}{\varrho} \right]^{\varrho} W^{-\varrho} \right\}^{\frac{\varrho-1}{\varrho} \frac{\phi}{\phi+\varrho-\varrho\phi}} (A_i)^{\frac{\varrho-1}{\phi+\varrho-\phi\varrho}} N_i^{\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}}, \\ \tilde{R}_{ij} &= P_j Q_j^{\frac{1}{\varrho}} \bar{Q}_{ij}^{\frac{\varrho-1}{\varrho}} \left\{ \left[P_i^{\varrho} Q_i + \sum_{j \in J_X} P_j^{\varrho} \tau_j^{1-\varrho} Q_j \right] \left[\frac{\phi(\varrho-1)}{\varrho} \right]^{\varrho} W^{-\varrho} \right\}^{\frac{\varrho-1}{\varrho} \frac{\phi}{\phi+\varrho-\phi\phi}} (A_i)^{\frac{\varrho-1}{\phi+\varrho-\phi\varrho}} N_i^{\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}}, \quad j \in J_X, j \notin J_F, \\ \tilde{R}_{ij} &= \left[P_j Q_j^{\frac{1}{\varrho}} \right] \left[(P_j/W_j) \frac{\phi(\varrho-1)}{\varrho} \right]^{\frac{\phi(\varrho-1)}{\phi+\varrho-\phi\varrho}} Q_j^{\frac{\varrho-1}{\varrho} \frac{\phi}{\phi+\varrho-\phi\varrho}} (A_j\sigma_{ij})^{\frac{\varrho-1}{\phi+\varrho-\phi\varrho}} N_j^{\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}}, \quad j \in J_F, j \notin J_X, \\ \tilde{R}_{ij} &= \left[P_j Q_j^{\frac{1}{\varrho}} \right] \left[(P_j/W_j) \frac{\phi(\varrho-1)}{\varrho} \right]^{\frac{\phi(\varrho-1)}{\phi+\varrho-\phi\varrho}} Q_j^{\frac{\varrho-1}{\varrho} \frac{\phi}{\phi+\varrho-\phi\varrho}} (A_j\sigma_{ij})^{\frac{\varrho-1}{\phi+\varrho-\phi\varrho}} N_j^{\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}} \\ &+ P_j Q_j^{\frac{1}{\varrho}} \bar{Q}_{ij}^{\frac{\varrho-1}{\varrho}} \left\{ \left[P_i^{\varrho} Q_i + \sum_{j \in J_X} P_j^{\varrho} \tau_j^{1-\varrho} Q_j \right] \left[\frac{\phi(\varrho-1)}{\varrho} \right]^{\varrho} W^{-\varrho} \right\}^{\frac{\varrho-1}{\varrho} \frac{\phi}{\phi+\varrho-\phi\varrho}} (A_i)^{\frac{\varrho-1}{\phi+\varrho-\phi\varrho}} N_i^{\frac{\gamma(\varrho-1)}{\phi+\varrho-\phi\varrho}}, \quad j \in J_X \cap J_F, \\ \tilde{R}_{ij} &= 0, \quad j \notin \{i\} \cup J_F \cup J_X. \end{split}$$

With these definitions, it's straightforward to show that the revenue share $\frac{R_j}{\sum_j R_j} = \frac{\tilde{R}_{ij}}{\sum_j \tilde{R}_{ij}}$. We can

further simplify the problem to

$$\begin{split} \max_{z,\lambda_{TH},\lambda_{LT}} \left\{ \left(1 - (1 - \theta)\tau_{i}\right) \left[\left(1 - \lambda \left(\frac{\gamma(\varrho - 1)}{\phi + \varrho - \phi\varrho}\right)\right) (\bar{R}_{ii} - \bar{C}_{ii}) + (1 - \lambda) \sum_{j \in J_{F}} \left(\frac{\gamma(\varrho - 1)}{\phi + \varrho - \phi\varrho}\right) (\bar{R}_{ij} - \bar{C}_{ij}) \right] \right. \\ \left. - (1 - (1 - \theta)\tau_{i}\right) \left[W_{i} \left(\mathcal{C}_{i,LT}(\lambda_{LT}) + \mathcal{C}_{i,TH}(\lambda_{TH})\right) \sum_{j \in J_{F} \cup \{i\}} \left(\frac{\gamma(\varrho - 1)}{\phi + \varrho - \phi\varrho}\right) (\bar{R}_{ij} - \bar{C}_{ij}) \right] \right. \\ \left. + (1 - (1 - \theta)\tau_{LT}) \left[\left(1 - (1 - \lambda_{LT}) \left(\frac{\gamma(\varrho - 1)}{\phi + \varrho - \phi\varrho}\right) (\bar{R}_{ij} - \bar{C}_{ij})\right) \right] \right. \\ \left. + (1 - (1 - \theta)\tau_{TH}) \left[\lambda_{TH} \sum_{j \in J_{F} \cup \{i\} \setminus \{LT\}} \left(\frac{\gamma(\varrho - 1)}{\phi + \varrho - \phi\varrho}\right) (\bar{R}_{ij} - \bar{C}_{ij}) \right] \right. \\ \left. + (1 - (1 - \theta)\tau_{TH}) \left[\lambda_{TH} \sum_{j \in J_{F} \cup \{i\} \setminus \{LT\}} \left(\frac{\gamma(\varrho - 1)}{\phi + \varrho - \phi\varrho}\right) (\bar{R}_{ij} - \bar{C}_{ij}) \right] \right] \right. \\ \left. + \sum_{j \in J_{F} \setminus \{LT\}} (1 - \tau_{j}) \left[\left(1 - \frac{\gamma(\varrho - 1)}{\phi + \varrho - \phi\varrho}\right) (\bar{R}_{ij} - \bar{C}_{ij}) \right] \right] z^{\frac{\gamma(\varrho - 1)}{\phi + \varrho - \phi\varrho}} - (1 - (1 - \theta)\tau_{i}) W_{i}z/A_{i} \\ \left. - \sum_{j \in \{i\} \cup J_{X} \cup J_{F}} \tau_{j}\theta \cdot \frac{\tilde{R}_{ij}}{\sum_{j} \tilde{R}_{ij}} \cdot \left\{ \sum_{k} \left(\bar{R}_{ik} - \bar{C}_{ik}\right) z^{\frac{\gamma(\varrho - 1)}{\phi + \varrho - \phi\varrho}} \right) (\bar{R}_{ij} - \bar{C}_{ij}) z^{\frac{\gamma(\varrho - 1)}{\phi + \varrho - \phi\varrho}} \right\}. \end{aligned}$$

As before, the shift shares λ_{TH} and λ_{LT} can be solved independently of z:

$$\lambda_{TH} = \left(\mathcal{C}_{i,LT}'\right)^{-1} \left[\frac{1}{W_i} \frac{\left(1-\theta\right)\left(\tau_i-\tau_{TH}\right)}{1-\left(1-\theta\right)\tau_i-\theta\sum_k \tau_k \frac{\bar{R}_{ik}}{\sum_j \bar{R}_{ij}}} \right],$$
$$\lambda_{LT} = \left(\mathcal{C}_{i,TH}'\right)^{-1} \left[\frac{1}{W_i} \frac{\left(1-\theta\right)\left(\tau_i-\tau_{LT}\right)}{1-\left(1-\theta\right)\tau_i-\theta\sum_k \tau_k \frac{\bar{R}_{ik}}{\sum_j \bar{R}_{ij}}} \right].$$

The optimal z is given by

$$z = \left\{ \left(\frac{\phi + \varrho - \phi \varrho}{\gamma(\varrho - 1)}\right) \left[\frac{\left(1 - (1 - \theta)\tau_i - \theta \sum_j \tau_j \frac{\tilde{R}_{ij}}{\sum_k \tilde{R}_{ik}}\right) W_i / A_i}{DENOM^{PR}}\right] \right\}^{\frac{\phi + \varrho - \phi \varrho}{\gamma \varrho + \phi \varrho - \gamma - \phi - \varrho}},$$
where $DENOM^{PR}$ is defined as

$$\begin{cases} \sum_{j \in J_F \cup \{i\}} (1 - (1 - \theta)\tau_j)(\bar{R}_{ij} - \bar{C}_{ij}) - \sum_{j \in J_F \cup \{i\}} (1 - \theta)(\tau_i - \tau_j) \left(\frac{\gamma(\varrho - 1)}{\phi + \varrho - \phi\varrho}\right)(\bar{R}_{ij} - \bar{C}_{ij}) \\ + (1 - \theta)(\tau_i - \tau_{LT})\lambda_{LT} \sum_{j \in J_F \cup \{i\}} \left(\frac{\gamma(\varrho - 1)}{\phi + \varrho - \phi\varrho}\right)(\bar{R}_{ij} - \bar{C}_{ij}) + (1 - \theta)(\tau_i - \tau_{TH})\lambda_{TH} \sum_{j \in J_F \cup \{i\}} \left(\frac{\gamma(\varrho - 1)}{\phi + \varrho - \phi\varrho}\right)(\bar{R}_{ij} - \bar{C}_{ij}) \\ - (1 - (1 - \theta)\tau_i)W_i\left(\mathcal{C}_{i,LT}(\lambda_{LT}) + \mathcal{C}_{i,TH}(\lambda_{TH})\right) \cdot \sum_{j \in J_F \cup \{i\}} \left(\frac{\gamma(\varrho - 1)}{\phi + \varrho - \phi\varrho}\right)(\bar{R}_{ij} - \bar{C}_{ij}) \\ - \sum_{j \in \{i\} \cup J_X \cup J_F} \tau_j \theta \cdot \frac{\tilde{R}_{ij}}{\sum_k \tilde{R}_{ik}} \cdot \left[\sum_{k \in J_F \cup \{i\}} \left(\bar{R}_{ik} - \bar{C}_{ik}\right) - W_i(C_{i,TH}(\lambda_{TH}) + C_{i,LT}(\lambda_{LT}))\sum_{j \in J_F \cup \{i\}} \left(\frac{\gamma(\varrho - 1)}{\phi + \varrho - \phi\varrho}\right)(\bar{R}_{ij} - \bar{C}_{ij})\right] \right\}.$$